Exercise 11e

Question 1.
Find the maximum or minimum values, if any, without using derivatives, of the function:

$$(5x-1)^2+4\cdot$$

Answer:
min. value = 4

Since the square of any no. Is positive, the given function has no maximum value.

The minimum value exists when the quantity $(5x-1)^2=0$

Therefore, minimum value=4

Question 2.
Find the maximum or minimum values, if any, without using derivatives, of the function:

$$-(x-3)^2+9$$

Answer:
max. value = 9

Since the quantity $(x-3)^2$ has a –ve sign, the max. Value it can have is 9.

Also hence it has no minimum value.

Question 3.
Find the maximum or minimum values, if any, without using derivatives, of the function:

$$-|x+4|+6$$

Answer:
max. value = 6

Since $|x+4|$ is non-negative for all x belonging to R.

Therefore the least value it can have is 0 .

Hence value of function is 6.

It has no minimumvalue as it can have infinitely many.

Question 4.
Find the maximum or minimum values, if any, without using derivatives, of the function:

$$\sin\ 2x + 5$$

Answer:
max. value = 4, min. value = 6

f(x)=sin2x+5

We know that,

-1≤sinΘ≤1

-1≤sin2x≤1

Adding 5 on both sides,

-1+5≤sin2x+5≤1+5

4≤sin2x+5≤6

Hence

max value of f(x)=2x+5 will be 6

Min value of f(x) =2x+5 will be 4

Question 5.
Find the maximum or minimum values, if any, without using derivatives, of the function:

$$|\sin\ 4x + 3|$$

Answer:
max. value = 4, min. value = 2

We know that

$-1 \leq \sin\Theta \leq 1$

$-1 \leq \sin 4x \leq 1$

Adding 3 on both sides,

We get

$-1+3 \leq \sin 4x+3 \leq 1+3$

$2 \leq |\sin 4x+3| \leq 4$

Hence min.Value is 2 and max value is 4

Question 6.
Find the point of local maxima or local minima or local minima and the corresponding local maximum and minimum values of each of the following functions:

$$f(x) = (x-3)^4$$

Answer:
local max. value is 0 at x = 3

$F'(x)=4(x-3)^3=0$

$\Rightarrow X=3$

.local max. Vaue is 0

Question 7.
Find the point of local maxima or local minima or local minima and the corresponding local maximum and minimum values of each of the following functions:

$$f(x) = x^2$$

Answer:
local min. value is 0 at x = 0

$F'(x)=2x=0$

$x=0$

$\therefore$ local min.value is 0

Question 8.
Find the point of local maxima or local minima or local minima and the corresponding local maximum and minimum values of each of the following functions:

$$f(x)=2x^3-21x^2+36x-20$$

Answer:
local max. value is –3 at x = 1 and local min. value is –128 at x = 6

$F'(x)=6x^2-42x+36=0$

$\Rightarrow 6(x-1)(x-6)=0$

$\Rightarrow x=1,6$

$F''(x)=12x-42$

$F''(1)<0$,1 is the pont of local max.

$F''(6)>0$, 6 is the point of localmin.

$F(1)=2-21+36-20=-3$

$F(6)=-128$

Question 9.
Find the point of local maxima or local minima or local minima and the corresponding local maximum and minimum values of each of the following functions:

$$f(x)=x^3-6x^2+9x+15$$

Answer:
local max. value is 19 at x = 1 and local min. value is 15 at x = 3

$F'(x)=3x^2-12x+9=0$

$\Rightarrow 3(x-3)(x-1)=0$

$\Rightarrow x=3,1$

$F''(x)=6x-12$

$F''(3)=18-12=6>0$, 3 is the of local min.

$F''(1)<0$, 1 is the point of local max.

$F(3)=15$

$F(1)=19$

Question 10.
Find the point of local maxima or local minima or local minima and the corresponding local maximum and minimum values of each of the following functions:

$$f(x)=x^4-62x^2+120x+9$$

Answer:
local max. value is 68 at x = 1 and local min. values are –1647 at x = –6 and –316 at x = 5

$F'(x)=4x^3-124x+120=0$

$\Rightarrow 4(x^3-31x+30)=0$

For x=1, the given eq is 0

∴x-1 is a factor,

$4(x-1)(x+6)(x-5)=0$

$\Rightarrow X=1,-6,5$

$F''(1)<0$, 1is the point of max.

$F''(-6)$ and $f''(5)>0$, -6 and 5 are point of min.

F(1)=68

F(-6)=-1647

F(5)=-316

Question 11.
Find the point of local maxima or local minima or local minima and the corresponding local maximum and minimum values of each of the following functions:

$$f(x) = -x^3 + 12x^2 - 5$$

Answer:
local max. value is 251 at x = 8 and local min. value is –5 at x = 0

'}(x)=$-3x^2$+24x=0

⇒ -3x(x-8)=0

⇒ x=0,8

$F''(x)=-6x+24$

$F''(0)>0$, 0 is the point of local min.

$F''(8)<0$, 8 is the point of local max.

F(8)=251 and f(0)=-5

Question 12.
Find the point of local maxima or local minima or local minima and the corresponding local maximum and minimum values of each of the following functions:

$$f(x) = (x-1)(x+2)^2$$

Answer:
local max. value is 0 at x = –2 and local min. value is –4 at x = 0

$f'(x)=(x-1)2(x+2)+(x+2)^2=0$

x=0,-2

$f''(0)>0$, 0 is the point of local min.

$f''(-2)<0$, -2 is the point of local max.

f(0)=-4

f(-2)=0

Question 13.
Find the point of local maxima or local minima or local minima and the corresponding local maximum and minimum values of each of the following functions:

$$f(x) = -(x-1)^3 (x+1)^2$$

Answer:

local max. value is 0 at each of the points x = 1 and x = –1 and local min. value is $\frac{-3456}{3125}$ at

$$x = -\frac{1}{5}$$

$F'(x)=-(x-1)^3 2(x+1)-3(x-1)^2(x+1)^2=0$

$$\Rightarrow x = 1, -1, -\frac{1}{5}$$

Since, $f^{ll}(1)$ and $f^{ll}(-1) <0$, 1 and -1 are the points of local max.

$F^{ll}(-\frac{1}{5})>0$, $-\frac{1}{5}$ is the point of local min.

F(1)=f(-1)=0

Also, $f\left(-\frac{1}{5}\right) = -\frac{3456}{3125}$

Question 14.

Find the point of local maxima or local minima or local minima and the corresponding local maximum and minimum values of each of the following functions:

$$f(x)=\frac{x}{2}+\frac{2}{x}, x>0$$

Answer:
local min. value is 2 at x = 2

$F'(x)=\frac{1}{2}-\frac{2}{x^2}=0$

$\Rightarrow x^2-4=0$

$\Rightarrow x=\pm 2$

But since x>0, x=2

$F''(2)=\frac{2}{x^3}$

$=\frac{2}{8}<0$

∴point of local mini. is 2

$F(2)=\frac{2}{2}+\frac{2}{2}=2$

Question 15.
Find the maximum and minimum values of $2x^3-24x+107$ on the interval $[-3,3]$.

Answer:
max. value is 139 at x = −2 and min. value is 89 at x = 3

$F'(x)=6x^2-24=0$

$6(x^2-4)=0$

$6(x^2-2^2)=0$

$6(x-2)(x+2)=0$

$X=2,-2$

Now, we shall evaluate the value of f at these points and the end points

$F(2)=2(2)^3-24(2)+107=75$

$F(-2)=2(-2)^3-24(-2)+107=139$

$F(-3)=2(-3)^3-24(-3)+107=125$

$F(3)=2(3)^3-24(3)+107=89$

Question 16.
Find the maximum and minimum values of $3x^4-8x^3+12x^2-48x+1$ on the interval $[1,4]$.

Answer:
max. value is 257 at x = 4 and min. value is –63 at x = 2

$F^{|}(x)=12x^3-24x^2+24x-48=0$

$12(x^3-2x^2+2x-4)=0$

Since for x=2, $x^3-2x^2+2x-4=0$, x-2 is a factor

On dividing x^3-2x^2+2x-4 by x-2, we get,

$12(x-2)(x^2+2)=0$

$X=2,4$

Now, we shall evaluate the value of f at these points and the end points

$F(1)=3(1)^4-8(1)^3+12(1)^2-48(1)+1=-40$

$F(2)= 3(2)^4-8(2)^3+12(2)^2-48(2)+1=-63$

$F(4)= 3(4)^4-8(4)^3+12(4)^2-48(4)+1=257$

Question 17.

Find the maximum and minimum of

$$f(x)=\left(\sin x+\frac{1}{2}\cos x\right) \text{ in } 0\le x\le\frac{\pi}{2}$$

Answer:

max. value is $\frac{3}{4}$ at $x=\frac{\pi}{6}$ and min. value is $\frac{1}{2}$ at $x=\frac{\pi}{2}$

$F^{I}(x)=\cos x-\frac{1}{2}\sin x=0$

ð 2 cos x=sin x

$$\Rightarrow\frac{\pi}{6}=\frac{\pi}{3}$$

$$f\left(\frac{\pi}{2}\right)=\sin\frac{\pi}{2}+\frac{1}{2}\cos\frac{\pi}{2}=\frac{1}{2}$$

$$f\left(\frac{\pi}{6}\right)=\sin\frac{\pi}{6}+\frac{1}{2}\cos\frac{\pi}{6}=\frac{1}{2}+\frac{\sqrt{3}}{4}$$

$$f\left(\frac{\pi}{3}\right)=\sin\frac{\pi}{3}+\frac{1}{2}\cos\frac{\pi}{3}=\frac{\sqrt{3}}{2}+\frac{1}{4}$$

Question 18.

Show that the maximum value of $x^{1/x}$ is $e^{1/e}$

Answer:

The given function is

$$Y=x^{\frac{1}{x}}$$

Now, taking logarithm from both sides, we get..

$$\log y=\frac{1}{x}\log x$$

Differentiating both sides w.r.t x....

$$\frac{1}{y}y' = -\frac{1}{x^2}\ln(x) + \frac{1}{x^2}$$

$$\Rightarrow y' = \frac{y}{x^2}(1 - \ln(x))$$

(1-ln(x))=0

ln(x)=1

x=e

hence the max. point is x=e

max value is $e^{\frac{1}{e}}$.

Question 19.

Show that $\left(x + \frac{1}{x}\right)$ has a maximum and minimum, but the maximum value is less than the minimum value.

Answer:

F(x)=x+$\frac{1}{x}$

Taking first derivative and equating it to zero to find extreme points.

F$'$(x)=1-$\frac{1}{x^2} = 0$

X^2=1

x=1,x=-1

now to determine which of these is min. And max. We use second derivative.

$f^{||}(x)=\frac{2}{x^3}$

$f^{||}(1)=2$ and $f^{||}(-1)=-2$

since $f^{||}(1)$ is +ve it is minimum point while $f^{||}(-1)$ is –ve it is maximum point

max value-> $f(-1)=-1+\frac{1}{-1}=-2$

min vaue-> $f(1)=1+\frac{1}{1}=2$

hence maximum value is less than minimum value

Question 20.
Find the maximum profit that a company can make, if the profit function is given by
$p(x)=41+24x-18x^2$.

Answer:
49

$$\frac{dp}{dx}=-24-36x$$

=0

⇒x=−23

Step 2

$\frac{d^2p}{dx^2}$=−36 is negative

Step 3

maximum profit $=p\left(-\frac{2}{3}\right)$

=49

Question 21.

An enemy jet is flying along the curve $y = (x^2 + 2)$. A soldier is placed at the point (3, 2). Find the nearest point between the soldier and the jet.

Answer:
(1, 3)

Let P(x,y) be the position of the jet and the soldier is placed at A(3,2)

AP=$\sqrt{(x-3)^2+(y-2)^2}$

As $y=x^2+2$ or $y-2=x^2$

∴ $AP^2=(x-3)^2 + x^4=z$ (say)

$$\frac{dz}{dx} = 2(x-3) + 4x^3$$

$$\frac{dz}{dx} = 0$$

$2x-6+4x^3=0$ ∴

Put x=1

2-6+4=0

x-1 is a factor ∴

And $\frac{d^2z}{dx^2} = 12x^2 + 2$

$\frac{dz}{dx} = 0$ or x=1

and $\frac{d^2z}{dx^2}$(at x=1)>0

z is minimum when x=1,y=1+2=3 ∴

Point is (1,3)

Question 22.
Find the maximum and minimum values of

$$f(x) = (-x + 2\sin x) \text{ on } [0, 2\pi].$$

Answer:

max. value is $\left(-\frac{\pi}{3} + \sqrt{3}\right)$ at $x = \frac{\pi}{3}$ and min. value is $\left(\frac{5\pi}{3} + \sqrt{3}\right)$ at $x = \frac{5\pi}{3}$

$f'(x) = -1 + 2\cos x = 0$

$$\Rightarrow \cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3}$$

By finding the general solution, we get $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$

Now, by finding the second derivative, we get that $f''(\frac{\pi}{3}) < 0$ and $f''(\frac{5\pi}{3}) > 0$

Therefore, max. value is $\left(-\frac{\pi}{3} + \sqrt{3}\right)$ at $x = \frac{\pi}{3}$ and min. value is $\left(\frac{5\pi}{3} + \sqrt{3}\right)$ at $x = \frac{5\pi}{3}$