Objective Questions

Question 1.
Mark (√) against the correct answer in the following:

$$\int_1^4 x\sqrt{x}\,dx = ?$$

A. 12.8

B. 12.4

C. 7

D. none of these

Answer:

$$y = \int_1^4 x\sqrt{x}\,dx$$

$$= \int_1^4 x^{\frac{3}{2}}\,dx$$

$$= \left(\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right)_1^4$$

$$= \frac{2}{5}\left(4^{\frac{5}{2}} - 1^{\frac{5}{2}}\right)$$

$$= \frac{2}{5}(32 - 1)$$

$$= \frac{62}{5}$$

=12.4

Question 2.

Mark (√) against the correct answer in the following:

$$\int_0^2 \sqrt{6x+4}\,dx = ?$$

A. $\frac{64}{9}$

B. 7

C. $\frac{56}{9}$

D. $\frac{60}{9}$

Answer:

$y = \int_0^2 \sqrt{6x+4}\,dx$

$$= \left(\frac{(6x+4)^{\frac{1}{2}+1}}{6\left(\frac{1}{2}+1\right)}\right)_0^2$$

$$= \frac{2}{6\times 3}\left(16^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)$$

$$= \frac{2}{6\times 3}(64-8)$$

$$= \frac{56}{9}$$

Question 3.

Mark (√) against the correct answer in the following:

$$\int_0^1 \frac{dx}{\sqrt{5x+3}} = ?$$

A. $\frac{2}{5}\left(\sqrt{8}-\sqrt{3}\right)$

B. $\frac{2}{5}\left(\sqrt{8}+\sqrt{3}\right)$

C. $\frac{2}{5}\sqrt{8}$

D. none of these

Answer:

$y = \int_0^1 \frac{dx}{\sqrt{5x+3}}$

$$= \left(\frac{(5x+3)^{\frac{-1}{2}+1}}{5\left(\frac{-1}{2}+1\right)}\right)_0^1$$

$$= \frac{2}{5}\left(8^{\frac{1}{2}} - 3^{\frac{1}{2}}\right)$$

$$= \frac{2}{5}(\sqrt{8}-\sqrt{3})$$

Question 4.

Mark (√) against the correct answer in the following:

$$\int_0^1 \frac{1}{\left(1+x^2\right)} dx = ?$$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. none of these

Answer:

$y = \int_0^1 \frac{1}{1+x^2} dx$

$= (\tan^{-1} x)_0^1$

$= \tan^{-1} 1 - \tan^{-1} 0$

$= \frac{\pi}{4} - 0$

$= \frac{\pi}{4}$

Question 5.
Mark (√) against the correct answer in the following:

$$\int_0^2 \frac{dx}{\sqrt{4-x^2}} = ?$$

A. 1

B. $\sin^{-1}\frac{1}{2}$

C. $\frac{\pi}{4}$

D. none of these

Answer:
$y = \int_0^2 \frac{dx}{\sqrt{4-x^2}}$

Use formula $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$

$y = \left(\sin^{-1}\frac{x}{2}\right)_0^2$

$= \sin^{-1} 1 - \sin^{-1} 0$

$= \frac{\pi}{2}$

Question 6.

Mark (√) against the correct answer in the following:

$$\int_{\sqrt{3}}^{\sqrt{8}} x\sqrt{1+x^2}\,dx = ?$$

A. $\frac{19}{3}$

B. $\frac{19}{6}$

C. $\frac{38}{3}$

D. $\frac{9}{4}$

Answer:

$y = \int_{\sqrt{3}}^{\sqrt{8}} x\sqrt{1+x^2}\,dx$

Let, $x^2 = t$

Differentiating both side with respect to t

$$2x\frac{dx}{dt} = 1$$

$$\Rightarrow xdx = \frac{1}{2}dt$$

At $x = \sqrt{3}$, $t = 3$

At $x = \sqrt{8}$, $t = 8$

$$y = \frac{1}{2}\int_{3}^{8} \sqrt{1+t}\,dt$$

$$= \frac{1}{2}\left(\frac{(1+t)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)}\right)_3^8$$

$$= \frac{1}{3}\left(9^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)$$

$$= \frac{1}{3}(27-8)$$

$$= \frac{19}{3}$$

Question 7.
Mark (√) against the correct answer in the following:

$$\int_0^1 \frac{x^3}{\left(1+x^8\right)} dx = ?$$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{8}$

D. $\frac{\pi}{16}$

Answer:
Let, $x^4 = t$

Differentiating both side with respect to t

$$4x^3 \frac{dx}{dt} = 1$$

$$\Rightarrow x^3 dx = \frac{1}{4} dt$$

At x = 0, t = 0

At x = 1, t = 1

$$y = \frac{1}{4}\int_0^1 \frac{1}{1+t^2}dt$$

$$= \frac{1}{4}(\tan^{-1} t)_0^1$$

$$= \frac{1}{4}(\tan^{-1} 1 - \tan^{-1} 0)$$

$$= \frac{\pi}{16}$$

Question 8.
Mark (√) against the correct answer in the following:

$$\int_1^e \frac{(\log x)^2}{x}dx = ?$$

A. $\frac{1}{3}$

B. $\frac{1}{3}e^3$

C. $\frac{1}{3}(e^3 - 1)$

D. none of these

Answer:
Let, log x = t

Differentiating both side with respect to t

$$\frac{1}{x}\frac{dx}{dt} = 1$$

$$\Rightarrow \frac{1}{x}\,dx = dt$$

At x = 1, t = 0

At x = e, t = 1

$$y = \int_0^1 t^2\,dt$$

$$= \left(\frac{t^3}{3}\right)_0^1$$

$$= \frac{1}{3}$$

Question 9.
Mark (√) against the correct answer in the following:

$$\int_{\pi/4}^{\pi/2} \cot x\,dx = ?$$

A. log 2

B. 2 log 2

C. $\frac{1}{2}\log 2$

D. none of these

Answer:

$$y = (\ln(\sin x))_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \ln(\sin\frac{\pi}{2}) - \ln(\sin\frac{\pi}{4})$$

$$= \ln 1 - \ln\frac{1}{\sqrt{2}}$$

$$=\frac{1}{2}\ln 2$$

Question 10.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/4} \tan^2 x\, dx = ?$$

A. $\left(1-\frac{\pi}{4}\right)$

B. $\left(1+\frac{\pi}{4}\right)$

C. $\left(1-\frac{\pi}{2}\right)$

D. $\left(1+\frac{\pi}{2}\right)$

Answer:

$$y = \int_0^{\frac{\pi}{4}} (\sec^2 x - 1)\, dx$$

$$= (\tan x - x)_0^{\frac{\pi}{4}}$$

$$= \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - (\tan 0 - 0)$$

$$= 1 - \frac{\pi}{4}$$

Question 11.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \cos^2 x\, dx = ?$$

A. $\frac{\pi}{2}$

B. π

C. $\frac{\pi}{4}$

D. 1

Answer:

$$y = \int_0^{\frac{\pi}{2}} \frac{1+\cos 2x}{2} dx$$

$$= \left(\frac{x}{2} + \frac{\sin 2x}{4}\right)_0^{\frac{\pi}{2}}$$

$$= \left(\frac{\frac{\pi}{2}}{2} + \frac{\sin \pi}{4}\right) - \left(\frac{0}{2} + \frac{\sin 0}{4}\right)$$

$$= \frac{\pi}{4}$$

Question 12.
Mark (√) against the correct answer in the following:

$$\int_{\pi/3}^{\pi/2} \operatorname{cosec} x \, dx = ?$$

A. $\frac{1}{2}\log 2$

B. $\frac{1}{2}\log 3$

C. - log 2

D. none of these

Answer:

$$y = (\ln(\operatorname{cosec} x - \cot x))_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$= \ln\left(\text{cosec}\,\frac{\pi}{2} - \cot\frac{\pi}{2}\right) - \ln\left(\text{cosec}\,\frac{\pi}{3} - \cot\frac{\pi}{3}\right)$

$= \ln(1-0) - \ln\left(\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right)$

$= \frac{1}{2}\log 3$

Question 13.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \cos^3 x\,dx = ?$$

A. 1

B. $\frac{3}{4}$

C. $\frac{2}{3}$

D. none of these

Answer:
$y = \int_0^{\frac{\pi}{2}} \cos x\,(1-\sin^2 x)\,dx$

Let, sin x = t

Differentiating both side with respect to t

$\text{Cos}\,x\frac{dx}{dt} = 1$

$\Rightarrow \cos x\,dx = dt$

At x = 0, t = 0

At x = $\frac{\pi}{2}$, t = 1

$$y = \int_0^1 1 - t^2 \; dt$$

$$= \left(t - \frac{t^3}{3}\right)_0^1$$

$$= 1 - \frac{1}{3}$$

$$= \frac{2}{3}$$

Question 14.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/4} \frac{e^{\tan x}}{\cos^2 x} dx = ?$$

A. (e – 1)

B. (e + 1)

C. $\left(\frac{1}{e} + 1\right)$

D. $\left(\frac{1}{e} - 1\right)$

Answer:
$y = \int_0^{\frac{\pi}{4}} e^{\tan x} sec^2 x \, dx$

Let, tan x = t

Differentiating both side with respect to t

$$sec^2 x \frac{dx}{dt} = 1$$

$$\Rightarrow \; sec^2 x \, dx = dt$$

At x = 0, t = 0

At x = $\frac{\pi}{4}$, t = 1

$$y = \int_0^1 e^t \, dt$$

$$= e^t\Big|_0^1$$

= $e^1 - e^0$

= e – 1

Question 15.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\cos x}{\left(1+\sin^2 x\right)} dx = ?$$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. π

D. none of these

Answer:
Let, sin x = t

Differentiating both side with respect to t

$$Cos\, x \frac{dx}{dt} = 1$$

$$\Rightarrow cos\, x\, dx = dt$$

At x = 0, t = 0

At x = $\frac{\pi}{2}$, t = 1

$$y = \int_0^1 \frac{1}{1+t^2} dt$$

$$= (tan^{-1} t)_0^1$$

= $\tan^{-1}1 - \tan^{-1}0$

= π/4

Question 16.
Mark (√) against the correct answer in the following:

$$\int_{1/\pi}^{2/\pi} \frac{\sin\left(1/x\right)}{x^2} dx = ?$$

A. 1

B. $\frac{1}{2}$

C. $\frac{3}{2}$

D. none of these

Answer:
Let, 1/x = t

Differentiating both side with respect to t

$$\frac{-1}{x^2}\frac{dx}{dt} = 1$$

$$\Rightarrow \frac{1}{x^2} dx = -dt$$

At x = 1/π, t = π

At x = 2/π, t = π/2

$$y = \int_{\pi}^{\frac{\pi}{2}} \sin t \; dt$$

$$= (-\cos t)_{\pi}^{\frac{\pi}{2}}$$

= 1

Question 17.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi} \frac{dx}{(1+\sin x)} = ?$$

A. $\frac{1}{2}$

B. 1

C. 2

D. 0

Answer:
$y = \int_0^{\pi} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx$

$$= \int_0^{\pi} \frac{1-\sin x}{\cos^2 x} dx$$

$$= \int_0^{\pi} \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} dx$$

$$= \int_0^{\pi} \sec^2 x \, dx - \int_0^{\pi} \frac{\sin x}{\cos^2 x} dx$$

Let, cos x = t

Differentiating both side with respect to t

$$-\sin x \frac{dx}{dt} = 1$$

$$\Rightarrow \sin x \, dx = -dt$$

At x = 0, t = 1

At x = π, t = -1

$$y = (\tan x)_0^{\pi} + \int_1^{-1} \frac{1}{t^2} dt$$

$$= (\tan \pi - \tan 0) + \left(\frac{t^{-1}}{-1}\right)_1^{-1}$$

=2

Question 18.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \left(\sqrt{\sin x} \cos x\right)^3 dx = ?$$

A. $\frac{2}{9}$

B. $\frac{2}{15}$

C. $\frac{8}{45}$

D. $\frac{5}{2}$

Answer:

$$y = \int_0^{\frac{\pi}{2}} sin^{\frac{3}{2}} x cos^3 x\, dx$$

$$y = \int_0^{\frac{\pi}{2}} sin^{\frac{3}{2}} x \cos x\, (1 - sin^2 x)\, dx$$

Let, sin x = t

Differentiating both side with respect to t

$$Cos\, x \frac{dx}{dt} = 1$$

⇒cos x dx=dt

At x = 0, t = 0

At x = π/2, t = 1

$$y = \int_0^1 t^{\frac{3}{2}} - t^{\frac{7}{2}}\, dt$$

$$= \left(\frac{t^{\frac{5}{2}}}{\frac{5}{2}} - \frac{t^{\frac{9}{2}}}{\frac{9}{2}} \right)_0^1$$

$$= \frac{2}{5} - \frac{2}{9}$$

$$= \frac{8}{45}$$

Question 19.
Mark (√) against the correct answer in the following:

$$\int_0^1 \frac{xe^x}{(1+x)^2} dx = ?$$

A. $\left(\frac{e}{2}-1\right)$

B. (e – 1)

C. e(e – 1)

D. none of these

Answer:

$y = \int_0^1 \frac{e^x(x+1-1)}{(1+x)^2}dx$

$$= \int_0^1 e^x\left(\frac{1}{1+x} - \frac{1}{(1+x)^2}\right)dx$$

Use formula $\int e^x(f(x) + f'(x))dx = e^x f(x)$

If $f(x) = \frac{1}{1+x}$

then $f'(x) = -\frac{1}{(1+x)^2}$

$$y = \left(\frac{e^x}{1+x}\right)_0^1$$

$$y = \frac{e}{2} - 1$$

Question 20.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx = ?$$

A. 0

B. $\frac{\pi}{4}$

C. $e^{\pi/2}$

D. $\left(e^{\pi/2} - 1\right)$

Answer:

$$y = \int_0^{\frac{\pi}{2}} e^x \left(\frac{1+\sin x}{2\cos^2\frac{x}{2}}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2\cos^2\frac{x}{2}} + \frac{\sin x}{2\cos^2\frac{x}{2}}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2\cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} e^x \left(\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right) dx$$

Use formula $\int e^x(f(x) + f'(x))dx = e^x f(x)$

If $f(x) = \tan\frac{x}{2}$ then $f'(x) = \frac{1}{2}\sec^2\frac{x}{2}$

$$y = \left(e^x \tan\frac{x}{2}\right)_0^{\frac{\pi}{2}}$$

$$= e^{\frac{\pi}{2}} \tan\frac{\frac{\pi}{2}}{2} - e^0 \tan\frac{0}{2}$$

$$= e^{\frac{\pi}{2}}$$

Question 21.

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/4} \sqrt{1+\sin 2x}\, dx = ?$$

A. 0

B. 1

C. 2

D. $\sqrt{2}$

Answer:

$y = \int_0^{\frac{\pi}{4}} \sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x}\, dx$

$$= \int_0^{\frac{\pi}{4}} \sin x + \cos x\, dx$$

$$= (-\cos x + \sin x)_0^{\frac{\pi}{4}}$$

$$= \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (-1 + 0)$$

y = 1

Question 22.

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \sqrt{1 + \cos 2x}\ dx = ?$$

A. $\sqrt{2}$

B. $\frac{3}{2}$

C. $\sqrt{3}$

D. 2

Answer:

$y = \int_0^{\frac{\pi}{2}} \sqrt{2\cos^2 x}\, dx$

$$= \int_0^{\frac{\pi}{2}} \sqrt{2} \cos x \, dx$$

$$= \sqrt{2}(\sin x)_0^{\frac{\pi}{2}}$$

=√2

Question 23.
Mark (√) against the correct answer in the following:

$$\int_0^1 \frac{(1-x)}{(1+x)} dx = ?$$

A. $\frac{1}{2}\log 2$

B. (2 log 2 + 1)

C. (2 log 2 – 1)

D. $\left(\frac{1}{2}\log 2 - 1\right)$

Answer:
$y = \int_0^1 \frac{1-x-1+1}{1+x} dx$

$$= \int_0^1 \frac{2}{1+x} - 1 \, dx$$

$$= (2\ln(1+x) - x)_0^1$$

= 2 ln 2 – 1

Question 24.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \sin^2 x \, dx = ?$$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer:

$y = \int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} dx$

$= \left(\frac{x}{2} - \frac{\sin 2x}{4}\right)_0^{\frac{\pi}{2}}$

$= \frac{\pi}{4} - \frac{\sin \pi}{4}$

$= \frac{\pi}{4}$

Question 25.

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/6} \cos x \cos 2x \, dx = ?$$

A. $\frac{1}{4}$

B. $\frac{5}{12}$

C. $\frac{1}{3}$

D. $\frac{7}{12}$

Answer:

$y = \int_0^{\frac{\pi}{6}} \cos x\,(1 - 2\sin^2 x)\,dx$

$$= \int_0^{\frac{\pi}{6}} \cos x - 2\cos x \sin^2 x\,dx$$

$$= (\sin x)_0^{\frac{\pi}{6}} - 2\int_0^{\frac{\pi}{6}} \cos x \sin^2 x\,dx$$

Let, sin x = t

Differentiating both side with respect to t

$$\mathrm{Cos}\,x\frac{dx}{dt} = 1$$

$$\Rightarrow \cos x\,dx = dt$$

At x = 0, t = 0

At x = π/6, t = 1/2

$$y = \sin\frac{\pi}{6} - \sin 0 - 2\int_0^{\frac{1}{2}} t^2\,dt$$

$$= \frac{1}{2} - 2\left(\frac{t^3}{3}\right)_0^{\frac{1}{2}}$$

$$= \frac{1}{2} - \frac{1}{12}$$

$$= \frac{5}{12}$$

Question 26.

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \sin x \sin 2x \, dx = ?$$

A. $\frac{2}{3}$

B. $\frac{3}{4}$

C. $\frac{5}{6}$

D. $\frac{3}{5}$

Answer:

$$y = \int_0^{\frac{\pi}{2}} \sin x (2 \sin x \cos x) \, dx$$

$$= 2 \int_0^{\frac{\pi}{2}} \sin^2 x \cos x \, dx$$

Let, sin x = t

Differentiating both side with respect to t

$$\text{Cos}\,x \frac{dx}{dt} = 1$$

$$\Rightarrow \cos x \, dx = dt$$

At x = 0, t = 0

At x = π/2, t = 1

$$y = 2\int_0^1 t^2\,dt$$

$$= 2\left(\frac{t^3}{3}\right)_0^1$$

$$= \frac{2}{3}$$

Question 27.

Mark (√) against the correct answer in the following:

$$\int_0^{\pi} (\sin 2x \cos 3x)\,dx = ?$$

A. $\frac{4}{5}$

B. $-\frac{4}{5}$

C. $\frac{5}{12}$

D. $-\frac{12}{5}$

Answer:

$y = \int_0^{\pi} (2\sin x \cos x)(4\cos^3 x - 3\cos x)\,dx$

Let, cos x = t

Differentiating both side with respect to t

$$-\sin x \frac{dx}{dt} = 1$$

$$\Rightarrow \sin x\,dx = -dt$$

At x = 0, t = 1

At x = π, t = -1

$$y = -\int_{1}^{-1} 8t^4 - 6t^2\,dt$$

$$= -\left(8\frac{t^5}{5} - 6\frac{t^3}{3}\right)_1^{-1}$$

$$= -\left[\left(\frac{-8}{5} + 2\right) - \left(\frac{8}{5} - 2\right)\right]$$

$$= -\frac{4}{5}$$

Question 28.
Mark (√) against the correct answer in the following:

$$\int_0^1 \frac{dx}{\left(e^x + e^{-x}\right)} = ?$$

A. $\left(1 - \frac{\pi}{4}\right)$

B. $\tan^{-1} e$

C. $\tan^{-1} e + \frac{\pi}{4}$

D. $\tan^{-1} e - \frac{\pi}{4}$

Answer:
$y = \int_0^1 \frac{e^x}{1+e^{2x}}dx$

Let $e^x = t$

Differentiating both side with respect to t

$$e^x \frac{dx}{dt} = 1$$

$$\Rightarrow e^x dx = dt$$

At x = 0, t = 1

At x = 1, t = e

$$y = \int_1^e \frac{1}{1+t^2} dt$$

$$= (\tan^{-1} t)_1^e$$

= $\tan^{-1}e - \tan^{-1}1$

= $\tan^{-1}e - \pi/4$

Question 29.
Mark (√) against the correct answer in the following:

$$\int_0^9 \frac{dx}{\left(1+\sqrt{x}\right)} = ?$$

A. (3 – 2 log 2)

B. (3 + 2 log 2)

C. (6 – 2 log 4)

D. (6 + 2 log 4)

Answer:
Let, $x = t^2$

Differentiating both side with respect to t

$$\frac{dx}{dt} = 2t$$

$$\Rightarrow dx = 2t\, dt$$

At x = 0, t = 0

At x = 9, t = 3

$$y = \int_0^3 \frac{2t}{1+t} dt$$

$$= 2\int_0^3 \frac{t+1-1}{1+t} dt$$

$$= 2\int_0^3 1 - \frac{1}{1+t} dt$$

$$= 2(t - \ln(1+t))_0^3$$

y = 2[(3 – ln 4) – (0 – ln 1)]

= 6 – 2 log 4

Question 30.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} x \cos x \, dx = ?$$

A. $\frac{\pi}{2}$

B. $\left(\frac{\pi}{2} - 1\right)$

C. $\left(\frac{\pi}{2} + 1\right)$

D. none of these

Answer:

Use integration by parts

$$\int \mathrm{I} \times \mathrm{II}\, dx = \mathrm{I} \times \int \mathrm{II}\ dx - \int \frac{d}{dx}\mathrm{I}\left(\int \mathrm{II}\, dx\right) dx$$

$$y = x\int_0^{\frac{\pi}{2}} \cos x\, dx - \int_0^{\frac{\pi}{2}} \frac{d}{dx}x\left(\int \cos x\, dx\right) dx$$

$$= (x \sin x)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x\, dx$$

$$= \frac{\pi}{2} - (-\cos x)_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} + (0 - 1)$$

$$= \frac{\pi}{2} - 1$$

Question 31.
Mark (√) against the correct answer in the following:

$$\int_0^1 \frac{dx}{\left(1 + x + x^2\right)} = ?$$

A. $\frac{\pi}{\sqrt{3}}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{3\sqrt{3}}$

D. none of these

Answer:
We have to convert denominator into perfect square

$$1 + x + x2 = x^2 + 2(x)\left(\frac{1}{2}\right) + \frac{1}{4} - \frac{1}{4} + 1$$

$$= \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$= \left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$y = \int_0^1 \frac{1}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx$$

Use formula $\int \frac{1}{x^2+a^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$

$$y = \left(\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\frac{x + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)_0^1$$

$$= \frac{2}{\sqrt{3}}\left(\tan^{-1}\frac{2}{\sqrt{3}}\left(\frac{3}{2}\right) - \tan^{-1}\frac{2}{\sqrt{3}}\left(\frac{1}{2}\right)\right)$$

$$= \frac{2}{\sqrt{3}}\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

$$= \frac{\pi}{3\sqrt{3}}$$

Question 32.

Mark (√) against the correct answer in the following:

$$\int_0^1 \sqrt{\frac{1-x}{1+x}} dx = ?$$

A. $\frac{\pi}{2}$

B. $\left(\frac{\pi}{2}-1\right)$

C. $\left(\frac{\pi}{2}+1\right)$

D. none of these

Answer:
Let, x = sin t

Differentiating both side with respect to t

$$\frac{dx}{dt} = \cos t \Rightarrow dx = \cos t \; dt$$

At x = 0, t = 0

At x = 1, t = π/2

$$y = \int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\sin t}{1+\sin t}} \cos t \, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\sin t}{1+\sin t} \times \frac{1-\sin t}{1-\sin t}} \cos t \, dt$$

$$= \int_0^{\frac{\pi}{2}} \frac{1-\sin t}{\cos t} \cos t \, dt$$

$$= \int_0^{\frac{\pi}{2}} 1 - \sin t \, dt$$

$$= (t + \cos t)_0^{\frac{\pi}{2}}$$

$$= \left(\frac{\pi}{2} + 0\right) - (0 + 1)$$

$$= \frac{\pi}{2} - 1$$

Question 33.
Mark (√) against the correct answer in the following:

$$\int_0^1 \frac{(1-x)}{(1+x)} dx = ?$$

A. (log 2 + 1)

B. (log 2 – 1)

C. (2 log 2 – 1)

D. (2 log 2 + 1)

Answer:
$$y = \int_0^1 \frac{1-x+1-1}{1+x} dx$$

$$= \int_0^1 \frac{2}{1+x} - 1\, dx$$

$$= (2\ln(1+x) - x)_0^1$$

= 2 log 2 – 1

Question 34.
Mark (√) against the correct answer in the following:

$$\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}} dx = ?$$

A. aπ

B. $\frac{a\pi}{2}$

C. 2 aπ

D. none of these

Answer:
Let, x = a sin t

Differentiating both side with respect to t

$$\frac{dx}{dt} = a\cos t \Rightarrow dx = a\cos t\ dt$$

At x = -a, t = - π/2

At x = a, t = π/2

$$y = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\frac{a - a\sin t}{a + a\sin t}}\, a\cos t\, dt$$

$$= a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\frac{1 - \sin t}{1 + \sin t} \times \frac{1 - \sin t}{1 - \sin t}}\cos t\, dt$$

$$= a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1 - \sin t}{\cos t}\cos t\, dt$$

$$= a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 - \sin t\, dt$$

$$= a(t + \cos t)_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= a\left[\left(\frac{\pi}{2} + 0\right) - \frac{\pi}{2} + 0\right)$$

= aπ

Question 35.
Mark (√) against the correct answer in the following:

$$\int_0^{\sqrt{2}} \sqrt{2-x^2}\,dx = ?$$

A. π

B. 2π

C. $\frac{\pi}{2}$

D. none of these

Answer:
Use formula $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$

$$y = \int_0^{\sqrt{2}} \sqrt{\left(\sqrt{2}\right)^2 - x^2}\,dx$$

$$= \left(\frac{x}{2}\sqrt{2 - x^2} + \frac{2}{2}\sin^{-1}\frac{x}{\sqrt{2}}\right)_0^{\sqrt{2}}$$

$$= \left(\frac{\sqrt{2}}{2}\sqrt{2-2} + \sin^{-1}\frac{\sqrt{2}}{\sqrt{2}}\right) - \left(0 + \sin^{-1}0\right)$$

$$= \frac{\pi}{2}$$

Question 36.
Mark (√) against the correct answer in the following:

$$\int_{-2}^{2} |x|\,dx = ?$$

A. 4

B. 3.5

C. 2

D. 0

Answer:
We know that

|x| = -x in [-2, 0)

|x| = x in [0, 2]

$$y = \int_{-2}^{0} |x|\,dx + \int_{0}^{2} |x|\,dx$$

$$= \int_{-2}^{0} -x\,dx + \int_{0}^{2} x\,dx$$

$$= \left(-\frac{x^2}{2}\right)_{-2}^{0} + \left(\frac{x^2}{2}\right)_{0}^{2}$$

y = 0 – (-2) + 2 – 0

= 4

Question 37.
Mark (√) against the correct answer in the following:

$$\int_{0}^{1} |2x - 1|\,dx = ?$$

A. 2

B. $\frac{1}{2}$

C. 1

D. 0

Answer:
We know that

|2x – 1| = -(2x – 1) in [0, 1/2)

|2x – 1| = (2x – 1) in [1/2, 1]

$$y = \int_0^{\frac{1}{2}} |2x - 1|\, dx + \int_{\frac{1}{2}}^{1} |2x - 1|\, dx$$

$$= \int_0^{\frac{1}{2}} -(2x - 1)\, dx + \int_{\frac{1}{2}}^{1} 2x - 1\, dx$$

$$= -(x^2 - x)_0^{\frac{1}{2}} + (x^2 - x)_{\frac{1}{2}}^{1}$$

$$= -\left[\left(\frac{1}{4} - \frac{1}{2}\right) - (0 - 0)\right] + \left[(1 - 1) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$y = \frac{1}{2}$$

Question 38.
Mark (√) against the correct answer in the following:

$$\int_{-2}^{1} |2x + 1| dx = ?$$

A. $\frac{5}{2}$

B. $\frac{7}{2}$

C. $\frac{9}{2}$

D. 0

Answer:
We know that

|2x + 1| = -(2x + 1) in [-2, -1/2)

|2x + 1| = (2x + 1) in [-1/2, 1]

$$y = \int_{-2}^{-\frac{1}{2}} |2x + 1|\,dx + \int_{-\frac{1}{2}}^{1} |2x + 1|\,dx$$

$$= \int_{-2}^{-\frac{1}{2}} -(2x + 1)\,dx + \int_{-\frac{1}{2}}^{1} 2x + 1\,dx$$

$$= -(x^2 + x)_{-2}^{-\frac{1}{2}} + (x^2 + x)_{-\frac{1}{2}}^{1}$$

$$= -\left[\left(\frac{1}{4} - \frac{1}{2}\right) - (4 - 2)\right] + \left[(1 + 1) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$y = \frac{9}{2}$$

Question 39.
Mark (√) against the correct answer in the following:

$$\int_{-2}^{1} \frac{|x|}{x} dx = ?$$

A. 3

B. 2.5

C. 1.5

D. none of these

Answer:
We know that

|x| = -x in [-2, 0)

|x| = x in [0, 1]

$$y = \int_{-2}^{0} \frac{|x|}{x} dx + \int_{0}^{1} \frac{|x|}{x} dx$$

$$= \int_{-2}^{0} \frac{-x}{x} dx + \int_{0}^{1} \frac{x}{x} dx$$

$$= \int_{-2}^{0} -1\, dx + \int_{0}^{1} 1\, dx$$

$$= (-x)_{-2}^{0} + (x)_{0}^{1}$$

= -(0 – (-2)) + (1 – 0)

= -1

Question 40.
Mark (√) against the correct answer in the following:

$$\int_{-a}^{a} x\,|\,x\,|\,dx = ?$$

A. 0

B. 2a

C. $\frac{2a^3}{3}$

D. none of these

Answer:
We know that

|x| = -x in [-a, 0) where a > 0

|x| = x in [0, a] where a > 0

$$y = \int_{-a}^{0} x|x|\, dx + \int_{0}^{a} x|x|\, dx$$

$$= \int_{-a}^{0} x(-x)\,dx + \int_{0}^{a} x(x)\,dx$$

$$= -\int_{-a}^{0} x^2\,dx + \int_{0}^{a} x^2\,dx$$

$$= -\left(\frac{x^3}{3}\right)_{-a}^{0} + \left(\frac{x^3}{3}\right)_{0}^{a}$$

$$= -\left(0-\left(\frac{-a^3}{3}\right)\right)+\left(\frac{a^3}{3}-0\right)$$

= 0

Question 41.
Mark (√) against the correct answer in the following:

$$\int_{0}^{\pi} |\cos x|\,dx = ?$$

A. 2

B. $\frac{3}{2}$

C. 1

D. 0

Answer:
Find the equivalent expression to |cos x| at 0≤x≤ π

In $0 \le x \le \frac{\pi}{2}$

=cos x

In $\frac{\pi}{2} \le x \le \pi$

=-cos x

$$\Rightarrow \int_{0}^{\frac{\pi}{2}} \cos x \, dx + \int_{\frac{\pi}{2}}^{\pi} -\cos x \, dx$$

$$\Rightarrow \sin\frac{\pi}{2} - \sin 0 - \cos\pi + \cos\frac{\pi}{2}$$

⇒1-0-(-1) +0=2

Question 42.
Mark (√) against the correct answer in the following:

$$\int_{0}^{2\pi} |\sin x| dx = ?$$

A. 2

B. 4

C. 1

D. none of these

Answer:
Find the equivalent expression to |sin x| at $0 \leq x \leq 2\pi$

In $0 \leq x \leq \pi$

|sin x| = sin x

In $\pi \leq x \leq 2\pi$

|sin x| = -sin x

$$\Rightarrow \int_{0}^{\pi} \sin x \, dx + \int_{\pi}^{2\pi} -\sin x dx$$
=-cos π-(-cos 0)+cos 2π-cos π

=-(-1)+1+1-(-1)

=2+2

=4

Question 43.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sin x}{(\sin x + \cos x)} dx = ?$$

A. π

B. $\frac{\pi}{2}$

C. 0

D. $\frac{\pi}{4}$

Answer:
We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$\therefore$ Here,$a = \frac{\pi}{2}$

$$f(x) = \frac{\sin x}{(\sin x + \cos x)}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$\frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos(\frac{\pi}{2} - x)} = \frac{\cos x}{\cos x + \sin x}$$

$$\therefore 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\cos x + \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore \ 2I = \frac{\pi}{2}$$

$$\therefore \ I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question 44.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\left(\sqrt{\cos x} + \sqrt{\sin x}\right)} dx = ?$$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. π

D. 0

Answer:
We know that,

$\therefore \ \int_0^a f(x) = \int_0^a f(a - x) = I$ ...(let)

∴ Here,

$a = \frac{\pi}{2}$;

$$f(x) = \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$$

$$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$\frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos(\frac{\pi}{2} - x)} + \sqrt{\sin(\frac{\pi}{2} - x)}} = \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore\ 2I = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question 45.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sin^4 x}{\left(\sin^4 x + \cos^4 x\right)} dx = ?$$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. 1

D. 0

Answer:
We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

∴ Here,

$a = \frac{\pi}{2}$;

$$f(x) = \frac{\sin^4 x}{\sin^4 x + \cos^4 x}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$\frac{\sin^4\left(\frac{\pi}{2} - x\right)}{\sin^4\left(\frac{\pi}{2} - x\right) + \cos^4\left(\frac{\pi}{2} - x\right)} = \frac{\cos^4 x}{\sin^4 x + \cos^4 x}$$

$$\therefore 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

Question 46.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\cos^{1/4} x}{\left(\sin^{1/4} x + \cos^{1/4} x\right)} dx = ?$$

A. 0

B. 1

C. $\frac{\pi}{4}$

D. none of these

Answer:
We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

∴ Here,

$a = \frac{\pi}{2}$;

$f(x) = \frac{\cos^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x}$

$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$\frac{\cos^{\frac{1}{4}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{1}{4}}}\left(\frac{\pi}{2} - x\right)\cos^{\frac{1}{4}}\left(\frac{\pi}{2} - x\right) = \sin^{\frac{1}{4}} x \sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x$

$$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x}{\sin^{\frac{1}{4}} x + \cos^{\frac{1}{4}} x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore\ 2I = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question 47.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sin^n x}{\left(\sin^n x + \cos^n x\right)} dx = ?$$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. 1

D. 0

Answer:
We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

∴ Here,

$$a = \frac{\pi}{2};$$

$$f(x) = \frac{\sin^n x}{\cos^n x + \sin^n x}$$

$$\therefore\ f(a - x) = f\left(\frac{\pi}{2} - x\right)$$

$$= \frac{\cos^n x}{\cos^n x + \sin^n x}$$

$$\therefore\ 2I = \int_0^{\frac{\pi}{2}} 1\, dx$$

$$\therefore\ 2I = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question 48.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx = ?$$

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. none of these

Answer:
We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

∴ Here,

$a = \frac{\pi}{2}$;

$f(x) = \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}}$

$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$= \frac{\sqrt{\tan x}}{\sqrt{\cot x} + \sqrt{\tan x}}$

$\therefore 2I = \int_0^{\frac{\pi}{2}} 1\, dx$

$\therefore 2I = \frac{\pi}{2}$

$\therefore I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

Question 49.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sqrt[3]{\tan x}}{\left(\sqrt[3]{\tan x} + \sqrt[3]{\cot x}\right)} dx = ?$$

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. π

Answer:
We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$$= \frac{\sqrt[3]{\tan x}}{\sqrt[3]{\cot x} + \sqrt[3]{\tan x}}$$

$$= \frac{\sqrt[3]{\frac{\sin x}{\cos x}}}{\sqrt[3]{\frac{\sin x}{\cos x}} + \sqrt[3]{\frac{\cos x}{\sin x}}}$$

$$= \frac{\sqrt[3]{\frac{\sin x}{\cos x}} * \left(\sqrt[3]{\sin x}\ \sqrt[3]{\cos x}\right)}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$$= \frac{\sin^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

∴ Here,

$a = \frac{\pi}{2}$;

$$f(x) = \frac{\sin^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$$= \frac{\cos^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x + \cos^{\frac{2}{3}} x}$$

$$\therefore\ 2I = \int_0^{\frac{\pi}{2}} 1dx$$

$$\therefore\ 2I = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question 50.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{1}{(1+\tan x)} dx = ?$$

A. 0

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. π

Answer:

$$\frac{1}{1+\tan x} = \frac{1}{1+\frac{\sin x}{\cos x}}$$

$$= \frac{1}{(\cos x + \sin x)\frac{1}{\cos x}}$$

$$= \frac{\cos x}{\cos x + \sin x}$$

So our integral becomes, $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x+\sin x} dx$

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

∴ Here,

$$a = \frac{\pi}{2}$$

$$f(x) = \frac{\sin x}{(\sin x + \cos x)}$$

$$\therefore f(a-x) = f\left(\frac{\pi}{2} - x\right)$$

$$= \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}$$

$$= \frac{\cos x}{\cos x + \sin x}$$

$$\therefore 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\cos x + \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

Question 51.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{1}{\left(1+\sqrt{\cot x}\right)} dx = ?$$

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. π

Answer:
So our integral becomes

$$\frac{1}{\sqrt[1]{\cot x}+1} = \frac{1}{\sqrt{\frac{\cos x}{\sin x}}+1}$$

$$= \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}$$

∴ Here,

$a = \frac{\pi}{2}$;

$$f(x) = \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}$$

$\therefore\ f(a-x) = f\left(\frac{\pi}{2} - x\right)$

$$= \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cos\left(\frac{\pi}{2} - x\right)} + \sqrt{\sin\left(\frac{\pi}{2} - x\right)}}$$

$$= \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore\ 2I = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question 52.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{1}{\left(1 + \tan^3 x\right)} dx = ?$$

A. $\frac{\pi}{4}$

B. 0

C. $\frac{\pi}{2}$

D. none of these

Answer:

$$\frac{1}{1+\tan^3 x} = \frac{\cos^3 x}{\sin^3 x+\cos^3 x}$$

∴ Here,

$$a = \frac{\pi}{2};$$

$$f(x) = \frac{\cos^3 x}{\sin^3 x+\cos^3 x}$$

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$$f(a-x) = \frac{\sin^3 x}{\sin^3 x+\cos^3 x}$$

$$\therefore 2I = \int_0^{\frac{\pi}{2}} 1dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question 53.

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sec^5 x}{\left(\sec^5 x + \operatorname{cosec}^5 x\right)} dx = ?$$

A. $\frac{\pi}{2}$

B. 0

C. $\frac{\pi}{4}$

D. π

Answer:
so our integral becomes,

$$\frac{\sec^5 x}{\sec^5 x + \operatorname{cosec}^5 x} = \frac{\frac{1}{\cos^5 x}}{\frac{1}{\cos^5 x} + \frac{1}{\sin^5 x}}$$

$$= \frac{\sin^5 x}{\sin^5 x + \cos^5 x}$$

Here $a = \frac{\pi}{2}$ and $f(x) = \frac{\sin^5 x}{\sin^5 x + \cos^5 x}$

$$f(a - x) = \frac{\cos^5 x}{\sin^5 x + \cos^5 x}$$

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a - x) = I$ ...(let)

$$\therefore 2I == \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore 2I = \frac{\pi}{2}$$

$\therefore\ I = \frac{\pi}{2.2}$

$= \frac{\pi}{4}$

Question 54.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\left(1+\sqrt{\cot x}\right)} dx = ?$$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. 0

D. 1

Answer:
So our integral becomes,

$$\frac{\sqrt{\cot x}}{1+\sqrt{\cot x}} = \frac{\sqrt{\frac{\cos x}{\sin x}}}{1+\sqrt{\frac{\cos x}{\sin x}}}$$

$$= \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}$$

We know that,

$\therefore\ \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

so, we know that,

∴ Here,

$a = \frac{\pi}{2}$;

$$f(a-x) = \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}}$$

$$\therefore\ f(x) = \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$\therefore\ 2I = \int_0^a f(x) + \int_0^a f(a-x)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore\ 2I = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question 55.
Mark (√) against the correct answer in the following:

$$\int_0^{\pi/2} \frac{\tan x}{(1+\tan x)} dx = ?$$

A. 0

B. 1

C. $\frac{\pi}{4}$

D. π

Answer:
So our integral becomes,

$$\frac{\tan x}{1+\tan x}=\frac{\sin x}{\cos x}\left(\frac{1}{1+\frac{\sin x}{\cos x}}\right)$$

$$=\frac{\sin x}{\sin x+\cos x}$$

We know that,

$\therefore\ \int_0^a f(x)=\int_0^a f(a-x)=I$ ...(let)

∴ Here,

$$a=\frac{\pi}{2}$$

$$f(x)=\frac{\sin x}{(\sin x+\cos x)}$$

$$\therefore\ f(a-x)=f\left(\frac{\pi}{2}-x\right)$$

$$=\frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}$$

$$=\frac{\cos x}{\cos x+\sin x}$$

$$\therefore\ 2I=\int_0^a f(x)+\int_0^a f(a-x)$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin x+\cos x}{\cos x+\sin x}dx$$

$$= \int_0^{\frac{\pi}{2}} 1 dx$$

$$\therefore \ 2I = \frac{\pi}{2}$$

$$\therefore \ I = \frac{\pi}{2.2}$$

$$= \frac{\pi}{4}$$

Question 56.
Mark (√) against the correct answer in the following:

$$\int_{-\pi}^{\pi} x^4 \sin x \, dx = ?$$

A. 2π

B. π

C. 0

D. none of these

Answer:
If f is an odd function,

$$\int_{-a}^{a} f(x) dx = 0$$

as, $\int_0^a f(x) dx = -\int_{-a}^0 f(x) dx$

here $f(x)=x^4\sin x$

we will see $f(-x)=(-x)^4\sin(-x)$

$=- x^4\sin x$

Therefore, f(x) is a odd function,

$$\int_{-\pi}^{\pi} x^4 \sin x dx = 0$$

Question 57.
Mark (√) against the correct answer in the following:

$$\int_{-\pi}^{\pi} x^3 \cos^3 x \, dx = ?$$

A. π

B. $\frac{\pi}{4}$

C. 2π

D. 0

Answer:
If f is an odd function,

$$\int_{-a}^{a} f(x)dx = 0$$

as, $\int_0^a f(x)\,dx = -\int_{-a}^{0} f(x)dx$

here f(x)=x^3 $\cos^3$ x

we will see f(-x)= $(-x)^3$ $\cos^3$(-x)

=$-x^3$ cos ^{3} x

Therefore, f(x) is a odd function,

$$\int_{-\pi}^{\pi} x^3 \cos^3 x = 0$$

Question 58.

Mark (√) against the correct answer in the following:

$$\int_{-\pi}^{\pi} \sin^5 x \, dx = ?$$

A. $\frac{3\pi}{4}$

B. 2π

C. $\frac{5\pi}{16}$

D. 0

Answer:
If f is an odd function,

$$\int_{-a}^{a} f(x)dx = 0$$

as, $\int_0^a f(x)dx = -\int_{-a}^0 f(x)dx$

$f(x)=\sin^5 x$

$f(-x)=\sin^5(-x)$

$=-\sin^5 x$

Therefore, f(x) is a odd function,

$$\int_{-\pi}^{\pi} \sin^5 x \, dx = 0$$

Question 59.
Mark (√) against the correct answer in the following:

$$\int_{-1}^{-2} x^3\left(1-x^2\right)dx = ?$$

A. $-\frac{40}{3}$

B. $\frac{40}{3}$

C. $\frac{5}{6}$

D. 0

Answer:

$$\int_{-1}^{-2} x^3(1-x^2)dx = \int_{-1}^{-2}(x^3-x^5)dx$$

$$= \left[\frac{x^4}{4} - \frac{x^6}{6}\right]$$

$$= \left[\frac{2^4}{4} - \frac{1^6}{4} - \frac{2^6}{6} + \frac{1^6}{6}\right]$$

$$= -\frac{27}{4}$$

Question 60.

Mark (√) against the correct answer in the following:

$$\int_{-a}^{a} \log\left(\frac{a-x}{a+x}\right)dx = ?$$

A. 2a

B. a

C. 0

D. 1

Answer:

If f is an odd function,

$$\int_{-a}^{a} f(x)dx = 0$$

as, $\int_0^a f(x)dx = -\int_{-a}^{0} f(x)dx$

$$f(x) \;= \log\left(\frac{a-x}{a+x}\right)$$

$$f(-x) \;= \log\frac{a-(-x)}{a-x}$$

$$= \log\frac{a+x}{a-x}$$

$$= -\log\frac{a-x}{a+x}$$

Hence it is a odd function

$$\int_{-a}^{a} \log\frac{a-x}{a+x} = 0$$

Question 61.
Mark (√) against the correct answer in the following:

$$\int_{-\pi}^{\pi} \left(\sin^{61} x + x^{123}\right) dx = ?$$

A. 2π

B. 0

C. $\frac{\pi}{2}$

D. 125π

Answer:
If f is an odd function,

$$\int_{-a}^{a} f(x)dx = 0$$

as, $\int_0^a f(x)dx = -\int_{-a}^0 f(x)dx$

$\sin^{61}x$ and $x^{123}$is an odd function,

so there integral is zero.

Question 62.
Mark (√) against the correct answer in the following:

$$\int_{-\pi}^{\pi} \tan x \, dx = ?$$

A. 2

B. $\frac{1}{2}$

C. -2

D. 0

Answer:
f(x)=tan x

f(-x) =tan(-x)

=-tan x

hence the function is odd,

therefore, I=0

Question 63.
Mark (√) against the correct answer in the following:

$$\int_{-1}^{1} \log\left(x + \sqrt{x^2+1}\right) dx = ?$$

A. $\log \frac{1}{2}$

B. log 2

C. $\frac{1}{2}\log 2$

D. 0

Answer:
By by parts,

$$\int \log\left(x+\sqrt{x^2+1}\right) = x\log\left(x+\sqrt{x^2+1}\right) - \int \frac{x}{\left(x+\sqrt{x^2+1}\right)\left(1+\frac{x}{\sqrt{x^2+1}}\right)}$$

$$= x\log(x+\sqrt{x^2+1}) - \int \frac{x}{\sqrt{x^2+1}} = x\log(x+\sqrt{x^2+1}) - \sqrt{x^2+1}$$

Question 64.
Mark (√) against the correct answer in the following:

$$\int_{-\pi/2}^{\pi/2} \cos x \, dx = ?$$

A. 0

B. 2

C. -1

D. none of these

Answer:
cosx is an even function so,

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = 2\int_{0}^{\frac{\pi}{2}} \cos x dx$$

=2(1-0)

=2

Question 65.

Mark (√) against the correct answer in the following:

$$\int_0^a \frac{\sqrt{x}}{\left(\sqrt{x}+\sqrt{a-x}\right)} dx = ?$$

A. $\frac{a}{2}$

B. 2a

C. $\frac{2a}{3}$

D. $\frac{\sqrt{a}}{2}$

Answer:

Here,

$$f(x) = \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}$$

$$f(a-x) = \frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}$$

We know that,

$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$ ...(let)

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}} dx$$

$$= \int_0^a dx$$

$$I = \frac{a}{2}$$

Question 66.

Mark (√) against the correct answer in the following:

$$\int_0^{\pi/4} \log(1+\tan x)\,dx = ?$$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{4}\log 2$

C. $\frac{\pi}{8}\log 2$

D. 0

Answer:

let $I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$$

$$\therefore f(a-x) = \log(1+\tan(\frac{\pi}{4}-x))$$

$$= \log\left(1+\frac{\left(\tan\frac{\pi}{4}-\tan x\right)}{1+\tan\frac{\pi}{4}\tan x}\right)$$

$$= \log(1+1(1-\tan x)\frac{1}{1+\tan x}$$

$$= \log\frac{2}{1+\tan x}$$

$$\therefore \int_0^a f(a-x) = I$$

$$= \int_0^{\frac{\pi}{4}} \log \frac{2}{1 + \tan x}\, dx$$

$$= \int_0^{\frac{\pi}{4}} \log 2 dx - \int_0^{\frac{\pi}{4}} (1 + \tan x) dx$$

$$\therefore\ I = \int_0^{\frac{\pi}{4}} \log 2\, dx - I$$

$$\therefore\ 2I = \frac{\pi}{4} \log 2$$

$$\therefore\ I = \frac{\pi}{8} \log 2$$

Question 67.

Mark (√) against the correct answer in the following:

$$\int_{-a}^{a} f(x) dx = ?$$

A. $2\int_0^a \{f(x) + f(-x)\} dx$

B. $2\int_0^a \{f(x) - f(-x)\} dx$

C. $\int_0^a \{f(x) + f(-x)\} dx$

D. none of these

Answer:

$$\therefore\ \int_{-a}^{a} f(x) dx$$

$$\therefore\ \int_{-a}^{0} f(x) dx + \int_0^a f(x) dx$$

$$\therefore \int_0^a f(-x)\,dx = \int_{-a}^0 f(x)\,dx$$

$$\therefore \int_0^a f(-x)\,dx + \int_0^a f(x)\,dx$$

Question 68.
Mark (√) against the correct answer in the following:

Let [x] denote the greatest integer less than or equal to x.

Then, $\int_0^{1.5} [x]\,dx = ?$

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. 2

D. 3

Answer:

$$\therefore \int_0^{1.5} [x]dx$$

$$= \int_0^1 [x]dx + \int_1^{1.5} [x]dx$$

$$= \int_0^1 0dx + \int_1^{1.5} 1.dx$$

$$= \frac{3}{2} - 1$$

$$= \frac{1}{2}$$

Question 69.
Mark (√) against the correct answer in the following:

Let [x] denote the greatest integer less than or equal to x.

Then, $\int_{-1}^{1}[x]dx = ?$

A. -1

B. 0

C. $\frac{1}{2}$

D. 2

Answer:
$$\int_{-1}^{1}[x]dx = \int_{-1}^{0}[x]dx + \int_{0}^{1}[x]dx$$

$$= \int_{-1}^{0} -1dx + \int_{0}^{1} 0dx$$

=-1-0+0

=-1

Question 70.
Mark (√) against the correct answer in the following:

$$\int_{1}^{2}\left|x^2 - 3x + 2\right|dx = ?$$

A. $\frac{-1}{6}$

B. $\frac{1}{6}$

C. $\frac{1}{3}$

D. $\frac{2}{3}$

Answer:
$$\int_1^2 |x^2 - 3x + 2|dx$$

$\therefore$ x^2-3x+2=0

(x-2)(x-1)=0

so, 2, and 1 itself are the limits so no breaking points for the integral,

$$\therefore \int_1^2 (-x^2 + 3x - 2)dx$$

$$= \left[\frac{-x^3}{3} + \frac{3x^2}{2} - 2x\right](1to2)$$

$$\therefore = \frac{1}{6}$$

Question 71.
Mark (√) against the correct answer in the following:

$$\int_{\pi}^{2\pi} |\sin x|dx = ?$$

A. 0

B. 1

C. 2

D. none of these

Answer:
$\therefore$ sin x=0

$\therefore$ x=0,π,2π....

So π, 2π are the limits so no breaking points for the integral,

$$\therefore \int_{\pi}^{2\pi} -\sin x dx = -\cos x(\pi \text{ to } 2\pi)$$

=2

Question 72.
Mark (√) against the correct answer in the following:

$$\int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{\left(1-x^2\right)^{3/2}} dx = ?$$

A. $\frac{1}{2}(\pi - \log 2)$

B. $\left(\frac{\pi}{2} - 2 \log 2\right)$

C. $\left(\frac{\pi}{4} - \frac{1}{2}\log 2\right)$

D. none of these

Answer:
put $\sin^{-1} x = t;$

$$dt = \frac{dx}{\sqrt{1-x^2}};$$

x=sin t

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

=t;

and $\sin^{-1} 0=0$

=t

Limit changes to,

$$\int_0^{\frac{\pi}{4}} \frac{tdt}{1-\sin^2 t} = \int_0^{\frac{\pi}{4}} t\sec^2 tdt$$

$$= t\tan t - \int_0^{\frac{\pi}{4}} \tan tdt$$

$$= [t\tan t + \log\cos t]\left(0 \text{ to } \frac{\pi}{4}\right)$$

$$= \frac{\pi}{4} - \frac{1}{2}\log 2$$

Question 73.
Mark (√) against the correct answer in the following:

$$\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = ?$$

A. $\frac{1}{2}(\pi - \log\ 2)$

B. $\left(\frac{\pi}{2} - \log\ 2\right)$

C. $(\pi - 2\ \log\ 2)$

D. none of these

Answer:
put x=tan y

dx=sec^{2}ydy

$$\int_0^{\frac{\pi}{4}} \sin^{-1}(\sin 2y)\sec^2 ydy$$

$$= 2\int_0^{\frac{\pi}{4}} y\sec^2 y dy$$

$$= 2[y\tan y - \int_0^{\frac{\pi}{4}} \tan y dy]$$

$$= 2[y\tan y + \log\cos y]\left(0 \text{ to } \frac{\pi}{4}\right)$$

$$= 2[\frac{\pi}{4} - \frac{1}{2}\log 2]$$

$$= \frac{\pi}{2} - \log 2$$