COLUMBIA UNIVERSITY EEME E6911 FALL ‘25

TOPICS IN CONTROL : PROBABILISTIC ROBOTICS

MAPPING

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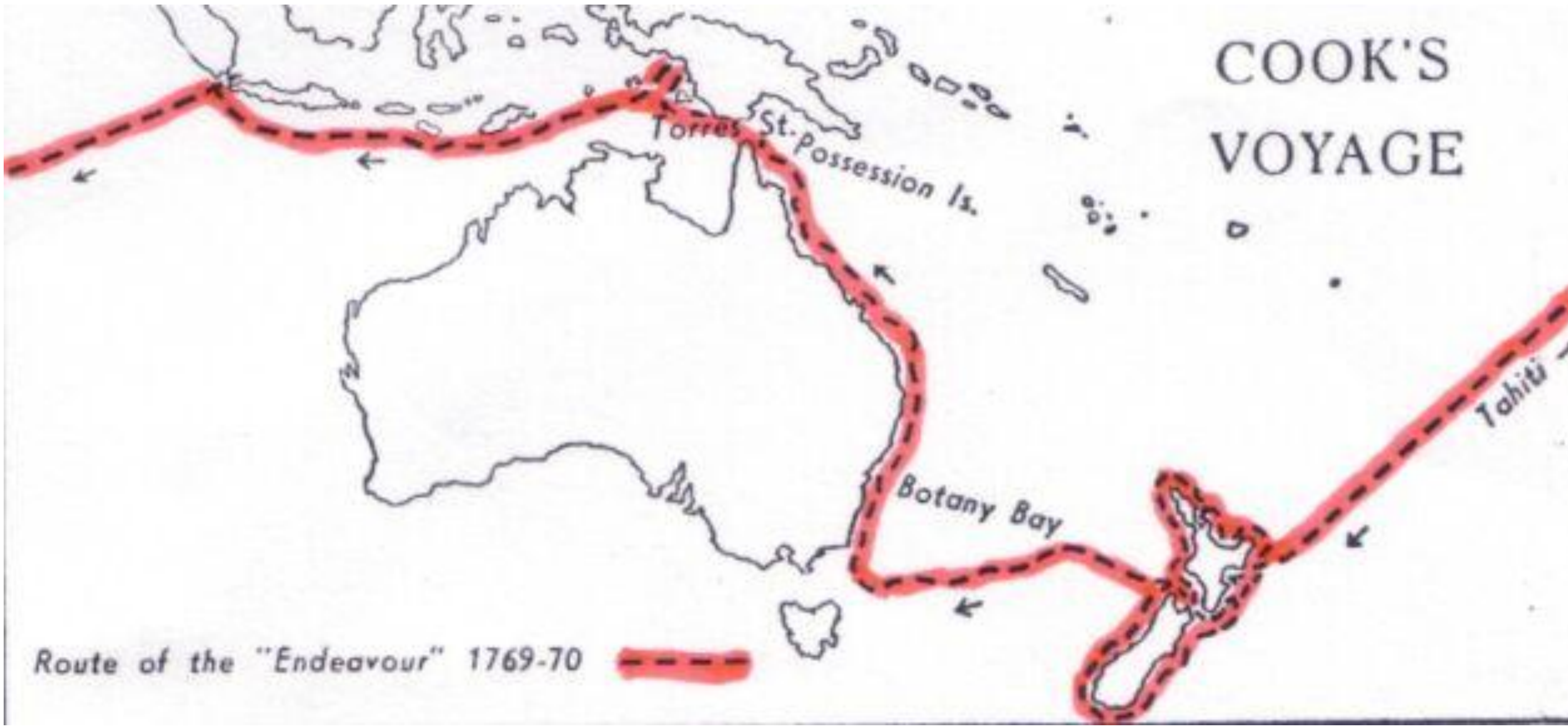
Mobile Robotics Problems

- Localization
- Mapping
- Simultaneous Localization and Mapping (SLAM)
- Navigation
- Exploration

Mapping and Localization

- Localization:
  - Map provided
  - Use sensors
  - Determine the pose
- Mapping
  - Pose provided
  - Use sensors
  - Determine the map

Mapping


COOK'S VOYAGE
Torres St.
Possession Is.
Botany Bay
Tahiti
Route of the "Endeavour" 1769-70

Mapping Problem

- Find the belief

$$p(M \mid z_{0..t}, x_{0..t})$$

- Set M is called the **occupancy grid**.
- $M = \{m_i\}$
- Variable $m_i \in \{0,1\}$ is called the **grid cell**.
- Grid cell is either occupied (1) or unoccupied (0).
- There are $2^{|M|}$ different maps.
- $|M|$ is in 100K-1M order of magnitude!

Mapping Problem – Marginals

- Find the belief

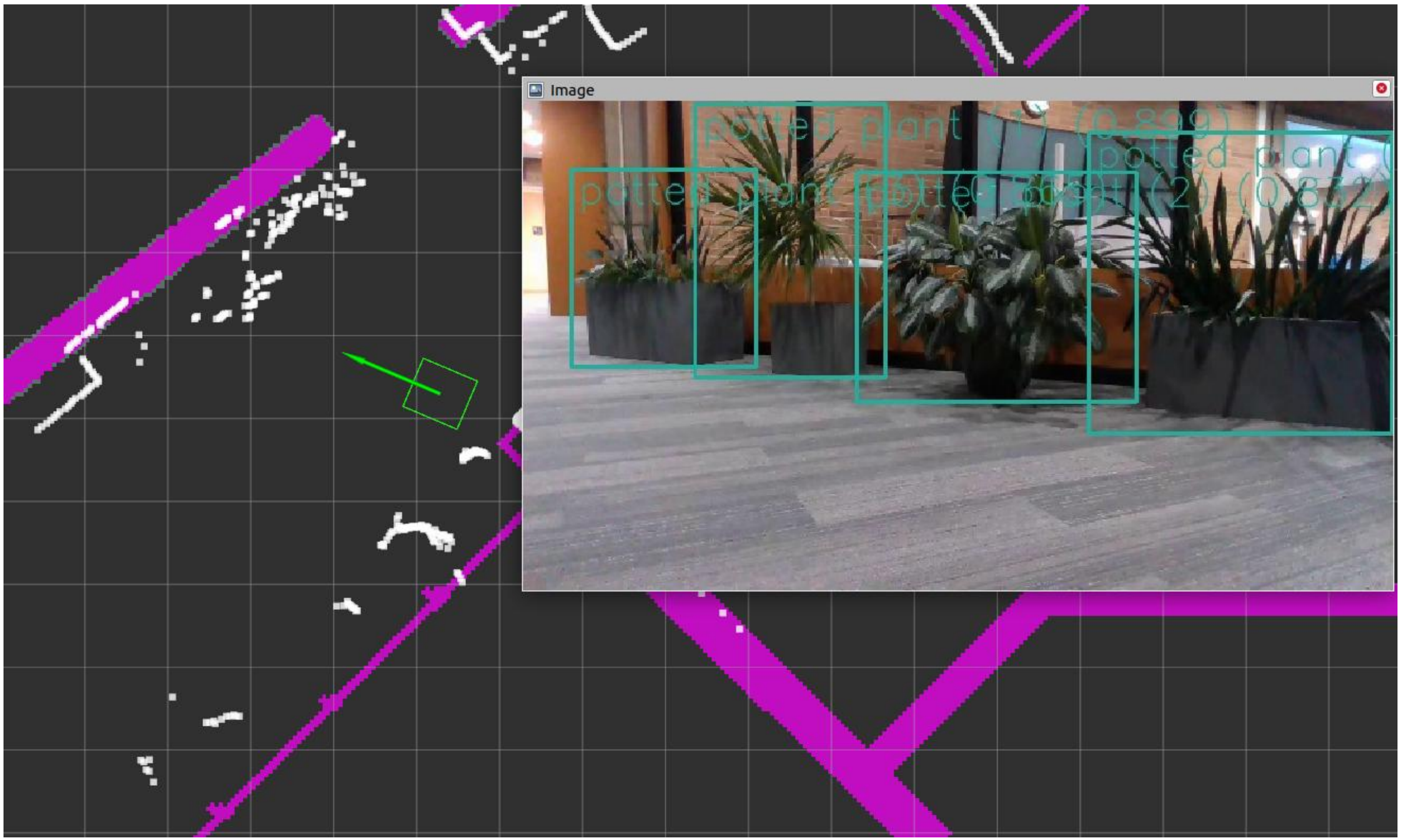
$$p(m_i \mid z_{0..t}, x_{0..t}) \quad \forall i$$

- Estimate each grid cell separately
- Not the same problem
- No way to express the dependency of neighboring cells
- Still, it works and scales linearly.

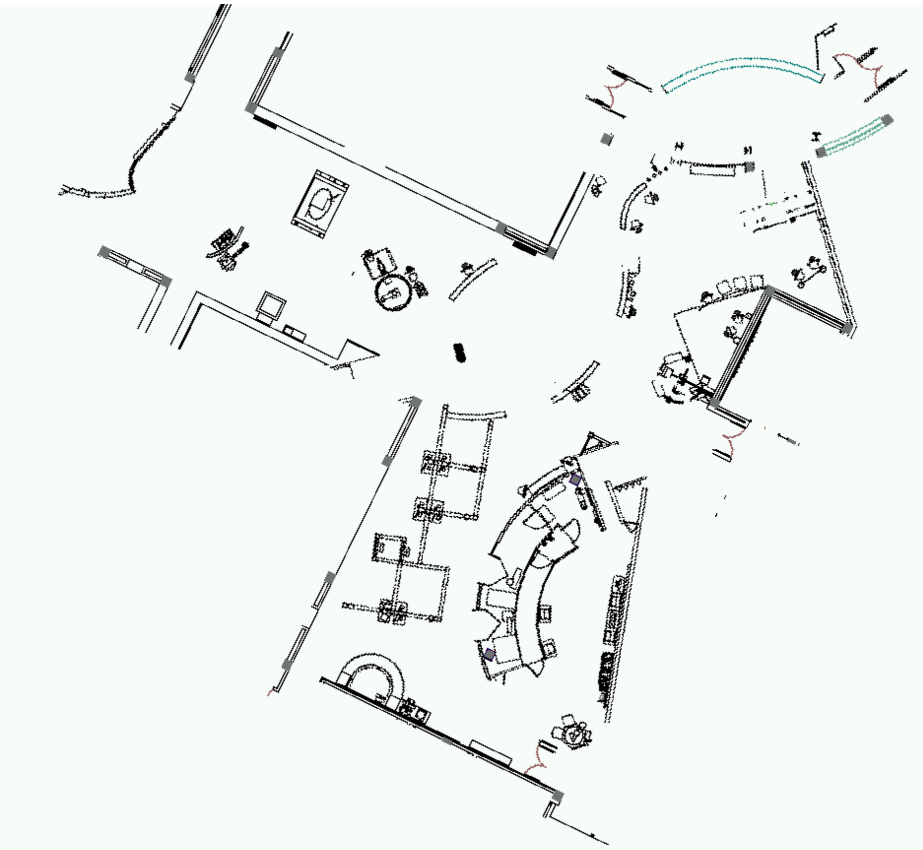
Mapping with Ranging Sensors

- If the cell is hit, increase the probability $p(m_i| \ldots)$.
- If the cell is not hit, reduce the probability.
- If there is an obstacle in front of the cell, leave it be.
- Measure again.
- Use the pose to transform hits to global frame.

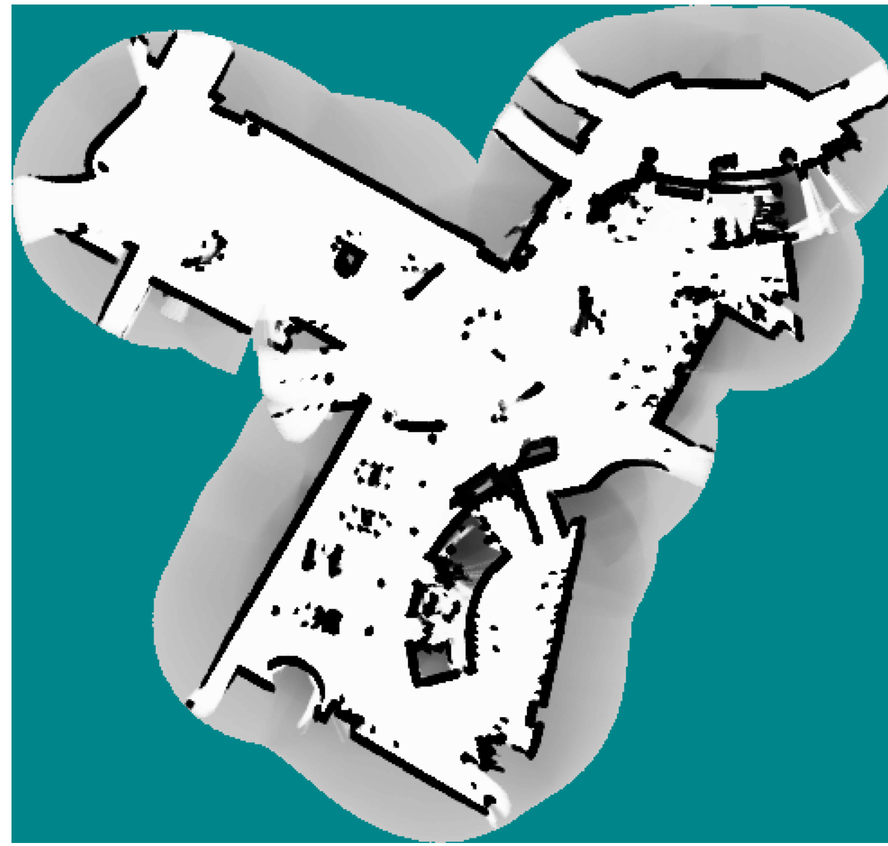
Mapping with Ranging Sensors



Tech Museum, San Jose



CAD map



occupancy grid map

Source: `www.probabilistic-robotics.org`

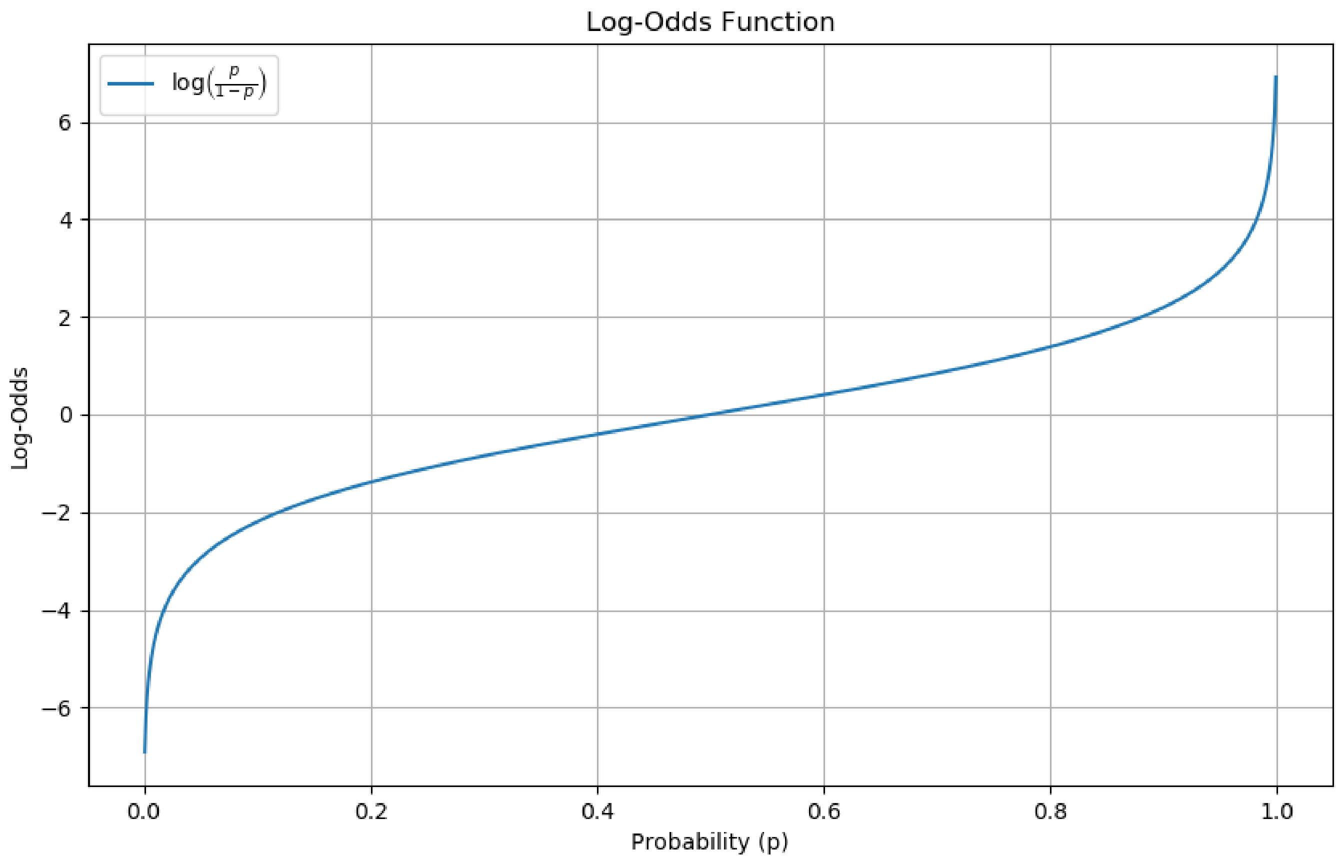
Log-Odds Metric

- Same as probability, but more convenient

$$l = \log \frac{p}{1-p}$$

$$p = 1 - \frac{1}{1 + \exp l}$$

Log-Odds Metric



Log-Odds Metric

- Additive with no bounds
- Monotonic
- Zero corresponds to probability 0.5
- Limits:
  - Probability 1 → Positive Infinity
  - Probability 0 → Negative Infinity

Log-Odds Update

$$r = \sqrt{(x_i - x)^2 + (y_i - x)^2}$$

Distance to cell

$$\phi = \text{atan2}(y_i - y, x_i - x) - \theta$$

Bearing to cell

$$k = \text{argmin}_j \left|\phi - \psi_j\right|$$

Corresponding beam

$$\Delta l = \begin{cases} l_0 & r > \min\left(z_{\max}, z_t^{(k)} + \frac{\alpha}{2}\right) \text{ or } |\phi - \psi_k| > \frac{\beta}{2} \\ l_+ & z_t^{(k)} < z_{\max} \text{ and } \left|r - z_t^{(k)}\right| < \frac{\alpha}{2} \\ l_- & r < z_t^{(k)} \end{cases}$$

Log-Odds Update

- What do the conditions mean?
  - $r > \min\left(z_{\max}, z_t^{(k)} + \frac{\alpha}{2}\right)$ - hit something in front of the cell (or cell not reachable)
  - $|\phi - \psi_k| > \frac{\beta}{2}$ - missed the cell
  - $z_t^{(k)} < z_{\max}$ - hit something valid
  - $\left|r - z_t^{(k)}\right| < \frac{\alpha}{2}$ - hit the cell
  - $r < z_t^{(k)}$ - hit something behind the cell