Ryan Darras, CS 5700 - HW11 5.(3, 9, 31)

<u>Problem 5.3</u>

Find a match in the following instance of the Post Correspondence Problem.

$$\left\{ \left[\frac{\mathtt{ab}}{\mathtt{abab}}\right], \left[\frac{\mathtt{b}}{\mathtt{a}}\right], \left[\frac{\mathtt{aba}}{\mathtt{b}}\right], \left[\frac{\mathtt{aa}}{\mathtt{a}}\right] \right\}$$

<u>Problem 5.3 Answer</u>

Heres 13… I wrote a program to solve it because it was fun. Fun fact, when I asked for the top 15 it ran for over a minute and couldn't find that amount.

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aa|aa|b|ab
a|a|a|abab

aa|aa|aa|b|aa|b|aba|b|ab|ab|ab
a|a|a|a|a|a|b|a|abab|abab|abab

aa|b|aba|b|aba|ab|aa|b|aba|b|ab|ab|ab
a|a|b|a|b|abab|a|a|b|a|abab|abab|abab

aa|aa|aa|b|aa|aa|aba|b|b|aa|b|ab|aba|b|ab|ab|ab|ab
a|a|a|a|a|a|b|a|a|a|a|abab|b|a|abab|abab|abab|abab

aa|b|aba|b|aba|ab|aa|aa|aba|b|b|aa|b|ab|aba|b|ab|ab|ab|ab
a|a|b|a|b|abab|a|a|b|a|a|a|a|abab|b|a|abab|abab|abab|abab

aa|b|aba|aa|aba|b|aa|b|ab|b|ab|aba|ab|aa|b|aba|b|ab|ab|ab
a|a|b|a|b|a|a|a|abab|a|abab|b|abab|a|a|b|a|abab|abab|abab

aa|b|aba|aa|aba|b|aa|b|ab|aa|b|aba|b|aba|b|ab|ab|ab|ab|ab
a|a|b|a|b|a|a|a|abab|a|a|b|a|b|a|abab|abab|abab|abab|abab

aa|aa|aa|b|aa|b|aba|b|b|aba|ab|aba|b|aba|b|ab|ab|ab|ab|ab
a|a|a|a|a|a|b|a|a|b|abab|b|a|b|a|abab|abab|abab|abab|abab

aa|b|aba|b|aba|ab|aa|b|aba|b|b|aba|ab|aba|b|aba|b|ab|ab|ab|ab|ab
a|a|b|a|b|abab|a|a|b|a|a|b|abab|b|a|b|a|abab|abab|abab|abab|abab

aa|b|aba|b|aba|b|aba|b|aba|ab|ab|b|ab|aba|ab|aa|b|aba|b|ab|ab|ab
a|a|b|a|b|a|b|a|b|abab|abab|a|abab|b|abab|a|a|b|a|abab|abab|abab

aa|b|aba|b|aba|b|aba|b|aba|ab|ab|aa|b|aba|b|aba|b|ab|ab|ab|ab|ab
a|a|b|a|b|a|b|a|b|abab|abab|a|a|b|a|b|a|abab|abab|abab|abab|abab

aa|b|aba|aa|aba|b|b|b|ab|aba|aba|b|aba|b|aba|b|ab|ab|ab|ab|ab|ab
a|a|b|a|b|a|a|a|abab|b|b|a|b|a|b|a|abab|abab|abab|abab|abab|abab

aa|aa|aa|b|aa|aa|aba|b|aa|aa|b|ab|b|aa|b|ab|aba|b|ab|ab|ab|ab
a|a|a|a|a|a|b|a|a|a|a|abab|a|a|a|abab|b|a|abab|abab|abab|abab
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Problem 5.9

Let T = {<M> | M is a TM that accepts w^R whenever it accepts w}. Show that T is undecidable.

Problem 5.9 Answer

We know that A_{tm} is undecidable, so use A_{tm} to create G.

G = “on valid input <M, w>

1) CreateTM F:
   a) F = “on valid input x:
      i) If x is in L(00*11), accept
      ii) If x is not in L(00*11*), run M on input w. If M accepts, accept; otherwise reject.
2) Run G on input <F>
3) If G accepts, accept; otherwise reject.”

Problem 5.31

Let

$$f(x) = \begin{cases} 3x+1 & \text{for odd } x \\ x/2 & \text{for even } x \end{cases}$$

for any natural number x. If you start with an integer x and iterate f, you obtain a sequence, $x, f(x), f(f(x)), \ldots$. Stop if you ever hit 1. For example, if $x = 17$, you get the sequence 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1. Extensive computer tests have shown that every starting point between 1 and a large positive integer gives a sequence that ends in 1. But the question of whether all positive starting points end up at 1 is unsolved; it is called the $3x + 1$ problem.

Suppose that A_{TM} were decidable by a TM H. Use H to describe a TM that is guaranteed to state the answer to the $3x + 1$ problem.

Problem 5.31 Answer

H = “On valid input <M>

1) For every natural number, run G.
   G = “On valid input x:
   1) If value is 1, accept; else continue to step 2.
   2) If x is odd, run 3x + 1 on H. Return to step 1
   3) If x is even, run x / 2 on H. Return to step 1
2) If G does not halt, reject; otherwise accept.

*Note: I realize that the above solution isn’t guaranteed to state the answer, but I can’t figure out how to get passed the fact that |N| is infinity. I thought about doing a proof by induction, but I don’t think that is the way to go.