Ryan Darras, CS 5070 - HW01 0.(2, 6, 8, 11)

Problem 0.2

Write formal descriptions of the following sets.

A. The set containing the numbers 1, 10, and 100.
B. The set containing all integers that are greater than 5.
C. The set containing all natural numbers that are less than 5.
D. The set containing the string aba.
E. The set containing the empty string.
F. The set containing nothing at all.

Problem 0.2 Answers

A. {1, 10, 100}
B. {x | x ε natural numbers & x > 5 }
C. {1, 2, 3, 4}
D. {aba}
E. {ε}
F. {}

Problem 0.6

Let X be the set {1,2,3,4,5} and Y be the set {6,7,8,9,10}. The unary function f: X→ Y and the binary function g: X × Y → Y are described in the following tables.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

A. What is the value of f(2)?
B. What are the range and domain of f?
C. What is the value of g(2, 10)?
D. What are the range and domain of g?
E. What is the value of g(4, f(4))?

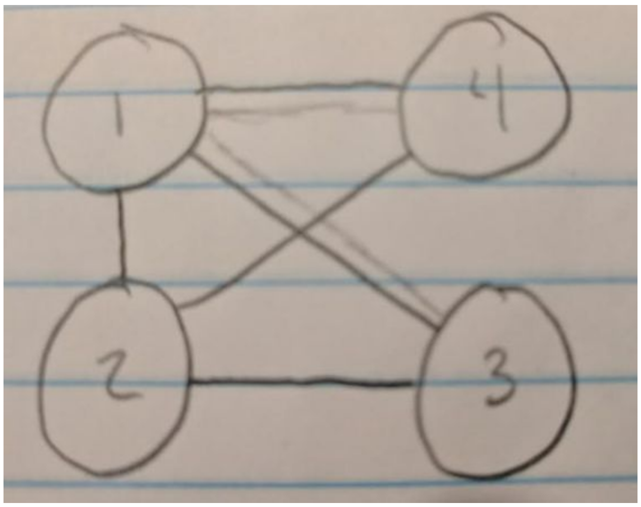
Problem 0.6 Answers

A. 7
B. Range = Y, Domain = X
C. 6
D. Range = Y, Domain = X × Y
E. 8

Problem 0.8

Consider the undirected graph G=(V,E) where V, the set of nodes, is{1,2,3,4} and E, the set of edges, is{{1,2}, {2,3}, {1,3}, {2,4}, {1,4}}. Draw the graph G. What are the degrees of each node? Indicate a path from node 3 to node 4 on your drawing of G.

Problem 0.8 Answers



Degrees: 1=3, 2=3, 3=2, 4=2

Path from 3-4 portrayed as the lighter lines. 3 to 1 to 4.

Problem 0.11

Let $S(n)=1+2+\cdots+ n$ be the sum of the first n natural numbers and let $C(n)=1^3 +2^3 + \cdots+ n^3$ be the sum of the first n cubes. Prove the following equalities by induction on n, to arrive at the curious conclusion that $C(n)=S^2(n)$ for every n.

A. $S(n)=(½)n(n + 1)$.
B. $C(n)=(¼)(n^4 + 2n^3 + n^2) = (¼)n^2(n + 1)^2$

Problem 0.11 Answers

A.

$S(n) = 1 + 2 + \ldots + n = (½)n(n + 1)$.

Basis step shows that $S(1)=(½)1(1+1)=1$ which is true.

Assume true for $n < k$. $S(k-1)=1 + 2 + \ldots + k-1 =((k-1)k)/2$ so

$S(k)=1 + 2 + \ldots + k - 1 + k = ((k-1)k)/2 + k = ((k-1)k + 2k)/2 = (k^2 + k)/2 = (k(k + 1))/2$.

Therefore $S(n) = 1 + 2 + \ldots + n$ is true for 1 and it is true for all values such that if it is true for one less than a number, then it is also true for that number, hence it is true for all numbers greater than 1.

B.

$C(n)=1^3 + 2^3 + \ldots + n^3 = (¼)n^2(n + 1)^2$

Basis step shows that $C(1)=(¼)(1)^2(1 + 1)^2 = 1$ which is true.

Assume true for $n<k$. $C(k-1)=1^3 + 2^3 + \ldots + (k-1)^3 = (¼)(k-1)^2(k)^2$ so

$C(k)=1^3 + 2^3 + \ldots + (k-1)^3 + k^3 = (¼)(k-1)^2(k)^2 + k = (¼)((k^2-2k+1)k^2 + 4k)$
$= (¼)((k^2+2k+1)k^2) = (¼)k^2(k+1)^2$.

Therefore $C(n)=1^3 + 2^3 + \ldots + n^3$ is true for 1 and it is true for all values such that if it is true for one less than a number, then it is also true for that number, hence it is true for all numbers greater than 1.