Exercise 10i

Question 1.

Find $\frac{dy}{dx}$, when

$\mathscr{X}$ = at^2, $\mathscr{Y}$ = 2at

Answer:

Theorem: y and x are given in a different variable that is t. We can find $\frac{dy}{dx}$ by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.

$\frac{dy}{dt} = \frac{d(2at)}{dt}$

= 2a. ......(1)

$\frac{dx}{dt} = \frac{d(at^2)}{dt}$

= 2at ......(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{2a}{2at}$

$= \frac{1}{t}$

Question 2.

Find $\frac{dy}{dx}$, when

$\mathscr{X}$ = a cos θ , $\mathscr{Y}$ = b sin θ

Answer:

y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{dbsin\theta}{d\theta}$ ($\frac{dsin\theta}{d\theta}$=cosθ)

= bcosθ. ......(1)

$\frac{dx}{d\theta} = \frac{d(acos\theta)}{d\theta}$ ($\frac{dcos\theta}{d\theta} = -sin\theta$)

= -asinθ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{bcos\theta}{-asin\theta}$ ($\frac{cos\theta}{sin\theta}$ = cotθ)

= $\frac{-bcot\theta}{a}$.

Question 3.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when

$\mathscr{X}$ = a cos^{2}θ , $\mathscr{Y}$ = b sin^{2}θ

Answer:

Theorem: y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{dbsin^2\theta}{d\theta}$

= b× 2sinθ × cosθ (using the chain rule $\frac{dsin^2\theta}{d\theta}$ = 2sinθ× $\frac{dsin\theta}{d\theta}$ = 2sinθ × cosθ)

= 2bsinθcosθ . ...(1)

$$\frac{dx}{d\theta} = \frac{dacos^2\theta}{d\theta}$$

= a × (2cosθ)× (-sinθ) (using chain rule $\frac{dcos^2\theta}{d\theta}$ = 2cosθ× $\frac{dcos\theta}{d\theta}$ = 2 cosθ × (-sinθ))

= -2asinθcosθ.

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{2b\sin\theta\cos\theta}{-2a\sin\theta\cos\theta}$

$= \frac{-b}{a}$.

Question 4.

Find $\frac{dy}{dx}$, when

$\mathscr{X}$ = a $\cos^3\theta$, $\mathscr{Y}$ = a $\sin^3\theta$

Answer:

Theorem: y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{da\sin^3\theta}{d\theta}$

= a× 3 $\sin^2\theta$ × cosθ (using the chain rule $\frac{d\sin^3\theta}{d\theta}$ = 3$\sin^2\theta$× $\frac{d\sin\theta}{d\theta}$ = 2$\sin^2\theta$ × cosθ)

= 3a$\sin^2\theta$cosθ . ...(1)

$$\frac{dx}{d\theta} = \frac{da\cos^3\theta}{d\theta}$$

= a × (3$\cos^2\theta$)× (-sinθ) (using chain rule $\frac{d\cos^2\theta}{d\theta}$ = 2cosθ× $\frac{d\cos\theta}{d\theta}$ = 2 cosθ × (-sinθ))

= -3asinθ$\cos^2\theta$.

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{3\,a\sin^2\theta\cos\theta}{-3a\sin\theta\cos^2\theta}$

$= \frac{-sin\theta}{cos\theta}$.

= -tanθ

Question 5.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when

$\mathscr{X}$ = a(1 – cos θ), $\mathscr{Y}$ = a(θ + sin θ)

Answer:

Theorem: y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{da(\theta+sin\theta)}{d\theta}$

= a× (1+cosθ) ....(1)

$$\frac{dx}{d\theta} = \frac{da(1-cos\theta)}{d\theta}$$

= asinθ. ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{a(1+co\theta)}{asin\theta}$

$= \frac{1+cos\theta}{sin\theta}$.

$= \frac{2\cos^2(\theta/2)}{2\sin(\theta/2)\cos(\theta/2)}$ (1+cosθ=2cos^{2}θ/2 and sinθ = 2sin(θ/2)cos(θ/2))

= cot(θ/2)

Question 6.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when

$\mathscr{X}$ = a log t, $\mathscr{Y}$ = b sin t

Answer:
Theorem: y and x are given in a different variable that is t. We can find $\frac{dy}{dx}$ by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.

$\frac{dy}{dt} = \frac{dbsint}{dt}$

= bcost ....(1)

$$\frac{dx}{dt} = \frac{d(a\,logt)}{dt}$$

= $\frac{a}{t}$ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{b\,cost}{a/t}$

= $\frac{bt\,cost}{a}$.

Question 7.
Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when

$\mathscr{X}$ = (log t + cos t), $\mathscr{Y}$ = (e^t + sin t)

Answer:
Theorem: *y* and *x* are given in a different variable that is *t* . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.

$\frac{dy}{dt} = \frac{d(e^t+sint)}{dt}$

= e^t + cost ....(1) $(\frac{de^t}{dt} = e^t)$

$$\frac{dx}{dt} = \frac{d(logt + \ cost)}{dt}$$

= $\frac{1}{t}$ - sint. ....(2) $(\frac{d\ logt}{dt} = \frac{1}{t})$

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{e^t + cost}{\frac{1}{t} - sint}$

= $\frac{t(e^t + cost)}{1 - tsint}$.

Question 8.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when

$\mathscr{X}$ = cos θ + cos 2θ, $\mathscr{Y}$ = sin θ + sin 2θ

Answer:

Theorem: *y* and *x* are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{d(sin\theta + sin2\theta)}{d\theta}$

= cosθ + cos2θ×2 ....(1) $(using\ chain\ rule\ \frac{d\ sin2\theta}{d\theta} = cos2\theta \times \frac{d\ 2\theta}{d\theta})$

$$\frac{dx}{d\theta} = \frac{d(cos\theta + cos2\theta)}{d\theta}$$

= -sinθ -2sin2θ ....(2) $(using\ chain\ rule\ \frac{d\ cos2\theta}{d\theta} = sin2\theta \times \frac{d\ 2\theta}{d\theta})$

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{cos\theta+2cos2\theta}{-(sin\theta+2sin2\theta)}$

Question 9.

Find $\frac{dy}{dx}$, when

$x = \sqrt{\sin 2\theta}$, $y = \sqrt{\cos 2\theta}$

Answer:

Theorem: y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dx}{d\theta} = \frac{d\sqrt{sin2\theta}}{d\theta}$

$= \frac{2cos2\theta}{2\sqrt{sin2\theta}}$ $(using\ chain\ rule\ \frac{d\sqrt{sin2\theta}}{d\theta} = \frac{1}{2\sqrt{sin2\theta}} \times \frac{d\,sin2\theta}{d\theta})$

$\frac{dx}{d\theta} = \frac{cos2\theta}{\sqrt{sin2\theta}}$ ....(1)

$\frac{dy}{d\theta} = \frac{d(\sqrt{cos2\theta})}{d\theta}$

$= \frac{-2sin2\theta}{2\sqrt{cos2\theta}}$ $(using\ chain\ rule\ \frac{d\sqrt{sin2\theta}}{d\theta} = \frac{1}{2\sqrt{sin2\theta}} \times \frac{d\,sin2\theta}{d\theta})$

$= \frac{-sin2\theta}{\sqrt{cos2\theta}}$ ....(2)

Dividing (2) and (2), we get

$\frac{dy}{dx} = -\frac{sin2\theta/\sqrt{cos2\theta}}{cos2\theta/\sqrt{sin2\theta}}$

$= -\frac{\sqrt{\sin^3 2\theta}}{\sqrt{\cos^3 2\theta}}$

= -(tan2θ)$^{3/2}$

Question 10.

Find $\frac{dy}{dx}$, when

$\mathscr{X} = e^{\theta}$ (sin θ + cos θ), $\mathscr{Y} = e^{\theta}$ (sin θ - θ cos θ)

Answer:

Theorem: y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$$\frac{dy}{d\theta} = \frac{d\,e^{\theta}\,(sin\theta - cos\theta)}{d\theta}$$

= e^{θ} (cosθ + sinθ) + (sinθ -cosθ)e^{θ} ....(1) {by using product rule, $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$}

$$\frac{dx}{d\theta} = \frac{d\ e^{\theta}(sin\theta + cos\theta)}{d\theta}$$

= e^{θ} (cosθ - sinθ) + e^{θ} (sinθ + cosθ) ....(2) {by using product rule, $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$}

Dividing (1) and (2), we get

$$\frac{dy}{dx} = \frac{e^{\theta}\,(2sin\theta)}{e^{\theta}\,(2cos\theta)}$$

=tanθ .

Question 11.

Find $\frac{dy}{dx}$, when

$\mathscr{X}$ = a (cos θ + θ sin θ), $\mathscr{Y}$ = a (sin θ - θ cos θ)

Answer:

Theorem: *y* and *x* are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{d\, a(sin\theta - \theta cos\theta)}{d\theta}$

= a(cosθ - (-θ sinθ + cosθ)) {by using product rule, $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ while differentiating θcosθ }

= a(θsinθ) ....(1)

$$\frac{dx}{d\theta} = \frac{d\ a(cos\theta + \theta sin\theta)}{d\theta}$$

= a(-sinθ +θ cosθ +sinθ) {by using product rule, $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ while differentiating θcosθ }

= a× θcosθ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{a \times \theta sin\theta}{a \times \theta cos\theta}$

= tanθ ANS

Question 12.

Find $\frac{dy}{dx}$, when

$\mathscr{X} = \frac{3at}{(1+t^2)}$, $\mathscr{Y} = \frac{3at^2}{(1+t^2)}$

Answer:

Theorem: *y* and *x* are given in a different variable that is *t* . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.

$\frac{dy}{dt} = \frac{d\frac{3at^2}{(1+t^2)}}{dt}$

$= \frac{(1+t^2)6at-3at^2(2t)}{(1+t^2)^2}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$}

$= \frac{6at+6at^3-6at^3}{(1+t^2)^2}$

$= \frac{6at}{(1+t^2)^2}$ ....(1)

$$\frac{dx}{dt} = \frac{d\,(\frac{3at}{1+t^2})}{d\theta}$$

$= \frac{(1+t^2)3a-3at(2t)}{(1+t^2)^2}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$}

$= \frac{3a+3at^2-6at^2}{(1+t^2)^2}$

$= \frac{3a-3at^2}{(1+t^2)^2}$ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{6at/(1+t^2)^2}{3a(1-t^2)/(1+t^2)^2}$

$= \frac{2t}{(1-t^2)}$

Question 13.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when

$\mathscr{X} = \frac{1-t^2}{1+t^2}$, $\mathscr{Y} = \frac{2t}{1+t^2}$

Answer:

Theorem: *y* and *x* are given in a different variable that is *t* . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.

$\frac{dy}{dt} = \frac{d\frac{2t}{(1+t^2)}}{dt}$

$= \frac{(1+t^2)2-2t(2t)}{(1+t^2)^2}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ }

$= \frac{2+2t^2-4t^2}{(1+t^2)^2}$

$= \frac{2-2t^2}{(1+t^2)^2}$ ....(1)

$$\frac{dx}{dt} = \frac{d\,(\frac{1-t^2}{1+t^2})}{d\theta}$$

$= \frac{(1+t^2)(-2t)-(1-t^2)(2t)}{(1+t^2)^2}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ }

$= \frac{-2t-2t^3-2t+2t^3}{(1+t^2)^2}$

$= \frac{-4t}{(1+t^2)^2}$ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{2-2t^2/(1+t^2)^2}{-4t/(1+t^2)^2}$

$= \frac{t^2-1}{(2t)}$

Question 14.

Find $\frac{dy}{dx}$, when

$\mathscr{X} = \cos^{-1}\frac{1}{\sqrt{1+t^2}}$, $\mathscr{Y} = \sin^{-1}\frac{t}{\sqrt{1+t^2}}$

Answer:

Theorem: *y* and *x* are given in a different variable that is *t* . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.

Let us assume u= $\frac{t}{\sqrt{(1+t^2)}}$

$\frac{dy}{dt} = \frac{d\sin^{-1}(u)}{dt}$

= $\frac{1}{\sqrt{(1-u^2)}} \times \frac{du}{dt}$

= $\frac{1}{\sqrt{(1-u^2)}} \times \frac{\sqrt{1+t^2}\times 1-t(2t/2\sqrt{(1+t^2)})}{\left(\sqrt{1+t^2}\right)^2}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ }

Putting value of u

= $\frac{\sqrt{(1+t^2)}}{1} \times \frac{1}{(1+t^2)^{\left(\frac{3}{2}\right)}}$

= $\frac{1}{1+t^2}$ ....(1)

Let assume v= $\frac{1}{\sqrt{(1+t^2)}}$

$$\frac{dx}{dt} = \frac{d\,(\cos^{-1} v)}{dv} \times \frac{dv}{dt}$$

= $\frac{-1}{\sqrt{(1-v^2)}} \times \left(\frac{-1}{\left(\sqrt{1+t^2}\right)^2}\right) \times \frac{2t}{2\sqrt{(1+t^2)}}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ }

Putting value of v

= $\frac{t\sqrt{(1+t^2)}}{t\times(1+t^2)^{\frac{3}{2}}}$

= $\frac{\sqrt{(1+t^2)}}{(1+t^2)^{\frac{3}{2}}}$

$= \frac{1}{(1+t^2)}$ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{1}{1+t^2} \times \frac{(1+t^2)}{1}$

= 1

Question 15.

If $\mathscr{X}$ = 2 cos t – 2 cos^{3}t, $\mathscr{Y}$ = sin t – 2 sin^{3}t, show that $\frac{\mathrm{dy}}{\mathrm{dx}}$ =cot t.

Answer:

Theorem: *y* and *x* are given in a different variable that is *t* . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.

$\frac{dy}{dt} = \frac{d(sint-2\sin^3 t)}{dt}$

= *cost – 6 sin^{2}t × cost* ....(1) (using chain rule)

$$\frac{dx}{dt} = \frac{d(2cost - 2\cos^3 t)}{dt}$$

= *-2sint + 6cos^{2}t ×sint* ....(2) (using chain rule)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{cost(1-6\sin^2 t)}{2sint\ (3\cos^2 t-1)}$

$= \frac{t(e^t+cost)}{1-tsint}$.

Question 16.

If $\mathscr{X} = \frac{1+\log t}{t^2}$ and $\mathscr{Y} = \frac{3+2\log t}{t}$ $\frac{dy}{dx}$ =t.

Answer:

Theorem: *y* and *x* are given in a different variable that is *t* . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.

$\frac{dy}{dt} = \frac{d(3+2logt)/t}{dt}$

$= \frac{t\left(\frac{2}{t}\right)-(3+2logt)\times 1}{t^2}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$}

$= -\ \frac{1+2logt}{t^2}$ ....(1)

$$\frac{dx}{dt} = \frac{d(1+logt)/t^2}{dt}$$

$= \frac{t^2\left(\frac{1}{t}\right)-(2t+2t\ logt)}{t^4}$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$}

$= -\frac{2logt+1}{t^3}$ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{-(1+2\ logt)/t^2}{-(1+2\ logt)/t^3}$

= t.

Question 17.

If $\mathscr{X}$ = a(θ – sin θ), $\mathscr{Y}$ = a(1 – cos θ), find $\frac{dy}{dx}$ at θ = $\frac{\pi}{2}$.

Answer:

Theorem: *y* and *x* are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{d\, a(1-cos\theta)}{d\theta}$

= asinθ ....(1)

$$\frac{dx}{d\theta} = \frac{d\ a(\theta - sin\theta\ )}{d\theta}$$

= a(1-cosθ) ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{asin\theta}{a\times(1-cos\theta)}$

Putting θ= π/2

$= \frac{\sin(\pi/2)}{1-\cos(\pi/2)}$

= 1.

Question 18.

If $\mathscr{X}$ = 2 cosθ – cos 2θ and $\mathscr{Y}$ = 2 sin θ – sin 2θ, show that $\frac{dy}{dx} = \tan\frac{3\theta}{2}$.

Answer:

Theorem: *y* and *x* are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{d\ (2sin\theta - sin2\theta)}{d\theta}$

= 2cosθ – 2cos2θ ....(1)

$$\frac{dx}{d\theta} = \frac{d\ (2cos\theta - cos2\theta)}{d\theta}$$

= -2sinθ + 2sin2θ ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{2cos\theta - 2cos2\theta}{2sin2\theta - 2sin\theta}$

$= \frac{cos\theta - cos2\theta}{sin2\theta - sin\theta}$

$= \frac{cos\theta - (2\cos^2\theta - 1)}{2sin\theta cos\theta - sin\theta}$ $\{sin2t = 2sintcost\}$ $\{cos2t = 2\cos^2 t - 1\}$

By factorising numerator, we get

$= \frac{(1 - cos\theta)(cos\theta + \frac{1}{2})}{2sin\theta(cos\theta - \frac{1}{2})}$

$= \frac{1 - cos\theta}{2sin\theta} \times \frac{cos\theta + \frac{1}{2}}{cos\theta - \frac{1}{2}}$ $\{\frac{1 - cos\theta}{sin\theta} = \tan(\frac{\theta}{2})\}$

$= \frac{\tan(\frac{\theta}{2})}{1} \times \frac{(2(1 - \tan^2(\frac{\theta}{2})) + (1 + \tan^2(\frac{\theta}{2}))}{2(1 - \tan^2(\frac{\theta}{2})) - (1 + \tan^2(\frac{\theta}{2}))}$

Foe simplicity let's take θ/2 as *x*.

$= \frac{tanx}{2} \times \frac{2 - 2\tan^2 x + 1 + \tan^2 x}{2 - 2\tan^2 x - 1 - \tan^2 x}$

$= \frac{tanx}{2} \times \frac{3 - \tan^2 x}{1 - 3\tan^2 x}$

$= \frac{3tanx - \tan^3 x}{1 - 3\tan^2 x}$ $\frac{3tanx - \tan^3 x}{1 - 3\tan^2 x} = \tan 3x$

$= \frac{tan3x}{2}$ $x = \frac{\theta}{2}$

$= \frac{\tan(\frac{3\theta}{2})}{2}$.

Question 19.

If $\mathscr{X} = \frac{\sin^3 t}{\sqrt{\cos 2t}}$, $\mathscr{Y} = \frac{\cos^3 t}{\sqrt{\cos 2t}}$, find $\frac{dy}{dx}$.

Answer:

Theorem: *y* and *x* are given in a different variable that is *t* . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{dt}$ and $\frac{dx}{dt}$ and then dividing them to get the required thing.

$$\frac{dx}{dt} = \frac{d\left(\frac{(\sin^3 t)}{(\sqrt{cos2t})}\right))}{dt}$$

$$= \frac{\sqrt{cos2t}\left(3\sin^2 t \times cost\right) - \sin^3 t\left(\frac{(-sin2t)}{\sqrt{cos2t}}\right))}{cos2t}$$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$}

$$= \frac{cos2t \times \left(3\sin^2 t \times cost\right) + \sin^3 t \times (2sintcost)}{(cos2t)^{\frac{3}{2}}} \quad \{sin2t = 2sintcost\}$$

$$= \frac{\sin^2 t\ cost(3cos2t + 2\sin^2 t)}{(cos2t)^{\frac{3}{2}}} \quad \{cos2t = 1 - 2\sin^2 t\}$$

$$= \frac{\sin^2 t\ cost(3 - 4\sin^2 t)}{(cos2t)^{\frac{3}{2}}}$$

$$= \frac{sint\ \text{cost}\ (3sint - 4\sin^3 t)}{2(cos2t)^{\frac{3}{2}}} \quad \{sin3t = 3sint - 4\sin^3 t\}$$

$$= \frac{sin2t \times sin3t}{(cos2t)^{\frac{3}{2}}} \text{ ....(1)}$$

$$\frac{dy}{dt} = \frac{d\,\frac{\cos^3 t}{\sqrt{cos2t}}}{dv}$$

$$= \frac{\sqrt{cos2t}\left(3\cos^2 t \times (-sint)\right) - \cos^3 t\left(\frac{(-sin2t)}{\sqrt{cos2t}}\right))}{cos2t}$$ {by using divide rule, $\frac{d(u/v)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$}

$$= \frac{cos2t \times \left(-3\cos^2 t \times sint\right) + \cos^3 t \times (2sintcost)}{(cos2t)^{\frac{3}{2}}} \quad \{sin2t = 2sintcost\}$$

$$= \frac{\cos^2 t\ sint(-3cos2t + 2\cos^2 t)}{(cos2t)^{\frac{3}{2}}} \quad \{cos2t = 2\cos^2 t - 1\}$$

$= \frac{\cos^2 t\, \sin t(3-4\cos^2 t)}{(\cos 2t)^{\frac{3}{2}}}$

$= \frac{\sin t \cos t\,(3\cos t-4\cos^3 t)}{(\cos 2t)^{\frac{3}{2}}}$ $\{\cos 3t = 4\cos^3 t - 3\cos t\}$

$= -\frac{\sin 2t \times \cos 3t}{2(\cos 2t)^{\frac{3}{2}}}$ ....(1)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{-\frac{\sin 2t \times \cos 3t}{2(\cos 2t)^{\frac{3}{2}}}}{\frac{\sin 2t \times \sin 3t}{(\cos 2t)^{\frac{3}{2}}}}$

$= -\cot 3t$

Question 20.

If $\mathscr{X}$ = (2 cos θ – cos 2θ) and = (2sin θ – sin 2θ), find $\left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{2}}$.

Answer:
here we have to find the double derivative, so to find double derivative we will just differentiate the first derivative once again with a similar method.

Theorem: *y* and *x* are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{d\,(2\sin\theta - \sin 2\theta)}{d\theta}$

= 2cosθ – 2cos2θ ....(1)

$\frac{dx}{d\theta} = \frac{d\,(2\cos\theta - \cos 2\theta)}{d\theta}$

= -2sinθ + 2sin2θ ....(2)

Dividing (1) and (2), we get

$$\frac{dy}{dx} = \frac{cos\theta - cos2\theta}{sin2\theta - sin\theta}$$

$= \tan(\frac{3\theta}{2})$ {as shown in question no. 18}

Let $\frac{dy}{dx} = f'$

$$\frac{d^2 y}{dx^2} = f''$$

⇒ To find f'' we will differentiate f' with θ and then divide with equation (2).

$$\frac{d\frac{dy}{dx}}{d\theta} = \frac{d\tan(\frac{3\theta}{2})}{d\theta}$$

$$= \frac{\sec^2(\frac{3\theta}{2})}{1} \times \frac{3}{2}$$

Now divide by equation (2).

$$\frac{d^2 y}{dx^2} = \frac{3\sec^2(\frac{3\theta}{2})}{4} \times \frac{1}{(sin2\theta - sin\theta)}$$

Putting θ = π/2

$$\frac{d^2 y}{dx^2} = \frac{3}{4} \times (-2)$$

$$= -\frac{3}{2}.$$

Question 21.

If $\mathscr{X}$ = a (θ – sin θ), $\mathscr{Y}$ = a(1+cos θ), find $\frac{d^2y}{dx^2}$.

Answer:
here we have to find the double derivative, so to find double derivative we will just differentiate the first derivative once again with a similar method.

Theorem: y and x are given in a different variable that is θ . We can find $\frac{dy}{dx}$ by finding $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ and then dividing them to get the required thing.

$\frac{dy}{d\theta} = \frac{d\, a(1+cos\theta)}{d\theta}$

$= \ -asin\theta$ ....(1)

$$\frac{dx}{d\theta} = \frac{d\ a(\theta - sin\theta\ )}{d\theta}$$

= a(1-cosθ) ....(2)

Dividing (1) and (2), we get

$\frac{dy}{dx} = \frac{-asin\theta}{a\times(1-cos\theta)}$

$= \frac{-2\sin(\frac{\theta}{2})\cos(\frac{\theta}{2})}{2\sin^2\frac{\theta}{2}}$ $\{sin2t\ = 2sintcost\}\ \{cos2t = 1 - 2\sin^2 t\}$

= -cot (θ/2)

⇒ To find f'' we will differentiate f' with θ and then divide with equation (2).

$\frac{d\frac{dy}{dx}}{d\theta} = \frac{cosec^2(\frac{\theta}{2})}{2} \times \frac{1}{a(1-cos\theta)}$

$= \frac{-1}{2\,a\sin^2(\frac{\theta}{2})\times(2\sin^2(\frac{\theta}{2}))}$ $\left\{1 - cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)\right\}$ $\{cosec^2\theta = \frac{1}{\sin^2\theta}\}$

$= \frac{1}{4a} cosec^4\left(\frac{\theta}{2}\right).$