CBSE SAMPLE PAPER-03

Class – XI

MATHEMATICS

Time allowed: 3 hours, Maximum Marks: 100

General Instructions:

a) All questions are compulsory.

b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.

c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

d) Use of calculators is not permitted.

Section A

1. Find the domain of the function $f(x)=\frac{1}{\sqrt{2-x^2}}$.

Sol: Domain of is in the open interval $(-2,2)$

2. If $A=\{y=\sin x, 0\le x<\frac{\pi}{4}\}\ and\ B=\{y=\cos x, 0\le x<\frac{\pi}{4}\}$

Sol: $(A\cap B)=\{\phi\}$

3. What is the maximum value of a $a=1-\sin x$ **if.**

Sol: Max value is 2.

4. Name the locus of points (M) **, the sum of whose distance from two given points is a constant.**

Sol: Ellipse

5. Check whether the three points (2, 0), (5, 3), (2, 6) are collinear.

Sol: Condition for colinearity is not satisfied here since

$$\begin{vmatrix} 2-2 & 0-6 \\ 5-2 & 3-6 \end{vmatrix} = \begin{vmatrix} 0 & -6 \\ 3 & -3 \end{vmatrix} \neq 0$$

6. Write the condition so that the equation $ax^2 + ay^2 + bx + cy + d = 0$ represents a circle.

Sol: $b^2 + c^2 - 4ad > 0$

Section B

7. Find the inverse of the function $f(x) = x^2 - x + 1$, $x > \frac{1}{2}$

Sol:

$$y = x^2 - x + 1$$

$$y = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$y - \frac{3}{4} = \left(x - \frac{1}{2}\right)^2$$

$$x = \frac{1}{2} + \sqrt{y - \frac{3}{4}}$$

$$f^{-1}(x) = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$$

8. Find the vertex, axis , Focus , Directrix and latus rectum of the parabola $8y^2 + 24x - 40y + 134 = 0$.

Sol:

$$\textit{Equation is } 8y^2 + 24x - 40y + 134 = 0$$

$$= 4y^2 + 12x - 20y + 67 = 0$$

This can be written as

$$y^2 - 5y = -3x - \frac{67}{4}$$

$$\left(y - \frac{5}{2}\right)^2 = -3x - \frac{67}{4} + \frac{25}{4} - 3\left(x + \frac{7}{2}\right)$$

$Let\, Y = y - \frac{5}{2}$

$X = x + \frac{7}{2}$

$Y^2 = -3X$

$This\ is\ of\ the\ form\ y^2 = -4ax$

$Latus\ rectum\ is = 3$

$Vertex\left(-\frac{7}{2}, \frac{5}{2}\right)$

$Axis\ y = \frac{5}{2}$

$Focus\ \left(-\frac{7}{2} - \frac{3}{4}, \frac{5}{2}\right)$

$Directrix: referred\ to\ New\ axis: X = a = \frac{3}{4}$

$Directrix referred\ to\ Old\ axis: \frac{3}{4} = x + \frac{7}{2}$

$x = \frac{3}{4} - \frac{7}{2}$

$x = -\frac{11}{4}$

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9. Express $\frac{7-4i}{3+2i}$ **in the form** $a+ib$.

Sol:

$\frac{7-4i}{3+2i} = \frac{7-4i}{3+2i} \times \frac{3-2i}{3+21}$

$\frac{13-26i}{13} = 1-2i$

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10. Solve the inequality $(x-2)((x-3) > 0$.

Sol: Either both factors are negative or both factors are positive to have this in equality. if $x < 2$ both factors are negative and if $x > 3$ both factors are positive. Hence the solution is $x \in \{(-\infty, 2) \cup (3, \infty)\}$

11. Find the general value of x if $\tan 5x = \dfrac{1}{\tan 2x}$.

Sol:

$\tan 5x = \cot 2x$

$\tan 5x = \tan(\frac{\pi}{2} - 2x)$

$5x = (\frac{\pi}{2} - 2x)$

$5x = n\pi + (\frac{\pi}{2} - 2x)$

$7x = n\pi + \frac{\pi}{2}$

$x = \frac{1}{7}(n\pi + \frac{\pi}{2})$

12. In a single throw of 2 dies what is the probability of getting a prime number on each die.

Sol: Total number of occurrence = $6 \times 6 = 36$
On each die there are 3 prime numbers $\{2, 3, 5\}$
Hence total number of favorable cases $3 \times 3 = 9$
Probability of getting a prime in each die = $\dfrac{9}{36} = \dfrac{1}{4}$

13. If $f(x) = x^3 - x; \phi(x) = \sin 2x$ Find the value $f[\phi(\frac{\pi}{12})]$.

Sol:

$$\phi(\frac{\pi}{12}) = \sin 2.(\frac{\pi}{12})$$
$$= \sin\frac{\pi}{6}$$
$$= \frac{1}{2}$$
$$f(x) = (\frac{1}{2})^3 - \frac{1}{2}$$
$$= \frac{1}{8} - \frac{1}{2}$$
$$= -\frac{3}{8}$$

14. If $\tan A = \frac{m}{m+1}$ *and* $\tan B = \frac{1}{2m+1}$ **prove that** $\tan A + \tan B + \tan A \tan B = 1$.

Sol:

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$= \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \frac{m}{m+1} \cdot \frac{1}{2m+1}} = 1$$
$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$
$$\tan A + \tan B + \tan A \tan B = 1$$

15. If $f: R \to R$ **is defined as follows:** $f(x) = \begin{cases} 1 & if\ x \in Q \\ -1 & if\ x \notin Q \end{cases}$ **Find**

$f(\sqrt{3}$, $f(3)$, $f(\sqrt{3}+1)$.

Sol:

$$f(\sqrt{3}) = -1$$

$f(3)=1$
$f(\sqrt{3}+1)=1$

16. Prove that the equation $sin\theta = x+\frac{1}{x}$ is impossible if x is real

Sol : Use the inequality

$AM \geq GM$

$AM\ between\ x, \frac{1}{x} = \frac{x+\frac{1}{x}}{2}$

$GM\ between\ x, \frac{1}{x} = \sqrt{x.\frac{1}{x}} = 1$

$\frac{x+\frac{1}{x}}{2} \geq 1$

$x+\frac{1}{x} \geq 2$

$Since\ \ -1 \leq \sin\theta \leq 1$

$\sin\theta = x+\frac{1}{x}\ is\ impossible$

17. Find the domain of the function for which

$f(x)=\phi(x);, if\ f(x)=3x^2+1, and\phi(x)=7x-1.$

Sol:

$f(x)=\phi(x)$

$f(x)=3x^2+1$

$\phi(x)=7x-1$

$3x^2+1=7x-1$

$3x^2-7x+2=0$

$(x-2)(3x-1)=0$

$x=2, x=\frac{1}{3}$

Hence f(x) and ϕ(x) are equal when the domain is in the set $\{\frac{1}{3}, 2\}$

18. Find the limit $\lim_{x\to 0}\frac{1-\cos x}{x}$ **.**

Sol:

$$\lim_{x\to 0}\frac{1-\cos x}{x}$$

$$=\lim_{x\to 0}\frac{1-(1-2\sin^2\frac{x}{2})}{x}$$

$$=\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{x}$$

$$=\lim_{x\to 0}\frac{\sin\frac{x}{2}}{2\frac{x}{2}}\sin\frac{x}{2}$$

$$=\frac{1}{2}.1.0$$

$$=0$$

19. Solve $2\sin^2 x+14\sin x\cos x+50\cos^2 x=26$

Sol:

$$2\sin^2 x+14\sin x\cos x+50\cos^2 x=26$$

$$=2\sin^2 x+14\sin x\cos x+50\cos^2 x=26(\sin^2 x+\cos^2 x)$$

$$=-24\sin^2 x+14\sin x\cos x+24\cos^2 x=0$$

$$=24\sin^2 x-14\sin x\cos x-24\cos^2 x=0$$

$$=24\tan^2 x-14\tan x-24=0$$

$$\tan x=\frac{14\pm\sqrt{196+2304}}{48}$$

$$\tan x = \frac{14 \pm \sqrt{2500}}{48}$$
$$\tan x = \frac{14 \pm 50}{48}$$
$$\tan x = \frac{64}{48} ; or ; -\frac{36}{48}$$
$$\tan x = \frac{4}{3} \text{ or } -\frac{3}{4}$$

Section C

20. Differentiate $\sin x$ from the first principle w.r.t. x.

Sol:

$$y = \sin x$$
$$y + \Delta y = \sin(x + \Delta x)$$
$$\Delta y = \sin(x + \Delta x) - y$$
$$\Delta y = \sin(x + \Delta x) - \sin x$$
$$\Delta y = 2\cos\frac{2x + \Delta x}{2}\sin\frac{\Delta x}{2}$$
$$\frac{\Delta y}{\Delta x} = \frac{2\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}}{\Delta x}$$
$$\frac{\Delta y}{\Delta x} = \frac{\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$
$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \cos x$$
$$\frac{dy}{dx} = \cos x$$

Note: As $\Delta x \to 0 ; \frac{\Delta x}{2} \; also \to 0$

21. Find the sum of n terms of the series $12+16+23+33+46......$

Sol: The successive First order of difference is $4, 7, 10, 13, ...$ this is an AP.

The second order difference is(Difference of the first difference) $3, 3, 3, ...$

Third order difference (Difference of second order differences) is all 0

n th term

$$T_n = T_1 + (n-1)\Delta T_1 + \frac{(n-1)(n-2)}{2!}\Delta T_2 + \frac{(n-1)(n-2)(n-3)}{3!}\Delta T_3$$

= $12+4(n-1)+3\frac{(n-1)(n-2)}{2}$

= $\frac{3n^2-n+22}{2}$

Sum = $\frac{1}{2}\left(3\ \Sigma n^2 - \Sigma n + 22n\right)$

= $\frac{1}{2}\left(3\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 22n\right)$

= $\frac{1}{2}\left(n^3+n^2+22n\right)$

22. Find the equation of a circle whose diameter is the line joining the points (x_1, y_1) and (x_2, y_2).

Sol: Let the point A be (x_1, y_1) and B be (x_2, y_2)

Let the point C be a point be (x, y) on the circle

Then AC and BC are perpendicular

Product of Slopes of line AC and BC =-1

$$\frac{y-y_1}{x-x_1}\cdot\frac{y-y_2}{x-x_2} = -1$$

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

23. Calculate the mean deviation about the mean from the following data

xi	5	7	9	10	12	15
fi	14	6	2	2	2	4

Sol:

x_i	f_i	$f_i x_i$	$\lvert x_i - 9\rvert$	$f_i \lvert x_i - 9\rvert$
5	14	70	4	56
7	6	42	2	12
9	2	18	0	0
10	2	20	1	2
12	2	24	3	6
15	4	60	6	24
	$N = \Sigma f_i = 26$	$\Sigma f_i\, x_i = 234$		$f_i \Sigma \lvert x_i - 9\rvert = 100$

$$Mean = \bar{X} = \frac{1}{N}(\Sigma f_i\, x_i) = \frac{234}{26} = 9$$

Mean Deviation $= M.D = \frac{1}{N}(\Sigma f_i\, |x_i - 9|) = \frac{100}{26} = 3.84$

24. How many numbers can be formed with the digits $1, 2, 3, 4, 3, 2, 1$ so that odd digits are in odd places and even digits are in even places.

Sol: The odd digits $1, 3, 3, 1$ can be arranged in their 4 places in $\frac{4!}{2!2!}$ ways

Even digits $2, 4, 2$ can be arranged in their 3 places in $\frac{3!}{2!}$

Hence the total number of arrangements = $\frac{4!}{2!2!}\times\frac{3!}{2!}=6\times3=18$ ways

25. Two engineers go for an interview for two vacancies in the same grade. The probability of engineer 1 (E1) getting selected is $\frac{1}{3}$ and that of engineer 2 (E2) is $\frac{1}{5}$. Find the probability that only one of them will be selected.

Sol: Probability of one of them getting selected $P(E_1 or E_2)$ = 1- (Probability of both getting selected + Probability of none getting selected)

= $1-[P(E_1\cap E_2)+P(E_1'\cap E_2')]$

= $1-(\frac{1}{3}\times\frac{1}{5}+\frac{2}{3}\times\frac{4}{5})$

= $1-(\frac{1}{15}+\frac{8}{15})$

= $1-\frac{9}{15}=\frac{6}{15}=\frac{2}{5}$

26. How many numbers are there between 1 and 1000(both included) that are not divisible by 2, 3, and 5?

Sol: Let A denote the set of numbers that are divisible by 2, B set of numbers that are divisible by 3, C set of numbers that are divisible by 5, D set of numbers that are divisible by both 2 and3 , E set of numbers that are divisible by both 2 and 5, F set of numbers that are divisible by 3 and 5, G set of numbers that are divisible by all the three numbers

$a+(n-1)d=T_n$

$n=\frac{T_n}{d}-\frac{a}{d}+1$

In this case $\frac{a}{d}=1$, Hence $n = integer\ \ part\ of\ \frac{T_n}{d}$

$$n(A) = [\frac{1000}{2}] = 500$$

$$n(B) = [\frac{1000}{3}] = 333$$

$$n(C) = [\frac{1000}{5}] = 200$$

$$n(D) = [\frac{1000}{2 \times 3}] = 166$$

$$n(E) = [\frac{1000}{2 \times 5}] = 100$$

$$n(F) = [\frac{1000}{3 \times 5}] = 66$$

$$n(G) = [\frac{1000}{2 \times 3 \times 5}] = 33$$

Numbers that are divisible by 2, 3, 5 are

$$\begin{aligned} & n(A \cup B \cup C) \\ & = n(A) + n(B) + n(C) - n(A \cup B) - n(A \cup C) - n(B \cup C) + n(A \cap B \cap C) \\ & = 500 + 333 + 200 + 1666 + 100 + 66 + 33 \\ & = 734 \end{aligned}$$

Numbers that are not divisible by 2, 3, 5 are

$$1000 - 734 = 266$$