NATIONAL UNIVERSITY OF SINGAPORE

CS2040 – DATA STRUCTURES AND ALGORITHMS

(Semester 1: AY2021/22)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. **Do NOT flip / turn over the test paper until you are told to do so**
2. Shade your **student number** in pages 1 AND 3 of the answer sheet. Do **NOT** write your name!
3. **Answer question 0** as instructed, or your submission will NOT be graded
4. Do NOT deface the answer sheet, or add staples to it. **COMPLETELY shade** the bubble for each answer on page 1 using a fairly **dark pencil**. You may write on page 2 using either pen or pencil
5. **Submit only the answer sheet** at the end of the assessment, unless you need more space for Q9a. It is your responsibility to ensure that you have submitted it, and submitted the correct answer sheet
6. If you fail to submit the correct answer sheet, fail to provide **correct particulars** or prevent the options from being **automatically detected** by software, we will consider it as if you did not submit your answers. In the best case, **marks will be deducted**
7. No extra time will be given at the end of the assessment for you to write your particulars, shade the answer sheet or write/transfer answers. You must do them **before** the end of the assessment
8. This paper consists of **10** questions including Q0. Not more than one option should be shaded per grid. The question paper comprises ten (**10**) printed pages including this front page. The answer sheet comprises two (**2**) printed pages
9. This is an open-hardcopy-notes examination but **WITHOUT** electronic materials
10. Marks allocated to each question are indicated. Total marks for the paper is **100**
11. The use of electronic **calculator** is **NOT** allowed

Sect Qn	***Max***	***Marks***
Q0	Min -100	
S1 Q1 - 4 ab	44	
S2 Q5 - 8 ab	44	
S3 Q9a XOR S3 Q9bcdefgh	12	
Total	**100**	

Question 0 **[0 for ENTIRE PAPER if not done satisfactorily!]**

Please read the following NUS Code of Student Conduct (in particular the part on Academic, Professional and Personal Integrity), as well as items B and C below.

(A) **I am aware of, and will abide by the NUS Code of Student Conduct (in particular the part on Academic, Professional and Personal Integrity as shown below) when attempting this assessment.**

- Academic, Professional and Personal Integrity
  1. The University is committed to nurturing an environment conducive for the exchange of ideas, advancement of knowledge and intellectual development. Academic honesty and integrity are essential conditions for the pursuit and acquisition of knowledge, and the University expects each student to maintain and uphold the highest standards of integrity and academic honesty at all times.
  2. The University takes a strict view of cheating in any form, deceptive fabrication, plagiarism and violation of intellectual property and copyright laws. Any student who is found to have engaged in such misconduct will be subject to disciplinary action by the University.
  3. It is important to note that all students share the responsibility of protecting the academic standards and reputation of the University. This responsibility can extend beyond each student's own conduct, and can include reporting incidents of suspected academic dishonesty through the appropriate channels. Students who have reasonable grounds to suspect academic dishonesty should raise their concerns directly to the relevant Head of Department, Dean of Faculty, Registrar, Vice Provost or Provost.

(B) **I have read and understood the rules of the assessments as stated below.**

1. Students should attempt the assessments on their own. There should be no discussions or communications, via face to face or communication devices, with any other person during the assessment.
2. Students should not reproduce any assessment materials, e.g. by photography, videography, screenshots, or copying down of questions, etc.

(C) **I understand that by breaching any of the rules above, I would have committed offences under clause 3(l) of the NUS Statute 6, Discipline with Respect to Students which is punishable with disciplinary action under clause 10 or clause 11 of the said statute.**

3) Any student who is alleged to have committed or attempted to commit, or caused or attempted to cause any other person to commit any of the following offences, may be subject to disciplinary proceedings:

l) plagiarism, giving or receiving unauthorised assistance in academic work, or other forms of academic dishonesty.

To answer Q0, shade the option on the answer sheet to declare that you have **read and will abide** by the NUS Code of Student Conduct (in particular, (a) Academic, Professional and Personal Integrity), (b) and (c).

Ans: `Read and AGREE`

Section 1 - Ivan has photographed and recorded magikarp[1] population in a canal over time. The magikarp population **Xt** at a given integer time period **t**, is a positive integer which can be as large as a few billion. Ivan provides you with the number of time periods **N** (**N** may be large), as well as the magikarp populations **X_1 X_2 X_3 ... X_N** as input

As an **example**, for the scenario below:

time **t**	1	2	3	4	5	6	7
popn **X_t**	1	4	3	2	1	2	2

Ivan will provide you with input, which is consumed as you read it (can't reverse and re-read):

```
7
1 4 3 2 1 2 2
```

There may be additional inputs given for some questions

Ivan has **four problems** for you to solve, each *independent* of the others. For each problem, solve it using the best ADT and algorithm and state your **ONE character** answer in:

- part (a) - the ADT required
- part (b) - the worst case time complexity (tight bound)

corresponding to the option you choose from among the choices given to you

For Each Part (a) of Q1-4

X No DS / ADT needed	**B** Balanced **B**ST	**D** **D**eque* *when Queue/Stack NOT possible	**L** **L**inked List*
M **M**in Priority Queue	**P** *Max* **P**riority Queue	**Q** **Q**ueue	**S** **S**tack

Choose the option that is most appropriate and allows you to achieve the best time complexity

Important Notes

- For Q1-4 parts (a), use **AT MOST ONE** **instance** **of AT MOST ONE ADT**. For example, if your answer is $\boxed{Q}$, you canNOT use 2 Queues, nor any other data structure besides that one Queue. However, each element in the ADT may be compound data as long as it uses O(1) space
- For full credit, no **option** should be used to answer more than **one question** in this section - In the event of repeated option, only the *highest mark* for that option will be awarded *once*
- As highlighted above, if you answer Deque or LinkedList when the best ADT involves using a Queue or Stack, you will get 0 marks for that question

For Each Part (b)

A O(log(log(N))	**B** O(log(N))	**C** $O(\sqrt{N})$	**D** O(N)	**E** O(N log N)	**F** $O(N^{1.5})$	**G** $O(N^{1.5}\log(N))$
H $O(N^2)$	**J** $O(N^2\log(N))$	**K** $O(N^3)$	**L** $O(N^4)$	**M** $O(2^N)$	**P** O(N!)	**R** $O(N^N)$

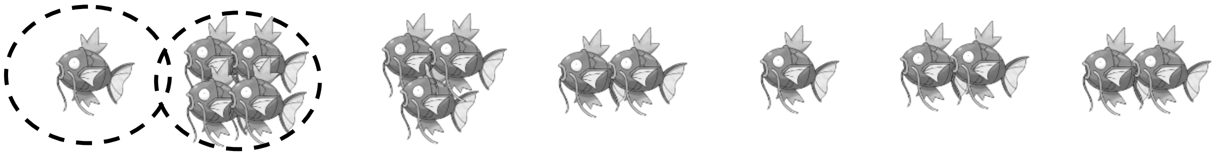
Unlike in part (a), it is possible for part (b) that the *same option may* (or may not) be the answer for *more than one question*, but a question will only have exactly one answer

Your choice in part (b) will cause the **question mark** to be **scaled** by a factor (i.e. multiplied), between 0.5 (completely wrong) and 1.0 (correct option with respect to *your chosen ADT* in part (a))

[1] Images of Magikarp on this and next 2 pages from https://www.pokemon.com/us/pokedex/Magikarp

Q1-4 **[44 marks == 4 x 11]**

Q1. **Elementary statistic** "Output the range (difference between the maximum and minimum) of the magikarp population. Using the example above, the range is $\boxed{3}$:



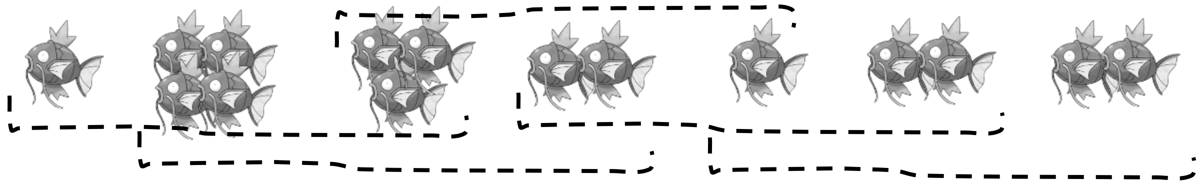
time **t**	1	2	3	4	5	6	7
pop $\mathbf{X}_t$	1	4	3	2	1	2	2

"

Q1(a) answer: ϵ {X, B, D, L, M, P, Q, S} Q1(b) answer: ϵ {A-R}

Q2. **Magikarp BOOM** "Along with the other inputs, you are also given an observation window length **L** (an integer where 1 ≤ **L** ≤ **N**). Output the **maximum** of the magikarp population in **EACH observation window**

Using the earlier example, if **L** = 3, the populations reported will be $\boxed{4\ 4\ 3\ 2\ 2}$:



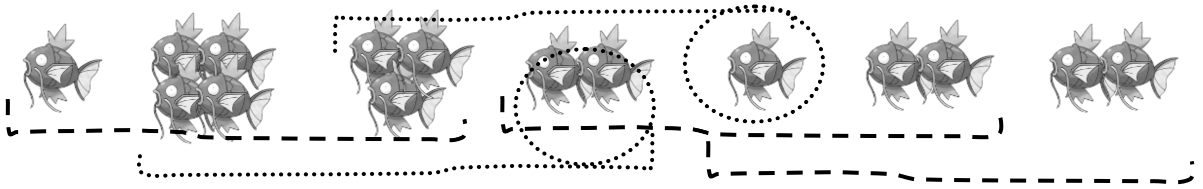
time **t**	1	2	3	4	5	6	7
pop $\mathbf{X}_t$	1	4	3	2	1	2	2

"

Q2(a) answer: ϵ {X, B, D, L, M, P, Q, S} Q2(b) answer: ϵ {A-R}

Q3. **Magi-less-karp** "Along with the other inputs, you are also given an observation window length **L** (an integer where 1 ≤ **L** ≤ **N**). Output the **number** of observation windows in which the last recorded population in the window is less than the average of that same window

Using the earlier example, if **L** = 3, the number of such windows is $\boxed{2}$:



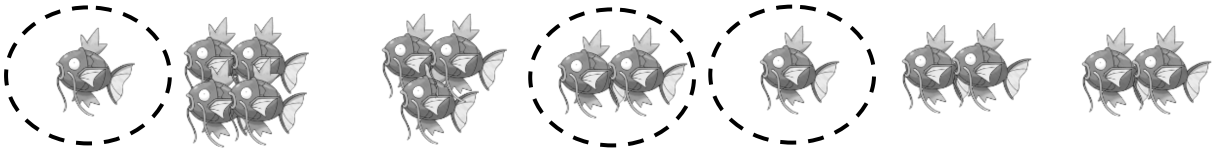
time **t**	1	2	3	4	5	6	7
pop $\mathbf{X}_t$	1	4	3	2	1	2	2

"

Q3(a) answer: ϵ {X, B, D, L, M, P, Q, S} Q3(b) answer: ϵ {A-R}

Q4. **Histori-karp** "Along with the other inputs, you are also given the number of photographs **P** (an integer where $\underline{1 \le \mathbf{P} \le \lfloor \sqrt{\mathbf{N}} \rfloor}$) to gather into an album. You want the photographs from time periods where magikarp populations are the lowest. The photographs may be in any order. Output a possible sequence of **P** distinct time periods for which the photographs can be taken from

Using the earlier example, if **P** = 3, one such possible sequence is `1 4 5`, out of 18 possible answers:



time **t**	1	2	3	4	5	6	7
pop $\mathbf{X}_t$	1	4	3	2	1	2	2

"

Q4(a) answer: `ϵ {X, B, D, L, M, P, Q, S}` Q4(b) answer: `ϵ {A-R}`

Section 2 - Fluffy gives you:

- a 2D rectangular Pokemon grid **G** of **N** pokemon (a 2D array; **N** may be large)
- a 18x18 2D array **E**

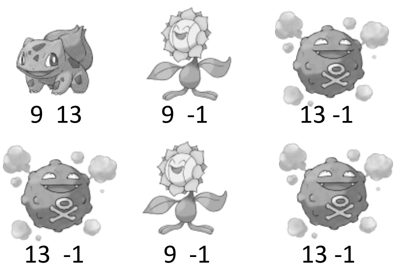
```
class Pokemon {
  public String name;
  public List<Integer> types;
  public int hp; // hit points
  // ctor, accessors/mutators
}
```

A Pokemon is defined above right. Each pokemon has *exactly one or two types*, a type encoded as an integer in 0..17 (since there are 18 different types). The pokemon type(s), when compared against the type(s) of another pokemon, affect how effective they are against each other

Koffing
[13]
70

In the example on the right, there is a pokemon named Koffing[2], having only 1 type - Poison which is encoded 13 - and having 70 hit points left (which can increase or decrease) as Koffing interacts with other pokemon. Such a pokemon object will be abstracted as shown on the far right

While a pokemon may appear repeatedly in **G**, each cell in **G** points to one distinct Pokemon object. As illustrated in the example below, there may be a few different Pokemon objects with the name "Koffing", possibly having different HP, but all Pokemon objects having the same name must have the same type(s)



G (Pokemon HPs hidden)

Bulbasaur [9, 13]	Sunflora [9]	Koffing [13]
Koffing [13]	Sunflora [9]	Koffing [13]

N = 6

[2] Pokemon images in section 2 adapted from https://bulbapedia.bulbagarden.net

As mentioned earlier, some pokemon types are *effective*[3] against other types. This information is stored in **E**, which is partially shown on the right

E[i][j] is a *boolean* which shows whether pokemon type i is effective against pokemon type j. For example, Koffing is effective against grass-typed pokemon because Koffing is of type 13 and grass type is encoded as 9

E (some rows and cols omitted)

i \ j	...	9	10	...	12	13	...
...							
9			true				
10						true	
...							
12							
13		true					
...							

Each pokemon (except those at the borders) are adjacent to 8 other pokemon (i.e. have 8 neighbours), which are the 8 pokemon nearest to them

Now with the inputs **G** and **E**, help Fluffy solve **four problems**, each *independent* of the others. For each problem, solve it using the best algorithm (using whatever ADTs as you need within your computation) and state your **ONE character** answer in:

- part (a) - the algorithm required
- part (b) - the worst case time complexity (tight bound)

corresponding to the option you choose from among the choices given to you

For Each Part (a) of Q5-8

T Binary **T**ree Traversal	**B** **B**readth First*	**D** **D**epth First*	**S** Topological **S**ort
	*can refer to either Search/Traversal		
J Di**J**kstra's SSSP	**F** Bellman-**F**ord	**P** **P**rim's MinST	**X** **P**rim's MaxST

Choose the option that is most appropriate and allows you to achieve the best time complexity

Important Note

- For full credit, no **option** should be used to answer more than **one question** in this section - In the event of repeated option, only the *highest mark* for that option will be awarded *once*

For Each Part (b) (options same as in Q1-4)

A O(log(log(N))	**B** O(log(N))	**C** $O(\sqrt{N})$	**D** O(N)	**E** O(N log N)	**F** $O(N^{1.5})$	**G** $O(N^{1.5} \log(N))$
H $O(N^2)$	**J** $O(N^2 \log(N))$	**K** $O(N^3)$	**L** $O(N^4)$	**M** $O(2^N)$	**P** O(N!)	**R** $O(N^N)$

Unlike in part (a), it is possible for part (b) that the *same option may* (or may not) be the answer for *more than one question*, but a question will only have exactly one answer

Your choice in part (b) will cause the **question mark** to be **scaled** by a factor (i.e. multiplied), between 0.5 (completely wrong) and 1.0 (correct option with respect to the *your chosen algorithm* in part (a))

[3] Disclaimer: This is simpler than the actual mechanism used in the real Pokemon

Q5-8 **[44 marks == 4 x 11]**

Q5. **3 will quarrel 4 will fight** (Pokemon HP, as well as input **E**, NOT used in this question)

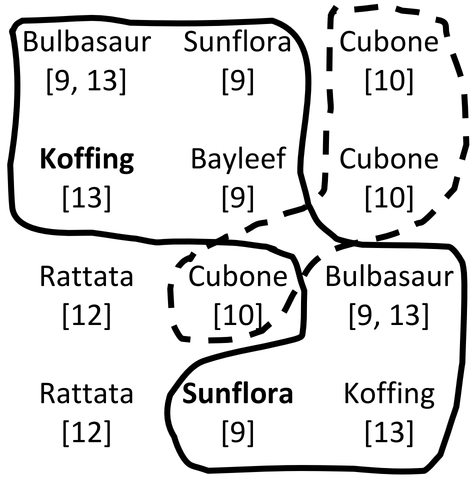
"A community is a maximal (i.e. includes as many cells as possibly can) group of cells in which for each cell **A** within the community, there must be some path to every other cell **B** in the same community such that every pair of adjacent pokemon on the path must share at least one type. Do note that a community must have 3 or more cells

Output the number of communities in **G**

As an **example**, in the 4x3 grid **G** on the right, there are 2 communities in **solid** and **dashed** outline

Within the **solid**-lined community, although **G**[1][0] Koffing and **G**[3][1] Sunflora are of different types, there is a path connecting them Koffing -> Bulbasaur -> Bayleef -> Bulbasaur -> Sunflora"

G (Pokemon HPs hidden)



Bulbasaur [9, 13]	Sunflora [9]	Cubone [10]
Koffing [13]	Bayleef [9]	Cubone [10]
Rattata [12]	Cubone [10]	Bulbasaur [9, 13]
Rattata [12]	**Sunflora** [9]	Koffing [13]

Q5(a) answer: `ϵ {T, B, D, S, J, F, P, X}` Q5(b) answer: `ϵ {A-R}`

Example for Q6-8

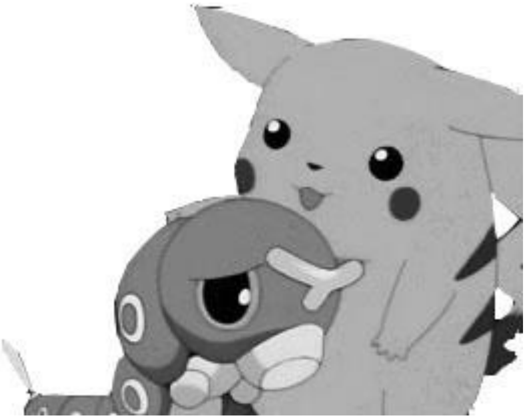
G (Pokemon HPs hidden)

Bulbasaur [9, 13]	**Caterpie** [0]	Cubone [10]	Cubone [10]
Golduck [17]	Bayleef [9]	Krabby [17]	Krabby [17]
Seel [17]	Cubone [10]	Bulbasaur [9, 13]	Charizard [7, 6]
Rattata [12]	**Pikachu** [3]	Psyduck [17]	Golduck [17]

E (some rows and cols omitted)

i \ j	...	7	...	17	...
...					
3		true		true	
...					

Assume type 3 is not effective against the other types



Hang in there[4]!

Q6 is on the next page

[4] Image adapted from https://pikachugifs.tumblr.com/post/83046476910/pikachu-comforting-caterpie

[Q6-8 have quite similar wording, so changes to Q8 and 9 with respect to Q7 are underlined]

Q6. **Pika finding Caterpie** "There is exactly 1 Pikachu and 1 Caterpie on the map, location unknown. Pika needs to move some number of cells to reach Caterpie. Each time, Pika can only move into an adjacent cell in which Pika's type (Pika has only one type) is **effective** against at least one of the types of the pokemon in that cell. Pika **loses 1 HP** for moving through a cell in any direction. Caterpie's cell is an exception

Output the lowest HP loss for Pika to reach Caterpie. Pika can always move onto an adjacent Caterpie's cell and it loses 0 HP doing so

Using the example on the previous page, the lowest HP loss is $\boxed{2}$ "

Q6(a) answer: ϵ {T, B, D, S, J, F, P, X} Q6(b) answer: ϵ {A-R}

Q7. **Brave Pika finding Caterpie** "There is exactly 1 Pikachu and 1 Caterpie on the map, location unknown. Pika needs to move some number of cells to reach Caterpie. Each time, Pika can only move into an adjacent cell. **Pika gains**[5] **1 HP** for moving through the adjacent cell for EACH of the other pokemon's type that Pika's type (Pika only has one type) is **effective** against (i.e. **Pika can gain 1-2 HP**). If Pika's type is **NOT effective** against any of the other pokemon's type(s), **Pika loses 3HP**. Caterpie's cell is an exception

Output the lowest **net HP loss** for Pika to reach Caterpie. Pika can always move onto an adjacent Caterpie's cell and it loses/gains 0 HP doing so. If it is possible for Pika to get **stuck** gaining HP instead, output "X"

Using the example on the previous page, the lowest HP loss is $\boxed{X}$

If both the leftmost and rightmost columns are removed, then the lowest HP loss is $\boxed{1}$

If all type 7 and type 17 pokemon are replaced by Bayleef [9], then the lowest HP loss is $\boxed{6}$ "

Q7(a) answer: ϵ {T, B, D, S, J, F, P, X} Q7(b) answer: ϵ {A-R}

Q8. **Tired Pika finding Caterpie** "There is exactly 1 Pikachu and 1 Caterpie on the map, location unknown. Pika needs to move some number of cells to reach Caterpie. Each time, Pika can only move into an adjacent cell in which Pika's type (Pika has only one type) is effective against at least one of the types of the pokemon in that cell. Pika loses 1 HP for moving up/down/left/right, **but loses 4.13 HP for moving diagonally**. Caterpie's cell is an exception

Output the lowest HP loss for Pika to reach Caterpie. Pika can always move onto an adjacent Caterpie's cell and it loses 0 HP doing so

Using the example on the previous page, the lowest HP loss is $\boxed{5.00}$ "

Q8(a) answer: ϵ {T, B, D, S, J, F, P, X} Q8(b) answer: ϵ {A-R}

[5] Disclaimer: This is different from what happens in battles in the real Pokemon

Section 3 - **Low Power** The power in National University of Computing has failed while **N** (large **N**) students are taking e-exam! Internet/network connectivity is lost (unfortunately, no one is using Examplify)

Each student encrypts and saves their work into a thumbdrive, passes the thumbdrive along with all thumbdrives that are with him/her to another student who still has at least one thumbdrive. Eventually, **N**-1 students have no thumbdrives and 1 student has **N** thumbdrives

Each student will only trust some other students with the thumbdrives. On average, a student will trust around 5 other students. You are given the freedom to decide which other student a student should pass the thumbdrives to, taking into account whom the student trusts. It is guaranteed that even with this constraint, all the **N** thumbdrives will definitely be able to reach 1 student

When all thumbdrives are passed from student x to student y, student x's anxiety increases (a value which is either 1, 2, or 3) but student y's anxiety remains at 0 till student y passes the thumbdrives to some other student (who must still have thumbdrive(s)). The amount of anxiety x feels is dependent on the pair of students, and NOT on the number of thumbdrives passed. The amount of anxiety student x feels when passing any number of thumbdrives to student y would be the same as the amount of anxiety that student y feels if student y were to pass any number of thumbdrives to student x, but could be different if he/she passes thumbdrives to student z instead

Q9 **[12 marks]**

While your laptop is still running, you are given as input an adjacency list (type ArrayList<LinkedList<Node>>), in which if a node in **adjList**[x] has attribute `to` == y, then the student x trusts student y, and the anxiety generated from student x passing the thumbdrive to student y is the attribute `anxiety`

```
class Node {
  public int from, to,
     anxiety; // 1,2 or 3
}
```

You are to return the maximum total anxiety that can possibly be generated from this unfortunate event

Important Notes

- You have the choice of writing short answers (<- *recommended*) **XOR** writing **Java** code
- If you do write both short answers and code, then each option will be marked independently, and you will be awarded the *lower* of the two options' marks
- If you choose the short answer option but your answer in Q9h is NOT a modification of the algorithm in Q9g, then you will NOT get full credit for Q9h
- Answers exceeding the line/char limit will be truncated
- Answers that do not follow the format for one-character answer parts will be disregarded
- Marks will be **awarded for the whole question**, and not a fixed amount for each part, so making an incorrect modification or modeling the graph wrongly may cause you to lose many marks

Grading bands:

- **≤ 5 marks** Worse time complexity than "classic algorithm without modification"
- **7 marks** "Classic algorithm without modification"
- **12 marks** Better than "classic algorithm without modification"

Code Option

Fill in the findMaxAnxiety **Java** function. You may write other helper functions below, or add any classes as necessary. Be neat, indent your code properly. The line limit excludes blank lines and the given function header

Q9a) **[line limit: 60]**

```
int findMaxAnxiety(ArrayList<LinkedList<Node>> adjList) {

    // write code here, ONLY IF not attempting short ans option



}
```

Short Answer Option

Q9b) Does it matter which student receives the **N** thumbdrives? (i.e. does that change the output?) `Y/N`

Q9c) Is the graph on which you run your algorithm connected `Y/N` ?

Q9d) Is the graph on which you run your algorithm directed `Y/N` ?

Q9e) Is the graph on which you run your algorithm weighted `Y/N` ?

Q9f) Is the graph on which you run your algorithm cyclic `Y/N` ?

Q9g) State the algorithm taught in class to be used, your answer being one letter from the list below:

`ϵ {T, S, B, D, J, F, P, X}`

T	Binary **T**ree Traversal	**S**	Topological **S**ort
B	**B**readth First*	**D**	**D**epth First*
*can refer to either Search/Traversal			
J	Di**J**kstra's SSSP	**F**	Bellman-**F**ord
P	**P**rim's MinST	**X**	**P**rim's MaxST

Q9h) Describe in English any modification to the algorithm in part (g), or anything that has to be avoided in the implementation to improve the efficiency of the algorithm **[char limit : 200]**

`SHORT ANSWER`

- End of paper -