Simple Models of Central Bank Digital Currency in Small Open Economies*

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April 13, 2021

Abstract

I develop a simple two-country RBC model that studies the operation of a Central Bank Digital Currency (CBDC) in a small open economy. I add an interest-bearing CBDC and eliminate cash from the household's set of liquidity services that can be used to purchase consumption goods and reduce transaction costs. Domestic risk-free interest rates are a function of average debt and world interest rates and CBDC interest rates are derived as a spread from the domestic risk-free rate or using a Taylor-type rule. I find that the steady-state CBDC share is decreasing in transaction efficiency and risk-free-CBDC spread, illustrate the relationship between the two interest-bearing assets in a CBDC economy (risk-free bonds and CBDC), and demonstrate the use of a cash-in-advance constraint. Furthermore, numerical simulations of exogenous shocks show that small open economies with CBDCs experience shocks with higher magnitude, although these effects can be dampened with flexible CBDC design.

*Email address: dwc236@cornell.edu. I thank Professor Eswar Prasad for advising this project and Bineet Mishra, Professor Eleonora Patacchini, Elisa Yu-Chun Cheng, and the economics thesis seminar at Cornell for helpful comments and suggestions.

1 Introduction

The introduction of new financial technologies has the capacity to shock both the cross-border aspect and the policy aspect of international monetary policy. While cross-border monetary policy transmission has been closely studied from both theoretical and empirical angles, the development of tools such as blockchain, advances in private cryptocurrencies such as Bitcoin, and the shift to cashless societies have made Central Bank Digital Currencies (CBDC) a realistic possibility instead of a pipe dream. In the context of these technological changes and a protracted period of low global interest rates, central banks are seriously considering CBDCs to increase financial inclusion and institute negative nominal interest rates by controlling the money supply.

This paper aims to characterize the transmission of economic shocks between open economies when countries use CBDC. I develop a simple theoretical two-country model in which one country implements CBDC. Cash-in-advance (CIA) constraints govern household consumption: the household can only use cash or CBDC to trade for consumption goods and cannot consume more than the assets they hold. The baseline economy uses only cash and bonds in which cash provides households lower transaction costs for the purchase of consumption goods. The CBDC economy introduces an interest-bearing substitute for cash such that there are two interest-bearing assets: bonds and CBDC. CBDC thus serves as a form of "e-cash" with a nonzero (either positive or negative) rate of return. Thus, CBDCs provide both liquidity services (like cash) and remuneration for holding the asset (like bonds) with the benefits of easier transaction (compared to cash) and greater safety (compared to bonds).

Given that small open economies are price-takers in the world economy, domestic interest rates are a function of the world interest rate and the debt level of each country. In contrast, the CBDC interest rate is calculated as a (typically) positive spread from the domestic risk-free rate or using a generalized Taylor-type rule. I find that consumption and the CBDC share of total assets are decreasing in the spread between the domestic risk-free interest rate and CBDC interest rate. On the other hand, the bond and capital share of total assets in the steady-state economy are increasing in this spread. Similarly, consumption and CBDC share are also decreasing in the rate of inflation.

In terms of dynamics, the primary findings are that a (1) CBDC economy is more volatile in its shock response and (2) the design of the interest-rate rule matters. I hypothesize that these effects are due to higher transaction efficiency for CBDC and the effect of another interest-bearing asset in the economy.

I arrive at a simple model by abstracting from several elements of monetary policy, asset design, and the open economy, including different exchange rate setups. Changing the design of the CBDC in this economy or the character of the open economy relationship may yield further insights into cross-border monetary policy. Adding these factors to the framework established in this paper could develop clearer explanations for interest rate transmission even in the absence of real-world data and economic facts.

The main contribution of this paper is to demonstrate the use of a CIA constraint in the open economy for multiple interest-bearing assets and provide intuition for the assets in a CBDC economy, which the current literature on CBDCs in open economies does not include. The result of this project adds to a small but growing literature on general equilibrium models for CBDCs as well as a rich body of research on open economies (Devereux and Engel 2002; Hnatkovska, Lahiri, and Vegh 2016), models of money (Kiyotaki and Wright 1993), and monetary policy in RBC models (Schmitt-Grohé and Uribe 2004).

The rest of the paper is structured as follows: I examine literature related to CBDC design, CBDC models, and open economy macroeconomics in Section (1.1), lay out model basics in Section (2), calibrate and simulate the model for quantitative exercises in Section (3), and conclude in Section (4).

1.1 Related Literature

1.1.1 CBDC Design

While this research does not substantively tackle the design issues of CBDC, background on the principles of digital currency and design issues is helpful for understanding model choices and results. Central Bank Digital Currency is "fiat currency issued by central banks in digital form" (Allen et al. 2020). While wholesale digital central bank money for financial institutions have been available in the form of electronic reserves, I focus on retail CBDC. Retail CBDC is differentiated from cash and reserves by wide accessibility, usability for retail transactions, interest-bearing nature, and structural separation from other forms of central bank money (Kumhof and Noone 2018). Other technical distinctions are also being debated, such as whether Distributed Ledger Technology would be used, whether the issuer could trace the digital currencies, whether CBDC would be issued against loans, the degree to which CBDC could be exchanged for reserves or deposits, and the relative amount of CBDC available.

Monetary authorities have several mechanisms to influence the interest rate on CBDCs. In systems with both reserves and CBDC, the no-arbitrage condition for households is $\mathrm{rr} = \mathrm{rc} + \mathrm{cy}_{hf}^{cbdc}(CBDC)$ where rr is the risk-free interest rate, rc is the interest rate on CBDC, and cy is the convenience yield on CBDC for households and firms. Kumhof and Noone (2018) explore when $\mathrm{rr} = \mathrm{rc}$, concluding that central banks are unlikely to issue enough CBDC to fulfill this condition given the large amount of assets needed for purchase and the consequent liquidity injection after allowing public access to reserves.

However, others have differing views of access to reserves. Before the 2007-09 Financial Crisis, commercial banks offered higher deposit rates because of their monopoly on electronic money. CBDC inserts a competitive alternative to commercial bank rates, allowing central banks greater leverage over interest rates in abnormal economic situations (Meaning et al. 2018). A scenario in which CBDC is introduced alongside reserves is functionally identical to a scenario in which CBDC takes the role of cash and reserves with the two assumptions that (1) there are different forms of money are interchangeable, or nearly so, and (2) electronic reserves offer similar utility (Brunnermeier and Niepelt 2019; Fernandez-Villaverde et al. 2020). Niepelt (2020) offers further intuition for this equivalence result between inside bank money and outside CBDC money: public access to reserves serves the same functions as bank money without redistribution or shifting incentives in the nonbank private sector. Moreover, replacing cash with CBDC would not require substantial liquidity injection because most central banks in the post-crisis era already hold excess reserves.

Bordo and Levin (2017) present three options for how CBDC can act as a secure store of value. First, CBDC could have a constant nominal value such that it acts as e-cash. Household CBDC holdings would be minimized with positive nominal interest rates on reserves. Monetary policy would be similar to status quo central bank operations; nominal negative interest rates would be difficult to achieve. Second, CBDC could have stable real value. The digital currency would be indexed to changes in the price level. When real interest rates become negative, households would shift to CBDC that earn zero real interest, thus imposing a *real* zero lower bound (ZLB). This real ZLB is even more constricting than the nominal ZLB of the present. Lastly, CBDC could be interest-bearing, paying positive interest in a growing economy with a stable price level. The interest rates on CBDC would then become a central bank's primary lever on monetary policy; the less cash available in the economy, the stronger this channel of monetary policy. This research studies the third scenario of an interest-bearing CBDC where the monetary authority controls the interest rate.

In addition to greater central bank control, policymakers may also choose to implement CBDC for a litany of ancillary reasons. CBDC can increase financial inclusion, which is especially important in emerging market economies, by using app-based frameworks to extend banking to underserved communities (Allen et al. 2020). CBDC can lower criminal activity by stopping counterfeiting, increasing compliance with anti-money laundering regulations, reducing transaction anonymity, and lowering the utility of stealing cash (Williamson 2019; Engert and Fung 2017). Finally, CBDCs can stabilize the banking system during economic shocks by increasing the pass-through of monetary policy and reassuring actors through the guarantee of central banking (Andolfatto 2020; Fernandez-Villaverde et al. 2020).

Since the Great Recession, several central banks have operated and continue to operate in protracted low-inflation environments and the world economy has largely experienced interlinked low interest rates (Amador et al. 2019). Given this macroeconomic environment and the possible benefits of CBDC, central banks are exploring digital currency adoption through pilot projects, research, and experimentation — 80 percent of central banks surveyed by the Bank of International Settlements have engaged in CBDC in some way and at least 36 central banks have published research on the subject (Auer, Cornelli, and Frost 2020). The People's Bank of China has already implemented a pilot project in large cities such as Shenzhen, Chengdu, and Beijing. Central banks in Uruguay, Ecuador, and Ukraine have finished their CBDC experiments with mixed results. Other central banks are on the verge of launching new CBDC projects. In this quickly-changing context, developing monetary policy frameworks to assess the effects of new financial technologies is a useful exercise for policymakers and academics with real-world implications for financial systems.

1.1.2 Models of CBDC

Models of CBDCs with various designs are still in the early stages of exploration. Barrdear and Kumhof (2016) research the implementation effects of an interest-bearing CBDC offset by government loans during creation. Their empirical simulation finds a CBDC injection equivalent to 30 percent of the GDP with parameters tuned to the pre-crisis United States resulted in steady-state output gains of 3%. In their conclusion, they ask "What could be the impact of CBDC on international liquidity and exchange rate dynamics?," an extension the current project addresses.

Piazzesi, Rogers, and Schneider (2019) study a banking system with New Keynesian price stickiness assumptions in which a CBDC earns a convenience yield while coexisting with short bonds. They conclude that monetary policy structure matters for transmission — a corridor system has stronger passthrough and there are no self-fulfilling fluctuations for policies that do not respond strongly to inflation. Andolfatto (2020) models a monopolistic banking sector, showing that CBDC introduction had no effect on bank lending activity and lending rates, though banks raised deposit rates through increased competition; additionally, the deposit base was expanded.

Because of the potential use of CBDC as a tool for increased financial inclusion, studying its adoption within economies with varying levels of formal and informal employment is another useful exercise. Oh and Zhang (2020) demonstrate a L-shaped relationship between CBDC acceptance and size of the informal economy. Positive policy rates on the CBDC and tax deductions may help improve adoption rates while reallocating households between formal and informal sectors of the economy.

Only two working papers investigate the effects of CBDC in open economies. In fact, most design papers focus solely on the domestic effects of introducing such an asset. Yet, ignoring the world economy in this interlinked era is foolhardy. George, Xie, and Alba (2020) extend Barrdear and Kumhof (2016) to a small open economy where the world economy is assumed to be large open economy. They replicate welfare gains for society but also show that there are uneven distributional effects of such gains between the financial sector and households. In addition, exchange rates and inflation are stabilized by the introduction of CBDC.

Ferrari, Mehl, and Stracca (2020) begin from the starting point of Eichenbaum, Johannsen, and Rebelo (2021) and develop the intuition for a new arbitrage condition facilitated by the introduction of a CBDC that is strictly preferred over other assets, given their liquidity service setup for the digital currency. Quantitatively, the CBDC economy experiences stronger exchange rate movements and deeper interlinkages to the world economy. Shocks manifest to a greater degree and foreign monetary policy autonomy is lessened. Different designs for CBDCs can dampen these effects and create greater welfare.

In light of the various approaches to model CBDC, the present project skips the question of implementation and adoption to focus on the salient question of operation.

2 Model

I sketch two-country models to form a theoretical basis before conducting analyses and experiments in later sections. I consider the Small-Open-Economy Real-Business-Cycle (SOE-RBC) framework studied in Uribe and Schmitt-Grohé (2017), which draw from Schmitt-Grohé and Uribe (2003) and Mendoza (1991). I add cash, CBDC, and a cash-in-advance (or liquidity-in-advance) constraint to the SOE-RBC model.

The home economy is a one-good small open economy that is inhabited by infinitely-lived representative households. Given the small open economy setup, the home economy is a price-taker and has little control over home interest rates. Production, employment, and capital use all occur within the household for simplicity; I abstract away from decentralization even though differentiating economic activity between the household and the marketplace does not change the equilibrium conditions. There are no restrictions on trade between the two economies.

I begin with an economy that has cash and bonds in Section 2.1 (hereafter the CB model) before moving to an economy that has CBDC and bonds in Section 2.2.

2.1 The Cash and Bonds Economy

The representative household in the home country maximizes lifetime expected welfare by choosing consumption goods c_t and labor n_t:

$$V = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t), 0 < \beta < 1 \tag{1}$$

subject to[1]

$$b_t + y_t + h_t + \psi(c_t, h_{t-1}) = \frac{1 + r_{t-1}}{1 + \pi_t} b_{t-1} + c_t + I_t + \Phi(k_{t+1} - k_t) + \frac{h_{t-1}}{1 + \pi_t} \tag{2}$$

$$c_t = \frac{h_{t-1}}{1 + \pi_t} \tag{3}$$

and a no-Ponzi constraint of the form

$$\lim_{j \to \infty} E_t \frac{b_{t+j}}{\prod_{s=0}^{j} (1 + r_s)} \leq 0$$

In the welfare function (1), β is the subjective discount factor. In the home budget constraint (2), b_t is the household's bond holding at the end of period t (Uribe and Schmitt-Grohé (2017) term this the debt position), h_t is the household's cash holding at the end of period t, r_t is the interest rates at which domestic households can borrow, y_t is domestic output, I_t is gross investment, k_t is physical capital, and $\Phi(\cdot)$ is a capital adjustment function. The gross inflation rate is defined as $1 + \pi_t = \frac{P_t}{P_{t-1}}$.

The capital adjustment function satisfies $\Phi(0) = \Phi'(0) = 0$ and expresses capital adjustment costs in terms of final goods. Capital adjustment costs are included to avoid excess investment volatility in response to world shocks and are a typical element of small open economy models.

To model demand for cash, I assume that households face transaction costs, which can be reduced through holdings of cash. The transactions technology is a strictly convex function of consumption and cash h_t: $\psi(c_t, h_{t-1})$. Increasing cash holdings decreases the transactions cost but the marginal reduction in transaction costs decline as cash holdings grow. The properties of the transaction costs function are that

1. $\psi \geq 0, \psi(0, h_{t-1}) = 0$

1. Nominal versions of the constraint equations are in Appendix A.

2. $\psi_c \geq 0, \psi_h \leq 0$

3. $\psi_{cc} \geq 0, \psi_{hh} \geq 0$

which follow Feenstra (1986). Cash, and later CBDC, are the only media of exchange explicitly modeled. Households with lower holdings of the transaction demand asset also have lower consumption.

The second constraint is the "cash-in-advance" (CIA) constraint in equation (3). The amount of consumption goods purchased cannot exceed the cash available to the household at the beginning of the period, following the timing convention of Svensson (1985) where goods markets open before asset markets.

The output function takes capital and labor as inputs and is a linearly homogenous function:

$$y_t = A_t F(k_t, n_t) \tag{4}$$

where A_t is productivity at time t. I assume that total factor productivity $a_t \equiv \log A_t$ follows an exogenous AR(1) process

$$a_t = \rho_a a_{t-1} + \epsilon_t^a \tag{5}$$

and is the only source of shock in the model. Note that $\epsilon_t^a \sim N(0, \sigma_a^2)$ is i.i.d. Capital stock evolves according to

$$k_t - (1-\delta)k_{t-1} = I_t \tag{6}$$

where $0 < \delta < 1$ is the depreciation rate of capital.

2.1.1 Equilibrium

First Order Conditions. I solve for first order conditions using typical Lagrangian techniques.[2] Full solutions are found in Appendix B. Household optimal choices are standard: households equate expected marginal utilities across time. Equation (8) shows that the labor supplied by household is related to consumption only through wages.

$$U_c(c_t, n_t) = \lambda_t(1 + \psi_c(c_t, h_{t-1})) + \lambda_t^{'} \tag{7}$$

$$-U_n(c_t, n_t) = \lambda_t A_t F_n(k_t, n_t) \tag{8}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \left(\frac{1 + r_t}{1 + \pi_{t+1}} \right) \tag{9}$$

$$\lambda_t[1 + \Phi_k(k_{t+1} - k_t)] = \beta E_t \lambda_{t+1}[A_{t+1}F_k(k_{t+1}, n_{t+1}) + 1 - \delta + \Phi_k(k_{t+2} - k_{t+1})] \tag{10}$$

$$\lambda_t + \beta E_t \lambda_{t+1}^{'} \frac{1}{1 + \pi_{t+1}} = \beta E_t \lambda_{t+1} \left(\frac{1}{1 + \pi_{t+1}} - \psi_h(c_{t+1}, h_t) \right) \tag{11}$$

Dividing the labor FOC (8) by the consumption FOC (7) to eliminate λ_t yields

$$-\frac{U_n(c_t, n_t)}{U_c(c_t, n_t)} = \frac{A_t F_n(k_t, n_t)}{1 + \psi_c(c_t, h_{t-1})} \tag{12}$$

2. Assuming that the CIA constraint does not hold in the equilibrium; that is, the Lagrangian multiplier associated with the constraint ($\lambda_t^{'}$) is 0.

which is the familiar intratemporal household Euler condition. As is typical, the marginal rate of substitution between leisure and consumption is increasing in hours worked. The marginal product of labor is decreasing in labor. Because transacting for consumption goods has a cost, the marginal product of labor is also decreasing in transaction costs.

Stationarity. Generic small open economy models are nonstationary — the calculation of debt and consumption rely on initial levels of different features of the economy. Given the dearth of data on CBDCs, this is an unworkable assumption. I adopt the first approach from Schmitt-Grohé and Uribe (2003), which is to model the interest rate on bonds as elastic to induce stationarity. Thus, the domestic interest rate is

$$r_t = r^* + p(\tilde{d}_t) \tag{13}$$

where r^* is the world interest rate (exogenously set as constant), $p(\cdot)$ is a country-specific interest rate premium function where $p' > 0$ by assumption, and $\tilde{d}_t$ is the cross-sectional average level of bond holdings. This debt elasticity approach has theoretical and empirical justifications beyond the scope of this work, which uses this approach for computational ease.[3]

Government. The government solves a simple budget constraint with an exogenous money supply growth parameter μ such that

$$h_{t-1} = \frac{1+\mu}{1+\pi_t} h_t \tag{14}$$

From the model environment above, an equilibrium can be defined with the exogenous productivity process A and world interest rate r^*.

Definition 1. *Given a set of realizations $\{A, r^*\}$ at time t, an equilibrium is a state-contingent set of allocations $\{c_t, n_t, b_t, h_t, k_t, I_t\}$ and prices $\{r_t, \pi_t\}$ such that*

1. *The allocations solve the problems faced by households at these prices.*
2. *All factor markets clear.*
3. *The government budget constraint in equation (14) is satisfied.*

In the CB model, the state variables are $\{b_t, r_t, k_t, A_t\}$.

2.1.2 Functional Forms

I assume the period utility function to be CRRA

$$U(c, n) = \frac{G(c, n)^{1-\sigma} - 1}{1-\sigma}, \sigma > 0 \tag{15}$$

3. See, for example, Chapters 5, 6, and 11 of Uribe and Schmitt-Grohé (2017).

over the subutility index

$$G(c,n) = c - \frac{h^{\omega}}{\omega}, \omega > 1 \tag{16}$$

which are GHH preferences (Greenwood, Hercowitz, and Huffman 1988) and eliminate the wealth effect on labor supply.

For production, I assume a Cobb-Douglas function:

$$F(k,n) = k^{\alpha}n^{1-\alpha}, 0 < \alpha < 1 \tag{17}$$

which implies unitary elasticity of substitution between the two inputs capital and labor.

I follow Mishra and Prasad (2021)[4] in assuming the transaction function to be

$$\psi(c_t, h_{t-1}) = \theta_1 \frac{c_t^{\gamma}}{h_{t-1}^{\eta}} \tag{18}$$

where $\gamma > 1$ is the transaction cost for the consumption good, $\eta > 1$ is the transaction efficiency of cash, and θ_1 is the level parameter. As the transaction efficiency of the medium of exchange increases, the transaction costs decrease.

The capital adjustment cost function is assumed to be

$$\Phi(x) = \frac{\phi}{2}x^2, \phi > 0 \tag{19}$$

where ϕ is the level parameter. The quadratic form of this function implies that net investment generates (positive) adjustment costs.

The country interest rate premium function p takes the form

$$p(b) = \psi_1(e^{b-\bar{b}} - 1) \tag{20}$$

where $\psi_1 > 0$ is the debt elasticity of interest rate and $\bar{b}$ is exogenously set. Thus, the country interest rate premium is an increasing and convex function of net external debt.

2.1.3 Steady State

In the deterministic steady state, all endogenous variables are constant over time. This creates a point of reference around which I simulate shocks and investigate equilibrium dynamics. Let the steady-state position be denoted by a bar over the variable with no time subscript; for example, the steady-state value of consumption holdings is $\bar{c}$.

In all equilibrium states, I assume that the cross-section average level of debt equals the individual level of debt (which is used interchangeably with bond holdings), as all representative agents are identical. This means that

$$d_t = \tilde{d}_t \tag{21}$$

and I can rewrite each of the first order conditions by substituting equation (13).

4. This is modified from the transaction function for an economy with both cash and CBDC.

In the steady state, then, equation (9) becomes

$$1 = \beta \left[1 + r^* + \psi_1 \left(e^{b-\bar{b}} - 1 \right) \right] (1 + \bar{\pi})^{-1}$$

Assume from the FOC for bonds that

$$\beta \left(\frac{1 + r^*}{1 + \bar{\pi}} \right) = 1$$

as is standard. This gives $b = \bar{b}$. Solving for each of the variables gives:

$$\bar{r^*} = \frac{(1 + \bar{\pi})}{\beta} - 1 \tag{22}$$

$$\bar{\kappa} \equiv \frac{\bar{k}}{\bar{n}} = \left(\frac{\beta^{-1} - 1 + \delta}{\alpha} \right)^{\frac{1}{\alpha - 1}} \tag{23}$$

$$\bar{n} = [(1 - \alpha)\bar{\kappa}^{\alpha}]^{\frac{1}{\omega - 1}} \tag{24}$$

$$\bar{k} = \bar{\kappa}\bar{n} \tag{25}$$

$$\bar{r} = \bar{r^*} \tag{26}$$

$$\bar{y} = A\bar{\kappa}^{\alpha}\bar{n} \tag{27}$$

$$\bar{I} = \delta\bar{k} \tag{28}$$

$$\bar{c}\bar{\pi} - \bar{c} + \theta_1 \bar{c}^{\gamma} \left(\frac{c}{1 + \bar{\pi}} \right)^{\eta} = \bar{b} \left(\frac{\bar{r}^* - \bar{\pi}}{1 + \bar{\pi}} \right) - \bar{\kappa}^{\alpha}\bar{n} - \delta\bar{k} \tag{29}$$

$$\bar{h} = \bar{c}(1 + \bar{\pi}) \tag{30}$$

$$\bar{\lambda} = \frac{\left(\bar{c} - \frac{\bar{n}^{\omega}}{\omega} \right)^{-\sigma}}{1 + \theta_1 \gamma \frac{\bar{c}^{\gamma - 1}}{\bar{h}^{\eta}}} \tag{31}$$

$$0 = \Delta\bar{h} = \frac{1 + \mu}{1 + \bar{\pi}} \Rightarrow \bar{\pi} = \mu \tag{32}$$

The steady-state condition in equation (30) assumes that the CIA constraint holds with equality, which is true when the nominal rate of interest is positive. Thus, real consumption is equal to real money balances. This assumption abstracts away from uncertainty, following Walsh (2010).[5]

The steady-state value for consumption in equation (29) is solved computationally. Holding cash levels constant produces equation (32), which shows that steady-state inflation is equal to the rate of growth of the money supply.

2.2 The CBDC and Bonds Economy

In this section, an interest-bearing CBDC issued by the government is used as an unit of account, medium of exchange, and store of value that wholly replaces cash. Households in the home country hold a combination of bonds and home CBDC and own the

5. Different situations in which the CIA constraint is and is not binding (e.g. further work on the neutrality of money) are left for future work.

firms. Households in the foreign country are not analyzed in detail, but they are assumed to hold a combination of interest-bearing bonds and cash, as in typical SOE-RBC models. Model setup and dynamics could change dramatically were the foreign household to adopt CBDC as well. Households solve canonical SOE-RBC problems with the exception that the central bank sets the interest rate on CBDC and controls the supply of CBDC.

Note that I add to or otherwise change the model discussed in Section 2.1 using red text, which is a convention also reflected in Section 3.[6]

The representative household maximizes

$$V = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t), 0 < \beta < 1 \tag{33}$$

subject to

$$b_t + y_t + d_t + \psi(c_t, d_{t-1}) = \frac{1+r_{t-1}}{1+\pi_t} b_{t-1} + c_t + I_t + \Phi(k_{t+1} - k_t) + \frac{1+r^d_{t-1}}{1+\pi_t} d_{t-1} \tag{34}$$

$$c_t = \frac{d_{t-1}}{1+\pi_t} \tag{35}$$

and a no-Ponzi constraint of the form

$$\lim_{j\to\infty} E_t \frac{d_{t+j}}{\prod_{s=0}^{j}(1+r^d_s)} \leq 0$$

In the home budget constraint (33), I remove the effect of cash holdings on the household's government constraint and add an interest-bearing asset d_t, which is the household's holding of CBDC at the end of period t. The interest rate on CBDC is $1 + r^d_t$ and its functional form is discussed below.

The transaction function and CIA constraint are functions of consumption and CBDC holdings (instead of consumption and cash holdings). The transaction function faces the same assumptions as above, although I assume that transacting with CBDC is more efficient, which is reflected through the parametrization of this model in Section (3.1).

While this model does not contain both cash and CBDC, the literature is exploring how to best model the difference in transactions demand for economies with multiple cash-like assets. Oh and Zhang (2020) assumes that CBDC has lower transaction cost but is monitored by the government, which may deter households who participate in the informal economy from using CBDC. In contrast, households use cash in Mishra and Prasad (2021) because cash allows for tax evasion — modeled by a penalty function — while CBDC does not.

The output and capital evolution functions are the same in both models.

2.2.1 Equilibrium

First Order Conditions. The first order conditions for the CBDC model are

6. When possible, I do not repeat equations and derivations from the CB model in the previous section and instead focus on new material.

$$U_c(c_t, n_t) = \lambda_t(1 + \psi_c(c_t, d_{t-1})) + \lambda_t^{'} \quad (36)$$
$$-U_n(c_t, n_t) = \lambda_t A_t F_n(k_t, n_t) \quad (37)$$
$$\lambda_t = \beta E_t \lambda_{t+1} \left(\frac{1 + r_t}{1 + \pi_{t+1}} \right) \quad (38)$$
$$\lambda_t[1 + \Phi_k(k_{t+1} - k_t)] = \beta E_t \lambda_{t+1}[A_{t+1} F_k(k_{t+1}, n_{t+1}) + 1 - \delta + \Phi_k(k_{t+2} - k_{t+1})] \quad (39)$$
$$\lambda_t + \beta E_t \lambda_{t+1}^{'} \frac{1}{1 + \pi_{t+1}} = \beta E_t \lambda_{t+1} \left(\frac{1 + r_t^d}{1 + \pi_{t+1}} - \psi_d(c_{t+1}, d_t) \right) \quad (40)$$

The household's intratemporal Euler condition is the same as in the CB model, with the exception that the transaction cost is for CBDC and not cash.

I again define an equilibrium with the exogenous productivity process A and world interest rate r^*.

Definition 2. *Given a set of realizations $\{A, r^*\}$ at time t, an equilibrium is a state-contingent set of allocations $\{c_t, n_t, b_t, d_t, k_t, I_t\}$ and prices $\{r_t, r_t^d, \pi_t\}$ such that*

1. *The allocations solve the problems faced by households at these prices.*
2. *All factor markets clear.*
3. *The government budget constraint or monetary authority interest rate rules are satisfied.*

In the CBDC model, the state variables are $\{b_t, r_t, k_t, A, d_t\}$.

2.2.2 Functional Forms

While the functional forms for utility, production, capital adjustment, and interest rate premium are the same across the CBDC and CB models, I assume the transaction function is

$$\psi(c_t, d_{t-1}) = \theta_1 \frac{c_t^{\gamma}}{d_{t-1}^{\zeta}} \quad (41)$$

where $\zeta > 1$ is the transaction efficiency of CBDC.

2.2.3 CBDC Interest Rate

I close the model by specifying the interest rate for CBDCs and a money supply rule. A multitude of methods exist to choose the interest rate. Mishra and Prasad (2021) exogenously chooses the steady-state rate of return on CBDC to be 5% per annum, Barrdear and Kumhof (2016) set price and quantity rules for CBDC while using an inflation forecast-based rule to set the risk-free interest rate, and Piazzesi, Rogers, and Schneider (2019) use a Taylor rule to set the CBDC interest rate. Were the CBDC interest rate to become the primary monetary policy lever, then governments may employ Taylor rules. If a CBDC were to become a secondary interest-bearing asset instead, the control of its rate may be different. Nonetheless, optimal monetary policy to maximize welfare is a design issue that has only been lightly explored.

First, I assume a money supply analogous to the government budget constraint equation in the previous section with a money supply growth parameter μ such that

$$d_{t-1} = \frac{1+\mu}{1+\pi_t} d_t \tag{42}$$

Second, I assume that the monetary authority sets the CBDC interest rate $1 + r_t^d$ by maintaining an exogenous, fixed (positive) spread between the CBDC interest rate and the risk-free interest rate r_t:

$$r_t^d = r_t - \theta_2 \tag{43}$$

where θ_2 is the size of this spread.

Third, I assume that the monetary authority sets the CBDC interest rate using a Taylor-type rule of the form (Taylor 1993):

$$i_t^d = \pi_t + \rho_m(\pi_t - \bar{\pi}) + (1-\rho_m)(y_t - \bar{y}) \tag{44}$$

This formulation does not account for a monetary shock. Parameter $\rho_m \in [0, 1)$ characterizes the persistence of the monetary policy rule.

I abstract away from the zero lower bound for the risk-free interest rate, although this is an area of continued work. By assumption, the nominal CBDC interest rate can be negative. Generally, governments would keep r_t^d low to ensure that CBDC would be used in everyday transactions instead of being hoarded for its high rate of return. If the return on CBDC is higher than bonds (e.g. $\theta_2 < 0$), households would not spend CBDC but would tradeoff consumption.

2.2.4 Steady State

With the previous assumption that

$$\beta \left(\frac{1+r^*}{1+\bar{\pi}} \right) = 1$$

the set of equations that governs the CBDC economy in the steady state is:

$$\bar{r^*} = \frac{(1+\bar{\pi})}{\beta} - 1 \tag{45}$$

$$\bar{\kappa} \equiv \frac{\bar{k}}{\bar{n}} = \left(\frac{\beta^{-1} - 1 + \delta}{\alpha} \right)^{\frac{1}{\alpha-1}} \tag{46}$$

$$\bar{n} = [(1-\alpha)\bar{\kappa}^{\alpha}]^{\frac{1}{\omega-1}} \tag{47}$$

$$\bar{k} = \bar{\kappa}\bar{n} \tag{48}$$

$$\bar{r} = \bar{r^*} \tag{49}$$

$$\bar{r}^d = \bar{r} - \theta_2 \tag{50}$$

$$\bar{y} = A\bar{\kappa}^{\alpha}\bar{n} \tag{51}$$

$$\bar{I} = \delta\bar{k} \tag{52}$$

$$\bar{c}\bar{\pi} - \bar{c}\left(\frac{\bar{r}^d - \bar{\pi}}{1+\bar{\pi}}\right) + \theta_1\bar{c}^{\gamma}\left(\frac{c}{1+\bar{\pi}}\right)^{\zeta} = \bar{b}\left(\frac{\bar{r}^* - \bar{\pi}}{1+\bar{\pi}}\right) - \bar{\kappa}^{\alpha}\bar{n} - \delta\bar{k} \tag{53}$$

$$\bar{d} = \bar{c}(1+\bar{\pi}) \tag{54}$$

$$\bar{\lambda} = \frac{\left(\bar{c} - \frac{\bar{n}^{\omega}}{\omega}\right)^{-\sigma}}{1 + \theta_1\gamma\frac{\bar{c}^{\gamma-1}}{\bar{d}^{\zeta}}} \tag{55}$$

$$0 = \Delta\bar{d} = \frac{1+\mu}{1+\bar{\pi}} \Rightarrow \bar{\pi} = \mu \tag{56}$$

Only a few equations have modified forms and parameters compared to the CB model. The steady-state rate of interest on CBDC is a function of the steady-state risk-free rate and an exogenous spread. Steady-state consumption in equation (53) adds the CBDC interest rate.

I also assume that steady-state CBDC holdings are equal to steady-state consumption multiplied by the rate of inflation. This may not be realistic in an economy with both cash and bonds, as households would hold CBDC for its nonzero rate of return when the nominal CBDC interest rate is positive. However, in an economy with only one interest-bearing cash-like asset, CBDC is the only way to acquire consumption goods for households and would thus be used.

Bonds and CBDC enter the household budget constraint in much the same way: both assets have rates of return. But, bonds are not in the CIA constraint and are unable to be exchanged for consumption goods. This is an important distinction for the quantitative analysis pursued below.

2.3 Model Comments

Much of the modeling strategy for the non-CBDC model and the CBDC model are similar. Given the tools of modern macroeconomics, we imagine CBDC to simply be an asset governed by the same ideas explored in the past few decades in general equilibrium models. From a mathematical perspective, this is entirely sensible. There is nothing special about CBDCs that cannot easily be represented in equations. The CBDC is simply an interest-bearing asset that can also be used for transactions; in this model, it can be exchanged for consumption goods. However, CBDC is a new innovation to (central) banking. Various behavioral and political economy factors will likely govern its use, such as heterogenous preferences for different forms of money for transaction ease, security, and anonymity (Agur, Ari, and Dell'Ariccia 2020). It remains to be seen the degree to which such factors will necessitate changes or complications in models of CBDC.

Compared to Ferrari, Mehl, and Stracca (2020), my approach differs in the explicit role given to cash-like assets to purchase today's consumption expenditures through the addition of the CIA constraint. In addition, I do not include the financial sector or complicated price or quantity rules for setting the interest rate on CBDCs. This helps to clarify some of the quantitative exercises in the next section, although my setup contains fewer areas to simulate shocks.

3 Numerical Analysis

In this section, I conduct quantitative experiments[7] to simulate the effect of CBDC in the open economy. Parameters are taken from the literature when possible. When the literature or data do not provide parameters, intuition for the values are given or statics in the steady state are displayed. Given the lack of data on CBDC, this analysis does not attempt to match or explain moments.

The domestic economy models Canada, which is a prototypical small open economy, following Galí and Monacelli (2005). Additionally, the Bank of Canada has heavily invested into research on CBDC policy (Davoodalhosseini 2018) and framed adoption as a contingency plan for periods of financial instability, which makes Canada a particularly useful case to study.

One useful metric for CBDC usage is the fraction of CBDC as an asset compared to the total assets in the economy. That is, I define

$$\bar{d}^{share} \equiv \frac{\bar{d}}{\bar{f}}$$

where $\bar{f} \equiv \bar{d} + \bar{k} + \bar{b}$ is the total assets held in steady state.

3.1 Calibration

3.1.1 Business Cycle Parameters

I follow Mendoza (1991) and Uribe and Schmitt-Grohé (2017) in finding parameters to fit the Canadian business cycle. I set the intertemporal elasticity of substitution at $\sigma = 2$, which is within the range found by empirical literature of $[2, 10]$ (Heipertz, Mihov, and Santacreu 2020). The discount rate is set at $\beta = 0.971$, which corresponds to an annualized world rate of return of $r^* = 4\%$. The depreciation rate of capital is set to $\delta = 0.1$. The capital elasticity of the production function, α, is set to 0.32 to match the labor share in Canada, which is 0.68.

To match the second moments of the Canadian economy, I accordingly set $\bar{b} = 0.7442$, which follows the assumption that there is a trade surplus large enough to service external debt. This is the key stationarity parameter from Uribe and Schmitt-Grohé (2017). I set the Frisch elasticity to $\omega = 1.455$, the capital adjustment cost to $\phi = 0.028$, and the debt elasticity of the risk-free interest rate to $\psi_1 = 0.000742$. The autocorrelation parameter of the TFP shock is $\rho = 0.42$, and the standard deviation of that shock is $\sigma_a = 0.0129$.

3.1.2 CBDC Parameters

I follow Mishra and Prasad (2021), which is one of the few papers to quantitatively study the CBDC problem, to set some of the CBDC-related parameters. The level parameter for the transaction function is set at $\theta_1 = 2$ and the transaction cost for consumption at

7. Replication code is available at https://github.com/darrenwchang/simple-cbdc. I use the `linearsolve` package for approximating DSGE models in `python 3.7.3` to linearize and solve the model. Additional information is available at https://github.com/letsgoexploring/linearsolve.

$\gamma = 2$. I assume that CBDC has a higher transaction efficiency than cash; in other words, $\zeta \geq \eta$. The baseline level of the transaction efficiency of CBDC is set to $\zeta = 1.75$ and the transaction efficiency of cash is set to $\eta = 1.05$. Money supply growth is set to $\mu = 0.02$. The Taylor rule is calibrated with a persistence parameter of $\rho_m = 0.75$.

The CBDC-risk-free parameter is calibrated to $\theta_2 = 0.01$. Existing data from the United States provides some insight into whether this is a realistic calibration, although there is no real-world data about these spreads. The 10-Year Treasury Constant Maturity Minus Federal Funds Rate data sourced from the Federal Reserve Bank of St. Louis[8] are released on a daily basis, and the average from 1990:Q1 to 2010:Q4 is 0.0157, which is a spread of 157 bp. On the other hand, the spread between the 3-Month Treasury Bill and the effective Federal Funds Rate is often (even mostly) negative as the policy rate is typically higher than the risk-free rate.[9]

The introduction of another interest-bearing asset into the economy would cause previously risk-free bonds to become relatively riskier. Therefore, the risk-free bonds would begin to model slightly more riskier assets, e.g. a longer-term security, while the CBDC asset would have lower rates because of its cash-like nature.

Table 1: Parameters for CB and CBDC Models

Parameter	**Description**	**Value**
	Business Cycle Parameters[1]	
β	Discount factor	0.971
σ	Intertemporal elasticity of substitution	2.000
δ	Depreciation rate of capital	0.100
α	Capital elasticity of production	0.320
$\bar{b}$	Average level of debt	0.744
ω	Frisch labor elasticity	1.455
ϕ	Capital adjustment cost	0.028
ψ_1	Debt elasticity of risk-free interest rate	0.000742
A	Productivity	1
ρ	Autocorrelation of TFP shock	0.420
σ_a	Standard deviation of TFP shock	0.0129
	CBDC Parameters[2]	
θ_1	Level parameter of transaction function	2.000
γ	Transaction cost for consumption	2.000
η[3]	Transaction efficiency of cash	1.050
ζ	Transaction efficiency of CBDC	1.750
θ_2	CBDC-risk-free spread	0.010
μ_h	Cash supply growth	0.020
μ_d	CBDC supply growth	0.030
ρ_m	Taylor rule persistence	0.750

[1] Follows Mendoza (1991) and Uribe and Schmitt-Grohé (2017).
[2] Follows Mishra and Prasad (2021).
[3] η is set to be 0.6 times ζ.

8. See https://fred.stlouisfed.org/series/T10YFF.
9. See https://fred.stlouisfed.org/series/T3MFF.

3.2 Statics

Table 2: Steady State Values for CB and CBDC Models

Variable	Description	CB Model Value	CBDC Model Value
$\bar{c}$	Consumption[1]	0.414	$4.92 * 10^{-7}$
$\bar{k}$	Capital	4.101	4.101
$\bar{h}$	Cash	0.418	-
$\bar{b}$	Bonds	0.744	0.744
$\bar{n}$	Labor	1.089	1.089
$\bar{y}$	Output	1.664	1.664
$\bar{I}$	Investment	0.410	0.410
$\bar{r}$	Risk-free interest	0.050	0.040
$\bar{r}^d$	CBDC interest	-	0.030
$\bar{d}$	CBDC	-	$4.97 * 10^{-7}$
$\bar{\pi}$	Inflation	0.02	0.03

[1] Solved using the `lm` method in the `SciPy.Optimize` package, which uses a modification of the Levenberg-Marquadt algorithm.

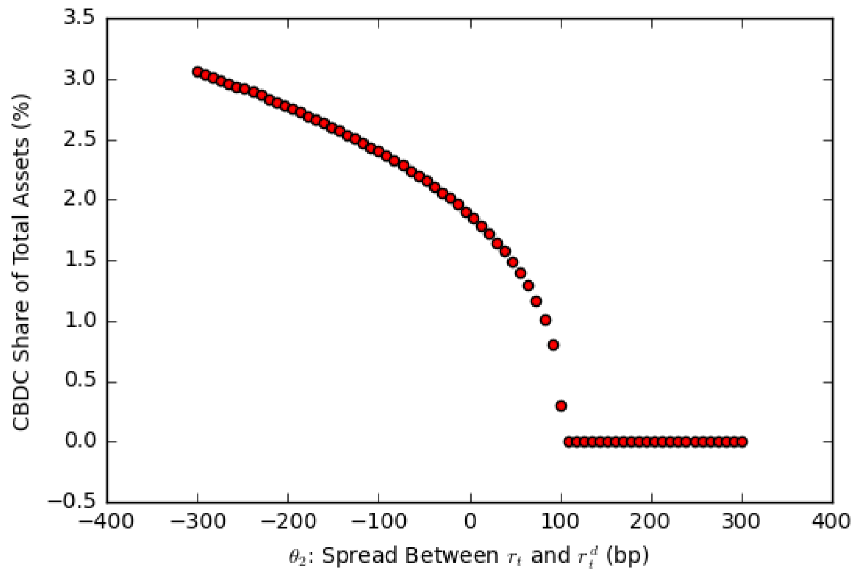
Based on the above parameters, the steady state values in each economy are in Table (2).

These values are not meaningful in and of themselves, but provide benchmarks for later experiments and dynamics. Many of the values are the same across the two economies, although the addition of a new interest-bearing asset is internalized by the consumption function and indeed reduces the value of steady-state consumption relative to the CB economy.

Next, I describe the paths of steady-state values when parameters are changed in the CBDC economy. I first experiment with the value of θ_2, the spread between the risk-free rate and the CBDC interest rate. I calculate steady-state values for CBDC share and bond holdings. CBDC, CBDC share, and consumption are all closely correlated in the steady state, based on the definition of the CIA constraint.

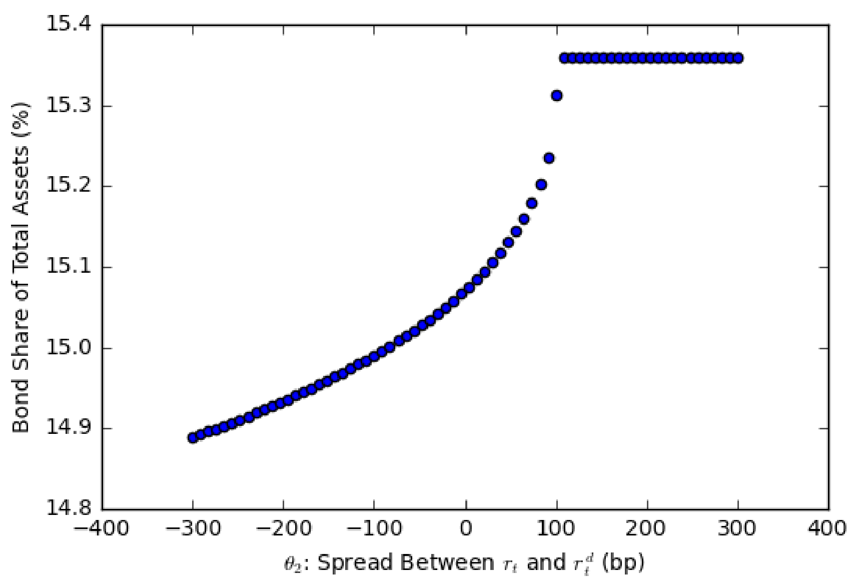
CBDC share is a decreasing convex function in θ_2 until $\theta_2 > 0.01$, past which the functions return a zero bound at this level of inflation. This is demonstrated in Figure (1).

Figure 1: CBDC Share and θ_2 Parameter



Note: This figure shows the relationship between the steady-state share of CBDC of total assets and the risk-free-CBDC spread.

Figure 2: Bond Share and θ_2 Parameter

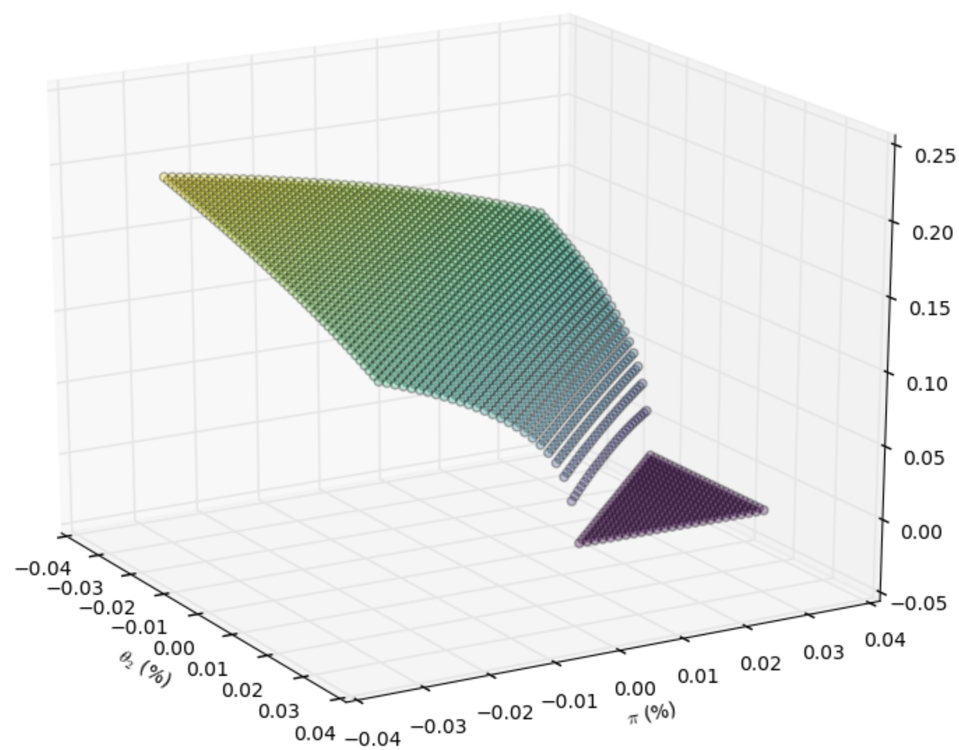


Note: This figure shows the relationship between the steady-state share of bonds of total assets and the risk-free-CBDC spread.

Bond share (and capital share) are increasing convex functions in θ_2, which is shown in Figure (2) although also zero-bounded at $\theta_2 > 0.01$. The asset shares in the CBDC economy follow conventional asset pricing wisdom. As the spread between $\bar{r}$ and $\bar{r}^d$ increases, CBDC becomes relatively less attractive for households to hold as a financial asset because the rate of return is lower. Conversely, as the spread between the two rates of interest decreases and even becomes negative, CBDC becomes relatively more attractive. In that case, bond holdings decrease and CBDC holdings increase as a percentage of total assets.

Figure (3) shows the effect of different parametrizations for inflation and risk-free-CBDC interest spread on steady-state consumption. Consumption is decreasing in both parameters and at a certain point becomes approximately 0, as households would rather hold assets than consume. Given the steady-state formulation, inflation and money supply growth are equivalent in the steady state. On the scatter plot, the height of the dots displays the steady-state level of consumption while inflation is on the x-axis and the risk-free-CBDC spread is on the y-axis. The purple dots indicate when consumption is approximately zero. Based on this figure, θ_2 and π are parameters that work in lockstep: decreasing either or both increases consumption.

Figure 3: Effects of Varying θ_2 and π on Consumption



Note: This figure displays values for steady-state consumption with varying levels for the inflation parameter π on the x-axis and the spread parameter θ_2, which is on the y-axis. Dots shaded in purple display when consumption is approximately zero. As steady-state consumption increases, dots are sequentially shaded in blue, green, and yellow.

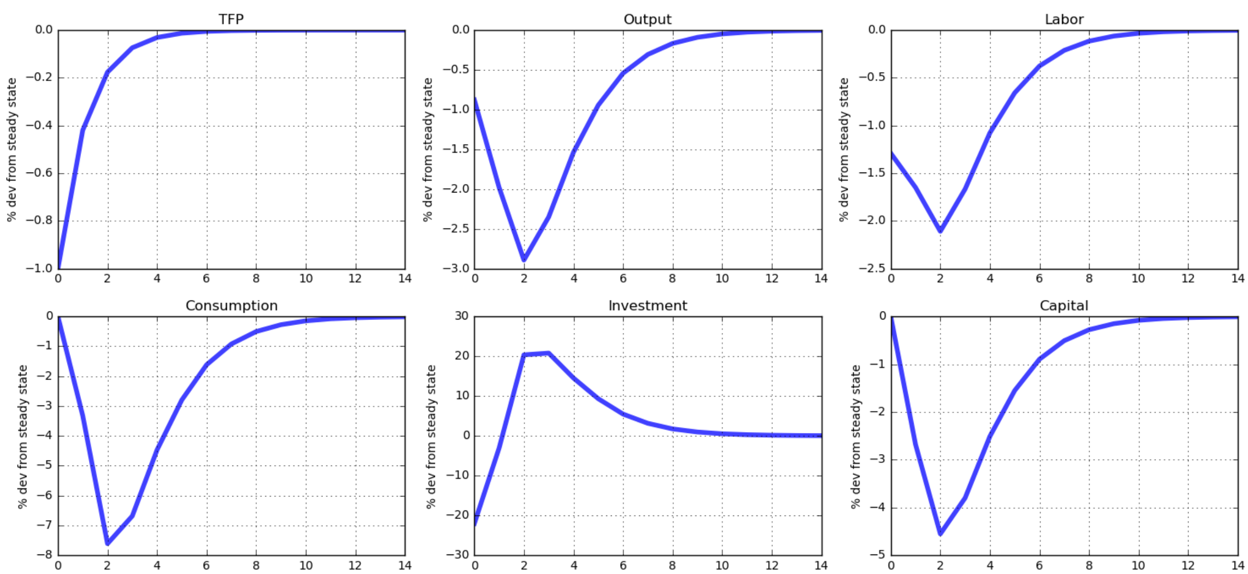
3.3 Impulse Responses

I now study contractionary productivity shocks that decrease productivity by 1 unit. As expected, the shock leads to a decline in domestic productivity and output that slowly dissipates as the economy recovers to the steady state.

Figures (4)-(7) should be considered together. Think of the CB model as a baseline model that mirrors status quo monetary dynamics. No country has adopted a true, interest-bearing CBDC — yet. In contrast, the CBDC model attempt to describe an economic situation in which CBDC becomes the only medium of exchange for consumption goods. In this way, two comparisons are being made. One is across each time period (after the shock) and the other is across the two models. Impulse responses for the CB model are shown in blue while impulse responses for the CBDC model are shown in red. All negative shocks occur in period $t = 0$.

Figure (4) displays the response to an unanticipated one time shock in an economy that has cash and bonds but no CBDC. These follow shocks to typical RBC-CIA models, like those seen in Walsh (2010). In particular, output, labor, consumption, and capital are negatively impacted by the TFP shock before recovering to steady-state values by about eight quarters, or two years. In contrast, investment first decreases before increasing and recovering to the steady state.

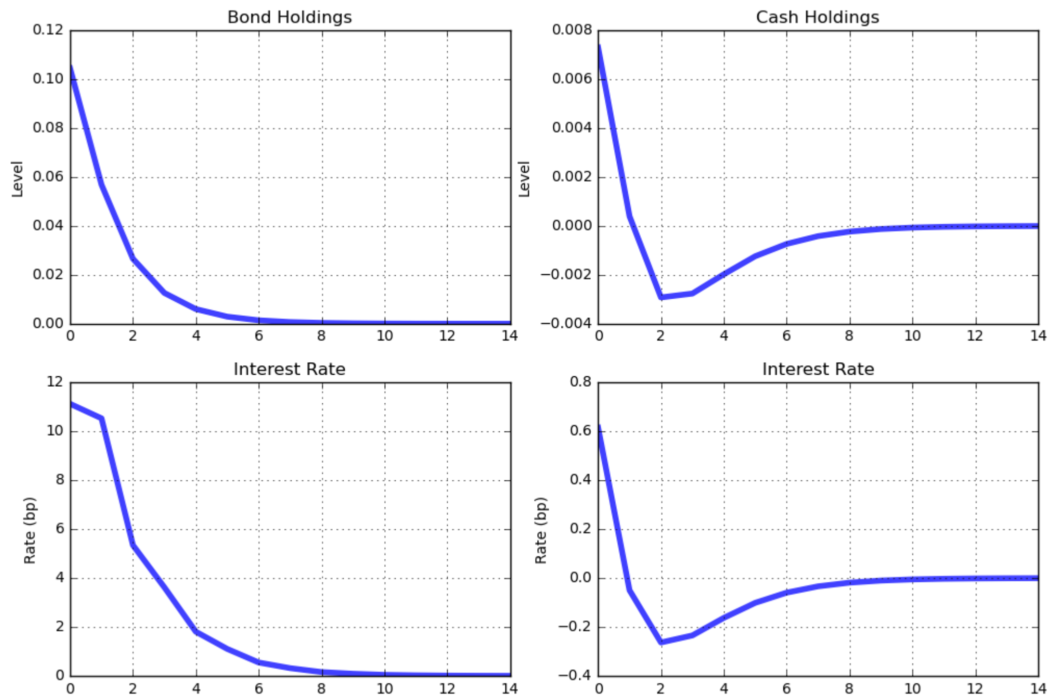
Figure 4: CB Model Impulse Responses following Negative TFP Shock



Note: These panels show impulse responses to a negative one unit TFP shock which occurs in period 0. All panels are presented in percentage deviations from the steady state.

Figure (5) shows impulse responses for variables related to interest-bearing assets and cash. As expected, the bond holdings increase after the negative TFP shock before decreasing to steady-state values. Cash holdings initially become negative after the shock before returning to the steady-state. The interest rate on bonds also increases before returning to lower levels that are closer to the steady state. At no time does $r_t < 0$. In each period, the calculation of r_t is constrained by the exogenously set value of r^*, which creates relatively large variability in the interest rate.

Figure 5: CB Model Impulse Responses: Cash, Bonds, and Interest

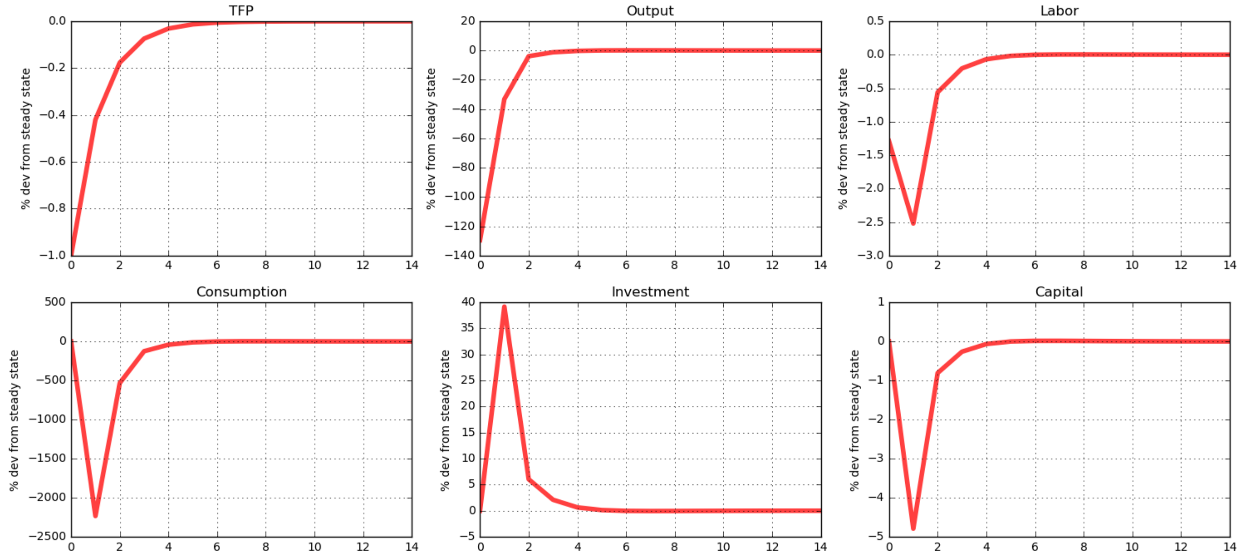


Note: These panels show impulse responses to a negative one unit TFP shock which occurs in period 0 for variables related to money and assets. The bond and cash holdings variables are represented as levels and the interest rate is represented in basis points.

The main quantitative result for the CBDC model is found in the following figures. In figure (6), macroeconomic variables are plotted after simulating a one unit negative TFP shock, as is done in figure (4). The interest rate spread rule is employed. The response of these variables in the CBDC model is somewhat qualitatively similar to the response in the CB model, as many of the major assumptions in the theoretical approach are the same.

However, the differences are important to note.The CBDC impulse responses appear to have larger variability and magnitude, especially for consumption. The steady-state consumption values are extremely small, so changes are quickly apparent in the impulse responses.

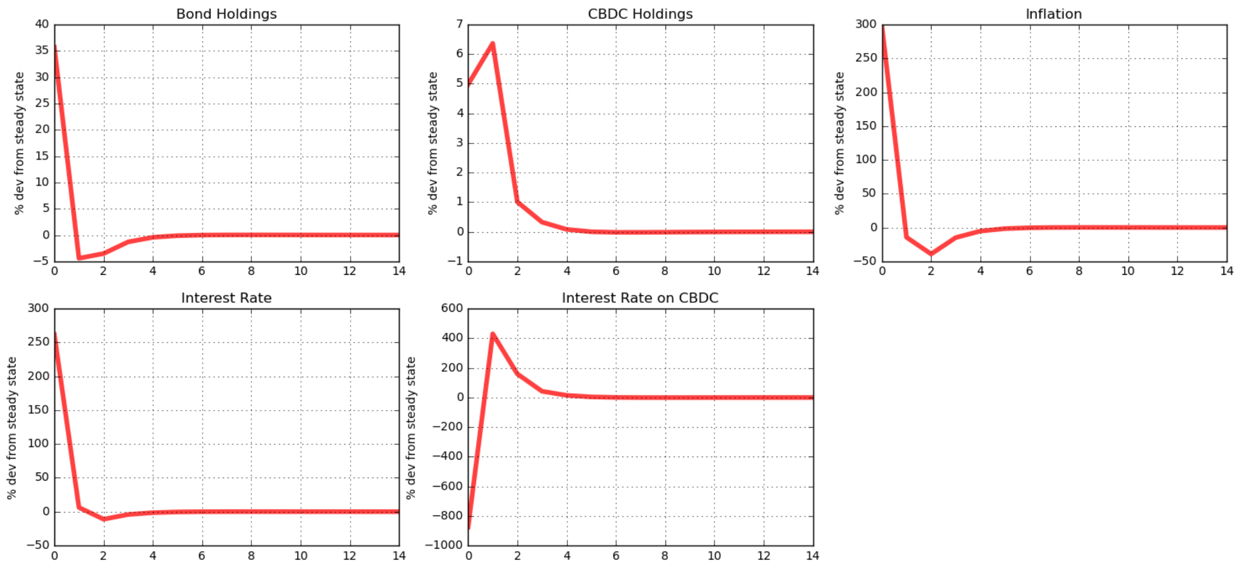
Figure 6: CBDC Model Impulse Responses following Negative TFP Shock (Spread Rule)



Note: These panels show impulse responses to a negative one unit TFP shock which occurs in period 0 with the spread rule. All panels are presented in percentage deviations from the steady state.

Figure (7) demonstrates the effect of the impulse response on different measures relating to interest-bearing assets and money. Because of the role of CBDC as a transaction technology, the negative shock to consumption is related to an increasing in CBDC holdings (as opposed to using CBDC for consumption). CBDC holdings follow the same pattern as the interest rate on CBDC (the right-most panel on the bottom row). The steady state value for the interest rate on bonds is higher than the steady state value for the interest rate on CBDC; as a result, bond holdings increase immediately while CBDC holdings lag. The two assets in this economy are highly correlated in both the interest rate and level, although the timing of this relationship is not obvious.

Figure 7: CBDC Model Impulse Responses: CBDC, Bonds, and Interest (Spread Rule)



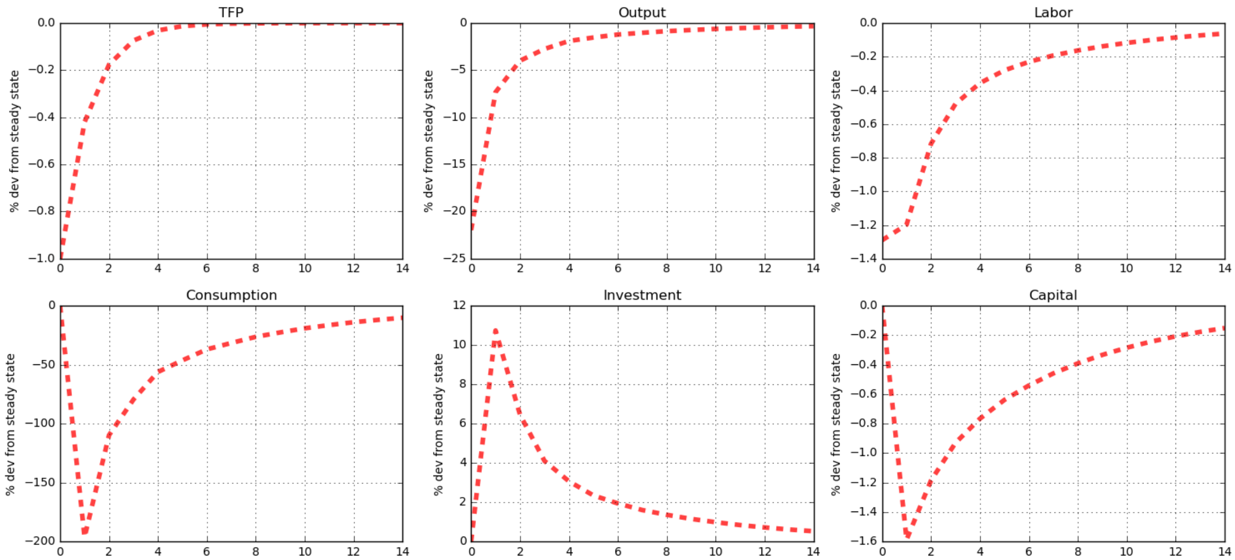
Note: These panels show impulse responses to a negative one unit TFP shock which occurs in period 0 for variables related to money and assets with the spread rule. The bond and CBDC holdings variables are represented as levels and the interest rate is represented in basis points.

Impulse responses are also constructed using the Taylor rule in figures (8) and (9). These impulse responses are shown with red dashed lines. Qualitatively, the impulse responses for the spread rule and the Taylor rule are similar in direction, although the Taylor rule impulse responses demonstrate more smoothing from period to period. The specific form of the Taylor equation that is estimated is a calibrated version of equation (44):

$$i_t^d = \pi_t + 0.75(\pi_t - \bar{\pi}) + 0.25(y_t - \bar{y}) \tag{57}$$

The dip in consumption following the productivity shock has a far smaller magnitude in the impulse responses with the Taylor rule, indicating that the flexibility of the interest rate dampens adverse economic outcomes.

Figure 8: CBDC Model Impulse Responses following Negative TFP Shock (Taylor Rule)



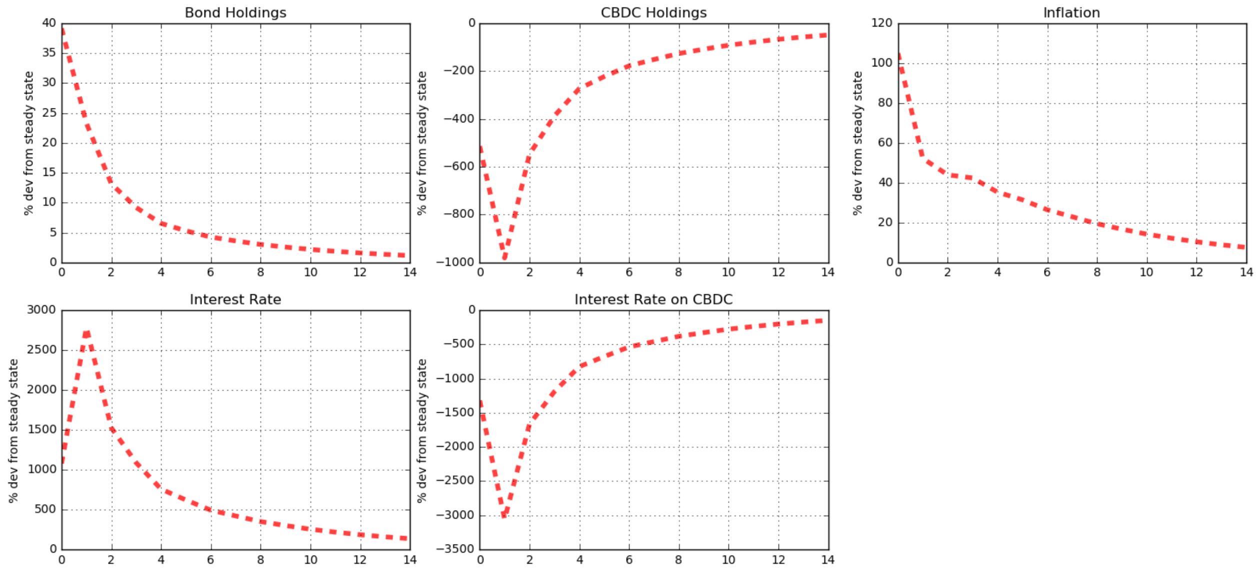
Note: These panels show impulse responses to a negative one unit TFP shock which occurs in period 0 with the Taylor-type rule. All panels are presented in percentage deviations from the steady state.

What is interesting to compare, though, are the assets in the CBDC economy with different types of interest rate rules.[10] With the Taylor rule, bond holdings and CBDC holdings have opposite (instead of lagged) effects, as do the interest rates. This is sensible: with the Taylor rule, the interest rate on CBDC is no longer derived from the risk-free interest rate. Instead, the interest rate on CBDC responds to the inflation and output levels of the domestic economy. Consumers flexibly respond to the interest rates and choose to hold whichever asset generates a higher return while also accounting for the transaction purpose of the CBDC.

This demonstrates that the two interest-bearing assets are effectively modeled in the presence of different interest rate rules. With the spread rule, the choice set of consumers for interest-bearing assets is limited. With the Taylor rule, the interest rates are allowed to fluctuate freely.

10. The IRF graphs are not presented on the same graph due to differences in scaling.

Figure 9: CBDC Model Impulse Responses: CBDC, Bonds, and Interest (Taylor Rule)



Note: These panels show impulse responses to a negative one unit TFP shock which occurs in period 0 for variables related to money and assets with the Taylor-type rule. All variables are expressed in percentage difference from the steady-state values.

How do these results fit with the current literature on CBDC and open economies? In many ways, the results are similar, although this paper chooses mathematically simpler approaches in several areas. Consequently, some of the quantitative suggestions appear unrealistic or have unclear interpretations. With a liquidity constraint role for money, I qualitatively and graphically show higher variability in impulse responses, which matches George, Xie, and Alba (2020).

Two findings extend Ferrari, Mehl, and Stracca (2020). First, I also demonstrate that CBDC increases the magnitude of the impulse response when compared to a baseline model with only one interest-bearing asset. Second, design matters. Even seemingly small changes in parametrization values or the choice of interest-rate rules can have large effects on the reaction of the domestic economy after a shock, as shown by the differences in the impulse responses when a Taylor rule is in place versus when a spread rule is in place.

This work assumes that world bonds have an exogenous interest rate that is higher than the CBDC interest rate (as long as $r^* - p(\bar{d}_t) - \theta_2 > 0$), meaning that households would not strictly prefer CBDC, as CBDC have a lower interest rate. Ferrari, Mehl, and Stracca (2020) write that greater interlinkages between the home economy and the world economy are due to a new arbitrage condition, which:

> ...defines the risk-free rate in the foreign economy as a mark-up on the remuneration of the CBDC (i.e. the interest rate on the CBDC adjusted for exchange rate risk). This is quite intuitive as households, for the same remuneration, strictly prefer to hold CBDC relative to a foreign bond given that the CBDC provides liquidity services.

While not yet explicitly discussed in mathematical terms in this paper, the "new" arbitrage condition is much less strong in the CBDC model pursued in this paper when using the spread-based exogenous interest rate rule.

4 Conclusion

In this paper, I develop (very) simple two-country general equilibrium models that build intuition for the operation of CBDCs in small open economies. Cash and CBDC are both used to obtain consumption goods, but have two key differences: higher transaction efficiency and the possibility of nonzero nominal rates of return for CBDC. Varying parameters in the steady state show that CBDC use is decreasing in the spread in interest rates between the risk-free bond rate and the CBDC rate. Quantitative exercises demonstrate small open economies with CBDCs and bonds experience higher magnitude shocks compared to small open economies that only use cash and bonds. Moreover, CBDC design matters: the economic response with a flexible Taylor-type rule is lower in magnitude and smoother than the response to a shock in the presence of a fixed interest-rate rule. This model provides a first-pass analysis of cross-border monetary policy for central bankers and policymakers prior to implementing CBDC.

While the beauty of this model is in its simplicity, there are several elements of the open economy macroeconomics that are unaddressed. These additional considerations are left for future work. Welfare calculations would be helpful to characterize the utility of consumers across economies, especially as others have shown that CBDCs generally increase welfare. The exchange rate and the behavior of open economies near the zero lower bound for the risk-free interest rate are other interesting possibilities. Exploring different ways to set the CBDC interest rate through, for example, by using exchange rate rules or other feedback-based money supply rules are also important to consider, especially for monetary authorities. Overall, this work uses strong assumptions to ponder a theoretical world economy with digital fiat currency.

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A Nominal Constraints

In the CB economy, the home nominal budget constraint is

$$P_t b_t + P_t y_t + H_t + P_t \psi(c_t, H_{t-1}) \leq P_t (1 + r_{t-1}) b_{t-1} + P_t(c_t + I_t) + P_t \Phi(k_{t+1} - k_t) + H_{t-1} \quad (58)$$

and the nominal liquidity constraint is

$$P_t c_t \leq H_{t-1} \quad (59)$$

In the CBDC economy, the home nominal budget constraint is

$$P_t b_t + P_t y_t + D_t + P_t \psi(c_t, D_{t-1}) \leq P_t (1 + r_{t-1}) b_{t-1} + P_t(c_t + I_t) + P_t \Phi(k_{t+1} - k_t) + \left(1 + r^d_{t-1}\right) D_{t-1} \quad (60)$$

and the nominal liquidity constraint is

$$P_t c_t \leq D_{t-1} \quad (61)$$

B Equilibrium Derivation

I solve the model using standard Lagrangian techniques. Let λ_t be the Lagrangian multiplier for the household budget constraint and $\lambda_t^{'}$ be the Lagrangian multiplier for the cash-in-advance constraint. Then, the Lagrangian is:

$$\begin{aligned}
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ U(c_t, n_t) + \lambda_t \Bigg[\frac{1 + r_{t-1}}{1 + \pi_t} b_{t-1} + c_t + I_t + \\
\Phi(k_{t+1} - k_t) + \frac{h_{t-1}}{1 + \pi_t} - b_t - y_t - h_t - \psi(c_t, h_{t-1}) \Bigg] \\
+ \lambda_t^{'} \left[\frac{h_{t-1}}{1 + \pi_t} - c_t \right]
\end{aligned} \quad (62)$$

In equilibrium, I assume that the CIA constraint is not binding; that is, the Lagrangian multiplier associated with the constraint ($\lambda_t^{'}$) is 0. Then, the household intertemporal conditions can be derived. These conditions are shown in the main text in equations (7) to (11). A similar solution method is used to obtain the equilibrium conditions for the CBDC economy.