Ryan Darras, CS 5070 - HW10 4.(6, 13, 21)

Problem 4.6

Let X be the set {1,2,3,4,5} and Y be the set {6,7,8,9,10}. We describe the functions f : X→ Y and g: X→ Y in the following tables. Answer each part and give a reason for each negative answer.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

n	$g(n)$
1	10
2	9
3	8
4	7
5	6

[A] **a.** Is f one-to-one?
b. Is f onto?
c. Is f a correspondence?

[A] **d.** Is g one-to-one?
e. Is g onto?
f. Is g a correspondence?

Problem 4.6a Answer

No, f(1) and f(3) are equal.

Problem 4.6b Answer

No, no input for f(n) results in 8, 9, or 10 from set Y.

Problem 4.6c Answer

No, f is neither one to one or onto, therefore it is not a correspondence.

Problem 4.6d Answer

Yes, no value of n in f(n) results in the same answer

Problem 4.6e Answer

Yes, every value of Y is reached.

Problem 4.6f Answer

Yes, both one to one and onto.

Problem 4.13

Let A = {⟨R,S⟩| R and S are regular expressions and L(R) ⊆ L(S)}. Show that A is decidable.

Problem 4.13 Answer

First construct a language L(C) = $\overline{L(S)}$ ∩ L(R). Then create DFA's R_{dfa} and S_{dfa} by converting R and S. We can use R_{dfa} and S_{dfa} to construct C_{dfa}. We know that since R is a subset of S, if we intersect the complement of S with R it will result in a language consisting of the empty set. Therefore, C_{dfa} should only accept the empty string. Knowing this, we can run E_{dfa} to check if C recognizes the empty set and accept it if it does. Otherwise, reject.

Problem 4.21

Let S = {⟨M⟩| M is a DFA that accepts w^R whenever it accepts w}. Show that S is decidable.

Problem 4.21 Answer

"On valid input <M>:

1) Construct an NFA M' such that L(M') = { w^R | w ε L(M) }
2) Create DFA M'_{dfa} using M'
3) Use EQ_{dfa} to compare L(M'_{dfa}) and L(M)
4) If L(M'_{dfa}) = L(M) accept; otherwise reject.

All steps can be done in a finite amount of time.