Exercise 10h

Question 1.

Differentiate x^6 with respect to $(1/\sqrt{x})$.

Answer:

Given : Let u = x^6 and v = $\frac{1}{\sqrt{x}}$

To differentiate : x^6 with respect to $(1/\sqrt{x})$.

Formula used : $\frac{d\,(x^n)}{dx} = n.x^{n-1}$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

Let u = x^6 and v = $\frac{1}{\sqrt{x}}$

Differentiating u with respect to x

$\frac{du}{dx} = 6x^5$

Differentiating v with respect to x

$\frac{dv}{dx} = \frac{-1}{2}\, x^{-\frac{3}{2}}$

$\frac{du}{dv} = \left.\frac{du}{dx}\middle/\frac{dv}{dx}\right.$

$\frac{du}{dv} = \left.6x^5\middle/\frac{-1}{2}\, x^{-\frac{3}{2}}\right.$

$\frac{du}{dv} = -12x^{5+\frac{3}{2}}$

$\frac{du}{dv} = -12x^{\frac{13}{2}}$

Ans. $-12x^{13/2}$

Question 2.
Differentiate log x with respect to $\cot x$.

Answer:
Given : Let u = log x and v = cot x

To differentiate : log xwith respect to cot x

Formula used : $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

$$\frac{d(\log x)}{dx} = \frac{1}{x}$$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

Let u = log x and v = cot x

Differentiating u with respect to x

$\frac{du}{dx} = \frac{1}{x}$

Differentiating v with respect to x

$\frac{dv}{dx} = -\operatorname{cosec}^2 x$

$\frac{du}{dv} = \frac{du}{dx} \Big/ \frac{dv}{dx}$

$\frac{du}{dv} = \frac{\frac{1}{x}}{-\operatorname{cosec}^2 x}$

$\frac{du}{dv} = \frac{-1}{x\,cosec^2 x}$

Question 3.

Differentiate $e^{\sin x}$ with respect to $\cos x$.

Answer:

Given : Let u = $e^{\sin x}$ and v = cos x

To differentiate : $e^{\sin x}$ with respect to cos x

Formula used : $\frac{d(e^x)}{dx} = e^x$

$\frac{d(\cos x)}{dx}$ = - sinx

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

Let u = $e^{\sin x}$ and v = cos x

Differentiating u with respect to x

$\frac{du}{dx} = \frac{d(e^{\sin x})}{dx} = \cos x.\ e^{\sin x}$

Differentiating v with respect to x

$\frac{dv}{dx} = -\sin x$

$\frac{du}{dv} = \frac{du}{dx} \Big/ \frac{dv}{dx}$

$\frac{du}{dv} = \frac{\cos x.\ e^{\sin x}}{-\sin x}$

$\frac{du}{dv} = -\,e^{\sin x}.\cot x$

Ans. $-e^{\sin x}\cot x$

Question 4.

Differentiate $\tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}$ with respect to $\cos^{-1}x^2$.

Answer:

Given : Let u = $\tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}$ and v = $\cos^{-1}x^2$.

To differentiate : $\tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}$ with respect to $\cos^{-1}x^2$.

Formula used : $\frac{d\,(x^n)}{dx} = n.x^{n-1}$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

$$\frac{d\,(\tan^{-1}x)}{dx} = \frac{1}{1+\ x^2}$$

$$\frac{d\,(\cos^{-1}x)}{dx} = \frac{-1}{\sqrt{1-\ x^2}}$$

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{vdu-udv}{v^2}$$

Let u = $\tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}$ and v = $\cos^{-1}x^2$.

Differentiating u with respect to x

$$\frac{du}{dx} = \frac{d(\tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}})}{dx} = \frac{1}{1+\frac{1-x^2}{1+x^2}} \cdot \frac{d\,(\sqrt{\frac{1-x^2}{1+x^2}})}{dx}$$

$$\frac{du}{dx} = \frac{1+x^2}{1+x^2+1-x^2} \cdot \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{-1}{2}} \cdot \frac{-2x(1+x^2)-2x(1-x^2)}{(1+x^2)^2}$$

$$\frac{du}{dx} = \frac{1+x^2}{2} \cdot \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{-1}{2}} \cdot \frac{-2x-2x^3-2x+2x^3}{(1+x^2)^2} = \frac{1+x^2}{2} \cdot \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{-1}{2}} \cdot \frac{-4x}{(1+x^2)^2}$$

$$\frac{du}{dx} = \left(\frac{1-x^2}{1+x^2}\right)^{\frac{-1}{2}} \cdot \frac{-x}{(1+x^2)} = \sqrt{\frac{1+x^2}{1-x^2}} \cdot \frac{-x}{(1+x^2)} = \frac{-x}{\sqrt{(1-x^2)(1+x^2)}} = \frac{-x}{\sqrt{1-x^4}}$$

$$\frac{du}{dx} = \frac{-x}{\sqrt{1-x^4}}$$

Differentiating v with respect to x

$$\frac{dv}{dx} = -\frac{1}{\sqrt{1-(x^2)^2}} \cdot \frac{d(x^2)}{dx} = \frac{-2x}{\sqrt{1-x^4}}$$

$$\frac{dv}{dx} = \frac{-2x}{\sqrt{1-x^4}}$$

$$\frac{du}{dv} = \frac{du}{dx} \Big/ \frac{dv}{dx}$$

$$\frac{du}{dv} = \frac{\frac{-x}{\sqrt{1-x^4}}}{\frac{-2x}{\sqrt{1-x^4}}} = \frac{1}{2}$$

$$\frac{du}{dv} = \frac{1}{2}$$

Ans. $\frac{1}{2}$

Question 5.

Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Answer:

Given : Let u = $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ and v = $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

To differentiate : $\tan^{-1}\frac{2x}{1-x^2}$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Formula used : $\frac{d\,(x^n)}{dx} = n.x^{n-1}$

$$\frac{d\,(\tan^{-1}x)}{dx} = \frac{1}{1+\ x^2}$$

$$\frac{d\,(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-\ x^2}}$$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{vdu-udv}{v^2}$$

Let u = $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ and v = $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Differentiating u with respect to x

$$\frac{du}{dx} = \frac{d(\tan^{-1}\frac{2x}{1-x^2})}{dx} = \frac{1}{1+(\frac{2x}{1-x^2})^2}\cdot\frac{d\,(\frac{2x}{1-x^2})}{dx} = \frac{1}{1+\frac{4x^2}{1+x^4-2x^2}}\cdot\frac{2\left(1-x^2\right)+2x(2x)}{(1-x^2)^2}$$

$$\frac{du}{dx} = \frac{(1-x^2)^2}{1+x^4-2x^2+4x^2}\cdot\frac{2-2x^2+4x^2}{(1-x^2)^2} = \frac{(1-x^2)^2}{1+x^4+2x^2}\cdot\frac{2+2x^2}{(1-x^2)^2} = \frac{2(1+x^2)}{(1+x^2)^2} = \frac{2}{(1+x^2)}$$

$$\frac{du}{dx} = \frac{2}{(1+x^2)}$$

Differentiating v with respect to x

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-(\frac{2x}{1+x^2})^2}}\cdot\frac{d(\frac{2x}{1+x^2})}{dx} = \frac{1+x^2}{\sqrt{1+x^4+2x^2-4x^2}}\cdot\frac{2\left(1+x^2\right)-2x(2x)}{(1+x^2)^2}$$

$$\frac{dv}{dx} = \frac{1+x^2}{\sqrt{1+x^4-2x^2}}\cdot\frac{2+2x^2-4x^2}{(1+x^2)^2} = \frac{1+x^2}{\sqrt{(1-x^2)^2}}\cdot\frac{2-2x^2}{(1+x^2)^2} = \frac{1+x^2}{1-x^2}\cdot\frac{2(1-x^2)}{(1+x^2)^2} = \frac{2}{1+x^2}$$

$\frac{dv}{dx} = \frac{2}{1+x^2}$

$\frac{du}{dv} = \frac{du}{dx} \Big/ \frac{dv}{dx}$

$\frac{du}{dv} = \frac{\frac{2}{(1+x^2)}}{\frac{2}{(1+x^2)}} = 1$

$\frac{du}{dv} = 1$

Ans. 1

Question 6.

Differentiate $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to $\cos^{-1}(2x^2 - 1)$.

Answer:

Given : Let u = $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ and v = $\cos^{-1}(2x^2 - 1)$.

To differentiate : $\tan^{-1}\frac{x}{\sqrt{1-x^2}}$ with respect to $\cos^{-1}(2x^2 - 1)$

Formula used : $\frac{d(x^n)}{dx} = n.x^{n-1}$

$$\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{d(\cos^{-1}x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{vdu - udv}{v^2}$

Let u = $\tan^{-1}\frac{x}{\sqrt{1-x^2}}$ and v = $\cos^{-1}(2x^2 - 1)$

Differentiating u with respect to x

$$\frac{du}{dx} = \frac{d(\tan^{-1}\frac{x}{\sqrt{1-x^2}})}{dx} = \frac{1}{1+(\frac{x}{\sqrt{1-x^2}})^2} \cdot \frac{d\,(\frac{x}{\sqrt{1-x^2}})}{dx} = \frac{1}{1+\frac{x^2}{1-x^2}} \cdot \frac{1(\sqrt{1-x^2})+x(\frac{-2x}{2\sqrt{1-x^2}})}{1-x^2}$$

$$\frac{du}{dx} = \frac{1-x^2}{1-x^2+x^2} \cdot \frac{\sqrt{1-x^2}-\frac{-x^2}{\sqrt{1-x^2}}}{1-x^2} = (1-x^2) \cdot \frac{\frac{1-x^2+x^2}{\sqrt{1-x^2}}}{(1-x^2)^2} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Differentiating v with respect to x

$$\frac{dv}{dx} = \frac{d\,[\cos^{-1}(2x^2-1)]}{dx} = \frac{-1}{\sqrt{1-(2x^2-1)^2}} \cdot \frac{d(2x^2-1)}{dx} = \frac{-1}{\sqrt{1-4x^4-1+4x^2}} \cdot 4x$$

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{4x^2-4x^4}} = \frac{-4x}{2x\sqrt{1-x^2}} = \frac{-2}{\sqrt{1-x^2}}$$

$$\frac{dv}{dx} = \frac{-2}{\sqrt{1-x^2}}$$

$$\frac{du}{dv} = \frac{du}{dx}\Big/\frac{dv}{dx}$$

$$\frac{du}{dv} = \frac{\frac{1}{\sqrt{1-x^2}}}{\frac{-2}{\sqrt{1-x^2}}} = \frac{-1}{2}$$

$$\frac{du}{dv} = \frac{-1}{2}$$

Ans. $\frac{-1}{2}$

Question 7.
Differentiate $\sin^3 x$ with respect to $\cos^3 x$.

Answer:
Given : Let u = $\sin^3 x$ and v = $\cos^3 x$

To differentiate : $\sin^3 x$ with respect to $\cos^3 x$

Formula used : $\frac{d(x^n)}{dx} = n.x^{n-1}$

$\frac{d(\sin x)}{dx}$ = cos x

$\frac{d(\cos x)}{dx}$ = - sin x

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

Let u = $\sin^3 x$ and v = $\cos^3 x$

Differentiating u with respect to x

$\frac{du}{dx} = 3\sin^2 x.\frac{d(\sin x)}{dx} = 3\sin^2 x \cos x$

$\frac{du}{dx} = 3\sin^2 x \cos x$

Differentiating v with respect to x

$\frac{dv}{dx} = 3\cos^2 x.\frac{d(\cos x)}{dx} = -3\cos^2 x \ \sin x$

$\frac{dv}{dx} = -3\cos^2 x \ \sin x$

$\frac{du}{dv} = \frac{du}{dx} \Big/ \frac{dv}{dx}$

$\frac{du}{dv} = \frac{3\sin^2 x\cos x}{-3\cos^2 x\sin x} = \frac{\sin x}{-\cos x} = -\tan x$

$\frac{du}{dv} = -\tan x$

Ans. $-\tan x$

Question 8.

Differentiate $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

Answer:

Given : Let u = $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and v = $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

To differentiate : $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

Formula used : $\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} = \tan 3\theta$

$\frac{d(x^n)}{dx} = n.x^{n-1}$

$\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$

$\frac{d(\cos^{-1} x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{vdu-udv}{v^2}$

Let u = $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and v = $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

Differentiating u with respect to x

$$\frac{du}{dx} = \frac{d\ \cos^{-1}\frac{1-x^2}{1+x^2}}{dx} = \frac{-1}{\sqrt{1-(\frac{1-x^2}{1+x^2})^2}} \cdot \frac{d(\frac{1-x^2}{1+x^2})}{dx} = \frac{-(1+x^2)}{\sqrt{(1+x^2)^2-(1-x^2)^2}} \cdot \frac{-2x(1+x^2)-2x(1-x^2)}{(1+x^2)^2}$$

$$\frac{du}{dx} = \frac{-(1+x^2)}{\sqrt{1+x^4+2x^2-1-x^4+2x^2}} \cdot \frac{-2x-2x^3-2x+2x^3}{(1+x^2)^2} = \frac{-(1+x^2)}{\sqrt{4x^2}} \cdot \frac{-4x}{(1+x^2)^2} = \frac{+2}{1+x^2}$$

$$\frac{du}{dx} = \frac{+2}{1+x^2}$$

For v = $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

Let x = tan θ

$$\tan^{-1}\frac{3x-x^3}{1-3x^2} = \tan^{-1}\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta} = \tan^{-1}(\tan 3\theta) = 3\theta = 3\tan^{-1}x$$

$$\tan^{-1}\frac{3x-x^3}{1-3x^2} = 3\tan^{-1}x$$

Differentiating v with respect to x ,

$$\frac{dv}{dx} = \frac{d(3\tan^{-1}x)}{dx} = \frac{3}{1+x^2}$$

$$\frac{dv}{dx} = \frac{3}{1+x^2}$$

$$\frac{du}{dv} = \frac{du}{dx}\Big/\frac{dv}{dx}$$

$$\frac{du}{dv} = \frac{\frac{+2}{1+x^2}}{\frac{+3}{1+x^2}} = \frac{2}{3}$$

$\frac{du}{dv} = \frac{2}{3}$

Ans. $\frac{2}{3}$

Question 9.

Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Answer:

Given : Let u = $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and v = $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

To differentiate : $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Formula used : $\frac{d(x^n)}{dx} = n.x^{n-1}$

$$\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

Let u = $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and v = $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Put x = cot θ or θ = $\cot^{-1} x$ in u

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = \tan^{-1}\frac{\sqrt{1+\cot^2\theta}-1}{\cot\theta} = \tan^{-1}\frac{\operatorname{cosec}\theta-1}{\cot\theta}$$

$$\tan^{-1}\frac{\operatorname{cosec}\theta-1}{\cot\theta} = \tan^{-1}\frac{\frac{1}{\sin\theta}-1}{\cot\theta} = \tan^{-1}\frac{\frac{1-\sin\theta}{\sin\theta}}{\cot\theta} = \tan^{-1}\frac{\frac{1-\sin\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}}$$

$$\tan^{-1}\frac{\frac{1-\sin\theta}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}} = \tan^{-1}\frac{1-\sin\theta}{\cos\theta}$$

We know that 1 - $\sin\theta = \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}$ - $2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ and $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$

1 - $\sin\theta = (\cos\frac{\theta}{2} - \sin\frac{\theta}{2})^2$

Substituting the above values in $\tan^{-1}\frac{1-\sin\theta}{\cos\theta}$,we get

$$\tan^{-1}\frac{1-\sin\theta}{\cos\theta} = \tan^{-1}\frac{(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})^2}{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}} = \tan^{-1}\frac{(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})^2}{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)(\cos\frac{\theta}{2}+\sin\frac{\theta}{2})}$$

$$\tan^{-1}\frac{1-\sin\theta}{\cos\theta} = \tan^{-1}\frac{(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})}{(\cos\frac{\theta}{2}+\sin\frac{\theta}{2})}$$

Dividing by $\cos\frac{\theta}{2}$ on numerator and denominator,we get

$$\tan^{-1}\frac{(\cos\frac{\theta}{2}-\sin\frac{\theta}{2})}{(\cos\frac{\theta}{2}+\sin\frac{\theta}{2})} = \tan^{-1}\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}} = \tan^{-1}\tan(\frac{\pi}{4} - \frac{\theta}{2}) = \frac{\pi}{4} - \frac{\theta}{2}$$

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = \frac{\pi}{4} - \frac{\theta}{2} = \frac{\pi}{4} - \frac{\cot^{-1}x}{2}$$

Differentiating u with respect to x

$$\frac{d(\tan^{-1}\frac{\sqrt{1+x^2}-1}{x})}{dx} = \frac{d(\frac{\pi}{4}-\frac{\cot^{-1}x}{2})}{dx} = \frac{1}{2(1+x^2)}$$

$$\frac{du}{dx} = \frac{1}{2(1+x^2)}$$

$v = \sin^{-1}\frac{2x}{1+x^2}$

Put x = tanθ

$V = \sin^{-1}\frac{2x}{1+x^2} = \sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta} = \sin^{-1}\frac{2\frac{\sin\theta}{\cos\theta}}{\sec^2\theta} = \sin^{-1}\frac{2\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}} = \sin^{-1}(2\sin\theta\cos\theta)$

$V = \sin^{-1}\frac{2x}{1+x^2} = \sin^{-1}(2\sin\theta\cos\theta) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1}x$

$V = \sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$

Differentiating v with respect to x

$\frac{dv}{dx} = \frac{2}{1+x^2}$

$\frac{du}{dv} = \frac{du}{dx}\Big/\frac{dv}{dx}$

$\frac{du}{dv} = \frac{\frac{1}{2(1+x^2)}}{\frac{2}{1+x^2}} = \frac{1}{4}$

$\frac{du}{dv} = \frac{1}{4}$

Ans. $\frac{1}{4}$

Question 10.

Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}(2x\sqrt{1-x^2})$ when $x \neq 0$.

Answer:

Given : Let u = $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ and v = $\cos^{-1}(2x\sqrt{1-x^2})$

To differentiate : $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}(2x\sqrt{1-x^2})$

Formula used : $\frac{d(x^n)}{dx} = n.x^{n-1}$

$$\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{d(\cos^{-1}x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

Let u = $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ and v = $\cos^{-1}(2x\sqrt{1-x^2})$

Substitute x = cosθ in u

u = $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \tan^{-1}\left(\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}\right) = \tan^{-1}\left(\frac{\sqrt{\sin^2\theta}}{\cos\theta}\right)$

u = $\tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = \tan^{-1}(\tan\theta) = \theta$

u = $\theta = \cos^{-1}x$

Differentiating u with respect to x

$$\frac{du}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

Substitute x = sinθ in v ,

v = $\cos^{-1}\left(2x\sqrt{1-x^2}\right) = \cos^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right) = \cos^{-1}\left(2\sin\theta\sqrt{\cos^2\theta}\right)$

v = $\cos^{-1}\left(2\sin\theta\sqrt{\cos^2\theta}\right) = \cos^{-1}\left(2\sin\theta\,.\cos\theta\right) = \cos^{-1}\left(\sin 2\theta\right)$

v = $\cos^{-1}\left(\sin 2\theta\right) = \cos^{-1}\left(\cos[\frac{\pi}{2} - 2\theta]\right) = \frac{\pi}{2} - 2\theta$

v = $\frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2\sin^{-1}x$

v = $\frac{\pi}{2} - 2\sin^{-1}x$

Differentiating v with respect to x

$$\frac{dv}{dx} = \frac{-2}{\sqrt{1-x^2}}$$

$$\frac{du}{dv} = \frac{du}{dx}\Big/\frac{dv}{dx}$$

$$\frac{du}{dv} = \frac{\frac{-1}{\sqrt{1-x^2}}}{\frac{-2}{\sqrt{1-x^2}}} = \frac{1}{2}$$

Ans. $-\frac{1}{2}$