Exercise 24

Question 1.

Find $\left(\vec{a} \times \vec{b}\right)$ and $\left|\vec{a} \times \vec{b}\right|$, when

$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

Answer:

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have

$\vec{a} = i - j + 2k$ and $\vec{b} = 2i + 3j - 4k$

$\Rightarrow a_1 = 1, a_2 = -1, a_3 = 2$ and $b_1 = 2, b_2 = 3, b_3 = -4$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = ((-1 \times -4) - 3 \times 2)i + (2 \times 2 - (-4) \times 1)j + (1 \times 3 - 2 \times (-1))k$

$\Rightarrow |a \times b| = \sqrt{(-2)^2 + 8^2 + 5^2}$

$\vec{a} \times \vec{b} = \left(-2\hat{i} + 8\hat{j} + 5\hat{k}\right)$ and $\left|\vec{a} \times \vec{b}\right| = \sqrt{93}$

Question 2.

Find $\left(\vec{a} \times \vec{b}\right)$ and $\left|\vec{a} \times \vec{b}\right|$, when

$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$

Answer:
$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 2i - j + 3k$ and $\vec{b} = 3i + 5j - 2k$

$\Rightarrow a_1 = 2, a_2 = -1, a_3 = 3$ and $b_1 = 3, b_2 = 5, b_3 = -2$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = ((-1 \times -2) - 5 \times 3)i + (3 \times 3 - (-2) \times 2)j + (2 \times 5 - 3 \times (-1))k$

$\Rightarrow |a \times b| = \sqrt{(-17)^2 + 13^2 + 7^2} = 13\sqrt{3}$

$\Rightarrow \vec{a} \times \vec{b} = (-17)i + (13)j + (7)k$

Question 3.
Find $\left(\vec{a} \times \vec{b}\right)$ and $\left|\vec{a} \times \vec{b}\right|$, when

$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

Answer:
$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = i - 7j + 7k$ and $\vec{b} = 3i - 2j + 2k$

$\Rightarrow a_1 = 1, a_2 = -7, a_3 = 7$ and $b_1 = 3, b_2 = -2, b_3 = 2$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = ((-7 \times 2) - (-2) \times 7)i + (7 \times 3 - 1 \times 2)j + ((-2) \times 1 - 3 \times (-7))k$

$\Rightarrow |a \times b| = \sqrt{(0)^2 + 19^2 + 19^2} = 19\sqrt{2}$

$\Rightarrow \vec{a} \times \vec{b} = (0)i + (19)j + (19)k$

Question 4.

Find $\left(\vec{a} \times \vec{b}\right)$ and $\left|\vec{a} \times \vec{b}\right|$, when

$\vec{a} = 4\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{k}$

Answer:

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 4i + j - 2k$ and $\vec{b} = 3i + 0j + k$

$\Rightarrow a_1 = 4, a_2 = 1, a_3 = -2$ and $b_1 = 3, b_2 = 0, b_3 = 1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (1 \times 1 - (0) \times -2)i + (-2 \times 3 - 1 \times 4)j + (4 \times 0 - 3 \times 1)k$

$\Rightarrow |a \times b| = \sqrt{1^2 + (-10)^2 + (-3)^2} = \sqrt{110}$

$\Rightarrow \vec{a} \times \vec{b} = i - 10j - 3k$

Question 5.
Find $\left(\vec{a} \times \vec{b}\right)$ and $\left|\vec{a} \times \vec{b}\right|$, when

$\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

Answer:
$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 3i + 4j + 0k$ and $\vec{b} = i + j + k$

$\Rightarrow a_1 = 3, a_2 = 4, a_3 = 0$ and $b_1 = 1, b_2 = 1, b_3 = 1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (4 \times 1 - 1 \times 0)i + (0 \times 1 - 1 \times 3)j + (3 \times 1 - 1 \times 4)k$

$\Rightarrow |a \times b| = \sqrt{4^2 + (-3)^2 + (-1)^2} = \sqrt{26}$

$\Rightarrow \vec{a} \times \vec{b} = 4i - 3j - k$

Question 6.
Find λ if $\left(2\hat{i} + 6\hat{j} + 14\hat{k}\right) \times \left(\hat{i} - \lambda\hat{j} + 7\hat{k}\right) = \vec{0}$.

Answer:
$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 2i + 6j + 14k$ and $\vec{b} = i - \lambda j + 7k$

$\Rightarrow a_1 = 2, a_2 = 6, a_3 = 14$ and $b_1 = 1, b_2 = \lambda, b_3 = 7$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (6 \times 7 - (-\lambda) \times 14)i + (14 \times 1 - 2 \times 7)j + (2 \times (-\lambda) - 1 \times 6)k$

$\Rightarrow \vec{a} \times \vec{b} = 0i + 0j + 0k$

$\Rightarrow 42 + 14\lambda = 0,$

$\Rightarrow \lambda = -3$

Question 7.

If $\vec{a} = \left(-3\hat{i} + 4\hat{j} - 7\hat{k}\right)$ and $\vec{b} = \left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$, find $\left(\vec{a} \times \vec{b}\right)$.

Verify that (i) $\vec{a}$ and $\left(\vec{a} \times \vec{b}\right)$ are perpendicular to each other

and (ii) $\vec{b}$ and $\left(\vec{a} \times \vec{b}\right)$ are perpendicular to each other.

Answer:

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = -3i + 4j - 7k$ and $\vec{b} = 6i + 2j - 3k$

$\Rightarrow a_1 = -3, a_2 = 4, a_3 = -7$ and $b_1 = 6, b_2 = 2, b_3 = -3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (4 \times (-3) - 2 \times (-7))i + ((-7) \times 6 - (-3) \times (-3))j + ((-3) \times 2 - 6 \times 4)k$

$\Rightarrow \vec{a} \times \vec{b} = 2i - 51j - 30k$

If $\vec{a}$ and $\vec{a} \times \vec{b}$ are perpendicular to each other then,

$\Rightarrow \vec{a}.\left(\vec{a} \times \vec{b}\right) = 0$

i.e.,

$\vec{a}.\left(\vec{a} \times \vec{b}\right) = (-6) - (204) + (210) = 0$

And in the similar way, we have,

$\vec{b}.\left(\vec{a} \times \vec{b}\right) = (12) - (102) + (90) = 0$

Hence proved.

Question 8.
Find the value of:

i. $\left(\hat{i} \times \hat{j}\right) \cdot \hat{k} + \hat{i} \cdot \hat{j}$ ii. $\left(\hat{j} \times \hat{k}\right) \cdot \hat{i} + \hat{j} \cdot \hat{k}$ iii. $\hat{i} \times \left(\hat{j} + \hat{k}\right) + \hat{j} \times \left(\hat{k} + \hat{i}\right) + \hat{k} \times \left(\hat{i} + \hat{j}\right)$

Answer:
i.

The value of $(i \times j).k + i.j$ is, ...As $i \times j = k$ and $i.j = 0$

$\Rightarrow (k).k + 0 = 1$

ii.

The value of $(j \times k).i + j.k$ is, … …$\text{As } j \times k = \; i \text{ and } j.k = 0$

$\Rightarrow (i).i + 0 = \; 1$

iii.

The value of $i \times (j + k) + j \times (k + i) + k \times (i + j)$ is, … …
$\text{As } i \times k = -j, i \times j = k, j \times k = i, j \times i = -k, k \times i = j, k \times j = -i$

$\Rightarrow k - j + i - k + j - i = 0$

Question 9.
Find the unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$ when

$\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$

Answer:
Let $\vec{r}$ be the vector which is perpendicular to $\vec{a} \;\&\; \vec{b}$ then we have,

$\vec{r} = k.(\vec{a} \times \vec{b})$ …where k is a scalor

Thus, we have r is a unit vector,

So,

We have,

$\vec{a} \times \vec{b} = \; (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 3i + j - 2k \text{ and } \vec{b} = 2i + 3j - k$

$\Rightarrow a_1 = 3, a_2 \; = \; 1, a_3 \; = -2 \text{ and } b_1 \; = 2, b_2 \; = 3, b_3 \; = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (1 \times -1 - 3 \times -2)i + (-2 \times 2 - (-1) \times 3)j + (3 \times 3 - 2 \times 1)k$$

$$\Rightarrow |a \times b| = \sqrt{(5)^2 + (-1)^2 + (7)^2} = 5\sqrt{3}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{5i - 1j + 7k}{5\sqrt{3}}$$

$$\Rightarrow \vec{r} = \pm\frac{5i - 1j + 7k}{5\sqrt{3}}$$

Question 10.
Find the unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$ when

$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

Answer:
Let $\vec{r}$ be the vector which is perpendicular to $\vec{a}$ & $\vec{b}$ then we have,

$\vec{r} = k.(\vec{a} \times \vec{b})$ ...where k is a scalar

Thus, we have r is a unit vector,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

have $\vec{a} = i - 2j + 3k$ and $\vec{b} = i + 2j - k$

$$\Rightarrow a_1 = 1, a_2 = -2, a_3 = 3 \text{ and } b_1 = 1, b_2 = 2, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-2 \times -1 - 2 \times 3)i + (3 \times 1 - (-1) \times 1)j + (1 \times 2 - (-2) \times 1)k$$

$$\Rightarrow |a \times b| = \sqrt{(-4)^2 + (4)^2 + (4)^2} = 4\sqrt{3}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{-4i+4j+4k}{4\sqrt{3}}$$

$$\Rightarrow \vec{r} = \pm\frac{-i+j+k}{\sqrt{3}}$$

Question 11.
Find the unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$ when

$\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{k}$

Answer:
Let $\vec{r}$ be the vector which is perpendicular to $\vec{a}$ & $\vec{b}$ then we have,

$\vec{r} = k.(\vec{a} \times \vec{b})$ ...where k is a scalar

Thus, we have r is a unit vector,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

have $\vec{a} = i + 3j - 2k$ and $\vec{b} = -i + 0j + 3k$

$\Rightarrow a_1 = 1, a_2 = 3, a_3 = -2$ and $b_1 = -1, b_2 = 0, b_3 = 3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (9 - 0)i + (2 - 3)j + (0 - (-3))k$

$\Rightarrow |a \times b| = \sqrt{(9)^2 + (-1)^2 + (3)^2} = \sqrt{91}$

$\Rightarrow \vec{a} \times \vec{b} = \frac{9i - j + 3k}{\sqrt{91}}$

$\Rightarrow \vec{r} = \pm \frac{9i - j + 3k}{\sqrt{91}}$

Question 12.
Find the unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$ when

$\vec{a} = 4\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - \hat{k}$

Answer:
Let $\vec{r}$ be the vector which is perpendicular to $\vec{a}$ & $\vec{b}$ then we have,

$\vec{r} = k.(\vec{a} \times \vec{b})$ ...where k is a scalar

Thus, we have r is a unit vector,

So,

We have,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 4i + 2j - k$ and $\vec{b} = i + 4j - k$

$\Rightarrow a_1 = 4, a_2 = 2, a_3 = -1$ and $b_1 = 1, b_2 = 4, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (2 \times -1 - (-1) \times 4)i + (-1 \times 1 - (-1) \times 4)j + (4 \times 4 - 1 \times 2)k$

$\Rightarrow |a \times b| = \sqrt{(2)^2 + (3)^2 + (14)^2} = \sqrt{209}$

$\Rightarrow \vec{a} \times \vec{b} = \frac{2i+3j+14k}{\sqrt{209}}$

$\Rightarrow \vec{r} = \pm \frac{2i+3j+14k}{\sqrt{209}}$

Question 13.
Find the unit vectors perpendicular to the plane of the vectors

$\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$

Answer:
Let $\vec{r}$ be the vector which is perpendicular to $\vec{a}$ & $\vec{b}$ then we have,

$\vec{r} = k.(\vec{a} \times \vec{b})$ ...where k is a scalar

Thus, we have r is a unit vector,

So,

We have,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 2i - 6j - 3k$ and $\vec{b} = 4i + 3j - k$

$\Rightarrow a_1 = 2, a_2 = -6, a_3 = -3$ and $b_1 = 4, b_2 = 3, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (-6 \times (-1) - 3 \times (-3))i + (-3 \times 4 - (-1) \times 2)j + (2 \times 3 - 4 \times (-6))k$

$\Rightarrow |a \times b| = \sqrt{(15)^2 + (-10)^2 + (30)^2} = \sqrt{1225}$

$\Rightarrow \vec{a} \times \vec{b} = \frac{3i-2j+6k}{7}$

$$\vec{r} = \pm \frac{3i - 2j + 6k}{7}$$

Question 14.
Find a vector of magnitude 6 which is perpendicular to both the vectors

$\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$.

Answer:
Let $\vec{r}$ be the vector which is perpendicular to $\vec{a}$ & $\vec{b}$ then we have,

$\vec{r} = k.(\hat{a} \times \hat{b})$ ...where k is a scalar

Thus, we have r is vector of magnitude 6,

So,

We have,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 4i - j + 3k$ and $\vec{b} = -2i + j - 2k$

$\Rightarrow a_1 = 4, a_2 = -1, a_3 = 3$ and $b_1 = -2, b_2 = 1, b_3 = -2$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (-1 \times (-2) - 1 \times (3))i + (3 \times (-2) - (-2) \times 4)j + (4 \times 1 - (-2) \times (-1))k$

$\Rightarrow |a \times b| = \sqrt{(-1)^2 + (2)^2 + (2)^2} = 3$

$\Rightarrow \hat{a} \times \hat{b} = \frac{-i+2j+2k}{3}$

$\vec{r} = \pm k.\dfrac{-i + 2j + 2k}{3}$

Here, as r is of magnitude 6 thus,

k = 6,

Thus, $\vec{r} = \pm 2(-i + 2j + 2k)$

Question 15.
Find a vector of magnitude 5 units, perpendicular to each of the vectors

$\left(\vec{a} + \vec{b}\right)and\left(\vec{a} - \vec{b}\right)$, where $\vec{a} = \left(\hat{i} + \hat{j} + \hat{k}\right)$ and $\vec{b} = \left(\hat{i} + 2\,\hat{j} + 3\,\hat{k}\right)$

Answer:
$\vec{a} + \vec{b} = 2i + 3j + 4k = \vec{l}$

$\vec{a} - \vec{b} = 0i - i - 2k = \vec{m}$

Let $\vec{r}$ be the vector which is perpendicular to $\vec{l}$ & $\vec{m}$ then we have,

$\vec{r} = k.(\hat{l} \times \hat{m})$ ...where k is a scalar

Thus, we have r is vector of magnitude 5,

So,

We have,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{l} = 2i + 3j + 4k$ and $\vec{m} = 0i - j - 2k$

$\Rightarrow a_1 = 2, a_2 = 3, a_3 = 4$ and $b_1 = 0, b_2 = -1, b_3 = -2$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{l} \times \vec{m} = (-2)i + (4)j + (-2)k$

$\Rightarrow |a \times b| = \sqrt{(-2)^2 + (4)^2 + (-2)^2} = \sqrt{24}$

$\Rightarrow \hat{a} \times \hat{b} = \frac{-i+2j-k}{\sqrt{6}}$

$$\vec{r} = \pm k.\frac{-i + 2j - k}{\sqrt{6}}$$

Here, as r is of magnitude 5 thus,

k = 5,

Thus, $\vec{r} = \pm 5(\frac{-i + 2j - k}{\sqrt{6}})$

Question 16.
Find an angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and $\left|\vec{a} \times \vec{b}\right| = \sqrt{3}$.

Answer:
We are given that $\overrightarrow{|a|} = 1$ and $\overrightarrow{|b|} = 2$.

And $|\vec{a} \times \vec{b}| = \sqrt{3}$,

So we have,

$|\vec{a} \times \vec{b}| = \overrightarrow{|a|}.\overrightarrow{|b|}\sin\theta = \sqrt{3}$

$\Rightarrow \overrightarrow{|a|}.\overrightarrow{|b|}\sin\theta = 1 \times 2 \times \sin\theta$

$\Rightarrow 2\sin\theta = \sqrt{3}$

$\Rightarrow \theta = \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

Question 17.
If $\vec{a} = (\hat{i} - \hat{j})$, $\vec{b} = (3\hat{j} - \hat{k})$ and $\vec{c} = (7\hat{i} - \hat{k})$, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and for which $\vec{c} \cdot \vec{d} = 1$.

Answer:
Given that

Let $\vec{d}$ be the vector which is perpendicular to a & $\vec{b}$ then we have,

$\vec{d} = k.(\hat{a} \times \hat{b})$ ...where k is a scalar

We have,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = i - j$ and $\vec{b} = 0i + 3j - k$

$\Rightarrow a_1 = 1, a_2 = -1, a_3 = 0$ and $b_1 = 0, b_2 = 3, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (1)i + (1)j + (3)k$

$\Rightarrow |a \times b| = \sqrt{(1)^2 + (1)^2 + (3)^2} = \sqrt{11}$

$\Rightarrow \hat{a} \times \hat{b} = \frac{i+j+3k}{\sqrt{11}}$

$\vec{d} = \pm k.\frac{i + j + 3k}{\sqrt{11}}$

Given that $\vec{c}.\vec{d} = 1$

$\vec{c} = 7i - k$

$\Rightarrow \vec{c}.\vec{d} = \frac{7k-3k}{\sqrt{11}} = 1,$

$\Rightarrow k = \frac{\sqrt{11}}{4}$

$\Rightarrow \vec{d} = \frac{i+j+3k}{4}$

Question 18.

If $\vec{a} = \left(4\hat{i} + 5\hat{j} - \hat{k}\right)$, $\vec{b} = \left(\hat{i} - 4\hat{j} + \hat{k}\right)$ and $\vec{c} = \left(3\hat{i} + \hat{j} - \hat{k}\right)$, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and for which $\vec{c} \cdot \vec{d} = 21$.

Answer:
Given that

Let $\vec{d}$ be the vector which is perpendicular to $a \ \& \ \vec{b}$ then we have,

$\vec{d} = k.(\hat{a} \times \hat{b})$ ...where k is a scalar

We have,

$\vec{a} \times \vec{b} = \ (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 4i + 5j - k$ and $\vec{b} = i - 4j + k$

⇒ $a_1 = 4, a_2 = 5, a_3 = -1$ and $b_1 = 1, b_2 = -4, b_3 = 1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

⇒ $\vec{a} \times \vec{b} = (1)i + (-5)j + (-21)k$

⇒ $|a \times b| = \sqrt{(1)^2 + (-5)^2 + (-21)^2} = \sqrt{467}$

⇒ $\hat{a} \times \hat{b} = \frac{i-5j-21k}{\sqrt{467}}$

$$\vec{d} = \pm k.\frac{i - 5j - 21k}{\sqrt{467}}$$

Given that $\vec{c}.\vec{d} = 21$

$\vec{c} = 3i + j - k$

$$\Rightarrow \vec{c}.\vec{d} = \frac{19k}{\sqrt{467}} = 21,$$

$$\Rightarrow k = \frac{\sqrt{467}}{19 \times 21}$$

$$\vec{d} = \frac{i - 5j - 21k}{319} \times \sqrt{467}$$

Question 19.
Prove that $\left|\vec{a} \times \vec{b}\right| = \left(\vec{a} \cdot \vec{b}\right) \tan \theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$.

Answer:
We know that $|\vec{a}.\vec{b}| = ||\vec{a}||\vec{b}|\cos\theta|$

And $|\vec{a} \times \vec{b}| = ||\vec{a}||\vec{b}|\sin\theta|$

So,

$$\tan\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}.\vec{b}|}$$

Hence, proved.

Question 20.
Write the value of p for which $\vec{a} = \left(3\hat{i} + 2\hat{j} + 9\hat{k}\right)$ and $\vec{b} = \left(\hat{i} + p\hat{j} + 3\hat{k}\right)$ are parallel vectors.

Answer:
As the vectors are parallel vectors so, $\vec{a} \times \vec{b} = 0$

Thus,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

have $\vec{a} = 3i + 2j + 9k$ and $\vec{b} = i + pj + 3k$

$\Rightarrow a_1 = 3, a_2 = 2, a_3 = 9$ and $b_1 = 1, b_2 = p, b_3 = 3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (6 - 9p)i + (0)j + (3p - 2)k = 0$

$\Rightarrow 6 - 9p = 0$

$\Rightarrow$ Thus, p = $\frac{2}{3}$.

Question 21.
Verify that $\vec{a} \times \left(\vec{b} + \vec{c}\right) = \left(\vec{a} + \vec{b}\right) + \left(\vec{a} \times \vec{c}\right)$, when

$\vec{a} = \hat{i} - \hat{j} - 3\hat{k}$, $\vec{b} = 4\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$

Answer:
To verify $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})$

We need to prove L.H.S = R.H.S

L.H.S we have,

Given, $\vec{a} = \hat{i} - \hat{j} - 3\hat{k}\ \vec{b} = 4\hat{i} - 3\hat{j} + \hat{k}\ \vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{a} \times (\vec{b} + \vec{c}) = (i - j - 3k) \times (6i - 4j + 3k)$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = i - j - 3k$ and $\vec{b} + \vec{c} = 6i - 4j + 3k$

$\Rightarrow a_1 = 1, a_2 = -1, a_3 = -3$ and $b_1 = 6, b_2 = -4, b_3 = 3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \overrightarrow{(b + c)} = (-3 - 12)i + (3 + 18)j + (-4 + 6)k$

$\Rightarrow (-15)i + (21)j + (2)k$

RHS is

$(\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) = (-10i + 13j + k) + (-5i + 8j + k)$

$\Rightarrow (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) = (-15)i + (21)j + (2)k$

Thus, LHS = RHS.

Question 22.
Verify that $\vec{a} \times \left(\vec{b} + \vec{c}\right) = \left(\vec{a} + \vec{b}\right) + \left(\vec{a} \times \vec{c}\right)$, when

$\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$.

Answer:
To verify $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})$

We need to prove L.H.S = R.H.S

L.H.S we have,

Given, $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}\ \vec{b} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{i} - \hat{j} + \hat{k}$

$\vec{a} \times (\vec{b} + \vec{c}) = (4i - j + k) \times (2i + 0j + 2k)$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 4i - j + k$ and $\vec{b} + \vec{c} = 2i + 0j + 2k$

$\Rightarrow a_1 = 4, a_2 = -1, a_3 = 1$ and $b_1 = 2, b_2 = 0, b_3 = 2$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \overrightarrow{(b + c)} = (-2)i + (-2)j + (2)k$

$\Rightarrow (-2)i + (-2)j + (2)k$

RHS is

$(\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) = (-2i - 3j + 5k) + (0i + j - 3k)$

$\Rightarrow (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) = (-2)i + (-2)j + (2)k$

Thus, LHS = RHS.

Question 23.
Find the area of the parallelogram whose adjacent sides are represented by the vectors:

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = -3\hat{i} - 2\hat{j} + \hat{k}$

Answer:
The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

Area = $|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = i + 2j + 3k$ and $\vec{b} = -3i - 2j + k$

$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3$ and $b_1 = -3, b_2 = -2, b_3 = 1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (8)i + (-10)j + (4)k$

$\Rightarrow |a \times b| = \sqrt{(8)^2 + (-10)^2 + (4)^2} = \sqrt{180}$

$\Rightarrow$ area $= 6\sqrt{5}$ sq units

Question 24.
Find the area of the parallelogram whose adjacent sides are represented by the vectors:

$\vec{a} = \left(3\hat{i} + \hat{j} + 4\hat{k}\right)$ and $\vec{b} = \left(\hat{i} - \hat{j} + \hat{k}\right)$

Answer:
The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

Area = $|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 3i + j + 4k$ and $\vec{b} = i - j + k$

$\Rightarrow a_1 = 3, a_2 = 1, a_3 = 4$ and $b_1 = 1, b_2 = -1, b_3 = 1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (5)i + (-1)j + (-4)k$

$\Rightarrow |a \times b| = \sqrt{(5)^2 + (-1)^2 + (-4)^2} = \sqrt{42}$

$\Rightarrow$ area $= \sqrt{42}$ sq units

Question 25.
Find the area of the parallelogram whose adjacent sides are represented by the vectors:

$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} - \hat{j}$

Answer:
The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

Area = $|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 2i + j + 3k$ and $\vec{b} = i - j + 0k$

$\Rightarrow a_1 = 2, a_2 = 1, a_3 = 3$ and $b_1 = 1, b_2 = -1, b_3 = 0$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (3)i + (3)j + (-3)k$

$\Rightarrow |a \times b| = \sqrt{(3)^2 + (3)^2 + (-3)^2} = 3\sqrt{3}$

$\Rightarrow$ $\text{area} = 3\sqrt{3}$ sq units

Question 26.
Find the area of the parallelogram whose adjacent sides are represented by the vectors:

$\vec{a} = 2\hat{i}$ and $\vec{b} = 3\hat{j}$

Answer:
The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

Area = $|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 2i + 0j + 0k$ and $\vec{b} = 0i + 3j + 0k$

$\Rightarrow a_1 = 2, a_2 = 0, a_3 = 0$ and $b_1 = 0, b_2 = 3, b_3 = 0$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (6)k$

$\Rightarrow |a \times b| = 6$

$\Rightarrow$ $\text{area} = 6$ sq units

Question 27.
Find the area of the parallelogram whose diagonal are represented by the vectors

$\vec{d}_1 = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{d}_2 = \hat{i} - 3\hat{j} + 4\hat{k}$

Answer:
The diagonals are $\vec{a} + \vec{b} = 3i + j - 2k$ & $\vec{a} - \vec{b} = i - 3j + 4k$

Thus, $\vec{a} = 2i - j + k, \vec{b} = i + 2j - 3k$

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

Area = $|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 2i - j + k$ and $\vec{b} = i + 2j - 3k$

⇒ $a_1 = 2, a_2 = -1, a_3 = 1$ and $b_1 = 1, b_2 = 2, b_3 = -3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

⇒ $\vec{a} \times \vec{b} = (3 - 2)i + 7j + (5)k$

⇒ $|a \times b| = \sqrt{(1)^2 + (7)^2 + (5)^2} = 5\sqrt{3}$

⇒

⇒ $\text{area} = 5\sqrt{3}$ sq units

Question 28.
Find the area of the parallelogram whose diagonal are represented by the vectors

$\vec{d}_1 = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{d}_2 = 3\hat{i} + 4\hat{j} - \hat{k}$

Answer:
The diagonals are $\vec{a} + \vec{b} = 2i - j + k$ & $\vec{a} - \vec{b} = 3i + 4j - k$

Thus, $\vec{a} = \frac{5}{2}i + \frac{3}{2}j$, $\vec{b} = -\frac{1}{2}i - \frac{5}{2}j + k$

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

Area = $|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have
, $\vec{a} = \frac{5}{2}i + \frac{3}{2}j$, $\vec{b} = -\frac{1}{2}i - \frac{5}{2}j + k$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = \left(\frac{3}{2}\right)i - \frac{5}{2}j + \left(-\frac{11}{2}\right)k$

$\Rightarrow |a \times b| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{5}{2}\right)^2 + \left(-\frac{11}{2}\right)^2} = \frac{1}{2}\sqrt{155}$

$\Rightarrow$

$\Rightarrow \text{area} = \frac{1}{2}\sqrt{155}$ sq units

Question 29.

Find the area of the parallelogram whose diagonal are represented by the vectors

$\vec{d}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{d}_2 = -\hat{i} + 2\hat{j}$.

Answer:

The diagonals are $\vec{a} + \vec{b} = i - 3j + 2k$ & $\vec{a} - \vec{b} = -i + 2j + 0k$

Thus, $\vec{a} = 0i - \frac{1}{2}j + k, \vec{b} = i - \frac{5}{2}j + k$

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

Area = $|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 0i - \frac{1}{2}j + k$ and $\vec{b} = i - \frac{5}{2}j + k$

$\Rightarrow a_1 = 0, a_2 = -\frac{1}{2}, a_3 = 1$ and $b_1 = 1, b_2 = -\frac{5}{2}, b_3 = 1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (2)i + 1j + \left(\frac{1}{2}\right)k$

$\Rightarrow |a \times b| = \sqrt{(2)^2 + (1)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{2}\sqrt{21}$

$\Rightarrow$

$\Rightarrow$ area $= \frac{\sqrt{21}}{2}$ sq units

Question 30.
Find the area of the triangle whose two adjacent sides are determined by the vectors

$\vec{a} = -2\hat{i} - 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - \hat{k}$

Answer:
The area of the triangle = $\frac{|\vec{a}\times\vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

Area = $\frac{|\vec{a}\times\vec{b}|}{2}$

$\vec{a}\times\vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = -2i + 0j - 5k$ and $\vec{b} = i - 2j - k$

$\Rightarrow a_1 = -2, a_2 = 0, a_3 = -5$ and $b_1 = 1, b_2 = -2, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a}\times\vec{b} = (8)i + (-10)j + (4)k$

$\Rightarrow |a \times b| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{165}$

$\Rightarrow$ area $= \frac{\sqrt{165}}{2}$ sq units

Question 31.

Find the area of the triangle whose two adjacent sides are determined by the vectors

$\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$.

Answer:

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

Area = $\frac{|\vec{a} \times \vec{b}|}{2}$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 3i + 4j + 0k$ and $\vec{b} = -5i + 7j + 0k$

$\Rightarrow a_1 = 3, a_2 = 4, a_3 = 0$ and $b_1 = -5, b_2 = 7, b_3 = 0$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (41)k$

$\Rightarrow |a \times b| = 41$

$\Rightarrow \text{area} = \frac{41}{2}$ sq units

Question 32.

Using vectors, find the area of ΔABC whose vertices are

A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5)

Answer:

Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = i + 2j + 3k$ and $\overrightarrow{AC} = 4j + 3k$

The area of the triangle = $\frac{|\vec{a}\times\vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

Area = $\frac{|\vec{a}\times\vec{b}|}{2}$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = i + 2j + 3k$ and $\overrightarrow{AC} = 4j + 3k$

$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3$ and $b_1 = 0, b_2 = 4, b_3 = 3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (-6)i + (-3)j + (4)k$

$\Rightarrow |a \times b| = \sqrt{(-6)^2 + (-3)^2 + (4)^2} = \sqrt{61}$

$\Rightarrow$ area $= \frac{\sqrt{61}}{2}$ sq units

Question 33.
Using vectors, find the area of ΔABC whose vertices are

A(1, 2, 3), B(2, –1, 4) and C(4, 5, Δ1) ((considering Δ1 as 1))

Answer:
Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = i - 3j + 1k$ and $\overrightarrow{AC} = 3i + 3j - 2k$

The area of the triangle = $\frac{|\vec{a}\times\vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

Area = $\frac{|\vec{a}\times\vec{b}|}{2}$

$\vec{a}\times\vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = i - 3j + k$ and $\overrightarrow{AC} = 3i + 3j - 2k$

$\Rightarrow a_1 = 1, a_2 = -3, a_3 = 1$ and $b_1 = 3, b_2 = 3, b_3 = -2$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a}\times\vec{b} = (3)i + (5)j + (12)k$

$\Rightarrow |a\times b| = \sqrt{(3)^2 + (5)^2 + (12)^2} = \sqrt{178}$

$\Rightarrow$ area $= \frac{\sqrt{178}}{2}$ sq units

Question 34.
Using vectors, find the area of ΔABC whose vertices are

A(3, –1, 2), B(1, –1, –3) and C(4, –3, 1)

Answer:
Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = -2i + 0j - 5k$ and $\overrightarrow{AC} = i - 2j - k$

The area of the triangle = $\frac{|\vec{a}\times\vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

Area = $\frac{|\vec{a}\times\vec{b}|}{2}$

$\vec{a}\times\vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = -2i - 5k$ and $\overrightarrow{AC} = i - 2j - k$

⇒ $a_1 = -2, a_2 = 0, a_3 = -5$ and $b_1 = 1, b_2 = -2, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

⇒ $\vec{a}\times\vec{b} = (-10)i + (-7)j + (4)k$

⇒ $|a \times b| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{165}$

⇒ $\text{area} = \frac{\sqrt{165}}{2}$ sq units

Question 35.
Using vectors, find the area of ΔABC whose vertices are

A(1, –1, 2), B(2, 1, –1) and C(3, –1, 2).

Answer:
Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = i + 2j - 3k$ and $\overrightarrow{AC} = 2i$

The area of the triangle = $\frac{|\vec{a}\times\vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

Area = $\frac{|\vec{a}\times\vec{b}|}{2}$

$\vec{a}\times\vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = i + 2j - 3k$ and $\overrightarrow{AC} = 2i$

⇒ $a_1 = 1, a_2 = 2, a_3 = 3$ and $b_1 = 0, b_2 = 4, b_3 = 3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

⇒ $\vec{a}\times\vec{b} = (-6) + (-4)k$

⇒ $|a\times b| = \sqrt{(-6)^2 + (-4)^2} = \sqrt{52}$

⇒ $\text{area} = \frac{\sqrt{52}}{2}$ sq units

Question 36.
Using vector method, show that the given points A, B, C are collinear:

A(3, −5, 1), B(−1, 0, 8) and C(7, −10, −6)

Answer:
Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = -4i + 5j + 7k$ and $\overrightarrow{AC} = 4i - 5j - 7k$

To prove that A, B, C are collinear we need to prove that

$\vec{a}\times\vec{b} = 0$.

So,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = i + 2j + 3k$ and $\overrightarrow{AC} = 4j + 3k$

$\Rightarrow a_1 = -4, a_2 = 5, a_3 = 7$ and $b_1 = 4, b_2 = -5, b_3 = -7$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (0)i + (0)j + (0)k$

$\Rightarrow |a \times b| = 0$

Question 37.
Using vector method, show that the given points A, B, C are collinear:

A(6, −7, −1), B(2, −3, 1) and C(4, −5, 0).

Answer:
Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = -4i + 4j + 2k$ and $\overrightarrow{AC} = -2i + 2j + k$

To prove that A, B, C are collinear we need to prove that

$\vec{a} \times \vec{b} = 0\cdot$

So,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = -4i + 4j + 2k$ and $\overrightarrow{AC} = -2i + 2j + k$

$\Rightarrow a_1 = -4, a_2 = 4, a_3 = 2$ and $b_1 = -2, b_2 = 2, b_3 = 1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (0)i + (0)j + (0)k$

$\Rightarrow |a \times b| = 0$

Thus, A, B and C are collinear.

Question 38.

Show that the point A, B, C with position vectors $\left(3\hat{i} - 2\hat{j} + 4\hat{k}\right)$, $\left(\hat{i} + \hat{j} + \hat{k}\right)$ and $\left(-\hat{i} + 4\hat{j} - 2\hat{k}\right)$ respectively are collinear.

Answer:

Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = -2i + 3j - 3k$ and $\overrightarrow{AC} = -4i + 6j - 6k$

To prove that A, B, C are collinear we need to prove that

$\vec{a} \times \vec{b} = 0\cdot$

So,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = -2i + 3j - 3k$ and $\overrightarrow{AC} = -4i + 6j - 6k$

⇒ $a_1 = -2, a_2 = 3, a_3 = -3$ and $b_1 = -4, b_2 = 6, b_3 = -6$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

⇒ $\vec{a} \times \vec{b} = (0)i + (0)j + (0)k$

⇒ $|a \times b| = 0$

Thus, A, B and C are collinear.

Question 39.
Show that the points having position vectors $\vec{a}, \vec{b}, \left(\vec{c} = 3\vec{a} - 2\vec{b}\right)$ are collinear, whatever be $\vec{a}, \vec{b}, \vec{c}$.

Answer:
Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = \vec{b} - \vec{a}$ and $\overrightarrow{AC} = \vec{c} - \vec{a} = 2\vec{a} + 2\vec{b}$

To prove that A, B, C are collinear we need to prove that

$\overrightarrow{AB} \times \overrightarrow{AC} = 0$.

So,

Here,

We

have $\overrightarrow{AB} = \vec{b} - \vec{a}$ and $\overrightarrow{AC} = 2\vec{a} + 2\vec{b}$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = (\overrightarrow{b} - \vec{a}) \times (2\vec{a} + 2\vec{b})$

$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = \vec{b} \times \overrightarrow{2a} + 0 - 0 - \vec{a} \times \overrightarrow{2b} = 0$

Thus, A, B and C are collinear.

Question 40.

Show that the points having position vector $\left(-2\vec{a} + 3\vec{b} + 5\vec{c}\right)$, $\left(\vec{a} + 2\vec{b} + 3\vec{c}\right)$ and $\left(7\vec{a} - \vec{c}\right)$ are collinear, whatever be $\vec{a}, \vec{b}, \vec{c}$.

Answer:

We have, $A = -2\vec{a} + 3\vec{b} + 5\vec{c}, B = \vec{a} + 2\vec{b} + 3\vec{c}, C = 7\vec{a} - \vec{c}$

Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = 3\vec{a} - \vec{b} - 2\vec{c}$ and $\overrightarrow{AC} = 9\vec{a} - 3\vec{b} - 6\vec{c}$

To prove that A, B, C are collinear we need to prove that

$\overrightarrow{AB} \times \overrightarrow{AC} = 0$.

So,

Here,

We

have

$\overrightarrow{AB} = 3\vec{a} - \vec{b} - 2\vec{c}$ and $\overrightarrow{AC} = 9\vec{a} - 3\vec{b} - 6\vec{c}$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = (3\vec{a} - \vec{b} - 2\vec{c}) \times (9\vec{a} - 3\vec{b} - 6\vec{c})$$

$$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = 0$$

Thus, A, B and C are collinear.

Question 41.
Find a unit vector perpendicular to the plane ABC, where the points A, B, C, are $(3, -1, 2)$, $(1, -1, -3)$ and $(4, -3, 1)$ respectively.

Answer:
A unit vector perpendicular to the plane ABC will be,

$$\pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

Through the vertices we get the adjacent vectors as,

$$\overrightarrow{AB} = -2i + 0j - 5k \text{ and } \overrightarrow{AC} = i - 2j - k$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

have $\overrightarrow{AB} = -2i + 0j - 5k \text{ and } \overrightarrow{AC} = i - 2j - k$

$$\Rightarrow a_1 = -2, a_2 = 0, a_3 = -5 \text{ and } b_1 = 1, b_2 = -2, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-10)i + (-7)j + (4)k$$

$\Rightarrow |a \times b| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{165}$

$\Rightarrow \text{unit vector} = \frac{-10i-7j+4k}{\sqrt{165}}$

Question 42.

If $\vec{a} = \left(\hat{i} + 2\,\hat{j} + 3\,\hat{k}\right)$ and $\vec{b} = \left(\hat{i} - 3\,\hat{k}\right)$ then find $\left|\vec{b} \times 2\,\vec{a}\right|$.

Answer:

$\vec{a} = i + 2j + 3k$ and $\vec{b} = i - 3k$

Then, $|\vec{b} \times \overrightarrow{2a}|$,

We have, $\vec{b} \times \vec{a} = (-2a_2.b_3 + 2b_2.a_3)i - (a_3.2b_1 - 2b_{3.}a_1)j - (a_1.2b_2 - 2b_1a_2)k$

Here,

We

have

$\vec{a} = i + 2j + 3k$ and $\vec{b} = i - 3k$

$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3 \text{ and } b_1 = 1, b_2 = 0, b_3 = -3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (-12)i + (12)j + (-4)k$

$\Rightarrow |a \times b| = \sqrt{(-12)^2 + (12)^2 + (-4)^2} = 4\sqrt{19}$

Question 43.

If $\left|\vec{a}\right| = 2$, $\left|\vec{b}\right| = 5$ and $\left|\vec{a} \times \vec{b}\right| = 8$, find $\vec{a} \cdot \vec{b}$.

Answer:

We have, $|\vec{a}|^2|\vec{b}|^2 = |\vec{a}\times\vec{b}|^2 + |\vec{a}.\vec{b}|^2$

So, $|\vec{a}.\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - |\vec{a}\times\vec{b}|^2$

$\Rightarrow |\vec{a}.\vec{b}|^2 = 10^2 - 8^2 = 6^2$

$\Rightarrow |\vec{a}.\vec{b}| = 6$

Question 44.
If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $(\vec{a}\times\vec{b}) = (3\hat{i}+2\hat{j}+6\hat{k})$, find the angle between $\vec{a}$ and $\vec{b}$.

Answer:
We have, $|\vec{a}|^2|\vec{b}|^2 = |\vec{a}\times\vec{b}|^2 + |\vec{a}.\vec{b}|^2$

$\Rightarrow \vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta$

$\Rightarrow |\vec{a}\times\vec{b}| = \sqrt{3^2+2^2+6^2} = 7$

$\Rightarrow 7 = 7\times 2\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \sin^{-1}\frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$