Exercise 19b

Question 1.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \frac{x-1}{y+2}$$

Answer:
(y + 2)dy = (x - 1)dx

Integrating on both sides,

$$\int (y + 2)dy = \int (x - 1)dx$$

$$\frac{y^2}{2} + 2y = \frac{x^2}{2} - x + C$$

$$y^2 + 4y - x^2 + 2x = C$$

Question 2.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \frac{x}{\left(x^2+1\right)}$$

Answer:
$$dy = \frac{x}{x^2 + 1}dx$$

Multiply and divide 2 in numerator and denominator of RHS,

$$y = \frac{1}{2}.\left(\frac{2x}{x^2 + 1}dx\right)$$

Integrating on both sides

$$y = \frac{1}{2}.\log(x^2 + 1) + C$$

Question 3.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

Answer:
$$\frac{1}{1 + y^2}dy = (1 + x)dx$$

Integrating on both sides

$$\int \frac{1}{1 + y^2}dy = \int (1 + x)dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^2}{2} + C$$

Question 4.
Find the general solution of each of the following differential equations:

$$(1+x^2)\frac{dy}{dx} = xy$$

Answer:
$$\frac{1}{y}.dy = \frac{x}{x^2 + 1}dx$$

Multiply and divide 2 in numerator and denominator of RHS,

$$\frac{1}{y}.dy = \frac{1}{2}.\left(\frac{2x}{x^2 + 1}dx\right)$$

Integrating on both sides

$$\log y = \frac{1}{2}.\log(1 + x^2) + \log C$$

$$\log y = \log\sqrt{1 + x^2} + \log C$$

$$\Rightarrow y = \sqrt{1 + x^2}.C_1$$

Question 5.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} + y = 1(y \neq 1)$$

Answer:

$$\frac{dy}{dx} = 1 - y$$

$$\frac{1}{1-y}dy = dx$$

Integrating on both sides

$$\int \frac{1}{1-y}dy = \int dx$$

$$\Rightarrow \log|1-y| = x + C$$

Question 6.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$

Answer:

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$$\frac{1}{\sqrt{1-y^2}}dy = -\frac{1}{\sqrt{1-x^2}}dx$$

Integrating on both sides

$$\int \frac{1}{\sqrt{1-y^2}}dy = \int -\frac{1}{\sqrt{1-x^2}}dx$$

$$\sin^{-1} y = \sin^{-1} x + C$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = C$$

Question 7.
Find the general solution of each of the following differential equations:

$$x\frac{dy}{dx} + y = y^2$$

Answer:
$$\Longrightarrow x.\frac{dy}{dx} + y = y^2$$

$$x.\frac{dy}{dx} = y^2 - y$$

$$\frac{1}{y^2 - y}dy = \frac{1}{x}dx$$

$$\frac{1}{y(y-1)}dy = \frac{1}{x}dx$$

Integrating on both the sides,

$$\int \frac{1}{y(y-1)}dy = \int \frac{1}{x}dx$$

LHS:

Let $\frac{1}{y(y-1)}dy = \frac{A}{y} + \frac{B}{(y-1)}$

$$\frac{1}{y(y-1)}dy = \frac{A(y-1)}{y} + \frac{By}{(y-1)}$$

$$1 = A(y-1) + By$$

$$1 = Ay + By - A$$

Comparing coefficients in both the sides,

A = - 1, B = 1

$$\frac{1}{y(y-1)}dy = -\frac{1}{y} + \frac{1}{(y-1)}$$

$$\int \frac{1}{y(y-1)}dy = \int \left[-\frac{1}{y} + \frac{1}{(y-1)}\right]dy$$

$$\int -\frac{1}{y}dy + \int \frac{1}{(y-1)}dy$$

$$-\log y + \log(y-1)$$

$$\Rightarrow \log\left(\frac{y-1}{y}\right)$$

RHS:

$$\int \frac{1}{x}dx$$

$$\int \frac{1}{x}dx = \log x + \log C$$

Therefore the solution of the given differential equation is

$$\log\left(\frac{y-1}{y}\right) = \log x + \log C$$

$$\frac{y-1}{y} = x.C$$

$$y-1 = yxC$$

$$\Rightarrow y = 1 + xyC$$

Question 8.
Find the general solution of each of the following differential equations:

x^2 (y + 1) dx + y^2 (x – 1) dy = 0

Answer:
$$x^2(y + 1)dx + y^2(x-1)dy = 0$$

$$x^2(y + 1)dx = -y^2(x-1)dy$$

$$x^2(y + 1)dx = y^2(1-x)dy$$

$$\frac{x^2}{(1-x)}dx = \frac{y^2}{y + 1}dy$$

Add and subtract 1 in numerators of both LHS and RHS,

$$\frac{x^2-1 + 1}{(1-x)}dx = \frac{y^2-1 + 1}{y + 1}dy$$

$$\frac{(x^2-1) + 1}{(1-x)}dx = \frac{(y^2-1) + 1}{y + 1}dy$$

By the identity, $(a^2-b^2) = (a + b).(a-b)$

$$\frac{(x + 1)(x-1) + 1}{(1-x)}dx = \frac{(y + 1)(y-1) + 1}{(y + 1)}dy$$

Splitting the terms,

$$-(x + 1)dx + \frac{1}{(1-x)}dx = (y-1)dy + \frac{1}{(y + 1)}dy$$

Integrating,

$$\int -(x + 1)dx + \int \frac{1}{(x-1)}dx = \int (y-1)dy + \int \frac{1}{(y + 1)}dy$$

$$-\left(\frac{x^2}{2} + x\right) + \log|x-1| = \left(\frac{y^2}{2} - y\right) + \log|1 + y| + C$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} + x - y + \log|x-1| + \log|1 + y| = C$$

Question 9.
Find the general solution of each of the following differential equations:

$$y\left(1-x^2\right)\frac{dy}{dx} = x\left(1+y^2\right)$$

Answer:

$$\frac{y}{1 + y^2}dy = \frac{x}{1-x^2}dx$$

Multiply 2 in both LHS and RHS,

$$\frac{2y}{1 + y^2}dy = \frac{2x}{1-x^2}dx$$

Integrating on both the sides,

$$\int \frac{2y}{1 + y^2}dy = \int \frac{2x}{1-x^2}dx$$

$$\log(1 + y^2) = -\log(1-x^2) + \log C$$

$$\log(1 + y^2) + \log(1-x^2) = \log C$$

$= (1 + y^2).(1 - x^2) = C$

Question 10.
Find the general solution of each of the following differential equations:

y log y dx – x dy = 0

Answer:
$y.\log y\, dx = xdx$

$$\frac{1}{x}dx = \frac{1}{y.\log y}dy$$

Integrating on both the sides,

$$\int \frac{1}{x}dx = \int \frac{1}{y.\log y}dy$$

LHS:

$$\int \frac{1}{x}dx = \log x$$

RHS:

$$\int \frac{1}{y.\log y}dy$$

Let $\log y = t$

So, $\frac{1}{y}dy = dt$

$$\int \frac{1}{y.\log y}dy = \int \frac{1}{t}dt$$

$= \log t$

$$= \log(\log y)$$

Therefore the solution of the given differential equation is

$$\log x = \log(\log y) + \log C$$

$$x = \log y . C$$

Question 11.
Find the general solution of each of the following differential equations:

$x(x^2 - x^2 y^2)$ dy + $y(y^2 + x^2y^2)$ dx = 0

Answer:
$$x . x^2(1 - y^2)dy + y . y^2(1 + x^2)dx = 0$$

$$x^3(1 - y^2)dy + y^3(1 + x^2)dx = 0$$

$$\frac{1 + x^2}{x^3}dx + \frac{1 - y^2}{y^3}dy = 0$$

$$\frac{1}{x^3}dx + \frac{1}{x}dx + \frac{1}{y^3}dy - \frac{1}{y}dy = 0$$

Integrating ,

$$\int \frac{1}{x^3}dx + \int \frac{1}{x}dx + \int \frac{1}{y^3}dy - \int \frac{1}{y}dy = C$$

$$\frac{x^{-3+1}}{-3+1} + \log x - \log y + \frac{y^{-3+1}}{-3+1} = C$$

$$-\frac{1}{2x^2} + -\frac{1}{2y^2} + \log x - \log y = C$$

$$-\frac{1}{2x^2} + -\frac{1}{2y^2} + \log\left(\frac{x}{y}\right) = C$$

Question 12.
Find the general solution of each of the following differential equations:

$(1 - x^2)\ dy + xy\ (1 - y)\ dx = 0$

Answer:
$(1-x^2)dy = -xy(1-y)dx$

$(1-x^2)dy = xy(y-1)dx$

$$\frac{1}{y(y-1)}dy = \frac{x}{1-x^2}dx$$

Integrating on both the sides,

$$\int \frac{1}{y(y-1)}dy = \int \frac{x}{1-x^2}dx$$

LHS:

Let $\frac{1}{y(y-1)}dy = \frac{A}{y} + \frac{B}{(y-1)}$

$$\frac{1}{y(y-1)}dy = \frac{A(y-1)}{y} + \frac{By}{(y-1)}$$

$1 = A(y-1) + By$

$\Rightarrow 1 = Ay + By - A$

Comparing coefficients in both the sides,

A = - 1, B = 1

$$\frac{1}{y(y-1)}dy = -\frac{1}{y} + \frac{1}{(y-1)}$$

$$\int \frac{1}{y(y-1)}dy = \int \left[-\frac{1}{y} + \frac{1}{(y-1)}\right]dy$$

$$\int -\frac{1}{y}dy + \int \frac{1}{(y-1)}dy$$

$$-\log y + \log(y-1)$$

$$= \log\left(\frac{y-1}{y}\right)$$

RHS:

$$\int \frac{x}{1-x^2}dx$$

Multiply and divide 2

$$\frac{1}{2}.\int \frac{2x}{1-x^2}dx$$

$$-\frac{1}{2}.\log(1-x^2) + \log C$$

$$-\log\sqrt{1-x^2} + \log C$$

Therefore the solution of the given differential equation is

$$\log\left(\frac{y-1}{y}\right) = -\log\sqrt{1-x^2} + \log C$$

$$-\log\left(\frac{y-1}{y}\right) = \log\sqrt{1-x^2} + \log C$$

$$\log\left(\frac{y}{y-1}\right) = \log\sqrt{1-x^2} + \log C$$

$$\frac{y}{y-1} = \sqrt{1-x^2}.C$$

$= y = (y-1).\sqrt{1-x^2}.C$

Question 13.

Find the general solution of each of the following differential equations:

$(1 - x^2)(1 - y)\, dx = xy\,(1 + y)\, dy$

Answer:

$$\frac{1-x^2}{x}dx = \frac{y(1+y)}{(1-y)}dy$$

$$\left[\frac{1}{x} - x\right]dx = \left[\frac{y + y^2}{1-y}\right]dy$$

$$\left[\frac{1}{x} - x\right]dx = \left[\frac{y}{1-y} + \frac{y^2}{1-y}\right]dy$$

Integrating on both the sides,

$$\int \left[\frac{1}{x} - x\right]dx = \int \left[\frac{y}{1-y} + \frac{y^2}{1-y}\right]dy$$

LHS:

$$\int \left[\frac{1}{x} - x\right]dx = \log x - \frac{x^2}{2}$$

RHS:

$$\int \frac{y}{1-y}dy = \int \frac{y-1+1}{1-y}dy$$

$$\int \frac{y-`1}{1-y}dy + \int \frac{1}{1-y}dy$$

$$\int -1.dy + \int \frac{1}{1-y}dy$$

$$-y + \log|1-y|$$

$$\int \frac{y^2}{1-y} dy$$

Add and subtract 1 in numerators of both LHS and RHS,

$$\frac{y^2 - 1 + 1}{(1-y)} dy$$

$$\frac{(y^2 - 1) + 1}{(1-y)} dy$$

By the identity, $(a^2 - b^2) = (a + b).(a - b)$

$$\frac{(y + 1)(y - 1) + 1}{(1-y)} dy$$

Splitting the terms,

$$-(y + 1)dy + \frac{1}{(1-y)} dy$$

Integrating,

$$\int -(y + 1)dy - \int \frac{1}{(y-1)} dy$$

$$-\left(\frac{y^2}{2} + y\right) + \log|y-1|$$

Therefore the solution of the given differential equation is

$$\log x - \frac{x^2}{2} = -y + \log|1-y| - \left(\frac{y^2}{2} + y\right) + \log|y-1|$$

$$= \log|x.(1-y)^2| = \frac{x^2}{2} - \frac{y^2}{2} - 2y + C$$

Question 14.
Find the general solution of each of the following differential equations:

$(y + xy)\ dx + (x - xy^2)\ dy = 0$

Answer:
$y(1 + x)dx + x(1 - y^2)dy = 0$

$$\frac{1 + x}{x}dx + \frac{1 - y^2}{y}dy = 0$$

$$\frac{1}{x}dx + 1.dx + \frac{1}{y}dy - ydy = 0$$

Integrating ,

$$\int \frac{1}{x}dx + \int 1.dx + \int \frac{1}{y}dy - \int ydy = C$$

$$\log|x| + x + \log|y| - \frac{y^2}{2} = C$$

$$= \log|xy| + x - \frac{y^2}{2} = C$$

Question 15.
Find the general solution of each of the following differential equations:

$(x^2 - yx^2)\ dy + (y^2 + xy^2)\ dx = 0$

Answer:
$x^2(1 - y)dy + y^2(1 + x)dx = 0$

$$\frac{1 + x}{x^2}dx + \frac{1 - y}{y^2}dy = 0$$

$$\frac{1}{x^2}dx + \frac{1}{x}dx + \frac{1}{y^2}dy - \frac{1}{y}dy = 0$$

Integrating,

$$\int \frac{1}{x^2}dx + \int \frac{1}{x}dx + \int \frac{1}{y^2}dy - \int \frac{1}{y}dy = C$$

$$-\frac{1}{x} + \log|x| - \frac{1}{y} - \log|y| = C$$

$$\log\left|\frac{x}{y}\right| = \frac{1}{x} + \frac{1}{y} + C$$

Question 16.
Find the general solution of each of the following differential equations:

$$\left(x^2y - x^2\right)dx + \left(xy^2 - y^2\right)dy = 0$$

Answer:
$x^2(y-1)dx + y^2(x-1)dy = 0$

$$\frac{x^2}{x-1}dx + \frac{y^2}{y-1}dy = 0$$

Add and subtract 1 in numerators ,

$$\frac{x^2 - 1 + 1}{(x-1)}dx + \frac{y^2 - 1 + 1}{(y-1)}dy$$

$$\frac{(x^2-1) + 1}{(x-1)}dx + \frac{(y^2-1) + 1}{(y-1)}dy$$

By the identity, $(a^2 - b^2) = (a+b).(a-b)$

$$\frac{(x+1)(x-1) + 1}{(x-1)}dx + \frac{(y+1)(y-1) + 1}{(y-1)}dy$$

Splitting the terms,

$$(x + 1)dx + \frac{1}{(x-1)}dx + (y + 1)dy + \frac{1}{(y-1)}dy$$

Integrating,

$$\int (x + 1)dx + \int \frac{1}{(x-1)}dx + \int (y + 1)dy + \int \frac{1}{(y-1)}dy = C$$

$$\frac{x^2}{2} + x + \log|x-1| + \frac{y^2}{2} + y + \log|y-1|$$

$$\frac{1}{2}.(x^2 + y^2) + (x + y) + \log|(x-1)(y-1)|$$

Question 17.
Find the general solution of each of the following differential equations:

$$x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$$

Answer:

$$\frac{x}{\sqrt{1 + x^2}}dx + \frac{y}{\sqrt{1 + y^2}}dy = 0$$

Integrating,

$$\int \frac{x}{\sqrt{1 + x^2}}dx + \int \frac{y}{\sqrt{1 + y^2}}dy$$

$$= C\ formula: \left\{\frac{d}{dx}\left(\sqrt{1 + x^2}\right) = \frac{2x}{2.\sqrt{1 + x^2}} = \frac{x}{\sqrt{1 + x^2}}\right\}$$

$$\sqrt{1 + x^2} + \sqrt{1 + y^2} = C$$

Question 18.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = e^{x+y} + x^2 e^y$$

Answer:

$$\frac{dy}{dx} = e^x.e^y + x^2.e^y$$

$$\frac{dy}{dx} = e^y(e^x + x^2)$$

$$\frac{1}{e^y}dy = (e^x + x^2)dx$$

Integrating on both the sides,

$$\int \frac{1}{e^y}dy = \int (e^x + x^2)dx$$

$$-e^{-y} = e^x + \frac{x^3}{3} + C$$

$$e^x + e^{-y} + \frac{x^3}{3} = C$$

Question 19.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \frac{3e^{2x} + 3d^{4x}}{e^x + e^{-x}}$$

Answer:
Considering 'd' as exponential 'e'

$$\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$$

$$\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + \frac{1}{e^x}}$$

$$\frac{dy}{dx} = \frac{(3e^{2x} + 3e^{4x}).e^x}{e^{2x} + 1}$$

$$\frac{dy}{dx} = \frac{3.e^{2x}(1 + e^{2x}).e^{x}}{e^{2x} + 1}$$

$$\frac{dy}{dx} = 3.e^{3x}$$

$$dy = 3.e^{3x}dx$$

Integrating on both the sides,

$$\int dy = \int 3.e^{3x}dx$$

$$y = 3.\frac{e^{3x}}{3} + C$$

$$y = e^{3x} + C$$

Question 20.
Find the general solution of each of the following differential equations:

$$3e^{x} \tan y\, dx + (1 - e^{x})\sec^{2} y\, dy = 0$$

Answer:
$\Rightarrow 3.e^{x}.\tan y\, dx = (e^{x} - 1)\sec^{2} y\, dy$

$$3.\frac{e^{x}}{e^{x} - 1}dx = \frac{\sec^{2} y}{\tan y}dy$$

$$3.\left[\frac{1}{\frac{e^{x} - 1}{e^{x}}}\right]dx = \frac{\sec^{2} y}{\tan y}dy$$

$$3.\left[\frac{1}{1 - e^{-x}}\right]dx = \frac{\sec^{2} y}{\tan y}dy$$

Integrating on both the sides,

$$\int 3.\left[\frac{1}{1-e^{-x}}\right]dx = \int \frac{\sec^2 y}{\tan y}dy$$

$$3.\log|1-e^{-x}| = \log|\tan y| + \log C$$ formula: $\left\{\frac{d}{dy}\tan y = \frac{1}{\tan y}.\sec^2 y\right\}$

$$\log(1-e^{-x})^3 = \log|\tan y| + \log C$$

$$\tan y = (1-e^{-x})^3.C$$

Question 21.
Find the general solution of each of the following differential equations:

$$e^y\left(1+x^2\right)dy - \frac{x}{y}dx = 0$$

Answer:
$$e^y(1 + x^2)dy = \frac{x}{y}dx$$

$$e^y.y\,dy = \frac{x}{1 + x^2}dx$$

Integrating on both the sides,

$$\int e^y.y\,dy = \int \frac{x}{1 + x^2}dx$$

LHS:

$$\int e^y.y\,dy$$

By ILATE rule,

$$\int e^y.y\,dy = y.\int e^y dy - \int\left[\frac{d}{dy}(y).\int e^y dy\right]dy$$

$$y.e^y - \int e^y\,dy$$

$$y.e^y - e^y$$

$$e^y(y-1)$$

RHS:

$$\int \frac{x}{1\ +\ x^2}dx$$

Multiply and divide by 2

$$\frac{1}{2}\int \frac{2x}{1\ +\ x^2}dx$$

$$\frac{1}{2}.\log|1\ +\ x^2|$$

$$\log\sqrt{1\ +\ x^2}$$

Therefore the solution of the given differential equation is

$$\Rightarrow e^y(y-1)\ =\ \log\sqrt{1\ +\ x^2}\ +\ C$$

Question 22.
Find the general solution of each of the following differential equations:

$$\frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{e}^{\mathrm{x}+\mathrm{y}} + \mathrm{e}^{\mathrm{x}-\mathrm{y}}$$

Answer:

$$\frac{dy}{dx}\ =\ e^x.e^y\ +\ e^x.e^{-y}$$

$$\frac{dy}{dx}\ =\ e^x(e^y\ +\ e^{-y})$$

$$\frac{1}{e^y + e^{-y}} dy = e^x dx$$

$$\frac{1}{e^y + \frac{1}{e^y}} dy = e^x dx$$

$$\frac{e^y}{(e^y)^2 + 1} dy = e^x dx$$

Integrating on both the sides,

$\int \frac{e^y}{(e^y)^2+1} dy = \int e^x dx$ formula: $\left\{\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}\right\}$

$= \Rightarrow \tan^{-1} e^{-y} = e^x + C$

Question 23.
Find the general solution of each of the following differential equations:

(e^y + 1) cos x dx + e^y sin x dy = 0

Answer:

$$\frac{\cos x}{\sin x} dx + \frac{e^y}{e^y + 1} dy = 0$$

$$\cot x\, dx + \frac{e^y}{e^y + 1} dy = 0$$

Integrating,

$$\int \cot x\, dx + \int \frac{e^y}{e^y + 1} dy = C$$

$$\log|\sin x| + \log|e^y + 1| = \log C$$

$$\log|\sin x.(e^y + 1)| = \log C$$

$$\Rightarrow \sin x.(e^y + 1) = C$$

Question 24.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx}+\frac{xy+y}{xy+x}=0$$

Answer:

$$\frac{dy}{dx} + \frac{y(1 + x)}{x(1 + y)} = 0$$

$$\frac{1 + y}{y}dy + \frac{1 + x}{x}dx = 0$$

$$\frac{1}{y}dy + 1.dy + \frac{1}{x}dx + 1.dx = 0$$

Integrating ,

$$\int \frac{1}{y}dy + \int 1.dy + \int \frac{1}{x}dx + \int 1.dx = C$$

$$\log|y| + y + \log|x| + x = C$$

$$\Rightarrow \log|xy| + x + y = C$$

Question 25.
Find the general solution of each of the following differential equations:

$$\sqrt{1-x^4}\,dy = x\,dx$$

Answer:

$$dy = \frac{x}{\sqrt{1 - x^4}}dx$$

Multiply and divide by 2,

$$dy = \frac{1}{2} \cdot \frac{2x}{\sqrt{1-x^4}} dx$$

$$dy = \frac{1}{2} \cdot \frac{2x}{\sqrt{1-(x^2)^2}} dx$$

Integrating on both the sides,

$\int dy = \frac{1}{2} . \int dy = \frac{1}{2} . \frac{2x}{\sqrt{1-x^4}} dx$ formula: $\left\{\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}\right\}$

$\Rightarrow y = \frac{1}{2} . \sin^{-1} x^2 + c$

Question 26.
Find the general solution of each of the following differential equations:

$$\operatorname{cosec} x \log y \frac{dy}{dx} + x^2 y = 0$$

Answer:

$$\frac{\log y}{y} dy + \frac{x^2}{\csc x} dx = 0$$

$$\frac{\log y}{y} dy + x^2 . \sin x \, dx = 0$$

Integrating ,

$$\int \frac{\log y}{y} dy + \int x^2 . \sin x \, dx = C$$

Consider the integral $\int \frac{\log y}{y} dy$

Let $\log y = t$

So, $\frac{1}{y} dy = dt$

$$\int \frac{\log y}{y} dy = \int t.dt$$

$$\frac{t^2}{2}$$

$$\frac{(\log y)^2}{2}$$

Consider the integral $\int x^2.\sin x\, dx$

By ILATE rule,

$$\int x^2.\sin x\, dx = x^2 \int \sin x\, dx - \int \left[\frac{d}{dx}(x^2) \int \sin x\, dx.\right] dx$$

$$-x^2.\cos x - \int \left[2x. \int \sin x\, dx\right] dx$$

$$-x^2 \cos x + 2 \int [x.\cos x]\, dx$$

Again by ILATE rule,

$$-x^2 \cos x + 2\left[x. \int \cos x\, dx - \int \left\{\frac{d}{dx} x. \int \cos x\, dx\right\} dx\right]$$

$$-x^2 \cos x + 2\left[x \sin x - \int \sin x\, dx\right]$$

$$-x^2 \cos x + 2\,[x \sin x + \cos x]$$

$$-x^2 \cos x + 2\,x \sin x + 2 \cos x$$

$$\cos x\,(2 - x^2) + 2x \sin x$$

Therefore the solution of the given differential equation is,

$$\frac{(\log y)^2}{2} + \cos x\,(2 - x^2) + 2x\sin x = C$$

Question 27.
Find the general solution of each of the following differential equations:

$$y\,dx + \left(1 + x^2\right)\tan^{-1}x\,dy = 0$$

Answer:

$$\frac{1}{\tan^{-1}x.(1 + x^2)}dx + \frac{1}{y}dy = 0$$

Integrating,

$$\int \frac{1}{\tan^{-1}x.(1 + x^2)}dx + \int \frac{1}{y}dy = C$$

Consider the integral $\int \frac{1}{\tan^{-1}x.(1 + x^2)}dx$

Let $\tan^{-1}x = t$

So, $\frac{1}{1 + x^2}dx = dt$

$$\int \frac{1}{\tan^{-1}x.(1 + x^2)}dx = \int \frac{1}{t}dt$$

$\log t$

$\log(\tan^{-1}x)$

Consider the integral $\int \frac{1}{y}dy$

$\log y$

Therefore the solution of the differential equation is

$$\log(\tan^{-1}x) + \log y = \log C$$

$$\tan^{-1}x . y = C$$

Question 28.
Find the general solution of each of the following differential equations:

$$\frac{1}{x} . \frac{dy}{dx} = \tan^{-1} x$$

Answer:
$dy = x . \tan^{-1} x \, dx$

Integrating on both the sides,

$$\int dy = \int x . \tan^{-1} x \, dx$$

$$y = \tan^{-1} x \int x \, dx - \int \left[\frac{d}{dx}(\tan^{-1} x) . \int x \, dx\right] dx \; \langle by\ ILATE\ rule \rangle$$

$$y = \tan^{-1} x . \frac{x^2}{2} - \int \left[\frac{1}{1 + x^2}\right] . \frac{x^2}{2} dx$$

$$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2} . \int \frac{x^2}{x^2 + 1} dx$$

$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2} \int \left[\frac{x^2 - 1 + 1}{x^2 + 1}\right]$ (adding and subtracting 1)

$$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2} \int \left[1 - \frac{1}{x^2 + 1}\right] dx$$

$$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2}[x - \tan^{-1} x] + C$$

$$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2} x + \frac{\tan^{-1} x}{2} + C$$

$$y = \frac{1}{2}.\tan^{-1} x\,(x^2 + 1) - \frac{1}{2}x + C$$

Question 29.
Find the general solution of each of the following differential equations:

$$e^x\sqrt{1-y^2}dx + \frac{y}{x}dy = 0$$

Answer:

$$e^x.x\,dx + \frac{y}{\sqrt{1-y^2}}dy = 0$$

Integrating,

$$\int e^x.x\,dx + \int \frac{y}{\sqrt{1-y^2}}dy = C$$

Consider the integral $\int e^x.x\,dx$

By ILATE rule,

$$\int e^x.x\,dx = x.\int e^x\,dx - \int \left[\frac{d}{dx}(x).\int e^x\,dx\right]dx$$

$$x.e^x - \int e^x\,dx$$

$$x.e^x - e^x$$

$$e^x(x-1)$$

Consider the integral $\int \frac{y}{\sqrt{1-y^2}}dy$

Its value is - $\sqrt{1-y^2}$ as $\frac{d}{dx}\left(\sqrt{1-y^2}\right) = \frac{-2y}{2\sqrt{1-y^2}} = \frac{-y}{\sqrt{1-y^2}}$

Therefore the solution of the given differential equation is

$e^x(x-1) - \sqrt{1-y^2} = C$

Question 30.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$$

Answer:

$$\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$$

$$dy = \frac{1-\cos x}{1+\cos x}dx$$

$\cos x$ can be written as $\cos x = \frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$

$$dy = \frac{1-\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}{1+\frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}dx$$

$$dy = \frac{\left[\frac{1+\tan^2\left(\frac{x}{2}\right)-\left(1-\tan^2\left(\frac{x}{2}\right)\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right]}{\frac{1+\tan^2\left(\frac{x}{2}\right)+\left(1-\tan^2\left(\frac{x}{2}\right)\right)}{1+\tan^2\left(\frac{x}{2}\right)}}dx$$

$$dy = \frac{1+\tan^2\left(\frac{x}{2}\right)-1+\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)+1-\tan^2\left(\frac{x}{2}\right)}dx$$

$$dy = \frac{2\tan^2\left(\frac{x}{2}\right)}{2}dx$$

$$dy = \tan^2\left(\frac{x}{2}\right)dx$$

Integrating on both the sides,

$$\int dy = \int \tan^2\left(\frac{x}{2}\right)dx$$

$y = \int\left[\sec^2\left(\frac{x}{2}\right) - 1\right]dx$ formula: $\{\sec^2 x - \tan^2 x = 1\}$

$y = 2.\tan\left(\frac{x}{2}\right) - x + C$ formula:$\left\{\frac{d}{dx}\tan\left(\frac{x}{2}\right) = \sec^2\left(\frac{x}{2}\right).\frac{1}{2}\right\}$

Question 31.
Find the general solution of each of the following differential equations:

$$(\cos x)\frac{dy}{dx} + \cos 2x = \cos 3x$$

Answer:
Given: $\frac{dy}{dx} + \frac{cos2x}{cosx} = \frac{cos3x}{cosx}$

$\Rightarrow \frac{dy}{dx} = \frac{cos(x+2x)-cos2x}{cosx}$

$\Rightarrow \frac{dy}{dx} = \frac{(cosxcos2x-sinxsin2x)-(2cos^2x-1)}{cosx}$

$\Rightarrow \frac{dy}{dx} = cos2x - \frac{2sinxcosxsinx}{cosx} - 2cosx + secx$

$\Rightarrow \frac{dy}{dx} = cos2x - 2sin^2x - 2cosx + secx$

$\Rightarrow y = \int(cos2x - 2sin^2x - 2cosx + secx)dx$

$\Rightarrow y = \int cos2xdx - \int 2sin^2xdx - \int 2cosxdx + \int secxdx$

$\Rightarrow y = \int cos2xdx - \int(1 - cos2x)dx - \int 2cosxdx + \int secxdx$

$\Rightarrow y = \frac{sin2x}{2} - 2sinx - x + log|secx + tanx| + c$

Question 32.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} + \frac{(1+\cos 2y)}{(1-\cos 2x)} = 0$$

Answer:
Given: $\frac{dy}{dx} + \frac{1+cos2y}{1-cos2x} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{2cos^2y}{2sin^2x}$

$\Rightarrow sec^2y\frac{dy}{dx} = -cosec^2x$

$\Rightarrow \int sec^2ydy = -\int cosec^2xdx$

$\Rightarrow tany = cotx + c$

Question 33.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} + \frac{\cos x \sin y}{\cos y} = 0$$

Answer:
Given: $\frac{dy}{dx} = -\frac{cosxsiny}{cosy}$

$\Rightarrow \frac{dy}{dx} = -cosxtany$

$\Rightarrow \int cotydy = -\int cosxdx$

$\Rightarrow \log|siny| = -sinx + c$

Question 34.
Find the general solution of each of the following differential equations:

cos x(1 + cos y)dx – sin y(1 + sin x)dy = 0

Answer:
Given: cosx(1+cosy)dx-siny(1+sinx)dy=0

Dividing the whole equation by (1+sinx)(1+cosy), we get,

$\Rightarrow \frac{\int \text{cosxdx}}{1+\text{sinx}} = \frac{\int \text{sinydy}}{1+\text{cosy}}$

⇒ log|1+sinx|+log|1+cosy|=logc

⇒ (1+sinx)(1+cosy)=c

Question 35.
Find the general solution of each of the following differential equations:

$\sin^3 x \, dx - \sin y \, dy = 0$

Answer:
Using $\sin^3 x = \frac{3sinx - sin3x}{4}$

We have,

$\Rightarrow \frac{3sinx - sin3x}{4} dx - sinydy = 0$

$\Rightarrow \frac{3}{4} sinxdx - \frac{sin3x}{4} dx - sinydy = 0$

$\Rightarrow \int \frac{3}{4} sinxdx - \int \frac{sin3x}{4} dx - \int sinydy = 0$

$\Rightarrow \frac{3}{4}(-cosx) + \frac{1}{12} cos3x + cosy = k$

$\Rightarrow 12cosy + cos3x - 9cosx = c$

Question 36.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx}+\sin(x+y)=\sin(x-y)$$

Answer:
$\frac{dy}{dx}+\sin(x+y)=\sin(x-y)$

$\Rightarrow \frac{dy}{dx}=\sin(x-y)-\sin(x+y)$

$\Rightarrow \frac{dy}{dx}=-2sinycosx$ (Using sin(A+B)-sin(A-B)=2sinBcosA)

$\Rightarrow -cosecydy=cosxdx$

$\Rightarrow -\int cosecydy=\int cosxdx$

$\Rightarrow -\log|cosecy-coty|=sinx+c$

⇒ sinx+log|cosecy-coty|+c=0

Question 37.
Find the general solution of each of the following differential equations:

$$\frac{1}{x}\cos^2 y\,dy+\frac{1}{y}\cos^2 x\,dx=0$$

Answer:
Given: $\frac{1}{x}cos^2ydy+\frac{1}{y}cos^2xdx=0$

$\Rightarrow ycos^2ydy+xcos^2xdx=0$

$\Rightarrow \frac{y}{2}(1+cos^2)dy+\frac{x}{2}(1+cos^2)dx=0$ (Using, $2cos^2a=1+cos2a$)

$\Rightarrow ydy + ycos^2ydy + xdx + xcos^2xdx = 0$

$\Rightarrow \frac{y^2}{2} + \frac{y}{2}sin^2y - \int \frac{sin^2y}{2}dy$

$\Rightarrow \frac{y^2}{2} + \frac{y}{2}sin^2y + \frac{cos^2}{4} + \frac{x^2}{2} + \frac{x}{2}sin^2x + \frac{sin^2}{4} = c$

Question 38.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \sin^3 x \cos^2 x + xe^x$$

Answer:
Here we have, $y = \int(\sin^3x\cos^2x + xe^x)\,dx$

$\Rightarrow \int \cos^2x(1-\cos^2x)\sin x dx + \int xe^x dx$

Taking cosx as t we have,

$\Rightarrow cosx = t,$

$\Rightarrow -sinxdx = dt,$

So we have,

$\Rightarrow y = \int cos^2xsinxdx - \int cos^4xsinxdx + \int xe^xdx$

$\Rightarrow y = -\int t^2dt - \int t^4(-dt) + \int xe^xdx$

$\Rightarrow y = -\frac{cos^3x}{3} + \frac{cos^5x}{5} + xe^x - e^x + c$

Question 39.
Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that y = 0 when x = 1.

Answer:
Given:

$$\frac{dy}{dx} = (1 + x)(1 + y)$$

$$\Rightarrow \frac{dy}{1+y} = (1 + x)dx$$

$$\Rightarrow log|y + 1| = (x + \frac{x^2}{2} + c)$$

$$\Rightarrow now, for\ y = 0\ and\ x = 1,$$

We have,

$$\Rightarrow 0 = 1 + \frac{1}{2} + c$$

$$\Rightarrow c = -\frac{3}{2}$$

$$\Rightarrow \log|y + 1| = \frac{x^2}{2} + x - \frac{3}{2}$$

Question 40.
Find the particular solution of the differential equation x(1 + y^2) dx -y(1 + x^2) dy = 0, given that y = 1 when x = 0.

Answer:

$$\frac{2xdx}{1+x^2} - \frac{2ydy}{1+y^2} = 0$$

$$\Rightarrow \frac{\log(1+x^2)}{1+y^2} = 0$$

$$\Rightarrow (1 + x^2) = c(1 + y^2)$$

$$\Rightarrow y = 1, x = 0$$

$$\Rightarrow 1 = c(2)$$

$$\Rightarrow c = \frac{1}{2}$$

$\Rightarrow 2(1+x^2) = 1+y^2$

$\Rightarrow 2+2x^2-1 = y^2$

$\Rightarrow 2x^2+1 = y^2$

$\Rightarrow y = \sqrt{2x^2+1}$

Question 41.

Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x+4y$, given that y = 0 when x = 0.

Answer:

$\log\left(\frac{dy}{dx}\right) = 3x+4y$

$\Rightarrow y = 0$

$\Rightarrow x = 0$

$\Rightarrow \frac{dy}{dx} = e^{3x}e^{4y}$

$\Rightarrow e^{-4y}dy = e^{3x}dx$

$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c$

$\Rightarrow$ For $y = 0, x = 0$, we have

$\Rightarrow -\frac{1}{4} = \frac{1}{3} + c$

$\Rightarrow c = -\frac{7}{12}$

$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$

Hence, the particular solution is:

$\Rightarrow 4e^{3x} + 3e^{-4x} = 7$

Question 42.
Solve the differential equation $(x^2 - yx^2)$
$dy + (y^2 + x^2y^2)\, dx = 0$, given that y = 1 when x = 1.

Answer:
$x^2(1-y)dy + y^2(1+x^2)dx = 0$

$\Rightarrow \frac{(1-y)}{y^2}dy + \frac{(1+x^2)}{x^2}dx = 0$

$\Rightarrow \int \frac{(1-y)}{y^2}dy + \int \frac{(1+x^2)}{x^2}dx = 0$

$\Rightarrow -\frac{1}{y} - logy - \frac{1}{x} + x = c$

For y=1,x=1, we have,

$\Rightarrow -1 - 0 - 1 + 1 = c$

$\Rightarrow c = -1$

Hence, the required solution is:

$\Rightarrow \frac{1}{y} + logy + \frac{1}{x} - x = 1$

Question 43.

Find the particular solution of the differential equation $e^x\sqrt{1-y^2}\,dx + \frac{y}{x}dy = 0,$ given that y = 1 when x = 0.

Answer:
Given:$e^x\sqrt{1-y^2dx} + \frac{y}{x}dy = 0$ Separating the variables we get,

$\Rightarrow xe^x dx + \frac{y}{\sqrt{1-y^2}}dy = 0$

$\Rightarrow \int xe^x dx + \int \frac{y}{\sqrt{1-y^2}} dy = 0$ Substituting $\sqrt{1-y^2} = t, 1 - y^2 = t^2, -2ydy = 2tdt$, we have,

$\Rightarrow xe^x - e^x - \frac{1}{2}\log\left|\sqrt{1-y^2}\right| = c$

For y=1 and x=0, we have,

$\Rightarrow 0 - 1 - 0 = c$

$\Rightarrow c = -1$

⇒ Hence, the particular solution will be:-

$\Rightarrow xe^x - e^x - \frac{1}{2}\log\left|\sqrt{1-y^2}\right| + 1 = 0$

Question 44.

Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x(2\log x + 1)}{(\sin y + y\cos y)}$, given that $y = \frac{\pi}{2}$ when x = 1.

Answer:

Given: $\frac{dy}{dx} = \frac{x(2logx+1)}{(siny+ycosy)}$

$\Rightarrow \int sinydy + \int ycosydy = \int 2xlogxdx + \int xdx$

Let $\int ycosydy = I$ Then,

$$\int ycosydy = (\int cosydy)\, y - \int ((\int ycosydy) . \frac{d}{dx} y) dy$$

And $\int xlogx = (\int xdx)logx - \int ((\int xdx)\frac{d}{dx}logx)dx$

We have,

$$\Rightarrow -cosy + ysiny + cosy = x^2logx - \frac{x^2}{2} + \frac{x^2}{2} + c$$

For $y = \frac{\pi}{2}, x = 1$ we have,

$$0 + \frac{\pi}{2} + 0 = 0 + c$$

$$c = \frac{\pi}{2}$$

$$\Rightarrow ysiny = x^2logx + \frac{\pi}{2}$$

Question 45.

Solve the differential equation $\frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{y}\sin 2\mathrm{x}$, given that y(0) = 1.

Answer:
We have,

$$\frac{dy}{dx} = ysin2x$$

$$\Rightarrow \frac{dy}{y} = sin2xdx$$

$$\Rightarrow logy = -\frac{cos2x}{2} + c$$

For y=1, x=0, we have,

c = $\frac{1}{2}$

$$\Rightarrow logy = \frac{1}{2}(1 - cos2x)$$

$$\Rightarrow logy = sin^2x$$

Thus,

The particular solution is:

$y = e^{sin^2 x}$

Question 46.

Solve the differential equation $(x+1)\frac{dy}{dx} = 2xy$, given that y(2) = 3.

Answer:

Given: $(x+1)\frac{dy}{dx} = 2xy$

$\Rightarrow \frac{dy}{y} = 2\frac{x}{x+1}dx$

$\Rightarrow logy = \int 2 - \frac{2}{x+1}dx$

$\Rightarrow logy = 2x - 2\log(x+1) + c$

For x=2 and y=3, we have,

c = 3log3 – 4

Hence, the particular solution is,

$\Rightarrow y(x+1)^2 = 27e^{2x-4}$

Question 47.

Solve $\frac{dy}{dx} = x(2\log x + 1)$, given that y = 0 when x = 2.

Answer:

we have, $\frac{dy}{dx} = 2xlogx + x$, Integrating we get,

$y = \int(2xlogx + x)dx,$

$$y = \int 2xlogxdx + xdx$$

$$y = \left(\int 2xdx\right)logx - \int [(\int 2xdx)\left(\frac{d}{dx} logx\right)] dx + \frac{x^2}{2} + c$$

given that y=0 when x=2

$$\Rightarrow y = x^2 logx - \frac{x^2}{2} + \frac{x^2}{2} + c$$

now putting x=2 and y=0,

$$\Rightarrow 0 = 4log2 + c$$

$$\Rightarrow c = -4log2$$

Thus, the solution is:

$$y = x^2 logx - 4log2$$

Question 48.

Solve $\left(x^3 + x^2 + x + 1\right)\frac{dy}{dx} = 2x^2 + x,$ given that y = 1 when x = 0.

Answer:

we have, $\left(x^3 + x^2 + x + 1\right)\frac{dy}{dx} = 2x^2 + x,$

$\left\{ \qquad \right\}$Given that: y=1 when x=0,

$$\Rightarrow (x^2 + 1)(x + 1)\frac{dy}{dx} = 2x^2 + x$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2+x+x^2}{(x^2+1)(x+1)} dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{x(x+1)+x^2+1-1}{(x^2+1)(x+1)} dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{x^2+1} dx + \frac{x^2+1-1}{(x^2+1)(x+1)} dx$$

$\Rightarrow dy = \frac{xdx}{x^2+1} + \frac{dx}{x+1} - \frac{dx}{(x^2+1)(x+1)}$

$\Rightarrow \int dy = \int \frac{xdx}{x^2+1} + \int \frac{dx}{x+1} - \int \frac{\frac{-1}{2}x+\frac{1}{2}}{x^2+1} dx + \int \frac{\frac{1}{2}}{x+1} dx$

$\Rightarrow y = \frac{1}{2}\log|x^2+1| + \log|x+1| + \frac{1}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1} x + \frac{1}{2}\log|x+1| + c$

$\Rightarrow y = \frac{3}{4}\log|x^2+1| + \frac{1}{2}\log|x+1| - \frac{1}{2}\tan^{-1} x + c$

For y=1, when x=0, we have,

$1 = 0 + 0 - 0 + c$

$\Rightarrow c = 1$

$$y = \frac{1}{2}\left\{\log|x+1| + \frac{3}{2}\log\left(x^2+1\right) - \tan^{-1} x\right\} + 1$$

Question 49.

Solve $\frac{dy}{dx} = y \tan x,$ given that y = 1 when x = 0.

Answer:

we have, $\frac{dy}{dx} = y \tan x,$

given that: y=1 when x=0

$\Rightarrow \frac{dy}{dx} = ytanx$

$\Rightarrow \frac{dy}{y} = tanx\, dx$

$\Rightarrow logy = losecx + c$

$\Rightarrow 0 = 0 + c$

$\Rightarrow ycosx = 1$ is the particular solution...

Question 50.

Solve $\frac{dy}{dx} = y^2 \tan 2x,$ given that y = 2 when x = 0.

Answer:

we have: $\frac{dy}{dx} = y^2 \tan 2x,$

Given that, y=2 when x=0

$\Rightarrow \frac{dy}{y^2} = tan2xdx$

$\Rightarrow \int \frac{dy}{y^2} = \int tan2xdx$ ...integrating both sides

$\Rightarrow -\frac{1}{y} = \frac{\log(sec2x)}{2}$

$\Rightarrow -\frac{1}{2} = 0 + c$

$\Rightarrow c = -\frac{1}{2}$

$\Rightarrow y(1 + logcos2x) = 2$ ...is the particular solution

Question 51.

Solve $\frac{dy}{dx} = y \cot 2x,$ given that y = 2 when $x = \frac{\pi}{4}.$

Answer:

we have $\frac{dy}{dx} = y \cot 2x,$

Given that, y=2 when x=$\frac{\pi}{2}$

$\Rightarrow \frac{dy}{y} = ycot2x$

$\Rightarrow \frac{dy}{y} = cot2xdx$

$\Rightarrow \int \frac{dy}{y} = \int cot2xdx$

$\Rightarrow logy = -\frac{\log(sin2x)}{2} + c$

$\Rightarrow log2 = 0$+c

$\Rightarrow$ Thus, $c = log2$

The particular solution is :- $log\frac{y}{\sqrt{sin2x}} = log2$

$\therefore y = 2\sqrt{sin2x}$

Question 52.

Solve (1 + x^2) sec2 y dy + 2x tan y dx = 0, given that $y = \frac{\pi}{4}$ when x = 1.

Answer:
we have, (1 + x^2) sec2 y dy + 2x tan y dx = 0,

Given that, $y = \frac{\pi}{4}$ when x=1

$\Rightarrow (1 + x^2)sec2ydy + 2xtanydx = 0$

$\Rightarrow \frac{sec^2y}{tany}dy + \frac{2x}{1+x^2}dx = 0$

$\Rightarrow \int \frac{sec^2y}{tany}dy + \int \frac{2x}{1+x^2}dx = 0$

$\Rightarrow logtany + \log(1 + x^2) = logc$

For $y = \frac{\pi}{4}, x = 1$

We have, $0 + log2 = logc,$

c = 2,

Hence the required particular solution is:-

$\therefore tany(1+x^2)=2$

Question 53.

Find the equation of the curve passing through the point $\left(0,\frac{\pi}{4}\right)$ whose differential equation is sin x cos y dx + cos x sin y dy = 0.

Answer:
we have, sin x cos y dx + cos x sin y dy = 0

$\Rightarrow sinx\ cosy\ dx + cosx\ siny\ dy = 0$

$\Rightarrow tanx\ dx + tany\ dy = 0$

$\Rightarrow \log secx + \log \mathrm{secy} = logc$

$\Rightarrow secx\ secy = c$

Given that, coordinates of point, $(0,\frac{\pi}{4})$

$\Rightarrow c=\sqrt{2}$

$\Rightarrow secy=\sqrt{2}cosx$

$\therefore y=cos^{-1}(\frac{1}{\sqrt{2}}secx)$ ...is the required particular solution

Question 54.
Find the equation of a curve which passes through the origin and whose differential equation is $\frac{\mathrm{dy}}{\mathrm{dx}}=\mathrm{e}^{\mathrm{x}}\sin\mathrm{x}$.

Answer:
Given, $\frac{dy}{dx}=e^x sinx$

$dy = e^x sinxdx$

$\Rightarrow \int dy = \int e^x sinxdx$

Let $I = \int e^x sinxdx$

$\Rightarrow I = \int e^x dx\ sinx\ - \int(\int e^x dx\ ).(\frac{d}{dx} sinx)\ dx$

$\Rightarrow I = e^x sinx - \int e^x cosxdx$

$\Rightarrow I = e^x sinx - \int e^x dx\ cosx\ - \int(\int e^x dx\ ).(\frac{d}{dx} cosx)\ dx$

$\Rightarrow I = e^x sinx - e^x cosx - \int e^x sinxdx$

$\Rightarrow 2I = e^x sinx - e^x cosx$

$\Rightarrow I = \frac{e^x sinx - e^x cosx}{2} + c$

$$\therefore\ y = \frac{e^x sinx - e^x cosx}{2} + c$$

For the curve passes through (0,0)

We have, c = $\frac{1}{2}$

$\therefore\ 2y - e^x sinx + e^x cosx = 1$

Question 55.
A curve passes through the point (0, -2) and at any point (x, y) of the curve, the product of the slope of its tangent and y-coordinate of the point is equal to the x-coordinate of the point. Find the equation of the curve.

Answer:
Given that the product of slope of tangent and y coordinate equals the x-coordinate i.e., $y\frac{dy}{dx} = x$

We have, $ydy = xdx$

$\Rightarrow \int y dy = \int x dx$

$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + c$

For the curve passes through (0, -2), we get c = 2,

Thus, the required particular solution is:-

$\therefore\ y^2 = x^2 + 4$

Question 56.
A curve passes through the point (-1, 1) and at any point (x, y) of the curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (-4, -3). Find the equation of the curve.

Answer:
Given : $\frac{dy}{dx} = \frac{2(y+3)}{x+4}$

$\Rightarrow \frac{dy}{y+3} = \frac{2dx}{x+4}$

$\Rightarrow \int \frac{dy}{y+3} = 2\int \frac{dx}{x+4}$

$\Rightarrow \log(y+3) = 2\log(x+4) + c$

The curve passes through (-2, 1)we have,

c = 0,

$\therefore\ y + 3 = (x+4)^2$

Question 57.
In a bank, principal increases at the rate fo r% per annum. Find the value of r if ` 100 double itself in 10 years.

(Given $\log_e 2$ = 0.6931)

Answer:
Given:
$$\frac{dp}{dt} = (\frac{r}{100}) \times p$$

Here, p is the principal, r is the rate of interest per annum and t is the time in years.

Solving the differential equation we get,

$$\frac{dp}{p} = \left(\frac{r}{100}\right) dt$$

$$\Rightarrow \int \frac{dp}{p} = \int \frac{r}{100} dt$$

$$\Rightarrow logp = \frac{rt}{100} + c$$

$$\Rightarrow p = e^{\frac{rt}{100}+c}$$

As it is given that the principal doubles itself in 10 years, so

Let the initial interest be p1 (for t = 0), after 10 years p1 becomes 2p1.

Thus, $p1 = e^c$ for (t = 0) ...(i)

$p = 2p1 = e^{\frac{r.(10)}{100}}.e^c$ ...(ii)

Substituting (i) in (ii), we get,

$$\Rightarrow 2\text{p1} = e^{\frac{r}{10}}.p1$$

$$\Rightarrow 2 = e^{\frac{r}{10}}$$

$$\Rightarrow log2 = \frac{r}{10}$$

$$\Rightarrow r = 10log2$$

$\Rightarrow r = 6.931$

∴ Rate of interest = 6.931

Question 58.
In a bank, principal increases at the rate of 5% per annum. An amount of ` 1000 is deposited in the bank. How much will it worth after 10 years?

(Given $e^{0.5}$ = 1.648)

Answer:
Given: rate of interest = 5%

P(initial) = Rs 1000

And,

$$\frac{dp}{dt} = \frac{5}{100} \times p$$

$$\Rightarrow \frac{dp}{p} = \frac{5}{100} dt$$

$$\Rightarrow \int \frac{dp}{p} = \int \frac{5}{100} dt$$

$$\Rightarrow \log p = \frac{5t}{100} + c$$

$$\Rightarrow p = e^{\frac{5t}{100} + c}$$

For t = 0, we have p = 1000

$$1000 = e^{c}$$

For t = 10 years we have, $p = e^{\frac{50}{100}}.1000$

$$p = 1000e^{1/2}$$

$$p = 1648$$

Thus, principal is Rs1648 for t = 10 years.

Question 59.
The volume of a spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of the balloon after t seconds.

Answer:
Given:

Volume V $= \frac{4\pi r^3}{3}$

$$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}$$

$\Rightarrow \frac{dV}{dt} = k$ (constant)

$$4\pi r^2 \frac{dr}{dt} = k$$

$\Rightarrow 4\pi r^2 dr = kdt$

$\Rightarrow \int 4\pi r^2 dr = \int kdt$

$\Rightarrow \frac{4\pi r^3}{3} = kt + c$

For t = 0, r = 3 and for t = 3, r = 6, So, we have,

$\Rightarrow \frac{4\pi(3)^3}{3} = 0 + c$

$\Rightarrow c = 36\pi$

$$\frac{4\pi(6)^3}{3} = k.(3) + 36\pi$$

$\Rightarrow \mathrm{k} = 84\pi$

So after t seconds the radius of the balloon will be,

$\Rightarrow \frac{4\pi r^3}{3} = 84\pi t + 36\pi$

$\Rightarrow 4\pi r^3 = 252\pi t + 108\pi$

$\Rightarrow r^3 = \frac{252\pi t + 108\pi}{4\pi}$

$\Rightarrow r^3 = 63t + 27$

$\Rightarrow r = \sqrt[3]{63t + 27}$

Hence, radius of the balloon as a function of time is

$\therefore\ r = (63t + 27)^{1/3}$

Question 60.
In a culture the bacteria count is 100000. The number is increased by 10% in 2 hours. In how many hours will the count reach 200000, if the rate of growth of bacteria is proportional to the number present?

Answer:
Let y be the bacteria count, then, we have,

rate of growth of bacteria is proportional to the number present

So,
$$\frac{dy}{dt} = cy$$

Where c is a constant,

Then, solving the equation we have,

$$\frac{dy}{y} = cdt$$

$$\int \frac{dy}{y} = \int cdt$$

$$logy = ct + k$$

Where k is constant of integration

$y = e^{ct+k}$

And we have for t = 0, y = 10000,

$10000 = e^k$ ...(i)

For t = 2hrs, y is increased by 10% i. e. y = 110000

$110000 = e^{c(2)}.e^k$

$\Rightarrow 110000 = e^{2c}.(100000)$ from (i)

$\Rightarrow e^{2c} = 1.1$

$\Rightarrow e^c = \sqrt{1.1}$

$\Rightarrow c = \frac{1}{2}\log(\frac{11}{10})$

When y = 200000, we have,

$200000 = e^{ct}.100000$

$\Rightarrow e^{ct} = 2$

$\Rightarrow (e^c)^t = 2$

$\Rightarrow tc = log2$

$\Rightarrow t = \frac{2log2}{log\frac{11}{10}}$

Hence, t = $\frac{2\log2}{\log\frac{11}{10}}$