Exercise 28f

Question 1.
Find the acute angle between the following planes :

(i) $\vec{r}\cdot\left(\hat{i}+\hat{j}-2\hat{k}\right)=5$ and $\vec{r}\cdot\left(2\hat{i}+2\hat{j}-\hat{k}\right)=9$

(ii) $\vec{r}\cdot\left(\hat{i}+2\hat{j}-\hat{k}\right)=6$ and $\vec{r}\cdot\left(2\hat{i}-\hat{j}-\hat{k}\right)+3=0$

(iii) $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=1$ and $\vec{r}\cdot\left(-\hat{i}+\hat{j}\right)=4$

(iv) $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+6\hat{k}\right)=8$ and $\vec{r}\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+7=0$

Answer:
To find the angle between two planes, we simply find the angle between the normal vectors of planes. So if n1 and n2 are normal vectors and θ is the angle between both then,

$$\cos\theta=\left|\frac{\vec{n}_1\overrightarrow{n_2}}{|\overrightarrow{n_1}||\vec{n}_2|}\right|$$

(i)On comparing with the standard equation of planes in vector form

$$\vec{r}.\vec{n}_1=d_1 \text{ and } \vec{r}.\vec{n}_2=d_2$$

$$\vec{n}_1=\hat{i}+\hat{j}-2\hat{k} \text{ and } \vec{n}_2=2\hat{i}+2\hat{j}-\hat{k}$$

Then

$$\cos\theta=\left|\frac{\left(\hat{i}+\hat{j}-2\hat{k}\right).\left(2\hat{i}+2\hat{j}-\hat{k}\right)}{\left|\hat{i}+\hat{j}-2\hat{k}\right|\left|2\hat{i}+2\hat{j}-\hat{k}\right|}\right|\Rightarrow\left|\frac{1.2+1.2+(-2).(-1)}{\left(\sqrt{1^2+1^2+(-2)^2}\right).\left(\sqrt{2^2+2^2+(-1)^2}\right)}\right|=\left|\frac{2+2+2}{\sqrt{1+1+4}\sqrt{4+4+1}}\right|$$

$$\Rightarrow\left|\frac{6}{\sqrt{6}.\sqrt{9}}\right|=\left|\frac{\sqrt{6}}{3}\right|$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{6}}{3}\right)$$

(ii) On comparing with the standard equation of planes in vector form

$$\vec{r}.\vec{n}_1 = d_1 \text{ and } \vec{r}.\vec{n}_2 = d_2$$

$$\vec{n}_1 = \hat{i} + \widehat{2j} - \hat{k} \text{ and } \vec{n}_2 = 2\hat{i} - \hat{j} - \hat{k}$$

Then

$$\cos\theta = \left|\frac{\left(\hat{i}+\widehat{2j}-\hat{k}\right).\left(2\hat{i}-\hat{j}-\hat{k}\right)}{\left|\hat{i}+2\hat{j}-\hat{k}\right|\left|2\hat{i}-\hat{j}-\hat{k}\right|}\right| \Rightarrow \left|\frac{1.2+2.(-1)+(-1).(-1)}{\left(\sqrt{1}^2+2^2+(-1)^2\right).\left(\sqrt{2^2+(-1)^2+(-1)^2}\right)}\right| = \left|\frac{2-2+1}{\sqrt{1}+4+1\sqrt{4}+1+1}\right|$$

$$\Rightarrow \left|\frac{1}{\sqrt{6}.\sqrt{6}}\right| = \left|\frac{1}{6}\right|$$

$$\theta = \cos^{-1}\left(\frac{1}{6}\right)$$

(iii) On comparing with the standard equation of planes in vector form

$$\vec{r}.\vec{n}_1 = d_1 \text{ and } \vec{r}.\vec{n}_2 = d_2$$

$$\vec{n}_1 = 2\hat{i} - 3\hat{j} + 4\hat{k} \text{ and } \vec{n}_2 = -\hat{i} + \hat{j}$$

Then

$$\cos\theta = \left|\frac{\left(2\hat{i}-3\hat{j}+4\hat{k}\right).\left(-\hat{i}+\hat{j}\right)}{\left|2\hat{i}-3\hat{j}+4\hat{k}\right|\left|-\hat{i}+\hat{j}\right|}\right| \Rightarrow \left|\frac{2.(-1)+(-3).1+4.0}{\left(\sqrt{2}^2+(-3)^2+4^2\right).\left(\sqrt{(-1)^2+1^2}\right)}\right| = \left|\frac{-2+(-3)}{\left(\sqrt{4}+9+16\right)\left(\sqrt{1}+1\right)}\right|$$

$$\Rightarrow \left|\frac{-5}{\sqrt{29}\sqrt{2}}\right| = \left|\frac{-5}{\sqrt{58}}\right|$$

$$\theta = \cos^{-1}\left(\frac{5}{\sqrt{58}}\right)$$

(iv)On comparing with the standard equation of planes in vector for

$$\vec{r}.\vec{n}_1 = d_1 \text{ and } \vec{r}.\vec{n}_2 = d_2$$

$$\vec{n}_1 = 2\hat{i} - 3\hat{j} + 6\hat{k} \text{ and } \vec{n}_2 = 3\hat{i} + 4\hat{j} - 12\hat{k}$$

Then

$$\cos\theta = \left|\frac{\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right).\left(3\hat{i} + 4\hat{j} - 12\hat{k}\right)}{\left|2\hat{i} - 3\hat{j} + 6\hat{k}\right|\left|3\hat{i} + 4\hat{j} - 12\hat{k}\right|}\right| \Rightarrow \left|\frac{2.3 + (-3).4 + 6.(-12)}{\left(\sqrt{2^2 + (-3)^2 + 6^2}\right).\left(\sqrt{3^2 + 4^2 + (-12)^2}\right)}\right|$$

$$= \left|\frac{6 + (-12) + (-72)}{\left(\sqrt{4 + 9 + 36}\right)\left(\sqrt{9 + 16 + 144}\right)}\right|$$

$$\Rightarrow \left|\frac{-78}{\sqrt{49}\sqrt{169}}\right| = \left|\frac{-78}{7.13}\right|$$

$$\theta = \cos^{-1}\left(\frac{6}{7}\right)$$

Question 2.
Show that the following planes are at right angles:

(i) $\vec{r} \cdot \left(4\hat{i} - 7\hat{j} - 8\hat{k}\right) = 5$ and $\vec{r} \cdot \left(3\hat{i} - 4\hat{j} + 5\hat{k}\right) + 10 = 0$

(ii) $\vec{r}\cdot\left(2\hat{i}+6\hat{j}+6\hat{k}\right)=13$ and $\vec{r}\cdot\left(3\hat{i}+4\hat{j}-5\hat{k}\right)+7=0$

Answer:
To show the right angle between two planes, we simply find the angle between the normal vectors of planes. So if n1 and n2 are normal vectors and θ is the angle between both then

$\cos\theta=\left|\dfrac{\vec{n}_1\overrightarrow{n_2}}{|\overrightarrow{n_1}||\vec{n}_2|}\right|$.for right angle θ=90°

Cos90°=0

$\vec{n}_1.\vec{n}_2=0$ (1)

(i)On comparing with standard equation

$$\vec{n}_1=4\hat{i}-7\hat{j}-8\hat{k}\text{ and }\vec{n}_2=3\hat{i}-4\hat{j}+5\hat{k}$$

$$\text{LHS}=\vec{n}_1.\vec{n}_2\Rightarrow\left(4\hat{i}-7\hat{j}-8\hat{k}\right).\left(3\hat{i}-4\hat{j}+5\hat{k}\right)=4.3+(-7).(-4)+(-8).5$$

$$\Rightarrow12+28-40=40-40\Rightarrow0=\text{RHS}$$

Hence proved planes at right angles.

(ii) On comparing with the standard equation of a plane

$$\vec{n}_1=2\hat{i}+6\hat{j}+6\hat{k}\text{ and }\vec{n}_2=3\hat{i}+4\hat{j}-5\hat{k}$$

$$\text{LHS}=\vec{n}_1.\vec{n}_2\Rightarrow\left(2\hat{i}+6\hat{j}+6\hat{k}\right).\left(3\hat{i}+4\hat{j}-5\hat{k}\right)=2.3+6.4+6.(-5)$$

$$\Rightarrow6+24-30=30-30\Rightarrow0=\text{RHS}$$

Hence proved planes at right angles.

Question 3.
Find the value of λ for which the given planes are perpendicular to each other:

(i) $\vec{r}\cdot\left(2\hat{i}-\hat{j}-\lambda\hat{k}\right)=7$ and $\vec{r}\cdot\left(3\hat{i}+2\hat{j}+2\hat{k}\right)=9$

(ii) $\vec{r}\cdot\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)=5$ and $\vec{r}\cdot\left(\hat{i}+2\hat{j}-7\hat{k}\right)+11=0$

Answer:
For planes perpendicular Cos90°=0

$\vec{n}_1.\vec{n}_2=0$ (1)

(i)On comparing with the standard equation of a plane

$$\vec{n}_1=2\hat{i}-\hat{j}-\lambda\hat{k}\text{ and }\vec{n}_2=3\hat{i}+2\hat{j}+2\hat{k}$$

$$\vec{n}_1.\vec{n}_2=\left(2\hat{i}-\hat{j}-\lambda\hat{k}\right).\left(3\hat{i}+2\hat{j}+2\hat{k}\right)=0$$

2.3 + (-1).2 + (-λ).2=0

6-2-2λ=0

2λ=4

λ=2

(ii) On comparing with the standard equation of a plane

$$\vec{n}_1=\lambda\hat{i}+2\hat{j}+3\hat{k}\text{ and }\vec{n}_2=\hat{i}+2\hat{j}-7\hat{k}$$

$$\vec{n}_1.\vec{n}_2=\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right).\left(\hat{i}+2\hat{j}-7\hat{k}\right)=0$$

λ.1 + 2.2 + 3.(-7)=0 λ + 4-21=0 λ=17

Question 4.

Find the acute angle between the following planes:

(i) 2X – y + z = 5 and x + y + 2z = 7

(ii) x + 2y + 2z = 3 and 2x - 3y + 6z = 8

(iii) x + y - z = 4 and x + 2y + z = 9

(iv) x + y - 2z = 6 and 2x - 2y + z = 11

Answer:
To find angle in Cartesian form, for standard equation of planes

$$A_1x + B_1y + C_1z + D_1 = 0 \text{ and } A_1x + B_2y + C_2z + D_2 = 0$$

$$\cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{\left(A_1^2 + B_1^2 + C_1^2\right)}\sqrt{\left(A_2^2 + B_2^2 + C_2^2\right)}}\right|$$

(i)On comparing with the standard equation of planes

$$A_1 = 2, B_1 = -1, C_1 = 1 \text{ and } A_2 = 1, B_2 = 1, C_2 = 2$$

$$\cos\theta = \left|\frac{2.1 + (-1).1 + 1.2}{\sqrt{2^2 + (-1)^2 + 1^2}\sqrt{1^2 + 1^2 + 2^2}}\right| \Rightarrow \left|\frac{2 + (-1) + 2}{\sqrt{4 + 1 + 1}\sqrt{1 + 1 + 4}}\right| = \left|\frac{3}{\sqrt{6}\sqrt{6}}\right|$$

$$= \frac{3}{6} \Rightarrow \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow \frac{\pi}{3}$$

(ii)On comparing with the standard equation of planes

$$A_1 = 1, B_1 = 2, C_1 = 2 \text{ and } A_2 = 2, B_2 = -3, C_2 = 6$$

$$\cos\theta = \left|\frac{1.2+2.(-3)+2.6}{\sqrt{1^2+2^2+2^2}\sqrt{2^2+(-3)^2+6^2}}\right| \Rightarrow \left|\frac{2+(-6)+12}{\sqrt{1+4+4}\sqrt{4+9+36}}\right| = \left|\frac{8}{\sqrt{9}\sqrt{49}}\right|$$

$$= \frac{8}{3.7} \Rightarrow \frac{8}{21}$$

$$\theta = \cos^{-1}\left(\frac{8}{21}\right)$$

(iii) On comparing with standard equation of planes

$$A_1 = 1, B_1 = 1, C_1 = -1 \text{ and } A_2 = 1, B_2 = 2, C_2 = 1$$

$$\cos\theta = \left|\frac{1.1+1.2+(-1).1}{\sqrt{1^2+1^2+(-1)^2}\sqrt{1^2+2^2+1^2}}\right| \Rightarrow \left|\frac{1+2+(-1)}{\sqrt{1+1+1}\sqrt{1+4+1}}\right| = \left|\frac{2}{\sqrt{3}\sqrt{6}}\right|$$

$$= \frac{\sqrt{2}}{3}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$$

(iv)On comparing with the standard equation of planes

$$A_1 = 1, B_1 = 1, C_1 = -2 \text{ and } A_2 = 2, B_2 = -2, C_2 = 1$$

$$\cos\theta = \left|\frac{1.2+1.(-2)+(-2).1}{\sqrt{1^2+1^2+(-2)^2}\sqrt{2^2+(-2)^2+1^2}}\right| \Rightarrow \left|\frac{2+(-2)+(-2)}{\sqrt{1+1+4}\sqrt{4+4+1}}\right| = \left|\frac{-2}{\sqrt{6}\sqrt{9}}\right|$$

$$= \frac{2}{\sqrt{6}.3}$$

$$\theta = \cos^{-1}\left(\frac{2}{3\sqrt{6}}\right)$$

Question 5.
Show that each of the following pairs of planes are at right angles:

(i) 3x + 4y - 5z = 7 and 2x + 6y + 6z + 7 = 0

(ii) x - 2y + 4z = 10 and 18x + 17y + 4z = 49

Answer:
To find angle in Cartesian form, for standard equation of planes

$$A_1x + B_1y + C_1z + D_1 = 0 \text{ and } A_1x + B_2y + C_2z + D_2 = 0 \ \cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{\left(A_1^2 + B_1^2 + C_1^2\right)}\sqrt{\left(A_2^2 + B_2^2 + C_2^2\right)}}\right|$$

For θ=90°, cos90°=0

$$A_1A_2 + B_1B_2 + C_1C_2 = 0$$

(i)On comparing with the standard equation of a plane

$$A_1 = 3, B_1 = 4, C_1 = -5 \text{ and } A_2 = 2, B_2 = 6, C_2 = 6$$

$$\text{LHS} = A_1A_2 + B_1B_2 + C_1C_2 \Rightarrow 3.2 + 4.6 + (-5).6 = 6 + 24 - 30$$

=0=RHS

Hence proved that the angle between planes is 90°.

(ii) On comparing with the standard equation of a plane

$$A_1 = 1, B_1 = -2, C_1 = 4 \text{ and } A_2 = 18, B_2 = 17, C_2 = 4$$

$$\text{LHS} = A_1A_2 + B_1B_2 + C_1C_2 \Rightarrow 1.18 + (-2).17 + 4.4 = 18 + (-34) + 16$$

=0=RHS

Hence proved that angle between planes is 90°.

Question 6.
Prove that the plane 2x + 2y + 4z = 9 is perpendicular to each of the planes x + 2y + 2z – 7 = 0 and 5x + 6y + 7z = 23.

Answer:
To show that planes are perpendicular

$$A_1A_2 + B_1B_2 + C_1C_2 = 0$$

Where A_1, B_1, C_1 are direction ratios of plane and A_2, B_2, C_2 are of other

plane.

2.1 + 2.2 + 4.2=2 + 4 + 8=14≠0

Hence, planes are not perpendicular.

Similarly for the other plane

2.5 + 2.6 + 2.7=10 + 12 + 14=36≠0

Hence, planes are not perpendicular.

Question 7.
Show that the planes 2x – 2y + 4z + 5 = 0 and 3x – 3y + 6z - 1 = 0 are parallel.

Answer:
To show that planes are parallel

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

On comparing with the standard equation of a plane

$$A_1 = 2, B_1 = -2, C_1 = 4 \text{and} A_2 = 3, B_2 = -3, C_2 = 6$$

$$\frac{A_1}{A_2}=\frac{2}{3},\frac{B_1}{B_2}=\frac{-2}{-3}\Rightarrow\frac{2}{3},\frac{C_1}{C_2}=\frac{4}{6}\Rightarrow\frac{2}{3}$$

So,

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=\frac{2}{3}$$

Hence proved that planes are parallel.

Question 8.
Find the value of λ for which the planes x – 4y + λz + 3 = 0 and 2x + 2y + 3z = 5 are perpendicular to each other.

Answer:
To find an angle in Cartesian form, for the standard equation of planes

$$A_1x+B_1y+C_1z+D_1=0\,\text{and}\,A_1x+B_2y+C_2z+D_2=0$$

$$\cos\theta=\left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{\left(A_1{}^2+B_1{}^2+C_1{}^2\right)}\sqrt{\left(A_2{}^2+B_2{}^2+C_2{}^2\right)}}\right|$$

For θ=90°, cos90°=0

$$A_1A_2+B_1B_2+C_1C_2=0$$

On comparing with the standard equation of the plane,

$$A_1=1,B_1=-4,C_1=\lambda\,\text{and}\,A_2=2,B_2=2,C_2=3$$

$$A_1A_2+B_1B_2+C_1C_2\Rightarrow 1.2+(-4).2+\lambda.3=0$$

2 + (-8) + 3λ=0

-6 + 3λ=0

λ=2

Question 9.
Write the equation of the plane passing through the origin and parallel to the plane 5x - 3y + 7z + 11 = 0.

Answer:
Let the equation of plane be

$$A_1x + B_1y + C_1z + D_1 = 0$$

Direction ratios of parallel planes are related to each other as

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = k\,(\text{constant})$$

Putting the values from the equation of a given parallel plane,

$$\frac{A_1}{5} = \frac{B_1}{-3} = \frac{C_1}{7} = k$$

$$A_1 = 5k, B_1 = -3k, C_1 = 7k$$

Putting in equation plane

$$5kx - 3ky + 7kz + D_1 = 0$$

As the plane is passing through (0,0,0), it must satisfy the plane,

$$5k.0 - 3k.0 + 7k.0 + D_1 = 0$$

$$D_1 = 0$$

5kx-3ky + 7kz=0

5x-3y + 7z=0

So, required equation of plane is 5x-3y + 7z=0.

Question 10.
Find the equation of the plane passing through the point (a, b, c) and parallel to the plane
$\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=2.$

Answer:
Let the equation of a plane

$\vec{r}.\left(x_1\hat{i}+y_1\hat{j}+z_1\hat{k}\right)=d$ (1)

Direction ratios of parallel planes are related to each other as

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=\lambda(\text{constant})$$

Putting the values from the equation of a given parallel plane,

$$\frac{x_1}{1}=\frac{x_1}{1}=\frac{z_1}{1}=\lambda$$

$$x_1=y_1=z_1=\lambda$$

Putting values in equation (1), we have

$\vec{r}.\left(\lambda\hat{i}+\lambda\hat{j}+\lambda\hat{k}\right)=d$ (2)

A plane passes through (a,b,c) then it must satisfy the equation of a plane

$$\left(a\hat{i}+b\hat{j}+c\hat{k}\right).\left(\lambda\hat{i}+\lambda\hat{j}+\lambda\hat{k}\right)=d$$

$$\lambda\left(a\hat{i}+b\hat{j}+c\hat{k}\right)\left(\hat{i}+\hat{j}+\hat{k}\right)=d$$

λ(a.1 + b.1 + c.1)=d

λ(a + b + c)=d

Putting value in equation (2)

$$\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right).\lambda=\lambda(a+\ b+\ c)$$

$$\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=a+\ b+\ c$$

So, required equtaion of plane is $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=a+\ b+\ c.$

Question 11.
Find the equation of the plane passing through the point (1, -2, 7) and parallel to the plane 5x + 4y - 11z = 6.

Answer:
Let the equation of plane be

$$A_1x+B_1y+C_1z+D_1=0$$

Direction ratios of parallel planes are related to each other as

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=k\,(\text{constant})$$

Putting the values from the equation of a given parallel plane,

$$\frac{A_1}{5}=\frac{B_1}{4}=\frac{C_1}{-11}=k$$

$$A_1=5k, B_1=4k, C_1=-11k$$

Putting in the equation of a plane

$$5kx+4ky-11kz+D_1=0$$

As the plane is passing through (1,-2,7), it must satisfy the plane,

$$5k.1+4k.(-2)-11k.7+D_1=0 \text{ (1)}$$

$$5k-8k-77k+D_1=0$$

$$D_1=80k$$

Putting value in equation (1), we have

$$5kx+4ky-11kz+80k=0$$

5x + 4y-11z + 80=0

So, the required equation of the plane is 5x + 4y-11z + 80=0.

Question 12.
Find the equation of the plane passing through the point A(-1, -1, 2) and perpendicular to each of the planes 3x + 2y - 3z = 1 and 5x – 4y + z = 5.

Answer:
Applying the condition of perpendicularity between planes

$$AA_1+BB_1+CC_1=0$$

Where A, B, C are direction ratios of plane and A_1, B_1, C_1 are of another plane.

$$3.A_1+2B_1-3C_1=0 \text{ (1)}$$

$$5.A_1-4B_1+C_1=0 \text{ (2)}$$

And plane passes through (-1,-1,2),

A(x + 1) + B(y + 1) + C(z-2)=0 (3)

On solving equation (1) and (2)

$$A=\frac{5B}{9} \text{ and } C=\frac{11B}{9}$$

Putting values in equation (3)

$$\frac{5B}{9}.(x+1)+B(y+1)+\frac{11B}{9}.(z-2)=0$$

B(5x + 5 + 9y + 9 + 11z-22)=0

5x + 9y + 11z-8=0

So, required equation of plane is 5x + 9y + 11z=8.

Question 13.
Find the equation of the plane passing through the origin and perpendicular to each of the planes x + 2y - z = 1 and 3x - 4y + z = 5.

Answer:
Applying condition of perpendicularity between planes,

$$AA_1+BB_1+CC_1=0$$

Where A, B, C are direction ratios of plane and A_1, B_1, C_1 are of other

plane.

$$1.A+2.B-1.C=0$$

$A+2B-C=0$ (1)

$$3.A-4.B+C=0$$

$3A-4B+C=0$ (2)

And plane passes through (0, 0, 0),

A(x-0) + B(y-0) + C(z-0)=0

Ax + By + Cz=0 (3)

On solving equation (1) and (2)

$$A = \frac{B}{2} \text{ and } C = \frac{5B}{2}$$

Putting values in equation(3)

$$\frac{B}{2}.x + By + \frac{5B}{2}.z = 0$$

B(x + 2y + 5z)=0

x + 2y + 5z=0

So, required equation of plane is x + 2y + 5z=0.

Question 14.
Find the equation of the plane that contains the point A(1, -1, 2) and is perpendicular to both the planes 3x + 3y – 2z = 5 and x + 2y - 3z = 8. Hence, find the distance of the point P(-2, 5, 5) from the plane obtained above.

Answer:
Applying condition of perpendicularity between planes,

$$AA_1 + BB_1 + CC_1 = 0$$

Where A, B, C are direction ratios of plane and A_1, B_1, C_1 are of other

plane.

$$3.A + 3.B - 2.C = 0$$

$3A + 3B - 2C = 0$ (1)

$$1.A + 2.B - 3C = 0$$

$A + 2B - 3C = 0$ (2)

And plane contains the point (1,-1,2),

A(x-1) + B(y + 1) + C(z-2)=0 (3)

On solving equation (1) and (2)

$$A = \frac{-5B}{7} \text{ and } C = \frac{3B}{7}$$

Putting values in equation (3)

$$\frac{-5B}{7}.(x-1)+B(y+1)+\frac{3B}{7}.(z-2)=0$$

$$B(-5(x-1)+7(y+1)+3(z-2))=0$$

-5x + 5 + 7y + 7 + 3z-6=0

-5x + 7y + 3z + 6=0

5x-7y-3z-6=0

For equation of plane Ax + By + Cz=D and point (x1,y1,z1), distance of a

point from a plane can be calculated as

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

$$\left|\frac{5.(-2)-7.5-3.5-6}{\sqrt{(5)^2+(-7)^2+(-3)^2}}\right| \Rightarrow \left|\frac{-10-35-15-6}{\sqrt{25+49+9}}\right| = \left|\frac{-66}{\sqrt{83}}\right| \Rightarrow \frac{66}{\sqrt{83}}$$

Question 15.
Find the equation of the plane passing through the points A(1, 1, 2) and B(2, -2, 2) and perpendicular to the plane 6x – 2y + 2z = 9.

Answer:
Plane passes through (1,1,2) and (2,-2,2),

A(x-1) + B(y-1) + C(z-2)=0 (1)

$A(x-2) + B(y + 2) + C(z-2)=0$ (2)

Subtracting (1) from (2),

$A(x-2-x + 1) + B(y + 2-y-1)=0$

$A-3B=0$ (3)

Now plane is perpendicular to $6x-2y + 2z=9$

$6A-2B + 2C=0$ (4)

Using (3) in (4)

$18A-2B + 2C=0$

$16B + 2C=0$

$C=-8B$

Putting values in equation (1)

$3B(x-1) + B(y + 2)-8B(z-2)=0$

$B(3x-3 + y + 2-8z + 16)=0$

$3x + y-8z + 15=0$

Question 16.
Find the equation of the plane passing through the points A(-1, 1, 1) and B(1, -1, 1) and perpendicular to the plane $x + 2y + 2z = 5$.

Answer:
Plane passes through (-1,1,1) and (1,-1,1),

$A(x + 1) + B(y-1) + C(z-1)=0$ (1)

$A(x-1) + B(y + 1) + C(z-1)=0$ (2)

Subtracting (1) from (2),

A(x-1-x-1) + B(y + 1-y + 1)=0

-2A + 2B=0

A=B (3)

Now plane is perpendicular to x + 2y + 2z=5

A + 2B + 2C=0 (4)

Using (3) in (4)

B + 2B + 2C=0

3B + 2C=0

$$C = \frac{-3}{2}B$$

Putting values in equation (1)

$$B(x+1)+B(y-1)+\frac{-3}{2}B(z-1)=0$$

B(2(x + 1) + 2(y-1)-3(z-1)=0

2x + 2y-3z + 2-2-3=0

2x + 2y-3z-3=0

Question 17.
Find the equation of the plane through the points A(3, 4, 2) and B(7, 0, 6) and perpendicular to the plane 2x – 5y = 15.

HINT: The given plane is 2x – 5y + 0z = 15

Answer:
Plane passes through (3,4,2) and (7,0,6),

A(x-3) + B(y-4) + C(z-2)=0 (1)

A(x-7) + B(y-0) + C(z-6)=0 (2)

Subtracting (1) from (2),

A(x-7-x + 3) + B(y-y + 4) + C(z-6-z + 2)=0

-4A + 4B-4C=0

A-B + C=0

B=A + C (3)

Now plane is perpendicular to 2x-5y=15

2A-5B=0 (4)

Using (3) in (4)

2A-5(A + C)=0

2A-5A-5C=0

-3A-5C=0

$$C = \frac{-3}{5}A$$

$$B = A + \frac{-3}{5}A \Rightarrow \frac{2}{5}A$$

Putting values in equation (1)

$$A\left(x-3\right)+\frac{2}{5}A\left(y-4\right)+\frac{-3}{5}A\left(z-2\right)=0$$

A(5(x-3) + 2(y-4)-3(z-2)=0

5x + 2y-3z-15-8 + 6=0

5x + 2y-3z-17=0

So, required equation of plane is 5x + 2y-3z-17=0.

Question 18.

Find the equation of the plane through the points A(2, 1, -1) and B(-1, 3, 4) and perpendicular to the plane x – 2y + 4z = 10. Also, show that the plane thus obtained contains the line

$$\vec{r} = \left(-\hat{i} + 3\hat{j} + 4\hat{k}\right) + \lambda\left(3\hat{i} - 2\hat{j} - 5\hat{k}\right)$$

Answer:

Plane passes through (2,1,-1) and (-1,3,4),

A(x-2) + B(y-1) + C(z + 1)=0 (1)

A(x + 1) + B(y-3) + C(z-4)=0 (2)

Subtracting (1) from (2),

A(x + 1-x + 2) + B(y-3-y + 1) + C(z-4-z-1)=0

3A-2B-5C=0 (3)

Now plane is perpendicular to x-2y + 4z=10

A-2B + 4C=0 (4)

Using (3) in (4)

2A-9C=0

$$C = \frac{2}{9}A$$

$$2B = A + 4.\frac{2}{9}A \Rightarrow \left(\frac{9+8}{9}\right)A = \frac{17}{9}A$$

$$B = \frac{17}{18}A$$

Putting values in equation (1)

$$A(x-2)+\frac{17}{18}A(y-1)+\frac{2}{9}A(z+1)=0$$

A(18(x-2) + 17(y-1) + 4(z + 1)=0

18x + 17y + 4z-36-17 + 4=0

18x + 17y + 4z-49=0

So, the required equation of plane is 18x + 17y + 4z-49=0

If plane contains $\vec{r}=-\hat{i}+3\hat{j}+4\hat{k}+(3\hat{i}-2\hat{j}-5\hat{k})$ then $(-1,3,4)$ satisfies plane and normal vector of plane is perpendicular to vector of line

LHS=18(-1) + 17.3 + 4.4-49

=-18 + 51 + 16-49

=-2 + 2=0=RHS

In vector form normal of plane

$$\vec{n}=18\hat{i}+17\hat{j}+4\hat{k}$$

LHS=18.3 + 17(-2) + 4.(-5)=54-34-20=0=RHS

Hence line is contained in plane.