NATIONAL UNIVERSITY OF SINGAPORE

CS2040 – DATA STRUCTURES AND ALGORITHMS

(Semester 1: AY2022/23)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. **Do NOT flip / turn over the test paper until you are told to do so**
2. Write your **student number** on pages 1, 3 AND 5 of the answer sheet. Do **NOT** write your name! Clearly **shade** your student number on page 1 too
3. Do NOT rearrange the answer sheet, or add/remove staples. **COMPLETELY shade** the bubble for each answer using a fairly **dark pencil**. You may answer the bonus question using pen or pencil
4. **Submit only the answer sheet** at the end of the assessment. It is your responsibility to ensure that you have submitted it, and submitted the correct answer sheet
5. If you fail to submit the correct answer sheet, fail to provide **correct particulars** or prevent the options from being **automatically detected** by software, we will consider it as if you did not submit your answers. In the best case, **marks will be deducted**
6. No extra time will be given at the end of the assessment for you to write your particulars, shade the answer sheet or write/transfer answers. You must do them **before** the end of the assessment
7. This paper consists of **17** questions. Not more than one option should be shaded per grid. The question paper comprises fourteen (**14**) printed pages including this front page and blank page 9. The answer sheet comprises five (**5**) printed pages, we will disregard the 6th page
8. This is an open-hardcopy-notes assessment but **WITHOUT** electronic materials
9. Marks allocated to each question are indicated. Total marks for the paper is **80**
10. The use of electronic **calculator** is **NOT** allowed

Sect Qn	***Max***	***Marks***
S1 Q1-7	32	
S2 Q8-12	15	
S3 Q13	8	
S3 Q14	8	
S3 Q15	10	
S3 Q16	6	
S3 Q17	1	
Total	**80**	

Section 1 – Shorts; Answer Questions 1-7 **[32 marks == 7 x 2 + 6 x 3]**

Question 1 – Warmup

L is a `java.util.LinkedList<Integer>`, **T** is a `java.util.TreeSet<Integer>` and **PQ** is a `java.util.PriorityQueue<Integer>`. Each of the 3 data structures contains **N** elements.

For **each** of **Q1a-g independent of one another**, the code snippet runs correctly, you are to choose the best answer for its time complexity:

◯ O(log(log(N))	◯ O(log(N))	◯ O($(\log(N))^2$)	◯ O($\sqrt{N}$)	◯ O($\sqrt{N}\log(N)$)	◯ O(N)
◯ O(N log(N))	◯ O($N^{1.5}$)	◯ O($N^{1.5}$ log(N))	◯ O(N^2)	◯ O(N^2 log(N))	◯ O(N^3)

Code Snippet Q1a

```
System.out.println(T.ceiling(T.first()));
```

Code Snippet Q1b

```
PQ.remove(key);
```

Code Snippet Q1c

```
System.out.println(T.headSet(key).size());
```

`headSet(key)` returns a view of the portion of this set whose elements are strictly less than `key`

Code Snippet Q1d

```
while (!PQ.isEmpty()) System.out.println(PQ.poll());
```

Code Snippet Q1e

```
for (int key : T) // internally uses TreeSet's iterator
  System.out.println(key);
```

Code Snippet Q1f

```
for (Integer key = T.last(); key != null; key = T.lower(key))
  System.out.println(key);
```

Code Snippet Q1g

```
for (int times = 1; times <= L.size(); times++)
   System.out.println(L.get(rand(0, L.size()-1)));
```

`rand(0, L.size()-1)` runs in O(1) time and returns some integer from 0 to L.size()-1 inclusive.

Now, each of **Q2-7 is INDEPENDENT** of one another, and **worth more marks** than each part in Q1…

Question 2

In an AVL tree, you are given 2 nodes **A** and **B** which are at the same *level*. The **maximum possible** *difference* in *heights* of the subtrees rooted at **A** and **B** is:

◯ 0	◯ 1	◯ 2	◯ 3	◯ 4
◯ up to twice the height of the shorter of the 2 subtrees			◯ (could be more than those listed)	

Question 3

You are given a reference to 2 nodes **A** and **B** in a BST. A Node is defined as:

```
class Node {
   int item, height; // height of subtree rooted at this Node
   Node left, right, parent;
}
```

You want to find the lowest possible Node that is an ancestor/parent of BOTH **A** and **B** in O(**h**) time or better, **h** being the height of the BST. Ali, Balu and Charlie each have a suggestion:

> Ali – Start from the root of the BST and navigate down ONE path. At each node, check whether **A** and **B** are in two different subtrees of the current node, or if the current node is already at **A** or **B**. If so, the current node is the answer. Otherwise, recurse in the direction where both nodes are in.
>
> Balu – Since we have a reference to both nodes, the answer can be found in O(1) time as it is a function of the height attributes of **A** and **B**.
>
> Charlie – Since we have a reference to both nodes, we can store a list of nodes along the path from **A** to root, and another list of nodes along the path from **B** to root. Examining these 2 lists one element at a time from the back will give us the answer.

Whose approach(es) work, if any at all?

◯ None	◯ Ali only	◯ Balu only	◯ Charlie only
◯ Ali & Balu only	◯ Ali & Charlie only	◯ Balu & Charlie only	◯ Ali, Balu and Charlie

Question 4

You have **N** *boxes* labelled 0..(**N**-1), each painted with some colour. Initially, there are **N** *groups*, one *box* in one *group*. You can repeatedly combine 2 *groups* together, one pair at a time, but all *boxes* in the combined *group* will be repainted to one colour (could be any colour, new or old). You want to keep track of the colour of each *box* efficiently.

Ivan says "UFDS is suitable here…

X – UFDS merges a pair of *groups* efficiently without having to mutate/shift many *boxes.*

Y – given a *box*, UFDS allows efficient finding and tracking of the *box's group's* properties.

Z – BUT for fairly efficient operations, **BOTH** union-by-rank **AND** path compression must be used, otherwise a sequence of **N** operations will run >> O(**N** log **N**) time (much worse than O(**N** log **N**) time)."

You don't quite agree with Ivan though… Or do you?

◯ Disagree with all 3	◯ Agree with X only	◯ Agree with Y only	◯ Agree with Z only
◯ Agree with X, Y only	◯ Agree with X, Z only	◯ Agree with Y, Z only	◯ Agree with X,Y,Z

Question 5

Tom says: "In general,

X – Dijkstra's SSSP algorithm will NOT work correctly on a graph with cycles.

Y – Prim's MinST algorithm will NOT work correctly on a graph with cycles.

Z – Bellman Ford's SSSP algorithm will NOT work correctly on a graph with cycles."

How about you?

◯ Disagree with all 3	◯ Agree with X only	◯ Agree with Y only	◯ Agree with Z only
◯ Agree with X, Y only	◯ Agree with X, Z only	◯ Agree with Y, Z only	◯ Agree with X,Y,Z

Question 6

You want to implement your OWN hash table that can map and store up to **N** keys (**N** is known beforehand and will not change) of type **K**, along with associated values of type **V**. You are able to find a *perfect hash function* hash(key).

Which of these statement(s), if any, are correct:

X – We can implement the table as a direct addressing table to store the **N** key-value pairs.

Y – There is no need for collision resolution, why bother chaining / probing?

Z – We can fix the table size at exactly **N** since there will be no need for expansion and rehashing.

◯ Disagree with all 3	◯ Agree with X only	◯ Agree with Y only	◯ Agree with Z only
◯ Agree with X, Y only	◯ Agree with X, Z only	◯ Agree with Y, Z only	◯ Agree with X,Y,Z

Question 7

You want to create your own social network e.g. Facebook, Instagram, … You have users as vertices and friendships / follows between users as edges. You design this graph **G** for the day when your program is used by billions of users worldwide…

Which of these statement(s), if any, are correct:

X – If we use an adjacency matrix to store **G**, we will likely need MASSIVE amounts of memory that can't fit onto one typical computer.

Y – If we use a **hash** map of **tree** maps to store **G** instead of an adjacency matrix, we will likely be better able to balance time vs space requirements.

Z – If we use a **hash** map of **hash** maps to store **G**, we should be able to run BFS, DFS, SSSP algorithms fairly efficiently.

◯ Disagree with all 3	◯ Agree with X only	◯ Agree with Y only	◯ Agree with Z only
◯ Agree with X, Y only	◯ Agree with X, Z only	◯ Agree with Y, Z only	◯ Agree with X,Y,Z

Section 2 – Read the scenario carefully and use it to answer Questions 8-12 **[15 marks]**

There are **N** cards placed in a line. Each card has a **positive integer number** written on it. Both **N** and the number on a card may be very large. The numbers on each card, in sequence, is given in an integer array `cards`.

You are also given 5 **independent** problems, as well as 10 possible solutions summarized by their **main** algorithm and/or **main** data structure(s) besides those given to you. For each of **Q8-12**, choose:

(a) ONE solution summary that is the **most suitable and most efficient**

◯ S1	◯ S2	◯ S3	◯ S4	◯ S5
◯ S6	◯ S7	◯ S8	◯ S9	◯ S10

and

(b) best answer for the time complexity of the chosen solution summary

◯ O(log(log(N))	◯ O(log(N))	◯ O($(\log(N))^2$)	◯ O($\sqrt{N}$)	◯ O($\sqrt{N}\log(N)$)	◯ O(N)
◯ O(N log(N))	◯ O($N^{1.5}$)	◯ O($N^{1.5}$ log(N))	◯ O(N^2)	◯ O(N^2 log(N))	◯ O(N^3)

Marking Scheme

- Any part (a) or part (b) with two or more options shaded will be treated as being completely wrong
- To discourage spamming of a solution, no **solution** (for part (a)) should be picked more than **once across questions** – If that happens, only the *highest mark* for that solution will be awarded *once* across the entire section
- The best answer for time complexity in part (b) may or may not be repeated across questions
- Each solution (for part (a)) is worth somewhere between 0-3 marks
- The choice of each part (b) will cause the awarded mark from part (a) to be scaled by a factor (i.e. multiplied), between 0.5 (completely wrong) and 1.0 (correct option with respect to *your* chosen ADT in part (a))

Solution Summaries

- S1: No data structure needed, algorithm involves combinatorics
- S2: Linked list problem
- S3: Stack problem
- S4: FIFO Queue, sliding window algorithm
- S5: Hash table problem
- S6: Balanced BST problem
- S7: Dijkstra's SSSP algorithm
- S8: Bellman-Ford SSSP algorithm
- S9: Prim's Minimum Spanning Tree algorithm
- S10: Kruskal's Minimum Spanning Tree algorithm

Question 8

Given `cards`, find and output the number of contiguous card subsequences, with at least 2 cards, that start and end with the same-numbered card.

E.g. if `cards` = {1, **2**, 1, 3, **1**, 2, *1*}, then the answer is 7, because there are 7 such subsequences:
{1, **2**, 1, 3, **1**, 2, *1*}, {1, **2**, 1, 3, **1**}, {1, 3, **1**, 2, *1*}, {1, 3, **1**}, {**1**, 2, *1*}, {1, **2**, 1}, {**2**, 1, 3, **1**, 2}

Question 9

Given `cards`, find and output the length of the longest contiguous card subsequence that is strictly-increasing.

E.g. if `cards` = {1, 4, 4, 1, 1, 1, 3, 5, 7, 9, 2, 3}, then the answer is 5, which is the length of:
{1, 3, 5, 7, 9}

Question 10

You are given `cards` and a positive integer `power`. You start off at the leftmost card. Each turn, from where you are currently at, you can jump exactly **k** cards left or right where **k** is the number on the card you are at, provided a card exists at that position you are about to jump to.

Each jump from card **A** to card **B** lowers your `power` by 1+|number on **A** - number on **B**| (i.e. 1 more than the absolute value of the difference in numbers). Your `power` never increases. You have to stop when you do not have enough `power` to make a jump.

Find and output the largest card number that you can possibly land on before you have to stop.

E.g. if `cards` = {3, 4, **3**, 2, **4**, 1, 5} and `power` = 9, then the answer is 5:
card with number 3 to card with number 2 lowers `power` by 2 to 7
card with number 2 to card with number 1 lowers `power` by 2 to 5
card with number 1 to card with number 5 lowers `power` by 5 to 0

E.g. if `cards` = {3, 4, **3**, 2, **4**, 1, 5} and `power` = 8, then the answer is 4:
card with number 3 to card with number 2 lowers `power` by 2 to 6
card with number 2 to card with number 1 lowers `power` by 2 to 4
card with number 1 to card with number **4** lowers `power` by 4 to 0
but unable to jump from card with number 1 to card with number 5 as
`power` needs to be at least 5

Question 11 – Looks similar to Q10 but different. Read carefully!

You are given `cards`. You start off at the leftmost card and with a positive integer **power** (which you are to decide on). Each turn, you can jump from where you are at **to any other card**.

Each jump from card **A** to card **B** costs 1+|number on **A** - number on **B**| (i.e. 1 more than the absolute value of the difference in numbers) on your **power**, but you will **restore your power to the initial value each jump**. You have to stop when you do not have enough **power** to make a jump.

Find and output the smallest **power** you need initially (again, **power** resets to this value after each jump made) such that you will be able to land on all cards in `cards`. It is alright for a card to be visited multiple times.

E.g. if `cards` = {3, 4, **3**, 2, **4**, 1, 5}, then the answer is 2:
card with number 3 to card with number **3** requires 1 `power`
card with number **3** to card with number 4 requires 2 `power`
card with number **3** to card with number 2 requires 2 `power`
card with number 4 to card with number **4** requires 1 `power`
card with number 2 to card with number 1 requires 2 `power`
card with number **4** to card with number 5 requires 2 `power`

Question 12

You are given `cards` and also the condition that the entire sequence of cards can be *folded*. A sequence of cards can be **folded** if:

- the sequence is empty
  or
- the first card is a different card from the last but has the same number on it, AND the remaining subsequence can be divided into 1 or more contiguous subsequences that can be **folded**

e.g. {3}, {3, 5}, {3, 5, 3} or {3, 5, 3, 5} all cannot be folded
e.g. {3, 3}, {3, 4, 4, 3}, {3, 4, 5, 5, 4, 3}, {3, 4, 4, 5, 5, 3} or {3, 4, 7, 7, 4, 5, 5, 3} can be folded

Find and output, for each element in sequence, the index of the corresponding first/last card that has the same number in each fold. If there are multiple possible answers, just output any valid answer.

E.g. if `cards` = {3, **4**, 7, **7**, 4, **5**, 5, **3**}, then the answer is "7 4 3 2 1 6 5 0", because:
3 at index 0 pairs up with **3** at index 7, enabling that contiguous subsequence to be folded
4 at index 1 pairs up with 4 at index 4, enabling that contiguous subsequence to be folded
7 at index 2 pairs up with **7** at index 3, enabling that contiguous subsequence to be folded
…

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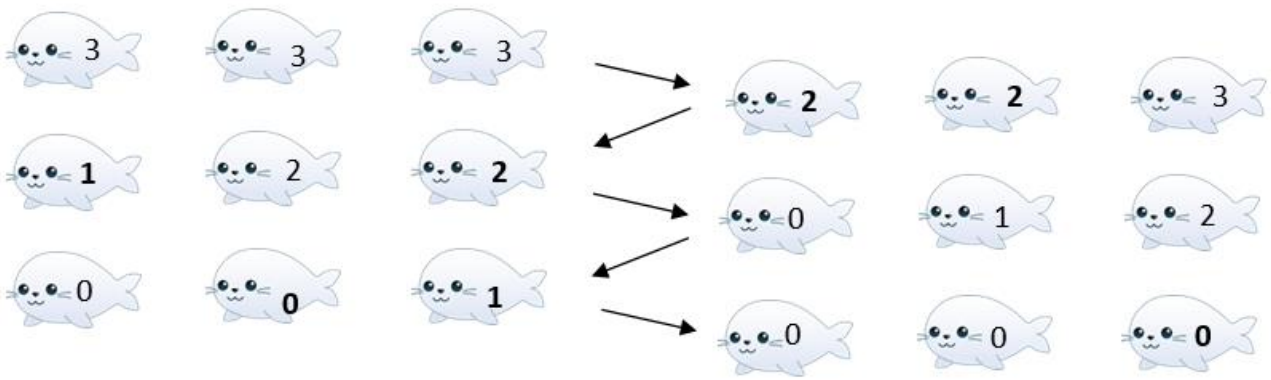
Section 3 – Read the scenario carefully and use it to answer Questions 13-17

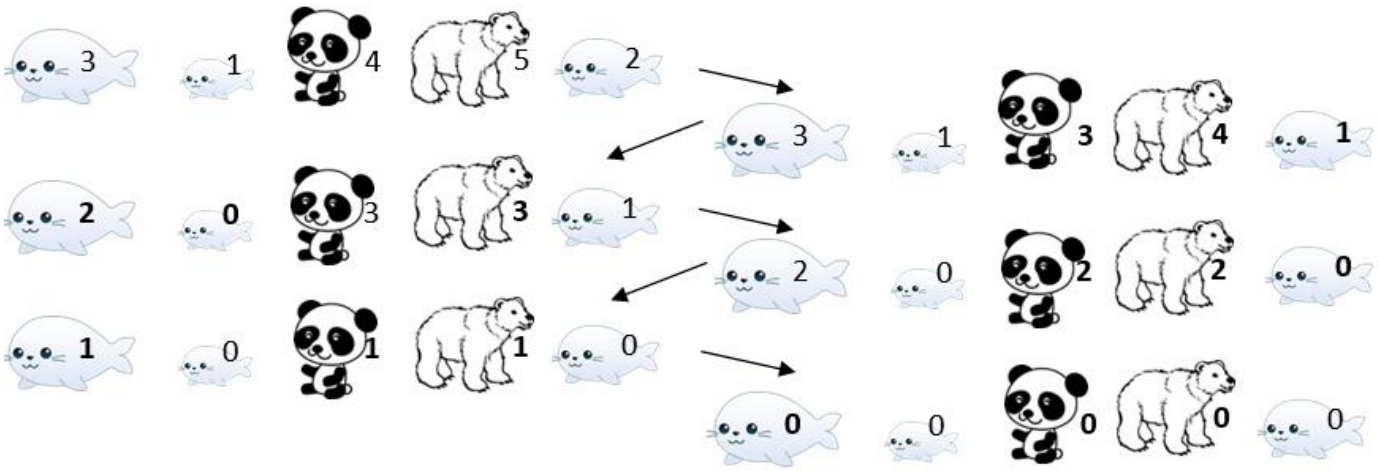
There are **N** animals[1] in a zoo and they are hungry! Each animal has a positive integer *hunger level* which could possibly be very very large (but never grows). You are tasked to satisfy the hunger of every animal using the *least* number of *feeding cycles*, i.e. all animals should end up with a *hunger level* of 0.

Each *feeding cycle*, you will be given a bucket of **k** (> 0) fish. Each of the **N** animals can be fed either 0 or 1 fish, but not more within that same *feeding cycle*. Being fed a fish reduces *hunger level* by 1.

As an **example**, for `hungers` = {3, 3, 3}, **k** = 2, the minimum number of *feeding cycles* required is **5**:



Another **example**, for `hungers` = {3, 1, 4, 5, 2}, **k** = 3, the minimum number of *feeding cycles* required is **5**:



[1] Images from http://clipart-library.com

Question 13 – Understanding the problem **[8 marks == 4 x 2]**

How many *feeding cycles* are minimally required to satisfy the hunger of every animal, for each of:

Q13a. `hungers` = {2, 3}, **k** = 5

Q13b. `hungers` = {3, 1, 4}, **k** = 3

Q13c. `hungers` = {3, 1, 4}, **k** = 2

Q13d. `hungers` = {1, 2, 1, 2, 1, 3, 1, 2, 1} (having length 9), **k** = 3

For each question in **Q13a-d**, choose the correct answer:

◯ 1	◯ 2	◯ 3	◯ 4	◯ 5	◯ 6	◯ 7	◯ 8	◯ 9	◯ 10

Question 14 – Graph Modelling **[8 marks == 4 x 2]**

Ivan suggests that this problem is actually a **shortest path problem**. A **vertex** holds the possible contents of the entire `hungers` array, starting from the initial state before any feeding cycle. An edge represents a *feeding cycle* in which at least 1 fish is fed to an animal, with the destination vertex being the contents of the array after the feed.

What are the characteristics of the graph mentioned above?

Q14a.	◯ Directed	◯ Undirected
Q14b.	◯ Weighted	◯ Unweighted
Q14c.	◯ Cyclic	◯ Acyclic

Q14d. What is the most suitable data structure that should be used to store the graph mentioned above, if any?

◯ Adjacency matrix	◯ Adjacency list	◯ Edge list	◯ No extra DS, implicit graph

Question 15 – Shortest Path Algorithm **[10 marks == 5 x 2]**

Assume that there exists a perfect hash function **`h`**`(a)` that can map a vertex (the contents of a possible `hungers` array) as mentioned in **Q14** to an integer hash. If there is a need to keep track of visited vertices, a hash map utilizing the hash function **`h`** will be used instead of an array. Assume that the integer hash can be read in O(1) time, no matter how large it is.

Also assume that you **do NOT have control** over the **ordering of the neighbours** of each vertex.

Which of these 4 algorithm(s) correctly solves the problem, ignoring efficiency? The problem may be solved by **AT LEAST 1** of these algorithms. For each of **Q15a-e**, shade the correct option:

◯ Yes	◯ No

Q15a. Run recursive DFS on the vertex containing the initial contents of `hungers`. Stop when a vertex containing all 0s is hit and output/return that number of hops made on that path. **DO** keep track of visited vertices as DFS usually does.

Q15b. Run recursive DFS on the vertex containing the initial contents of `hungers`. Stop when a vertex containing all 0s is hit and output/return that number of hops made on that path. Due to the graph's characteristics, we **DON'T** keep track of visited vertices. If there are multiple paths found at a vertex, return the one with the least number of hops.

Q15c. Run BFS on the vertex containing the initial contents of `hungers`. Stop when a vertex containing all 0s is hit and output/return that number of hops made on that path. **DO** keep track of visited vertices as BFS usually does.

Q15d. Run BFS on the vertex containing the initial contents of `hungers`. Stop when a vertex containing all 0s is hit and output/return that number of hops made on that path. Due to the graph's characteristics, we **DON'T** keep track of visited vertices.

Q15e. Just for this part, assume that **k** = 2 to simplify the analysis. Out of the shortest path algorithms you have **chosen among Q15a-d only**, what is the time complexity of the **best** shortest path algorithm? [**Reminder**: ***N*** *is the number of animals and* ***k*** *is the maximum number of animals that can be fed per feeding cycle*]

◯ O(**N**)	◯ O($\mathbf{N}^2$)	◯ O($\mathbf{N}^3$)	◯ O($\mathbf{N}^4$)	◯ Other worse polynomial time in **N**
◯ Exponential in **N**		◯ Factorial in **N**		◯ Could be worse than O(**N**!) even when **k** = 2

Question 16 – Simulation **[6 marks == 3 x 2]**

Remember that k is now not necessarily 2 (back to just k > 0), and no longer a graph problem

There are more efficient algorithms to solve this problem. Suppose we **simulate** each *feeding cycle* using different algorithms/data structures to find the answer to the problem, which we will call **A**. In each *feeding cycle*, only the best choice (or one of the many best choices if exist) will be made.

[**Reminder:** *As mentioned earlier, **A** could be very very large*]

For each of **Q16a-c** independent of one another, the algorithm correctly solves the problem. Choose the time complexity of using the respective algorithm to find the answer:

◯ O(**N**)	◯ O(**N** log**N**)	◯ O($\mathbf{N}^2$)	◯ O($\mathbf{N}^2$ log**N**)	◯ O(**AN**)	◯ O(**AN** log**N**)
◯ O(**A** $\mathbf{N}^2$)	◯ O(**A** $\mathbf{N}^2$ log**N**)	◯ O($\mathbf{A}^2$ **N**)	◯ O($\mathbf{A}^2$ **N**log**N**)	◯ O($\mathbf{A}^2\mathbf{N}^2$)	◯ O($\mathbf{A}^2\mathbf{N}^2$ log **N**)

Q16a.

While at least one animal is still hungry
    Sort `hungers` (you decide which algo, choose the best)
    For i = 1..**k**, terminating early if out of bounds or a zero is found
        Decrement ith-from-the-right element in `hungers`
Output num of while loop passes

Q16b.

Create **binary heap** (you decide min/max, choose the best) with all hunger levels in `hungers`
Create arraylist of `fedAnimals`, currently empty
While at least one animal is still hungry
    For i = 1..**k**, terminating early if heap is empty
        Dequeue top element of the heap, decrement, and add to the back of `fedAnimals`
    Enqueue all non-zero elements from `fedAnimals` back into the heap
    Clear `fedAnimals`
Output number of while loop passes

Q16c.

Create a general **binary search tree** with all hunger levels in `hungers`, where
        each element is a (hunger, frequency) pair
Create arraylist of `fedAnimals`, currently empty
While at least one animal is still hungry
    Let **i** = **k** and repeat till **i** == 0 or tree is empty
        Update and possibly remove some element in the BST
        Add a (new hunger, frequency) pair to the back of `fedAnimals`
    Add/update all non-zero-hunger elements (pairs) from `fedAnimals` back into the BST
    Clear `fedAnimals`
Output number of while loop passes

Question 17 – Most Efficient Algorithm **[1 mark (+3 bonus marks?)]**

Remember that k is now not necessarily 2 (back to just k > 0)

Legend has it that there is still a more efficient algorithm to solve this problem!

What is the time complexity of the **most efficient** algorithm that can be used to solve this problem?
[**Reminder**: ***N*** *is the number of animals. The time complexity should NOT depend on* ***k*** *the maximum number of animals and/or* ***A*** *the answer to the problem (min number of feeding cycles needed)*]

◯ O(log**N**)	◯ O($\sqrt{\mathbf{N}}$))	◯ O(**N**)	◯ O(**N** log**N**)
◯ O($\mathbf{N}^{1.5}$)	◯ O($\mathbf{N}^2$)	◯ O($\mathbf{N}^2$ log**N**)	◯ O($\mathbf{N}^3$)

[Bonus]

Implement the **most efficient** algorithm in **Java** for 3 **bonus** marks:

```
long solve(int[] hungers, int k)
```

[**WARNING: Do not waste time on this part**, you will receive no credit at all for a solution that is incorrect or that is less efficient than the intended answer]

Box to answer is in answer sheet

- End of paper -