Chapter - 9 Mechanical Properties Of Solids

Properties of solids

- Solids have a definite shape and size.
- Solids are crystalline or amorphous.
- The density of solids is slightly higher than their liquid states.

Deforming force: –

A force which changes the size, shape or volume .

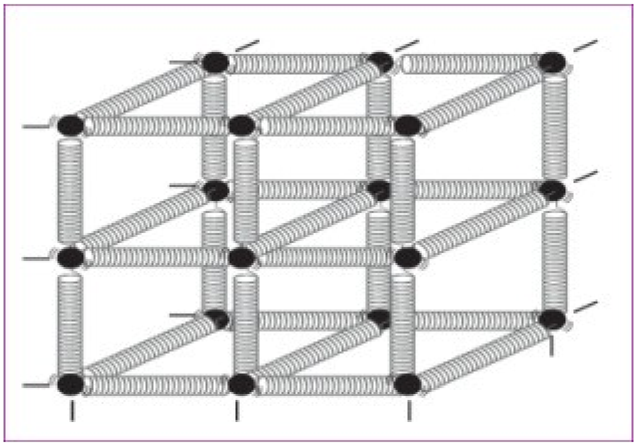
When a body is subjected to a deforming force, a restoring force is developed in the body. Elasticity and elastic deformation:

- The property of a body, by virtue of which it tends to regain its original size and shape when the applied force is removed, is known as elasticity and the deformation caused is known as elastic deformation.
- Example of perfectly elastic body- quartz Plasticity :
- Substances which do not retain their state when deforming forces are removed are called plastic and this property is called plasticity.
- Eg of perfectly plastic body- putty

ELASTIC BEHAVIOUR OF SOLIDS

- When a solid is deformed, the atoms or molecules are displaced from their equilibrium positions causing a change in the inter atomic (or intermolecular) distances.
- When the deforming force is removed, the inter atomic forces tend to drive them back to their original positions.

- Thus the body regains its original shape and size.



STRESS:

- The restoring force per unit area is known as stress.
- If F is the force applied and A is the area of cross section of the body, magnitude of the stress = F/A
- The SI unit of stress is N m-2 or pascal (Pa)
- Its dimensional formula is

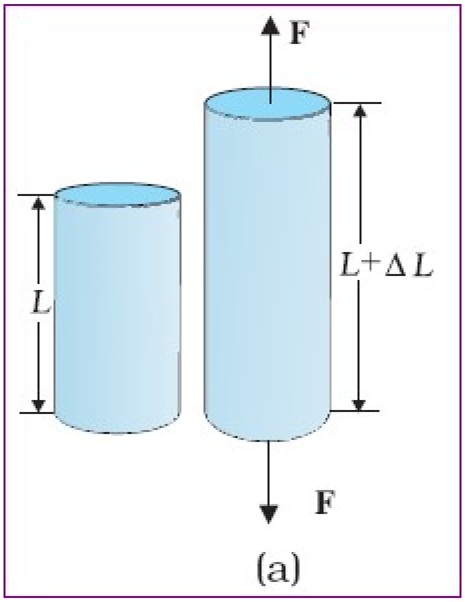
$$[\ ML^{-1}T^{-2}\ ]$$

Types of stress

- Longitudinal stress or linear stress
- Normal stress or hydraulic stress
- Shearing stress or tangential stress

Longitudinal stress or linear stress

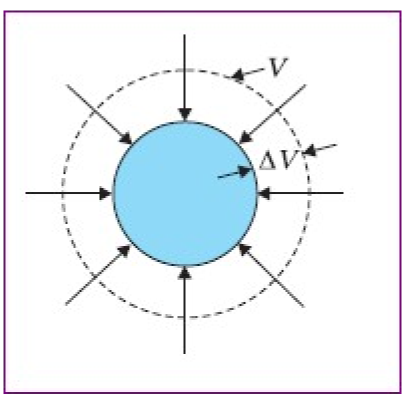
- This stress produces a change in length.
- The change in length may be elongation(tensile stress) or compression (compressive stress)



(a)

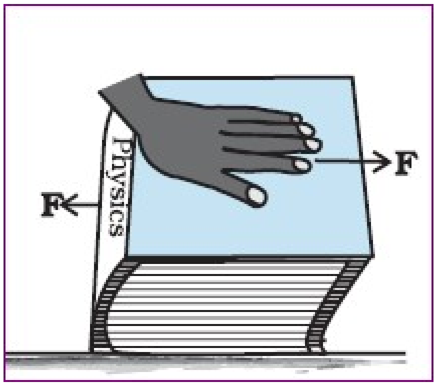
Normal stress or hydraulic stress or volume stress

• This stress produces a change in volume



Shearing stress or tangential stress

- This stress produces a change in shape



STRAIN:

- It is the ratio of change in dimension to the original dimension.
- It has no unit and dimensions.

Longitudinal (Linear) strain:

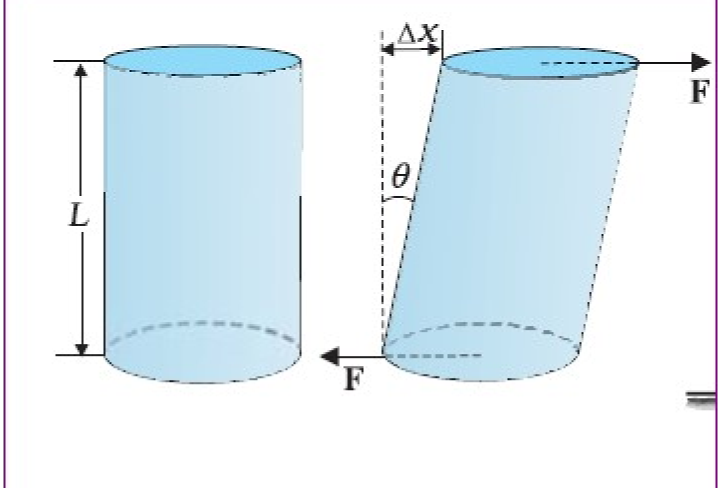
- It is the ratio of change in the length (ΔL) to the original length(L) of the body .
- Longitudinal strain = $\Delta L/L$

Volume strain:

- It is the ratio of change in volume (ΔV) to the original volume (V)
- Volume strain = $\Delta V/ V$

Shearing strain :

- It is the angle turned by a straight line assumed on the body which was originally perpendicular to the tangential force.



(b)

$$\text{Shearing strain} \quad \frac{x}{L} = \tan\theta$$

- Usually θ is very small, tan θ is nearly equal to angle θ.
- Thus, shearing strain = tan θ ≈ θ

HOOKE'S LAW:

- For small deformations the stress is directly proportional to strain.

Thus,

$$\text{stress} \propto \text{strain}$$
$$\text{stress} = k \times \text{strain}$$

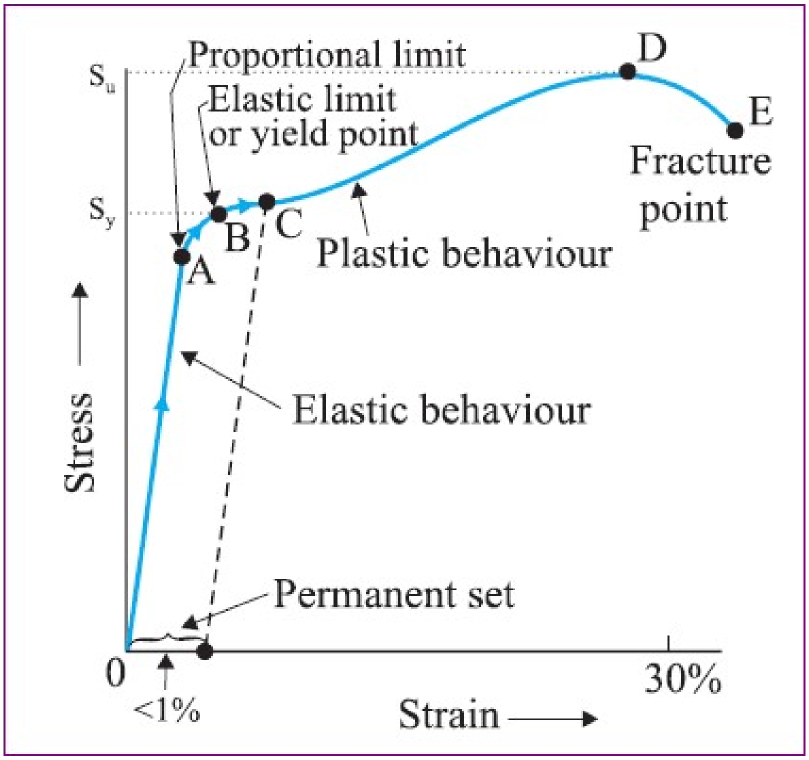
- Where k is the proportionality constant and is known as modulus of elasticity.

$$\boxed{K = \frac{\text{Stress}}{\text{Strain}}}$$

- Modulus of elasticity depends on, nature of the material of the body and temperature.
- It is independent of the dimensions of the body.
- S.I unit of ‘k’ is Nm-2 or Pascal [Pa]

Stress – Strain Curve:

- A graph drawn with strain along x-axis and strain along y-axis.

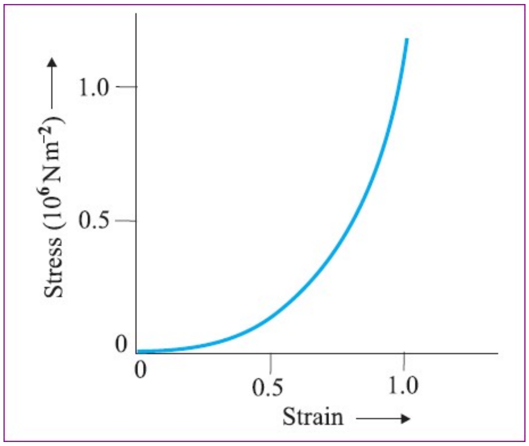


- The point B in the curve is known as yield point (also known as elastic limit) and the corresponding stress is known as yield strength (Sy) of the material.
- The point D on the graph is the ultimate tensile strength (Su) of the material.
- If the ultimate strength and fracture points D and E are close, the material is said to be brittle.
- If they are far apart, the material is said to be ductile.

Elastomers:

- Substances which can be stretched to cause large strains are called elastomers.
- Eg: tissue of aorta, rubber etc

Stress-strain curve for the elastic tissue of Aorta



MODULUS OF ELASTICITY

Young's Modulus

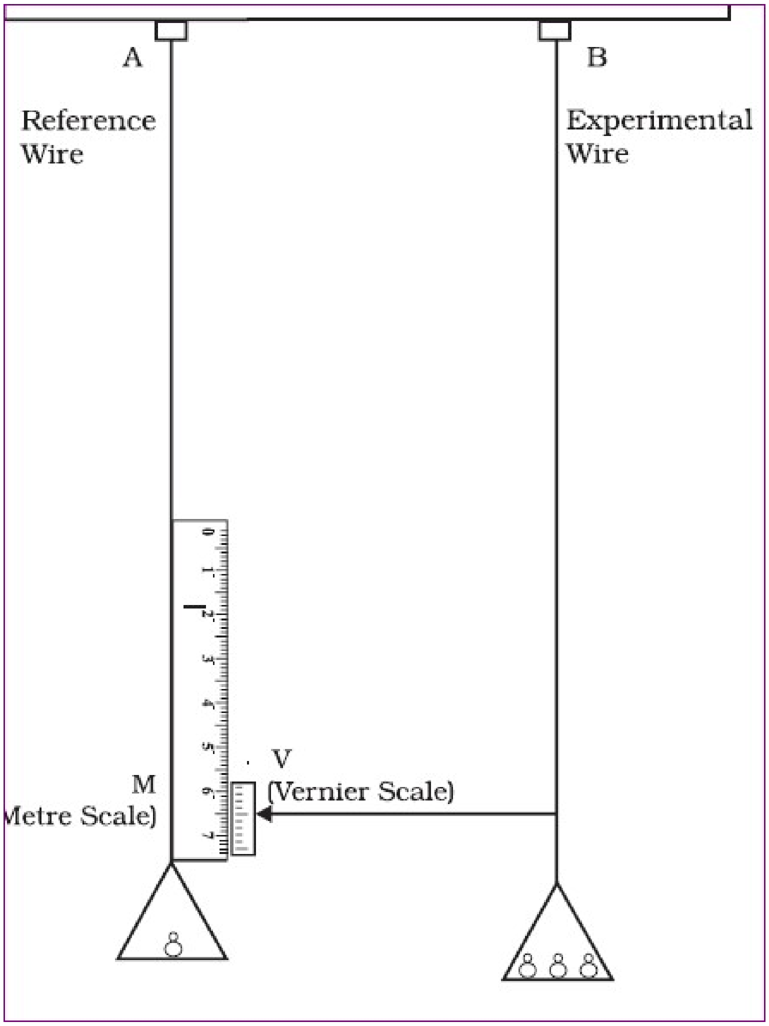
- The ratio of tensile (or compressive) stress (σ) to the longitudinal strain (ε) is defined as Young's modulus and is denoted by the symbol Y.

$$Y = (F/A)/(\Delta L/L)$$
$$= (F \times L)/(A \times \Delta L)$$

- The unit of Young's modulus is N m-2 or Pascal (Pa).

Substance	Density ρ (kg m^{-3})	Young's modulus Y (10^9 N m^{-2})	Ultimate strength, S_u (10^6 N m^{-2})	Yield strength S_y (10^6 N m^{-2})
Aluminium	2710	70	110	95
Copper	8890	110	400	200
Iron (wrought)	7800-7900	190	330	170
Steel	7860	200	400	250
Glass#	2190	65	50	—
Concrete	2320	30	40	—
Wood#	525	13	50	—
Bone#	1900	9	170	—
Polystyrene	1050	3	48	—

Determination of Young's Modulus of the Material of a Wire



• The Young's modulus of the material of the experimental wire is given by

$$Y = \frac{\sigma}{\varepsilon} = \frac{Mg}{\pi r^2} \cdot \frac{L}{\Delta L}$$
$$= Mg \times L/(\pi r^2 \times \Delta L)$$

Shear Modulus (Modulus of Rigidity)

- The ratio of shearing stress to the corresponding shearing strain is called the shear modulus of the material and is represented by G.

$$\boxed{\begin{aligned} G &= (F/A)/(\Delta x/L) \\ &= (F\times L)/(A\times \Delta x) \end{aligned}}$$

$$\boxed{\begin{aligned} G &= (F/A)/\theta \\ &= F/(A\times \theta) \end{aligned}}$$

- The shearing stress σs can also be expressed as

$$\boxed{\sigma_s = G \times \theta}$$

• SI unit of shear modulus is N m-2 or Pa.

- For most materials G ≈ Y/3.

Bulk Modulus

- The ratio of hydraulic (Normal) stress to the corresponding hydraulic (volume) strain is called bulk modulus. It is denoted by symbol B.

$$\boxed{B = -p/(\Delta V/V)}$$

- SI unit of bulk modulus is the same as that of pressure.

Compressibility

- The reciprocal of the bulk modulus is called compressibility and is denoted by k.
- It is defined as the fractional change in volume per unit increase in pressure.

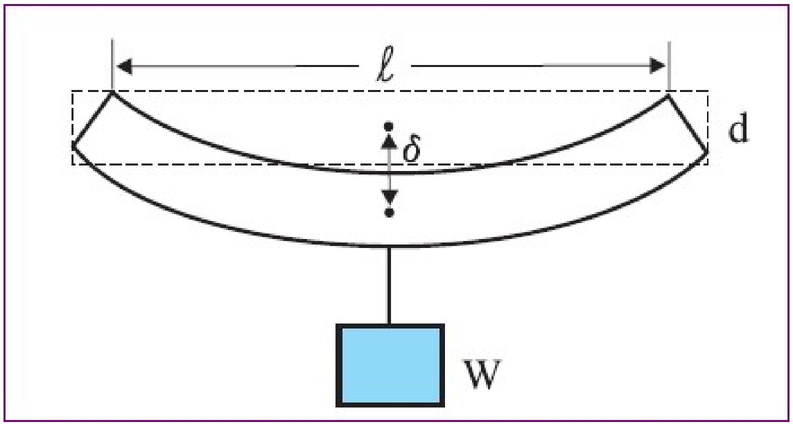
$$\boxed{k = (1/B) = -(1/\Delta p) \times (\Delta V/V)}$$

- The bulk moduli for solids are much larger than for liquids, which are again much larger than the bulk modulus for gases(air).
- Thus solids are least compressible whereas gases are most compressible.

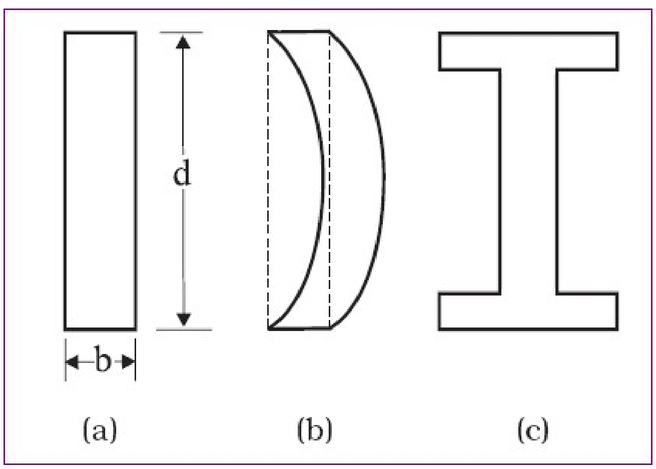
APPLICATIONS OF ELASTIC BEHAVIOUR OF MATERIALS

1. In designing a building, the structural design of the columns, beams and supports.
    - A bar of length l, breadth b, and depth d when loaded at the centre by a load W sags by an amount given by

$$\boxed{\delta = W\, l^3/(4bd^3 Y)}$$



- For a given material, increasing the depth d rather than the breadth b is more effective in reducing the bending.
- The bending of deep bars is called buckling.
- To avoid buckling I shaped beams are used.



- I shaped section provides a large load bearing surface and enough depth to prevent bending.
- This shape reduces the weight of the beam, without sacrificing the strength. Hence reduces the cost.

2. To find the thickness required for a metal rope, to be used in cranes to pull

up heavy objects.

• The ropes are always made of a number of thin wires braided together for flexibility and strength

3. The maximum height of the mountain is limited by the elastic properties of the rock which hold the mountain.

• The maximum height is calculated to be around 10 kms.