Ryan Darras, CS 5070 - HW07 2.(30(a,d), 34, 42, 48)

*Used a combination of https://www.cefns.nau.edu/~edo/Classes/CS315_WWW/Tools/fsm.html and paint.net to create images

Problem 2.30

Use the pumping lemma to show that the following languages are not context free.

Problem 2.30 A Answer

$L = \{0^n 1^n 0^n 1^n \mid n \geq 0\}$

Proof by contradiction:

Assume L is a CFL. Let p be the pumping length for L. Select the string $s = 0^p 1^p 0^p 1^p$. We can see that s is a member of L. We can show that this string cannot be pumped because no matter how we divide s into uvxyz one of the three lemma conditions are violated. The pumping lemma states that $|vy| > 0$, which means either v or y is nonempty. If v or y contain only one type of alphabet symbol (0 or 1) then when pumped it would change the relationship of our string such that we wouldn't have p number of 0's followed by p 1's, followed by p 0's, followed by p 1's. In the case that v and y contain both elements of the alphabet (0 and 1), then no matter how we partition into uvxyz we will end up getting patterns of 010101 etc when we pump it. Hence, L is not pumpable, which means that L is not context free.

Problem 2.30 D Answer

$L = \{ t_1 \# t_2 \# \cdots \# t_k \mid k \geq 2$, each $t_i \in \{a,b\}^*$, and $t_i = t_j$ for some $i \,!\!= j \}$

Let string $s = a^p b^p \# a^p b^p$. If vxy contains #, and # is in either v or y, when we pump down to uv^0xy^0z then we get a string that doesn't have # in it, which is therefore not in the language. If x is #, then we know that v is a subset of b^p and that y is a subset of a^p because $|vxy| <= p$. When we pump down, we notice that we either lose b's from the left side of #, or we lose a's from the right side, or a combination of both. Thus when we pump, left side != right side which means that s isn't a string in language L. Hence L is not a context free language by the pumping lemma.

Problem 2.34

Consider the language B = L(G), where G is the grammar given in Exercise 2.13. The pumping lemma for context-free languages, Theorem 2.34, states the existence of a pumping length p for B. What is the minimum value of p that works in the pumping lemma? Justify your answer.

2.13 Grammar: Let G =(V,Σ,R,S) be the following grammar. V = {S,T,U}; Σ={0,#}; and R is the set of rules:
S → TT | U
T → 0T | T0 | #
U → 0U00 | #

Problem 2.34 Answer

Pumpable string s can be seen as ##0, formed by taking the TT route of S where the first T gives #, and the 2nd T gives T0. Currently at #T0, this T gives # giving ##0.
Let u, v, z = ε, x = ##, and y = 0. When pumped, we get ##00, ##000, ##0000, etc. Thus the pumping length p is |##0| = 3.

Problem 2.42

Let Y ={w | w = t_1#t_2#···#t_k for k≥0, each $t_i \in 1^*$, and t_i != t_j whenever i != j}. Here Σ={1,#}. Prove that Y is not context free.

Problem 2.42 Answer

Let s = 1^p#1^{p+1}. If # is contained in either v or y, when we pump the string we will get something like 1^p#1^p#1^p#1^{p+1} in which $t_0 = t_1$. So let us consider # being contained in x. Because |vxy| <= p, we need to consider u = 1, v = 1^{p-1}, x = #, y = ε, z = 1^{p+1}. When we go to pump the string, we notice that on the first pump v = 1^p. Including u on that, we notice that $t_0 = t_1$ where they both equal 1^p. In other words, we don't have any memory to store what number of 1's have been used for each t_i which makes Y not context free.

Problem 2.48

Let Σ={0,1}. Let C_1 be the language of all strings that contain a 1 in their middle third. Let C_2 be the language of all strings that contain two 1s in their middle third.
So $C_1 = \{ xyz \mid x,z \in \Sigma^* \text{ and } y \in \Sigma^*1\Sigma^*, \text{ where } |x| = |z| \geq |y| \}$ and
$C_2 = \{ xyz \mid x,z \in \Sigma^* \text{ and } y \in \Sigma^*1\Sigma^*1\Sigma^*, \text{ where } |x| = |z| \geq |y| \}$.

Problem 2.48 A Answer

Show that C_1 is a CFL.
Let s be a string broken down into $u = \Sigma^{p-1}$, $v = \Sigma^1$, $x = 1$, $y = \Sigma^1$ $z = \Sigma^{p-1}$. When we pump s, v and y will become increasingly larger on both sides of the 1. Since we follow the rules of the pumping lemma $|vxy| <= p$ and $|vy| > 0$ this follows the pumping lemma, and s is in the language because either side of the 1 is evenly increased resulting in this 1 being in the middle third of the string.

Problem 2.48 B Answer

Show that C_2 is not a CFL.
Let $s = 0^{p+3}10^p10^{p+3}$. Break it down into $u = 0^{p+2}$, $v = 0^1$, $x = 10^p1$, $y = 0^1$, $z = 0^{p+2}$. We see that when we pump this, we follow the rule of the language that s contains 2 1's in the middle third of its language, however we end up breaking the rules of the pumping lemma such that $|vyx| <= p$.