Exercise 06 – November 20, 2024

1. A company claims their light bulbs last 1000 hours on average. A sample of 10 bulbs yields the following lifespans (in hours):

   `950, 960, 970, 980, 1020, 1030, 990, 1010, 1000, 995`

   Test whether the mean lifespan differs significantly from 1000 hours using α = 0.05

   Solution:
   Null Hypothesis (H_0): The mean lifespan is 1000 hours (μ = 1000)

   Alternative Hypothesis (H_1): The mean lifespan is not 1000 hours ($\mu \neq$ 1000)

   Sample Mean = $\frac{950 + 960 + 970 + 980 + 1020 + 1030 + 990 + 1010 + 1000 + 995}{10}$ = 990.5

   Standard Deviation (s): s ≈ 25.87

   Formula for t-statistic:

   $$t = \frac{990.5 - 1000}{25.87/\sqrt{10}} \approx \frac{-9.5}{8.18} \approx -1.16$$

   Compare t-statistic with Critical Value
   Degrees of freedom: n – 1 = 10 – 1 = 9.
   At α = 0.05 (two-tailed), the critical t-value is approximately ±2.262 (from the t-distribution table).

   Since t = −1.16 falls within the range [−2.262, 2.262], we fail to reject the null hypothesis.

2. A fitness coach measures the weight of 8 clients before and after a 6-week training program.

Client	Before (kg)	After (kg)	Difference (d)
1	85	82	-3
2	78	75	-3
3	90	85	-5
4	76	74	-2
5	88	85	-3
6	81	78	-3
7	79	76	-3
8	92	89	-3

   Conduct a paired t-test to determine if the training program significantly reduced weight. Use α = 0.05

   Solution:
   Null Hypothesis (H_0): The training program has no effect on weight ($\mu_d = 0$).
   Alternative Hypothesis (H_1): The training program reduces weight ($\mu_d < 0$).

Calculate Mean and Standard Deviation of Differences (d):

$$\bar{d} = \frac{\sum d}{n} = \frac{-3-3-5-2-3-3-3-3}{8} = \frac{-25}{8} = -3.125$$

$$s_d = \sqrt{\frac{\sum(d_i - \bar{d})^2}{n-1}} \approx 0.835$$

Formula for t-statistic:

$$t = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{-3.125}{0.835/\sqrt{8}} = \frac{-3.125}{0.295} \approx -10.59$$

Degrees of Freedom (df): df = n – 1 = 8 – 1 = 7
Critical t-value for $\alpha = 0.05$ (one-tailed): 1.895
Since −10.59 < 1.895, we reject H_0
Conclusion: The training program significantly reduced weight.

3. A nutritionist wants to test if a new diet plan (Group A) significantly improves weight loss compared to a standard diet plan (Group B).

The following data was collected:

Group	Sample Size (n)	Mean Weight Loss (x)	Standard Deviation (s)
Group A (New)	25	8 kg	2
Group B (Standard)	25	6 kg	2.5

Perform an independent t-test to determine if the new diet plan significantly improves weight loss at a significant level of $\alpha = 0.05$

Solution:
State Hypotheses

- Null Hypothesis (H_0): The mean weight loss for both groups is equal ($\mu_A = \mu_B$)
- Alternative Hypothesis (H_1): The new diet plan leads to greater weight loss ($\mu_A > \mu_B$)

Calculate the t-statistic:

$$t = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

Substitute the values:

$$t = \frac{8-6}{\sqrt{\frac{2^2}{25} + \frac{2.5^2}{25}}}$$

First, calculate the variances divided by sample sizes:

$$\frac{2^2}{25} = 0.16, \quad \frac{2.5^2}{25} = 0.25$$

Sum them:

$$\sqrt{0.16 + 0.25} = \sqrt{0.41} \approx 0.64$$

Now calculate t:

$$t = \frac{8 - 6}{0.64} = \frac{2}{0.64} \approx 3.13$$

Degrees of Freedom and Critical t-value

Degrees of freedom: $df = n_A + n_B - 2 = 25 + 25 - 2 = 48$

At $\alpha = 0.05$ (one-tailed), the critical t-value for df = 48 is approximately 1.679

Compare the t-statistic to the critical value t = 3.13 > 1.679

Since the calculated t-value exceeds the critical t-value, we reject the null hypothesis.

The new diet plan leads to significantly greater weight loss than the standard diet plan.

https://www.meracalculator.com/math/t-distribution-critical-value-table.php