

Exercise 33a

Question 1.

Graph the solution sets of the following inequations:

$$x + y \geq 4$$

Answer:

Given $x + y \geq 4$

$$\Rightarrow y \geq 4 - x$$

Consider the equation $y = 4 - x$.

Finding points on the coordinate axes:

If $x = 0$, the y value is 4 i.e, $y = 4$

\Rightarrow the point on the Y axis is A(0,4)

If $y = 0$, $0 = 4 - x$

$$\Rightarrow x = 4$$

The point on the X axis is B(4,0)

Plotting the points on the graph: fig. 1a

Now consider the inequality $y \geq 4 - x$

Here we need the y value greater than or equal to $4 - x$

\Rightarrow the required region is above point A.

Therefore the graph of the inequation $x + y \geq 4$ is fig. 1b

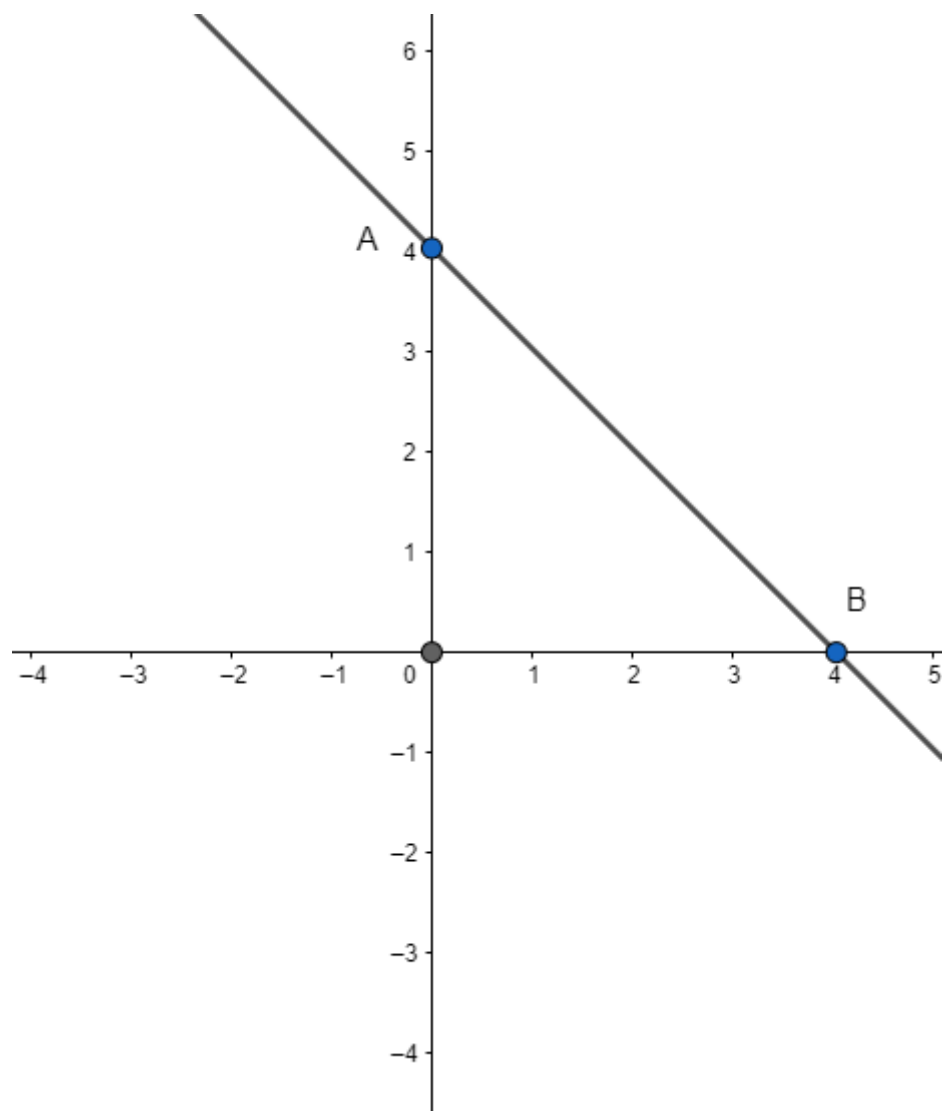


Fig 1a

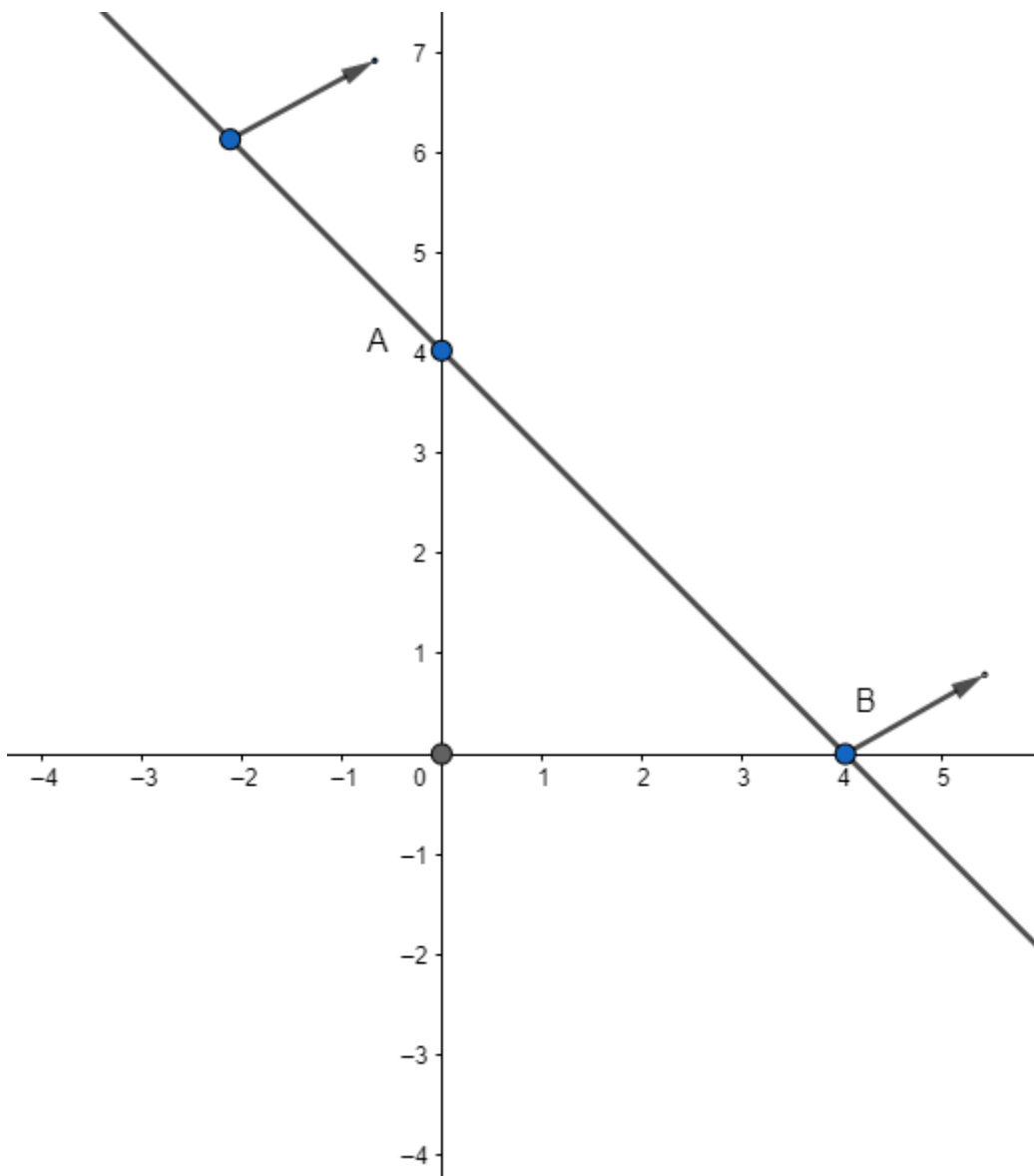


Fig 1b

Question 2.

Graph the solution sets of the following inequations:

$$x - y \leq 3$$

Answer:

Given $x - y \leq 3$

$$\Rightarrow -y \leq 3 - x$$

Multiplying by minus on both the sides, we'll get

$$y \geq -3 + x$$

$$y \geq x - 3$$

Consider the equation $y = x - 3$.

Finding points on the coordinate axes:

If $x = 0$, the y value is -3 i.e, $y = -3$

\Rightarrow the point on the Y axis is $A(0, -3)$

If $y = 0$, $0 = x - 3$

$\Rightarrow x = 3$

The point on the X axis is $B(3,0)$

Plotting the points on the graph: fig. 2a

Now consider the inequality $y \geq x - 3$

Here we need the y value greater than or equal to $x - 3$

\Rightarrow the required region is above point A .

Therefore the graph of the inequation $x + y \geq 4$ is fig. 2b

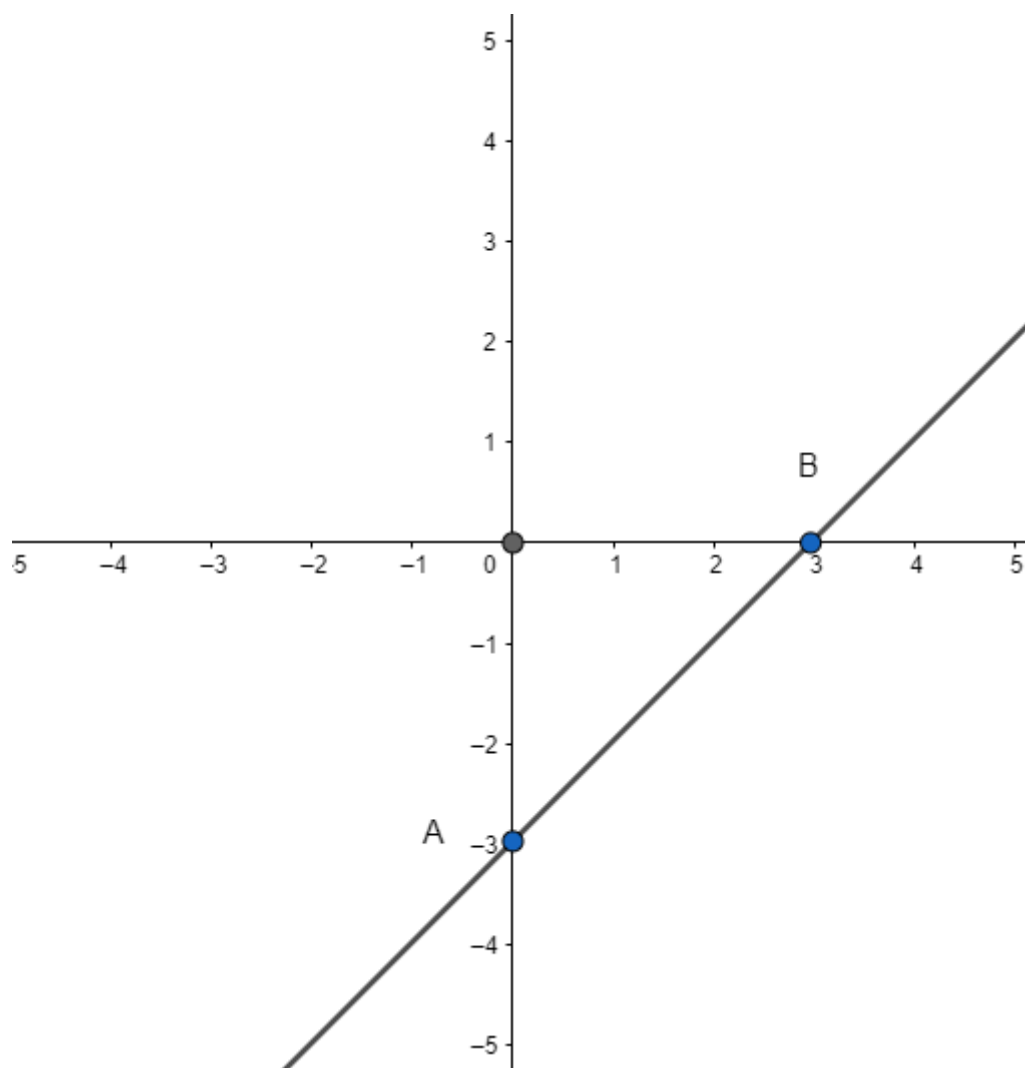


Fig 2a

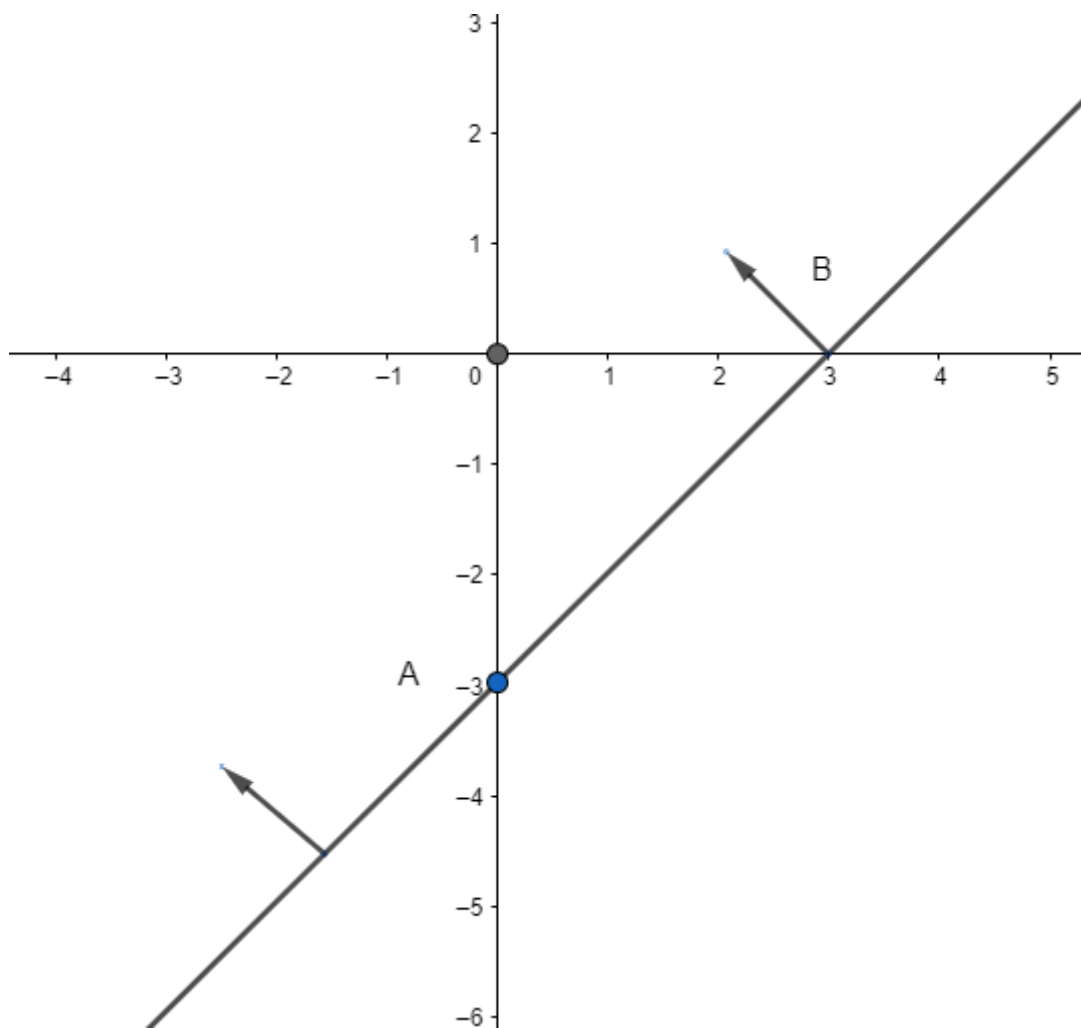


Fig 2b

Question 3.

Graph the solution sets of the following inequations:

$$x + 2y > 1$$

Answer:

Given $x + 2y > 1$

$$\Rightarrow 2y > 1 - x$$

$$\Rightarrow y > \frac{1}{2} - \frac{x}{2}$$

Consider the equation $y = \frac{1}{2} - \frac{x}{2}$

Finding points on the coordinate axes:

If $x = 0$, the y value is $\frac{1}{2}$ i.e., $y = \frac{1}{2}$

\Rightarrow the point on the Y axis is $A(0, \frac{1}{2})$

If $y = 0$, $x = 1$

The point on the X axis is $B(1,0)$

Plotting the points on the graph: fig. 3a

Now consider the inequality $y > \frac{1}{2} - \frac{x}{2}$

Here we need the y value greater than $\frac{1}{2} - \frac{x}{2}$

\Rightarrow the required region is above point A .

Also, the line AB is represented in dotted line. This is done because $y \neq \frac{1}{2} - \frac{x}{2}$

Therefore the graph of the inequation $y > \frac{1}{2} - \frac{x}{2}$ is fig. 3b

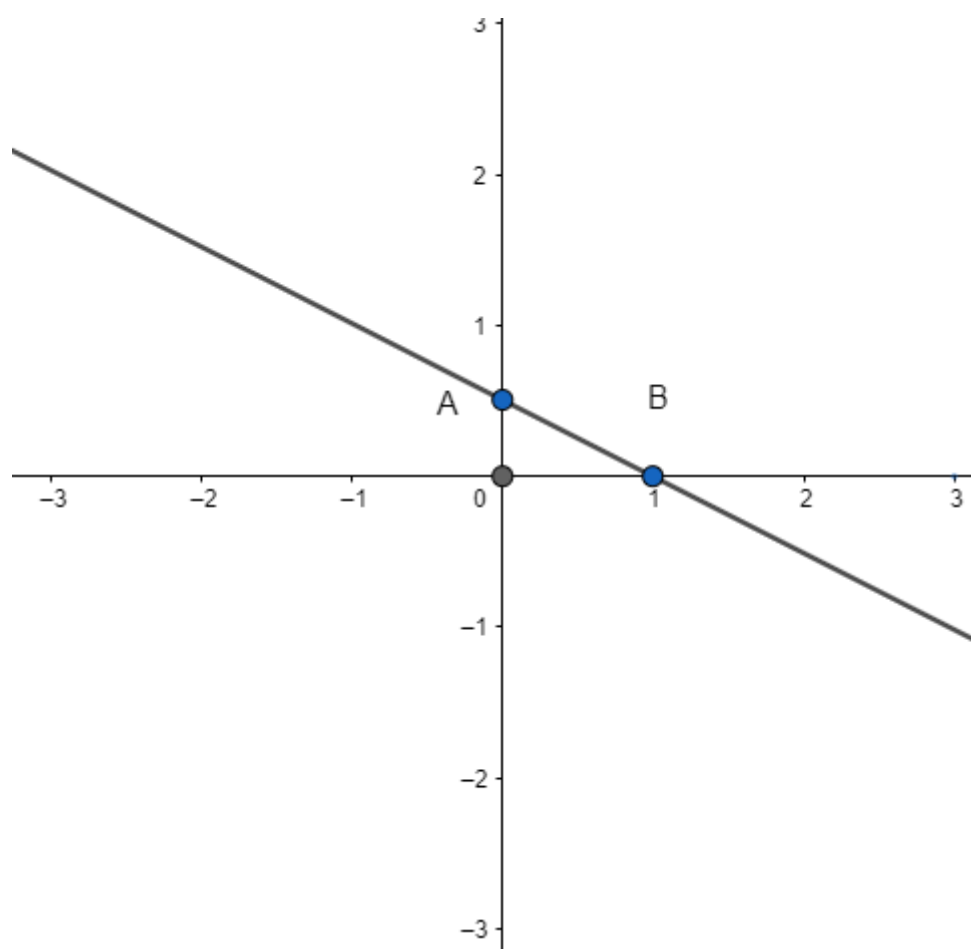


Fig 3a

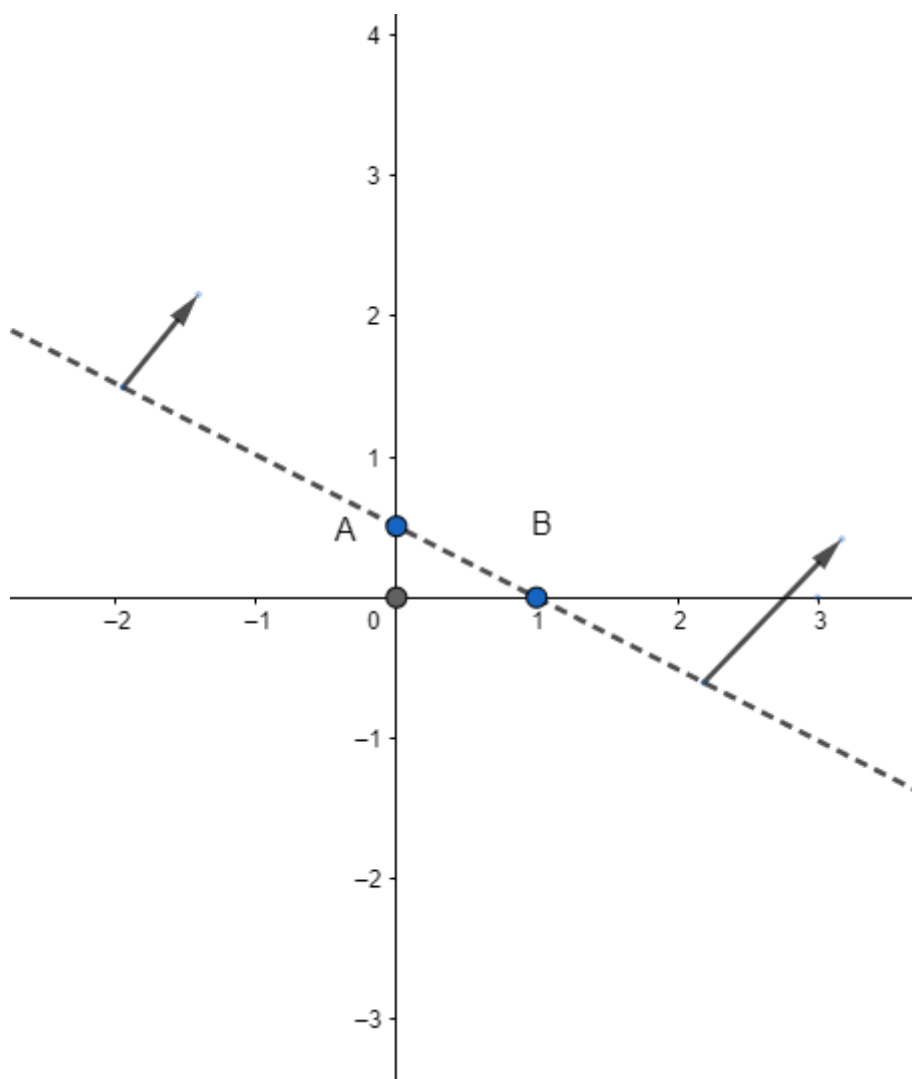


Fig 3b

Question 4.

Graph the solution sets of the following inequations:

$$2x - 3y < 4$$

Answer:

Given $2x - 3y < 4$

$$\Rightarrow 2x - 4 < 3y$$

$$\Rightarrow 3y > 2x - 4$$

$$\Rightarrow y > \frac{2}{3}x - \frac{4}{3}$$

Consider the equation $y = \frac{2}{3}x - \frac{4}{3}$

Finding points on the coordinate axes:

If $x = 0$, the y value is $\frac{1}{2}$ i.e., $y = -\frac{4}{3}$

\Rightarrow the point on the Y axis is $A(0, -\frac{4}{3})$

If $y = 0$, $x = 2$

The point on the X axis is $B(2,0)$

Plotting the points on the graph: fig. 4a

Now consider the inequality $y > \frac{2}{3}x - \frac{4}{3}$

Here we need the y value greater than $\frac{2}{3}x - \frac{4}{3}$

\Rightarrow the required region is above point A .

Also, the line AB is represented in dotted line. This is done because $y \neq \frac{2}{3}x - \frac{4}{3}$

Therefore the graph of the inequation $y > \frac{2}{3}x - \frac{4}{3}$ is fig. 4b

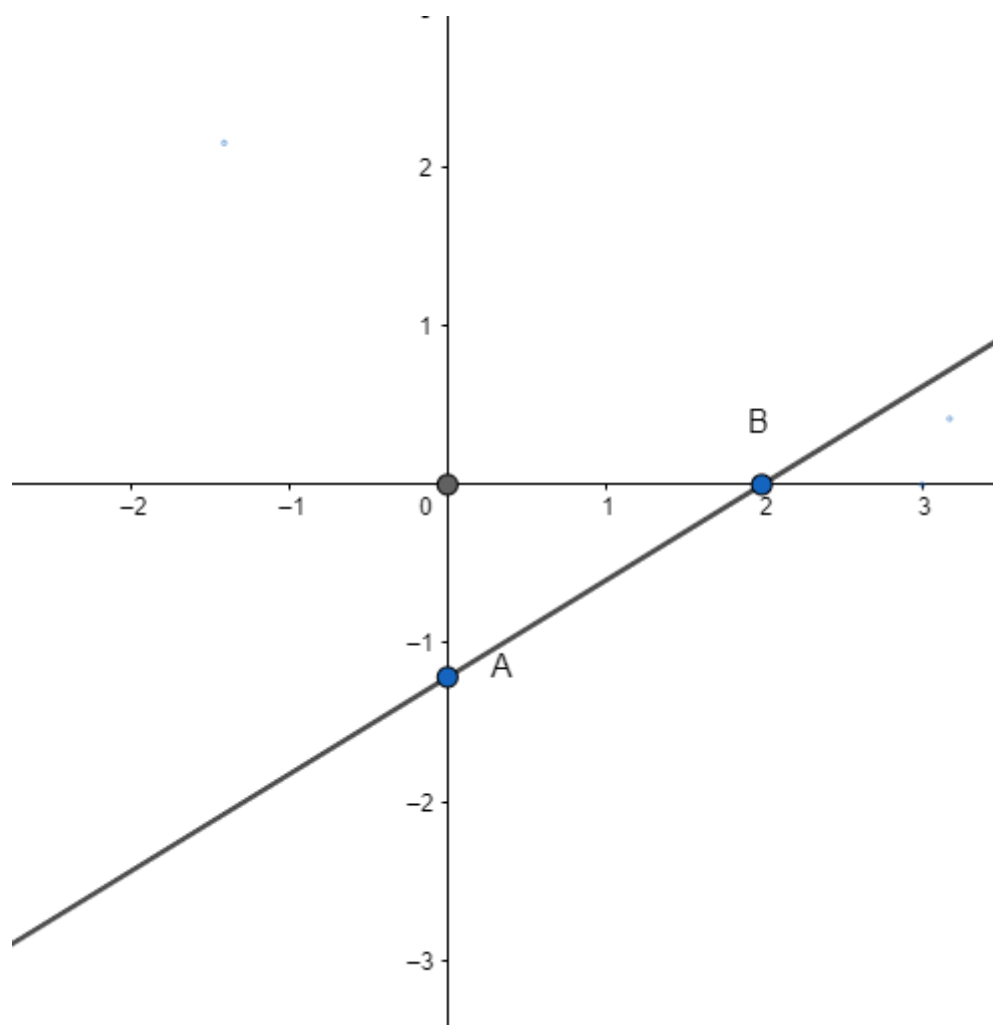


Fig 4a

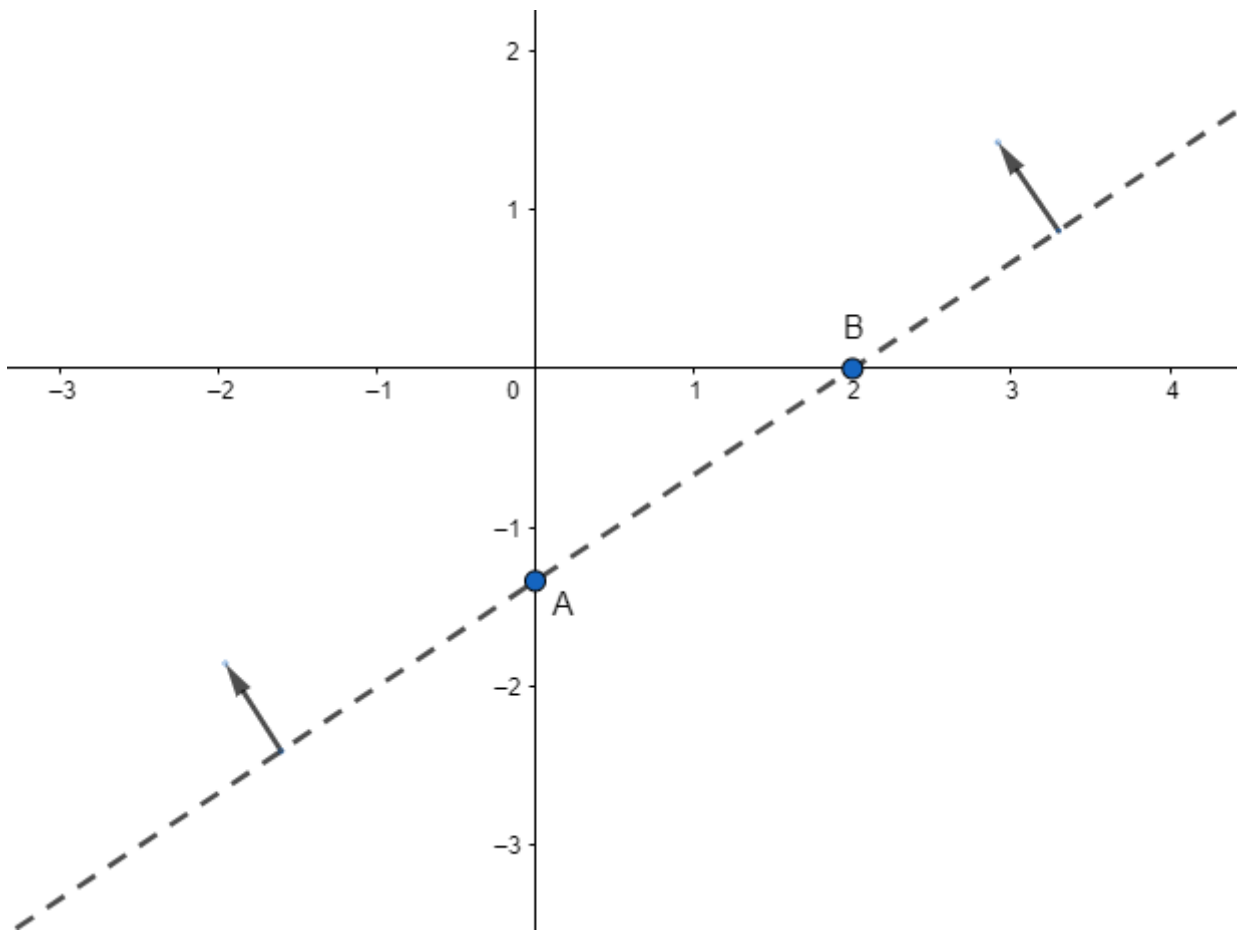


Fig 4b

Question 5.

Graph the solution sets of the following inequations:

$$x \geq y - 2$$

Answer:

Given $x \geq y - 2$

$$\Rightarrow y \leq x + 2$$

Consider the equation $y = x + 2$

Finding points on the coordinate axes:

If $x = 0$, the y value is 2 i.e, $y = 2$

\Rightarrow the point on the Y axis is $A(0,2)$

If $y = 0$, $0 = x + 2$

$$\Rightarrow x = -2$$

The point on the X axis is B(- 2,0)

Plotting the points on the graph: fig. 5a

Now consider the inequality $y \leq x + 2$

Here we need the y value less than or equal to $x + 2$

\Rightarrow the required region is below point A.

Therefore the graph of the inequation $x \geq y - 2$ is fig. 5b

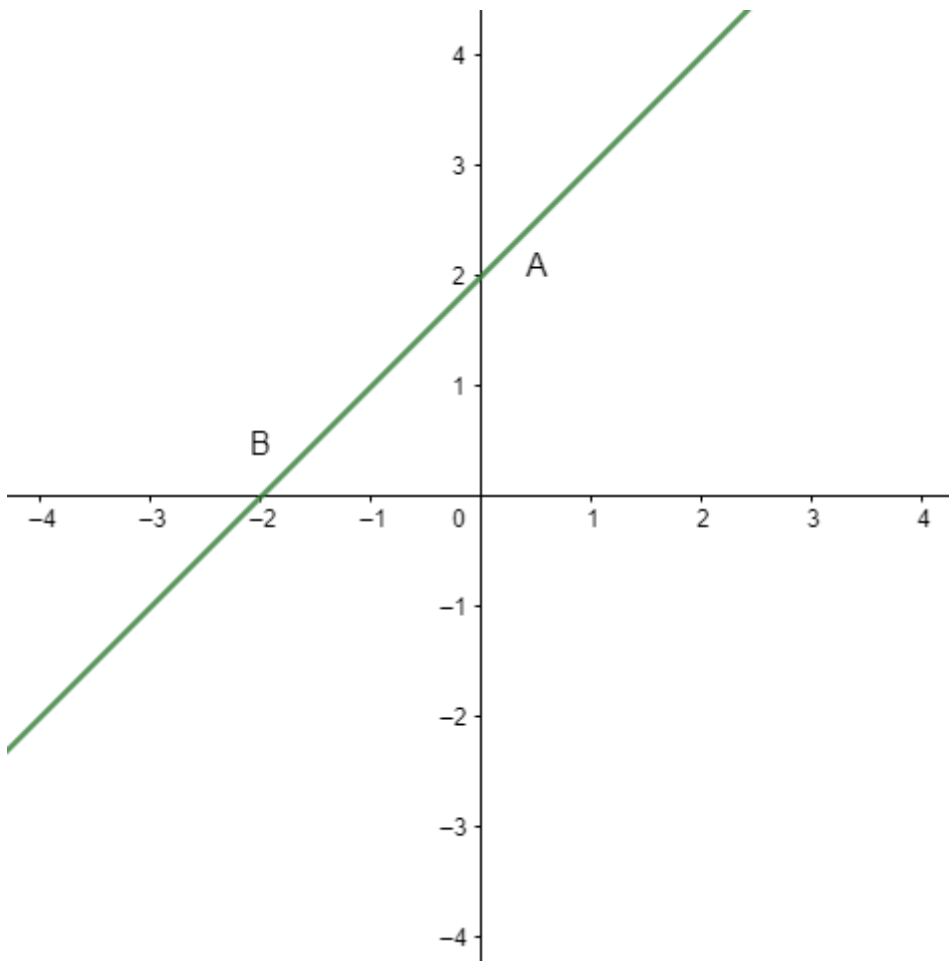


fig 5a

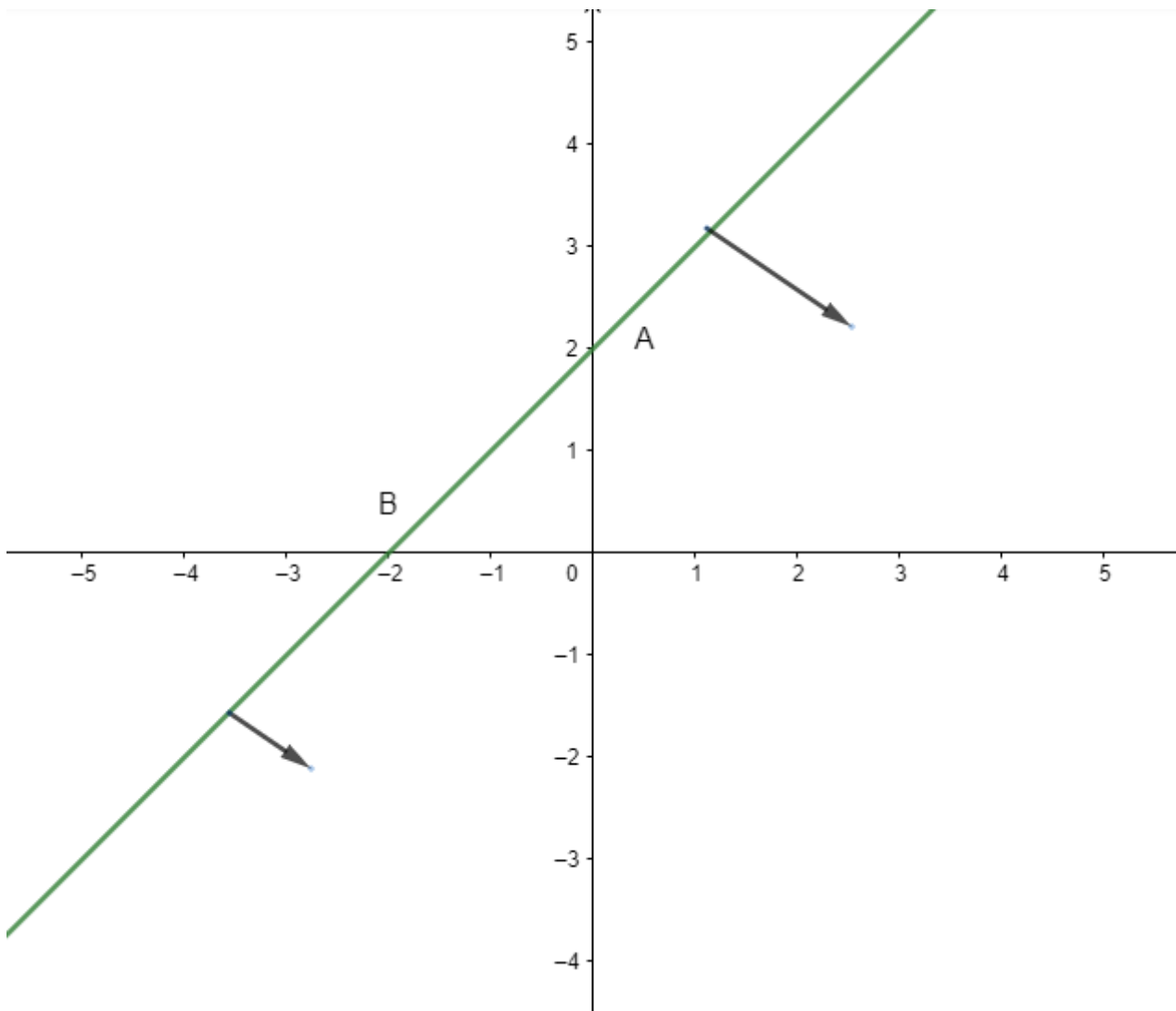


Fig 5b

Question 6.

Graph the solution sets of the following inequations:

$$y - 2 \leq 3x$$

Answer:

Given $y - 2 \leq 3x$

$$\Rightarrow y \leq 3x + 2$$

Consider the equation $y = 3x + 2$

Finding points on the coordinate axes:

If $x = 0$, the y value is 2 i.e, $y = 2$

⇒ the point on Y axis is A(0,2)

If $y = 0$, $0 = 3x + 2$

$$\Rightarrow x = -\frac{3}{2}$$

The point on the X axis is B($-\frac{3}{2}$, 0)

Plotting the points on the graph: fig. 6a

Now consider the inequality $y \leq 3x + 2$

Here we need the y value less than or equal to $3x + 2$

⇒ the required region is below point A.

Therefore the graph of the inequation $y \leq 3x + 2$ is fig. 5b

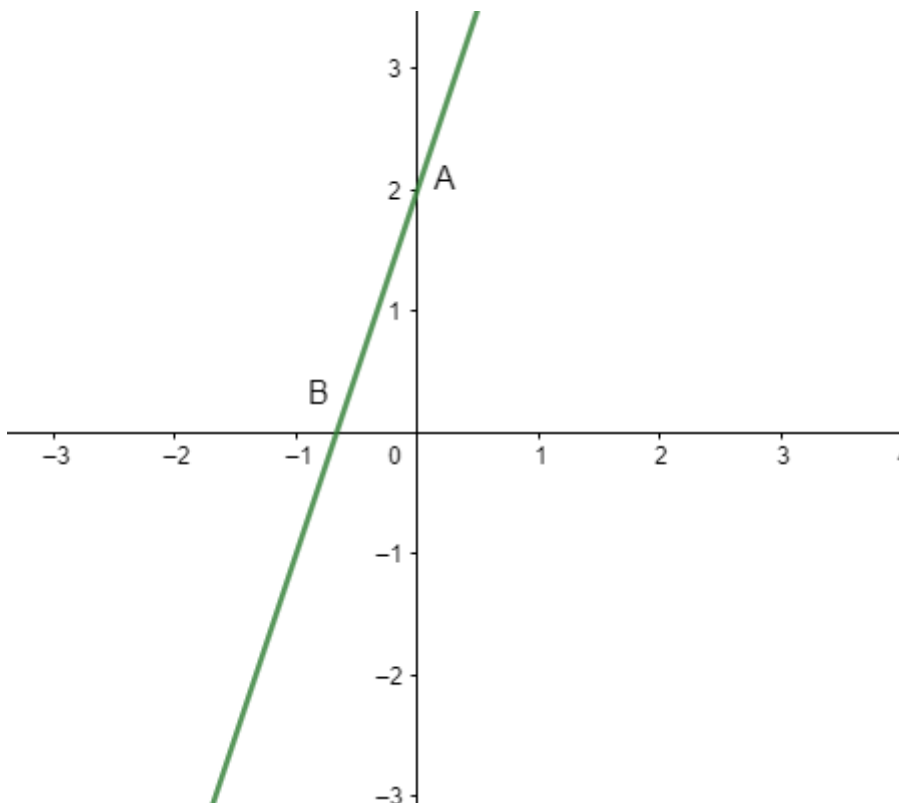


Fig 6a

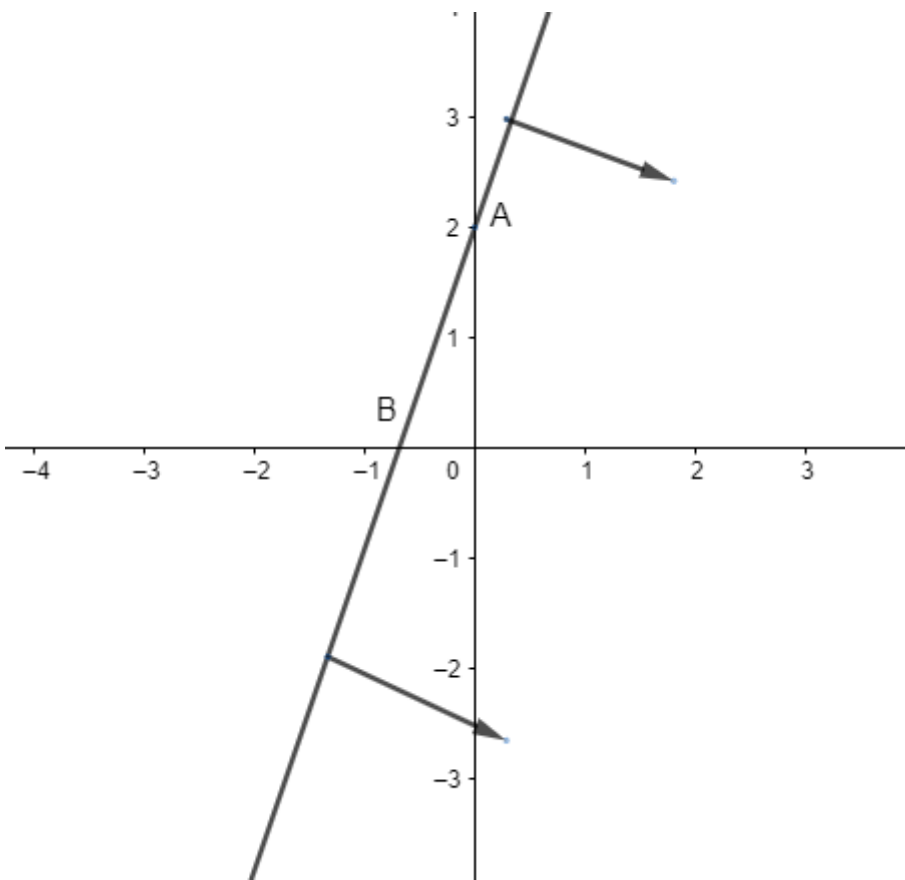


Fig 6b

Question 7.

Solve each of the following systems of simultaneous inequations:

$$2x + y > 1 \text{ and } 2x - y \geq -3$$

Answer:

Consider the inequation $2x + y > 1$:

$$\Rightarrow y > 1 - 2x$$

Consider the equation $y = 1 - 2x$

Finding points on the coordinate axes:

If $x = 0$, the y value is 1 i.e, $y = 1$

\Rightarrow the point on Y axis is A(0,1)

If $y = 0$, $0 = x + 2$

$$\Rightarrow x = \frac{1}{2}$$

The point on the X axis is $B(\frac{1}{2}, 0)$

Plotting the points on the graph: fig. 7a

Now consider the inequality $y > 1 - 2x$

Here we need the y value greater than $x + 2$

\Rightarrow the required region is below point A.

Therefore the graph of the inequation $y > 1 - 2x$ is fig. 7b

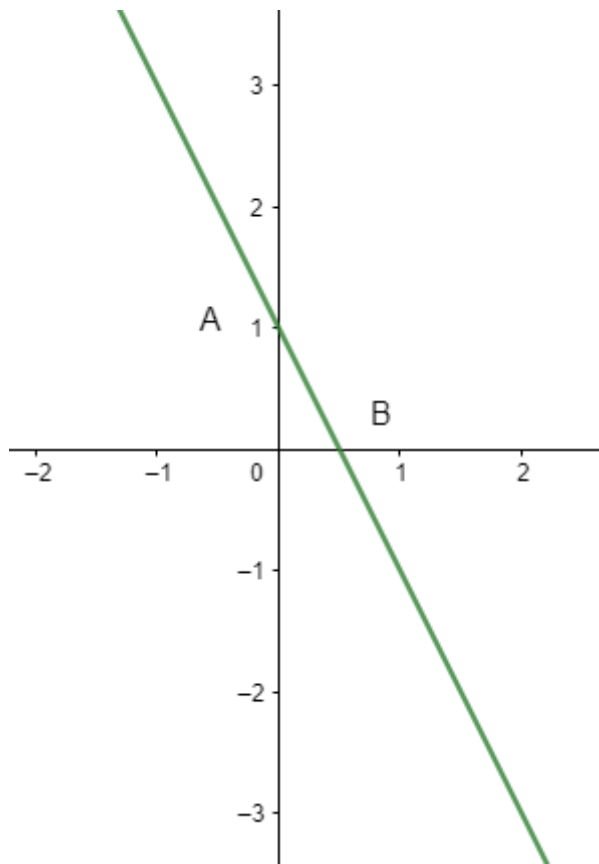


Fig 7a

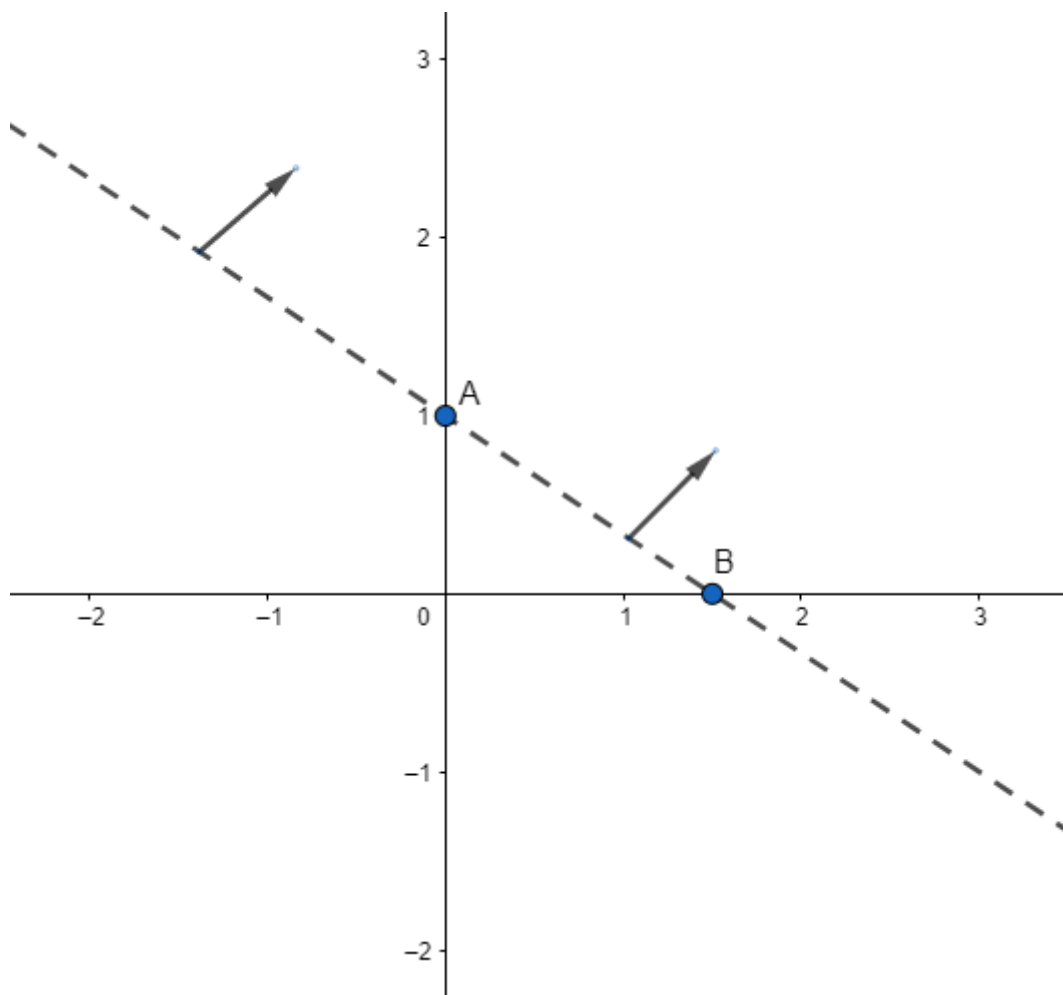


Fig 7b

Consider the inequation $2x - y \geq 3$

$$\Rightarrow y \leq 2x - 3$$

Consider the equation $y = 2x - 3$

Finding points on the coordinate axes:

If $x = 0$, the y value is -3 i.e, $y = -3$

\Rightarrow the point on the Y axis is $C(0, -3)$

If $y = 0$, $0 = 2x + 3$

$$\Rightarrow x = -\frac{3}{2}$$

The point on the X axis is $D(\frac{3}{2}, 0)$

Plotting the points on the graph: fig. 7c

Now consider the inequality $y \leq 2x - 3$

Here we need the y value less than or equal to $2x - 3$

\Rightarrow the required region is below point C.

Therefore the graph of the inequation $y \leq 2x - 3$ is fig. 7d

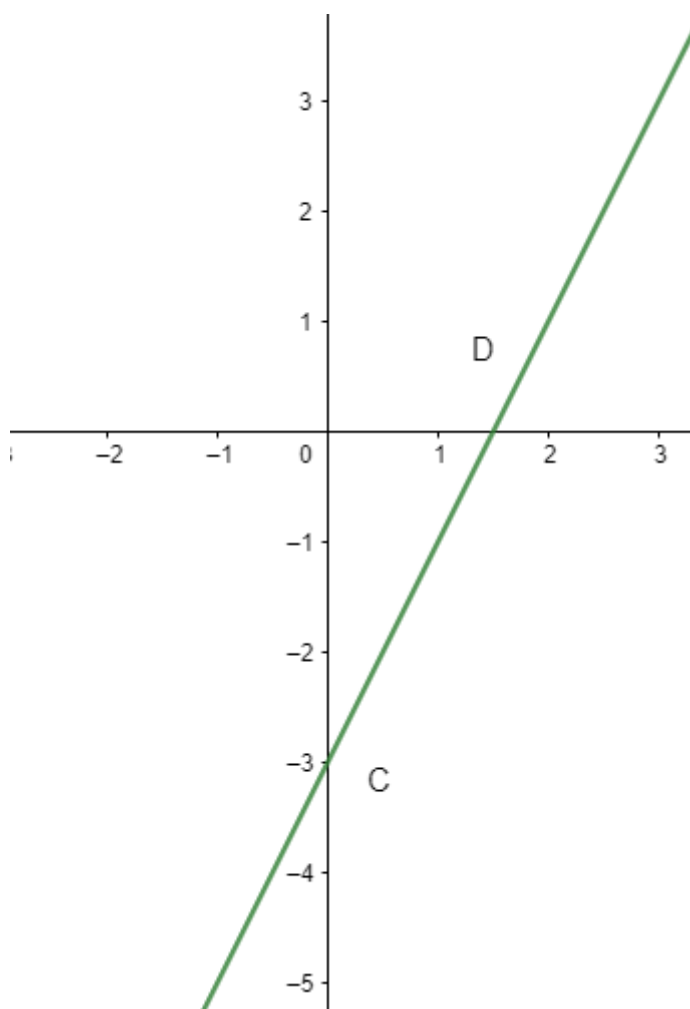


Fig 7c

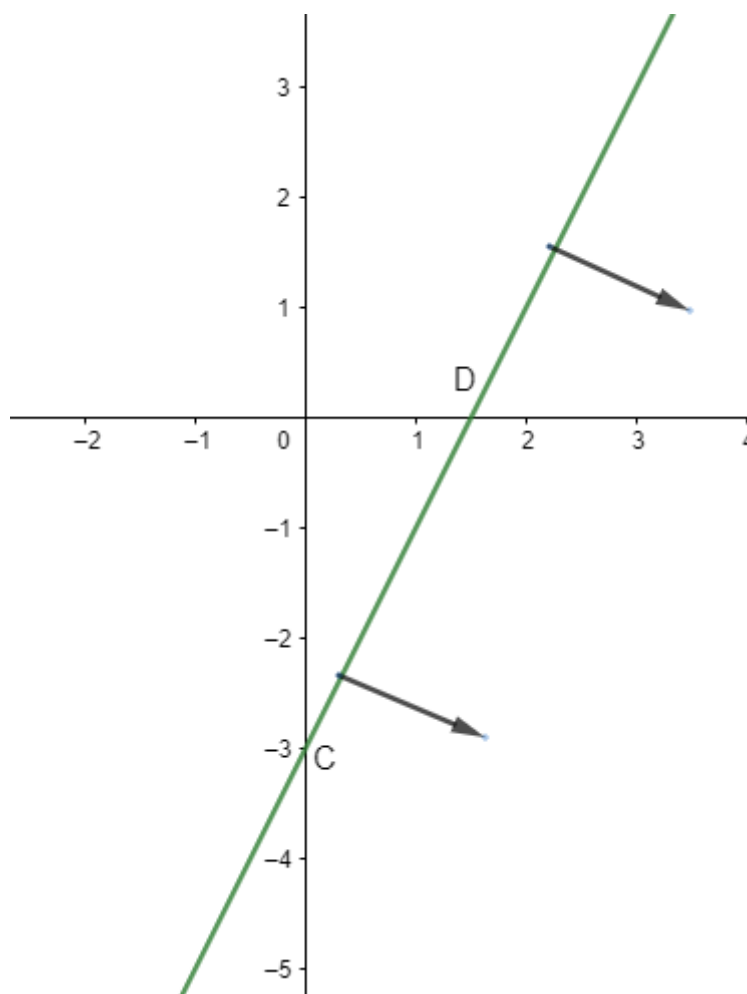
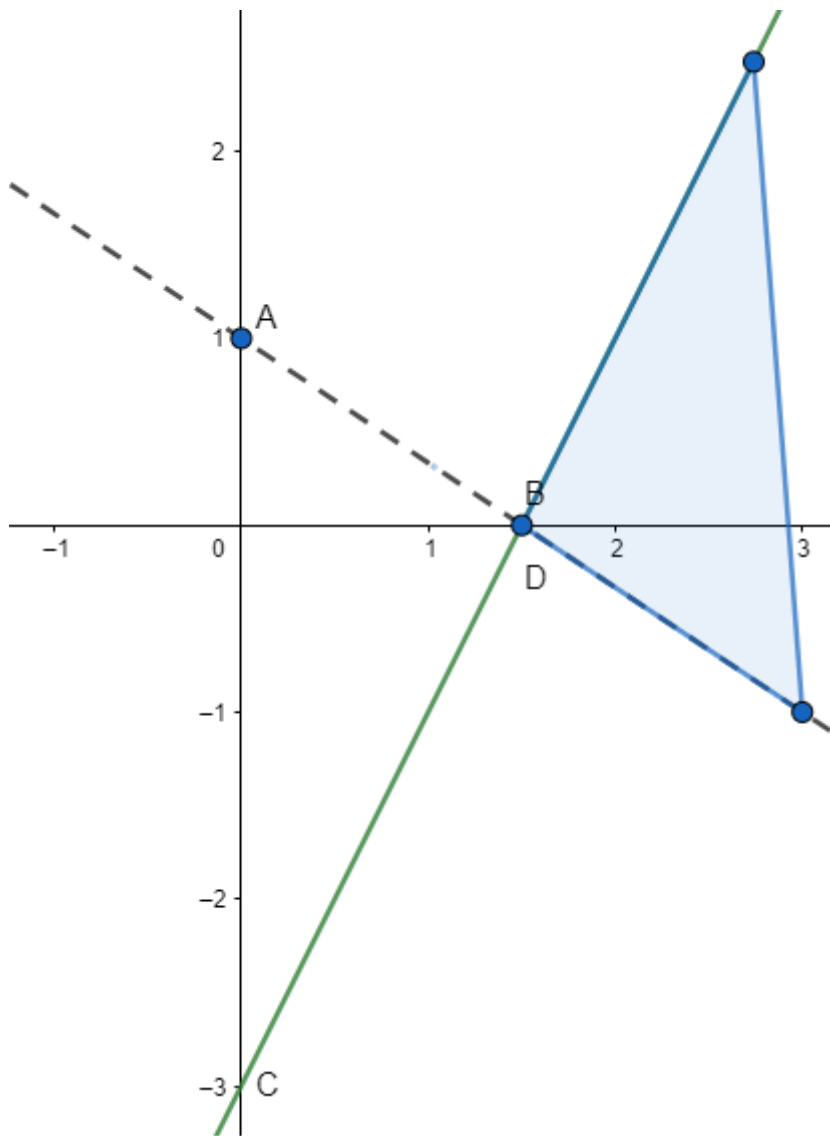


Fig 7d

Combining the graphs 7c and 7d, we'll get,



The solution of the system of simultaneous inequations is the intersection region of the solutions of the two given inequations.

Question 8.

Solve each of the following systems of simultaneous inequations:

$$x - 2y \geq 0, 2x - y \leq -2$$

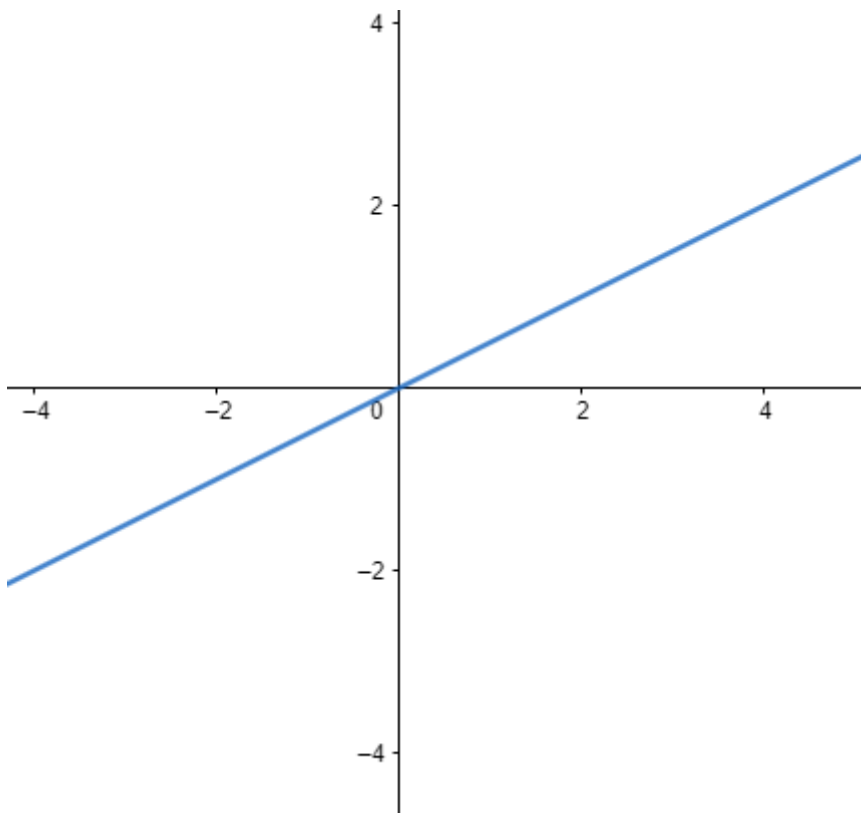
Answer:

Consider the inequation $x - 2y \geq 0$:

$$\Rightarrow x \geq 2y$$

$$\Rightarrow y \leq \frac{x}{2}$$

consider the equation $y = \frac{x}{2}$. This equation's graph is a straight line passing through origin.



Now consider the inequality $y \leq \frac{x}{2}$

Here we need the y value less than or equal to $\frac{x}{2}$

\Rightarrow the required region is below the origin.

Therefore the graph of the inequation $y \leq \frac{x}{2}$ is fig.8a

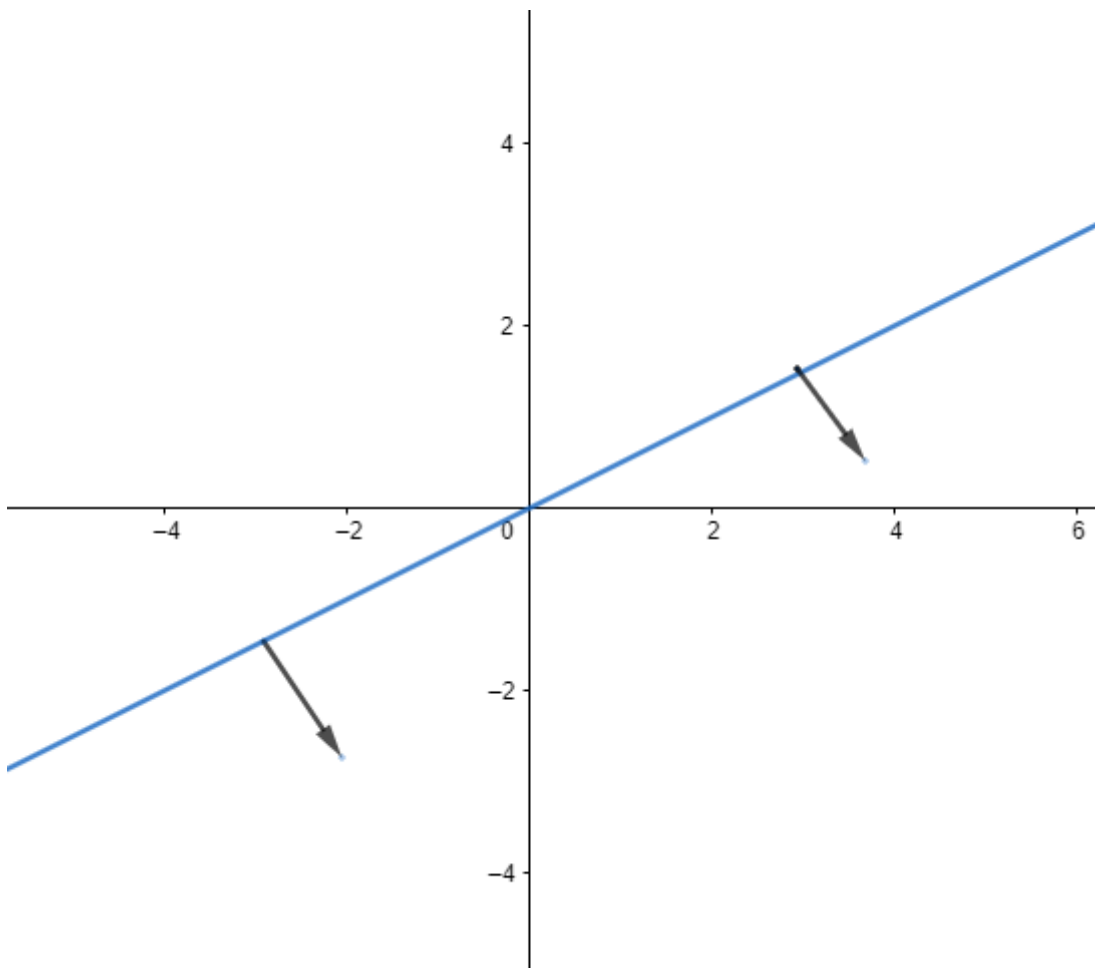


Fig 8a

Consider the inequation $2x - y \leq -2$:

$$\Rightarrow y \geq 2x + 2$$

Consider the equation $y = 2x + 2$

Finding points on the coordinate axes:

If $x = 0$, the y value is 2 i.e, $y = 2$

\Rightarrow the point on the Y axis is $A(0,2)$

If $y = 0$, $0 = 2x + 2$

$$\Rightarrow x = -1$$

The point on the X axis is $B(-1,0)$

Plotting the points on the graph: fig. 8b.

Now consider the inequality $y \geq 2x + 2$

Here we need the y value greater than or equal to $2x + 2$

\Rightarrow the required region is above point A.

Therefore the graph of the inequation $y \geq 2x + 2$ is fig. 8c

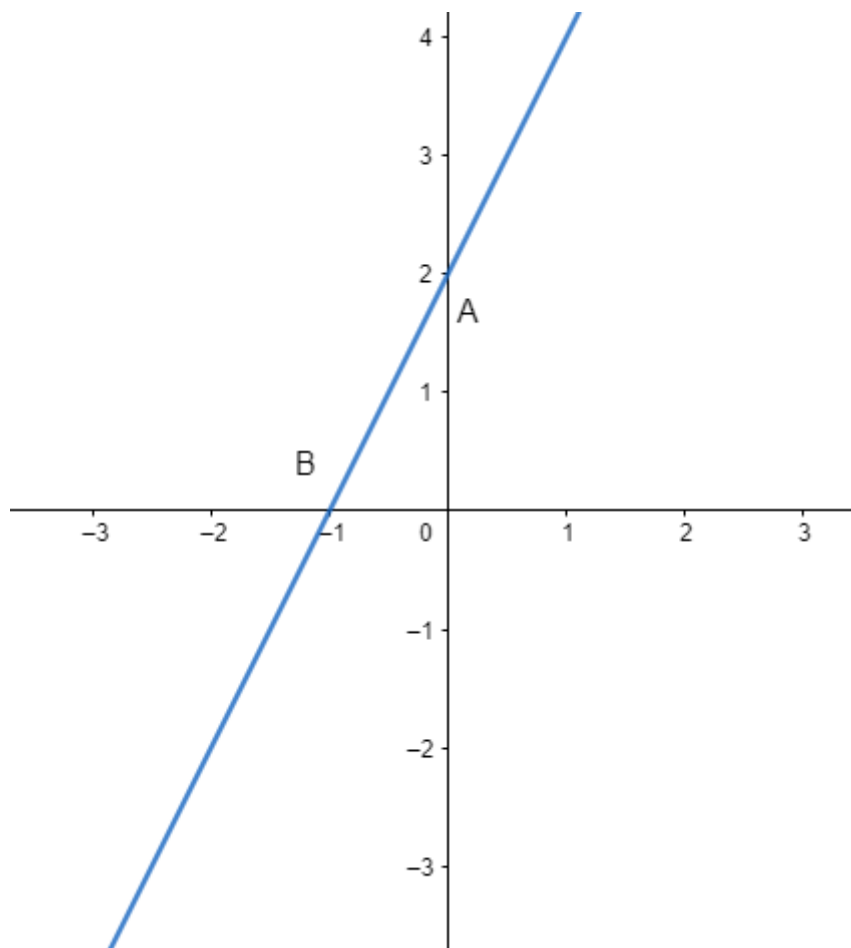


Fig 8b

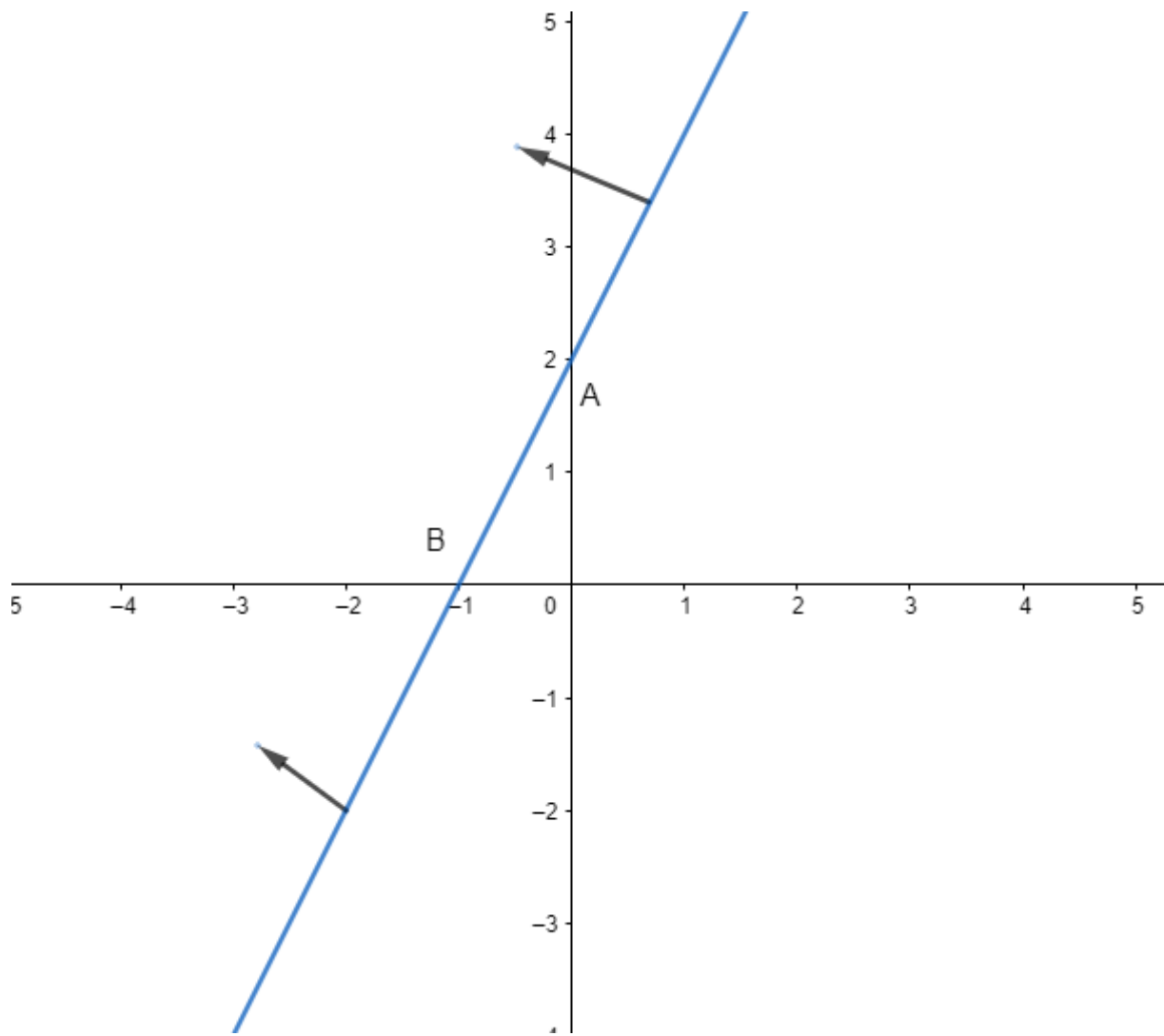
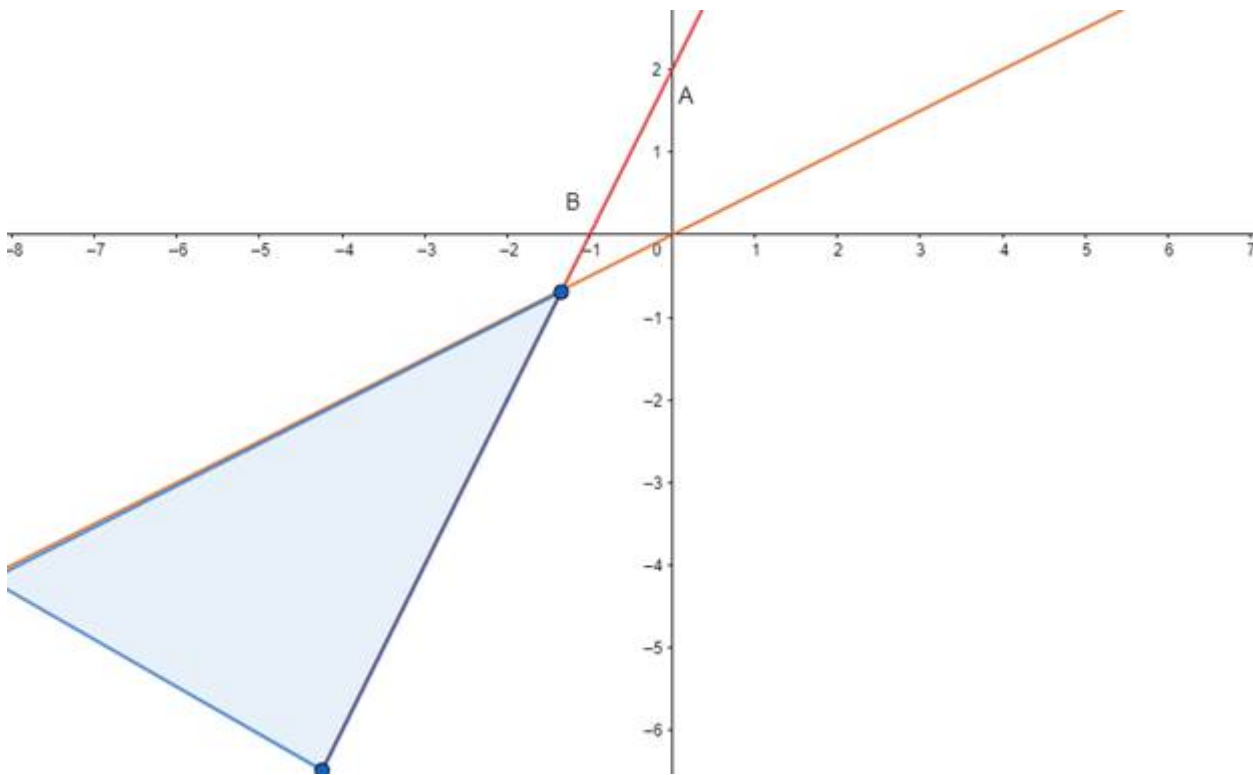


Fig 8c

Combining the graphs of 8a and 8c, we'll get



The solution of the system of simultaneous inequations is the intersection region of the solutions of the two given inequations.

Question 9.

Solve each of the following systems of simultaneous inequations:

$$3x + 4y \geq 12, x \geq 0, y \geq 1 \text{ and } 4x + 7y \leq 28$$

Answer:

Consider the inequation $3x + 4y \geq 12$:

$$\Rightarrow 4y \geq 12 - 3x$$

$$\Rightarrow y \geq 3 - \frac{3}{4}x$$

Consider the equation $y = 3 - \frac{3}{4}x$

Finding points on the coordinate axes:

If $x = 0$, the y value is 3 i.e, $y = 3$

\Rightarrow the point on the Y axis is A(0,3)

If $y = 0$, $0 = 3 - \frac{3}{4}x$

$\Rightarrow x = 4$

The point on the X axis is B(4,0)

Now consider the inequality $y \geq 3 - \frac{3}{4}x$

Here we need the y value greater than or equal to $y \geq 3 - \frac{3}{4}x$

\Rightarrow the required region is above point A.

Therefore the graph of the inequation $y \geq 3 - \frac{3}{4}x$ is fig. 9a

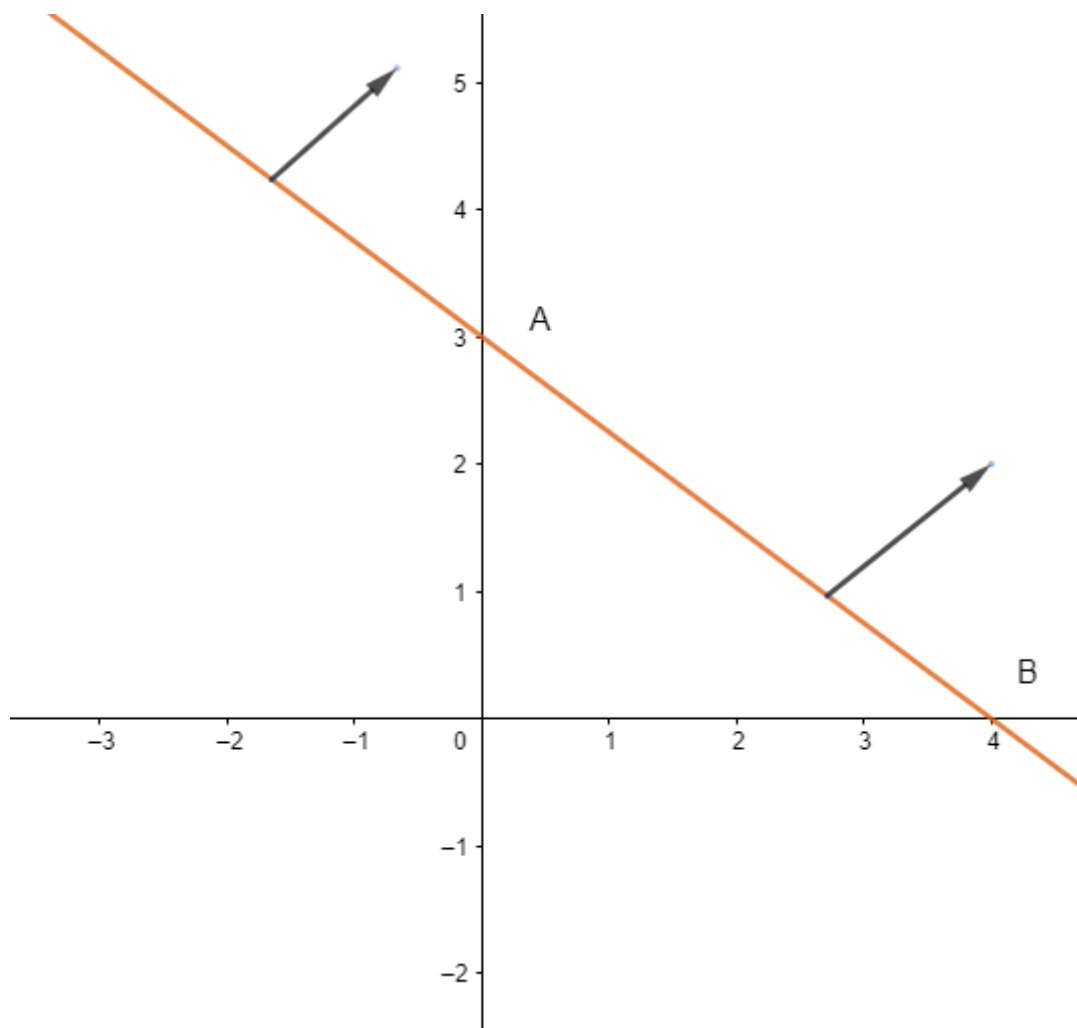


Fig 9a

Consider the inequation $4x + 7y \leq 28$

$$\Rightarrow 7y \leq 28 - 4x$$

$$\Rightarrow y \leq 4 - \frac{4}{7}x$$

Consider the equation $y = 4 - \frac{4}{7}x$

Finding points on the coordinate axes:

If $x = 0$, the y value is 4 i.e, $y = 4$

\Rightarrow the point on the Y axis is $C(0,4)$

$$\text{If } y = 0, 0 = 4 - \frac{4}{7}x$$

$$\Rightarrow x = 7$$

The point on the X axis is $D(7,0)$

Now consider the inequality $y \leq 4 - \frac{4}{7}x$

Here we need the y value less than or equal to $4 - \frac{4}{7}x$

\Rightarrow the required region is below point C .

Therefore the graph of the inequation $y \leq 4 - \frac{4}{7}x$ is fig. 9b

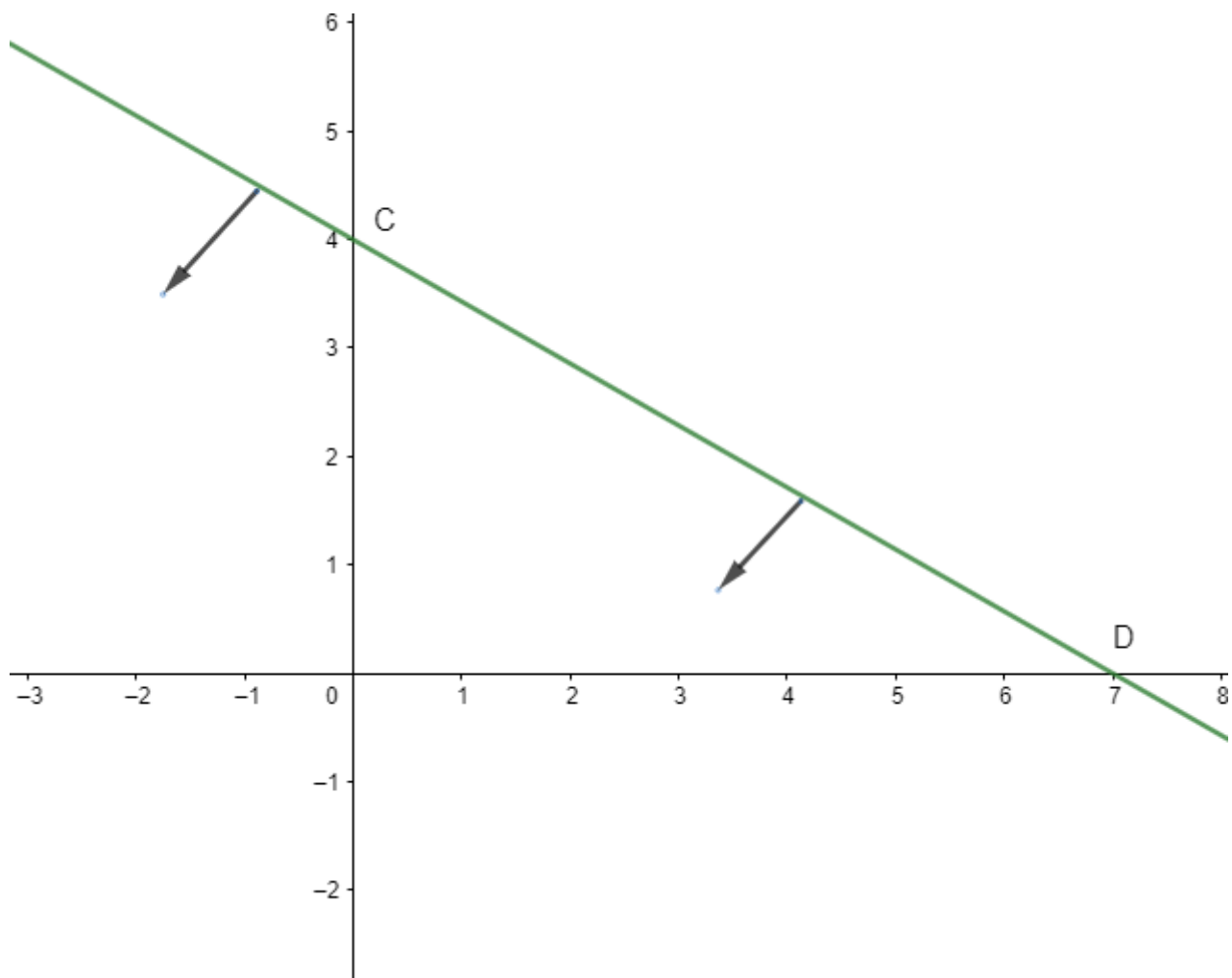
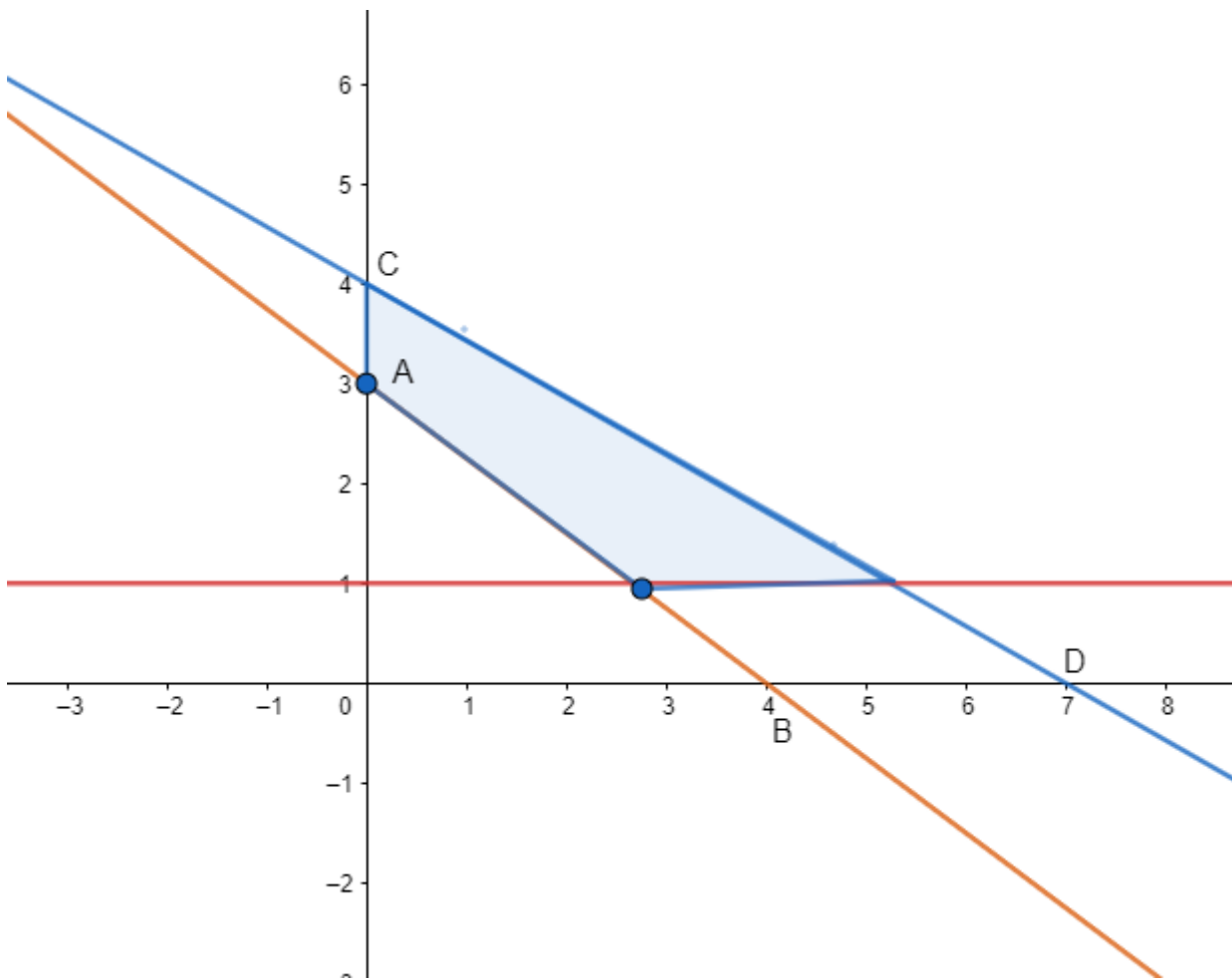


Fig 9b

$x \geq 0$ is the region right side of Y - axis.

$y \geq 1$ is the region above the line $y = 1$

Combining all the above results in a single graph , we'll get



The solution of the system of simultaneous inequations is the intersection region of the solutions of the two given inequations.

Question 10.

Show that the solution set of the following linear constraints is empty:

$$x - 2y \geq 0, 2x - y \leq -2, x \geq 0 \text{ and } y \geq 0$$

Answer:

Consider the inequation $x - 2y \geq 0$:

$$\Rightarrow x \geq 2y$$

$$\Rightarrow y \leq \frac{x}{2}$$

consider the equation $y = \frac{x}{2}$. This equation's graph is a straight line passing through origin.

Now consider the inequality $y \leq \frac{x}{2}$

Here we need the y value less than or equal to $\frac{x}{2}$

⇒ the required region is below origin.

Therefore the graph of the inequation $y \leq \frac{x}{2}$ is fig.10a

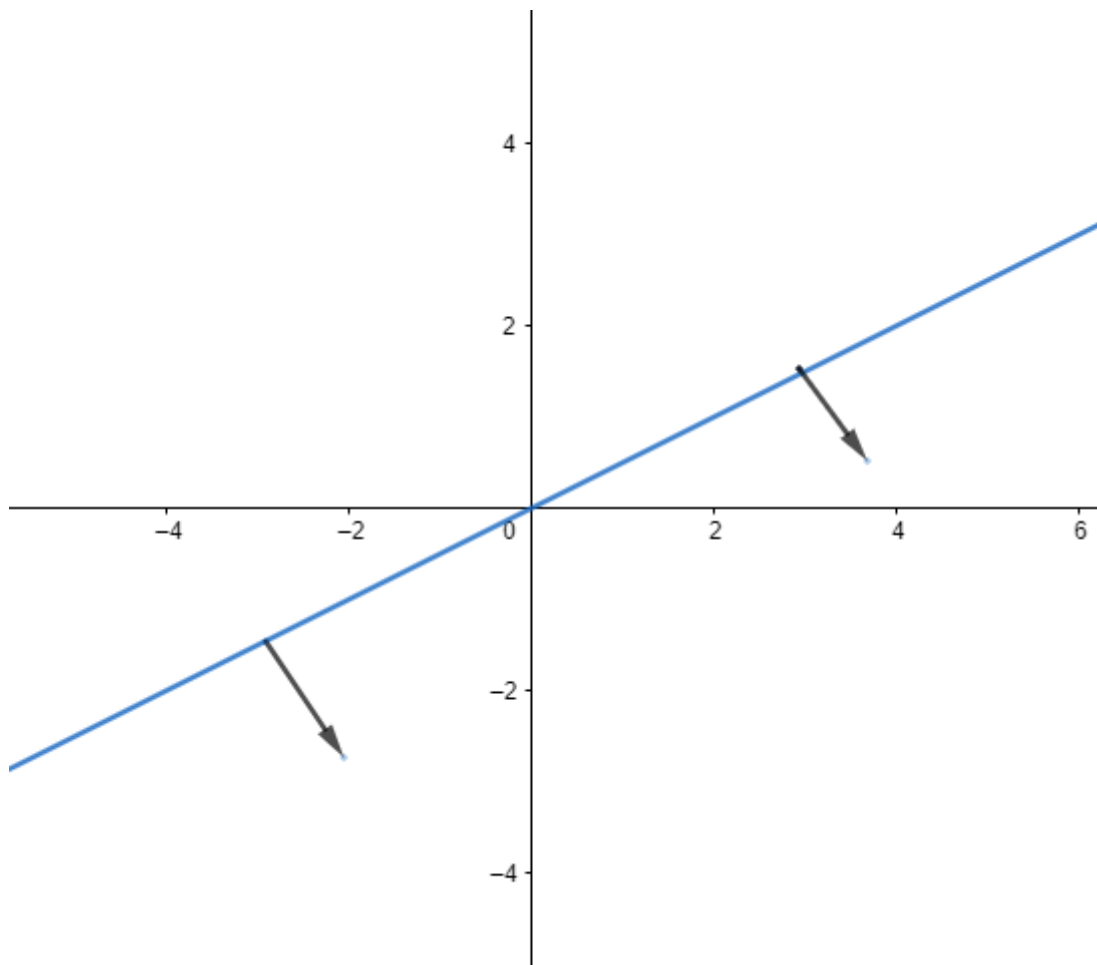


Fig 10a

Consider the inequation $2x - y \leq -2$:

$$\Rightarrow y \geq 2x + 2$$

Consider the equation $y = 2x + 2$

Finding points on the coordinate axes:

If $x = 0$, the y value is 2 i.e, $y = 2$

⇒ the point on Y axis is A(0,2)

If $y = 0$, $0 = 2x + 2$

⇒ $x = -1$

The point on X axis is B(- 1,0)

Now consider the inequality $y \geq 2x + 2$

Here we need the y value greater than or equal to $2x + 2$

⇒ the required region is above point A.

Therefore the graph of the inequation $y \geq 2x + 2$ is fig. 10b

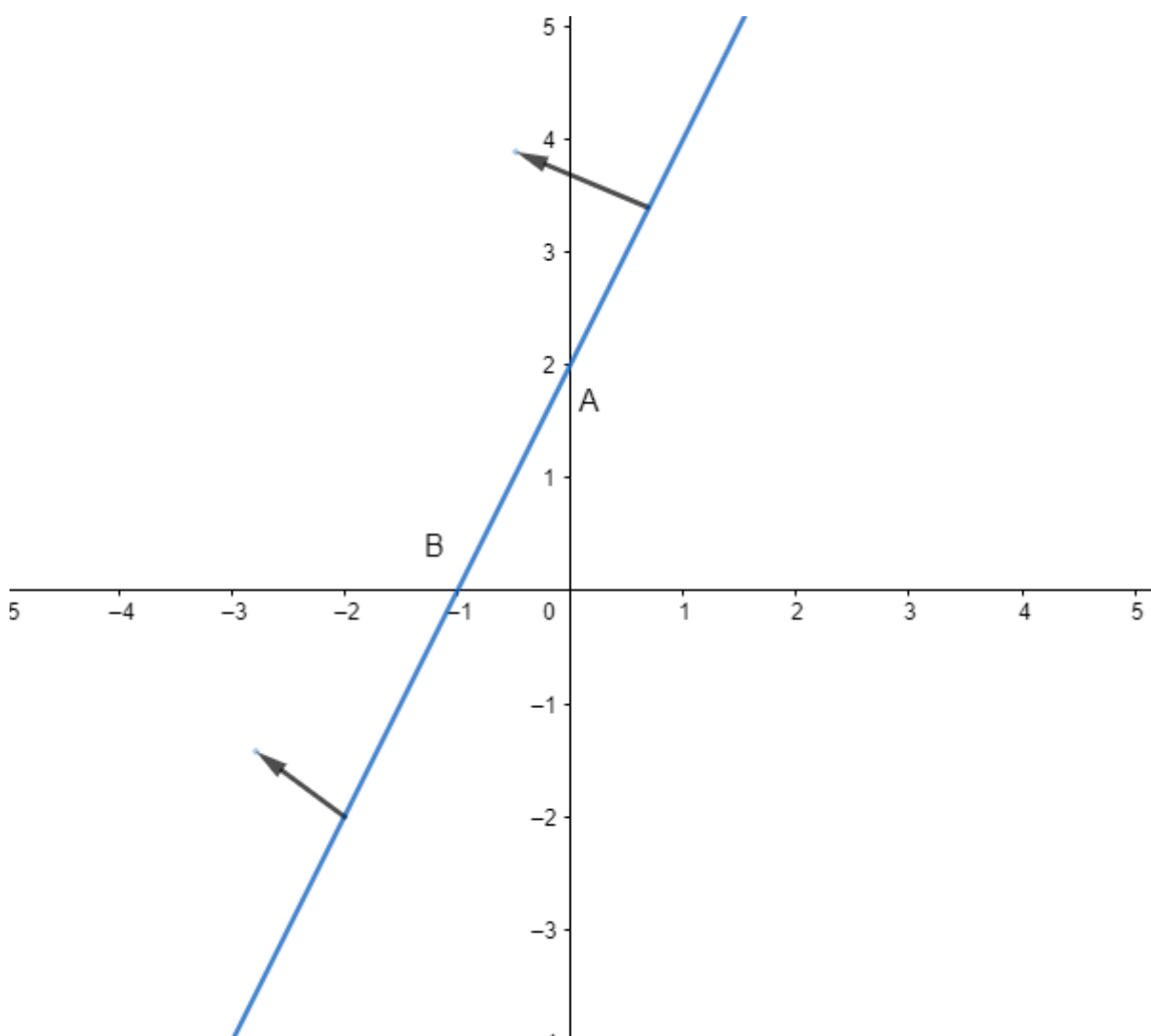
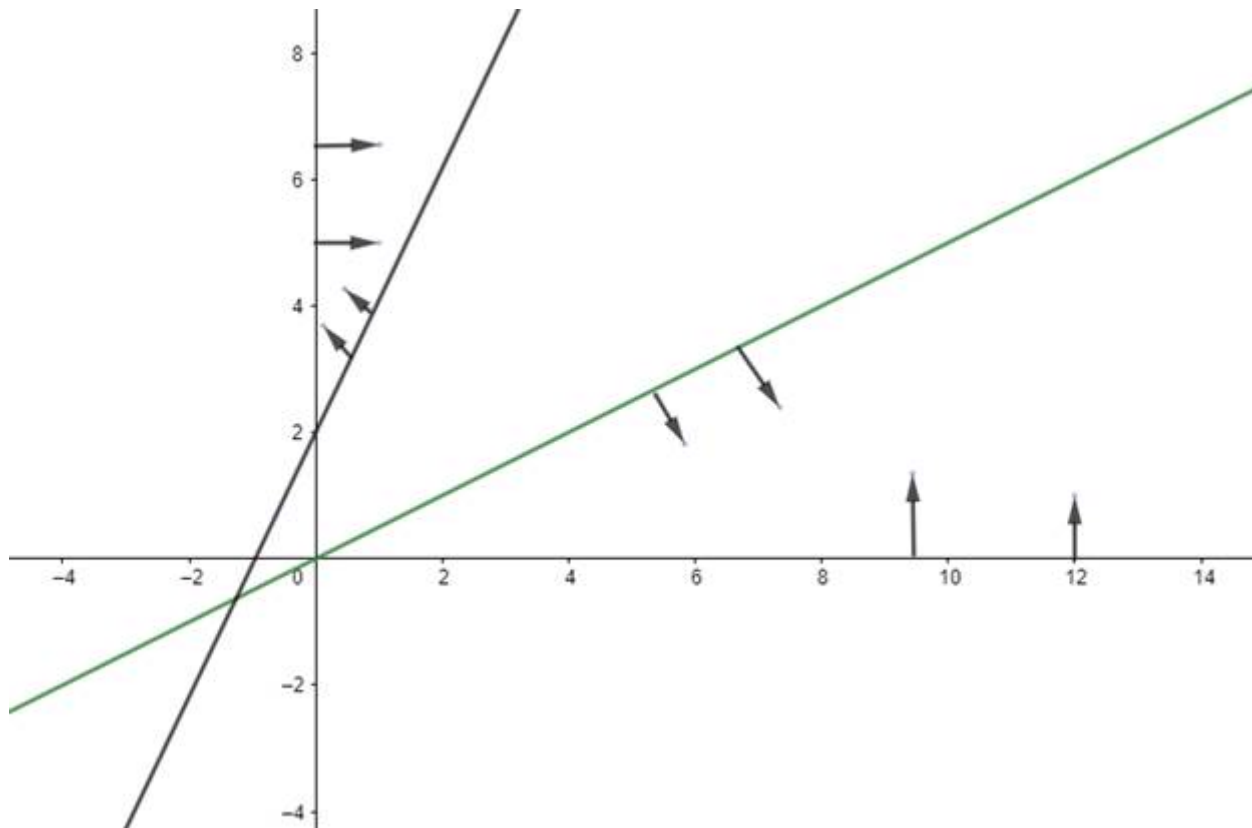


Fig 10b

$y \geq 0$ is the region above X - axis

$x \geq 0$ is the region right side of Y - axis

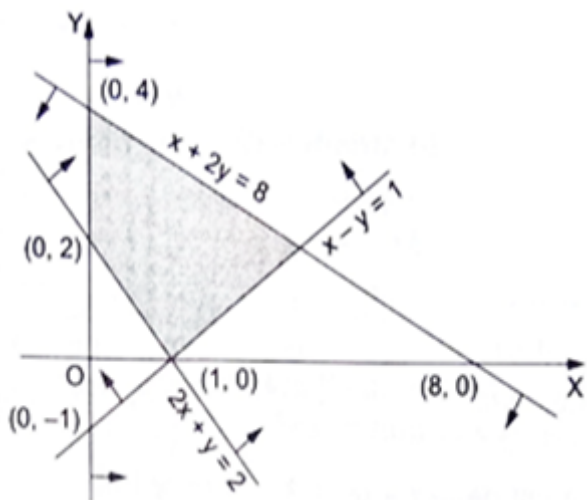
Combining the above results, we'll get



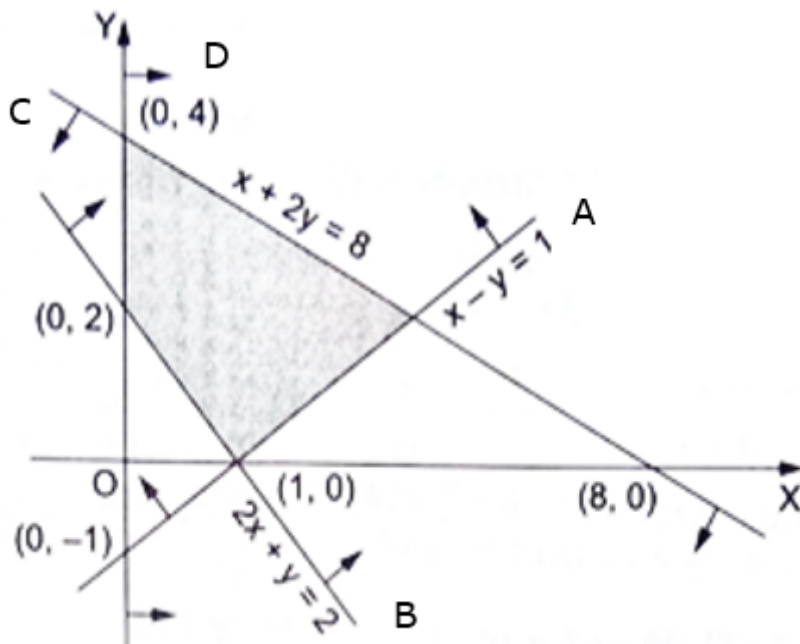
As they is no common area of intersection , there is no solution for the given set of simultaneous inequations.

Question 11.

Find the linear constraints for which the shaded area in the figure given is the solution set.



Answer:



Consider A:

Given line $x - y = 1$

$$\Rightarrow y = x - 1$$

As the region given in the figure is above the y - intercept's coordinates (0, - 1),

$$\Rightarrow y \geq x - 1$$

$$\Rightarrow x - y \leq 1$$

Consider B:

Given line $2x + y = 2$

$$\Rightarrow y = 2 - 2x$$

As the region given in the figure is above the y - intercept's coordinates (0,2),

$$\Rightarrow y \geq 2 - 2x$$

$$\Rightarrow 2x + y \geq 2$$

Consider C:

Given line $x + 2y = 8$

$$\Rightarrow 2y = 8 - x$$

$$\Rightarrow y = 4 - \frac{x}{2}$$

As the region given in the figure is below the y - intercept's coordinates (0,4),

$$\Rightarrow y \leq 4 - \frac{x}{2}$$

$$\Rightarrow 2y \leq 8 - x$$

$$\Rightarrow x + 2y \leq 8$$

Consider D:

It is the region right side of the Y - axis.

It is $x \geq 0$.

All the results derived:

$$x - y \leq 1$$

$$2x + y \geq 2$$

$$x + 2y \leq 8$$

$$x \geq 0$$