THE CONDITIONAL HETEROSCEDASTICITY OF THE YEN–DOLLAR EXCHANGE RATE

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SUMMARY

This paper examines the conditional heteroscedasticity of the yen–dollar exchange rate. A model is constructed by extending the asymmetric power autoregressive conditional heteroscedasticity model to a process that is fractionally integrated. It is found that, unlike the equity markets, the appreciation and depreciation shocks of the yen against the dollar have similar effects on future volatilities. Although the results reject both the stable and the integrated models, our analysis of the response coefficients of the past shocks and the application of the models to the estimation of the capital requirements for trading the currencies show that there are no substantial differences between the fractionally integrated models and the stable models. © 1998 John Wiley & Sons, Ltd.

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1. INTRODUCTION

This paper examines the conditional heteroscedasticity of the yen–dollar exchange rate. The persistence in the exchange rate volatility is studied using a fractionally integrated conditional heteroscedasticity model. This model is constructed by extending the asymmetric power autoregressive conditional heteroscedasticity (APARCH) model of Ding, Granger, and Engle (1993) to a process that is fractionally integrated, as defined by Baillie, Bollerslev, and Mikkelsen (1996). Baillie *et al*. (1996) argued that the exchange rate volatility may be more persistent than what is captured by ordinary ARCH and GARCH type of models. Their analysis of the mark–dollar exchange rate using a fractionally integrated GARCH (FIGARCH) model showed that the FIGARCH specification was preferred to both the stable GARCH and the integrated GARCH (IGARCH) models.

Work in modelling the conditional volatility of stock prices (see, e.g. Engle and Ng, 1993) has found that the stock volatility responds asymmetrically to positive versus negative shocks. Typically, stock volatility tends to rise higher in response to negative shocks as opposed to positive shocks. In this paper we investigate whether there is any asymmetry in the conditional volatility of exchange rate.

The remainder of this paper is organized as follows. In Section 2 we describe the conditional heteroscedasticity models and the estimation results. In Section 3 we examine the stability of the model parameters and report the results of the analysis of the response functions of the past shocks. The models are applied to estimate the margin requirements for the trading in currency futures. Finally, some conclusions are given in Section 4.

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2. THE DATA AND THE ESTIMATION RESULTS

The data consist of daily observations of the yen–dollar exchange rate (in yen per dollar) from 1978 January through 1994 June, totalling 4067 observations. We denote the differenced logarithmic exchange rate by r_t, which is the continuously compounded rate of depreciation (appreciation if negative) of the yen against the dollar. r_t is found to be negatively skewed and leptokurtic. The Box–Pierce portmanteau statistics of the $|r_t|$ and r_t^2 series suggest that there may be conditional heteroscedasticity in r_t.[1]

To model the conditional heteroscedasticity in the data we adopt the APARCH model proposed by Ding *et al.* (1993). Denoting $\varepsilon_t = r_t - \mu_t$, where μ_t is the conditional mean of r_t, as the residual of the differenced logarithmic exchange rate, we consider the APARCH(1, 1) model given by the following equations:

$$\begin{aligned} \varepsilon_t &= \sigma_t e_t \\ \sigma_t^\delta &= \eta + \alpha(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^\delta + \beta\sigma_{t-1}^\delta \end{aligned} \tag{1}$$

where e_t are independently and identically distributed (IID) with unit variance, η is positive, α, β, and δ are non-negative and $|\gamma| < 1$. When $\gamma > 0$ (<0), negative (positive) shocks give rise to higher volatility than positive (negative) shocks. We further assume that the conditional mean is characterized by a first-order autoregressive process specified as: $\mu_t = \mu + \rho r_{t-1}$, with $|\rho| < 1$.

We estimate the GARCH(1, 1) and APARCH(1, 1) models using the quasi-maximum likelihood estimation (QMLE) method. To obtain robust inference about the estimated models, we compute the robust standard errors as suggested by Bollerslev and Wooldridge (1992). The results are summarized in Table I. The models apparently pass the diagnostics satisfactorily.[2] The estimated values of α and β are very similar across the models, and this is especially true for the estimates of β. The autoregressive parameter $\hat{\rho}$ is not statistically significant. The asymmetry parameter $\hat{\gamma}$ is positive, but not statistically significant. This is in contrast to the results for the equity markets. For the APARCH models the null hypotheses of $\delta = 1$ and $\delta = 2$ are not rejected. Thus, the data do not provide sufficient information to distinguish the conditional standard-deviation model from the conditional variance model.

To test for the persistence of the conditional heteroscedasticity models, we examine the Wald statistic for the linear constraint $\alpha + \beta = 1$. This hypothesis is rejected for all models, indicating that the conditional heteroscedasticity is not integrated.[3] However, as the estimated values of $\alpha + \beta$ are quite close to one, there is perhaps a need to examine closely the possibility of long-memory persistence in the conditional volatility. To this effect, we consider the general APARCH(p, q) model represented by the following conditional variance equation:

$$\sigma_t^\delta = \eta + \alpha(L)(|\varepsilon_t| - \gamma\varepsilon_t)^\delta + \beta(L)\sigma_t^\delta \tag{2}$$

[1] Details of the results can be found in Tse (1996).

[2] $Q_1(M)$ and $Q_2(M)$ are the portmanteau statistics based on the autocorrelation coefficients of the standardized residuals and squared standardized residuals, respectively, where M is the number of lag terms. The formulae of Q_1 and Q_2 can be found in Li and Mak (1994) and Tse (1996). These statistics take account of the fact that the residuals are estimated. They are asymptotically distributed as χ^2_M on the null.

[3] Lumsdaine's (1995) Monte Carlo results show that the Wald test for an IGARCH(1, 1) model has empirical size near its nominal level. The performance of the test for the IAPARCH(1, 1) model, however, remains unknown. Although IGARCH models are known to be strictly stationary but not covariance stationary (see Baillie *et al.*, 1996), similar results for the IAPARCH models are not available.

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Table I. Estimated GARCH and APARCH models

	GARCH model		APARCH model		
μ	−0·0100	−0·0104	−0·0129	−0·0128	−0·0091
	(0·0111)	(0·0111)	(0·0108)	(0·0108)	(0·0112)
ρ		−0·0132		−0·0178	
		(0·0171)		(0·0174)	
η	0·0352	0·0348	0·0459	0·0460	0·0401
	(0·0106)	(0·0104)	(0·0147)	(0·0147)	(0·0129)
α	0·0763	0·0771	0·0859	0·0873	0·0845
	(0·0135)	(0·0135)	(0·0160)	(0·0160)	(0·0164)
γ			0·1546	0·1484	
			(0·1033)	(0·1012)	
δ			1·4483	1·4327	1·6369
			(0·3529)	(0·3535)	(0·3617)
β	0·8558	0·8560	0·8549	0·8543	0·8585
	(0·0279)	(0·0276)	(0·0304)	(0·0303)	(0·0296)
ML	−534·0250	−529·8763	−529·0367	−525·0021	−532·7100
$Q_1(4)$	4·1898	5·0970	3·8178	5.3627	4.3212
$Q_1(8)$	8·1629	9·2050	7·6615	9·3554	8·2932
$Q_2(4)$	1·5383	1·4597	2·4379	2·3518	2·2205
$Q_2(8)$	6·7295	6·5979	9·0479	9·0104	7·9538

Notes: The figures in parentheses below the parameter estimates are the robust standard errors. ML is the maximized likelihood, Q_1 tests for the serial correlation in the standardized residuals, and Q_2 tests for the serial correlation in the squares of the standardized residuals. The number of lag terms taken in calculating the Q statistics are given in the parentheses. $Q_i(M)$ for $i = 1, 2$, are approximately distributed as χ^2_M under the null.

where $\alpha(L) = \sum_{i=1}^{p} \alpha_i L^i$, $\beta(L) = \sum_{i=1}^{q} \beta_i L^i$ and L is the lag operator. The constraints $\eta > 0$, $\alpha_i \geqslant 0$ for $i = 1, \ldots, p$, $-1 < \gamma < 1$, and $\beta_i \geqslant 0$ for $i = 1, \ldots, q$, are assumed. If we define $g(\varepsilon_t) = |\varepsilon_t| - \gamma\varepsilon_t$ and $\xi_t = g(\varepsilon_t)^\delta - \sigma_t^\delta$, equation (2) can be written as:

$$(1 - \alpha(L) - \beta(L))g(\varepsilon_t)^\delta = \eta + (1 - \beta(L))\xi_t \tag{3}$$

To modify the model so that the effect of a shock on the conditional volatility may exhibit long-memory persistence, we assume that there exists a polynomial $\phi(L)$ such that the conditional variance equation is given by:

$$(1 - L)^d \phi(L) g(\varepsilon_t)^\delta = \eta + (1 - \beta(L))\xi_t \tag{4}$$

where $0 \leqslant d \leqslant 1$ and the roots of the equation $\phi(L) = 0$ are outside the unit circle. On rearranging the terms, equation (4) can be written as follows:

$$\sigma_t^\delta = \frac{\eta}{1 - \beta(1)} + [1 - (1 - \beta(L))^{-1}\phi(L)(1 - L)^d]g(\varepsilon_t)^\delta \tag{5}$$

In the case when both $1 - \beta(L)$ and $\phi(L)$ are polynomials of degree 1 and we let $\beta(L) = \beta L$ and $\phi(L) = 1 - \phi L$, we obtain the FIAPARCH$(1, d, 1)$ model with $\sigma_t^\delta = \omega + \lambda(L)(|\varepsilon_t| - \gamma\varepsilon_t)^\delta$,

[4] The coefficients λ_i in the expansion can be calculated using the following recursions: $\lambda_1 = \phi - \beta + d$, and $\lambda_i = \beta\lambda_{i-1} + [(i - 1 - d)/i - \phi]\delta_{i-1}$ for $i = 2, \ldots, \infty$, where $\delta_i = \delta_{i-1}(i - 1 - d)/i$, with $\delta_1 = d$. We impose, however, a finite truncation at lag 1000 in estimating the parameters.

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where $\omega = \eta/(1-\beta)$ and $\lambda(L) = \sum_{i=1}^{\infty} \lambda_i L = 1 - (1-\beta L)^{-1}(1-\phi L)(1-L)^{\mathrm{d}}$.[4] Thus, the conditional standard deviation has an infinite series representation in terms of $g(\varepsilon_t)^{\delta}$.[5] The parameters of the FIAPARCH(1, d, 1) model can be estimated using the QMLE approach, with appropriate assumptions about the start-up conditions.[6] In the calibration below, presample values of $g(\varepsilon_t)^{\delta}$ are replaced by the sample mean of $(|\hat{\varepsilon}_t| - \hat{\gamma}\hat{\varepsilon}_t)^{\hat{\delta}}$, with $\hat{\gamma}$ and $\hat{\delta}$ taken from Table I.

Table II summarizes the estimation results of the FIGARCH and FIAPARCH models. It can be checked that the estimates satisfy the inequalities in footnote 6. The estimates of d are statistically significantly different from zero as well as from one. This result gives support to the modelling of fractionally integrated conditional heteroscedasticity. The autoregressive parameter $\hat{\rho}$ is found to be insignificant for all models. As in the APARCH models, the asymmetry parameter $\hat{\gamma}$ of the FIAPARCH models is not statistically significant. Thus, unlike the equity markets,

Table II. Estimated FIGARCH and FIAPARCH models

	FIGARCH model			FIAPARCH model		
μ	−0·0110	−0·0114	−0·0106	−0·0138	0·0135	−0·0090
	(0·0111)	(0·0111)	(0·0111)	(0·0108)	(0·0108)	(0·0112)
ρ		−0·0127			−0·0197	
		(0·0171)			(0·0175)	
ω	0·0928	0·0915	0·1228	0·1393	0·1397	0·1630
	(0·0478)	(0·0471)	(0·0344)	(0·0575)	(0·0571)	(0·0539)
ϕ	0·1524	0·1529		0·2042	0·2057	
	(0·1847)	(0·1827)		(0·1429)	(0·1409)	
γ				0·2101	0·2067	
				(0·1177)	(0·1162)	
δ				1·3067	1·2828	1·6071
				(0·3824)	(0·3826)	(0·3715)
β	0·2873	0·2893	0·1210	0·3758	0·3800	0·1499
	(0·2103)	(0·2083)	(0·0458)	(0·1773)	(0·1748)	(0·0612)
d	0·2070	0·2088	0·1908	0·2625	0·2666	0·2299
	(0·0520)	(0·0522)	(0·0385)	(0·0733)	(0·0734)	(0·0555)
ML	−538·0160	−534·2311	−538·5992	−529·5264	−525·7670	−536·6248
$Q_1(4)$	4·4859	5·4590	4·5158	3·9420	5·8976	4·6109
$Q_1(8)$	8·5604	9·6689	8·5567	7·7820	9·8924	8·6042
$Q_2(4)$	1·2489	1·1593	1·3616	4·6452	4·3455	1·8549
$Q_2(8)$	6·4417	6·3238	7·1844	8·1145	8·0189	8·0900

Notes: The figures in parentheses below the parameter estimates are the robust standard errors. ML is the maximized likelihood, Q_1 tests for the serial correlation in the standardized residuals, and Q_2 tests for the serial correlation in the squares of the standardized residuals. The number of lag terms taken in calculating the Q statistics are given in the parentheses. $Q_i(M)$ for $i = 1, 2$, are approximately distributed as χ^2_M under the null.

[5] For the FIGARCH(1, d, 1) model Baillie *et al.* (1996) obtained an infinite series representation in terms of ε_t^2. Although ε_t^2 is replaced by $g(\varepsilon_t)^{\delta}$ in the FIAPARCH(1, d, 1) model, the coefficients λ_i are the same. Thus, in the FIAPARCH model the effects of the past residuals show the same kind of hyperbolic decay as found in the FIGARCH model. Using a dominance argument, Baillie *et al.* (1996) show that the FIGARCH process is strictly stationary and ergodic. However, whether this result holds for the FIAPARCH model is an open question.

[6] For the FIAPARCH model to be well defined, all the coefficients of the infinite series representation must be non-negative. We impose the following sufficient conditions, found in Bollerslev and Mikkelsen (1996), to ensure the non-negativity of λ_i: $\beta - d \leqslant \phi \leqslant (2-d)/3$ and $d[\phi - (1-d)/2] \leqslant \beta(\phi - \beta + d)$. It should be noted, however, that the results of Nelson and Cao (1992) suggested that weaker sufficient conditions may be obtained if desired.

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there is no asymmetry in the conditional volatility of the exchange rate. Both the conditional standard deviation and the conditional variance forms cannot be rejected. The estimates of ϕ in the FIGARCH and FIAPARCH models are similar, and they are found to be statistically insignificant. For the FIGARCH model, $\hat{\beta}$ is insignificant when it is estimated jointly with ϕ. This conclusion is, however, reversed when ϕ is left out of the equation. In contract, $\hat{\beta}$ in the FIAPARCH model is found to be statistically significant.

3. FURTHER ANALYSIS AND SOME APPLICATIONS

The results reported in the previous section are based on more than 16 years of daily data. It would be interesting to examine if the parameters of the models remain stable over the whole sample period. To this end, we divide the sample into two subperiods, called Period I and Period II, of equal numbers of observations. We consider the models for which the statistically insignificant parameters in the conditional variance equation are deleted. The remaining parameters are allowed to differ over the two subperiods and are reestimated jointly over the whole sample. The results are presented in Table III. The t-statistics for the equality of the (separate) parameters over the two subperiods show that there is no evidence of parametric instability. This is confirmed by the joint tests based on the likelihood ratio statistics.

To provide additional comparison for the stable and fractionally integrated models, we examine the coefficients of their infinite series representations. For the FIGARCH and FIAPARCH models the coefficients λ_i are given in footnote 4. The GARCH model has an infinite series

Table III. Parametric stability tests

	Period	GARCH	APARCH	FIGARCH	FIAPARCH
μ	I	0·0023	0·0022	0·0019	0·0021
	II	−0·0220	−0·0185	−0·0229	−0·0177
		(0·9911)	(1·0172)	(1·1433)	(0·7841)
$\eta(\omega)$	I	0·0363	0·0374	0·1256	0·1419
	II	0·0360	0·0435	0·1630	0·2601
		(0·0144)	(0·2383)	(0·5530)	(1·1649)
α	I	0·0883	0·0902		
	II	0·0655	0·0781		
		(0·6468)	(0·3242)		
d	I			0·2046	0·2226
	II			0·1677	0·2120
				(0·4613)	(0·1089)
δ	I		1·9097		1·8736
	II		1·4995		1·3123
			(0·8088)		(1·1085)
β	I	0·8423	0·8440	0·1225	0·1384
	II	0·8641	0·8643	0·1070	0·1277
		(0·3254)	(0·3088)	(0·1650)	(0·0938)
LR		2·892	3·521	2·614	4·673

Notes: Period I is from 1978/1/3 to 1986/4/15. Period II is from 1986/4/17 to 1994/6/29. The figures in parentheses are the absolute values of the t-statistics for separate parametric equality. LR is the likelihood ratio statistic for joint parametric equality.

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Table IV. Daily maintenance margin (in %) at 0·5% failure rate

Position/volatility	APARCH	FIAPARCH	GARCH	FIGARCH
Long/mean	2·3225	2·3276	2·4000	2·3669
Long/maximum	4·6497	4·5590	5·2053	5·0176
Short/mean	1·9746	1·9620	1·9602	1·9905
Short/maximum	4·0415	3·9248	4·3642	4·3267

Notes: Daily maintenance margins are calculated for long and short positions at the mean and maximum conditional standard deviations calculated from the sample.

representation with $\lambda_i = \alpha\beta^{i-1}$. This expansion also applies to the APARCH model.[7] It is found that the λ_i coefficients are very similar. As expected, the stable GARCH and APARCH models produce λ_i coefficients that decay to zero faster than those of the fractionally integrated models. The differences between the models, however, appear to be small.[8]

To further compare the estimated models, we consider the application of these models to the estimation of the minimum capital requirements for the trading in the yen and the dollar. Assuming a given failure probability c, Hsieh (1993) derived the minimum margin requirements for the trading of yen futures. If f_t is the futures price and K_t^L is the daily maintenance margin, it is required that $\Pr(f_t - f_{t+1} > K_t^L)$, i.e. $\Pr(f_{t+1}/f_t < 1 - K_t^L/f_t)$. Assuming that $\varepsilon_t = \log(f_{t+1}/f_t) = \sigma_t\varepsilon_t$ follows a conditional heteroscedasticity process, it can be shown that the minimum capital requirement as a fraction of the initial contract value is given by $\kappa_t^L = 1 - \exp(\sigma_{t+1|t}e_c)$, where e_c is the $100c$ percentile of e_t and $\sigma_{t+1|t}$ is the standard deviation of ε_{t+1} conditional upon the information up to time t. For a short position the minimum capital requirement is $\kappa_t^s = \exp(\sigma_{t+1|t}e_{1-c}) - 1$. Table IV summarizes the minimum capital requirements assuming a failure probability of 0.5%, with $\sigma_{t+1|t}$ taken as the mean and the maximum values of the estimated conditional standard deviations of the residuals calculated from the sample and e_c as the empirical $100c$ percentile of the standardized residuals.[9] The results show that the margin requirement estimates given by the GARCH models exceed those obtained from the APARCH models. Also, the fractionally integrated models provide lower margin requirement estimates than their stable counterparts. Overall, however, there are no substantial differences in the margin requirement estimates.

4. CONCLUSIONS

We have examined the conditional volatility of the yen–dollar exchange rate using a long time series of daily observations. It is found that, unlike the volatilities in the equity markets, the appreciation and depreciation shocks in the yen–dollar exchange rates have similar effects on the future volatilities.

We consider fractionally integrated processes that allow the effects of the residuals to decay at a slow hyperbolic rate not found in a stable conditional heteroscedasticity model. The results reject

[7] It should be noted that, unlike the FIGARCH model, $\lambda_i L^i$ operates on $g(\varepsilon_t)^\delta$ in the FIAPARCH model rather than on ε_t^2. Thus, the net effects of the lagged residuals on the conditional variance also depend on $\hat{\gamma}$ and $\hat{\delta}$. As these parameter estimates vary across the models, the net effects would have to take account of these estimates. Nonetheless, the pattern of λ_i gives some indications of the dependence of the conditional volatility on the lagged residuals.

[8] See Tse (1996) for the details. These results are for the models with γ and ϕ imposed to be zero. The graphical plots for the models where γ and ϕ are both included are qualitatively similar.

[9] These results are based on the models with ρ imposed to be zero. We have used spot exchange rates rather than future prices. This exercise, however, is to compare the implications of the conditional heteroscedasticity models.

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both the hypotheses of a stable and an integrated model. Our analysis of the response coefficients of the infinite series representations shows, however, that there are no substantial differences between the stable and the fractionally integrated models. Application of the models to the calculation of the margin requirements also shows that the models produce very similar results.

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