

MATHEMATICS

Textbook for Class X



1062



एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

1062 – MATHEMATICS

Textbook for Class X

ISBN 81-7450-634-9

First Edition

December 2006 Pausa 1928

Reprinted

October 2007 Kartika 1929
January 2009 Pausa 1930
December 2009 Agrahayana 1931
November 2010 Kartika 1932
January 2012 Pausa 1933
November 2012 Kartika 1934
November 2013 Kartika 1935
November 2014 Agrahayana 1936
December 2015 Agrahayana 1937
December 2016 Pausa 1938
December 2017 Pausa 1939
January 2019 Pausa 1940
August 2019 Shrawana 1941
January 2021 Pausa 1942
November 2021 Agrahayana 1943

PD 570T RSP

© National Council of Educational
Research and Training, 2006

₹ 160.00

Printed on 80 GSM paper with
NCERT watermark

Published at the Publication Division by
the Secretary, National Council of
Educational Research and Training,
Sri Aurobindo Marg, New Delhi 110 016
and printed at Green World
Publications (India) Pvt. Ltd., Mander
Mode, Bamrauli, Allahabad (U.P.)
211003

ALL RIGHTS RESERVED

- ❑ No part of this publication may be reproduced, stored in a retrieval system or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise without the prior permission of the publisher.
- ❑ This book is sold subject to the condition that it shall not, by way of trade, be lent, re-sold, hired out or otherwise disposed of without the publisher's consent, in any form of binding or cover other than that in which it is published.
- ❑ The correct price of this publication is the price printed on this page. Any revised price indicated by a rubber stamp or by a sticker or by any other means is incorrect and should be unacceptable.

OFFICES OF THE PUBLICATION DIVISION, NCERT

NCERT Campus
Sri Aurobindo Marg
New Delhi 110 016 Phone : 011-26562708

108, 100 Feet Road
Hosdakere Halli Extension
Banashankari III Stage
Bengaluru 560 085 Phone : 080-26725740

Navjivan Trust Building
P.O. Navjivan
Ahmedabad 380 014 Phone : 079-27541446

CWC Campus
Opp. Dhankal Bus Stop
Panihati
Kolkata 700 114 Phone : 033-25530454

CWC Complex
Maligaon
Guwahati 781 021 Phone : 0361-2674869

Publication Team

Head, Publication Division : Anup Kumar Rajput
Chief Editor : Shweta Uppal
Chief Production Officer : Arun Chitkara
Chief Business Manager : Vipin Dewan
Editor : Bijnan Sutar
Production Assistant : Sunil Kumar

Illustrations

Joel Gill and Arvinder Chawla

Cover

Arvinder Chawla

Foreword

The National Curriculum Framework, 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that, given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

The National Council of Educational Research and Training (NCERT) appreciates the hard work done by the textbook development committee responsible for this book. We wish to thank the Chairperson of the advisory group in Science and Mathematics, Professor J.V. Narlikar and the Chief Advisors for this book, Professor P. Sinclair of IGNOU, New Delhi and Professor G.P. Dikshit (Retd.) of Lucknow University, Lucknow for guiding the work of this committee. Several teachers

contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. We are especially grateful to the members of the National Monitoring Committee, appointed by the Department of Secondary and Higher Education, Ministry of Human Resource Development under the Chairpersonship of Professor Mrinal Miri and Professor G.P. Deshpande, for their valuable time and contribution. As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

New Delhi
15 November 2006

Director
National Council of Educational
Research and Training

Preface

Through the years, from the time of the Kothari Commission, there have been several committees looking at ways of making the school curriculum meaningful and enjoyable for the learners. Based on the understanding developed over the years, a National Curriculum Framework (NCF) was finalised in 2005. As part of this exercise, a National Focus Group on Teaching of Mathematics was formed. Its report, which came in 2005, highlighted a constructivist approach to the teaching and learning of mathematics.

The essence of this approach is that children already know, and do some mathematics very naturally in their surroundings, before they even join school. The syllabus, teaching approach, textbooks etc., should build on this knowledge in a way that allows children to enjoy mathematics, and to realise that mathematics is more about a way of reasoning than about mechanically applying formulae and algorithms. The students and teachers need to perceive mathematics as something natural and linked to the world around us. While teaching mathematics, the focus should be on helping children to develop the ability to particularise and generalise, to solve and pose meaningful problems, to look for patterns and relationships, and to apply the logical thinking behind mathematical proof. And, all this in an environment that the children relate to, without overloading them.

This is the philosophy with which the mathematics syllabus from Class I to Class XII was developed, and which the textbook development committee has tried to realise in the present textbook. More specifically, while creating the textbook, the following broad guidelines have been kept in mind.

- The matter needs to be linked to what the child has studied before, and to her experiences.
- The language used in the book, including that for ‘word problems’, must be clear, simple and unambiguous.
- Concepts/processes should be introduced through situations from the children’s environment.
- For each concept/process give several examples and exercises, but not of the same kind. This ensures that the children use the concept/process again and again, but in varying contexts. Here ‘several’ should be within reason, not overloading the child.
- Encourage the children to see, and come out with, diverse solutions to problems.

- As far as possible, give the children motivation for results used.
- All proofs need to be given in a non-didactic manner, allowing the learner to see the flow of reason. The focus should be on proofs where a short and clear argument reinforces mathematical thinking and reasoning.
- Whenever possible, more than one proof is to be given.
- Proofs and solutions need to be used as vehicles for helping the learner develop a clear and logical way of expressing her arguments.
- All geometric constructions should be accompanied by an analysis of the construction and a proof for the steps taken to do the required construction. Accordingly, the children would be trained to do the same while doing constructions.
- Add such small anecdotes, pictures, cartoons and historical remarks at several places which the children would find interesting.
- Include optional exercises for the more interested learners. These would not be tested in the examinations.
- Give answers to all exercises, and solutions/hints for those that the children may require.
- Whenever possible, propagate constitutional values.

As you will see while studying this textbook, these points have been kept in mind by the Textbook Development Committee. The book has particularly been created with the view to giving children space to explore mathematics and develop the abilities to reason mathematically. Further, two special appendices have been given — Proofs in Mathematics, and Mathematical Modelling. These are placed in the book for interested students to study, and are only optional reading at present. These topics may be considered for inclusion in the main syllabi in due course of time.

As in the past, this textbook is also a team effort. However, what is unusual about the team this time is that teachers from different kinds of schools have been an integral part at each stage of the development. We are also assuming that teachers will contribute continuously to the process in the classroom by formulating examples and exercises contextually suited to the children in their particular classrooms. Finally, we hope that teachers and learners would send comments for improving the textbook to the NCERT.

PARVIN SINCLAIR
G.P. DIKSHIT
Chief Advisors
Textbook Development Committee

Textbook Development Committee

CHAIRPERSON, ADVISORY GROUP IN SCIENCE AND MATHEMATICS

J.V. Narlikar, *Emeritus Professor*, Inter-University Centre for Astronomy & Astrophysics (IUCAA), Ganeshkhind, Pune University, Pune

CHIEF ADVISORS

P. Sinclair, *Professor of Mathematics*, IGNOU, New Delhi

G.P. Dikshit, *Professor (Retd.)*, Lucknow University, Lucknow

CHIEF COORDINATOR

Hukum Singh, *Professor and Head (Retd.)*, DESM, NCERT, New Delhi

MEMBERS

Anjali Lal, *PGT*, DAV Public School, Sector-14, Gurgaon

A.K. Wazalwar, *Professor and Head*, DESM, NCERT

B.S. Upadhyaya, *Professor*, RIE, Mysore

Jayanti Datta, *PGT*, Salwan Public School, Gurgaon

Mahendra Shanker, *Lecturer (S.G.) (Retd.)*, NCERT

Manica Aggarwal, Green Park, New Delhi

N.D. Shukla, *Professor (Retd.)*, Lucknow University, Lucknow

Ram Avtar, *Professor (Retd.) & Consultant*, DESM, NCERT

Rama Balaji, *TGT*, K.V., MEG & Centre, St. John's Road, Bangalore

S. Jagdeeshan, *Teacher and Member*, Governing Council, Centre for Learning, Bangalore

S.K.S. Gautam, *Professor (Retd.)*, DESM, NCERT

Vandita Kalra, *Lecturer*, Sarvodaya Kanya Vidyalaya, Vikaspuri District Centre, Delhi

V.A. Sujatha, *TGT*, Kendriya Vidyalaya No. 1, Vasco, Goa

V. Madhavi, *TGT*, Sanskriti School, Chankyapuri, New Delhi

MEMBER-COORDINATOR

R.P. Maurya, *Professor*, DESM, NCERT, New Delhi

Acknowledgements

The Council gratefully acknowledges the valuable contributions of the following participants of the Textbook Review Workshop:

Mala Mani, *TGT*, Amity International School, Sector-44, Noida; Meera Mahadevan, *TGT*, Atomic Energy Central School, No. 4, Anushakti Nagar, Mumbai; Rashmi Rana, *TGT*, D.A.V. Public School, Pushpanjali Enclave, Pitampura, Delhi; Mohammad Qasim, *TGT*, Anglo Arabic Senior Secondary School, Ajmeri Gate, Delhi; S.C. Rauto, *TGT*, Central School for Tibetans, Happy Valley, Mussoorie; Rakesh Kaushik, *TGT*, Sainik School, Kunjpura, Karnal; Ashok Kumar Gupta, *TGT*, Jawahar Navodaya Vidyalaya, Dudhnoi, Distt. Goalpara; Sankar Misra, *TGT*, Demonstration Multipurpose School, RIE, Bhubaneswar; Uaday Singh, *Lecturer*, Department of Mathematics, B.H.U., Varanasi; B.R. Handa, *Emeritus Professor*, IIT, New Delhi; Monika Singh, *Lecturer*, Sri Ram College (University of Delhi), Lajpat Nagar, New Delhi; G. Sri Hari Babu, *TGT*, Jawahar Navodaya Vidyalaya, Sirpur, Kagaz Nagar, Adilabad; Ajay Kumar Singh, *TGT*, Ramjas Sr. Secondary School No. 3, Chandni Chowk, Delhi; Mukesh Kumar Agrawal, *TGT*, S.S.A.P.G.B.S.S. School, Sector-V, Dr Ambedkar Nagar, New Delhi.

Special thanks are due to Professor Hukum Singh, *Head (Retd.)*, DESM, NCERT for his support during the development of this book.

The Council acknowledges the efforts of Deepak Kapoor, *Incharge*, Computer Station; Purnendu Kumar Barik, *Copy Editor*; Naresh Kumar and Nargis Islam, *D.T.P. Operators*; Yogita Sharma, *Proof Reader*.

The Contribution of APC-Office, administration of DESM, Publication Department and Secretariat of NCERT is also duly acknowledged.

Contents

<i>Foreword</i>	<i>iii</i>
<i>Preface</i>	<i>v</i>
1. Real Numbers	1
1.1 Introduction	1
1.2 Euclid's Division Lemma	2
1.3 The Fundamental Theorem of Arithmetic	7
1.4 Revisiting Irrational Numbers	11
1.5 Revisiting Rational Numbers and Their Decimal Expansions	15
1.6 Summary	18
2. Polynomials	20
2.1 Introduction	20
2.2 Geometrical Meaning of the Zeroes of a Polynomial	21
2.3 Relationship between Zeroes and Coefficients of a Polynomial	28
2.4 Division Algorithm for Polynomials	33
2.5 Summary	37
3. Pair of Linear Equations in Two Variables	38
3.1 Introduction	38
3.2 Pair of Linear Equations in Two Variables	39
3.3 Graphical Method of Solution of a Pair of Linear Equations	44
3.4 Algebraic Methods of Solving a Pair of Linear Equations	50
3.4.1 Substitution Method	50
3.4.2 Elimination Method	54
3.4.3 Cross-Multiplication Method	57
3.5 Equations Reducible to a Pair of Linear Equations in Two Variables	63
3.6 Summary	69
4. Quadratic Equations	70
4.1 Introduction	70
4.2 Quadratic Equations	71

4.3	Solution of a Quadratic Equation by Factorisation	74
4.4	Solution of a Quadratic Equation by Completing the Square	76
4.5	Nature of Roots	88
4.6	Summary	91
5.	Arithmetic Progressions	93
5.1	Introduction	93
5.2	Arithmetic Progressions	95
5.3	n th Term of an AP	100
5.4	Sum of First n Terms of an AP	107
5.5	Summary	116
6.	Triangles	117
6.1	Introduction	117
6.2	Similar Figures	118
6.3	Similarity of Triangles	123
6.4	Criteria for Similarity of Triangles	129
6.5	Areas of Similar Triangles	141
6.6	Pythagoras Theorem	144
6.7	Summary	154
7.	Coordinate Geometry	155
7.1	Introduction	155
7.2	Distance Formula	156
7.3	Section Formula	162
7.4	Area of a Triangle	168
7.5	Summary	172
8.	Introduction to Trigonometry	173
8.1	Introduction	173
8.2	Trigonometric Ratios	174
8.3	Trigonometric Ratios of Some Specific Angles	181
8.4	Trigonometric Ratios of Complementary Angles	187
8.5	Trigonometric Identities	190
8.6	Summary	194

9. Some Applications of Trigonometry	195
9.1 Introduction	195
9.2 Heights and Distances	196
9.3 Summary	205
10. Circles	206
10.1 Introduction	206
10.2 Tangent to a Circle	207
10.3 Number of Tangents from a Point on a Circle	209
10.4 Summary	215
11. Constructions	216
11.1 Introduction	216
11.2 Division of a Line Segment	216
11.3 Construction of Tangents to a Circle	220
11.4 Summary	222
12. Areas Related to Circles	223
12.1 Introduction	223
12.2 Perimeter and Area of a Circle — A Review	224
12.3 Areas of Sector and Segment of a Circle	226
12.4 Areas of Combinations of Plane Figures	231
12.5 Summary	238
13. Surface Areas and Volumes	239
13.1 Introduction	239
13.2 Surface Area of a Combination of Solids	240
13.3 Volume of a Combination of Solids	245
13.4 Conversion of Solid from One Shape to Another	248
13.5 Frustum of a Cone	252
13.6 Summary	258
14. Statistics	260
14.1 Introduction	260
14.2 Mean of Grouped Data	260
14.3 Mode of Grouped Data	272

14.4 Median of Grouped Data	277
14.5 Graphical Representation of Cumulative Frequency Distribution	289
14.6 Summary	293
15. Probability	295
15.1 Introduction	295
15.2 Probability — A Theoretical Approach	296
15.3 Summary	312
Appendix A1 : Proofs in Mathematics	313
A1.1 Introduction	313
A1.2 Mathematical Statements Revisited	313
A1.3 Deductive Reasoning	316
A1.4 Conjectures, Theorems, Proofs and Mathematical Reasoning	318
A1.5 Negation of a Statement	323
A1.6 Converse of a Statement	326
A1.7 Proof by Contradiction	329
A1.8 Summary	333
Appendix A2 : Mathematical Modelling	334
A2.1 Introduction	334
A2.2 Stages in Mathematical Modelling	335
A2.3 Some Illustrations	339
A2.4 Why is Mathematical Modelling Important?	343
A2.5 Summary	344
Answers/Hints	345