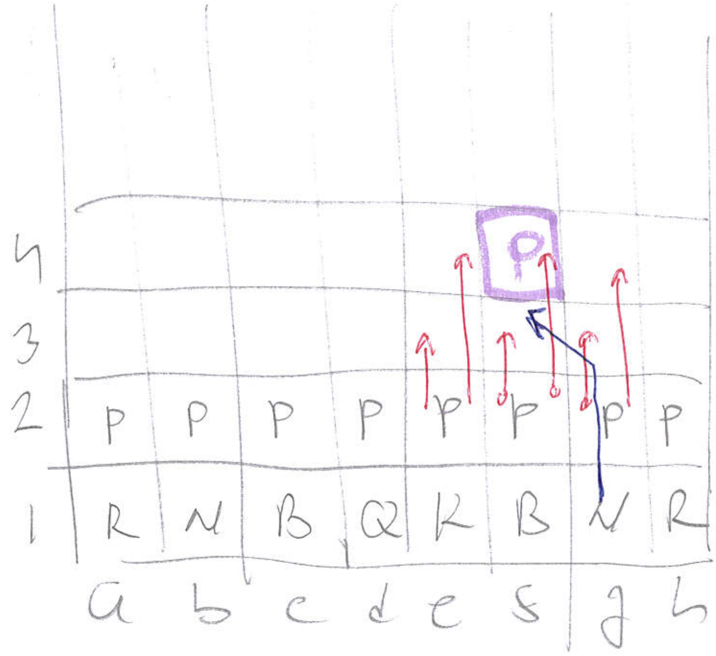
① Only 7 moves could have been played if the camera sees $z_0 = (P: f4)$:

e3, e4, f3, f4,
g3, g4, Nf3




Measurement model:

$p(z_0 | e3) = p_{miss} \cdot p_{pp} = 0.05 \cdot 0.8 = 0.04$

$p(z_0 | e4) = p_{miss} \cdot p_{pp} = 0.04$

$p(z_0 | f3) = p_{miss} \cdot p_{pp} = 0.04$

$p(z_0 | f4) = p_{hit} \cdot p_{pp} = 0.6 \cdot 0.8 = 0.48$

$p(z_0 | g3) = p_{miss} \cdot p_{pp} = 0.04$

$p(z_0 | g4) = p_{miss} \cdot p_{pp} = 0.04$

$p(z_0 | Nf3) = p_{miss} \cdot p_{pq} = 0.05 \cdot 0.2 = 0.01$

Calculate posteriors

$$p(e3|z_0) = \eta\, p(z_0|e_3)\, p(e_3) = \eta\, 0.04 \cdot \frac{0.01}{16}$$

one of 16 unlikely moves

$$p(e4|z_0) = \eta\, p(z_0|e_4)\, p(e_4) = \eta\, 0.04 \cdot 0.7$$

Most likely move from prior knowledge

$$p(f3|z_0) = \eta\, p(z_0|f_3)\, p(f_3) = \eta\, 0.04 \cdot \frac{0.01}{16}$$

$$p(f4|z_0) = \eta\, p(z_0|f_4)\, p(f_4) = \eta\, 0.48 \cdot \frac{0.01}{16}$$

$$p(g_3|z_0) = \eta\, p(z_0|g_3)\, p(g_3) = \eta\, 0.04 \cdot \frac{0.01}{16}$$

$$p(g_4|z_0) = \eta\, p(z_0|g_4)\, p(g_4) = \eta\, 0.04 \cdot \frac{0.01}{16}$$

$$p(Nf3|z_0) = \eta\, p(z_0|Nf3)\, p(Nf3) = \eta\, 0.01 \cdot 0.04$$

Now add up and normalize and calculate $\eta$

$$\frac{1}{\eta} = 0.04 \cdot \frac{0.01}{16} \cdot 4 + 0.48 \cdot \frac{0.01}{16} + 0.04 \cdot 0.07 + 0.01 \cdot 0.04$$

$$\eta \approx 34.722$$

New belief

$p(e3) = 0.00087$

$p(e4) = 0.98593$

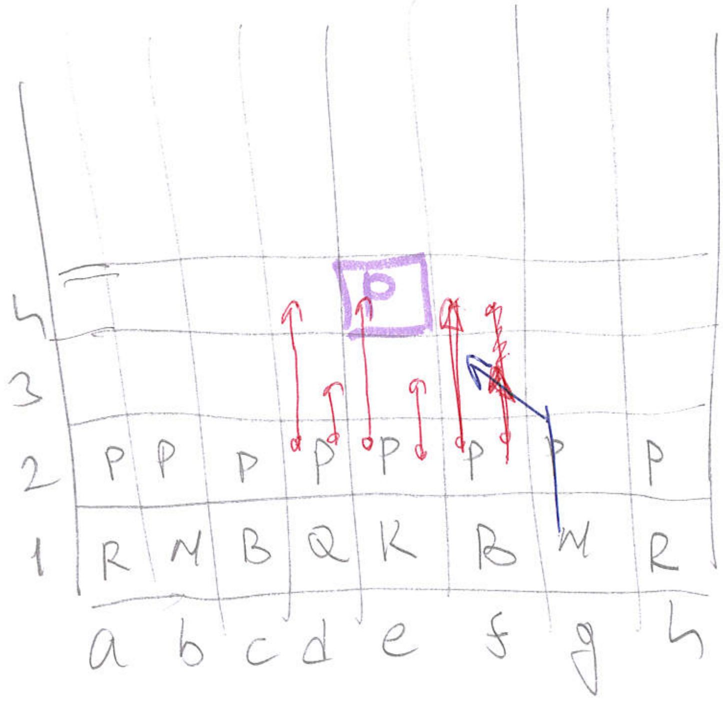
$p(f3) = 0.00087$

$p(f4) = 0.0104$

$p(g3) = 0.0087$

$p(g4) = 0.0087$

$p(Nf3) = 0.00019$




Intersection of these are possible moves given $Z_1$ measurement there are only 5 of them:

e3, e4, f3, f4, Nf3

Measurement model

$p(z1|e3) = p_{miss} \cdot p_{pp} = 0.04$

$p(z1|e4) = p_{hit} \cdot p_{pp} = 0.48$

$p(z1|f3) = p_{miss} \cdot p_{pp} = 0.04$

$p(z1|Nf3) = p_{miss} \cdot p_{pq} = 0.01$

Posteriors

$p(e3|z1) = \eta\, p(z1|e3)\, p(e3) = \eta\, 0.04 \cdot 0.00087$

$p(e4|z1) = \eta\, p(z1|e4)\, p(e4) = \eta\, 0.48 \cdot 0.98593$

$p(f3|z1) = \eta\, p(z1|f3)\, p(f3) = \eta\, 0.04 \cdot 0.00087$

$p(f4|z1) = \eta\, p(z1|f4)\, p(f4) = \eta\, 0.04 \cdot 0.0104$

$p(Nf3|z1) = \eta\, p(z1|Nf3)\, p(Nf3) = \eta = 0.01 \cdot 0.00019$

... do the numbers $\longrightarrow \eta = 2.11$

Beliefs

$p(e3) = 7.35 \times 10^{-5}$, $p(e4) = 0.99895$

$p(f3) = 7.35 \times 10^{-5}$, $p(f4) = 0.0009$

$p(Nf3) = 4 \times 10^{-6}$

(2) $\alpha[n] = \alpha[n-1] + \Delta t\, \omega_1[n]$

$\beta[n] = \beta[n-1] + \Delta t\, \omega_1[n] + \Delta t\, \omega_2[n]$

State: Input:

$x = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ $u = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$

$\Sigma_x$ is $2 \times 2$ $\Sigma_u$ is $2 \times 2$

$A = I_{2\times 2}$ $B = \begin{bmatrix} \Delta t & 0 \\ \Delta t & \Delta t \end{bmatrix}$

$Z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ a\cos\alpha + b\cos\beta \\ a\sin\alpha + b\sin\beta \end{bmatrix}$ $a = b = 1$

$G = \left[\frac{\partial z}{dx}\right]_{4\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -a\sin\alpha & -b\sin\beta \\ a\cos\alpha & b\cos\beta \end{bmatrix}$

State in radians

$$x = \begin{bmatrix} 30^\circ \\ 60^\circ \end{bmatrix} = \begin{bmatrix} 0.5236 \\ 1.0419 \end{bmatrix}$$

$$\Sigma_x = \begin{bmatrix} 0.0076 & -0.011 \\ -0.011 & 0.0304 \end{bmatrix}$$

Input in rad/s

$$u = \begin{bmatrix} 1.047 \\ 0.524 \end{bmatrix}$$

$$\Sigma_u = \begin{bmatrix} 0.0109 & 0 \\ 0 & 0.0109 \end{bmatrix}$$

1 rev = $2\pi$ rad

1 min = 60 sec

$$1\ \text{rpm} = \frac{2\pi}{60} \frac{\text{rad}}{\text{s}}$$

Prediction

$$\bar{x} = Ax + Bu = \begin{bmatrix} 1.047 \\ 1.833 \end{bmatrix}$$

$$\Sigma_{\bar{x}} = A\Sigma_x A^T + B\Sigma_u B^T = \Sigma_x + B\Sigma_u B^T$$

$$\Sigma_{\bar{x}} = \begin{bmatrix} 0.0103 & -0.008 \\ -0.008 & 0.036 \end{bmatrix}$$

Expected measurement:

$$\bar{z} = \begin{bmatrix} 1.047 \\ 1.833 \\ 0.241 \\ 1.832 \end{bmatrix}$$

Actual measurement:

$$z = \begin{bmatrix} 1.047 \\ 1.222 \\ 0.8 \\ 1.8 \end{bmatrix}$$

$$\Sigma_z = \begin{bmatrix} 0.0076 & 0.003 & 0 & 0 \\ 0.003 & 0.0076 & 0 & 0 \\ 0 & 0 & 0.01 & 0.003 \\ 0 & 0 & 0.003 & 0.01 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -0.866 & -0.966 \\ 0.5 & -0.259 \end{bmatrix}$$

<u>Kalman gain</u>

$$K = \Sigma_{\bar{x}} H^T \left( H \Sigma_{\bar{x}} H^T + \Sigma_z \right)^{-1}$$

$$K = \begin{bmatrix} 0.416 & -0.223 & -0.235 & 0.206 \\ -0.293 & 0.591 & -0.304 & -0.023 \end{bmatrix}$$

## Posterior

$$x = \begin{bmatrix} 1.049 \\ 2.362 \end{bmatrix} = \begin{bmatrix} 60^\circ \\ 135^\circ \end{bmatrix} \quad \Sigma_x = \begin{bmatrix} 0.0024 & -0.0004 \\ -0.0004 & 0.0036 \end{bmatrix}$$

## Improvement

$$\sigma_x = \sqrt{0.0024} = 0.04 = 2.29^\circ \rightarrow \text{factor of } 2$$

$$\sigma_x = \sqrt{0.0036} = 0.06 = 3.43^\circ \rightarrow \text{factor of } 3$$

Big discrepancy b/c end effector measurement and potentiometer measurement for ~~a~~ for 3 are inconsistent!

(3)

$$x = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \qquad z = \begin{bmatrix} r \\ \theta \\ \phi \end{bmatrix}$$

<u>State</u> Position in Cartesian coordinates

Measurement in polar coordinates

$$u = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

<u>Input</u>: Velocity in Cartesian coordinates

$\Sigma_u$ is $3 \times 3$

$\Sigma_x$ is $3 \times 3$

$\Sigma_z$ is $3 \times 3$

<u>Measurement model</u>

$$r = \sqrt{p_x^2 + p_y^2 + p_z^2}$$

$$\theta = a\cos \frac{p_z}{r} = a\cos \frac{p_z}{\sqrt{p_x^2 + p_y^2 + p_z^2}}$$

$$\phi = \operatorname{atan2}(p_y, p_x)$$

This is non-linear $g(x)$

Prediction model

$A = I$ $\quad\quad\quad \bar{x} = Ax + Bu$

$B = \Delta t I$

No Jacobian is needed b/c the filter is UKF. State dimension is $n = 3$, so
$2n+1 = 7$ sigma-points are needed

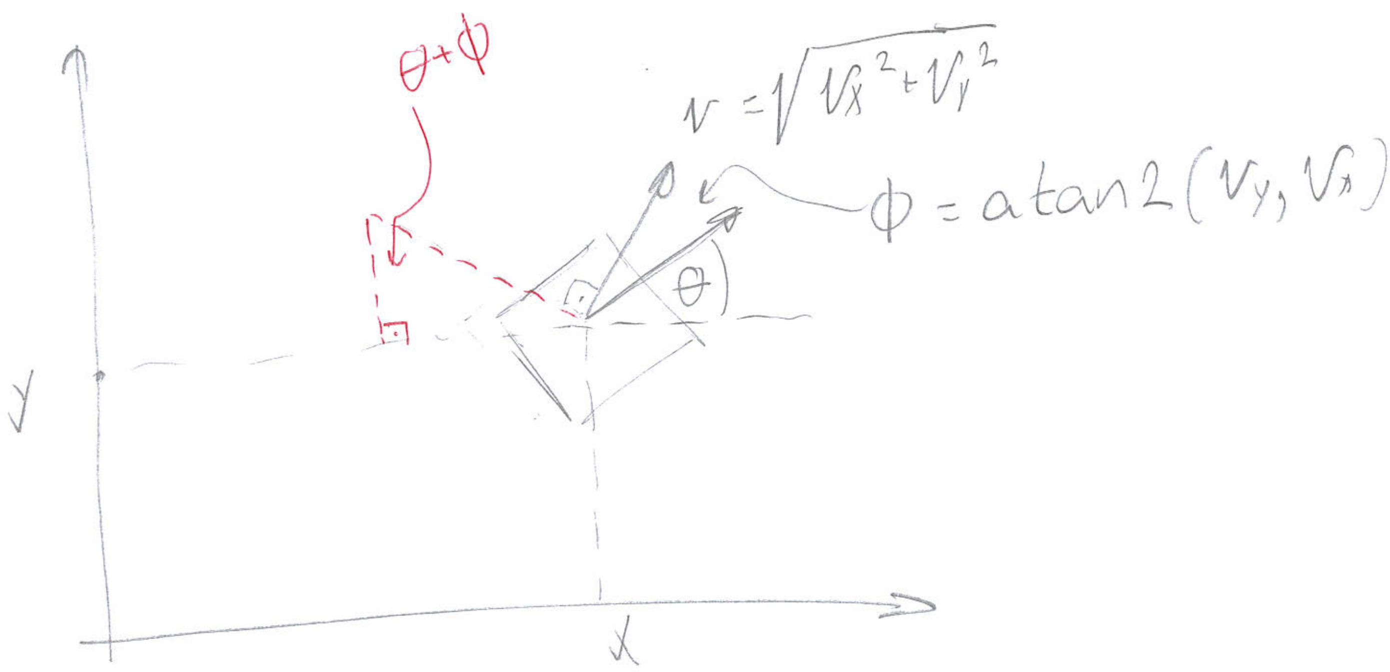
$F(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:

$\bar{\mu}_t = A\mu_{t-1} + Bu_t = \mu_{t-1} + \Delta t u_t$

$\bar{\Sigma}_t = A\Sigma_{t-1}A^T + B\Sigma_u B^T = \Sigma_{t-1} + \Delta t^2 \Sigma_u$

$\downarrow$ the rest continues from step 6 in Table 3.4 in Thrun

4




$$\phi = \operatorname{atan2}(v_y, v_x)$$

$$v = \sqrt{v_x^2 + v_y^2}$$

Center of rotation

$$x_c = x - \frac{\sqrt{v_x^2+v_y^2}}{\omega}\sin(\theta + \operatorname{atan2}(v_y, v_x))$$

$$y_c = y + \frac{\sqrt{v_x^2+v_y^2}}{\omega}\cos(\theta + \operatorname{atan2}(v_y, v_x))$$

$$x' = x_c + \frac{\sqrt{v_x^2+v_y^2}}{\omega}\sin(\theta + \operatorname{atan2}(v_y, v_x) + \omega\Delta t)$$

$$y' = y_c - \frac{\sqrt{v_x^2+v_y^2}}{\omega}\cos(\theta + \operatorname{atan2}(v_y, v_x) + \omega\Delta t)$$

$$\theta' = \theta + \omega\Delta t$$

The next step is to find Jacobians

$$\left[\frac{dg}{dx}\right] \text{ and } \left[\frac{dg}{du}\right]$$