

Objective Questions

Question 1.

Mark (✓) against the correct answer in the following:

$f : \mathbb{N} \rightarrow \mathbb{N} : f(x) = 2x$ is

- A. one - one and onto
- B. one - one and into
- C. many - one and onto
- D. many - one and into

Answer:

$$f(x) = 2x$$

For One - One

$$f(x_1) = 2x_1$$

$$f(x_2) = 2x_2$$

put $f(x_1) = f(x_2)$ we get

$$2x_1 = 2x_2$$

Hence, if $f(x_1) = f(x_2)$, $x_1 = x_2$

Function f is one - one

For Onto

$$f(x) = 2x$$

let $f(x) = y$, such that $y \in \mathbb{N}$

$$2x = y$$

$$\Rightarrow x = \frac{y}{2}$$

If $y = 1$

$$x = \frac{1}{2} = 0.5$$

which is not possible as $x \in \mathbb{N}$

Hence, f is not onto., f is into

Hence, option b is correct

Question 2.

Mark (\surd) against the correct answer in the following:

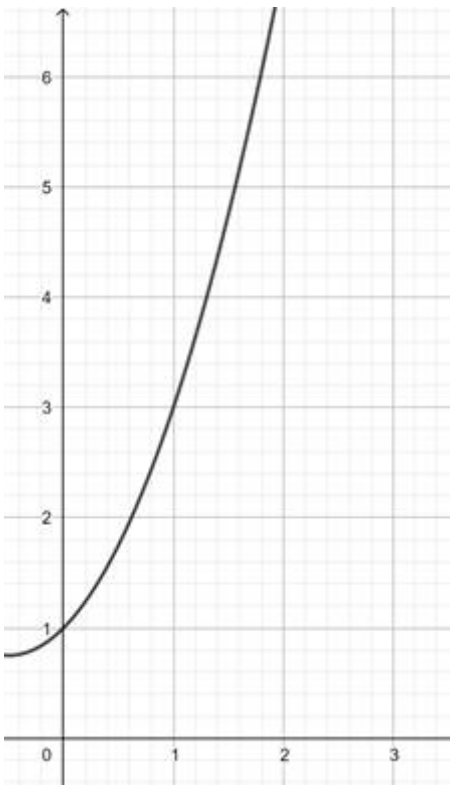
$f : \mathbb{N} \rightarrow \mathbb{N} : f(x) = x^2 + x + 1$ is

- A. one - one and onto
- B. one - one and into
- C. many - one and onto
- D. many - one and into

Answer:

In the given range of \mathbb{N} $f(x)$ is monotonically increasing.

$\therefore f(x) = x^2 + x + 1$ is one one.



But Range of $f(n) = [0.75, \infty) \neq \mathbb{N}(\text{codomain})$

Hence, $f(x)$ is not onto.

Hence, the function $f : \mathbb{N} \rightarrow \mathbb{N} : f(x) = (x^2 + x + 1)$ is one - one but not onto. i.e. into

Question 3.

Mark (\checkmark) against the correct answer in the following:

$f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = x^2$ is

- A. one - one and onto
- B. one - one and into
- C. many - one and onto
- D. many - one and into

Answer:

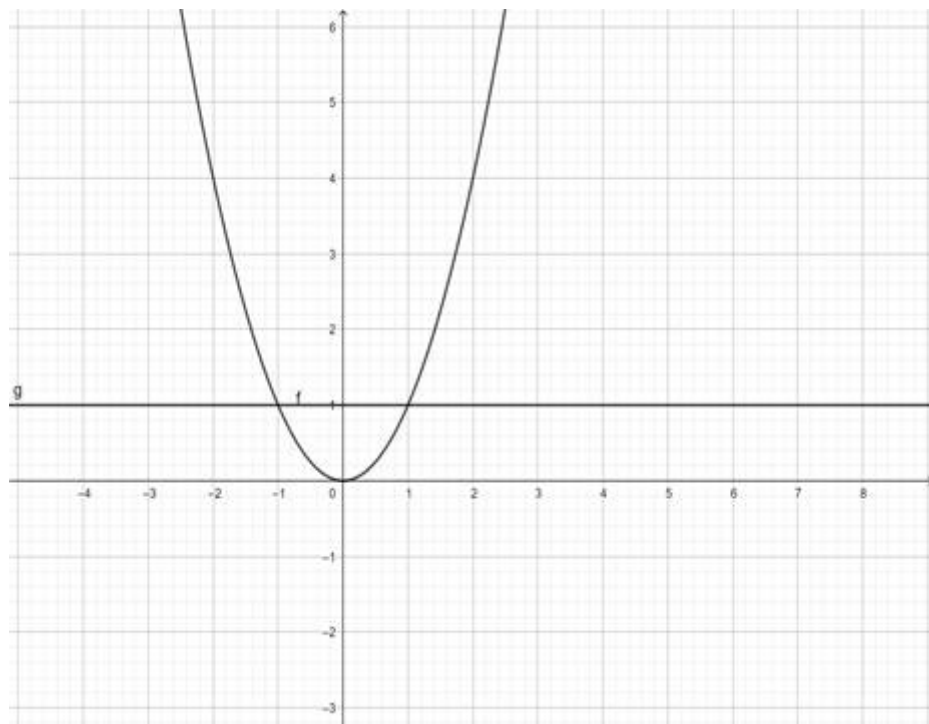
$$f(x) = x^2$$

$$\Rightarrow y = x^2$$

in this range the lines cut the curve in 2 equal valued points of y , therefore, the function $f(x) = x^2$ is many - one .

Range of $f(x) = (0, \infty) \neq \mathbb{R}$ (codomain)

$\therefore f(x)$ is into



$\therefore f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = x^2$ is many - one into

Question 4.

Mark (\checkmark) against the correct answer in the following:

$f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = x^3$ is

- A. one - one and onto
- B. one - one and into
- C. many - one and onto
- D. many - one and into

Answer:

$$f(x) = x^3$$

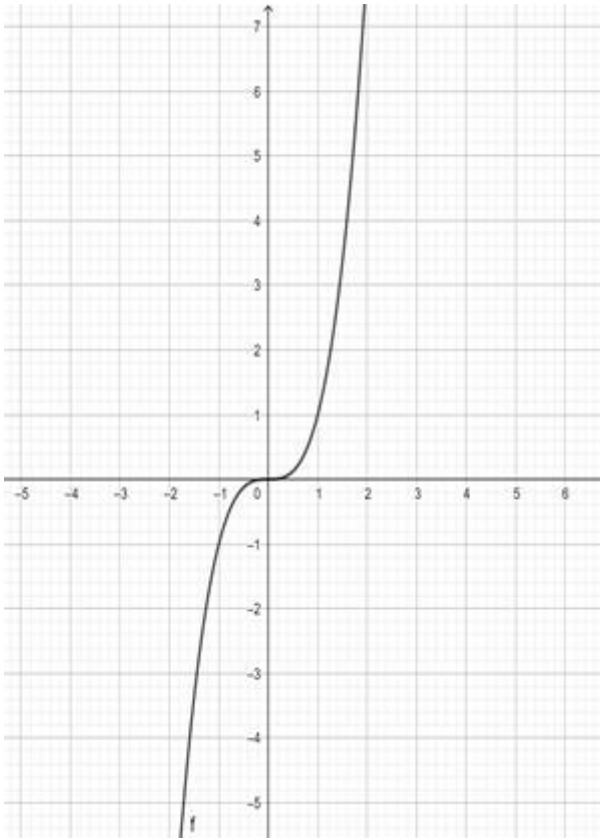
Since the function $f(x)$ is monotonically increasing from the domain $\mathbb{R} \rightarrow \mathbb{R}$

$\therefore f(x)$ is one - one

Range of $f(x) = (-\infty, \infty) = \mathbb{R}$ (codomain)

$\therefore f(x)$ is into

$\therefore f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = x^3$ is one - one into.



Question 5.

Mark (\checkmark) against the correct answer in the following:

$f : \mathbb{R}^+ \rightarrow \mathbb{R}^+ : f(x) = e^x$ is

- A. many - one and into
- B. many - one and onto
- C. one - one and into
- D. one - one and onto

Answer:

$f(x) = e^x$

Since the function $f(x)$ is monotonically increasing from the domain $\mathbb{R}^+ \rightarrow \mathbb{R}^+$

$\therefore f(x)$ is one -one

Range of $f(x) = (1, \infty) = \mathbb{R}^+$ (codomain)

$\therefore f(x)$ is onto

$\therefore f : \mathbb{R}^+ \rightarrow \mathbb{R}^+ : f(x) = e^x$ is one - one onto.

Question 6.

Mark (\checkmark) against the correct answer in the following:

$$f : \left[\frac{-\pi}{2}, \frac{\pi}{2} \right] \rightarrow [-1, 1] : f(x) = \sin x \text{ is}$$

- A. one - one and into
- B. one - one and onto
- C. many - one and into
- D. many - one and onto

Answer:

$$f : \left[\frac{-\pi}{2}, \frac{\pi}{2} \right] \rightarrow [-1, 1] : f(x) = \sin x$$

Here in this range, the function is NOT repeating its value,

Therefore it is one - one.

Range = Codomain

\therefore Function is onto

Hence, option B is the correct choice.

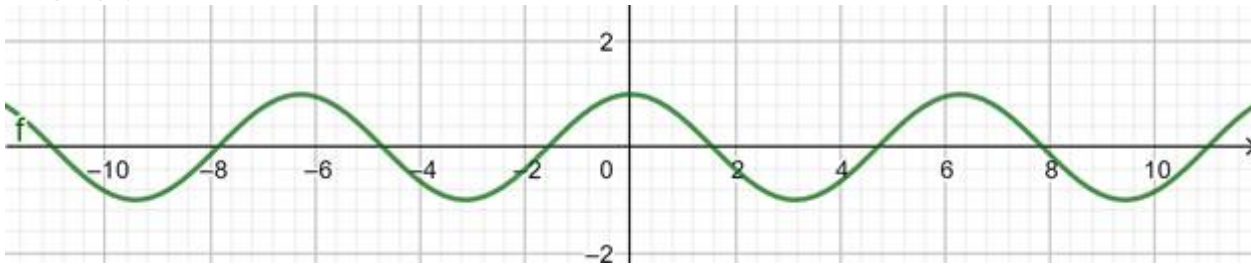
Question 7.

Mark (\checkmark) against the correct answer in the following:

$$f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = \cos x \text{ is}$$

- A. one - one and into
- B. one - one and onto
- C. many - one and into
- D. many - one and onto

Answer:



$$f(x) = \cos x$$

$$y = \cos x$$

Here in this range the lines cut the curve in many equal valued points of y therefore the function $f(x) = \cos x$ is not one - one.

$$\Rightarrow f(x) = \text{many one}$$

$$\text{Range of } f(x) = [-1, 1] \neq \mathbb{R}(\text{codomain})$$

$\therefore f(x)$ is not onto.

$$\Rightarrow f(x) = \text{into}$$

Hence, $f(x) = \cos x$ is many one and into

Ans: (c) many - one and into

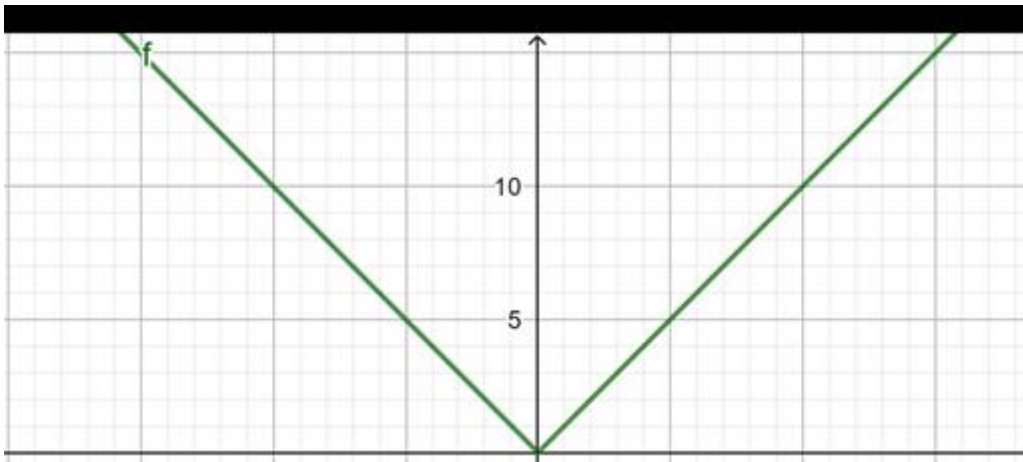
Question 8.

Mark (\surd) against the correct answer in the following:

$$f : \mathbb{C} \rightarrow \mathbb{R} : f(z) = |z| \text{ is}$$

- A. one - one and into
- B. one - one and onto
- C. many - one and into
- D. many - one and onto

Answer:



Here in this range the lines cut the curve in 2 equal valued points of y therefore the function $f(z) = |z|$ is not one - one

$\Rightarrow f(z) = \text{many one.}$

Range of $f(z) = [0, \infty) \neq \mathbb{R}(\text{codomain})$

$\therefore f(z)$ is not onto.

$\Rightarrow f(z) = \text{into}$

Hence, $f(z) = |z|$ is many one and into

Question 9.

Mark (\checkmark) against the correct answer in the following:

Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Then $f : A \rightarrow A : f(x) = \frac{(x-2)}{(x-3)}$ is

- A. one - one and into
- B. one - one and onto
- C. many - one and into
- D. many - one and onto

Answer:

$$f : A \rightarrow A : f(x) = \frac{(x-2)}{(x-3)}$$

In this function

$x = 3$ and $y = 1$ are the asymptotes of this curve and these are not included in the functions of the domain and range respectively therefore the function $f(x)$ is one-one since there are no different values of x which has same value of y .

and the function has no value at $y = 1$ here range = codomain

$\therefore f(x)$ is onto

Question 10.

Mark (\checkmark) against the correct answer in the following:

Let A and B be two non - empty sets and let

$f : (A \times B) \rightarrow (B \times A) : f(a, b) = (b, a)$. Then, f is

- A. one - one and into
- B. one - one and onto
- C. many - one and into
- D. many - one and onto

Answer:

SINCE, $f(a, b) = (b, a)$. There is no same value of y at different values of x \therefore function is one one

$\therefore \text{Range}(A \times B) \neq \text{Codomain}(B \times A)$

\Rightarrow function is into

Question 11.

Mark (\checkmark) against the correct answer in the following:

Let $f : \mathbb{Q} \rightarrow \mathbb{Q} : f(x) = (2x + 3)$. Then, $f^{-1}(y) = ?$

- A. $(2y - 3)$
- B. $\frac{1}{(2y - 3)}$
- C. $\frac{1}{2}(y - 3)$

D. none of these

Answer:

$f(x) = 2x + 3$

$$\Rightarrow y = 2x + 3$$

$$x \Leftrightarrow y$$

$$\Rightarrow x = 2y + 3$$

$$\Rightarrow x - 3 = 2y$$

$$\Rightarrow \frac{x-3}{2} = y$$

$$x \Leftrightarrow y$$

$$\Rightarrow \frac{y-3}{2} = x$$

Question 12.

Mark (✓) against the correct answer in the following:

Let $f : \mathbb{R} - \left\{ \frac{-4}{3} \right\} \rightarrow -\left\{ \frac{4}{3} \right\} : f(x) = \frac{4x}{(3x+4)}$. Then $f^{-1}(y) = ?$

A. $\frac{4y}{(4-3y)}$

B. $\frac{4y}{(4y+3)}$

C. $\frac{4y}{(3y-4)}$

D. None of these

Answer:

$$f(x) = \frac{4x}{3x+4}$$

$$\Rightarrow y = \frac{4x}{3x+4}$$

$$x \Leftrightarrow y$$

$$\Rightarrow x = \frac{4y}{3y+4}$$

$$\Rightarrow 3yx + 4x = 4y$$

$$\Rightarrow y(3x - 4) = -4x$$

$$\Rightarrow y = \frac{4x}{4-3x}$$

$$x \Leftrightarrow y$$

$$\Rightarrow x = \frac{4y}{4-3y}$$

Question 13.

Mark ($\sqrt{}$) against the correct answer in the following:

If $f(x) = \frac{(4x+3)}{(6x-4)}$, $x \neq \frac{2}{3}$ then $(f \circ f)(x) = ?$

A. x

B. $(2x - 3)$

C. $\frac{4x-6}{3x+4}$

D. None of these

Answer:

$$f(x) = \frac{4x+3}{6x-4}$$

$$\Rightarrow f(f(x)) = \frac{4f(x)+3}{6f(x)-4} = (f \circ f)(x)$$

$$\Rightarrow f(f(x)) = \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}$$

$$\Rightarrow f(f(x)) = \frac{16x + 12 + 18x - 12}{24x + 18 - 24x + 16} = \frac{34x}{34} = x$$

Question 14.

Mark (✓) against the correct answer in the following:

If $f(x) = (x^2 - 1)$ and $g(x) = (2x + 3)$ then $(g \circ f)(x) = ?$

- A. $(2x^2 + 3)$
- B. $(3x^2 + 2)$
- C. $(2x^2 + 1)$
- D. None of these

Answer:

$$f(x) = (x^2 - 1)$$

$$g(x) = (2x + 3)$$

$$\therefore (g \circ f)(x) = g(f(x))$$

$$\Rightarrow g(f(x)) = 2f(x) + 3$$

$$\Rightarrow g(f(x)) = 2((x^2 - 1)) + 3 = 2x^2 - 2 + 3 = 2x^2 + 1$$

Question 15.

Mark (✓) against the correct answer in the following:

If $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ then $f(x) = ?$

- A. x^2
- B. $(x^2 - 1)$
- C. $(x^2 - 2)$
- D. None of these

Answer:

$$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

$$\Rightarrow f(x) = x^2 - 2$$

Question 16.

Mark (✓) against the correct answer in the following:

If $f(x) = \frac{1}{(1-x)}$ then $(f \circ f \circ f)(x) = ?$

A. $\frac{1}{(1-3x)}$

B. $\frac{x}{(1+3x)}$

C. x

D. None of these

Answer:

$$f(x) = \frac{1}{1-x}$$

$$\Rightarrow (f \circ f \circ f)(x) = f(f(f(x)))$$

$$\Rightarrow f(f(x)) = \frac{1}{1-f(x)} = \frac{1}{1-\frac{1}{1-x}} = \frac{1-x}{1-x-1} = \frac{x-1}{x} = 1 - \frac{1}{x}$$

$$\Rightarrow f(f(f(x))) = \frac{1}{1-f(f(x))} = \frac{1}{1-\left(1-\frac{1}{x}\right)} = \frac{1}{\frac{1}{x}} = x$$

Question 17.

Mark (✓) against the correct answer in the following:

If $f(x) = \sqrt[3]{3-x^3}$ then $(f \circ f)(x) = ?$

A. $x^{\frac{1}{3}}$

B. x

C. $\left(1-x^{\frac{1}{3}}\right)$

D. None of these

Answer:

$$f(x) = \sqrt[3]{3 - x^3}$$

$$\Rightarrow f(f(x)) = \sqrt[3]{3 - f(x)^3} = \sqrt[3]{3 - \left(\sqrt[3]{3 - x^3}\right)^3}$$

$$\Rightarrow f(f(x)) = \sqrt[3]{3 - (3 - x^3)}$$

$$\Rightarrow f(f(x)) = \sqrt[3]{x^3} = x$$

Question 18.

Mark ($\sqrt{}$) against the correct answer in the following:

If $f(x) = x^2 - 3x + 2$ then $(f \circ f)(x) = ?$

A. x^4

B. $x^4 - 6x^3$

C. $x^4 - 6x^3 + 10x^2$

D. None of these

Answer:

$$f(x) = x^2 - 3x + 2$$

$$\Rightarrow f(x) = x^2 - 2x - x + 2 = x(x - 2) - 1(x - 2)$$

$$\Rightarrow f(x) = (x - 2)(x - 1)$$

$$\Rightarrow f(x) = (x - 2)(x - 1)$$

$$\Rightarrow f(f(x)) = (f(x) - 2)(f(x) - 1)$$

$$\Rightarrow f(f(x)) = ((x - 2)(x - 1) - 2)((x - 2)(x - 1) - 1)$$

$$\Rightarrow f(f(x)) = (x^2 - 3x + 2 - 2)(x^2 - 3x + 2 - 1)$$

$$\Rightarrow f(f(x)) = (x^2 - 3x)(x^2 - 3x + 1)$$

$$\Rightarrow f(f(x)) = x^4 - 3x^3 + x^2 - 3x^3 + 9x^2 - 3x$$

$$\Rightarrow f(f(x)) = x^4 - 6x^3 + 10x^2 - 3x$$

Question 19.

Mark (✓) against the correct answer in the following:

If $f(x) = 8x^3$ and $g(x) = x^{1/3}$ then $(g \circ f)(x) = ?$

A. x

B. $2x$

C. $\frac{x}{2}$

D. $3x^2$

Answer:

$$f(x) = 8x^3$$

$$g(x) = x^{1/3}$$

$$\Rightarrow (g \circ f)(x) = (f(x))^{\frac{1}{3}} = (8x^3)^{\frac{1}{3}} = 2x$$

Question 20.

Mark (✓) against the correct answer in the following:

If $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$ then $\{h \circ (g \circ f)\} \left(\sqrt{\frac{\pi}{4}} \right) = ?$

A. 0

B. 1

C. $\frac{1}{x}$

D. $\frac{1}{2} \log \frac{\pi}{4}$

Answer:

$f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$

$$\Rightarrow g(f(x)) = \tan(f(x)) = \tan(x^2)$$

$$\Rightarrow h(g(f(x))) = \log(g(f(x))) = \log(\tan(x^2))$$

$$\Rightarrow h\left(g\left(f\left(\sqrt{\frac{\pi}{4}}\right)\right)\right) = \log\left(\tan\left(\sqrt{\frac{\pi}{4}}\right)\right) = \log\left(\tan\left(\frac{\pi}{4}\right)\right) = \log(1) = 0$$

Question 21.

Mark (\surd) against the correct answer in the following:

If $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$ then $(g \circ f) = ?$

A. $\{(3, 1), (1, 3), (3, 4)\}$

B. $\{(1, 3), (3, 1), (4, 3)\}$

C. $\{(3, 4), (4, 3), (1, 3)\}$

D. $\{(2, 5), (5, 2), (1, 5)\}$

Answer:

$$g = \{(2, 3), (5, 1), (1, 3)\}$$

$$(g \circ f) = \{(\text{dom}(f), 3), (\text{dom}(f), 1), (\text{dom}(f), 3)\}$$

$$\Rightarrow (g \circ f) = \{(1, 3), (3, 1), (4, 3)\}$$

Question 22.

Mark (\surd) against the correct answer in the following:

Let $f(x) = \sqrt{9 - x^2}$. Then, $\text{dom}(f) = ?$

A. $[-3, 3]$

B. $[-\infty, -3]$

C. $[3, \infty)$

D. $(-\infty, -3] \cup (4, \infty)$

Answer:

$$F(x) = \sqrt{9 - x^2}$$

$$\sqrt{9 - x^2} \text{ should be } \geq 0$$

$$\Rightarrow 9 - x^2 \geq 0$$

$$\Rightarrow x^2 \leq 9$$

$$\Rightarrow -3 \leq x \leq 3$$

$$\therefore \text{dom}(f) = [-3, 3]$$

Question 23.

Mark (\checkmark) against the correct answer in the following:

$$\text{Let } f(x) = \sqrt{\frac{x-1}{x+4}}. \text{ Then, dom } (f) = ?$$

A. $[1, 4)$

B. $[1, 4]$

C. $(-\infty, 4]$

D. $(-\infty, 1] \cup (4, \infty)$

Answer:

$$f(x) = \sqrt{\frac{x-1}{x-4}}$$

$$\sqrt{\frac{x-1}{x-4}} \geq 0$$

$$\Rightarrow x - 1 \geq 0$$

$$\Rightarrow x \geq 1$$

And $x \neq 4$

$$x > 4 \text{ and } x \leq 1$$

$$\Rightarrow \text{dom}(f) = (-\infty, 1] \cup (4, \infty)$$

Question 24.

Mark (\checkmark) against the correct answer in the following:

Let $f(x) = e^{\sqrt{x^2-1}} \cdot \log(x-1)$. Then, $\text{dom}(f) = ?$

A. $(-\infty, 1]$

B. $[-1, \infty)$

C. $(1, \infty)$

D. $(-\infty, -1] \cup (1, \infty)$

Answer:

$$f(x) = e^{\sqrt{x^2-1}} \log(x-1)$$

$$x-1 > 0$$

$$\Rightarrow x > 1$$

And

$$\Rightarrow x^2 - 1 \geq 0$$

$$\Rightarrow x^2 \geq 1$$

$$\Rightarrow -1 \leq x \leq 1$$

Taking the intersection we get

$$\text{Dom}(f) = (1, \infty)$$

Question 25.

Mark (✓) against the correct answer in the following:

Let $f(x) = \frac{x}{(x^2 - 1)}$. Then, $\text{dom}(f) = ?$

- A. \mathbb{R}
- B. $\mathbb{R} - \{1\}$
- C. $\mathbb{R} - \{-1\}$
- D. $\mathbb{R} - \{-1, 1\}$

Answer:

$$f(x) = \frac{x}{x^2 - 1}$$

$$x^2 - 1 \neq 0$$

$$x \neq (1, -1)$$

$$\therefore \text{Dom}(f) = \mathbb{R} - \{-1, 1\}$$

Question 26.

Mark (✓) against the correct answer in the following:

Let $f(x) = \frac{\sin^{-1} x}{x}$. Then, $\text{dom}(f) = ?$

- A. $(-1, 1)$
- B. $[-1, 1]$
- C. $[-1, 1] - \{0\}$
- D. none of these

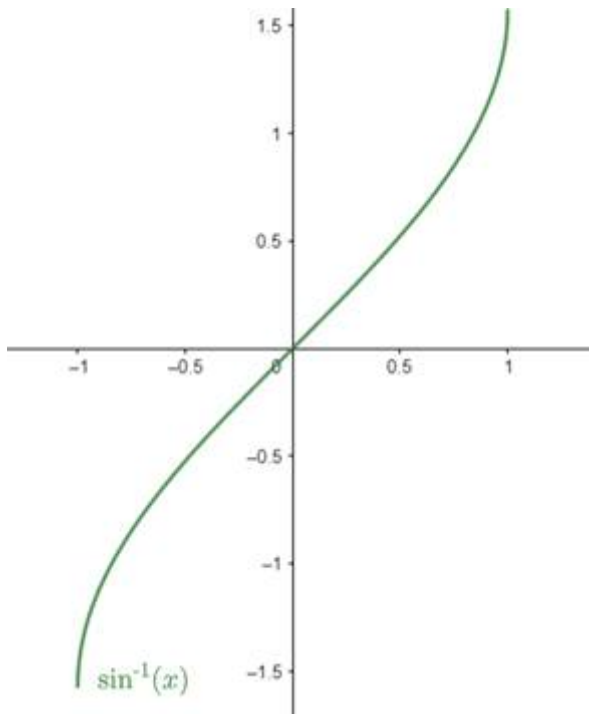
Answer:

$$\text{Given: } f(x) = \frac{\sin^{-1} x}{x}$$

From $f(x)$, $x \neq 0$

Now, domain of $\sin^{-1}x$ is $[-1, 1]$ as the values of $\sin^{-1}x$ lies between -1 and 1 .

We can see that from this graph:



Domain of $f(x) = \sin^{-1}(x)$ is $[-1, 1]$

Hence, B is the correct answer.

Question 27.

Mark (✓) against the correct answer in the following:

Let $f(x) = \cos^{-1} 2x$. Then, $\text{dom}(f) = ?$

A. $[-1, 1]$

B. $\left[\frac{-1}{2}, \frac{1}{2}\right]$

C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

D. $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

Answer:

$$f(x) = \cos^{-1} 2x.$$

domain of $\cos^{-1} x = [-1, 1]$

on multiplying by an integer the domain decreases by same number

$$\Rightarrow \text{domain of } \cos^{-1} 2x = [-1/2, 1/2]$$

Question 28.

Mark (✓) against the correct answer in the following:

Let $f(x) = \cos^{-1}(3x - 1)$. Then, $\text{dom}(f) = ?$

A. $\left(0, \frac{2}{3}\right)$

B. $\left[0, \frac{2}{3}\right]$

C. $\left[\frac{-2}{3}, \frac{2}{3}\right]$

D. None of these

Answer:

$$f(x) = \cos^{-1}(3x - 1).$$

$$\text{domain of } \cos^{-1} x = [-1, 1]$$

on multiplying by an integer the domain decreases by same number

$$\Rightarrow \text{domain of } \cos^{-1} 3x = [-1/3, 1/3]$$

$$\Rightarrow \text{domain of } \cos^{-1}(3x - 1) = [1/3 - 1/3, 1/3 + 1/3] = [0, 2/3]$$

Question 29.

Mark (✓) against the correct answer in the following:

Let $f(x) = \sqrt{\cos x}$. Then, $\text{dom}(f) = ?$

A. $\left[0, \frac{\pi}{2}\right]$

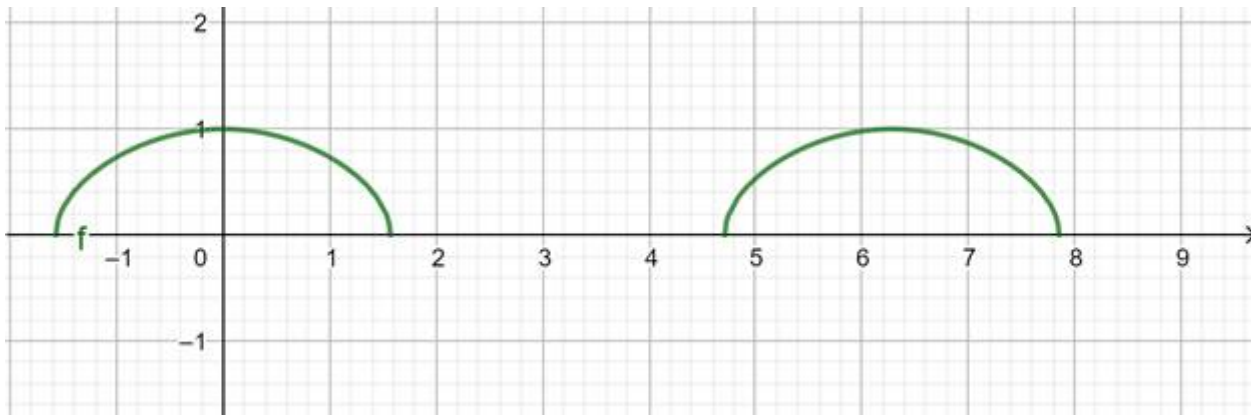
B. $\left[\frac{3\pi}{2}, 2\pi\right]$

C. $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

D. none of these

Answer:

$$f(x) = \sqrt{\cos x}$$



As per the diagram

We can imply that domain of $\sqrt{\cos x}$

is $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

Question 30.

Mark (✓) against the correct answer in the following:

Let $f(x) = \sqrt{\log (2x - x^2)}$. Then, $\text{dom} (f) = ?$

A. (0, 2)

B. [1, 2]

C. $(-\infty, 1]$

D. None of these

Answer:

$$f(x) = \sqrt{\log (2x - x^2)}$$

$$2x - x^2 > 1$$

$$\Rightarrow x^2 - 2x + 1 < 0$$

$$\Rightarrow (x - 1)^2 < 0$$

$$\Rightarrow x - 1 < 0$$

$$\Rightarrow x < 1$$

$$\log(2x - x^2) > 0$$

$$\Rightarrow 2x - x^2 > e^0 = 1$$

$$\Rightarrow x < 1$$

$$\text{Dom}(f) = (-\infty, 1)$$

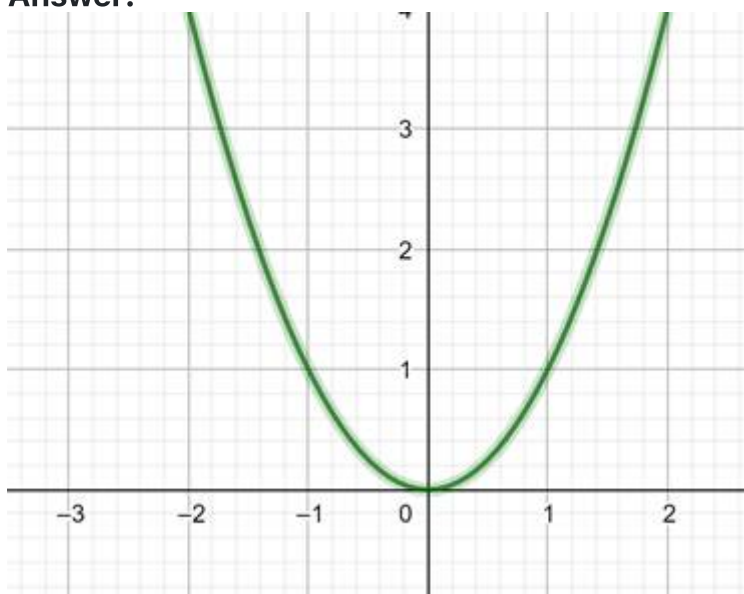
Question 31.

Mark (\checkmark) against the correct answer in the following:

Let $f(x) = x^2$. Then, dom (f) and range (f) are respectively.

- A. \mathbb{R} and \mathbb{R}
- B. \mathbb{R}^+ and \mathbb{R}^+
- C. \mathbb{R} and \mathbb{R}^+
- D. \mathbb{R} and $\mathbb{R} - \{0\}$

Answer:



According to sketched graph of x^2

Domain of $f(x) = R$

And Range of $f(x) = R^+$

Question 32.

Mark (✓) against the correct answer in the following:

Let $f(x) = x^3$. Then, dom (f) and range (f) are respectively

A. R and R

B. R^+ and R^+

C. R and R^+

D. R^+ and R

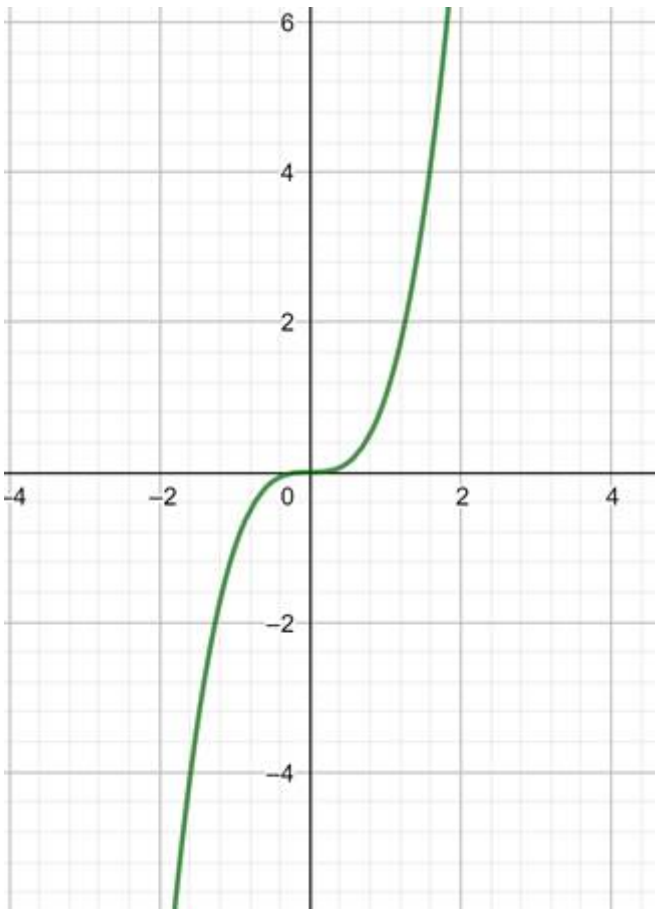
Answer:

According to sketched graph of x^3

Domain of $f(x) = R$

And Range of $f(x) = R$

Since x^3 is a, monotonically increasing function



Question 33.

Mark (✓) against the correct answer in the following:

Let $f(x) = \log(1-x) + \sqrt{x^2-1}$. Then, $\text{dom}(f) = ?$

- A. $(1, \infty)$
- B. $(-\infty, -1]$
- C. $[-1, 1)$
- D. $(0, 1)$

Answer:

$$\log(1-x) + \sqrt{x^2-1}$$

$$1-x > 0$$

$$x < 1$$

$$x^2 - 1 \geq 0$$

$$x^2 \geq 1$$

$$\Rightarrow -1 \leq x \leq 1$$

Taking intersection of the ranges we get

$$\text{Dom } (f) = (b) (-\infty, -1]$$

Question 34.

Mark (✓) against the correct answer in the following:

Let $f(x) = \frac{1}{(1-x^2)}$. Then, range (f) = ?

A. $(-\infty, 1]$

B. $[1, \infty)$

C. $[-1, 1]$

D. none of these

Answer:

$$f(x) = \frac{1}{1-x^2}$$

$$\Rightarrow y = \frac{1}{1-x^2}$$

$$\Rightarrow y - yx^2 = 1$$

$$\Rightarrow y - 1 = yx^2$$

$$\Rightarrow x = \sqrt{\frac{y-1}{y}}$$

$$\Rightarrow \frac{y-1}{y} \geq 0$$

$$\Rightarrow y \geq 1$$

$$\therefore \text{range } (f) = [1, \infty)$$

Question 35.

Mark (✓) against the correct answer in the following:

Let $f(x) = \frac{x^2}{(1+x^2)}$. Then, range (f) = ?

- A. $[1, \infty)$
- B. $[0, 1)$
- C. $[-1, 1]$
- D. $(0, 1]$

Answer:

$$f(x) = \frac{x^2}{1+x^2}$$

$$\Rightarrow y = \frac{x^2}{1+x^2}$$

$$\Rightarrow y + yx^2 = x^2$$

$$\Rightarrow y = x^2(1-y)$$

$$\Rightarrow x = \sqrt{\frac{y}{1-y}}$$

$$\frac{y}{1-y} \geq 0$$

$$\Rightarrow y \geq 0$$

And

$$1-y > 0$$

$$\Rightarrow y < 1$$

Taking intersection we get

$$\text{range } (f) = [0, 1)$$

Question 36.

Mark (✓) against the correct answer in the following:

The range of $f(x) = x + \frac{1}{x}$ is

- A. $[-2, 2]$
- B. $[2, \infty)$
- C. $(-\infty, -2]$
- D. none of these

Answer:

$$f(x) = x + \frac{1}{x}$$

For this type

Range is

$$-2 \leq y \leq 2$$

Question 37.

Mark (✓) against the correct answer in the following:

The range of $f(x) = a^x$, where $a > 0$ is

- A. $[-\infty, 0]$
- B. $[-\infty, 0)$
- C. $[0, \infty)$
- D. $(0, \infty)$

Answer:

$$f(x) = a^x$$

when $x < 0$

$$0 < a^x < 1$$

When $x \geq 0$

$$a^x > 0$$

Therefore range of $f(x) = a^x = (0, \infty)$

Question 38.

Mark (✓) against the correct answer in the following:

$$\text{Let } f : \mathbb{N} \rightarrow \mathbb{N} : f(x) = \begin{cases} \frac{1}{2}(n+1), & \text{when } n \text{ is odd} \\ \frac{n}{2}, & \text{when } n \text{ is even.} \end{cases}$$

Then, f is

- A. one - one and into
- B. one - one and onto
- C. many - one and into
- D. many - one and onto

Answer:

$$f(1) = 1$$

$$f(2) = 1$$

$$f(3) = 2$$

$$f(4) = 2$$

$$f(5) = 3$$

$$f(6) = 3$$

Since at different values of x we get same value of y $\therefore f(n)$ is many -one

And range of $f(n) = \mathbb{N} = \mathbb{N}(\text{codomain})$

\therefore the function $f: \mathbb{N} \rightarrow \mathbb{Z}$, defined by

$$f : \mathbb{N} \rightarrow \mathbb{N} : f(x) = \begin{cases} \frac{1}{2}(n+1), & \text{when } n \text{ is odd} \\ \frac{n}{2}, & \text{when } n \text{ is even.} \end{cases}$$

is both many - one and onto.

Question 39.

Mark (✓) against the correct answer in the following:

Let $f : \mathbb{N} \rightarrow \mathbb{X} : f(x) = 4x^2 + 12x + 15$. Then, $f^{-1}(y) = ?$

A. $\frac{1}{2}(\sqrt{y-4} + 3)$

B. $\frac{1}{2}(\sqrt{y-6} - 3)$

C. $\frac{1}{2}(\sqrt{y-4} + 5)$

D. None of these

Answer:

$$f(x) = 4x^2 + 12x + 15$$

$$\Rightarrow y = 4x^2 + 12x + 15$$

$$\Rightarrow y = (2x + 3)^2 + 6$$

$$\Rightarrow \sqrt{y - 6} = 2x + 3$$

$$\Rightarrow \frac{1}{2}(\sqrt{y - 6} - 3) = x$$

$$f^{-1}(y) = \frac{1}{2}(\sqrt{y - 6} - 3)$$