

## State-Space Based Simulink Modeling of BLDC Motor and its Speed Control using Fuzzy PID Controller

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**Abstract:** In this paper, an efficient simulation model for Fuzzy logic controlled Brushless DC Motor drives using Matlab / Simulink is presented. Here Model of BLDC motor is based on State-space and its speed loop control is based on Fuzzy logic PID controller. A three phase inverter model is implemented for Motor commutation in six steps and PWM based motor Current Control with help of three Hall Sensors which are placed at 120 electrical degrees apart around the motor shaft. Dynamic performances (i.e. Torque and Speed) and currents and voltages of the inverter components are analyzed for this model. The modeling, simulation and two loop control of BLDC have been done in MATLAB\SIMULINK software.

**Key Words:** BLDC Motor, H-bridge Inverter, PWM Controller, State-Space Model and PID & Fuzzy logic controller

### I. Introduction

Conventional DC motors have many attractive properties such as high efficiency and linear torque-speed characteristics. The control of DC motor is also simple and does not require much complex hardware. However, the main drawback of the dc motor is the need of periodic maintenance. The Brushes of the mechanical commutator eventually wear out and need to be replaced. The mechanical commutator has other undesirable effects such as sparks, acoustic noise and carbon particles coming from the brushes.

With rapid developments in power electronics, power semiconductor technologies, modern control theory for motors and manufacturing technology for high performance magnetic materials, the Brushless DC (BLDC) motors have been widely used in many applications. BLDC Motor have many advantages over conventional DC motors like: Long operating life, High dynamic response, High efficiency, Better Speed vs. Torque characteristic, Noiseless operational Higher speed range and Higher Torque-Weight ratio [1].

A BLDC Motor is a permanent synchronous motor that uses position detectors and an inverter to control the armature currents. Its armature is in the stator and the magnets are on the rotor and its operating characteristic resembles those of a DC motor. Instead of using a mechanical commutator as in the conventional DC Motor, the BLDC motor employs electronic commutation which makes it a virtually maintenance free. The BLDC motor is driven by DC voltage but current commutation is done by solid-state switches. The commutation instants are determined by the rotor position and the position of the rotor is determined either by position sensors like Hall sensor, position encoder and resolver etc. or by sensorless techniques. There are two main types of BLDC Motors: Trapezoidal type and Sinusoidal type. The trapezoidal motor is a more attractive alternative for most applications due to its simplicity, lower price and higher efficiency. Here State-Space based trapezoidal motor has been taken for modeling and simulation in MATLAB\SIMULINK.

In this paper, a detail simulation model of BLDC Motor with fuzzy logic PID controller and H-bridge inverter Drives is presented. MATLAB/fuzzy logic toolbox is used to design PID Fuzzy Logic Controller (FLC), which is integrated with BLDC Motor Model in Simulink in closed loop for speed control of Motor. Several simulation results are shown to confirm the performance and the validity of the proposed model. The model based on system-level simulation makes the simulation faster while it is able to provide greater details of the BLDC motor drive



system. Overall system modelling and simulation is divided into three parts: State-Space Model of BLDC, H-bridge Inverter and Fuzzy PID based Motion controller.

## II. State-Space Based Modelling of BLDC Motor

In this paper, a three phase, 2 poles and Y connected trapezoidal back-EMF type BLDC motor is modelled. Trapezoidal back-EMF is referring that mutual inductance between stator and rotor has trapezoidal shape [3]. Therefore abc phase variable model is more applicable than d-q axis. With the intention of simplifying equations and overall model the following assumptions are made:

- Stator resistance and self inductance of all phases are equal and constant and mutual inductance is taken zero.
- Hysteresis and eddy current losses are eliminated.
- All semiconductor switches are ideal.

The mathematical model of BLDC Motor is comprises into two parts: Electrical and Mechanical equations.

### A. Electrical Equations:

$$V_a = R * i_a + L * \frac{di_a}{dt} + emf_a \quad (1)$$

$$V_b = R * i_b + L * \frac{di_b}{dt} + emf_b \quad (2)$$

$$V_c = R * i_c + L * \frac{di_c}{dt} + emf_c \quad (3)$$

$$emf_a = K_e * w_m * F(\theta_e)$$

$$emf_b = K_e * w_m * F(\theta_e - 2\pi/3)$$

$$emf_c = K_e * w_m * F(\theta_e + 2\pi/3)$$

$$T_e = [emf_a * i_a + emf_b * i_b + emf_c * i_c] / w_m$$

### B. Mechanical Equation

$$T_e = B * w_m + J * \frac{dw_m}{dt} + T_l \quad (4)$$

Where k=a, b , c

$V_k$  = phase voltages

$i_k$  = phase currents

$emf_k$  = phases Back – emf

$R, L$  = phase resistance and inductance of stator winding

$T_e$  = Total electromagnetic torque and  $T_l$  = Load at motor

$w_m$ =angular speed of rotor

$\theta_e$  = Electrical angle of rotor and  $\theta_m$ =Mechanical angle of rotor

P=no. of pole on rotor

$K_e, J$  =motor Back-emf constant (V/rad/sec) and moment of inertia (Kg-m2).

$B$ =Dynamic Frictional Torque constant (Nm/rad/sec)



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Electrical and mechanical angle of rotor can be related as

$$\theta_e = \frac{p}{2} \theta_m$$

$F(\theta_e)$  = Back-emf reference function which has trapezoidal shape and max magnitude in  $\pm 1$  and can be represented as

$$F(\theta_e) = \begin{cases} 1, & 0 \leq \theta_e \leq 2\pi/3 \\ 1 - \frac{6}{\pi}(\theta_e - 2\pi/3), & 2\pi/3 \leq \theta_e \leq \pi \\ -1, & \pi \leq \theta_e \leq 5\pi/3 \\ -1 + \frac{6}{\pi}(\theta_e - 5\pi/3), & 5\pi/3 \leq \theta_e \leq 2\pi \end{cases}$$

After solving above four equations, the complete State-Space Model can be represented as

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{di_b}{dt} \\ \frac{dw_m}{dt} \\ \frac{d\theta_m}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & 0 & 0 & 0 \\ 0 & -\frac{R}{L} & 0 & 0 \\ 0 & 0 & -\frac{B}{J} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ w_m \\ \theta_m \end{bmatrix} + \begin{bmatrix} \frac{2}{3L} & \frac{1}{3L} & 0 \\ -\frac{1}{3L} & \frac{1}{3L} & 0 \\ 0 & 0 & \frac{1}{J} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{ab} - emf_{ab} \\ V_{bc} - emf_{bc} \\ T_e - T_l \end{bmatrix}$$

And

$$\begin{bmatrix} i_a \\ i_b \\ i_c \\ w_m \\ \theta_m \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ w_m \\ \theta_m \end{bmatrix}$$

Where,  $V_{ab}=V_a-V_b$ ,  $emf_{ab} = emf_a - emf_b$  and  $V_{bc}=V_b-V_c$ ,  $emf_{bc} = emf_b - emf_c$

Simulink block model for State-Space model of BLDC is shown in figure (1).

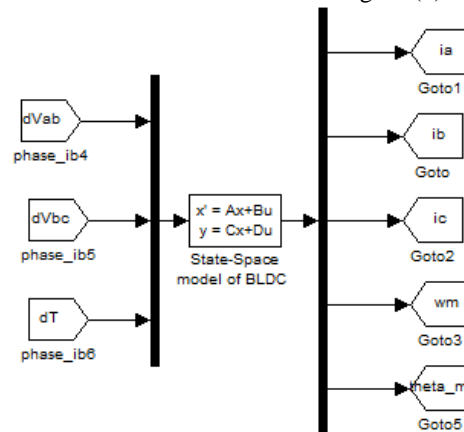
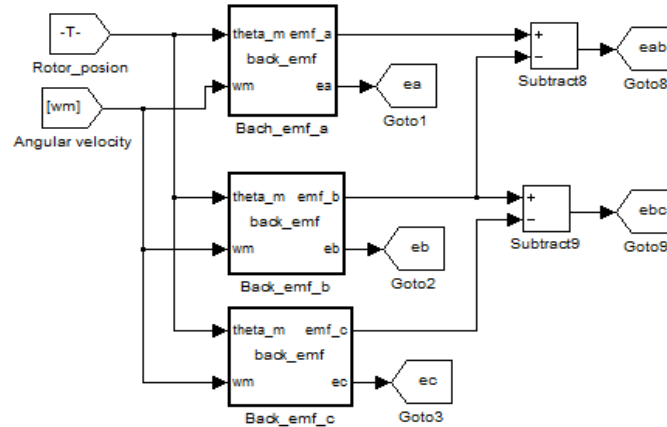


Figure 1: State-Space Model of BLDC Motor Simulink block

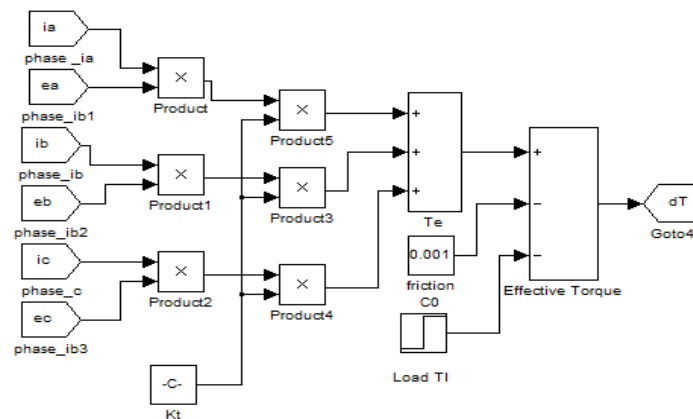
Where,  $dV_{ab}=V_{ab}-emf_{ab}$ ,  $dV_{bc}=V_{bc}-emf_{bc}$  and  $dT=T_e-T_l$ .

Back-emf and Electrical torque are modelled in Simulink as per above equations are shown in figure (2) and Figure (3) respectively.



**Figure 2: Back-Emf generation Simulink block**

Where ea, eb, ec are back-emf generating function like  $F(\theta_e)$  function.



**Figure 3: Effective Electrical torque generation Simulink block**

Where C0 is static frictional torque of BLDC Motor

### III. Motor Driver (Voltage Source Inverter)

BLDC Motor Driver is DMOSFET/IGBT based device which includes all the circuitry needed to drive a three phase BLDC motor. In this configuration, three hall sensors are placed at 120 electrical degrees apart around the motor shaft, detects the rotor position in three phase motor. There are eight possible combinations for three hall sensors inputs out of which six combinations are valid with 120 electrical degree sensor phasing. For any Hall sensor input combination logic there is one output configuration used to drive the motor. Rate of variation for three phase switching signals depends on the average output voltage of PWM signal. Higher magnitude of PWM signal allows higher current to flow through the motor which increases the speed of motor. Therefore, Motor driver perform two tasks: (a) Electronic commutation and (b) Implement PWM Current controller. A BLDC motor with driver circuitry is shown in figure (4).

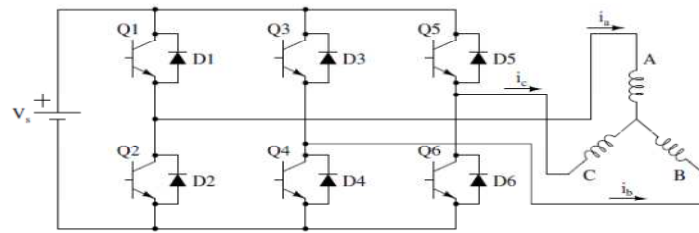


Figure 4: BLDC Motor with driver circuitry

In BLDC motor, the rotor position was guided by the electronic switches which to ensure proper commutation occurs and the motor rotates. In every 60 of rotation, the Hall sensor changed its state and each combination of hall sensors states represents a specific rotor position. The BLDC motor uses six step inverter operations for commutation part. Six-step commutation is a cost-effective due to its simple and relatively inexpensive feedback and drive dives. In six-step inverter operation, only two winding are energized at a time. Each step rotates at 60 electrical degrees, which six paths make a full 360 degree rotation. One full 360 degree loop is able to control the current, due to the fact that there is only one current path. A six step switching sequence for electronic commutation is shown in Table.1. A Simulink Model of Motor drives comprises two parts: Reference Current Generation and PWM Current Controller.

Rotor position ( $\theta_e$ ) (deg)	Hall sensor state			Switch closed		Phase Current		
	H1	H2	H3			A	B	C
0-60	1	0	0	Q1	Q4	+	-	off
60-120	1	1	0	Q1	Q6	+	off	-
120-180	0	1	0	Q3	Q6	off	+	-
180-240	0	1	1	Q3	Q2	-	+	off
240-300	0	0	1	Q5	Q2	-	off	+
300-360	1	0	1	Q5	Q4	off	-	+

Table 1: Six step switching sequence for Commutation

**A. Reference Current Generation:** The magnitude of reference current  $i_{ref}$  is determined by reference torque  $T_{ref}$  which is the output of PID or Fuzzy PID motion controller.

$$i_{ref} = \frac{T_{ref}}{K_t}$$

Where  $K_t$  is Torque constant. Depending upon rotor position with help of Hall sensor, Reference current generation block generate three phase reference currents ( $i_{aref}, i_{bref}, i_{cref}$ ). According to rotor position, three phase reference current generation are listed below Table 2.

Rotor position ( $\theta_e$ )(deg)	$i_{aref}$	$i_{bref}$	$i_{cref}$
0-60	$i_{ref}$	- $i_{ref}$	0
60-120	$i_{ref}$	0	- $i_{ref}$
120-180	0	$i_{ref}$	- $i_{ref}$
180-240	- $i_{ref}$	$i_{ref}$	0
240-300	- $i_{ref}$	0	$i_{ref}$
300-360	0	- $i_{ref}$	$i_{ref}$

Table 2: Three phase reference current generation sequence

Simulink Model Block for Three phase Reference Current generation is shown in figure (5)

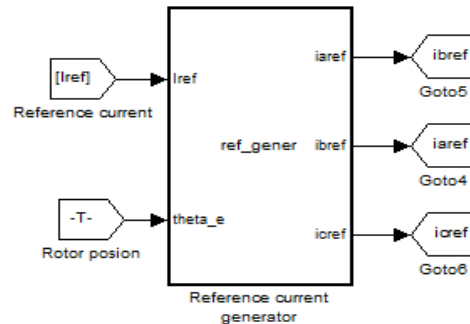


Figure 5: Three phase reference generation Simulink block model

**B. PWM Current Controller:** After generation of three phase reference currents, these are sent to PWM Current Controller block which is also known as Hysteresis Current Controller. In Current Controller block, these three phase currents are compared with actual three phase currents and current error are generated. Then these current errors are then sent to hysteresis current controller for generation of switching signals for Motor driver. Hysteresis band PWM is basically an instantaneous feedback current control method of PWM, where the actual current continuously tracks the command current within hysteresis band. The current exceed upper band limit the upper switch is OFF and lower switch is ON. As the current exceed lower band limit upper switch is ON and lower switch is OFF. From the Hysteresis block the switching function are modelled as per operation of PWM inverter [2] [3].

The switching logic is given below

- Case 1. If  $i_{aer} \geq h_b$  then Q1 is ON and Q4 is OFF. Therefore,  $V_a = V_s/2$
- Case 2. If  $i_{aer} < h_b$  then Q1 is OFF and Q4 is ON. Therefore,  $V_a = -V_s/2$
- Case 3. If  $i_{ber} \geq h_b$  then Q3 is ON and Q6 is OFF. Therefore,  $V_b = V_s/2$ .
- Case 4. If  $i_{ber} < h_b$  then Q3 is OFF and Q6 is ON. Therefore,  $V_b = -V_s/2$ .
- Case 5. If  $i_{cer} \geq h_b$  then Q5 is ON and Q2 is OFF. Therefore,  $V_c = V_s/2$ .
- Case 6. If  $i_{cer} < h_b$  then Q5 is OFF and Q2 is ON. Therefore,  $V_c = -V_s/2$ .

Where  $V_a$ ,  $V_b$  and  $V_c$  are taken to mid-point of DC supply voltage and  $h_b$  is the hysteresis band around the three phase reference currents. The Simulink Bloch model of Current Controller is shown in Figure (6).

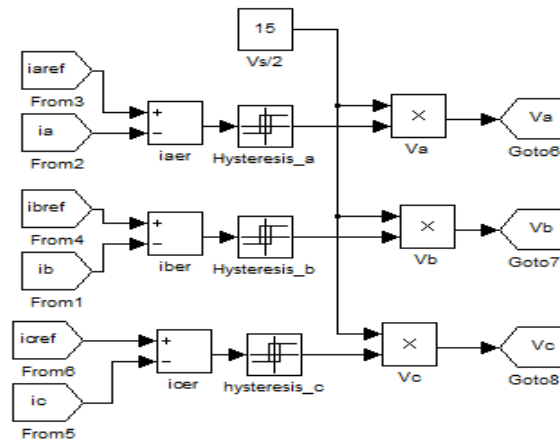


Figure 6: Hysteresis Current Controller block Simulink Model

#### IV. Fuzzy PID based Motion Controller



Fuzzy Control incorporates ambiguous human logic into computer programs. It suits control problems that cannot be easily represented by mathematical models. Because of its unconventional approach, design of such controllers leads to faster development/ implementation cycles. A closed loop control of a plant is shown in figure (7).

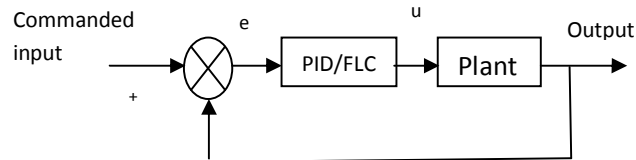


Figure 7: A Closed loop Control of Plant

According to PID Control law [4] [5]:

$$u(t) = K \left( e(t) + \frac{1}{T_i} \int_0^t e(t) dt + T_d \frac{de(t)}{dt} \right)$$

It can be represented in discrete domain

$$u(k) = K \left[ e(k) + \frac{1}{T_i} (e(k) + e(k-1) + \dots) \Delta T + T_d \left[ \frac{e(k) - e(k-1)}{\Delta T} \right] \right] \quad (5)$$

Where,  $t = k\Delta T$ ,  $\Delta T$ =sampling time and  $k$ =sampling instant.

From equation (5), we can obtain a relation between current and previous output value like as

$$u(k) = u(k-1) + \Delta u(k)$$

Where,  $\Delta u(k)$  is incremented controller output which is functions of  $e$  and  $\Delta e$  like as  $\Delta u(k) = f(e, \Delta e)$ .

**Fuzzy PID Controller:** A Fuzzy PID which is based on Mamdani Type takes two inputs ( $e, \Delta e$ ) and according to rule generates one control output  $u$ . A linguistic variable which implies input and output have been classified as: Negative Large (NL), Negative Medium (NM), Negative Small (NS), Zero (Z) Positive Small (PS), Positive Medium (PM) and Positive Large (PL). Inputs and Output are all normalized in interval of  $[-1, 1]$ . The membership function for Inputs and output is shown in figure (8). From set of input linguistic variables, a set of rules can be formed which is presented in Table 3.

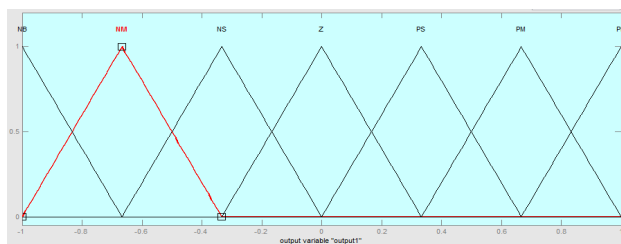


Figure 8: Membership function of inputs  $e, \Delta e$  and output  $u$

$e \backslash \Delta e$	NL	NM	NS	Z	PS	PM	PL
NL	NL	NL	NL	NL	NM	NS	Z
NM	NL	NL	NL	NM	NS	Z	PS
NS	NL	NL	NM	NS	Z	PS	PM
Z	NL	NM	NS	Z	PS	PM	PL
PS	NM	NS	Z	PS	PM	PL	PL
PM	NS	Z	PS	PM	PL	PL	PL
PL	Z	PS	PM	PL	PL	PL	PL

Table 3: Rules for Fuzzy PID Controller

A Simulink Block model for Fuzzy PID is given in figure (9).

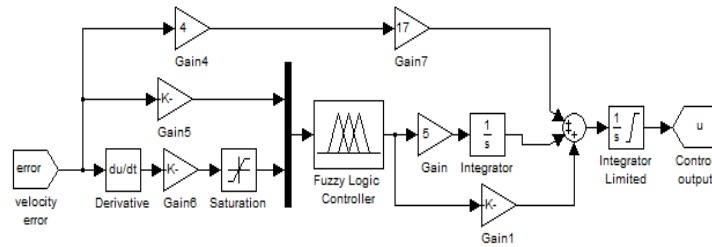


Figure 9: Fuzzy PID FLC Simulink Block model

## V. Simulation Result

A closed loop Speed control of BLDC Motor by Fuzzy PID is simulated with above BLDC model. Here Speed control and Current control a loop are cascaded and is presented in figure (10). According to velocity error, PID/FLC Controller generate reference current which is sent to PWM current Controller for generation of inverter switching signals. A Faulhaber BLDC motor (Series-**2444024B**)[10] and Motor driver rated current 6A peak have been taken for simulation. The List of parameters of motor is given in Table 4. Motor is set to 10000rpm speed and Simulated Performance of Motor at no load and loaded condition are analysed.

Nominal Voltage (Vs)	28V
Stator winding resistance (terminal)R	2.1 ohm
Stator winding inductance(terminal)L	180 uh
Friction Torque B	0.000035 mNm/rpm
Back-emf constant Ke	1.026 mV/rpm
Rotor inertia J	6.5 gcm <sup>2</sup>
No load Current	0.184A

Table 4: List of Motor parameters

**A. Results at No Load Condition:** A Plot of Stator winding phase currents for duration of 20ms is shown in figure (11)

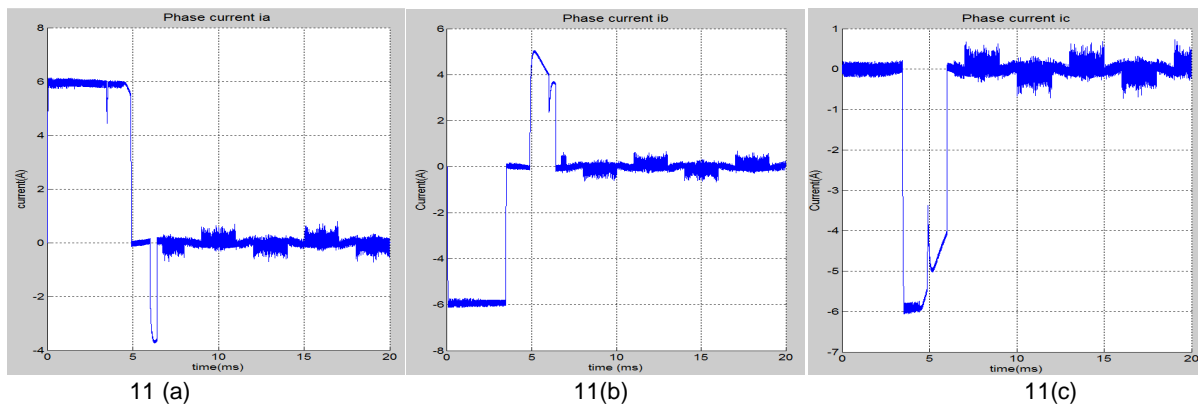


Figure 11: Stator Winding Current (a) Phase A (b) Phase B (c) Phase C



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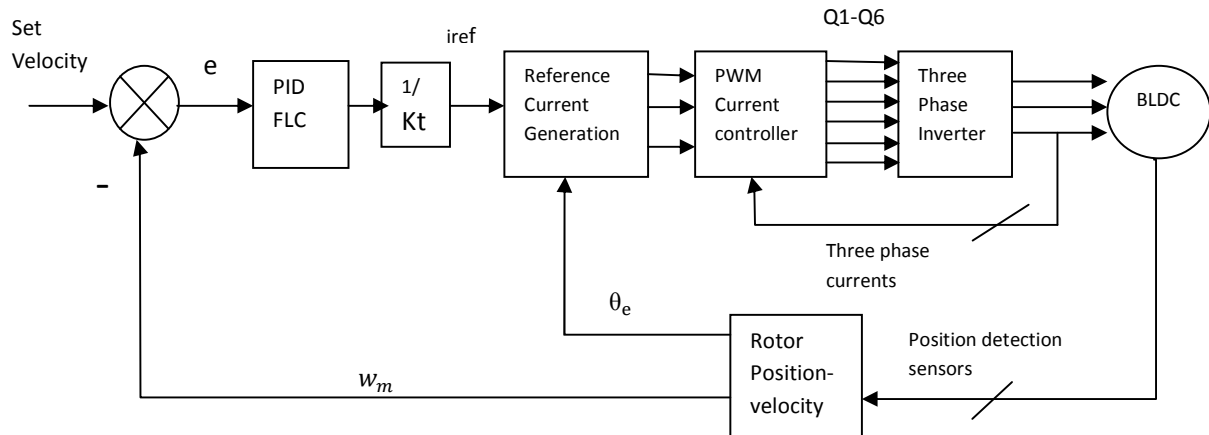
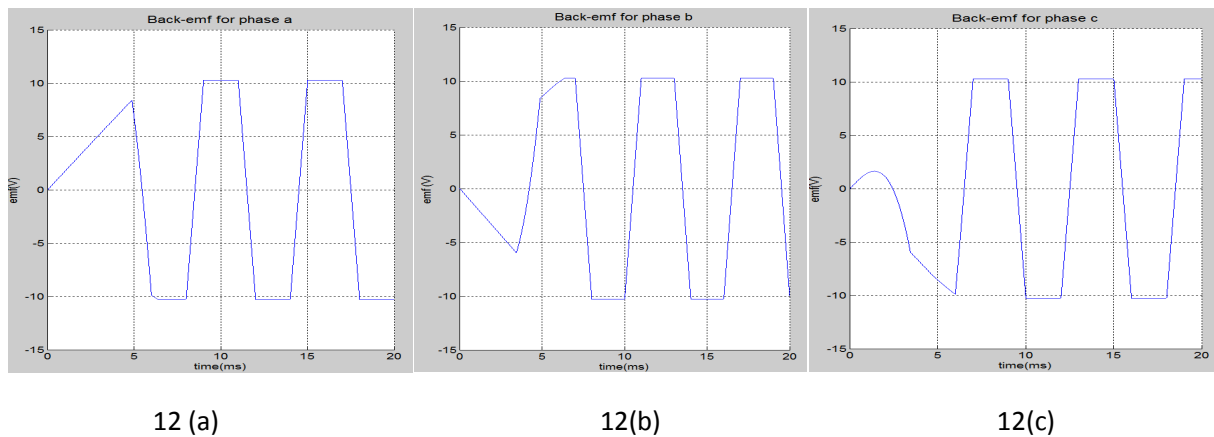


Figure 10: Closed loop Speed Control of BLDC Motor

A plot of generated Back-emf in phase winding is shown in figure (12)



12 (a)

12(b)

12(c)

Figure 12: Stator winding Back-emf (a) Phase A (b) Phase B (c) Phase C

A Motor Rotor speed response and total effective electrical torque generated by Simulated Model at no load are shown in figure (13.a) and (13.b) respectively. Rise time of obtained speed response is 6.3ms.

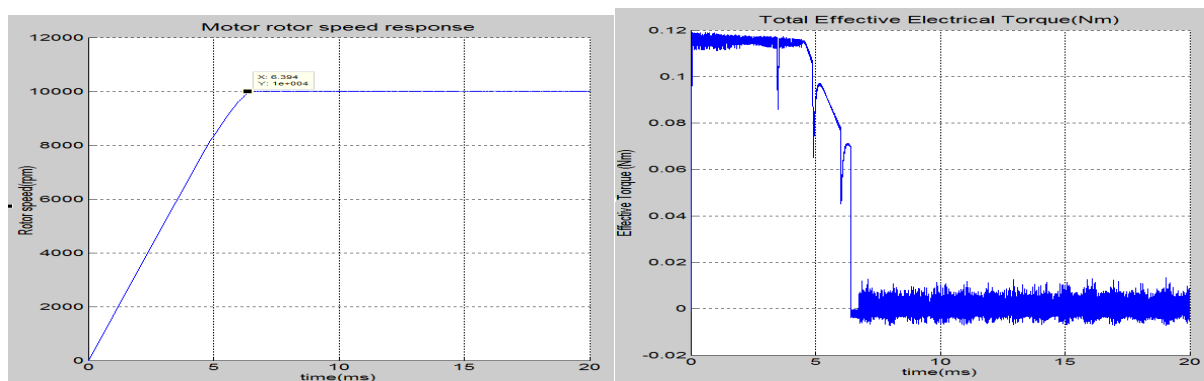
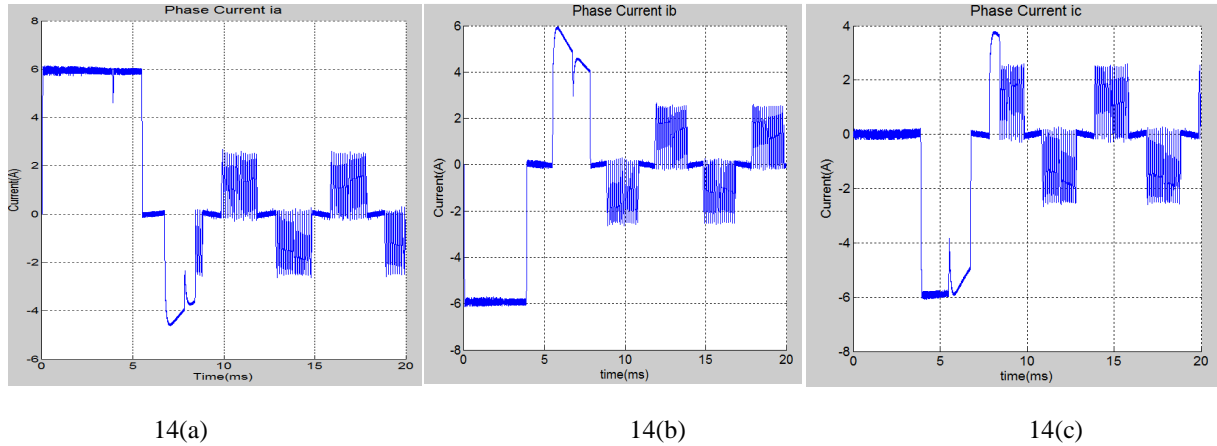


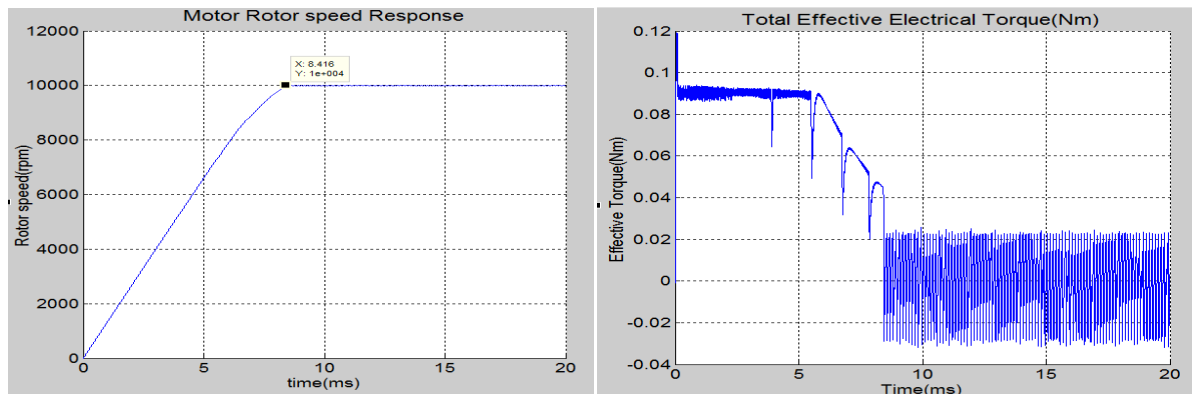
Figure 13.a: Motor Rotor Speed Response Plot

Figure 13.b: Total Effective Electrical Torque

**B. Results at Loaded condition:** A 25mNm Load is applied at Motor rotor shaft. The Stator Phase Currents, Speed response and Total effective electrical torque generated plots at loaded condition are shown in figure (14)-(16) respectively.



**Figure 14: Stator winding Phase Current (a) Phase A (b) Phase B (c) Phase C**



**Figure15: Motor Rotor Speed Response**

**Figure 16: Total Effective Electrical Torque**

From above Simulation results, It is very clear that at no load condition, Current is **0.2A** (near to no load current) and Effective torque is very less. At loaded condition, Current, Effective torque and speed response Rise time are nearby **2.5A**, **25mNm** and **8.4ms** respectively which are very close to desired results.

## V. Conclusion

This paper presents the complete Speed Control of BLDC Motor in Matlab/Simulink environment. Here BLDC motor modelled in State-Space is very efficiently controlled by Fuzzy PID Controller. Besides, Fuzzy reasoning algorithm designed to control BLDC to get the optimum performance under the unstable rotor turning situation or sudden load change, the proposed Fuzzy PID controller has good adaptability and strong robustness whenever the system is disturbed. The Simulation of Dynamic performances (Speed, Torque and Phase Currents etc.) of Motor at both loaded and not loaded conditions are analysed and compared. It can be said that Matlab/Simulink is a good tool for modelling and simulation of Fuzzy logic controlled Brushless DC motor. The presented model can be made equivalent to actual system model by incorporating more details of motor and Drivers parameters. By adopting

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Hybrid controller, Adaptive Fuzzy PID controller, artificial neural networks or Neural-Fuzzy controller, performance of BLDC Motor can be improved.

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