CS-4700/5700 Automata, Computability, and Formal Languages
HX01

True/False (w/justification)

Identify whether each of the following statements is **True** or **False**. In either case, you must provide a brief (two or three short sentences) justification for your answer. In general, the grading awards 20% for correctly indicated T/F and 80% for a sound justification. However, if the justification contradicts a correct T/F choice, then the points associated with it may be reduced.

1. In an NFA, an input string is accepted as long as at least one possible processing path passed through an accepting state at some point.
2. The pumping lemma for regular languages can be used to prove that a language is regular.
3. There are some NFAs that do not have an equivalent DFA.
4. The union of two non-regular languages cannot be regular.
5. The intersection of two non-regular languages cannot be regular.
6. The union of a regular language and a non-regular language cannot be regular.
7. The complement of a non-regular language can be regular.
8. Any language consisting of a finite number of strings is regular.
9. If an NFA accepts language L, then an NFA that accepts the complement of L can be constructed by making all of the accept states non-accept states and vice-versa.
10. If two DFAs recognize the same language, then they must be the same except for state labeling.
11. It is not possible to have two topologically different DFAs that recognize the same language and have the same number of states.
12. In order for a finite automaton to recognize a language, the language must be finite in size.
13. A minimal DFA recognizing a given language is unique (sans state labeling).
14. The regular expression associated with a minimal DFA is unique.
15. The intersection of two regular languages may or may not be a regular language.
16. If the union of a regular language with language L is regular, then L is regular.
17. Some regular expressions, such as $L = a^n b^n$, do not represent regular languages.
18. If an expression is regular, then the language it represents is regular.
19. Regular languages are only closed under the operations of union, concatenation, and star.
20. Two strings are distinguishable by language L if a DFA recognizing L places them in different states.
21. The size of the largest set of strings that are pairwise distinguishable by language L is equal to the number of states in the minimal DFA that recognizes L.

22. Any two strings that are both in a language cannot be distinguished by that language.
23. A generalized NFA may only have epsilon transitions leaving the start state.
24. There is no order of precedence for the regular operations.
25. All DFAs are deciders while not all NFAs are deciders.

Long Answer

1. Is the language L regular? If it is, provide either an FA that recognizes it or a regular expression that generates it. If it is not, prove so using the pumping lemma for regular languages. You may also invoke the closure of regular languages under union, intersection, star, and complementation.

   L = { ω | $\omega \in \{a, b\}^*$ *and* ω *is NOT a palindrome*}

2. Construct a DFA that recognizes the following language over the alphabet {a,b}.

   L = {w | w has an even number of **a**'s, each of which is immediately followed by a **b**}

3. Write a regular expression that generates the language over {0,1}* that contains every string except 11 and 111. You may use Σ = (a U b) in your expression.

4. Draw the GNFA for the DFA M = {Q, Σ, δ, q_0, F} where
   Q = {q_0, q_1}; Σ= {a, b}; F = {q_1}
   δ(q_0, a) = q_0; δ(q_0, b) = q_1; δ(q_1, a) = q_1; δ(q_1, b) = q_0;

5. Consider the strings over the decimal digits that represent an unsigned base-10 integer that is processed beginning with the most-significant digit. Draw a DFA that accepts this string if the integer is evenly divisible by 5.

6. Draw a 3-state NFA recognizing the language L = 1*(0011*)*.

7. Draw a finite automaton that accepts the language over the alphabet {a,b} in which all strings have an even length and an odd number of a's.

8. Construct a DFA that recognizes the following language over the alphabet {a,b}.

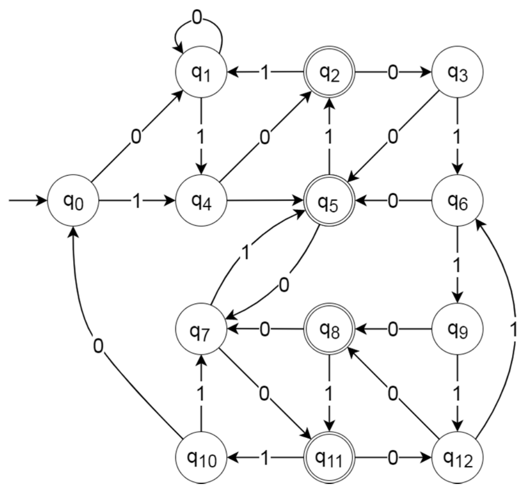
   L = {w | w has an even number of a's, no two of which are consecutive}

9. Describe how to transform any NFA into an equivalent one that has at most one accept state.

10. What is the minimum pumping length for the language 1*01*01*?

11. Construct a minimal DFA, using shortlex labels, that accepts those strings that represent base-3 numbers divisible by 6 when read least-significant digit first.

12. Find the minimal DFA, using shortlex labels, for the following DFA.



13. Consider the language

L = { $\omega \mid \omega \in \{0,1,2,3\}^*$ $where\ \omega\ is\ a\ base-4\ number\ divisible\ by\ 6\}$

What are the class representatives for the equivalence classes for L?

14. Consider the language

L = { $\omega \mid \omega \in \{0 \ldots 9\}^*$ $where\ \omega\ is\ a\ decimal\ number\ divisible\ by\ 10\}$

What are the class representatives for the equivalence classes for L?

15. Consider the language

L = { $\omega \mid \omega \in \{a,b\}^*$ $where\ either\ n_a(\omega)\ or\ n_b(w) is\ odd, but\ not\ both\}$

What are the class representatives for the equivalence classes for L?