COLUMBIA UNIVERSITY EEME E6911 FALL ‘25

# TOPICS IN CONTROL : PROBABILISTIC ROBOTICS

# LOCALIZATION

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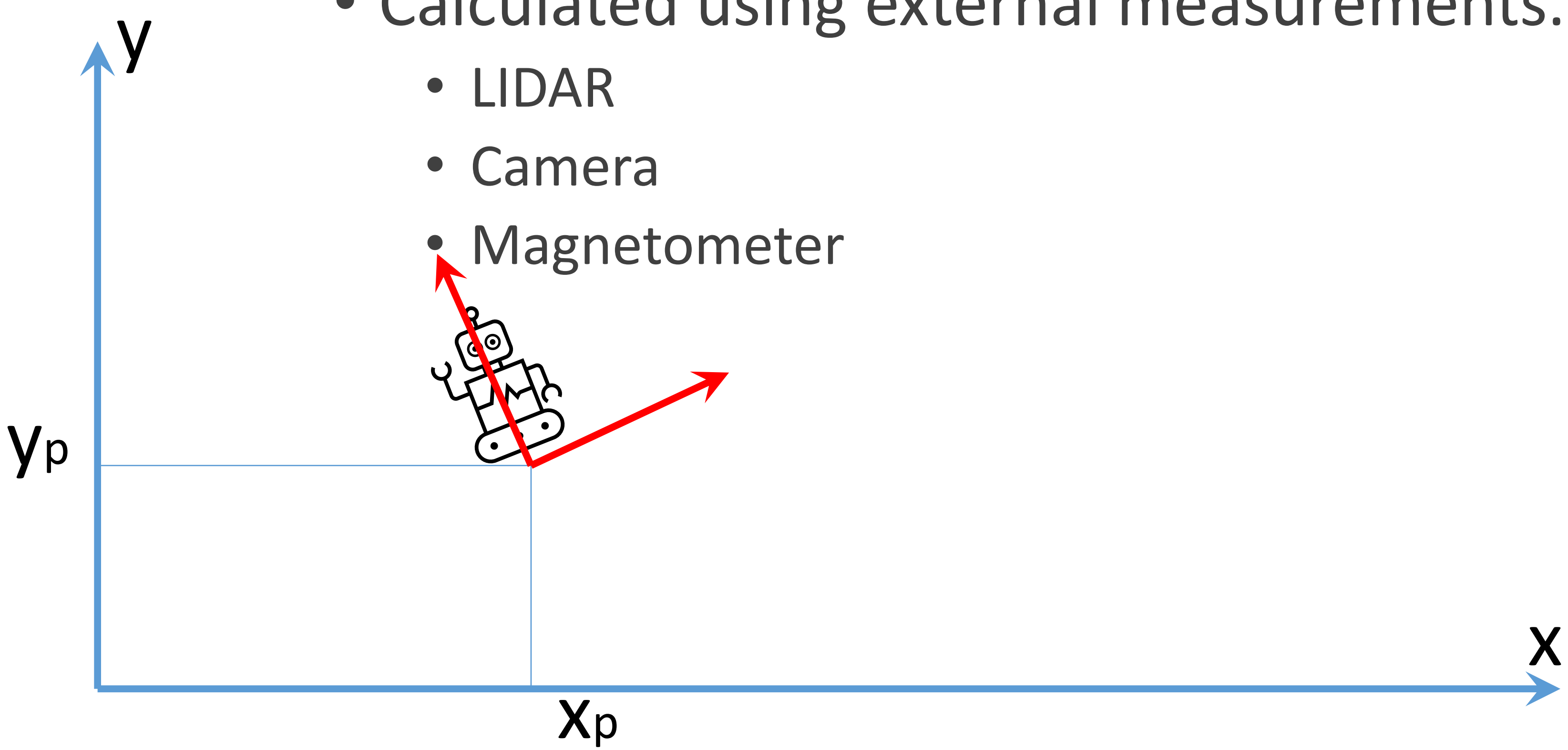
# Fixed-Frame Pose

- Relative to a fixed frame of reference.
  - Origin established by convention.
- Calculated using external measurements.
  - LIDAR
  - Camera
  - Magnetometer



# Odometry **vs.** Localization

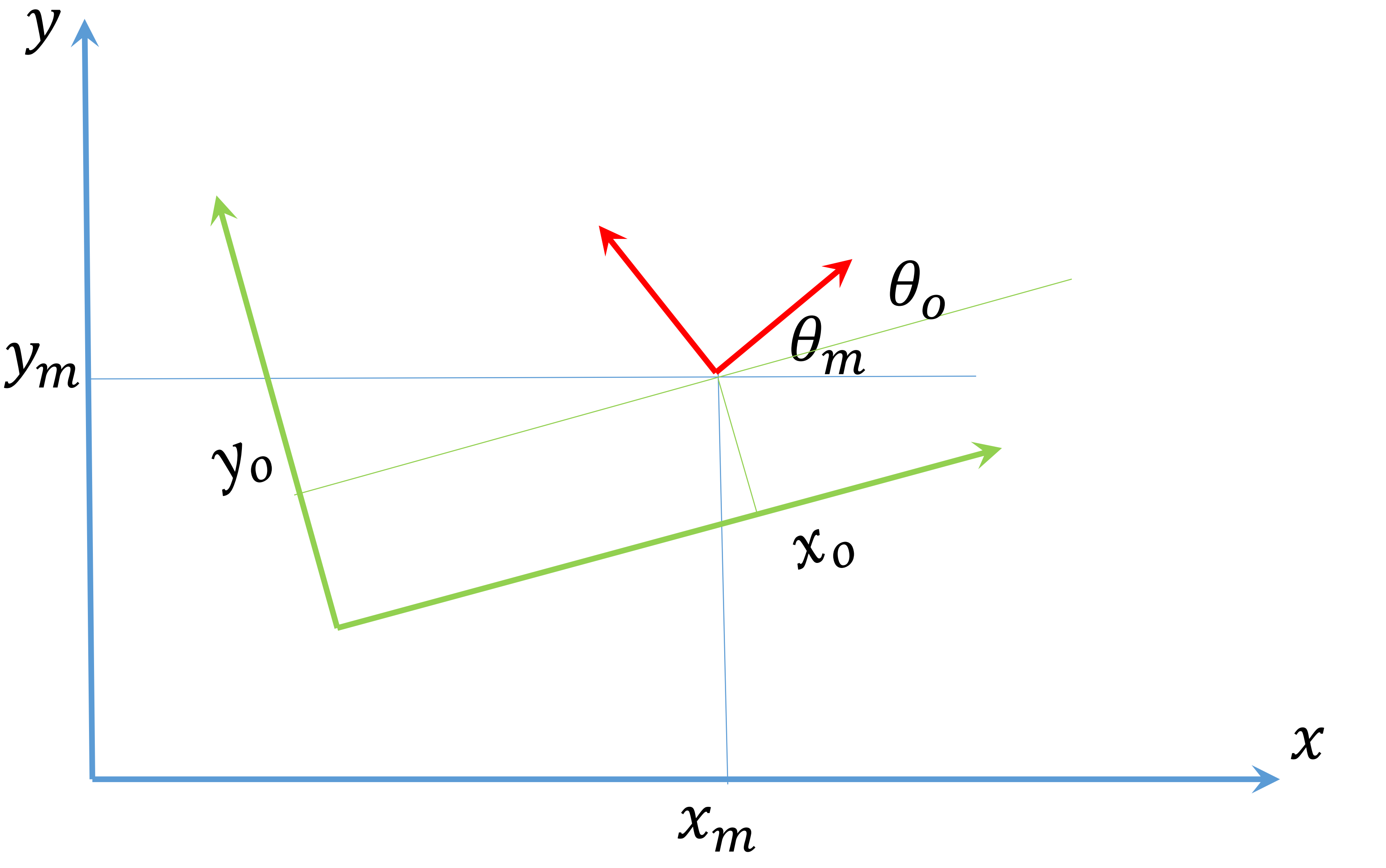
**Odometry**

- Floating frame
- Continuous
- Drifts.
- Uniform sampling rate.
- Low computation.
- Cheap sensors.

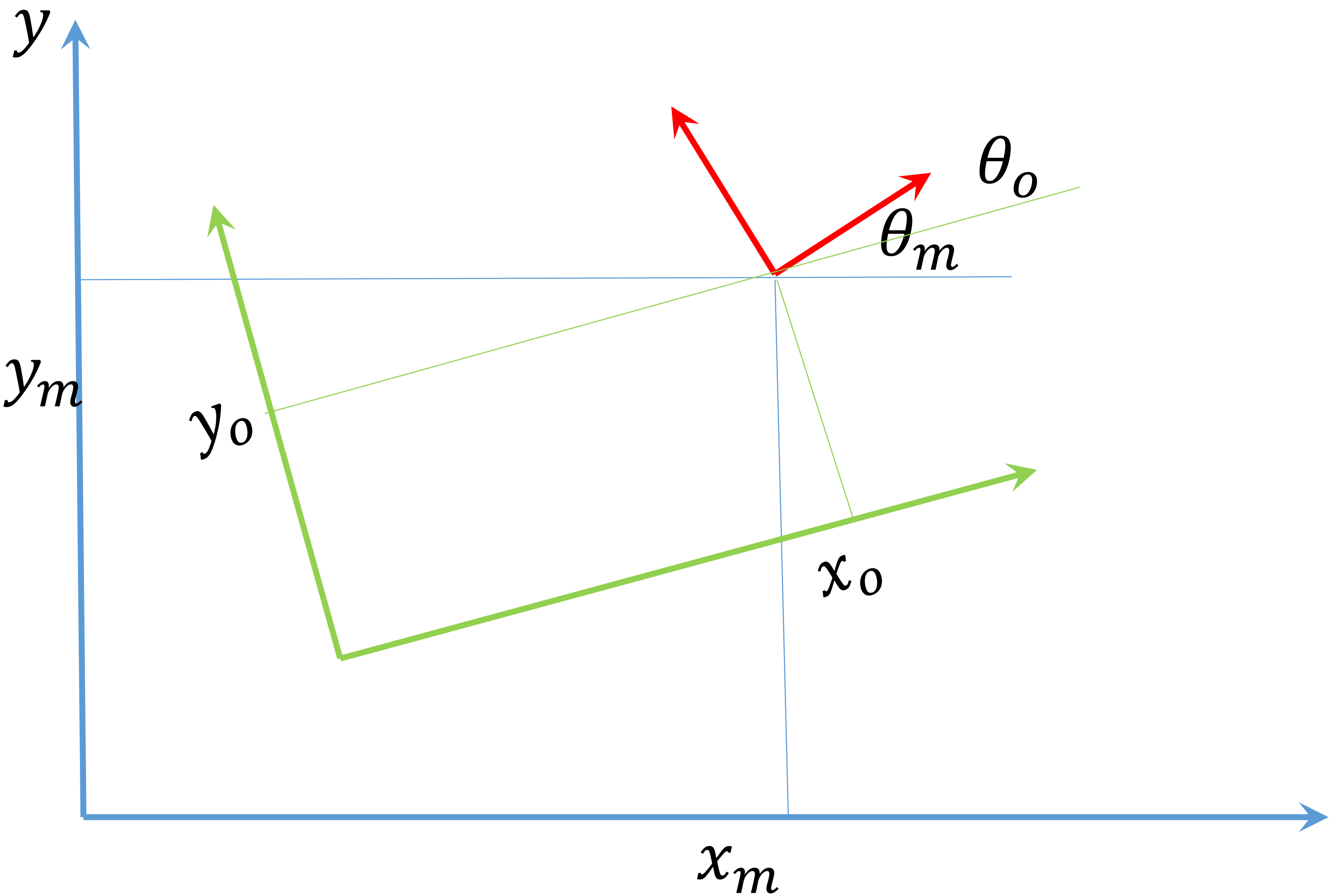
**Localization**

- Fixed frame.
- Can have discontinuities.
- Stable over time.
- Sparse sampling rate.
- Can be compute-intensive.
- Sensors can be expensive.

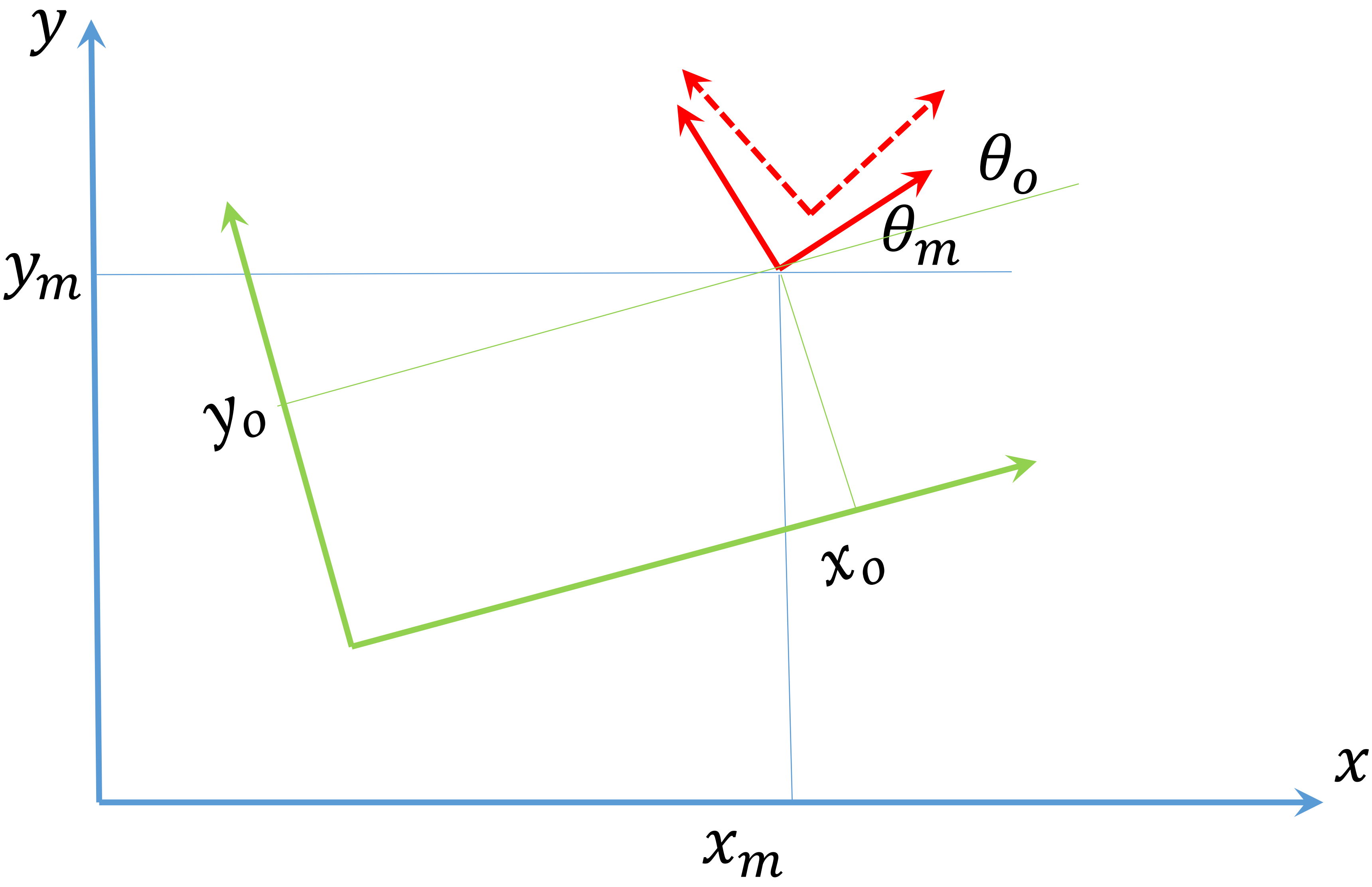
# Odometry **with** Localization



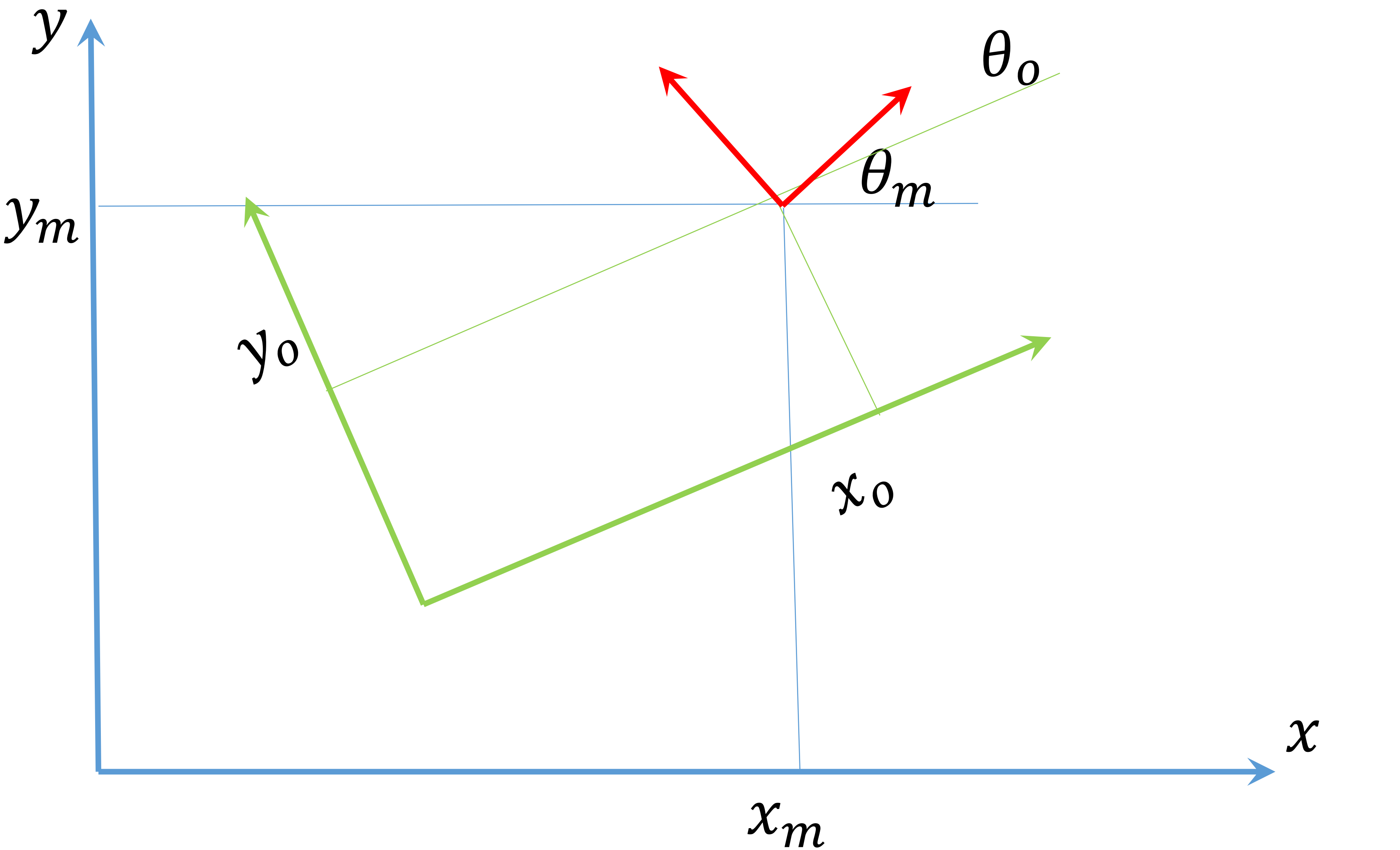
# Odometry **with** Localization



# Odometry **with** Localization



# Odometry **with** Localization



# Odometry with Localization

Video ….

# Dead Reckoning Period

$$\boldsymbol{T}_{MB}(t) = \boldsymbol{T}_{MO}(t_0)\boldsymbol{T}_{OB}(t)$$

Odometry origin at time $t_0$

Elapsed odometry since time $t_0$

- Both poses are uncertain, so propagate the covariance accordingly.
- Odometry origin:
  - Uncertainty determined last time the robot was localized (time $t_0$) – discussed next.
- Elapsed odometry (always in local frame):
  - Accumulated uncertainty since time $t_0$.

# Localization

$$\boldsymbol{T}_{MB}(t_1) \neq \boldsymbol{T}_{MO}(t_0)\boldsymbol{T}_{OB}(t_1)$$

Location determined at $t_1$

They differ because of odometry drift!

Location extrapolated since time $t_0$

$$\boldsymbol{T}_{MO}(t_1) = \boldsymbol{T}_{MB}(t_1)\boldsymbol{T}_{OB}^{-1}(t_1)$$

New origin, corrected at time at $t_1$

# Localization

$$\boldsymbol{T}_{MO}(t_1) = \boldsymbol{T}_{MB}(t_1)\boldsymbol{T}_{OB}^{-1}(t_1)$$

This is uncertain, use covariance

Treat this one as **deterministic!**

- We want covariance of the new origin yield covariance of localization
- After determining the new origin, accumulated odometry covariance resets
  - N.B. All uncertainty is now contained in localization!
  - Future uncertainty is only what accumulates until the next localization

# Covariance

$$\mathbf{\Sigma}_{MO}(t_{\mathbf{1}}) = \begin{bmatrix} \boldsymbol{R}_{MO}(t_1) & 0 \\ 0 & \boldsymbol{I} \end{bmatrix} \left( Adj\boldsymbol{T}_{OB}(t_1)\mathbf{\Sigma}_{MB}^{l}(t_1)(Adj\boldsymbol{T}_{OB}(t_1))^T \right) \begin{bmatrix} \boldsymbol{R}_{MO}(t_1) & 0 \\ 0 & \boldsymbol{I} \end{bmatrix}^T$$

$$\mathbf{\Sigma}_{\boldsymbol{MB}}^{\boldsymbol{l}}(t_{\mathbf{1}}) = \begin{bmatrix} \boldsymbol{R}_{M\boldsymbol{B}}(t_1) & 0 \\ 0 & \boldsymbol{I} \end{bmatrix} \mathbf{\Sigma}_{MB}(t_1) \begin{bmatrix} \boldsymbol{R}_{M\boldsymbol{B}}(t_1) & 0 \\ 0 & \boldsymbol{I} \end{bmatrix}^T$$

# Localization means to …

- Determine the transform between map (fixed frame) and the body (robot) frame.
- Determine the origin pose of odometry frame in fixed frame.
- Determine the probability distribution of the pose, given the previous pose, control inputs, sensor measurements, and the map

$$bel(x_t) = p\left(x_t \middle| x_{0..t-1,} u_{0..t}, z_{0..t}, m\right), \forall x$$

# Markov Localization

- All history contained in the previous pose!
- Bayes filter, applied to every possible pose

$$\overline{bel}(x_t) = \int p\left(x_t \middle| x_{t-1,} u_t, m\right) bel(x_{t-1}) dx, \forall x$$

$$bel(x_t) = \eta p\left(z_t \middle| x_{t,} m\right) \overline{bel}(x_t), \forall x$$

# States of Localization

- Tracking:
  - Belief distribution has at least one mode
  - Modes are close to true location
  - Next measurement will give us meaningful update
- Lost:
  - Belief has uniform distribution across all possible poses
  - Relaxed: modes (if any) are under some threshold that defines useful localization
- Kidnapped:
  - Modes do **not** reflect the true location.