Objective Questions

Question 1.
Mark (√) against the correct answer in each of the following:

A unit vector in the direction of the vector $\vec{a} = \left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$ is

A. $\left(\hat{i} - \frac{3}{2}\hat{j} + 3\hat{k}\right)$

B. $\left(\frac{2}{5}\hat{i} - \frac{3}{5}\hat{j} + \frac{6}{5}\hat{k}\right)$

C. $\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right)$

D. none of these

Answer:
Tip – A vector in the direction of another vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\lambda(a\hat{i} + b\hat{j} + c\hat{k})$ and the unit vector is given by $\frac{\lambda(a\hat{i}+b\hat{j}+c\hat{k})}{\sqrt{(a\lambda)^2+(b\lambda)^2+(c\lambda)^2}}$

So, a vector parallel to $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is given by $\lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$ where λ is an arbitrary constant.

Now, $|\vec{a}| = \sqrt{2^2 + 3^2 + 6^2} = 7$

Hence, the required unit vector

$$= \frac{\lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)}{\sqrt{(2\lambda)^2 + (3\lambda)^2 + (6\lambda)^2}}$$

$$= \frac{\lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)}{\lambda\sqrt{2^2 + 3^2 + 6^2}}$$

$$= \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

Question 2.
Mark (√) against the correct answer in the following:

A unit vector in the direction of the vector $\vec{a} = \left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$ is

A. $\left(\hat{i} - \frac{3}{2}\hat{j} + 3\hat{k}\right)$

B. $\left(\frac{2}{5}\hat{i} - \frac{3}{5}\hat{j} + \frac{6}{5}\hat{k}\right)$

C. $\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right)$

D. none of these

Answer:
Given vector $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

Property : The unit vector corresponding to the vector $a\hat{i} + b\hat{j} + c\hat{k} = \frac{a\hat{i}+b\hat{j}+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$

Therefore the unit vector corresponding to the vector $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

is

$$\hat{a} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{2^2 + (-3)^2 + 6^2}}$$

$$\hat{a} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{4 + 9 + 16}}$$

$$\hat{a} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{49}}$$

$$\hat{a} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$$

$\hat{a} = \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

Question 3.
Mark (√) against the correct answer in each of the following:

Two adjacent sides of a triangle are represented by the vectors $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$. The area of the triangle is

A. 41 sq units

B. 37 sq units

C. $\frac{41}{2}$ sq units

D. none of these

Answer:
Given - Two adjacent sides of a triangle are represented by the vectors $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$

To find – Area of the triangle

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Area of triangle $= \frac{1}{2}|\vec{a} \times \vec{b}|$ and magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -5 & 7 & 0 \end{vmatrix}$

$= \hat{k}(21 + 20)$

$= 41\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{41^2} = 41$

i.e. the area of the parallelogram = $\frac{41}{2}$ sq. units

Question 4.
Mark (√) against the correct answer in each of the following:

A unit vector in the direction of the vector $\vec{a} = \left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$ is

A. $\left(\hat{i} - \frac{3}{2}\hat{j} + 3\hat{k}\right)$

B. $\left(\frac{2}{5}\hat{i} - \frac{3}{5}\hat{j} + \frac{6}{5}\hat{k}\right)$

C. $\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right)$

D. none of these

Answer:
Tip – A vector in the direction of another vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\lambda(a\hat{i} + b\hat{j} + c\hat{k})$ and the unit vector is given by $\frac{\lambda(a\hat{i}+b\hat{j}+c\hat{k})}{\sqrt{(a\lambda)^2+(b\lambda)^2+(c\lambda)^2}}$

So, a vector parallel to $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is given by $\lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$ where λ is an arbitrary constant.

Now, $|\vec{a}| = \sqrt{2^2 + 3^2 + 6^2} = 7$

Hence, the required unit vector

$$= \frac{\lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)}{\sqrt{(2\lambda)^2 + (3\lambda)^2 + (6\lambda)^2}}$$

$$=\frac{\lambda(2\hat{i}-3\hat{j}+6\hat{k})}{\lambda\sqrt{2^2+3^2+6^2}}$$

$$=\frac{2}{7}\hat{i}-\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}$$

Question 5.
Mark (√) against the correct answer in each of the following:

Two adjacent sides of a triangle are represented by the vectors $\vec{a}=3\hat{i}+4\hat{j}$ and $\vec{b}=-5\hat{i}+7\hat{j}$. The area of the triangle is

A. 41 sq units

B. 37 sq units

C. $\frac{41}{2}$ sq units

D. none of these

Answer:
Given - Two adjacent sides of a triangle are represented by the vectors $\vec{a}=3\hat{i}+4\hat{j}$ and $\vec{b}=-5\hat{i}+7\hat{j}$

To find – Area of the triangle

Formula to be used - $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$ where $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

Tip – Area of triangle $=\frac{1}{2}|\vec{a}\times\vec{b}|$ and magnitude of a vector $\vec{p}=x\hat{i}+y\hat{j}+z\hat{k}$ is given by $|\vec{p}|=\sqrt{x^2+y^2+z^2}$

Hence,

$\vec{a}\times\vec{b}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -5 & 7 & 0 \end{vmatrix}$$

$$= \hat{k}(21 + 20)$$

$$= 41\hat{k}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{41^2} = 41$$

i.e. the area of the parallelogram = $\frac{41}{2}$ sq. units

Question 6.
Mark (√) against the correct answer in each of the following:

The direction cosines of the vector $\vec{a} = \left(-2\hat{i} + \hat{j} - 5\hat{k}\right)$ are

A. -2, 1, -5

B. $\frac{1}{3}, \frac{-1}{6}, \frac{-5}{6}$

C. $\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}}$

D. $\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$

Answer:
Formula to be used – The direction cosines of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $-2\hat{i} + \hat{j} - 5\hat{k}$ is given by

$$\left(\frac{-2}{\sqrt{2^2 + 1^2 + 5^2}}, \frac{1}{\sqrt{2^2 + 1^2 + 5^2}}, \frac{-5}{\sqrt{2^2 + 1^2 + 5^2}}\right)$$

$$= \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$$

Question 7.
Mark (√) against the correct answer in each of the following:

The direction cosines of the vector $\vec{a} = \left(-2\hat{i} + \hat{j} - 5\hat{k}\right)$ are

A. -2, 1, -5

B. $\frac{1}{3}, \frac{-1}{6}, \frac{-5}{6}$

C. $\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}}$

D. $\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$

Answer:
Formula to be used – The direction cosines of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $-2\hat{i} + \hat{j} - 5\hat{k}$ is given by

$$\left(\frac{-2}{\sqrt{2^2+1^2+5^2}}, \frac{1}{\sqrt{2^2+1^2+5^2}}, \frac{-5}{\sqrt{2^2+1^2+5^2}}\right)$$

$$= \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$$

Question 8.
Mark (√) against the correct answer in the following:

The direction cosines of the vector $\vec{a} = \left(-2\hat{i} + \hat{j} - 5\hat{k}\right)$ are

A. -2, 1, -5

B. $\frac{1}{3}, \frac{-1}{6}, \frac{-5}{6}$

C. $\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}}$

D. $\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$

Answer:
Given vector $\vec{r} = -2\hat{\imath} + 1\hat{\jmath} - 5\hat{k}$

Property: for the vector $\hat{\imath} + b\hat{\jmath} + c\hat{k}$,

1) Direction ratios dr's are a,b,c

2) Direction cosines dc's are $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$

Therefore the dc's of the vector $-2\hat{\imath} + 1\hat{\jmath} - 5\hat{k} = \frac{-2}{\sqrt{(-2)^2+1^2+(-5)^2}}, \frac{1}{\sqrt{(-2)^2+1^2+(-5)^2}}, \frac{-5}{\sqrt{(-2)^2+1^2+(-5)^2}}$

$= \frac{-2}{\sqrt{4+1+25}}, \frac{1}{\sqrt{4+1+25}}, \frac{-5}{\sqrt{4+1+25}}$

$= \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$

Question 9.
Mark (√) against the correct answer in the following:

If A(1, 2, -3) and B(-1, -2, 1) are the end points of a vector $\overrightarrow{AB}$ then the direction cosines of $\overrightarrow{AB}$ are

A. -2, -4, 4

B. $\frac{-1}{2}, -1, 1$

C. $\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$

D. none of these

Answer:
Given A(1,2,-3) and B(-1,-2,1)

Property: The position vector of the of the vector joining two points (x_1,y_1,z_1)and (x_2,y_2,z_2) is $(x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$

So, the position vector of the line joining A and B is

$$\overrightarrow{AB} = (-1 - 1)\hat{\imath} + (-2 - 2)\hat{\jmath} + [1 - (-1)\}\hat{k}$$

$$\overrightarrow{AB} = -2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$$

Property: for the vector $\hat{\imath} + b\hat{\jmath} + c\hat{k}$, Direction cosines dc's are $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$

Therefore the Dc's of the vector $\overrightarrow{AB} = \frac{-2}{\sqrt{(-2)^2+(-4)^2+4^2}}, \frac{-4}{\sqrt{(-2)^2+(-4)^2+4^2}}, \frac{4}{\sqrt{(-2)^2+(-4)^2+4^2}}$

$$= \frac{-2}{\sqrt{4+16+16}}, \frac{-4}{\sqrt{4+16+16}}, \frac{4}{\sqrt{4+16+16}}$$

$$= \frac{-2}{\sqrt{36}}, \frac{-4}{\sqrt{36}}, \frac{4}{\sqrt{36}}$$

$$= -\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}$$

$$= -\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$$

Question 10.
Mark (√) against the correct answer in each of the following:

If A(1, 2, -3) and B(-1, -2, 1) are the end points of a vector $\overrightarrow{AB}$ then the direction cosines of $\overrightarrow{AB}$ are

A. -2, -4, 4

B. $\frac{-1}{2}, -1, 1$

C. $\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$

D. none of these

Answer:
Given - A(1, 2, -3) and B(-1, -2, 1) are the end points of a vector $\overrightarrow{AB}$

Tip – If P(a_1,b_1,c_1) and Q(a_2,b_2,c_2) be two points then the vector $\overrightarrow{PQ}$ is represented by $(a_2 - a_1)\hat{i} + (b_2 - b_1)\hat{j} + (c_2 - c_1)\hat{k}$

Hence, $\overrightarrow{AB} = (-1 - 1)\hat{i} + (-2 - 2)\hat{j} + (1 + 3)\hat{k} = -2\hat{i} - 4\hat{j} + 4\hat{k}$

Formula to be used – The direction cosines of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $-2\hat{i} - 4\hat{j} + 4\hat{k}$ is given by

$$\left(\frac{-2}{\sqrt{2^2 + 4^2 + 4^2}}, \frac{-4}{\sqrt{2^2 + 4^2 + 4^2}}, \frac{4}{\sqrt{2^2 + 4^2 + 4^2}}\right)$$

$$= \left(\frac{-2}{6}, \frac{-4}{6}, \frac{4}{6}\right)$$

$$= \frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$$

Question 11.
Mark (√) against the correct answer in each of the following:

If A(1, 2, -3) and B(-1, -2, 1) are the end points of a vector $\overrightarrow{AB}$ then the direction cosines of $\overrightarrow{AB}$ are

A. -2, -4, 4

B. $\frac{-1}{2}, -1, 1$

C. $\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$

D. none of these

Answer:
Given - A(1, 2, -3) and B(-1, -2, 1) are the end points of a vector $\overrightarrow{AB}$

Tip – If P(a_1,b_1,c_1) and Q(a_2,b_2,c_2) be two points then the vector $\overrightarrow{PQ}$ is represented by $(a_2 - a_1)\hat{\imath} + (b_2 - b_1)\hat{\jmath} + (c_2 - c_1)\hat{k}$

Hence, $\overrightarrow{AB} = (-1-1)\hat{\imath} + (-2-2)\hat{\jmath} + (1+3)\hat{k} = -2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$

Formula to be used – The direction cosines of a vector $a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $-2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$ is given by

$$\left(\frac{-2}{\sqrt{2^2+4^2+4^2}}, \frac{-4}{\sqrt{2^2+4^2+4^2}}, \frac{4}{\sqrt{2^2+4^2+4^2}}\right)$$

$$= \left(\frac{-2}{6}, \frac{-4}{6}, \frac{4}{6}\right)$$

$$= \frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$$

Question 12.
Mark (√) against the correct answer in each of the following:

If a vector makes angle α, β and γ with the x-axis, y-axis and z-axis respectively then the value of ($\sin^2\alpha + \sin^2\beta + \sin^2\gamma$) is

A. 1

B. 2

C. 0

D. 3

Answer:
Given - A vector makes angle α, β and γ with the x-axis, y-axis and z-axis respectively.

To Find - $(\sin^2\alpha + \sin^2\beta + \sin^2\gamma)$

Formula to be used - $\cos^2\alpha + \cos^2\beta + \cos^2\gamma=1$

Hence,

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$=(1-\cos^2\alpha) +(1-\cos^2\beta) +(1-\cos^2\gamma)$

$= 3-(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$=3-1$

$=2$

Question 13.
Mark (√) against the correct answer in each of the following:

If a vector makes angle α, β and γ with the x-axis, y-axis and z-axis respectively then the value of $(\sin^2\alpha + \sin^2\beta + \sin^2\gamma)$ is

A. 1

B. 2

C. 0

D. 3

Answer:
Given - A vector makes angle α, β and γ with the x-axis, y-axis and z-axis respectively.

To Find - $(\sin^2\alpha + \sin^2\beta + \sin^2\gamma)$

Formula to be used - $\cos^2\alpha + \cos^2\beta + \cos^2\gamma=1$

Hence,

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$=(1-\cos^2\alpha) +(1-\cos^2\beta) +(1-\cos^2\gamma)$

$= 3-(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$=3-1$

$=2$

Question 14.
Mark (√) against the correct answer in the following:

If a vector makes angle α, β and γ with the x-axis, y-axis and z-axis respectively then the value of $(\sin^2\alpha + \sin^2\beta + \sin^2\gamma)$ is

A. 1

B. 2

C. 0

D. 3

Answer:
Given α , β and γ are the angles made by the vector with X,Y and z axes respectively

$\Rightarrow \cos\alpha, \cos\beta, \cos\gamma$ are the direction cosines .

As we know that if $\cos\alpha, \cos\beta, \cos\gamma$ are the direction cosines , then the relation between them is $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

We also know that $\cos^2\theta = 1 - \sin^2\theta$

So we can write $(1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$

$\Rightarrow 3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Question 15.
Mark (√) against the correct answer in each of the following:

The vector $(\cos\alpha\cos\beta)\hat{i} + (\cos\alpha\cos\beta)\hat{j} + (\sin\alpha)\hat{k}$ is a

A. null vector

B. unit vector

C. a constant vector

D. none of these

Answer:
Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

A unit vector is a vector whose magnitude = 1.

Formula to be used - $\sin^2\theta + \cos^2\theta = 1$

Hence, magnitude of $(\cos\alpha\cos\beta)\hat{i} + (\cos\alpha\sin\beta)\hat{j} + (\sin\alpha)\hat{k}$ will be given by $\sqrt{(\cos\alpha\cos\beta)^2 + (\cos\alpha\sin\beta)^2 + (\sin\alpha)^2}$

$= \sqrt{\cos^2\alpha(\cos^2\beta + \sin^2\beta) + \sin^2\alpha}$

$= \sqrt{\cos^2\alpha + \sin^2\alpha}$

= 1 i.e a unit vector

Question 16.
Mark (√) against the correct answer in the following:

The vector $(\cos\alpha\cos\beta)\hat{i} + (\cos\alpha\cos\beta)\hat{j} + (\sin\alpha)\hat{k}$ is a

A. null vector

B. unit vector

C. a constant vector

D. none of these

Answer:
Given vector

$\cos\alpha\cos\beta\,\hat{i} + \cos\alpha\sin\beta\,\hat{j} + \sin\alpha\,\hat{k}$

UNIT VECTOR: the vector with magnitude as 1.

Property: The magnitude of the vector $a\hat{i} + b\hat{j} + c\hat{k} = \sqrt{a^2 + b^2 + c^2}$

The magnitude of the given vector is $\sqrt{(\cos\alpha\cos\beta)^2 + (\cos\alpha\sin\beta)^2 + \sin^2\alpha}$

$=\sqrt{\cos^2\alpha(\cos^2\beta + \sin^2\beta) + \sin^2\alpha}$

$=\sqrt{\cos^2\alpha + \sin^2\alpha}$

=1

As the magnitude of the given vector is 1, it is a UNIT VECTOR.

Question 17.
Mark (√) against the correct answer in each of the following:

The vector $(\cos\alpha\cos\beta)\hat{i} + (\cos\alpha\cos\beta)\hat{j} + (\sin\alpha)\hat{k}$ is a

A. null vector

B. unit vector

C. a constant vector

D. none of these

Answer:
Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

A unit vector is a vector whose magnitude = 1.

Formula to be used - $\sin^2\theta + \cos^2\theta = 1$

Hence, magnitude of $(\cos\alpha\cos\beta)\hat{i} + (\cos\alpha\sin\beta)\hat{j} + (\sin\alpha)\hat{k}$ will be given by $\sqrt{(\cos\alpha\cos\beta)^2 + (\cos\alpha\sin\beta)^2 + (\sin\alpha)^2}$

$= \sqrt{\cos^2\alpha(\cos^2\beta + \sin^2\beta) + \sin^2\alpha}$

$= \sqrt{\cos^2\alpha + \sin^2\alpha}$

= 1 i.e a unit vector

Question 18.
Mark (√) against the correct answer in the following:

What is the angle which the vector $\left(\hat{i}+\hat{j}+\sqrt{2}\,\hat{k}\right)$ makes with the z-axis?

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{2\pi}{3}$

Answer:
Given vector is $1\hat{i} + 1\hat{j} + \sqrt{2}\hat{k}$

Property: for the vector $\hat{i} + b\hat{j} + c\hat{k}$, Direction cosines dc's are $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$

Therefore the dc's of the given vector is $\frac{1}{\sqrt{1^2+1^2+\sqrt{2}^2}}, \frac{1}{\sqrt{1^2+1^2+\sqrt{2}^2}}, \frac{\sqrt{2}}{\sqrt{1^2+1^2+\sqrt{2}^2}}$

$= \frac{1}{\sqrt{1+1+2}}, \frac{1}{\sqrt{1+1+2}}, \frac{\sqrt{2}}{\sqrt{1+1+2}}$

$= \frac{1}{\sqrt{4}}, \frac{1}{\sqrt{4}}, \frac{\sqrt{2}}{\sqrt{4}}$

$= \frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$

Let the angle made by the vector with the Z axis be γ.

we got that the cosine of the angle γ is $\frac{1}{\sqrt{2}}$

$\Rightarrow \cos\gamma = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos\gamma = \cos\left(\frac{\pi}{4}\right)$

$\Rightarrow \gamma = \frac{\pi}{4}$

Question 19.
Mark (√) against the correct answer in each of the following:

What is the angle which the vector $\left(\hat{i} + \hat{j} + \sqrt{2}\hat{k}\right)$ makes with the z-axis?

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{2\pi}{3}$

Answer:
Formula to be used – The direction cosines of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by
$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ is given by

$$\left(\frac{1}{\sqrt{1^2 + 1^2 + \left(\sqrt{2}\right)^2}}, \frac{1}{\sqrt{1^2 + 1^2 + \left(\sqrt{2}\right)^2}}, \frac{\sqrt{2}}{\sqrt{1^2 + 1^2 + \left(\sqrt{2}\right)^2}}\right)$$

$$= \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}$$

$$= \frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$$

The direction cosine of z-axis = $\frac{1}{\sqrt{2}}$ i.e. $\cos\theta = \frac{1}{\sqrt{2}}$ where θ is the angle the vector makes with the z-axis.

$$\therefore \theta = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

Question 20.
Mark (√) against the correct answer in each of the following:

What is the angle which the vector $\left(\hat{i} + \hat{j} + \sqrt{2}\hat{k}\right)$ makes with the z-axis?

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{2\pi}{3}$

Answer:
Formula to be used – The direction cosines of a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ is given by

$$\left(\frac{1}{\sqrt{1^2 + 1^2 + (\sqrt{2})^2}}, \frac{1}{\sqrt{1^2 + 1^2 + (\sqrt{2})^2}}, \frac{\sqrt{2}}{\sqrt{1^2 + 1^2 + (\sqrt{2})^2}}\right)$$

$$= \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}$$

$= \frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$

The direction cosine of z-axis = $\frac{1}{\sqrt{2}}$ i.e. $\cos\theta = \frac{1}{\sqrt{2}}$ where θ is the angle the vector makes with the z-axis.

$\therefore \theta = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$

Question 21.
Mark (√) against the correct answer in the following:

if $\vec{a}$ and $\vec{b}$ are vectors such that $\left|\vec{a}\right| = \sqrt{3}, \left|\vec{b}\right| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{2\pi}{3}$

Answer:
Given $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$

And $\vec{a}.\vec{b} = \sqrt{6}$

Let angle between the vectors $\vec{a}$ and $\vec{b}$ be θ

Using the dot product property of the vectors,

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Substituting the given values in the equation,

$\sqrt{6} = \sqrt{3} \times 2 \times \cos\theta$

$\Rightarrow \cos\theta = \frac{\sqrt{6}}{\sqrt{3} \times 2}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{4}$

Question 22.
Mark (√) against the correct answer in each of the following:

f $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{2\pi}{3}$

Answer:
Given - $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{6}$

To find – Angle between $\vec{a}$ and $\vec{b}$.

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Hence, $\sqrt{6} = 2\sqrt{3}\cos\theta$ i.e. $\cos\theta = \frac{1}{\sqrt{2}}$ $\therefore \theta = \frac{\pi}{4}$

Question 23.

Mark (√) against the correct answer in each of the following:

f $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{2\pi}{3}$

Answer:

Given - $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{6}$

To find – Angle between $\vec{a}$ and $\vec{b}$.

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Hence, $\sqrt{6} = 2\sqrt{3}\cos\theta$ i.e. $\cos\theta = \frac{1}{\sqrt{2}}$ $\therefore \theta = \frac{\pi}{4}$

Question 24.

Mark (√) against the correct answer in the following:

If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a} \cdot \vec{b} = -1$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{2\pi}{3}$

Answer:
Given

Given $|\vec{a}| = \sqrt{2}, |\vec{b}| = \sqrt{2}$

And $\vec{a}.\vec{b} = -1$

Let angle between the vectors $\vec{a}$ and $\vec{b}$ be θ

Using the dot product property of the vectors,

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Substituting the given values in the equation,

$-1 = \sqrt{2} \times \sqrt{2} \times \cos\theta$

$\Rightarrow \cos\theta = -\frac{1}{2}$

$\Rightarrow -\cos\theta = \frac{1}{2}$

$\Rightarrow \cos(\pi - \theta) = \cos\frac{\pi}{3}$

$\Rightarrow \pi - \theta = \frac{\pi}{3}$

$\Rightarrow \theta = \frac{2\pi}{3}$

Question 25.
Mark (√) against the correct answer in each of the following:

If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a} \cdot \vec{b} = -1$ then the angle between $\vec{a}$

and $\vec{b}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{2\pi}{3}$

Answer:
Given - $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a}.\vec{b} = -1$

To find – Angle between $\vec{a}$ and $\vec{b}$.

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Hence, $-1 = \sqrt{2}\sqrt{2}\cos\theta$ i.e. $\cos\theta = \frac{1}{2}$ $\therefore \theta = \frac{\pi}{3}$

Question 26.
Mark (√) against the correct answer in each of the following:

If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a} \cdot \vec{b} = -1$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{2\pi}{3}$

Answer:
Given - $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a}.\vec{b} = -1$

To find – Angle between $\vec{a}$ and $\vec{b}$.

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Hence, $-1 = \sqrt{2}\sqrt{2}\cos\theta$ i.e. $\cos\theta = \frac{1}{2}$ $\quad\therefore\ \theta = \frac{\pi}{3}$

Question 27.
Mark (√) against the correct answer in each of the following:

The angle between the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$ is

A. $\cos^{-1}\frac{5}{7}$

B. $\cos^{-1}\frac{3}{5}$

C. $\cos^{-1}\frac{3}{\sqrt{14}}$

D. none of these

Answer:
Given - $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$

To find – Angle between $\vec{a}$ and $\vec{b}$.

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Tip – Magnitude of a vector $\vec{a} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $\vec{a}.\vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k}).(3\hat{i} - 2\hat{j} + \hat{k})$=3+4+3=10

$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$|\vec{b}| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$

Hence, $10 = \sqrt{14}\sqrt{14}\cos\theta$ i.e. $\cos\theta = \frac{10}{14} = \frac{5}{7}$

$\therefore \theta = \cos^{-1}\frac{5}{7}$

Question 28.
Mark (√) against the correct answer in each of the following:

The angle between the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$ is

A. $\cos^{-1}\frac{5}{7}$

B. $\cos^{-1}\frac{3}{5}$

C. $\cos^{-1}\frac{3}{\sqrt{14}}$

D. none of these

Answer:
Given - $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

To find – Angle between $\vec{a}$ and $\vec{b}$.

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $\vec{a}.\vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k}).(3\hat{i} - 2\hat{j} + \hat{k})$=3+4+3=10

$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$|\vec{b}| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$

Hence, $10 = \sqrt{14}\sqrt{14}\cos\theta$ i.e. $\cos\theta = \frac{10}{14} = \frac{5}{7}$

$\therefore \theta = \cos^{-1}\frac{5}{7}$

Question 29.
Mark (√) against the correct answer in the following:

The angle between the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$ is

A. $\cos^{-1}\frac{5}{7}$

B. $\cos^{-1}\frac{3}{5}$

C. $\cos^{-1}\frac{3}{\sqrt{14}}$

D. none of these

Answer:
Given vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and

Magnitude $|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$

$\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

Magnitude of $|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$

Property:

$\vec{r_1} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{r_2} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$\vec{r_1}.\vec{r_2} = (x_1.x_2)\hat{i} + (y_1.y_2)\hat{j} + (z_1.z_2)\hat{k}$

Then

$\vec{a}.\vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k}).(3\hat{i} - 2\hat{j} + \hat{k})$

= (1 x 3)+(-2 x -2)+(3 x 1)

=3+4+3

= 10

Let angle between the vectors $\vec{a}$ and $\vec{b}$ be θ

Using the dot product property of the vectors,

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Substituting the given values in the equation,

$10 = \sqrt{14} \times \sqrt{14} \times \cos\theta$

$\Rightarrow \cos\theta = \frac{10}{14}$

$\Rightarrow \cos\theta = \frac{5}{7}$

$\Rightarrow \theta = \cos^{-1}\frac{5}{7}$

Question 30.
Mark (√) against the correct answer in the following:

If $\vec{a} = (\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{b} = (3\hat{i} - \hat{j} + 2\hat{k})$ then the angle between $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer:
Given vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - 1\hat{j} + 2\hat{k}$

$\vec{a} + \vec{b} = 4\hat{i} + \hat{j} - \hat{k}$

$\vec{a} - \vec{b} = -2\hat{i} + 3\hat{j} - 5\hat{k}$

$(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = (4\hat{i} + \hat{j} - \hat{k}).(-2\hat{i} + 3\hat{j} - 5\hat{k})$

= -8+3+5

=0

As $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 0$, then the cosine of angle between the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is 0.

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}$.

Question 31.
Mark (√) against the correct answer in each of the following:

If $\vec{a} = (\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{b} = (3\hat{i} - \hat{j} + 2\hat{k})$ then the angle between $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer:
Given - $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$

To find – Angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}| \cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $\vec{a} + \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} - \hat{j} + 2\hat{k}) = 4\hat{i} + \hat{j} - \hat{k}$

and $\vec{a} - \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) - (3\hat{i} - \hat{j} + 2\hat{k}) = -2\hat{i} + 3\hat{j} - 5\hat{k}$

$\therefore (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = (4\hat{i} + \hat{j} - \hat{k}).(-2\hat{i} + 3\hat{j} - 5\hat{k}) = -8 + 3 + 5 = 0$

$|\vec{a} + \vec{b}| = \sqrt{4^2 + 1^2 + 1^2} = \sqrt{18}$

$|\vec{a} - \vec{b}| = \sqrt{2^2 + 3^2 + 5^2} = \sqrt{38}$

Hence, $0 = \sqrt{18}\sqrt{38} \cos\theta$ i.e. $\cos\theta = 0$

$\therefore \theta = \frac{\pi}{2}$

Question 32.

Mark (√) against the correct answer in each of the following:

If $\vec{a} = \left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$ and $\vec{b} = \left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ then the angle between $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right)$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer:
Given - $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$

To find – Angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $\vec{a} + \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} - \hat{j} + 2\hat{k}) = 4\hat{i} + \hat{j} - \hat{k}$

and $\vec{a} - \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) - (3\hat{i} - \hat{j} + 2\hat{k}) = -2\hat{i} + 3\hat{j} - 5\hat{k}$

$\therefore (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = (4\hat{i} + \hat{j} - \hat{k}).(-2\hat{i} + 3\hat{j} - 5\hat{k}) = -8 + 3 + 5 = 0$

$|\vec{a} + \vec{b}| = \sqrt{4^2 + 1^2 + 1^2} = \sqrt{18}$

$|\vec{a} - \vec{b}| = \sqrt{2^2 + 3^2 + 5^2} = \sqrt{38}$

Hence, $0 = \sqrt{18}\sqrt{38}\cos\theta$ i.e. $\cos\theta = 0$

$\therefore \theta = \frac{\pi}{2}$

Question 33.

Mark (√) against the correct answer in each of the following:

If $\vec{a} = \left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$ and $\vec{b} = \left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ then the angle between $\left(2\vec{a} + \vec{b}\right)$ and $\left(\vec{a} + 2\vec{b}\right)$ is

A. $\cos^{-1}\left(\frac{21}{40}\right)$

B. $\cos^{-1}\left(\frac{31}{50}\right)$

C. $\cos^{-1}\left(\frac{11}{30}\right)$

D. none of these

Answer:

Given - $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

To find – Angle between $2\vec{a} + \vec{b}$ and $\vec{a} + 2\vec{b}$.

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – Magnitude of a vector $\vec{a} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $2\vec{a} + \vec{b} = 2(\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) + (3\hat{\imath} - \hat{\jmath} + 2\hat{k}) = 5\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

and $\vec{a} + 2\vec{b} = (\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) + 2(3\hat{\imath} - \hat{\jmath} + 2\hat{k}) = 7\hat{\imath} + \hat{k}$

$\therefore (2\vec{a} + \vec{b}).(\vec{a} - 2\vec{b}) = (5\hat{\imath} + 3\hat{\jmath} - 4\hat{k}).(7\hat{\imath} + \hat{k}) = 35 - 4 = 31$

$|2\vec{a} + \vec{b}| = \sqrt{5^2 + 3^2 + 4^2} = \sqrt{50}$

$|\vec{a} - 2\vec{b}| = \sqrt{7^2 + 1^2} = \sqrt{50}$

Hence, $31 = \sqrt{50}\sqrt{50}\cos\theta$ i.e. $\cos\theta = \frac{31}{50}$

$\therefore \theta = \cos^{-1}\frac{31}{50}$

Question 34.
Mark (√) against the correct answer in the following:

If $\vec{a} = \left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$ and $\vec{b} = \left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ then the angle between $\left(2\vec{a} + \vec{b}\right)$ and $\left(\vec{a} + 2\vec{b}\right)$ is

A. $\cos^{-1}\left(\frac{21}{40}\right)$

B. $\cos^{-1}\left(\frac{31}{50}\right)$

C. $\cos^{-1}\left(\frac{11}{30}\right)$

D. none of these

Answer:
Given vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - 1\hat{j} + 2\hat{k}$

$2\vec{a} = 2\hat{i} + 4\hat{j} - 6\hat{k}$

$2\vec{b} = 6\hat{i} - 2\hat{j} + 4\hat{k}$

Let the vector $2\vec{a} + \vec{b}$ be $\vec{U}$

$\vec{U} = 2\vec{a} + \vec{b} = 2\hat{i} + 4\hat{j} - 6\hat{k} + 3\hat{i} - 1\hat{j} + 2\hat{k}$

$\vec{U} = 2\vec{a} + \vec{b} = 5\hat{i} + 3\hat{j} - 4\hat{k}$

$|\vec{U}| = \sqrt{5^2 + 3^2 + (-4)^2} = \sqrt{25 + 9 + 16} = \sqrt{50}$

Let the vector $2\vec{b} + \vec{a}$ be $\vec{V}$

$\vec{V} = \vec{a} + 2\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k} + 6\hat{i} - 2\hat{j} + 4\hat{k}$

$\vec{V} = \vec{a} + 2\vec{b} = 7\hat{i} + 0\hat{j} + \hat{k}$

$|\vec{V}| = \sqrt{7^2 + 0^2 + 1^2} = \sqrt{49 + 1} = \sqrt{50}$

$\vec{U}.\vec{V} = (5\hat{i} + 3\hat{j} - 4\hat{k}).(7\hat{i} + 0\hat{j} + \hat{k})$

= (5 $\times$ 7) + 0 -(4 $\times$ 1)

=35-4

=31

Let angle between the vectors $\vec{U}$and $\vec{V}$ be θ

Using the dot product property of the vectors,

$\vec{U}.\vec{V} = |\vec{U}||\vec{V}|\cos\theta$

Substituting the given values in the equation,

$31 = \sqrt{50} \times \sqrt{50} \times \cos\theta$

$\Rightarrow \cos\theta = \frac{31}{50}$

$\Rightarrow \theta = \cos^{-1}\frac{31}{50}$

Question 35.
Mark (√) against the correct answer in each of the following:

If $\vec{a} = (\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{b} = (3\hat{i} - \hat{j} + 2\hat{k})$ then the angle between $(2\vec{a} + \vec{b})$ and $(\vec{a} + 2\vec{b})$ is

A. $\cos^{-1}\left(\frac{21}{40}\right)$

B. $\cos^{-1}\left(\frac{31}{50}\right)$

C. $\cos^{-1}\left(\frac{11}{30}\right)$

D. none of these

Answer:
Given - $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$

To find – Angle between $2\vec{a} + \vec{b}$ and $\vec{a} + 2\vec{b}$.

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $2\vec{a} + \vec{b} = 2(\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} - \hat{j} + 2\hat{k}) = 5\hat{i} + 3\hat{j} - 4\hat{k}$

and $\vec{a} + 2\vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + 2(3\hat{i} - \hat{j} + 2\hat{k}) = 7\hat{i} + \hat{k}$

$\therefore (2\vec{a} + \vec{b}).(\vec{a} - 2\vec{b}) = (5\hat{i} + 3\hat{j} - 4\hat{k}).(7\hat{i} + \hat{k}) = 35 - 4 = 31$

$|2\vec{a} + \vec{b}| = \sqrt{5^2 + 3^2 + 4^2} = \sqrt{50}$

$|\vec{a} - 2\vec{b}| = \sqrt{7^2 + 1^2} = \sqrt{50}$

Hence, $31 = \sqrt{50}\sqrt{50}\cos\theta$ i.e. $\cos\theta = \frac{31}{50}$

$\therefore \theta = \cos^{-1}\frac{31}{50}$

Question 36.

Mark (√) against the correct answer in each of the following:

If $\vec{a}=\left(2\hat{i}+4\hat{j}-\hat{k}\right)$ and $\vec{b}=\left(3\hat{i}-2\hat{j}+\lambda\hat{k}\right)$ be such that $\vec{a}\perp\vec{b}$ then λ = ?

A. 2

B. -2

C. 3

D. -3

Answer:
Given - $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k},\ \vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$ and $\vec{a} \perp \vec{b}$

To find – Value of λ

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – For perpendicular vectors, $\theta = \frac{\pi}{2}$ i.e. $\cos\theta = 0$ i.e. the dot product=0

Hence, $\vec{a}.\vec{b} = 0$

∴ $\left(2\hat{i} + 4\hat{j} - \hat{k}\right).\left(3\hat{i} - 2\hat{j} + \lambda\hat{k}\right) = 0$

⇨ $6 - 8 - \lambda = 0$

i.e. $\lambda = -2$

Question 37.
Mark (√) against the correct answer in each of the following:

If $\vec{a}=\left(2\hat{i}+4\hat{j}-\hat{k}\right)$ and $\vec{b}=\left(3\hat{i}-2\hat{j}+\lambda\hat{k}\right)$ be such that $\vec{a}\perp\vec{b}$ then λ = ?

A. 2

B. -2

C. 3

D. -3

Answer:
Given - $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$ and $\vec{a} \perp \vec{b}$

To find – Value of λ

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – For perpendicular vectors, $\theta = \frac{\pi}{2}$ i.e. $\cos\theta = 0$ i.e. the dot product=0

Hence, $\vec{a}.\vec{b} = 0$

$\therefore (2\hat{i} + 4\hat{j} - \hat{k}).(3\hat{i} - 2\hat{j} + \lambda\hat{k}) = 0$

$\Rightarrow 6 - 8 - \lambda = 0$

i.e. $\lambda = -2$

Question 38.
Mark (√) against the correct answer in the following:

If $\vec{a} = (2\hat{i} + 4\hat{j} - \hat{k})$ and $\vec{b} = (3\hat{i} - 2\hat{j} + \lambda\hat{k})$ be such that $\vec{a} \perp \vec{b}$ then λ = ?

A. 2

B. -2

C. 3

D. -3

Answer:
Given vectors $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$

Also given that $\vec{a} \perp \vec{b}$

Let the angle between the vectors $\vec{a}$ and $\vec{b}$ be θ.

$\Rightarrow \theta = \frac{\pi}{2}$

$= \cos\theta = 0$

$\Rightarrow \vec{a}.\vec{b} = 0$

So, $(2\hat{i} + 4\hat{j} - \hat{k}).(3\hat{i} - 2\hat{j} + \lambda\hat{k}) = 0$

$\Rightarrow (2 \times 3) + (4 \times -2) + (-1 \times \lambda) = 0$

$\Rightarrow 6 - 8 - \lambda = 0$

$\Rightarrow \lambda = -2$

Question 39.
Mark (√) against the correct answer in the following:

What is the projection of $\vec{a} = (2\hat{i} - \hat{j} + \hat{k})$ on $\vec{b} = (\hat{i} - 2\hat{j} + \hat{k})$?

A. $\frac{2}{\sqrt{3}}$

B. $\frac{4}{\sqrt{5}}$

C. $\frac{5}{\sqrt{6}}$

D. none of these

Answer:
Given vectors $\vec{a} = 2\hat{i} - 1\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 1\hat{k}$

Property:

Projection of the vector $\vec{a}$ on $\vec{b}$ is $\vec{a}.\frac{\vec{b}}{|\vec{b}|} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

Therefore the projection of $\vec{a}$ on $\vec{b}$ is $\frac{(2\hat{i} - 1\hat{j} + \hat{k}).(\hat{i} - 2\hat{j} + 1\hat{k})}{\sqrt{1^2 + (-2)^2 + 1^2}}$

$=\frac{(2x1)+(-1x-2)+(1x1)}{\sqrt{1+4+1}}$

$=\frac{2+2+1}{\sqrt{6}}$

$=\frac{5}{\sqrt{6}}$

Question 40.
Mark (√) against the correct answer in each of the following:

What is the projection of $\vec{a}=\left(2\hat{i}-\hat{j}+\hat{k}\right)$ on $\vec{b}=\left(\hat{i}-2\hat{j}+\hat{k}\right)$?

A. $\frac{2}{\sqrt{3}}$

B. $\frac{4}{\sqrt{5}}$

C. $\frac{5}{\sqrt{6}}$

D. none of these

Answer:
Given - $\vec{a}=2\hat{\imath}-\hat{\jmath}+\hat{k},\ \vec{b}=\hat{\imath}-2\hat{\jmath}+\hat{k}$

To find – Projection of $\vec{a}$ on $\vec{b}$ i.e. $\vec{a}\cos\theta$

Formula to be used - $\vec{p}.\vec{q}=|\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – If $\vec{p}$ and $\vec{q}$ are two vectors, then the projection of $\vec{p}$ on $\vec{q}$ is defined as $\vec{p}\cos\theta$

Magnitude of a vector $\vec{p}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$ is given by $|\vec{p}|=\sqrt{x^2+y^2+z^2}$

So,

$\vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow (2\hat{i} - \hat{j} + \hat{k}).(\hat{i} - 2\hat{j} + \hat{k}) = \sqrt{1^2 + 2^2 + 1^2}|\vec{a}|\cos\theta$

$\Rightarrow |\vec{a}|\cos\theta = \dfrac{2 + 2 + 1}{\sqrt{6}}$

$\Rightarrow |\vec{a}|\cos\theta = \dfrac{5}{\sqrt{6}}$

Question 41.
Mark (√) against the correct answer in each of the following:

What is the projection of $\vec{a} = (2\hat{i} - \hat{j} + \hat{k})$ on $\vec{b} = (\hat{i} - 2\hat{j} + \hat{k})$?

A. $\dfrac{2}{\sqrt{3}}$

B. $\dfrac{4}{\sqrt{5}}$

C. $\dfrac{5}{\sqrt{6}}$

D. none of these

Answer:
Given - $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

To find – Projection of $\vec{a}$ on $\vec{b}$ i.e. $\vec{a}\cos\theta$

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – If $\vec{p}$ and $\vec{q}$ are two vectors, then the projection of $\vec{p}$ on $\vec{q}$ is defined as $\vec{p}\cos\theta$

Magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow (2\hat{\imath} - \hat{\jmath} + \hat{k}).(\hat{\imath} - 2\hat{\jmath} + \hat{k}) = \sqrt{1^2 + 2^2 + 1^2}|\vec{a}|\cos\theta$

$$\Rightarrow |\vec{a}|\cos\theta = \frac{2+2+1}{\sqrt{6}}$$

$$\Rightarrow |\vec{a}|\cos\theta = \frac{5}{\sqrt{6}}$$

Question 42.
Mark (√) against the correct answer in the following:

If $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - \vec{b}\right|$, then

A. $\left|\vec{a}\right| = \left|\vec{b}\right|$

B. $\vec{a} \parallel \vec{b}$

C. $\vec{a} \perp \vec{b}$

D. none of these

Answer:
Given $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

Squaring on both the sides,

$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a}.\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a}.\vec{b})$$

$\Rightarrow$ 4. $\vec{a}.\vec{b}$=0

$\Rightarrow \vec{a}.\vec{b}$=0

$\Rightarrow \vec{a} \perp \vec{b}$

Question 43.
Mark (√) against the correct answer in each of the following:

If $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|$, then

A. $\left|\vec{a}\right|=\left|\vec{b}\right|$

B. $\vec{a} \parallel \vec{b}$

C. $\vec{a} \perp \vec{b}$

D. none of these

Answer:
Given - $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

Tip – If $\vec{a}$ and $\vec{b}$ are two vectors then $|\vec{a} \pm \vec{b}| = \sqrt{a^2 + b^2 \pm 2ab\cos\theta}$

Hence,

$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

⇨ $\sqrt{a^2 + b^2 + 2ab\cos\theta} = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

⇨ $a^2 + b^2 + 2ab\cos\theta = a^2 + b^2 - 2ab\cos\theta$

⇨ $4ab\cos\theta = 0$

⇨ $\cos\theta = 0$

i.e. $\theta = \frac{\pi}{2}$

So, $\vec{a} \perp \vec{b}$

Question 44.

Mark (√) against the correct answer in each of the following:

If $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|$, then

A. $\left|\vec{a}\right|=\left|\vec{b}\right|$

B. $\vec{a} \parallel \vec{b}$

C. $\vec{a} \perp \vec{b}$

D. none of these

Answer:
Given - $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

Tip – If $\vec{a}$ and $\vec{b}$ are two vectors then $|\vec{a} \pm \vec{b}| = \sqrt{a^2 + b^2 \pm 2ab\cos\theta}$

Hence,

$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

⇨ $\sqrt{a^2 + b^2 + 2ab\cos\theta} = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

⇨ $a^2 + b^2 + 2ab\cos\theta = a^2 + b^2 - 2ab\cos\theta$

⇨ $4ab\cos\theta = 0$

⇨ $\cos\theta = 0$

i.e. $\theta = \frac{\pi}{2}$

So, $\vec{a} \perp \vec{b}$

Question 45.
Mark (√) against the correct answer in each of the following:

If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors then $\left(3\vec{a}+2\vec{b}\right)\cdot\left(5\vec{a}-6\vec{b}\right)=?$

A. 3

B. 5

C. 6

D. 12

Answer:
Given - $\vec{a}$ and $\vec{b}$ are two mutually perpendicular unit vectors i.e. $|\vec{a}| = |\vec{b}| = 1$

To Find – $(3\vec{a} + 2\vec{b}).(5\vec{a} - 6\vec{b})$

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}| \cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip - $\vec{a} \perp \vec{b}$

$\therefore |\vec{a}||\vec{b}| \cos\theta = |\vec{a}||\vec{b}| \cos\frac{\pi}{2} = 0$

$\therefore \vec{a}.\vec{b} = \vec{b}.\vec{a} = 0$

Hence,

$(3\vec{a} + 2\vec{b}).(5\vec{a} - 6\vec{b})$

$= 15|\vec{a}|^2 + 10\vec{b}.\vec{a} - 18\vec{a}.\vec{b} - 12|\vec{b}|^2$

$= 15 - 12$

$= 3$

Question 46.
Mark (√) against the correct answer in each of the following:

If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors then $(3\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 6\vec{b}) = ?$

A. 3

B. 5

C. 6

D. 12

Answer:
Given - $\vec{a}$ and $\vec{b}$ are two mutually perpendicular unit vectors i.e. $|\vec{a}| = |\vec{b}| = 1$

To Find – $(3\vec{a} + 2\vec{b}).(5\vec{a} - 6\vec{b})$

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip - $\vec{a} \perp \vec{b}$

$\therefore |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\cos\frac{\pi}{2} = 0$

$\therefore \vec{a}.\vec{b} = \vec{b}.\vec{a} = 0$

Hence,

$(3\vec{a} + 2\vec{b}).(5\vec{a} - 6\vec{b})$

$= 15|\vec{a}|^2 + 10\vec{b}.\vec{a} - 18\vec{a}.\vec{b} - 12|\vec{b}|^2$

$= 15 - 12$

$= 3$

Question 47.
Mark (√) against the correct answer in the following:

If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors then $(3\vec{a} + 2\vec{b})\cdot(5\vec{a} - 6\vec{b}) = ?$

A. 3

B. 5

C. 6

D. 12

Answer:
Given $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors

$\Rightarrow|\vec{a}| = |\vec{b}| = 1$

And angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$ and $\vec{a}.\vec{b}$=0

Asking to find $(3\vec{a} + 2\vec{b}).(5\vec{a} - 6\vec{b})$

Multiplying ,

$=(3\times5)\ |\vec{a}|^2 - (3\times6)\ (\vec{a}.\vec{b}) + (2\times5)(\vec{b}.\vec{a}) - (2\times6)\ |\vec{b}|^2$

$= 15|\vec{a}|^2 - 18(\vec{a}.\vec{b}) + 10(\vec{a}.\vec{b}) - 12|\vec{b}|^2$ [reason: dot product is commutative i.e, $\vec{a}.\vec{b} = \vec{b}.\vec{a}$]

$=15-8(\vec{a}.\vec{b})-12$

$=15-12$ [reason: $\vec{a}.\vec{b}$=0]

= 3

Question 48.
Mark (√) against the correct answer in the following:

If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors then $\left(3\vec{a} + 2\vec{b}\right)\cdot\left(5\vec{a} - 6\vec{b}\right) = ?$

A. 3

B. 5

C. 6

D. 12

Answer:
Given vectors $\vec{a} = 3\hat{\imath} + 1\hat{\jmath} - 2\hat{k}$ and $\vec{b} = \hat{\imath} + \lambda\hat{\jmath} - 3\hat{k}$

Also given $\vec{a} \perp \vec{b}$

As they are perpendicular, $\vec{a}.\vec{b}$=0

⇒ $(3\hat{i} + 1\hat{j} - 2\hat{k}).(\hat{i} + \lambda\hat{j} - 3\hat{k}) = 0$

⇒ (3 × 1) + (1× λ) + (-2 × -3) =0

⇒ 3+ λ+6=0

⇒ $\lambda = -9$

Question 49.
Mark (√) against the correct answer in each of the following:

If the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ are perpendicular to each other then λ = ?

A. -3

B. -6

C. -9

D. -1

Answer:
Given - $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{a} \perp \vec{b}$

To find – Value of λ

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}| \cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – For perpendicular vectors, $\theta = \frac{\pi}{2}$ i.e. $\cos\theta = 0$ i.e. the dot product=0

Hence, $\vec{a}.\vec{b} = 0$

∴ $(3\hat{i} + \hat{j} - 2\hat{k}).(\hat{i} + \lambda\hat{j} - 3\hat{k}) = 0$

⇨ $3 + \lambda + 6 = 0$

i.e. $\lambda = -9$

Question 50.
Mark (√) against the correct answer in each of the following:

If the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ are perpendicular to each other then λ = ?

A. -3

B. -6

C. -9

D. -1

Answer:
Given - $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{a} \perp \vec{b}$

To find – Value of λ

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – For perpendicular vectors, $\theta = \frac{\pi}{2}$ i.e. $\cos\theta = 0$ i.e. the dot product=0

Hence, $\vec{a}.\vec{b} = 0$

$\therefore (3\hat{i} + \hat{j} - 2\hat{k}).(\hat{i} + \lambda\hat{j} - 3\hat{k}) = 0$

⇨ $3 + \lambda + 6 = 0$

i.e. $\lambda = -9$

Question 51.
Mark (√) against the correct answer in each of the following:

If θ is the angle between two unit vectors $\hat{a}$ and $\hat{b}$ then $\frac{1}{2}\left|\hat{a} - \hat{b}\right| = ?$

A. $\cos\frac{\theta}{2}$

B. $\sin\frac{\theta}{2}$

C. $\tan\frac{\theta}{2}$

D. none of these

Answer:
Given - $\hat{a}$ and $\hat{b}$ are two unit vectors with an angle θ between them

To find - $\frac{1}{2}|\hat{a}-\hat{b}|$

Formula used - If $\vec{a}$ and $\vec{b}$ are two vectors then $|\vec{a} \pm \vec{b}| = \sqrt{a^2 + b^2 \pm 2ab\cos\theta}$

$\cos 2\theta = 1 - 2\sin^2\theta$

Tip - $|\hat{a}|^2 = |\hat{b}|^2 = 1$ & $\hat{a}.\hat{b} = 1$

Hence,

$$\frac{1}{2}|\hat{a}-\hat{b}|$$

$$= \frac{1}{2}\sqrt{|\hat{a}|^2 + |\hat{b}|^2 + 2ab\cos\theta}$$

$$= \frac{1}{2}\sqrt{2 + 2\cos\theta}$$

$$= \frac{1}{\sqrt{2}}\sqrt{1 + \cos\theta}$$

$$= \frac{1}{\sqrt{2}} \times \sqrt{2\sin^2\frac{\theta}{2}}$$

$$= \sin\frac{\theta}{2}$$

Question 52.
Mark (√) against the correct answer in the following:

If θ is the angle between two unit vectors $\hat{a}$ and $\hat{b}$ then $\frac{1}{2}\left|\hat{a}-\hat{b}\right|=?$

A. $\cos\frac{\theta}{2}$

B. $\sin\frac{\theta}{2}$

C. $\tan\frac{\theta}{2}$

D. none of these

Answer:
Given $\hat{a}$ and $\hat{b}$ are unit vectors

Let θ be the angle between them.

Asking us to find the value of $\frac{1}{2}|\,\hat{a}-\hat{b}\,|$

Let this value de T

$\Rightarrow T=\frac{1}{2}|\,\hat{a}-\hat{b}\,|$

Squaring on both the sides

$T^2=\frac{1}{4}\left|(\hat{a})^2+(\hat{b})^2-2.(\hat{a}.\hat{b})\right|$

$T^2=\frac{1}{4}\left|1+1-2.(\hat{a}.\hat{b})\right|$

$T^2=\frac{1}{4}\left|2-2.(\hat{a}.\hat{b})\right|$

$T^2=\frac{2}{4}\left|1-(\hat{a}.\hat{b})\right|$

$T^2 = \frac{1}{2}\left|1 - (\hat{a}.\hat{b})\right|$

$T^2 = \frac{1}{2}\left|1 - (|\hat{a}||\hat{b}|)\cos\theta)\right|$

$T^2 = \frac{1}{2}.|1 - (1.\cos\theta)|$

$(1 - \cos\theta)$ can be written as $2.\sin^2\frac{\theta}{2}$

$\Rightarrow T^2 = \frac{1}{2}.|1 - (1.\cos\theta)|$

$= T^2 = \frac{1}{2}.|2.\sin^2\frac{\theta}{2}|$

$T^2 = \sin^2\frac{\theta}{2}$

$\Rightarrow T = \sin\frac{\theta}{2}$

Question 53.
Mark (√) against the correct answer in each of the following:

If θ is the angle between two unit vectors $\hat{a}$ and $\hat{b}$ then $\frac{1}{2}\left|\hat{a} - \hat{b}\right| = ?$

A. $\cos\frac{\theta}{2}$

B. $\sin\frac{\theta}{2}$

C. $\tan\frac{\theta}{2}$

D. none of these

Answer:
Given - $\hat{a}$ and $\hat{b}$ are two unit vectors with an angle θ between them

To find - $\frac{1}{2}|\hat{a}-\hat{b}|$

Formula used - If $\vec{a}$ and $\vec{b}$ are two vectors then $|\vec{a} \pm \vec{b}| = \sqrt{a^2 + b^2 \pm 2ab\cos\theta}$

$\cos 2\theta = 1 - 2\sin^2\theta$

Tip - $|\hat{a}|^2 = |\hat{b}|^2 = 1$ & $\hat{a}.\hat{b} = 1$

Hence,

$$\frac{1}{2}|\hat{a}-\hat{b}|$$

$$= \frac{1}{2}\sqrt{|\hat{a}|^2 + |\hat{b}|^2 + 2ab\cos\theta}$$

$$= \frac{1}{2}\sqrt{2 + 2\cos\theta}$$

$$= \frac{1}{\sqrt{2}}\sqrt{1 + \cos\theta}$$

$$= \frac{1}{\sqrt{2}} \times \sqrt{2\sin^2\frac{\theta}{2}}$$

$$= \sin\frac{\theta}{2}$$

Question 54.
Mark (√) against the correct answer in each of the following:

If $\vec{a} = (\hat{i} - \hat{j} + 2\hat{k})$ and $\vec{b} = (2\hat{i} + 3\hat{j} - 4\hat{k})$ then $|\vec{a} \times \vec{b}| = ?$

A. $\sqrt{174}$

B. $\sqrt{87}$

C. $\sqrt{93}$

D. none of these

Answer:
Given - $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ are two vectors.

To find - $|\vec{a} \times \vec{b}|$

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 3 & -4 \end{vmatrix}$

$= \hat{i}(4 - 6) + \hat{j}\,(4 + 4) + \hat{k}(3 + 2)$

$= -2\hat{i} + 8\hat{j} + 5\hat{k}$

$\therefore\ |\vec{a} \times \vec{b}| = \sqrt{2^2 + 8^2 + 5^2} = \sqrt{93}$

Question 55.
Mark (√) against the correct answer in each of the following:

If $\vec{a} = (\hat{i} - \hat{j} + 2\hat{k})$ and $\vec{b} = (2\hat{i} + 3\hat{j} - 4\hat{k})$ then $|\vec{a} \times \vec{b}| = ?$

A. $\sqrt{174}$

B. $\sqrt{87}$

C. $\sqrt{93}$

D. none of these

Answer:
Given - $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ are two vectors.

To find - $|\vec{a} \times \vec{b}|$

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 3 & -4 \end{vmatrix}$

$= \hat{i}(4 - 6) + \hat{j}\,(4 + 4) + \hat{k}(3 + 2)$

$= -2\hat{i} + 8\hat{j} + 5\hat{k}$

$\therefore\ |\vec{a} \times \vec{b}| = \sqrt{2^2 + 8^2 + 5^2} = \sqrt{93}$

Question 56.
Mark (√) against the correct answer in the following:

If $\vec{a} = (\hat{i} - \hat{j} + 2\hat{k})$ and $\vec{b} = (2\hat{i} + 3\hat{j} - 4\hat{k})$ then $|\vec{a} \times \vec{b}| = ?$

A. $\sqrt{174}$

B. $\sqrt{87}$

C. $\sqrt{93}$

D. none of these

Answer:
Given vectors $\vec{a} = \hat{\imath} - 1\hat{\jmath} + 2\hat{k}$ and $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 3 & -4 \end{vmatrix}$

$= \hat{\imath}[(-1 \times -4)-(2 \times 3)] - \hat{\jmath}[(1 \times -4)-(2 \times 2)] + \hat{k}[(1 \times 3)-(2 \times -1)]$

$=\hat{\imath}[4 - 6] - \hat{\jmath}[-4 - 4] + \hat{k}[3 + 2]$

$=-2\hat{\imath} + 8\hat{\jmath} + 5\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{(-2)^2 + 8^2 + 5^2} = \sqrt{4 + 64 + 25} = \sqrt{93}$

Question 57.
Mark (√) against the correct answer in the following:

If $\vec{a} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$ and $\vec{b} = \left(\hat{i} - 3\hat{k}\right)$ then $\left|\vec{b} \times 2\vec{a}\right| = ?$

A. $10\sqrt{3}$

B. $5\sqrt{17}$

C. $4\sqrt{19}$

D. $2\sqrt{23}$

Answer:
Given vectors $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = \hat{\imath} - 3\hat{k}$

Asking us to find, $|\vec{b} \times 2\vec{a}|$

$2\vec{a} = 2\hat{\imath} - 4\hat{\jmath} + 6\hat{k}$

$\vec{b} \times 2\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -3 \\ 2 & -4 & 6 \end{vmatrix}$

$= \hat{i}[0-(-4 \times -3)] - \hat{j}[(1 \times 6)-(2 \times -3)] + \hat{k}[(1 \times -4)-0]$

$= \hat{i}(-12) - \hat{j}(6+6) + \hat{k}(-4)$

$= -12\hat{i} + 12\hat{j} - 4\hat{k}$

$|\vec{b} \times 2\vec{a}| = \sqrt{(-12)^2 + 12^2 + (-4)^2} = \sqrt{414 + 144 + 16} = \sqrt{304}$

$= \sqrt{16.19}$

$= 4.\sqrt{19}$

Question 58.
Mark (√) against the correct answer in each of the following:

If $\vec{a} = (\hat{i} - 2\hat{j} + 3\hat{k})$ and $\vec{b} = (\hat{i} - 3\hat{k})$ then $|\vec{b} \times 2\vec{a}| = ?$

A. $10\sqrt{3}$

B. $5\sqrt{17}$

C. $4\sqrt{19}$

D. $2\sqrt{23}$

Answer:
Given - $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{k}$ are two vectors.

To find - $|\vec{b} \times 2\vec{a}|$

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$\vec{b} \times 2\vec{a}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -3 \\ 2 & -4 & 6 \end{vmatrix}$$

$= \hat{i}(12) + \hat{j}(-6 - 6) + \hat{k}(-4)$

$= 12\hat{i} - 12\hat{j} - 4\hat{k}$

$\therefore |\vec{b} \times 2\vec{a}| = \sqrt{12^2 + 12^2 + 4^2} = \sqrt{304} = 4\sqrt{19}$

Question 59.
Mark (√) against the correct answer in each of the following:

If $\vec{a} = (\hat{i} - 2\hat{j} + 3\hat{k})$ and $\vec{b} = (\hat{i} - 3\hat{k})$ then $|\vec{b} \times 2\vec{a}| = ?$

A. $10\sqrt{3}$

B. $5\sqrt{17}$

C. $4\sqrt{19}$

D. $2\sqrt{23}$

Answer:
Given - $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{k}$ are two vectors.

To find - $|\vec{b} \times 2\vec{a}|$

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$\vec{b} \times 2\vec{a}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -3 \\ 2 & -4 & 6 \end{vmatrix}$$

$= \hat{i}(12) + \hat{j}\,(-6 - 6) + \hat{k}(-4)$

$= 12\hat{i} - 12\hat{j} - 4\hat{k}$

$\therefore |\vec{b} \times 2\vec{a}| = \sqrt{12^2 + 12^2 + 4^2} = \sqrt{304} = 4\sqrt{19}$

Question 60.
Mark (√) against the correct answer in each of the following:

If $|\vec{a}| = 2, |\vec{b}| = 7$ and $(\vec{a} \times \vec{b}) = (3\hat{i} + 2\hat{j} + 6\hat{k})$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{3\pi}{4}$

Answer:

Given - $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

To find – Angle between $\vec{a}$ and $\vec{b}$

Formula to be used - $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$

Tip – $|\vec{p} \times \vec{q}| = \left||\vec{p}||\vec{q}|\sin\theta\hat{n}\right| = |\vec{p}||\vec{q}|\sin\theta$ & magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence, $\left|\vec{a} \times \vec{b}\right| = \left|3\hat{i} + 2\hat{j} + 6\hat{k}\right| = \sqrt{3^2 + 2^2 + 6^2} = 7$

$\therefore 7 = 2 \times 7\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

Question 61.
Mark (√) against the correct answer in the following:

If $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 7$ and $\left(\vec{a} \times \vec{b}\right) = \left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{3\pi}{4}$

Answer:
Given

$|\vec{a}| = 2$

And $|\vec{b}| = 7$

$\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{3^2 + 2^2 + 6^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7$

Let the angle between the vector be θ

As we know that,

$|\vec{a} \times \vec{b}| = |\vec{a}|.|\vec{b}| \sin\theta$

Substituting the values,

$7 = 2 \times 7 \times \sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

Question 62.
Mark (√) against the correct answer in each of the following:

If $|\vec{a}| = 2, |\vec{b}| = 7$ and $(\vec{a} \times \vec{b}) = (3\hat{i} + 2\hat{j} + 6\hat{k})$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{3\pi}{4}$

Answer:
Given - $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$

To find – Angle between $\vec{a}$ and $\vec{b}$

Formula to be used - $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$

Tip – $|\vec{p} \times \vec{q}| = \left||\vec{p}||\vec{q}|\sin\theta\hat{n}\right| = |\vec{p}||\vec{q}|\sin\theta$ & magnitude of a vector $\vec{p} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence, $\left|\vec{a} \times \vec{b}\right| = \left|3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}\right| = \sqrt{3^2 + 2^2 + 6^2} = 7$

$\therefore 7 = 2 \times 7\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

Question 63.
Mark (√) against the correct answer in the following:

If $\left|\vec{a}\right| = \sqrt{26}, \left|\vec{b}\right| = 7$ and $\left|\vec{a} \times \vec{b}\right| = 35$ then $\vec{a} \cdot \vec{b} = ?$

A. 5

B. 7

C. 13

D. 12

Answer:
Given

$|\vec{a}| = \sqrt{26}$

And $|\vec{b}| = 7$

$|\vec{a} \times \vec{b}| = 35$ and $|\vec{a} . \vec{b}| = ?$

As we know that,
$|\vec{a}.\vec{b}| = |\vec{a}||\vec{b}| \cos\theta$ and $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

Adding and subtracting the above equations,

$$|\vec{a}.\vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 \cos^2\theta + |\vec{a}|^2|\vec{b}|^2 \sin^2\theta$$

$$|\vec{a}.\vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 (\cos^2\theta + \sin^2\theta)$$

$$|\vec{a}.\vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 (1)$$

Substituting the given values, we get

$$|\vec{a}.\vec{b}|^2 + 35^2 = \sqrt{26}^2 7^2$$

$$|\vec{a}.\vec{b}|^2 + 1225 = 26.49$$

$$|\vec{a}.\vec{b}|^2 + 1225 = 1274$$

$$|\vec{a}.\vec{b}|^2 = 1274 - 1225$$

$$|\vec{a}.\vec{b}|^2 = 49$$

$$|\vec{a} . \vec{b}| = 7$$

Question 64.

Mark (√) against the correct answer in each of the following:

If $\left|\vec{a}\right| = \sqrt{26}, \left|\vec{b}\right| = 7$ and $\left|\vec{a} \times \vec{b}\right| = 35$ then $\vec{a} \cdot \vec{b} = ?$

A. 5

B. 7

C. 13

D. 12

Answer:
Given – $|\vec{a}| = \sqrt{26}$, $|\vec{b}| = 7$ and $|\vec{a} \times \vec{b}| = 35$

To find - $\vec{a}.\vec{b}$

Formula to be used - $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$ & $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ & $\vec{q}$ are any two vectors

Tip – $|\vec{p} \times \vec{q}| = \left||\vec{p}||\vec{q}|\sin\theta\hat{n}\right| = |\vec{p}||\vec{q}|\sin\theta$

So,

$|\vec{a} \times \vec{b}| = 35$

⇨ $|\vec{a}||\vec{b}|\sin\theta = 35$

⇨ $\sin\theta = \frac{35}{7\sqrt{26}} = \frac{5}{\sqrt{26}}$

$\therefore \cos\theta = \sqrt{1 - \left(\frac{5}{\sqrt{26}}\right)^2} = \frac{1}{\sqrt{26}}$

$\therefore \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta = \sqrt{26} \times 7 \times \frac{1}{\sqrt{26}} = 7$

Question 65.

Mark (√) against the correct answer in each of the following:

If $|\vec{a}| = \sqrt{26}, |\vec{b}| = 7$ and $|\vec{a} \times \vec{b}| = 35$ then $\vec{a} \cdot \vec{b} = ?$

A. 5

B. 7

C. 13

D. 12

Answer:
Given – $|\vec{a}| = \sqrt{26}$, $|\vec{b}| = 7$ and $|\vec{a} \times \vec{b}| = 35$

To find - $\vec{a}.\vec{b}$

Formula to be used - $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$ & $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ & $\vec{q}$ are any two vectors

Tip – $|\vec{p} \times \vec{q}| = \left||\vec{p}||\vec{q}|\sin\theta\hat{n}\right| = |\vec{p}||\vec{q}|\sin\theta$

So,

$|\vec{a} \times \vec{b}| = 35$

⇨ $|\vec{a}||\vec{b}|\sin\theta = 35$

⇨ $\sin\theta = \frac{35}{7\sqrt{26}} = \frac{5}{\sqrt{26}}$

$\therefore \cos\theta = \sqrt{1 - \left(\frac{5}{\sqrt{26}}\right)^2} = \frac{1}{\sqrt{26}}$

$\therefore \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta = \sqrt{26} \times 7 \times \frac{1}{\sqrt{26}} = 7$

Question 66.

Mark (√) against the correct answer in the following:

Two adjacent sides of a || gm are represented by the vectors $\vec{a} = \left(3\hat{i} + \hat{j} + 4\hat{k}\right)$ and $\vec{b} = \left(\hat{i} - \hat{j} + \hat{k}\right)$. The area of the || gm is

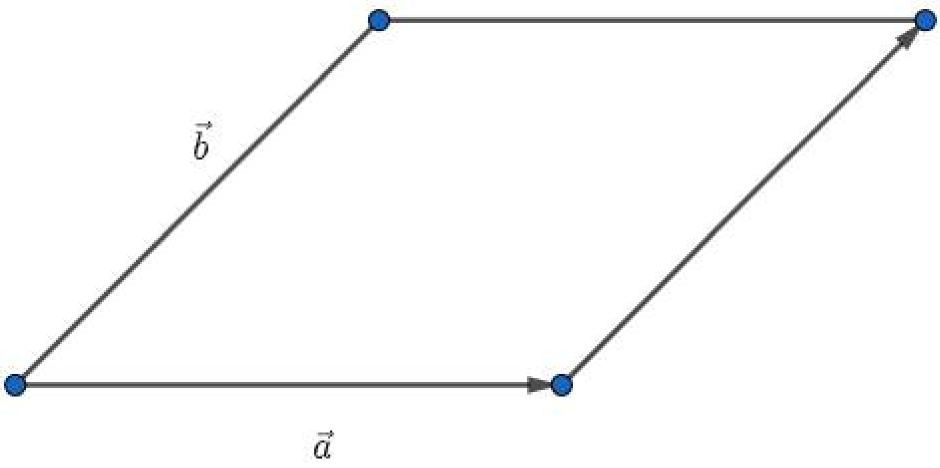
A. $\sqrt{42}$ sq units

B. 6 sq units

C. $\sqrt{35}$ sq units

D. none of these

Answer:



Given the adjacent sides of the parallelogram

$\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

Property: The area of the parallelogram with the adjacent sides are $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}|$

Therefore the area of the parallelogram is

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$= \hat{i}[1 - (-4)] - \hat{j}[3 - 4] + \hat{k}[-3 - 1]$

$= 5\hat{i} + \hat{j} - 4\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{5^2 + 1^2 + (-4)^2} = \sqrt{25 + 1 + 16} = \sqrt{42}$ sq.units

Question 67.
Mark (√) against the correct answer in each of the following:

Two adjacent sides of a || gm are represented by the vectors $\vec{a} = \left(3\hat{i} + \hat{j} + 4\hat{k}\right)$ and $\vec{b} = \left(\hat{i} - \hat{j} + \hat{k}\right)$. The area of the || gm is

A. $\sqrt{42}$ sq units

B. 6 sq units

C. $\sqrt{35}$ sq units

D. none of these

Answer:
Given - Two adjacent sides of a || gm are represented by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

To find – Area of the parallelogram

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Area of ||gm $= |\vec{a} \times \vec{b}|$ and magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$\vec{a} \times \vec{b}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(-4 - 1) + \hat{j}(4 - 3) + \hat{k}(-3 - 1)$$

$= -5\hat{i} + \hat{j} - 4\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{5^2 + 1^2 + 4^2} = \sqrt{42}$

i.e. the area of the parallelogram = $\sqrt{42}$ sq. units

Question 68.
Mark (√) against the correct answer in the following:

The diagonals of a || gm are represented by the vectors $\vec{d_1} = (3\hat{i} + \hat{j} - 2\hat{k})$ and $\vec{d_2} = (\hat{i} - 3\hat{j} + 4\hat{k})$. The area of the || gm is

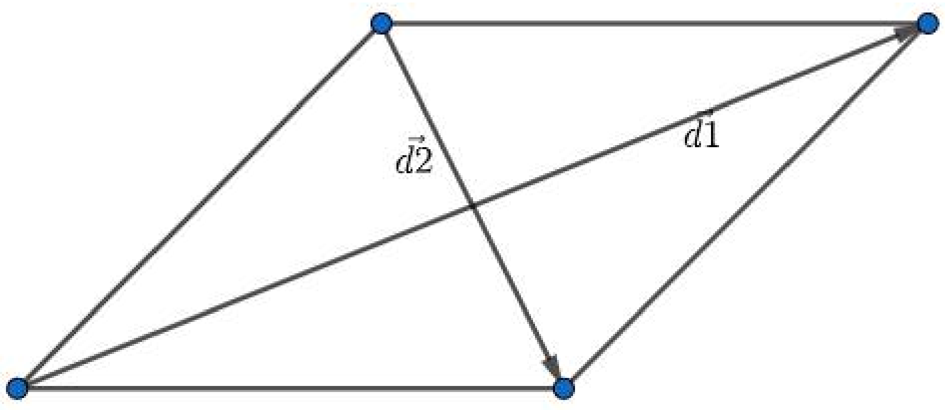
A. $7\sqrt{3}$ sq units

B. $5\sqrt{3}$ sq units

C. $3\sqrt{5}$ sq units

D. none of these

Answer:



Given diagonals of the parallelogram $\vec{d_1} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{d_2} = \hat{i} - 3\hat{j} + 4\hat{k}$

Area of the parallelogram as $\vec{d_1}$ and $\vec{d_2}$ as diagonals is $\frac{1}{2}|\vec{d_1} x \vec{d_2}|$

$\vec{d_1} x \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$

$= \hat{i}[4-6] - \hat{j}[12-(-2)] + \hat{k}[-9-1]$

$= -2\hat{i} - 14\hat{j} - 10\hat{k}$

$|\overrightarrow{d_1} \times \overrightarrow{d_2}| = \sqrt{(-2)^2 + (-14)^2 + (-10)^2} = \sqrt{4+196+100} = \sqrt{300} = 10 \times \sqrt{3}$

Therefore the area of the parallelogram is $\frac{1}{2}|\overrightarrow{d_1} x \overrightarrow{d_2}| = \frac{1}{2} \times\ 10 \times \sqrt{3}$

$=. 5\sqrt{3}$ sq units

Question 69.
Mark (√) against the correct answer in each of the following:

The diagonals of a || gm are represented by the vectors $\overrightarrow{d_1} = (3\hat{i} + \hat{j} - 2\hat{k})$ and $\overrightarrow{d_2} = (\hat{i} - 3\hat{j} + 4\hat{k})$. The area of the || gm is

A. $7\sqrt{3}$ sq units

B. $5\sqrt{3}$ sq units

C. $3\sqrt{5}$ sq units

D. none of these

Answer:
Given - Two diagonals of a || gm are represented by the vectors $\overrightarrow{d_1} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{d_2} = \hat{i} - 3\hat{j} + 4\hat{k}$

To find – Area of the parallelogram

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Area of ||gm $= \frac{1}{2}|\overrightarrow{d_1} \times \overrightarrow{d_2}|$ and magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$$\overrightarrow{d_1} \times \overrightarrow{d_2}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$$= \hat{i}(4 - 6) + \hat{j}(-2 - 12) + \hat{k}(-9 - 1)$$

$$= -2\hat{i} - 14\hat{j} - 10\hat{k}$$

$$\therefore \left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right| = \sqrt{2^2 + 14^2 + 10^2} = \sqrt{300}$$

i.e. the area of the parallelogram = $\frac{1}{2} \times \sqrt{300} = 5\sqrt{3}$ sq. units

Question 70.
Mark (√) against the correct answer in the following:

Two adjacent sides of a triangle are represented by the vectors $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$. The area of the triangle is

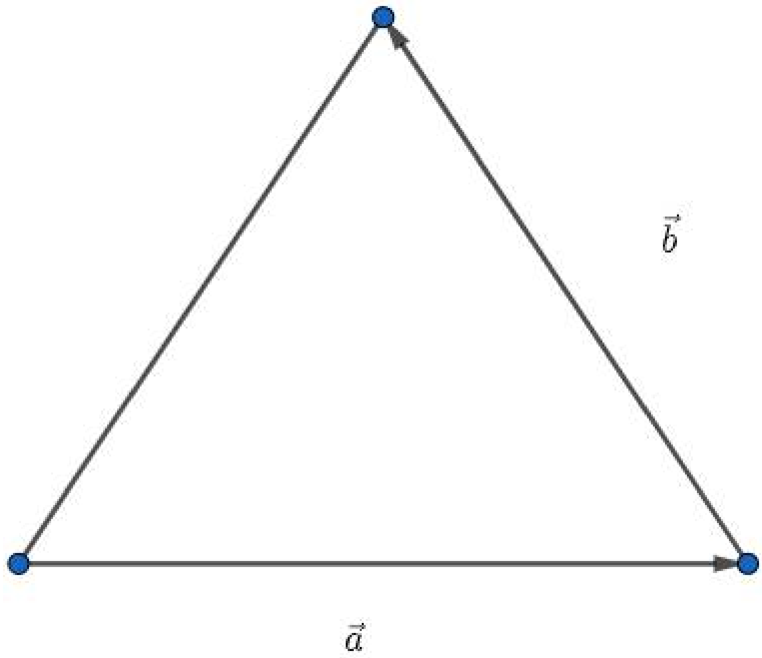
A. 41 sq units

B. 37 sq units

C. $\frac{41}{2}$ sq units

D. none of these

Answer:



Given the adjacent sides of the triangle are $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$

Property: The area of the triangle with the sides $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}|\vec{a}\text{x}\vec{b}|$

$\vec{a}\text{x}\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -5 & 7 & 0 \end{vmatrix}$

$= \hat{k}[21 - (-20)]$

$=41\hat{k}$

$|\vec{a}\text{x}\vec{b}|=41$

Therefore area of the triangle $=\frac{1}{2} \times 41 = \frac{41}{2}$ sq. units

Question 71.
Mark (√) against the correct answer in each of the following:

The unit vector normal to the plane containing $\vec{a} = (\hat{i} - \hat{j} - \hat{k})$ and $\vec{b} = (\hat{i} + \hat{j} + \hat{k})$ is

A. $(\hat{j} - \hat{k})$

B. $\left(-\hat{j}+\hat{k}\right)$

C. $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$

D. $\frac{1}{\sqrt{2}}\left(-\hat{i}+\hat{k}\right)$

Answer:
Given - $\vec{a}=\hat{\imath}-\hat{\jmath}-\hat{k}$ & $\vec{b}=\hat{\imath}+\hat{\jmath}+\hat{k}$

To find – A unit vector perpendicular to the two given vectors.

Formula to be used - $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$ where $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$ and $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

Tip – A vector perpendicular to two given vectors is their cross product.

The unit vector of any vector $a\hat{\imath}+b\hat{\jmath}+c\hat{k}$ is given by $\frac{(a\hat{\imath}+b\hat{\jmath}+c\hat{k})}{\sqrt{a^2+b^2+c^2}}$

Hence,

$\vec{a}\times\vec{b}$

$=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & -1 & -1\\ 1 & 1 & 1\end{vmatrix}$

$=-2\hat{\jmath}+2\hat{k}$, which the vector perpendicular to the two given vectors.

The required unit vector $=\frac{-2\hat{\jmath}+2\hat{k}}{\sqrt{2^2+2^2}}=\frac{1}{\sqrt{2}}(-\hat{\jmath}+\hat{k})$

Question 72.

Mark (√) against the correct answer in each of the following:

The unit vector normal to the plane containing $\vec{a}=\left(\hat{i}-\hat{j}-\hat{k}\right)$ and $\vec{b}=\left(\hat{i}+\hat{j}+\hat{k}\right)$ is

A. $\left(\hat{j}-\hat{k}\right)$

B. $\left(-\hat{j}+\hat{k}\right)$

C. $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$

D. $\frac{1}{\sqrt{2}}\left(-\hat{i}+\hat{k}\right)$

Answer:
Given - $\vec{a}=\hat{\imath}-\hat{\jmath}-\hat{k}\ \ \&\ \ \vec{b}=\hat{\imath}+\hat{\jmath}+\hat{k}$

To find – A unit vector perpendicular to the two given vectors.

Formula to be used - $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$ where $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$ and $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

Tip – A vector perpendicular to two given vectors is their cross product.

The unit vector of any vector $a\hat{\imath}+b\hat{\jmath}+c\hat{k}$ is given by $\frac{(a\hat{\imath}+b\hat{\jmath}+c\hat{k})}{\sqrt{a^2+b^2+c^2}}$

Hence,

$\vec{a}\times\vec{b}$

$$=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & -1 & -1\\ 1 & 1 & 1\end{vmatrix}$$

$=-2\hat{\jmath}+2\hat{k}$, which the vector perpendicular to the two given vectors.

The required unit vector $= \frac{-2\hat{j}+2\hat{k}}{\sqrt{2^2+2^2}} = \frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$

Question 73.
Mark (√) against the correct answer in the following:

The unit vector normal to the plane containing $\vec{a} = \left(\hat{i}-\hat{j}-\hat{k}\right)$ and $\vec{b} = \left(\hat{i}+\hat{j}+\hat{k}\right)$ is

A. $\left(\hat{j}-\hat{k}\right)$

B. $\left(-\hat{j}+\hat{k}\right)$

C. $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$

D. $\frac{1}{\sqrt{2}}\left(-\hat{i}+\hat{k}\right)$

Answer:
Given the plane is passing through $\vec{a} = \hat{i}-\hat{j}-\hat{k}$ and $\vec{b} = \hat{i}+\hat{j}+\hat{k}$

Property: The normal to the plane passing through $\vec{a}$ and $\vec{b}$ is $\vec{a}x\vec{b}$

Here ,

$\vec{a}x\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$

$=\hat{i}[-1-(-1)]-\hat{j}[1-(-1)]+\hat{k}[1-(-1)]$

$= -2\hat{j}+2\hat{k}$

As it is a unit normal vector,

$\Rightarrow \vec{a}x\vec{b}$ is divided by its magnitude.

Therefore the unit normal vector is $\frac{-2\hat{j}+2\hat{k}}{\sqrt{(-2)^2+2^2}}$

$= \frac{-2\hat{j}+2\hat{k}}{\sqrt{4+4}}$

$=\frac{-2\hat{j}+2\hat{k}}{\sqrt{8}}$

$=\frac{-2\hat{j}+2\hat{k}}{2\sqrt{2}}$

$=\frac{-\hat{j}+\hat{k}}{\sqrt{2}}$

Question 74.
Mark (√) against the correct answer in the following:

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ then $\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=?$

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. $\frac{3}{2}$

D. $\frac{-3}{2}$

Answer:
Given $\vec{a},\vec{b},\vec{c}$ are unit vectors and $\vec{a}+\vec{b}+\vec{c}=0$

$|\vec{a}| = 1, |\vec{b}| = 1, |\vec{c}| = 1$

Let the angle between $\vec{a}$ and $\vec{b}$ be θ

We can write the given relation as $\vec{a}+\vec{b}=-\vec{c}$

Squaring on both the sides

$(\vec{a}+\vec{b})^2 = \vec{c}^2$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a}.\vec{b}) = |\vec{c}|^2$

$\Rightarrow$ 1+1+$2(\vec{a}.\vec{b})$=1

$\Rightarrow 2(\vec{a}.\vec{b})$=-1

$\Rightarrow \vec{a}.\vec{b}$=-$\frac{1}{2}$

Similarly we can prove that $\vec{b}.\vec{c} = 0$ and $\vec{c}.\vec{a} = 0$

Asking us to find the value of $(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

$= -\frac{1}{2} - \frac{1}{2} - \frac{1}{2}$

$= -\frac{3}{2}$

Question 75.
Mark (√) against the correct answer in each of the following:

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ then $\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=?$

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. $\frac{3}{2}$

D. $\frac{-3}{2}$

Answer:
Given - $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors and $(\vec{a} + \vec{b} + \vec{c}) = 0$

To find - $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

Tip – $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

So,

$(\vec{a} + \vec{b} + \vec{c})^2 = 0$

⇨ $|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

⇨ $3 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

⇨ $(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = \frac{-3}{2}$

Question 76.
Mark (√) against the correct answer in each of the following:

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then $\left(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}\right) = ?$

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. $\frac{3}{2}$

D. $\frac{-3}{2}$

Answer:
Given - $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors and $(\vec{a} + \vec{b} + \vec{c}) = 0$

To find - $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

Tip – $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

So,

$(\vec{a}+\vec{b}+\vec{c})^2=0$

⇨ $|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

⇨ $3+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

⇨ $(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=\frac{-3}{2}$

Question 77.
Mark (√) against the correct answer in the following:

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular unit vectors then $\left[\vec{a}+\vec{b}+\vec{c}\right]=?$

A. 1

B. $\sqrt{2}$

C. $\sqrt{3}$

D. 2

Answer:
Given $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular unit vectors

$|\vec{a}|=1, |\vec{b}|=1, |\vec{c}|=1$

And $\vec{a}.\vec{b}=0$, $\vec{b}.\vec{c}=0$, $\vec{c}.\vec{a}=0$

Let the value of $\vec{a}+\vec{b}+\vec{c}=$T

Squaring on both the sides,

$(\vec{a}+\vec{b}+\vec{c})^2 = T^2$

$\Rightarrow (\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c}) = T^2$

$\Rightarrow |\vec{a}|^2 + (\vec{a}.\vec{b}) + (\vec{a}.\vec{c}) + |\vec{b}|^2 + (\vec{b}.\vec{a}) + (\overrightarrow{b.\vec{c}}) + |\vec{c}|^2 + (\vec{c}.\vec{a}) + (\vec{c}.\vec{b}) = T^2$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = T^2$

$\Rightarrow$1+1+1= T^2

$\Rightarrow T^2 = 3$

$\Rightarrow$T=$\sqrt{3}$

Question 78.
Mark (√) against the correct answer in each of the following:

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular unit vectors then $\left[\vec{a}+\vec{b}+\vec{c}\right] = ?$

A. 1

B. $\sqrt{2}$

C. $\sqrt{3}$

D. 2

Answer:
Given - $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors

To find - $[\vec{a}+\vec{b}+\vec{c}]$

Tip – $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ & $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

So,

$(\vec{a}+\vec{b}+\vec{c})^2$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

=3

$\therefore [\vec{a} + \vec{b} + \vec{c}] = \sqrt{3}$

Question 79.
Mark (√) against the correct answer in each of the following:

If $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular unit vectors then $\left[\vec{a} + \vec{b} + \vec{c}\right] = ?$

A. 1

B. $\sqrt{2}$

C. $\sqrt{3}$

D. 2

Answer:
Given - $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors

To find - $[\vec{a} + \vec{b} + \vec{c}]$

Tip – $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ & $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

So,

$(\vec{a} + \vec{b} + \vec{c})^2$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

=3

$\therefore [\vec{a} + \vec{b} + \vec{c}] = \sqrt{3}$

Question 80.

Mark (√) against the correct answer in the following:

$$\left[\hat{i}\ \hat{j}\ \hat{k}\right] = ?$$

A. 0

B. 1

C. 2

D. 3

Answer:
Asking us to find the value of $[\hat{i}\,\hat{j}\,\hat{k}]$

$[\hat{i}\,\hat{j}\,\hat{k}]=\hat{i}.\,(\hat{j} \times \hat{k})$ or $(\hat{i} \times \hat{j}).\hat{k}$

The value of $\hat{j} \times \hat{k} = \hat{i}$ and $\hat{i} \times \hat{j} = \hat{k}$

$\Rightarrow \hat{i}.\,(\hat{j} \times \hat{k}) = \hat{i}.\,(\hat{i})$ or $(\hat{i} \times \hat{j}).\hat{k} = \hat{k}.\hat{k}$

=1 =1

Question 81.
Mark (√) against the correct answer in each of the following:

$$\left[\hat{i}\ \hat{j}\ \hat{k}\right] = ?$$

A. 0

B. 1

C. 2

D. 3

Answer:
To find - $[\hat{i}\ \ \hat{j}\ \ \hat{k}]$

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b} \times \hat{c})$

$\therefore [\hat{i}\ \ \hat{j}\ \ \hat{k}]$

$= \hat{i} . (\hat{j} \times \hat{k})$

$= \hat{i}. \hat{i}$

$= |\hat{i}|^2$

$= 1$

Question 82.
Mark (√) against the correct answer in each of the following:

$\left[\hat{i}\ \hat{j}\ \hat{k}\right] = ?$

A. 0

B. 1

C. 2

D. 3

Answer:
To find - $[\hat{i}\ \ \hat{j}\ \ \hat{k}]$

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a} . (\hat{b} \times \hat{c})$

$\therefore [\hat{i}\ \ \hat{j}\ \ \hat{k}]$

$= \hat{i} . (\hat{j} \times \hat{k})$

$= \hat{i}. \hat{i}$

$= |\hat{i}|^2$

$= 1$

Question 83.

Mark (√) against the correct answer in each of the following:

If $\vec{a} = (2\hat{i} - 3\hat{j} + 4\hat{k})$, $\vec{b} = (\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{c} = (3\hat{i} - \hat{j} - 2\hat{k})$ be the coterminous edges of a parallelepiped then its volume is

A. 21 cubic units

B. 14 cubic units

C. 7 cubic units

D. none of these

Answer:
Given – The three coterminous edges of a parallelepiped are $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$,

$\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ & $\vec{c} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$

To find – The volume of the parallelepiped

Formula to be used - $[\hat{a} \;\; \hat{b} \;\; \hat{c}] = \hat{a}\,.(\hat{b} \times \hat{c})$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

Tip - The volume of the parallelepiped $= |[\hat{a} \;\; \hat{b} \;\; \hat{c}]|$

Hence,

$[\hat{a} \;\; \hat{b} \;\; \hat{c}]$

$= \hat{a}\,.(\hat{b} \times \hat{c})$

$= (2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}).\{(\hat{\imath} + 2\hat{\jmath} - \hat{k}) \times (3\hat{\imath} - \hat{\jmath} - 2\hat{k})\}$

$= (2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}).\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$

$= (2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}).(-5\hat{\imath} - \hat{\jmath} - 7\hat{k})$

$= -10 + 3 - 28$

$= -35$

The volume = 35 sq units

Question 84.
Mark (√) against the correct answer in each of the following:

If $\vec{a} = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right)$, $\vec{b} = \left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\vec{c} = \left(3\hat{i} - \hat{j} - 2\hat{k}\right)$ be the coterminous edges of a parallelepiped then its volume is

A. 21 cubic units

B. 14 cubic units

C. 7 cubic units

D. none of these

Answer:
Given – The three coterminous edges of a parallelepiped are $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$,

$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ & $\vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$

To find – The volume of the parallelepiped

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b} \times \hat{c})$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip - The volume of the parallelepiped $= \left|\,[\hat{a}\ \ \hat{b}\ \ \hat{c}]\right|$

Hence,

$[\hat{a}\ \ \hat{b}\ \ \hat{c}]$

$= \hat{a}\,.(\hat{b} \times \hat{c})$

$= (2\hat{i} - 3\hat{j} + 4\hat{k}).\{(\hat{i} + 2\hat{j} - \hat{k}) \times (3\hat{i} - \hat{j} - 2\hat{k})\}$

$= (2\hat{i} - 3\hat{j} + 4\hat{k}).\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$

$= (2\hat{i} - 3\hat{j} + 4\hat{k}).(-5\hat{i} - \hat{j} - 7\hat{k})$

$= -10 + 3 - 28$

$= -35$

The volume = 35 sq units

Question 85.
Mark (√) against the correct answer in the following:

If $\vec{a} = (2\hat{i} - 3\hat{j} + 4\hat{k})$, $\vec{b} = (\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{c} = (3\hat{i} - \hat{j} - 2\hat{k})$ be the coterminous edges of a parallelepiped then its volume is

A. 21 cubic units

B. 14 cubic units

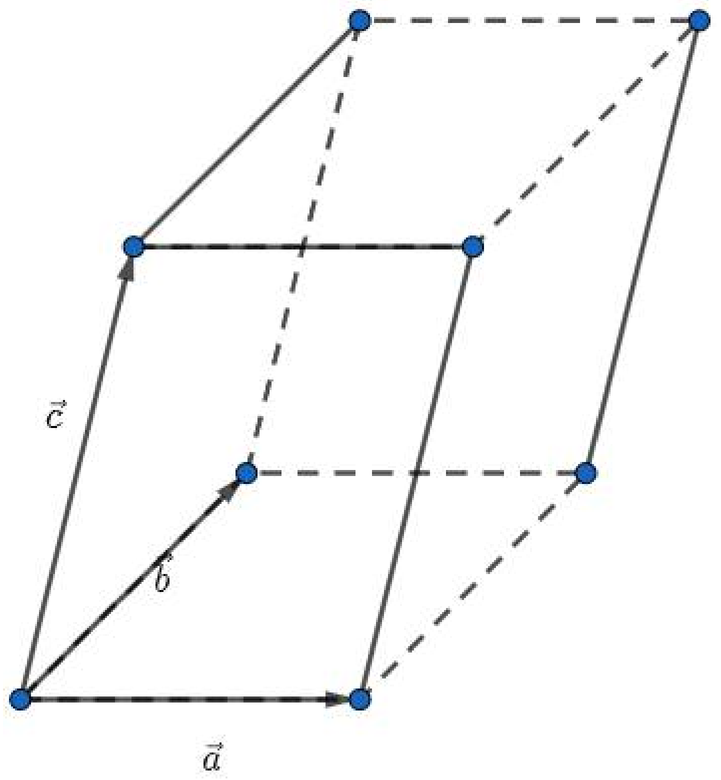
C. 7 cubic units

D. none of these

Answer:
Given $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

And $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

$\vec{c}=3\hat{i} - \hat{j} - 2\hat{k}$ are the coterminous edges of the parallelepiped.



Property:

If $\vec{a}, \vec{b}, \vec{c}$ are the coterminous edges of the parallelepiped, the the volume of the parallelepiped is [$\vec{a}\,\vec{b}\,\vec{c}$]

[$\vec{a}\,\vec{b}\,\vec{c}$] is the scalar triple product.

[$\vec{a}\,\vec{b}\,\vec{c}$] = |$\vec{a}$.($\vec{b}$ x $\vec{c}$)|

$\vec{b}$ x $\vec{c}$=$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$

=$\hat{\imath}$[-4-1]- $\hat{\jmath}$[-2-(-3)]+ $\hat{k}$[-1-6]

=-5$\hat{\imath}$-$\hat{\jmath}$-7$\hat{k}$

$\vec{a}$.($\vec{b}$ x $\vec{c}$)=($2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$).(-5$\hat{\imath}$-$\hat{\jmath}-7\hat{k}$)

= -10+3-28

= -35

$|\vec{a}.(\vec{b} \times \vec{c})|$=35 cubic units

OR

$$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$$

=2 [-4-1]- (-3)[-2-(-3)]+ 4[-1-6]

= -35

Therefore the volume of the parallelepiped with the given coterminous edges is 35 cubic units

Question 86.
Mark (√) against the correct answer in the following:

If the volume of a parallelepiped having $\vec{a} = \left(5\hat{i} - 4\hat{j} + \hat{k}\right), \vec{b} = \left(4\hat{i} + 3\hat{j} + \lambda\hat{k}\right)$ and $\vec{c} = \left(\hat{i} - 2\hat{j} + 7\hat{k}\right)$ as conterminous edges, is 216 cubic units then the value of λ is

A. $\frac{5}{3}$

B. $\frac{4}{3}$

C. $\frac{2}{3}$

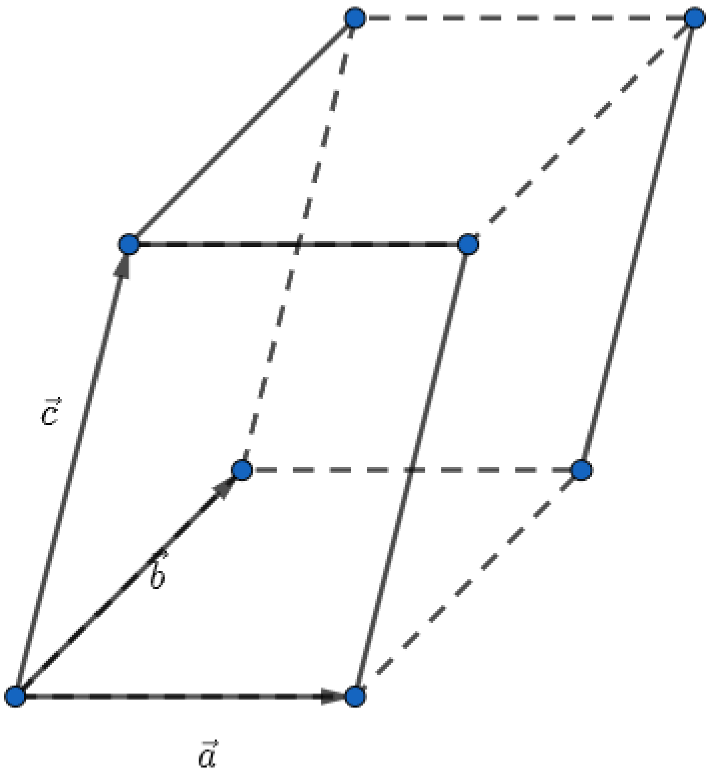
D. $\frac{1}{3}$

Answer:
Given volume of the parallelepiped is 216 cubic units

Given $\vec{a} = 5\hat{i} - 4\hat{j} + \hat{k}$

And $\vec{b} = 4\hat{i} + 3\hat{j} - \lambda\hat{k}$

$\vec{c}=\hat{\imath}-2\hat{\jmath}+7\hat{k}$ are the coterminous edges of the parallelepiped.



$[\vec{a}\ \vec{b}\ \vec{c}]$ =216

$$\Rightarrow 216=\begin{vmatrix} 5 & -4 & 1 \\ 4 & 3 & \lambda \\ 1 & -2 & 7 \end{vmatrix}$$

$\Rightarrow$ 216=5[21-(-2 λ)]-(-4)[28- λ]+1[-8-3]

$\Rightarrow$ 216=5[21+2 λ]+4[28-λ]+1[-11]

$\Rightarrow$ 216= 105 +10 λ +112 -4 λ -11

$\Rightarrow$ 216-105-112+11=6 λ

$\Rightarrow$ 6 λ =10

$\Rightarrow \lambda=\frac{10}{6}$

$\Rightarrow \lambda = \frac{5}{3}$

Question 87.

Mark (√) against the correct answer in each of the following:

If the volume of a parallelepiped having $\vec{a} = \left(5\hat{i} - 4\hat{j} + \hat{k}\right), \vec{b} = \left(4\hat{i} + 3\hat{j} + \lambda\hat{k}\right)$ and $\vec{c} = \left(\hat{i} - 2\hat{j} + 7\hat{k}\right)$ as conterminous edges, is 216 cubic units then the value of λ is

A. $\frac{5}{3}$

B. $\frac{4}{3}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

Answer:

Given – The three coterminous edges of a parallelepiped are $\vec{a} = 5\hat{\imath} - 4\hat{\jmath} + \hat{k},$

$\vec{b} = 4\hat{\imath} + 3\hat{\jmath} + \lambda\hat{k}$ & $\vec{c} = \hat{\imath} - 2\hat{\jmath} + 7\hat{k}$

To find – The value of λ

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b} \times \hat{c})$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

Tip - The volume of the parallelepiped =$|\ [\hat{a}\ \ \hat{b}\ \ \hat{c}]|$

Hence,

$[\hat{a}\ \ \hat{b}\ \ \hat{c}]$

$= \hat{a}.(\hat{b} \times \hat{c})$

$= (5\hat{i} - 4\hat{j} + \hat{k}).\{(4\hat{i} + 3\hat{j} + \lambda\hat{k}) \times (\hat{i} - 2\hat{j} + 7\hat{k})\}$

$= (5\hat{i} - 4\hat{j} + \hat{k}).\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & \lambda \\ 1 & -2 & 7 \end{vmatrix}$

$= (5\hat{i} - 4\hat{j} + \hat{k}).\left((21 + 2\lambda)\hat{i} + (\lambda - 28)\hat{j} - 11\hat{k}\right)$

=5(21+2λ)-4(λ-28)-11

=206+6λ

The volume =206+6λ

But, the volume = 216 sq units

So, 206+6λ=216 ⇨λ=$\frac{10}{6} = \frac{5}{3}$

Question 88.
Mark (√) against the correct answer in each of the following:

If the volume of a parallelepiped having $\vec{a} = (5\hat{i} - 4\hat{j} + \hat{k}), \vec{b} = (4\hat{i} + 3\hat{j} + \lambda\hat{k})$ and $\vec{c} = (\hat{i} - 2\hat{j} + 7\hat{k})$ as conterminous edges, is 216 cubic units then the value of λ is

A. $\frac{5}{3}$

B. $\frac{4}{3}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

Answer:

Given – The three coterminous edges of a parallelepiped are $\vec{a} = 5\hat{\imath} - 4\hat{\jmath} + \hat{k}$,

$\vec{b} = 4\hat{\imath} + 3\hat{\jmath} + \lambda\hat{k}$ & $\vec{c} = \hat{\imath} - 2\hat{\jmath} + 7\hat{k}$

To find – The value of λ

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b} \times \hat{c})$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

Tip - The volume of the parallelepiped =$\left|\,[\hat{a}\ \ \hat{b}\ \ \hat{c}]\right|$

Hence,

$[\hat{a}\ \ \hat{b}\ \ \hat{c}]$

$= \hat{a}\,.(\hat{b} \times \hat{c})$

$= (5\hat{\imath} - 4\hat{\jmath} + \hat{k}).\{(4\hat{\imath} + 3\hat{\jmath} + \lambda\hat{k}) \times (\hat{\imath} - 2\hat{\jmath} + 7\hat{k})\}$

$= (5\hat{\imath} - 4\hat{\jmath} + \hat{k}).\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 3 & \lambda \\ 1 & -2 & 7 \end{vmatrix}$

$= (5\hat{\imath} - 4\hat{\jmath} + \hat{k}).\left((21 + 2\lambda)\hat{\imath} + (\lambda - 28)\hat{\jmath} - 11\hat{k}\right)$

=5(21+2λ)-4(λ-28)-11

=206+6λ

The volume =206+6λ

But, the volume = 216 sq units

So, 206+6λ=216 ⇨λ=$\frac{10}{6}=\frac{5}{3}$

Question 89.

Mark (√) against the correct answer in the following:

It is given that the vectors $\vec{a}=\left(2\hat{i}-2\hat{k}\right), \vec{b}=\hat{i}+(\lambda+1)\hat{j}$ and $\vec{c}=\left(4\hat{i}+2\hat{k}\right)$ are coplanar.

Then, the value of λ is

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. 2

D. -1

Answer:

Given $\vec{a}=2\hat{\imath}-2\hat{k}$

And $\vec{b}=1\hat{\imath}+(1+\lambda)\hat{\jmath}$

$\vec{c}$=$4\hat{\imath}+2\hat{k}$ are the coplanar.

If three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then $[\vec{a}\,\vec{b}\,\vec{c}]=0$

$[\vec{a}\,\vec{b}\,\vec{c}]=\begin{vmatrix}2 & 0 & -2\\ 1 & 1+\lambda & 0\\ 4 & 0 & 2\end{vmatrix}$=0

⇒2[2(1+ λ)]-2[-4(1+ λ)=0

⇒4(1+ λ)+8(1+ λ)=0

⇒12(1+ λ)=0

⇒λ=-1

Question 90.
Mark (√) against the correct answer in each of the following:

It is given that the vectors $\vec{a} = \left(2\hat{i} - 2\hat{k}\right), \vec{b} = \hat{i} + (\lambda + 1)\hat{j}$ and $\vec{c} = \left(4\hat{i} + 2\hat{k}\right)$ are coplanar. Then, the value of λ is

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. 2

D. 1

Answer:
Given – The vectors $\vec{a} = 2\hat{i} - 2\hat{k}, \vec{b} = \hat{i} + (\lambda + 1)\hat{j}$ & $\vec{c} = 4\hat{i} + 2\hat{k}$ are coplanar

To find – The value of λ

Formula to be used - $[\hat{a} \ \ \hat{b} \ \ \hat{c}] = \hat{a}.(\hat{b} \times \hat{c})$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – For vectors to be coplanar, $[\hat{a} \ \ \hat{b} \ \ \hat{c}] = 0$

Hence,

$[\hat{a} \ \ \hat{b} \ \ \hat{c}] = 0$

⇨ $\hat{a}.(\hat{b} \times \hat{c}) = 0$

⇨ $(2\hat{i} - 2\hat{k}).\{(\hat{i} + (\lambda + 1)\hat{j}) \times (4\hat{i} + 2\hat{k})\} = 0$

⇨ $(2\hat{i} - 2\hat{k}).\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \lambda + 1 & 0 \\ 4 & 0 & 2 \end{vmatrix} = 0$

$\Rightarrow (2\hat{\imath} - 2\hat{k}).\left(2(\lambda + 1)\hat{\imath} - 2\hat{\jmath} - 4(\lambda + 1)\hat{k}\right) = 0$

$\Rightarrow$ 4(λ-1)+8(λ-1)=0

$\Rightarrow$ 12(λ-1)=0 i.e. λ= 1

Question 91.
Mark (√) against the correct answer in each of the following:

It is given that the vectors $\vec{a} = \left(2\hat{i} - 2\hat{k}\right), \vec{b} = \hat{i} + (\lambda + 1)\hat{j}$ and $\vec{c} = \left(4\hat{i} + 2\hat{k}\right)$ are coplanar. Then, the value of λ is

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. 2

D. 1

Answer:
Given – The vectors $\vec{a} = 2\hat{\imath} - 2\hat{k}, \vec{b} = \hat{\imath} + (\lambda + 1)\hat{\jmath}\ \&\ \vec{c} = 4\hat{\imath} + 2\hat{k}$ are coplanar

To find – The value of λ

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b} \times \hat{c})$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

Tip – For vectors to be coplanar, $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = 0$

Hence,

$[\hat{a}\ \ \hat{b}\ \ \hat{c}] = 0$

$\Rightarrow \hat{a}\,.\left(\hat{b} \times \hat{c}\right) = 0$

⇨ $(2\hat{i}-2\hat{k}).\{(\hat{i}+(\lambda+1)\hat{j})\times(4\hat{i}+2\hat{k})\}=0$

⇨ $(2\hat{i}-2\hat{k}).\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \lambda+1 & 0\\ 4 & 0 & 2\end{vmatrix}=0$

⇨ $(2\hat{i}-2\hat{k}).(2(\lambda+1)\hat{i}-2\hat{j}-4(\lambda+1)\hat{k})=0$

⇨ 4(λ-1)+8(λ-1)=0

⇨ 12(λ-1)=0 i.e. λ= 1

Question 92.
Mark (√) against the correct answer in each of the following:

Which of the following is meaningless?

A. $\vec{a}\cdot(\vec{b}\times\vec{c})$

B. $\vec{a}\times(\vec{b}\cdot\vec{c})$

C. $(\vec{a}\times\vec{b})\cdot\vec{c}$

D. none of these

Answer:
Tip - $[\hat{a}\ \ \hat{b}\ \ \hat{c}]=\hat{a}.(\hat{b}\times\hat{c})=\hat{b}.(\hat{c}\times\hat{a})=\hat{c}.(\hat{a}\times\hat{b})=(\hat{a}\times\hat{b}).\hat{c}$ since, dot product is commutative

Hence, $\hat{a}\times(\hat{b}.\hat{c})$ is meaningless.

Question 93.
Mark (√) against the correct answer in each of the following:

Which of the following is meaningless?

A. $\vec{a}\cdot(\vec{b}\times\vec{c})$

B. $\vec{a}\times(\vec{b}\cdot\vec{c})$

C. $\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}$

D. none of these

Answer:
Tip - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b}\times\hat{c}) = \hat{b}\,.(\hat{c}\times\hat{a}) = \hat{c}\,.(\hat{a}\times\hat{b}) = (\hat{a}\times\hat{b}).\hat{c}$ since, dot product is commutative

Hence, $\hat{a}\ \times(\hat{b}.\hat{c})$ is meaningless.

Question 94.
Mark (√) against the correct answer in the following:

Which of the following is meaningless?

A. $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$

B. $\vec{a}\times\left(\vec{b}\cdot\vec{c}\right)$

C. $\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}$

D. none of these

Answer:
Option B is meaningless

Reason:

The term $(\vec{b}.\vec{c})$ is a scalar term and $\vec{a}$ is a vector. Cross product can only be applied in between the vectors . It is meaning less if used in between scalars or between scalar and vector.

Question 95.
Mark (√) against the correct answer in each of the following:

$\vec{a}\cdot\left(\vec{a}\times\vec{b}\right)=?$

A. 0

B. 1

C. a^2b

D. meaningless

Answer:
Tip – The cross product of two vectors is the vector perpendicular to both the vectors.

$\therefore \vec{a} \times \vec{b}$ gives a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

Now,

$\vec{a}.(\vec{a} \times \vec{b})$

$= |\vec{a}||\vec{b}|\cos\theta$

$= |\vec{a}||\vec{b}|\cos\frac{\pi}{2}$

$= 0$

Question 96.
Mark (√) against the correct answer in the following:

$\vec{a} \cdot (\vec{a} \times \vec{b}) = ?$

A. 0

B. 1

C. a^2b

D. meaningless

Answer:
Asking us to find $\vec{a}.(\vec{a}x\vec{b})$

By the definition of the scalar triple product,

$\vec{a}.(\vec{a}x\vec{b}) = (\vec{a}.\vec{b}) \cdot \vec{a} - (\vec{a}.\vec{a})\vec{b}$

Also $(\vec{a}.\vec{b}) \cdot \vec{a} = (\vec{a}.\vec{a})\vec{b}$ [reason : dot product is associative]

$\Rightarrow\vec{a}.(\vec{a}x\vec{b}) = (\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{a})\vec{b}$

=0

Question 97.
Mark (√) against the correct answer in each of the following:

$\vec{a}\cdot\left(\vec{a}\times\vec{b}\right) = ?$

A. 0

B. 1

C. a^2b

D. meaningless

Answer:
Tip – The cross product of two vectors is the vector perpendicular to both the vectors.

$\therefore \vec{a}\times\vec{b}$ gives a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

Now,

$\vec{a}.(\vec{a}\times\vec{b})$

$= |\vec{a}||\vec{b}|\cos\theta$

$= |\vec{a}||\vec{b}|\cos\frac{\pi}{2}$

$= 0$

Question 98.
Mark (√) against the correct answer in each of the following:

For any three vectors $\vec{a}, \vec{b}, \vec{c}$ the value of $\left[\vec{a}-\vec{b}\ \vec{b}-\vec{c}\ \vec{c}-\vec{a}\right]$ is

A. $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

B. 1

C. 0

D. none of these

Answer:
Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}.(\hat{b}\times\hat{c}) = \hat{b}.(\hat{c}\times\hat{a})$ for any three arbitrary vectors

$$\therefore [\hat{a}-\hat{b}\ \ \hat{b}-\hat{c}\ \ \hat{c}-\hat{a}]$$

$$= (\hat{a}-\hat{b}).\{(\hat{b}-\hat{c})\times(\hat{c}-\hat{a})\}$$

$$= (\hat{a}-\hat{b}).(\hat{b}\times\hat{c}-\hat{c}\times\hat{c}-\hat{b}\times\hat{a}+\hat{c}\times\hat{a})$$

$$= (\hat{a}-\hat{b}).(\hat{b}\times\hat{c}-\hat{b}\times\hat{a}+\hat{c}\times\hat{a})$$

$$= [\hat{a}.(\hat{b}\times\hat{c})-\hat{b}(\hat{b}\times\hat{c})-\hat{a}.(\hat{b}\times\hat{a})+\hat{b}(\hat{b}\times\hat{a})+\hat{a}.(\hat{c}\times\hat{a})-\hat{b}.(\hat{c}\times\hat{a})]$$

$$= [\hat{a}\ \hat{b}\ \hat{c}]-[\hat{a}\ \hat{b}\ \hat{c}] = 0$$

Question 99.
Mark (√) against the correct answer in the following:

For any three vectors $\vec{a}, \vec{b}, \vec{c}$ the value of $\left[\vec{a}-\vec{b}\ \vec{b}-\vec{c}\ \vec{c}-\vec{a}\right]$ is

A. $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

B. 1

C. 0

D. none of these

Answer:
Asking us to find the value of $[\vec{a}-\vec{b}\ \vec{b}-\vec{c}\ \vec{c}-\vec{a}]$

$$[\vec{a}-\vec{b} \quad \vec{b}-\vec{c} \quad \vec{c}-\vec{a}] = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} . [\vec{a} \quad \vec{b} \quad \vec{c}]$$

Coefficients Of $\vec{a}$ | coefficients of $\vec{b}$ | coefficients of $\vec{c}$

$$= \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} . [\vec{a}\,\vec{b}\,\vec{c}]$$

=1[1]-(-1)[-1]

=1-1

=0

Question 100.
Mark (√) against the correct answer in each of the following:

For any three vectors $\vec{a}, \vec{b}, \vec{c}$ the value of $\left[\vec{a}-\vec{b}\ \vec{b}-\vec{c}\ \vec{c}-\vec{a}\right]$ is

A. $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

B. 1

C. 0

D. none of these

Answer:
Formula to be used - $[\hat{a} \ \ \hat{b} \ \ \hat{c}] = \hat{a}.(\hat{b} \times \hat{c}) = \hat{b}.(\hat{c} \times \hat{a})$ for any three arbitrary vectors

$\therefore [\hat{a}-\hat{b} \ \ \hat{b}-\hat{c} \ \ \hat{c}-\hat{a}]$

$= (\hat{a}-\hat{b}).\{(\hat{b}-\hat{c}) \times (\hat{c}-\hat{a})\}$

$= (\hat{a}-\hat{b}).(\hat{b} \times \hat{c} - \hat{c} \times \hat{c} - \hat{b} \times \hat{a} + \hat{c} \times \hat{a})$

$$= (\hat{a} - \hat{b}).(\hat{b} \times \hat{c} - \hat{b} \times \hat{a} + \hat{c} \times \hat{a})$$

$$= [\hat{a}.(\hat{b} \times \hat{c}) - \hat{b}(\hat{b} \times \hat{c}) - \hat{a}.(\hat{b} \times \hat{a}) + \hat{b}(\hat{b} \times \hat{a}) + \hat{a}.(\hat{c} \times \hat{a}) - \hat{b}.(\hat{c} \times \hat{a})]$$

$$= [\hat{a}\ \hat{b}\ \hat{c}] - [\hat{a}\ \hat{b}\ \hat{c}] = 0$$