In this lecture we introduced graphical models. We covered the fundamental ideas and notation of graphical models, simple parametric models and fitting tools, and regularized graphical models. The various concepts were supplemented with examples and applications.

1 An Application of the Feed-Forward Neural Network in Approximate Bayesian Computation by Miranda Fix

Miranda started class by presenting an application of neural networks in ABC. She provided neural network cupcakes for her birthday. Happy birthday Miranda!

ABC (approximate Bayesian computation): general set up

- Goal: compute the posterior distribution of a mulitidimensional parameter ϕ
- Typically used when likelihood has no closed form
- Most basic algorithm:
  1. compute summary statistic, $\mathbf{S}$, from observed data
  2. for i in 1:nsim, draw $\phi^{(i)}$ from prior distribution
  3. simulate new data using $\phi^{(i)}$
  4. compute summary statistic, $\mathbf{S}^{(i)}$, from simulated data
  5. keep $\phi^{(i)}$ if $d(\mathbf{S}, \mathbf{S}^{(i)}) < \delta$, where d is some distance metric and δ is a specified tolerance

Miranda then discussed two methods used to approximate ϕ with the aim of speeding up the sampling algorithm:

1. local linear model
2. feed forward neural network

2 Graphical Models: Basics

- Core idea for graphical models: condition independence relations
- A graph is a pair $G = \{V, E\}$, where
  1. V is a set of vertices
  2. E is a set of edges
- For graphical models, each vertex corresponds to a random variable and each edge corresponds to some aspect of their joint distribution

- In particular, the absence of an edge encodes condition independence; i.e., $\{j,k\} \notin E \Rightarrow X_j \perp X_k|\text{rest}$
- A couple of ways to think about graphical models
    1. a hierarchical Bayesian model corresponds to a directed acyclic graph (DAG)
    2. Let $x = (X_1, \dots, X_p)^\top \sim \mathbb{P}$, then a graph G for $\mathbb{P}$ has p vertices (aka nodes)
    3. Important to note that, in general, there ISN'T a 1-1 map between $\mathbb{P}$ and G; instead every graph generates a class of distributions

3 Statistical Graphical Models

1. **Non-parametric statistical graphical models**
    - idea: factor the graph in some way
    - undirected (Markov) graphical models allow a decomposition into clique potentials
    - example: directed acyclic graph can be factored into conditional distributions
2. **Parametric statistical graphical models**
    - Goal: given a sample $x_1, \dots, x_n \sim \mathbb{P}$, we wish to estimate (or less ambitiously, constrain) the graph G
    - Under the assumption that the data follow a MVN distribution, $X_j \perp X_k|\text{rest} \Leftrightarrow \Omega_{jk} = 0$, where $\Omega = \Sigma^{-1}$ is the precision matrix
    - Can use MLE to compute $\hat{\Omega}$, and then do hypothesis tests to determine where $\Omega_{jk} = 0$.
    - As usual, use the MLE at your own risk, especially if n isn't extremely large relative to p
    - Example: stocks values from the S & P 500. Estimate Ω and plot the resulting graph for various thresholds on the size of the entry in $\hat{\Omega}$.

4 SIN

- a psuedo-acronym for partitioning graph vertices in a(n)
    1. significant set S
    2. indeterminate set I
    3. non-significant N
- can be thought of as a way for controlling the overall error rate for incorrect edge inclusion
- see notes, and sources therein, for 'SIN' algorithm details
- available in R package "SIN"

5 Regularized Statistical Graphical Models

There are two common methods for estimation when $p > n$. Both methods are available in the R package "huge".

1. parallel lasso

- For each $j = 1, \ldots, p$, regress X_i on all other variables using lasso
- Put an edge between X_i and X_j if each appears in the active set of the other variable

2. graphical lasso
   - Take the usual (MVN) log likelihood and penalize it
   - i.e., $\min -\frac{1}{2}\left(\log|\Omega| - n\text{trace}(\Omega S)\right) + \lambda||\Omega_1||$

6 An Exploration of Darren's Vocabulary

- germane: *(adjective)* relevant to a subject under consideration
- implicit: *(adjective)* implied though not plainly expressed
- edifying: *(adjective)* providing moral or intellectual instruction
- innocuous: *(adjective)* not harmful or offensive
- recapitulate: *(verb)* summarize and state again the main points of
- codify: *(verb)* arrange (laws or rules) into a systematic code
- obfuscate: *(verb)* render obscure, unclear, or unintelligible
- amalgamation: *(noun)* the action, process, or result of combining or uniting