Rolled-over Credit Cycles[*]

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February 7, 2022

Abstract

Canonical macroeconomic and financial models require credit to be equal to its fundamental component, i.e., the net present value of the net flows to creditors. According to this conventional view, credit booms are expected to precede increased flows to creditors. However, data suggests the opposite. To rationalize the novel empirical findings, we develop a model with financial frictions and heterogeneous firms, allowing firms to roll over a fraction of credits indefinitely. We show that an increase in indefinite rollover credit raises the aggregate credit and output while depressing the credit's fundamental component through firms' precautionary savings.

Keywords: Rational Bubbles, Firm Dynamics, Heterogeneous Firms, Financial Frictions
JEL Classification: E32, E44, E47

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1   Introduction

Credit booms and busts accompanied by business cycles are reoccurring global events.[1] The depth and the length of the Great Recession (re-)motivate expanding theoretical literature to uncover the mechanism for credit cycles. While previous studies differ in many dimensions, they typically share the following consensus view: Credit is a promise of *future* net outflows to creditors in exchange for net inflows *today*. Built on this paradigm, canonical models feature obtained credits that are equal to its fundamental component— the net present value (NPV) of future net flows to creditors.[2] It follows that credit booms are expected to be explained by increases in the fundamental component and precede rises in the net flows to creditors.

This paper challenges the conventional wisdom described above. We document novel evidence that credit booms are followed by drops in the net flows to creditors, albeit the interest rates barely change. In other words, credit booms coincide with declining fundamental components of credit. Motivated by the empirical observations, we revisit the question: What is the source of credit cycles? What are the theoretical mechanisms generating these puzzling empirical facts that were not present in a canonical model? We develop a quantitative model with financial frictions and heterogeneous firms to address these questions. Importantly, firms can persistently roll over a fraction of their debts. Hence, firms can obtain some of the credits *without* exchanging future net outflows to lenders. Instead, future repayments to creditors are financed through future credits. We show that financial frictions, firm heterogeneity, and the newly introduced indefinite rolled-over credits are important to account for the empirical findings.

The empirical part of the paper presents evidence that, contrary to the conventional wisdom, a credit boom is followed by *decreased* net flows to creditors. The findings are based on a Vector Autoregressive (VAR) model using aggregate data from the Financial Accounts of the United States for all corporate and non-corporate non-financial firms. The estimated VAR provides estimates for the effects of an *innovation* to the total liabilities (or total debt) on real GDP,[3] and importantly, on *future* net flows to creditors. In the baseline specification, we compute *the innovation* as the linear combination of reduced-form residuals that maximize the expected fluctuations in the total liabilities at the horizons

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[1]For example, see Schularick and Taylor (2012).

[2]This is also known as the no-Ponzi-game condition.

[3]In the data, we consider the total liabilities in the baseline. Results are robust to the use of total debt. The total liabilities are equal to the total debt in the model. Whenever it is not ambiguous, we use total liabilities and total debt interchangeably.

corresponding to the business cycle frequency, in the spirit of Uhlig (2003, 2004). A positive innovation that increases the total liabilities *today* are accompanied by an expansion in the aggregate production. Surprisingly, the net flow to creditors *drops* in the subsequent periods until it converges to its value in the absence of shocks—the **Fact 1**. Real interest rates are not affected.

A linearized aggregate debt decomposition equation, together with the impulse response functions estimated using the VAR, enable us to compute the responses of the fundamental component of the total liabilities. Following a credit boom, the constructed fundamental component *decreases* significantly both economically and statistically—the **Fact 2**.

We then build a model to provide a structural interpretation of the newly uncovered facts. The model features standard elements of a workhorse model of firm heterogeneity and financial frictions.[4] Our key innovation is to relax the conventionally imposed no-Ponzi-game condition: We allow firms to roll over a fraction of their existing debts indefinitely. Firms cannot take an infinite amount of these indefinite rollover debts. Instead, the maximum amounts of the indefinite rollover debts are exogenously determined by market sentiment. Furthermore, indefinite rollover debts are consistent with rational expectations. The remaining fraction of debts can be rolled over but not indefinitely. These components are the traditional debts investigated in the literature.

We propose a novel financial shock called rollover shocks, defined as an unexpected change (driven by market sentiment) in the size of indefinite rollover debts. Suppose a firm expects to receive x from financial intermediaries by renewing an indefinite rollover debt. Following a rollover shock of the size ϵ, the firm receives $x + \epsilon$ from the financial intermediaries. A positive (negative) ϵ implies an increase (a loss) of the debts to be indefinitely rolled over.

We show that a positive rollover shock is equivalent to an increase in firms' net worth, which has two contrasting effects. On the one hand, increasing a firm's net worth may reduce its default probability, and therefore, the borrowing cost. This channel induces firms to increase the fundamental component of credit. On the other hand, increasing net worth allows firms to borrow less (or save more) to insure against future shocks, leading to a cut in the fundamental component. For a firm with moderate capital stock and net worth, the motive to expand investment is overwhelming, and therefore, a positive rollover shock increases the fundamental component. In contrast, for a firm with abundant capital and

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[4]See Khan and Thomas (2013), Arellano et al. (2019), Ottonello and Winberry (2020).

net worth, the precautionary motive of self-insurance overrides the motive to expand investment. Thus, a positive rollover shock reduces the fundamental component for those firms. The overall effect of the proposed financial shock on the aggregate amount of fundamental components depends on the *distribution* of firms.

We calibrate the model to match firms' life cycle and financial characteristics in the United States. The model predicts that a positive rollover shock stimulates aggregate output and TFP. More importantly, it generates a credit boom followed by *declining* net flows to creditors and fundamental credit. In contrast, other shocks, such as shocks to the collateral values, productivity, and news shocks (an anticipated increase in productivity), fail to rationalize the empirical findings.

Literature Review   Our paper contributes to the extensive literature on credit cycles. Empirical works document booms and busts in credit markets, see e.g., Mian and Sufi (2009), Claessens et al. (2012), Covas and Haan (2011), and Schularick and Taylor (2012). The theoretical line of research seeks to account for the procyclicality of credit and the magnitude of credit fluctuations, by investigating the importance of financial frictions and financial shocks. Examples of such include Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Jermann and Quadrini (2012), Gorton and Ordonez (2014), Guerrieri and Lorenzoni (2017), and Perri and Quadrini (2018). Our paper is closely related to Azariadis et al. (2016), who argue that the procyclicality of credit is mostly driven by unsecured credit instead of secured credit. Their model features multiple self-fulfiling equilibria. Those previous theoretical works exclude indefinite rollover and feature credit fundamental component as the only component of credits. Hence these models imply that credit fluctuations are completely aligned with the fundamental component.[5]

Indefinite rollovers can be viewed as rational bubbles in credit markets. Therefore the paper relates to a growing literature of rational bubbles.[6] Caballero and Krishnamurthy (2006), Farhi and Tirole (2012), Martin and Ventura (2012), Hirano and Yanagawa (2017), Miao and Wang (2018), Ikeda and Phan (2019), and Asriyan et al. (2020) argue that bubbles can relax financial constraints and increase investment and output. Tang and Zhang (2021) study asset price bubbles in an environment with competitive markets and hetero-

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[5]Azariadis et al. (2016) mention the similarity between unsecured debts and bubbles since the size of unsecured debts depends on market sentiment. Section 3.3 show that indefinite rollovers in our model are essentially pyramid schemes (rational bubbles) and can also be viewed as unsecured credits. However, unlike our indefinite rollover component, unsecured debts in Azariadis et al. (2016) are backed by expected future net flows to creditors, and therefore, those debts belong to the fundamental components.

[6]Martin and Ventura (2018) provide a review for the literature.

geneous firms, where bubbles appear in a frictionless equity market instead of a frictional credit market. Our paper is closely related to the work by Martin and Ventura (2016), who develop a stylized model of a representative firm, stochastic credit bubbles,[7] and collateral constraints. However, it can be shown that in their model, the fundamental credit component is always procyclical and move side by side with the non-fundamental credit component (credit bubbles) when the economy is financially constrained.[8] Whereas in our model, because of firm heterogeneity and precautionary saving, positive shocks in the non-fundamental credit crowd out fundamental credit while generating a protracted increase in the output.

The indefinite rollover that we introduced to the credit market is related to the literature on public debt. Early works include, e.g., Barro (1984), Aschauer (1985), Seater and Mariano (1985), Hamilton and Flavin (1986), and Bohn (1998, 2007). Recently, Blanchard (2019) argues that the current economic condition ($r < g$) permits the U.S. government to persistently roll over its public debt. Cochrane (2019, 2021) conduct a variance decomposition of the value of government debt according to the conventional view—without allowing for indefinite debt rollover. Jiang et al. (2019) document the over valuation of the U.S. government debt: the size of government debt exceeds the NPV of fiscal surpluses. Moreover, Jiang et al. (2019) show that a rich, but traditional, asset pricing model cannot generate their empirical findings. Models with bubbles in the public debt, see e.g., Hellwig and Lorenzoni (2009), Domeij and Ellingsen (2018), Miao and Su (2021), Reis (2021) and Brunnermeier et al. (2022), can rationalize the public debt over-valuation puzzle. The key innovation of the current paper, apart from it being a study on firms' debts, is that we document and study the rationale behind the *co-movement* between the total debt and its fundamental component. The theoretical interpretation and the mechanisms for the *co-movement* presented in the current paper are novel to the literature on public debts. They are potentially applicable to the studies of public debts.

Our paper closely relates to a vast body of research on firm heterogeneity, particularly the studies involving financial frictions. Cooley and Quadrini (2001), Cabral and Mata (2003), Clementi and Hopenhayn (2006), Hennessy and Whited (2007) study the effects of financial frictions on firm life cycles. Recent works have studied the role of financial fric-

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[7]See the works of Blanchard and Watson (1982), and Weil (1987) for references of stochastic bubbles.

[8]Martin and Ventura (2016) do not perform the decomposition of credit and study the cyclicality of the fundamental component of credit. However, it can be easily derived that the fundamental component in their model is always proportional to capital income, regardless of whether the economy is financially constrained.

tions in the transmission of business cycles. Some examples of these works are Gilchrist et al. (2014), Crouzet (2018), Begenau and Salomao (2019), Jeenas (2019), Crouzet and Mehrotra (2020), and Salomao and Varela (2021). In particular, our paper builds upon the theoretical works by Khan and Thomas (2013), Arellano et al. (2019), Ottonello and Winberry (2020). The current paper contributes to this line of research by incorporating indefinite rollover and by illustrating how firm heterogeneity affects the propagation of rollover shocks that ultimately generate the puzzling reduced-form empirical evidence.

The remainder of the paper is organized as follows. Section 2 presents our empirical findings. Section 3 develops the model. Section 4 provides the predictions of the model. Section 5 concludes.

2 Empirical Analysis

Section 2.1 develops a simple framework that guides the interpretation of empirical findings. Section 2.2 documents the data used in the paper. Section 2.3 discusses the empirical strategy. Section 2.4 presents the empirical findings.

2.1 The Accounting Framework

The starting point is the cash flow identify at $t+1$:

$$R_{t+1}D_t = S_{t+1} + D_{t+1}, \tag{1}$$

where, D_t is the real market value of total non-financial firms' debt held by the market at the end of period t, R_{t+1} denotes the real return of holding the portfolio D_t, and S_t denotes the surplus of non-financial firms: real gross saving plus changes in the market value of total equity minus total investment. By moving D_{t+1} to the left hand side of the equation, it is straightforward to see that surpluses are equivalent to net flows to creditors ($R_{t+1}D_t - D_{t+1}$). Henceforth, we use the term surpluses and flows to creditors interchangeably. The total investment includes both the capital investment and the financial investment. Appendix B.1 provides detailed explanations about how to derive the equation (1) in a market with bonds of the different maturities.

Solving equation (1) forward obtains:

$$D_t = E_t \sum_{h=1}^{\infty} \left(\prod_{j=1}^{h} R_{t+j}^{-1} \right) S_{t+j} + \lim_{h \to \infty} E_t \left(\prod_{j=1}^{h} R_{t+j}^{-1} \right) D_{t+h}. \tag{2}$$

Note that we have merely used the cash flow identity. The above equation can be re-written as:

$$D_t = F_t + D_t^{\infty}, \tag{3}$$

where

$$F_t \equiv E_t \sum_{h=1}^{\infty} \left(\prod_{j=1}^{h} R_{t+j}^{-1} \right) S_{t+j} \tag{4}$$

$$D_t^{\infty} \equiv \lim_{h \to \infty} E_t \left(\prod_{j=1}^{h} R_{t+j}^{-1} \right) D_{t+h}. \tag{5}$$

F_t denotes the fundamental component of the debt that is backed by future surpluses. D_t^{∞} is the limiting value of the debt or the debt component that can be rolled over indefinitely.

Based on the simple model, we now re-state the objective of the empirical part of the paper. The existing literature (see e.g., Bernanke and Gertler 1989, Kiyotaki and Moore 1997, Jermann and Quadrini 2012, and Azariadis et al. 2016) typically assumes that $D_t = F_t$. According to this view, acquiring a new credit today should be accompanied by surpluses in the future. We assess this assumption in the data.

2.2 Data

We use data from the Financial Accounts of the United States, previously known as the flow of funds accounts. The Financial Accounts are parts of the U.S. system of national economic accounts. The Financial Accounts record the acquisition of assets throughout the U.S. economy, document the sources of the funds used to acquire those assets, and measure the value of total assets and liabilities. We combine flows of funds data for non-financial corporate business (Table F.103) and non-financial non-corporate business (Table F.104).

The total liabilities are the sum of the total liabilities (excluding equities) for non-financial corporate and non-financial non-corporate firms. The surplus (S_t) is the dif-

ference between the total *inflows* and the total *outflows*. The total *inflows* are the gross saving less net capital transfers paid for both corporate business(Table F.103 line 8) and non-corporate non-financial business (Table F.104 line 2) plus changes in equities (F.103 line 43 and F.104 line 32). The *outflows* consist of the total capital expenditures (Table F.103 line 8 plus Table F.104 line 4) and new acquisition of financial assets (Table F.103 line 15 plus Table F.104 line 8). The constructed S_t measure flows to creditors who are asset holders of the total liabilities of non-financial corporate and non-corporate firms in the U.S.

2.3 Empirical Strategy

We consider a VAR model consisting of the following variables: real GDP (y_t), total liabilities (d_t), surplus ($\widetilde{s}_t$), and real interest rate (r_t) constructed using Moody's Baa corporate bond yield. All nominal variables are deflated by the GDP deflator to obtain their real values. To interpret the results in percentage terms, Real GDP and the total liabilities enter the VAR in log. The surplus is transformed into the percentage deviation from its sample mean instead of taking a log because it contains negative values.

Let $Y_t \equiv [d_t, \widetilde{s}_t, y_t, r_t]'$, the reduced-form VAR model is:

$$Y_t = B(L)Y_t + U_t, \tag{6}$$

where L is the lag operator, B(L) is the matrix of lag order polynomials, and $U_t \equiv [u_t^d, u_t^s, u_t^y, u_t^r]$ contains the reduced-form residuals.

The VAR model can be rewritten in the companion form:

$$\mathbb{Y}_t = \mathbb{A}\mathbb{Y}_{t-1} + \mathbb{U}_t,$$

where

$$\mathbb{Y}_t \equiv \begin{bmatrix} Y_t \\ \vdots \\ Y_{t-p+1} \end{bmatrix}, \mathbb{A} \equiv \begin{bmatrix} B_1 & B_2 & \cdots & B_{p-1} & B_p \\ I_K & 0 & & 0 & 0 \\ 0 & I_K & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_K & 0 \end{bmatrix}, \mathbb{U}_t \equiv \begin{bmatrix} U_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

K and p denote the number of variables and the number of lags, respectively.

The VAR model can inform us, on average, what will happen in the future, given the current economic states. Formally, given the parameter estimates ($\hat{B}$s) and the data available at time t ($\mathbb{Y}_t$), the $E_t\mathbb{Y}_{t+h}$ can be computed as:

$$E_t(\mathbb{Y}_{t+h}|\mathbb{Y}_t) = \mathbb{A}^h\mathbb{Y}_t.$$

For a given choice of reduced-form residuals, the average changes in $\mathbb{Y}$ at h periods ahead ($E_t\mathbb{Y}_{t+h}$)—impulse responses functions (IRFs)—can be easily computed.

The central objective of the empirical part of the paper is to document what happens to future surpluses ($E_t s_{t+h}$) when the current level of debt (d_t) increases. In the baseline, we consider the Max-share innovations. These innovations are constructed as the linear combination of reduced-form residuals that maximizes the fluctuations in d_t at the business cycle frequency.

It is important to note that we do not intend to impose structural interpretations on these innovations in the empirical part of the paper. Instead, they serve as sources of fluctuations that help us to evaluate the co-movement and the cyclical properties of debt and future surpluses (flows to creditors).

2.4   Empirical Results

Figure 1 plots the responses to a one standard deviation increase in the Max-share innovation to the total liabilities. The shaded areas indicate the 95% confidence bands constructed using the bootstrap method. On average, a rise in debt is associated with an increase in real GDP. This pro-cyclicality of firms' debt is well-documented in the literature, see e.g., Jermann and Quadrini (2012) and Covas and Haan (2011). We replicate these findings using an alternative empirical strategy based on a VAR.

Interestingly, an innovation that drives up real debt is not followed by increases in *future* surplus. In contrast, flows to creditors drop in the short run. This fact, called Fact 1, is the novel observation that the current paper documents. Formally, in Figure 1, one should compare the responses of real debt at time t with the surplus's responses at time $t + h$. Note that the real interest rate changes insignificantly, both economically and statistically.

Robustness checks   The baseline findings are robust to: (i) the use pre-2008 recession sample (Figure 5); (ii) conditional on innovations to the total debts instead of the total
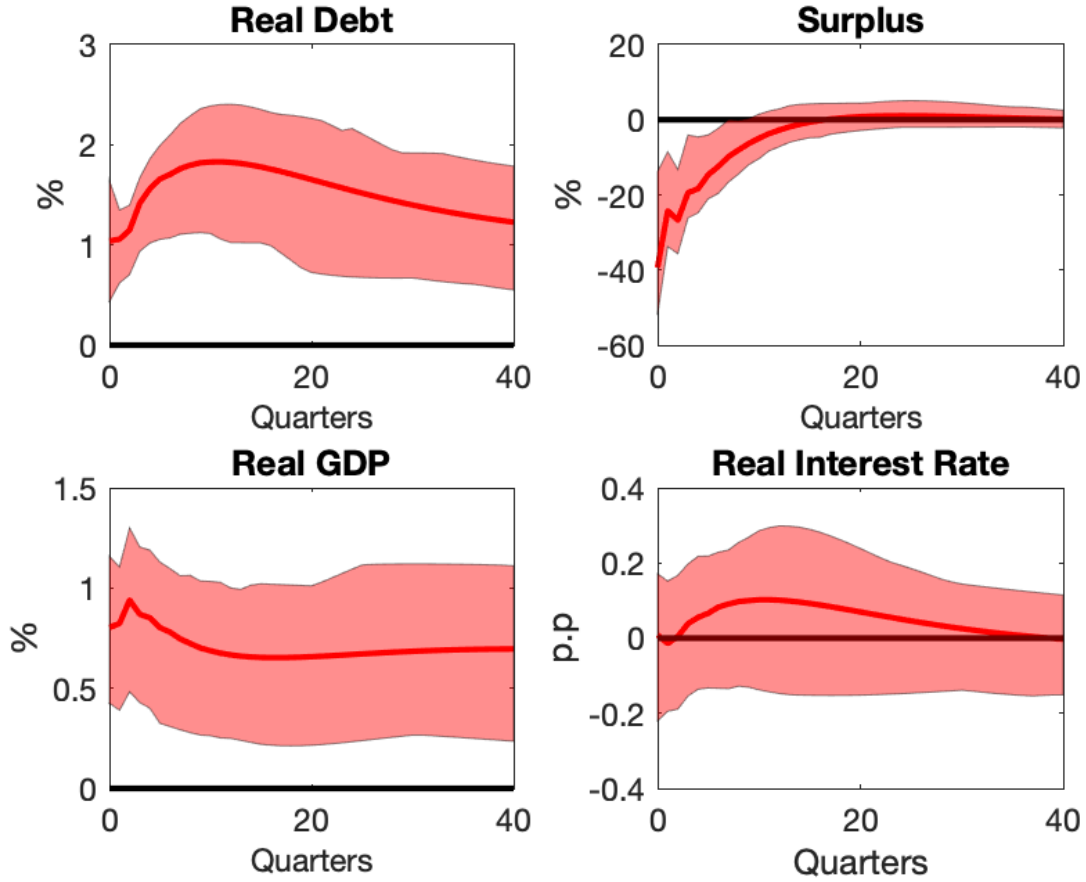
Figure 1: Responses to Total Liabilities Innovations (Max-share)

liabilities (Figure 6); and (iii) conditional on alternative innovations that are constructed using zero restrictions (Figure 7).

The Fundamental Component In the next, we compute the responses of the fundamental component of total liabilities. Linearize equation (4) to obtain:

$$\widehat{f}_t = \sum_{h=0}^{\infty} \rho^h \left[(1 - \rho)\widetilde{s}_{t+h+1} + \widehat{r}_{t+h+1} \right], \tag{7}$$

where $\rho \equiv \frac{1}{sd+1}$ and sd denotes the surplus-to-debt ratio on the balanced growth path (BGP). See Appendix B.1 for detailed derivations.

The estimated VAR provides IRFs of $\widetilde{s}_{t+h}$ and r_{t+h}. Together with equation (7), one can compute the IRFs of $\widehat{f}_t$. Figure 2 plots the results. The constructed fundamental
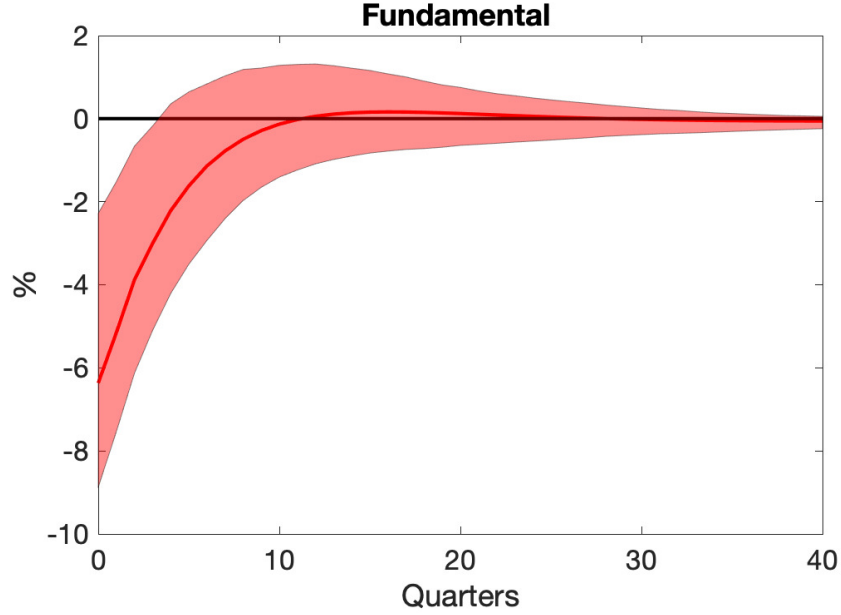
Figure 2: Responses of the NPV of Surpluses to Total Liabilities Innovations (Max-share)

component decreases significantly in response to a positive innovation to total liabilities. To compute the results displayed in Figure 2, we calibrate ρ to be equal to 0.95. Assigning alternative values to ρ does not alter the result qualitatively: see Figure 8.

In theory, the IRFs of the real interest rate matter for constructing the responses of fundamental component. However, as it can be seen in Figure 1, the real interest rate constructed using Baa corporate bond yield barely changes. This result is not due to the choice of interest rate. Figure 9 plots the responses of the fundamental component constructed based on alternative VARs that include other measures of real interest rates: the US corporate index effective yield, the BBB US corporate index effective yield, and the CCC & lower US high yield index effective yield, respectively. The right panels show that alternative real interest rates react less than 0.01 points in response to one standard deviation increase in innovation to total liabilities. The left panels confirm that the responses of the fundamental component are not affected by the use of alternative interest rates.

In the remaining paper, we argue that changes in the D_t^∞ component explain these empirical findings.

3 Model

Our model describes a dynamic economy with entry and exit of heterogeneous firms. Firms have access to a frictional credit market, where they issue defaultable state-uncontingent debt. The model is similar to the cannonical firm dynamics models with financial frictions,[9] except that we relax no-Ponzi-game condition for firm optimization problem: Firms are allowed to rollover a fraction of debts indefinitely.

3.1 Households

Time is discrete and infinite. There exists no aggregate uncertainty. The economy is populated with overlapping generations of risk-neutral households, who maximize their lifetime utility

$$U_{it} = E_t \sum_{\tau=0}^{\infty} \beta^\tau C_{it+\tau} \tag{8}$$

where $\beta \in (0,1)$ is the discount factor, and $C_{it} > 0$ denotes the consumption of goods.[10]

Every period the population is joined by a new cohort of individuals. The size of the new cohort is given by M_t, which increases over time at the rate g

$$M_{t+1} = (1+g) M_t, \tag{9}$$

Hence the entire population grows at g as well.

3.2 Firms

Firms use capital to produce a homogeneous product. The product is sold in a frictionless market. The price of the product is normalized to one. The production technology displays decreasing returns to scale

$$y_{jt} = A_t \varphi_{jt} k_{jt}^\alpha, \tag{10}$$

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[9]See Khan and Thomas (2013), Arellano et al. (2019), Ottonello and Winberry (2020).

[10]The setup is similar to a typical perpetual youth model, as developed by Blanchard (1985), who introduced a constant probability of death of individuals. In our model, individuals survive forever, yet the idea is similar: to have overlapping generations that survive indefinitely. See also Olivier (2000) and Gali (2021) for examples of perpetual youth and rational bubbles.

with $\alpha \in (0,1)$. A_t denotes a common productivity component which is identical across all firms. φ_{jt} represents idiosyncratic productivity that is independent and identically distributed (i.i.d) across different firms. k_{jt} denotes predetermined capital stock, of which δk_{jt} is depreciated in the process of production. For any existing firm j, the idiosyncratic productivity φ_{jt} follows a Markov process

$$\log \varphi_{jt+1} = \rho \log \varphi_{jt} + \varepsilon_{jt+1}, \tag{11}$$

where $\rho \in (0,1)$, $\varepsilon_{jt} \sim N\left(0,\sigma^2\right)$ $\forall t$, $\forall j$. Production incurs a fixed operation cost c^f. The operation profit of a firm j, before interest payment, is given by

$$\pi_{jt} \equiv y_{jt} - c^f. \tag{12}$$

Every period, the new-born households have the opportunity to create new firms that start to produce in the subsequent period. Each new-born individual has the access to a unit continuum of blueprints, which have the potential to become firms. We name these blueprints potential entrants. Each potential entrant draws its idiosyncratic productivity from the distribution

$$\varphi_{jt} \sim \log N\left(-m \frac{\sigma}{\sqrt{1-\rho^2}}, n \frac{\sigma^2}{1-\rho^2} \right),$$

where m and n capture the possible difference between the initial productivity draw and the idiosyncratic productivity process.[11] We label entrants as those potential entrants who choose to enter.

3.3 Creditors

Firms can issue one-period state-noncontingent debts, but cannot issue new equity. The debts are purchased by a representative, perfectly competitive financial intermediary that receives deposits from households. A firm j that promises to repay b_{jt+1} at $t+1$ receives $q_{jt}b_{jt+1}$ at t. The average debt price schedule q_{jt} is pinned down by the following break-even condition

$$q_{jt}b_{jt+1} = \frac{1}{R_{t+1}}\left[\left(p_{jt+1}^n b_{jt+1} + \left(1 - p_{jt+1}^n\right) \gamma_{t+1} k_{jt+1} \right) \right], \tag{13}$$

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[11]Potential entrants do not produce until they become firms in the subsequent period. Hence one may label the initial draw as the signal about future productivity upon entering the market.

where R_{t+1} denotes the interest rate, γ_{t+1} denotes the pledgeability of capital stock k_{jt+1}, p_{t+1}^n denotes the probability of repaying.

The dividend of a surviving firm is given by

$$d_{jt} = \pi_{jt} - b_{jt} - I_{jt} + q_{jt}b_{jt+1}. \tag{14}$$

Rearrange Equation (14)

$$b_{jt} = \pi_{jt} - d_{jt} - I_{jt} + q_{jt}b_{jt+1}.$$

Define the net flow to creditors (f_{jt}) as

$$f_{jt} \equiv \begin{cases} \pi_{jt} - d_{jt} - I_{jt}, & \text{if repay} \\ \gamma_t k_{jt}, & \text{if default} \end{cases}.$$

The break-even condition (13) can be rewritten into

$$q_{jt}b_{jt+1} = \frac{1}{R_{t+1}}E_t\left(f_{jt+1} + q_{jt+1}b_{jt+2}\right). \tag{15}$$

Solving (15) forward leads to

$$q_{jt}b_{jt+1} = \Sigma_{\tau=t}^{\infty}\left(\Pi_{i=t}^{\tau}\frac{1}{R_{i+1}}\right)E_t f_{j\tau+1} + \lim_{\tau\to\infty}\left(\Pi_{i=t}^{\tau}\frac{1}{R_{i+1}}\right)E_t q_{j\tau}b_{j\tau+1}, \tag{16}$$

which corresponds to the debt decomposition aforementioned in Section 2.1.

No-Ponzi-game Condition   In the literature of macroeconomics and finance, it is conventionally assumed that, for all t, firms are subject to

$$\lim_{\tau\to\infty}\left(\Pi_{i=t}^{\tau}\frac{1}{R_{i+1}}\right)E_t q_{j\tau}b_{j\tau+1} = 0. \tag{17}$$

Equation (17) is typically refered to as No-Ponzi-game condition. (15) and (17) imply that

$$q_{jt}b_{jt+1} = \Sigma_{\tau=t}^{\infty}\left(\Pi_{i=t}^{\tau}\frac{1}{R_{i+1}}\right)E_t f_{j\tau+1}.$$

As long as the no-Ponzi-game condition (17) holds, the income of debt issuance is equal to the net present value of future net flows to creditors.

Indefinite Debt Rollover We relax the no-Ponzi-game condition (17). Particularly, we allow b_{jt} to be composed of two types of debt

$$b_{jt} = F_{jt} + B_{jt}.$$

F_{jt} denotes secured debt from which the lenders can retrieve $\gamma_{t+1}k_{jt+1}$ if firm j default at time $t + 1$. The income from issuing F_{jt+1} is equal to

$$q_{jt}^F F_{jt+1} = \frac{1}{R_{t+1}} \left[\left(p_{jt+1}^n F_{jt+1} + \left(1 - p_{jt+1}^n \right) \gamma_{t+1}k_{jt+1} \right) \right], \tag{18}$$

where q_{jt}^F denotes the price of F_{jt+1}. B_{jt} denotes unsecured debt, from which the lenders cannot retrieve any collateral upon default. The income from the issuance is equal to

$$q_{jt}^B B_{jt+1} = \frac{1}{R_{t+1}} p_{jt+1}^n B_{jt+1}, \tag{19}$$

where q_{jt}^B denotes the price of B_{jt+1}.

Moreover, the issuance of F_{jt+1} is subject to no-Ponzi-game condition, i.e.,

$$\lim_{\tau \to \infty} \left(\Pi_{i=t}^\tau \frac{1}{R_{i+1}} \right) E_t q_{j\tau}^F F_{j\tau+1} = 0. \tag{20}$$

As opposed to F_{jt+1}, the issuance of B_{jt+1} is not subject to no-Ponzi-game condition. Instead, we impose an alternative restriction: for any incumbent firm j

$$q_{jt}^B B_{jt+1} \leq B_{jt}. \tag{21}$$

(21) implies that incumbent firm j can fully roll over its unsecured debt B_{jt}. In addition, we assume that potential entrants can issue

$$q_{jt}^B B_{jt+1} \leq s_{jt} B_0, \tag{22}$$

where $s_{jt} \in \{0,1\}$ is i.i.d. across potential entrants. $s_{jt} = 1$ with the probability p_B. $1 - p_B$ of potential entrants are subject to no-Ponzi-game condition, and can only issue the secured debt F_{jt+1}.

For clarification purposes, we denote F_{jt} and B_{jt} as two specific types of debts. Note that B_{jt} is unsecured and thus entirely backed by the future rollover. In Section 3.6, we

demonstrate that F_{jt} and B_{jt} can indeed be viewed as two components of the total debt b_{jt} in a more generic scenario. Provided that a firm can indefinitely roll over some of those debts, no matter whether these debts are secured by collateral, the firm faces an optimization problem as if it issues only two types of debts, F_{jt} and B_{jt}.

3.4 Timing

Each period t consists of multiple stages for a firm to take actions. At the beginning of a period, all shocks are realized. Firms then choose whether to default on their outstanding debts. If firms decide to default, they lose all the capital stock and exit the market immediately, before production takes place. A fraction of the capital stock, $\gamma_t k_{jt}$, is retrieved and repaid to their creditors. The rest of the capital stock is lost in the liquidation process. Those firms who decide not to default produce according to the production function (11).

After producing, firms repay their debt to the creditors, choose the dividends for their shareholders, and decide how much to invest for the future production. The investment funds come from internal resource, and external finance via one-period state-uncontingent debts. Firms are prohibited from financing through equity. In other words, they can not pay negative dividends to their shareholders. Meanwhile, entrants enter the economy and make investments. All entrants start with zero capital stock and can only borrow through the frictional debt market.

It is important to note that firms only default if paying negative dividends is inevitable. The value of default is equal to zero, and thus firms choose to default as long as the value of repaying debt is negative. However, the value of repaying debt is equal to the sum of contemporary and discounted future dividends, which are always non-negative. Therefore firms only default if producing inevitably leads to negative dividends.[12]

The optimization problems of existing firms and potential entrants are formalized in Section 3.5.

3.5 Equilibrium Analysis

Firms' Optimization Problem Define net worth x_{jt} as

$$x_{jt} \equiv \pi_{jt} + (1 - \delta) k_{jt} - F_{jt} - B_{jt}.$$

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[12]The same reasoning also implies that firms do not voluntarily exit the market (as in the model of Hopenhayn (1994) or Clementi and Palazzo (2016), for example) after repaying their debt, since the present value of future dividends is always non-negative.

Net worth is equal to the operation profit nets the debt repayment (b_{jt} that is the sum of F_{jt} and B_{jt}), plus the undepreciated capital. To minimize the notational burdens, hereafter we suppress the subscripts of variables in functions. We use λ to denote idiosyncratic state variables other than idiosyncratic productivity φ. The value function of an incumbent firm is given by[13]

$$V_t(\varphi,\lambda) = \max_{F',B',k'} \left\{ x - k' + q_t^F(\varphi,F',B',k') F' + q_t^B(\varphi,F',B',k') B' + \frac{1}{R_{t+1}} \int V_{t+1}(\varphi',\lambda') \, dJ(\varphi'|\varphi) \right\}, \quad (23)$$

$$x - k' + q_t^F(\varphi,F',B',k') F' + q_t^B(\varphi,F',B',k') B' \geq 0, \quad (24)$$

$$q_t^B(\varphi,F',B',k') B' \leq B, \quad (25)$$

where $V_t(\varphi,\lambda)$ denotes the value function of an incumbent firm, and $J(\varphi'|\varphi)$ represents the transition probability of φ. The choice of control variables is subject to (24), the non-negative-dividend constraint, and (25), the limit of B'. Note that every firm uses $\frac{1}{R}$ as the discount factor for future dividends. Firstly, every single firm generates the same expected return, otherwise the market for shares cannot clear as risk-neutral investors only purchase the shares with highest expected return. Secondly, the expected return on shares is equal to the interest rate the financial intermediary pays on deposits. On the one hand, if the expected return on shares is higher than the interest rate, there is no lending and the credit market cannot clear. On the other hand, if the expected return on shares is less than the interest rate, the market for shares cannot clear as investors have no incentive to hold shares.[14]

Similarly, the value function of a firm entry is equal to

$$V_t^e(\varphi,s) = \max_{F',B',k'} \left\{ -k' + q_t^F(\varphi,F',B',k') F' + q_t^B(\varphi,F',B',k') B' + \frac{1}{R_{t+1}} \int V_{t+1}(\varphi',\lambda') \, dJ(\varphi'|\varphi) \right\}, \quad (26)$$

$$-k' + q_t^F(\varphi,F',B',k') F' + q_t^B(\varphi,F',B',k') B' \geq 0, \quad (27)$$

$$q_t^B(\varphi,F',B',k') B' \leq sB_0. \quad (28)$$

where $V_t^e(\varphi,s)$ denotes the value function of an entrant. $s \in \{0,1\}$ denotes the state determining whether a firm receives indefinite rollover credit: $s = 1$ implies that the firm can issue an indefinite rollover debt B_0, otherwise ($s = 0$) the firm's total debt is subject to the

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[13]Throughout this paper, we assume that there is no equity price bubble. See Tang and Zhang (2021) for a model of equity price bubbles and firm dynamics.

[14]Throughout this paper, we assume that there exists unique solution to policy functions and value functions. Following the typical procedure in the literature, we verify that our iterative numerical solution leads to the same results, given different initial guesses. We also assume that the probability of repayment is greater than zero over the state space, which can also be verified in the numerical exercise.

no-Ponzi-game condition. A potential entrant enters the market if and only if

$$V_t^e(\varphi, s) \geq 0.$$

Proposition 1. $\forall t$, *it is optimal for an incumbent firm j to choose*

$$q_{jt}^B B_{jt+1} = B_{jt}, \tag{29}$$

and for an entrant j to choose

$$q_{jt}^B B_{jt+1} = s_{jt} B_0. \tag{30}$$

Proof. See Appendix C.1. $\qquad\square$

The intuition for Proposition 1 is the following. Firms can persistently roll over a fraction of their debt, B, without using their own resources (f in Equation 16) to repay B. Therefore, firms always maximize borrowing this type of debt.

Equation (25) implies that incumbents can only roll over the existing B but can not issue extra amount of B. Without constraints (25) and (28), firms would borrow unlimited amount of B. The existing macroeconomic models typically impose the no-Ponzi-game condition so that the optimization problem is well defined. We impose (25) and (28) as an alternative for the no-Ponzi-game condition.

The debt decomposition equation (16) can be rewritten into

$$q_{jt}^F F_{jt+1} + q_{jt}^B B_{jt+1} = \Sigma_{\tau=t}^\infty \left(\Pi_{i=t}^\tau \frac{1}{R_{i+1}} \right) E_t f_{j\tau+1} + \lim_{\tau\to\infty} \left(\Pi_{i=t}^\tau \frac{1}{R_{i+1}} \right) E_t \left(q_{j\tau}^F F_{j\tau+1} + q_{j\tau}^B B_{j\tau+1} \right). \tag{31}$$

According to Equation (19) and (29)

$$q_{jt}^B B_{jt+1} = \frac{1}{R_{t+1}} E_t B_{jt+1} = \frac{1}{R_{t+1}} E_t q_{jt+1}^B B_{jt+2}. \tag{32}$$

Solving (32) forward

$$q_{jt}^B B_{jt+1} = \lim_{\tau\to\infty} \left(\Pi_{i=t}^\tau \frac{1}{R_{i+1}} \right) E_t q_{j\tau}^B B_{j\tau+1}. \tag{33}$$

Combining (31) and (33), we get

$$q_{jt}^F F_{jt+1} = \Sigma_{\tau=t}^\infty \left(\Pi_{i=t}^\tau \frac{1}{R_{i+1}} \right) E_t f_{j\tau+1}. \tag{34}$$

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Equation (34) implies that $q_{jt}^F F_{jt+1}$ is equal to the net present value of future net flow to creditors, which we label as fundamental credit, or the fundamental component. Equation (33) demonstrates that $q_{jt}^B B_{jt+1}$ is not backed by future flows to creditors, but instead by future credits. We label $q_{jt}^B B_{jt+1}$ as the indefinite rollover component.

Reformulate Firms' Optimization Problem   Define adjusted net worth

$$\tilde{x}_{jt} \equiv x_{jt} + q_{jt}^B B_{jt+1} = \pi_{jt} + (1-\delta) k_{jt} - F_{jt}. \tag{35}$$

Proposition 1 enables us to reformulate the optimization problems. Firstly, the idiosyncratic states can be fully summarized by $\varphi, \tilde{x}$. Therefore we can replace λ with $\tilde{x}$. Secondly, since B' can always be rolled over, it does not affect the expected dividend upon production and the probability of default. Thus we can omit B' in the price schedule function. Henceforth we redefine the price schedule function by $q_t^F(\varphi, F', k')$, removing B' from the original debt price schedule function $q_t^F(\varphi, F', B', k')$ in (23)-(24).

In a nutshell, (23)-(25) can be rewritten into

$$V_t(\varphi, \tilde{x}) = \max_{F', k'} \left\{ \tilde{x} - k' + q_t^F(\varphi, F', k') F' + \frac{1}{R_{t+1}} \int V_{t+1}(\varphi', \tilde{x}') dJ(\varphi'|\varphi) \right\}, \tag{36}$$
$$\tilde{x} - k' + q_t^F(\varphi, F', k') F' \geq 0. \tag{37}$$

(26)-(28) can be rewritten into

$$V_t^e(\varphi, s) = V_t(\varphi, sB_0). \tag{38}$$

(36)-(37) describe a typical financially-constrained optimization problem in the literature.[15] We follow a standard numerical method to solve for the value functions and policy functions. The details are presented in Appendix C.2.

Dynamic Equilibrium   When consumers defined in Section 3.1 maximize their lifetime utility, it is immediate that for every t, the equilibrium interest rate R_t is equal to $\frac{1}{\beta}$: if $R_t > \frac{1}{\beta}$, the market of consumption goods cannot clear as no one consumes; if $R_t < \frac{1}{\beta}$, the credit and equity market cannot clear as no one invests.

A dynamic equilibrium should include value functions, policy functions, prices, and the measure of firms, such that (i) firms and households optimize, (ii) the law of motion

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[15] For example, see Ottonello and Winberry (2020)

of firms' distribution is consistent with firms' decision making, and (iii) all markets clear. We define the equilibrium of our model in Appendix C.3.

3.6 Discussions

Debt Decomposition So far we have studied a scenario where indefinite debt rollovers arise in the form of unsecured debts. We now demonstrate that F and B can be viewed as two components of firms' total credit b, which consists of various types of secured or unsecured debts. Without loss of generality, suppose a firm issues two debts, Debt X and Debt Y, which are priced by

$$q_{jt}^X b_{jt+1}^X = \frac{1}{R_{t+1}} \left[\left(p_{jt+1}^n b_{jt+1}^X + \left(1 - p_{jt+1}^n\right) \gamma_{t+1}^X k_{jt+1} \right) \right], \tag{39}$$

$$q_{jt}^Y b_{jt+1}^Y = \frac{1}{R_{t+1}} \left[\left(p_{jt+1}^n b_{jt+1}^Y + \left(1 - p_{jt+1}^n\right) \gamma_{t+1}^Y k_{jt+1} \right) \right]. \tag{40}$$

b^X and b^Y denote the face value, and q_{jt}^X and q_{jt}^Y denote the price, of Debt X and Y respectively. If firm j defaults at $t+1$, the creditors of Debt X retrieve collateral $\gamma_{t+1}^X k_{jt+1}$, and the creditors of Debt Y retrieve collateral $\gamma_{t+1}^Y k_{jt+1}$. $\gamma_t^X + \gamma_t^Y = \gamma_t$ for all t.

In addition, we assume that Debt X is subject to no-Ponzi-game condition, whereas the existing amount of Debt Y can be persistently rolled over such that

$$q_{jt}^Y b_{jt+1}^Y \leq b_{jt+1}^Y \tag{41}$$

Similar to Proposition 1, it is always optimal for firms to fully rollover the Debt Y, i.e., $q_{jt}^Y b_{jt+1}^Y = b_{jt+1}^Y$, instead of repaying the debt with firms' surplus.[16] Therefore we can rewrite (40) into

$$q_{jt}^Y b_{jt+1}^Y = \frac{1}{R_{t+1}} E_t \left(f_{jt+1}^Y + q_{jt+1}^Y b_{jt+2}^Y \right), \tag{42}$$

where

$$f_{jt}^Y = \begin{cases} 0, & \text{if repay} \\ \gamma_t^Y k_{jt}. & \text{if default} \end{cases}.$$

---

[16] The same reasoning in Appendix C.1 applies to any debt that can be indefinitely rolled over, no matter whether the debt is secured by collateral.

(15)-(42), we obtain

$$q_{jt}^X b_{jt+1}^X = \frac{1}{R_{t+1}} E_t \left(f_{jt+1}^X + q_{jt+1}^X b_{jt+2}^X \right),$$
(43)

where

$$f_{jt}^X = \begin{cases} \pi_{jt} - d_{jt} - I_{jt}, & \text{if repay} \\ \gamma_t^X k_{jt}, & \text{if default} \end{cases}.$$

f_{jt}^X and f_{jt}^Y can be viewed as the net payment from the borrower to the creditors of Debt X and Y respectively.

The total income from issuing debts, $q_{jt}^X b_{jt+1}^X + q_{jt}^Y b_{jt+1}^Y$, can be decomposed into two components, $\tilde{F}_{jt}$ and $\tilde{Y}_{jt}$

$$\tilde{F}_{jt} \equiv q_{jt}^X b_{jt+1}^X + \Sigma_{\tau=t}^\infty \left(\Pi_{i=t}^\tau \frac{1}{R_{i+1}} \right) E_t f_{j\tau+1}^Y = \Sigma_{\tau=t}^\infty \left(\Pi_{i=t}^\tau \frac{1}{R_{i+1}} \right) E_t f_{j\tau+1}.$$
(44)

$$\tilde{B}_{jt} \equiv q_{jt}^Y b_{jt+1}^Y - \Sigma_{\tau=t}^\infty \left(\Pi_{i=t}^\tau \frac{1}{R_{i+1}} \right) E_t f_{j\tau+1}^Y = \lim_{\tau\to\infty} \left(\Pi_{i=t}^\tau \frac{1}{R_{i+1}} \right) E_t q_{j\tau}^Y b_{j\tau+1}^Y.$$
(45)

(44) is derived by imposing the no-Ponzi-game condition $\lim_{\tau\to\infty} \left(\Pi_{i=t}^\tau \frac{1}{R_{i+1}} \right) E_t q_{j\tau}^X b_{j\tau+1}^X = 0$. It is immediate that $\tilde{F}_{jt} = q_{jt}^F F_{jt+1}$ and $\tilde{B}_{jt} = q_{jt}^B B_{jt+1}$ for all t. Furthermore,

$$\tilde{B}_{jt} = \frac{1}{R_{t+1}} E_t \lim_{\tau\to\infty} \left(\Pi_{i=t+1}^\tau \frac{1}{R_{i+1}} \right) E_t q_{j\tau}^B b_{j\tau+1}^B. = \frac{1}{R_{t+1}} E_t \tilde{B}_{jt+1}.$$
(46)

The decomposition above pertains to firms that issue any $n \in N+$ types of debts, regardless of whether the debts are secured or not. Provided that a firm can indefinitely roll over a fraction of its debts, it solves the optimization problem as if it issues only the two types of debts, F and B, which are defined in Section 3.3. We can thus always decompose the total credit into a fundamental component, which is equal to the net present value of the net flows to creditors, and an indefinite rollover component, which is backed by the expectation of future rollover.

The Sustainability of Bubbles As firms can permanently roll over a fraction of their debts, there exists no upper bound for the firm-level credit/output ratio. However, in an equilibrium, the aggregate credit/GDP ratio should be limited, otherwise the resources in the economy are insufficient to sustain the credit market. In the remaining of this section, we demonstrate that as long as $\beta(1+g) > 1$, indefinite rollovers are sustainable on a

balanced growth path.

The aggregate indefinite rollover B_A can be decomposed into

$$B_A = B_I + B_N,$$

where B_I represents the aggregate indefinite rollover of incumbent firms, B_N represents the aggregate indefinite rollover of entrants. According to Equation (19) and (29), B_I follows

$$
\begin{aligned}
B_I' &= \int \left[\left(q_*^B(\varphi,\tilde{x}) \right)^{-1} B \right] \cdot [p_*^n(\varphi,\tilde{x})] \, d\eta(\varphi,\tilde{x},B) \\
&= \int \left[\left(\frac{1}{R} \cdot p_*^n(\varphi,\tilde{x}) \right)^{-1} B \right] \cdot [p_*^n(\varphi,\tilde{x})] \, d\eta(\varphi,\tilde{x},B) \\
&= R \int B \cdot d\eta(\varphi,B,k') = R B_A,
\end{aligned}
\tag{47}
$$

where $\eta(\varphi,\tilde{x},B)$ denotes the measure of the incumbents and entrants,[17] $q_*^B(\varphi,\tilde{x})$ and $p_*^n(\varphi,\tilde{x})$ respectively denote the price of indefinite rollover component and the probability of repayment along a BGP, when firms choose the optimal k' and F' given state variables φ and $\tilde{x}$. The law of motion for the B_A is described by

$$B_A' = R B_A + p_B B_0 N'. \tag{48}$$

Here, $B_N = p_B B_0 N$, where N denotes the number of entrants. The ratio $\frac{B_A'}{Y'}$ follows

$$
\frac{B_A'}{Y'} = R \frac{B_A}{Y} \frac{Y}{Y'} + \frac{p_B B_0 N'}{Y'} = \frac{R}{1+g} \frac{B_A}{Y} + \frac{p_B B_0 N_0}{Y_0},
\tag{49}
$$

where $\frac{N_0}{Y_0}$ denotes the number of entrants normalized by the aggregate output along a BGP. Equation (49) uses the fact that along a BGP, the number of entrants, the aggregate output Y, and the aggregate indefinite rollover B_A all increase at the rate g. Insofar as $\beta(1+g) > 1$, Equation (49) implies that along a BGP, $\frac{B_A'}{Y'}$ stays at the fixed point $\left(1 - \frac{R}{1+g}\right)^{-1} \frac{p_B B_0 N_0}{Y_0}$.

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[17] Here entrants can be viewed as incumbents with $\tilde{x} = s B_0$ and $B = s B_0$, where $s = 1$ with prob p_B, and $s = 0$ with prob $1 - p_B$.

Parameter	Description	Value
α	Decreasing returns to scale	0.65
δ	Depreciation rate	0.10
β	Discount factor	0.98
g	Growth rate	3.02%
A	Common TFP component	1
ρ	Idiosy. shock persistence	0.70
σ	Idiosy. shock volatility	0.38

Table 1: Fixed Parameters

4 Results

4.1 Calibration

We calibrate the model along a BGP, in annual frequency. Table 1 lists the fixed parameter values. We choose the decreasing returns to scale $\alpha = 0.65$, and the capital depreciation rate $\delta = 0.1$. We set the discount factor $\beta = 0.98$, implying that $R = 1.02$. The growth rate is set to $g = 3.02\%$, which corresponds to the average growth rate of real GDP in the U.S. over the sample. Without loss of generality, we set common TFP component $A = 1$. Finally, regarding idiosyncratic productivity shocks, we choose $\rho = 0.7$ and $\sigma = 0.38$ to match the estimates by İmrohoroğlu and Tüzel (2014):.

Table 2 presents our estimates of the remaining parameters. We target three types of moments. Firstly, we target moments regarding business dynamism. We use the data from the Business Dynamics Statistics (BDS) to compute the share of new establishments, the exit rate of new establishments, and the share of establishments of age 4. Secondly, we target the investment behavior of firms: the mean investment rate as reported by Crouzet and Mehrotra (2020), who use the extensive micro data underlying the Quarterly Financial Reports. Finally, we target firms' financial characteristics. We compute the mean spread, which is measured by the yield on BAA rated corporate bonds relative to a ten-year Treasury bond. Additionally, we calculate the aggregate leverage ratio, using the data . We also take the mean leverage ratio of firms, as reported by Crouzet and Mehrotra (2020). Panel A lists the estimated parameter values that minimize the difference between the targets and their model-implied counterparts. Panel B reports the target values and model fitness.

Overall, our model fits most of the selected moments well. The model tends to over-predict the mean investment rate and under-predict the mean leverage ratio. One poten-

Panel A: Calibrated Parameters

Parameter	Description	Value
m	Mean shift of entrants' prod.	0.086
n	Std shift of entrants' prod.	0.017
b_0	Initial bubble component	234.3
c_f	Fixed cost of production	2.766
γ	Pledgeability of capital stock	0.914
p_b	Fraction of potential entrants with IR	0.033

Panel B: Targets and Model Fit

Moment	Data	Model
Share of new establishments	0.087	0.061
Share of four-year-old establishments	0.045	0.030
Exit rate of new establishments	0.244	0.198
Mean investment rate	0.263	0.554
Average spread	0.024	0.013
Mean leverage ratio	0.344	0.339
Aggregate leverage ratio	0.436	0.447

Table 2: Calibration and Model Fit

tial explanation is the lack of adjustment cost in our model. Hence in the model young firms can quickly accumulate capital stock and lower the borrowing cost, leading to a high mean investment rate and a low mean leverage ratio relative to the data.

4.2 Impulse Responses

The structural model can be used as a lab to assess effects of various shocks. We compute the transition dynamics following unexpected shocks. The shocks are realized at $t = 0$, prior to which the economy is along its BGP. We assume the economy follows a perfect foresight equilibrium path from $t = 1$ and returns to BGP at $t = 100$.

Rollover Shocks   Consider a shock to firms' indefinite rollover components. According to Equation (29), in the absence of aggregate shocks, incumbent firm j is supposed to receive $q_{jt}^B B_{jt+1} = B_{jt}$ to renew the existing B_{jt} at t. We assume that at $t = 0$, firms receive a shock at their borrowing constraint (21)

$$q_{jt}^B B_{jt+1} \leq (1 + \epsilon^B) B_{jt}, \tag{50}$$

where ϵ^B represents a surprise increase in the indefinite rollover component: optimizing firms set $q_{jt}^B B_{jt+1} = (1 + \epsilon^B) B_{jt}$.[18] Along the perfect foresight equilibrium path afterwards, incumbent firms can only rollover the existing indefinite rollover component, i.e., $q_{jt}^B B_{jt+1} = B_{jt}$.

Such a shock is indeed equivalent to an unexpected increase in firms' adjusted net worth. For incumbent firms

$$\tilde{x}_{jt} \equiv x_{jt} + q_{jt}^B B_{jt+1} = \pi_{jt} + (1 - \delta) k_{jt} - F_{jt} + q_{jt}^B B_{jt+1} - B_{jt},$$

where k_{jt}, F_{jt}, and B_{jt} are predetermined, and π_{jt} is determined by φ_{jt} and k_{jt}. In the absence of rollover shocks, $q_{jt}^b B_{jt+1} = B_{jt}$. As to the incumbents with $B_{jt} > 0$, the rollover shock at $t = 0$ induces $q_{jt}^b B_{jt+1} - B_{jt} = \epsilon^B B_{jt} > 0$, which implies an unexpected increase by $\epsilon^B B_{jt}$ in $\tilde{x}_{jt}$.

Figure 3 plots the impulse responses to the rollover shock. The shock relaxes financial constraints for firms and thus stimulates aggregate output, TFP, and credit. Relative to the output increase, the credit expansion is disproportionately large. As opposed to conventional wisdom, the credit boom coincides with a persistent drop in the net flow to creditors (surplus) and fundamental credit. Despite that the fundamental credit declines following the rollover shock, the total credit increases as a consequence of the rising indefinite rollover.

Additionally, we plot the NPV of the aggregate surpluses, as what we do in Section 2.4. Note that the NPV of the aggregate surpluses is not necessarily equal to the aggregate fundamental credit, unless there exist no firm entry and exit. The NPV of the aggregate surpluses can thus be viewed as the fundamental component of the economy if we view the economy as a representative firm. Figure 3 shows that, albeit the magnitudes are different, the impulse responses of the NPV are qualitatively analogous to the impulse responses of the aggregate fundamental credit.

As aforementioned, the rollover shock is equivalent to an increase in adjusted net worth. Figure 4 plots the policy functions of firms along the BGP, as functions of adjusted net worth $\tilde{x}$. Capital investment k' is increasing in $\tilde{x}$, whereas fundamental credit $q_{jt}^F F_{jt+1}$ is non-monotone in $\tilde{x}$. As to firms with low $\tilde{x}$, collateral assets are scarce and the increase in $\tilde{x}$ helps boost fundamental credit to further increase investment. On the contrary, firms with high $\tilde{x}$ are likely have accumulated abundant capital stock. These firms are prone
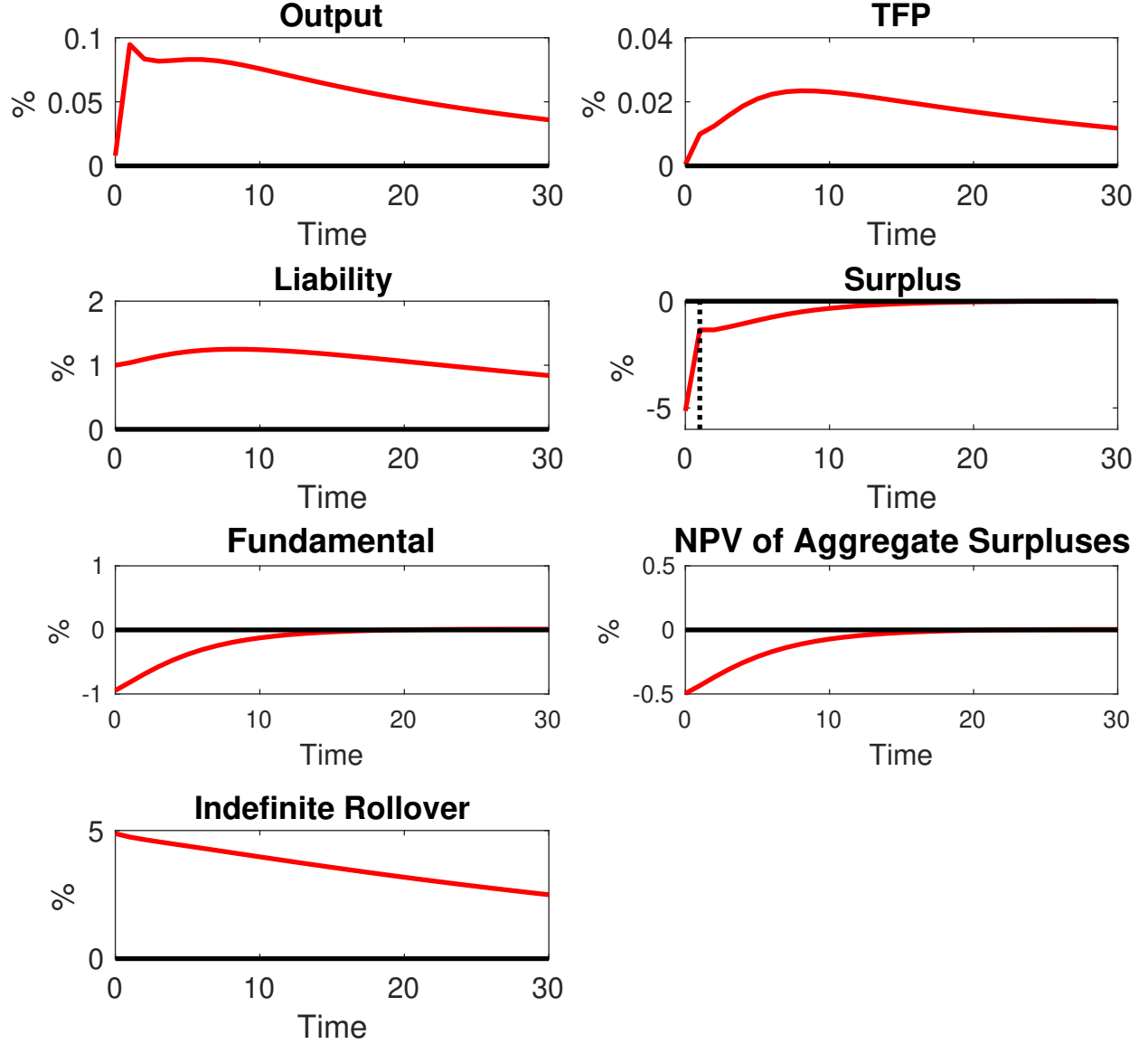
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[18] See Appendix C.1.

Figure 3: IRFs to Indefinite Rollover Shocks

to insure against idiosyncratic risk through saving the newly gained adjusted net worth to lower fundamental credit. Hence the shock has heterogeneous effects over firms given their level of adjusted net worth. The overall effects of the rollover shock depend on the distribution of φ, $\tilde{x}$, and B across firms.

Rollover shocks belong to the broader class of financial shocks that change firms' financing conditions and borrowing capacity. We compare the rollover shock with a typical financial shock: a shock to pledgeability γ. We assume γ increases unexpectedly by ϵ at $t = 0$, and the increment decays at the rate $\rho_\gamma = 0.5$ per period. At $t = 100$, γ reverts to the
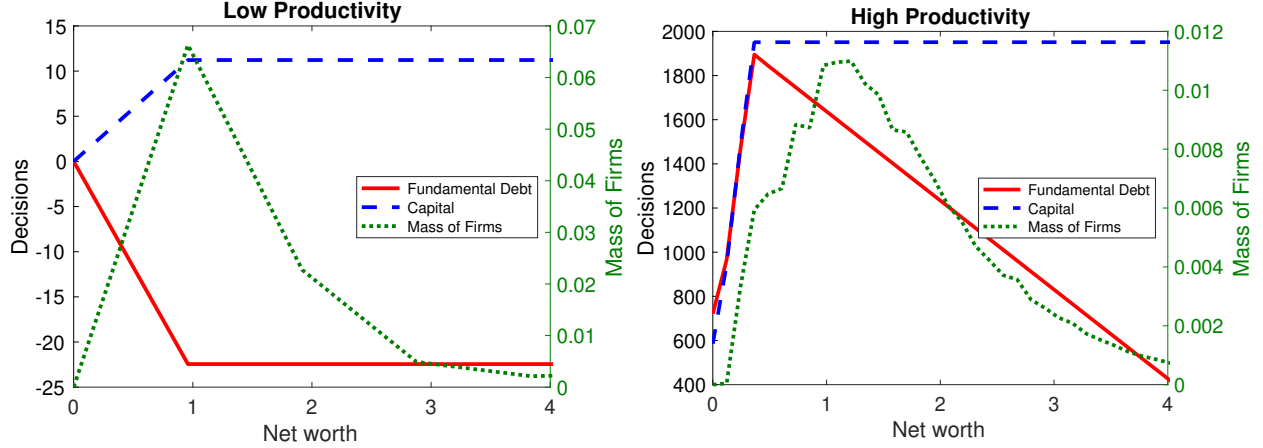
Figure 4: Policy Functions

BGP level.

Figure 10 plots the impulse responses to the pledgeability shock. The pledgeability shock boosts the aggregate output, TFP, and credit. However, unlike the rollover shock, the pledgeability shock also raises the net flows to creditors and the fundamental credit. The increase in the total credit is entirely driven by the increase in fundamental credit. These results are robust to alternative values of ρ_γ. In our model, the implications of the pledgeability shock mimic those of a pledgeability shock in a canonical model of financial frictions.

Productivity Shocks We now turn to the effects of real shocks. Firstly consider a shock to the common TFP component A_t. At $t = 0$, A_t increases by ϵ_A. The innovation falls at the annual rate $\rho_A = 0.5$ from $t = 1$ to $t = 99$ and reverts to zero at $t = 100$. The impulse responses of the productivity shock are displayed in Figure 11. The productivity shock immediately stimulates the aggregate output, TFP, and credit, as well as fundamental credit. On impact, the shock dampens the net flows to creditors, in exchange for a sequence of positive net flows in the future. This corresponds to the increase of the fundamental credit. Note that there is barely any fluctuation in the non-fundamental component.

Next we study news shocks, i.e., anticipated changes in future A_t. We assume $A_t = 1$ at $t = 0$, before it rises to $A_t = 1 + \epsilon_A$ at $t = 1$. From $t = 1$ to $t = 99$ the rise in A_t decays at $\rho_A = 0.5$ per year before it fully perishes at $t = 100$. Albeit the productivity rise occurs at $t = 1$, it is expected at $t = 0$. The lines in Figure 12 depicts the impulse responses. The effects of the news shock are analogous to the productivity shock with a

delay. The increase in aggregate credit is likewise completely driven by the increase of the fundamental credit.

4.3 Interpreting the Reduced-form Empirical Evidence

In this section, we use the structural model as a laboratory. We apply the empirical strategies adopted in the empirical section using the data simulated from our structural model to validate the empirical strategies and provide plausible structural interpretations to the resulting reduced-form estimates.

Following Boppart et al. (2018), we use impulse responses as numerical derivatives to simulate data using the model. The simulated data are generated separately based on rollover, productivity, and pledgeability shocks. Small measurement errors are added to simulated variables to avoid the issue of dynamic singularity. Each simulated data set consists of the same variables of the same sample size as in the empirical study.

The red lines in Figure 13 plot the IRFs to a positive debt innovation, estimated by applying our empirical strategy to 1000 simulated data based on rollover shocks. The solid red lines are the median IRFs out of the 1000 simulation, and the dotted red lines indicate the 90% probability intervals. For comparison, the dashed blue lines plot the effects of a positive rollover shock in the model, called the true IRFs. Interestingly, the IRFs plotted in Figure 13 are similar to these obtained in the data, see Figure 13.

Figure 14 plots the simulation results based on productivity shocks. As discussed intensively above, consistent with the conventional wisdom, those traditional shocks fail to reproduce the empirical findings.

Overall, our analysis suggests that rollover shocks are responsible for puzzling empirical observations.

5 Conclusion

This paper assesses the conventional wisdom that a firm's credit equals its fundamental credit, i.e., the net present value of future net flows to creditors. Data reveals that a credit boom is followed by a drop in net flows to creditors, implying that fundamental credits have limited contribution to the aggregate credit fluctuations.

Such finding is at odds with canonical models that exclude indefinite rollover and require credits equal to the fundamental component. We develop an otherwise standard

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model of firm heterogeneity and financial frictions by incorporating indefinite rollover. We show that shocks to indefinite rollover lead to procyclical aggregate credit and counter-cyclical fundamental credit. We demonstrate that the theoretical results are driven by firms' precautionary saving motive arising from financial frictions.

References

Arellano, C., Y. Bai, and P. J. Kehoe (2019). Financial Frictions and Fluctuations in Volatility. *Journal of Political Economy 127*(5), 2049–2103.

Aschauer, D. A. (1985). Fiscal policy and aggregate demand. *The American Economic Review 75*(1), 117–127.

Asriyan, V., L. Fornaro, A. MartÃn, and J. Ventura (2020). Monetary Policy for a Bubbly World. *Review of Economic Studies forthcoming*.

Azariadis, C., L. Kaas, and Y. Wen (2016). Self-Fulfilling Credit Cycles. *The Review of Economic Studies 83*(4), 1364â1405.

Barro, R. J. (1984). The behavior of us deficits. Technical report, National Bureau of Economic Research.

Begenau, J. and J. Salomao (2019). Firm financing over the business cycle. *The Review of Financial Studies 32*(4), 1235â1274.

Bernanke, B. and M. Gertler (1989). Agency costs, net worth, and business fluctuations. *The American Economic Review 79*(1), 14–31.

Blanchard, O. (2019). Public debt and low interest rates. *American Economic Review 109*(4), 1197–1229.

Blanchard, O. J. and M. W. Watson (1982). Bubbles, Rational Expectations and Financial Markets .

Bohn, H. (1998). The behavior of us public debt and deficits. *the Quarterly Journal of economics 113*(3), 949–963.

Bohn, H. (2007). Are stationarity and cointegration restrictions really necessary for the intertemporal budget constraint? *Journal of Monetary Economics 54*(7), 1837–1847.

Boppart, T., K. Mitman, and P. Krusell (2018). Exploiting mit shocks in heterogeneous-agent economies: The impulse response as a numerical derivative. *Journal of Economic Dynamics and Control 89*, 68–92.

Brunnermeier, M. K., S. A. Merkel, and Y. Sannikov (2022). Debt as safe asset. Working Paper 29626, National Bureau of Economic Research.

Caballero, R. J. and A. Krishnamurthy (2006). Bubbles and Capital Flow Volatility: Causes and Risk Management. *Journal of Monetary Economics 53*(1), 35–53.

Cabral, L. M. B. and J. Mata (2003). On the evolution of the firm size distribution: Facts and theory. *American Economic Review 93*(4), 1075–1090.

Claessens, S., A. Kose, and M. E. Terrones. (2012). How Do Business and Financial Cycles Interact? *Journal of International Economics 87*(1), 178–190.

Clementi, G. L. and H. A. Hopenhayn (2006). A theory of financing constraints and firm dynamics. *The Quarterly Journal of Economics 121*(1), 229â265.

Cochrane, J. H. (2019). The value of government debt. Technical report, National Bureau of Economic Research.

Cochrane, J. H. (2021). The Fiscal Roots of Inflation. *Review of Economic Dynamics*.

Cooley, T. F. and V. Quadrini (2001). Financial markets and firm dynamics. *American Economic Review 91*(5), 1286–1310.

Covas, F. and W. J. D. Haan (2011). The Cyclical Behavior of Debt and Equity Finance. *American Economic Review 101*(1), 877–899.

Crouzet, N. (2018). Aggregate Implications of Corporate Debt Choices. *Review of Economic Studies 85*(3), 1635–1682.

Crouzet, N. and N. Mehrotra (2020). Small and Large Firms over the Business Cycle. *American Economic Review 110*(11), 3549–3601.

Domeij, D. and T. Ellingsen (2018). Rational bubbles and public debt policy: A quantitative analysis. *Journal of Monetary Economics 96*, 109–123.

Farhi, E. and J. Tirole (2012). Bubbly Liquidity. *Review of Economic Studies 79*(2), 678–706.

Gilchrist, S., J. W. Sim, and E. Zakrajšek (2014). Uncertainty, financial frictions, and investment dynamics. *NBER Working Papers*.

Gorton, G. and G. Ordonez (2014). Collateral crises. *American Economic Review 104*(2), 343–378.

Guerrieri, V. and G. Lorenzoni (2017). Credit crises, precautionary savings, and the liquidity trap. *The Quarterly Journal of Economics 132*(3), 1427–1467.

Hamilton, J. D. and M. A. Flavin (1986). On the Limitations of Government Borrowing: A Framework for Empirical Testing. *The American Economic Review 76*(4), 808–819.

Hellwig, C. and G. Lorenzoni (2009). Bubbles and Self-enforcing Debt. *Econometrica 77*(4), 1137–1164.

Hennessy, C. A. and T. M. Whited (2007). How costly is external financing? evidence from a structural estimation. *Journal of Finance 62*(4), 1705–1745.

Hirano, T. and N. Yanagawa (2017). Asset Bubbles, Endogenous Growth, and Financial Frictions. *The Review of Economic Studies 84*(1), 406–443.

Ikeda, D. and T. Phan (2019, July). Asset Bubbles and Global Imbalances. *American Economic Journal: Macroeconomics 11*(3), 209–251.

İmrohoroğlu, A. and S. Tüzel (2014). Firm-Level Productivity, Risk, and Return. *Management Science 60*(8), 2073–2090.

Jeenas, P. (2019). Firm balance sheet liquidity, monetary policy shocks, and investment dynamics.

Jermann, U. J. and V. Quadrini (2012). Macroeconomic Effects of Financial Shocks. *American Economic Review 102*(1), 238–271.

Jiang, Z., H. Lustig, S. Van Nieuwerburgh, and M. Z. Xiaolan (2019). The us public debt valuation puzzle. Technical report, National Bureau of Economic Research.

Khan, A. and J. K. Thomas (2013). Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity. *Journal of Political Economy 121*(6), 1055–1107.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*(2), 211–248.

Martin, A. and J. Ventura (2012, May). Economic Growth with Bubbles. *American Economic Review 102*(6), 3033–58.

Martin, A. and J. Ventura (2016). Managing Credit Bubbles. *Journal of the European Economic Association 14*(3), 753–789.

Martin, A. and J. Ventura (2018, August). The Macroeconomics of Rational Bubbles: A User's Guide. *Annual Review of Economics 10*(1), 505–539.

Mian, A. and A. Sufi (2009). The Consequences of Mortgage Credit Expansion: Evidence from the U.S. Mortgage Default Crisis. *The Quarterly Journal of Economics 124*(4), 1449â1496.

Miao, J. and D. Su (2021). Fiscal and monetary policy interactions in a model with low interest rates. Technical report.

Miao, J. and P. Wang (2018, September). Asset Bubbles and Credit Constraints. *American Economic Review 108*(9), 2590–2628.

Ottonello, P. and T. Winberry (2020). Financial Heterogeneity and the Investment Channel of Monetary Policy. *Econometrica 88*(6), 2473–2502.

Perri, F. and V. Quadrini (2018). International Recessions. *American Economic Review 108*(4-5), 935–984.

Reis, R. (2021). The constraint on public debt when $r < g$ but $g < m$.

Salomao, J. and L. Varela (2021). Exchange rate exposure and firm dynamics. *The Review of Economic Studies Forthcoming*.

Schularick, M. and A. M. Taylor (2012, April). Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 1870-2008. *American Economic Review 102*(2), 1029–61.

Seater, J. J. and R. S. Mariano (1985). New tests of the life cycle and tax discounting hypotheses. *Journal of Monetary Economics 15*(2), 195–215.

Tang, H. and D. Zhang (2021). Bubbly Firm Dynamics and Aggregate Fluctuations .

Uhlig, H. (2003). What Drives GNP? *Unpublished manuscript, Euro Area Business Cycle Network*.

Uhlig, H. (2004). Do technology Shocks Lead to a Fall in Total Hours Worked? *Journal of the European Economic Association 2*(2-3), 361–371.

Weil, P. (1987). Confidence and the real value of money in an overlapping generations economy. *The Quarterly Journal of Economics 102*(1), 1–22.
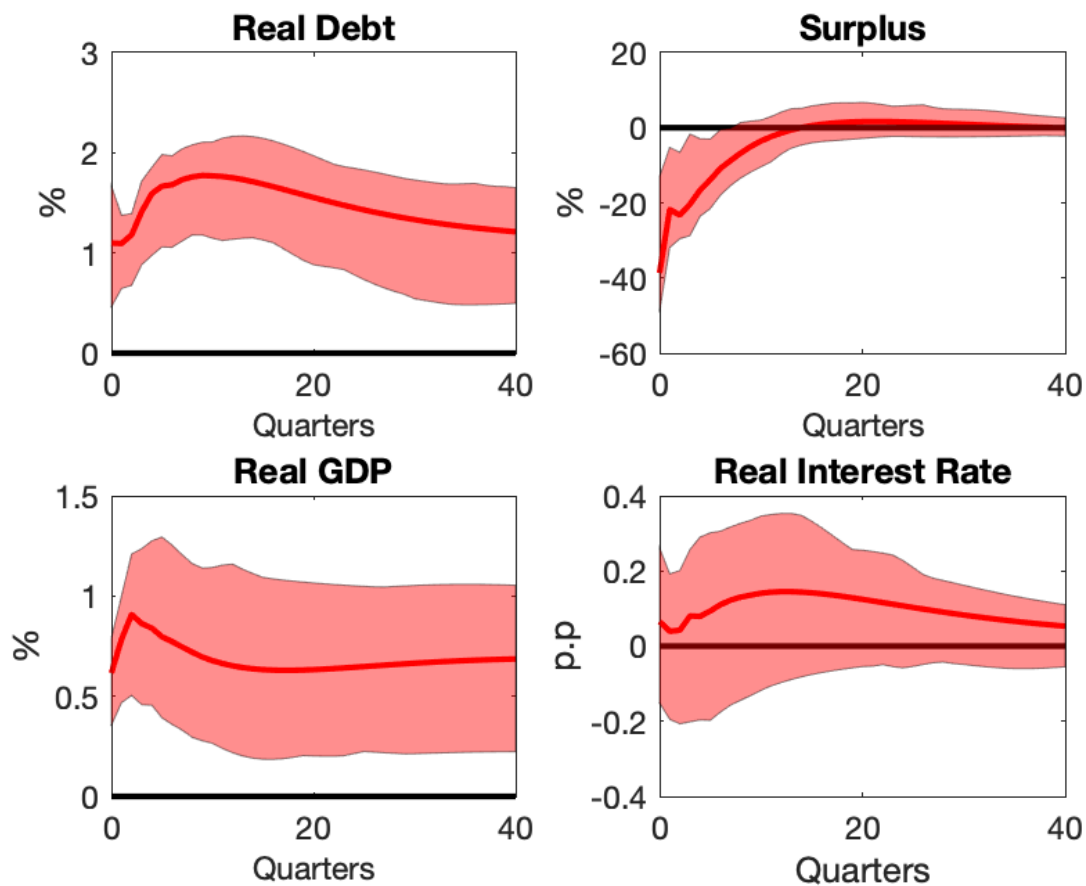
A Figures

Figure 5: Responses to Total Liabilities Innovation (Max-share): Pre-2007Q4 Sample
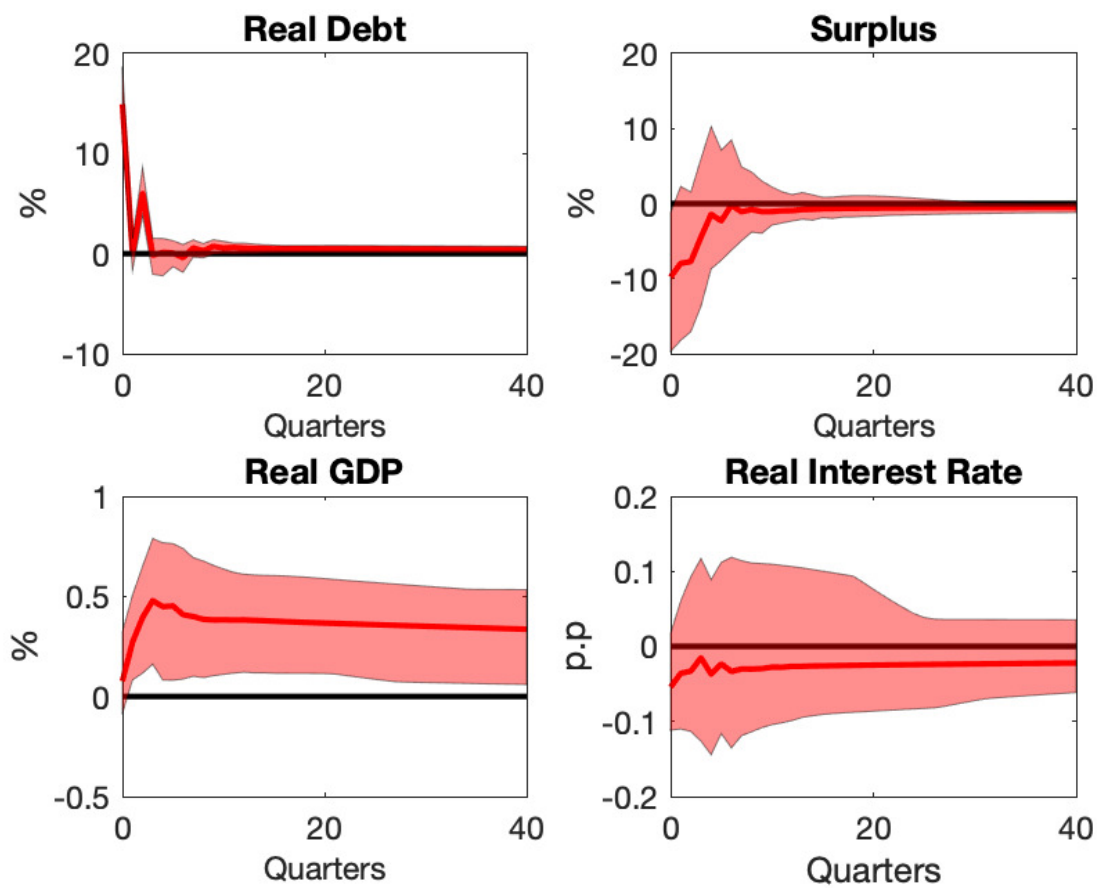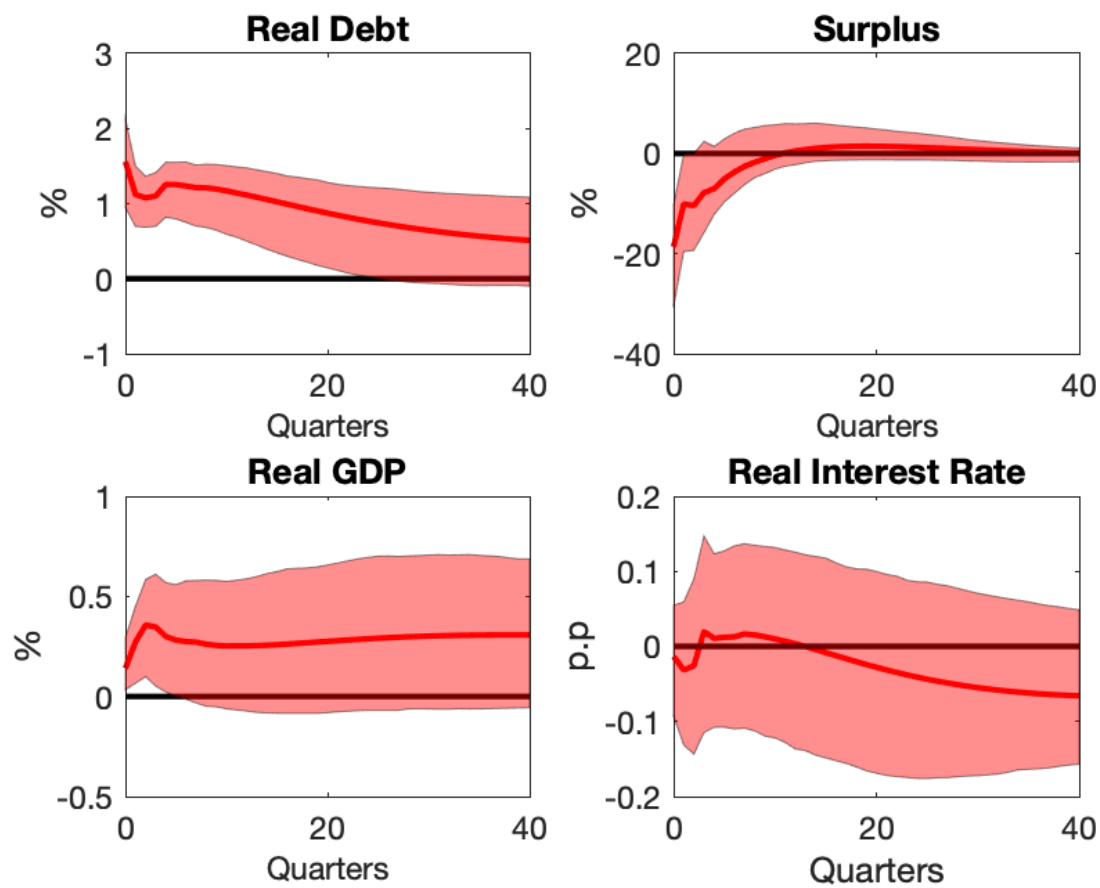
Figure 6: Responses to Total Debt Innovation (Max-share)

Figure 7: Responses to Total Liabilities Innovation (Zero Restrictions)
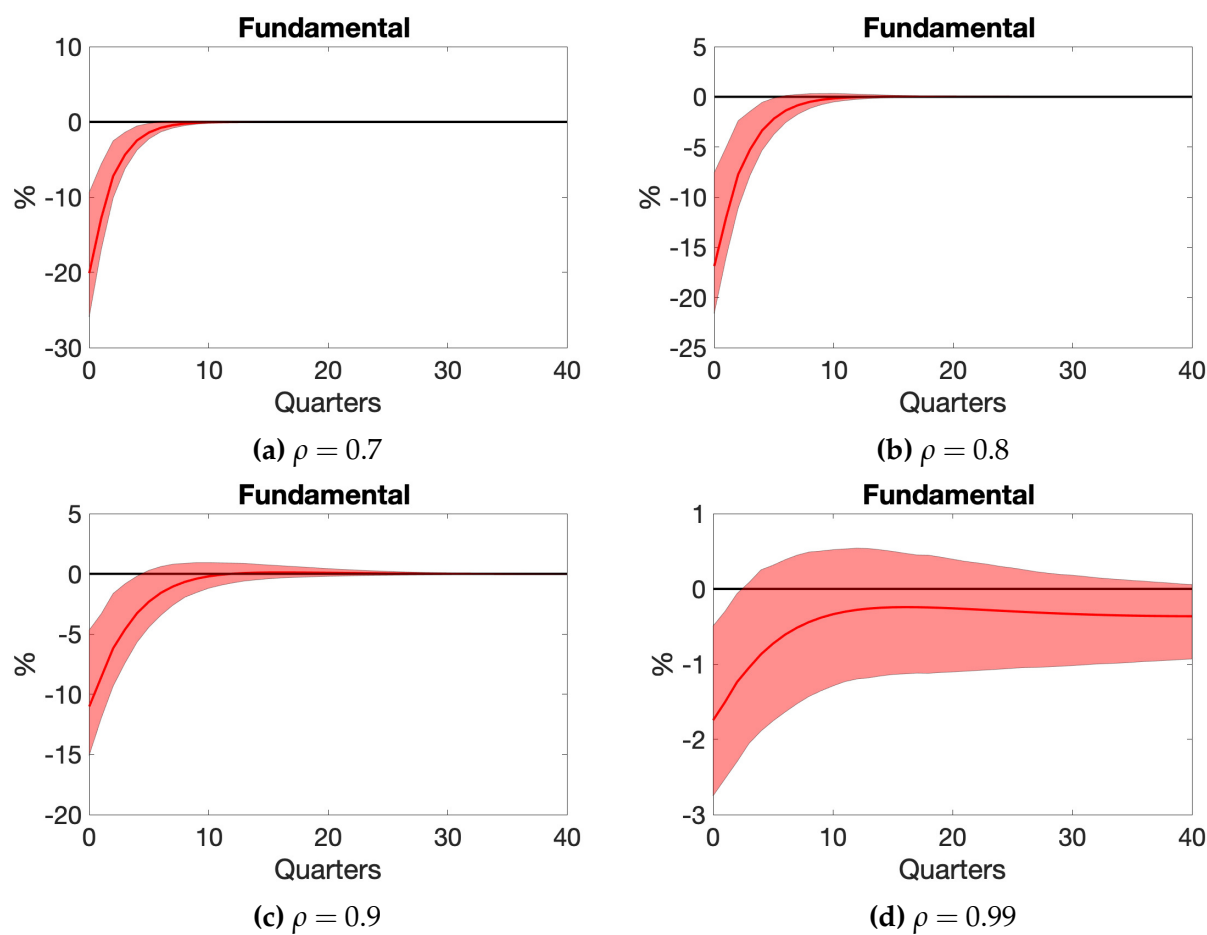
(a) $\rho = 0.7$

(b) $\rho = 0.8$

(c) $\rho = 0.9$

(d) $\rho = 0.99$

Figure 8: This figure plots responses of the NPV of surpluses to total liabilities innovations under alternative calibrations of ρ
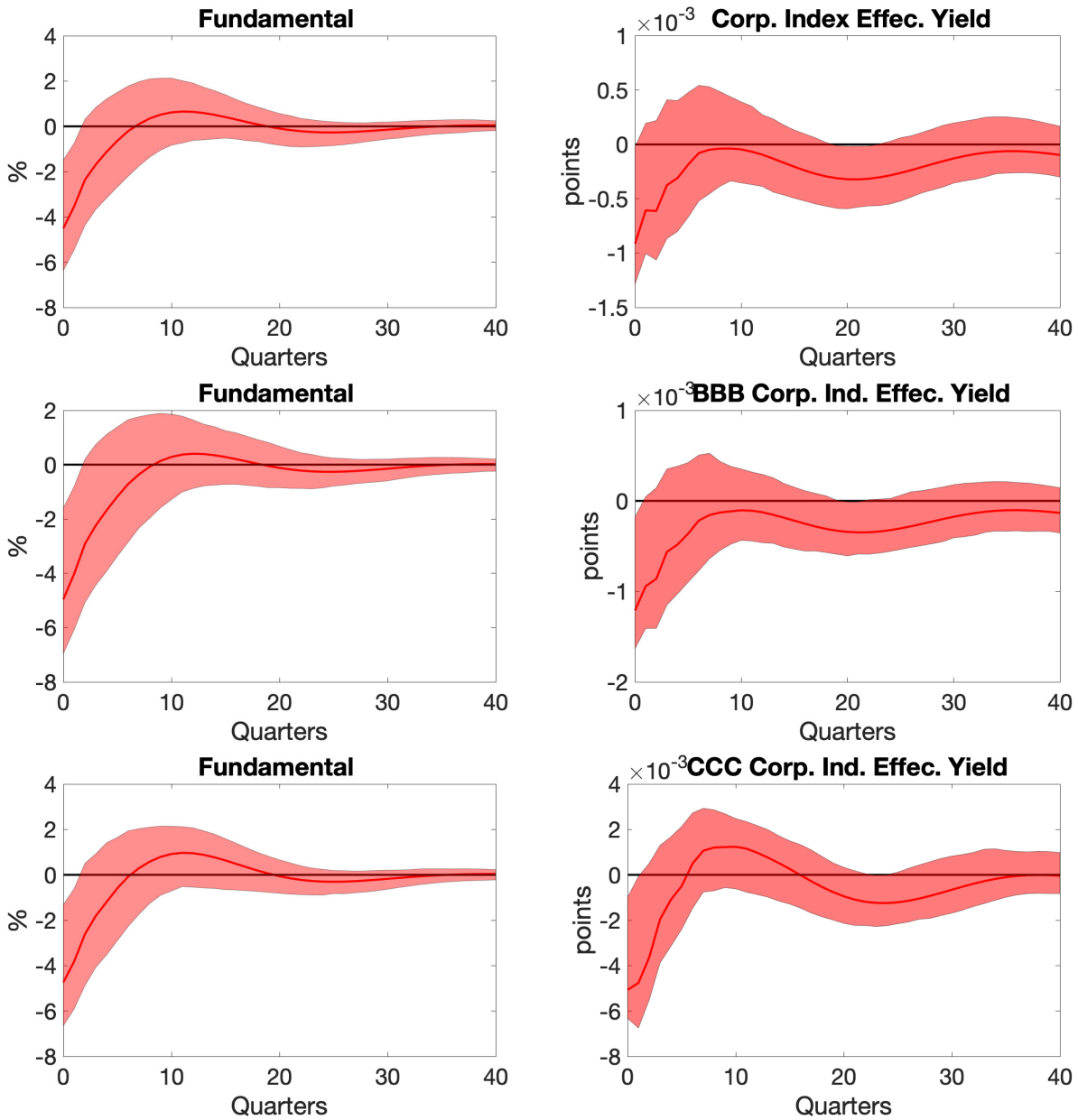
Figure 9: This figure plots responses of the NPV of surpluses to total liabilities innovations constructed based on estimated VARs that include alternative measures of real interest rates.
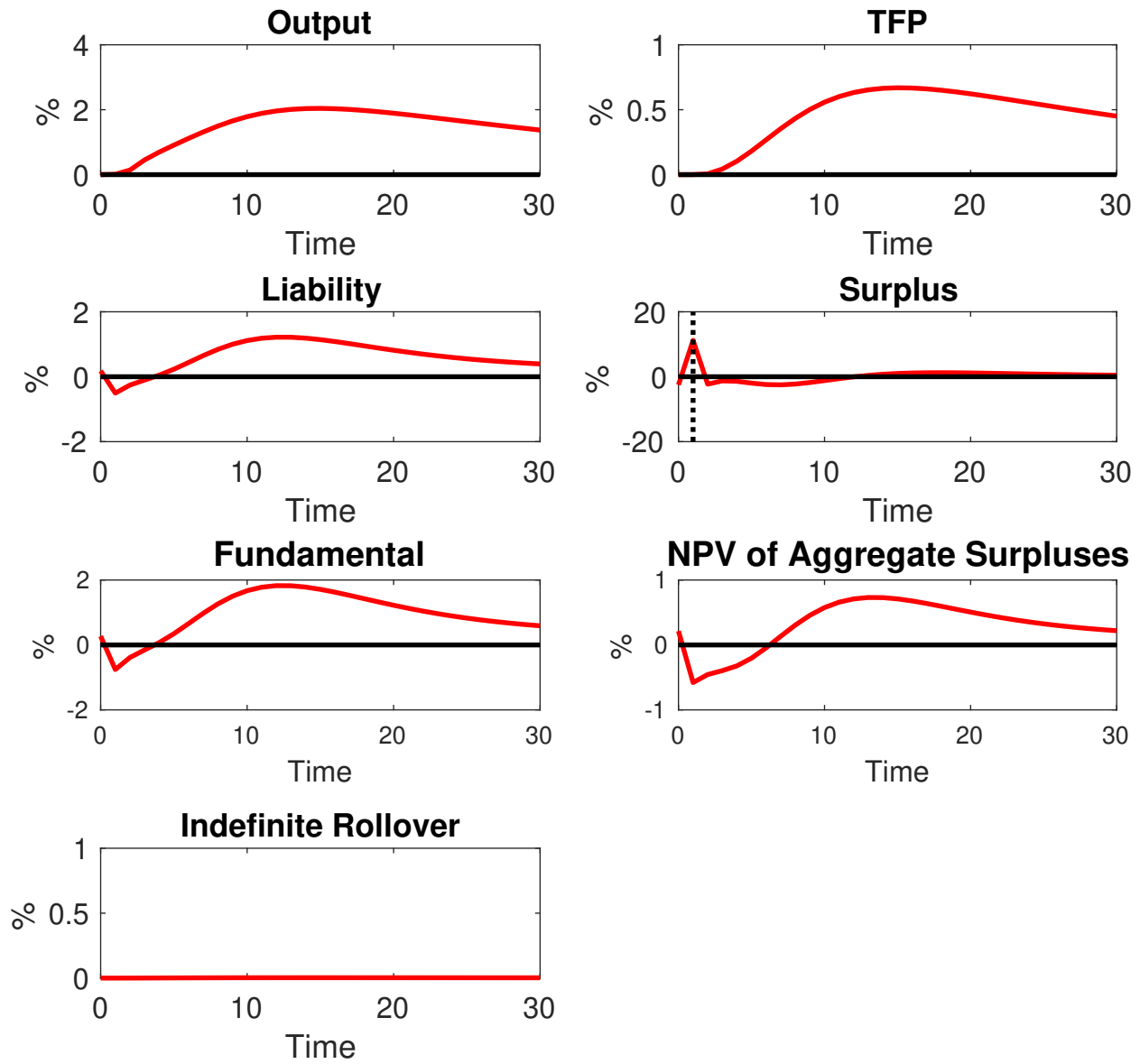
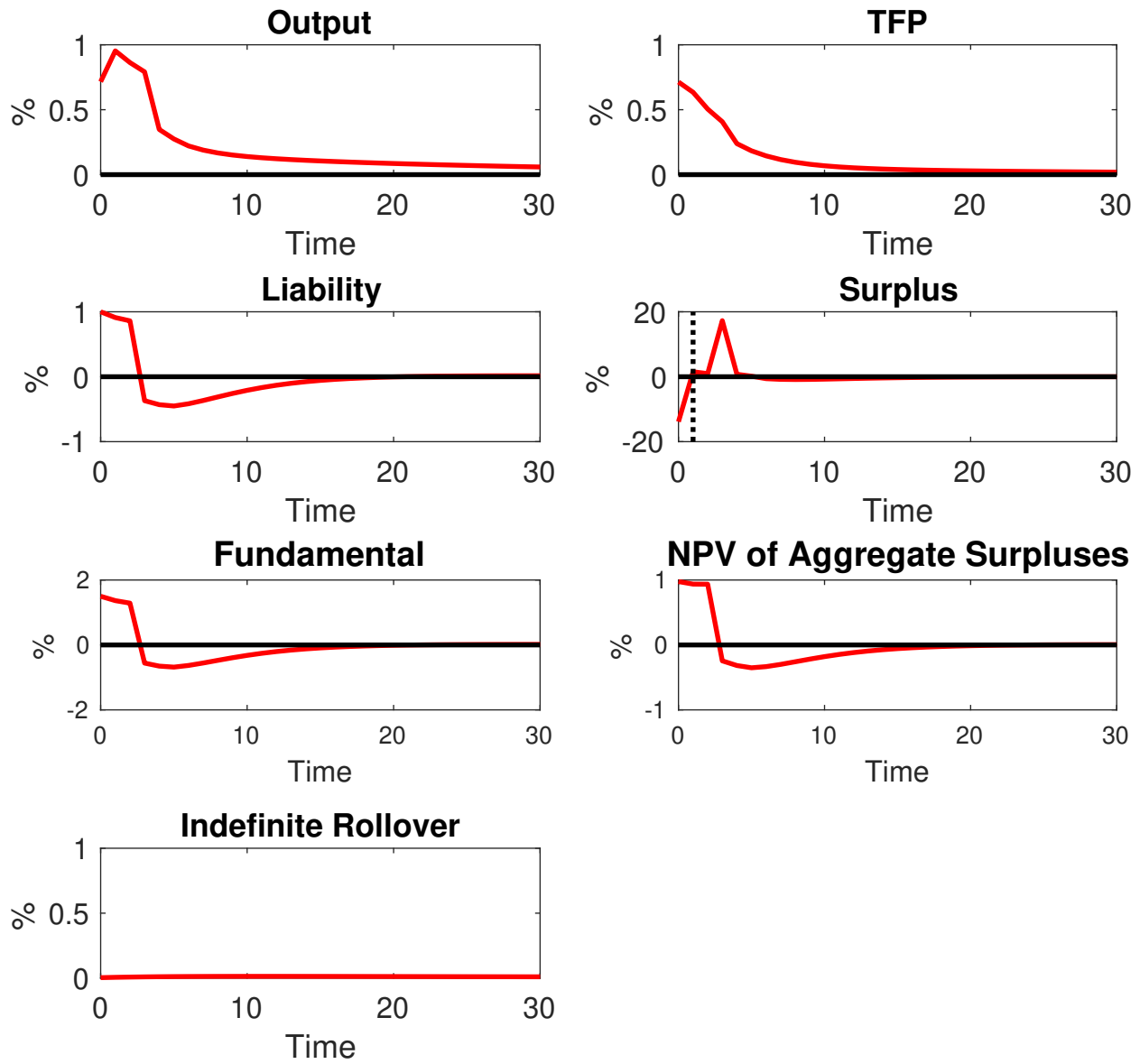**Figure 10:** IRFs to Pledgeability Shocks

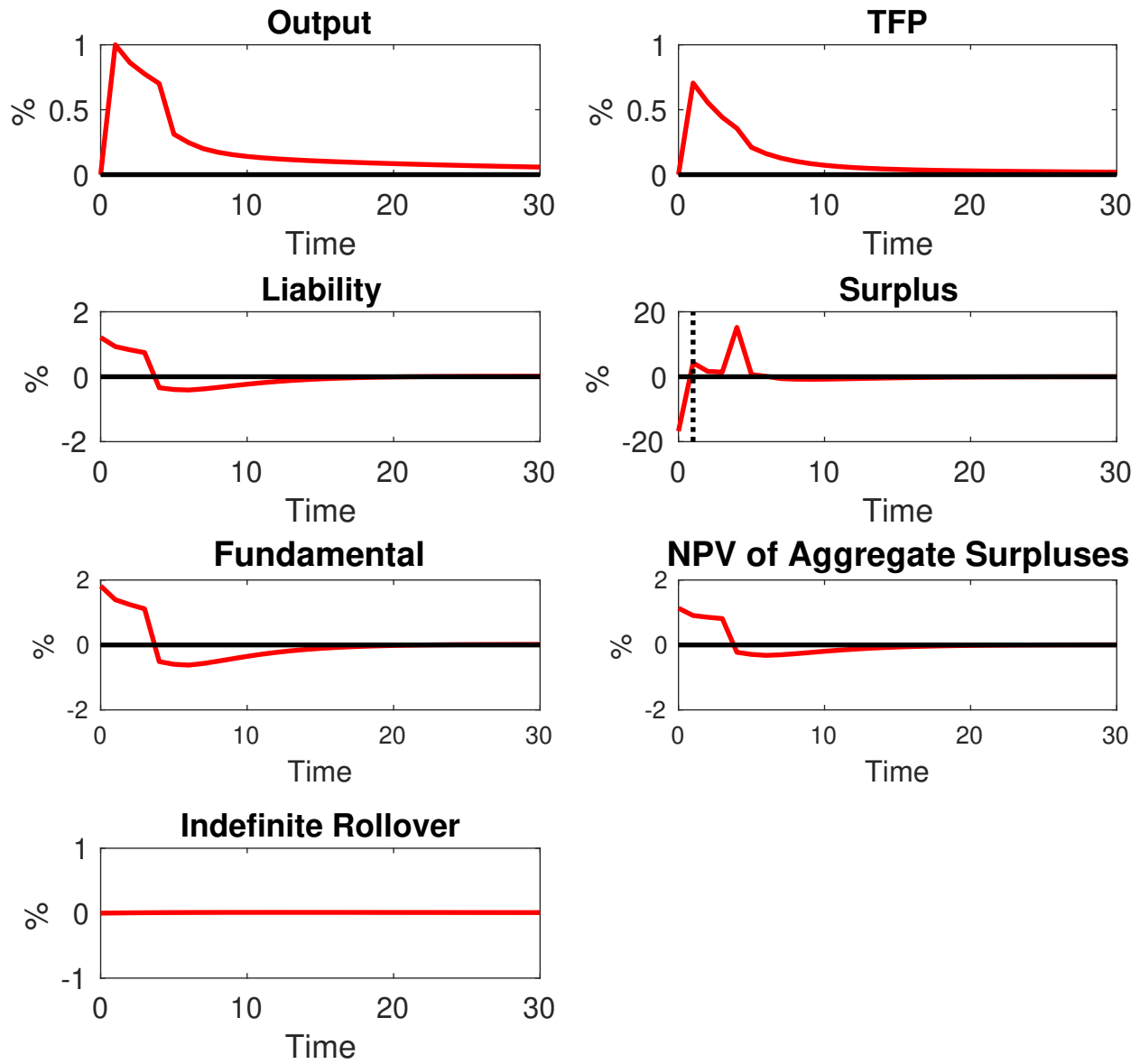**Figure 11:** IRFs to Productivity Shocks
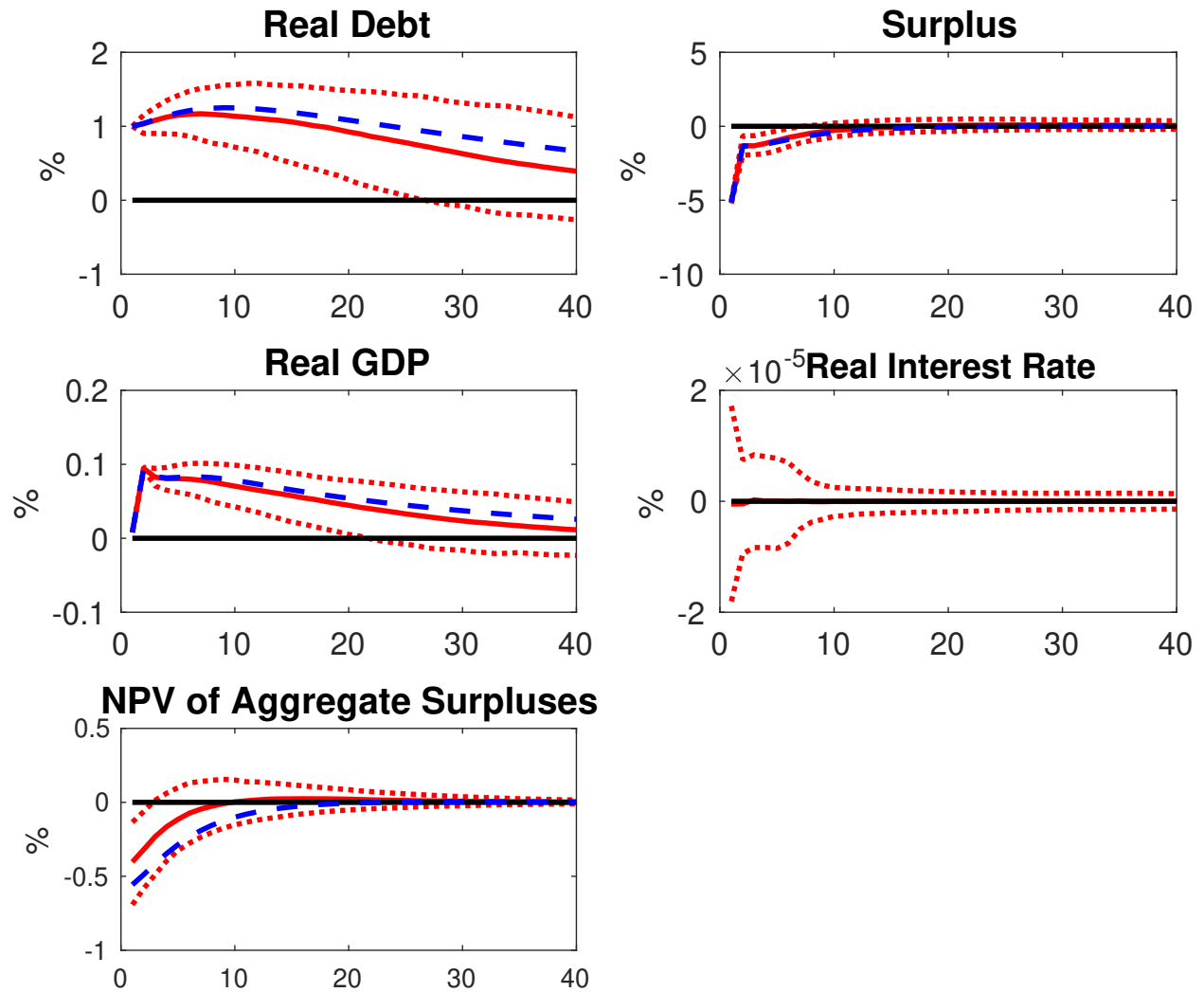
Figure 12: IRFs to News Shocks

Figure 13: Replication of VAR (Max-share) Results: Model Simulated Data conditional on Rollover Shocks
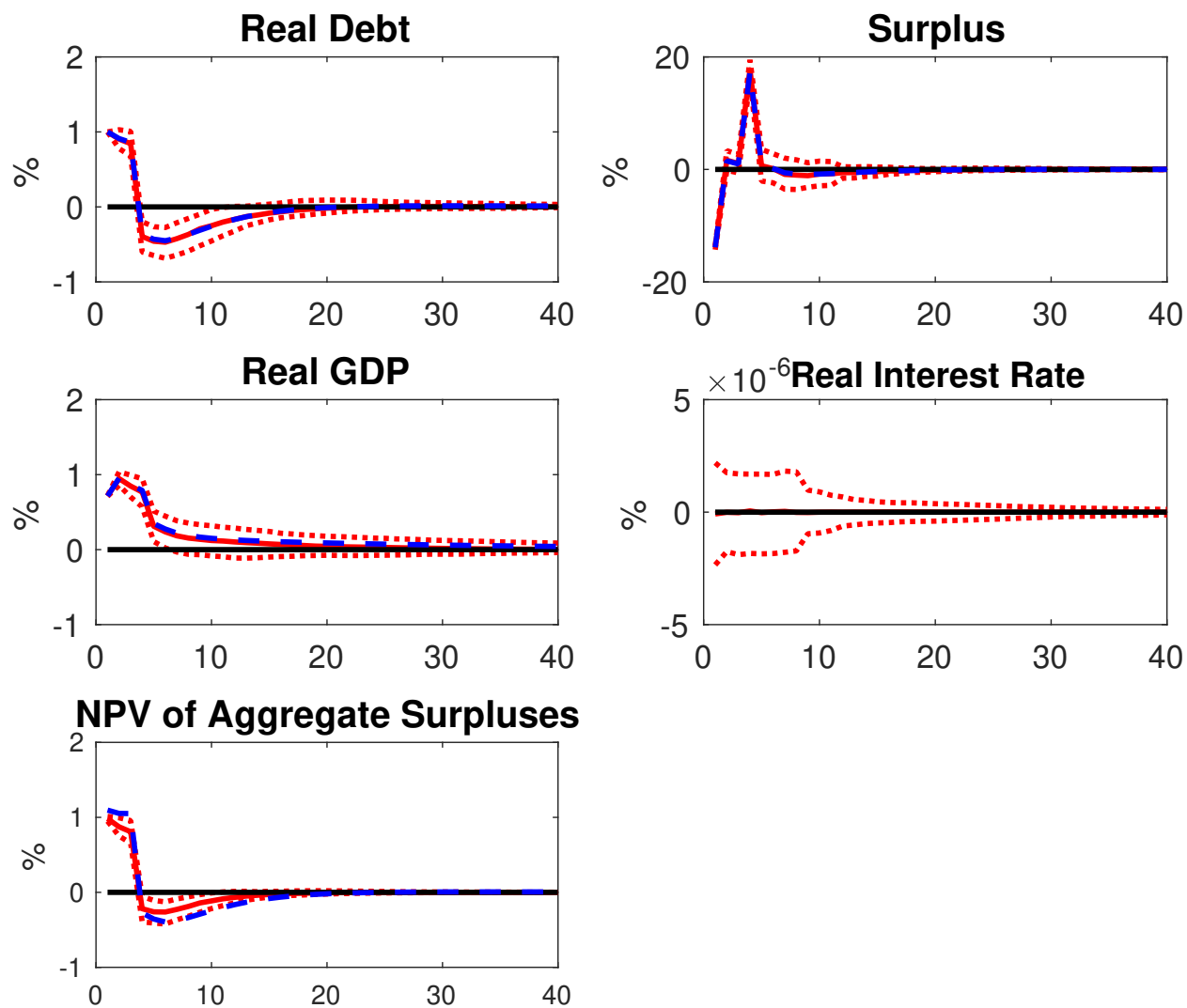
Figure 14: Replication of VAR Results: Model Simulated Data conditional on Productivity Shocks

B   Appendix for Empirical Analysis

B.1   Cash Flow Identity

This section extends Cochrane (2021) and Hamilton and Flavin (1986) to model the corporate bonds market. We collect all non-financial firms' zero-coupon debt of a given maturity into group j. Let $d_{j,t}$ denote the market value of a debt of the group j at the end of period t, and $q_{j,t}$ is the price of the debt j at time t. Let D_t denote the real market value of total non-financial firms' debt held by the market at the end of period t: $D_t \equiv \sum_j q_{j,t} d_{j,t}$.

Investors' return (R_{t+1}) of holding the portfolio of non-financial firms' debt is due to changes in prices (taking into account potential default). The return of such portfolio is defined as:

$$R_{t+1} \equiv \frac{\sum_j q_{j,t+1} d'_{j,t}}{\sum_j q_{j,t} d_{j,t}}, \tag{51}$$

where $d'_{j,t}$ is the the the amount of debt issued at time t that are not defaulted.

The derivation starts with the following cash flow identity:

$$\sum_j q_{j,t+1} d'_{j,t} = S_{t+1} + \sum_j q_{j,t+1} d_{j,t+1}, \tag{52}$$

where S_{t+1} denotes the surplus of non-financial firms: gross saving plus changes in total equity (market value) minus total investment. The total investment includes both the capital investment and the financial investment.

Equation (51) implies $\sum_j q_{j,t+1} d'_{j,t} = R_{t+1} \sum_j q_{j,t} d_{j,t}$, plus this into the cash flow identity:

$$R_{t+1} \sum_j q_{j,t} d_{j,t} = S_{t+1} + \sum_j q_{j,t+1} d_{j,t+1}. \tag{53}$$

Using the notation of the real market value of total non-financial firms' debt leads to the equation (1) in the main text:

$$R_{t+1} D_t = S_{t+1} + D_{t+1}.$$

Note that allowing for non-zero coupon debt does not change the final equation, see Hamilton and Flavin (1986) for an alternative derivation that allows for coupon payments. Moreover, interest payment does not enter directly into the right hand side of

the equation (1), see Hamilton and Flavin (1986) for detailed explanations in the context of government bond.

Linearization To construct the effects of debt innovations on its fundamental component, we first linearize the equation (1), then substitute it forward to obtain the linearized fundamental as the discounted sum of future surpluses and interest rates.

Divide (1) by its counter-part at the BGP and take log on both hand side of the equation:

$$\widehat{r}_{t+1} + \widehat{d}_t = log(\frac{S_{t+1}}{S+D} + \frac{D_{t+1}}{S+D}), \tag{54}$$

where S and D are the surplus and debt on the BGP and a variable with hat denotes the log deviation of the variable from its BGP. Because surplus can be negative in the data, we linearize the equation (1) in surplus in percentage deviation from its sample mean. To do so, first rewrite the previous equation as

$$\widehat{r}_{t+1} + \widehat{d}_t = log\left(\widetilde{s}_{t+1}\frac{S}{S+D} + \frac{D_{t+1}}{D}\frac{D}{S+D} + \frac{S}{S+D}\right), \tag{55}$$

where $\widetilde{s}_{t+1} \equiv \frac{S_{t+1} - S}{S}$ is the surplus in percentage deviation from its sample mean. Replace the right hand side by its first order Taylor approximation and abstract from the constant terms:

$$\widehat{r}_{t+1} + \widehat{d}_t = \frac{S}{S+D}\widetilde{s}_{t+1} + \frac{D}{S+D}\widehat{d}_{t+1}, \tag{56}$$

To be able implement the equation to the data, we expand the equation around the point where surplus and debt grows at the common rate.[19] The previous equation can be rewritten as:

$$\widehat{r}_{t+1} + \widehat{d}_t = (1-\rho)\widetilde{s}_{t+1} + \rho\widehat{d}_{t+1}, \tag{57}$$

where $\rho \equiv \frac{1}{sd+1}$ and sd denotes the surplus-to-debt ratio on the BGP. Substitute equation (57) forward, and impose the no-Ponzi-game condition to obtain the fundamental com-

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[19]As it is argued by Cochrane (2021): "There is nothing wrong with expanding about (this point). The point of expansion need not be the sample mean".

ponent of the debt:

$$\widehat{f}_t = \sum_{h=0}^{\infty} \rho^h \left[(1-\rho) \widetilde{s}_{t+h+1} + \widehat{r}_{t+h+1} \right], \tag{58}$$

C   Model Appendix

C.1   Proof of Proposition 1

We prove this proposition for incumbent firms. The same reasoning can be applied to the optimization problem of potential entrants. Suppose that the optimal path for firm j can be represented by policy functions $k_t^* \left(\varphi_{jt}, \lambda_{jt} \right)$, $F_t^* \left(\varphi_{jt}, \lambda_{jt} \right)$, $B_t^* \left(\varphi_{jt}, \lambda_{jt} \right)$ for $\forall t$, where $k_t^* \left(\varphi_{jt}, \lambda_{jt} \right)$, $F_t^* \left(\varphi_{jt}, \lambda_{jt} \right)$, $B_t^* \left(\varphi_{jt}, \lambda_{jt} \right)$ respectively denote policy functions for k_{jt+1}, F_{jt+1}, B_{jt+1}.[20] Moreover, at τ, $q_\tau^B \left(\varphi_{j\tau}, K_{j\tau+1}, F_{j\tau+1}, B_{j\tau+1} \right) B_\tau^* \left(\varphi_{j\tau}, \lambda_{j\tau} \right) < B_{j\tau}$. As in the main text, the corresponding value of firm j is denoted by $V_t \left(\varphi_{jt}, \lambda_{jt} \right)$. We label this path as Path 1. Note that idiosyncratic states can be sufficiently summarized by φ_{jt}, x_{jt} and B_{jt}. Therefore hereafter we replace λ_{jt} with x_{jt} and B_{jt}.

We need to prove that, when choosing $q_\tau^B \left(\varphi_{j\tau}, K_{j\tau+1}, F_{j\tau+1}, B_{j\tau+1} \right) B_{j\tau+1} = B_{j\tau}$, the net present value of expected dividends is at least as great as $V_\tau \left(\varphi_{j\tau}, x_{j\tau}, B_{j\tau} \right)$. Consider an alternative path: at τ, the firm chooses $q_\tau^B \left(\varphi_{j\tau}, K_{j\tau+1}, F_{j\tau+1}, B_{j\tau+1} \right) B_{j\tau+1} = B_{j\tau}$, $F_{j\tau+1} = F_\tau^* \left(\varphi_{j\tau}, x_{j\tau}, B_{j\tau} \right) + B_\tau^* \left(\varphi_{j\tau}, x_{j\tau}, B_{j\tau} \right) - B_{j\tau+1}$, $k_{j\tau+1} = k_\tau^* \left(\varphi_{j\tau}, x_{j\tau}, B_{j\tau} \right)$; at $\tau + 1$, the firm chooses $B_{j\tau+2} = B_{\tau+1}^* \left(\varphi_{j\tau+1}, x_{j\tau+1}, B_\tau^* \left(\varphi_{j\tau}, x_{j\tau}, B_{j\tau} \right) \right)$, $F_{j\tau+2} = F_{\tau+1}^* \left(\varphi_{j\tau+1}, x_{j\tau+1}, B_\tau^* \left(\varphi_{j\tau}, x_{j\tau}, B_{j\tau} \right) \right)$, $k_{j\tau+2} = k_{\tau+1}^* \left(\varphi_{j\tau+1}, x_{j\tau+1}, B_\tau^* \left(\varphi_{j\tau}, x_{j\tau}, B_{j\tau} \right) \right)$; $\forall t > \tau + 1$, $k_{jt+1} = k_t^* \left(\varphi_{jt}, x_{jt}, B_{jt} \right)$, $F_{jt+1} = F_t^* \left(\varphi_{jt}, x_{jt}, B_{jt} \right)$, $B_{jt+1} = B_t^* \left(\varphi_{jt}, x_{jt}, B_{jt} \right)$. We label this path as Path 2.

It is straightforward to prove that, compared with the supposely optimal Path 1, Path 2 generates exactly the same sequence of dividends (as well as the same default probability) for all $t \geq \tau$, regardless of the realization of φ_{jt}, μ_t. In fact, $\forall t \geq \tau + 1$, control variables k_{jt+1}, F_{jt+1}, B_{jt+1} are the same across two paths for any realization of φ_{jt}, μ_t. It is obvious that the sequence of F_{jt} satisfies no-Ponzi-game condition.

Moreover, we can verify that along Path 2, at $\tau + 1$, $q_{jt+1}^B B_{j\tau+2} \leq B_{j\tau+1}$. Along Path 2,

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[20]Throughout this paper, we assume that there exsit unique solution to policy functions and value functions. We follow the typical procedure in the literature by verifying that our iterative numerical solution leads to the same results, given different initial guesses. We also assume that the probability of repayment is greater than zero over the state space, which can also be verified in the numerical exercise.

$B_{j\tau+1} = \dfrac{B_{j\tau}}{q_\tau^B\left(\varphi_{j\tau}, K_{j\tau+1}, F_{j\tau+1}, B_{j\tau+1}\right)}$. According to Equation (19), the price schedule

$$q_\tau^B\left(\varphi_{j\tau}, K_{j\tau+1}, F_{j\tau+1}, B_{j\tau+1}\right)$$

is pinned down by the default probability at $\tau + 1$. Since the sequence of default probabilities are the same across the two paths, along Path 2 we have

$$B_{j\tau+1} = \dfrac{B_{j\tau}}{q_\tau^B\left(\varphi_{j\tau}, K_{j\tau+1}, F_{j\tau+1}, B_{j\tau+1}\right)} > B_\tau^*\left(\varphi_{j\tau}, x_{j\tau}, B_{j\tau}\right).$$

Along Path 1, at $\tau + 1$, $q_{jt+1}^B B_{j\tau+2} \leq B_{j\tau+1} = B_\tau^*\left(\varphi_{j\tau}, x_{j\tau}, B_{j\tau}\right)$. Note that $q_{jt+1}^B B_{j\tau+2}$ are the same across Path 1 and Path 2. We can thus verify that along Path 2, $q_{jt+1}^B B_{j\tau+2} \leq B_\tau^*\left(\varphi_{j\tau}, x_{j\tau}, B_{j\tau}\right) < B_{j\tau+1}$.

To sum up, when choosing $q_\tau^B\left(\varphi_{j\tau}, K_{j\tau+1}, F_{j\tau+1}, B_{j\tau+1}\right) B_{j\tau+1} = B_{j\tau}$, the net present value of expected dividends is at least as great as $V_\tau\left(\varphi_{j\tau}, x_{j\tau}, B_{j\tau}\right)$. Note that we construct Path 2 so that it is simply as good as the supposedly optimal path, yet Path 2 is not necessarily the actual optimal path.

An incumbent firm faces the constraint $q_\tau^B\left(\varphi_{j\tau}, K_{j\tau+1}, F_{j\tau+1}, B_{j\tau+1}\right) B_{j\tau+1} \leq B_{j\tau}$. The proof above applies to the optimization problem of potential entrants if we replace the right-hand-side upper bound $B_{j\tau}$ with B_0. In fact, the reasoning also works if we replace the upper bound $B_{j\tau}$ with any number: If firm j faces the constraint

$$q_\tau^B\left(\varphi_{j\tau}, K_{j\tau+1}, F_{j\tau+1}, B_{j\tau+1}\right) B_{j\tau+1} \leq \bar{B},$$

where $\bar{B}$ represents an arbitrary upper bound for $q_\tau^B\left(\varphi_{j\tau}, K_{j\tau+1}, F_{j\tau+1}, B_{j\tau+1}\right) B_{j\tau+1}$, it is optimal that $q_\tau^B\left(\varphi_{j\tau}, K_{j\tau+1}, F_{j\tau+1}, B_{j\tau+1}\right) B_{j\tau+1} = \bar{B}$. It is always optimal to maximize the borrowing in indefinite rollover debts B.

C.2   Numerical Appendix

Before we proceed to the algorithm, it is useful to establish some results regarding the optimal behavior of firms. The optimization problem defined in (36) and (37) is well investigated by the literature:[21] 1) there exist a cutoff level of adjusted networth, $\underline{x}\left(\varphi\right)$,

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[21]See the discussions of Ottonello and Winberry (2020). Khan and Thomas (2013), and Arellano et al. (2019) study a more complicated version of this problem, with aggregate states included in the state space. Ottonello and Winberry (2020) and Jeenas (2019) solve the problem at steady states. Like us, they can remove the aggregate states from the state space.

below which firms default; 2) there exist a cutoff level of adjusted networth, $\bar{x}(\varphi)$, above which firms become financially unconstrained, i.e., firms can perpetually choose the unconstrained level of capital; 3) firms with $\tilde{x} \in [\underline{x}(\varphi), \bar{x}(\varphi)]$ are financially constrained and set their dividends to zero.

Balanced Growth Path   Along a BGP, state space can be fully summarized by idiosyncratic productivity φ, and adjusted net worth $\tilde{x}$, which is defined by (35). We follow the standard numerical approach to solve the problem.

1. Solve for $p^n(\varphi, k', F')$, the probability of repaying in the subsequent period, $q^F(\varphi, k', F')$, debt price schedule, and $\underline{x}(\varphi)$, the cutoff level of adjusted net worth $\tilde{x}$ for firms to default. Make an initial guesses for function $p^n(\varphi, k', F')$. Note that B' does not affect the probability of repaying, since B can always be rolled over and does not affect the adjusted net worth in the subsequent period. Then characterize the corresponding debt price schedule $q^F(\varphi, k', F')$ from Equation (18). Next calculate the maximum amount of $q^F(\varphi, k', F') F' - k'$ for various k' and F'. Denote the amount by $\underline{M}(\varphi)$. The cutoff level $\underline{x}(\varphi)$ is equal to $-\underline{M}(\varphi)$. Given idiosyncratic productivity level φ, if the adjusted net worth $\tilde{x} < \underline{x}(\varphi)$, the dividend of the firm is inevitably negative, and the firm has to default. We can then update the probability $p^n(\varphi, k', F')$. Iterate this process until $p^n(\varphi, k', F')$ and $\underline{x}(\varphi)$ converge.

2. Solve for $v_c^u(\varphi)$, the continuation value function of financially unconstrained firms, $\bar{x}(\varphi)$, the cutoff level of adjusted net worth $\tilde{x}$ for firms to be financially unconstrained, and $k^u(\varphi)$ and $F^u(\varphi)$, the policy functions for financially unconstrained firms. $k^u(\varphi)$ can be solved by equating the marginal return on investment to risk-free interest rate. Unconstrained firms are indeed indifferent over F' as long as they remain unconstrained in the subsequent periods with probability one. To resolve the indeterminacy, we follow Khan and Thomas (2013) by assigning the minimum saving policy to $F^u(\varphi)$ such that unconstrained firms borrow maximum amount of $F^u(\varphi)$ (or equivalently, save the minimum amount) that ensures them to maintain unconstrained in the future, regardless of the realization of states. $F^u(\varphi)$ follows

$$F^u(\varphi) = \min_{\varphi'} \left\{ \tilde{x}(\varphi', \varphi) - k^u(\varphi') + \frac{1}{R} F^u(\varphi') \right\},$$

where $\tilde{x}(\varphi', \varphi) \equiv A\varphi'(k^u(\varphi))^\alpha - c^f + (1+\delta)k^u(\varphi)$. $F^u(\varphi)$ can be solved by iterative guessing and verifying. The cutoff level $\bar{x}(\varphi)$ is equal to $k^u(\varphi) - \frac{1}{R}F^u(\varphi)$. Given id-

iosyncratic productivity level φ, if the adjusted net worth $\tilde{x} \geq \bar{x}(\varphi)$, firms can not only choose unconstrained level of k' and F', but also remain unconstrained permanently by persistently following the unconstrained policy functions. The continuation value $v_c^u(\varphi)$ follows

$$v_c^u(\varphi) = -k^u(\varphi) + \frac{1}{R}F^u(\varphi) + \frac{1}{R}\sum_{i=1}^{N_\varphi} \omega(\varphi_i|\varphi)[\tilde{x}(\varphi_i,\varphi) + v_c^u(\varphi_i)],$$

where $\omega(\varphi_i|\varphi)$ denotes the transition probability from φ to φ_i, N_φ represents the number of grids for φ. Note that in the Appendix we denote the transition probability by discrete distribution since functions are defined on discrete grids in the algorithm. We use Tauchen method to approximate the AR1 process (11) to get the transition probability $\omega(\varphi_i|\varphi)$. $v_c^u(\varphi)$ can be solved by iterative guessing and verifying.

3. Solve for $V(\varphi,\tilde{x})$, the value function of incumbent firms, $v^c(\varphi,\tilde{x})$, the value function of financially constrained firms, and $k^c(\varphi,\tilde{x})$ and $F^c(\varphi,\tilde{x})$, the policy functions for financially unconstrained firms. Constrained firms issue zero dividend since the return from self-financing the firms is higher than risk-free rate, which is also the investment return the shareholders can get on dividends. Therefore we use bisection method to characterize $F(\varphi,\tilde{x},k')$, the level of fundamental credit that satisfies zero-dividend given φ, $\tilde{x}$, k'. Substitute F' in debt price schedule $q^F(\varphi,k',F')$ with $F(\varphi,\tilde{x},k')$ to get $q_c^F(\varphi,\tilde{x},k')$, the debt price schedule for constrained firms given φ, $\tilde{x}$, k'. The value function of financially constrained firms $v^c(\varphi,\tilde{x})$ follows

$$v^c(\varphi,\tilde{x}) = \max_{k'}\left\{\tilde{x} - k' + q_c^F(\varphi,\tilde{x},k')F(\varphi,\tilde{x},k') + \frac{1}{R}\sum_{i=1}^{N_\varphi}\omega(\varphi_i|\varphi)V(\varphi',\tilde{x}')\right\},$$

where $V(\varphi,\tilde{x})$ follows

$$V(\varphi,\tilde{x}) = \mathbb{1}\{\tilde{x} \geq \bar{x}(\varphi)\}(\tilde{x} + v_c^u(\varphi)) + \mathbb{1}\{\tilde{x} < \underline{x}(\varphi)\}v^c(\varphi,\tilde{x}).$$

$v^c(\varphi,\tilde{x})$ and $V(\varphi,\tilde{x})$ can be solved by iterative guessing and verifying. Use grid search to pin down $k^c(\varphi,\tilde{x})$, the optimal k' for constrained firms. Substitute k' in $F(\varphi,\tilde{x},k')$ with $k^c(\varphi,\tilde{x})$ to obtain $F^c(\varphi,\tilde{x})$.

As for any firm, policy functions can be represented by

$$k^*(\varphi,\tilde{x}) = \mathbb{1}\{\tilde{x} \geq \bar{x}(\varphi)\}k^u(\varphi) + \mathbb{1}\{\tilde{x} < \underline{x}(\varphi)\}k^c(\varphi,\tilde{x}),$$

$$F^* (\varphi, \tilde{x}) = \mathbb{1} \{\tilde{x} \geq \bar{x} (\varphi)\} F^u (\varphi) + \mathbb{1} \{\tilde{x} < \underline{x} (\varphi)\} F^c (\varphi, \tilde{x}).$$

Substitute k' and F' in $p^n (\varphi, k', F')$ with policy functions $k^* (\varphi, \tilde{x})$ and $F^* (\varphi, \tilde{x})$ respectively to get $p^n (\varphi, \tilde{x})$, the probability of repaying conditional on φ and $\tilde{x}$.[22] $p^n (\varphi, \tilde{x})$ is used to characterize the law of motion for B, according to (19) and (29). Now given the law of motion for B, policy functions $k^* (\varphi, \tilde{x})$ and $F^* (\varphi, \tilde{x})$, cutoff levels $\bar{x} (\varphi)$ and $\underline{x} (\varphi)$, and the transition matrix of φ, to construct the transition matrix Γ, of φ, $\tilde{x}$, B. Along BGP, the relative distribution of firms remain unchanged. The detrended stationary distribution vector $\tilde{\eta}_t$ follows the law of motion

$$\tilde{\eta}_{t+1} = \frac{1}{1+g} \Gamma \times \tilde{\eta}_t + \tilde{\zeta}, \tag{59}$$

where $\tilde{\zeta}$ denotes the detrended distribution vector of entrants. Iterate (59) until $\tilde{\eta}_t$ converges to get the stationary distribution.

Transition Dynamics We characterize the perfect foresight equilibrium paths along which: i) unexpected shocks arrive at $t = 0$; ii) prior to $t = 0$ and after $t = 99$, the economy is along BGP. Use backward induction to compute: i) the value functions value functions $V_t (\varphi, \tilde{x})$ and $V_t^e (\varphi)$, ii) policy functions $k_t^* (\varphi, \tilde{x})$, $F_t^* (\varphi, \tilde{x})$ and $B_t^* (\varphi, \tilde{x}, B)$, iii) and cutoff of defaulting $\underline{x}_t (\varphi)$, from $t = 99$ to $t = 0$. Then construct transition matrix Γ_t and the detrended distribution vector of entrants $\tilde{\zeta}_t$ for every t from $t = 0$ to $t = 99$. Finanlly, iterate forward the law of motion

$$\tilde{\eta}_{t+1} = \frac{1}{1+g} \Gamma_t \times \tilde{\eta}_t + \tilde{\zeta}_{t+1}, \tag{60}$$

to get the detrended distribution vector of firms for every t from $t = 1$ to $t = 99$.

C.3 Define equilibrium

A recursive competitive bubbly equilibrium consists of: i) value functions $V_t (\varphi, \tilde{x})$ and $V_t^e (\varphi)$ ii) policy functions $k_t^* (\varphi, \tilde{x})$, $F_t^* (\varphi, \tilde{x})$ and $B_t^* (\varphi, \tilde{x}, B)$, iii) the measure of entrants $\zeta_t(\varphi, B)$, the measure of surviving firms $\eta_t(\varphi, \tilde{x}, B)$, iv) the initial bubble component B_0, v) debt price schedule $q_t^F (\varphi, F', k')$, and vi) the investment return R_t, such that for all t,

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[22]$p^n (\varphi, \tilde{x}) = 1$ if $\tilde{x} \geq \bar{x} (\varphi)$.

1. $V_t(\varphi_t, \tilde{x}_t)$ solves the optimization problem of incumbent firms, which is described by

$$V_t(\varphi, \tilde{x}) \;=\; \max_{F', k'} \left\{ \tilde{x} - k' + q_t^F(\varphi, F', k') F' + \frac{1}{R_{t+1}} \int V_{t+1}(\varphi', \tilde{x}') \, dJ(\varphi'|\varphi) \right\}, \quad (61)$$

$$\tilde{x} - k' + q_t^F(\varphi, F', k') F' \geq 0, \quad (62)$$

where $k_t^*(\varphi, \tilde{x})$ and $F_t^*(\varphi, \tilde{x})$ solve the optimal k' and F' respectively. $B_t^*(\varphi, \tilde{x}, B)$ follows

$$B_t^*(\varphi, \tilde{x}, B) = \frac{R_{t+1}}{p_{t+1}^n(\varphi, F', k')} B, \quad (63)$$

where $p_{t+1}^n(\varphi, F', k')$ denotes the probability of repaying at $t + 1$, in which $k' = k_t^*(\varphi, \tilde{x})$ and $F' = F_t^*(\varphi, \tilde{x})$.

2. The value of firm entry is given by

$$V_t^e(\varphi) = V_t(\varphi, B_0).$$

$k_t^*(\varphi, B_0)$, $F_t^*(\varphi, B_0)$, and $B_t^*(\varphi, B_0, B_0)$ solve the optimal k' and F' respectively for entrants.

3. The measure of entrants is given by

$$\zeta_t(\varphi) = M_t \int \mathbb{1}\{V_t^e(\varphi) \geq 0\} \, d\omega(\varphi),$$

where $\omega(\varphi)$ denotes the distribution of initial φ.

4. The measure of surviving firms is given by

$$\eta_{t+1}(\varphi', \tilde{x}', B') = \int \mathbb{1}\{\tilde{x}' \geq \underline{x}_t(\varphi')\} \times \mathbb{1}\{k' = k_t^*(\varphi, \tilde{x})\} \times \mathbb{1}\{F' = F_t^*(\varphi, \tilde{x})\} \times \mathbb{1}\{B' = B_t^*(\varphi, \tilde{x}, B)\}$$

$$\times \mathbb{1}\left\{\tilde{x}' = A\varphi'(k')^\alpha - c^f + (1 - \delta)k' - F'\right\} \times j(\varphi'|\varphi) \, d\eta_t(\varphi, \tilde{x}, B)$$

$$+ \int \mathbb{1}\{\tilde{x}' \geq \underline{x}_t(\varphi')\} \times \mathbb{1}\{k' = k_t^*(\varphi, B_0)\} \times \mathbb{1}\{F' = F_t^*(\varphi, B_0)\} \times \mathbb{1}\{B' = B_t^*(\varphi, B_0, B_0)\}$$

$$\times \mathbb{1}\left\{\tilde{x}' = A\varphi'(k')^\alpha - c^f + (1 - \delta)k' - F'\right\} \times j(\varphi'|\varphi) \, d\zeta_t(\varphi)$$

where $j(\varphi'|\varphi)$ denotes the density of the transition probability $J(\varphi'|\varphi)$, $\underline{x}(\varphi)$ denotes the cutoff adjusted net worth of defaulting.

5. The investment return is equal to

$$R_t = \frac{1}{\beta}.$$

6. The debt price schedule is characterized by

$$q_t^F \left(\varphi, F', k' \right) = \frac{1}{R_{t+1}} \left[\left(p_{t+1}^n \left(\varphi, F', k' \right) F' + \left(1 - p_{t+1}^n \left(\varphi, F', k' \right) \right) \eta_{t+1} k' \right) \right].$$