Objective Questions

Question 1.

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} \cos 70^{\circ} & \sin 20^{\circ} \\ \sin 70^{\circ} & \cos 20^{\circ} \end{vmatrix} = ?$$

- A. 1
- B. 0
- C. cos 50°
- D. sin 50°

Answer:

To find: Value of sin70° sin20° cos20°

Formula used: (i) $\cos \theta = \sin (90 - \theta)$

We have, | cos 70° sin 20° | sin 70° cos 20° |

On expanding the above,

 $\Rightarrow \{\cos 70^{\circ}\} \{\cos 20^{\circ}\} - \{\sin 70^{\circ}\} \{\sin 20^{\circ}\}$

On applying formula $\cos \theta = \sin (90 - \theta)$

$$\Rightarrow$$
 {sin (90 - 70)} {sin (90 - 20)} - {sin 70°} {sin 20°}

= 0

Question 2.

$$\begin{vmatrix} \cos 15^{\circ} & \sin 15^{\circ} \\ \sin 15^{\circ} & \cos 15^{\circ} \end{vmatrix} = ?$$

- A. 1
- B. $\frac{1}{2}$
- c. $\frac{\sqrt{3}}{2}$

D. none of these

Answer:

To find: Value of sin 15° cos 15° cos 15°

Formula used: (i) $\cos (A + B) = \cos A \cos B - \sin A \sin B$

We have, | cos 15° sin 15° | sin 15° cos 15° |

On expanding the above,

$$\Rightarrow \{\cos 15^{\circ}\} \{\cos 15^{\circ}\} - \{\sin 15^{\circ}\} \{\sin 15^{\circ}\}$$

On applying formula $\cos (A + B) = \cos A \cos B - \sin A \sin B$

$$= \cos (15 + 15)$$

$$=\frac{\sqrt{3}}{2}$$

Question 3.

$$\begin{vmatrix} \sin 23^{\circ} & -\sin 7^{\circ} \\ \cos 23^{\circ} & \cos 7^{\circ} \end{vmatrix} = ?$$

A.
$$\frac{\sqrt{3}}{2}$$

B.
$$\frac{1}{2}$$

C. sin 16°

D. cos 16°

Answer:

Formula used: (i) sin(A + B) = sin A cos B + cos A sin B

On expanding the above,

$$\Rightarrow$$
 (sin 23°) (cos 7°) - (cos 23°) (-sin 7°)

$$\Rightarrow$$
 (sin 23°) (cos 7°) + (cos 23°) (sin 7°)

On applying formula sin(A + B) = sin A cos B + cos A sin B

$$= \sin (23 + 7)$$

$$=\frac{1}{2}$$

Question 4.

$$\begin{vmatrix} a+ib & c+id \\ -c+id & a-id \end{vmatrix} = ?$$

A.
$$(a^2 + b^2 - c^2 - d^2)$$

B.
$$(a^2 - b^2 + c^2 - d^2)$$

C.
$$(a^2 + b^2 + c^2 + d^2)$$

D. none of these

Answer:

To find: Value of $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

Formula used: $i^2 = -1$

We have, $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

On expanding the above,

$$\Rightarrow$$
 (a + ib) (a - ib) - (-c + id) (c + id)

$$\Rightarrow$$
 (a² - iab + iba - i²b²) - (-c² - icd + icd + i²d²)

$$\Rightarrow$$
 {a² - iab + iba - (-1)b²} - {-c² - icd + icd + (-1)d²}

$$\Rightarrow$$
 {a² - iab + iba + 1b²} - {-c² - icd + icd - 1d²}

$$\Rightarrow$$
 a² + b² + c² + d²

Question 5.

Mark the tick against the correct answer in the following:

If ω is a complex root of unity then $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = ?$

- A. 1
- B. -1
- C. 0
- D. none of these

Answer:

To find: Value of
$$\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix}$$

Formula used: $\omega^3 = 1$

We have,
$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

On expanding the above along 1st column

$$\Rightarrow 1 \begin{vmatrix} \omega^2 & \mathbf{1} \\ \mathbf{1} & \omega \end{vmatrix} - \omega \begin{vmatrix} \omega & \omega^2 \\ \mathbf{1} & \omega \end{vmatrix} + \omega^2 \begin{vmatrix} \omega & \omega^2 \\ \omega^2 & \mathbf{1} \end{vmatrix}$$

$$\Rightarrow \left[\mathbf{1}\!\left\{\!\left(\omega^2\right)\!\left(\omega\right)\!\!-\!\left(\mathbf{1}\right)\!\left(\mathbf{1}\right)\right\}\right] - \left[\omega\!\left\{\!\left(\omega\right)\!\left(\omega\right)\!\!-\!\left(\omega^2\right)\!\left(\mathbf{1}\right)\right\}\right] + \left[\omega^2\!\left\{\!\left(\omega\right)\!\left(\mathbf{1}\right)\!\!-\!\left(\omega^2\right)\!\left(\omega^2\right)\right\}\right]$$

$$\Rightarrow \left[\mathbf{1}\{\omega^3 - \mathbf{1}\}\right] - \left[\omega\{\omega^2 - \omega^2\}\right] + \left[\omega^2\{\omega - \omega^4\}\right] \dots \text{ (i)}$$

As
$$\omega^{3} = 1$$
,

$$\Rightarrow \omega^3.\omega = 1.\omega$$

$$\Rightarrow \omega^4 = \omega$$

Using the above obtained value of ω^4 in eqn. (i)

$$\Rightarrow \left[\mathbf{1}\left\{\omega^{3}\mathbf{-1}\right\}\right]\mathbf{-}\left[\omega\left\{\omega^{2}\mathbf{-}\omega^{2}\right\}\right]+\left[\omega^{2}\left\{\omega\mathbf{-}\omega\right\}\right]$$

$$\Rightarrow$$
 1{ ω^3 -1}

$$\Rightarrow \omega^3-1$$

$$\Rightarrow 1 - 1 = 0$$

Question 6.

Mark the tick against the correct answer in the following:

If ω is a complex cube root of unity then the value of $\begin{vmatrix} 1 & \omega & 1+\omega\\ 1+\omega & 1 & \omega\\ \omega & 1+\omega & 1 \end{vmatrix}$ is

- A. 2
- B. 4
- C. 0
- D. -3

Answer

To find: Value of
$$\begin{bmatrix} 1 & \omega & 1+\omega \\ 1+\omega & 1 & \omega \\ \omega & 1+\omega & 1 \end{bmatrix}$$

Formula used: (i) $\omega^3 = 1$

(ii)
$$1+\omega+\omega^2 = 0$$

We have,
$$\begin{bmatrix} 1 & \omega & 1+\omega \\ 1+\omega & 1 & \omega \\ \omega & 1+\omega & 1 \end{bmatrix}$$

On expanding the above along 1st column

$$\Rightarrow 1 \begin{vmatrix} \omega^2 & 1 \\ 1 & \omega \end{vmatrix} - \omega \begin{vmatrix} \omega & \omega^2 \\ 1 & \omega \end{vmatrix} + \omega^2 \begin{vmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{vmatrix}$$

$$\Rightarrow \left[\mathbf{1}\{\left(\omega^{2}\right)(\omega)-(\mathbf{1})(\mathbf{1})\}\right]-\left[\omega\{(\omega)(\omega)-\left(\omega^{2}\right)(\mathbf{1})\}\right]+\left[\omega^{2}\{(\omega)\mathbf{1}-\left(\omega^{2}\right)\left(\omega^{2}\right)\}\right]$$

$$\Rightarrow \left[\mathbf{1}\left\{\omega^{3}\mathbf{-1}\right\}\right]\mathbf{-}\left[\omega\left\{\omega^{2}\mathbf{-}\omega^{2}\right\}\right]\mathbf{+}\left[\omega^{2}\left\{\omega\mathbf{-}\omega^{4}\right\}\right]...\ (i)$$

As
$$\omega^{3} = 1$$
,

$$\Rightarrow \omega^3.\omega = 1.\omega$$

$$\Rightarrow \omega^4 = \omega$$

Using the above obtained value of ω^4 in eqn. (i)

$$\Rightarrow \left[\mathbf{1}\{\omega^3 - \mathbf{1}\}\right] - \left[\omega\{\omega^2 - \omega^2\}\right] + \left[\omega^2\{\omega - \omega\}\right]$$

$$\Rightarrow$$
 1{ ω^3 -1}

$$\Rightarrow \omega^3-1$$

$$\Rightarrow 1 - 1 = 0$$

Question 7.

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix} = ?$$

To find: Value of
$$\begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$$

We have,
$$\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$$

Applying $R_1 \rightarrow R_3 - R_1$

Applying $R_2 \rightarrow R_1 - R_2$

Taking 4 common from R₁

Applying $R_1 \rightarrow R_1 - R_2$

Taking -2 common from R₁

$$\Rightarrow (4)(-2)\begin{vmatrix} 1 & 0 & -2 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$$

Applying $R_1 \rightarrow 9R_1$

$$\Rightarrow \frac{-8}{9} \begin{vmatrix} 9 & 0 & -18 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \frac{-8}{9} \begin{vmatrix} 9 & 0 & -18 \\ 4 & 3 & 0 \\ 0 & 16 & 43 \end{vmatrix}$$

Taking 9 common from R₁

$$\Rightarrow -8 \begin{vmatrix} 1 & 0 & -2 \\ 4 & 3 & 0 \\ 0 & 16 & 43 \end{vmatrix}$$

Expanding along R₁

$$\Rightarrow$$
 -8 [1[(3)(43)-(16)(0)] - 0 [(4)(43)-(0)(0)] - 2 [(4)(16)-(3)(0)]]

$$\Rightarrow$$
 -8 [[(129)-(0)] - 2 [(64)-(0)]]

Question 8.

Mark the tick against the correct answer in the following:

$$\Rightarrow \begin{vmatrix} 1 & 2 & 6 \\ 2 & 6 & 24 \\ 6 & 24 & 120 \end{vmatrix}$$

Taking 2 common from R₂

Taking 6 common from R₃

Applying $R_2 \rightarrow R_2 - R_1$

Applying $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow 12 \begin{vmatrix} 1 & 2 & 6 \\ 0 & 1 & 6 \\ 0 & 2 & 14 \end{vmatrix}$$

Expanding column 1

$$\Rightarrow$$
 12 [1{(1)(14)-(6)(2)}]

$$\Rightarrow$$
 12 [1{(14)-(12)}]

$$\Rightarrow 24$$

Question 9.

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a - b & b - c & c - a \\ b - c & c - a & a - b \\ c - a & a - b & b - c \end{vmatrix} = ?$$

A.
$$(a + b + c)$$

B.
$$3(a + b + c)$$

- C. 3abc
- D. 0

Applying
$$R_1 \rightarrow R_1 + R_2$$

Applying
$$R_1 \rightarrow R_1 + R_3$$

If every element of a row is 0 then the value of the determinant will be 0

Question 10.

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = ?$$

- A. 0
- B. 1
- C. -1
- D. none of these

Answer:

Applying
$$R_2 \rightarrow R_2 - 2R_1$$

Applying
$$R_3 \rightarrow R_3 - 3R_1$$

$$\Rightarrow \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & p-2 \\ 0 & 3 & 3p-2 \end{vmatrix}$$

Expanding along C₁

$$\Rightarrow$$
 [1{(1)(3p-2)-(3)(p-2)}]

Question 11.

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = ?$$

A.
$$(a - b) (b - c) (c - a)$$

B.
$$-(a - b) (b - c) (c - a)$$

C.
$$(a - b) (b - c) (c - a) (a + b + c)$$

D. abc
$$(a - b)(b - c)(c - a)$$

To find: Value of
$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

We have,
$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Applying
$$C_2 \rightarrow C_2 - C_1$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 1 \\ a & b-a & c \\ a^3 & b^3-a^3 & c^3 \end{vmatrix}$$

Applying
$$C_3 \rightarrow C_3 - C_1$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & b^3-a^3 & c^3-a^3 \end{vmatrix}$$

We know,
$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & (b-a)(b^2+ab+a^2) & (c-a)(c^2+ca+a^2) \end{vmatrix}$$

Taking (b-a) common from C₂

$$\Rightarrow (b-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & c-a \\ a^3 & (b^2+ab+a^2) & (c-a)(c^2+ca+a^2) \end{vmatrix}$$

Taking (c-a) common from C₂

$$\Rightarrow (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^3 & (b^2+ab+a^2) & (c^2+ca+a^2) \end{vmatrix}$$

Expanding along C₁

$$\Rightarrow$$
 (b - a) (c - a)[1{(1)(c² + ca + a²) - (b² + ab + a²)(1)}]

$$\Rightarrow$$
 (b - a) (c - a)[c² + ca + a² - b² - ab - a²]

$$\Rightarrow$$
 (b - a) (c - a)[c² - b² + ca - ab]

$$\Rightarrow$$
 (b - a) (c - a)[(c - b) (c + b) + a(c - b)]

$$\Rightarrow (b - a) (c - a)[(a + b + c)(c - b)]$$

$$\Rightarrow$$
 (a - b) (b - c) (c - a) (a + b + c)

Question 12.

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} \sin \alpha & \cos \alpha & \sin (\alpha + \delta) \\ \sin \beta & \cos \beta & \sin (\beta + \delta) \\ \sin \gamma & \cos \gamma & \sin (\gamma + \delta) \end{vmatrix} = ?$$

C.
$$\sin (\alpha + \delta) + \sin (\beta + \delta) + \sin (\gamma + \delta)$$

D. none of these

Answer:

To find: Value of
$$\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}$$

Formula Used: sin(A+B) = sinAcosB+cosAsinB

We have,
$$\begin{array}{cccc} sina & cosa & sin(a+\delta) \\ sin\beta & cos\beta & sin(\beta+\delta) \\ sin\gamma & cos\gamma & sin(\gamma+\delta) \end{array}$$

Applying
$$C_1 \rightarrow \cos(\delta)C_1$$

$$\Rightarrow \begin{vmatrix} \sin \alpha \cos \delta & \cos \alpha & \sin(\alpha + \delta) \\ \sin \beta \cos \delta & \cos \beta & \sin(\beta + \delta) \\ \sin \gamma \cos \delta & \cos \gamma & \sin(\gamma + \delta) \end{vmatrix}$$

Applying
$$C_2 \rightarrow \sin(\delta)C_2$$

$$\Rightarrow$$
 sina cosδ cosa sinδ sin(a+δ)
 \Rightarrow sinβ cosδ cosβ sinδ sin(β+δ)
sinγ cosδ cosγ sinδ sin(γ+δ)

We know, sin(A+B) = sinAcosB+cosAsinB

$$\Rightarrow \begin{array}{c} sina\;cos\delta\;\;cosa\;sin\delta\;\;sina\;cos\delta\;+\;cosa\;sin\delta\\ sin\beta\;cos\delta\;\;\;cos\beta\;sin\delta\;\;\;sin\beta\;cos\delta\;+\;cos\beta\;sin\delta\\ sin\gamma\;cos\delta\;\;\;cos\gamma\;sin\delta\;\;\;sin\gamma\;cos\delta\;+\;cos\gamma\;sin\delta \end{array}$$

Applying
$$C_3 \rightarrow C_3 - C_1$$

$$\Rightarrow$$
 sina cosδ cosa sinδ sina cosδ + cosa sinδ - sina cosδ \Rightarrow sinβ cosδ cosβ sinδ sinβ cosδ + cosβ sinδ - sinβ cosδ sinγ cosδ cosγ sinδ sinγ cosδ + cosγ sinδ - sinγ cosδ

$$\Rightarrow \begin{vmatrix} \sin \alpha \cos \delta & \cos \alpha \sin \delta & \cos \alpha \sin \delta \\ \sin \beta \cos \delta & \cos \beta \sin \delta & \cos \beta \sin \delta \\ \sin \gamma \cos \delta & \cos \gamma \sin \delta & \cos \gamma \sin \delta \end{vmatrix}$$

$$= 0$$

When two columns are identical then the value of determinant is 0

Question 13.

Mark the tick against the correct answer in the following:

If a, b, c be distinct positive real numbers then the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

- A. positive
- B. negative
- C. a perfect square
- D. 0

Answer:

Applying
$$C_1 \rightarrow C_1 + C_2 + C_3$$

Taking (a+b+c) common from R₁

$$\Rightarrow (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$

Expanding along R₁

$$\Rightarrow$$
 (a+b+c)[1{(b)(c)-(a)(a)} - 1{(b)(b)-(c)(a)} + 1{(a)(b)-(c)(c)}]

$$\Rightarrow$$
 (a+b+c)[1{bc-a²} - 1{b²-ca} + 1{ba - c²}]

$$\Rightarrow$$
 (a+b+c)[bc - a² -b² + ca + ab - c²]

$$\Rightarrow$$
 -(a+b+c)[c² + a² + b² - ca - bc - ba]

$$\Rightarrow -\frac{1}{2}(a+b+c) 2[c^2+a^2+b^2-ca-bc-ba]$$

$$\Rightarrow -\frac{1}{2}(a+b+c)[2c^2+2a^2+2b^2-2ca-2bc-2ba]$$

$$\Rightarrow -\frac{1}{2}(a+b+c)[c^2+a^2-2ca+c^2+b^2-2bc+a^2+b^2-2ba]$$

$$\Rightarrow -\frac{1}{2}(a+b+c)[(c-a)^2+(c-b)^2+(a-b)^2]$$

Clearly, we can see that the answer is negative

Question 14.

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} x + y & x & x \\ 5x + 4y & 4x & 2x \\ 10x + 8y & 8x & 3x \end{vmatrix} = ?$$

A. 0

D. none of these

Answer:

To find: Value of
$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$$

We have,
$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$$

Applying $R_2 \rightarrow 2R_2$

$$\Rightarrow \frac{1}{2} \begin{vmatrix} x+y & x & x \\ 10x+8y & 8x & 4x \\ 10x+8y & 8x & 3x \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_3$

$$\Rightarrow \frac{1}{2} \begin{vmatrix} x+y & x & x \\ 0 & 0 & x \\ 10x+8y & 8x & 3x \end{vmatrix}$$

Applying $R_1 \rightarrow 8R_1$

$$\Rightarrow \frac{1}{2 \times 8} \begin{vmatrix} 8x + 8y & 8x & 8x \\ 0 & 0 & x \\ 10x + 8y & 8x & 3x \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \frac{1}{16} \begin{vmatrix} 8x + 8y & 8x & 8x \\ 0 & 0 & x \\ 2x & 0 & -5x \end{vmatrix}$$

Expanding along R₂

$$\Rightarrow \frac{1}{16} [x\{(2x)(8x) - (8x+8y)(0)\}]$$

$$\Rightarrow \frac{1}{16} \left[x\{16x^2\} \right]$$

$$\Rightarrow x^3$$

Question 15.

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = ?$$

A.
$$(a - 1)$$

B.
$$(a - 1)^2$$

C.
$$(a - 1)^3$$

D. none of these

Answer:

To find: Value of
$$\begin{vmatrix} a^2+2a & 2a+1 & 1\\ 2a+1 & a+2 & 1\\ 3 & 3 & 1 \end{vmatrix}$$

We have,
$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

Applying
$$R_1 \rightarrow R_1 - R_2$$

$$\Rightarrow \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

Applying
$$R_2 \rightarrow R_2 - R_3$$

$$\Rightarrow \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Expanding along C₃

$$\Rightarrow$$
 [1{(a²-1)(a-1) - (a-1)(2a - 2)}]

$$\Rightarrow$$
 [1{(a-1)(a+1)(a-1) - (a-1)2(a - 1)}]

$$\Rightarrow [\{(a+1)(a-1)^2 - 2(a-1)^2\}]$$

$$\Rightarrow$$
 [{(a-1)² (a+1-2)}]

$$\Rightarrow [{(a-1)^2 (a-1)}]$$

$$\Rightarrow$$
 (a-1)³

Question 16.

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} = ?$$

C. 0

D. none of these

Applying
$$R_3 \rightarrow R_3 - 2R_2$$

⇒
$$\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 0 & a & a+b \end{vmatrix}$$

Applying
$$R_2 \rightarrow R_2 - 3R_1$$

Expanding along C₁

$$\Rightarrow [a\{(a) (a+b) - (a)(2a+b)\}]$$

$$\Rightarrow [a\{(a^2 + ab) - (2a^2 + ab)\}]$$

$$\Rightarrow$$
 [a{a² + ab - 2a² - ab}]

$$\Rightarrow [a\{-a^2\}]$$

$$\Rightarrow$$
 -a³

Question 17.

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix} = ?$$

A.
$$(a + b + c) (a - c)$$

B.
$$(a + b + c) (b - c)$$

C.
$$(a + b + c) (a - c)^2$$

D.
$$(a + b + c) (b - c)^2$$

To find: Value of
$$\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$$

Applying
$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$\Rightarrow \begin{vmatrix} 2(a+b+c) & a+b+c & a+b+c \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$$

$$\Rightarrow (a+b+c)\begin{vmatrix} 2 & 1 & 1 \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$$

Expanding along R₁

$$\Rightarrow$$
 (a+b+c)[2{(c) (c) - (b) (a)} -1{(c+a)(c)-(a+b)(a)} + 1{(c+a)(b)-(a+b)(c)}]

$$\Rightarrow$$
 (a+b+c)[2{c² - ab} -1{c²+ac-a²-ab} + 1{bc+ba-ac-bc}]

$$\Rightarrow$$
 (a+b+c)[2c² - 2ab - c² - ac + a² + ab + ba - ac]

$$\Rightarrow$$
 (a+b+c)[c² + a² - 2ac]

$$\Rightarrow$$
 (a+b+c)(c - a)²

Question 18.

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = ?$$

A.
$$(x + y)$$

B.
$$(x - y)$$

D. none of these

To find: Value of
$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$$

We have,
$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$$

Applying
$$R_1 \rightarrow R_2 - R_1$$

$$\Rightarrow \begin{vmatrix} 0 & -x & 0 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$$

Expanding along R₁

$$\Rightarrow [x\{(1)(1+y)-(1)(1)\}]$$

$$\Rightarrow [x\{1+y-1\}]$$

$$\Rightarrow xy$$

Question 19.

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = ?$$

A.
$$(a - b) (b - c) (c - a)$$

B.
$$-(a - b) (b - c) (c - a)$$

C.
$$(a + b) (b + c) (c + a)$$

D. None of these

To find: Value of
$$\begin{vmatrix} bc & b+c & 1 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$$

Applying
$$R_1 \rightarrow R_2 - R_1$$

Taking (b - a) common

$$\Rightarrow (b-a) \begin{vmatrix} c & 1 & 0 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$$

Applying
$$R_2 \rightarrow R_2 - R_3$$

$$\Rightarrow (b-a) \begin{vmatrix} c & 1 & 0 \\ ca-ab & c-b & 0 \\ ab & a+b & 1 \end{vmatrix}$$

$$\Rightarrow (b-a) \begin{vmatrix} c & 1 & 0 \\ a(c-b) & c-b & 0 \\ ab & a+b & 1 \end{vmatrix}$$

Taking (c - b) common

$$\Rightarrow$$
 (b-a) (c-b) $\begin{vmatrix} c & 1 & 0 \\ a & 1 & 0 \\ ab & a+b & 1 \end{vmatrix}$

Expanding along C_3

$$\Rightarrow$$
 (b - a) (c - b) [1{(c) (1) - (a) (1)}]

$$\Rightarrow$$
 (b - a) (c - b) (c - a)

$$\Rightarrow$$
 (a - b) (b - c) (c - a)

Question 20.

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = ?$$

- A. 4abc
- B. 2(a + b + c)
- C. (ab + bc + ca)
- D. none of these

Answer:

Applying
$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$\Rightarrow \begin{vmatrix} b+c+b+c & a+c+a+c & a+b+a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Taking 2 common

$$\Rightarrow 2 \begin{vmatrix} b+c & a+c & a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying
$$R_1 \rightarrow R_1 - R_2$$

$$\Rightarrow 2 \begin{vmatrix} c & 0 & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Expanding along R₁

$$\Rightarrow$$
 2 [c{(c + a) (a + b) - (b) (c)} + a{(b)(c) - (c) (c + a)}]

$$\Rightarrow$$
 2 [c{(ac + cb +a² + ab - bc} + a{(bc - c² - ac)}]

$$\Rightarrow$$
 2 [c{(ac + a² + ab)} + a{(bc - c² - ac)}]

$$\Rightarrow$$
 2 [ac² + ca² + abc + abc - ac² - a²c]

Question 21.

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix} = ?$$

$$A.a+b+c$$

B.
$$2(a + b + c)$$

D.
$$a^2b^2c^2$$

Applying
$$R_2 \rightarrow R_2 - R_1$$

$$\Rightarrow \begin{vmatrix} a & 1 & b+c \\ b-a & 0 & a-b \\ c & 1 & a+b \end{vmatrix}$$

Taking (a - b) common

$$\Rightarrow (a-b) \begin{vmatrix} a & 1 & b+c \\ -1 & 0 & a-b \\ c & 1 & a+b \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow (a-b) \begin{vmatrix} a & 1 & b+c \\ -1 & 0 & a-b \\ c-a & 0 & a-c \end{vmatrix}$$

Taking (c-a) common

$$\Rightarrow (b-a)(c-a)\begin{vmatrix} a & 1 & b+c \\ -1 & 0 & a-b \\ 1 & 0 & -1 \end{vmatrix}$$

Expanding along R₁

$$= (b-a)(c-a)[0-1(1-(a-b))+(b+c)(0)]$$

$$= (b - a)(c - a)(-1 + a - b)$$

$$= (b - a)(c - a)(a - b - 1)$$

$$= (b - a)(ac - bc - c - a^2 + ab + a)$$

$$= (abc - b^2c - bc - a^2b + ab^2 + ab - a^2c + abc + ac + a^3 + a^2b + a^2)$$

= 4abc

Question 22.

$$\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix} = ?$$

C.
$$x^2 - 2$$

D.
$$x^2 + 2$$

Answer:

To find: Value of
$$\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$$

We have,
$$\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$$

Applying
$$R_1 \rightarrow R_2 - R_1$$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$$

Applying
$$R_2 \rightarrow R_3 - R_2$$

$$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 4 & 5 & 6 \\ x+7 & x+10 & x+14 \end{vmatrix}$$

Expanding along R₁

$$\Rightarrow \left[2\{(5)(x+14)-(6)(x+10)\}-3\{(4)(x+14)-(6)(x+7)\}+4\ \{(4)(x+10)-(5)(x+7)\}\right]$$

$$\Rightarrow [2\{5x + 70 - 6x - 60\} - 3\{4x + 56 - 6x - 42\} + 4\{4x + 40 - 5x - 35\}]$$

$$\Rightarrow [2\{10 - x\} - 3\{14 - 2x\} + 4\{5 - x\}]$$

$$\Rightarrow$$
 [20 - 2x - 42 + 6x + 20 - 4x]

Question 23.

Mark the tick against the correct answer in the following:

If
$$\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0 \text{ then } x = ?$$

- A. 0
- B. 6
- C. -6
- D. 9

Answer:

To find: Value of x

We have,
$$\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$$

Applying $R_1 \rightarrow 2R_1$

$$\Rightarrow \begin{vmatrix} 10 & 6 & -2 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$$

Applying $R_1 \rightarrow R_1 - R_3$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$$

Expanding along R₁

$$\Rightarrow [1\{(x)(-2) - (6)(2)\}] = 0$$

$$\Rightarrow [1\{-2x - 12\}] = 0$$

$$\Rightarrow$$
 -2x-12 = 0

$$\Rightarrow$$
 -2x = 12

$$\Rightarrow x = -6$$

Question 24.

Mark the tick against the correct answer in the following:

The solution set of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0 \text{ is }$

D. none of these

Answer:

To find: Value of x

We have,
$$\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

Applying $R_1 \rightarrow 2R_1$

$$\Rightarrow \begin{vmatrix} 2x & 6 & 14 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

Applying $R_1 \rightarrow R_1 - R_3$

$$\Rightarrow \begin{vmatrix} 2x-7 & 0 & 14-x \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

Expanding along R₁

$$\Rightarrow [(2x-7)\{(x)(x) - (6)(2)\} + (14-x)\{(2)(6) - (x)(7)\} = 0$$

$$\Rightarrow [(2x-7)\{x^2-12\} + (14-x)\{12-7x\}] = 0$$

$$\Rightarrow [2x^3 - 24x - 7x^2 + 84 + 168 - 98x - 12x + 7x^2] = 0$$

$$\Rightarrow [2x^3 - 134x + 252] = 0$$

$$\Rightarrow [x^3 - 67x + 126] = 0$$

By Hit and trial x = -2, 3, -7

Question 25.

Mark the tick against the correct answer in the following:

The solution set of the equation $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 2x-64 \end{vmatrix} = 0 \text{ is}$

- A. {4}
- B. {2, 4}
- C. {2, 8}
- D. {4, 8}

Answer:

To find: Value of x

We have,
$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

Applying
$$C_2 \rightarrow C_2 - 2C_1$$

$$\Rightarrow \begin{vmatrix} x-2 & 1 & 3x-4 \\ x-4 & -1 & 3x-16 \\ x-8 & -11 & 3x-64 \end{vmatrix} = 0$$

Applying $C_3 \rightarrow C_3 - 3C_1$

$$\Rightarrow \begin{vmatrix} x-2 & 1 & 2 \\ x-4 & -1 & -4 \\ x-8 & -11 & -40 \end{vmatrix} = 0$$

Expanding along R₁

$$\Rightarrow [x-2\{(-1)(-40)-(-4)(-11)\}-1\{(x-4)(-40)-(-4)(x-8)\}+2\{(x-4)(-11)-(-1)(x-8)\}=0$$

$$\Rightarrow [(x-2)\{40-44\} -1\{(-40x + 160 + 4x - 32\} + 2\{-11x + 44 + x - 8\}] = 0$$

$$\Rightarrow [(x-2)\{-4\} -1 \{(-36x + 128\} + 2 \{-10x+36\}] = 0$$

$$\Rightarrow$$
 [-4x + 8 + 36x - 128 - 20x + 72] = 0

$$\Rightarrow$$
 12x - 48 = 0

$$\Rightarrow x = 4$$

Question 26.

Mark the tick against the correct answer in the following:

The solution set of the equation $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0 \text{ is}$

D. None of these

Answer:

To find: Value of x

We have,
$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Applying $R_1 \rightarrow R_1 - R_2$

$$\Rightarrow \begin{vmatrix} 2x & -2x & 0 \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_3$

$$\Rightarrow \begin{vmatrix} 2x & -2x & 0 \\ 0 & 2x & -2x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Taking 2 common from R₁

$$\Rightarrow 2 \begin{vmatrix} x & -x & 0 \\ 0 & 2x & -2x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Taking 2 common from R₂

$$\Rightarrow 2 \times 2 \begin{vmatrix} x & -x & 0 \\ 0 & x & -x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

Applying $R_3 \rightarrow R_1 + R_3$

$$\Rightarrow 4 \begin{vmatrix} x & -x & 0 \\ 0 & x & -x \\ a & a-2x & a+x \end{vmatrix} = 0$$

Expanding along R₁

$$\Rightarrow 4[x\{(x)(a+x)-(-x)(a-2x)\}]-(-x)\{(0)(a+x)-(-x)(a)\}]=0$$

$$\Rightarrow 4[x\{ax + x^2 + ax - 2x^2\}] - (-x)\{ax\}] = 0$$

$$\Rightarrow 4[x{2ax - x^2}] + ax^2] = 0$$

$$\Rightarrow 4[2ax^2 - x^3 + ax^2] = 0$$

$$\Rightarrow$$
 - x^2 + 3ax = 0

$$\Rightarrow$$
 -x(x - 3a) = 0

$$\Rightarrow$$
 x = 0, or x = 3a

Question 27.

Mark the tick against the correct answer in the following:

The solution set of the equation $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0 \text{ is}$

$$A. \left\{ \frac{2}{3}, \frac{8}{3} \right\}$$

B.
$$\left\{ \frac{2}{3}, \frac{11}{3} \right\}$$

$$C.\left\{\frac{3}{2},\frac{8}{3}\right\}$$

D. None of these

Answer:

To find: Value of x

We have,
$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

Applying $R_1 \rightarrow R_1 - R_2$

$$\Rightarrow \begin{vmatrix} 3x-11 & 11-3x & 0 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

Applying $R_2 \rightarrow R_2 - R_3$

$$\Rightarrow \begin{vmatrix} 3x-11 & 11-3x & 0 \\ 0 & 3x-11 & 11-3x \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

Expanding along R₁

$$\Rightarrow (3x-11)\{(3x-11)(3x-8) - (3)(11-3x)\} - (11-3x)\{(0)((3x-8) - (11-3x)(3)\} = 0$$

$$\Rightarrow (3x-11)\{(3x-11)(3x-8+3)\} - (11-3x)\{-(11-3x)(3)\} = 0$$

$$\Rightarrow (3x-11)^2(3x-5)\} + (3x-11)\{(3x-11)(3)\} = 0$$

$$\Rightarrow$$
 (3x-11)²(3x-5)} + (3x-11)²(3)} = 0

$$\Rightarrow$$
 (3x-11)²(3x-5+3) = 0

$$\Rightarrow (3x-11)^2(3x-2) = 0$$

$$\Rightarrow$$
 x = $\frac{11}{3}$, Or, x = $\frac{2}{3}$

Question 28.

Mark the tick against the correct answer in the following:

The vertices of a a ABC are A(-2, 4), B(2, -6) and C(5, 4). The area of a ABC is

- A. 17.5 sq units
- B. 35 sq units
- C. 32 sq units
- D. 28 sq units

Answer:

To find: Area of ABC

Given: A(-2,4), B(2,-6) and C(5,4)

Formula used:
$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

We have, A(-2,4), B(2,6) and C(5,4)

$$\Delta = \frac{1}{2} \begin{vmatrix} -2 & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

Expanding along R₁

$$\Rightarrow \frac{1}{2} \left[-2\{(-6)(1)-(4)(1)-4\{(2)(1)-(5)(1)\}+1\{(2)(4)-(5)(-6)\} \right]$$

$$\Rightarrow \frac{1}{2} \left[-2\{-6-4\} - 4\{2-5\} + 1\{8+30\} \right]$$

$$\Rightarrow \frac{1}{2} \left[-2\{-10\} - 4\{-3\} + 1\{38\} \right]$$

$$\Rightarrow \frac{1}{2} [20 + 12 + 38]$$

$$\Rightarrow \frac{1}{2}$$
 [70]

Question 29.

Mark the tick against the correct answer in the following:

If the points A(3, -2), B(k, 2) and C(8, 8) are collinear then the value of k is

- A. 2
- B. -3
- C. 5
- D. -4

Answer:

To find: Area of ABC

Given: A(3,-2), B(k,2) and C(8,8)

The formula used:
$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

We have, A(3,-2), B(k,2) and C(8,8)

$$\Delta = \frac{1}{2} \begin{vmatrix} 3 & -2 & 1 \\ k & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix}$$

Expanding along R₁

$$\Rightarrow \frac{1}{2} \left[3 \big\{ (2)(1) \text{-} (8)(1) \big\} \text{-} (\text{-}2) \big\{ (k)(1) \text{-} (8)(1) \big\} + 1 \big\{ (k)(8) \text{-} (2)(8) \big\} \right] = 0$$

$$\Rightarrow \frac{1}{2} [3\{2-8\} + 2\{k-8\} + 1\{8k-16\}] = 0$$

$$\Rightarrow$$
 -18 +2k - 16 + 8k -16 = 0

$$\Rightarrow$$
 10k -50 = 0

$$\Rightarrow$$
 k = 5