# Fourier Transform

- generate a multispectral signal with noise

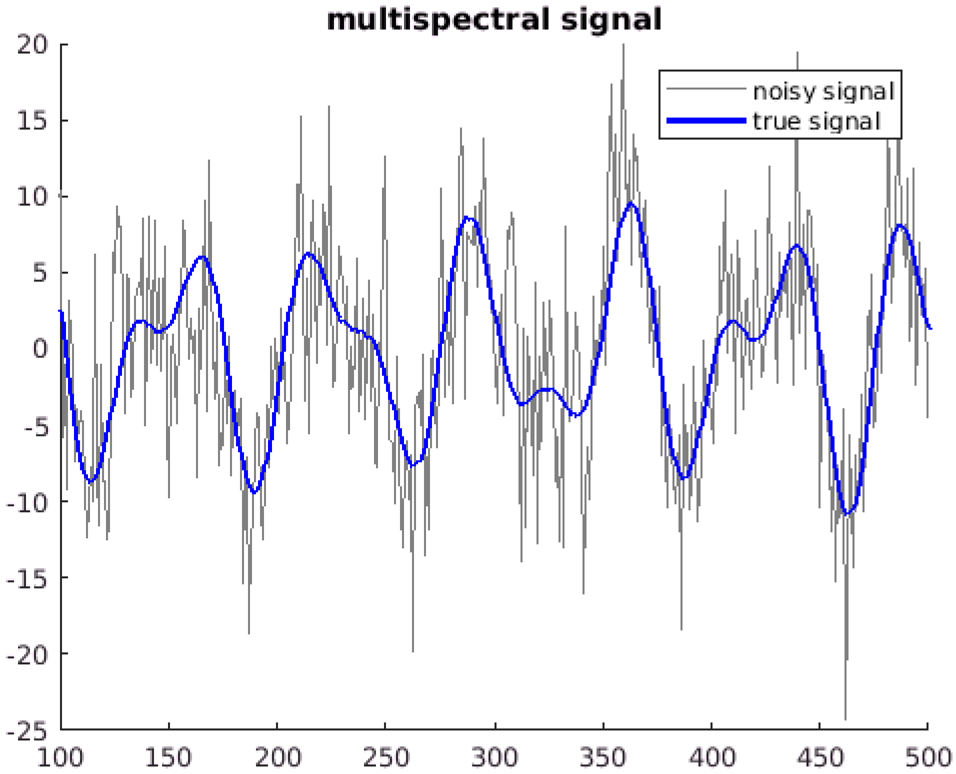
```
sample_rate = 1024; %hz
N = sample_rate*2; %number of samples
t = (0:N-1)/sample_rate; %time vector

%create some random frequencies and amplitudes
freqs = [ floor(1+30*rand(3,1)')];
amplitudes = zeros(size(freqs));
signal = zeros(size(t));

for i=1:length(freqs)
    amplitudes(i) = randi(6);
    signal = signal + amplitudes(i)*sin(2*pi*freqs(i)*t);  %give some random amplitude
end

noisy = signal + randn(size(signal))*5;
```

- create strings for annotation

```
str1 = sprintf('freq: %d, amplitude: %d', freqs(1), amplitudes(1));
str2 = sprintf('freq: %d, amplitude: %d', freqs(2), amplitudes(2));
str3 = sprintf('freq: %d, amplitude: %d', freqs(3), amplitudes(3));
```

### Plot the true and noisy signals

```
figure(1); clf;
hold on;
noisy_plot = plot(noisy);
set(noisy_plot, 'color', [.5 .5 .5]);
true_plot = plot(signal, 'b', 'linew', 2);
xlim([100 500]);
title("multispectral signal");
legend(["noisy signal", "true signal"]);
hold off;
```

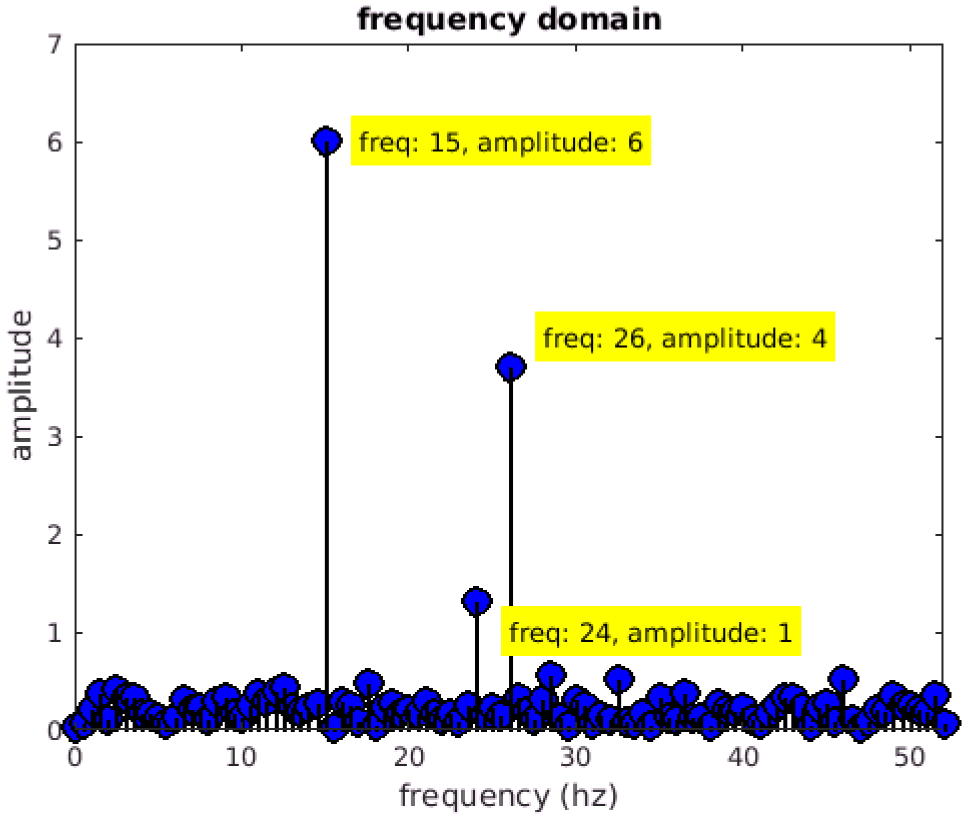


## Apply FFT

```
signal_fft = fft(noisy);
signal_amp = 2*abs(signal_fft)/N;
hz_vals = linspace(0,sample_rate/2, floor(N/2)+1);

inv_signal = ifft(signal_fft); %apply inverse fft to reconstruct signal
```

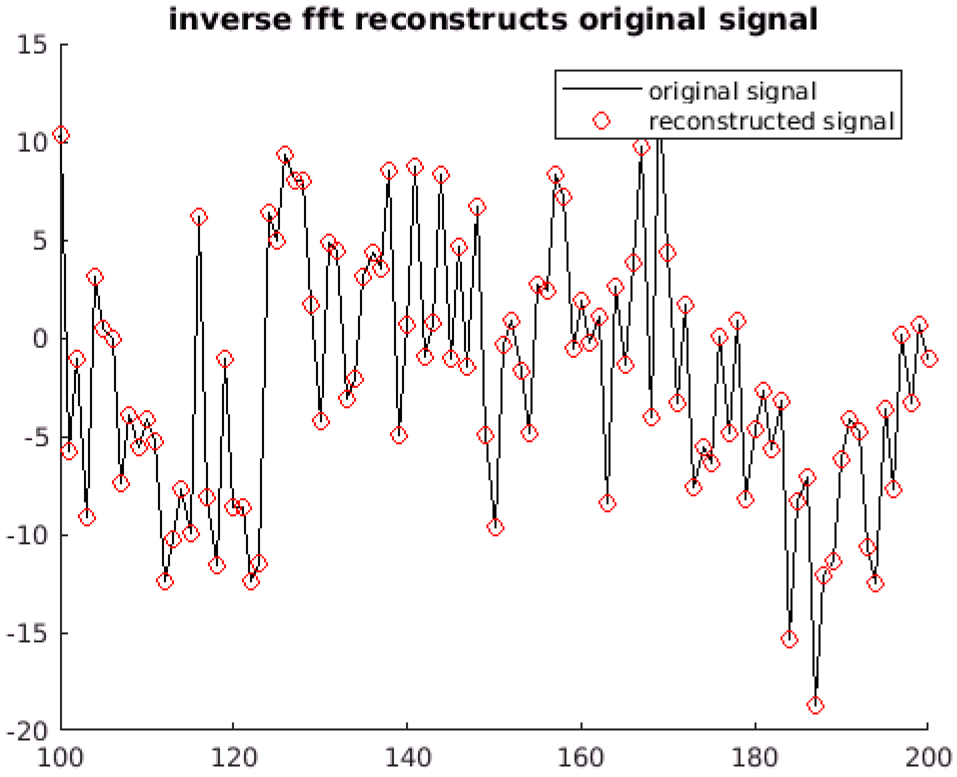
## Plot the frequency spectrum

```
figure(2); clf;
stem(hz_vals, signal_amp(1:length(hz_vals)), 'ko', 'linew',2, 'markersize', 10, 'markerfacecol
set(gca, 'xlim', [0 max(freqs)*2]);
t1 = text(freqs(1)+2, amplitudes(1), str1, 'BackgroundColor', 'yellow');
t2 = text(freqs(2)+2, amplitudes(2), str2, 'BackgroundColor', 'yellow');
t3 = text(freqs(3)+2, amplitudes(3), str3, 'BackgroundColor', 'yellow');
xlabel("frequency (hz)");
ylabel("amplitude");
title("frequency domain");
hold off;
```



### Plot the inverse fft

```
figure(3); clf;
hold on;
orig_plot = plot(noisy, 'k-');
inv_plot = plot(inv_signal, 'ro');
xlim([100 200]);
title("inverse fft reconstructs original signal");
legend(["original signal", "reconstructed signal"]);
hold off;
```



## Another Example

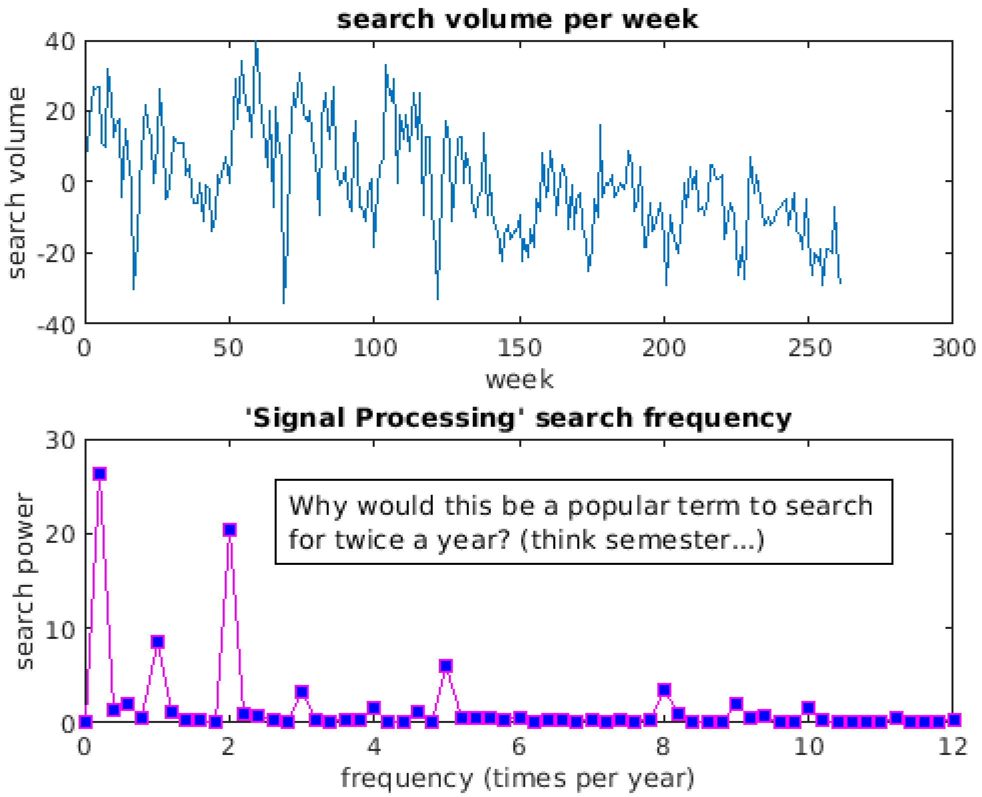
- normalized number of searches per week with term "signal processing"

```
clear all;
% data downloaded from https://trends.google.com/trends/explore?date=today%205-y&geo=US&q=sign
searches = [69 77 87 86 87 71 70 92 83 73 76 78 56 75 68 60 30 44 58 69 82 76 73 60 71 86 72 5
N = length(searches);

%normalize the data
searches  = searches - mean(searches);

%square amplitude to get power
powers = abs(fft(searches)/N).^2;

%create freq bins
hz_vals = linspace(0,52,N);
```

- plot the search volume and power spectrum

```
figure(4); clf;
subplot(211);
plot(searches);
title("search volume per week");
xlabel("week");
ylabel("search volume");
```

```
subplot(212);
plot(hz_vals, powers, 'ms-', 'markerfacecolor', 'b');
xlabel("frequency (times per year)");
ylabel("search power");
title("'Signal Processing' search frequency");
set(gca, 'xlim', [0 12]);
str = 'Why would this be a popular term to search for twice a year? (think semester...)';
annotation('textbox',[.3 .3 .55 .1],'String',str);
```



### Use windowing (welches method) to increase the signal to noise ratio

- https://en.wikipedia.org/wiki/Welch%27s_method

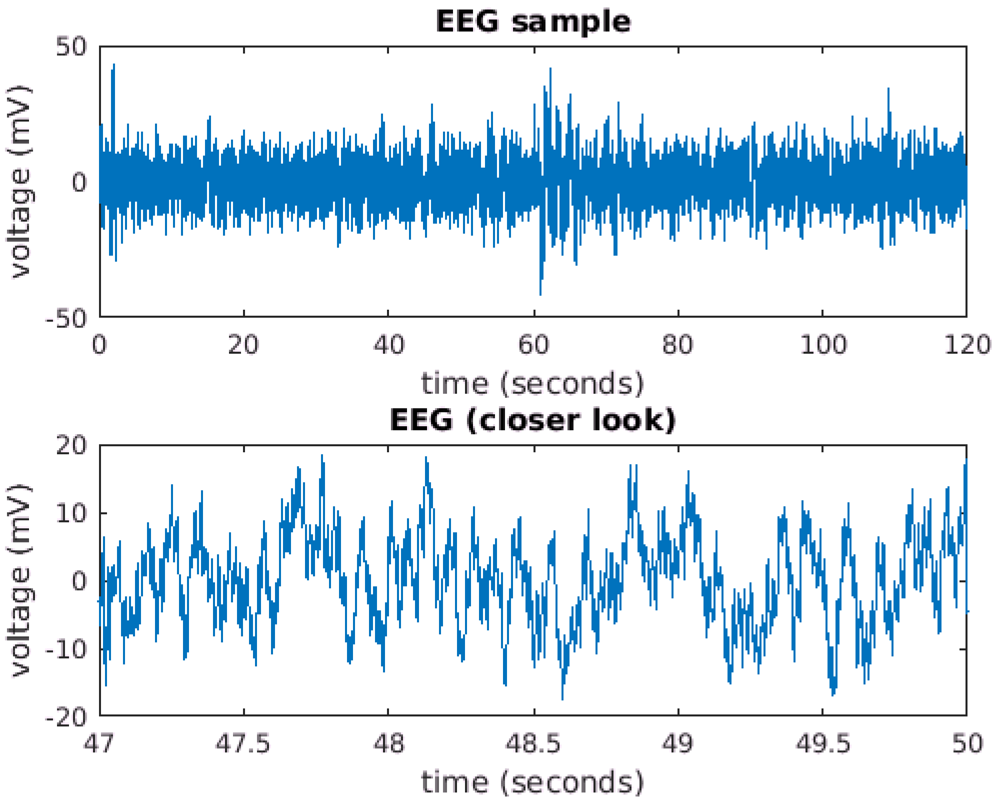
```
clear all;
load ../datasets/EEGrestingState.mat;
whos;
```

```
  Name          Size                 Bytes  Class     Attributes

  eegdata       1x122880            491520  single
  srate         1x1                      8  double
```

```
N = length(eegdata);
t = (0:N-1)/srate;
%get the power spectrum
powers = abs(fft(eegdata)/N).^2;
hz_vals = linspace(0, srate/2, floor(N/2)+1);
```

- Plot the data

```
figure(5); clf;
subplot(211);
eeg_plot = plot(t, eegdata);
xlabel("time (seconds)");
ylabel("voltage (mV)");
title("EEG sample");

subplot(212);
eeg_plot = plot(t, eegdata);
xlim([47 50]);
xlabel("time (seconds)");
ylabel("voltage (mV)");
title("EEG (closer look)");
```



## Use Welches Method

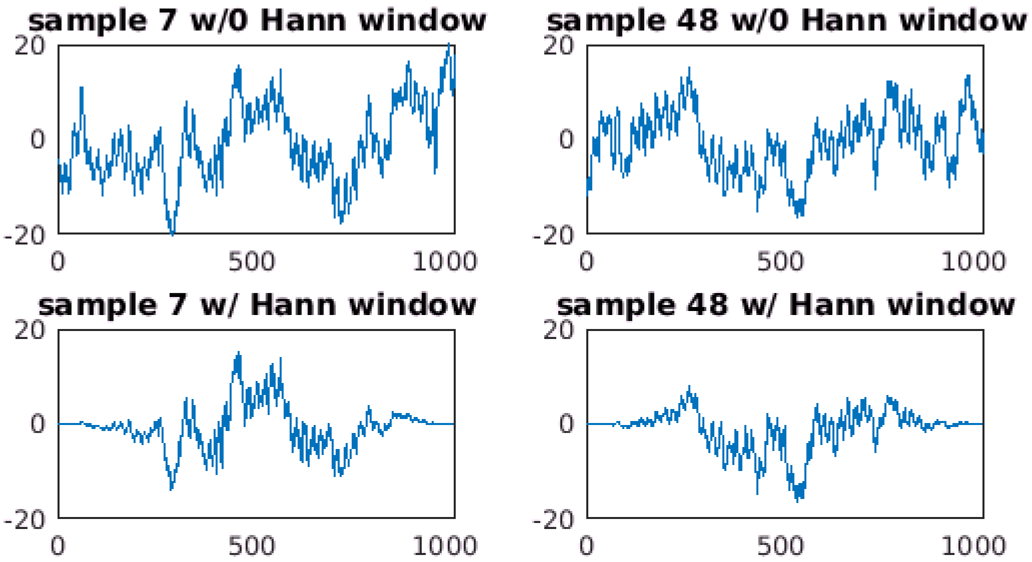
- manaully first
- then us matlab function
- Hann window minimizes the edge effects

```
%window size is 1 sec, overlapping at .5 seconds
window_len = 1*srate; %seconds* sampling rate
%overlapping points in subsequent windows
num_overlap = round(srate/2);
window_starts = 1:num_overlap:N-window_len;
```

```
window_hz = linspace(0, srate/2, floor(window_len/2)+1);
%https://en.wikipedia.org/wiki/Hann_function
window_hann = .5 - cos(2*pi*linspace(0,1,window_len))./2;
window_powers = zeros(1, length(window_hz));
%for plotting
samples = zeros(4, window_len);
j = 1;

for i=1:length(window_starts)
    piece = eegdata(window_starts(i):window_starts(i)+window_len-1);
    if i == 7 || i == 48
        samples(j,:) = piece;
        j = j + 1;
    end
    piece = piece .* window_hann;
    power = abs(fft(piece)/window_len).^2;
    window_powers = window_powers + power(1:length(window_hz));
    %get some samples to plot
    if i == 7 || i == 48
        samples(j,:) = piece;
        j = j + 1;
    end
end

window_powers = window_powers / (window_len * 2);
```

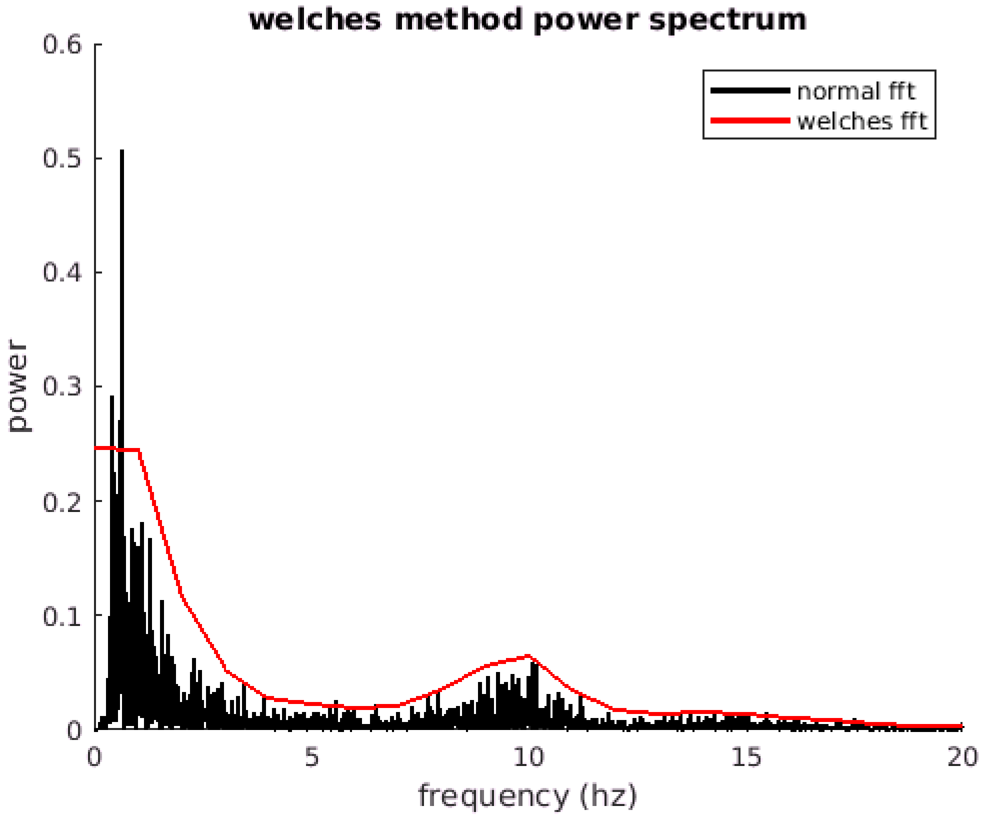
- Plot some sample windows

```
f6 = figure(6); clf;
f6.Position = [0,0,600,300];
subplot(221);
plot(samples(1,:));
title("sample 7 w/0 Hann window");
xlim([0 1024]); ylim([-20 20]);

subplot(223);
plot(samples(2,:));
title("sample 7 w/ Hann window");
xlim([0 1024]); ylim([-20 20]);

subplot(222);
plot(samples(3,:));
title("sample 48 w/0 Hann window");
xlim([0 1024]); ylim([-20 20]);

subplot(224);
plot(samples(4,:));
title("sample 48 w/ Hann window");
xlim([0 1024]); ylim([-20 20]);
```



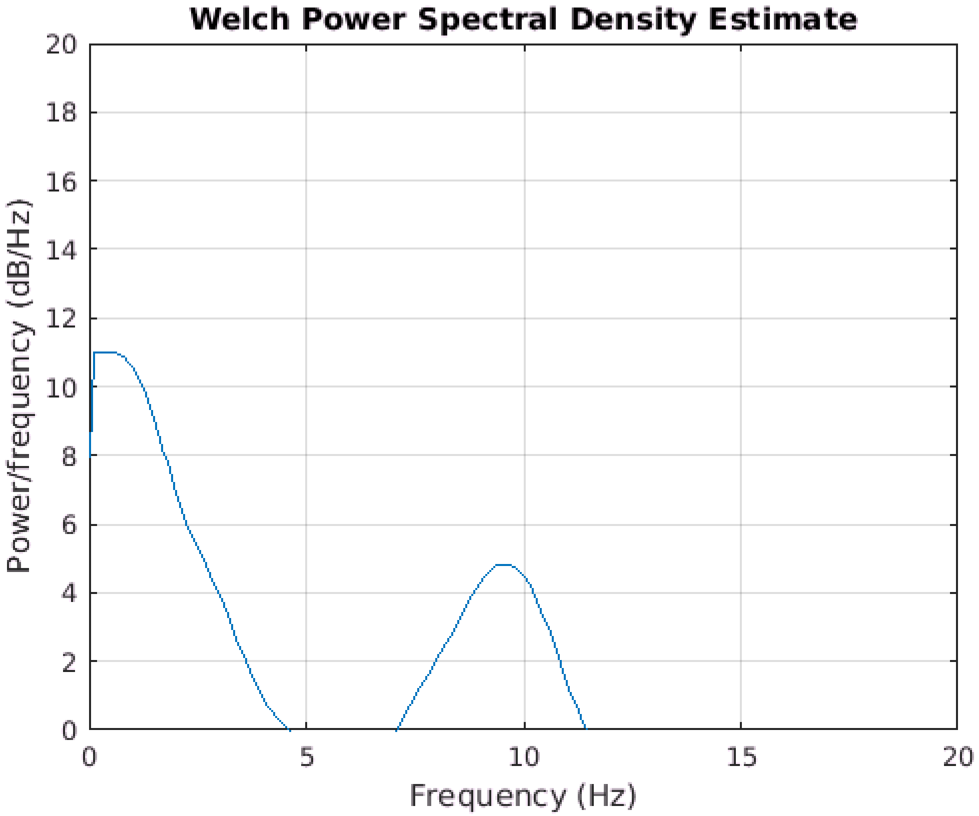
- Plot the power spectrum of welches method
- compare to normal

```
figure(7); clf;
hold on;
plot(hz_vals, powers(1:length(hz_vals)), 'k', 'linew', 2);
plot(window_hz, window_powers, 'r', 'linew', 2);
xlim([0 20]);
title("welches method power spectrum");
xlabel("frequency (hz)");
ylabel("power");
legend(["normal fft", "welches fft"]);
```



- use matlab function

```
figure(8); clf;
pwelch(eegdata, window_hann, round(window_len/4), srate*10, srate);
xlim([0 20]);
ylim([0 20]);
```


Welch Power Spectral Density Estimate
Power/frequency (dB/Hz)
20
18
16
14
12
10
8
6
4
2
0
0
5
10
15
20
Frequency (Hz)