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CONDITIONAL HETEROSCEDASTICITY OF EXCHANGE RATES: FURTHER RESULTS BASED ON THE FRACTIONALLY INTEGRATED APPROACH

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SUMMARY

A recent article (Tse, 1998) published in this journal analysed the conditional heteroscedasticity of the yen–dollar exchange rate based on the fractionally integrated asymmetric power ARCH model. In this paper, we present replication results using Tse's (1998) yen–dollar series. We also examine the robustness of Tse's (1998) findings across different currencies, sample periods and non-nested GARCH-type models. Unlike Tse (1998), we find some evidence of asymmetric conditional volatility for daily returns of currencies measured against the dollar or the yen. Copyright © 2004 John Wiley & Sons, Ltd.

1. INTRODUCTION

This paper presents replication results of the conditional volatility of the yen–dollar exchange rate based on the fractionally integrated asymmetric power ARCH (FIAPARCH) model by Tse (1998). It also examines the robustness of Tse's (1998) findings across: (a) four other currencies, including the Canadian dollar (CND), the British pound (GBP), the Malaysian ringgit (MYR) and the Singapore dollar (SGD); (b) two additional sample periods, one from 2 January 1986 to 30 June 1997 and the other from 2 January 1986 to 21 February 2003; and finally (c) two non-nested GARCH-type models.[1] The rest of the paper is organized as follows. Section 2 briefly describes the models and the estimation procedure, while Section 3 presents the data sets and estimation results. Finally, Section 4 concludes.

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[1] We thank both the referees for these suggestions. One referee specifically suggests that we should focus on long memory and asymmetry.

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2. MODELLING CONDITIONAL VOLATILITY

The daily returns (as a percentage) of nominal exchange rates S_t on a continuously compounding basis are computed as

$$y_t = \log(S_t/S_{t-1}) \times 100 \tag{1}$$

We assume that the conditional mean equation is characterized by an autoregressive (AR) process of order p:

$$y_t = \xi_0 + \sum_{a=1}^{p} \xi_a y_{t-a} + \varepsilon_t \tag{2}$$

and

$$\varepsilon_t = h_t^{1/2} e_t \tag{3}$$

where the e_t are independently and identically distributed variables with mean 0 and variance 1, and the conditional variance h_t is dependent on all past information up to time $t-1$. Table I displays different specifications of the conditional variance. All parameters of the conditional mean and variance equations are estimated simultaneously using Bollerslev and Wooldridge's (1992) quasi-maximum likelihood estimation technique, assuming normally distributed errors. Further details of the different GARCH-type specifications and the estimation procedure are available in Ho and Tsui (2003).

3. DATA AND RESULTS

Three data sets sampled in different periods are used in this paper. They are displayed as follows:

Data set	Exchange rates	Sample period
I	JPY/USD [Tse's (1998) data set]	I: 3 January 1978–29 June 1994
II	JPY/USD CND/USD, GBP/USD, MYR/USD, SGD/USD CND/JPY, GBP/JPY, MYR/JPY, SGD/JPY	II: 2 January 1986–30 June 1997
III	JPY/USD CND/USD, GBP/USD, SGD/USD CND/JPY, GBP/JPY, SGD/JPY	III: 2 January 1986–21 February 2003

For details of the data sets, see Ho and Tsui (2003). Based on parsimony and residual diagnostics, we find that either a constant or an AR(1) filter is adequate for the conditional mean equation, which is consistent with Tse's (1998) empirical findings. Some estimation results for the AGARCH and APARCH-type models are reported in Table II. Owing to space constraints, we do not report the estimates of the conditional mean equation.

3.1. Data Set I

In the case of Tse's (1998) yen–dollar series, we obtain replication results for the GARCH, APARCH, FIGARCH and FIAPARCH models that match with those reported by Tse (1998) almost

Table I. Specifications of the conditional variance equation

Model	Equation	Reference
GARCH	$h_t = \eta + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$	Bollerslev (1986)
Asymmetric GARCH (AGARCH)	$h_t = \eta + \alpha(\varepsilon_{t-1} - \gamma)^2 + \beta h_{t-1}$	Engle (1990)
Asymmetric power ARCH (APARCH)	$h_t^{\delta/2} = \eta + \alpha(\lvert\varepsilon_{t-1}\rvert - \gamma\varepsilon_{t-1})^\delta + \beta h_{t-1}^{\delta/2}$	Ding *et al.* (1993)
Fractionally integrated GARCH (FIGARCH)	$h_t = \frac{\eta}{1-\beta} + \lambda(L)\varepsilon_t^2$	Baillie *et al.* (1996)
Fractionally integrated AGARCH (FIAGARCH)	$h_t = \frac{\eta}{1-\beta} + \lambda(L)(\varepsilon_t - \gamma)^2$	Ho and Tsui (2003)
Fractionally integrated APARCH (FIAPARCH)	$h_t^{\delta/2} = \frac{\eta}{1-\beta} + \lambda(L)(\lvert\varepsilon_t\rvert - \gamma\varepsilon_t)^\delta$	Tse (1998)

Note: $\lambda(L) = \sum_{a=1}^{\infty} \lambda_a L^a = 1 - (1-\beta L)^{-1}(1-\phi L)(1-L)^d$ is the lag polynomial of the response coefficients.

up to the fourth decimal place. See tables 2–4 of Ho and Tsui (2003) and tables I and II of Tse (1998). The estimated values of the fractional differencing parameter d are statistically different from 0 and 1, thereby providing support for the modelling of fractionally integrated conditional heteroscedasticity. However, all the estimated values of γ are insignificant at the 5% level, indicating the absence of asymmetric conditional volatility[2] in the four models. Nevertheless, when we consider the non-nested alternatives, the AGARCH and the FIAGARCH models, there is some evidence of asymmetric volatility.

3.2. Data Set II

Table II presents estimation results using the exchange rates of the CND, GBP, MYR and SGD measured against either the dollar or the yen based on different GARCH-type models. The main findings are summarized as follows.

(a) For the JPY/USD, the estimates are reasonably consistent with those in Tse (1998).
(b) For the currencies against the dollar, the estimated values of the fractional differencing parameter d for CND, GBP, JPY and MYR are different from 0 and 1 and statistically significant at the 5% level, regardless of the fractionally integrated GARCH-type models.[3] Similar findings apply to the exchange rates of the CND, GBP, MYR and SGD measured against the yen, respectively.
(c) For the currencies against the dollar, unlike Tse (1998), we detect significant evidence of asymmetry for the JPY based on the AGARCH and FIAGARCH models, and for the MYR using the APARCH and FIAPARCH models. However, there is more support for asymmetric effects when the currencies are measured against the yen. This is particularly evident for the

[2] One of the referees has asked us about the economic interpretation of asymmetric volatility. Among others, Tse and Tsui (1997) have proposed that the unbalanced reaction of heterogeneous expectations of market traders to foreign exchange shocks may be the cause of asymmetric volatility. They use this argument to explain the asymmetric effects of shocks to future volatilities in the MYR. Due to space constraints, we will skip the detailed discussion here.

[3] As discussed in Davidson (2004), the length of memory of the FIGARCH model is increasing as d approaches 0. Most of our estimates of d for the FIGARCH model are between 0.2 and 0.4, thereby suggesting that there could still be some support for long memory in the conditional volatility. Indeed, Davidson (2004) warned that researchers should distinguish the different nature of modelling 'long memory' in the conditional mean and that in the conditional variance. We are grateful to one of the referees for pointing this out.

Table II. Estimated GARCH-type models using data set II

Parameter	η	$\alpha(\phi)$	γ	δ	β	d	LL	η	$\alpha(\phi)$	γ	β	d	LL
Panel A: APARCH model								Panel C: AGARCH model					
CND/USD	0.0057	0.0727	−0.2055	1.4428	0.9052	—	2458.9790	0.0020	0.0562	−0.0585	0.9163	—	2455.1561
	(0.0082)	(0.0351)	(0.1677)	(0.4056)	(0.0603)			(0.0018)	(0.0259)	(0.0507)	(0.0488)		
GBP/USD	0.0029	0.0373	−0.0232	1.9049	0.9583	—	−145.4231	0.0017	0.0342	0.0853	0.9618	—	−144.4318
	(0.0023)	(0.0117)	(0.0857)	(0.3415)	(0.0109)			(0.0017)	(0.0081)	(0.0944)	(0.0098)		
MYR/USD	0.0039	0.1680	−0.1368	1.8769	0.7912	—	3256.4123	0.0033	0.1715	−0.0241	0.7792	—	3251.5588
	(0.0021)	(0.0359)	(0.0665)	(0.2694)	(0.0485)			(0.0014)	(0.0357)	(0.0237)	(0.0469)		
SGD/USD	0.0055	0.1083	−0.0114	1.4412	0.8781	—	2857.6447	0.0021	0.0909	0.0165	0.8814	—	2853.3784
	(0.0037)	(0.0328)	(0.0911)	(0.2894)	(0.0444)			(0.0012)	(0.0343)	(0.0294)	(0.0487)		
CND/JPY	0.0156	0.0417	−0.2382	1.6681	0.9383	—	−597.1897	0.0108	0.0378	−0.3608	0.9353	—	−593.0552
	(0.0116)	(0.0222)	(0.1631)	(0.4206)	(0.0310)			(0.0066)	(0.0168)	(0.1422)	(0.0291)		
GBP/JPY	0.0096	0.0459	−0.1763	1.6853	0.9402	—	−219.1053	0.0071	0.0435	−0.1748	0.9385	—	−218.5810
	(0.0058)	(0.0147)	(0.1414)	(0.3845)	(0.0174)			(0.0040)	(0.0120)	(0.1238)	(0.0184)		
MYR/JPY	0.0154	0.0456	−0.2406	1.4473	0.9357	—	−254.5947	0.0095	0.0391	−0.2519	0.9350	—	−251.7829
	(0.0140)	(0.0209)	(0.2069)	(0.5746)	(0.0322)			(0.0055)	(0.0140)	(0.1056)	(0.0264)		
SGD/JPY	0.0213	0.0547	−0.3146	1.1983	0.9216	—	−46.0114	0.0093	0.0401	−0.2650	0.9299	—	−45.7007
	(0.0177)	(0.0243)	(0.1746)	(0.4036)	(0.0401)			(0.0054)	(0.0144)	(0.0989)	(0.0274)		
JPY/USD	0.0158	0.0436	0.1537	1.6063	0.9349	—	−338.8506	0.0118	0.0395	0.2776	0.9304	—	−333.6465
	(0.0127)	(0.0227)	(0.1741)	(0.4712)	(0.0340)			(0.0075)	(0.0181)	(0.1219)	(0.0340)		
Panel B: FIAPARCH (1,d,1) model								Panel D: FIAGARCH (1,d,1) model					
CND/USD	0.0084	0.4157	−0.2284	1.7469	0.6139	0.3022	2469.9413	0.0034	0.4027	−0.0923	0.5577	0.2616	2466.7656
	(0.0102)	(0.1380)	(0.1310)	(0.3853)	(0.1544)	(0.1202)		(0.0036)	(0.1836)	(0.0540)	(0.2362)	(0.0986)	
GBP/USD	0.0192	0.2222	−0.0453	2.0341	0.5436	0.3320	−144.8890	0.0189	0.2165	0.0406	0.5605	0.3513	−144.8203
	(0.0207)	(0.0767)	(0.0817)	(0.3390)	(0.1258)	(0.0883)		(0.0103)	(0.0722)	(0.0715)	(0.0978)	(0.0559)	
MYR/USD	0.0035	0.3634	−0.1646	1.8949	0.5340	0.4132	3302.8204	0.0026	0.3613	−0.0219	0.5211	0.4270	3294.1919
	(0.0027)	(0.1156)	(0.0632)	(0.1860)	(0.1100)	(0.1431)		(0.0014)	(0.1226)	(0.0245)	(0.1207)	(0.1783)	
SGD/USD	0.0041	0.4201	−0.0891	1.5864	0.8374	0.6572	2878.8444	0.0015	0.5335	−0.0138	0.7923	0.4983	2871.5342
	(0.0047)	(0.4728)	(0.0809)	(0.2497)	(0.2551)	(0.7470)		(0.0008)	(0.1446)	(0.0319)	(0.1172)	(0.2405)	
CND/JPY	0.0998	0.3974	−0.3463	1.4661	0.5421	0.2491	−594.0169	0.0317	0.2609	−0.3872	0.3719	0.1929	−589.4990
	(0.1840)	(0.6391)	(0.2240)	(0.3711)	(0.6849)	(0.1286)		(0.0477)	(0.4122)	(0.1237)	(0.4160)	(0.0502)	
GBP/JPY	0.0398	0.1489	−0.1120	2.1837	0.3201	0.2143	−213.9887	0.0424	0.1426	−0.1501	0.3527	0.2563	−213.5282
	(0.0325)	(0.1313)	(0.0985)	(0.4300)	(0.1617)	(0.0878)		(0.0216)	(0.1195)	(0.1073)	(0.1326)	(0.0544)	
MYR/JPY	0.1263	0.1851	−0.3099	1.4276	0.3493	0.2412	−252.9286	0.0398	0.0963	−0.2960	0.2434	0.2047	−249.6198
	(0.0789)	(0.2389)	(0.1799)	(0.4699)	(0.2642)	(0.0619)		(0.0304)	(0.1653)	(0.0947)	(0.1705)	(0.0441)	
SGD/JPY	0.0835	0.4129	−0.3517	1.1271	0.6087	0.3063	−41.7112	0.0199	0.3575	−0.2944	0.4880	0.2090	−44.0199
	(0.0516)	(0.1341)	(0.1579)	(0.2994)	(0.1788)	(0.1042)		(0.0234)	(0.1975)	(0.1002)	(0.2157)	(0.0586)	
JPY/USD	0.1213	0.2289	0.2254	1.4871	0.4036	0.2443	−339.7164	0.0447	0.1288	0.3080	0.2800	0.2009	−332.7807
	(0.0931)	(0.3068)	(0.1879)	(0.4513)	(0.3496)	(0.0834)		(0.0379)	(0.1929)	(0.1052)	(0.1987)	(0.0481)	

Note: All standard errors (in parentheses) are the heteroscedastic-consistent Bollerslev–Wooldridge (1992) standard errors. LL refers to the log-likelihood value.

CND, MYR and SGD based on the AGARCH and FIAGARCH specifications, and also for the SGD based on the FIAPARCH structure.

(d) As regards the model adequacy by log-likelihood values, the CND/USD, MYR/USD, SGD/USD and SGD/JPY series under the FIAPARCH specification have the highest log-likelihood values. However, the FIAGARCH structure is preferred for the CND/JPY, GBP/JPY, MYR/JPY and JPY/USD series among all the competing models.

3.3. Data Set III

The estimation results are consistent with those reported for data set II for the same currencies and the GARCH-type models.

4. CONCLUSION

We have successfully replicated results based on Tse's (1998) yen–dollar series. There is evidence of fractionally integrated conditional heteroscedasticity, but no support for asymmetric conditional volatility based on the APARCH and FIAPARCH models. Unlike Tse (1998), however, we find evidence of asymmetric volatility in the non-nested AGARCH and FIAGARCH models. Regardless of the sample periods and currencies against the dollar or the yen, we find support for long memory of conditional volatility in the fractionally integrated GARCH-type models. In contrast, the evidence of asymmetric conditional volatility is rather mixed. We find stronger support for asymmetric effects in the currencies measured against the yen.

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