Reconstructing Mass-Conserved Water Surfaces using Shape from Shading and Optical Flow

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Abstract. This paper introduces a method for reconstructing water from real video footage. Using a single input video, the proposed method produces a more informative reconstruction from a wider range of possible scenes than the current state of the art. The key is the combination of vision algorithms and physics laws. Shape from shading is used to capture the change of the water's surface, from which a vertical velocity gradient field is calculated. Such a gradient field is used to constrain the tracking of horizontal velocities by minimizing an energy function as a weighted combination of mass-conservation and intensity-conservation. Hence the final reconstruction contains a dense velocity field that is incompressible in 3D. The proposed method is efficient and performs consistently well across water of different types.

1 Introduction

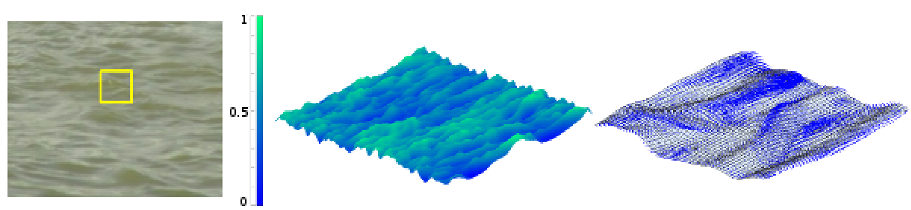
In recent years, fruitful progress has been made in reconstructing complex objects and scenes from images or videos, for example: faces [6], human bodies [1] hair [16], trees [21] [20] and fluids [2]. Among them, water brings unique challenges, a solution to which is of great interest to many research areas such as mechanical engineering [19] and computer graphics [23]. Traditional vision techniques are found to work less well in these cases. Major challenges include: a water surface generally lacks visually salient features; its complex dynamics, including topological changes, yield extreme difficulties for tracking; ground truth data is difficult to acquire – even active acquisition systems such as laser scanners will fail due to the over complicated reflection and refraction conditions.

This paper advances the current art of image based water reconstruction to work with a single input video captured in ordinary outdoor conditions, where the water is of a large scale and appears opaque. In these cases the traditional refraction and reflection based techniques as well as sophisticated experimental setups are impractical.

The proposed method is not only more flexible than previous methods of modelling the surface geometry but also reconstructs extra information in the form of a dense grid of 3D velocities. The key is the combination of shape from shading and optical flow using a physical constraint. First, shape from shading is used to estimate the geometry of the water surface for each frame. Although this is an unusual method for reconstructing reflective and refractive materials, we will demonstrate in our experiments that the opaque appearance of large bodies of water outdoors, and our choice of shape from shading algorithm cause this method to produce a convincing result (figure 2). We then produce a vertical velocity gradient field calculated from the change of the recovered surface over time. This vertical gradient is coupled with the law of mass-conservation to constrain the tracking of horizontal velocities on the water surface. The final vertical velocity is recovered from the tracked horizontal velocities, producing the dense 3D velocity field.

Compared to the existing state of the art, the proposed method has the following advantages:

- It is designed to work with a single input video recorded by an ordinary capturing device. All the example videos are recorded by a digital video camera in an outdoor environment, where the water is of a large scale and appears opaque.
- It is more informative as not only the surface geometry is recovered, but so is a dense 3D velocity field.
- The recovered velocities comply with the conservation of mass in 3D.
- It is practically efficient and stable. No complex optimization schemes are used and experiments show it performs consistently well across different scenarios with fixed parameters.



(a) (b) (c)

Fig. 1. The proposed single video based water reconstruction method. **a:** One frame from the input video. **b:** The fluid surface is recovered using combined shape from shading and optical flow. The surface geometry is demonstrated in 3D. **c:** Details of the 3D velocities and geometry inside the yellow box shown in (a). All height field results are normalised to [0, 1] for visualization.

2 Related Work

The proposed method aims to reconstruct both the geometry and the velocity of the water. Two major research areas will be reviewed: water surface geometry reconstruction and fluid tracking.

2.1 Surface Geometry Reconstruction

Various types of physical properties have been used to reconstruct the water surface geometry, for example, refraction [13, 3, 12] and reflection [23], as well as others [10] [8].

Murase [13] reconstructs a water surface from the apparent motion of a refracted pattern. The distortion of an underwater pattern is tracked by optical flow, from which the water's surface normal is calculated using a refraction model. The water surface is then recovered by 2D integration of the surface normal. Balschbach *et al.* [3] also use a refraction approach, but based on a shape from shading technique where multiple illuminations are used to better determine surface gradients. Morris and Kutulakos [12] show that refractive index is not indispensable by assuming light is refracted only once. Their system reconstructs the water surface by minimizing the refractive disparity. These refraction based methods are generally called "shape from distortion" and they work well for transparent water. The disadvantages are they can not work with opaque liquids and specially designed devices are required to capture the distortion of a known pattern being located underneath the surface of the water. These methods are not suitable for outdoor conditions where water often appears opaque.

Shape from stereo techniques have been explored to reconstruct liquids that are opaque. Wang *et al.* [23] dye water with white paint and light patterns are projected onto its surface. A depth field is first reconstructed by dense reconstruction and then refined using physically-based constraints. This method shows very accurate reconstructions of surface details. Ihrke *et al.* [10] dissolve the chemical Fluorescein in the water and measures the thickness of the water from the amplitude of the emitted light. The visual hull of the water surface is then calculated by utilizing weighted minimal surfaces using the thickness measurements as constraints. Hilsenstein [8] reconstructs water waves from thermographic image sequences acquired from a pair of infrared cameras. As a viable approach, infrared stereo reduces the problem associated with transparency, specular reflection and lack of texture at visible wavelengths. These techniques all require sophisticated equipment and complex experimental setups.

Missing from the literature is a solution for reconstructing water surfaces from a single video captured in an ordinary outdoor environment, as demonstrated by Figure 1 (a). In this case, nothing can be put under or dissolved in the water. The water is almost opaque, where refraction based approaches are impracticable but reflection based approaches tend to gain performance. This paper demonstrates shape from shading is able to perform consistently well across different types of such water surfaces.

2.2 Fluid Tracking

Although surface geometry is important, it does not contain the full set of water properties. It only describes the change of the water surface height over time, while horizontal velocities are missing. Various types of trackers are proposed to acquire the fluid flow field.

Traditional 2D tracking algorithms such as Horn-Schunck optical flow [9] are found to perform less well for water where the conservation of intensity rarely holds. As an improvement, Nakajima *et al.* [14] propose an energy function as a weighted combination of conservation of intensity, conservation of mass, and momentum equations. The resulting flow complies with physical properties of fluids in 2D. Doshi and Bors [5] use a robust kernel which adapts to the local data geometry in the diffusion stage of the Navier-Stokes formulation. The kernel ensures that smoothing occurs along the structure of the motion field while maintaining the general optical flow structure and the main optical flow features. Sakaino [18] proposes a method to model abrupt image flow change. Flow is modelled using a number of base waves and their coefficients are found to match the input sequence. Although these methods significantly improve 2D flow tracking, their physical constraints are not designed to work in 3D.

Papadakis *et al.* [15], and Heas and Memin [7] estimate 3D motions of a stratified atmosphere by minimizing an object function that describes the dynamics of an interacting stack of atmospheric layers. Li [11] treats the image as a wavefront surface and derives a general brightness constraint to model brightness variation in terms of fluid dynamics of the velocity potential. The gradient of the 3D velocity potential describes the actual motion flow. The general brightness constraint separates the flow dynamic from the brightness variation, hence one can replace the fluid dynamics model with other physical models and reuse the same solution process.

The method proposed in this paper recovers 3D velocities, producing a more informative reconstruction than previous 2D tracking algorithms. Compared with [15, 7, 11] the novelty of the proposed method is the combination of shape from shading and optical flow. Surface information acquired from the former is used as a prior to improve the performance of the latter, where physical rules are incorporated. The method is efficient and performs consistently well across different types of water captured in an outdoor environment.

3 Reconstructing 3D Mass-Conserved Water

The proposed method reconstructs the surface geometry and a dense 3D velocity field of water captured with a single video camera. The key is the law of mass-conservation, which is used as a physical link between the change of the surface height and the horizontal velocities. The proposed method uses shape from shading to acquire the change of surface height over time. It is then used as a prior to constrain the optical flow tracking. The final water surface will be reconstructed back from the horizontal velocities.

The rest of this section will first introduce the water model and the law of mass-conservation; then demonstrate shape from shading in acquiring surface geometry for a wide range of water; the physically constrained fluid tracking is explained at the end.

3.1 Conservation of Mass

A height field $h(x, y, t)$ is used to represent the water surface at time t. A vector $\boldsymbol{u} = (u, v, w)$ is used to represent the 3D velocity for each point on the surface. The law of mass-conservation constrains the 3D divergence of the velocity to zero, which leads to

$$\frac{\partial w}{\partial z} = -(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) \tag{1}$$

We first shown how the vertical velocity w can be approximated from the divergence of the horizontal velocities (u, v). By assuming the horizontal velocities do not vary along the z-direction, the right-hand side of this equation does not depend on z, so $\frac{\partial w}{\partial z}$ is a constant along the z-direction. This means the vertical velocity w is a linear function of the water depth z. The velocity at the bottom of the water comes from the boundary condition $\boldsymbol{u} \cdot \boldsymbol{n} = 0$ where $\boldsymbol{n}$ is the normal of the water bed. By further assuming a flat bottom, we have $\boldsymbol{n} = (0, 0, 1)$ hence w needs to be zero to satisfy the boundary conditions. Integrating $\frac{\partial w}{\partial z}$ along z-direction gives the vertical velocity:

$$w = h\frac{\partial w}{\partial z} = -h(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) \tag{2}$$

The vertical velocity can also be calculated from the material derivative of the surface height with respect to time:

$$w = \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\partial h}{\partial x}u + \frac{\partial h}{\partial y}v + \frac{\partial h}{\partial t} \tag{3}$$

Here we simplify the fluid dynamic by not considering the advection part $\frac{\partial h}{\partial x}u + \frac{\partial h}{\partial y}v$. Hence the Eulerian measurement of the surface change is used as an approximation of the vertical velocity $w \approx \frac{\partial h}{\partial t} = h(x, y, t+1) - h(x, y, t)$. This significantly simplifies the later optimization process and experiments show the results are generally plausible.

The evolution of water surface can then be directly linked to horizontal velocities via:

$$h(x, y, t+1) - h(x, y, t) = -h(x, y, t)(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) \tag{4}$$

Accurate horizontal velocities are expected to satisfy the surface change over time based on equation 4. The rest of this section first demonstrates shape from shading can be used to acquire a prior for water surface and then explains how to use such a prior to improve the tracking of horizontal velocities.

3.2 Recovering The Water Surface Using Shape from Shading

Shape from shading deals with the recovery of shape from a gradual variation of shading in the image, see Zhang *et al.* [24] for a detailed survey. A general assumption made by shape from shading techniques is that the scene follows the Lambertian model, in which the grey level at a pixel in the image depends on the light source direction and the surface normal. For specular surfaces, this assumption holds less well and more complex reflection/refraction models [4] are expected to be needed.

Although water is expected to be a highly reflective and refractive substance, we show that shape from shading can provide a high quality reconstruction of an outdoor water surface. Figure 2 shows eight scenes captured in ordinary outdoor conditions with their shape from shading recovered surfaces underneath (using Tsai *et al.*'s method [22]). One important reason for shape from shading to perform so well is that the water in these scenes appears visually opaque because of its depth and the suspension of dirt, mud and air. Also the particular shape from shading algorithm [22] used here is reported to have good performance with specular surfaces.

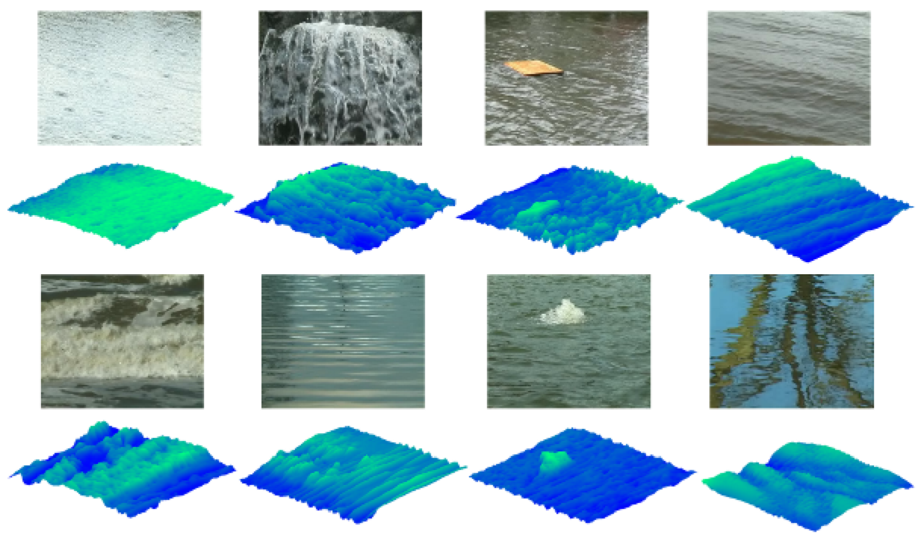


Fig. 2. Despite distortions from strong reflections (bottom right), experiments show shape from shading performs consistently well in recovering water surfaces of different types.

Our experiments also show shape from shading can work for dynamic water with very few adaptations. Videos are low-pass filtered to remove noise, such as extreme bright or dark points. A height field $h(x, y, t)$ is then individually recovered for each frame t to represent the water surface. For a T-frames video of resolution M by N, the average height of each surface $\frac{1}{MN}\sum_{i=1}^{N}\sum_{j=1}^{M} h(i, j, t)$ is rectified to the same level $\frac{1}{TMN}\sum_{k=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{M} h(i, j, k)$ to remove the affect of global luminance change:

$$h'(x,y,t) = h(x,y,t) - \frac{1}{MN}\sum_{i=1}^{N}\sum_{j=1}^{M} h(i,j,t) + \frac{1}{TMN}\sum_{k=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{M} h(i,j,k) \quad (5)$$

An example is shown in figure 3, where the shape from shading surface successfully follows the movement of the water in the video. However, surface geometry is not a completely informative description for water, but can be used to constrain optical flow to obtain the velocities of the surface using the law of mass conservation.

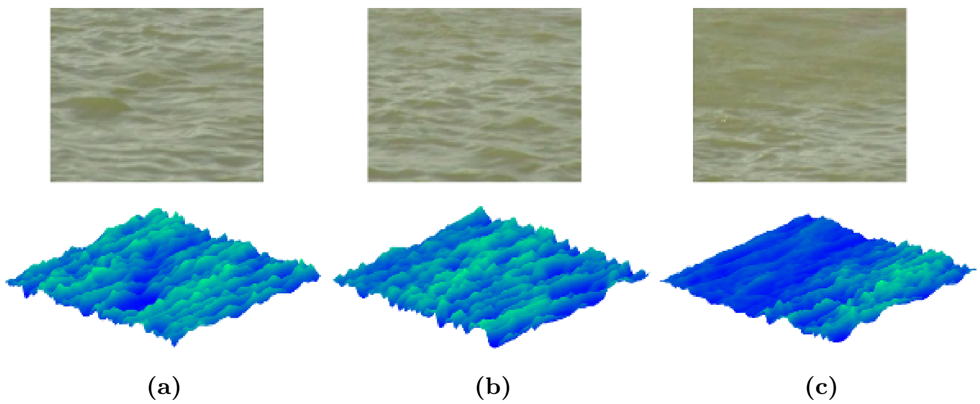


Fig. 3. Experiments show shape from shading is able to reconstruct the change of the fluid surface over time. **a - c**: different frames in the sequence and their shape from shading reconstructions.

3.3 Combined Shape from Shading and Optical Flow

The general idea is to use shape from shading water surfaces to constrain the tracking of horizontal velocities based on the conservation of mass. As explained in section 3.1, the vertical velocity w is approximated as the Eulerian derivatives of the shape from shading surfaces with respect to time. Its gradient along the z-direction $\frac{\partial w}{\partial z}$ is consequently calculated as $\frac{h(x,y,t+1)-h(x,y,t)}{h(x,y,t)}$. The horizontal velocities (u, v) are then whatever it takes to make the water incompressible.

The objective energy function is a weighted combination of intensity-conservation, mass-conservation and smoothness:

$$E = \int\int [(I_x u + I_y v + I_t)^2 + \alpha^2(|\nabla u|^2 + |\nabla v|^2) + \beta^2(u_x + v_y + w_z)^2] dxdy \quad (6)$$

$(I_x u + I_y v + I_t)^2$ and $|\nabla u|^2 + |\nabla v|^2$ are the intensity-conservation term and smoothness terms from the Horn-Schunck [9] optical flow. $(u_x + v_y + w_z)^2$ is the mass-conservation term that describes the 3D divergence of the velocity. In practice, w is calculated by subtracting the current shape from shading surface from its successor. Then w_z is calculated as $\frac{w}{h}$. The following Euler-Lagrangian equations are used to minimize the objective function 6:

$$I_x(I_x u + I_y v + I_t) - \alpha^2 \triangle u - \beta^2 (u_{xx} + v_{xy} + w_{xz}) = 0 \tag{7}$$

$$I_y(I_x u + I_y v + I_t) - \alpha^2 \triangle v - \beta^2 (u_{xy} + v_{yy} + w_{yz}) = 0 \tag{8}$$

In practice $\triangle u$, $\triangle v$, u_{xx} and v_{yy} are approximated numerically using finite differences: $\widetilde{u}(x,y) - u(x,y) = \frac{u(x-1,y)+u(x+1,y)+u(x,y-1)+u(x,y+1)}{4} - u(x,y)$, $\widetilde{v}(x,y) - v(x,y) = \frac{v(x-1,y)+v(x+1,y)+v(x,y-1)+v(x,y+1)}{4} - v(x,y)$, $\overline{u}(x,y) - u(x,y) = \frac{u(x-1,y)+u(x+1,y)}{2} - u(x,y)$, $\overline{v}(x,y) - v(x,y) = \frac{v(x,y-1)+v(x,y+1)}{2} - v(x,y)$. The Lagrange multipliers α^2 and β^2 are fixed to 1000 across all scenes. The solution of equations 7 and 8 is found using the Gauss Seidel method. The resulting horizontal velocity (u, v) is then used to calculate the final vertical velocity w and the change of the water surface using equation 4. Due to the mass conservation constraint the surface produced from these new vertical velocities is very similar to the shape from shading surfaces, which have been shown to model the real water dynamics well.

4 Experiment

To evaluate the quality of our method we compare our method with several state of the art flow estimators on different water scenes. Our hypothesis is that our method will track the horizontal flow of the fluid more plausibly than previous methods, the major improvement being that our result conforms with the movement of fluid in 3D. We compare both the appearance of the tracked horizontal velocities alone, and the surface reconstructed using mass-conservation.

This paper chooses the classical Horn-Schunck [9] optical flow and the more contemporary physics-based flow tracker [14] to compare with. These two methods, like ours, both minimize an energy function as the weighted combination of some energy terms such as the intensity-conservation term and the smoothness term. The difference is Nakajima *et al.*'s [14] method contains extra terms for 2D momentum equations and 2D mass-conservation; the proposed method in this paper contains an extra term for mass-conservation in 3D and Horn-Schunck [9] flow does not employ any physical constraint. In this paper, same weight coefficients (Lagrangian multipliers) are used to combine different energy terms and they are fixed across all the experiment sequences.

Figure 4 shows the horizontal flow fields acquired using the three different methods. The flow produced by the Horn-Schunck [9] method clearly oversmooths the velocities and only captures the global flow of the different image regions. The flow field produced by Nakajima *et al.* [14] improves on this but still oversmooths the finer details of the water movement. As demonstrated, our

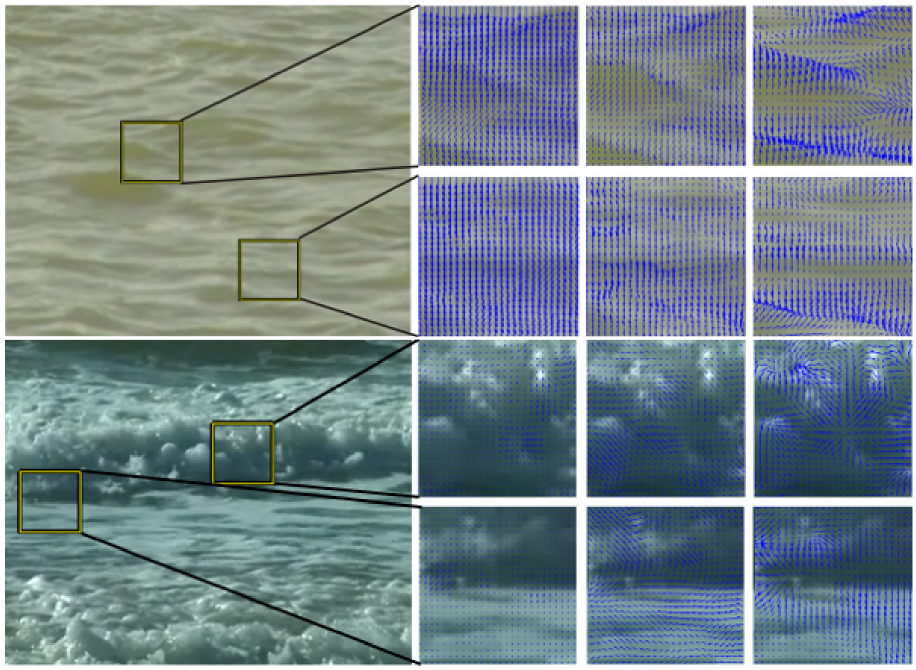


Fig. 4. Results of different methods. Our method successfully captures the sharp velocity features, while previous methods tend to over smooth the flow.

method manages to create a flow field which captures the detailed sharp features of the flow successfully.

We have produced reconstructions of the surface geometry using the velocities by both the Horn-Schunk [9] and Nakajima *et al.* [14] methods. Figure 5 shows the surface geometries produced using these vertical velocities and an initial height at frame 1 produced by shape from shading. This experiment evaluates how well the velocities produced by each algorithm comply with the movement of fluid in 3D. Our results show that both methods tend to "halt" the water surface due to the lack of vertical velocity. As error accumulates in time, the water surface drifts away from its real appearance in the video. These results are due to the lack of a 3D physical constraint and therefore the vertical velocities calculated using the 3D law of mass-conservation are incorrect.

The robustness of the proposed method is tested on a wide range of water sequences. 40 water sequences from the Dyntex database [17] are used. These include water of calm, wavy and turbulent motion. The sequences are filmed outdoors with an ordinary digital video camera with a fixed tripod. A common property of these videos is the water generally appears opaque which allows the shape from shading surface a veridical prior to constrain the optical flow tracker. Results show the proposed method performs consistently across these test sequences. Some of the reconstructed water surfaces and velocity fields are

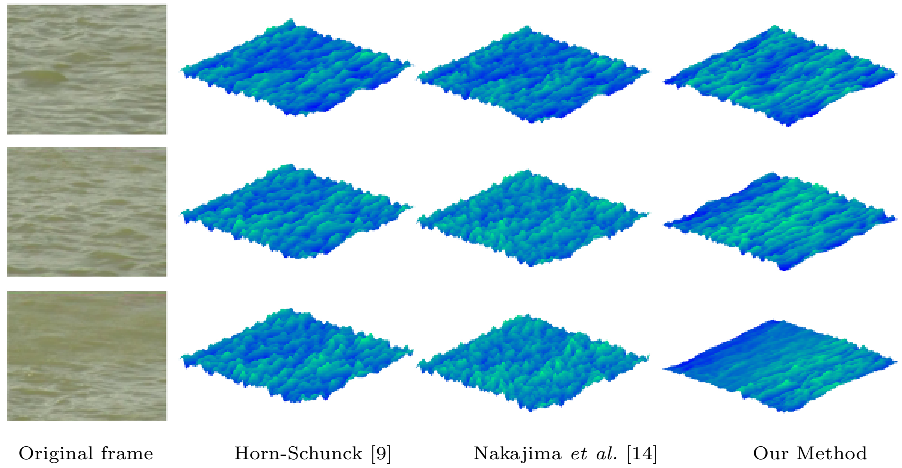


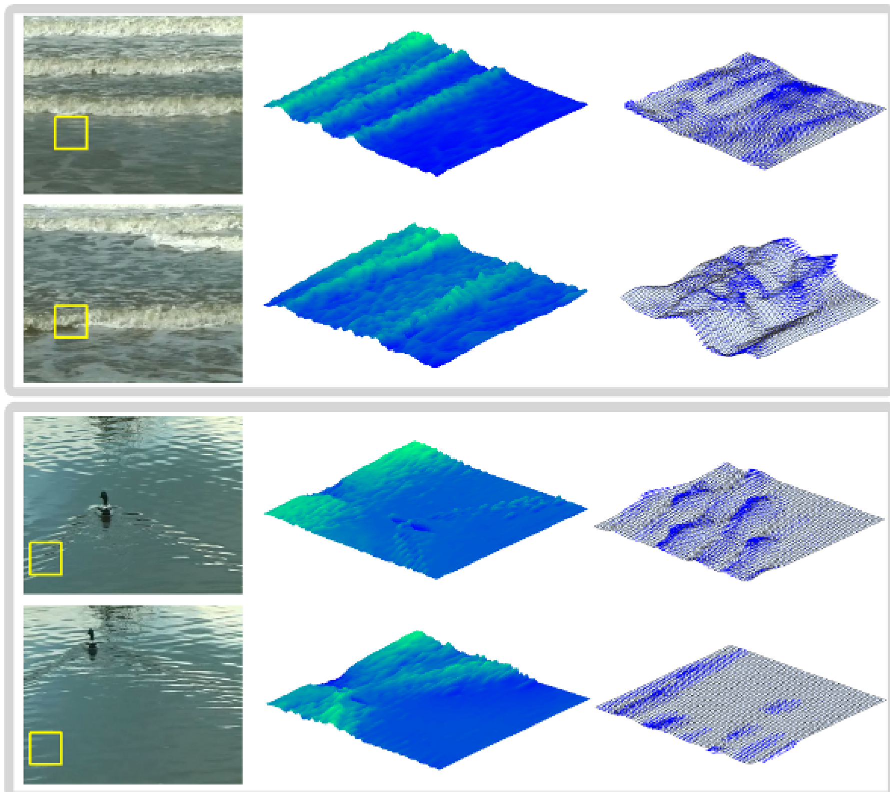
Fig. 5. Results of reconstructions produced from horizontal velocities given by different flow estimators. The Horn-Schunck and Nakajima reconstructions are "halted" and noisy, while the proposed method is significantly better.

shown in figure 6. The fluid dynamics caused by objects interfering, such as an animal swimming, can also be well captured.

An advantage of the proposed method is its efficiency. Solving equations 7 and 8 is a linear optimization process without any extra complexity compared to the classic Horn-Schunck [9] optical flow. A C++ implementation of the whole system, including shape from shading and flow estimation, is able to process over 10 frames of resolution 352×288 per second on an Intel quad-core processor, which makes realtime applications practically possible.

There are several limitations of the proposed method. First, it strongly depends on the surface prior acquired from shape from shading. Although it has been shown in this paper that shape from shading works consistently well over a wide range of water that has opaque and Lambertian properties, failure modes can appear when the water is transparent or highly specular. In this case the refraction/reflection will distort the reconstructed surface. A good example is shown in the last picture of figure 2 where the reflection of the trees yield valleys on the surface. Currently the proposed method simply uses a low-pass filter to remove the extreme bright or dark pixels in the image, this can be replace by better specular/shadow removal methods. Also, the height field representation works efficiently well for calm water surfaces but does not well describe complex scenes such as splashing and breaking waves. In these cases a more sophisticated fluid representation is needed to handle the topological change.

In summary an important characteristic of the reconstruction is it is physically sound, as the velocity field complies with the conservation of mass in 3D. Compared to previous flow estimators our method captures sharp velocity features and reconstructs a water surface that successfully models the change of the water surface geometry. Our method works fully automatically and requires



(a) **(b)** **(c)**

Fig. 6. Results of different water surfaces. **a:** the original input video frame. **b:** the mass-conserved surface reconstructions. **c:** 3D velocities and geometry of the surface inside the yellow box shown in (a). Each pair of results are two frames from the same video sequence.

only a single input video. It has been tested on a wide range of scenes and found to perform consistently (figure 6).

5 Conclusion

This paper studied the problem of image-based water reconstruction for a single input video that is captured in ordinary outdoor conditions. In this case the water is of a large scale, appears opaque and traditional refraction and reflection based reconstruction techniques are impractical. One important discovery is the capability of shape from shading to recover different water surfaces of this kind. Consistent performance is demonstrated by experimenting on a wide range of scenes. Based on this discovery, the paper proposes a method for reconstructing water by combining shape from shading and optical flow. It essentially uses the vertical velocity acquired from shape from shading to constrain the optical flow tracking of horizontal velocities. The advantages of the proposed method are: 1) it works fully automatically and requires only basic input resources; 2) the reconstruction is more informative as it contains not only the surface geometry profile but also a 3D velocity field; 3) the recovered velocities are mass-conserved in 3D; 4) it is efficient and generally stable, as tested by a wide range of water. We also discussed several failure modes where the water is highly specular. Interesting future avenues include finding better solutions for removing shadows and highlights from the water surface and integrating more sophisticated fluid dynamics, for example the full Naiver-Stokes equations.

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