Little Slice of Pi

Saturday, October 2, 2021 12:04 PM

Variables the will be* apparent in each function:

k, sum, ε, T

↳ T represents each term

* these values, except ε, are **NOT** constant

Finding e with Taylor Series

$$e = \sum_{k=0}^{\infty} \frac{1}{k!}$$

computing the terms: (generalized)

$$\underbrace{\frac{x^k}{k!} = \frac{x^{k-1}}{(k-1)!} \cdot \frac{x}{k}}_{\text{could be calculated w/ a while loop}}$$

← In the case of e, x would be 1

Initial values:

k = 0

sum = 0

T = 1, since $\frac{1}{0!} = \frac{1}{1} = 1$

could be calculated w/ a while loop → while (|T| > ε) {

we use abs() for T because some series have alternating values

sum += T ← adding each term

k++

T ×= $\frac{1}{k}$ ← goes to the next term while also performing factorial since we are multiplying k

}

Example:

Sum = 1 + 1 + $\frac{1}{2}$ + $\frac{1}{6}$ ...

T = $1 \cdot \frac{1}{1} \cdot \frac{1}{2} \cdot \frac{1}{3}$ ...

factorial magic!

we'll be using this while-loop structure a lot

Finding π with Madhava

$$\sqrt{12} \sum_{k=0}^{\infty} \frac{(-3)^{-k}}{2k+1} \rightarrow \frac{\pi}{\sqrt{12}}$$

How can we iteratively find T w/out using power function?

$$\frac{(-3)^{-k}}{2k+1} \div \frac{(-3)^{-(k-1)}}{2(k-1)+1} \qquad (-3)^{-(k-1)} \rightarrow -3^{-k} \cdot -3$$

$$\frac{\cancel{(-3)^{-k}} \cdot (\overset{2k-1}{2k-2+1})}{(2k+1) \cdot (\cancel{-3^{-k}} \cdot -3)} = \boxed{\frac{1-2k}{6k+3}}$$

while (|T| > ε) {

sum += T

k++

T ×= $\frac{1-2k}{6k+3}$

}

Initial values:

k = 0

sum = 0

T = 1

Sum × = $\sqrt{12}$

→ how we get this w/out using math library?

→ Using Newton's square root function

Just like our calculations with e, we can a close approximation using a while loop

Slice of Pi (Part 2)

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Finding π with Euler

$$\boxed{\sum_{k=1}^{\infty} \frac{1}{k^2}} \rightarrow \frac{\pi^2}{6} \qquad p(n) = \sqrt{6 \sum_{k=1}^{n} \frac{1}{k^2}}$$

Initial values:

$k = 1$

$sum = 0$

$T = \frac{1}{1^2} = 1$

```
while( |T| > ε ){
    sum += T
    k++
    T = 1/k²
}
sum x= 6
sum = √sum
```

$T = \frac{1}{k^2}$ → we don't need to iteratively find the next term because this function is simple

sum = √sum → will most rely on Newton square root

Finding π with Viete

$$\boxed{\prod_{k=1}^{\infty} \frac{a_i}{2}} \rightarrow \frac{2}{\pi}$$

where $a_1 = \sqrt{2}$

$a_k = \sqrt{2 + a_{k-1}}$

$$\hookrightarrow \frac{\sqrt{2}}{2} \times \frac{\sqrt{2+\sqrt{2}}}{2} \times \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \ldots$$

Initial Values:

$k = 1$

$product = \frac{\sqrt{2}}{2}$

$T = \frac{\sqrt{2}}{2}$

These values come from plugging in a_1

```
while( |T| > ε ){
    k++
    T = 2T + 2
    T = √T / 2
    product x= T
}
```

$$\left.\begin{array}{l} T = 2T + 2 \\ T = \frac{\sqrt{T}}{2} \end{array}\right\} \frac{\sqrt{2T+2}}{2}$$

Slice of Pi (Part 3)

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Finding Square-root using Newton-Raphson

↳ This part is provided to us by Dr. Long.
Thank you, Dr Long. Here is art:

Anyway, we'll be calling this function everytime we need to find the square-root of some value.
} Madhava
Euler
Viete

Finding π with Bailey Borwein Plouffe (BBP)

$$p(n) = \sum_{k=0}^{n} 16^{-k} \cdot \frac{(k(120k+151)+47)}{k(k(k(512k+1024)+712)+194)+15}$$

$$\frac{16^{-k}(k(120k+151)+47)\cdot((k-1)((k-1)((k-1)(512(k-1)+1024)+712)+194)+15)}{16^{-(k-1)}(k(k(k(512k+1024)+712)+194)+15)\cdot((k-1)(120(k-1)+151)+47)}$$

$$\frac{(120k^2+151k+47)\cdot(512k^4-1024k^3+712k^2-206k+21}{16\cdot(512k^4+1024k^3+712k^2+194k+15)\cdot(120k^2-89k+16)}$$

$$\left.\frac{k(k(k(k(k(61440k-45568)-45120)+34664)+4878)-6511)+987}{16k(k(k(k(k(61440k-77316)+2496)-23704)-4074)+1769)+3840}\right\} \text{Horner normal}$$

↳ This is what we are multiplying T with

```
while (|T| > ε){
  sum += T
  k++
  T = k*(k*(k*(k*(k*(61440k-45568)-45120)+34664)+4878)-6511+987
      / 16k*(k*(k*(k*(k*(61440k-77316)+2496)-23704)-4074)+1769)+3840
}
```

↳ There is probably an easier way to show this, so we will come back to this later.

— After Monday's lecture (10/4/2021), Dr. Long showed his code for BBP which looked insanely better than mine.
So again, thank you Dr. Long, I will be using that.

Slice of Pi (Part 4)

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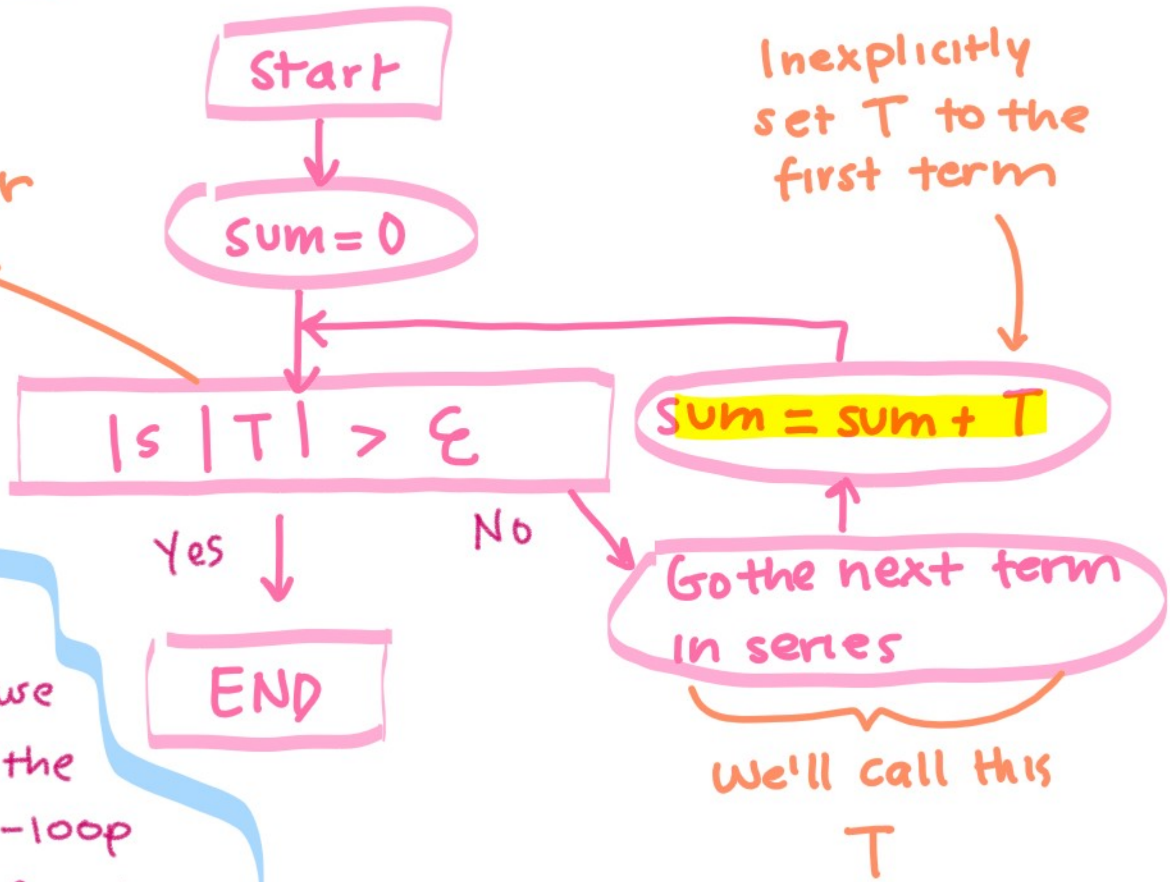
In summary :

where the value is either e or π



Writting this flowchat because it represents the general while-loop I use in the functions.

→ this would be product = product x T when we use the Viete series

Things to note :

- Although I use a while-loop for each function, I am unsure if a for-loop would be better.