SCHOOL OF ECONOMICS AND MANAGEMENT

Computer Lab Report 2

NEKN34 - Time-Series Analysis

Axel Löveråsen

Daniel Arredondo

Sebastian Brugger

Lecturer: Joakim Westerlund

Teaching Assistant: Pelle Almgren

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1 Introduction

Scientific research both in economics and in general are very important for the development of society and as grounds for decision makers of public institutions. In order to be considered reliable the results of scientific studies must be replicable in order to verify their findings. In a 2019 paper Mueller-Langer et al., 2019 examine the practice of replication studies in economics. The author investigates the characteristics of replicated studies by looking at a sample taken from the top 50 economics journals from 1974 to 2014. The study finds that replication studies are relatively rare in economics with only a small fraction of papers being chosen for replication. The authors identifies a number of factors that may influence the selection of papers for replications, such as the original papers importance, the availability of data and the interest of the replication researcher. Mueller-Langer et al., 2019 also find that replication studies within economics are typically more successful in replicating the original results when the replication study is carried out by or in close collaboration with the original authors. Finally, the study suggests that replication studies can play an important role in improving quality and reliability of research in economics.

With this as a starting point, this paper is meant to provide a narrow replication of results on the conditional heteroskedasticity of the yen-dollar exchange rate modeled by ARCH models such as was done by Tse, 1998, and later replicated by Tsui and Ho, 2004. The two Models replicated are a GARCH(1,1) and an APARCH(1,1). In section two, those two models will be introduced and in section three, we will present the results of our replication analysis. In the last section we will conclude our findings.

2 Data and Models

2.1 Data description

The data set used in this report includes daily yen-dollar exchange rate spanning from January 4, 1971 to January 13, 2021, totaling 12767 daily observations. Note that this data set is not from the same source as the one used by Tse, 1998 or Tsui and Ho, 2004[1].

We compute the daily returns on nominal exchange rates S_t in the same way Tse, 1998 and Tsui and Ho, 2004 did. The daily returns , denoted r_t and represented as a percentage, are calculated as

$$r_t = log(\frac{S_t}{S_{t-1}}) \times 100. \tag{1}$$

We notice that daily returns r_t exhibits a negative skew and leptokurtic behavior given that skew and kurtosis values are -0.657 and 12.8 respectively.

[1]The data has been downloaded from "www.macrotrends.net".

We will use two different time periods from the data set in this replication study. First we will replicate the calculations perform by Tse, 1998 and set Period 1 to be from 1978-01-03 to 1994-06-29. Note that the data set we are using in this analysis contains 4142 observations within this time frame, which is more than the observed 4067 observations within the study of Tse, 1998. In Period 2 we will be using the span of our entire data set from 1971-01-04 until 2021-01-13. These two periods will be used to validate the conclusion of Tse, 1998 and to confirm whether his conclusion holds or not over a different time period, as discussed by Tsui and Ho, 2004.

2.2 Model Description

Introduced by Engle, 1982 the auto regressive conditional heteroskedasticity model (ARCH) is a common model choice when estimating and forecasting volatility. A lot of financial and macroeconomic variables exhibit heteroskedasticity in the form of volatility clustering. Subsequently meaning that the variance of a time series is time varying with periods of high and low volatility clustered together. The main feature of the model is its ability to capture this time varying nature of the variance. The ARCH model uses an AR(p) process in order to forecast the error term based on its past values. The problem is that in practice the ARCH often requires a large number of lags to capture the time variation in the volatility.

The Generalized ARCH (GARCH) model introduced by Bollerslev, 1986 expands the ARCH model by including an MA(q) component, allowing for a more flexible lag structure where the conditional variance is able to depend on its own lags, thereby reducing the number of lags required. Another extension of the ARCH model as proposed by Ding et al., 1993 is the Asymmetric Power ARCH (APARCH). The APARCH models extend GARCH models by incorporating the idea of asymmetry in the volatility response to positive and negative shocks. Specifically, APARCH models use a power function to model the asymmetric response, which allows for different exponents for positive and negative shocks. This can be useful in modeling financial time series data, where volatility often responds differently to positive and negative market shocks Francq and Zakoïan, 2019.

The mathematical equation of our GARCH model is as follows:

$$h_t = \eta + \alpha\epsilon_{t-1}^2 + \beta h_{t-1}, \tag{2}$$

and the equation behind our APARCH model is as follows:

$$h_t^\delta = \eta + \alpha(|\epsilon_{t-1}| - \gamma\epsilon_{t-1})^\delta + \beta h_{t-1}^\delta, \tag{3}$$

where η is positive, α, β and δ are non-negative. The asymmetry parameter γ in equation

3 indicates the strength of negative and positive shocks and is bounded to $|\gamma| < 1$. δ is the exponent in equation 3. The parameter γ indicates the asymmetrical volatility depending on whether the value is positive or negative. A positive value indicates that negative shocks lead to higher volatility in comparison to positive shocks, and vice versa.

In our analysis we will use the package "rugarch" in a R environment (created by Ghalanos, 2022) to control for the parameters. Robust standard errors, based on the method of White, 1982, will be given with the parameter estimates. We will replicate the GARCH / APARCH models investigated by Tse, 1998. He uses two GARCH(1,1) models; the first is a model using a time-independent mean; the second GARCH(1,1) model utilizes an AR(1) model with ρ being the coefficient to estimate the time-dependent mean, such that $\mu_t = \mu + \rho r_{t-1}$. Note, the parameter ρ is assumed to be bounded such that $|\rho| < 1$. The APARCH(1,1) comes in three variants. The first variant will again have a time-independent mean. The second model controls for all three additional parameters (ρ, γ, and δ) in the APARCH equation. The third model assumes γ to have a true vale of zero, which represents symmetry in our model.

3 Analysis

We continue with the comparison of our computations to the results found in Tse, 1998. The results for the GARCH and APARCH models for Period 1 cam be seen in Table 1. Our results have been laid out similarly to Tse, 1998. The first column represents the first GARCH(1,1) model without a conditional (time-dependent) mean; the second column represents a GARCH(1,1) with a conditional mean. The last three columns are APARCH(1,1) models which follow the same structure as introduced before, fitting coefficients μ, η, α and β along with varying other coefficients; first APARCH model additionally fits γ and δ; the second model fits all three parameters ρ, γ and δ; the third model additionally fits ρ and δ and has the assumption that γ is zero. Compare with figure 1 in the appendix.

3.1 Period 1

Table 1 represents the estimates for Period 1. The findings in Tse, 1998 was that he found that when there are sudden changes in the yen-dollar exchange rate, such as appreciation or depreciation shocks, they have a similar impact on future volatilities, which is unlike, for example, the volatility in the stock market. We attempt to replicate his findings on his selected time period. We similarly estimate that $\hat{\gamma}$ is statistically insignificant. Parameter $\hat{\rho}$ appears to be statistically relevant for both GARCH and APARCH model types, but we estimate the value close to zero, which contradicts the findings of Tse, 1998. Additionally, our estimated value for $\hat{\delta}$ also contradicts Tse's findings, whose value is closer to two and

Table 1: Estimated GARCH and APARCH models using period 1.

	GARCH model		APARCH model		
μ	-0.011152	-0.011651	-0.010229	-0.010451	-0.010552
	(0.011635)	(0.011712)	(0.011741)	(0.011824)	(0.009827)
ρ		0.030433		0.030789	
		(0.017822)		(0.017612)	
η	0.028731	0.028626	0.031077	0.030690	0.031598
	(0.009409)	(0.009418)	(0.010578)	(0.010430)	(0.010152)
α	0.085076	0.084966	0.089668	0.089279	0.090198
	(0.018434)	(0.018667)	(0.019563)	(0.019731)	(0.020239)
γ			-0.011873	-0.017644	
			(0.063822)	(0.064295)	
δ			1.726988	1.735037	1.707727
			(0.353668)	(0.358162)	(0.337301)
β	0.851442	0.851720	0.858513	0.859166	0.857956
	(0.034220)	(0.034517)	(0.034428)	(0.034489)	(0.036371)
LLH	-4033	-4032	-4032	-4031	-4032

Note that figures in parenthesis represent the robust standard errors of the parameter estimates. LLH represents the log-liklihood of the model.

thus implies the model follows closer to a conditional variance model. Generally, the parameters are pretty similar to each other within the model types, matching up to the 2nd or 3rd decimal place. There are slightly bigger differences between the GARCH and APARCH models, with the exception of the parameters ρ and β. These small differences in the parameters could be explained by the nature of the different data sets used.

3.2 Period 2

Table 2 represents the estimates for Period 2, Where we are looking at the entire data set. Our findings contradict what we found by replicating the time period of Tse, 1998. This statement holds for most parameters, and will be explained parameter by parameter as follows.

- First, we notice that the mean is closer to zero, yet insignificant.
- $\hat{\rho}$ is still close to zero, but now only significant for the APARCH(1,1).
- $\hat{\eta}$ is again closer to zero, and still significant. Here we find different values for the model types. While $\hat{\eta}$ in the APARCH(1,1) is more then 10 times greater than $\hat{\eta}$ for the GARCH(1,1) models, the APARCH(1,1) has the smallest value, if the asymmetry parameter $\hat{\gamma}$ is set to zero.

Table 2: Estimated GARCH and APARCH models using period 2.

	GARCH model		APARCH model		
μ	0.004027	0.007159	−0.001306	−0.001868	0.004231
	(0.005466)	(0.046487)	(0.006331)	(0.006817)	(0.006159)
ρ		0.020806		0.022830	
		(0.135103)		(0.012669)	
η	0.001429	0.001820	0.021262	0.021191	0.000703
	(0.000000)	(0.000045)	(0.007885)	(0.007805)	(0.000001)
α	0.085063	0.105278	0.138714	0.138789	0.073551
	(0.002802)	(0.683104)	(0.035553)	(0.035388)	(0.002581)
γ			0.117517	0.120620	
			(0.051451)	(0.052211)	
δ			1.419565	1.420900	2.282057
			(0.226886)	(0.223639)	(0.000531)
β	0.913937	0.891912	0.853034	0.852939	0.916747
	(0.000260)	(0.017704)	(0.033353)	(0.033256)	(0.000207)
LLH	−10945	−10947	−11029	−11026	−10999

Note that figures in parenthesis represent the robust standard errors of the parameter estimates. LLH represents the log-liklihood of the model.

- $\hat{\alpha}$ now has quite some variation over the different models and is insignificant in the GARCH(1,1) with time-dependent mean. The coefficient is once again the greatest for the first two APARCH(1,1) models and the smallest for the last of this model type.

- $\hat{\gamma}$ is ultimately different from zero, and statistically significant, which contradicts the conclusion of Tse, 1998. This means, that shocks on the yen-dollar exchange rate in the long run seem to function similar to the stock market.

- $\hat{\delta}$ is closer to 1 for the first two APARCH(1,1) and greater two in the last model, and significant. While Tse, 1998 couldn't find evidence to this distinguish the conditional standard-deviation model from the conditional variance model, with more data this seems feasible for the last model.

- $\hat{\beta}$ is fairly constant for all 5 models, yet vary a bit more than in table 1

- In table 1, the GARCH(1,1) ended up being the least likely model. The log-likelihood values are all very close too each other, so much so that one could not argue that these models perform worse or better. Now, with more data, the simple GARCH(1,1) seems to have the best likelihood, followed by the second GARCH(1,1) model with time-dependent mean.

4 Conclusion

We have attempted to replicate the results based on Tse's (1998) yen-dollar series. We achieved similar results while using a different data set but equivalent time period to Tse's analysis. Furthermore, by including additional observations in our data set as represented by Period 2, we conclude that there lies a small measure of asymmetry. Thus, shocks on the yen-dollar exchange rate appear to have asymmetrical volatility.

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A Appendix

Table I. Estimated GARCH and APARCH models

	GARCH model		APARCH model		
μ	−0·0100	−0·0104	−0·0129	−0·0128	−0·0091
	(0·0111)	(0·0111)	(0·0108)	(0·0108)	(0·0112)
ρ		−0·0132		−0·0178	
		(0·0171)		(0·0174)	
η	0·0352	0·0348	0·0459	0·0460	0·0401
	(0·0106)	(0·0104)	(0·0147)	(0·0147)	(0·0129)
α	0·0763	0·0771	0·0859	0·0873	0·0845
	(0·0135)	(0·0135)	(0·0160)	(0·0160)	(0·0164)
γ			0·1546	0·1484	
			(0·1033)	(0·1012)	
δ			1·4483	1·4327	1·6369
			(0·3529)	(0·3535)	(0·3617)
β	0·8558	0·8560	0·8549	0·8543	0·8585
	(0·0279)	(0·0276)	(0·0304)	(0·0303)	(0·0296)
ML	−534·0250	−529·8763	−529·0367	−525·0021	−532·7100
$Q_1(4)$	4·1898	5·0970	3·8178	5.3627	4.3212
$Q_1(8)$	8·1629	9·2050	7·6615	9·3554	8·2932
$Q_2(4)$	1·5383	1·4597	2·4379	2·3518	2·2205
$Q_2(8)$	6·7295	6·5979	9·0479	9·0104	7·9538

Notes: The figures in parentheses below the parameter estimates are the robust standard errors. ML is the maximized likelihood, Q_1 tests for the serial correlation in the standardized residuals, and Q_2 tests for the serial correlation in the squares of the standardized residuals. The number of lag terms taken in calculating the Q statistics are given in the parentheses. $Q_i(M)$ for $i = 1, 2$, are approximately distributed as χ^2_M under the null.

Figure 1: The original table 1 from Tse, 1998 for easier comparison with our table 1.