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A NOTE ON 'NON-SECRET ENCRYPTION'

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A possible implementation is suggested of J H Ellis's proposed method of encryption involving no sharing of secret information (key lists, machine set-ups, pluggings etc) between sender and receiver.

Note on "Non-Secret Encryption"

1. In [1] J H Ellis describes a theoretical method of encryption which does not necessitate the sharing of secret information between the sender and receiver. The following describes a possible implementation of this.

a. The receiver picks 2 primes P, Q satisfying the conditions
   i. P does not divide Q-1.
   ii. Q does not divide P-1.

   He then transmits $N = PQ$ to the sender.

b. The sender has a message, consisting of numbers $C_1, C_2, \ldots C_r$ with $0 < C_i < N$

   He sends each, encoded as D_i where $D_i = C_i^N$ reduced modulo N.

c. To decode, the receiver finds, by Euclid's Algorithm, numbers P', Q' satisfying
   i. $P P' = 1 \pmod{Q - 1}$
   ii. $Q Q' = 1 \pmod{P - 1}$

   Then $C_i = D_i^{P'} \pmod{Q}$ and $C_i = D_i^{Q'} \pmod{P}$ and so C_i can be calculated.

Processes Involved

2. There is an algorithm, involving work of the order of log M, to test if M is prime, which usually works but can fail to give an answer. Hence as the density of primes is $(\log M)^{-1}$, picking primes is a process of order $(\log M)^k$ where k is a small integer.

3. Also, computing $C_i^N \pmod{N}$ is of order $(\log N)^{k'}$ and the computation of $D_i^{P'}$ and $D_i^{Q'}$ even smaller; hence coding and decoding is a process requiring work of order $(\log N)^k$ where k will be about 2 or 3.

4. However, factorising N is a process requiring work of order $N^{1/4} (\log N)^k$, where k is a small integer (alternatively computing C from $C^N \pmod{N}$ requires work of order N if the factorization of N is not known); so decoding for an interceptor of the communication is a process of order about $N^{1/4}$.

Reference [1] The possibility of Non-Secret digital encryption. J H Ellis, CESG Research Report, January 1970.

Note: There is no loss of security in transmitting $C_1 \ldots C_r$ all using the same N. Even if the enemy can guess a crib for eg $C_1 \ldots C_{r-1}$, this gives no information of use in decoding D_r etc. He could in any case provide himself with as many pairs (C_i, D_i) as he pleases, since the encryption process is known to him as well as to the transmitter!