Exercise 13b

Question 1.
Evaluate the following integrals:

(i) $\int \sin^2 x \, dx$

(ii) $\int \cos^2 x \, dx$

Answer:
i) $\int sin^2 x dx$

$\Rightarrow \int sin^2 x dx$

Now, we know that 1-cos2x=2sin^{2}x

So, applying this identity in the given integral, we get,

$$\int sin^2 x dx = \int \frac{(1 - cos2x)dx}{2}$$

$\Rightarrow \frac{1}{2}(\int dx - \int cos2xdx)$

$$\Rightarrow \frac{x}{2} - \frac{sin2x}{2 \times 2} + c$$

$$\Rightarrow \frac{x}{2} - \frac{sin2x}{4} + c$$

Ans: $\int sin^2 x dx = \frac{x}{2} - \frac{sin2x}{4} + c$

ii) $\int cos^2 x \, dx$

$\Rightarrow \int cos^2 x \, dx$

Now, we know that 1+cos2x=2cos^{2}x

So, applying this identity in the given integral, we get,

$$\int cos^2x\,dx = \int \frac{(1 + cos2x)dx}{2}$$

$\Rightarrow \frac{1}{2}(\int dx + \int cos2xdx)$

$$\Rightarrow \frac{x}{2} + \frac{sin2x}{2 \times 2} + c$$

$$\Rightarrow \frac{x}{2} + \frac{sin2x}{4} + c$$

Ans: $\int cos^2x\,dx = \frac{x}{2} + \frac{sin2x}{4} + c$

Question 2.
Evaluate the following integrals:

(i) $\int \cos^2(x/2)dx$

(ii) $\int \cot^2(x/2)dx$

Answer:
(i) $\int \cos^2(x/2)dx$

$\Rightarrow \int cos^2\left(\frac{x}{2}\right)dx$

Now, we know that 1+cosx=2cos^2 (x/2)

So, applying this identity in the given integral, we get,

$$\int cos^2(\frac{x}{2})\,dx = \int \frac{(1 + cosx)dx}{2}$$

$\Rightarrow \frac{1}{2}(\int dx + \int cosxdx)$

$\Rightarrow \frac{x}{2}+\frac{sin2x}{2}+c$

$\Rightarrow \frac{x}{2}+\frac{sin2x}{2}+c$

Ans: $\frac{x}{2}+\frac{sin2x}{2}+c$

ii) $\int \cot^2 (x/2) dx$

$\Rightarrow \int cot^2\left(\frac{x}{2}\right)dx$

Now, we know that cosec^{2}x-cot^2 x=1

So, applying this identity in the given integral we get,

$\Rightarrow \int cot^2\left(\frac{x}{2}\right)dx = \int (cosec^2\left(\frac{x}{2}\right)-1)\,dx$

$\Rightarrow \int (cosec^2\left(\frac{x}{2}\right)-1)\,dx = \int cosec^2\left(\frac{x}{2}\right)dx - \int 1dx$

$\Rightarrow \int cosec^2\left(\frac{x}{2}\right)dx - \int 1dx = \frac{-cotx}{\frac{1}{2}} - x + c$

⇒-2cotx-x+c

$\Rightarrow \int cot^2\left(\frac{x}{2}\right)dx =$-2cotx-x+c

Ans: -2cotx-x+c

Question 3.
Evaluate the following integrals:

(i) $\int \sin^2 nx\,dx$

(ii) $\int \sin^5 x\,dx$

Answer:

i) $\int sin^2 nxdx$

$\Rightarrow \int sin^2 nxdx$

Now, we know that 1-cos2nx=2sin^{2}nx

So, applying this identity in the given integral, we get,

$$\int sin^2 nxdx = \int \frac{(1 - cos2nx)dx}{2}$$

$\Rightarrow \frac{1}{2}(\int dx - \int cos2nxdx)$

$$\Rightarrow \frac{x}{2} - \frac{sin2nx}{2n \times 2} + c$$

$$\Rightarrow \frac{x}{2} - \frac{sin2x}{4n} + c$$

Ans: $\int sin^2 nxdx = \frac{x}{2} - \frac{sin2nx}{4n} + c$

(ii) $\int \sin^5 x\, dx$

We know that 1-cos^{2}x=sin^{2}x

$\Rightarrow \int sin^5 xdx = \int (1 - cos^2 x)^2 sinxdx$

$\Rightarrow$Put cosx=t

$\Rightarrow$-sinxdx=dt

$\Rightarrow \int (1 - cos^2 x)^2 sinxdx = -\int (1 - t^2)^2 dt$

$\Rightarrow -\int (1 - t^2)^2 dt = -\int (1 + t^4 - 2t^2)dt$

$\Rightarrow -\int dt + \int 2t^2 dt - \int t^4 dt$

$\Rightarrow -t + \frac{2t^3}{3} - \frac{t^5}{5} + c$

Resubstituting the value of t=cosx we get,

$\Rightarrow -cosx + \frac{2cos^3x}{3} - \frac{cos^5x}{5} + c$

Ans: $-cosx + \frac{2cos^3x}{3} - \frac{cos^5x}{5} + c$

Question 4.
Evaluate the following integrals:

$$\int \cos^3(3x+5)dx$$

Answer:
Substitute 3x+5=u

$\Rightarrow$3dx=du

$\Rightarrow$dx=du/3

$\Rightarrow \int cos^3(3x+5)dx = \frac{1}{3}\int cos^3(u)du$

Now We know that 1-cos^{2}x=sin^{2}x ,

$\Rightarrow \frac{1}{3}\int cos^3(u)du = \frac{1}{3}\int (1 - sin^2(u))cosu\, du$

$\Rightarrow$Substitute sinu=t

$\Rightarrow$cosu du=dt

$\Rightarrow \frac{1}{3}\int (1 - sin^2(u))cosu\, du = \frac{1}{3}\int (1 - t^2)\ dt$

$\Rightarrow \frac{1}{3}\int dt - \frac{1}{3}\int t^2 dt$

$\Rightarrow \frac{t}{3} - \frac{t^3}{3 \times 3} + c$

$\Rightarrow \frac{t}{3} - \frac{t^3}{9} + c$

Resubstituting the value of t=sinu and u=3x+5 we get,

$\Rightarrow \frac{\sin(3x+5)}{3} - \frac{sin^3(3x+5)}{9} + c$

Ans: $\frac{\sin(3x+5)}{3} - \frac{sin^3(3x+5)}{9} + c$

Question 5.
Evaluate the following integrals:

$$\int \sin^7(3-2x)dx$$

Answer:
$\Rightarrow -\int sin^7(2x-3)dx$

Substitute 2x-3=u

$\Rightarrow$ 2dx=du

$\Rightarrow$dx=du/2

$\Rightarrow -\left(\frac{1}{2}\right)\int sin^7(u)du$

$\Rightarrow$ We know that 1-cos^{2}x=sin^{2}x

$\Rightarrow -\left(\frac{1}{2}\right)\int \left(1 - cos^2(u)\right)^3 sinu\ du$

$\Rightarrow$Put cosu=t

$\Rightarrow$-sinxdu=dt

$$\Rightarrow\left(\frac{1}{2}\right)\int(1-t^2)^3dt$$

$$\Rightarrow\left(\frac{1}{2}\right)\int(1-t^6-3t^2+3t^4)dt$$

$$\Rightarrow\left(\frac{1}{2}\right)[\int dt-\int t^6dt-\int 3t^2dt+\int 3t^4dt]$$

$$\Rightarrow\left(\frac{1}{2}\right)\left[t-\frac{t^7}{7}-\frac{3t^3}{3}+\frac{3t^5}{5}\right]+c$$

$$\Rightarrow\left(\frac{1}{2}\right)\left[t-\frac{t^7}{7}-t^3+\frac{3t^5}{5}\right]+c$$

Resubstituting the value of t=cosu and u=2x-3 we get

$$\Rightarrow\left(\frac{1}{2}\right)\left[\cos(2x-3)-\frac{cos^7(2x-3)}{7}-cos^3(2x-3)+\frac{3cos^5(2x-3)}{5}\right]+c$$

$$\Rightarrow\frac{\cos(2x-3)}{2}-\frac{cos^7(2x-3)}{14}-\frac{cos^3(2x-3)}{2}+\frac{3cos^5(2x-3)}{10}+c$$

Now as we know cos(-x)=cosx

$$\Rightarrow\frac{\cos(2x-3)}{2}-\frac{cos^7(2x-3)}{14}-\frac{cos^3(2x-3)}{2}+\frac{3cos^5(2x-3)}{10}+c$$

$$=\frac{\cos(3-2x)}{2}-\frac{cos^7(3-2x)}{14}-\frac{cos^3(3-2x)}{2}+\frac{3cos^5(3-2x)}{10}+c$$

Ans: $\frac{\cos(3-2x)}{2}-\frac{cos^7(3-2x)}{14}-\frac{cos^3(3-2x)}{2}+\frac{3cos^5(3-2x)}{10}+c$

Question 6.
Evaluate the following integrals:

(i) $\left(\frac{1-\cos 2x}{1+\cos 2x}\right)dx$

(ii) $\left(\frac{1+\cos 2x}{1-\cos 2x}\right)dx$

Answer:

(i) $\left(\frac{1-\cos 2x}{1+\cos 2x}\right)dx$

$\Rightarrow \int \frac{1-cos2x}{1+cos2x} dx$

1-cos2x=2sin^{2}x and 1+cos2x=2cos^{2}x

$\Rightarrow \int \frac{1-cos2x}{1+cos2x} dx = \int \frac{2sin^2 x}{2cos^2 x} dx$

$\Rightarrow \int tan^2 x \, dx$

Now sec^{2}x-1=tan^{2}x

$\Rightarrow \int (sec^2 x - 1) dx$

$\Rightarrow \int sec^2 x dx - \int dx$

⇒tanx-x+c

Ans: tanx-x+c

(ii) $\left(\frac{1+\cos 2x}{1-\cos 2x}\right)dx$

$\Rightarrow \int \frac{1+cos2x}{1-cos2x} dx$

1-cos2x=2sin^{2}x and 1+cos2x=2cos^{2}x

$\Rightarrow \int \frac{1+cos2x}{1-cos2x} dx = \int \frac{2cos^2 x}{2sin^2 x} dx$

$\Rightarrow \int cot^2 x \, dx$

Now cosec^{2}x-1=cot^{2}x

$\Rightarrow \int (cosec^2 x - 1) dx$

$\Rightarrow \int cosec^2 x dx - \int dx$

⇒-cotx-x+c

Ans: -cotx-x+c

Question 7.
Evaluate the following integrals:

(i) $\int \frac{1-cosx}{1+cosx} dx$

(ii) $\int \frac{1+cosx}{1-cosx} dx$

Answer:
i) $\int \frac{1-cosx}{1+cosx} dx$

$\Rightarrow \int \frac{1-cosx}{1+cosx} dx$

1-cosx=2sin^{2}x/2 and 1+cosx=2cos^{2}x/2

$\Rightarrow \int \frac{1-cosx}{1+cosx} dx = \int \frac{2sin^2(\frac{x}{2})}{2cos^2(\frac{x}{2})} dx$

$\Rightarrow \int tan^2(\frac{x}{2}) dx$

Now sec^2(x/2)-1=tan2(x/2)

$\Rightarrow \int \left(sec^2(\frac{x}{2}) - 1\right) dx$

$\Rightarrow \int sec^2(\frac{x}{2}) dx - \int dx$

⇒2tan(x/2)-x+c

Ans: 2tan(x/2)-x+c

(ii) $\int \frac{1+cosx}{1-cosx} dx$

$\Rightarrow \int \frac{1+cosx}{1-cosx} dx$

1-cosx=2sin^{2}x/2 and 1+cosx=2cos^{2}x/2

$\Rightarrow \int \frac{1+cosx}{1-cosx} dx = \int \frac{2cos^2(\frac{x}{2})}{2sin^2(\frac{x}{2})} dx$

$\Rightarrow \int cot^2(\frac{x}{2}) dx$

Now cosec^2 (x/2)-1=cot^2 (x/2)

$\Rightarrow \int \left(cosec^2(\frac{x}{2}) - 1\right) dx$

$\Rightarrow \int cosec^2(\frac{x}{2}) dx - \int dx$

⇒-2cot(x/2)-x+c

Ans: ⇒-2cot(x/2)-x+c

Question 8.
Evaluate the following integrals:

$$\int \sin 3x \cos 4x \ dx$$

Answer:
$\Rightarrow \int sin3x \ cos4x \ dx$

Applying the formula: sinx×cosy=1/2(sin(x+y)-sin(y-x))

$\Rightarrow \frac{1}{2} \int (sin7x - sinx) dx$

$\Rightarrow \frac{1}{2} \int sin7x \ dx - \frac{1}{2} \int sinx \ dx$

$\Rightarrow \frac{-cos7x}{14} + \frac{cosx}{2} + c$

Ans: $\frac{-cos7x}{14} + \frac{cosx}{2} + c$

Question 9.
Evaluate the following integrals:

$$\int \cos 4x \cos 3x \, dx$$

Answer:
$\Rightarrow \int cos4x\ cos3x\ dx$

Applying the formula: cosx×cosy=1/2(cos(x+y)+cos(x-y))

$\Rightarrow \frac{1}{2} \int (cos7x + cosx) dx$

$\Rightarrow \frac{1}{2} \int cos7x\ dx + \frac{1}{2} \int cosx\ dx$

$\Rightarrow \frac{sin7x}{14} + \frac{sinx}{2} + c$

Ans: $\frac{sin7x}{14} + \frac{sinx}{2} + c$

Question 10.
Evaluate the following integrals:

$$\int \sin 4x \sin 8x \, dx$$

Answer:
$\Rightarrow \int sin4x\ sin8x\ dx$

Applying the formula: sinx×siny=1/2(cos(y-x)-cos(y+x))

$\Rightarrow \frac{1}{2} \int (cos4x - cos12x) dx$

$\Rightarrow \frac{1}{2}\int cos4x\, dx - \frac{1}{2}\int cos12x\, dx$

$\Rightarrow \frac{sin4x}{8} - \frac{sin12x}{24} + c$

Ans: $\frac{sin4x}{8} - \frac{sin12x}{24} + c$

Question 11.
Evaluate the following integrals:

$$\int \sin\ 6x\ \cos\ x\ dx$$

Answer:
$\Rightarrow \int sin6x\ cosx\ dx$

Applying the formula: sinx×cosy=1/2(sin(y+x)-sin(y-x))

$\Rightarrow \frac{1}{2}\int (sin7x - \sin(-5x))dx$

$\Rightarrow \frac{1}{2}\int sin7x\, dx + \frac{1}{2}\int sin5x\, dx$

$\Rightarrow \frac{-cos7x}{14} - \frac{cosx}{10} + c$

Ans: $\frac{-cos7x}{14} - \frac{cosx}{10} + c$

Question 12.
Evaluate the following integrals:

$$\int \sin\ x\sqrt{1+\cos\ 2x}\, dx$$

Answer:
we know that 1+cos2x=2cos^{2}x

So, applying this identity in the given integral we get,

$\Rightarrow \int sinx\sqrt{1 + cos2x}\, dx$

$\Rightarrow\int sinx\sqrt{(2cos^2x)}dx$

$\Rightarrow\sqrt{2}\int sinxcosxdx$

Let sinx =t

⇒ cosx dx=dt

$\Rightarrow\sqrt{2}\int tdt$

$\Rightarrow\sqrt{2}\frac{t^2}{2}+c=\frac{t^2}{\sqrt{2}}+c$

Resubstituting the value of t=sinx we get

$\Rightarrow\frac{sin^2x}{\sqrt{2}}+c$

Ans: $\frac{sin^2x}{\sqrt{2}}+c$

Question 13.
Evaluate the following integrals:

$$\int \cos^4 x\,dx$$

Answer:
$\Rightarrow\int cos^2xcos^2xdx$

$\Rightarrow\int(\frac{1+cos2x}{2})(\frac{1+cos2x}{2})dx\ldots(\frac{1+cos2x}{2}=cos^2x)$

$\Rightarrow\frac{1}{4}\int(1+cos2x)^2dx$

$\Rightarrow\frac{1}{4}\int(1+cos^22x+2cos2x)dx$

$\Rightarrow\frac{1}{4}[\int 1dx\ +\int cos^2 2xdx+\int 2cos2x\ dx$

$\Rightarrow \frac{1}{4}[x + \int \frac{(1+cos4x)dx}{2} + 2\frac{sin2x}{2}]$ ...(1+cos4x=2cos^{2}x)

$\Rightarrow \frac{1}{4}[x + \frac{1}{2}(\int dx + \int cos4xdx) + sin2x] + c$

$\Rightarrow [\frac{x}{4} + \frac{1}{2} \times \frac{1}{4}\left(\int dx + \int cos4xdx\right) + \frac{sin2x}{4}\right] + c$

$\Rightarrow [\frac{x}{4} + \left(\frac{x}{8} + \frac{sin4x}{32}\right) + \frac{sin2x}{4}\right] + c$

$\Rightarrow \frac{3x}{8} + \frac{sin4x}{32} + \frac{sin2x}{4} + c$

Ans: $\frac{3x}{8} + \frac{sin4x}{32} + \frac{sin2x}{4} + c$

Question 14.
Evaluate the following integrals:

$$\int \cos 2x \cos 4x \cos 6x\, dx$$

Answer:
$\Rightarrow \int cos2x\, cos4x \cos 6xdx$

$\Rightarrow \frac{1}{2}\int (cos6x + cos2x)cos6xdx$

$\Rightarrow \frac{1}{2}\int cos^2 6xdx + \frac{1}{2}\int cos2xcos6xdx$

$$\Rightarrow \frac{1}{2}\int cos^2 6xdx + \frac{1}{4}\int (cos8x + cos4x)dx$$

$$\Rightarrow \frac{1}{2}\int cos^2 6xdx + \frac{1}{4}\int cos8x\, dx + \frac{1}{4}\int cos4x\, dx$$

$$\Rightarrow \frac{1}{2}\int \frac{(1+cos12x)dx}{2} + \frac{1}{4}\frac{sin8x}{8} + \frac{1}{4}\frac{sin4x}{4} + c$$

$$\Rightarrow \frac{1}{4}\left(x + \frac{sin12x}{12}\right) + \frac{sin8x}{32} + \frac{sin4x}{16} + c$$

$\Rightarrow \frac{x}{4} + \frac{sin12x}{48} + \frac{sin8x}{32} + \frac{sin4x}{16} + c$

Ans: $\frac{x}{4} + \frac{sin12x}{48} + \frac{sin8x}{32} + \frac{sin4x}{16} + c$

Question 15.
Evaluate the following integrals:

$$\int \sin^3 x \cos x \, dx$$

Answer:
Let sinx =t

⇒ cosx dx =dt

$\Rightarrow \int sin^3 x \, cosx \, dx = \int t^3 dt$

$\Rightarrow \frac{t^4}{4} + c$

Resubstituting the value of t=sinx we get

$\Rightarrow \frac{sin^4 x}{4} + c$

Ans: $\frac{sin^4 x}{4} + c$

Question 16.
Evaluate the following integrals:

$$\int \sec^4 x \, dx$$

Answer:
$\Rightarrow \int sec^4 dx = \int sec^2 x sec^2 x dx$

$\Rightarrow \int sec^2 x(1 + tan^2 x)dx$

$\Rightarrow$Put tanx=t $\Rightarrow$sec^{2}dx=dt

$\Rightarrow \int (1 + t^2) dt$

$\Rightarrow t + \frac{t^3}{3} + c$

Resubstituting the value of t=tanx we get

$\Rightarrow tanx + \frac{tan^3 x}{3} + c$

Ans: $tanx + \frac{tan^3 x}{3} + c$

Question 17.
Evaluate the following integrals:

$$\int \cos^3 x \sin^4 x \, dx$$

Answer:
$\Rightarrow \int cos^3 x sin^4 x \, dx$

$\Rightarrow \int cosx \, sin^4 x cos^2 x dx$

$\Rightarrow \int cosx \, sin^4 x (1 - sin^2 x) dx$

Put sinx=t

$\Rightarrow$cosxdx=dt

$\Rightarrow \int t^4 (1 - t^2) dt$

$\Rightarrow \int t^4 dt - \int t^6 \, dt$

$\Rightarrow \frac{t^5}{5} - \frac{t^7}{7} + c$

Resubstituting the value of t=sinx we get,

$\Rightarrow \frac{sin^5 x}{5} - \frac{sin^7 x}{7} + c$

Ans: $\frac{sin^5 x}{5} - \frac{sin^7 x}{7} + c$

Question 18.
Evaluate the following integrals:

$$\int \cos^4 x \sin^3 x \, dx$$

Answer:
$\Rightarrow \int cos^4 x sin^3 x \, dx$

$\Rightarrow \int sinx \, sin^2 x cos^4 x dx$

$\Rightarrow \int sinx \, cos^4 x(1 - cos^2 x)dx$

Put cosx=t

⇒-sinxdx=dt

$\Rightarrow \int t^4(t^2 - 1)dt$

$\Rightarrow \int t^6 dt - \int t^4 \, dt$

$\Rightarrow \frac{t^7}{7} - \frac{t^5}{5} + c$

Resubstituting the value of t=sinx we get,

$\Rightarrow \frac{cos^7 x}{7} - \frac{cos^5 x}{5} + c$

Ans: $\frac{cos^7 x}{7} - \frac{cos^5 x}{5} + c$

Question 19.

Evaluate the following integrals:

$$\int \sin^{2/3} x \cos^3 x \, dx$$

Answer:

$\Rightarrow \int cos^3 x sin^{\frac{2}{3}} x \, dx$

$\Rightarrow \int cosx \, cos^2 x sin^{\frac{2}{3}} x dx$

$\Rightarrow \int cosx \, (1 - sin^2 x) sin^{\frac{2}{3}} x dx$

Put sinx=t

⇒cosxdx=dt

$\Rightarrow \int t^{\frac{2}{3}} (1 - t^2) dt$

$\Rightarrow \int t^{\frac{2}{3}} dt - \int t^{\frac{8}{3}} dt$

$$\Rightarrow \frac{t^{\frac{5}{3}}}{\frac{5}{3}} - \frac{t^{\frac{11}{3}}}{\frac{11}{3}} + c$$

Resubstituting the value of t=sinx we get

$$\Rightarrow \frac{3sin^{\frac{5}{3}} x}{5} - \frac{3sin^{\frac{11}{3}} x}{11} + c$$

Ans: $\frac{3sin^{\frac{5}{3}} x}{5} - \frac{3sin^{\frac{11}{3}} x}{11} + c$

Question 20.

Evaluate the following integrals:

$$\int \cos^{3/5} x \sin^3 x \, dx$$

Answer:

$\Rightarrow \int sin^3 x cos^{\frac{3}{5}} x\, dx$

$\Rightarrow \int sinx\, sin^2 x cos^{\frac{3}{5}} x dx$

$\Rightarrow \int sinx\,(1 - cos^2 x) cos^{\frac{3}{5}} x dx$

Put cosx=t

⇒-sinxdx=dt

$\Rightarrow \int t^{\frac{3}{5}}(t^2 - 1)dt$

$\Rightarrow \int t^{\frac{13}{5}} dt - \int t^{\frac{3}{5}} dt$

$$\Rightarrow \frac{t^{\frac{18}{5}}}{\frac{18}{5}} - \frac{t^{\frac{8}{5}}}{\frac{8}{5}} + c$$

Resubstituting the value of t=cosx we get

$$\Rightarrow \frac{5cos^{\frac{18}{5}} x}{18} - \frac{5cos^{\frac{8}{5}} x}{8} + c$$

Ans: $\frac{5cos^{\frac{18}{5}} x}{18} - \frac{5cos^{\frac{8}{5}} x}{8} + c$

Question 21.
Evaluate the following integrals:

$$\int cosec^4 2x\ dx$$

Answer:

$$\Rightarrow \int cosec^4 2x dx$$

$$\Rightarrow \int cosec^2 2x cosec^2 2x dx$$

$$\Rightarrow \int cosec^2 2x\,(1 + cot^2 2x) dx$$

$\Rightarrow$cot2x=t $\Rightarrow$-2cosec^2 2xdx=dt

$$\Rightarrow -1/2 \int (1 + t^2) dt$$

$$\Rightarrow -1/2 \int dt - 1/2 \int t^2 dt$$

$$\Rightarrow -(\frac{1}{2})\mathrm{t} - \frac{t^3}{6} + c$$

Resubstituting the value of t=cotx we get

$$\Rightarrow -\frac{\mathrm{cotx}}{2} - \frac{cot^3 x}{6} + c$$

Ans: $-\frac{\mathrm{cotx}}{2} - \frac{cot^3 x}{6} + c$

Question 22.
Evaluate the following integrals:

$$\int \frac{\cos 2\mathrm{x}}{\cos \mathrm{x}} \mathrm{dx}$$

Answer:

$$\Rightarrow \int \frac{cos2x}{cosx} dx = \int \frac{2cos^2 x - 1}{cosx} dx$$

$$\Rightarrow \int \frac{2cos^2 x}{cosx} dx - \int \frac{1}{cosx} dx$$

$$\Rightarrow \int 2cosxdx - \int secxdx$$

$\Rightarrow 2\sin x - \log|\sec x + \tan x| + c$

Ans: 2sinx-log|secx+tanx|+c

Question 23.
Evaluate the following integrals:

$$\int \frac{\cos x}{\cos(x+\alpha)} dx$$

Answer:
$$\Rightarrow \int \frac{cosx}{\cos(x+\alpha)} dx = \int \frac{\cos((x+\alpha)-\alpha)}{\cos(x+\alpha)} dx$$

$$\Rightarrow \int \frac{\cos(x+\alpha)cos\alpha + \sin(x+\alpha)\, sin\alpha}{\cos(x+\alpha)} dx$$

$$\Rightarrow \int cos\alpha dx + \int \tan(x+\alpha)\, sin\alpha\, dx$$

Now α is a constant

$\Rightarrow x\cos\alpha - \sin\alpha\log|\cos(x+\alpha)| + c$

Ans:xcos α-sin αlog|cos(x+ α)|+c

Question 24.
Evaluate the following integrals:

$$\int \cos^3 x \sin 2x\, dx$$

Answer:
$$\Rightarrow \int sin2x\, cos^3 xdx$$

$$\Rightarrow \int 2sinxcosxcos^3 xdx$$

$$\Rightarrow \int 2sinx\,cos^4 x dx$$

Now put cosx=t

⇒-sinxdx=dt

$$\Rightarrow -2\int t^4 dt$$

$$\Rightarrow -2 \times \frac{t^5}{5} + c$$

Resubstituting the value of t= cosx we get,

$$\Rightarrow \frac{-2cos^5 x}{5} + c$$

Ans: $\frac{-2cos^5 x}{5} + c$

Question 25.
Evaluate the following integrals:

$$\int \frac{\cos^9 x}{\sin x} dx$$

Answer:

$$\Rightarrow \int \frac{cos^9 x}{sinx} dx$$

$$\Rightarrow \int \frac{cos^9 x}{sin^2 x} sinx dx$$

$$\Rightarrow \int \frac{cos^9 x}{1 - cos^2 x} sinx dx$$

Put cosx =t

⇒ -sinxdx=dt

$$\Rightarrow \int \frac{t^9}{t^2-1} dt$$

Now put t^2-1=a

⇒2tdt=da

And $t^8=(a+1)^4$

$$\Rightarrow \frac{1}{2}\int \frac{(a+1)^4}{a} da$$

$$\Rightarrow \frac{1}{2}\int (a^3+4a^2+6a+\frac{1}{a}+4) da$$

$$\Rightarrow \frac{1}{2}\left(\frac{a^4}{4}+\frac{4a^3}{3}+\frac{6a^2}{2}+lna+4a\right)+c$$

$$\Rightarrow \left(\frac{a^4}{8}+\frac{2a^3}{3}+\frac{3a^2}{2}+\frac{lna}{2}+2a\right)+c$$

Resubstituting the value of $a=t^2-1$ and t=cosx ⇒$a=\cos^2x-1=-\sin^2x$ we get

$$\Rightarrow \left(\frac{(-sin^2x)^4}{8}+\frac{2(-sin^2x)^3}{3}+\frac{3(-sin^2x)^2}{2}+\frac{\ln|(-sin^2x)|}{2}+2(-sin^2x)\right)+c$$

$$\Rightarrow \left(\frac{sin^8x}{8}-\frac{2sin^6x}{3}+\frac{3sin^4x}{2}+\frac{2\ln|(-sinx)|}{2}-2sin^2x\right)+c$$

$$\Rightarrow \left(\frac{sin^8x}{8}-\frac{2sin^6x}{3}+\frac{3sin^4x}{2}+\ln(sinx)-2sin^2x\right)+c$$

Ans: $\left(\frac{sin^8x}{8}-\frac{2sin^6x}{3}+\frac{3sin^4x}{2}+\ln(sinx)-2sin^2x\right)+c$

Question 26.

Evaluate the following integrals:

$$\int \cos^4 2x\, dx$$

Answer:

$\Rightarrow \int cos^2 2x cos^2 2x dx$

$\Rightarrow \int (\frac{1+cos4x}{2})(\frac{1+cos4x}{2}) dx$ ...$(\frac{1+cos4x}{2} = cos^2 2x)$

$\Rightarrow \frac{1}{4}\int (1 + cos4x)^2 dx$

$\Rightarrow \frac{1}{4}\int (1 + cos^2 4x + 2cos4x) dx$

$\Rightarrow \frac{1}{4}[\int 1dx \ + \int cos^2 4xdx + \int 2cos4x\ dx$

$\Rightarrow \frac{1}{4}[x + \int \frac{(1+cos8x)dx}{2} + 2\frac{sin4x}{4}]$ ...(1+cos8x=2cos^2 4x)

$\Rightarrow \frac{1}{4}[x + \frac{1}{2}\left(\int dx + \int cos8xdx\right) + (\frac{sin4x}{2})\right] + c$

$\Rightarrow [\frac{x}{4} + \frac{1}{2} \times \frac{1}{4}\left(\int dx + \int cos8xdx\right) + \frac{sin4x}{8}\right] + c$

$\Rightarrow [\frac{x}{4} + \left(\frac{x}{8} + \frac{sin8x}{64}\right) + \frac{sin4x}{8}\right] + c$

$\Rightarrow \frac{3x}{8} + \frac{sin8x}{64} + \frac{sin4x}{8} + c$

Ans: $\frac{3x}{8} + \frac{sin8x}{64} + \frac{sin4x}{8} + c$

Question 27.
Evaluate the following integrals:

$$\int \frac{\sin^2 x}{(1+\cos x)^2} dx$$

Answer:
Doing tangent half angle substitution we get,

$$\Rightarrow \int \frac{sin^2 x}{(1+cos^2 x)} dx = \int \frac{\left(\frac{2tan\frac{x}{2}}{1+tan^2\frac{x}{2}}\right)^2}{\left[1+\left(\frac{1-tan^2\frac{x}{2}}{1-tan^2\frac{x}{2}}\right)\right]^2}$$

Substitute u=tan(x/2)

$\Rightarrow 2du=sec^2(x/2)dx$

$\Rightarrow dx=\frac{2du}{u^2+1}$

$\Rightarrow 2\int \frac{u^2}{1+u^2} du$

$\Rightarrow 2\int \frac{1+u^2}{1+u^2} du - 2\int \frac{1}{1+u^2} du$

$\Rightarrow 2\int du - \tan^{-1} u + c$

$\Rightarrow 2u - \tan^{-1} u + c$

Resubstituting the values we get,

$$\Rightarrow 2\tan\frac{x}{2} - \tan^{-1}\tan\frac{x}{2} + c$$

$$\Rightarrow 2\tan\frac{x}{2} - \frac{x}{2} + c$$

Ans: $2\tan\frac{x}{2} - \frac{x}{2} + c$

Question 28.

Evaluate the following integrals:

$$\int \frac{dx}{(3\cos x + 4\sin x)}$$

Answer:

$$\int \frac{dx}{3cosx + 4sinx} = \int \frac{dx}{3\left(\frac{1 - tan^2\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right) + 4\left(\frac{2tan\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right)}$$

$$\Rightarrow \int \frac{sec^2\frac{x}{2}dx}{3 + 8tan\frac{x}{2} - 3tan^2\frac{x}{2}}$$

Let $\tan\frac{x}{2}=t$

$$\therefore \frac{1}{2}sec^2\frac{x}{2}dx = dt$$

$$\Rightarrow \int \frac{2dt}{3 + 8t - 3t^2} = \frac{2}{3}\int \frac{dt}{1 + \frac{8}{3}t - t^2} = \frac{2}{3}\int \frac{dt}{1 - \left(t - \frac{4}{3}\right)^2 + \frac{16}{9}}$$

$$\Rightarrow \frac{2}{3}\int \frac{dt}{\frac{25}{9} - \left(t - \frac{4}{3}\right)^2} = \frac{2}{3}\int \frac{dt}{\left(\frac{5}{3}\right)^2 - \left(t - \frac{4}{3}\right)^2}$$

$$\Rightarrow \frac{2}{3} \times \frac{1}{2 \times \frac{5}{3}} \ln\left|\frac{\frac{5}{3} + \left(t - \frac{4}{3}\right)}{\frac{5}{3} - \left(t - \frac{4}{3}\right)}\right| + c = \frac{1}{5}\ln\left|\frac{1 + 3t}{9 - 3t}\right| + c$$

Resubstituting the value of t we get

$$\Rightarrow \frac{1}{5}\ln\left|\frac{1 + 3tan\frac{x}{2}}{9 - 3tan\frac{x}{2}}\right| + c$$

Ans: $\frac{1}{5}\ln\left|\frac{1+3tan\frac{x}{2}}{9-3tan\frac{x}{2}}\right| + c$

Question 29.
Evaluate the following integrals:

$$\int \frac{dx}{(a\cos x + b\sin x)^2},\ a > 0 \text{ and } b > 0$$

Answer:

$$\int \frac{dx}{(acosx + bsinx)^2}$$

Taking bcosx common from the denominator we get,

$$\int \frac{dx}{b^2cos^2x(\frac{a}{b} + tanx)^2}$$

$$\Rightarrow \frac{1}{b^2}\int \frac{sec^2xdx}{(\frac{a}{b} + tanx)^2}$$

Let (a/b)+tanx=t

$$\therefore sec^2xdx = dt$$

$$\Rightarrow \frac{1}{b^2}\int \frac{dt}{t^2} = \frac{-1}{b^2} \times \frac{1}{t} = \frac{-1}{b^2t} + c$$

Resubstituting the value of t = (a/b)+tanx we get

$$\Rightarrow \frac{-1}{b^2(\frac{a}{b} + tanx)} + c = \frac{-1}{ab + b^2tanx} + c$$

Ans: $\frac{-1}{ab+b^2tanx} + c$

Question 30.

Evaluate the following integrals:

$$\int \frac{dx}{(\cos x - \sin x)}$$

Answer:

$$\int \frac{dx}{cosx - sinx} = \int \frac{dx}{\left(\frac{1 - tan^2\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right) - \left(\frac{2tan\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right)}$$

$$\Rightarrow \int \frac{sec^2\frac{x}{2}dx}{1 - 2tan\frac{x}{2} - tan^2\frac{x}{2}}$$

Let $\tan\frac{x}{2}=t$

$$\therefore \frac{1}{2}sec^2\frac{x}{2}dx = dt$$

$$\Rightarrow \int \frac{2dt}{1 - 2t - t^2} = -2\int \frac{dt}{t^2 + 2t - 1} = -2\int \frac{dt}{(t+1)^2 - 2}$$
$$= -2\int \frac{dt}{(t+1)^2 - (\sqrt{2})^2}$$

$\Rightarrow -2 \times \frac{1}{2\times\sqrt{2}}\ln\left|\frac{t+1-\sqrt{2}}{t+1+\sqrt{2}}\right| + c$ resubstituting the value of t we get

$$\Rightarrow \frac{-1}{\sqrt{2}}\ln\left|\frac{tan\frac{x}{2} + 1 - \sqrt{2}}{tan\frac{x}{2} + 1 + \sqrt{2}}\right| + c = \frac{-1}{\sqrt{2}}\ln\left|\tan\left(\frac{\pi}{8} - \frac{x}{2}\right)\right| + c$$

Ans: $\frac{-1}{\sqrt{2}}\ln\left|\tan\left(\frac{\pi}{8} - \frac{x}{2}\right)\right| + c$

Question 31.

Evaluate the following integrals:

$$\int (2\tan x - 3\cot x)^2 dx$$

Answer:

$$\int (2tanx - 3cotx)^2 dx$$

$$\Rightarrow \int (4tan^2x + 9cot^2x - 12tanxcotx)dx$$

$$\Rightarrow \int (4(sec^2x - 1) + 9(cosec^2x - 1) - 12)dx$$

$$\Rightarrow \int 4sec^2x\,dx + \int 9cosec^2xdx - \int 25dx$$

$$\Rightarrow 4\tan x - 9\cot x - 25x + c$$

Ans: 4tanx-9cotx-25x+c

Question 32.

Evaluate the following integrals:

$$\int \sin x \sin 2x \sin 3x\, dx$$

Answer:

$\Rightarrow \int sinx\ sin2xsin3x\ dx$

Applying the formula: sinx×siny=1/2(cos(y-x)-cos(y+x))

$\Rightarrow \frac{1}{2}\int (cos2x - cos4x)sin2xdx$

$\Rightarrow \frac{1}{2}\int sin2xcos2x\ dx - \frac{1}{2}\int sin2xcos4x\ dx$

$\Rightarrow \frac{1}{4}\int sin4x\ dx - \frac{1}{4}\int (sin6x - sin2x)dx$

$\Rightarrow \frac{-cos4x}{16} + \frac{cos6x}{24} - \frac{cos2x}{8} + c$

Ans: $\frac{-cos4x}{16} + \frac{cos6x}{24} - \frac{cos2x}{8} + c$

Question 33.
Evaluate the following integrals:

$$\int\left(\frac{1-\cot x}{1+\cot x}\right)dx$$

Answer:

$$\Rightarrow \int \frac{1-cotx}{1+cotx}dx = \int \frac{1-\frac{cosx}{sinx}}{1+\frac{cosx}{sinx}}dx$$

$$\Rightarrow \int \frac{sinx-cosx}{sinx+cosx}dx = -\int \frac{cosx-sinx}{sinx+cosx}dx$$

$$\Rightarrow -\int \frac{d(sinx+cosx)}{sinx+cosx}$$

$$\Rightarrow -\log|\sin x+\cos x|+c$$

Ans: -log(sinx+cosx)+c

Question 34.
Evaluate the following integrals:

$$\int\frac{dx}{(2\sin x+\cos x+3)}$$

Answer:

$$\int \frac{dx}{cosx+2sinx+3} = \int \frac{dx}{\left(\frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}\right)+2\left(\frac{2tan\frac{x}{2}}{1+tan^2\frac{x}{2}}\right)+3}$$

$$\Rightarrow \int \frac{sec^2\frac{x}{2}dx}{3+1+3tan^2\frac{x}{2}+4tan\frac{x}{2}-tan^2\frac{x}{2}}$$

Let $tan\frac{x}{2}$=t

$\therefore \frac{1}{2} sec^2 \frac{x}{2} dx = dt$

$$\Rightarrow \int \frac{2dt}{4 + 4t + 2t^2} = \int \frac{dt}{2 + 2t + t^2} = \frac{2}{3} \int \frac{dt}{(t+1)^2 + 2 - 1}$$

$$\Rightarrow \int \frac{dt}{(t+1)^2 + 1} = \int \frac{dt}{(1)^2 + (t+1)^2}$$

$$\Rightarrow \tan^{-1}(t+1) + c$$

Resubstituting the value of t we get

$$\Rightarrow \tan^{-1}(tan\frac{x}{2} + 1) + c$$

Ans: $\tan^{-1}(tan\frac{x}{2} + 1) + c$