Exercise 28d

Question 1.
Show that the planes $2x - y + 6z = 5$ and $5x - 2.5y + 15z = 12$ are parallel.

Answer:
Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are either same or proportional to each other

Therefore ,

Plane 1 : - $2x - y + 6z = 5$

Normal vector (Plane 1) = $(2i - j + 6k)$ .....(1)

Plane 2 : - $5x - 2.5y + 15z = 12$

Normal vector (Plane 2) = $(5i - 2.5j + 15k)$ .....(2)

Multiply equation(1) by 5 and equation(2) by 2

Normal vector (Plane 1) = $5(2i - j + 6k)$

$= 10i - 5j + 30k$

Normal vector (Plane 2) = $2(5i - 2.5j + 15k)$

$= 10i - 5j + 30k$

Since, both normal vectors are same .Therefore both planes are parallel

Question 2.

Find the vector equation of the plane through the point $\left(3\hat{i}+4\hat{j}-\hat{k}\right)$ and parallel to the plane $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+5\hat{k}\right)+5=0.$

Answer:

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane r . (2i – 3j + 5k) + 5 = 0

Normal vector = (2i - 3j + 5k)

∴ Normal vector of required plane = (2i - 3j + 5k)

Equation of required plane r . (2i - 3j + 5k) = d

In cartesian form 2x – 3y + 5z = d

Plane passes through point (3,4, - 1) therefore it will satisfy it.

2(3) – 3(4) + 5(- 1) = d

6 – 12 - 5 = d

d = - 11

Equation of required plane r . (2i - 3j + 5k) = - 11

r . (2i - 3j + 5k) + 11 = 0

Question 3.
Find the vector equation of the plane passing through the point (a, b, b) and parallel to the plane $\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 2$.

There is a error in question ...... the point should be (a,b,c) instead of (a,b,b) to get the required answer.

Answer:
Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane r . (i + j + k) = 2

Normal vector = (i + j + k)

∴ Normal vector of required plane = (i + j + k)

Equation of required plane r . (i + j + k) = d

In cartesian form x + y + z = d

Plane passes through point (a,b,c) therefore it will satisfy it.

(a) + (b) + (c) = d

d = a + b + c

Equation of required plane r . (i + j + k) = a + b + c

Question 4.
Find the vector equation of the plane passing through the point (1, 1, 1) and parallel to the plane $\vec{r} \cdot \left(2\hat{i} - \hat{j} + 2\hat{k}\right) = 5.$

Answer:
Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane r . (2i - j + 2k) = 5

Normal vector = (2i - j + 2k)

∴ Normal vector of required plane = (2i - j + 2k)

Equation of required plane r . (2i - j + 2k) = d

In cartesian form 2x - y + 2z = d

Plane passes through point (1,1,1) therefore it will satisfy it.

2(1) - (1) + 2(1) = d

d = 2 – 1 + 2 = 3

Equation of required plane r . (2i - j + 2k) = 3

Question 5.
Find the equation of the plane passing through the point (1, 4, - 2) and parallel to the plane 2x – y + 3z + 7 = 0.

Answer:

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane $2x - y + 3z + 7 = 0$

Normal vector = $(2i - j + 3k)$

$\therefore$ Normal vector of required plane = $(2i - j + 3k)$

Equation of required plane $r . (2i - j + 3k) = d$

In cartesian form $2x - y + 3z = d$

Plane passes through point (1,4, - 2) therefore it will satisfy it.

$2(1) - (4) + 3(- 2) = d$

$d = 2 - 4 - 6 = - 8$

Equation of required plane $2x - y + 3z = - 8$

$2x - y + 3z + 8 = 0$

Question 6.
Find the equations of the plane passing through the origin and parallel to the plane $2x - 3y + 7z + 13 = 0$.

Answer:
Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane $2x – 3y + 7z + 13 = 0$

Normal vector = $(2i - 3j + 7k)$

∴ Normal vector of required plane = $(2i - 3j + 7k)$

Equation of required plane $r . (2i - 3j + 7k) = d$

In cartesian form $2x - 3y + 7z = d$

Plane passes through point (0,0,0) therefore it will satisfy it.

$2(0) - (0) + 3(0) = d$

$d = 0$

Equation of required plane $2x - 3y + 7z = 0$

Question 7.
Find the equations of the plane passing through the point (- 1, 0, 7) and parallel to the plane $3x – 5y + 4z = 11$.

Answer:
Formula : Plane = $r . (n) = d$

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane $3x - 5y + 4z = 11$

Normal vector = $(3i - 5j + 4k)$

$\therefore$ Normal vector of required plane = $(3i - 5j + 4k)$

Equation of required plane $r \cdot (3i - 5j + 4k) = d$

In cartesian form $3x - 5y + 4z = d$

Plane passes through point $(-1,0,7)$ therefore it will satisfy it.

$3(-1) - 5(0) + 4(7) = d$

$d = -3 + 28 = 25$

Equation of required plane $3x - 5y + 4z = 25$

Question 8.
Find the equations of planes parallel to the plane $x - 2y + 2z = 3$ which are at a unit distance from the point (1, 2, 3).

Answer:
Formula : Plane = $r \cdot (n) = d$

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same

Therefore ,

Parallel Plane $x - 2y + 2z - 3 = 0$

Normal vector = (i - 2j + 2k)

∴ Normal vector of required plane = (i - 2j + 2k)

Equation of required planes r . (i - 2j + 2k) = d

In cartesian form x – 2y + 2y = d

It should be at unit distance from point (1,2,3)

$$\text{Distance} = \frac{|(1\times1)+(2\times-2)+(3\times2)-(d)|}{\sqrt{(1)^2+(-2)^2+(2)^2}}$$

$$= \frac{|1-4+6-d|}{\sqrt{1+4+4}}$$

$$= \frac{|3-d|}{\sqrt{9}}$$

$$1 = \frac{\pm(3-d)}{3}$$

$$3 = \pm(3-d)$$

For + sign = > 3 = 3 - d = > d = 0

For - sign = > 3 = - 3 + d = > d = 6

Therefore equations of planes are : -

For d = 0 For d = 6

x – 2y + 2y = d x – 2y + 2y = d

x – 2y + 2y = 0 x – 2y + 2y = 6

Required planes = x – 2y + 2y = 0

x – 2y + 2y – 6 = 0

Question 9.
Find the distance between the planes x + 2y + 3z + 7 = 0 and 2x + 4y + 6z + 7 = 0.

Answer:
Formula : The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Plane 2:ax + by + cz + d2 = 0 is given by the formula

$$Distance = \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

where (d_1, d_2) are constants of the planes

Therefore ,

First Plane x + 2y + 3z + 7 = 0

2(x + 2y + 3z + 7) = 0

2x + 4y + 6z + 14 = 0 ...... (1)

Second plane 2x + 4y + 6z + 7 = 0 ...... (2)

Using equation (1) and (2)

Distance between both planes $= \frac{|7-(14)|}{\sqrt{(2)^2 + (4)^2 + (6)^2}}$

$$= \frac{|-7|}{\sqrt{4 + 16 + 36}}$$

$$= \frac{|-7|}{\sqrt{56}}$$

$$= \frac{7}{\sqrt{56}}\ units$$