# Exercise 27d

**Question 1.**
Find the shortest distance between the given lines.

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}),$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}).$$

**Answer:**
**Given equations:**

$$\bar{r} = (\hat{\imath} + \hat{\jmath}) + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$$

$$\bar{r} = (2\hat{\imath} + \hat{\jmath} - \hat{k}) + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$$

**To Find:** d

**Formula:**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \bar{a}_1)}{|\overline{b_1} \times \overline{b_2}|}\right|$$

**Answer:**

For given lines,

$\bar{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$\bar{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

Here,

$\bar{a_1} = \hat{i} + \hat{j}$

$\overline{b_1} = 2\hat{i} - \hat{j} + \hat{k}$

$\overline{a_2} = 2\hat{i} + \hat{j} - \hat{k}$

$\overline{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$$= \hat{\imath}(-2+5) - \hat{\jmath}(4-3) + \hat{k}(-10+3)$$

$$\therefore \overline{b_1} \times \overline{b_2} = 3\hat{\imath} - \hat{\jmath} - 7\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{3^2 + (-1)^2 + (-7)^2}$$

$$= \sqrt{9+1+49}$$

$$= \sqrt{59}$$

$$\overline{a_2} - \overline{a_1} = (2-1)\hat{\imath} + (1-1)\hat{\jmath} + (-1-0)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = \hat{\imath} + 0\hat{\jmath} - \hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(3\hat{\imath} - \hat{\jmath} - 7\hat{k}\right).\left(\hat{\imath} + 0\hat{\jmath} - \hat{k}\right)$$

$$= (3 \times 1) + ((-1) \times 0) + ((-7) \times (-1))$$

= 3 + 0 + 7

= 10

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{10}{\sqrt{59}}\right|$$

**Question 2.**

Find the shortest distance between the given lines.

$$\vec{r} = \left(-4\hat{i} + 4\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right),$$

$$\vec{r} = \left(-3\hat{i} - 8\hat{j} - 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 3\hat{k}\right)$$

**Answer:**
**<u>Given equations</u>:**

$$\bar{r} = \left(-4\hat{\imath} + 4\hat{\jmath} + \hat{k}\right) + \lambda\left(\hat{\imath} + \hat{\jmath} - \hat{k}\right)$$

$$\bar{r} = \left(-3\hat{\imath} - 8\hat{\jmath} - 3\hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{\jmath} + 3\hat{k}\right)$$

**<u>To Find</u>:** d

**<u>Formula</u>:**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

__**Answer**__:

For given lines,

$\bar{r} = \left(-4\hat{i} + 4\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right)$

$\bar{r} = \left(-3\hat{i} - 8\hat{j} - 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 3\hat{k}\right)$

Here,

$\overline{a_1} = -4\hat{i} + 4\hat{j} + \hat{k}$

$\overline{b_1} = \hat{i} + \hat{j} - \hat{k}$

$\overline{a_2} = -3\hat{i} - 8\hat{j} - 3\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + 3\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & 3 & 3 \end{vmatrix}$$

$= \hat{\imath}(3+3) - \hat{\jmath}(3+2) + \hat{k}(3-2)$

$\therefore \overline{b_1} \times \overline{b_2} = 6\hat{\imath} - 5\hat{\jmath} + \hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{6^2 + (-5)^2 + 1^2}$

$= \sqrt{36 + 25 + 1}$

$= \sqrt{62}$

$\overline{a_2} - \overline{a_1} = (-3+4)\hat{\imath} + (-8-4)\hat{\jmath} + (-3-1)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{\imath} - 12\hat{\jmath} - 4\hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(6\hat{\imath} - 5\hat{\jmath} + \hat{k}\right).\left(\hat{\imath} - 12\hat{\jmath} - 4\hat{k}\right)$

$= (6 \times 1) + ((-5) \times (-12)) + (1 \times (-4))$

= 6 + 60 – 4

= 62

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{62}{\sqrt{62}}\right|$$

$$d = \sqrt{62}\, units$$

**Question 3.**

Find the shortest distance between the given lines.

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right),$$

$$\vec{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right).$$

**Answer:**
**<u>Given equations</u>:**

$$\bar{r} = \left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right) + \lambda\left(\hat{\imath} - 3\hat{\jmath} + 2\hat{k}\right)$$

$$\bar{r} = \left(4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{\jmath} + \hat{k}\right)$$

**<u>To Find</u>:** d

**<u>Formula</u>:**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

**<u>Answer</u>:**

For given lines,

$\bar{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$

$\bar{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$

Here,

$\overline{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overline{b_1} = \hat{i} - 3\hat{j} + 2\hat{k}$

$\overline{a_2} = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + \hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$= \hat{i}(-3-6) - \hat{j}(1-4) + \hat{k}(3+6)$

$\therefore \overline{b_1} \times \overline{b_2} = -9\hat{i} + 3\hat{j} + 9\hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-9)^2 + 3^2 + 9^2}$

$= \sqrt{81 + 9 + 81}$

$= \sqrt{171}$

$\overline{a_2} - \overline{a_1} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right) . (\overline{a_2} - \overline{a_1}) = \left(-9\hat{i} + 3\hat{j} + 9\hat{k}\right) . \left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$

$= ((-9) \times 3) + (3 \times 3) + (9 \times 3)$

= - 27 + 9 + 27

= 9

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right) . (\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{9}{\sqrt{171}}\right|$$

$$\therefore d = \frac{9}{\sqrt{19} . \sqrt{9}}$$

$$\therefore d = \frac{3}{\sqrt{19}}$$

$\therefore d = \frac{3\sqrt{19}}{19}$

**Question 4.**
Find the shortest distance between the given lines.

$$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right),$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right).$$

**Answer:**
**Given equations:**

$$\bar{r} = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$$

$$\bar{r} = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$$

**To Find:** d

**Formula:**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

**<u>Answer</u>:**

For given lines,

$$\bar{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

Here,

$$\overline{a_1} = \hat{i} + 2\hat{j} + \hat{k}$$

$$\overline{b_1} = \hat{i} - \hat{j} + \hat{k}$$

$$\overline{a_2} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\overline{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= \hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1+2)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -3\hat{i} + 0\hat{j} + 3\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-3)^2 + 0^2 + 3^2}$$

$$= \sqrt{9+0+9}$$

$$= \sqrt{18}$$

$= 3\sqrt{2}$

$$\overline{a_2} - \overline{a_1} = (2-1)\hat{i} + (-1-2)\hat{j} + (-1-1)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = \hat{i} - 3\hat{j} - 2\hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = \left(-3\hat{i} + 0\hat{j} + 3\hat{k}\right).(\hat{i} - 3\hat{j} - 2\hat{k})$$

$$= ((-3) \times 1) + (0 \times (-3)) + (3 \times (-2))$$

$= - 3 + 0 – 6$

$= - 9$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{-9}{3\sqrt{2}}\right|$$

$$\therefore d = \frac{3}{\sqrt{2}}$$

$$\therefore d = \frac{3\sqrt{2}}{2}$$

**Question 5.**
Find the shortest distance between the given lines.

$$\vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right),$$

$$\vec{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu\left(-2\hat{i} + 3\hat{j} + 8\hat{k}\right).$$

**Answer:**
**<u>Given equations</u>:**

$$\bar{r} = \left(\hat{\imath} + 2\hat{\jmath} - 4\hat{k}\right) + \lambda\left(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}\right)$$

$$\bar{r} = \left(3\hat{\imath} + 3\hat{\jmath} - 5\hat{k}\right) + \mu\left(-2\hat{\imath} + 3\hat{\jmath} + 8\hat{k}\right)$$

**<u>To Find</u>:** d

**<u>Formula</u>:**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \bar{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

**Answer:**

For given lines,

$$\bar{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$$\bar{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu\left(-2\hat{i} + 3\hat{j} + 8\hat{k}\right)$$

Here,

$$\bar{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\overline{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$\overline{a_2} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$

$\overline{b_2} = -2\hat{\imath} + 3\hat{\jmath} + 8\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix}$$

$= \hat{\imath}(24 - 18) - \hat{\jmath}(16 + 12) + \hat{k}(6 - 6)$

$\therefore \overline{b_1} \times \overline{b_2} = 6\hat{\imath} - 28\hat{\jmath} + 0\hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{6^2 + (-28)^2 + 0^2}$

$= \sqrt{36 + 784 + 9}$

$= \sqrt{820}$

$\overline{a_2} - \overline{a_1} = (3 - 1)\hat{\imath} + (3 - 2)\hat{\jmath} + (-5 + 4)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = \left(6\hat{\imath} - 28\hat{\jmath} + 0\hat{k}\right).\left(2\hat{\imath} + \hat{\jmath} - \hat{k}\right)$

$= (6 \times 2) + ((-28) \times 1) + (0 \times (-1))$

= 12 – 28 + 0

= - 16

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{-16}{\sqrt{820}}\right|$$

$$d = \frac{16}{\sqrt{820}}\ units$$

**Question 6.**
Find the shortest distance between the given lines.

$$\vec{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k}),$$

$$\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k}).$$

**Answer:**
**Given equations:**

$$\bar{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$$

$$\bar{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$$

**To Find:** d

**Formula:**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \bar{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

**<u>Answer</u>:**

For given lines,

$$\bar{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$$

$$\bar{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$$

Here,

$$\bar{a_1} = 6\hat{i} + 3\hat{k}$$

$$\overline{b_1} = 2\hat{i} - \hat{j} + 4\hat{k}$$

$\overline{a_2} = -9\hat{i} + \hat{j} - 10\hat{k}$

$\overline{b_2} = 4\hat{i} + \hat{j} + 6\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 4 \\ 4 & 1 & 6 \end{vmatrix}$$

$= \hat{i}(-6 - 4) - \hat{j}(12 - 16) + \hat{k}(2 + 4)$

$\therefore \overline{b_1} \times \overline{b_2} = -10\hat{i} + 4\hat{j} + 6\hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-10)^2 + 4^2 + 6^2}$

$= \sqrt{100 + 16 + 36}$

$= \sqrt{152}$

$\overline{a_2} - \overline{a_1} = (-9 - 6)\hat{i} + (1 - 0)\hat{j} + (6 - 3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = -15\hat{i} + \hat{j} + 3\hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(-10\hat{i} + 4\hat{j} + 6\hat{k}\right).\left(-15\hat{i} + \hat{j} + 3\hat{k}\right)$

$= ((-10) \times (-15)) + (4 \times 1) + (6 \times 3)$

= 150 + 4 + 18

= 172

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{172}{\sqrt{152}}\right|$$

$$\therefore d = \frac{172}{2\sqrt{38}}$$

$$\therefore d = \frac{86}{\sqrt{38}}$$

$$d = \frac{86}{\sqrt{38}} \; units$$

**Question 7.**
Find the shortest distance between the given lines.

$$\vec{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k},$$

$$\vec{r} = (1+s)\hat{i} + (3s-7)\hat{j} + (2s-2)\hat{k}.$$

**Answer:**
**Given equations:**

$$\bar{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k}$$

$$\bar{r} = (1+s)\hat{i} + (3s-7)\hat{j} + (2s-2)\hat{k}$$

**To Find:** d

**Formula:**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

<u>**Answer**</u>:

Given lines,

$$\bar{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k}$$

$$\bar{r} = (1+s)\hat{i} + (3s-7)\hat{j} + (2s-2)\hat{k}$$

Above equations can be written as

$$\bar{r} = \left(3\hat{i} + 4\hat{j} - 2\hat{k}\right) + t\left(-\hat{i} + 2\hat{j} + \hat{k}\right)$$

$$\bar{r} = \left(\hat{i} - 7\hat{j} - 2\hat{k}\right) + s\left(\hat{i} + 3\hat{j} + 2\hat{k}\right)$$

Here,

$$\overline{a_1} = 3\hat{i} + 4\hat{j} - 2\hat{k}$$

$$\overline{b_1} = -\hat{i} + 2\hat{j} + \hat{k}$$

$$\overline{a_2} = \hat{i} - 7\hat{j} - 2\hat{k}$$

$$\overline{b_2} = \hat{i} + 3\hat{j} + 2\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$$= \hat{i}(4 - 3) - \hat{j}(-2 - 1) + \hat{k}(-3 - 2)$$

$$\therefore \overline{b_1} \times \overline{b_2} = \hat{i} + 3\hat{j} - 5\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{1^2 + 3^2 + (-5)^2}$$

$$= \sqrt{1 + 9 + 25}$$

$$= \sqrt{35}$$

$$\overline{a_2} - \overline{a_1} = (1 - 3)\hat{i} + (-7 - 4)\hat{j} + (-2 + 2)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = -2\hat{i} - 11\hat{j} + 0\hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(\hat{\imath} + 3\hat{\jmath} - 5\hat{k}\right).\left(-2\hat{\imath} - 11\hat{\jmath} + 0\hat{k}\right)$$

$$= (1 \times (-2)) + (3 \times (-11)) + ((-5) \times 0)$$

= - 2 – 33 + 0

= - 35

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{-35}{\sqrt{35}}\right|$$

$$\therefore d = \sqrt{35}$$

$$d = \sqrt{35}\, units$$

**Question 8.**
Find the shortest distance between the given lines.

$$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (\lambda + 1)\hat{k},$$

$$\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}.$$

**Answer:**
**Given equations:**

$$\bar{r} = (\lambda - 1)\hat{\imath} + (\lambda + 1)\hat{\jmath} - (\lambda + 1)\hat{k}$$

$$\bar{r} = (1 - \mu)\hat{\imath} + (2\mu - 1)\hat{\jmath} + (\mu + 2)\hat{k}$$

**To Find:** d

**Formula:**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

<u>**Answer**</u>**:**

Given lines,

$$\bar{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (\lambda + 1)\hat{k}$$

$$\bar{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$$

Above equations can be written as

$$\bar{r} = \left(-\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right)$$

$$\bar{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + s\left(-\hat{i} + 2\hat{j} + \hat{k}\right)$$

Here,

$$\overline{a_1} = -\hat{i} + \hat{j} - \hat{k}$$

$$\overline{b_1} = \hat{i} + \hat{j} - \hat{k}$$

$$\overline{a_2} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\overline{b_2} = -\hat{i} + 2\hat{j} + \hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(1 + 2) - \hat{j}(1 - 1) + \hat{k}(2 + 1)$$

$$\therefore \overline{b_1} \times \overline{b_2} = 3\hat{i} - 0\hat{j} + 3\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{3^2 + 0^2 + 3^2}$$

$$= \sqrt{9 + 0 + 9}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

$\overline{a_2} - \overline{a_1} = (1+1)\hat{\imath} + (-1-1)\hat{\jmath} + (2+1)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = \left(3\hat{\imath} - 0\hat{\jmath} + 3\hat{k}\right).\left(2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}\right)$

$= (3 \times 2) + (0 \times (-2)) + (3 \times 3)$

= 6 + 0 + 9

= 15

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{15}{3\sqrt{2}}\right|$$

$$\therefore d = \frac{5}{\sqrt{2}}$$

$$\therefore d = \frac{5\sqrt{2}}{2}$$

$$d = \frac{5\sqrt{2}}{2}\ units$$

**Question 9.**

Compute the shortest distance between the lines $\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} - \hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} - \hat{j} - \hat{k}\right)$. Determine whether these lines intersect or not.

**Answer:**

**Given equations:**

$$\bar{r} = (\hat{\imath} - \hat{\jmath}) + \lambda(2\hat{\imath} - \hat{k})$$

$$\bar{r} = (2\hat{\imath} - \hat{\jmath}) + \mu(\hat{\imath} - \hat{\jmath} - \hat{k})$$

**To Find:** d

**Formula:**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\bar{b_1}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b_2}$ is given by,

$$d = \left|\frac{(\bar{b_1} \times \bar{b_2}).(\bar{a_2} - \bar{a_1})}{|\bar{b_1} \times \bar{b_2}|}\right|$$

**Answer:**

For given lines,

$$\bar{r} = (\hat{\imath} - \hat{\jmath}) + \lambda(2\hat{\imath} - \hat{k})$$

$$\bar{r} = (2\hat{\imath} - \hat{\jmath}) + \mu(\hat{\imath} - \hat{\jmath} - \hat{k})$$

Here,

$$\bar{a_1} = \hat{\imath} - \hat{\jmath}$$

$$\bar{b_1} = 2\hat{\imath} - \hat{k}$$

$$\bar{a_2} = 2\hat{\imath} - \hat{\jmath}$$

$$\bar{b_2} = \hat{\imath} - \hat{\jmath} - \hat{k}$$

Therefore,

$$\bar{b_1} \times \bar{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 0 & -1 \\ 1 & -1 & -1 \end{vmatrix}$$

$$= \hat{\imath}(0 - 1) - \hat{\jmath}(-2 + 1) + \hat{k}(-2 - 0)$$

$$\therefore \bar{b_1} \times \bar{b_2} = -\hat{\imath} + \hat{\jmath} - 2\hat{k}$$

$$\therefore |\bar{b_1} \times \bar{b_2}| = \sqrt{(-1)^2 + 1^2 + (-2)^2}$$

$$= \sqrt{1 + 1 + 4}$$

$= \sqrt{6}$

$\overline{a_2} - \overline{a_1} = (2-1)\hat{\imath} + (-1+1)\hat{\jmath} + (0-0)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (-\hat{\imath} + \hat{\jmath} - 2\hat{k}).(\hat{\imath} + 0\hat{\jmath} + 0\hat{k})$

$= ((-1) \times 1) + (1 \times 0) + ((-2) \times 0)$

= - 1 + 0 + 0

= - 1

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{-1}{\sqrt{6}}\right|$$

$$\therefore d = \frac{1}{\sqrt{6}}$$

$$\therefore d = \frac{\sqrt{6}}{6}$$

$$d = \frac{\sqrt{6}}{6}\ units$$

As d ≠ 0

Hence, the given lines do not intersect.

**Question 10.**

Show that the lines $\vec{r} = \left(3\hat{i} - 15\hat{j} + 9\hat{k}\right) + \lambda\left(2\hat{i} - 7\hat{j} + 5\hat{k}\right)$, and $\vec{r} = \left(-\hat{i} + \hat{j} + 9\hat{k}\right) + \mu$ $\left(2\hat{i} + \hat{j} - 3\hat{k}\right)$ do not intersect.

**Answer:**

**Given equations:**

$$\bar{r} = \left(3\hat{\imath} - 15\hat{\jmath} + 9\hat{k}\right) + \lambda\left(2\hat{\imath} - 7\hat{\jmath} + 5\hat{k}\right)$$

$$\bar{r} = \left(-\hat{\imath} + \hat{\jmath} + 9\hat{k}\right) + \mu\left(2\hat{\imath} + \hat{\jmath} - 3\hat{k}\right)$$

**To Find:** d

**Formula:**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

<u>**Answer**</u>**:**

For given lines,

$$\bar{r} = \left(3\hat{i} - 15\hat{j} + 9\hat{k}\right) + \lambda\left(2\hat{i} - 7\hat{j} + 5\hat{k}\right)$$

$$\bar{r} = \left(-\hat{i} + \hat{j} + 9\hat{k}\right) + \mu\left(2\hat{i} + \hat{j} - 3\hat{k}\right)$$

Here,

$$\overline{a_1} = 3\hat{i} - 15\hat{j} + 9\hat{k}$$

$$\overline{b_1} = 2\hat{i} - 7\hat{j} + 5\hat{k}$$

$$\overline{a_2} = -\hat{i} + \hat{j} + 9\hat{k}$$

$$\overline{b_2} = 2\hat{i} + \hat{j} - 3\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$$= \hat{i}(21 - 5) - \hat{j}(-6 - 10) + \hat{k}(2 + 14)$$

$\therefore \overline{b_1} \times \overline{b_2} = 17\hat{i} + 16\hat{j} + 16\hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{17^2 + 16^2 + 17^2}$

$= \sqrt{289 + 256 + 289}$

$= \sqrt{834}$

$\overline{a_2} - \overline{a_1} = (-1 - 3)\hat{i} + (1 + 15)\hat{j} + (9 - 9)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = -4\hat{i} + 16\hat{j} + 0\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (17\hat{i} + 16\hat{j} + 16\hat{k}).(-4\hat{i} + 16\hat{j} + 0\hat{k})$

$= (17 \times (-4)) + (16 \times 16) + (16 \times 0)$

= - 68 + 256 + 0

= 188

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{188}{\sqrt{834}}\right|$$

$$\therefore d = \frac{188}{\sqrt{834}} \text{units}$$

As d ≠ 0

Hence, the given lines do not intersect.

**Question 11.**

Show that the lines $\vec{r} = \left(2\hat{i} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \mu$ $\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ intersect.

Also, find their point of intersection.

**Answer:**
**Given equations:**

$$\bar{r} = \left(2\hat{\imath} - 3\hat{k}\right) + \lambda\left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right)$$

$$\bar{r} = \left(2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right)$$

**To Find:** d

**Formula:**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

<u>**Answer**</u>**:**

For given lines,

$\bar{r} = \left(2\hat{\imath} - 3\hat{k}\right) + \lambda\left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right)$

$\bar{r} = \left(2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right)$

Here,

$\overline{a_1} = 2\hat{\imath} - 3\hat{k}$

$\overline{b_1} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\overline{a_2} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$

$\overline{b_2} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$= \hat{i}(12 - 9) - \hat{j}(4 - 6) + \hat{k}(3 - 4)$

$\therefore \overline{b_1} \times \overline{b_2} = 3\hat{i} + 2\hat{j} - \hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{3^2 + 2^2 + (-1)^2}$

$= \sqrt{9 + 4 + 1}$

$= \sqrt{14}$

$\overline{a_2} - \overline{a_1} = (2 - 2)\hat{i} + (6 - 0)\hat{j} + (3 + 3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 0\hat{i} + 6\hat{j} + 6\hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(3\hat{i} + 2\hat{j} - \hat{k}\right).\left(0\hat{i} + 6\hat{j} + 6\hat{k}\right)$

$= (3 \times 0) + (2 \times 6) + ((-1) \times 6)$

= 0 + 12 – 6

= 6

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{6}{\sqrt{14}}\right|$$

$$\therefore d = \frac{6}{\sqrt{14}} \text{units}$$

As d ≠ 0

Hence, the given lines do not intersect.

**Question 12.**

Show that the lines $\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ and $\vec{r} = \left(4\hat{i} + \hat{j}\right) + \mu$ $\left(5\hat{i} + 2\hat{j} + \hat{k}\right)$ intersect.

Also, find their point of intersection.

**Answer:**

**<u>Given equations</u>:**

$$\bar{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$$

$$\bar{r} = (4\hat{\imath} + \hat{\jmath}) + \mu(5\hat{\imath} + 2\hat{\jmath} + \hat{k})$$

**<u>To Find</u>:** d

**<u>Formula</u>:**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

**<u>Answer</u>:**

For given lines,

$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

$\bar{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$

Here,

$\overline{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overline{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\overline{a_2} = 4\hat{i} + \hat{j}$

$\overline{b_2} = 5\hat{i} + 2\hat{j} + \hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(3-8) - \hat{j}(2-20) + \hat{k}(4-15)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -5\hat{i} + 18\hat{j} - 11\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-5)^2 + 18^2 + (-11)^2}$$

$$= \sqrt{25 + 324 + 121}$$

$$= \sqrt{470}$$

$$\overline{a_2} - \overline{a_1} = (4-1)\hat{i} + (1-2)\hat{j} + (0-3)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = 3\hat{i} - \hat{j} - 3\hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right) . (\overline{a_2} - \overline{a_1}) = \left(-5\hat{i} + 18\hat{j} - 11\hat{k}\right) . \left(3\hat{i} - \hat{j} - 3\hat{k}\right)$$

$$= ((-5) \times 3) + (18 \times (-1)) + ((-11) \times (-3))$$

= - 15 – 18 + 33

= 0

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{\left(\overline{b_1} \times \overline{b_2}\right) . (\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|} \right|$$

$$\therefore d = \left| \frac{0}{\sqrt{470}} \right|$$

$$\therefore d = 0 \text{ units}$$

As d = 0

Hence, the given lines not intersect each other.

Now, to find point of intersection, let us convert given vector equations into Cartesian equations.

For that substituting $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in given equations,

$$\therefore L1 : x\hat{i} + y\hat{j} + z\hat{k} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\therefore L2 : x\hat{i} + y\hat{j} + z\hat{k} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$$

$$\therefore L1 : (x - 1)\hat{i} + (y - 2)\hat{j} + (z - 3)\hat{k} = 2\lambda\hat{i} + 3\lambda\hat{j} + 4\lambda\hat{k}$$

$$\therefore L2 : (x - 4)\hat{i} + (y - 1)\hat{j} + (z - 0)\hat{k} = 5\mu\hat{i} + 2\mu\hat{j} + \mu\hat{k}$$

$$\therefore L1 : \frac{x - 1}{2} = \frac{y - 2}{3} = \frac{z - 3}{4} = \lambda$$

$$\therefore L2 : \frac{x - 4}{5} = \frac{y - 1}{2} = \frac{z - 0}{1} = \mu$$

General point on L1 is

$x_1 = 2\lambda+1$ , $y_1 = 3\lambda+2$ , $z_1 = 4\lambda+3$

let, $P(x_1, y_1, z_1)$ be point of intersection of two given lines.

Therefore, point P satisfies equation of line L2.

$$\therefore \frac{2\lambda + 1 - 4}{5} = \frac{3\lambda + 2 - 1}{2} = \frac{4\lambda + 3 - 0}{1}$$

$$\therefore \frac{2\lambda - 3}{5} = \frac{3\lambda + 1}{2}$$

$\Rightarrow 4\lambda - 6 = 15\lambda + 5$

$\Rightarrow 11\lambda = -11$

$\Rightarrow \lambda = -1$

Therefore, $x_1 = 2(-1)+1$ , $y_1 = 3(-1)+2$ , $z_1 = 4(-1)+3$

$\Rightarrow x_1 = -1$ , $y_1 = -1$ , $z_1 = -1$

Hence point of intersection of given lines is (-1, -1, -1).

**Question 13.**
Find the shortest distance between the lines $L_1$ and $L_2$ whose vector equations are

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$.

**HINT:** The given lines are parallel.

**Answer:**
**Given equations:**

$$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

**To Find:** d

**Formula:**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two parallel lines :**

The shortest distance between the parallel lines $\bar{r} = \bar{a_1} + \lambda\bar{b}$ and

$\bar{r} = \overline{a_2} + \lambda\bar{b}$ is given by,

$$d = \left|\frac{|(\overline{a_2} - \bar{a_1}) \times \bar{b}|}{|\bar{b}|}\right|$$

**Answer:**

For given lines,

$$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Here,

$$\bar{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\overline{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$\overline{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

As $\overline{b_1} = \overline{b_2} = \overline{b}\ (\text{say})$, given lines are parallel to each other.

Therefore,

$\overline{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\therefore |\overline{b}| = \sqrt{2^2 + 3^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

$= 7$

$\overline{a_2} - \overline{a_1} = (3-1)\hat{i} + (3-2)\hat{j} + (-5+4)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + \hat{j} - \hat{k}$

$$(\overline{a_2} - \overline{a_1}) \times \overline{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$= \hat{i}(6+3) - \hat{j}(12+2) + \hat{k}(6-2)$

$\therefore (\overline{a_2} - \overline{a_1}) \times \overline{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$

$\therefore |(\overline{a_2} - \overline{a_1}) \times \overline{b}| = \sqrt{9^2 + (-14)^2 + 4^2}$

$= \sqrt{81 + 196 + 16}$

$= \sqrt{293}$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{|(\overline{a_2} - \overline{a_1}) \times \overline{b}|}{|\overline{b}|}\right|$$

$$\therefore d = \left|\frac{\sqrt{293}}{7}\right|$$

$$d = \frac{\sqrt{293}}{7}\ units$$

**Question 14.**
Find the distance between the parallel lines $L_1$ and $L_2$ whose vector equations are $\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$, and $\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$.

**Answer:**
**<u>Given equations</u>:**

$$\bar{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$$

$$\bar{r} = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(\hat{\imath} - \hat{\jmath} + \hat{k})$$

**<u>To Find</u>:** d

**<u>Formula</u>:**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two parallel lines :**

The shortest distance between the parallel lines $\bar{r} = \overline{a_1} + \lambda\bar{b}$ and

$\bar{r} = \overline{a_2} + \lambda\bar{b}$ is given by,

$$d = \left| \frac{\left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right|}{|\bar{b}|} \right|$$

**<u>Answer</u>:**

For given lines,

$$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$$

Here,

$$\overline{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overline{a_2} = 2\hat{i} - \hat{j} - \hat{k}$$

$\bar{b} = \hat{i} - \hat{j} + \hat{k}$

$\therefore |\bar{b}| = \sqrt{1^2 + (-1)^2 + 1^2}$

$= \sqrt{1 + 1 + 1}$

$= \sqrt{3}$

$\bar{a_2} - \bar{a_1} = (2 - 1)\hat{i} + (-1 - 2)\hat{j} + (-1 - 3)\hat{k}$

$\therefore \bar{a_2} - \bar{a_1} = \hat{i} - 3\hat{j} - 4\hat{k}$

$$(\bar{a_2} - \bar{a_1}) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -4 \\ 1 & -1 & 1 \end{vmatrix}$$

$= \hat{i}(-3 - 4) - \hat{j}(1 + 4) + \hat{k}(-1 + 3)$

$\therefore (\bar{a_2} - \bar{a_1}) \times \bar{b} = -7\hat{i} - 5\hat{j} + 2\hat{k}$

$\therefore |(\bar{a_2} - \bar{a_1}) \times \bar{b}| = \sqrt{(-7)^2 + (-5)^2 + 2^2}$

$= \sqrt{49 + 25 + 4}$

$= \sqrt{78}$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{|(\bar{a_2} - \bar{a_1}) \times \bar{b}|}{|\bar{b}|} \right|$$

$$\therefore d = \left| \frac{\sqrt{78}}{\sqrt{3}} \right|$$

$\therefore d = \sqrt{26}$

$d = \sqrt{26}\ units$

**Question 15.**
Find the vector equation of a line passing through the point (2, 3, 2) and parallel to the line $\vec{r} = \left(-2\hat{i} + 3\hat{j}\right) + \lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$. Also, find the distance between these lines.

HINT: The given line is

$$L_1 : \vec{r} = \left(-2\hat{i} + 3\hat{j}\right) + \lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right).$$

The required line is

$$L_2 : \vec{r} = \left(2\hat{i} + 3\hat{j} + 2\hat{k}\right) + \mu\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right).$$

Now, find the distance between the parallel lines $L_1$ and $L_2$.

**Answer:**
**<u>Given</u>:** point A ≡ (2, 3, 2)

Equation of line : $\bar{r} = (-2\hat{i} + 3\hat{j}) + \lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$

**<u>To Find</u>:** i) equation of line

ii) distance d

**<u>Formulae</u>:**

**1. Equation of line :**

Equation of line passing through point A ($a_1$, $a_2$, $a_3$) and parallel to vector $\bar{b} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by

$$\bar{r} = \bar{a} + \lambda\bar{b}$$

Where, $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

**2. Cross Product :**

If $\bar{a}\ \&\ \bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**3. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**4. Shortest distance between two parallel lines :**

The shortest distance between the parallel lines $\bar{r} = \bar{a_1} + \lambda\bar{b}$ and

$\bar{r} = \overline{a_2} + \lambda\bar{b}$ is given by,

$$d = \left|\frac{\left|(\overline{a_2} - \bar{a_1}) \times \bar{b}\right|}{|\bar{b}|}\right|$$

**<u>Answer</u>:**

As the required line is parallel to the line

$$\bar{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$$

Therefore, the vector parallel to the required line is

$\bar{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

Given point A ≡ (2, 3, 2)

$\therefore \bar{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

Therefore, equation of line passing through A and parallel to $\bar{b}$ is

$\bar{r} = \bar{a} + \mu\bar{b}$

$\therefore \bar{r} = \left(2\hat{i} + 3\hat{j} + 2\hat{k}\right) + \mu\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$

Now, to calculate distance between above line and given line,

$\bar{r} = \left(2\hat{i} + 3\hat{j} + 2\hat{k}\right) + \mu\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$

$\bar{r} = (-2\hat{i} + 3\hat{j}) + \lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$

Here,

$\overline{a_1} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

$\overline{a_2} = -2\hat{i} + 3\hat{j}$

$\bar{b} \; = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$\therefore \left|\bar{b}\right| = \sqrt{2^2 + (-3)^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

= 7

$\overline{a_2} - \overline{a_1} = (-2 - 2)\hat{i} + (3 - 3)\hat{j} + (0 - 2)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = -4\hat{i} + 0\hat{j} - 2\hat{k}$

$$(\overline{a_2} - \overline{a_1}) \times \overline{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 0 & -2 \\ 2 & -3 & 6 \end{vmatrix}$$

$$= \hat{i}(0-6) - \hat{j}(-24+4) + \hat{k}(12-0)$$

$$\therefore (\overline{a_2} - \overline{a_1}) \times \overline{b} = -6\hat{i} + 20\hat{j} + 12\hat{k}$$

$$\therefore \left|(\overline{a_2} - \overline{a_1}) \times \overline{b}\right| = \sqrt{(-6)^2 + 20^2 + 12^2}$$

$$= \sqrt{36 + 400 + 144}$$

$$= \sqrt{580}$$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left|(\overline{a_2} - \overline{a_1}) \times \overline{b}\right|}{|\overline{b}|}\right|$$

$$\therefore d = \left|\frac{\sqrt{580}}{7}\right|$$

$$\therefore d = \frac{\sqrt{580}}{7}$$

$$d = \frac{\sqrt{580}}{7}\ units$$

**Question 16.**
Write the vector equation of each of the following lines and hence determine the distance between them :

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} \text{ and } \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}.$$

HINT: The given lines are

$$L_1 : \vec{r} = \left(-2\hat{i} + 3\hat{j}\right) + \lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$$

$$L_2 : \vec{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + 2\mu\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

Now, find the distance between the parallel lines $L_1$ and $L_2$.

**Answer:**
**<u>Given</u>:** Cartesian equations of lines

$$L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

$$L2 : \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$$

**<u>To Find</u>:** i) vector equations of given lines

ii) distance d

**<u>Formulae</u>:**

**1. Equation of line :**

Equation of line passing through point A ($a_1$, $a_2$, $a_3$) and having direction ratios ($b_1$, $b_2$, $b_3$) is

$$\bar{r} = \bar{a} + \lambda\bar{b}$$

Where, $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

And $\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

**2. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**3. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**4. Shortest distance between two parallel lines :**

The shortest distance between the parallel lines $\bar{r} = \bar{a_1} + \lambda\bar{b}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b}$ is given by,

$$d = \left| \frac{|(\bar{a_2} - \bar{a_1}) \times \bar{b}|}{|\bar{b}|} \right|$$

<u>**Answer**</u>**:**

Given Cartesian equations of lines

$$L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

Line L1 is passing through point (1, 2, -4) and has direction ratios (2, 3, 6)

Therefore, vector equation of line L1 is

$$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

And

$$L2 : \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$$

Line L2 is passing through point (3, 3, -5) and has direction ratios (4, 6, 12)

Therefore, vector equation of line L2 is

$$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(4\hat{i} + 6\hat{j} + 12\hat{k})$$

$$\therefore \bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + 2\mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Now, to calculate distance between the lines,

$$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + 2\mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Here,

$$\overline{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\overline{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\overline{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\overline{b_2} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

As $\overline{b_1} = \overline{b_2} = \bar{b}\ (\text{say})$, given lines are parallel to each other.

Therefore,

$$\bar{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$\therefore |\bar{b}| = \sqrt{2^2 + 3^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

= 7

$\overline{a_2} - \overline{a_1} = (3 - 1)\hat{\imath} + (3 - 2)\hat{\jmath} + (-5 + 4)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$$(\overline{a_2} - \overline{a_1}) \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$= \hat{\imath}(6 + 3) - \hat{\jmath}(12 + 2) + \hat{k}(6 - 2)$

$\therefore (\overline{a_2} - \overline{a_1}) \times \bar{b} = 9\hat{\imath} - 14\hat{\jmath} + 4\hat{k}$

$\therefore |(\overline{a_2} - \overline{a_1}) \times \bar{b}| = \sqrt{9^2 + (-14)^2 + 4^2}$

$= \sqrt{81 + 196 + 16}$

$= \sqrt{293}$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{|(\overline{a_2} - \overline{a_1}) \times \bar{b}|}{|\bar{b}|} \right|$$

$$\therefore d = \left| \frac{\sqrt{293}}{7} \right|$$

$$d = \frac{\sqrt{293}}{7}\ units$$

**Question 17.**
Write the vector equation of the following lines and hence find the shortest distance between them :

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and } \frac{x-2}{3}=\frac{y-3}{4}=\frac{z-5}{5}.$$

**Answer:**
**<u>Given</u>:** Cartesian equations of lines

$$L1 : \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

$$L2 : \frac{x-2}{3}=\frac{y-3}{4}=\frac{z-5}{5}$$

**<u>To Find</u>:** i) vector equations of given lines

ii) distance d

**<u>Formulae</u>:**

**1. Equation of line :**

Equation of line passing through point A ($a_1$, $a_2$, $a_3$) and having direction ratios ($b_1$, $b_2$, $b_3$) is

$$\bar{r}=\bar{a}+\lambda\bar{b}$$

Where, $\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

And $\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

**2. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**3. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**4. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left| \frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \bar{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|} \right|$$

**<u>Answer</u>:**

Given Cartesian equations of lines

$$L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

Line L1 is passing through point (1, 2, 3) and has direction ratios (2, 3, 4)

Therefore, vector equation of line L1 is

$$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

And

$$L2 : \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$$

Line L2 is passing through point (2, 3, 5) and has direction ratios (3, 4, 5)

Therefore, vector equation of line L2 is

$$\bar{r} = (3\hat{i} + 3\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$$

Now, to calculate distance between the lines,

$$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\bar{r} = (3\hat{i} + 3\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$$

Here,

$$\overline{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overline{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\overline{a_2} = 3\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\overline{b_2} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-1)^2 + 2^2 + (-1)^2}$

$= \sqrt{1 + 4 + 1}$

$= \sqrt{6}$

$\overline{a_2} - \overline{a_1} = (3 - 1)\hat{\imath} + (3 - 2)\hat{\jmath} + (5 - 3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = \left(-\hat{\imath} + 2\hat{\jmath} - \hat{k}\right).\left(2\hat{\imath} + \hat{\jmath} + 2\hat{k}\right)$

$= ((-1) \times 2) + (2 \times 1) + ((-1) \times 2)$

= - 2 + 2 - 2

= -2

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{-2}{\sqrt{6}}\right|$$

$$\therefore d = \frac{2}{\sqrt{3}.\sqrt{2}}$$

$$\therefore d = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\therefore d = \sqrt{\frac{2}{3}}$$

$$d = \sqrt{\frac{2}{3}}\ units$$

**Question 18.**
Find the shortest distance between the lines given below:

$$\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2} \text{ and } \frac{x-1}{2} = \frac{y+1}{2}\frac{z+1}{-2}.$$

**Answer:**
**Given:** Cartesian equations of lines

$$L1 : \frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2}$$

$$L2 : \frac{x-1}{2} = \frac{y+1}{2} = \frac{z+1}{-2}$$

**To Find:** distance d

**Formulae:**

**1. Equation of line :**

Equation of line passing through point A ($a_1$, $a_2$, $a_3$) and having direction ratios ($b_1$, $b_2$, $b_3$) is

$$\bar{r} = \bar{a} + \lambda\bar{b}$$

Where, $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

**2. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**3. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**4. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \bar{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

**<u>Answer</u>:**

Given Cartesian equations of lines

$$L1 : \frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2}$$

Line L1 is passing through point (1, -2, 3) and has direction ratios (-1, 1, -2)

Therefore, vector equation of line L1 is

$$\bar{r} = (\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + \lambda(-\hat{\imath} + \hat{\jmath} - 2\hat{k})$$

And

$$L2 : \frac{x-1}{2} = \frac{y+1}{2} = \frac{z+1}{-2}$$

Line L2 is passing through point (1, -1, -1) and has direction ratios (2, 2, -2)

Therefore, vector equation of line L2 is

$$\bar{r} = (\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$$

Now, to calculate distance between the lines,

$$\bar{r} = (\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + \lambda(-\hat{\imath} + \hat{\jmath} - 2\hat{k})$$

$$\bar{r} = (\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$$

Here,

$$\overline{a_1} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$$

$$\overline{b_1} = -\hat{\imath} + \hat{\jmath} - 2\hat{k}$$

$$\overline{a_2} = \hat{\imath} - \hat{\jmath} - \hat{k}$$

$$\overline{b_2} = 2\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 1 & -2 \\ 2 & 2 & -2 \end{vmatrix}$$

$$= \hat{\imath}(-2 + 4) - \hat{\jmath}(2 + 4) + \hat{k}(-2 - 2)$$

$\therefore \overline{b_1} \times \overline{b_2} = 2\hat{i} - 6\hat{j} - 4\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{2^2 + (-6)^2 + (-4)^2}$

$= \sqrt{4 + 36 + 16}$

$= \sqrt{56}$

$\overline{a_2} - \overline{a_1} = (1 - 1)\hat{i} + (-1 + 2)\hat{j} + (-1 - 3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 0\hat{i} + \hat{j} - 4\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1}) = (2\hat{i} - 6\hat{j} - 4\hat{k}) . (0\hat{i} + \hat{j} - 4\hat{k})$

$= (2 \times 0) + ((-6) \times 1) + ((-4) \times (-4))$

= 0 - 6 + 16

= 10

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{10}{\sqrt{56}} \right|$$

$$\therefore d = \frac{10}{\sqrt{56}}$$

$$d = \frac{10}{\sqrt{56}}\ units$$

**Question 19.**
Find the shortest distance between the lines given below:

$$\frac{x-12}{-9}=\frac{y-1}{4}=\frac{z-5}{2} \text{ and } \frac{x-23}{-6}=\frac{y-10}{-4}=\frac{z-25}{3}.$$

**HINT:** Change the given equations in vector form.

**Answer:**
**Given:** Cartesian equations of lines

$$L1:\frac{x-12}{-9}=\frac{y-1}{4}=\frac{z-5}{2}$$

$$L2:\frac{x-23}{-6}=\frac{y-10}{-4}=\frac{z-23}{3}$$

**To Find:** distance d

**Formulae:**

**1. Equation of line :**

Equation of line passing through point A ($a_1$, $a_2$, $a_3$) and having direction ratios ($b_1$, $b_2$, $b_3$) is

$$\bar{r}=\bar{a}+\lambda\bar{b}$$

Where, $\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

And $\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

**2. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**3. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**4. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

**Answer:**

Given Cartesian equations of lines

$$L1 : \frac{x - 12}{-9} = \frac{y - 1}{4} = \frac{z - 5}{2}$$

Line L1 is passing through point (12, 1, 5) and has direction ratios (-9, 4, 2)

Therefore, vector equation of line L1 is

$$\bar{r} = \left(12\hat{i} + \hat{j} + 5\hat{k}\right) + \lambda\left(-9\hat{i} + 4\hat{j} + 2\hat{k}\right)$$

And

$$L2 : \frac{x-23}{-6} = \frac{y-10}{-4} = \frac{z-23}{3}$$

Line L2 is passing through point (23, 10, 23) and has direction ratios (-6, -4, 3)

Therefore, vector equation of line L2 is

$$\bar{r} = \left(23\hat{i} + 10\hat{j} + 23\hat{k}\right) + \mu\left(-6\hat{i} - 4\hat{j} + 3\hat{k}\right)$$

Now, to calculate distance between the lines,

$$\bar{r} = \left(12\hat{i} + \hat{j} + 5\hat{k}\right) + \lambda\left(-9\hat{i} + 4\hat{j} + 2\hat{k}\right)$$

$$\bar{r} = \left(23\hat{i} + 10\hat{j} + 23\hat{k}\right) + \mu\left(-6\hat{i} - 4\hat{j} + 3\hat{k}\right)$$

Here,

$$\overline{a_1} = 12\hat{i} + \hat{j} + 5\hat{k}$$

$$\overline{b_1} = -9\hat{i} + 4\hat{j} + 2\hat{k}$$

$$\overline{a_2} = 23\hat{i} + 10\hat{j} + 23\hat{k}$$

$$\overline{b_2} = -6\hat{i} - 4\hat{j} + 3\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -9 & 4 & 2 \\ -6 & -4 & 3 \end{vmatrix}$$

$$= \hat{i}(12+8) - \hat{j}(-27+12) + \hat{k}(36+24)$$

$$\therefore \overline{b_1} \times \overline{b_2} = 20\hat{i} + 15\hat{j} + 60\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{20^2 + 15^2 + 60^2}$$

$= \sqrt{400 + 225 + 3600}$

$= \sqrt{4225}$

= 65

$\overline{a_2} - \overline{a_1} = (23 - 12)\hat{i} + (10 - 1)\hat{j} + (23 - 5)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 11\hat{i} + 9\hat{j} + 18\hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = \left(20\hat{i} + 15\hat{j} + 60\hat{k}\right).\left(11\hat{i} + 9\hat{j} + 18\hat{k}\right)$

$= (20 \times 11) + (15 \times 9) + (60 \times 18)$

= 220 + 135 + 1080

= 1435

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{1435}{65}\right|$$

$$\therefore d = \frac{287}{13}$$

$$d = \frac{287}{13}\ units$$