Exercise 28h

Question 1.
Find the vector and Cartesian equations of the plane passing through the origin and parallel to the vectors $\left(\hat{i}+\hat{j}-\hat{k}\right)$ and $\left(3\hat{i}-\hat{k}\right)$.

Answer:
Given - $\vec{r} = \hat{i} + \hat{j} - \hat{k}$ & $\vec{r'} = 3\hat{i} - \hat{k}$ are two lines to which a plane is parallel and it passes through the origin.

To find – The equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

$\therefore \vec{r} \times \vec{r'}$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \hat{i}(-1-0) + \hat{j}(-3+1) + \hat{k}(0-3)$$

$$= -\hat{i} - 2\hat{j} - 3\hat{k}$$

The plane passes through origin (0, 0, 0).

Formula to be used – If a line passes through the point (a, b, c) and has the direction ratios as (a′, b′, c′), then its vector equation is given by $\vec{r} = \left(a\hat{i} + b\hat{j} + c\hat{k}\right) + \lambda\left(a'\hat{i} + b'\hat{j} + c'\hat{k}\right)$ where λ is any scalar constant

The required plane will be

$$\vec{r} = \left(0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}\right) + \lambda'\left(-\hat{i} - 2\hat{j} - 3\hat{k}\right)$$

$$\Rightarrow \vec{r} = \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

The vector equation : $\mathbf{\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 0}$

The Cartesian equation : **x + 2y + 3z = 0**

Question 2.
Find the vector equation of a plane passing through the point (1, 2, 3) and parallel to the lines whose direction ratios are 1, - 1, - 2, and - 1, 0, 2.

Answer:
Given – The lines have direction ratios of (1, - 1, - 2) and (- 1, 0, 2). The plane parallel to these lines passes through (1, 2, 3)

To find – The vector equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

$\vec{R} = \hat{\imath} - \hat{\jmath} - 2\hat{k}$ & $\vec{R'} = -\hat{\imath} + 2\hat{k}$, where the two vectors represent the directions

$\therefore \vec{R} \times \vec{R'}$

$$= \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & -2 \\ -1 & 0 & 2 \end{bmatrix}$$

$$= \hat{\imath}(-2 - 0) + \hat{\jmath}(2 - 2) + \hat{k}(0 - 1)$$

$$= -2\hat{\imath} - \hat{k}$$

The equation of the plane maybe represented as - 2x - z + d = 0

Now, this plane passes through the point (1, 2, 3)

Hence,

(- 2) × 1 - 3 + d = 0

⇒ d = 5

The Cartesian equation of the plane : - 2x - z + 5 = 0 i.e. 2x + z = 5

The vector equation : $\vec{\mathbf{r}}.\left(2\hat{\mathbf{i}} + \hat{\mathbf{k}}\right) = 5$

Question 3.
Find the vector and Cartesian equations of the plane passing through the point (3, - 1, 2) and parallel to the lines $\vec{r} = \left(-\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}-5\hat{j}-\hat{k}\right)$ and $\vec{r} = \left(\hat{i}-3\hat{j}+\hat{k}\right)+\mu\left(-5\hat{i}+4\hat{j}\right)$.

Answer:
Given - $\vec{r} = \left(-\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} - 5\hat{j} - \hat{k}\right)$ & $\vec{r} = \left(\hat{i} - 3\hat{j} + \hat{k}\right) + \mu(-5\hat{i} + 4\hat{j})$. A plane is parallel to both these lines and passes through (3, - 1, 2).

To find – The equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

$\vec{R} = 2\hat{i} - 5\hat{j} - \hat{k}$ & $\overrightarrow{R'} = -5\hat{i} + 4\hat{j}$, where the two vectors represent the directions

$\therefore \vec{R} \times \overrightarrow{R'}$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & -1 \\ -5 & 4 & 0 \end{bmatrix}$$

$$= \hat{i}(0 + 4) + \hat{j}(5 - 0) + \hat{k}(8 - 25)$$

$$= 4\hat{i} + 5\hat{j} - 17\hat{k}$$

The equation of the plane maybe represented as 4x + 5y - 17z + d = 0

Now, this plane passes through the point (3, - 1, 2)

Hence,

4 × 3 + 5 × (- 1) – 17 × 2 + d = 0

⇒ d = 27

The Cartesian equation of the plane : **4x + 5y - 17z + 27 = 0**

The vector equation : $\vec{\mathbf{r}}.\left(\mathbf{4\hat{i} + 5\hat{j} - 17\hat{k}}\right) + \mathbf{27} = \mathbf{0}$

Question 4.
Find the Cartesian and vector equations of a plane passing through the point (1, 2, - 4) and parallel to the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$ and $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z}{-1}$.

Answer:
Given - $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$ & $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z}{-1}$. A plane is parallel to both these lines and passes through (1, 2, - 4).

To find – The equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

The direction ratios of the given lines are (2, 3, 6) and (1, 1, - 1)

$\therefore \vec{R} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ & $\vec{R'} = \hat{i} + \hat{j} - \hat{k}$

$\therefore \vec{R} \times \vec{R'}$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{bmatrix}$$

$$= \hat{i}(-3 - 6) + \hat{j}(6 + 2) + \hat{k}(2 - 3)$$

$$= -9\hat{i} + 8\hat{j} - \hat{k}$$

The equation of the plane maybe represented as - 9x + 8y - z + d = 0

Now, this plane passes through the point (1, 2, - 4)

Hence,

(- 9) × 1 + 8 × 2 - (- 4) + d = 0

$\Rightarrow$ d = - 11

The Cartesian equation of the plane : - 9x + 8y - z - 11 = 0 i.e. **9x - 8y + z + 11 = 0**

The vector equation : $\vec{\mathbf{r}}.\left(\mathbf{9\hat{i} - 8\hat{j} + \hat{k}}\right) + \mathbf{11} = \mathbf{0}$

Question 5.

Find the vector equation of the plane passing through the point $\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and parallel to the vectors $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $\left(\hat{i}-\hat{j}+\hat{k}\right)$.

Answer:

Given - $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k}$ & $\vec{r'} = \hat{i} - \hat{j} + \hat{k}$ are two lines to which a plane is parallel and it passes through the point $3\hat{i} + 4\hat{j} + 2\hat{k}$

To find – The equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

$\therefore \vec{r} \times \vec{r'}$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$$

$$= \hat{i}(2 + 3) + \hat{j}(3 - 1) + \hat{k}(-1 - 2)$$

$$= 5\hat{i} + 2\hat{j} - 3\hat{k}$$

The equation of the plane maybe represented as 5x + 2y - 3z + d = 0

Now, this plane passes through the point (3, 4, 2)

Hence,

5 × 3 + 2 × 4 – 3 × 2 + d = 0

$\Rightarrow$ d = - 17

The Cartesian equation of the plane : 5x + 2y - 3z - 17 = 0 i.e. 5x + 2y - 3z = 17

The vector equation : $\vec{r}.\left(5\hat{i} + 2\hat{j} - 3\hat{k}\right) = 17$