Exercise 11a

Question 1.
The side of a square is increasing at the rate of 0.2 cm/s. Find the rate of increase of the perimeter of the square.

Answer:
Let the side of the square be a

Rate of change of side= $\frac{da}{dt} = 0.2\ \mathrm{cm/s}$

Perimeter of the square = 4a

Rate of change of perimeter = $4\frac{da}{dt} = 4 \times 0.2$

$$\frac{dP}{dt} = 0.8\ \mathrm{cm/s}$$

Question 2.
The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

Answer:
Let the radius of the circle be r

$$\frac{dr}{dt} = 0.7\mathrm{cm/s}$$

Circumference of the circle= $2\pi r$

Rate of change of circumference $= 2\pi\frac{dr}{dt}$

$$= 2 \times 3.14 \times 0.7$$

$$\frac{dC}{dt} = 4.4\ \mathrm{cm/s}$$

Question 3.
The radius of a circle is increasing uniformly at the rate of 0.3 centimetre per second. At what rate is the area increasing when the radius is 10 cm?

(Take $\pi = 3.14$.)

Answer:
Let the radius of the circle be r

$$\frac{dr}{dt} = 0.3cm/s$$

Area of the circle=πr^2

Rate of change of Area $= 2\pi r \frac{dr}{dt}$

$$= 2 \times 3.14 \times 10 \times 0.3$$

$$\frac{dA}{dt} = 18.84 \ cm^2/s$$

Question 4.
The side of a square sheet of metal is increasing at 3 centimetres per minute. At what rate is the area increasing when the side is 10 cm long?

Answer:
Let the side of the square be a

Rate of change of side= $\frac{da}{dt} = 3$ cm/s

Area of the square = a^2

Rate of change of Area = $2a\frac{da}{dt} = 2 \times 10 \times 3$

$$\frac{dA}{dt} = 60 \ cm^2/s$$

Question 5.
The radius of a circular soap bubble is increasing at the rate of 0.2 cm/s. Find the rate of increase of its surface area when the radius is 7 cm.

Answer:
Soap bubble will be in the shape of a sphere

Let the radius of the soap bubble be r

$$\frac{dr}{dt} = 0.2cm/s$$

Surface area of the soap bubble=$4\pi r^2$

Rate of change of Surface area $= 8\pi r \frac{dr}{dt}$

$$= 8 \times 3.14 \times 7 \times 0.2$$

$$\frac{dS}{dt} = 35.2\ cm^2/s$$

Question 6.
The radius of an air bubble is increasing at the rate of 0.5 centimetre per second. At what rate is the volume of the bubble increasing when the radius is 1 centimetre?

Answer:
Soap bubble will be in the shape of a sphere

Let the radius of the soap bubble be r

$$\frac{dr}{dt} = 0.5cm/s$$

Volume of the soap bubble=$\frac{4}{3}\pi r^3$

Rate of change of Volume $= 4\pi r^2 \frac{dr}{dt}$

$$= 4 \times 3.14 \times 1^2 \times 0.5$$

$$\frac{dV}{dt} = 6.28\ cm^3/s$$

Question 7.
The volume of a spherical balloon is increasing at the rate of 25 cubic centimetres per second. Find the rate of change of its surface at the instant when its radius is 5 cm.

Answer:
Let the radius of the balloon be r

Let the volume of the spherical balloon be V

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$25\ cm^3/s = 4 \times \pi \times 5^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi}$$

Surface area of the bubble $= 4\pi r^2$

Rate of change of Surface area $= 8\pi r \frac{dr}{dt}$

$$= 8 \times \pi \times 5 \times \frac{1}{4\pi}$$

$$\frac{dS}{dt} = 10\ cm^2/s$$

Question 8.
A balloon which always remains spherical is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15 cm.

Answer:
When we pump a balloon its volume changes.

Let the radius of the balloon be r

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$900\ \mathrm{cm}^3/\mathrm{s} = 4 \times \pi \times 15^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{900}{4 \times 3.14 \times 225}$$

$$\frac{dr}{dt} = 0.32\ \mathrm{cm/s}$$

Question 9.
The bottom of a rectangular swimming tank is 25 m by 40 m. Water is pumped into the tank at the rate of 500 cubic metres per minute. Find the rate at which the level of water in the tank is rising.

Answer:
Let the volume of the water tank be V

$$V = l \times b \times h$$

$$V = 25 \times 40 \times h$$

$$\frac{dV}{dt} = 1000 \times \frac{dh}{dt}$$

$$500 = 1000 \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.5\ \mathrm{m/min}$$

Question 10.
A stone is dropped into a quiet lake and waves move in circles at a speed of 3.5 cm per second. At the instant when the radius of the circular wave is 7.5 cm. how fast is the enclosed area increasing? (Take π = 22/7.)

Answer:
Let the radius of the circle be r

$$\frac{dr}{dt} = 3.5 \text{ cm/s}$$

Area of the circle=πr^2

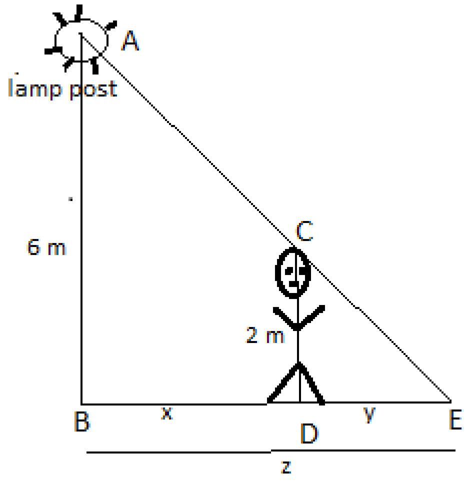
Rate of change of Area $= 2\pi r \frac{dr}{dt}$

$$= 2 \times 3.14 \times 7.5 \times 3.5$$

$$= 165 \text{ cm}^2/\text{s}$$

Question 11.
A 2-m tall man walks at a uniform speed of a uniform speed of 5 km per hour away from a 6-metre-high lamp post. Find the rate at which the length of his shadow increases.

Answer:



ABE and CDE are similar triangles.

So,

$$\frac{AB}{BE} = \frac{CD}{DE}$$

$$\frac{0.006}{x+y} = \frac{0.002}{y}$$

$$6y = 2(x+y)$$

$$6\frac{dy}{dt} = 2(\frac{dx}{dt} + \frac{dy}{dt})$$

$$6\frac{dy}{dt} = 2(5 + \frac{dy}{dt})$$

$$6\frac{dy}{dt} = 10 + 2\frac{dy}{dt}$$

$$4\frac{dy}{dt} = 10$$

$$\frac{dy}{dt} = 2.5\text{km/h}$$

Question 12.
An inverted cone has a depth of 40 cm and a base of radius 5 cm. Water is poured into it at a rate of 1.5 cubic centimetres per minute. Find the rate at which the level of water in the cone is rising when the depth is 4 cm.

Answer:
Let the volume of the cone be V

$$\frac{dV}{dt} = 1.5\ \text{cm}^3/\text{s}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi 5^2 h$$

$$V = \frac{25}{3}\pi h$$

$$\frac{dV}{dt} = \frac{25}{3}\pi\frac{dh}{dt}$$

$$\frac{15}{10} = \frac{25}{3}\pi\frac{dh}{dt}$$

Question 13.
Sand is pouring from a pipe at the rate of 18 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is one-sixth of the radius of the base. How fast is the height of the sand cone increasing when its height is 3 cm?

Answer:

$$h = \frac{1}{6}r$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(6h)^2 h$$

$$V = 12\pi h^3$$

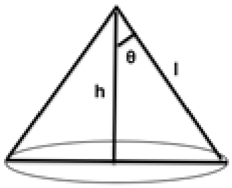
$$\frac{dV}{dt} = 36\pi h^2\frac{dh}{dt}$$

$$18 = 36 \times 9 \times \pi \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{18\pi}\text{cm/s}$$

Question 14.
Water is dripping through a tiny hole at the vertex in the bottom of a conical funnel at a uniform rate of 4 cm^3/s. When the slant height of the water is 3 cm, find the rate of decrease of the slant height of the water, given that the vertical angle of the funnel is 120°.

Answer:



Let the volume of the cone be V

$$\frac{dV}{dt} = 4cm^3/s$$

$$V = \frac{1}{3}\pi r^2 h$$

$$\cos Q = \frac{h}{l} = \cos 120 = \cos(180 - 60) = -\frac{1}{2}$$

$$\sin Q = \frac{r}{l} = \sin 120 = \sin(180 - 60) = \sin 60 = \frac{\sqrt{3}}{2}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{\sqrt{3}}{2}l\right)^2 (-\frac{1}{2}l)$$

$$V = -\frac{3}{24}\pi l^3$$

$$\frac{dV}{dt} = -\frac{9}{24}\pi l^2 \frac{dl}{dt}$$

$$4 = -\frac{3}{8}\pi 3^2 \frac{dl}{dt}$$

$$-\frac{32}{27\pi}\ cm/s = \frac{dl}{dt}$$

Question 15.

Oil is leaking at the rate of 15 mL/s from a vertically kept cylindrical drum containing oil. If the radius of the drum is 7 cm and its height is 60 cm, find the rate at which the level of the oil is changing when the oil level is 18 cm.

Answer:

$$\frac{dV}{dt} = 15 \text{ mL/s}$$

$$\frac{d}{dt}(\pi r^2 h) = 15$$

$$\frac{d}{dt}(\pi 7^2 h) = 15$$

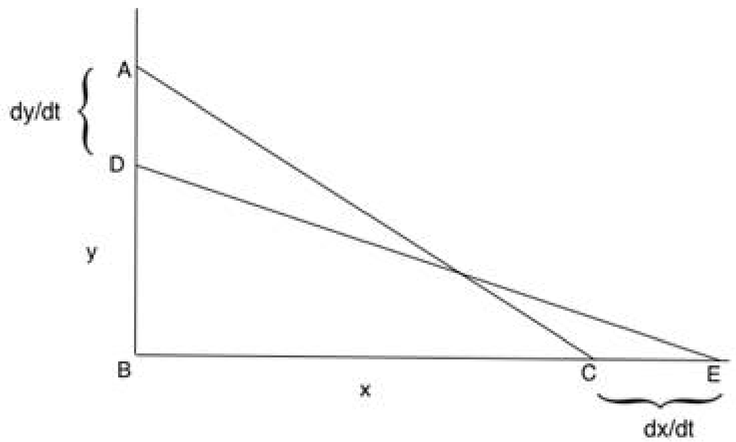
$$49\pi \frac{dh}{dt} = 15$$

$$\frac{dh}{dt} = \frac{15}{49\pi}$$

Question 16.

A 13-m long ladder is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 m/s. How fast is its height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

Answer:



Let the original ladder be AC and the pulled ladder be DE

Let AB=y and BC=x

Applying Pythagoras Theorem in ABC

$x^2 + y^2 = 13^2$ ...(1)

$5^2 + y^2 = 13^2$

$y = 12\text{cm}$

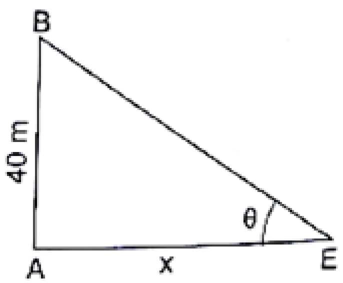
Differentiating both sides of eqn (1) wrt to t

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

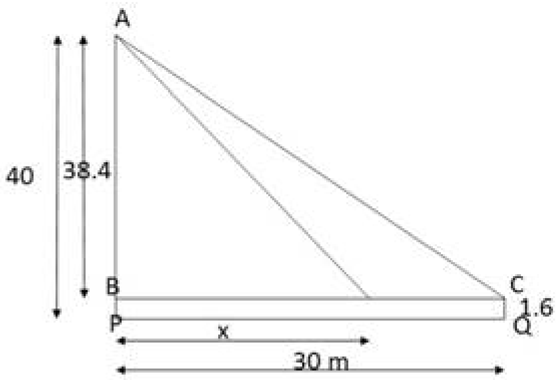
$$5.2 + 12\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{10}{12} = -\frac{5}{6}\text{cm/s}$$

Question 17.
A man is moving away from a 40-m high tower at a speed of 2 m/s. Find the rate is which the angle of elevation of the top of the tower is changing when he is at a distance of 30 metres from the foot of the tower. Assume that the eye level of the man is 1.6 m from the ground.



Answer:



$$\frac{dx}{dt} = -2 \text{ cm/s}$$

$$\tan Q = \frac{38.4}{x}$$

$$Q = \tan^{-1}\frac{38.4}{x}$$

$$\frac{dQ}{dt} = \frac{1}{1 + \frac{38.4^2}{x^2}}\left(-\frac{1}{x^2}\right).38.4$$

$$\frac{dQ}{dt} = \frac{x^2}{x^2 + 1474.56}\left(-\frac{1}{x^2}\right).38.4$$

$$\frac{dQ}{dt} = -\frac{1}{30^2 + 1474.56}.38.4.\frac{dx}{dt}$$

$$\frac{dQ}{dt} = -\frac{1}{30^2 + 1474.56}.38.4 \times 2$$

$$\frac{dQ}{dt} = -0.032 \text{ radian/second}$$

Question 18.
Find an angle x which increases twice as fast as its sine.

Answer:
ATQ,

$$\frac{dx}{dt} = 2\frac{d}{dt}(\sin x)$$

$$\frac{dx}{dt} = 2\cos x\frac{dx}{dt}$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}$$

Question 19.
The radius of a balloon is increasing at the rate of 10 m/s. At what rate is the surface area of the balloon increasing when the radius is 15 cm?

Answer:
$$\frac{dr}{dt} = 10 \text{ m/s}$$

$$S = 4\pi r^2$$

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dS}{dt} = 8\pi.15.10$$

$$\frac{dS}{dt} = 1200\pi \text{ cm}^2/\text{s}$$

Question 20.
An edge of a variable cube is increasing at the rate of 5 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?

Answer:
$$\frac{da}{dt} = 5\text{cm/s}$$

$$V = a^3$$

$$\frac{dV}{dt} = 3a^2 \frac{da}{dt}$$

$$\frac{dV}{dt} = 3.10^2.5$$

$$\frac{dV}{dt} = 1500 \text{ cm}^3/\text{s}$$

Question 21.

The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which the area is increasing when the side is 10 cm.

Answer:

$$\frac{da}{dt} = 2cm/s$$

$$A = \frac{\sqrt{3}}{4}a^2$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4}2a\frac{da}{dt}$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2}.10.2$$

$$\frac{dA}{dt} = 10\sqrt{3}\ cm^2/s$$