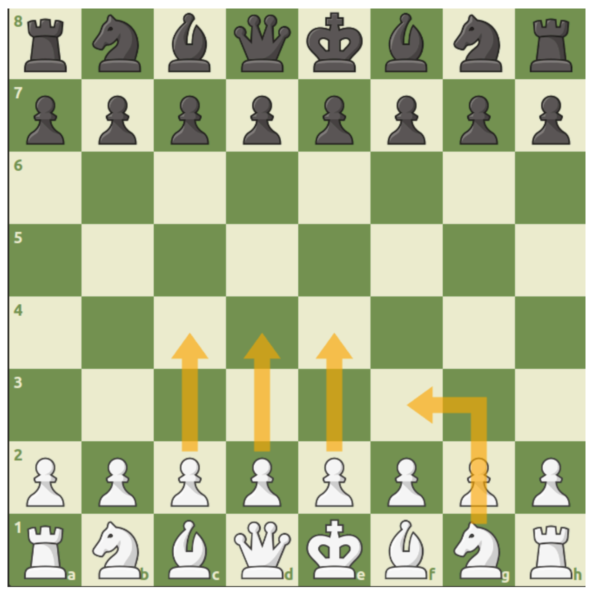
Problem 1: Your team is building a robot that uses a camera and an arm to play chess with physical pieces. Your project is the vision system that overlooks the board and recognizes the game position. The robot scans squares, one square at the time and evaluates which piece (if any) is present on that square. This measurement feeds the Bayes filter that fuses prior position belief with the measurement and generates the new position belief.

The error model is the following:

1) The robot can correctly identify the piece **p** on a square with probability $P_{pp}=0.8$.
2) The robot can get confused and believe that it identified a piece **p**, where in fact there is a different piece **q** on that square with probability $P_{pq}=1-P_{pp}$.
3) Empty square is always identified correctly and the camera just moves on.
4) The robot may believe that it focused on the target square, where in fact it focused on one of the 8 neighboring squares. The probability of correctly focusing on the target square is $P_{hit}=0.6$ and the probability of incorrectly focusing (P_{miss}) is uniformly distributed across 8 neighboring squares, such that $P_{hit}+8P_{miss}=1$.

The game has just started and the robot is playing black. The robot has the following prior knowledge:

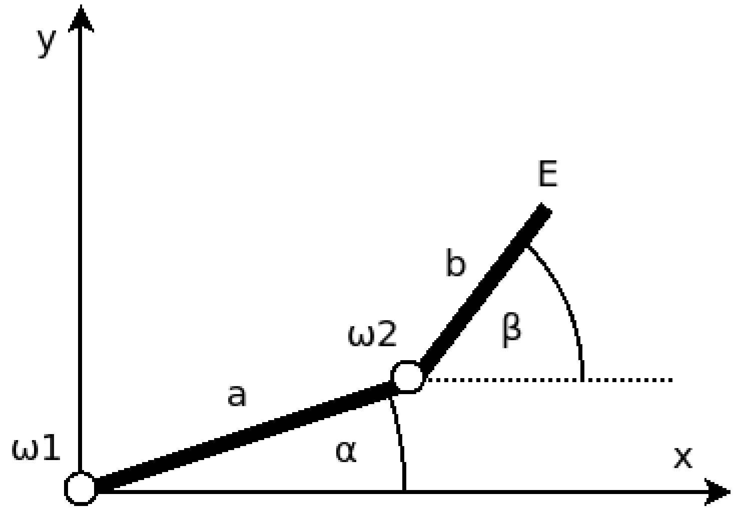


1) There are 20 legal moves for white: each pawn advancing by one or two squares (16 moves) and each knight jumping inward or outward (4 moves).
2) The robot has access to the database of previously played games and knows that the the most common opening moves for white are (see figure):
    a) **e4** with probability 0.7 (King's pawn game),
    b) **d4** with probability 0.2 (Queen's pawn game),
    c) **c4** with probability 0.05 (English opening), and
    d) **Nf3** with probability 0.04 (Reti opening).

    The remaining probability of 0.01 is uniformly distributed across the remaining 16 moves.

The camera starts the scan and initially spots the pawn on square **f4**, so the measurement is z0=(**pawn**, **f4**). Continuing the scan, the camera next spots the pawn on **e4**: z1=(**pawn**, **e4**). Write all finite (non-zero) probabilities for the first measurement, conditioned on the first move that white could have played: *P(z=z0 | x=<move>).* Use the Bayes filter to update the belief of which move was played and calculate and write all finite belief-probabilities. Continue the calculations to update the belief with information from the second measurement (*z1*) and list all non-zero belief-probabilities. What is the robot's belief, *P(<move>)* after the two measurements? Show all work.

Problem 2: Consider a 2D two-bar mechanism shown in the figure on the right. The two joints are motorized and they rotate at angular velocities ω_1 and ω_2. The system is equipped with shaft encoders that can measure the angular velocity of each joint, potentiometers that can measure the position of the joints, and a vision sensor that can measure the position of the end effector E in the x-y plane. The angles that the two bars form with the x-axis α and β are the robot configuration and bar lengths a and b are known system parameters and both are 1 meter long. Design the extended Kalman filter that estimates the robot configuration using all available sensors.



Write the motion model and the measurement model and specify all model matrices and Jacobians where appropriate. Specify the dimension of the state covariance matrix, input covariance matrix and measurement covariance matrix.

Assume that the prior state of the system is the following:

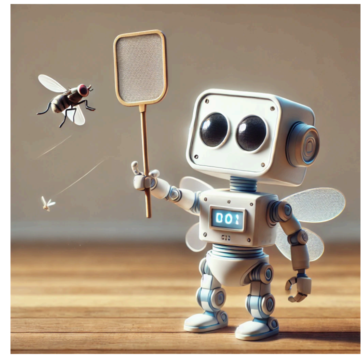
- $\alpha = 30^o$ is known with the standard deviation of 5 degrees.
- $\beta = 60^o$ is known with the standard deviation of 10 degrees.
- Correlation coefficient between α and β is -0.7.

The measurement arrives 0.5 s after the above prior has been recorded and sensors are showing the following readings:

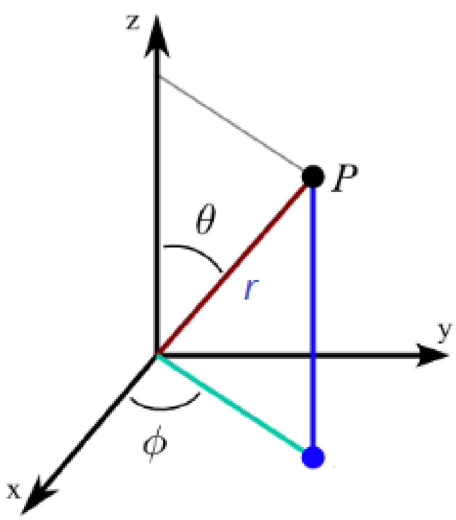
- The shaft encoders are measuring that the mean velocity since the last sample was 10 rpm on joint 1 and 5 rpm on joint 2, both with standard deviation of 1 rpm. The two measurements are independent.
- The potentiometers are measuring 60 and 70 degrees for angles α and β respectively, with standard deviation of 5 degrees and correlation coefficient of 0.4.
- The end effector position is measured to be at $x = 0.8$ and $y = 1.8$ meters with standard deviation of 10 cm and correlation coefficient 0.3.
- There is no correlation between potentiometers and the end-effector sensors.

Calculate the configuration estimate that the Extended Kalman Filter will produce (mean and covariance) after the update and determine the factor by which the standard deviation of α and β angle-estimate has improved. Show numerical results for each step in your work.

Problem 3: You have partnered with your friend from Columbia Business School on a startup that builds home robots. Your business-major partner did some market research and decided that the next feature to implement on the robot should be the ability to swat a fly.



To accomplish this task the robot must estimate the location of the fly in its local coordinate system. The robot has sensors that can determine the velocity of the fly in Euclidean coordinate system (vector $[v_x, v_y, v_z]^T$) and also measure the distance *r* to the fly along with the bearing angles θ and ϕ that determine the direction from which the fly is coming. (i.e., the polar coordinates of the fly location in the robot-local coordinate system) as shown in the figure below.



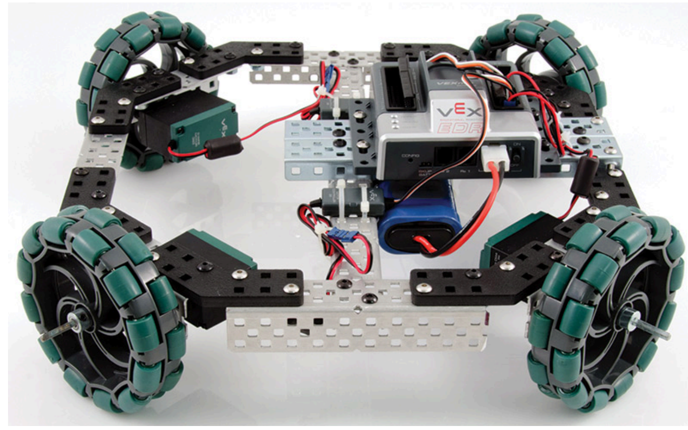
Your senior colleague suggested that the motion model for this system is linear, but that the measurement model is non-linear, so the recommended approach is to design the processing steps that combine those of the classical Kalman Filter and the Unscented Kalman Filter as appropriate.

a) Write the motion and measurement model for this system and specify the system state, inputs, and measurements. State the dimensionality of all vectors and covariance matrices.

b) Using the pseudocode notation that the textbook uses, present the algorithm that implements the filter.

c) How many sigma-points does the unscented transformation need for this system and in which stage of the filter?

Problem 4: Consider a wheeled robot with a drivetrain that can independently control its longitudinal motion (x-component of the linear velocity) and its lateral motion (y-component of the linear velocity) as well as its angular velocity.



Derive the odometry function *g(x, u)* governs the relationship between the current pose, previous pose and the current twist. Track the motion along the arc. Assuming that the function *g(x, u)* is to be used in the prediction step of the EKF-localization algorithm, explain in words what would be the next step in deriving the prediction model. Be precise, elaborate, and complete in your answer, but you can stop short of doing the actual calculations or derivations.