STAT675 – Homework 2
Due: October 7

1. Derive the lasso, ridge, and least squares solutions under orthogonal design and contrast them.

2. Let
$$\mathcal{T} = \left\{ \max_{1 \le j \le p} 2|\epsilon^\top x_j|/n \le \lambda_0 \right\}$$
and assume $\mathbb{X}$ is standardized (that is, $\hat{\Sigma} = \mathbb{X}^\top \mathbb{X}/n$ has 1's on diagonal). Show that if $\lambda_0 = 2\sigma\sqrt{(t^2 + 2\log p)/n}$ and $\epsilon \sim N(0, \sigma^2 I)$, then
$$\mathbb{P}(\mathcal{T}) \ge 1 - 2e^{-t^2/2}$$
and hence that $\mathcal{T}$ is 'large'

3. Show that on $\mathcal{T}$ with $\lambda \ge 2\lambda_0$ and $S^* = \{j : \beta_j^* \ne 0\}$
$$2\left|\left|\mathbb{X}(\hat{\beta} - \beta^*)\right|\right|_2^2/n + \lambda\left|\left|\hat{\beta}_{S_*^c}\right|\right|_1 \le 3\lambda\left|\left|\hat{\beta}_{S_*} - \beta_{S_*}^*\right|\right|_1$$

4. Suppose that the compatibility condition holds for S_* with constant ϕ_*. Then on $\mathcal{T}$ and for $\lambda \ge 2\lambda_0$
$$n^{-1}\left|\left|\mathbb{X}(\hat{\beta} - \beta_*)\right|\right|_2^2 + \lambda\left|\left|\hat{\beta} - \beta_*\right|\right|_1 \le 4\lambda^2\frac{|S_*|}{\phi_*}$$

5. From the `UCI` repository, use HARNESS to infer relevant predictors in the Wisconsin breast cancer data set:

   `http://archive.ics.uci.edu/ml/datasets/Breast+Cancer+Wisconsin+%28Prognostic%29`