Exercise 27c

Question 1.
Find the angle between each of the following pairs of lines:

$\vec{r} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j} - 5\hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

Answer:
Given – $\overrightarrow{L_1} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$

& $\overrightarrow{L_2} = \left(2\hat{i} - \hat{j} - 5\hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,-1,-2)

Direction ratios of L_2 = (3,-5,-4)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2+b^2+c^2} \times \sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$= \cos^{-1}\left(\frac{1 \times 3 + (-1) \times (-5) + (-2) \times (-4)}{\sqrt{1^2 + 1^2 + 2^2}\sqrt{3^2 + 5^2 + 4^2}}\right)$

$= \cos^{-1}\left(\frac{3 + 5 + 8}{\sqrt{6}\sqrt{50}}\right)$

$= \cos^{-1}\left(\frac{16}{5\sqrt{6}\sqrt{2}}\right)$

$= \mathbf{\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)}$

Question 2.
Find the angle between each of the following pairs of lines:

$\vec{r} = \left(3\hat{i} - 4\hat{j} + 2\hat{k}\right) + \lambda\left(\hat{i} + 3\hat{k}\right)$ and $\vec{r} = 5\hat{i} + \mu\left(-\hat{i} + \hat{j} + \hat{k}\right)$

Answer:
Given – $\overrightarrow{L_1} = \left(3\hat{i} - 4\hat{j} + 2\hat{k}\right) + \lambda\left(\hat{i} + 3\hat{k}\right)$

& $\overrightarrow{L_2} = (5\hat{i}) + \mu\left(-\hat{i} + \hat{j} + \hat{k}\right)$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,0,3)

Direction ratios of L_2 = (-1,1,1)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}}\right)$

The angle between the lines

$= \cos^{-1}\left(\frac{1 \times (-1) + 0 \times 1 + 3 \times 1}{\sqrt{1^2 + 0^2 + 3^2}\sqrt{1^2 + 1^2 + 1^2}}\right)$

$= \cos^{-1}\left(\frac{-1 + 3}{\sqrt{10}\sqrt{3}}\right)$

$= \cos^{-1}\left(\frac{2}{\sqrt{30}}\right)$

$= \mathbf{\cos^{-1}\left(\frac{\sqrt{30}}{15}\right)}$

Question 3.

Find the angle between each of the following pairs of lines:

$\vec{r} = \left(\hat{i} - 2\hat{j}\right) + \lambda\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ and $\vec{r} = 3\hat{k} + \mu\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

Answer:
Given – $\overrightarrow{L_1} = (\hat{i} - 2\hat{j}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$

& $\overrightarrow{L_2} = (3\hat{k}) + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (2,-2,1)

Direction ratios of L_2 = (1,2,-2)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2+b^2+c^2} \times \sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$= \cos^{-1}\left(\frac{2 \times 1 + (-2) \times 2 + 1 \times (-2)}{\sqrt{2^2 + 2^2 + 1^2}\sqrt{1^2 + 2^2 + 2^2}}\right)$

$= \cos^{-1}\left(\frac{2 - 4 - 2}{3 \times 3}\right)$

$= \mathbf{\cos^{-1}\left(-\frac{4}{9}\right)}$

Question 4.
Find the angle between each of the following pairs of lines:

$\frac{x-1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$ and $\frac{x+3}{3} = \frac{y-2}{5} = \frac{z+5}{4}$

Answer:

Given – $\overrightarrow{L_1} = \frac{x-1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

& $\overrightarrow{L_2} = \frac{x+3}{3} = \frac{y-2}{5} = \frac{z+5}{4}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,1,2)

Direction ratios of L_2 = (3,5,4)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times 3 + 1\times 5 + 2\times 4}{\sqrt{1^2+1^2+2^2}\sqrt{3^2+5^2+4^2}}\right)$$

$$= \cos^{-1}\left(\frac{3+5+8}{\sqrt{6}\times\sqrt{50}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)}$$

Question 5.
Find the angle between each of the following pairs of lines:

$$\frac{x-4}{4} = \frac{y+1}{4} = \frac{z-6}{5} \text{ and } \frac{x-5}{1} = \frac{2y+5}{-2} = \frac{z-3}{1}$$

Answer:
Given – $\overrightarrow{L_1} = \frac{x-4}{4} = \frac{y+1}{3} = \frac{z-6}{5}$

& $\overrightarrow{L_2} = \frac{x-5}{1} = \frac{y+5/2}{-1} = \frac{z-3}{1}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (4,3,5)

Direction ratios of L_2 = (1,-1,1)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{4\times 1+3\times(-1)+5\times 1}{\sqrt{4^2+3^2+5^2}\sqrt{1^2+1^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{4-3+5}{5\sqrt{2}\times\sqrt{3}}\right)$$

$$= \cos^{-1}\left(\frac{6}{5\sqrt{6}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{2\sqrt{6}}{15}\right)}$$

Question 6.
Find the angle between each of the following pairs of lines:

$$\frac{3-x}{-2}=\frac{y+5}{1}=\frac{1-z}{3} \text{ and } \frac{x}{3}=\frac{1-y}{-2}=\frac{z+2}{-1}$$

Answer:
Given – $\overrightarrow{L_1} = \frac{x-3}{2}=\frac{y+5}{1}=\frac{z-1}{-3}$

& $\overrightarrow{L_2} = \frac{x}{3}=\frac{y-1}{2}=\frac{z+2}{-1}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (2,1,-3)

Direction ratios of L_2 = (3,2,-1)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{2\times 3+1\times 2+(-3)\times(-1)}{\sqrt{2^2+1^2+3^2}\sqrt{3^2+2^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{6+2+3}{\sqrt{14}\times\sqrt{14}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{11}{14}\right)}$$

Question 7.
Find the angle between each of the following pairs of lines:

$\frac{x}{1}=\frac{z}{-1}, y=0$ and $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$

Answer:
Given – $\overrightarrow{L_1} = \frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$

& $\overrightarrow{L_2} = \frac{x}{3}=\frac{y}{4}=\frac{z}{5}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (1,0,-1)

Direction ratios of L_2 = (3,4,5)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times 3+0\times 4+(-1)\times 5}{\sqrt{1^2+0^2+1^2}\sqrt{3^2+4^2+5^2}}\right)$$

$$= \cos^{-1}\left(\frac{3-5}{5\sqrt{2}\times\sqrt{2}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{1}{5}\right)}$$

Question 8.
Find the angle between each of the following pairs of lines:

$\frac{5-x}{3}=\frac{y+3}{-2}, z=5$ and $\frac{x-1}{1}=\frac{1-y}{3}=\frac{z-5}{2}$

Answer:
Given – $\overrightarrow{L_1} = \frac{x-5}{-3}=\frac{y+3}{-2}=\frac{z-5}{0}$

& $\overrightarrow{L_2} = \frac{x-1}{1}=\frac{y-1}{-3}=\frac{z-5}{2}$

To find – Angle between the two pair of lines

Direction ratios of L_1 = (-3,-2,0)

Direction ratios of L_2 = (1,-3,2)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{(-3)\times 1+(-2)\times(-3)+0\times 2}{\sqrt{3^2+2^2+0^2}\sqrt{1^2+3^2+2^2}}\right)$$

$$= \cos^{-1}\left(\frac{-3+6}{\sqrt{13}\times\sqrt{14}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{3}{\sqrt{182}}\right)}$$

Question 9.

Show that the lines $\frac{x-3}{2}=\frac{y+1}{-3}=\frac{z-2}{4}$ and $\frac{x+2}{2}=\frac{y-4}{4}=\frac{z+5}{2}$ are perpendicular to each other.

Answer:

Given – $\overrightarrow{L_1} = \frac{x-3}{2}=\frac{y+1}{-3}=\frac{z-2}{4}$

& $\overrightarrow{L_2} = \frac{x+2}{2}=\frac{y-4}{4}=\frac{z+5}{2}$

To prove – The lines are perpendicular to each other

Direction ratios of L_1 = (2,-3,4)

Direction ratios of L_2 = (2,4,2)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{2\times 2+(-3)\times 4+4\times 2}{\sqrt{2^2+3^2+4^2}\sqrt{2^2+4^2+2^2}}\right)$$

$$= \cos^{-1}\left(\frac{4-12+8}{\sqrt{29}\times\sqrt{24}}\right)$$

$= \cos^{-1}(0)$

$= \frac{\pi}{2}$

Hence, **the lines are perpendicular to each other.**

Question 10.

If the lines $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{5}$ are perpendicular to each other then find the value of λ.

Answer:

Given – $\overrightarrow{L_1} = \frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$

& $\overrightarrow{L_2} = \frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{z-6}{-5}$

To find – The value of λ

Direction ratios of L_1 = (-3,2λ,2)

Direction ratios of L_2 = (3λ,1,-5)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

Since the lines are perpendicular to each other,

The angle between the lines

$\Rightarrow \cos^{-1}\left(\frac{(-3)\times 3\lambda + 2\lambda\times 1 + 2\times(-5)}{\sqrt{3^2+(2\lambda)^2+2^2}\sqrt{(3\lambda)^2+1^2+5^2}}\right) = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}\left(\frac{-9\lambda + 2\lambda - 10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right) = \frac{\pi}{2}$

$$\Rightarrow \cos^{-1}\left(\frac{-7\lambda-10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right)=\frac{\pi}{2}$$

$$\Rightarrow \left(\frac{-7\lambda-10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right)=\cos\frac{\pi}{2}=0$$

$$\Rightarrow -7\lambda-10=0$$

$$\Rightarrow \boldsymbol{\lambda=-\frac{10}{7}}$$

Question 11.
Show that the lines x = - y = 2z and x + 2 = 2y – 1 = - z + 1 are perpendicular to each other.

HINT: The given lines are $\frac{x}{2}=\frac{y}{-2}=\frac{z}{1}$ and $\frac{x+2}{1}=\frac{y-1/2}{1}=\frac{z-1}{-2}$.

Answer:
Given – $\overrightarrow{L_1} = \frac{x}{2}=\frac{y}{-2}=\frac{z}{1}$

& $\overrightarrow{L_2} = \frac{x+2}{2}=\frac{y-1/2}{1}=\frac{z-1}{-2}$

To prove – The lines are perpendicular to each other

Direction ratios of L_1 = (2,-2,1)

Direction ratios of L_2 = (2,1,-2)

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$=\cos^{-1}\left(\frac{2\times 2+(-2)\times 1+1\times(-2)}{\sqrt{2^2+2^2+1^2}\sqrt{1^2+1^2+2^2}}\right)$$

$$= \cos^{-1}\left(\frac{4-2-2}{\sqrt{29}\times\sqrt{24}}\right)$$

$$= \cos^{-1}(0)$$

$$= \frac{\pi}{2}$$

Hence, **the lines are perpendicular to each other.**

Question 12.
Find the angle between two lines whose direction ratios are

i. 2, 1, 2 and 4, 8, 1

ii. 5, -12, 13 and -3, 4, 5

iii. 1, 1, 2 and $\left(\sqrt{3}-1\right),\left(-\sqrt{3}-1\right),4$

iv. a, b, c and (b – c), (c – a), (a – b)

Answer:
(i): Given – Direction ratios of L_1 = (2,1,2) & Direction ratios of L_2 = (4,8,1)

To find – Angle between the two pair of lines

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{2\times 4+1\times 8+2\times 1}{\sqrt{2^2+1^2+2^2}\sqrt{4^2+8^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{8+8+2}{3\times 9}\right)$$

$$= \cos^{-1}\left(\frac{18}{27}\right)$$

$$= \cos^{-1}\left(\frac{2}{3}\right)$$

(ii): Given – Direction ratios of L_1 = (5,-12,13) & Direction ratios of L_2 = (-3,4,5)

To find – Angle between the two pair of lines

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{5\times(-3)+(-12)\times 4+13\times 5}{\sqrt{5^2+12^2+13^2}\sqrt{3^2+4^2+5^2}}\right)$$

$$= \cos^{-1}\left(\frac{-15-48+65}{13\sqrt{2}\times 5\sqrt{2}}\right)$$

$$= \cos^{-1}\left(\frac{2}{130}\right)$$

$$= \cos^{-1}\left(\frac{1}{65}\right)$$

(iii) Given – Direction ratios of L_1 = (1,1,2) & Direction ratios of L_2 = ($\sqrt{3}$-1,-$\sqrt{3}$-1,4)

To find – Angle between the two pair of lines

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times(\sqrt{3}-1)+1\times(-\sqrt{3}-1)+2\times 4}{\sqrt{1^2+1^2+2^2}\sqrt{(\sqrt{3}-1)^2+(-\sqrt{3}-1)^2+4^2}}\right)$$

$$= \cos^{-1}\left(\frac{\sqrt{3}-1-\sqrt{3}-1+8}{\sqrt{6}\sqrt{24}}\right)$$

$$= \cos^{-1}\left(\frac{1}{2}\right)$$

$$= \mathbf{\frac{\pi}{3}}$$

(iv) Given – Direction ratios of L_1 = (a,b,c) & Direction ratios of L_2 = ((b-c),(c-a),(a-b))

To find – Angle between the two pair of lines

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{a\times(b-c)+b\times(c-a)+c\times(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}\right)$$

$$= \cos^{-1}\left(\frac{0}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}\right)$$

$$= \cos^{-1}(0)$$

$$= \mathbf{\frac{\pi}{2}}$$

Question 13.
If A(1, 2, 3), B(4, 5, 7), C(-4, 3, -6) and D(2, 9, 2) are four given points then find the angle between the lines AB and CD.

Answer:
Given -

A = (1,2,3)

B = (4,5,7)

C = (-4,3,-6)

D = (2,9,2)

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((4-1),(5-2),(7-3))

=(3,3,4)

Similarly, the direction ratios of the line CD can be given by

((2+4),(9-3),(2+6))

=(6,6,8)

To find – Angle between the two pair of lines AB and CD

Tip – If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{3\times 6+3\times 6+4\times 8}{\sqrt{3^2+3^2+4^2}\sqrt{6^2+6^2+8^2}}\right)$$

$$= \cos^{-1}\left(\frac{18+18+32}{\sqrt{34}\times 2\sqrt{34}}\right)$$

$$= \cos^{-1}\left(\frac{68}{2 \times 34}\right)$$

$$= \cos^{-1} 1$$

$$= \mathbf{0}$$