Objective Questions I

Question 1.
Mark (√) against the correct answer in each of the following:

$\int (2x+3)^5 \, dx = ?$

A. $\frac{(2x+3)^6}{6} + C$

B. $\frac{(2x+3)^4}{8} + C$

C. $\frac{(2x+3)^6}{12} + C$

D. none of these

Answer:
Given = $\int (2x+3)^5$

Let, 2x + 3 = z

⇒ 2dx = dz

So,

$$\int (2x+3)^5 dx$$

$$= \int \frac{z^5}{2} dz$$

$$= \frac{1}{2}\frac{z^6}{6} + c$$ where c is the integrating constant.

$$= \frac{z^6}{12} + c$$

$$= \frac{(2x+3)^6}{12} + c$$

Question 2.
Mark (√) against the correct answer in each of the following:

$$\int (3-5x)^7 dx = ?$$

A. $-5(3 - 5x)^6 + C$

B. $\frac{(3-5x)^8}{-40} + C$

C. $\frac{-5(3-5x)^8}{8} + C$

D. none of these

Answer:
Given = $\int (3-5x)^7$

Let, 3 – 5x = z

⇒ -5dx = dz

So,

$$\int (3-5x)^7 dx$$

$$= -\int \frac{z^7}{5} dz$$

$$= -\frac{1}{5}\frac{z^8}{8} + c$$ where c is the integrating constant.

$$= -\frac{z^8}{40} + c$$

$$= -\frac{(3-5x)^8}{40} + c$$

Question 3.
Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{(2-3x)^4} dx = ?$$

A. $\frac{1}{15(2-3x)^5} + C$

B. $\frac{1}{-12(2-3x)^3} + C$

C. $\frac{1}{9(2-3x)^3} + C$

D. none of these

Answer:

Given = $\int \frac{1}{(2-3x)^4}$

Let, 2 – 3x = z

⇒ -3dx = dz

So,

$$\int \frac{1}{(2-3x)^4} dx$$

$$= \int \frac{1}{z^4}\left(\frac{dz}{-3}\right)$$

$$= -\frac{1}{3}\int \frac{dz}{z^4}$$

where c is the integrating constant.

$$= -\frac{1}{3}\int z^{-4} dz$$

$$= -\frac{1}{3}\frac{z^{-3}}{-3} + c$$

$$= \frac{1}{9(2-3x)^3} + c$$

Question 4.
Mark (√) against the correct answer in each of the following:

$$\int \sqrt{ax+b}\, dx = ?$$

A. $\frac{2(ax+b)^{3/2}}{3a} + C$

B. $\frac{3(ax+b)^{3/2}}{2a} + C$

C. $\frac{1}{2\sqrt{ax+b}} + C$

D. none of these

Answer:
Given = $\int \sqrt{ax+b}$

Let, ax + b = z^2

⇒ adx = 2zdz

So,

$$\int \sqrt{ax+b}dx$$

$$= \int z \frac{2zdz}{a}$$

$$= \frac{2}{a}\int z^2 dz$$

$$= \frac{2}{a}\frac{z^3}{3} + c$$ where c is the integrating constant.

$$= \frac{2}{3a}z^3 + c$$

$$= \frac{2(ax+b)^{3/2}}{3a} + c$$

Question 5.
Mark (√) against the correct answer in each of the following:

$$\int \sec^2(7-4x)dx = ?$$

A. $\frac{1}{4}\tan(7-4x)+C$

B. $\frac{-1}{4}\tan(7-4x)+C$

C. 4 tan (7 – 4x) + C

D. - 4 tan (7 – 4x) + C

Answer:
Given = $\int \sec^2(7-4x)$

Let, 7 – 4x = z

⇒ -4dx = dz

So,

$$\int \sec^2(7-4x)dx$$

$$= \int \sec^2 z \frac{dz}{-4}$$

$$= -\frac{1}{4}\int \sec^2 z dz$$ where c is the integrating constant.

$$= -\frac{1}{4}\tan z + c$$

$$= -\frac{1}{4}\tan(7-4x) + c$$

Question 6.
Mark (√) against the correct answer in each of the following:

$$\int \cos 3x \, dx = ?$$

A. $-\frac{1}{3}\sin 3x + C$

B. $\frac{1}{3}\sin 3x + C$

C. 3 sin 3x + C

D. -3 sin 3x + C

Answer:
Given = $\int \cos 3x$

So, $\int \cos 3x dx = \frac{\sin 3x}{3} + c$ where c is the integrating constant.

Question 7.
Mark (√) against the correct answer in each of the following:

$$\int e^{(5-3x)} dx = ?$$

A. $-3e^{(5-3x)} + C$

B. $\frac{1}{3}e^{(5-3x)} + C$

C. $\frac{e^{(5-3x)}}{-3} + C$

D. none of these

Answer:
Given = $\int e^{(5-3x)}$

Let, 5 – 3x = z

⇒ -3dx = dz

So,

$$\int e^{(5-3x)}dx$$
$$= \int e^z \frac{dz}{-3}$$
$$= -\frac{1}{3}\int e^z dz$$ where c is the integrating constant.
$$= -\frac{1}{3}e^z + c$$
$$= -\frac{1}{3}e^{(5-3x)} + c$$

Question 8.
Mark (√) against the correct answer in each of the following:

$$\int e^{(3x+4)}dx = ?$$

A. $\frac{3}{(\log 2)}.2^{(3x+4)} + C$

B. $\frac{2^{(3x+4)}}{3(\log 2)+C}$

C. $\dfrac{2^{(3x+4)}}{2(\log 3)}+C$

D. none of these

Answer:
Given = $\int e^{(3x+4)}$

Let, 3x + 4 = z

⇒ 3dx = dz

So,

$$\int e^{(3x+4)}dx$$

$$=\int e^{z}\frac{dz}{3}$$

$$=\frac{1}{3}\int e^{z}dz$$

$$=\frac{1}{3}e^{z}+c$$

$$=\frac{1}{3}e^{(3x+4)}+c$$

where c is the integrating constant.

Question 9.
Mark (√) against the correct answer in each of the following:

$$\int \tan^{2}\frac{x}{2}dx=?$$

A. $\tan\dfrac{x}{2}-x+C$

B. $\tan\frac{x}{2}+x+C$

C. $2\tan\frac{x}{2}+x+C$

D. $2\tan\frac{x}{2}-x+C$

Answer:

Given = $\int \tan^2\frac{x}{2}$

Let, $\frac{x}{2}=z$

⇒ dx = 2dz

So,

$$\int \tan^2\frac{x}{2}dx$$
$$=2\int \tan^2 z dz$$
$$=2\int \frac{\sin^2 z}{\cos^2 z}dz$$
$$=2\int \frac{1-\cos^2 z}{\cos^2 z}dz$$
$$=2\int (\sec^2 z-1)dz$$

$$=2\left[\tan z - z\right]+c$$
$$=2[\tan\frac{x}{2}-\frac{x}{2}]+c$$ where c is the integrating constant.

Question 10.

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{1-\cos x}\, dx = ?$$

A. $-\sqrt{2}\cos\frac{x}{2} + C$

B. $-2\sqrt{2}\cos\frac{x}{2} + C$

C. $\frac{-1}{2}\cos\frac{x}{2} + C$

D. $\frac{-1}{\sqrt{2}}\cos\frac{x}{2} + C$

Answer:

Given = $\int \sqrt{1-\cos x}$

So,

$$\int \sqrt{1-\cos x}\,dx$$

$$= \int \sqrt{1-\cos x}\,\frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}}dx$$

$$= \int \frac{\sqrt{1-\cos^2 x}}{\sqrt{1+\cos x}}dx$$

$$= \int \frac{\sin x}{\sqrt{1+\cos x}}dx$$

Let 1 + cosx = u^2

So, -sinxdx = 2udu

$$-\int \frac{2u}{u}du = -2\int du = -2u + c = -2\sqrt{1+\cos x} + c$$

where c is the integrating constant.

Question 11.
Mark (√) against the correct answer in each of the following:

$$\int \sqrt{1+\sin x}\, dx = ?$$

A. $-\sqrt{2} \sin\left(\frac{\pi}{4}-\frac{x}{2}\right)+C$

B. $\sqrt{2} \sin\left(\frac{\pi}{4}-\frac{x}{2}\right)+C$

C. $-2\sqrt{2} \sin\left(\frac{\pi}{4}-\frac{x}{2}\right)+C$

D. none of these

Answer:
Given = $\int \sqrt{1+\sin x}$

So,

$$\int \sqrt{1+\sin x}\, dx$$

$$= \int \sqrt{1+\sin x}\, \frac{\sqrt{1-\sin x}}{\sqrt{1-\sin x}} dx$$

$$= \int \frac{\sqrt{1-\sin^2 x}}{\sqrt{1-\sin x}} dx$$

$$= \int \frac{\cos x}{\sqrt{1-\sin x}} dx$$

Let 1 - sinx = u^2

So, -cosxdx = 2udu

$$-\int \frac{2u}{u} du = -2\int du = -2u + c = -2\sqrt{1-\sin x} + c$$

where c is the integrating constant.

Question 12.
Mark (√) against the correct answer in each of the following:

$$\int \sin^3 x \, dx = ?$$

A. $-\frac{3}{4}\cos x + \frac{\cos 3x}{12} + C$

B. $\frac{3}{4}\cos x + \frac{\cos 3x}{12} + C$

C. $-\frac{3}{4}\cos x - \frac{\cos 3x}{12} + C$

D. none of these

Answer:
Given = $\int \sin^3 x dx$

So,

$$\int \sin^3 x dx$$
$$= \int \sin^2 x \sin x dx$$
$$= \int \left(1 - \cos^2 x\right) \sin x dx$$

Let cosx = u

So, -sinxdx = du

$$-\int\left(1-u^2\right)du$$

$$=-\int du+\int u^2 du$$

$$=-u+\frac{u^3}{3}+c$$

$$=-\cos x+\frac{\cos^3 x}{3}+c$$

where c is the integrating constant.

Question 13.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\log x}{x}dx=?$$

A. $\frac{1}{2}(\log x)^2+C$

B. $-\frac{1}{2}(\log x)^2+C$

C. $\frac{2}{x^2}+C$

D. $\frac{-2}{x^2}+C$

Answer:
Given = $\int \frac{\log x}{x}$

Let, logx = u

So, $\frac{1}{x}dx=du$

So,

$$\int \frac{\log x}{x} dx$$

$$= \int u du$$

$$= \frac{u^2}{2} + c$$

$$= \frac{(\log x)^2}{2} + c$$

where c is the integrating constant.

Question 14.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\sec^2(\log x)}{x} dx = ?$$

A. log (tan x) + C

B. - log (tan x) + C

C. tan (tan x) + C

D. - tan (log x) + C

Answer:

Given = $\int \frac{\sec^2(\log x)}{x}$

Let, logx = z

$$\Rightarrow \frac{dx}{x} = dz$$

So,

$$\int \frac{\sec^2(\log x)}{x} dx$$
$$= \int \sec^2 z dz$$
$$= \tan z + c$$
$$= \tan(\log x) + c$$

where c is the integrating constant.

Question 15.
Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{x(\log\ x)} dx = ?$$

A. $\log|x| + C$

B. $\frac{-2}{x^2} + C$

C. $(\log x)^2 + C$

D. $\log|\log x| + C$

Answer:
Given = $\int \frac{1}{x(\log x)}$

Let, logx = z

$\Rightarrow \frac{dx}{x} = dz$

So,

$$\int \frac{1}{x(\log x)} dx$$

$$= \int \frac{1}{z} dz$$

$$= \log z + c$$

$$= \log(\log x) + c$$

where c is the integrating constant.

Question 16.

Mark (√) against the correct answer in each of the following:

$$\int e^{x^3} x^2 dx = ?$$

A. $e^{x^3} + C$

B. $\frac{1}{3} e^{x^3} + C$

C. $\frac{1}{6} e^{x^3} + C$

D. none of these

Answer:

Given = $\int e^{x^3} x^2$

Let, $x^3 = z$

$\Rightarrow 3x^2 dx = dz$

$\Rightarrow x^2 dx = \frac{dz}{3}$

So,

$$\int e^{x^3} x^2 dx$$

$$= \frac{1}{3}\int e^z dz$$

$$= \frac{1}{3}e^z + c$$

$$= \frac{1}{3}e^{x^3} + c$$

where c is the integrating constant.

Question 17.
Mark (√) against the correct answer in each of the following:

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = ?$$

A. $e^{\sqrt{x}} + C$

B. $\frac{1}{2}e^{\sqrt{x}} + C$

C. $2e^{\sqrt{x}} + C$

D. none of these

Answer:
Given = $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}$

Let, $x = z^2$

$\Rightarrow dx = 2zdz$

So,

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$$= \int \frac{e^z}{z} 2z dz$$

$$= 2\int e^z dz$$

$$= 2e^z + c$$

$$= 2e^{\sqrt{x}} + c$$

where c is the integrating constant.

Question 18.
Mark (√) against the correct answer in each of the following:

$$\int \frac{e^{\tan^{-1} x}}{\left(1+x^2\right)} dx = ?$$

A. $\frac{e^{\tan^{-1} x}}{x} + C$

B. $e^{\tan^{-1} x} + C$

C. $e^x \tan^{-1} x + C$

D. none of these

Answer:

Given = $\int \frac{e^{\tan^{-1} x}}{\left(1+x^2\right)}$

Let, $\tan^{-1}x = z$

$\Rightarrow \frac{1}{1+x^2} dx = dz$

So,

$$\int \frac{e^{\tan^{-1} x}}{\left(1+x^2\right)} dx$$

$$= \int e^z dz$$

$$= e^z + c$$

$$= e^{\tan^{-1} x} + c$$

where c is the integrating constant.

Question 19.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx = ?$$

A. $2\cos\sqrt{x} + C$

B. $-2\cos\sqrt{x} + C$

C. $-\frac{\cos\sqrt{x}}{2} + C$

D. $\frac{\cos\sqrt{x}}{2} + C$

Answer:
Given = $\int \frac{\sin\sqrt{x}}{\sqrt{x}}$

Let, $x = z^2$

$\Rightarrow dx = 2zdz$

So,

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$$

$$= \int \frac{\sin z}{z} 2z dz$$

$$= 2\int \sin z dz$$

$$= -2\cos z + c$$

$$= -2\cos\sqrt{x} + c$$

where c is the integrating constant.

Question 20.
Mark (√) against the correct answer in each of the following:

$$\int \left(\sqrt{\sin x}\right) \cos x \, dx = ?$$

A. $\frac{2}{3}(\cos x)^{3/2} + C$

B. $\frac{3}{2}(\cos x)^{3/2} + C$

C. $\frac{2}{3}(\sin x)^{3/2} + C$

D. $\frac{3}{2}(\sin x)^{3/2} + C$

Answer:
Given = $\int \left(\sqrt{\sin x}\right) \cos x$

Let, sinx = z^2

⇒ cosxdx = 2zdz

So,

$$\int\left(\sqrt{\sin x}\right)\cos x dx$$

$$= 2\int z^2 dz$$

$$= 2\frac{z^3}{3} + c$$

$$= \frac{2}{3}\sin^{3/2} x + c$$

where c is the integrating constant.

Question 21.
Mark (√) against the correct answer in each of the following:

$$\int\frac{1}{\left(1+x^2\right)\sqrt{\tan^{-1} x}}$$

A. $\frac{1}{2}\log\left|\tan^{-1} x\right| + C$

B. $2\sqrt{\tan^{-1} x} + C$

C. $\frac{1}{2\sqrt{\tan^{-1} x}} + C$

D. none of these

Answer:

Given = $\int\frac{1}{\left(1+x^2\right)\sqrt{\tan^{-1} x}}$

Let, $\tan^{-1}x = z^2$

$\Rightarrow \frac{1}{1+x^2}dx = 2z dz$

So,

$$\int \frac{1}{\left(1+x^2\right)\sqrt{\tan^{-1} x}} dx$$

$$= \int \frac{2z}{z} dz$$

$$= 2\int dz$$

$$= 2z + c$$

$$= 2\sqrt{\tan^{-1} x} + c$$

where c is the integrating constant.

Question 22.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\cot x}{\log(\sin x)} dx = ?$$

A. log |cot x| + C

B. log |cot x cosec x| + C

C. log |log sin x| + C

D. none of these

Answer:
Given = $\int \frac{\cot x}{\log(\sin x)}$

Let, sinx = z

⇒ cosxdx = dz

So,

$$\int \frac{\cot x}{\log(\sin x)} dx$$
$$= \int \frac{\cos x}{\sin x \log(\sin x)} dx$$
$$= \int \frac{dz}{z \log z}$$

Let, logz = u

$$\Rightarrow \frac{1}{z} dz = du$$

So,

$$\int \frac{dz}{z \log z}$$
$$= \int \frac{du}{u}$$
$$= \log u + c$$
$$= \log|\log z| + c$$

where c is the integrating constant.

Question 23.
Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{x \cos^2(1 + \log x)} dx = ?$$

A. tan (1 + log x) + C

B. cot (1 + log x) + C

C. sec (1 + log x) + C

D. none of these

Answer:

Given = $\int \frac{1}{x\cos^2(1+\log x)}$

Let, 1 + logx = z

$\Rightarrow \frac{1}{x}dx = dz$

So,

$$\int \frac{1}{x\cos^2(1+\log x)}dx$$
$$= \int \frac{dz}{\cos^2 z}$$
$$= \int \sec^2 z dz$$
$$= \tan z + c$$
$$= \tan(1+\log x) + c$$

where c is the integrating constant.

Question 24.
Mark (√) against the correct answer in each of the following:

$$\int \frac{x^2 \tan^{-1} x^3}{(1+x^6)} dx = ?$$

A. $\frac{1}{3}(\tan^{-1} x^3) + C$

B. log $|\tan^{-1} x^3| + C$

C. $\frac{1}{6}(\tan^{-1} x^3)^2 + C$

D. none of these

Answer:

Given = $\int \frac{x^2 \tan^{-1} x^3}{(1+x^6)} dx$

Let, $\tan^{-1}x^3 = z$

$$\Rightarrow \frac{1}{1+x^6} \times 3x^2 dx = dz$$

$$\Rightarrow \frac{x^2}{1+x^6} dx = \frac{dz}{3}$$

So,

$$\frac{1}{3}\int z dz$$

$$= \frac{1}{3}\frac{z^2}{2} + c$$

$$= \frac{z^2}{6} + c$$

$$= \frac{(\tan^{-1} x^3)^2}{6} + c$$

where c is the integrating constant.

Question 25.
Mark (√) against the correct answer in each of the following:

$$\int \sec^5 x \tan x \, dx = ?$$

A. $5 \tan^5 x + C$

B. $\frac{1}{5} \tan^5 x + C$

C. $5 \log |\cos x| + C$

D. none of these

Answer:

Given = $\int \sec^5 x \tan x$

So, $\int \sec^5 \tan x dx = \int \sec^4 x (\sec x \tan x) dx$

Let, secx = z

⇒ secxtanxdx = dz

$$\int \sec^4 x (\sec x \tan x) dx$$
$$= \int z^4 dz$$
$$= \frac{z^5}{5} + c$$
$$= \frac{\sec^5 x}{5} + c$$

where c is the integrating constant.

Question 26.

Mark (√) against the correct answer in each of the following:

$\int \operatorname{cosec}^3 (2x+1) \cot (2x+1) dx = ?$

A. $\frac{1}{4} \operatorname{cosec}^4 (2x+1) + C$

B. $-\frac{1}{3} \operatorname{cosec}^3 (2x+1) + C$

C. $-\frac{1}{6} \operatorname{cosec}^3 (2x+1) + C$

D. $\frac{1}{2} \operatorname{cosec} (2x+1) \cot (2x+1) + C$

Answer:

Given = $\int \operatorname{cosec}^3(2x+1)\cot(2x+1)$

So,

$$\int \operatorname{cosec}^3(2x+1)\cot(2x+1)dx$$
$$= \int \operatorname{cosec}^2(2x+1)\operatorname{cosec}(2x+1)\cot(2x+1)dx$$

Let, cosec(2x + 1) = z

⇒ -2cosec(2x + 1)cot(2x + 1)dx = dz

$$\int \operatorname{cosec}^2(2x+1)\operatorname{cosec}(2x+1)\cot(2x+1)dx$$
$$= \int z^2 \frac{dz}{-2} =$$
$$= -\frac{1}{2}\frac{z^3}{3} + c$$
$$= -\frac{\operatorname{cosec}^6(2x+1)}{6} + c$$

where c is the integrating constant.

Question 27.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\tan\left(\sin^{-1} x\right)}{\sqrt{1-x^2}} dx = ?$$

A. log |sec ($\sin^{-1}$ x)| + C

B. log |cos ($\sin^{-1}$ x)| + C

C. tan ($\sin^{1}$ x) + C

D. none of these

Answer:

Given = $\int \frac{\tan\left(\sin^{-1} x\right)}{\sqrt{1-x^2}}$

Let, $\sin^{-1}x = z$

$$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = dz$$

So,

$$\int \frac{\tan\left(\sin^{-1} x\right)}{\sqrt{1-x^2}} dx$$
$$= \int \tan z dz$$
$$= \log\left|\sec z\right| + c$$
$$= \log\left|\sec\left(\sin^{-1} x\right)\right| + c$$

where c is the integrating constant.

Question 28.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\tan\left(\log x\right)}{x} dx = ?$$

A. x tan (log x) + C

B. log |tan x| + C

C. log |cos (log x)| + C

D. - log |cos (log x)| + C

Answer:
Given = $\int \frac{\tan\left(\log x\right)}{x}$

Let, logx = z

$$\Rightarrow \frac{1}{x}dx = dz$$

So,

$$\int \frac{\tan(\log x)}{x}dx$$
$$= \int \tan z dz$$
$$= \log|\sec z| + c$$
$$= \log|\sec(\log x)| + c$$
$$= -\log|\cos(\log x)| + c$$

where c is the integrating constant.

Question 29.
Mark (√) against the correct answer in each of the following:

$$\int e^x \cot(e^x)dx = ?$$

A. cot (e^x) + C

B. log |sin e^x| + C

C. log |cosec e^x| + C

D. none of these

Answer:
Given = $\int e^x \cot(e^x)dx$

Let, $e^x = z$

$\Rightarrow e^x dx = dz$

So,

$$\int e^x \cot\left(e^x\right)dx$$

$$= \int \cot z dz$$

$$= \log\left|\sin z\right| + c$$

$$= \log\left|\sin\left(e^x\right)\right| + c$$

where c is the integrating constant.

Question 30.
Mark (√) against the correct answer in each of the following:

$$\int \frac{e^x}{\sqrt{1+e^x}} dx = ?$$

A. $2\sqrt{1+e^x} + C$

B. $\frac{1}{2}\sqrt{1+e^x} + C$

C. $\frac{1}{\sqrt{1+e^x}} + C$

D. none of these

Answer:

Given = $\int \frac{e^x}{\sqrt{1+e^x}}$

Let, $1 + e^x = z^2$

$\Rightarrow e^x dx = 2z dz$

So,

$$\int \frac{e^x}{\sqrt{1+e^x}} dx$$

$$= \int \frac{2zdz}{z}$$

$$= 2\int dz$$

$$= 2z + c$$

$$= 2\sqrt{1+e^x} + c$$

where c is the integrating constant.

Question 31.
Mark (√) against the correct answer in each of the following:

$$\int \frac{x}{\sqrt{1-x^2}} dx = ?$$

A. $\sin^{-1} x + C$

B. $\sin^{-1} \sqrt{x} + C$

C. $\sqrt{1-x^2} + C$

D. $-\sqrt{1-x^2} + C$

Answer:

Given = $\int \frac{x}{\sqrt{1-x^2}} dx$

Let, $1 - x^2 = z^2$

⇒ $-2xdx = 2zdz$

So,

$$\int \frac{x}{\sqrt{1-x^2}}dx$$

$$= -\int \frac{zdz}{z}$$

$$= -\int dz$$

$$= -z + c$$

$$= -\sqrt{1-x^2} + c$$

where c is the integrating constant.

Question 32.

Mark (√) against the correct answer in each of the following:

$$\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx = ?$$

A. tan (xe^x) + C

B. cot (xe^x) + C

C. ex^x tan x + C

D. none of these

Answer:

Given = $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$

Let, $xe^x = z$

⇒ $e^x(1 + x)dx = dz$

So,

$$\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$$

$$= \int \frac{dz}{\cos^2 z}$$

$$= \int \sec^2 z dz$$

$$= \tan z + c$$

$$= \tan(xe^x) + c$$

where c is the integrating constant.

Question 33.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(e^x + e^{-x})} = ?$$

A. cot-1 (e^x) + C

B. tan-1 (e^x) + C

C. log |e^x + 1| + C

D. none of these

Answer:
Given =

$$\int \frac{dx}{(e^x + e^{-x})}$$

$$= \int \frac{e^x}{(e^x+1)}dx$$

Let, $e^x + 1 = z$

⇒ $e^x dx = dz$

So,

$$\int \frac{e^x dx}{(e^x + 1)}$$

$$= \int \frac{dz}{z}$$

$$= \log|z| + c$$

$$= \tan|e^x + 1| + c$$

where c is the integrating constant.

Question 34.
Mark (√) against the correct answer in each of the following:

$$\int \frac{2^x}{1-4^x} dx = ?$$

A. $\sin^{-1}(2^x) + C$

B. $(\log e^2) \sin^{-1}(2^x) + C$

C. $(\log e^2) \cos^{-1}(2^x) + C$

D. $\log_2 e) \sin^{-1}(2^x) + C$

Answer:
Given =

$$\int \frac{2^x dx}{1-4^x}$$

$$= \int \frac{2^x}{1-(2^x)^2} dx$$

Let, $2^x = z$

$\Rightarrow 2^x(\log 2)dx = dz$

So,

$$\int \frac{2^x dx}{1-\left(2^x\right)^2}$$

$$= \frac{1}{\log 2}\int \frac{dz}{1-z^2}$$

$$= \frac{1}{\log 2}\sin^{-1} z + c$$

$$= \frac{\sin^{-1} 2x}{\log 2} + c$$

where c is the integrating constant.

Question 35.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(e^x - 1\right)} = ?$$

A. log $|e^x - 1| + C$

B. log $|1 - e^{-x}| + C$

C. log $|e^x - 1| + C$

D. none of these

Answer:
Given =

$$\int \frac{dx}{e^x - 1}$$

$$= -\int \frac{-1 + e^x - e^x}{e^x - 1} dx$$

$$= -\int \frac{e^x - 1}{e^x - 1} dx + \int \frac{e^x}{e^x - 1} dx$$

$$= -\int dx + \int \frac{e^x}{e^x - 1} dx$$

Let, $e^x - 1 = z$

$\Rightarrow e^x dx = dz$

So,

$$-\int dx + \int \frac{e^x}{e^x - 1} dx$$
$$= -x + \int \frac{dz}{z}$$
$$= -x + \log z + c$$
$$= -x + \log\left|e^x - 1\right| + c$$

where c is the integrating constant.

Question 36.
Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{\left(\sqrt{x} + x\right)} dx = ?$$

A. $\log\left|1 + \sqrt{x}\right| + C$

B. $2\log\left|1 + \sqrt{x}\right| + C$

C. $\frac{1}{\sqrt{x}} \tan^{-1}\sqrt{x} + C$

D. none of these

Answer:
Given =

$$\int \frac{dx}{\left(\sqrt{x} + x\right)}$$
$$= \int \frac{1}{\sqrt{x}} \frac{1}{\left(1 + \sqrt{x}\right)} dx$$

Let, $1+\sqrt{x}=z$

$$\Rightarrow \frac{1}{2\sqrt{x}}dx = dz$$

So,

$$\int \frac{1}{\sqrt{x}} \frac{1}{\left(1+\sqrt{x}\right)} dx$$
$$= 2\int \frac{dz}{z}$$
$$= 2\log|z| + c$$
$$= 2\tan\left|1+\sqrt{x}\right| + c$$

where c is the integrating constant.

Question 37.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(1+\sin x)} = ?$$

A. tan x + sec x + C

B. tan x – sec x + C

C. $\frac{1}{2}\tan\frac{x}{2}+C$

D. none of these

Answer:
Given

$$\int \frac{dx}{1+\sin x}$$

$$= \int \frac{dx}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \int \frac{dx}{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}$$

$$= \int \frac{\sec^2 \frac{x}{2} dx}{\left(\tan\frac{x}{2} + 1\right)^2}$$

Let, $\tan\frac{x}{2} + 1 = z$

$$\Rightarrow \frac{1}{2}\sec^2 \frac{x}{2} dx = dz$$

So,

$$\int \frac{2dz}{z^2}$$

$$= -\frac{2}{z} + c$$

$$= -\frac{2}{\tan\frac{x}{2} + 1} + c$$

where c is the integrating constant.

Question 38.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{(1+\sin x)} dx = ?$$

A. x + tan x – sec x + C

B. x – tan x – sec x + C

C. x – tan x + sec x + C

D. none of these

Answer:
Given

$$\int \frac{\sin x}{1+\sin x} dx$$

$$= \int dx - \int \frac{dx}{1+\sin x}$$

$$= x - \int \frac{dx}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2\sin \frac{x}{2} \cos \frac{x}{2}}$$

$$= x - \int \frac{dx}{\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2}$$

$$= x - \int \frac{\sec^2 \frac{x}{2} dx}{\left(\tan \frac{x}{2} + 1\right)^2}$$

Let, $\tan \frac{x}{2} + 1 = z$

$$\Rightarrow \frac{1}{2}\sec^2 \frac{x}{2} dx = dz$$

So,

$$x - \int \frac{2dz}{z^2}$$

$$= x + \frac{2}{z} + c$$

$$= x + \frac{2}{\tan\frac{x}{2} + 1} + c$$

where c is the integrating constant.

Question 39.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{(1 - \sin x)} dx = ?$$

A. - x + sec x – tan x + C

B. x + cos x – sin x + C

C. - log |1 – sin x| + C

D. none of these

Answer:
Given

$$\int \frac{\sin x}{1 - \sin x} dx$$

$$= -\int dx + \int \frac{dx}{1 - \sin x}$$

$$= -x + \int \frac{dx}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= -x + \int \frac{dx}{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^2}$$

$$= -x + \int \frac{\sec^2 \frac{x}{2} dx}{\left(\tan \frac{x}{2} - 1\right)^2}$$

Let, $\tan \frac{x}{2} - 1 = z$

$\Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dz$

So,

$$-x + \int \frac{2dz}{z^2}$$
$$= -x - \frac{2}{z} + c$$
$$= -x - \frac{2}{\tan \frac{x}{2} + 1} + c$$

where c is the integrating constant.

Question 40.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(1 + \cos x)} = ?$$

A. $\frac{1}{2} \tan \frac{x}{2} + C$

B. $-\cot \frac{x}{2} + C$

C. $\tan \frac{x}{2} + C$

D. none of these

Answer:
Given

$$\int \frac{dx}{1+\cos x}$$

$$= \int \frac{dx}{1+2\cos^2\frac{x}{2}-1}$$

$$= \frac{1}{2}\int \frac{dx}{\cos^2\frac{x}{2}}$$

$$= \frac{1}{2}\int \sec^2\frac{x}{2}dx$$

$$= \frac{1}{2}2\tan\frac{x}{2}+c$$

$$= \tan\frac{x}{2}+c$$

where c is the integrating constant.

Question 41.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(1-\cos x)} = ?$$

A. $\frac{1}{(x-\sin x)}+C$

B. log |x – sin x| + C

C. $\log\left|\tan\frac{x}{2}\right|+C$

D. $-\cot\frac{x}{2}+C$

Answer:
Given

$$\int \frac{dx}{1-\cos x}$$

$$= \int \frac{dx}{1-1+2\sin^2 \frac{x}{2}}$$

$$= \frac{1}{2}\int \frac{dx}{\sin^2 \frac{x}{2}}$$

$$= \frac{1}{2}\int \operatorname{cosec}^2 \frac{x}{2} dx$$

$$= -\frac{1}{2} 2\cot\frac{x}{2} + c$$

$$= -\cot\frac{x}{2} + c$$

where c is the integrating constant.

Question 42.

Mark (√) against the correct answer in each of the following:

$$\int \left\{ \frac{1-\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)} \right\} dx = ?$$

A. $2\log\left|\sec\frac{x}{2}\right| + C$

B. $2\log\left|\cos\frac{x}{2}\right| + C$

C. $2\log\left|\sec\left(\frac{\pi}{4}-\frac{x}{2}\right)\right| + C$

D. $2\log\left|\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)\right| + C$

Answer:
Given

$$\int \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\,dx$$

$$= \int \frac{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}\,dx$$

$$= \int \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\,dx$$

Let, $\cos\frac{x}{2}+\sin\frac{x}{2}=z$

$$\Rightarrow \left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)dx = dz$$

So,

$$\int \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\,dx$$

$$= \int \frac{dz}{z}$$

$$= \log z + c$$

$$= \log\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+c$$

where c is the integrating constant.

Question 43.
Mark (√) against the correct answer in each of the following:

$$\int \sqrt{e^x}\, dx = ?$$

A. $\sqrt{e^x} + C$

B. $2\sqrt{e^x} + C$

C. $\frac{1}{2}\sqrt{e^x} + C$

D. none of these

Answer:
Given

$$\int \sqrt{e^x} dx$$

$$= \int \left(e^x\right)^{\frac{1}{2}} dx$$

$$= \int e^{\frac{1}{2}x} dx$$

$$= 2e^{\frac{1}{2}x} + c$$

$$= 2\sqrt{e^x} + c$$

where c is the integrating constant.

Question 44.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos x}{(1+\cos x)} dx = ?$$

A. $x + \tan\frac{x}{2} + C$

B. $-x + \tan\frac{x}{2} + C$

C. $x - \tan\frac{x}{2} + C$

D. none of these

Answer:
Given

$$\int \frac{\cos x dx}{1+\cos x}$$
$$= \int \frac{1+\cos x - 1}{1+\cos x} dx$$
$$= \int dx - \int \frac{dx}{1+\cos x}$$
$$= x - \tan\frac{x}{2} + c$$

[From Question no. 40] where c is the integrating constant.

Question 45.
Mark (√) against the correct answer in each of the following:

$$\int \sec^2 x \ \text{cosec}^2 x \, dx = ?$$

A. tan x – cot x + C

B. tan x + cot x + C

C. - tan x + cot x + C

D. none of these

Answer:
Given

$$\int \sec^2 x \operatorname{cosec}^2 x dx$$

$$= \int \frac{1}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

$$= \tan x - \cot x + c$$

where c is the integrating constant.

Question 46.
Mark (√) against the correct answer in each of the following:

$$\int \frac{(1 - \cos 2x)}{(1 + \cos 2x)} dx = ?$$

A. tan x + x + C

B. tan x – x + C

C. - tan x + x + C

D. none of these

Answer:
Given

$$\int \frac{(1-\cos 2x)}{(1+\cos 2x)} dx$$

$$= \int \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} dx$$

$$= \int \tan^2 \frac{x}{2} dx$$

$$= \int \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

$$= 2\tan \frac{x}{2} - x + c$$

where c is the integrating constant.

Question 47.
Mark (√) against the correct answer in each of the following:

$$\int \frac{(1+\cos x)}{(1-\cos x)} dx = ?$$

A. $-2\cot \frac{x}{2} - x + C$

B. $-2\cot \frac{x}{2} + x + C$

C. $2\cot \frac{x}{2} + x + C$

D. none of these

Answer:
Given

$$\int \frac{(1+\cos 2x)}{(1-\cos 2x)} dx$$

$$= \int \frac{2\cos^2 \frac{x}{2}}{2\sin^2 \frac{x}{2}} dx$$

$$= \int \cot^2 \frac{x}{2} dx$$

$$= \int \left(\operatorname{cosec}^2 \frac{x}{2} - 1 \right) dx$$

$$= -2\cot \frac{x}{2} - x + c$$

where c is the integrating constant.

Question 48.
Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{\sin^2 x \cos^2 x} dx = ?$$

A. tan x + cot x + C

B. tan x – cot x + C

C. - tan x + cot x + C

D. none of these

Answer:
Given

$$\int \frac{1}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

$$= \tan x - \cot x + c$$

where c is the integrating constant.

Question 49.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = ?$$

A. cot x + tan x + C

B. - cot x + tan x + C

C. cot x – tan x + C

D. - cot x – tan x + C

Answer:
Given

$$\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$$
$$= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} dx$$
$$= \int \frac{1}{\sin^2 x} dx - \int \frac{1}{\cos^2 x} dx$$
$$= \int \operatorname{cosec}^2 x dx - \int \sec^2 x dx$$

$$= -\tan x - \cot x + c$$

where c is the integrating constant.

Question 50.
Mark (√) against the correct answer in each of the following:

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx = ?$$

A. sin x + x cos α + C

B. 2sin x + x cos α + C

C. 2 sin x + 2x cos α + C

D. none of these

Answer:
Given

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx$$

$$= \int \frac{-2\sin\left(\frac{2x+2\alpha}{2}\right)\sin\left(\frac{2x-2\alpha}{2}\right)}{-2\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \int \frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \int \frac{2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\times 2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= 2\int 2\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$= 2\int \cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\left(\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right)$$

$$= 2\int (\cos x + \cos \alpha) dx$$

$$= 2[\sin x + x\cos\alpha] + c$$

where c is the integrating constant.

Question 51.

Mark (√) against the correct answer in each of the following:

$$\int \tan^{-1}\left\{\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}\right\} dx = ?$$

A. $2x^2 + C$

B. $\frac{x^2}{2} + C$

C. $\frac{2}{\left(1+x^2\right)} + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \;; 1 + \cos 2x = 2\cos^2 x \;;\; 1 - \cos 2x = 2\sin^2 x$

Therefore ,

$$\Rightarrow \int \tan^{-1}\sqrt{\frac{1-\cos 2x}{1+\cos 2x}} dx = \int \tan^{-1}\sqrt{\frac{2\sin^2 x}{2\cos^2 x}} dx = \int \tan^{-1}\tan x \, dx$$

$$\Rightarrow \int x \, dx = \frac{x^2}{2} + c$$

Question 52.

Mark (√) against the correct answer in each of the following:

$$\int \tan^{-1}(\sec x + \tan x) dx = ?$$

A. $\frac{\pi x}{4} + \frac{x^2}{4} + C$

B. $\frac{\pi x}{4} - \frac{x^2}{4} + C$

C. $\frac{1}{\left(1+x^2\right)} + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $1 + \sin x = (\cos\frac{x}{2} + \sin\frac{x}{2})^2$

$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$; $\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$

Therefore ,

$$\Rightarrow \int \tan^{-1}(\sec x + \tan x)\, dx = \int \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right) dx$$

$$\Rightarrow \int \tan^{-1}\frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}} dx = \int \tan^{-1}\frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{(\cos\frac{x}{2}+\sin\frac{x}{2})(\cos\frac{x}{2}-\sin\frac{x}{2})} dx$$

$$\Rightarrow \int \tan^{-1}\frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^1}{(\cos\frac{x}{2}-\sin\frac{x}{2})} dx = \int \tan^{-1}\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}} dx$$

(Multiply by $\sec\frac{x}{2}$ in numerator and denominator)

$$\Rightarrow \int \tan^{-1}\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}} dx = \int \tan^{-1}\frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{\tan\frac{\pi}{4}-\tan\frac{\pi}{4}\tan\frac{x}{2}} dx = \int \tan^{-1}\tan(\frac{\pi}{4}+\frac{x}{2})\, dx$$

$$\Rightarrow \int (\frac{\pi}{4}+\frac{x}{2}) dx = \frac{\pi x}{4} + \frac{x^2}{4} + c$$

Question 53.

Mark (√) against the correct answer in each of the following:

$$\int \frac{(1+\sin x)}{(1-\sin x)} dx = ?$$

A. 2 tan x + x – 2sec x + C

B. 2 tan x – x + 2 sec x + C

C. 2 tan x – x – 2sec x + C

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int sec^2 x dx = tan\ x$

Therefore ,

$$\Rightarrow \int \frac{1+\sin x(1+\sin x)}{1-\sin x(1+\sin x)} dx$$

$$\Rightarrow \int \frac{(1+\sin x)^2}{1-sin^2 x} dx = \int \frac{1+sin^2 x+2\sin x}{cos^2 x} dx$$

$$\Rightarrow \int \frac{1}{cos^2 x} dx + 2\int \frac{\sin x}{cos^2 x} dx + \int \frac{sin^2 x}{cos^2 x} dx$$

$$\Rightarrow \int sec^2 x\, dx + 2\int \frac{\sin x}{cos^2 x} dx + \int tan^2 x\, dx$$

$$\Rightarrow \int sec^2 x\, dx + 2\int \frac{\sin x}{cos^2 x} dx + \int (-1 + sec^2 x)\, dx$$

$$\Rightarrow 2\int sec^2 x\, dx + 2\int \frac{\sin x}{cos^2 x} dx - \int 1\, dx$$

Put cos x = t

Therefore -> sin x dx = - dt

$$\Rightarrow 2\tan x - 2\int \frac{dt}{t^2} - x + c$$

$$\Rightarrow 2\tan x + 2\frac{1}{t} - x + c$$

$$\Rightarrow 2\tan x + 2\sec x - x + c$$

Question 54.

Mark (√) against the correct answer in each of the following:

$$\int \frac{x^4}{(1+x^2)} dx = ?$$

A. $\frac{x^3}{3}+x+\tan^{-1}x+C$

B. $\frac{-x^3}{3}+x-\tan^{-1}x+C$

C. $\frac{x^3}{3}-x+\tan^{-1}x+C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1}+c$; $\int sec^2 x dx = tan\,x$; $\int \frac{1}{1+x^2}dx = \tan^{-1}x + c$

Therefore ,

$\Rightarrow \int \frac{x^4+1-1}{1+x^2}dx$

$\Rightarrow \int \frac{x^4-1}{1+x^2}dx + \int \frac{1}{1+x^2}dx = \int \frac{(1+x^2)(x^2-1)}{1+x^2}dx + \int \frac{1}{1+x^2}dx$

$\Rightarrow \int x^2 - 1\,dx + \int \frac{1}{1+x^2}dx$

$\Rightarrow \frac{x^3}{3} - x + \tan^{-1}x + c$

Question 55.

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)}dx = ?$$

A. x cos 2α - sin 2α . log |sin (x + α)| + C

B. x cos 2α + sin 2α . log|sin (x + α)| + C

C. x cos 2α + sin α . log |sin (x + α)| + C

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{\sin(x+\propto-2\propto)}{\sin(x+\propto)} dx$

$\Rightarrow \int \frac{\sin(x+\propto)\cos(-2\propto)+\cos(x+\propto)\sin(-2\propto)}{\sin(x+\propto)} dx$

$\Rightarrow \int \cos(2\propto)\, dx - \sin 2\propto \int \cot(x+\propto)\, dx$

$\Rightarrow \cos(2\propto)\, x - \sin 2\propto \log|\sin(x+\propto)| + c$

Question 56.
Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{\left(\sqrt{x+3}-\sqrt{x+2}\right)} dx = ?$$

A. $\frac{2}{3}(x+3)^{3/2} - \frac{2}{3}(x+3)^{3/2} + C$

B. $\frac{2}{3}(x+3)^{3/2} + \frac{2}{3}(x+3)^{3/2} + C$

C. $\frac{3}{2}(x+3)^{3/2} - \frac{3}{2}(x+3)^{3/2} + C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{(\sqrt{x+3}+\sqrt{x+2})}{(\sqrt{x+3}-\sqrt{x+2})(\sqrt{x+3}+\sqrt{x+2})} dx$ (Rationalizing the denominator)

$\Rightarrow \int (\sqrt{x+3} + \sqrt{x+2}) dx$

$\Rightarrow \int \sqrt{x+3}\, dx + \int \sqrt{x+2}\, dx$

$\Rightarrow \frac{2(x+3)^{\frac{3}{2}}}{3} + \frac{2(x+2)^{\frac{3}{2}}}{3} + c$

Question 57.
Mark (√) against the correct answer in each of the following:

$$\int \frac{(1+\tan x)}{(1-\tan x)} dx = ?$$

A. - log |cos x – sin x| + C

B. log |cos x – sin x| + C

C. log |cos x + sin x | + C

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}} dx$ (Rationalizing the denominator)

$\Rightarrow \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$

Put cos x - sin x = t

(- sin x - cos x) dx = dt

(sin x + cos x) dx = -dt

$\Rightarrow \int \frac{-dt}{t} = -\log t + c$

$\Rightarrow -\log|\cos x - \sin x| + c$

Question 58.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{x\sqrt{x^6 - 1}} = ?$$

A. $\frac{1}{3}\sec^{-1} x^3 + C$

B. $\frac{1}{3}\text{cosec}^{-1} x^3 + C$

C. $\frac{1}{3}\cot^{-1} x^3 + C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

Therefore ,

Put $x^3 = t$,$3x^2 dx = dt$

$\Rightarrow \int \frac{dt}{x \times 3x^2 \sqrt{t^2-1}} = \int \frac{dt}{3t\sqrt{t^2-1}}$

$\Rightarrow \frac{1}{3} \int \frac{dt}{t\sqrt{t^2-1}}$

$\Rightarrow \frac{1}{3} \sec^{-1} t + c$

$\Rightarrow \frac{1}{3} \sec^{-1} x^3 + c$

Question 59.
Mark (√) against the correct answer in each of the following:

$$\int \frac{3x^2}{\left(1+x^6\right)} dx = ?$$

A. $\sin^{-1} x^3 + C$

B. $\cos^{-1} x^3 + C$

C. $\tan^{-1} x^3 + C$

D. $\cot^{-1} x^3 + C$

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^3 = t3x^2 dx = dt$

$\Rightarrow \int \frac{dt}{1+t^2}$

$\Rightarrow \tan^{-1} t + c$

$\Rightarrow \tan^{-1} x^3 + c$

Question 60.
Mark (√) against the correct answer in each of the following:

$$\int \left\{(2x+1)\sqrt{x^2+x+1}\right\} dx = ?$$

A. $\frac{3}{2}\left(x^2+x+1\right)^{3/2}+C$

B. $\frac{2}{3}\left(x^2+x+1\right)^{3/2}+C$

C. $\frac{3}{2}\left(2x+1\right)^{3/2}+C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1}+c$; $\int \frac{1}{x\sqrt{x^2-1}}dx = \sec^{-1}x + c$

Therefore ,

Put $x^2+x+1=t$, $(2x+1)dx = dt$

$\Rightarrow \int \sqrt{t}dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}}+c$

$\Rightarrow \frac{2}{3}t^{\frac{3}{2}}+c$

$\Rightarrow \frac{2}{3}(x^2+x+1)^{\frac{3}{2}}+c$

Question 61.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left\{\sqrt{2x+3}+\sqrt{2x+3}\right\}}=?$$

A. $\frac{1}{18}\left(2x+3\right)^{3/2}+\frac{1}{18}\left(2x-3\right)^{3/2}+C$

B. $\frac{1}{18}\left(2x+3\right)^{3/2}-\frac{1}{18}\left(2x-3\right)^{3/2}+C$

C. $\frac{1}{12}\left(2x+3\right)^{3/2}-\frac{1}{12}\left(2x-3\right)^{3/2}+C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int cot\ x = log\ (sin\ x) + c$

Therefore ,

$\Rightarrow \int \frac{(\sqrt{2x+3}-\sqrt{2x-3})}{(\sqrt{2x+3}+\sqrt{2x-3})(\sqrt{2x+3}-\sqrt{2x-3})} dx$ (Rationalizing the denominator)

$\Rightarrow \int \frac{\sqrt{2x+3}-\sqrt{2x-3}}{6} dx$

$\Rightarrow \frac{1}{6} \int \sqrt{2x+3}\, dx - \frac{1}{6} \int \sqrt{2x-3}\, dx$

$\Rightarrow \frac{2(2x+3)^{\frac{3}{2}}}{3\times6\times2} - \frac{2(2x-3)^{\frac{3}{2}}}{3\times6\times2} + c$

$\Rightarrow \frac{(2x+3)^{\frac{3}{2}}}{18} - \frac{(2x-3)^{\frac{3}{2}}}{18} + c$

Question 62.

Mark (√) against the correct answer in each of the following:

$\int \tan x\ dx = ?$

A. log |cos x| + C

B. - log |cos x| + C

C. log |sin x| + C

D. - log |sin x| + C

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \frac{\sin x}{\cos x} dx$

Put cos x = t -sin x dx = dt

$\Rightarrow \int \frac{-dt}{t}$

$\Rightarrow -\log t + c$

$\Rightarrow -\log|\cos x| + c$

Question 63.
Mark (√) against the correct answer in each of the following:

$\int \sec x \, dx = ?$

A. log |sec x - tan x| + C

B. - log |sec x + tan x| + C

C. log |sec x + tan x| + C

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} dx$

$\Rightarrow \int \frac{sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$

Put $\sec x + \tan x = t$, $(sec^2 x + \sec x \tan x)$dx = dt

$\Rightarrow \int \frac{dt}{t}$

$\Rightarrow \log t + c$

$\Rightarrow \log|\sec x + \tan x| + c$

Question 64.
Mark (√) against the correct answer in each of the following:

$\int \operatorname{cosec} x \, dx = ?$

A. log |cosec x - cot x| + C

B. - log |cosec x - cot x| + C

C. log |cosec x + cot x| + C

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\Rightarrow \int \operatorname{cosec} x \frac{\operatorname{cosec} x - \cot x}{\operatorname{cosec} x - \cot x} dx$

$\Rightarrow \int \frac{cosec^2 x - \operatorname{cosec} x \cot x}{\operatorname{cosec} x - \cot x} dx$

Put $\operatorname{cosec} x - \cot x = t$, $(cosec^2 x - \operatorname{cosec} x \cot x)$dx = dt

$\Rightarrow \int \frac{dt}{t}$

$\Rightarrow \log t + c$

$\Rightarrow \log|\operatorname{cosec} x - \cot x| + c$

Question 65.
Mark (√) against the correct answer in each of the following:

$\int \frac{(1+\sin x)}{(1+\cos x)} dx = ?$

A. $\tan\frac{x}{2} + 2\log\left|\cos\frac{x}{2}\right| + C$

B. $-\tan\frac{x}{2} + 2\log\left|\cos\frac{x}{2}\right| + C$

C. $\tan\frac{x}{2} - 2\log\left|\cos\frac{x}{2}\right| + C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int sec^2 x dx = tan\ x$

Therefore ,

$\Rightarrow \int \frac{1+\sin x}{2cos^2\frac{x}{2}} dx$

$\Rightarrow \int \frac{1}{2cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2cos^2\frac{x}{2}} dx = \frac{1}{2}\int sec^2\frac{x}{2} dx + \int \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} dx$

$\Rightarrow \frac{1}{2}\tan\frac{x}{2} \times 2 + \int tan\ \frac{x}{2} dx$

$$\Rightarrow \tan\frac{x}{2} + 2\left(-\log\cos\frac{x}{2}\right) + c$$

$$\Rightarrow \tan\frac{x}{2} - 2\log|\cos\frac{x}{2}| + c$$

Question 66.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\tan\ x}{(\sec\ x + \cos\ x)} dx = ?$$

A. $\tan^{-1}$ (cos x) + C

B. - $\tan^{-1}$ (cos x) + C

C. $\cot^{-1}$ (cos x) + C

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\int sec^2 x dx = tan\ x$

Therefore ,

$$\Rightarrow \int \frac{\sec x \tan x}{sec^2 x + 1} dx$$

Put sec x = t (sec x tan x) dx = dt

$$\Rightarrow \int \frac{dt}{1+t^2} = \tan^{-1} t + c$$

$$\Rightarrow \tan^{-1} \sec x + c$$

$$\Rightarrow -\tan^{-1}(\cos x) + c$$

Question 67.
Mark (√) against the correct answer in each of the following:

$$\int \sqrt{\frac{1+x}{1-x}} dx = ?$$

A. $\sin^{-1}x + \sqrt{1-x^2} + C$

B. $\sin^{-1}x + \left(1+x^2\right) + C$

C. $\sin^{-1}x - \sqrt{1-x^2} + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \sec^2 x dx = \tan x$

Therefore ,

$$\Rightarrow \int \sqrt{\frac{(1+x)^2}{(1+x)(1-x)}} dx$$

$$\Rightarrow \int \frac{1+x}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{1-x^2}} dx + \int \frac{x}{\sqrt{1-x^2}} dx$$

Put $1 - x^2 = t$ -2x dx = dt

$$\Rightarrow \sin^{-1}x - \frac{1}{2}\int \frac{1}{\sqrt{t}} dt + c$$

$$\Rightarrow \sin^{-1}x - \frac{1}{2}\frac{\sqrt{t}}{\frac{1}{2}} + c$$

$$\Rightarrow \sin^{-1}x - \sqrt{t} + c \quad = \quad \sin^{-1}x - \sqrt{1-x^2} + c$$

Question 68.

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{x^2} e^{-1/x} dx = ?$$

A. $e^{-1/x} + C$

B. $-e^{-1/x} + C$

C. $\dfrac{e^{-1/x}}{x}+C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int sec^2 x dx = tan\ x$

Therefore ,

Put $-\frac{1}{x} = t \frac{1}{x^2} dx = dt$

$\Rightarrow \int e^t dt$

$\Rightarrow e^t + c$

$\Rightarrow e^{-\frac{1}{x}} + c$

Question 69.
Mark (√) against the correct answer in each of the following:

$$\int \frac{x^3}{\left(1+x^8\right)} dx = ?$$

A. $\tan^{-1} x^4 + C$

B. $4 \tan^{-1} x^4 + C$

C. $\frac{1}{4} \tan^{-1} x^4 + C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $x^4 = t 4x^3 dx = dt$

$\Rightarrow \frac{1}{4}\int \frac{1}{1+t^2}\,dt$

$\Rightarrow \frac{1}{4}\tan^{-1} t + c$

$\Rightarrow \frac{1}{4}\tan^{-1} x^4 + c$

Question 70.
Mark (√) against the correct answer in each of the following:

$$\int \frac{(x+1)(x+\log x)^2}{x}\,dx = ?$$

A. $\frac{1}{3}(x+\log\ x)^3 + C$

B. $\frac{x^2}{2} + x + C$

C. $\frac{x^3}{3} + \frac{x^2}{2} + x + C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + c$

Therefore ,

Put $x^1 + \log x = t(1+\frac{1}{x})dx = dt \Rightarrow (\frac{x+1}{x})dx = dt$

$\Rightarrow \int t^2 dt$

$\Rightarrow \frac{t^3}{3} + c$

$\Rightarrow \frac{(x+\log x)^3}{3} + c$

Question 71.
Mark (√) against the correct answer in each of the following:

$$\int \frac{2x \tan^{-1} x^2}{\left(1+x^4\right)} dx = ?$$

A. $(\tan^{-1}x^2)^2 + C$

B. $2 \tan^{-1} x^2 + C$

C. $\frac{1}{2}\left(\tan^{-1} x^2\right)^2 + C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $\tan^{-1} x^2 = t (\frac{1}{1+(x^2)^2} \times 2x) dx = dt \Rightarrow (\frac{2x}{1+x^4}) dx = dt$

$\Rightarrow \int t^1 dt$

$\Rightarrow \frac{t^2}{2} + c$

$\Rightarrow \frac{(\tan^{-1} x^2)^2}{2} + c$

Question 72.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(2-3x)} = ?$$

A. $- 3 \log |2 – 3x| + C$

B. $-\frac{1}{3} \log |2-3x| + C$

C. $- \log |2 – 3x| + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x^1} dx = \log x + c$

Therefore ,

Put $2 - 3x = t-3dx = dt$

$\Rightarrow -\frac{1}{3}\int \frac{1}{t} dt$

$\Rightarrow -\frac{1}{3}\log t + c$

$\Rightarrow -\frac{1}{3}\log|2 - 3x| + c$

Question 73.

Mark (√) against the correct answer in each of the following:

$\int x\sqrt{x^2 - 1}\, dx = ?$

A. $\frac{2}{3}(x^2 - 1)^{3/2} + C$

B. $\frac{1}{3}(x^2 - 1)^{3/2} + C$

C. $\frac{1}{\sqrt{x^2 - 1}} + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{x^1} dx = \log x + c$

Therefore ,

Put $x^2 - 1 = t2xdx = dt$

$\Rightarrow \int \sqrt{t}dt$

$\Rightarrow \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c \Rightarrow \frac{t^{\frac{3}{2}}}{3} + c$

$\Rightarrow \frac{(x^2-1)^{\frac{3}{2}}}{3} + c$

Question 74.
Mark (√) against the correct answer in each of the following:

$\int e^{(5-3x)}dx = ?$

A. $\frac{3^{(5-3x)}}{3(\log 3)} + C$

B. $\frac{3^{(4-3x)}}{(\log 3)} + C$

C. $-3^{(5-3x)} \log 3 + C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int a^x dx = \frac{a^x}{\log a} + c$

Therefore ,

Put $5 - 3x = t - 3dx = dt$

$\Rightarrow -\frac{1}{3}\int 3^t dt$

$\Rightarrow -\frac{1}{3} \times \frac{3^t}{\log 3} + c \Rightarrow -\frac{1}{3} \times \frac{3^{(5-3x)}}{\log 3} + c$

$\Rightarrow -\frac{3^{(5-3x)}}{3\log 3} + c$

Question 75.
Mark (√) against the correct answer in each of the following:

$\int e^{\tan x} \sec^2 x \, dx = ?$

A. $e^{\tan x} + \tan x + C$

B. $e^{\tan x} . \tan x + C$

C. $e^{\tan x} + C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \; \int e^x dx = e^x + c$

Therefore ,

Put $\tan x = t sec^2 x dx = dt$

$\Rightarrow \int e^t dt$

$\Rightarrow e^t + c \Rightarrow e^{\tan x} + c$

Question 76.
Mark (√) against the correct answer in each of the following:

$\int e^{\cos^2 x} \sin 2x \, dx = ?$

A. $e^{\cos^2 x} + C$

B. $-e^{\cos^2 x} + C$

C. $e^{\sin^2 x} + C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \; ; \; \int e^x dx = e^x + c$

Therefore ,

Put $cos^2 x = t \Rightarrow 2\cos x\,(-\sin x)dx = dt \Rightarrow -\sin 2x\ \ dx = dt$

$\Rightarrow -\int e^t dt$

$\Rightarrow -e^t + c \Rightarrow -e^{cos^2 x} + c$

Question 77.
Mark (√) against the correct answer in each of the following:

$\int x \sin^3 x^2 \cos x^2 dx = ?$

A. $\frac{1}{4}\sin^4 x^2 + C$

B. $\frac{1}{8}\sin^4 x^2 + C$

C. $\frac{1}{2}\sin^4 x^2 + C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int e^x dx = e^x + c$

Therefore ,

Put $\sin x^2 = t \Rightarrow 2x\cos x^2\, dx = dt$

$\Rightarrow \frac{1}{2}\int t^3 dt$

$\Rightarrow \frac{1}{2}\frac{t^4}{4} + c \Rightarrow \frac{t^4}{8} + c$

$\Rightarrow \frac{(\sin x^2)^4}{8} + c$

Question 78.

Mark (√) against the correct answer in each of the following:

$$\int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}} dx = ?$$

A. $\sin\left(e^{\sqrt{x}}\right) + C$

B. $\frac{1}{2}\sin\left(e^{\sqrt{x}}\right) + C$

C. $2\sin\left(e^{\sqrt{x}}\right) + C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c \, ; \int e^x dx = e^x + c$

Therefore ,

Put $\sin e^{\sqrt{x}} = t \Rightarrow (\cos e^{\sqrt{x}}) \times (e^{\sqrt{x}}) \times (\frac{1}{2\sqrt{x}}) dx = dt$

$\Rightarrow \int 2dt$

$\Rightarrow 2t + c \Rightarrow 2\sin e^{\sqrt{x}} + c$

Question 79.
Mark (√) against the correct answer in each of the following:

$$\int x^2 \sin x^3 dx = ?$$

A. $\cos x^3 + C$

B. $-\cos x^3 + C$

C. $-\frac{1}{3}\cos x^3 + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$\Rightarrow \frac{1}{3}\int \sin t \, dt$

$\Rightarrow -\frac{1}{3}\cos t + c \Rightarrow -\frac{1}{3}\cos x^3 + c$

Question 80.

Mark (√) against the correct answer in each of the following:

$$\int \frac{(x+1)e^x}{\cos^2(xe^x)} dx = ?$$

A. tan (xe^x) + C

B. - tan (xe^x) + C

C. cot (xe^x) + C

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

Put $xe^x = t \Rightarrow (e^x + xe^x)dx = dt \Rightarrow e^x(1+x)dx = dt$

$\Rightarrow \int \frac{dt}{\cos^2 t} \Rightarrow \int \sec^2 t \, dt = \tan t + c$

$\Rightarrow \tan(xe^x) + c$

Question 81.

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{x\sqrt{x^4-1}} dx = ?$$

A. $\sec^{-1} x^2 + C$

B. $\frac{1}{2}\sec^{-1} x^2 + C$

C. $\operatorname{cosec}^{-1} x^2 + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

Put $x^2 = t \Rightarrow 2x dx = dt$

$$\Rightarrow \int \frac{1}{x\sqrt{t^2-1}} \times \frac{dt}{2x} \Rightarrow \frac{1}{2}\int \frac{1}{t\sqrt{t^2-1}} dt$$

$$\Rightarrow \frac{1}{2}\sec^{-1} t + c \Rightarrow \frac{1}{2}\sec^{-1} x^2 + c$$

Question 82.

Mark (√) against the correct answer in each of the following:

$$\int x\sqrt{x-1}\, dx = ?$$

A. $\frac{2}{3}(x-1)^{3/2} + C$

B. $\frac{2}{5}(x-1)^{5/2} + C$

C. $\frac{2}{5}(x-1)^{5/2} + \frac{3}{2}(x-1)^{3/2} + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

Put $x - 1 = t$ ⇒ x = t + 1 ⇒ $dx = dt$

$\Rightarrow \int (t+1) \times \sqrt{t} dt \Rightarrow \int t^{\frac{3}{2}} dt + \int t^{\frac{1}{2}} dt$

$\Rightarrow \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c \Rightarrow \frac{2t^{\frac{5}{2}}}{5} + \frac{2t^{\frac{3}{2}}}{3} + c$

$\Rightarrow \frac{2(x-1)^{\frac{5}{2}}}{5} + \frac{2(x-1)^{\frac{3}{2}}}{3} + c$

Question 83.

Mark (√) against the correct answer in each of the following:

$\int x\sqrt{x^2 - x}\, dx = ?$

A. $\frac{1}{3}(x^2 - 1)^{3/2} + C$

B. $\frac{2}{3}(x^2 - 1)^{3/2} + C$

C. $\frac{1}{\sqrt{x^2 - 1}} + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

$\Rightarrow \int x\sqrt{x^2 - 1} dx$

Put $x^2 - 1 = t \Rightarrow 2xdx = dt$

$$\Rightarrow \int \sqrt{t}\frac{dt}{2} \Rightarrow \frac{1}{2}\int \frac{t^{\frac{3}{2}}}{\frac{3}{2}} dt$$

$$\Rightarrow \frac{t^{\frac{3}{2}}}{3} + c \Rightarrow \frac{(x^2-1)^{\frac{3}{2}}}{3} + c$$

$$\Rightarrow \frac{1}{3}(x^2-1)^{\frac{3}{2}} + c$$

Question 84.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(1+\sqrt{x}\right)} = ?$$

A. $\sqrt{x} - \log\left|1+\sqrt{x}\right| + C$

B. $\sqrt{x} + \log\left|1+\sqrt{x}\right| + C$

C. $2\sqrt{x} - 2\log\left|1+\sqrt{x}\right| + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

$$\Rightarrow \int \frac{1}{1+\sqrt{x}} dx$$

Put $x = t^2 \Rightarrow dx = 2tdt$

$$\Rightarrow \int \frac{2t}{1+t} dt \Rightarrow 2\int \frac{t}{1+t} dt \Rightarrow 2\int \frac{t+1-1}{1+t} dt \Rightarrow 2\int dt - 2\int \frac{1}{1+t} dt$$

$$\Rightarrow 2t - 2\log(1+t) + c \Rightarrow 2\sqrt{x} - 2\log(1+\sqrt{x}) + c$$

Question 85.

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{e^x - 1}\, dx$$

A. $\frac{3}{2}\left(e^x - 1\right)^{3/2} + C$

B. $\frac{1}{2}\left(e^x - 1\right)^{1/2} + C$

C. $\frac{2}{3}\left(e^x - 1\right)^{3/2} + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$$\Rightarrow \int \sqrt{e^x - 1} dx$$

Put $e^x - 1 = t \Rightarrow e^x dx = dt$

$$\Rightarrow \int \sqrt{t} \frac{dt}{1+t} \Rightarrow \int \frac{\sqrt{t}}{1+t} dt$$

Put $t = z^2$ dt = 2z dz

$$\Rightarrow \int \frac{2z^2}{1+z^2} dz \Rightarrow \int \frac{2+2z^2-2}{1+z^2} dz \Rightarrow 2\int \frac{1+z^2}{1+z^2} dz - 2\int \frac{1}{1+z^2} dz$$

$$\Rightarrow 2\int dz - 2\int \frac{1}{1+z^2} dz \Rightarrow 2z - 2\tan^{-1} z + c$$

$$\Rightarrow 2\sqrt{t} - 2\tan^{-1}\sqrt{t} + c \Rightarrow 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + c$$

Question 86.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{(\sin x - \cos x)} dx = ?$$

A. $\frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x| + C$

B. $\frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + C$

C. log |sin x – cos x| + C

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int e^x dx = e^x + c$

Therefore ,

We can write $\sin x = \frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]$

$$\Rightarrow \int \frac{\frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]}{(\sin x - \cos x)} dx$$

$$\Rightarrow \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)} dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx$$

$$\Rightarrow \frac{1}{2}\int dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx \Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx$$

Put $(\sin x - \cos x) = t (\sin x + \cos x)\ dx = dt$

$$\Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{1}{t} dt \Rightarrow \frac{x}{2} + \frac{1}{2}\log t + c \Rightarrow \frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + c$$

Question 87.

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(1-\tan x)} = ?$$

A. $\frac{1}{2}\log|\sin x - \cos x| + C$

B. $\frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + C$

C. $\frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x| + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int e^x dx = e^x + c$

Therefore ,

$$\Rightarrow \int \frac{1}{1-\frac{\sin x}{\cos x}} dx \Rightarrow \int \frac{\cos x}{\cos x - \sin x} dx$$

We can write $\cos x = \frac{1}{2}[(\cos x - \sin x) + (\sin x + \cos x)]$

$$\Rightarrow \int \frac{\frac{1}{2}[(\cos x - \sin x) + (\sin x + \cos x)]}{(\cos x - \sin x)} dx$$

$$\Rightarrow \frac{1}{2}\int \frac{(\cos x - \sin x)}{\cos x - \sin x} dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx$$

$$\Rightarrow \frac{1}{2}\int dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx \Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{(\sin x + \cos x)}{\cos x - \sin x} dx$$

Put $(\cos x - \sin x) = t (\sin x + \cos x)\ dx = -dt$

$$\Rightarrow \frac{x}{2} - \frac{1}{2}\int \frac{1}{t} dt \Rightarrow \frac{x}{2} - \frac{1}{2}\log t + c \Rightarrow \frac{1}{2}x - \frac{1}{2}\log|\cos x - \sin x| + c$$

Question 88.

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(1-\cot x)} = ?$$

A. log |sin x – cos x| + C

B. $\frac{1}{2}\log|\sin x - \cos x| + C$

C. $\frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x| + C$

D. $\frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + C$

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c\ ;\ \int e^x dx = e^x + c$

Therefore ,

$$\Rightarrow \int \frac{1}{1-\frac{\cos x}{\sin x}} dx \Rightarrow \int \frac{\sin x}{\sin x - \cos x} dx$$

We can write $\sin x = \frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]$

$$\Rightarrow \int \frac{\frac{1}{2}[(\sin x - \cos x) + (\sin x + \cos x)]}{(\sin x - \cos x)} dx$$

$$\Rightarrow \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)} dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx$$

$$\Rightarrow \frac{1}{2}\int dx + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx \Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)} dx$$

Put $(\sin x - \cos x) = t(\sin x + \cos x)\ dx = dt$

$$\Rightarrow \frac{x}{2} + \frac{1}{2}\int \frac{1}{t} dt \Rightarrow \frac{x}{2} + \frac{1}{2}\log t + c \Rightarrow \frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + c$$

Question 89.

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}} dx = ?$$

A. $\sin^{-1} (\tan x) + C$

B. $\cos^{-1} (\sin x) + C$

C. tan-1 (cos x) + C

D. $\tan^{-1} (\sin x) + C$

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

Put $\tan x = t \Rightarrow sec^2 x\, dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{1-t^2}} dt \Rightarrow \sin^{-1} t + c$

$\Rightarrow \sin^{-1}(\tan x) + c$

Question 90.

Mark (√) against the correct answer in each of the following:

$$\int \frac{(x^2+1)}{(x^4+1)} dx = ?$$

A. $\frac{1}{\sqrt{2}} \tan^{-1}\left(x - \frac{1}{x}\right) + C$

B. $\frac{1}{\sqrt{2}} \cot^{-1}\left\{\left(x - \frac{1}{x}\right)\right\} + C$

C. $\frac{1}{\sqrt{2}} \tan^{-1}\left\{\frac{1}{\sqrt{2}}\left(x - \frac{1}{x}\right)\right\} + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

Therefore ,

$$\Rightarrow \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}} dx \Rightarrow \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-2+2} dx \Rightarrow \int \frac{1+\frac{1}{x^2}}{(x-\frac{1}{x})^2+2} dx$$

Put $x - \frac{1}{x} = t \Rightarrow (1 + \frac{1}{x^2})\, dx = dt$

$$\Rightarrow \int \frac{1}{t^2+2} dt \Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}} + c$$

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}[\frac{1}{\sqrt{2}}(x - \frac{1}{x})] + c$$

Question 91.

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin^6 x}{\cos^8} dx = ?$$

A. $\frac{1}{7}\tan^7 x + C$

B. $\frac{1}{7}\sec^7 x + C$

C. log|cos^6 x| + C

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$$\Rightarrow \int \frac{\sin^6 x}{\cos^6 x \cos^2 x} dx \Rightarrow \int \frac{\tan^6 x}{\cos^2 x} dx \Rightarrow \int \tan^6 x \sec^2 x dx$$

Put $\tan x = t \Rightarrow sec^2 x\, dx = dt$

$\Rightarrow \int t^6 dt \Rightarrow \frac{t^7}{7} + c$

$\Rightarrow \frac{(\tan x)^7}{7} + c$

Question 92.
Mark (√) against the correct answer in each of the following:

$\int \sec^5 x \tan x\, dx = ?$

A. $\frac{1}{5}\tan^5 x + C$

B. $\frac{1}{5}\sec^5 x + C$

C. 5 log |cos x| + C

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sec^4 x \sec x \tan x\, dx$

Put $\sec x = t \Rightarrow \sec x \tan x\, dx = dt$

$\Rightarrow \int t^4 dt \Rightarrow \frac{t^5}{5} + c$

$\Rightarrow \frac{(\sec x)^5}{5} + c$

Question 93.
Mark (√) against the correct answer in each of the following:

$\int \tan^5 x\, dx = ?$

A. $\frac{1}{6}\tan^6 x + C$

B. $\frac{1}{4}\tan^4 x + \frac{1}{2}\tan^2 x + \log|\sec x| + C$

C. $\frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + \log|\sec x| + C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int tan^3 x\, tan^2 x dx \Rightarrow \int tan^3 x(sec^2 x - 1)dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int tan^3 x dx \Rightarrow \int tan^3 x sec^2 x dx - \int tan^1 x\, tan^2 x dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int \tan x\,(sec^2 x - 1)dx$

$\Rightarrow \int tan^3 x sec^2 x dx - \int \tan x\, sec^2 x dx + \int \tan x\, dx$

Put $\tan x = t \Rightarrow sec^2 x dx = dt$

$\Rightarrow \int t^3 dt - \int t^1 dt + \log|\sec x| \Rightarrow \frac{t^4}{4} - \frac{t^2}{2} + \log|\sec x| + c$

$\Rightarrow \frac{(\tan x)^4}{4} - \frac{(\tan x)^2}{2} + \log|\sec x| + c$

Question 94.
Mark (√) against the correct answer in each of the following:

$\int \sin^3 x \cos^3 x\, dx = ?$

A. $-\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C$

B. $\frac{1}{4}\cos^4 x - \frac{1}{6}\cos^6 x + C$

C. $\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int \cos x\,(cos^2 x\; sin^3 x)dx \Rightarrow \int \cos x\,((1 - sin^2 x)\; sin^3 x)dx$

$\Rightarrow \int \cos x\,(sin^3 x - sin^5 x)dx \Rightarrow \int sin^3 x cos\, xdx - \int sin^5 x \cos x\, dx$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$

$\Rightarrow \int t^3 dt - \int t^5 dt \Rightarrow \frac{t^4}{4} - \frac{t^6}{6} + c$

$\Rightarrow \frac{(\sin x)^4}{4} - \frac{(\sin x)^6}{6} + c$

Question 95.
Mark (√) against the correct answer in each of the following:

$\int \sec^4 x \tan x\, dx = ?$

A. $\frac{1}{2}\sec^2 x + \frac{1}{4}\sec^4 x + C$

B. $\frac{1}{2}\tan^2 x + \frac{1}{4}\tan^4 x + C$

C. $\frac{1}{2}\sec x + \log|\sec x + \tan x| + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sec^2 x sec^2 x \tan x \, dx \Rightarrow \int (1 + tan^2 x) sec^2 x \tan x \, dx$

$\Rightarrow \int sec^2 x \tan x \, dx + \int tan^3 x sec^2 x dx$

Put $\tan x = t \Rightarrow sec^2 x dx = dt$

$\Rightarrow \int t^1 dt + \int t^3 dt \Rightarrow \frac{t^2}{2} + \frac{t^4}{4} + c$

$\Rightarrow \frac{(\tan x)^2}{2} + \frac{(\tan x)^4}{4} + c$

Question 96.

Mark (√) against the correct answer in each of the following:

$$\int \frac{\log \tan x}{\sin x \cos x} dx = ?$$

A. log {log (tan x)| + C

B. $\frac{1}{2}(\log \tan x)^2 + C$

C. log (sin x cos x) + C

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sec^2 x sec^2 x \tan x \, dx \Rightarrow \int (1 + tan^2 x) sec^2 x \tan x \, dx$

$\Rightarrow \int sec^2 x \tan x \, dx + \int tan^3 x sec^2 x dx$

Put $\log(\tan x) = t \Rightarrow \frac{1}{\tan x} sec^2 x dx = dt \Rightarrow \frac{1}{\sin x \cos x} dx = dt$

$\Rightarrow \int t^1 dt \Rightarrow \frac{t^2}{2} + c$

$\Rightarrow \frac{(\log|\tan x|)^2}{2} + c$

Question 97.
Mark (√) against the correct answer in each of the following:

$\int \sin^3(2x+1)dx = ?$

A. $\frac{1}{8}\sin^4(2x+1)+C$

B. $\frac{1}{2}\cos(2x+1)+\frac{1}{3}\cos^3(2x+1)+C$

C. $-\frac{1}{2}\cos(2x+1)+\frac{1}{6}\cos^3(2x+1)+C$

D. none of these

Answer:
Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$\Rightarrow \int sin^2(2x+1)\sin(2x+1)\,dx \Rightarrow \int (1 - cos^2(2x+1))\sin(2x+1)\,dx$

$\Rightarrow \int \sin(2x+1)\,dx - \int cos^2(2x+1)\sin(2x+1)\,dx$

Put $\cos(2x+1) = t \Rightarrow -2\sin(2x+1)\,dx = dt$

$\Rightarrow -\int \frac{dt}{2} - (-\frac{1}{2})\int t^2 dt \Rightarrow -\frac{1}{2}\int dt + \frac{1}{2}\int t^2 dt$

$\Rightarrow -\frac{1}{2}t + \frac{1}{2}\frac{t^3}{3} + c \Rightarrow -\frac{1}{2}t + \frac{t^3}{6} + c$

$\Rightarrow -\frac{1}{2}\cos(2x+1)+\frac{[\cos(2x+1)]^3}{6}+c$

Question 98.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\sqrt{\tan x}}{\sin x+\cos x}dx=?$$

A. $2\sqrt{\tan x}+C$

B. $2\sqrt{\cot x}+C$

C. $2\sqrt{\sec x}+C$

D. none of these

Answer:
Formula :- $\int x^n dx=\frac{x^{n+1}}{n+1}+c\ ;\ \int \frac{1}{1+x^2}dx=\tan^{-1}x+c$

Therefore ,

$$\Rightarrow \int \frac{\sqrt{\tan x}}{\sin x\times\cos x}dx \Rightarrow \int \frac{\sqrt{\tan x}}{\frac{\tan x}{\sec x}\times\frac{1}{\sec x}}dx \Rightarrow \int \frac{sec^2 x}{\sqrt{\tan x}}dx$$

Put $\tan x=t \Rightarrow sec^2 x dx=dt$

$$\Rightarrow \int \frac{dt}{\sqrt{t}} \Rightarrow \frac{\sqrt{t}}{\frac{1}{2}}+c \Rightarrow 2\sqrt{t}+c$$

$$\Rightarrow 2\sqrt{\tan x}+c$$

Question 99.
Mark (√) against the correct answer in each of the following:

$$\int \frac{(\cos+\sin x)}{(1-\sin 2x)}dx=?$$

A. log |sin x – cos x| + C

B. $\frac{1}{(\cos x - \sin x)} + C$

C. log |cos x + sin x| + C

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

Therefore ,

$$\Rightarrow \int \frac{\cos x + \sin x}{\cos^2 x + \sin^2 x - \sin 2x} dx \Rightarrow \int \frac{\cos x + \sin x}{(\cos x - \sin x)^2} dx$$

Put $\cos x - \sin x = t \Rightarrow (\cos x + \sin x) dx = -dt$

$$\Rightarrow \int \frac{-dt}{t^2} \Rightarrow \frac{1}{t} + c \Rightarrow \frac{1}{\cos x - \sin x} + c$$

Question 100.

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{e^x - 1}\, dx = ?$$

A. $\frac{2}{3}(e^x - 1)^{3/2} + C$

B. $\frac{1}{2} \cdot \frac{e^x}{\sqrt{e^x - 1}} + C$ e

C. $2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + C$

D. none of these

Answer:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$\Rightarrow \int \sqrt{e^x - 1} dx$

Put $e^x - 1 = t \Rightarrow e^x dx = dt$

$\Rightarrow \int \sqrt{t} \frac{dt}{1+t} \Rightarrow \int \frac{\sqrt{t}}{1+t} dt$

Put $t = z^2$ dt = 2z dz

$\Rightarrow \int \frac{2z^2}{1+z^2} dz \Rightarrow \int \frac{2+2z^2-2}{1+z^2} dz \Rightarrow 2\int \frac{1+z^2}{1+z^2} dz - 2\int \frac{1}{1+z^2} dz$

$\Rightarrow 2\int dz - 2\int \frac{1}{1+z^2} dz \Rightarrow 2z - 2\tan^{-1} z + c$

$\Rightarrow 2\sqrt{t} - 2\tan^{-1}\sqrt{t} + c \Rightarrow 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + c$

Question 101.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{\sin^3 x \cos x}} = ?$$

A. $2\sqrt{\tan x} + C$

B. $2\sqrt{\cot x} + C$

C. $-2\sqrt{\tan x} + C$

D. $\frac{-2}{\sqrt{\tan x}} + C$

Answer:
Let $I = \int \frac{dx}{\sqrt{\sin^3 x \cos x}}$

Now multiplying and dividing by $\cos^2 x$, we get,

$$I = \int \frac{dx}{\sqrt{\sin^3 x \times \cos x}} \times \frac{1}{\cos^2 x} \times \cos^2 x$$

$$I = \int \frac{(\sec^2 x)}{\sqrt{\frac{\sin^3 x}{\cos^3 x}}} dx$$

$$I = \int \frac{\sec^2 x}{\sqrt{\tan^3 x}} dx$$

Let tan x = t

Differentiating both sides, we get,

$\sec^2 x \, dx = dt$

Therefore,

$$I = \int \frac{dt}{t^{3/2}}$$

Integrating, we get,

$$I = \frac{t^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + C$$

$$I = \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + C$$

$$I = -\frac{2}{\sqrt{t}} + C$$

$$I = -\frac{2}{\sqrt{\tan x}} + C$$