CBSE SAMPLE PAPER-02
Class – XI
MATHEMATICS

Time allowed: 3 hours,Maximum Marks: 100

General Instructions:
a) All questions are compulsory.
b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d) Use of calculators is not permitted.

Section A

1. Identify a function $f(x)$ so that $f(x).f(y) = f(x+y)$
Sol: $f(x) = a^x$
$f(y) = a^y$ $\begin{aligned} f(x).f(y) &= a^x.a^y \\ &= a^{x+y} = f(x).f(y) \end{aligned}$

2. If A = {(x, y) : y = ax, x $\in$ R} and B = {(x, y) : y = a-x, x $\in$ R} then what is $(A \cap B)$
Sol: When $x = 0, y = 1$ in both cases. Hence
$(A \cap B) = \{0, 1\}$

3. If R is a relation from a set A containing p elements to a set B containing q elements the find the number of subsets of $A \times B$
Sol: 2^{pq}

4. Check whether the given lines are parallel or perpendicular.
$ax - by + c = 0 \quad and \quad \frac{ax}{2} - \frac{by}{2} + d = 0$
Sol: They are parallel since

$$\begin{vmatrix} a & -b \\ \frac{a}{2} & \frac{-b}{2} \end{vmatrix} = 0$$

5. Find the area of the triangle whose vertices are (2,0),(5,3),(2,6)

Sol: Area of a triangle

$\frac{1}{2}\begin{vmatrix} 2-2 & 0-6 \\ 5-2 & 3-6 \end{vmatrix}$ = $\frac{1}{2}\begin{vmatrix} 0 & -6 \\ 3 & -3 \end{vmatrix}$

$= 9$

6. Write the equation of a circle with center (0,0) and radius 5.

Sol: $x^2 + y^2 = 25$

Section B

7. Solve $\cos 3x = -\frac{1}{2}$

Sol:

$$\cos 3x = \cos \frac{2\pi}{3}$$

$$3x = 2n\pi \pm \frac{2\pi}{3}$$

$$x = \frac{2n\pi}{3} \pm \frac{2\pi}{9}, n \in Z$$

8. Prove by mathematical induction that $1 + 2 + 3 + \ldots\ldots\ldots + n = \frac{n(n+1)}{2}$**.**

Sol: Let $P(n)$ be the statement given by

$1 + 2 + 3 + \ldots\ldots\ldots + n = \frac{n(n+1)}{2}$

$P(1) = \frac{1(1+1)}{2}$ = 1, True

Let it be true for n=m

1 + 2 + 3 + ... + m = $\frac{m(m+1)}{2}$

1 + 2 + 3 + ... + m + (m + 1) = $\frac{m(m+1)}{2}$ + (m + 1)

P(m + 1) = $\frac{m(m+1)}{2}$ + (m + 1)

P(m + 1) = $\frac{m^2+3m+2}{2}$

P(m + 1) = $\frac{(m+1)(m+2)}{2}$

Thus P(m) is true $\Rightarrow$ P(m + 1) is True

9. Find the square root of $\sqrt{-8i}$.

Sol: Let $\sqrt{z}$ = $\sqrt{-8i}$

$$\sqrt{z} = \pm\left\{\frac{\sqrt{|z| - Re(z)}}{\sqrt{2}}\right\} - i\left\{\frac{\sqrt{|z| - Re(z)}}{\sqrt{2}}\right\}, Im(z) < 0$$

$$\sqrt{-8i} = \pm\left\{\frac{\sqrt{8+0}}{\sqrt{2}} - i\frac{\sqrt{8-0}}{\sqrt{2}}\right\}, Im(z) < 0$$

$$= \pm(2 - 2i)$$

10. Solve the inequality $\frac{2x+5}{x-2} \geq 3$.

Sol: $\frac{2x+5}{x-2} - 3 \geq 0$

11. Find the value of x if 12Cx = 12Cx+4.

Sol: x + x + 4 = 12

2x = 8

x = 4

12. Three cars are there in a race. Car A is 3 times as likely to win as car B. Car B is twice as likely to win as car C. What is the probability of winning each car.

Sol: Let p be the probability of winning Car C,

$P(C)$

P(C) = p

P(B) = 2p

P(A) = 6p

P(A) + P(B) + P(C) = 1

p + 2p + 6p = 1

9p = 1

p = $\frac{1}{9}$

P(C) = $\frac{1}{9}$

P(B) = $\frac{2}{9}$

P(A) = $\frac{6}{9}$

13. If $f(x)$ is a function that contains 3 in its domain and range and satisfy the relation $f(f(x)).(1+f(x)) = -f(x)$ find f(3)

Sol: Let a satisfy the relation $f(a) = 3$

f(f(a)).(1 + f(a)) = -f(a)

f(3).(4) = - 3

f(3) = -$\frac{3}{4}$

14. If $\tan A = \frac{1}{3}$ and $\tan B = \frac{1}{2}$ prove that $\sin 2(A+B) = 1$.

Sol:

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{1}{3} + \frac{1}{2}}{1 - \frac{1}{3}.\frac{1}{2}} = 1$$

$$A + B = 45$$

$$2(A+B) = 90$$

$$\sin 90 = 1$$

15. Find two numbers such that their arithmetic mean is 15 and Geometric mean is 9 without using the identity $(\mathbf{a}+\mathbf{b})^2 = (\mathbf{a}-\mathbf{b})^2 + 4\mathbf{ab}$

Sol: Form a quadratic equation sum of whose roots are 30 and product of the roots is 81

x^2 - x(30) + 81 = 0

x^2 - 3x - 27x + 81 = 0

x(x - 3) - 27(x - 3)

(x - 3)(x - 27) = 0

Hence the numbers are 3 and 27

16. Let $f : R \rightarrow R$ be a function given by f(x) = x^2 + 2 find f-1(27)

Sol: Let $f : R \rightarrow R$ be a function given by f(x) = x^2 + 2 find f-1(27)

f(x) = x^2 + 2

x^2 + 2 = 27

x^2 = 25

x = ± 5

f-1(27) = {-5, 5}

17. Find the domain and range of the function $f(x) = \frac{x-a}{a+1-x}$ where a is a positive integer.

Sol: The function is defined for all values of x where the denominator is not equal to zero

$a + 1 - x \neq 0$

Hence domain = $R - \{(a + 1)\}$

Range of f

Let $y = f(x)$

$$y = \frac{x-a}{a+1-x}$$

$$(a+1)y - xy = x - a$$

$$x(y+1) = (a+1)y + a$$

$$x = \frac{(a+1)y+2}{y+1}$$

Range of $f = R - \{-1\}$

18. Find the limit of $\lim_{x \to 0} \frac{\sqrt{a+x} - \sqrt{a}}{x}$

Sol: Rationalize the numerator

$$\lim_{x \to 0} \frac{\sqrt{a+x} - \sqrt{a}}{x}$$

$$= \lim_{x \to 0} \frac{(\sqrt{a+x} - \sqrt{a})(\sqrt{a+x} + \sqrt{a})}{x(\sqrt{a+x} + \sqrt{a})}$$

$$= \lim_{x \to 0} \frac{x}{x(\sqrt{a+x} + \sqrt{a})}$$

$$= \frac{1}{2\sqrt{a}}$$

19. Find the sign and value of the expression $\sin 75^0 + \cos 75^0$

Sol:

$$\sin 75^0 + \cos 75^0$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}} \sin 75^0 + \frac{1}{\sqrt{2}} \cos 75^0\right.$$

$$= \sqrt{2}(\cos 45^0 \sin 75^0 + \sin 45^0 \cos 75^0)$$

$$= \sqrt{2} \sin(75^0 + 45^0)$$

$$= \sqrt{2} \sin 120^0$$

Hence sign is positive and value is $\frac{\sqrt{2}\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$

Section C

20. In how many ways can 3 students from Class 12, 4 from class 11, 4 from class 10 and 2 from class 9 be seated in a row so that those of the same classes sit together. Also find the number of ways they can be arranged in at a round table.

Sol: There are 4 groups and four groups can be arranged in 4! ways. Class 12 can be arranged in 3! ways , Class 11 can be arranged in 4! Class 10 can be arranged in 4!. Class 9 can be arranged in ways. Hence Total number of ways that they can be arranged in a row 4! $\times$ 3! $\times$ 4! $\times$ 4! $\times$ 2! = 165888

In a circular seating arrangement the four groups can be arranged only in 3! ways only.

Hence the total number of ways that they can be seated at a round table = 3! $\times$ 3! $\times$ 4! $\times$ 4! $\times$ 2! = 41472

21. A circle represented by the equation $(\mathbf{x}-\mathbf{a})^{2}+(\mathbf{y}-\mathbf{b})^{2}=\mathbf{r}^{2}$
This makes two complete revolutions along the positive direction of the x axis. Find the equation of the circle in the new position.
Sol: The new coordinates of the centre in the new position are

$$(a+4\pi r\,,b)$$

$$\{x-(a+4\pi r)\}^2+(y-b)^2=r^2$$

22. Show that the equation $\mathbf{x}^{2}+4\mathbf{y}^{2}+4\mathbf{x}+16\mathbf{y}+16=0$ represents an ellipse.
Sol:

$$x^2+4y^2+4x+16y+16=0$$
$$x^2+4x+4+4y^2+16y+16=4$$
$$(x+2)^2+4(y+2)^2=4$$
$$\frac{(x+2)^2}{2^2}+\frac{(y+2)^2}{1^2}=1$$

This equation represents an ellipse.

23. Calculate the mean deviation about the mean from the following data

xi	2	15	17	23	27
fi	12	6	12	9	5

Sol:

x_i	f_i	$f_i x_i$	$\|x_i - 15\|$	$f_i \|x_i - 15\|$
2	12	24	13	156
15	6	90	0	0
17	12	204	2	24
23	9	207	8	72
27	5	135	12	60
	$N = \Sigma f_i = 44$	$\Sigma f_i\, x_i = 660$		$f_i \Sigma \|x_i - 15\| = 312$

Mean = $\bar{X} = \frac{1}{N}(\Sigma f_i\, x_i)$

$= \frac{660}{44} = 15$

Mean Deviation = M.D = $\frac{1}{N}(\Sigma f_i\ |x_i - 15|)$

$= \frac{312}{44} = 7.0909$

24. If the ratio of the roots of the equation $x^2 + px + q = 0$ is the same as $x^2 + p_1 x + q_1 = 0$ then prove that $p^2 q_1 = p_1^2 q$

Sol: Let the ratios be a : b

x^2 + px + q = 0

aα + bβ = - p

aβ + bβ = - p1

a$\alpha\times$ bα = q

a$\beta \times b\beta$ = q1

(a + b)α = - p

(a + b)β = - p1

abα2 = q

abβ2 = q1

$\frac{(a+b)^2 \alpha^2}{(a+b)^2 \beta^2} = \frac{p^2}{p_1^2}$

$$\frac{\alpha^2}{\beta^2} = \frac{p^2}{p_1^2}$$
$$\frac{\alpha^2}{\beta^2} = \frac{q}{q_1}$$
$$\frac{p^2}{p_1^2} = \frac{q}{q_1}$$

$$p^2 q_1 = p_1^2 q$$

25. Prove that $a.a^{\frac{1}{2}}.a^{\frac{1}{4}}.a^{\frac{1}{8}}.......\infty = a^2$.

Sol:

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\infty$

$= \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}}$ = 2

$a.a^{\frac{1}{2}}.a^{\frac{1}{4}}.a^{\frac{1}{8}}.......\infty$ = a^2

26. In a survey of 700 students in a medical college 200 went for regular entrance coaching, 295 attended only correspondence coaching, 115 attended both regular and correspondence coaching. Find how many got admission without any entrance coaching.

Sol: It is given that

n (U) = 700 ,n(A) = 200 ,n(B) = 295 ,n(A⋂B) = 115

We need to find out

$$n(A' \cap B')$$

$$n(A' \cap B') = n(A \cup B)'$$
$$= n(U) - n(A \cup B)$$
$$= n(U) - \{n(A) + n(B) - n(A \cap B)\}$$
$$= 700 - \{200 + 295 - 115\}$$
$$= 320$$