Exercise 8a

Question 1.
Show that each one of the following systems of equations is inconsistent.

x + 2y = 9;

2x + 4y = 7.

Answer:
To prove: Set of given lines are inconsistent.

Given set of lines are : -

x + 2y = 9

2x + 4y = 7

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & 2\\2 & 4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}9\\7\end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix}1 & 2\\0 & 0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}9\\-11\end{bmatrix}$$

Again converting into equation form, we get

x + 2y = 9

0x + 0y = - 11

∴ 0 = - 11

which is not true

∴ x + 2y = 9

2x + 4y = 7 are inconsistent.

Question 2.
Show that each one of the following systems of equations is inconsistent.

2x + 3y = 5;

6x + 9y = 10.

Answer:
To prove: Set of given lines are inconsistent.

Given set of lines are : -

2x + 3y = 5

6x + 9y = 10

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$R_2 - 3R_1$

$$\begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$$

Again converting into equation form, we get

2x + 3y = 5

0x + 0y = - 5

∴ 0 = - 5

which is not true

∴ 2x + 3y = 5

6x + 9y = 10 are inconsistent.

Question 3.
Show that each one of the following systems of equations is inconsistent.

4x – 2y = 3;

6x – 3y = 5.

Answer:
To prove: Set of given lines are inconsistent.

Given set of lines are : -

4x – 2y = 3

6x – 3y = 5

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$4R_2 – 6R_1$

$$\begin{bmatrix} 4 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Again converting into equation form, we get

4x – 2y = 3

0x + 0y = 2

∴ 0 = 2

which is not true

∴ 4x – 2y = 3

6x – 3y = 5 are inconsistent.

Question 4.
Show that each one of the following systems of equations is inconsistent.

6x + 4y = 5;

9x + 6y = 8.

Answer:
To prove: Set of given lines are inconsistent.

Given set of lines are : -

6x + 4y = 5

9x + 6y = 8

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$2R_2 – 3R_1$

$$\begin{bmatrix} 6 & 4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

Again converting into equation form, we get

6x + 4y = 5

0x + 0y = 3

∴ 0 = 3

which is not true

∴ 6x + 4y = 5

9x + 6y = 8 are inconsistent.

Question 5.
Show that each one of the following systems of equations is inconsistent.

x + y – 2z = 5;

x – 2y + z = - 2;

- 2x + y + z = 4.

Answer:
To prove: Set of given lines are inconsistent.

Given set of lines are : -

x + y – 2z = 5;

x – 2y + z = - 2;

- 2x + y + z = 4

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

$R_2 - R_1$

$R_3 + 2R_1$

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \\ 14 \end{bmatrix}$$

$R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \\ 7 \end{bmatrix}$$

Converting back into equation form we get,

x + y – 2z = 5;

0x – 3y + 3z = - 7;

0x + 0y + 0z = 7

∴ 0 = 7

Which is not true.

∴x + y – 2z = 5;

x – 2y + z = - 2;

- 2x + y + z = 4

are inconsistent.

Question 6.
Show that each one of the following systems of equations is inconsistent.

2x – y + 3z = 1;

3x – 2y + 5z = - 4;

5x – 4y + 9z = 14.

Answer:
To prove: Set of given lines are inconsistent.

Given set of lines are : -

2x – y + 3z = 1;

3x – 2y + 5z = - 4;

5x – 4y + 9z = 14

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & -1 & 3 \\ 3 & -2 & 5 \\ 5 & -4 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 14 \end{bmatrix}$$

$2R_2 – 3R_1$

$2R_3 - 5R_1$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & -1 & 1 \\ 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -11 \\ 23 \end{bmatrix}$$

$R_3 - 3R_2$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -11 \\ 56 \end{bmatrix}$$

Converting back into equation form we get,

2x – y + 3z = 1;

0x – 1y + 1z = - 11;

0x + 0y + 0z = 56

∴ 0 = 56

Which is not true.

∴2x – y + 3z = 1;

3x – 2y + 5z = - 4;

5x – 4y + 9z = 14

are inconsistent.

Question 7.
Show that each one of the following systems of equations is inconsistent.

x + 2y + 4z = 12;

y + 2z = - 1;

3x + 2y + 4z = 4.

Answer:
To prove: Set of given lines are inconsistent.

Given set of lines are : -

x + 2y + 4z = 12;

y + 2z = - 1;

3x + 2y + 4z = 4

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & 2 & 4\\0 & 1 & 2\\3 & 2 & 4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}12\\-1\\4\end{bmatrix}$$

$R_3 - 3R_1$

$$\begin{bmatrix}1 & 2 & 4\\0 & 1 & 2\\0 & -4 & -8\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}12\\-1\\-32\end{bmatrix}$$

$R_3 + 4R_2$

$$\begin{bmatrix}1 & 2 & 4\\0 & 1 & 2\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}12\\-1\\-36\end{bmatrix}$$

Converting back into equation form we get,

x + 2y + 4z = 12;

y + 2z = - 1;

0x + 0y + 0z = - 36

∴ 0 = - 36

Which is not true.

∴2x – y + 3z = 1;

3x – 2y + 5z = - 4;

5x – 4y + 9z = 14

are inconsistent.

Question 8.
Show that each one of the following systems of equations is inconsistent.

3x – y – 2z = 2;

2y – z = - 1;

3x – 5y = 3.

Answer:
To prove: Set of given lines are inconsistent.

Given set of lines are : -

3x – y – 2z = 2;

2y – z = - 1;

3x – 5y = 3

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

$R_3 - R_1$

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 0 & -4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$R_3 + 2R_2$

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$$

Converting back into equation form we get,

3x – y – 2z = 2;

2y – z = - 1;

0x + 0y + 0z = - 1

∴ 0 = - 1

Which is not true.

∴3x – y – 2z = 2;

2y – z = - 1;

3x – 5y = 3

are inconsistent.

Question 9.
Solve each of the following systems of equations using matrix method.

5x + 2y = 4;

7x + 3y = 5.

Answer:
To find: - x , y

Given set of lines are : -

5x + 2y = 4;

7x + 3y = 5.

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$5R_2 - 7R_1$

$$\begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

Again converting into equation form, we get

5x + 2y = 4;

y = - 3

5x + 2× - 3 = 4

5x = 10

X = 2

∴ x = 2 , y = - 3

Question 10.
Solve each of the following systems of equations using matrix method.

3x + 4y – 5 = 0;

x - y + 3 = 0.

Answer:
To find: - x , y

Given set of lines are : -

3x + 4y – 5 = 0;

x - y + 3 = 0

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$3R_2 – R_1$

$$\begin{bmatrix} 3 & 4 \\ 0 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -14 \end{bmatrix}$$

Again converting into equation form, we get

3x + 4y = 5

- 7 y = - 14

Y = 2

3x + 4y = 5

3x + 4×2 = 5

3x = - 3

X = - 1

∴ x = - 1 , y = 2

Question 11.
Solve each of the following systems of equations using matrix method.

x + 2y = 1;

3x + y = 4.

Answer:
To find: - x , y

Given set of lines are : -

x + 2y = 1

3x + y = 4

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1\\4\end{bmatrix}$$

$R_2 - 3R_1$

$$\begin{bmatrix}1 & 2\\0 & -5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$$

Again converting into equation form, we get

x + 2y = 1

- 5y = 1

$Y = -\frac{1}{5}$

$x + 2\times -\frac{1}{5} = 1$

$x + -\frac{2}{5} = 1$

$x = 1 + \frac{2}{5}$

$X = \frac{7}{5}$

$\therefore x = \frac{7}{5}, y = -\frac{1}{5}$

Question 12.
Solve each of the following systems of equations using matrix method.

5x + 7y + 2 = 0;

4x + 6y + 3 = 0.

Answer:
To find: - x , y

Given set of lines are : -

5x + 7y + 2 = 0;

4x + 6y + 3 = 0.

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$5R_2 - 4R_1$

$$\begin{bmatrix}5 & 7\\0 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}-2\\-7\end{bmatrix}$$

Again converting into equation form, we get

5x + 7y = - 2

2y = - 7

Y = $-\frac{7}{2}$

5x + 7× $-\frac{7}{2}$ = - 2

5x = - 2 + $\frac{49}{2}$

5x = $\frac{45}{2}$

X = $\frac{9}{2}$

$\therefore x = \frac{9}{2}, y = -\frac{7}{2}$

Question 13.
Solve each of the following systems of equations using matrix method.

2x - 3y + 1 = 0;

x + 4y + 3 = 0.

Answer:
To find: - x , y

Given set of lines are : -

2x - 3y + 1 = 0;

x + 4y + 3 = 0

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$$

$2R_2 - R_1$

$$\begin{bmatrix} 2 & -3 \\ 0 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$$

Again converting into equation form we get

2x - 3y = - 1

11 y = - 5

$Y = -\frac{5}{11}$

$2x - 3\times -\frac{5}{11} = -1$

$2x = -1 - \frac{15}{11}$

$X = -\frac{13}{11}$

$\therefore x = -\frac{13}{11}, y = -\frac{5}{11}$

Question 14.
Solve each of the following systems of equations using matrix method.

4x - 3y = 3;

3x - 5y = 7.

Answer:
To find: - x , y

Given set of lines are : -

4x - 3y = 3;

3x - 5y = 7

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$4R_2 - 3R_1$

$$\begin{bmatrix} 4 & -3 \\ 0 & -11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 19 \end{bmatrix}$$

Again converting into equation form, we get

4x – 3y = 3

- 11y = 19

$Y = -\frac{19}{11}$

$4x - 3 \times -\frac{19}{11} = 3$

$4x = 3 - \frac{57}{11}$

$4x = -\frac{24}{11}$

$X = -\frac{6}{11}$

$\therefore x = -\frac{6}{11}, y = -\frac{19}{11}$

Question 15.
Solve each of the following systems of equations using matrix method.

2x + 8y + 5z = 5;

x + y + z = - 2;

x + 2y - z = 2.

Answer:
To find: - x , y , z

Given set of lines are : -

2x + 8y + 5z = 5;

x + y + z = - 2;

x + 2y - z = 2

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & 8 & 5 \\ 1 & 1 & 1 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}$$

$2R_2 - R_1$

$2R_3 - R_1$

$$\begin{bmatrix} 2 & 8 & 5 \\ 0 & -6 & -3 \\ 0 & -4 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -9 \\ -1 \end{bmatrix}$$

$3R_3 - 2R_2$

$$\begin{bmatrix} 2 & 8 & 5 \\ 0 & -6 & -3 \\ 0 & 0 & -15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -9 \\ 15 \end{bmatrix}$$

Again converting into equations, we get

2x + 8y + 5z = 5

- 6y - 3z = - 9

- 15z = 15

Z = - 1

- 6y - 3 × - 1 = - 9

- 6y = - 9 - 3

Y = 2

2x + 8×2 + 5× - 1 = 5

2x = 5 - 16 + 5

X = - 3

$\therefore x = -3, y = 2, z = -1$

Question 16.
Solve each of the following systems of equations using matrix method.

x – y + z = 1;

2x + y – z = 2;

X – 2y – z = 4.

Answer:
To find: - x , y , z

Given set of lines are : -

x – y + z = 1;

2x + y – z = 2;

X – 2y – z = 4

Converting following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$R_2 - 2R_1$

$R_3 - R_1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -3 \\ 0 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

$3R_3 + R_2$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -3 \\ 0 & 0 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 9 \end{bmatrix}$$

Again converting into equations we get

X – y + z = 1

3y - 3z = 0

- 9z = 9

Z = - 1

Y = z

Y = - 1

X + 1 - 1 = 1

X = 1

$\therefore x = 1, y = -1, z = -1$

Question 17.
Solve each of the following systems of equations using matrix method.

3X + 4y + 7z = 4;

2x – y + 3z = - 3;

x + 2y – 3z = 8.

Answer:
To find: - x , y , z

Given set of lines are : -

3X + 4y + 7z = 4;

2x – y + 3z = - 3;

x + 2y – 3z = 8

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 8 \end{bmatrix}$$

$3R_2 - 2R_1$

$3R_3 - R_1$

$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & -11 & -5 \\ 0 & 2 & -16 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -17 \\ 20 \end{bmatrix}$$

$11R_3 + 2R_2$

$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & -11 & -5 \\ 0 & 0 & -186 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -17 \\ 186 \end{bmatrix}$$

Again converting into equations we get

3x + 4y + 7z = 4

- 11y - 5z = - 17

- 186z = 186

Z = - 1

- 11y + 5 = - 17

- 11y = - 22

Y = 2

3x + 4×2 + 7× - 1 = 4

3x = 4 - 8 + 7

X = 1

$\therefore x = 1, y = 2, z = -1$

Question 18.
Solve each of the following systems of equations using matrix method.

x + 2y + z = 7;

x + 3z = 11;

2x – 3y = 1.

Answer:
To find: - x , y , z

Given set of lines are : -

x + 2y + z = 7;

x + 3z = 11;

2x – 3y = 1

Converting following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

$R_2 - R_1$

$R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 2 \\ 0 & -7 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -13 \end{bmatrix}$$

$R_3 + R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 2 \\ 0 & -9 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -9 \end{bmatrix}$$

Again converting into equations we get

X + 2y + z = 7

- 2y + 2z = 4

- 9y = - 9

Y = 1

- 2×1 + 2z = 4

2z = 6

Z = 3

X + 2×1 + 3 = 7

X = 7 - 2 - 3

X = 2

$\therefore x = 2, y = 1, z = 3$

Question 19.
Solve each of the following systems of equations using matrix method.

2x - 3y + 5z = 16;

3x + 2y – 4z = - 4

x + y – 2z = - 3.

Answer:
To find: - x , y , z

Given set of lines are : -

2x - 3y + 5z = 16;

3x + 2y – 4z = - 4

x + y – 2z = - 3

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -4 \\ -3 \end{bmatrix}$$

$2R_2 – 3R_1$

$2R_3 –R_1$

$$\begin{bmatrix} 2 & -3 & 5 \\ 0 & 13 & -23 \\ 0 & 5 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -56 \\ -22 \end{bmatrix}$$

$13R_3 - 5R_2$

$$\begin{bmatrix} 2 & -3 & 5 \\ 0 & 13 & -23 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -56 \\ -6 \end{bmatrix}$$

Again converting into equations, we get

2x - 3y + 5z = 16

13y - 23z = - 56

- 2z = - 6

Z = 3

13y – 23×3 = - 56

13y = - 56 + 69

Y = 1

2x - 3×1 + 5×3 = 16

2x = 16 + 3 - 15

2x = 4

X = 2

$\therefore x = 2, y = 1, z = 3$

Question 20.
Solve each of the following systems of equations using matrix method.

x + y + z = 4;

2x – y + z = - 1;

2x + y – 3z = - 9.

Answer:
To find: - x , y , z

Given set of lines are : -

x + y + z = 4;

2x – y + z = - 1;

2x + y – 3z = - 9.

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ -9 \end{bmatrix}$$

$R_2 - 2R_1$

$R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & -1 \\ 0 & -1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -9 \\ -17 \end{bmatrix}$$

$3R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & -1 \\ 0 & 0 & -14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -9 \\ -42 \end{bmatrix}$$

Again converting into equations, we get

X + y + z = 4

- 3y - z = - 9

- 14z = - 42

Z = 3

- 3y - 3 = - 9

- 3y = - 6

Y = 2

X + 2 + 3 = 4

X = 4 - 5

X = - 1

$\therefore x = -1, y = 2, z = 3$

Question 21.
Solve each of the following systems of equations using matrix method.

2x - 3y + 5z = 11;

3x + 2y - 4z = - 5;

x + y – 2z = - 3.

Answer:
To find: - x , y , z

Given set of lines are : -

2x - 3y + 5z = 11;

3x + 2y - 4z = - 5;

x + y – 2z = - 3.

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$2R_2 - 3R_1$

$2R_3 - R_1$

$$\begin{bmatrix} 2 & -3 & 5 \\ 0 & 13 & -23 \\ 0 & 5 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -43 \\ -17 \end{bmatrix}$$

$13R_3 - 5R_2$

$$\begin{bmatrix} 2 & -3 & 5 \\ 0 & 13 & -23 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -43 \\ -6 \end{bmatrix}$$

Again converting into equations we get

$2x - 3y + 5z = 11$

$13y - 23z = -43$

$-2z = -6$

$Z = 3$

$13y - 23 \times 3 = -43$

$13y = -43 + 69$

$13y = 26$

$Y = 2$

$2x - 3 \times 2 + 5 \times 3 = 11$

$2x = 11 + 6 - 15$

$X = 1$

$\therefore x = 1, y = 2, z = 3$

Question 22.
Solve each of the following systems of equations using matrix method.

x + y + z = 1;

x – 2y + 3z = 2;

5x – 3y + z = 3.

Answer:
To find: - x , y , z

Given set of lines are : -

x + y + z = 1;

x – 2y + 3z = 2;

5x – 3y + z = 3.

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 5 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$R_2 – R_1$

$R_3 – 5R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 2 \\ 0 & -8 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

$R_3 + 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 2 \\ 0 & -14 & -0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Again converting into equations we get

X + y + z = 1

- 3y + 2z = 1

- 14 y = 0

Y = 0

- 3 × 0 + 2z = 1

Z = $\frac{1}{2}$

X + 0 + $\frac{1}{2}$ = 1

X = $\frac{1}{2}$

$\therefore x = \frac{1}{2}, y = 0, z = \frac{1}{2}$

Question 23.
Solve each of the following systems of equations using matrix method.

x + y + z = 6;

x + 2z = 7;

3x + y + z = 12.

Answer:
To find: - x , y , z

Given set of lines are : -

x + y + z = 6;

x + 2z = 7;

3x + y + z = 12

Converting following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & 1 & 1\\1 & 0 & 2\\3 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}6\\7\\12\end{bmatrix}$$

$R_2 - R_1$

$R_3 - 3R_1$

$$\begin{bmatrix}1 & 1 & 1\\0 & -1 & 1\\0 & -2 & -2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}6\\1\\-6\end{bmatrix}$$

$R_3 + 2R_2$

$$\begin{bmatrix}1 & 1 & 1\\0 & -1 & 1\\0 & -4 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}6\\1\\-4\end{bmatrix}$$

Again converting into equations we get

X + y + z = 6

- y + z = 1

- 4 y = - 4

Y = 1

- 1 + z = 1

Z = 2

X + 1 + 2 = 6

X = 6 - 3

X = 3

$\therefore x = 3, y = 1, z = 2$

Question 24.
Solve each of the following systems of equations using matrix method.

2x + 3y + 3z = 5;

x – 2y + z = - 4;

3x – y – 2z = 3.

Answer:
To find: - x , y , z

Given set of lines are : -

2x + 3y + 3z = 5;

x – 2y + z = - 4;

3x – y – 2z = 3

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$2R_2 – R_1$

$2R_3 – 3R_1$

$$\begin{bmatrix} 2 & 3 & 3 \\ 0 & -7 & -1 \\ 0 & -11 & -13 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -13 \\ -9 \end{bmatrix}$$

$R_3 - 13R_2$

$$\begin{bmatrix} 2 & 3 & 3 \\ 0 & -7 & -1 \\ 0 & 80 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -13 \\ 160 \end{bmatrix}$$

Again converting into equations we get

2X + 3y + 3z = 5

- 7y - z = - 13

80 y = 160

Y = 2

- 7×2 - z = - 13

Z = - 1

2x + 3×2 + 3× - 1 = 5

2x = 5 - 6 + 3

X = 1

$\therefore x = 1, y = 2, z = -1$

Question 25.
Solve each of the following systems of equations using matrix method.

4x - 5y – 11z = 12;

X – 3y + z = 1;

2x + 3y – 7z = 2.

Answer:
To find: - x , y , z

Given set of lines are : -

4x - 5y - 11z = 12

X – 3y + z = 1;

2x + 3y – 7z = 2

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 4 & -5 & -11 \\ 1 & -3 & 1 \\ 2 & 3 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 1 \\ 2 \end{bmatrix}$$

$4R_2 – R_1$

$2R_3 – R_1$

$$\begin{bmatrix} 4 & -5 & -11 \\ 0 & -7 & 15 \\ 0 & 11 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \\ -8 \end{bmatrix}$$

$5R_3 + R_2$

$$\begin{bmatrix} 4 & -5 & -11 \\ 0 & -7 & 15 \\ 0 & 48 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \\ -48 \end{bmatrix}$$

Again converting into equations we get

4x – 5y - 11z = 12

- 7y + 15z = - 8

48 y = - 48

Y = - 1

7 + 15z = - 8

15z = - 15

Z = - 1

4x + 5 + 11 = 12

4x = 12 - 5 - 11

4x = - 4

X = - 1

$\therefore x = -1, y = -1, z = -1$

Question 26.
Solve each of the following systems of equations using matrix method.

x – y + 2z = 7;

3x + 4y – 5z = - 5:

2x – y + 3z = 12.

Answer:
To find: - x , y , z

Given set of lines are : -

x - y + 2z = 7

3x + 4y – 5z = - 5

2x – y + 3z = 12

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$R_2 - 3R_1$

$R_3 - 2R_1$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 7 & -11 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -26 \\ -2 \end{bmatrix}$$

$7R_3 - R_2$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 7 & -11 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -26 \\ 12 \end{bmatrix}$$

Again converting into equations we get

x – y + 2z = 7

7y - 11 z = - 26

4z = 12

Z = 3

7y – 11 ×3 = - 26

7y = - 26 + 33

7y = 7

Y = 1

X – 1 + 2×3 = 7

X = 7 + 1 - 6

X = 2

$\therefore x = 2, y = 1, z = 3$

Question 27.

Solve each of the following systems of equations using matrix method.

6X - 9y – 20z = - 4;

4x – 15y + 10z = - 1;

2x - 3y - 5z = - 1.

Answer:
To find: - x , y , z

Given set of lines are : -

6x - 9y - 20z = - 4

4x – 15y + 10z = - 1

2x - 3y - 5z = - 1

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 6 & -9 & -20 \\ 4 & -15 & 10 \\ 2 & -3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ -1 \\ -1 \end{bmatrix}$$

$3R_2 – 2R_1$

$3R_3 – R_1$

$$\begin{bmatrix} 6 & -9 & -20 \\ 0 & -27 & 70 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 5 \\ 1 \end{bmatrix}$$

Again converting into equations, we get

6x - 9y - 20 z = - 4

- 27y + 70z = 5

5z = 1

$Z = \frac{1}{5}$

$-27y + 70 \times \frac{1}{5} = 5$

$-27y = 5 - 14$

$-27y = -9$

$Y = \frac{1}{3}$

$6x - 9 \times \frac{1}{3} - 20 \times \frac{1}{5} = -4$

$6x = -4 + 3 + 4$

$X = \frac{1}{2}$

$\therefore x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$

Question 28.
Solve each of the following systems of equations using matrix method.

3x - 4y + 2z = - 1;

2x + 3y + 5z = 7;

X + z = 2.

Answer:
To find: - x , y , z

Given set of lines are : -

3x - 4y + 2z = - 1

2x + 3y + 5z = 7;

x + z = 2

Converting the following equations in matrix form,

$AX = B$

$$\begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

$3R_2 - 2R_1$

$3R_3 - R_1$

$$\begin{bmatrix} 3 & -4 & 2 \\ 0 & 17 & 11 \\ 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 23 \\ 7 \end{bmatrix}$$

$11R_3 - R_2$

$$\begin{bmatrix} 3 & -4 & 2 \\ 0 & 17 & 11 \\ 0 & 27 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 23 \\ 54 \end{bmatrix}$$

Again converting into equations, we get

$3x - 4y + 2z = -1$

$17y + 11z = 23$

$27y = 54$

$Y = 2$

$17 \times 2 + 11z = 23$

$11z = 23 - 34$

$Z = -1$

$3x - 4 \times 2 + 2x - 1 = -1$

$3x = -1 + 8 + 2$

3x = 9

X = 3

$\therefore x = 3, y = 2, z = -1$

Question 29.
Solve each of the following systems of equations using matrix method.

X + y - z = 1;

3x + y – 2z = 3;

X – y – z = - 1.

Answer:
To find: - x , y , z

Given set of lines are : -

x + y - z = 1

3x + y - 2z = 3

x - y - z = - 1

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

$R_2 - 3R_1$

$R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

Again converting into equations we get

X + y – z = 1

- 2y + z = 0

- 2y = - 2

Y = 1

- 2 + z = 0

Z = 2

X + 1 - 2 = 1

X = 2

$\therefore x = 2, y = 1, z = 2$

Question 30.
Solve each of the following systems of equations using matrix method.

2x + y - z = 1;

x – y + z = 2;

3x + y – 2z = - 1.

Answer:
To find: - x , y , z

Given set of lines are : -

2x + y - z = 1

x - y + z = 2

3x + y - 2z = - 1

Converting the following equations in matrix form,

$AX = B$

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$2R_2 - R_1$

$2R_3 - 3R_1$

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & -3 & 3 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}$$

$3R_3 - R_2$

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & -3 & 3 \\ 0 & 0 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -18 \end{bmatrix}$$

Again converting into equations we get

$2x + y - z = 1$

$- 3y + 3z = 3$

$- 6z = - 18$

$Z = 3$

$- 3y + 3 \times 3 = 3$

$- 3y = 3 - 9$

$- 3y = - 6$

$Y = 2$

$2x + 2 - 3 = 1$

2x = 1 + 1

X = 1

∴ x = 1 , y = 2 , z = 3

Question 31.
Solve each of the following systems of equations using matrix method.

X + 2y + z = 4;

- x + y + z = 0;

x - 3y + z = 4.

Answer:
To find: - x , y , z

Given set of lines are : -

x + 2y + z = 4

- x + y + z = 0

x - 3y + z = 4

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

$R_2 + R_1$

$R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 2 \\ 0 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$$

Again converting into equations we get

X + 2y + z = 4

3y + 2z = 4

- 5y = 0

Y = 0

0 + 2z = 4

Z = 2

X + 0 + 2 = 4

X = 2

∴ x = 2 , y = 0 , z = 2

Question 32.
Solve each of the following systems of equations using matrix method.

x - y – 2z = 3;

x + y = 1;

x + z = - 6.

Answer:
To find: - x , y , z

Given set of lines are : -

x - y - 2z = 3

x + y = 1

x + z = - 6

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -6 \end{bmatrix}$$

$R_2 - R_1$

$R_3 - R_1$

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 2 & 2 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -9 \end{bmatrix}$$

$2R_3 - R_2$

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 2 & 2 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -16 \end{bmatrix}$$

Again converting into equations we get

X + y - 2z = 3

2y + 2z = - 2

4z = - 16

Z = - 4

2y – 8 = - 2

2y = - 2 + 8

2y = 6

Y = 3

X - 3 + 8 = 3

X = - 2

∴ x = - 2 , y = 3 , z = - 4

Question 33.
Solve each of the following systems of equations using matrix method.

5x - y = - 7;

2x + 3z = 1;

3y – z = 5.

Answer:
To find: - x , y , z

Given set of lines are : -

5x - y = - 7

2x + 3z = 1

3y - z = 5

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 5 & -1 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 1 \\ 5 \end{bmatrix}$$

$5R_2 – 2R_1$

$$\begin{bmatrix} 5 & -1 & 0 \\ 0 & 2 & 15 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 19 \\ 5 \end{bmatrix}$$

$2R_3 - 3R_2$

$$\begin{bmatrix} 5 & -1 & 0 \\ 0 & 2 & 15 \\ 0 & 0 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 19 \\ -47 \end{bmatrix}$$

Again converting into equations we get

5x – y = - 7

2y + 15z = 19

- 47z = - 47

Z = 1

2y + 15 = 19

2y = 19 - 15

Y = 2

5x – 2 = - 7

5x = - 5

X = - 1

∴ x = - 1 , y = 2 , z = 1

Question 34.
Solve each of the following systems of equations using matrix method.

x - 2y + z = 0;

y – z = 2;

2x – 3z = 10.

Answer:
To find: - x , y , z

Given set of lines are : -

x - 2y + z = 0

y - z = 2

2x - 3z = 10

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 2 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 10 \end{bmatrix}$$

$R_3 - 2R_1$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 4 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 10 \end{bmatrix}$$

$R_3 - 4R_2$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$$

Again converting into equations we get

X - 2y + z = 0

Y – z = 2

- z = 2

Z = - 2

Y + 2 = 2

Y = 0

X + 0 - 2 = 0

X = 2

∴ x = 2 , y = 0 , z = - 2

Question 35.
Solve each of the following systems of equations using matrix method.

x - y = 3;

2x + 3y + 4z = 17;

y + 2z = 7.

Answer:
To find: - x , y , z

Given set of lines are : -

x - y = 3

2x + 3y + 4z = 17

y + 2z = 7

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}3\\ 17\\ 7\end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix}1 & -1 & 0\\ 0 & 5 & 4\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}3\\ 11\\ 7\end{bmatrix}$$

$2R_3 - R_2$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 5 & 4 \\ 0 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \\ 3 \end{bmatrix}$$

Again converting into equations we get

X – y = 3

5y + 4z = 11

- 3y = 3

Y = - 1

5× - 1 + 4z = 11

4z = 16

Z = 4

X + 1 = 3

X = 2

∴ x = 2 , y = - 1 , z = 4

Question 36.
Solve each of the following systems of equations using matrix method.

4x + 3y + 2z = 60;

x + 2y + 3z = 45;

6x + 2y + 3z = 70.

Answer:
To find: - x , y , z

Given set of lines are : -

4x + 3y + 2z = 60

x + 2y + 3z = 45

6x + 2y + 3z = 70

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

$4R_2 - R_1$

$2R_3 - 3R_1$

$$\begin{bmatrix} 4 & 3 & 2 \\ 0 & 5 & 10 \\ 0 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 120 \\ -40 \end{bmatrix}$$

Again converting into equations, we get

4x + 3y + 2z = 60

5y + 10z = 120

- 5y = - 40

Y = 8

5×8 + 10z = 120

10z = 120 - 40

10z = 80

Z = 8

4x + 3×8 + 2×8 = 60

4x = 60 – 24 - 16

4x = 20

X = 5

∴ x = 5 , y = 8 , z = 8

Question 37.

If $A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$, find A^{-1}.

Using A^{-1}, solve the following system of equations:

2x – 3y + 5z = 11;

3x + 2y – 4z = - 5;

x + y – 2z = - 3.

Answer:

Given,

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj(A)$$

The determinant of matrix A is

$$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$$

= 2(2 × - 2 – (- 4)×1) + 3(3× - 2 – (- 4)×1) + 5(3×1 – 2×1)

= 2(- 4 + 4) + 3(- 6 + 4) + 5(3 – 2)

= 2(0) + 3(- 2) + 5(1)

= - 6 + 5

= - 1

|A| ≠ 0

∴ A^{-1} is possible.

$A^T = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$

$\text{Adj}(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adj(A)$

$A^{-1} = \frac{1}{-1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

Given set of lines are : -

2x – 3y + 5z = 11

3x + 2y – 4z = - 5

x + y – 2z = - 3

Converting following equations in matrix form,

AX = B

Where $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Pre - multiplying by A^{-1}

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \times 11 - 5 \times 1 - 3 \times -2 \\ -2 \times 11 - 5 \times 9 - 3 \times -23 \\ -1 \times 11 - 5 \times 5 - 3 \times -13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore x = 1 , y = 2 , z = 3$

Question 38.

If $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{pmatrix}$, find A^{-1}.

Using A^{-1}, solve the following

system of linear equations:

$2x + y + z = 1;$

$X - 2y - z = \frac{3}{2};$

$3y - 5z = 9.$

HINT: Here $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{pmatrix}$,

$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 3/2 \\ 9 \end{pmatrix}$.

Answer:
Given,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$$

$A^{-1} = \frac{1}{|A|} adj(A)$

The determinant of matrix A is

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{vmatrix}$$

$= 2(- 2\times - 5 - (- 1)\times 3) - (1\times - 5 - (- 1)\times 0) + (1\times 3 - (- 2)\times 0)$

$= 2(10 + 3) - (- 5) + (3)$

$= 26 + 5 + 3$

$= 34$

$|A| \neq 0$

$\therefore A^{-1}$ is possible.

$$A^T = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & 3 \\ 1 & -1 & -5 \end{bmatrix}$$

$$Adj(A) = \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

$A^{-1} = \frac{1}{|A|} adj(A)$

$A^{-1} = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$

Given set of lines are : -

$2x + y + z = 1$

$X - 2y - z = \frac{3}{2}$

$3y - 5z = 9$

Converting the following equations in matrix form,

$AX = B$

Where $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$

Pre - multiplying by A^{-1}

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 1 \times 13 + \frac{3}{2} \times 8 + 9 \times 1 \\ 1 \times 5 + \frac{3}{2} \times -10 + 9 \times 3 \\ 1 \times 3 + \frac{3}{2} \times -6 + 9 \times -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13 + 12 + 9 \\ 5 - 15 + 27 \\ 3 - 9 - 45 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{3}{2} \end{bmatrix}$$

$\therefore$ x = 1 , y = $\frac{1}{2}$, z = - $\frac{3}{2}$

Question 39.

If $A = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{pmatrix}$ and

B = $\begin{pmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{pmatrix}$, find *AB*.

Hence, solve the system of equations:

x – 2y = 10,

2x + y + 3z = 8 and

- 2y + z = 7.

HINT: $AB = (11)I = A\left(\frac{1}{11}B\right) = I$

$A^{-1} = \left(\frac{1}{11}\right)B$.

Answer:

Given,

A = $\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$

AB = $\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$

$$AB = \begin{bmatrix} 7\times1-2\times-2-4\times0 & 2\times1+1\times-2+2\times0 & -6\times1-3\times-2+5\times0 \\ 7\times2-2\times1-4\times3 & 2\times2+1\times1+2\times3 & -6\times2-3\times1+5\times3 \\ 7\times0-2\times-2-4\times1 & 2\times0+1\times-2+2\times1 & -6\times0-3\times-2+5\times1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 7+4+0 & 2-2+0 & -6+6+0 \\ 14-2-12 & 4+1+6 & -12-3+15 \\ 0+4-4 & 0-2+2 & 0+6+5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$AB = 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$AB = 11I$

Pre - multiplying by A^{-1}

$A^{-1}AB = 11\,A^{-1}I$

$IB = 11\,A^{-1}$

$B = 11\,A^{-1}$

$A^{-1} = \frac{1}{11}B$

Given set of lines are : -

$x - 2y = 10$

$x + y + 3z = 8$

$-2y + z = 7$

Converting following equations in matrix form,

$AX = C$

Where A = $\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$,X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, C = $\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

Pre - multiplying by A^{-1}

$A^{-1}AX = A^{-1}C$

$IX = A^{-1}C$

$X = A^{-1}C$

$X = \frac{1}{11}BC$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 10\times 7 + 8\times 2 + 7\times -6 \\ 10\times -2 + 8\times 1 + 7\times -3 \\ 10\times -4 + 8\times 8 + 7\times 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + -16 + 35 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

$\therefore$ x = 4 , y = - 3 , z = 1

Question 40.

$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$,

$\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$

Ans. x = $\frac{1}{2}$, y = $\frac{1}{3}$, z = $\frac{1}{5}$

Answer:
To find: - x , y , z

Given set of lines are : -

$\frac{2}{x}-\frac{3}{y}+\frac{3}{z}$ = 10

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ = 10,

$\frac{3}{x}-\frac{1}{y}+\frac{2}{z}$ = 13

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$

$2R_2 - R_1$

$2R_3 - 3R_1$

$$\begin{bmatrix} 2 & -3 & 3 \\ 0 & 5 & -1 \\ 0 & 7 & -5 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ -4 \end{bmatrix}$$

$R_3 - 5R_2$

$$\begin{bmatrix} 2 & -3 & 3 \\ 0 & 5 & -1 \\ 0 & -18 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ -54 \end{bmatrix}$$

Again converting into equations we get

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$$

$$\frac{5}{y} - \frac{1}{z} = 10$$

$$-\frac{18}{y} = -54$$

$$y = \frac{1}{3}$$

$$5 \times 3 - \frac{1}{z} = 10$$

$$-\frac{1}{z} = 10 - 15$$

$Z = \frac{1}{5}$

$$\frac{2}{x} - 3 \times 3 + 3 \times 5 = 10$$

$\frac{2}{x} = 10 + 9 - 15 = 4$

$X = \frac{1}{2}$

$\therefore x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$

Question 41.

$\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=4$; $\frac{2}{x}+\frac{1}{y}-\frac{3}{z}=0$;

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=2$. (x, y, z ≠ 0)

Answer:
To find: - x , y , z

Given set of lines are : -

$\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=4$

$\frac{2}{x}+\frac{1}{y}-\frac{3}{z}=0$

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=2$

Converting following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}\frac{1}{x}\\ \frac{1}{y}\\ \frac{1}{z}\end{bmatrix}=\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$$

$R_2 - 2R_1$

$R_3 - R_1$

$$\begin{bmatrix}1 & -1 & 1\\ 0 & 3 & -5\\ 0 & 2 & 0\end{bmatrix}\begin{bmatrix}\frac{1}{x}\\ \frac{1}{y}\\ \frac{1}{z}\end{bmatrix}=\begin{bmatrix}4\\ -8\\ -2\end{bmatrix}$$

Again converting into equations we get

$$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4$$

$$\frac{3}{y} - \frac{5}{z} = -8$$

$$\frac{2}{y} = -2$$

$$y = -1$$

$$3 \times -1 - \frac{5}{z} = -8$$

$$-\frac{5}{z} = -8 + 3$$

Z = 1

$$\frac{1}{x} - 1 \times -1 + 1 \times 1 = 4$$

$$\frac{1}{x} = 4 - 1 - 1$$

$X = \frac{1}{2}$

$\therefore x = \frac{1}{2}$, $y = 1$, $z = 1$

Question 42.
The sum of three numbers is 2. If twice the second number is added to the sum of first and third, we get 1. On adding the sum of second and third numbers to five times the first, we get 6. Find the three numbers by using matrices.

Answer:
Let the three numbers be x, y and z.

According to the question,

X + y + z = 2

X + 2y + z = 1

5x + y + z = 6

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & 1 & 1\\1 & 2 & 1\\5 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}2\\1\\6\end{bmatrix}$$

$R_2 - R_1$

$R_3 - R_1$

$$\begin{bmatrix}1 & 1 & 1\\0 & 1 & 0\\4 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}2\\-1\\5\end{bmatrix}$$

Converting back into the equations we get

X + y + z = 2

Y = - 1

4x = 5

$X = \frac{5}{4}$

$\frac{5}{4} - 1 + z = 2$

$Z = 2 - \frac{5}{4} + 1$

$Z = \frac{7}{4}$

∴ The numbers are $\frac{5}{4}, \frac{7}{4}, -1$.

Question 43.
The cost of 4 kg potato, 3 kg wheat and 2 kg of rice is ₹ 60. The cost of 1 kg potato, 2 kg wheat and 3 kg of rice is ₹45. The cost of 6 kg potato, 2 kg wheat and 3 kg of rice is ₹70. Find the cost of each item per kg by matrix method.

Answer:
Let the price of 1kg potato, wheat and rice be x, y and z respectively.

According to the question,

4x + 3y + 2z = 60

X + 2y + 3z = 45

6x + 2y + 3z = 70

Converting into matrix form

AX = B

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

$4R_2 - R_1$

$2R_3 - 3R_1$

$$\begin{bmatrix} 4 & 3 & 2 \\ 0 & 5 & 10 \\ 0 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 120 \\ -40 \end{bmatrix}$$

Converting back into the equations we get

4x + 3y + 2z = 60

5y + 10z = 120

- 5y = - 40

Y = 8

5×8 + 10z = 120

10z = 120 – 40

Z = 8

4x + 3×8 + 2×8 = 60

4x = 60 - 24 - 16

4x = 20

X = 5

∴ The cost of 1 kg potatoes, wheat and rice is Rs.5, Rs.8 and Rs. 8 respectively.

Question 44.
An amount of ₹ 5000 is put into three investments at 6%, 7% and 8% per annum respectively. The total annual income from these investments is ₹358. If the total annual income from first two investments is ₹70more

than the income from the third, find the amount of each investment by the matrix method.

HINT: Let these investments be ₹x, ₹y and ₹z, respectively.

Then, x + y + z = 5000, ...(i)

$\frac{6x}{100}+\frac{7y}{100}+\frac{8z}{100}$ = 358 ⇒

6x + 7y + 8z = 35800 ...(ii)

And, $\frac{6x}{100}+\frac{7y}{100}=\frac{8z}{100}+70$

⇒6x + 7y - 8z = 7000. ...(iii)

Answer:
Let these investments be ₹x, ₹y and ₹z, respectively.

Then, x + y + z = 5000

$\frac{6x}{100}+\frac{7y}{100}+\frac{8z}{100}$ = 358

6x + 7y + 8z = 35800

And, $\frac{6x}{100}+\frac{7y}{100}=\frac{8z}{100}+70$

6x + 7y - 8z = 7000.

Representing in the matrix form,

AX = B

$$\begin{bmatrix}1 & 1 & 1\\ 6 & 7 & 8\\ 6 & 7 & -8\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}5000\\ 35800\\ 7000\end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix}1 & 1 & 1\\ 6 & 7 & 8\\ 0 & 0 & -16\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}5000\\ 35800\\ -28800\end{bmatrix}$$

$R_2 - 6R_1$

$$\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 2\\ 0 & 0 & -16\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}5000\\ 5800\\ -28800\end{bmatrix}$$

Converting back into the equations we get

X + y + z = 5000

Y + 2z = 5800

- 16z = - 28800

Z = 1800

Y + 2×1800 = 5800

Y = 5800 - 3600

Y = 2200
x + 2200 + 1800 = 5000

X = 5000 – 4000

X = 1000

Amount of 1000 , 2200 , 1800 were invested in the investments of 6% , 7%, 8% respectively.

Question 45.
Two schools *A* and *B* want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school *A* wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with total award money of ₹ 1,600. School *B* wants to spend ₹ 2,300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is ₹ 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.

HINT: By the given data, we have

$$\left.\begin{aligned} 3x+2y+z&=1600\\ 4x+y+3z&=2300\\ x+y+z&=900 \end{aligned}\right\}$$

Answer:
Let the amount x, y and z be considered for sincerity, truthfulness and helpfulness.

According to the questions,

3x + 2y + z = 1600

4x + y + 3z = 2300

X + y + z = 900

Converting into the matrix form

AX = B

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$R_1 - 3R_3$

$R_2 - 4R_3$

$$\begin{bmatrix} 0 & -1 & -2 \\ 0 & -3 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1100 \\ -1300 \\ 900 \end{bmatrix}$$

$2R_2 - R_1$

$$\begin{bmatrix} 0 & -1 & -2 \\ 0 & -5 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1100 \\ -1500 \\ 900 \end{bmatrix}$$

Converting back into the equations we get

- y - 2z = - 1100

- 5y = - 1500

X + y + z = 900

Y = 300

- 300 - 2z = - 1100

- 2z = - 800

Z = 400

X + 300 + 400 = 900

X = 900 - 700

X = 200

₹ 200 for sincerity, ₹ 300 for truthfulness and ₹ 400 for helpfulness. One more value may be like honesty, kindness, etc.