Chapter - 10 Mechanical Properties Of Fluids

Fluids

- A fluid is something which can flow.
- A fluid has no definite shape of its own.
- The fluids shape changes by application of very small shear stress.
- Liquids and gases are fluids.

Thrust

• The normal force acting on a surface is called thrust.

Fluid Pressure

- Pressure is defined as the normal force acting per unit area of a surface.
- Or pressure is the thrust per unit area.

$$P = \frac{F}{A}$$

- Where F – Force, A –Area
- Pressure is a scalar quantity.
- The S I unit is Nm-2 or Pascal (Pa)
- The dimensions are [ML-1T-2]
- A common unit of pressure is the atmosphere (atm), i.e. the pressure

exerted by the atmosphere at sea level .

$$\boxed{1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}}$$

Density

- Density is the mass per unit volume of a fluid.

$$\boxed{\rho = \frac{m}{V}}$$

- A liquid is largely incompressible and its density is nearly constant at all pressures.
- Density of gases changes with pressure.
- The dimensions of density are[ML-3]
- Its SI unit is kg m-3.
- The density of water at 40 C is

$$\boxed{1.0 \times 10^3 \text{ kg m}^{-3}}$$

Fluid	ρ (kg m^{-3})
Water	1.00×10^3
Sea water	1.03×10^3
Mercury	13.6×10^3
Ethyl alcohol	0.806×10^3
Whole blood	1.06×10^3
Air	1.29
Oxygen	1.43
Hydrogen	9.0×10^{-2}
Interstellar space	$\approx 10^{-20}$

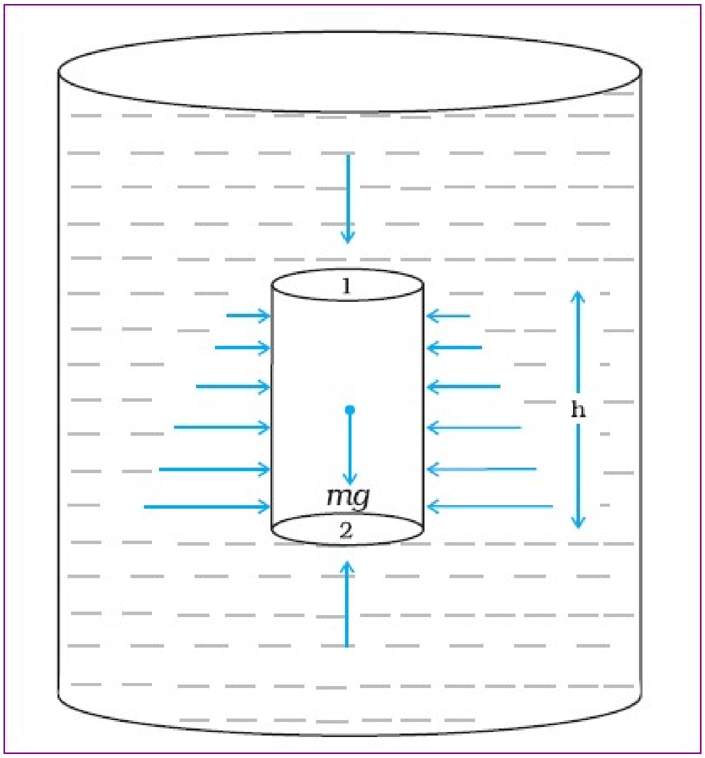
Relative Density

- The relative density of a substance is the ratio of its density to the density of water at 40 C .
- It is a dimensionless positive scalar quantity.

Pascal's Law

• The pressure in a fluid at rest is the same at all points if they are at the same height.

Variation of Pressure with Depth



- Consider a cylindrical element of fluid having base area ‘A’ and height ‘h’.
- As the fluid is at rest the resultant horizontal forces should be zero and the resultant vertical forces should balance the weight of the element.
- If mg is weight of the fluid in the cylinder we have

$$P_2A - P_1A = mg$$

- That is

$$\boxed{(P_2 - P_1)\ A = mg}$$

- if ρ is the density of the fluid, we have the mass of fluid $m = \rho V = \rho h A$
- Thus

$$\boxed{P_2 - P_1 = \rho g h}$$

- If the top of the fluid element considered is open to the atmosphere, then P1= Pa, and let P2=P, therefore

$$\boxed{P = P_a + \rho g h}$$

- Thus pressure increases with depth.
- The liquid pressure is the same at all points at the same horizontal level.

Gauge pressure

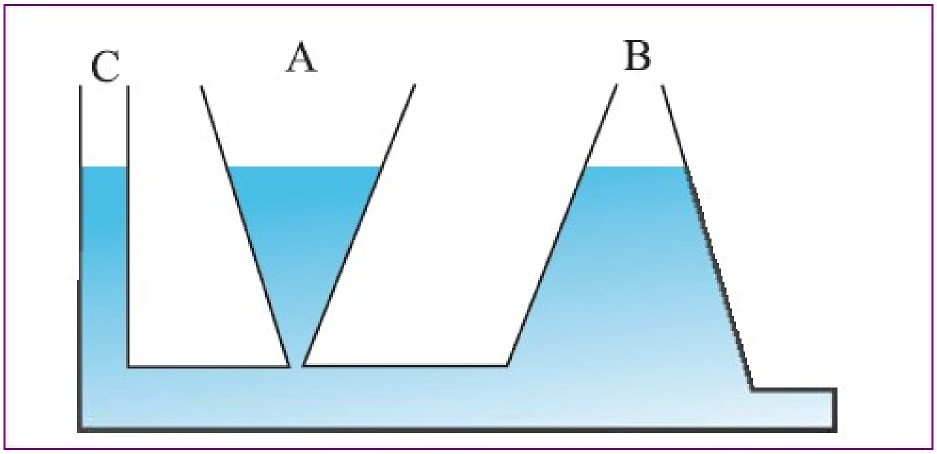
• The excess of pressure, P − Pa, at depth h is called a gauge pressure at that point.

P P− =a ρgh

Hydrostatic paradox

- The liquid level is independent of the shape of the container.

- Hydrostatic paradox is a consequence of Pascal’s law.



Atmospheric pressure

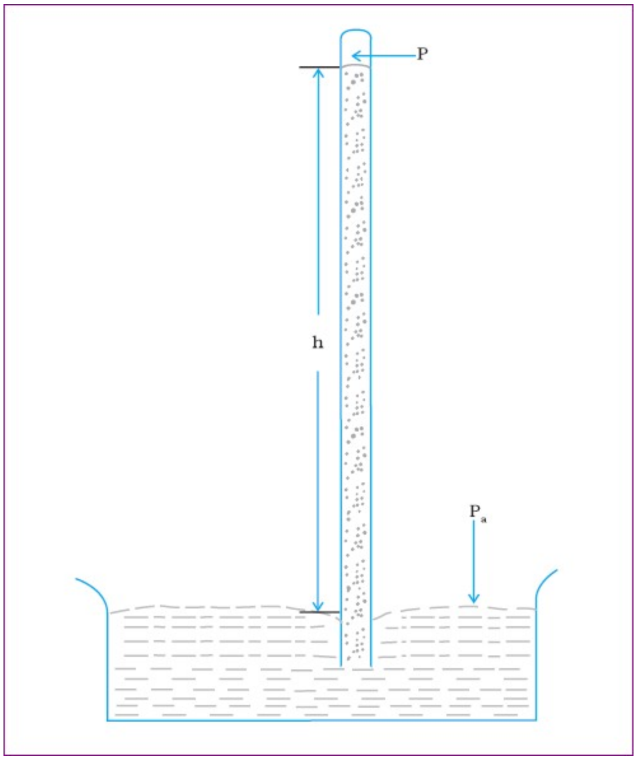
- The pressure of the atmosphere at any point is equal to the weight of a column of air of unit cross sectional area extending from that point to the top of the atmosphere
- At sea level, the atmospheric pressure is **1.013×105 Pa.**
- It is maximum on the surface and it decreases with altitude.

Measurement of Pressure

- Instruments used to measure pressure are called Manometers.
- Barometer is a device used to measure atmospheric pressure.

Mercury Barometer

- Italian scientist Evangelista Torricelli devised mercury barometer.
- It is a long glass tube closed at one end and filled with mercury is inverted into a trough of mercury.



- The space above the mercury column in the tube contains only mercury vapour whose pressure P is so small that it may be neglected (P=0)
- The pressure inside the column at point A must equal the pressure at point B, which is at the same level.
- Pressure at B = atmospheric pressure = Pa.
- Thus

$$\boxed{P_a = \rho g h}$$

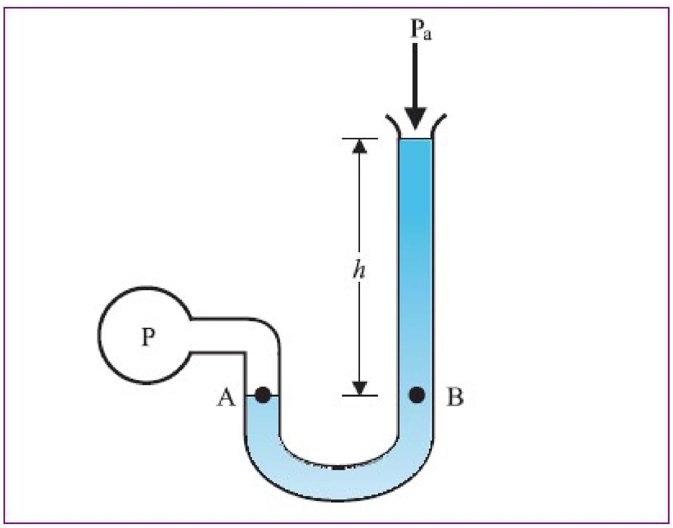
- Where ρ is the density of mercury and h is the height of the mercury column in the tube.
- At sea level h= 76 cm and is equivalent to 1 atm.
- Mercury is used in the barometer because of the following reasons.
  - It has a shining surface.
  - High Density (So we can reduce the height of the tube)

- Does not wet glass.

Open-tube manometer

- An open-tube manometer is a useful instrument for measuring pressure differences.
- It consists of a U-tube containing a suitable liquid i.e. a low density liquid (such as oil) for measuring small pressure differences and a high density liquid (such as mercury) for large pressure differences.
- One end of the tube is open to the atmosphere and other end is connected to the system whose pressure we want to measure

.



- The pressure P at A is equal to pressure at point B.
- Here the gauge pressure is measured and is proportional to manometer height h.

$P\ P - = a\ \rho gh$

Units of Pressure

- The SI unit of pressure is pascal [Pa]
- A common unit of pressure is the atmosphere [atm] , 1atm= 1.013 ×105 Pa
- Another unit of pressure is torr (named after Torricelli).
- 1 torr is the pressure equivalent of 1mm of Hg. , 1 torr = 133 Pa
- The mm of Hg and torr are used in medicine and physiology.
- In meteorology, a common unit is bar and mill bar, 1bar = 105Pa. Applications of Pascal’s law

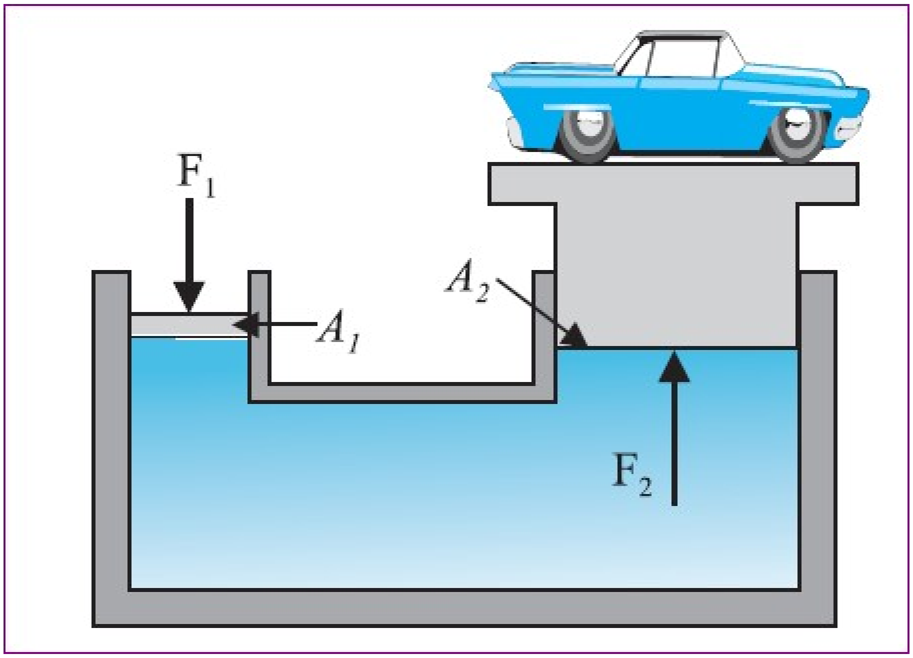
Hydraulic Machines

Pascal’s law for transmission of fluid pressure

- Whenever external pressure is applied on any part of a fluid contained in a vessel, it is transmitted undiminished and equally in all directions.
- Hydraulic lift and hydraulic brakes are based on the Pascal’s law.

Hydraulic Lift

- In a hydraulic lift two pistons are separated by the space filled with a liquid.



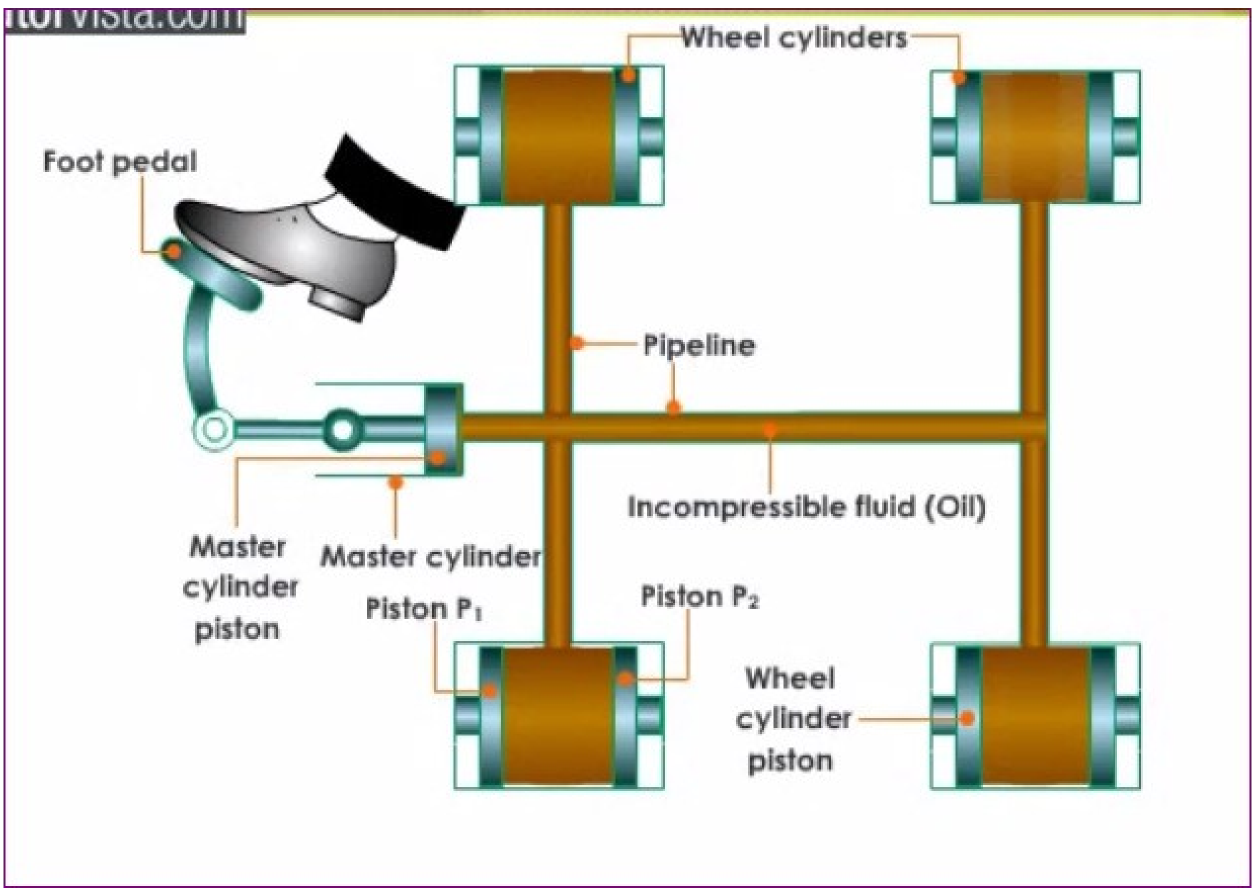
- A piston of small cross section A1 is used to exert a force F1 directly on the liquid.
- The pressure on the first piston is **P =F1/A1**
- According to Pascal's law this pressure is transmitted throughout the liquid.
- Then the upward force on the second piston is ,

$$F_2 = PA_2 = \frac{F_1 A_2}{A_1}$$

- Therefore, the piston is capable of supporting a large force (large weight of, say a car, or a truck, placed on the platform)
- Thus, the applied force has been increased by a factor of A2/A1 , this factor is the mechanical advantage of the device.
- By changing the force at A1, the platform can be moved up or down.

Hydraulic brakes

- When we apply a little force on the pedal with our foot the master piston moves inside the master cylinder, and the pressure caused is transmitted through the brake oil to act on a piston of larger area.
- A large force acts on the piston and is pushed down expanding the brake shoes against brake lining.
- Thus a small force on the pedal produces a large retarding force on the wheel.
- The pressure set up by pressing pedal is transmitted equally to all cylinders attached to the four wheels so that the braking effort is equal on all wheels.



Archimedes' Principle

- The buoyant force on a body which is immersed totally or partially in a fluid is equal to the weight of the liquid displaced by the body.
  Buoyant force :
- The resultant upward thrust acting on a body immersed in a fluid.
- For totally immersed objects the volume of the fluid displaced by the object is equal to its own volume.

If the density of the immersed object is more than that of the fluid, the object will sink as the weight of the body is more than the upward thrust.

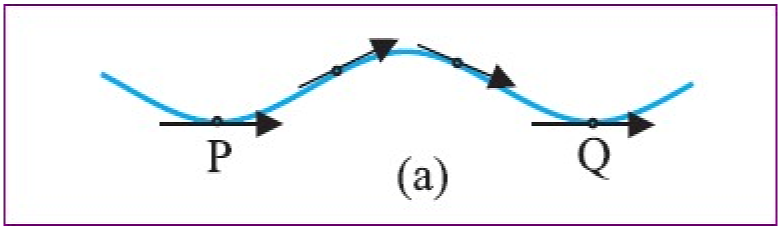
- If the density of the object is less than that of the fluid, it floats in the fluid partially submerged

Fluid dynamics

• The study of the fluids in motion is known as fluid dynamics.

Stream line flow (Steady flow)

- The flow of the fluid is said to be steady if at any given point, the velocity of each passing fluid particle remains constant in time.
- But the velocities at different points may differ.
- Each particle follows a smooth path, and the paths of the particles do not cross each other.
- Steady flow is achieved at low flow speeds.
- The path taken by a fluid particle under a steady flow is a streamline.
- A streamline is a curve whose tangent at any point is in the direction of the fluid velocity at that point.



- A steady flow is also called stream line flow.

EQUATION OF CONTINUITY

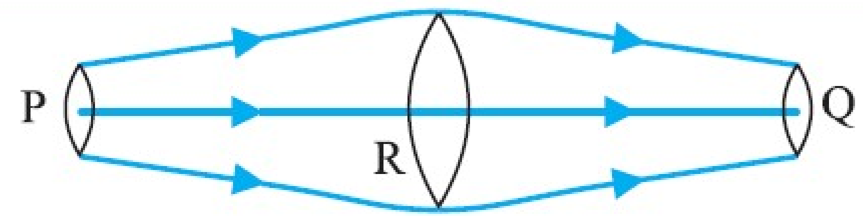
- Equation of continuity is a statement of conservation of mass, in the flow of incompressible fluid.
- For a flowing fluid,

$$\boxed{Av = \text{constant}}$$

Where A- area of cross section, v- speed of fluid flow

- Av gives the volume flux or flow rate and remains constant throughout the pipe of flow.
- Thus, at narrower portions where the streamlines are closely spaced, velocity increases and at wider portions, velocity decreases.

Derivation



(b)

- The figure shows the stream line flow of an incompressible fluid through a pipe of varying cross sections.
- The number of fluid particles crossing the surfaces as indicated at P, R and Q is the same.
- The area of cross-sections at these points are AP , AR and AQ,
- The speed of fluid particles are vP , vR and vQ
- Thus the mass of fluid particles ΔmP crossing at AP in the time interval Δt is

- Thus the mass of fluid particles Δm_P crossing at A_P in the time interval Δt is

$$\Delta m_P = \rho_P A_P v_P \Delta t$$

(Since mass = density X volume)

- Similarly

$$\Delta m_Q = \rho_Q A_Q v_Q \Delta t$$

- And

$$\Delta m_R = \rho_R A_R v_R \Delta t$$

- According to conservation of mass, the mass of liquid flowing out equals the mass flowing in,

$$\Delta m_P = \Delta m_R = \Delta m_Q$$

- Thus

$$\rho_Q A_Q v_Q \Delta t = \rho_R A_R v_R \Delta t = \rho_P A_P v_P \Delta t$$

- For flow of incompressible fluids

$$\boxed{\rho_P = \rho_R = \rho_Q}$$

- Therefore

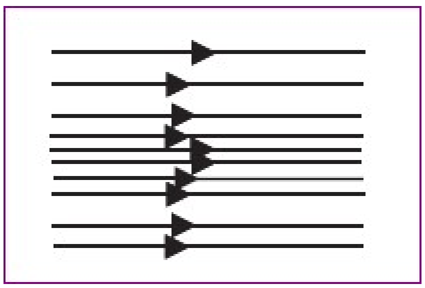
$$\boxed{A_P v_P = A_R v_R = A_Q v_Q}$$

- This equation is the equation of continuity.
- In general

$$\boxed{Av = \text{constant}}$$

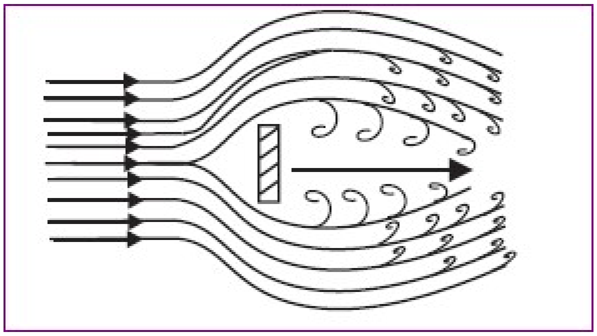
Laminar flow

• In this flow the velocities at different points in the fluid may have different magnitudes but their directions are parallel.



Turbulent flow

- Beyond a limiting value, called critical speed, the flow loses steadiness and becomes turbulent.
- This occurs when a fast flowing stream encounters rocks, small foamy whirlpool like regions called water rapids are formed.
- In a turbulent flow the velocity of the fluids at any point in space varies rapidly and randomly with time.



Critical velocity

• The maximum velocity of a liquid up to which its flow is streamline is called critical velocity.

Examples of Turbulence

- The smoke rising from a burning stack of wood, oceanic currents are turbulent.
- Twinkling of stars is the result of atmospheric turbulence.
- The wakes in the water and in the air left by cars, aeroplanes and boats are also turbulent.

Disadvantages of turbulence

- Turbulence dissipates kinetic energy usually in the form of heat.
- Racing cars and planes are engineered to precision in order to minimise turbulence.

Uses of turbulence

- Turbulence promotes mixing and increases the rates of transfer of mass, momentum and energy.
- The blades of a kitchen mixer induce turbulent flow and provide thick

milk shakes as well as beat eggs into a uniform texture.

REYNOLDS NUMBER

- Reynolds Number is a dimensionless number, whose value gives one an approximate idea whether the flow would be turbulent .

$$\boxed{R_e = \rho v d/\eta}$$

- Where ρ – the density of the fluid, v- flow speed, d- the dimension of the pipe, η – the viscosity of the fluid.
- This number is defined by Osborne Reynolds.
- For a Laminar or streamline flow Re < 1000
- Turbulent flow , Re > 2000
- Unsteady flow ,1000<Re < 2000
- Reynolds number is useful in designing of ships, submarines, racing cars and aero planes.
- Reynolds number can also be written as

$$\boxed{\begin{array}{l} R_e = \rho v^2 \;/\; (\eta v/d) = \rho A v^2 \;/\; (\eta A v/d) \\ = \text{ inertial force/force of viscosity.} \end{array}}$$

- Thus Re represents the ratio of inertial force (force due to inertia i.e. mass of moving fluid or due to inertia of obstacle in its path) to viscous force..

BERNOULLI'S PRINCIPLE

- The total energy of an incompressible non viscous fluid in a steady flow from one point to another is a constant.

- It is a statement of conservation of energy.

Applicability of Bernoulli's theorem

- The fluids must be non viscous.
- Fluids must be incompressible
- This law does not hold for non steady or turbulent flow- since the velocity and pressure constantly changes with time.

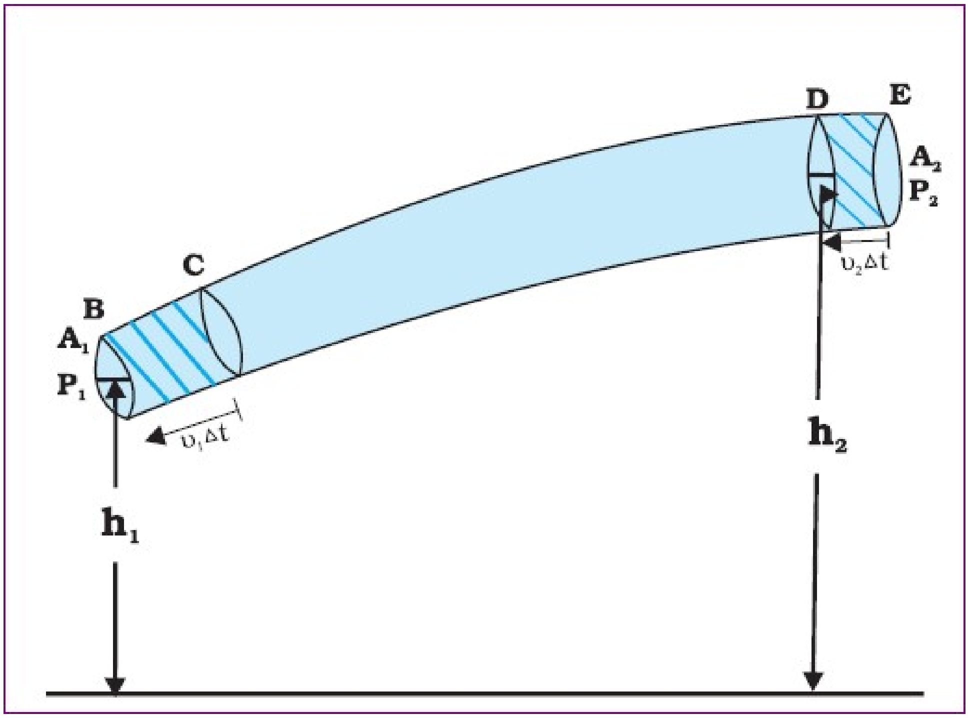
BERNOULLI'S EQUATION

- Bernoulli's equation is a general expression that relates the pressure difference between two points in a pipe to both velocity changes (kinetic energy change) and elevation (height) changes (potential energy change).
- Swiss Physicist Daniel Bernoulli developed this relationship.

$$\boxed{P + \left(\frac{1}{2}\right)\rho v^2 + \rho g h = \text{constant}}$$

- Where ρ- density of fluid, v- speed, Ppressure.
- The Bernoulli's relation may be stated as follows:
- As we move along a streamline the sum of the pressure (P), the kinetic energy per unit volume and the potential energy per unit volume (ρgh) remains a constant.

Derivation



- The figure shows, a fluid moving in a pipe of variable area of cross section and different heights.
- v1 is the speed at B and v2 at D, then fluid initially at B has moved a distance v1Δt to C
- At the same interval Δt the fluid initially at D moves to E, a distance equal to v2Δt.

Total work done on the fluid

- The work done on the fluid at left end (BC) is

$$W_1 = P_1 A_1 (v_1 \Delta t) = P_1 \Delta V.$$

- The work done by the fluid at the other end (DE) is

$$W_2 = P_2 A_2 (v_2 \Delta t) = P_2 \Delta V$$

- Thus the work done on the fluid at DE is

$W2 =- \Delta P\ V2$

- The total work done on the fluid is

$$W_1 - W_2 = (P_1 - P_2)\ \Delta V$$

- Part of this work goes into changing the kinetic energy of the fluid, and part goes into changing the gravitational potential energy.

Change in potential energy

- If the density of the fluid is ρ, the mass passing through the pipe in time Δt is

$$\Delta m = \rho A_1 v_1 \Delta t = \rho \Delta V$$

- Potential energy at height h1 is **$U1 = \rho gh\ V1\Delta$**
- Potential energy at height h2 is **$U2 = \rho gh\ V2\Delta$**

• Thus, change in gravitational potential energy is

$$\boxed{\Delta U = \rho g \Delta V\ (h_2 - h_1)}$$

Change in kinetic energy

- The kinetic energy at the first end is

$$K_1 = \frac{1}{2}mv_1^{\ 2} = \frac{1}{2}\rho v_1^{\ 2}\Delta V$$

- The kinetic energy at the last end is

$$K_2 = \frac{1}{2}mv_2^{\ 2} = \frac{1}{2}\rho v_2^{\ 2}\Delta V$$

- Thus the change in kinetic energy is

$$\boxed{\Delta K = \left(\frac{1}{2}\right) \rho\ \Delta V\ (v_2^{\ 2} - v_1^{\ 2})}$$

Work-energy theorem

- Applying Work-Energy Theorem we get

$$\boxed{(P_1 - P_2)\ \Delta V = \left(\frac{1}{2}\right) \rho\ \Delta V\ (v_2^{\ 2} - v_1^{\ 2}) + \rho g \Delta V\ (h_2 - h_1)}$$

- Dividing each term by ΔV

$$\boxed{(P_1 - P_2) = \left(\frac{1}{2}\right) \rho \ (v_2^2 - v_1^2) + \rho g \ (h_2 - h_1)}$$

- Rearranging the above terms

$$\boxed{P_1 + \left(\frac{1}{2}\right) \rho v_1^2 + \rho g h_1 = P_2 + \left(\frac{1}{2}\right) \rho v_2^2 + \rho g h_2}$$

- This is Bernoulli's equation
- In general

$$\boxed{P + \left(\frac{1}{2}\right) \rho v^2 + \rho g h = \text{constant}}$$

Flow-through a horizontal pipe

- If the pipe is horizontal h1=h2 ,then ΔU =0
- Bernoulli's Theorem becomes ,

$$\boxed{P + \frac{1}{2} \rho V^2 = \text{Constant}}$$

Bernoulli's equation for a stationary fluid

- When fluid is at rest the velocity is zero

- Thus the equation becomes

$$\boxed{P_1 + \rho g h_1 = P_2 + \rho g h_2}$$

- Or

$$\boxed{(P_1 - P_2) = \rho g\ (h_2 - h_1)}$$

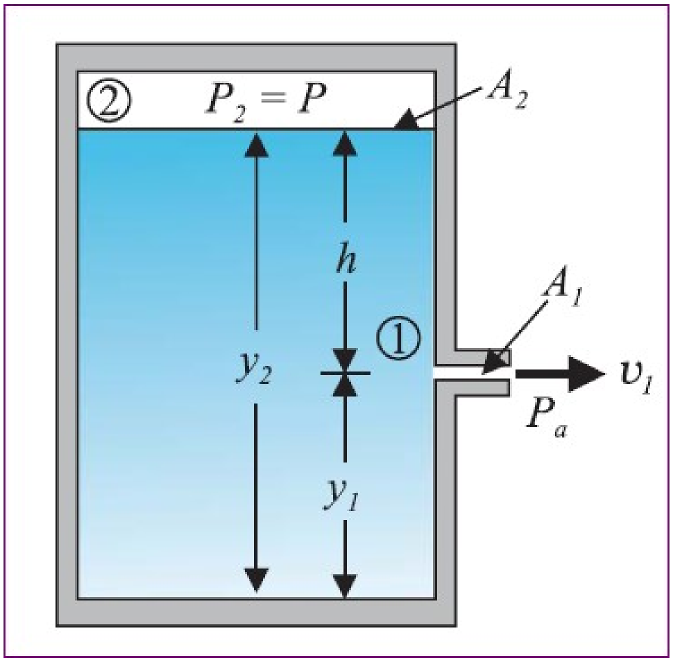
- This is Pascal's law

APPLICATIONS OF BERNOULLI'S THEOREM

Application 1

Speed of Efflux

- The word efflux means fluid outflow
- Consider a tank containing a liquid of density ρ with a small hole in its side at a height y1 from the bottom.
- The air above the liquid, whose surface is at height y2, is at pressure P.



- From the equation of continuity

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{A_1}{A_2} v_1$$

- If the cross sectional area of the tank is larger than that of the hole then$v2$ =0 .
- Applying Bernoulli's Equation (here P1=Pa , P2=P)

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^{\,2} = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^{\,2}$$

- That is

$$\boxed{P_a + \rho g y_1 + \frac{1}{2}\rho v_1^{\,2} = P + \rho g h_2}$$

$$\boxed{\frac{1}{2}\rho v_1^{\,2} = (P - P_a) + \rho g (y_2 - y_1)}$$

$$\boxed{\frac{1}{2}\rho v_1^{\,2} = (P - P_a) + \rho g h}$$

- Where y2 – y1 = h

$$\boxed{v_1 = \sqrt{\frac{2}{\rho}(P - P_a) + 2gh}}$$