Exercise 21

Question 1.
Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + \frac{1}{x}.y = x^2$$

Answer:
Given Differential Equation :

$\frac{dy}{dx} + \frac{1}{x}.y = x^2$ ....eq(1)

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x}$ and Q = x^2

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{x} dx}$$

$$= e^{\log x} \text{.........}\left(\because \int \frac{1}{x} dx = \log x\right)$$

= x ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(x) = \int x^2.(x)dx + c$$

$$\therefore xy = \int x^3 dx + c$$

$$\therefore xy = \frac{x^4}{4} + c \text{.........}\left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$$

$$\therefore y = \frac{x^3}{4} + \frac{c}{x}$$

Question 2.
Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx}+2y=x^2$$

Answer:
Given Differential Equation :

$$x\frac{dy}{dx}+2y=x^2$$

Formula :

i) $\int \frac{1}{x}dx=\log x$

ii) $\int x^n dx=\frac{x^{n+1}}{n+1}+c$

iii) $a\log b=\log b^a$

iv) $a^{\log_a b}=b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx}+Py=Q$$

The general solution is given by,

$$y.(I.F.)=\int Q.(I.F.)dx+c$$

Where integrating factor,

$\mathrm{I.F.} = e^{\int P\, dx}$

<u>Answer</u> :

Given differential equation is

$$x\frac{dy}{dx} + 2y = x^2$$

Dividing the above equation by x,

$\frac{dy}{dx} + \frac{2}{x}.y = x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{2}{x}$ and Q = x

Therefore, integrating factor is

$\mathrm{I.F.} = e^{\int P\, dx}$

$= e^{\int \frac{2}{x} dx}$

$= e^{2\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= e^{\log x^2}$ ....$(\because a\log b = \log b^a)$

$= x^2$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(x^2) = \int x.(x^2)dx + c$$

$$\therefore x^2y = \int x^3 dx + c$$

$$\therefore x^2y = \frac{x^4}{4} + c \cdots\cdots\cdots \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$$

$$\therefore y = \frac{x^2}{4} + \frac{c}{x^2}$$

Question 3.

Find the general solution for each of the following differential equations.

$$2x\frac{dy}{dx} + y = 6x^3$$

Answer:

Given Differential Equation :

$$2x\frac{dy}{dx} + y = 6x^3$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$2x\frac{dy}{dx} + y = 6x^3$$

Dividing the above equation by 2x,

$\frac{dy}{dx} + \frac{1}{2x}.y = 3x^2$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{2x}$ and Q = $3x^2$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$= e^{\int \frac{1}{2x} dx}$

$= e^{\frac{1}{2}\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= e^{\log \sqrt{x}}$ ....$(\because a \log b = \log b^a)$

= √x.........$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sqrt{x}) = \int 3x^2.(\sqrt{x})dx + c$$

$$\therefore \sqrt{x}.y = \int 3x^{5/2}dx + c$$

$$\therefore \sqrt{x}.y = 3\frac{x^{7/2}}{7/2} + c \text{ ....} \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$$

Dividing the above equation by √x

$$\therefore y = \frac{6}{7}x^3 + \frac{c}{\sqrt{x}}$$

$$\therefore y = \frac{6}{7}x^3 + \frac{c}{\sqrt{x}}$$

Question 4.
Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} + y = 3x^2 - 2, x > 0$$

Answer:

Given Differential Equation :

$$x\frac{dy}{dx} + y = 3x^2 - 2$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} + y = 3x^2 - 2$$

Dividing the above equation by x,

$\frac{dy}{dx}+\frac{1}{x}.y=\frac{3x^2-2}{x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P=\frac{1}{x}$ and $Q=\frac{3x^2-2}{x}$

Therefore, the integrating factor is

$$I.F.=\ e^{\int P\,dx}$$

$$=e^{\int \frac{1}{x}dx}$$

$=e^{\log x}$ ....$\left(\because \int \frac{1}{x}dx=\log x\right)$

= x.........$\left(\because a^{\log_a b}=b\right)$

General solution is

$$y.(I.F.)=\int Q.(I.F.)dx\ +c$$

$$\therefore y.(x)=\int\left(\frac{3x^2-2}{x}\right).(x)dx\ +c$$

$$\therefore xy=\int(3x^2-2)dx\ +c$$

$\therefore xy=3\frac{x^3}{3}-2x+c$ ....$\left(\because \int x^n dx=\frac{x^{n+1}}{n+1}+c\right)$

Dividing the above equation by x

$$\therefore y = x^2 - 2 + \frac{c}{x}$$

$$\therefore y = x^2 - 2 + \frac{c}{x}$$

Question 5.
Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} - y = 2x^3$$

Answer:
Given Differential Equation :

$$x\frac{dy}{dx} - y = 2x^3$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} - y = 2x^3$$

Dividing the above equation by x,

$\frac{dy}{dx} - \frac{1}{x}.y = 2x^2$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and Q = $2x^2$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x}dx}$$

$= e^{-\log x}$ ....$\left(\because \int \frac{1}{x}dx = \log x\right)$

$= e^{\log\frac{1}{x}}$ ....$(\because a\log b = \log b^a)$

$= \frac{1}{x}$.........$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int 2x^2.\left(\frac{1}{x}\right)dx + c$$

$$\therefore \frac{y}{x} = \int 2xdx + c$$

$$\therefore \frac{y}{x} = 2\frac{x^2}{2} + c \text{.........} \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$$

Multiplying above equation by x

$$\therefore y = x^3 + cx$$

$$\therefore y = x^3 + cx$$

Question 6.
Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} - y = x + 1$$

Answer:
<u>Given Differential Equation</u> :

$$x\frac{dy}{dx} - y = x + 1$$

<u>Formula</u> :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$x\frac{dy}{dx} - y = x + 1$$

Dividing above equation by x,

$\frac{dy}{dx} - \frac{1}{x}.y = \frac{x+1}{x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = \frac{x+1}{x}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x}\,dx}$$

$$= e^{-\log x} \text{.........}\left(\because \int \frac{1}{x}dx = \log x\right)$$

$$= e^{\log\frac{1}{x}} \text{.........}(\because a\log b = \log b^{a})$$

$$= \frac{1}{x} \text{.........}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int \left(\frac{x+1}{x}\right).\left(\frac{1}{x}\right)dx + c$$

$$\therefore \frac{y}{x} = \int \left(\frac{x+1}{x^2}\right)dx + c$$

$$\therefore \frac{y}{x} = \int \left(\frac{1}{x} + \frac{1}{x^2}\right)dx + c$$

$$\therefore \frac{y}{x} = \int \left(\frac{1}{x} + x^{-2}\right)dx + c$$

$$\therefore \frac{y}{x} = \log x + \frac{x^{-1}}{-1} + c \text{.........}\left(\because \int x^n dx = \frac{x^{n+1}}{n+1} \;\&\; \int \frac{1}{x}dx = \log x\right)$$

$$\therefore \frac{y}{x} = \log x - \frac{1}{x} + c$$

Multiplying above equation by x,

$$\therefore y = x \log x - 1 + cx$$

$$\therefore y = x \log x - 1 + cx$$

Question 7.
Find the general solution for each of the following differential equations.

$$(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{(1+x^2)}$$

Answer:
Given Differential Equation :

$$(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{(1+x^2)}$$

Formula :

i) $\int \frac{f'(x)}{f(x)} dx = \log f(x)$

ii) $\int \frac{1}{(1+x^2)} dx = \tan^{-1} x$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{(1+x^2)}$$

Dividing above equation by $(1+x^2)$,

$\frac{dy}{dx} + \frac{2x}{(1+x^2)}.y = \frac{1}{(1+x^2)^2}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{2x}{(1+x^2)}$ and $Q = \frac{1}{(1+x^2)^2}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{2x}{(1+x^2)}dx}$$

Let, $f(x) = (1+x^2)$ & $f'(x) = 2x$

$= e^{\log(1+x^2)}$ ....$\left(\because \int \frac{f'(x)}{f(x)}dx = \log f(x)\right)$

$$= (1 + x^2) \text{ ....} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(1 + x^2) = \int \frac{1}{(1 + x^2)^2}.(1 + x^2)dx + c$$

$$\therefore y.(1 + x^2) = \int \frac{1}{(1 + x^2)}dx + c$$

$$\therefore y.(1 + x^2) = \tan^{-1}x + c \text{ ....} \left(\because \int \frac{1}{(1+x^2)}dx = \tan^{-1}x\right)$$

Therefore, general solution is

$$y.(1 + x^2) = tan^{-1}x + c$$

Question 8.
Find the general solution for each of the following differential equations.

$$(1 - x^2)\frac{dy}{dx} + xy = x\sqrt{1 - x^2}$$

Answer:
Given Differential Equation :

$$(1 - x^2)\frac{dy}{dx} + xy = x\sqrt{1 - x^2}$$

Formula :

i) $\int \frac{f'(x)}{f(x)}dx = \log f(x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$(1 - x^2)\frac{dy}{dx} + xy = x\sqrt{1 - x^2}$$

Dividing above equation by $(1 – x^2)$,

$$\frac{dy}{dx} + \frac{x}{(1 - x^2)}.y = \frac{x\sqrt{1 - x^2}}{(1 - x^2)}$$

$\frac{dy}{dx} + \frac{x}{(1-x^2)}.y = \frac{x}{\sqrt{1-x^2}}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{x}{(1-x^2)}$ and $Q = \frac{x}{\sqrt{1-x^2}}$

Therefore, integrating factor is

$$\mathrm{I.F.} = e^{\int P\,dx}$$

$$= e^{\int \frac{x}{(1-x^2)} dx}$$

$$= e^{\frac{-1}{2}\int \frac{-2x}{(1-x^2)} dx}$$

Let $(1 - x^2) = f(x)$

Therefore $f'(x) = -2x$

$$\therefore \int \frac{f'(x)}{f(x)} dx = \int \frac{-2x}{(1-x^2)} dx = \log f(x) = \log(1-x^2) \text{......eq(2)}$$

$$\therefore \mathrm{I.F.} = e^{\frac{-1}{2}\log(1-x^2)}$$

$$= e^{\log(1-x^2)^{-1/2}} \text{.........}(\because a\log b = \log b^a)$$

$$= e^{\log\left(\frac{1}{\sqrt{1-x^2}}\right)}$$

$$= \frac{1}{\sqrt{1-x^2}} \text{.........}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$$\therefore y.\left(\frac{1}{\sqrt{1-x^2}}\right) = \int \left(\frac{x}{\sqrt{1-x^2}}\right).\left(\frac{1}{\sqrt{1-x^2}}\right) dx + c$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \int \frac{x}{(1-x^2)} dx + c$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \frac{-1}{2} \int \frac{-2x}{(1-x^2)} dx + c$$

$\therefore \frac{y}{\sqrt{1-x^2}} = \frac{-1}{2} \log(1-x^2) + c$ ....from eq(2)

Multiplying above equation by $\sqrt{1-x^2}$,

$$\therefore y = \frac{-1}{2}\sqrt{1-x^2}\log(1-x^2) + c\sqrt{1-x^2}$$

Question 9.
Find the general solution for each of the following differential equations.

$$(1-x^2)\frac{dy}{dx} + xy = ax$$

Answer:
Given Differential Equation :

$$(1-x^2)\frac{dy}{dx} + xy = ax$$

Formula :

i) $\int \frac{f'(x)}{f(x)} dx = \log f(x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(1 - x^2)\frac{dy}{dx} + xy = ax$$

Dividing above equation by $(1 – x^2)$,

$$\frac{dy}{dx} + \frac{x}{(1-x^2)}.y = \frac{ax}{(1-x^2)}$$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{x}{(1-x^2)}$ and $Q = \frac{ax}{(1-x^2)}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{x}{(1-x^2)}dx}$$

$$= e^{\frac{-1}{2}\int\frac{-2x}{(1-x^2)}dx}$$

Let $(1 - x^2) = f(x)$

Therefore $f'(x) = -2x$

$$\therefore \int \frac{f'(x)}{f(x)}dx = \int \frac{-2x}{(1-x^2)}dx = \log f(x) = \log(1-x^2)$$

$$\therefore \mathrm{I.F.} = e^{\frac{-1}{2}\log(1-x^2)}$$

$$= e^{\log(1-x^2)^{-1/2}} \text{.........} (\because a\log b = \log b^a)$$

$$= e^{\log\left(\frac{1}{\sqrt{1-x^2}}\right)}$$

$$= \frac{1}{\sqrt{1-x^2}} \text{.........} (\because a^{\log_a b} = b)$$

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$$\therefore y.\left(\frac{1}{\sqrt{1-x^2}}\right) = \int \left(\frac{ax}{(1-x^2)}\right).\left(\frac{1}{\sqrt{1-x^2}}\right)dx + c$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \int \frac{ax}{(1-x^2)^{3/2}}dx + c \text{......eq(2)}$$

Let

$$I = \int \frac{ax}{(1-x^2)^{3/2}}dx$$

Put $(1 - x^2) = t$

$$\therefore -2x\,dx = dt$$

$$\therefore \mathrm{x\,dx} = \frac{-\mathrm{dt}}{2}$$

$$\therefore \mathrm{I} = \int \frac{\mathrm{a}}{\mathrm{t}^{3/2}} \cdot \frac{-\mathrm{dt}}{2}$$

$$\therefore \mathrm{I} = \frac{-\mathrm{a}}{2} \int \mathrm{t}^{-3/2}\mathrm{dt}$$

$$\therefore \mathrm{I} = \frac{-\mathrm{a}}{2} \cdot \frac{\mathrm{t}^{-1/2}}{-1/2}$$

$$\therefore \mathrm{I} = \mathrm{a}.\frac{1}{\sqrt{\mathrm{t}}}$$

$$\therefore \mathrm{I} = \frac{\mathrm{a}}{\sqrt{1-\mathrm{x}^2}}$$

Substituting I in eq(2)

$$\therefore \frac{\mathrm{y}}{\sqrt{1-\mathrm{x}^2}} = \frac{\mathrm{a}}{\sqrt{1-\mathrm{x}^2}} + \mathrm{c}$$

Multiplying above equation by $\sqrt{1-\mathrm{x}^2}$,

$$\therefore y = a + c\sqrt{1-x^2}$$

Question 10.
Find the general solution for each of the following differential equations.

$$(\mathrm{x}^2+1)\frac{\mathrm{dy}}{\mathrm{dx}} - 2\mathrm{xy} = (\mathrm{x}^2+1)(\mathrm{x}^2+2)$$

Answer:
Given Differential Equation :

$$(x^2+1)\frac{dy}{dx}-2xy=(x^2+1)(x^2+2)$$

Formula :

i) $\int \frac{f'(x)}{f(x)}dx=\log f(x)$

ii) $a\log b=\log b^a$

iii) $a^{\log_a b}=b$

iv) $\int 1dx=x$

v) $\int \frac{1}{1+x^2}dx=\tan^{-1}x$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx}+Py=Q$$

General solution is given by,

$$y.(I.F.)=\int Q.(I.F.)dx+c$$

Where, integrating factor,

$$I.F.=e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(x^2+1)\frac{dy}{dx}-2xy=(x^2+1)(x^2+2)$$

Dividing above equation by (1 + x^2),

$\frac{dy}{dx} + \frac{-2x}{(1+x^2)} . y = (x^2 + 2)$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-2x}{(1+x^2)}$ and $Q = (x^2 + 2)$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-2x}{(1+x^2)} dx}$$

$$= e^{-\int \frac{2x}{(1+x^2)} dx}$$

Let (1 + x^2) = f(x)

Therefore f′(x) = 2x

$$\therefore \int \frac{f'(x)}{f(x)} dx = \int \frac{2x}{(1+x^2)} dx = \log f(x) = \log(1+x^2)$$

$$\therefore I.F. = e^{-\log(1+x^2)}$$

$= e^{\log(1+x^2)^{-1}}$ ....($\because a\log b = \log b^a$)

$$= e^{\log\left(\frac{1}{(1+x^2)}\right)}$$

$= \frac{1}{(1+x^2)}$ ....($\because a^{\log_a b} = b$)

General solution is

$$y.(\text{I.F.}) = \int \text{Q.(I.F.)}dx + c$$

$$\therefore y.\left(\frac{1}{(1+x^2)}\right) = \int (2+x^2).\left(\frac{1}{(1+x^2)}\right)dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int \frac{2+x^2}{1+x^2}dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int \frac{1+x^2+1}{1+x^2}dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int \left(\frac{1+x^2}{1+x^2}+\frac{1}{1+x^2}\right)dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int \left(1+\frac{1}{1+x^2}\right)dx + c$$

$$\therefore \frac{y}{(1+x^2)} = x + \tan^{-1}x + c$$

.........$\left(\because \int 1dx = x \ \& \ \int \frac{1}{1+x^2}dx = \tan^{-1}x\right)$

$$\therefore y = (1+x^2)(x+\tan^{-1}x+c)$$

Therefore general solution is

$$y = (1+x^2)(x+tan^{-1}x+c)$$

Question 11.
Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 2y = 6e^x$$

Answer:
<u>Given Differential Equation</u> :

$$\frac{dy}{dx} + 2y = 6e^{x}$$

<u>Formula</u> :

i) $\int 1 dx = x$

ii) $\int e^{kx} dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$\frac{dy}{dx} + 2y = 6e^{x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2$ and $Q = 6e^x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 2dx}$$

$$= e^{2\int 1dx}$$

$$= e^{2x} \text{.........}(\because \int 1dx = x)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(e^{2x}) = \int (6e^x).(e^{2x})dx + c$$

$$\therefore y.(e^{2x}) = 6\int e^{3x}dx + c$$

$$\therefore y.(e^{2x}) = 6\frac{e^{3x}}{3} + c \text{.........}\left(\because \int e^{kx}dx = \frac{e^{kx}}{k}\right)$$

$$\therefore y.(e^{2x}) = 2e^{3x} + c$$

Dividing above equation by (e^{2x}),

$$\therefore y = \frac{2e^{3x}}{e^{2x}} + \frac{c}{e^{2x}}$$

$$\therefore y = 2e^{(3x-2x)} + ce^{-2x}$$

$\therefore y = 2e^{x} + ce^{-2x}$

Therefore general solution is

$$y = 2e^{x} + ce^{-2x}$$

Question 12.
Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 3y = e^{-2x}$$

Answer:
Given Differential Equation :

$$\frac{dy}{dx} + 3y = e^{-2x}$$

Formula :

i) $\int 1dx = x$

ii) $\int e^{kx}dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$\mathrm{I.F.} = e^{\int P\,dx}$

Answer :

Given differential equation is

$\frac{dy}{dx} + 3y = e^{-2x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 3$ and $Q = e^{-2x}$

Therefore, integrating factor is

$\mathrm{I.F.} = e^{\int P\,dx}$

$= e^{\int 3dx}$

$= e^{3\int 1dx}$

$= e^{3x}$ ....($\because \int 1dx = x$)

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$$\therefore y.(e^{3x}) = \int (e^{-2x}).(e^{3x})dx + c$$

$$\therefore y.(e^{3x}) = \int e^{x}dx + c$$

$\therefore y.(e^{3x}) = e^{x} + c$ ....$\left(\because \int e^{kx}dx = \frac{e^{kx}}{k}\right)$

Dividing above equation by (e^{3x}),

$$\therefore y = \frac{e^{x}}{e^{3x}} + \frac{c}{e^{3x}}$$

$$\therefore y = e^{(x-3x)} + ce^{-3x}$$

$$\therefore y = e^{-2x} + ce^{-3x}$$

Therefore general solution is

$$y = e^{-2x} + ce^{-3x}$$

Question 13.
Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 8y = 5e^{-3x}$$

Answer:
Given Differential Equation :

$$\frac{dy}{dx} + 8y = 5e^{-3x}$$

Formula :

i) $\int 1dx = x$

ii) $\int e^{kx}dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$\frac{dy}{dx} + 8y = 5e^{-3x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 8$ and $Q = 5e^{-3x}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 8dx}$$

$$= e^{8\int 1dx}$$

$= e^{8x}$ ....$(\because \int 1dx = x)$

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$$\therefore y.(e^{8x}) = \int (5e^{-3x}).(e^{8x})dx + c$$

$$\therefore y.(e^{8x}) = 5\int e^{5x}dx + c$$

$$\therefore y.(e^{8x}) = 5\frac{e^{5x}}{5} + c \,\ldots\ldots\ldots \left(\because \int e^{kx}dx = \frac{e^{kx}}{k}\right)$$

$$\therefore y.(e^{8x}) = e^{5x} + c$$

Dividing above equation by (e^{8x}),

$$\therefore y = \frac{e^{5x}}{e^{8x}} + \frac{c}{e^{8x}}$$

$$\therefore y = e^{(5x-8x)} + ce^{-8x}$$

$$\therefore y = e^{-3x} + ce^{-8x}$$

Therefore general solution is

$$y = e^{-3x} + ce^{-8x}$$

Question 14.
Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} - y = (x-1)e^{x}, x > 0$$

Answer:
<u>Given Differential Equation</u> :

$$x\frac{dy}{dx} - y = (x-1)e^x$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int e^x \left(f(x) + f'(x)\right) dx = e^x . f(x)$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} - y = (x-1)e^x$$

Dividing above equation by x,

$\frac{dy}{dx} - \frac{1}{x}y = \frac{(x-1)}{x}e^x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = \frac{(x-1)}{x}e^x$

Therefore, integrating factor is

$$\mathrm{I.F.} = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x}dx}$$

$$= e^{-\log x} \text{.........}\left(\because \int \frac{1}{x}dx = \log x\right)$$

$$= e^{\log x^{-1}} \text{.........}(\because a\log b = \log b^a)$$

$$= \frac{1}{x} \text{.........}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int \left(\frac{(x-1)}{x}e^x\right).\left(\frac{1}{x}\right)dx + c$$

$\therefore \frac{y}{x} = \int \left(\frac{x-1}{x^2}e^x\right)dx + c$ ....eq(2)

Let,

$$I = \int \left(\frac{x-1}{x^2} e^x\right) dx$$

$$\therefore I = \int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$$

Let $f(x) = \frac{1}{x} \therefore f'(x) = \frac{-1}{x^2}$

$$\therefore I = e^x . \frac{1}{x} \text{.........} \left(\because \int e^x (f(x) + f'(x)) dx = e^x . f(x)\right)$$

Substituting I in eq(2),

$$\therefore \frac{y}{x} = e^x . \frac{1}{x} + c$$

Multiplying above equation by x,

$$\therefore y = e^x + cx$$

Therefore general solution is

$$y = e^x + cx$$

Question 15.
Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} - y \tan x = e^x \sec x$$

Answer:
Given Differential Equation :

$$\frac{dy}{dx} - y \tan x = e^x \sec x$$

Formula :

i) $\int \tan x\, dx = \log(\sec x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int e^x dx = e^x$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

<u>Answer</u> :

Given differential equation is

$\frac{dy}{dx} - y\tan x = e^x \sec x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -\tan x$ and $Q = e^x \sec x$

Therefore, integrating factor is

$$\mathrm{I.F.} = e^{\int P\,dx}$$

$$= e^{\int -\tan x\,dx}$$

$$= e^{-\log(\sec x)} \text{ ....} (\because \int \tan x\,dx = \log(\sec x)\,)$$

$$= e^{\log(\sec x)^{-1}} \text{ ....} (\because a\log b = \log b^a)$$

$$= e^{\log(\cos x)}$$

$$= \cos x \text{ ....} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$$\therefore y.(\cos x) = \int (e^x \sec x).(\cos x)dx + c$$

$$\therefore y.(\cos x) = \int \left(e^x.\frac{1}{\cos x}\right).(\cos x)dx + c$$

$$\therefore y.(\cos x) = \int e^x dx + c$$

$$\therefore y.(\cos x) = e^x + c \text{ ....} (\because \int e^x dx = e^x)$$

Therefore general solution is

$$y.(\cos x) = e^x + c$$

Question 16.

Find the general solution for each of the following differential equations.

$$(x\log x)\frac{dy}{dx}+y=2\log x$$

Answer:
Given Differential Equation :

$$(x\log x)\frac{dy}{dx}+y=2\log x$$

Formula :

i) $\int \frac{f'(x)}{f(x)}dx=\log(f(x))$

ii) $a^{\log_a b}=b$

iii) $\int u.v\,dx=u.\int v\,dx-\int\left(\frac{du}{dx}.\int v\,dx\right)dx$

iv) $\frac{d}{dx}(\log x)=\frac{1}{x}$

v) $\int \frac{1}{x}dx=\log x$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx}+Py=Q$$

General solution is given by,

$$y.(I.F.)=\int Q.(I.F.)dx+c$$

Where, integrating factor,

$\mathrm{I.F.} = e^{\int P\,dx}$

Answer :

Given differential equation is

$$(x\log x)\frac{dy}{dx} + y = 2\log x$$

Dividing above equation by (x.log x),

$\frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x\log x}$ and $Q = \frac{2}{x}$

Therefore, integrating factor is

$\mathrm{I.F.} = e^{\int P\,dx}$

$= e^{\int \frac{1}{x\log x}dx}$

$= e^{\int \frac{1/x}{\log x}dx}$

Let, $f(x) = \log x \therefore f'(x) = 1/x$

$\therefore \mathrm{I.F.} = e^{\log(\log x)}$ ....$\left(\because \int \frac{f'(x)}{f(x)}dx = \log\left(f(x)\right)\right)$

$= \log x$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\log x) = \int \left(\frac{2}{x}\log x\right) dx + c$$

$\therefore y.(\log x) = 2\int\left(\frac{1}{x}\log x\right) dx + c$ ....eq(2)

Let,

$$I = \int \frac{1}{x}.\log x\, dx$$

Let, $u = \log x \;\&\; v = \frac{1}{x}$

$$\therefore I = \log x \int \frac{1}{x} dx - \int \left(\frac{d}{dx}(\log x). \int \frac{1}{x} dx\right) dx$$

$$.........\left(\because \int u.v\, dx = u.\int v\, dx - \int\left(\frac{du}{dx}.\int v\, dx\right) dx\right)$$

$$\therefore I = \log x.\log x - \int \left(\frac{1}{x}.\log x\right) d$$

$$.........\left(\because \frac{d}{dx}(\log x) = \frac{1}{x} \;\&\; \int \frac{1}{x} dx = \log x\right)$$

$$\therefore I = (\log x)^2 - I$$

$$\therefore 2I = (\log x)^2$$

$$\therefore I = \frac{1}{2}(\log x)^2$$

Substituting I in eq(2),

$$\therefore y.(\log x) = 2.\frac{1}{2}(\log x)^2 + c$$

$$y.(\log x) = (\log x)^2 + c$$

Question 17.
Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} + y = x\log x$$

Answer:
Given Differential Equation :

$$x\frac{dy}{dx} + y = x\log x$$

Formula :

i) $\int \frac{1}{x}dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx$

iv) $\frac{d}{dx}(\log x) = \frac{1}{x}$

v) $\int x^n\,dx = \frac{x^{n+1}}{n+1}$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$x\frac{dy}{dx} + y = x\log x$$

Dividing above equation by x,

$\frac{dy}{dx} + \frac{1}{x}y = \log x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x}$ and $Q = \log x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{x}dx}$$

$= e^{\log x}$ ....$\left(\because \int \frac{1}{x}dx = \log x\right)$

$= x$ ....$(\because a^{\log_a b} = b)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$\therefore y.(x) = \int (x \log x)dx + c$ ....eq(2)

Let,

$$I = \int (x \log x)dx$$

Let, $u = \log x \;\&\; v = x$

$$\therefore I = \log x \int x\,dx - \int \left(\frac{d}{dx}(\log x).\int x\,dx\right)dx$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right)dx\right)$

$$\therefore I = \log x.\frac{x^2}{2} - \int \left(\frac{1}{x}.\frac{x^2}{2}\right)dx$$

.........$\left(\because \frac{d}{dx}(\log x) = \frac{1}{x} \;\&\; \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$

$$\therefore I = \log x.\frac{x^2}{2} - \frac{1}{2}\int (x)\,dx$$

$\therefore I = \log x.\frac{x^2}{2} - \frac{1}{2}\left(\frac{x^2}{2}\right)$ ....$\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$

$$\therefore I = \frac{x^2}{2}.\log x - \frac{x^2}{4}$$

Substituting I in eq(2),

$$\therefore xy = \frac{x^2}{2}.\log x - \frac{x^2}{4} + c$$

Multiplying above equation by 4,

$$\therefore 4xy = 2x^2.\log x - x^2 + 4c$$

Therefore general equation is

$$4xy = 2x^2.\log x - x^2 + 4c$$

Question 18.
Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

Answer:
Given Differential Equation :

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx$

v) $\frac{d}{dx}(\log x) = \frac{1}{x}$

vi) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

vii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dx}$$

<u>Answer</u> :

Given differential equation is

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

Dividing above equation by x,

$\frac{dy}{dx} + \frac{2}{x}y = x \log x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{2}{x}$ and $Q = x \log x$

Therefore, integrating factor is

$\mathrm{I.F.} = e^{\int P\,dx}$

$= e^{\int \frac{2}{x}\,dx}$

$= e^{2\int \frac{1}{x}\,dx}$

$= e^{2\log x}$ ....$\left(\because \int \frac{1}{x}dx = \log x\right)$

$= e^{\log x^2}$ ....$(\because a\log b = \log b^a)$

$= x^2$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$$\therefore y.(x^2) = \int (x^2.x\log x)dx + c$$

$\therefore y.(x^2) = \int (x^3\log x)dx + c$ ....eq(2)

Let,

$$I = \int (x^3\log x)dx$$

Let, $u = \log x \;\&\; v = x^3$

$$\therefore I = \log x \int x^3\,dx - \int \left(\frac{d}{dx}(\log x).\int x^3\,dx\right)dx$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right)dx\right)$

$$\therefore I = \log x.\frac{x^4}{4} - \int\left(\frac{1}{x}.\frac{x^4}{4}\right)dx$$

$$\text{.........}\left(\because \frac{d}{dx}(\log x) = \frac{1}{x} \ \& \ \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

$$\therefore I = \log x.\frac{x^4}{4} - \frac{1}{4}\int(x^3)\,dx$$

$$\therefore I = \log x.\frac{x^4}{4} - \frac{1}{4}\left(\frac{x^4}{4}\right)\text{.........}\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

$$\therefore I = \frac{x^4}{4}.\log x - \frac{x^4}{16}$$

Substituting I in eq(2),

$$\therefore x^2y = \frac{x^4}{4}.\log x - \frac{x^4}{16} + c$$

Dividing above equation by x^2,

$$\therefore y = \frac{x^2}{4}.\log x - \frac{x^2}{16} + \frac{c}{x^2}$$

$$\therefore y = \frac{x^2}{16}(4\log x - 1) + \frac{c}{x^2}$$

Therefore general equation is

$$y = \frac{x^2}{16}(4\log x - 1) + \frac{c}{x^2}$$

Question 19.
Find the general solution for each of the following differential equations.

$$(1+x)\frac{dy}{dx} - y = e^{3x}(1+x)^2$$

Answer:

Given Differential Equation :

$$(1+x)\frac{dy}{dx}-y=e^{3x}(1+x)^2$$

Formula :

i) $\int \frac{1}{px+q}dx=\frac{1}{p}\log(px+q)$

ii) $a\log b=\log b^a$

iii) $a^{\log_a b}=b$

iv) $\int e^{kx}dx=\frac{1}{k}e^{kx}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx}+Py=Q$$

General solution is given by,

$$y.(I.F.)=\int Q.(I.F.)dx+c$$

Where, integrating factor,

$$I.F.=e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(1+x)\frac{dy}{dx}-y=e^{3x}(1+x)^2$$

Dividing above equation by (1+x),

$\frac{dy}{dx}-\frac{1}{(1+x)}y=e^{3x}(1+x)$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P=\frac{-1}{(1+x)}$ and $Q=e^{3x}(1+x)$

Therefore, integrating factor is

$$\mathrm{I.F.}=\ e^{\int P\,dx}$$

$$=e^{\int\frac{-1}{(1+x)}dx}$$

$$=e^{-\int\frac{1}{(1+x)}dx}$$

$$=e^{-\log(1+x)}\ ....\left(\because\ \int\frac{1}{px+q}dx=\frac{1}{p}\log(px+q)\right)$$

$$=e^{\log\frac{1}{(1+x)}}\ ....(\because a\log b=\log b^a)$$

$$=\frac{1}{(1+x)}\ ....\left(\because a^{\log_a b}=b\right)$$

General solution is

$$y.(\mathrm{I.F.})=\int Q.(\mathrm{I.F.})dx\ +c$$

$$\therefore y.\left(\frac{1}{(1+x)}\right) = \int e^{3x}(1+x)\left(\frac{1}{(1+x)}\right)dx + c$$

$$\therefore y.\left(\frac{1}{(1+x)}\right) = \int e^{3x}dx + c$$

$$\therefore y.\left(\frac{1}{(1+x)}\right) = \frac{1}{3}e^{3x} + c \text{ ....}\left(\because \int e^{kx}\,dx = \frac{1}{k}e^{kx}\right)$$

Multiplying above equation by (1+x),

$$\therefore y = \frac{1}{3}(1+x)e^{3x} + c(1+x)$$

Therefore general equation is

$$y = \frac{1}{3}(1+x)e^{3x} + c(1+x)$$

Question 20.
Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + \frac{4x}{(x^2+1)}y + \frac{1}{(x^2+1)^2} = 0$$

Answer:
<u>Given Differential Equation</u> :

$$\frac{dy}{dx} + \frac{4x}{(x^2+1)}y + \frac{1}{(1+x^2)^2} = 0$$

<u>Formula</u> :

i) $\int \frac{f'(x)}{f(x)}dx = \log\left(f(x)\right)$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int 1\, dx = x$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

<u>Answer</u> :

Given differential equation is

$$\frac{dy}{dx} + \frac{4x}{(x^2+1)}y + \frac{1}{(1+x^2)^2} = 0$$

$\therefore \frac{dy}{dx} + \frac{4x}{(x^2+1)}y = \frac{-1}{(1+x^2)^2}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{4x}{(x^2+1)}$ and $Q = \frac{-1}{(1+x^2)^2}$

Therefore, integrating factor is

$\mathrm{I.F.} = e^{\int P\,dx}$

$= e^{\int \frac{4x}{(x^2+1)} dx}$

$= e^{2\int \frac{2x}{(x^2+1)} dx}$

Let, $f(x) = (x^2+1)$ & $f'(x) = 2x$

$\therefore \mathrm{I.F.} = e^{2\log(x^2+1)}$ ....$\left(\because \int \frac{f'(x)}{f(x)} dx = \log\left(f(x)\right)\right)$

$= e^{\log(1+x^2)^2}$ ....$(\because a\log b = \log b^a)$

$= (1+x^2)^2$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$$\therefore y.(1+x^2)^2 = \int \frac{-1}{(1+x^2)^2}(1+x^2)^2 dx + c$$

$$\therefore y.(1+x^2)^2 = \int -1 dx + c$$

$\therefore y.(1+x^2)^2 = -x + c$ ....$(\because \int 1\,dx = x)$

Dividing above equation by $(1+x^2)^2$,

$$\therefore y = \frac{-x}{(1+x^2)^2} + \frac{c}{(1+x^2)^2}$$

Therefore general equation is

$$y=\frac{-x}{(1+x^2)^2}+\frac{c}{(1+x^2)^2}$$

Question 21.
Find the general solution for each of the following differential equations.

$$(y+3x^2)\frac{dx}{dy}=x$$

Answer:
Given Differential Equation :

$$(y+3x^2)\frac{dx}{dy}=x$$

Formula :

i) $\int \frac{1}{x}dx=\log x$

ii) $a\log b=\log b^a$

iii) $a^{\log_a b}=b$

iv) $\int 1\,dx=x$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx}+Py=Q$$

General solution is given by,

$$y.(I.F.)=\int Q.(I.F.)dx+c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$(y + 3x^2)\frac{dx}{dy} = x$$

$$\therefore \frac{dy}{dx} = \frac{(y + 3x^2)}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x} + 3x$$

$\therefore \frac{dy}{dx} - \frac{y}{x} = 3x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = 3x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x}\,dx}$$

$= e^{-\log x}$ ....$\left(\because \int \frac{1}{x}dx = \log x\right)$

$= e^{\log\left(\frac{1}{x}\right)}$ ....$(\because a\log b = \log b^a)$

$= \frac{1}{x}$ ....$(\because a^{\log_a b} = b)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int 3x.\left(\frac{1}{x}\right) dx + c$$

$$\therefore \frac{y}{x} = \int 3dx + c$$

$$\therefore \frac{y}{x} = 3\int 1dx + c$$

$$\therefore \frac{y}{x} = 3x + c \text{ ....}(\because \int 1\, dx = x)$$

Multiplying above equation by x,

$$\therefore y = 3x^2 + cx$$

Therefore general equation is

$$y = 3x^2 + cx$$

Question 22.
Find the general solution for each of the following differential equations.

$$xdy - (y + 2x^2)dx = 0$$

Answer:
Given Differential Equation :

$$xdy - (y + 2x^2)dx = 0$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int 1\, dx = x$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$$xdy - (y + 2x^2)dx = 0$$

$$\therefore xdy = (y + 2x^2)dx$$

$$\therefore \frac{dy}{dx} = \frac{(y + 2x^2)}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x} + 2x$$

$\therefore \frac{dy}{dx} - \frac{y}{x} = 2x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = 2x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x}\,dx}$$

$= e^{-\log x}$ ....$\left(\because \int \frac{1}{x}dx = \log x\right)$

$= e^{\log\left(\frac{1}{x}\right)}$ ....$(\because a\log b = \log b^a)$

$= \frac{1}{x}$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int 2x.\left(\frac{1}{x}\right)dx + c$$

$$\therefore \frac{y}{x} = \int 2dx + c$$

$$\therefore \frac{y}{x} = 2\int 1dx + c$$

$$\therefore \frac{y}{x} = 2x + c \text{ ....} (\because \int 1\,dx = x)$$

Multiplying above equation by x,

$$\therefore y = 2x^2 + cx$$

Therefore general equation is

$$y = 2x^2 + cx$$

Question 23.
Find the general solution for each of the following differential equations.

$$xdy + (y - x^3)dx = 0$$

Answer:
Given Differential Equation :

$$xdy + (y - x^3)dx = 0$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int x^n\,dx = \frac{x^{n+1}}{n+1}$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$xdy + (y - x^3)dx = 0$$

$$\therefore xdy = -(y - x^3)dx$$

$$\therefore xdy = (x^3 - y)dx$$

$$\therefore \frac{dy}{dx} = \frac{(x^3 - y)}{x}$$

$$\therefore \frac{dy}{dx} = x^2 - \frac{y}{x}$$

$\therefore \frac{dy}{dx} + \frac{y}{x} = x^2$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x}$ and $Q = x^2$

Therefore, integrating factor is

$$\mathrm{I.F.} = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{x}dx}$$

$$= e^{\log x} \;\ldots\ldots\ldots \left(\because \int \frac{1}{x}dx = \log x\right)$$

$$= x \;\ldots\ldots\ldots \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$$\therefore y.(x) = \int x^2.(x)dx + c$$

$$\therefore xy = \int x^3 dx + c$$

$$\therefore xy = \frac{x^4}{4} + c \;\ldots\ldots\ldots \left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

Dividing above equation by x,

$$\therefore y = \frac{x^3}{4} + \frac{c}{x}$$

Therefore general equation is

$$y = \frac{x^3}{4} + \frac{c}{x}$$

Question 24.

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 2y = \sin x$$

Answer:
Given Differential Equation :

$$\frac{dy}{dx} + 2y = \sin x$$

Formula :

i) $\int 1\, dx = x$

ii) $\int u.v\, dx = u. \int v dx - \int \left(\frac{du}{dx}. \int v\, dx\right) dx$

iii) $\int e^{kx}\, dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(\sin x) = \cos x$

v) $\frac{d}{dx}(\cos x) = \sin x$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.) dx + c$$

Where, integrating factor,

$\mathrm{I.F.} = e^{\int P\,dx}$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2y = \sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2$ and $Q = \sin x$

Therefore, integrating factor is

$\mathrm{I.F.} = e^{\int P\,dx}$

$= e^{\int 2\,dx}$

$= e^{2\int 1\,dx}$

$= e^{2x}$ ....$(\because \int 1\,dx = x)$

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$\therefore y.(e^{2x}) = \int \sin x.(e^{2x})dx + c$ ....eq(2)

Let,

$$I = \int \sin x.(e^{2x})dx$$

Let, u=sin x and v=e^{2x}

$$I = \sin x.\int e^{2x}dx - \int\left(\frac{d}{dx}(\sin x).\int e^{2x}\,dx\right)dx$$

$$\text{.........}\left(\because\ \int u.v\,dx = u.\int v dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$$

$$= \sin x.\frac{e^{2x}}{2} - \int\left(\cos x.\frac{e^{2x}}{2}\right)dx$$

$$\text{.........}\left(\because\ \int e^{kx}\,dx = \frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(\sin x) = \cos x\right)$$

$$= \sin x.\frac{e^{2x}}{2} - \frac{1}{2}\int(\cos x.e^{2x})dx$$

Again, let u=cos x and v=e^{2x}

$$\therefore I = \sin x.\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos x.\int e^{2x}dx - \int\left(\frac{d}{dx}(\cos x).\int e^{2x}\,dx\right)dx\right\}$$

$$\text{.........}\left(\because\ \int u.v\,dx = u.\int v dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$$

$$\therefore I = \sin x.\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos x.\frac{e^{2x}}{2} - \int\left((-\sin x).\frac{e^{2x}}{2}\right)dx\right\}$$

$$\text{.........}\left(\because\ \int e^{kx}\,dx = \frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(\cos x) = \sin x\right)$$

$$\therefore I = \sin x.\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos x.\frac{e^{2x}}{2} + \int\left(\sin x.\frac{e^{2x}}{2}\right)dx\right\}$$

$$\therefore I = \sin x.\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos x.\frac{e^{2x}}{2} + \frac{1}{2}\int(\sin x.e^{2x})dx\right\}$$

$$\therefore \mathrm{I} = \sin x.\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos x.\frac{e^{2x}}{2} + \frac{\mathrm{I}}{2}\right\}$$

$$\therefore \mathrm{I} = \sin x.\frac{e^{2x}}{2} - \cos x.\frac{e^{2x}}{4} - \frac{\mathrm{I}}{4}$$

$$\therefore \mathrm{I} + \frac{\mathrm{I}}{4} = \sin x.\frac{e^{2x}}{2} - \cos x.\frac{e^{2x}}{4}$$

$$\therefore \frac{5\mathrm{I}}{4} = \sin x.\frac{e^{2x}}{2} - \cos x.\frac{e^{2x}}{4}$$

Multiplying above equation by 4,

$$\therefore 5\mathrm{I} = 2\sin x.e^{2x} - \cos x.e^{2x}$$

$$\therefore 5\mathrm{I} = e^{2x}(2\sin x - \cos x)$$

$$\therefore \mathrm{I} = \frac{e^{2x}}{5}(2\sin x - \cos x)$$

Substituting I in eq(2),

$$\therefore y.(e^{2x}) = \frac{e^{2x}}{10}(2\sin x - \cos x) + c$$

Dividing above equation by e^{2x},

$$\therefore y = \frac{1}{5}(2\sin x - \cos x) + ce^{-2x}$$

Therefore general equation is

$$y = \frac{1}{5}(2\sin x - \cos x) + ce^{-2x}$$

Question 25.

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + y = \cos x - \sin x$$

Answer:
Given Differential Equation :

$$\frac{dy}{dx} + y = \cos x - \sin x$$

Formula :

i) $\int 1\,dx = x$

ii) $\int e^{x}.\left(f(x) + f'(x)\right)dx = e^{x}.f(x)$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + y = \cos x - \sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{Py} = \mathrm{Q}$$

Where, $\mathrm{P} = 1$ and $\mathrm{Q} = \cos \mathrm{x} - \sin \mathrm{x}$

Therefore, integrating factor is

$$\mathrm{I.F.} = \mathrm{e}^{\int \mathrm{P\,dx}}$$

$$= \mathrm{e}^{\int 1\,\mathrm{dx}}$$

$$= \mathrm{e}^{\mathrm{x}} \,\ldots\ldots\ldots (\because \int 1\,\mathrm{dx} = \mathrm{x})$$

General solution is

$$\mathrm{y.(I.F.)} = \int \mathrm{Q.(I.F.)dx} + \mathrm{c}$$

$$\therefore \mathrm{y.(e^x)} = \int (\cos \mathrm{x} - \sin \mathrm{x}).(\mathrm{e^x})\mathrm{dx} + \mathrm{c}$$

Let, f(x)=cos x => f ′(x) = -sin x

$$\therefore \mathrm{y.(e^x)} = (\mathrm{e^x}).\cos \mathrm{x} + \mathrm{c}$$

$$\ldots\ldots\ldots\left(\because \int \mathrm{e^x}.\big(\mathrm{f(x)} + \mathrm{f'(x)}\big)\,\mathrm{dx} = \mathrm{e^x}.\mathrm{f(x)}\right)$$

Dividing above equation by e^x,

$$\therefore \mathrm{y} = \cos \mathrm{x} + \frac{\mathrm{c}}{\mathrm{e^x}}$$

Therefore general equation is

$$y = \cos x + ce^{-x}$$

Question 26.
Find the general solution for each of the following differential equations.

$$\sec x \frac{dy}{dx} - y = \sin x$$

Answer:
Given Differential Equation :

$$\sec x \frac{dy}{dx} - y = \sin x$$

Formula :

i) $\int \cos x \, dx = \sin x$

ii) $\int u.v \, dx = u.\int v \, dx - \int \left(\frac{du}{dx}.\int v \, dx\right) dx$

iii) $\int e^{kx} \, dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(kx) = k$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$\mathrm{I.F.} = e^{\int P\,dx}$

Answer :

Given differential equation is

$$\sec x \frac{dy}{dx} - y = \sin x$$

Dividing above equation by sec x,

$$\frac{dy}{dx} - \frac{1}{\sec x} y = \frac{\sin x}{\sec x}$$

$\therefore \frac{dy}{dx} - \cos x\,.y = \sin x\,.\cos x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -\cos x$ and $Q = \sin x\,.\cos x$

Therefore, integrating factor is

$\mathrm{I.F.} = e^{\int P\,dx}$

$= e^{\int -\cos x\,dx}$

$= e^{-\sin x}$ ....($\because \int \cos x\,dx = \sin x$)

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$\therefore y.\left(e^{-\sin x}\right) = \int (\sin x\,.\cos x).\left(e^{-\sin x}\right)dx + c$ ....eq(2)

Let,

$$I = \int (\sin x . \cos x).\left(e^{-\sin x}\right) dx$$

Put sin x=t => cos x.dx=dt

$$\therefore I = \int e^{-t}.t\, dt$$

$$\therefore I = t.\int e^{-t}\, dt - \int \left(\frac{d}{dt}(t).\int e^{-t}\, dt\right)\, dt$$

$$\ldots\ldots\ldots\left(\because\ \int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right)\, dx\right)$$

$$\therefore I = -t.e^{-t} - \int \left((1).(-e^{-t})\right)\, dt$$

$$\ldots\ldots\ldots\left(\because\ \int e^{kx}\, dx = \frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(kx) = k\right)$$

$$\therefore I = -t.e^{-t} + (-e^{-t})\ \ldots\ldots\ldots\left(\because\ \int e^{kx}\, dx = \frac{e^{kx}}{k}\right)$$

$$\therefore I = -\sin x . e^{-\sin x} - e^{-\sin x}$$

Substituting I in eq(2),

$$\therefore y.\left(e^{-\sin x}\right) = -\sin x . e^{-\sin x} - e^{-\sin x} + c$$

$$\therefore y.\left(e^{-\sin x}\right) = -e^{-\sin x}(\sin x + 1) + c$$

$$\therefore y.\left(e^{-\sin x}\right) = c - e^{-\sin x}(\sin x + 1)$$

Dividing above equation by $e^{-\sin x}$,

$$\therefore y = \frac{c}{e^{-\sin x}} - (\sin x + 1)$$

Therefore general equation is

$$y = ce^{-\sin x} - (\sin x + 1)$$

Question 27.
Find the general solution for each of the following differential equations.

$$(1 + x^2)\frac{dy}{dx} + 2xy = \cot x$$

Answer:
Given Differential Equation :

$$(1 + x^2)\frac{dy}{dx} + 2xy = \cot x$$

Formula :

i) $\int \frac{f'(x)}{f(x)} dx = \log(f(x))$

ii) $a^{\log_a b} = b$

iii) $\int \cot x \, dx = \log|\sin x|$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$\mathrm{I.F.} = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$(1+x^2)\frac{dy}{dx} + 2xy = \cot x$$

Dividing above equation by $(1+x^2)$,

$$\therefore \frac{dy}{dx} + \frac{2x}{(1+x^2)}y = \frac{\cot x}{(1+x^2)} \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{2x}{(1+x^2)}$ and $Q = \frac{\cot x}{(1+x^2)}$

Therefore, integrating factor is

$$\mathrm{I.F.} = e^{\int P\,dx}$$

$$= e^{\int \frac{2x}{(1+x^2)}dx}$$

Let, $f(x) = (1+x^2) \Rightarrow f'(x) = 2x$

$$= e^{\log(1+x^2)} \text{ ....} \left(\because \int \frac{f'(x)}{f(x)}dx = \log\left(f(x)\right)\right)$$

$$= (1+x^2) \text{ ....} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(1+x^2) = \int \frac{\cot x}{(1+x^2)}.(1+x^2)dx + c$$

$$\therefore y.(1+x^2) = \int \cot x\,dx + c$$

$$\therefore y.(1+x^2) = \log|\sin x| + c \,\ldots\ldots\ldots (\because \int \cot x\,dx = \log|\sin x|)$$

Therefore, general solution is

$$y.(1+x^2) = \log|\sin x| + c$$

Question 28.
Find the general solution for each of the following differential equations.

$$(\sin x)\frac{dy}{dx} + (\cos x)y = \cos x \sin^2 x$$

Answer:
Given Differential Equation :

$$\sin x\frac{dy}{dx} + (\cos x)y = \cos x.\sin^2 x$$

Formula :

v) $\int \cot x\,dx = \log(\sin x)$

vi) $a^{\log_a b} = b$

vii) $\int x^n\,dx = \frac{x^{n+1}}{n+1}$

viii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$\sin x \frac{dy}{dx} + (\cos x)y = \cos x . \sin^2 x$$

Dividing above equation by sin x,

$$\therefore \frac{dy}{dx} - \frac{\cos x}{\sin x} y = \frac{\cos x . \sin^2 x}{\sin x}$$

$\therefore \frac{dy}{dx} + (\cot x)y = \cos x . \sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = \sin x . \cos x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \cot x\,dx}$$

$$= e^{\log(\sin x)}$$ ....$(\because \int \cot x\,dx = \log(\sin x))$

= sin x ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sin x) = \int (\sin x . \cos x).(\sin x)dx + c$$

$\therefore y.(\sin x) = \int (\sin^2 x. \cos x)dx + c$ ....eq(2)

Let,

$$I = \int (\sin^2 x. \cos x)dx$$

Put sin x=t => cos x.dx=dt

$$\therefore I = \int t^2 dt$$

$\therefore I = \frac{t^3}{3}$ ....$\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$

$$\therefore I = \frac{\sin^3 x}{3}$$

Substituting I in eq(2),

$$\therefore y.(\sin x) = \frac{\sin^3 x}{3} + c$$

Therefore, general solution is

$$y.(\sin x) = \frac{sin^3 x}{3} + c$$

Question 29.
Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 2y \cot x = 3x^2 \operatorname{cosec}^2 x$$

Answer:
Given Differential Equation :

$$\frac{dy}{dx} + 2y(\cot x) = 3x^2 \operatorname{cosec}^2 x$$

Formula :

i) $\int \cot x \, dx = \log(\sin x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$\frac{dy}{dx} + 2y(\cot x) = 3x^2 \operatorname{cosec}^2 x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2\cot x$ and $Q = 3x^2 \operatorname{cosec}^2 x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 2\cot x\,dx}$$

$= e^{2\log(\sin x)}$ ....($\because \int \cot x\,dx = \log(\sin x)$)

$= e^{\log(\sin x)^2}$ ....($\because a\log b = \log b^a$)

$= \sin^2 x$ ....($\because a^{\log_a b} = b$)

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sin^2 x) = \int (3x^2 \text{cosec}^2 x).(\sin^2 x)dx + c$$

$$\therefore y.(\sin^2 x) = \int \left(3x^2 \frac{1}{\sin^2 x}\right).(\sin^2 x)dx + c$$

$$\therefore y.(\sin^2 x) = 3\int (x^2)dx + c$$

$$\therefore y.(\sin^2 x) = 3\frac{x^3}{3} + c \text{.........}\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

$$\therefore y.(\sin^2 x) = x^3 + c$$

Therefore, general solution is

$$y.(sin^2 x) = x^3 + c$$

Question 30.
Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} - y = 2x^2 \sec x$$

Answer:
Given Differential Equation :

$$x\frac{dy}{dx} - y = 2x^2 \sec x$$

Formula :

vi) $\int \cot x\,dx = \log(\sin x)$

vii) $a\log b = \log b^a$

viii) $a^{\log_a b} = b$

ix) $\int x^n\,dx = \frac{x^{n+1}}{n+1}$

x) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$x\frac{dy}{dx} - y = 2x^2 \sec x$ ....eq(1)

Dividing above equation by x,

$$\therefore \frac{dy}{dx} - \frac{1}{x}y = 2x\sec x$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = 2x\sec x$

Therefore, integrating factor is

$$\mathrm{I.F.} = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x}\,dx}$$

$$= e^{-\log x} \text{.........}\left(\because \int \frac{1}{x}dx = \log x\right)$$

$$= e^{\log x^{-1}} \text{.........}(\because a\log b = \log b^{a})$$

$$= \frac{1}{x} \text{.........}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int (2x\sec x).\left(\frac{1}{x}\right)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = 2\int \sec x\,dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = 2\log|\sec x + \tan x| + c$$

$$\text{.........}(\because \int \sec x\,dx = \log|\sec x + \tan x|)$$

Multiplying above equation by x,

$$\therefore y = 2x\log|\sec x + \tan x| + cx$$

Therefore, general solution is

$$y = 2x\log|\sec x + \tan x| + cx$$

Question 31.

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} = y\tan x - 2\sin x$$

Answer:
Given Differential Equation :

$$\frac{dy}{dx} = y\tan x - 2\sin x$$

Formula :

i) $\int \tan x\, dx = \log|\sec x|$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $2\sin x . \cos x = \sin 2x$

v) $\int \sin x\, dx = -\cos x$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$$\frac{dy}{dx} = y\tan x - 2\sin x$$

$\frac{dy}{dx} - y\tan x = -2\sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -\tan x$ and $Q = -2\sin x$

Therefore, integrating factor is

$$\mathrm{I.F.} = e^{\int P\,dx}$$

$$= e^{\int -\tan x\,dx}$$

$= e^{-\log|\sec x|}$ ....$\left(\because \int \tan x\,dx = \log|\sec x|\right)$

$= e^{\log|\sec x|^{-1}}$ ....$\left(\because a\log b = \log b^{a}\right)$

$$= e^{\log\left(\frac{1}{\sec x}\right)}$$

$$= e^{\log(\cos x)}$$

$= \cos x$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\cos x) = \int (-2\sin x).(\cos x)dx + c$$

$$\therefore y.(\cos x) = -\int (2\sin x).(\cos x)dx + c$$

$$\therefore y.(\cos x) = -\int (\sin 2x)\,dx + c \text{ ....}(\because 2\sin x.\cos x = \sin 2x)$$

$$\therefore y.(\cos x) = \frac{\cos 2x}{2} + c \text{ ....}(\because \int \sin x\,dx = -\cos x)$$

Multiplying above equation by 2,

$$\therefore 2y.(\cos x) = \cos 2x + 2c$$

$\therefore 2y.(\cos x) = \cos 2x + C$ where, C=2c

Therefore, general solution is

$$2y.(\cos x) = \cos 2x + C$$

Question 32.
Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} = y\cot x = \sin 2x$$

Answer:
Given Differential Equation :

$$\frac{dy}{dx} + y\cot x = \sin 2x$$

Formula :

i) $\int \cot x \, dx = \log|\sin x|$

ii) $a^{\log_a b} = b$

iii) $\int u.v \, dx = u.\int v \, dx - \int \left(\frac{du}{dx}.\int v \, dx\right) dx$

iv) $\int \sin x \, dx = -\cos x$

v) $\frac{d}{dx}(\sin x) = \cos x$

vi) $2 \sin x . \cos x = \sin 2x$

vii) $\cos 2x = (\cos^2 x - \sin^2 x)$

viii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + y\cot x = \sin 2x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = \sin 2x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \cot x\,dx}$$

$$= e^{\log|\sin x|} \,\ldots\ldots\ldots (\because \int \cot x\,dx = \log|\sin x|)$$

$$= \sin x \,\ldots\ldots\ldots \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sin x) = \int (\sin 2x).(\sin x)dx + c$$ ....eq(2)

Let,

$$I = \int (\sin 2x).(\sin x)dx$$

Let, u=sin 2x & v=sin x

$$\therefore I = \sin 2x.\int \sin x\ dx - \int \left(\frac{d}{dt}(\sin 2x).\int \sin x\ dx\right) dx$$

$$\ldots\ldots\ldots \left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx\right)$$

$$\therefore I = -\sin 2x.\cos x - \int \left((2\cos 2x).(-\cos x)\right) dx$$

$$\ldots\ldots\ldots\left(\because \int \sin x\, dx = -\cos x \;\&\; \frac{d}{dx}(\sin x) = \cos x\right)$$

$$\therefore I = -\sin 2x.\cos x + 2\int \left((\cos 2x).(\cos x)\right) dx$$

Again let, u=cos 2x & v=cos x

$$\therefore I = -\sin 2x.\cos x$$
$$+ 2\left\{\cos 2x.\int \cos x\ dx - \int\left(\frac{d}{dt}(\cos 2x).\int \cos x\ dx\right) dx\right\}$$

$$\ldots\ldots\ldots\left(\because \int u.v\, dx = u.\int v\, dx - \int\left(\frac{du}{dx}.\int v\, dx\right) dx\right)$$

$$\therefore I = -\sin 2x.\cos x + 2\left\{\cos 2x.\sin x - \int \left((-2\sin 2x).(\sin x)\right) dx\right\}$$

$$\therefore I = -\sin 2x.\cos x + 2\{\cos 2x.\sin x + 2\int\left((\sin 2x).(\sin x)\right) dx\}$$

$$\therefore I = -\sin 2x.\cos x + 2\{\cos 2x.\sin x + 2I\}$$

$$\therefore I = -\sin 2x.\cos x + 2\cos 2x.\sin x + 4I$$

$$\therefore I - 4I = -2\sin x\cos x.\cos x + 2(\cos^2 x - \sin^2 x).\sin x$$

$$\ldots\ldots\ldots(\because \sin 2x = 2\sin x.\cos x \;\& \cos 2x = (\cos^2 x - \sin^2 x))$$

$$\therefore -3I = -2\sin x\cos^2 x + 2\sin x\cos^2 x - 2\sin^3 x$$

$$\therefore -3I = -2\sin^3 x$$

$$\therefore I = \frac{2}{3}\sin^3 x$$

Substituting I in eq(2),

$$\therefore \mathrm{y}.(\sin \mathrm{x}) = \frac{2}{3}\sin^3 \mathrm{x} + \mathrm{c}$$

Therefore, general solution is

$$y.(\sin x) = \frac{2}{3} sin^3 x + c$$

Question 33.
Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 2y\tan x = \sin x$$

Answer:
Given Differential Equation :

$$\frac{dy}{dx} + 2y\tan x = \sin x$$

Formula :

i) $\int \tan x\,dx = \log|\sec x|$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int \left(\frac{-1}{x^2}\right) dx = \frac{1}{x}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2y\tan x = \sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2\tan x$ and $Q = \sin x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 2\tan x\,dx}$$

$= e^{2\log|\sec x|}$ ....$(\because \int \tan x\,dx = \log|\sec x|)$

$= e^{\log|\sec x|^2}$ ....$(\because a\log b = \log b^a)$

$= \sec^2 x$ ....$(\because a^{\log_a b} = b)$

$$= \frac{1}{\cos^2 x}$$

General solution is

$$y.(\mathrm{I.F.}) = \int \mathrm{Q}.(\mathrm{I.F.})dx + c$$

$\therefore y.\left(\frac{1}{\cos^2 x}\right) = \int (\sin x).\left(\frac{1}{\cos^2 x}\right)dx + c$ ....eq(2)

Let,

$$\mathrm{I} = \int (\sin x).\left(\frac{1}{\cos^2 x}\right)dx$$

Put, cos x=t => -sin x dx = dt

$$\therefore \mathrm{I} = \int \left(\frac{-1}{t^2}\right)dt$$

$\therefore \mathrm{I} = \frac{1}{t}$ ....$\left(\because \int \left(\frac{-1}{x^2}\right)dx = \frac{1}{x}\right)$

$$\therefore \mathrm{I} = \frac{1}{\cos x}$$

Substituting I in eq(2),

$$\therefore y.\left(\frac{1}{\cos^2 x}\right) = \frac{1}{\cos x} + c$$

Multiplying above equation by $\cos^2 x$,

$$\therefore y = \cos x + c(\cos^2 x)$$

Therefore, general solution is

$$y = \cos x + c(cos^2 x)$$

Question 34.
Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + y\cot x = x^2 \cot x + 2x$$

Answer:
Given Differential Equation :

$$\frac{dy}{dx} + y\cot x = x^2 \cot x + 2x$$

Formula :

i) $\int \cot x\, dx = \log|\sin x|$

ii) $a^{\log_a b} = b$

iii) $\int u.v\, dx = u.\int v\, dx - \int\left(\frac{du}{dx}.\int v\, dx\right) dx$

iv) $\int \cos x\, dx = \sin x$

v) $\frac{d}{dx}(x^n) = nx^{n-1}$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + y\cot x = x^2\cot x + 2x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = x^2\cot x + 2x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \cot x\,dx}$$

$= e^{\log|\sin x|}$ ....$(\because \int \cot x\,dx = \log|\sin x|)$

= sin x ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sin x) = \int (x^2\cot x + 2x).(\sin x)dx + c$$

$$\therefore y.(\sin x) = \int (x^2\cot x.\sin x + 2x\sin x)\,dx + c$$

$$\therefore y.(\sin x) = \int \left(x^2 \frac{\cos x}{\sin x}.\sin x + 2x\sin x\right) dx + c$$

$$\therefore y.(\sin x) = \int (x^2 \cos x + 2x\sin x)\, dx + c$$

$$\therefore y.(\sin x) = \int x^2 \cos x\ dx + \int 2x\sin x\, dx + c \text{ ....eq(2)}$$

Let,

$$I = \int x^2 \cos x\ dx$$

Let, $u=x^2$ and $v=\cos x$

$$\therefore I = x^2.\int \cos x\ dx - \int \left(\frac{d}{dt}(x^2).\int \cos x\ dx\right) dx$$

$$\text{.........}\left(\because \int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx\right)$$

$$\therefore I = x^2.\sin x - \int 2x.\sin x\, dx$$

$$\text{.........}\left(\because \int \cos x\, dx = \sin x\ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$$

Substituting I in eq(2),

$$\therefore y.(\sin x) = x^2.\sin x - \int 2x.\sin x\, dx + \int 2x\sin x\, dx + c$$

$$\therefore y.(\sin x) = x^2.\sin x + c$$

Dividing above equation by sin x,

$$\therefore y = x^2 + \frac{c}{\sin x}$$

Therefore, general solution is

$$y = x^2 + c(cosec\,x)$$

Question 35.
Find a particular solution satisfying the given condition for each of the following differential equations.

$x\frac{dy}{dx} + y = x^3$, given that $\mathscr{Y}$ =1 when $\mathscr{X}$ =2

Answer:
Given Differential Equation :

$$x\frac{dy}{dx} + y = x^3$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int x^n\, dx = \frac{x^{n+1}}{n+1}$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$\mathrm{I.F.} = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$x\frac{dy}{dx} + y = x^3$$

Dividing above equation by x,

$\therefore \frac{dy}{dx} + \frac{1}{x}.y = x^2$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x}$ and $Q = x^2$

Therefore, integrating factor is

$$\mathrm{I.F.} = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{x}dx}$$

$$= e^{\log x} \text{.........}\left(\because \int \frac{1}{x}dx = \log x\right)$$

$$= x \text{.........}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$$\therefore y.(x) = \int x^2.(x)dx + c$$

$$\therefore xy = \int x^3 dx + c$$

$$\therefore xy = \frac{x^4}{4} + c \text{.........} \left(\because \int x^n\, dx = \frac{x^{n+1}}{n+1}\right)$$

Dividing above equation by x,

$$\therefore y = \frac{x^3}{4} + \frac{c}{x}$$

Therefore general equation is

$$y = \frac{x^3}{4} + \frac{c}{x}$$

For particular solution put y=1 and x=2 in above equation,

$$\therefore 1 = \frac{2^3}{4} + \frac{c}{2}$$

$$\therefore 1 = \frac{8}{4} + \frac{c}{2}$$

$$\therefore 1 = 2 + \frac{c}{2}$$

$$\therefore \frac{c}{2} = -1$$

$$\therefore c = -2$$

Therefore, particular solution is

$$y = \frac{x^3}{4} - \frac{2}{x}$$

Question 36.
Find a particular solution satisfying the given condition for each of the following differential equations.

$\frac{dy}{dx} + y \cot x = 4x \cos ec x$, given that $\mathscr{Y}$ = 0 when $\mathscr{X} = \frac{\pi}{2}$.

Answer:
Given Differential Equation :

$$\frac{dy}{dx} + y.\cot x = 4x \operatorname{cosec} x$$

Formula :

i) $\int \cot x \, dx = \log|\sin x|$

ii) $a^{\log_a b} = b$

iii) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + y.\cot x = 4x \operatorname{cosec} x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = 4x \operatorname{cosec} x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \cot x\,dx}$$

$= e^{\log|\sin x|}$ ....$\left(\because \int \cot x\,dx = \log|\sin x|\right)$

= sin x ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sin x) = \int (4x \operatorname{cosec} x).(\sin x)dx + c$$

$$\therefore y.(\sin x) = 4\int \left(x \frac{1}{\sin x}\right).(\sin x)dx + c$$

$$\therefore y.(\sin x) = 4\int (x)\,dx + c$$

$$\therefore y.(\sin x) = 4\frac{x^2}{2} + c \text{.........} \left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

$$\therefore y.(\sin x) = 2x^2 + c$$

Therefore general equation is

$$y.(\sin x) = 2x^2 + c$$

For particular solution put y=0 and $x = \frac{\pi}{2}$ in above equation,

$$\therefore 0 = 2\frac{\pi^2}{4} + c$$

$$\therefore 0 = \frac{\pi^2}{2} + c$$

$$\therefore c = -\frac{\pi^2}{2}$$

Therefore, particular solution is

$$y.(\sin x) = 2x^2 - \frac{\pi^2}{2}$$

Question 37.
Find a particular solution satisfying the given condition for each of the following differential equations.

$\frac{dy}{dx} + 2xy = x$, given that $\mathscr{Y}$ = 0 when $\mathscr{X}$ =0.

Answer:
Given Differential Equation :

$$\frac{dy}{dx} + 2xy = x$$

Formula :

i) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

ii) $\int \left(e^{kx}\right) dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2xy = x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2x$ and $Q = x$

Therefore, integrating factor is

$$\mathrm{I.F.} = e^{\int P\,dx}$$

$$= e^{\int 2x\,dx}$$

$$= e^{2\frac{x^2}{2}} \text{.........} \left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

$$= e^{x^2}$$

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$$\therefore y.\left(e^{x^2}\right) = \int (x).\left(e^{x^2}\right)dx + c$$

$$\therefore y.\left(e^{x^2}\right) = \frac{1}{2}\int (2x).\left(e^{x^2}\right)dx + c \text{ ....eq(2)}$$

Let,

$$I = \int (2x).\left(e^{x^2}\right)dx$$

Put, x^2=t => 2x dx = dt

$$\therefore I = \int (e^t)dt$$

$$\therefore I = e^t \text{ ....} \left(\because \int \left(e^{kx}\right)dx = \frac{e^{kx}}{k}\right)$$

$$\therefore I = e^{x^2}$$

Substituting I in eq(2),

$$\therefore y.\left(e^{x^2}\right) = \frac{1}{2}.e^{x^2} + c$$

Therefore, general solution is

$$y.\left(e^{x^2}\right)=\frac{1}{2}.e^{x^2}+c$$

For particular solution put y=0 and x=0 in above equation,

$$\therefore 0=\frac{1}{2}.e^0+c$$

$$\therefore 0=\frac{1}{2}+c$$

$$\therefore c=-\frac{1}{2}$$

Substituting c in general solution,

$$y.\left(e^{x^2}\right)=\frac{1}{2}.e^{x^2}-\frac{1}{2}$$

Multiplying above equation by $\frac{2}{e^{x^2}}$

$$\therefore 2y=1-e^{-x^2}$$

Therefore, particular solution is

$$2y=1-e^{-x^2}$$

Question 38.
Find a particular solution satisfying the given condition for each of the following differential equations.

$\frac{dy}{dx}+2y=e^{-2x}\sin x$, given that $\mathscr{Y}$ = 0, when $\mathscr{X}$ = 0.

Answer:

Given Differential Equation :

$$\frac{dy}{dx} + 2y = e^{-2x}.\sin x$$

Formula :

i) $\int 1\, dx = x$

ii) $\int (\sin x) dx = -\cos x$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2y = e^{-2x}.\sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $\mathrm{P} = 2$ and $\mathrm{Q} = \mathrm{e}^{-2\mathrm{x}}.\sin \mathrm{x}$

Therefore, integrating factor is

$$\mathrm{I.F.} = \mathrm{e}^{\int \mathrm{P}\,\mathrm{dx}}$$

$$= \mathrm{e}^{\int 2\,\mathrm{dx}}$$

$$= \mathrm{e}^{2\mathrm{x}} \text{.........}(\because \int 1\,\mathrm{dx} = \mathrm{x})$$

General solution is

$$\mathrm{y}.(\mathrm{I.F.}) = \int \mathrm{Q}.(\mathrm{I.F.})\mathrm{dx} + \mathrm{c}$$

$$\therefore \mathrm{y}.(\mathrm{e}^{2\mathrm{x}}) = \int (\mathrm{e}^{-2\mathrm{x}}.\sin \mathrm{x}).(\mathrm{e}^{2\mathrm{x}})\mathrm{dx} + \mathrm{c}$$

$$\therefore \mathrm{y}.(\mathrm{e}^{2\mathrm{x}}) = \int \left(\frac{1}{\mathrm{e}^{2\mathrm{x}}}.\sin \mathrm{x}\right).(\mathrm{e}^{2\mathrm{x}})\mathrm{dx} + \mathrm{c}$$

$$\therefore \mathrm{y}.(\mathrm{e}^{2\mathrm{x}}) = \int (\sin \mathrm{x})\mathrm{dx} + \mathrm{c}$$

$$\therefore \mathrm{y}.(\mathrm{e}^{2\mathrm{x}}) = -\cos \mathrm{x} + \mathrm{c} \text{.........}(\because \int (\sin \mathrm{x})\mathrm{dx} = -\cos \mathrm{x})$$

Therefore, general solution is

$$y.(e^{2x}) = -\cos x + c$$

For particular solution put y=0 and x=0 in above equation,

$$\therefore 0 = -\cos 0 + \mathrm{c}$$

$$\therefore 0 = -1 + \mathrm{c}$$

$$\therefore \mathrm{c} = 1$$

Substituting c in general solution,

$$y.(e^{2x}) = -\cos x + 1$$

Therefore, particular solution is

$$y.(e^{2x}) = -\cos x + 1$$

Question 39.
Find a particular solution satisfying the given condition for each of the following differential equations.

$(1+x^2)\dfrac{dy}{dx} + 2xy = 4x^2$, given that y = 0 when x = 0.

Answer:
Given Differential Equation :

$$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$$

Formula :

i) $\int \frac{f(x)}{f'(x)} dx = \log f(x)$

ii) $\int x^n\, dx = \frac{x^{n+1}}{n+1}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$$

Dividing above equation by $(1+x^2)$,

$\therefore \frac{dy}{dx} + \frac{2x}{(1+x^2)}y = \frac{4x^2}{(1+x^2)}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{2x}{(1+x^2)}$ and $Q = \frac{4x^2}{(1+x^2)}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{2x}{(1+x^2)}dx}$$

Let, $f(x) = (1+x^2) \therefore f'(x) = 2x$

$\therefore I.F. = e^{\log(1+x^2)}$ ....$\left(\because \int \frac{f(x)}{f'(x)}dx = \log f(x)\right)$

$$= (1 + x^2)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(1 + x^2) = \int \left(\frac{4x^2}{(1 + x^2)}\right).(1 + x^2)dx + c$$

$$\therefore y.(1 + x^2) = 4\int x^2 dx + c$$

$$\therefore y.(1 + x^2) = 4\frac{x^3}{3} + c \text{.........}\left(\because \int x^n\, dx = \frac{x^{n+1}}{n+1}\right)$$

Therefore, general solution is

$$y.(1 + x^2) = 4\frac{x^3}{3} + c$$

For particular solution put y=0 and x=0 in above equation,

$$\therefore 0 = 0 + c$$

$$\therefore c = 0$$

Substituting c in general solution,

$$\therefore y.(1 + x^2) = 4\frac{x^3}{3}$$

Dividing above equation by $(1+x^2)$,

$$\therefore y = \frac{4x^3}{3(1 + x^2)}$$

Therefore, particular solution is

$$y=\frac{4x^3}{3(1+x^2)}$$

Question 40.
Find a particular solution satisfying the given condition for each of the following differential equations.

$x\frac{dy}{dx}-y=\log x$, given that $\mathscr{Y}$ = 0 when $\mathscr{X}$ = 1.

Answer:
Given Differential Equation :

$$x\frac{dy}{dx}-y=\log x$$

Formula :

i) $\int\frac{1}{x}dx=\log x$

ii) $a\log b=\log b^a$

iii) $a^{\log_a b}=b$

iv) $\int u.v\,dx=u.\int v\,dx-\int\left(\frac{du}{dx}.\int v\,dx\right)dx$

v) $\int e^{kx}\,dx=\frac{e^{kx}}{k}$

vi) $\frac{d}{dx}(kx)=k$

vii) $\log 1=0$

viii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$x\frac{dy}{dx} - y = \log x$$

Dividing above equation by x,

$\therefore \frac{dy}{dx} - \frac{1}{x}y = \frac{\log x}{x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = \frac{\log x}{x}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x}\,dx}$$

$$= e^{-\log(x)} \,......... \left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= e^{\log x^{-1}} \,.........(\because \ a\log b = \log b^{a})$$

$$= e^{\log\left(\frac{1}{x}\right)}$$

$$= \frac{1}{x} \,......... \left(\because \ a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int \left(\frac{\log x}{x}\right).\left(\frac{1}{x}\right) dx + c \text{ ....eq(2)}$$

Let,

$$I = \int \left(\frac{\log x}{x}\right).\left(\frac{1}{x}\right) dx$$

Put, log x =t => x=e^t

Therefore, (1/x) dx = dt

$$\therefore I = \int \left(\frac{t}{e^t}\right) dt$$

$$\therefore I = \int t.e^{-t} dt$$

Let, u=t and v=e^{-t}

$$\therefore I = t.\int e^{-t} dt - \int \left(\frac{d}{dt}(t).\int e^{-t} dt\right) dt$$

$$......... \left(\because \int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx\right)$$

$$\therefore \mathrm{I} = -\mathrm{t}.\mathrm{e}^{-\mathrm{t}} - \int \left((1).(-\mathrm{e}^{-\mathrm{t}})\right) \mathrm{dt}$$

$$\text{.........}\left(\because \int \mathrm{e}^{\mathrm{kx}}\,\mathrm{dx} = \frac{\mathrm{e}^{\mathrm{kx}}}{\mathrm{k}} \;\&\; \frac{\mathrm{d}}{\mathrm{dx}}(\mathrm{kx}) = \mathrm{k}\right)$$

$$\therefore \mathrm{I} = -\mathrm{t}.\mathrm{e}^{-\mathrm{t}} - \mathrm{e}^{-\mathrm{t}} \text{ ....}\left(\because \int \mathrm{e}^{\mathrm{kx}}\,\mathrm{dx} = \frac{\mathrm{e}^{\mathrm{kx}}}{\mathrm{k}}\right)$$

$$\therefore \mathrm{I} = -\frac{\log \mathrm{x}}{\mathrm{x}} - \frac{1}{\mathrm{x}}$$

Substituting I in eq(2),

$$\therefore \mathrm{y}.\left(\frac{1}{\mathrm{x}}\right) = -\frac{\log \mathrm{x}}{\mathrm{x}} - \frac{1}{\mathrm{x}} + \mathrm{c}$$

Multiplying above equation by x,

$$\therefore \mathrm{y} = -\log \mathrm{x} - 1 + \mathrm{cx}$$

Therefore, general solution is

$$y = -\log x - 1 + cx$$

For particular solution put y=0 and x=1 in above equation,

$$\therefore 0 = -\log 1 - 1 + \mathrm{c}$$

$$\therefore \mathrm{c} = 1 \text{ ....}(\because \log 1 = 0)$$

Substituting c in general solution,

$$\therefore \mathrm{y} = -\log \mathrm{x} - 1 + \mathrm{x}$$

$$\therefore \mathrm{y} = \mathrm{x} - \log \mathrm{x} - 1$$

Therefore, particular solution is

$y = x - \log x - 1$

Question 41.
Find a particular solution satisfying the given condition for each of the following differential equations.

$\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$, given that y = 1 when x = 0.

Answer:
Given Differential Equation :

$$\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$$

Formula :

i) $\int \tan x \, dx = \log|\sec x|$

ii) $a^{\log_a b} = b$

iii) $\int u.v \, dx = u.\int v \, dx - \int \left(\frac{du}{dx}.\int v \, dx\right) dx$

iv) $\int \sec x . \tan x \, dx = \sec x$

v) $\frac{d}{dx}(x^n) = nx^{n-1}$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \tan x$ and $Q = 2x + x^2 \tan x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \tan x\,dx}$$

$= e^{\log|\sec x|}$ ....$(\because \int \tan x\,dx = \log|\sec x|)$

$= \sec x$ ....$(\because a^{\log_a b} = b)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sec x) = \int (2x + x^2 \tan x).(\sec x)dx + c$$

$$\therefore y.(\sec x) = \int (x^2 \tan x . \sec x + 2x \sec x)\ dx\ + c$$

$\therefore y.(\sec x) = \int x^2 \tan x . \sec x\ \ dx\ + \int 2x \sec x\ dx\ + c$ ....eq(2)

Let,

$$I = \int x^2 \tan x . \sec x\ \ dx$$

Let, $u=x^2$ and v= tan x. sec x

$$\therefore I = x^2 . \int \sec x . \tan x\ \ dx\ - \int \left(\frac{d}{dt}(x^2) . \int \sec x . \tan x\ \ dx\right)\ dx$$

.........$\left(\because\ \int u.v\ dx = u.\int v\ dx - \int \left(\frac{du}{dx}.\int v\ dx\right)\ dx\right)$

$$\therefore I = x^2 . \sec x - \int 2x. \sec x\ \ dx$$

.........$\left(\because\ \int \sec x . \tan x\, dx = \sec x\ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$

Substituting I in eq(2),

$$\therefore y.(\sec x) = x^2 . \sec x - \int 2x. \sec x\ \ dx\ + \int 2x \sec x\, dx\ + c$$

$$\therefore y.(\sec x) = x^2 . \sec x + c$$

$$\therefore y.\left(\frac{1}{\cos x}\right) = x^2 . \left(\frac{1}{\cos x}\right) + c$$

Multiplying above equation by cos x,

$$\therefore y = x^2 + c.(\cos x)$$

Therefore, general solution is

$$y = x^2 + c.(\cos x)$$

For particular solution put y=1 and x=0 in above equation,

$$\therefore 1 = 0 + \mathrm{c}$$

$$\therefore \mathrm{c} = 1$$

Substituting c in general solution,

$$\therefore \mathrm{y} = \mathrm{x}^2 + \cos \mathrm{x}$$

Therefore, particular solution is

$$y = x^2 + \cos x$$

Question 42.
A curve passes through the origin and the slope of the tangent to the curve at any point ($\mathscr{X}$,) is equal to the sum of the coordinates of the point. Find the equation of the curve.

Answer:
Formula :

i) $\int 1\,\mathrm{dx} = \mathrm{x}$

ii) $\int \mathrm{u.v\,dx} = \mathrm{u}.\int \mathrm{v\,dx} - \int\left(\frac{\mathrm{du}}{\mathrm{dx}}.\int \mathrm{v\,dx}\right)\mathrm{dx}$

iii) $\int \mathrm{e}^{\mathrm{kx}}\,\mathrm{dx} = \frac{\mathrm{e}^{\mathrm{kx}}}{\mathrm{k}}$

iv) $\frac{\mathrm{d}}{\mathrm{dx}}(\mathrm{x}^{\mathrm{n}}) = \mathrm{nx}^{\mathrm{n}-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

The slope of the tangent to the curve $= \frac{dy}{dx}$

The slope of the tangent to the curve is equal to the sum of the coordinates of the point.

$$\therefore \frac{dy}{dx} = x + y$$

Therefore differential equation is

$$\therefore \frac{dy}{dx} = x + y$$

$\therefore \frac{dy}{dx} - y = x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -1$ and $Q = x$

Therefore, integrating factor is

$\mathrm{I.F.} = e^{\int P\,dx}$

$= e^{\int -1\,dx}$

$= e^{-x}$ ....$(\because \int 1\,dx = x)$

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$\therefore y.(e^{-x}) = \int (x).(e^{-x})dx + c$ ....eq(2)

Let,

$$I = \int (x).(e^{-x})dx$$

Let, u=x and v= e^{-x}

$$\therefore I = x.\int e^{-x}\,dx - \int \left(\frac{d}{dx}(x).\int e^{-x}\,dx\right)dx$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right)dx\right)$

$$\therefore I = -x.e^{-x} - \int (1).(-e^{-x})\,dx$$

.........$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k} \ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$

$\therefore I = -x.e^{-x} - e^{-x}$ ....$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\right)$

Substituting I in eq(2),

$\therefore y.(e^{-x}) = -x.e^{-x} - e^{-x} + c$

Dividing above equation by e^{-x},

$$\therefore y = -x - 1 + c.e^{x}$$

Therefore, general solution is

$$y + x + 1 = c.e^{x}$$

The curve passes through origin , therefore the above equation satisfies for x=0 and y=0,

$$\therefore 0 + 0 + 1 = c.e^{0}$$

$$\therefore c = 1$$

Substituting c in general solution,

$$\therefore y + x + 1 = e^{x}$$

Therefore, equation of the curve is

$$y + x + 1 = e^{x}$$

Question 43.
A curve passes through the point (0, 2) and the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5. Find the equation of the curve.

Answer:
Formula :

i) $\int 1\,dx = x$

ii) $\int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right)dx$

iii) $\int e^{kx}\,dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n) = nx^{n-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

The slope of the tangent to the curve $= \frac{dy}{dx}$

The sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at the given point by 5.

$$\therefore 5 + \frac{dy}{dx} = x + y$$

Therefore differential equation is

$$\therefore 5 + \frac{dy}{dx} = x + y$$

$\therefore \frac{dy}{dx} - y = x - 5$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -1$ and $Q = x - 5$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int -1\,dx}$$

$$= e^{-x} \text{.........} (\because \int 1\,dx = x)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$\therefore y.(e^{-x}) = \int (x-5).(e^{-x})dx + c$ ....eq(2)

Let,

$$I = \int (x-5).(e^{-x})dx$$

Let, u=x-5 and v= e^{-x}

$$\therefore I = (x-5).\int e^{-x}\,dx - \int \left(\frac{d}{dt}(x-5).\int e^{-x}\,dx\right) dx$$

$$\text{.........}\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx\right)$$

$$\therefore I = -(x-5).e^{-x} - \int (1).(-e^{-x})\,dx$$

$$\text{.........}\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k} \;\&\; \frac{d}{dx}(x^n) = nx^{n-1}\right)$$

$$\therefore I = -(x-5).e^{-x} - e^{-x} \text{.........}\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\right)$$

Substituting I in eq(2),

$$\therefore y.(e^{-x}) = -(x-5).e^{-x} - e^{-x} + c$$

Dividing above equation by e^{-x},

$$\therefore y = -(x-5) - 1 + c.e^{x}$$

$$\therefore y = -x + 5 - 1 + c.e^{x}$$

$$\therefore y = -x + 4 + c.e^{x}$$

Therefore, general solution is

$$y = -x + 4 + c.e^{x}$$

The curve passes through point (0,2) , therefore the above equation satisfies for x=0 and y=2,

$$\therefore 2 = -0 + 4 + c.e^{0}$$

$$\therefore c = -2$$

Substituting c in general solution,

$$\therefore y = -x + 4 - 2e^{x}$$

Therefore, equation of the curve is

$$y = 4 - x - 2e^{x}$$

Question 44.
Find the general solution for each of the following differential equations.

$$ydx - (x + 2y^2)dy = 0$$

Answer:
Given Differential Equation :

$$y dx - (x + 2y^2) dy = 0$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.) dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dy}$$

Answer :

Given differential equation is

$$y\,dx - (x + 2y^2)\,dy = 0$$

$$\therefore y\,dx = (x + 2y^2)\,dy$$

$$\therefore \frac{dx}{dy} = \frac{(x + 2y^2)}{y}$$

$$\therefore \frac{dx}{dy} = \frac{x}{y} + 2y$$

$\therefore \frac{dx}{dy} - \frac{1}{y}.x = 2y$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{-1}{y}$ and $Q = 2y$

Therefore, integrating factor is

$$\mathrm{I.F.} = e^{\int P\,dy}$$

$$= e^{\int \frac{-1}{y}\,dy}$$

$= e^{-\log y}$ ....$\left(\because \int \frac{1}{x}dx = \log x\right)$

$= e^{\log \frac{1}{y}}$ ....$(\because a \log b = \log b^a)$

$= \frac{1}{y}$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$x.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dy + c$$

$$\therefore x.\left(\frac{1}{y}\right) = \int (2y).\left(\frac{1}{y}\right) dy + c$$

$$\therefore \frac{x}{y} = \int (2)dy + c$$

$$\therefore \frac{x}{y} = 2y + c \text{ ....}(\because \int 1dx = x)$$

Multiplying above equation by y,

$$\therefore x = 2y^2 + cy$$

Therefore, general solution is

$$\therefore x = 2y^2 + cy$$

Question 45.
Find the general solution for each of the following differential equations.

$$ydx + (x - y^2)dy = 0$$

Answer:
Given Differential Equation :

$$ydx + (x - y^2)dy = 0$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int 1dx = x$

iv) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dy}$$

<u>Answer</u> :

Given differential equation is

$$ydx + (x - y^2)dy = 0$$

$$\therefore ydx = -(x - y^2)dy$$

$$\therefore ydx = (y^2 - x)dy$$

$$\therefore \frac{dx}{dy} = \frac{(y^2 - x)}{y}$$

$$\therefore \frac{dx}{dy} = -\frac{x}{y} + y$$

$\therefore \frac{dx}{dy} + \frac{1}{y}.x = y$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{1}{y}$ and $Q = y$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dy}$$

$$= e^{\int \frac{1}{y} dy}$$

$$= e^{\log y} \ldots\ldots\ldots \left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= y \ldots\ldots\ldots \left(\because a^{\log_a b} = b\right)$$

General solution is

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

$$\therefore x.(y) = \int (y).(y)dy + c$$

$$\therefore xy = \int y^2\,dy + c$$

$$\therefore xy = \frac{y^3}{3} + c \ldots\ldots\ldots (\because \int 1dx = x)$$

Dividing above equation by y,

$$\therefore x = \frac{1}{3}y^2 + \frac{c}{y}$$

Therefore, general solution is

$$x = \frac{1}{3}y^2 + \frac{c}{y}$$

Question 46.
Find the general solution for each of the following differential equations.

$$ydx + (x - y^2)dy = 0$$

Answer:
Given Differential Equation :

$$ydx + (x - y^2)dy = 0$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int 1 dx = x$

iv) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dy}$$

Answer :

Given differential equation is

$$\mathrm{ydx} + (\mathrm{x} - \mathrm{y}^2)\mathrm{dy} = 0$$

$$\therefore \mathrm{ydx} = -(\mathrm{x} - \mathrm{y}^2)\mathrm{dy}$$

$$\therefore \mathrm{ydx} = (\mathrm{y}^2 - \mathrm{x})\mathrm{dy}$$

$$\therefore \frac{\mathrm{dx}}{\mathrm{dy}} = \frac{(\mathrm{y}^2 - \mathrm{x})}{\mathrm{y}}$$

$$\therefore \frac{\mathrm{dx}}{\mathrm{dy}} = -\frac{\mathrm{x}}{\mathrm{y}} + \mathrm{y}$$

$\therefore \frac{\mathrm{dx}}{\mathrm{dy}} + \frac{1}{\mathrm{y}}.\mathrm{x} = \mathrm{y}$ ....eq(1)

Equation (1) is of the form

$$\frac{\mathrm{dx}}{\mathrm{dy}} + \mathrm{Px} = \mathrm{Q}$$

Where, $\mathrm{P} = \frac{1}{\mathrm{y}}$ and $\mathrm{Q} = \mathrm{y}$

Therefore, integrating factor is

$$\mathrm{I.F.} = e^{\int \mathrm{P}\,\mathrm{dy}}$$

$$= e^{\int \frac{1}{\mathrm{y}}\mathrm{dy}}$$

$= e^{\log \mathrm{y}}$ ....$\left(\because \int \frac{1}{\mathrm{x}}\mathrm{dx} = \log \mathrm{x}\right)$

$= \mathrm{y}$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

$$\therefore x.(y) = \int (y).(y)dy + c$$

$$\therefore xy = \int y^2\, dy + c$$

$$\therefore xy = \frac{y^3}{3} + c \text{ ....}(\because \int 1dx = x)$$

Dividing above equation by y,

$$\therefore x = \frac{1}{3}y^2 + \frac{c}{y}$$

Therefore, general solution is

$$x = \frac{1}{3}y^2 + \frac{c}{y}$$

Question 47.
Find the general solution for each of the following differential equations.

$$(x + 3y^3)\frac{dy}{dx} = y, (y > 0)$$

Answer:
Given Differential Equation :

$$(x + 3y^3)\frac{dy}{dx} = y$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int x^n dx = \frac{x^{n+1}}{n+1}$

v) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dy}$$

<u>Answer</u> :

Given differential equation is

$$(x + 3y^3)\frac{dy}{dx} = y$$

$$\therefore \frac{dx}{dy} = \frac{(x + 3y^3)}{y}$$

$$\therefore \frac{dx}{dy} = \frac{x}{y} + 3y^2$$

$\therefore \frac{dx}{dy} - \frac{1}{y}.x = 3y^2$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{-1}{y}$ and $Q = 3y^2$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dy}$$

$$= e^{\int \frac{-1}{y}\,dy}$$

$$= e^{-\log y} \text{.........} \left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= e^{\log \frac{1}{y}} \text{.........} (\because a \log b = \log b^a)$$

$$= \frac{1}{y} \text{.........} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

$$\therefore x.\left(\frac{1}{y}\right) = \int (3y^2).\left(\frac{1}{y}\right)dy + c$$

$$\therefore \frac{x}{y} = 3\int (y)dy + c$$

$$\therefore \frac{x}{y} = \frac{3y^2}{2} + c \text{.........} \left(\because \int x^n dx = \frac{x^{n+1}}{n+1}\right)$$

Multiplying above equation by y,

$$\therefore x = \frac{3}{2}y^3 + cy$$

Therefore, general solution is

$$x = \frac{3}{2}y^3 + cy$$

Question 48.
Find the general solution for each of the following differential equations.

$$(x + y)\frac{dy}{dx} = 1$$

Answer:
Given Differential Equation :

$$(x + y)\frac{dy}{dx} = 1$$

Formula :

i) $\int 1 dx = x$

ii) $\int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx$

iii) $\int e^{kx}\, dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n) = nx^{n-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dy}$$

<u>Answer</u> :

Given differential equation is

$$(x + y)\frac{dy}{dx} = 1$$

$$\therefore \frac{dx}{dy} = x + y$$

$\therefore \frac{dx}{dy} - x = y$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = -1$ and $Q = y$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dy}$$

$$= e^{\int -1\,dy}$$

$= e^{-y}$ ....$(\because \int 1 dx = x)$

General solution is

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

$\therefore x.(e^{-y}) = \int (y).(e^{-y})dy + c$ ....eq(2)

Let,

$$I = \int (y).(e^{-y})dy$$

Let, u=y and v= e^{-y}

$$\therefore I = y.\int e^{-y}\,dy - \int \left(\frac{d}{dy}(y).\int e^{-y}\,dy\right) dy$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx\right)$

$$\therefore I = -y.e^{-y} - \int (1).(-e^{-y})\,dy$$

.........$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k} \;\&\; \frac{d}{dx}(x^n) = nx^{n-1}\right)$

$\therefore I = -y.e^{-y} - e^{-y}$ ....$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\right)$

Substituting I in eq(2),

$$\therefore x.(e^{-y}) = -y.e^{-y} - e^{-y} + c$$

$$\therefore x.(e^{-y}) + y.e^{-y} + e^{-y} = c$$

$$\therefore e^{-y}(x + y + 1) = c$$

Therefore, general solution is

$$e^{-y}(x+y+1)=c$$

Question 49.
Find the general solution for each of the following differential equations.

$$(x+y+1)\frac{dy}{dx}=1$$

Answer:
Given Differential Equation :

$$(x+y+1)\frac{dy}{dx}=1$$

Formula :

i) $\int 1dx=x$

ii) $\int u.v\,dx=u.\int v\,dx-\int\left(\frac{du}{dx}.\int v\,dx\right)dx$

iii) $\int e^{kx}\,dx=\frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n)=nx^{n-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dx}{dy}+Px=Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dy}$$

Answer :

Given differential equation is

$$(x+y+1)\frac{dy}{dx} = 1$$

$$\therefore \frac{dx}{dy} = x+y+1$$

$\therefore \frac{dx}{dy} - x = y+1$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = -1$ and $Q = y+1$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dy}$$

$$= e^{\int -1\,dy}$$

$= e^{-y}$ ....$(\because \int 1dx = x)$

General solution is

$$\mathrm{x.(I.F.)} = \int \mathrm{Q.(I.F.)dy} + \mathrm{c}$$

$\therefore \mathrm{x.(e^{-y})} = \int (\mathrm{y}+1).(\mathrm{e^{-y}})\mathrm{dy} + \mathrm{c}$ ....eq(2)

Let,

$$\mathrm{I} = \int (\mathrm{y}+1).(\mathrm{e^{-y}})\mathrm{dy}$$

Let, u=y+1 and v= e^{-y}

$$\therefore \mathrm{I} = (\mathrm{y}+1).\int \mathrm{e^{-y}\,dy} - \int \left(\frac{\mathrm{d}}{\mathrm{dy}}(\mathrm{y}+1).\int \mathrm{e^{-y}\,dy}\right)\mathrm{dy}$$

.........$\left(\because \int \mathrm{u.v\,dx} = \mathrm{u.}\int \mathrm{v\,dx} - \int \left(\frac{\mathrm{du}}{\mathrm{dx}}.\int \mathrm{v\,dx}\right)\mathrm{dx}\right)$

$$\therefore \mathrm{I} = -(\mathrm{y}+1).\mathrm{e^{-y}} - \int (1).(-\mathrm{e^{-y}})\,\mathrm{dy}$$

.........$\left(\because \int \mathrm{e^{kx}\,dx} = \frac{\mathrm{e^{kx}}}{\mathrm{k}}\ \&\ \frac{\mathrm{d}}{\mathrm{dx}}(\mathrm{x^n}) = \mathrm{nx^{n-1}}\right)$

$\therefore \mathrm{I} = -(\mathrm{y}+1).\mathrm{e^{-y}} - \mathrm{e^{-y}}$ ....$\left(\because \int \mathrm{e^{kx}\,dx} = \frac{\mathrm{e^{kx}}}{\mathrm{k}}\right)$

Substituting I in eq(2),

$$\therefore \mathrm{x.(e^{-y})} = -(\mathrm{y}+1).\mathrm{e^{-y}} - \mathrm{e^{-y}} + \mathrm{c}$$

$$\therefore \mathrm{x.(e^{-y})} = -\mathrm{e^{-y}}(\mathrm{y}+1+1) + \mathrm{c}$$

$$\therefore \mathrm{x.(e^{-y})} = -\mathrm{e^{-y}}(\mathrm{y}+2) + \mathrm{c}$$

$$\therefore \mathrm{x.(e^{-y})} = \mathrm{c} - \mathrm{e^{-y}}(\mathrm{y}+2)$$

Dividing above equation by e^{-y}

$\therefore x = ce^y - (y + 2)$

Therefore, general solution is

$x = ce^y - (y + 2)$

Question 50.

Solve $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$, given that $\mathscr{X}$ = 0 when $\mathscr{Y}$ = 0.

Answer:

Given Equation: $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$

Re-arranging, we get,

$$\frac{1}{2e^{-y} - 1}dy = \frac{dx}{(x + 1)}$$

$$\frac{e^y}{2 - e^y}dy = \frac{dx}{(x + 1)}$$

Let $2 - e^y = t$

$-e^y dy = dt$

Therefore,

$$\frac{dt}{t} = \frac{dx}{x + 1}$$

Integrating both sides, we get,

$\log t = \log(x + 1) + C$

$\log (2 - e^y) = \log (x + 1) + C$

At x = 0, y = 0.

Therefore,

log(2) = log(1) + C

Therefore,

C = log 2

Now, we have,

log (2 – e^y) – log (x + 1) – log 2 = 0

$$y = \log\left|\frac{2x+1}{x+1}\right|$$

Question 51.

Solve $(1+y^2)dx + (x - e^{-\tan^{-1}y})dy = 0$, given that when y =o, then x = 0.

Answer:

Given Differential Equation :

$$(1+y^2)dx + \left(x - e^{-\tan^{-1}y}\right)dy = 0$$

Formula :

i) $\int \frac{1}{(1+x^2)}dx = \tan^{-1}x$

ii) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dy}$$

Answer :

Given differential equation is

$$(1+y^2)dx + \left(x - e^{-\tan^{-1}y}\right)dy = 0$$

$$\therefore (1+y^2)dx = -\left(x - e^{-\tan^{-1}y}\right)dy$$

$$\therefore (1+y^2)dx = \left(e^{-\tan^{-1}y} - x\right)dy$$

$$\therefore \frac{dx}{dy} = \frac{\left(e^{-\tan^{-1}y} - x\right)}{(1+y^2)}$$

$$\therefore \frac{dx}{dy} = \frac{e^{-\tan^{-1}y}}{(1+y^2)} - \frac{x}{(1+y^2)}$$

$\therefore \frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{e^{-\tan^{-1}y}}{(1+y^2)}$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{1}{(1+y^2)}$ and $Q = \frac{e^{-\tan^{-1}y}}{(1+y^2)}$

Therefore, integrating factor is

$\mathrm{I.F.} = e^{\int P\,dy}$

$= e^{\int \frac{1}{(1+y^2)}\,dy}$

$= e^{\tan^{-1}y}$ ....$\left(\because \int \frac{1}{(1+x^2)}dx = \tan^{-1}x\right)$

General solution is

$$x.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dy + c$$

$$\therefore x.\left(e^{\tan^{-1}y}\right) = \int \left(\frac{e^{-\tan^{-1}y}}{(1+y^2)}\right).\left(e^{\tan^{-1}y}\right)dy + c$$

$$\therefore x.\left(e^{\tan^{-1}y}\right) = \int \left(\frac{1}{e^{\tan^{-1}y}.(1+y^2)}\right).\left(e^{\tan^{-1}y}\right)dy + c$$

$$\therefore x.\left(e^{\tan^{-1}y}\right) = \int \frac{1}{(1+y^2)}dy + c$$

$\therefore x.\left(e^{\tan^{-1}y}\right) = \tan^{-1}y + c$ ....$\left(\because \int \frac{1}{(1+x^2)}dx = \tan^{-1}x\right)$

Putting x=0 and y=0

$\therefore 0 = 0 + c$

$\therefore c = 0$

Therefore, general solution is

$$x.\left(e^{tan^{-1}y}\right) = tan^{-1}y$$