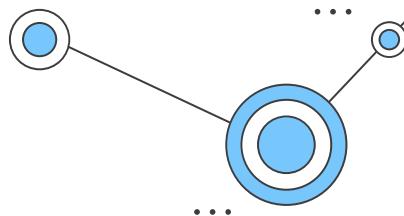


COE Price Prediction & Elasticity Analysis

Lim Kang Heng
Govtech Case Study - Section 1 (Question 2)

Bottom Line Up Front



Study Context & Objective

Quarterly quota adjustments by LTA are a key policy lever, yet the price response of COE premiums to marginal quota changes is not well understood. This study evaluates:

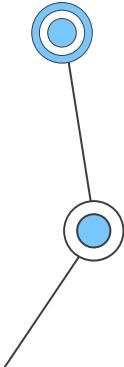
- Whether COE prices can be reliably forecasted ([data source](#)).
- How sensitive prices are to quota changes for Categories A and B.

Study Approach: SARIMA Forecasting Model Exogenous Features (SARIMAX). The process includes:

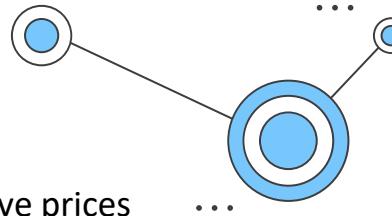
- Feature Engineering, Train-Test Split, Time-series Cross Validation & Grid Search for Hyperparameter Tuning

Key Findings:

- COE prices exhibit strong persistence, with current prices closely tracking past auction outcomes.
- The forecasting models perform well (4% MAPE), capturing medium-term price dynamics without overfitting short-term auction noise.
- Small, marginal quota increases have **limited** short-run impact on prices, once demand pressure and price dynamics are accounted for.



Model Features



Motivation: To convert auction mechanics into economically meaningful signals that drive prices

1. **Oversubscription Ratio** - Captures competitiveness of each auction

$$\text{Oversubscription Ratio} = \frac{\text{Bid Received}}{\text{Quota}}$$

2. **Quota** - Policy lever under LTA's control. Subsequently used for elasticity interpretation
3. **Lagged Prices** - Captures expectations and inertia between bidding exercises
4. **Month of Year** - Captures cyclical patterns (Apply sin/cos encoding - Dec & Jan treated as adjacent periods)

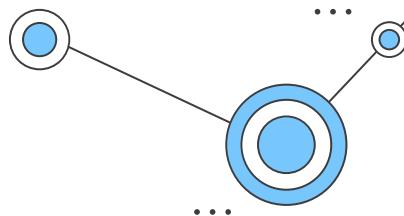


Apply log-transformations where applicable to stabilize variance and reduce effect of extreme values

Hyperparameter Tuning

Model: SARIMAX($p, d, q, P, D, Q, 12$)

Monthly Seasonality



Tuning Method:

1. Train-test Separation - Train set (2010 - 2021), test set (2022 - 2025)
2. Time-series Cross Validation within train set
3. Grid Search Process* - Selects parameters with the lowest cross-validated average RMSE

Model Efficacy:

1. Performance Measure - Performance evaluated on the held-out test set (gives realistic estimate of future performance)

Separate tuning for Cat A and Cat B - Reflects different buyer profiles & demand sensitivities



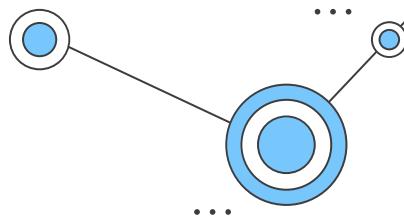
* Parameters tuned: $p, q \in \{0, 1, 2\}$, $P, D, Q, d \in \{0, 1\}$.

Category A Pricing Forecast Model & Performance

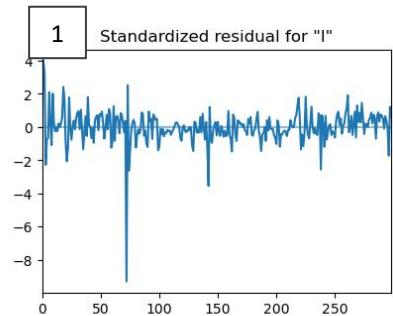
No Seasonal Factors

Optimised Model: SARIMAX(1, 0, 1, 0, 0, 0, 12)

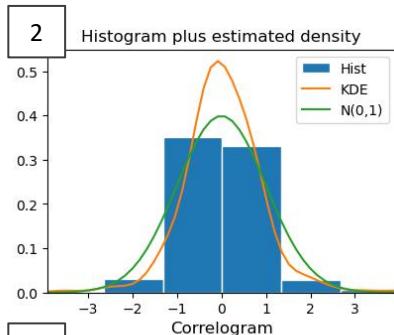
Model Diagnostic Check: Model Well-specified & Adequate for Forecasting



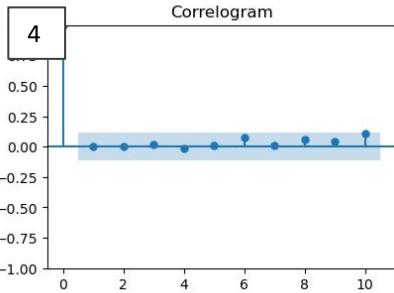
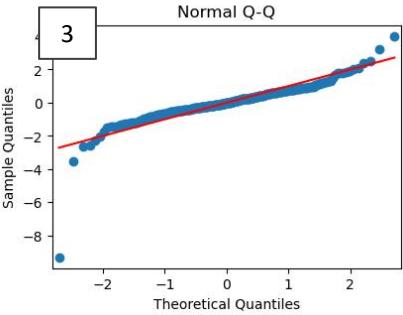
1: Random Fluctuations;
No Obvious Trending /
Persistence



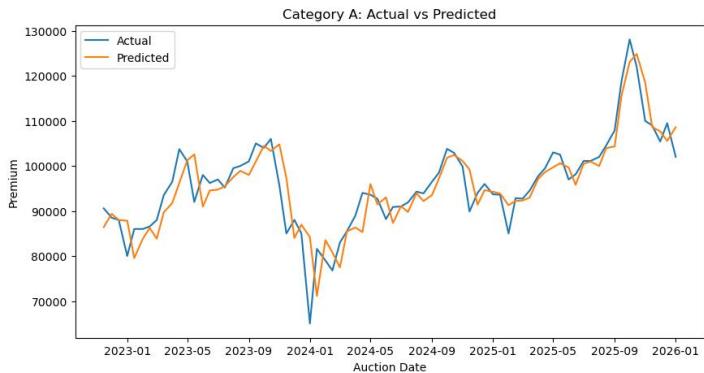
2 & 3: Errors
Approximately Normal



4: No Significant
Autocorrelations



Model Predictions Performance:



RMSE: 4,806 SGD

MAE: 3,397 SGD

MAPE: 4 % (predictions within 4% of actual prices)

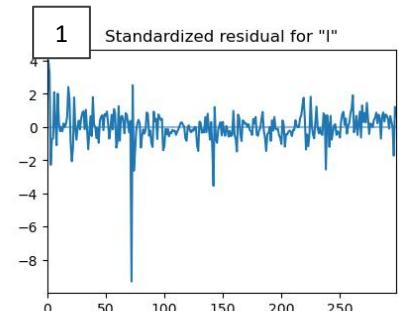
Category B Pricing Forecast Model & Performance

No Seasonal Factors

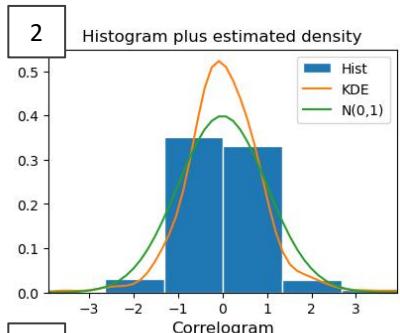
Optimised Model: SARIMAX(1, 0, 2, 0, 0, 0, 12)

Model Diagnostic Check: Model Well-specified & Adequate for Forecasting (Same Conclusion as Cat A Model)

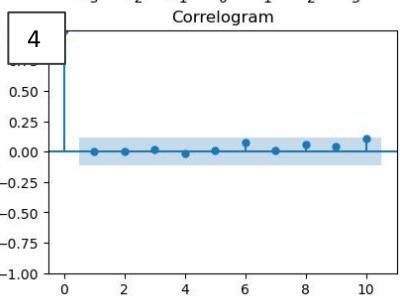
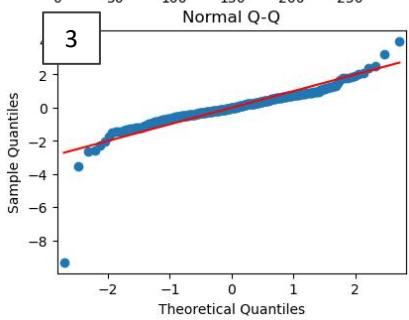
1: Random Fluctuations;
No Obvious Trending /
Persistence



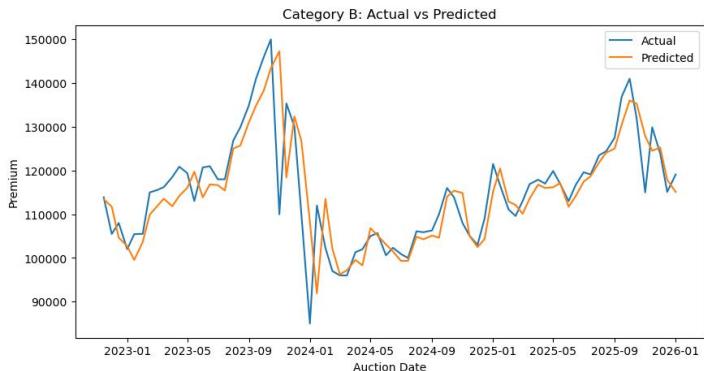
2 & 3: Errors
Approximately Normal



4: No Significant
Autocorrelations



Model Predictions Performance:



RMSE: 7,388 SGD

MAE: 4,606 SGD

MAPE: 4 % (predictions within 4% of actual prices)

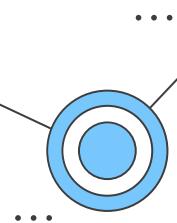
Price Elasticity of COE Quotas

COE Price ↙

Quota ↑

Oversubscription Rate ↑

$$\text{General Model Equation: } \ln(P_t) = \beta_q \ln(Q_t) + \beta_o \ln(O_t) + \dots + \text{ARMA terms} + \varepsilon_t$$



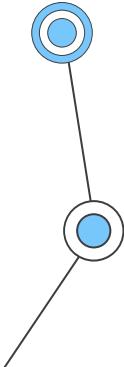
Exogenous Parameters	Weight Value (A / B)	Statistical Significance (A / B)
$\ln(Q_t)$	0.00442 / 0.00495	No / No
$\ln(O_t)$	0.201 / 0.237	Yes / Yes
$\ln(P_{t-1})$	0.995 / 0.992	Yes / Yes
$\ln(O_{t-1})$	- 0.133 / - 0.0795	Yes / Yes

→ **Interpretation:** The short-run sensitivity of COE prices to marginal quota changes is very small (approx. zero).

→ **Interpretation:** COE prices exhibit strong persistence from previous auction outcomes.

Price Elasticity of COE Quotas:

- A 1% increase in quota is associated with about only 0.004% increase in premiums (for both Cat A and Cat B)
- In other words, with 1,100 existing quotas for Cat A, adding 200 quotas (18%) would raise premiums by 0.07%, from 18,500 SGD to 18,513 SGD (i.e., tens of dollars increase)



Study Limitations, Future Considerations & Conclusion

1. Forecast Accuracy-optimised Model

The SARIMAX specification prioritises predictive accuracy, resulting in strong price persistence that limits the precision of short-run quota elasticity estimates.

2. Model-implied Elasticity

Estimated quota sensitivity reflects historical correlations captured by the model and should be interpreted as policy sensitivity rather than a causal effect.

3. External Policy & Market Shocks

Shifts in market sentiment or policy expectations can cause price spikes that are difficult to predict using historical data alone.

Concluding Remarks - COE prices are primarily driven by expectations and price persistence, with marginal quota adjustments having limited short-run effects.