Exercise 11b

Question 1.
Using differentials, find the approximate values of:

find the approximate values of $\sqrt{37}$.

Answer:
Let y =$\sqrt{}$x.

Let x =36 and Δx = 1.

As y = $\sqrt{}$x.

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{1}{2\sqrt{x}}.\Delta x$

$\Rightarrow \Delta y = \frac{1}{2\sqrt{36}}.1$

$\Rightarrow \Delta y = \frac{1}{12}$

∴ Δy = 0.08

Also,

Δy = f(x+Δx)-f(x)

$\therefore 0.08 = \sqrt{36+1} - \sqrt{36}$

⇒ 0.08 = $\sqrt{}$37 – 6

$\Rightarrow \sqrt{37} = 6.08$

Question 2.
Using differentials, find the approximate values of:

Find the approximate values of $\sqrt[3]{29}$.

Answer:
Let $y = \sqrt[3]{x}$.

Let x =27 and Δx = 2.

As $y = \sqrt[3]{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{3}x^{\frac{-2}{3}}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{1}{3}x^{\frac{-2}{3}}.\Delta x$

$\Rightarrow \Delta y = \frac{1}{3}27^{\frac{-2}{3}}.2$

$\Rightarrow \Delta y = \frac{2}{27}$

$\therefore \Delta y = 0.074$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore 0.074 = \sqrt[3]{27+2} - \sqrt[3]{27}$

$\Rightarrow 0.074 = \sqrt[3]{29} - 3$

$\Rightarrow \sqrt[3]{29} = 3.074$

Question 3.
Using differentials, find the approximate values of:

Find the approximate values of $\sqrt{27}$

Answer:
Let y =√x.

Let x =25 and Δx = 2.

As y = √x.

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{1}{2\sqrt{x}}.\Delta x$

$\Rightarrow \Delta y = \frac{1}{2\sqrt{25}}.2$

$\Rightarrow \Delta y = \frac{1}{5}$

∴ Δy = 0.2

Also,

Δy = f(x+Δx)-f(x)

$\therefore 0.2 = \sqrt{25+2} - \sqrt{25}$

⇒ 0.2 = √27 – 5

$\Rightarrow \sqrt{27} = 5.2$

Question 4.
Using differentials, find the approximate values of:

Find the approximate values of $\sqrt{0.24}$

Answer:
Let $y = \sqrt{x}$.

Let $x = 0.25$ and $\Delta x = -0.01$.

As $y = \sqrt{x}$.

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{1}{2\sqrt{x}}.\Delta x$

$\Rightarrow \Delta y = \frac{1}{2\sqrt{0.25}}.(-0.01)$

$\Rightarrow \Delta y = -0.01$

$\therefore \Delta y = -0.01$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore -0.01 = \sqrt{0.25 - 0.01} - \sqrt{0.25}$

$\Rightarrow -0.01 = \sqrt{0.24} - 0.5$

$\Rightarrow \sqrt{0.24} = 0.49$

Question 5.
Using differentials, find the approximate values of:

Find the approximate values of $\sqrt{49.5}$

Answer:
Let y =$\sqrt{x}$.

Let x =19 and Δx = 0.5.

As y = $\sqrt{x}$.

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{1}{2\sqrt{x}}.\Delta x$

$\Rightarrow \Delta y = \frac{1}{2\sqrt{49}}.0.5$

$\Rightarrow \Delta y = \frac{0.5}{14}$

$\therefore \Delta y = 0.0357$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore 0.0357 = \sqrt{49+0.5} - \sqrt{49}$

$\Rightarrow 0.0357 = \sqrt{49.5} - 7$

⇒ $\sqrt{49.5} = 7.0357$.

Question 6.
Using differentials, find the approximate values of:

Find the approximate values of $\sqrt[4]{15}$

Answer:
Let $y = \sqrt[4]{x}$.

Let x =16 and Δx = 1.

As $y = \sqrt[4]{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{4}x^{\frac{-3}{4}}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{1}{4}x^{\frac{-3}{4}}.\Delta x$

$\Rightarrow \Delta y = \frac{1}{4}16^{\frac{-3}{4}}.(-1)$

$\Rightarrow \Delta y = \frac{-1}{32}$

∴ Δy = -0.03125

Also,

Δy = f(x+Δx)-f(x)

$\therefore -0.03125 = \sqrt[4]{16-1} - \sqrt[4]{16}$

$\Rightarrow -0.03125 = \sqrt[4]{15} - 2$

$\Rightarrow \sqrt[4]{15} = 1.96875$

Question 7.

find the approximate values of $\dfrac{1}{(2.002)^2}$

Answer:

Let $y = \frac{1}{x^2}$

Let x =2 and Δx = 0.002.

As $y = \frac{1}{x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{x^3}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{-2}{x^3}.\Delta x$

$\Rightarrow \Delta y = \frac{-2}{8}.(0.002)$

$\Rightarrow \Delta y = \frac{-0.5}{1000}$

∴ Δy = -0.0005

Also,

Δy = f(x+Δx)-f(x)

$\therefore -0.0005 = \frac{1}{(2.002)^2} - \frac{1}{2^2}$

$\Rightarrow -0.0005 = \frac{1}{(2.002)^2} - 0.25$

$\Rightarrow \frac{1}{(2.002)^2} = 0.2495$

Question 8.
find the approximate values of $\log_e 10.02$, given that $\log_e 10 = 2.3026$

Answer:
Let $y = \log_e x$

Let x =10 and Δx = 0.02.

As $y = \log_e x$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx} \Delta x$

$\therefore \Delta y = \frac{1}{x} . \Delta x$

$\Rightarrow \Delta y = \frac{1}{10} . (0.02)$

$\Rightarrow \Delta y = \frac{0.02}{10}$

∴ Δy = 0.002

Also,

Δy = f(x+Δx)-f(x)

∴ 0.002 = $\log_e(10+0.02) - \log_e(10)$

⇒ 0.002 = $\log_e(10.02) - 2.3026$

⇒ $\log_e(10.02) = 2.3046$.

Question 9.
find the approximate values of $\log_{10}(4.04)$, it being given that $\log_{10}4 = 0.6021$ and $\log_{10}e = 0.4343$

Answer:
Let $y = \log_{10} x$

$\therefore y = \frac{\log_e x}{\log_e 10}$

$\therefore y = 0.4343 \log_e x$

Let x =4 and $\Delta x = 0.04$.

As $y = 0.4343 \log_e x$

$\Rightarrow \frac{dy}{dx} = \frac{0.4343}{x}$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx}\Delta x$

$\therefore \Delta y = \frac{0.4343}{x}.\Delta x$

$\Rightarrow \Delta y = \frac{0.4343}{4}.(0.04)$

$\Rightarrow \Delta y = \frac{0.017372}{4}$

$\therefore \Delta y = 0.004343$

Also,

$\Delta y = f(x+\Delta x)-f(x)$

$\therefore 0.004343 = \log_e(4+0.04) – \log_e(4)$

$\Rightarrow 0.004343 = \log_e(4.04) – 0.6021$

$\Rightarrow \log_e(4.04) = 0.606443.$

Question 10.
find the approximate values of cos 61°, it being given that sin 60° = 0.86603 and 1° = 0.01745 radian

Answer:
Let y = cosx

Let x =60° and Δx = 1° .

As y =cos x

$\Rightarrow \frac{dy}{dx} = \sin x$

We, know

$\Rightarrow \Delta y = \frac{dy}{dx} \Delta x$

$\therefore \Delta y = \sin x . \Delta x$

$\Rightarrow \Delta y = \sin(60)(0.01745)$

$\Rightarrow \Delta y = (0.86603)(0.01745$

$\therefore \Delta y = 0.01511$

Also,

Δy = f(x+Δx)-f(x)

∴ 0.01511 = cos(60+1) – cos(60)

⇒ 0.01511 = cos61° – 0.5

⇒ cos61° = 0.48489

Question 11.

If y = sin x and x changes from $\frac{\pi}{2}$ to $\frac{22}{14}$, what is the approximate change in y?

Answer:
Given x is π /2

Value of π is 22/7

22/14 is π /2

Hence there will be no change.

Question 12.
A circular metal plate expands under heating so that its radius increases by 2%. Find the approximate increase in the area of the plate, if the radius of the plate before heating is 10 cm.

Answer:
Let the radius of the plate 10cm.

Radius increases by 2% by heating

∴ After increment = $\frac{2}{100} \times 10 = 0.2$

Change in radius dr = 0.2

∴ New radius = 10+0.2 = 10.2cm

Area of circular plate = $A=\pi r^2$

∴ Change in Area = $\frac{dA}{dr}$

$\Rightarrow \frac{dA}{dr} = 2\pi r dr$

$\Rightarrow \frac{dA}{dr} = 2 \times \pi \times 10 \times 0.2$

$\Rightarrow \frac{dA}{dr} = 4\pi\ cm^2$

Question 13.
If the length of a simple pendulum is decreased by 2%, find the percentage decrease in its period T, where $T = 2\pi\sqrt{\frac{l}{g}}$.

Answer:
The formula for time period -

$$\therefore T = 2\pi\sqrt{\frac{l}{g}}$$

Here 2,π,g have no dimensions. So we can eliminate them.

Now $\frac{\Delta T}{T} = \frac{1}{2} \times \frac{\Delta L}{L}$

By representing in percentage error

$$\Rightarrow \frac{\Delta T}{T} \times 100\% = \frac{1}{2} \times \frac{\Delta L}{L} \times 100\%$$

$$\Rightarrow \frac{\Delta T}{T} \times 100\% = \frac{1}{2} \times \frac{\Delta L}{L} \times 100\%$$

$$\Rightarrow \frac{\Delta T}{T}\% = \frac{1}{2} \times 2\%$$

$$\Rightarrow \frac{\Delta T}{T}\% = 1\%$$

Hence the time period becomes 1 %.

Question 14.
The pressure p and the volume V of a gas are connected by the relation, $pV^{1/4} = k$, where k is a constant. Find the percentage increase in the pressure, corresponding to a diminution of 0.5% in the volume.

Answer:
Given: $pv^{1/4} = k$

%decrease in the volume = 1/2%s

$\therefore \frac{\Delta V}{V} \times 100 = \frac{-1}{2}$

$pv^{1/4} = k$

taking log on both sides

$\log[pv^{1/4}] = \log a$

log P + 1.4logV = log a

Differentiating both the sides we get

$\Rightarrow \frac{1}{P} dP + \frac{1.4}{V} dV = 0$

$\Rightarrow \frac{dP}{P} + 1.4 \frac{dV}{V} = 0$

Multiplying both sides by 100.

$\Rightarrow \frac{dP}{p} \times 100 + 1.4 \times \frac{dV}{V} \times 100 = 0$

$\Rightarrow \frac{dP}{P} \times 100 + 1.4 \left(\frac{-1}{2}\right) = 0$

$\Rightarrow \frac{dP}{P} \times 100 = 0.7$

%error in P = 0.7%.

Question 15.
The radius of a sphere shrinks from 10 cm to 9.8 cm. Find approximately the decrease in (i) volume, and (ii) surface area.

Answer:
Decrease in radius = dr = 10-9.8

$\therefore$ dr = 0.2

Volume of the sphere is given by = $V = \frac{4}{3} \pi r^3$

Change in volume = $dV = 4\pi r^2 dr$

$\therefore dV = 4\pi(10)^2 \times 0.2$

$\Rightarrow dV = 80\pi\ cm^3$

Surface area of the sphere is given by = $A = 4\pi r^2$

Change in volume = $dA = 8\pi r dr$

$\therefore$ dA = 8π×10× 0.2

$\therefore$ dA = 16π.

Question 16.
If there is an error of 0.1% in the measurement of the radius of a sphere, find approximately the percentage error in the calculation of the volume of the sphere.

Answer:
Volume of the sphere is given by = $V = \frac{4}{3}\pi r^2$

Change in volume = $dV = 4\pi r^2 dr$

Given: Δr = 0.1

$\Rightarrow \Delta r.\frac{dV}{dr} = 4\pi r^2 \Delta r$

$\Rightarrow \Delta V = 4\pi r^2 \Delta r$

Percentage error

$\Rightarrow \frac{\Delta V}{V} = \frac{4\pi r^2}{\frac{4\pi r^3}{3}} \times 0.1$

= 0.3%

Question 17.

Show that the relative error in the volume of a sphere, due to an error in measuring the diameter, is three times the relative error in the diameter.

Answer:

Let d be the diameter r be the radius and V be the volume of Sphere

Volume of the sphere is given by = $V = \frac{4}{3}\pi r^2$

$$\Rightarrow V = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \frac{1}{6}\pi D^3$$

Let Δd be the error in d and the corresponding error in V be ΔV.

$$\therefore \Delta V = \frac{dV}{dd}\Delta d = \frac{1}{2}\pi d^2 \Delta D$$

$$\therefore \frac{\Delta V}{V} = \frac{\frac{1}{2}\pi d^2 \Delta D}{\frac{1}{6}\pi D^3} = 3\frac{\Delta D}{D}$$

Hence Proved