Exercise 10d

Question 1.
Differentiate each of the following w.r.t x:

$$\sin^{-1}\left\{\sqrt{\frac{1-\cos x}{2}}\right\}$$

Answer:

To find: Value of $\sin^{-1}\left\{\sqrt{\frac{1-\cos x}{2}}\right\}$

Formula used: (i) $\cos\theta = 2\sin^2\frac{\theta}{2}$

We have, $\sin^{-1}\left\{\sqrt{\frac{1-\cos x}{2}}\right\}$

$\Rightarrow \sin^{-1}\left\{\sqrt{\frac{2\sin^2\frac{x}{2}}{2}}\right\}$

$\Rightarrow \sin^{-1}\left\{\sqrt{\sin^2\frac{x}{2}}\right\}$

$\Rightarrow \sin^{-1}\left\{\sin\frac{x}{2}\right\}$

$\Rightarrow \frac{x}{2}$

Now, we can see that $\sin^{-1}\left\{\sqrt{\frac{1-\cos x}{2}}\right\} = \frac{x}{2}$

Now differentiating ,

$\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx}$

$\Rightarrow \frac{1}{2}\ \frac{dx}{dx}$

$\Rightarrow \frac{1}{2}$

Ans) $\frac{1}{2}$

Question 2.
Differentiate each of the following w.r.t x:

$$\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$$

Answer:
To find: Value of $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

Formula used: (i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

We have, $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sin x}{2\cos^2\frac{x}{2}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)$

$\Rightarrow \tan^{-1}\left(\tan\frac{x}{2}\right)$

$\Rightarrow \frac{x}{2}$

Now, we can see that $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right) = \frac{x}{2}$

Now differentiating ,

$\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx}$

$\Rightarrow \frac{1}{2}\frac{dx}{dx}$

$\Rightarrow \frac{1}{2}$

Ans) $\frac{1}{2}$

Question 3.
Differentiate each of the following w.r.t x:

$$\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

Answer:
To find: Value of $\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

Formula used: (i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

We have, $\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

$\Rightarrow \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{\sin x}\right)$

$$\Rightarrow \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow \frac{x}{2}$$

Now, we can see that $\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right) = \frac{x}{2}$

Now differentiating ,

$$\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx}$$

$$\Rightarrow \frac{1}{2}\frac{dx}{dx}$$

$$\Rightarrow \frac{1}{2}$$

Ans) $\frac{1}{2}$

Question 4.
Differentiate each of the following w.r.t x:

$$\cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$$

Answer:

To find: Value of $\cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$

Formula used: (i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

We have, $\cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$

$$\Rightarrow \cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\ \sqrt{\frac{1+\cos x}{1+\cos x}}\right)$$

$$\Rightarrow \cot^{-1}\left(\sqrt{\frac{(1+\cos x)^2}{1-\cos^2 x}}\right)$$

$$\Rightarrow \cot^{-1}\left(\sqrt{\frac{(1+\cos x)^2}{\sin^2 x}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow \frac{x}{2}$$

Now, we can see that $\cot^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right) = \frac{x}{2}$

Now differentiating ,

$$\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx}$$

$$\Rightarrow \frac{1}{2}\frac{dx}{dx}$$

$$\Rightarrow \frac{1}{2}$$

Ans) $\frac{1}{2}$

Question 5.
Differentiate each of the following w.r.t x:

$$\tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)$$

Answer:
To find: Value of $\tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$

Formula used: (i) $\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$

We have, $\tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$

Dividing numerator and denominator by cosx

$$\Rightarrow \tan^{-1}\left(\frac{\frac{\cos x+\sin x}{\cos x}}{\frac{\cos x-\sin x}{\cos x}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan x\,\tan\frac{\pi}{4}}\right)$$

$$\Rightarrow \tan^{-1}\left(\tan\left(\frac{\pi}{4}+x\right)\right)$$

$$\Rightarrow \frac{\pi}{4}+x$$

Now, we can see that $\tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right) = \frac{\pi}{4}+x$

Now differentiating ,

$$\Rightarrow \frac{d\left(\frac{\pi}{4}+x\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\pi}{4}\right)}{dx}+\frac{dx}{dx}$$

$\Rightarrow 0 + 1$

$\Rightarrow 1$

Ans) 1

Question 6.
Differentiate each of the following w.r.t x:

$$\cot^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$$

Answer:
To find: Value of $\cot^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

Formula used: (i) $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$

We have, $\cot^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

Dividing numerator and denominator by cosx

$$\Rightarrow \cot^{-1}\left(\frac{\frac{\cos x-\sin x}{\cos x}}{\frac{\cos x+\sin x}{\cos x}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan x\tan\frac{\pi}{4}}\right)$$

$$\Rightarrow \cot^{-1}\left(\tan\left(\frac{\pi}{4}-x\right)\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-x\right)\right)\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\left(\frac{\pi}{4}+x\right)\right)$$

$$\Rightarrow \frac{\pi}{4}+x$$

Now, we can see that $\cot^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)=\frac{\pi}{4}+x$

Now differentiating ,

$$\Rightarrow \frac{d\left(\frac{\pi}{4}+x\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\pi}{4}\right)}{dx} + \frac{dx}{dx}$$

$\Rightarrow 0 + 1$

$\Rightarrow 1$

Ans) 1

Question 7.
Differentiate each of the following w.r.t x:

$$\cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right)$$

Answer:

To find: Value of $\cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right)$

Formula used: (i) $1 - \cos \theta = 2\sin^2 \frac{\theta}{2}$

(ii) $1 + \cos \theta = 2\cos^2 \frac{\theta}{2}$

We have, $\cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right)$

$$\Rightarrow \cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{2\sin^2 \frac{3x}{2}}}\right)$$

$$\Rightarrow \cot^{-1}\left(\sqrt{\frac{2\cos^2 \frac{3x}{2}}{2\sin^2 \frac{3x}{2}}}\right)$$

$$\Rightarrow \cot^{-1}\left(\sqrt{\cot^2\left(\frac{3x}{2}\right)}\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\left(\frac{3x}{2}\right)\right)$$

$$\Rightarrow \frac{3x}{2}$$

Now, we can see that $\cot^{-1}\left(\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\right) = \frac{3x}{2}$

Now differentiating ,

$$\Rightarrow \frac{d\left(\frac{3x}{2}\right)}{dx}$$

$$\Rightarrow \frac{3}{2}\frac{dx}{dx}$$

$$\Rightarrow \frac{3}{2}$$

Ans) $\frac{3}{2}$

Question 8.
Differentiate each of the following w.r.t x:

$$\sec^{-1}\left(\frac{1+\tan^2 x}{1-\tan^2 x}\right)$$

Answer:

To find: Value of $\sec^{-1}\left(\frac{1+\tan^2 x}{1-\tan^2 x}\right)$

Formula used: (i) $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

We have, $\sec^{-1}\left(\frac{1+\tan^2 x}{1-\tan^2 x}\right)$

Dividing numerator and denominator by $1+\tan^2 x$

$$\Rightarrow \sec^{-1}\left(\frac{\left(\frac{1+tan^2 x}{1+tan^2 x}\right)}{\left(\frac{1-tan^2 x}{1+tan^2 x}\right)}\right)$$

$$\Rightarrow \sec^{-1}\left(\frac{1}{\left(\frac{1-tan^2 x}{1+tan^2 x}\right)}\right)$$

$$\Rightarrow \sec^{-1}\left(\frac{1}{\cos 2x}\right)$$

$$\Rightarrow \sec^{-1}(\sec 2x)$$

$\Rightarrow 2x$

Now, we can see that $\sec^{-1}\left(\frac{1+\tan^2 x}{1-\tan^2 x}\right) = 2x$

Now differentiating ,

$$\Rightarrow \frac{d(2x)}{dx}$$

$$\Rightarrow 2\frac{dx}{dx}$$

$\Rightarrow 2$

Ans) 2

Question 9.
Differentiate each of the following w.r.t x:

$$\sin^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$$

Answer:
To find: Value of $\sin^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$

Formula used: (i) cos $2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

We have, $\sin^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$

$\Rightarrow \sin^{-1}(\cos 2x)$

$\Rightarrow \sin^{-1}\left(\sin\left(\frac{\pi}{2}-2x\right)\right)$

$\Rightarrow \frac{\pi}{2}-2x$

Now, we can see that $\sin^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right) = \frac{\pi}{2}-2x$

Now differentiating ,

$\Rightarrow \frac{d\left(\frac{\pi}{2}-2x\right)}{dx}$

$\Rightarrow \frac{d\left(\frac{\pi}{2}\right)}{dx} - 2\frac{dx}{dx}$

$\Rightarrow 0 - 2$

$\Rightarrow$ -2

Ans) -2

Question 10.
Differentiate each of the following w.r.t x:

$$\operatorname{cosec}^{-1}\left(\frac{1+\tan^2 x}{2\tan x}\right)$$

Answer:
To find: Value of $\operatorname{cosec}^{-1}\left(\frac{1+\tan^2 x}{2\tan x}\right)$

Formula used: (i) sin 2θ = $\frac{2\tan\theta}{1+\tan^2\theta}$

We have, $\operatorname{cosec}^{-1}\left(\frac{1+\tan^2 x}{2\tan x}\right)$

Dividing Numerator and Denominator with $1+\tan^2 x$

$$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{\left(\frac{1+\tan^2 x}{1+\tan^2 x}\right)}{\left(\frac{2\tan x}{1+\tan^2 x}\right)}\right)$$

$$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{(1)}{\left(\frac{2\tan x}{1+\tan^2 x}\right)}\right)$$

$$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{1}{\sin 2x}\right)$$

$$\Rightarrow \operatorname{cosec}^{-1}(\operatorname{cosec} 2x)$$

$\Rightarrow$ 2x

Now, we can see that $\operatorname{cosec}^{-1}\left(\frac{1+\tan^2 x}{2\tan x}\right) = 2x$

Now differentiating ,

$$\Rightarrow \frac{d(2x)}{dx}$$

$$\Rightarrow 2\frac{dx}{dx}$$

$\Rightarrow 2$

Ans) 2

Question 11.
Differentiate each of the following w.r.t x:

$$\cot^{-1}(\operatorname{cosec} x + \cot x)$$

Answer:
To find: Value of $\cot^{-1}(\operatorname{cosec} x + \cot x)$

Formula used: (i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

We have, $\cot^{-1}(\operatorname{cosec} x + \cot x)$

$$\Rightarrow \cot^{-1}\left(\frac{1}{\sin x} + \frac{\cos x}{\sin x}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{\sin x}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow \frac{x}{2}$$

Now, we can see that $\cot^{-1}(\operatorname{cosec} x + \cot x) = \frac{x}{2}$

Now differentiating ,

$$\Rightarrow \frac{d\left(\frac{x}{2}\right)}{dx}$$

$$\Rightarrow \frac{1}{2}\frac{dx}{dx}$$

$$\Rightarrow \frac{1}{2}$$

Ans) $\frac{1}{2}$

Question 12.
Differentiate each of the following w.r.t x:

$$\tan^{-1}(\cot x) + \cot^{-1}(\tan x)$$

Answer:
To find: Value of $\tan^{-1}(\cot x) + \cot^{-1}(\tan x)$

The formula used: (i) $\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$

(ii) $\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

We have, $\tan^{-1}(\cot x) + \cot^{-1}(\tan x)$

$\Rightarrow \tan^{-1}\left[\tan\left(\frac{\pi}{2} - x\right)\right] + \cot^{-1}\left[\cot\left(\frac{\pi}{2} - x\right)\right]$

$\Rightarrow \left(\frac{\pi}{2} - x\right) + \left(\frac{\pi}{2} - x\right)$

$\Rightarrow \pi - 2x$

Now, we can see that $\tan^{-1}(\cot x) + \cot^{-1}(\tan x) = \pi - 2x$

Now differentiating ,

$\Rightarrow \frac{d(\pi - 2x)}{dx}$

$\Rightarrow \frac{d\pi}{dx} - \frac{d2x}{dx}$

$\Rightarrow -2$

Ans) -2

Question 13.
Differentiate each of the following w.r.t x:

$\sin^{-1}\left\{\sqrt{1-x^2}\right\}$

Answer:
To find: Value of $\sin^{-1}\{\sqrt{1-x^2}\}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\{\sqrt{1-x^2}\}$

⇒ Putting x = cosθ

$\theta = \cos^{-1}x$ ... (i)

Putting x = cosθ in the equation

$\Rightarrow \sin^{-1}\{\sqrt{1-\cos^2\theta}\}$

$\Rightarrow \sin^{-1}\left(\sqrt{\sin^2\theta}\right)$

$\Rightarrow \sin^{-1}(\sin\theta)$

$\Rightarrow \theta$

$\Rightarrow \frac{d\theta}{dx}$

$\Rightarrow \frac{d(\cos^{-1}x)}{dx}$ [From (i)]

$\Rightarrow -\frac{1}{\sqrt{1-x^2}}$

Ans) $-\frac{1}{\sqrt{1-x^2}}$

Question 14.

Differentiate each of the following w.r.t x:

$$\sin^{-1}\left(\sqrt{\frac{1-x}{2}}\right)$$

Answer:

To find: Value of $\sin^{-1}\left(\sqrt{\frac{1-x}{2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\left(\sqrt{\frac{1-x}{2}}\right)$

⇒ Putting x = cosθ

$\theta = \cos^{-1}x$ ... (i)

Putting x = cosθ in the equation

$\Rightarrow \sin^{-1}\left(\sqrt{\frac{1-\cos\theta}{2}}\right)$

$\Rightarrow \sin^{-1}\left(\sqrt{\sin^2\frac{\theta}{2}}\right)$

$\Rightarrow \sin^{-1}\left(\sin\frac{\theta}{2}\right)$

$\Rightarrow \frac{\theta}{2}$

Now, we can see that $\sin^{-1}\left(\sqrt{\frac{1-x}{2}}\right) = \frac{\theta}{2}$

$\Rightarrow \theta = \cos^{-1}x$

$$\Rightarrow \frac{d\left(\frac{\theta}{2}\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\cos^{-1}x}{2}\right)}{dx}$$

$$\Rightarrow -\frac{1}{2\sqrt{1-x^2}}$$

Ans) $-\frac{1}{2\sqrt{1-x^2}}$

Question 15.
Differentiate each of the following w.r.t x:

$$\cos^{-1}\left\{\sqrt{\frac{1+x}{2}}\right\}$$

Answer:

To find: Value of $\cos^{-1}\left(\sqrt{\frac{1+x}{2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\cos^{-1}\left(\sqrt{\frac{1+x}{2}}\right)$

$\Rightarrow$ Putting $x = \cos\theta$

$\theta = \cos^{-1}x$ ... (i)

Putting x = cosθ in the equation

$$\Rightarrow \cos^{-1}\left(\sqrt{\frac{1+\cos\theta}{2}}\right)$$

$$\Rightarrow \cos^{-1}\left(\sqrt{\cos^2\frac{\theta}{2}}\right)$$

$$\Rightarrow \cos^{-1}\left(\cos\frac{\theta}{2}\right)$$

$$\Rightarrow \frac{\theta}{2}$$

Now, we can see that $\cos^{-1}\left(\sqrt{\frac{1+x}{2}}\right) = \frac{\theta}{2}$

$\Rightarrow \theta = \cos^{-1}x$

$$\Rightarrow \frac{d\left(\frac{\theta}{2}\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\cos^{-1}x}{2}\right)}{dx}$$

$$\Rightarrow -\frac{1}{2\sqrt{1-x^2}}$$

Ans) $-\frac{1}{2\sqrt{1-x^2}}$

Question 16.
Differentiate each of the following w.r.t x:

$$\cos^{-1}\left\{\sqrt{1-x^2}\right\}$$

Answer:

To find: Value of $\cos^{-1}\left(\sqrt{1-x^2}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\cos^{-1}\left(\sqrt{1-x^2}\right)$

⇒ Putting x = sinθ

θ = $\sin^{-1}x$ ... (i)

Putting x = sinθ in the equation

⇒ $\cos^{-1}\left(\sqrt{1-(sin\theta)^2}\right)$

⇒ $\cos^{-1}\left(\sqrt{1-\sin^2\theta}\right)$

⇒ $\cos^{-1}(\cos\theta)$

⇒ θ

Now, we can see that $\cos^{-1}\left(\sqrt{1-x^2}\right)$ = θ

⇒ θ = $\sin^{-1}x$

⇒ $\frac{d(\theta)}{dx}$

⇒ $\frac{d(\sin^{-1}x)}{dx}$

$\Rightarrow \frac{1}{\sqrt{1-x^2}}$

Ans) $\frac{1}{\sqrt{1-x^2}}$

Question 17.
Differentiate each of the following w.r.t x:

$$\sin^{-1}\left\{2x\sqrt{1-x^2}\right\}$$

Answer:
To find: Value of $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$\Rightarrow$ Putting x = sinθ

$\theta = \sin^{-1}x$ ... (i)

Putting x = sinθ in the equation

$\Rightarrow \sin^{-1}\left(2\sin\theta\sqrt{1-(\sin\theta)^2}\right)$

$\Rightarrow \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

$\Rightarrow \sin^{-1}(2\sin\theta\cos\theta)$

$\Rightarrow \sin^{-1}(\sin2\theta)$

$\Rightarrow 2\theta$

$\Rightarrow 2\sin^{-1}x$

Now, we can see that $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1}x$

Now Differentiating

$$\Rightarrow \frac{d2\theta}{dx} = \frac{d(2\sin^{-1}x)}{dx}$$

$$\Rightarrow 2\frac{d(\theta)}{dx}$$

$$\Rightarrow 2\frac{d(\sin^{-1}x)}{dx}$$

$$\Rightarrow 2\frac{1}{\sqrt{1-x^2}}$$

Ans) $\frac{2}{\sqrt{1-x^2}}$

Question 18.
Differentiate each of the following w.r.t x:

$$\sin^{-1}(3x - 4x^3)$$

Answer:
To find: Value of $\sin^{-1}(3x - 4x^3)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}(3x - 4x^3)$

⇒ Putting x = sinθ

θ = $\sin^{-1}x$ ... (i)

Putting x = sinθ in the equation

$$\Rightarrow \sin^{-1}(3\sin\theta - 4(\sin\theta)^3)$$

$$\Rightarrow \sin^{-1}(3\sin\theta - 4\sin^3\theta)$$

$$\Rightarrow \sin^{-1}(\sin 3\theta)$$

⇒ 3θ

Now, we can see that $\sin^{-1}(3x - 4x^3) = 3\theta$

Now Differentiating

$$\Rightarrow \frac{d3\theta}{dx} = \frac{d(3\sin^{-1}x)}{dx}$$

$$\Rightarrow 3\frac{d(\sin^{-1}x)}{dx}$$

$$\Rightarrow 3\frac{1}{\sqrt{1-x^2}}$$

Ans) $\frac{3}{\sqrt{1-x^2}}$

Question 19.
Differentiate each of the following w.r.t x:

$$\sin^{-1}(1-2x^2)$$

Answer:

To find: Value of $\sin^{-1}(1 - 2x^2)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}(1 - 2x^2)$

⇒ Putting $x = \sin\theta$

$\theta = \sin^{-1}x$ ... (i)

Putting $x = \sin\theta$ in the equation

$\Rightarrow \sin^{-1}\left(1 - 2(\sin\theta)^2\right)$

$\Rightarrow \sin^{-1}(1 - 2\sin^2\theta)$

$\Rightarrow \sin^{-1}(\cos 2\theta)$

$\Rightarrow \sin^{-1}\left(\sin\left(\frac{\pi}{2} - 2\theta\right)\right)$

$\Rightarrow \frac{\pi}{2} - 2\theta$

Now, we can see that $\sin^{-1}(1 - 2x^2) = \frac{\pi}{2} - 2\theta$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\pi}{2} - 2\theta\right)}{dx} = \frac{d\left(\frac{\pi}{2}\right)}{dx} - \frac{d2\theta}{dx}$$

$$\Rightarrow 0 - \frac{d2\theta}{dx}$$

$$\Rightarrow -2\frac{d\sin^{-1}x}{dx}$$

$$\Rightarrow \frac{-2}{\sqrt{1-x^2}}$$

Ans) $\frac{-2}{\sqrt{1-x^2}}$

Question 20.
Differentiate each of the following w.r.t x:

$$\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$$

Answer:
To find: Value of $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

$\Rightarrow$ Putting x = sinθ

$\theta = \sin^{-1}x$ ... (i)

Putting x = sinθ in the equation

$$\Rightarrow \sec^{-1}\left(\frac{1}{\sqrt{1-(\sin\theta)^2}}\right)$$

$$\Rightarrow \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$$

$$\Rightarrow \sec^{-1}\left(\frac{1}{\sqrt{\cos^2\theta}}\right)$$

$$\Rightarrow \sec^{-1}\left(\frac{1}{\cos\theta}\right)$$

$$\Rightarrow \sec^{-1}(\sec\theta)$$

$$\Rightarrow \theta$$

Now, we can see that $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right) = \theta$

Now Differentiating

$$\Rightarrow \frac{d\theta}{dx}$$

$$\Rightarrow \frac{d(\sin^{-1}x)}{dx}$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}}$$

Ans) $\frac{1}{\sqrt{1-x^2}}$

Question 21.
Differentiate each of the following w.r.t x:

$$\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

Answer:

To find: Value of $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

⇒ Putting x = sinθ

θ = $\sin^{-1}x$ ... (i)

Putting x = sinθ in the equation

$$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-(\sin\theta)^2}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos^2\theta}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$\Rightarrow \tan^{-1}(\tan\theta)$$

⇒ θ

Now, we can see that $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) = \theta$

Now Differentiating

$$\Rightarrow \frac{d\theta}{dx}$$

$$\Rightarrow \frac{d(\sin^{-1}x)}{dx}$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}}$$

Ans) $\frac{1}{\sqrt{1-x^2}}$

Question 22.
Differentiate each of the following w.r.t x:

$$\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$$

Answer:

To find: Value of $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$

⇒ Putting x = sinθ

θ = $\sin^{-1}x$ … (i)

Putting x = sinθ in the equation

$$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{1+\sqrt{1-(\sin\theta)^2}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{1+\sqrt{1-\sin^2\theta}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{1+\sqrt{\cos^2\theta}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right)$$

$$\Rightarrow \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$\Rightarrow \frac{\theta}{2}$$

Now, we can see that $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right) = \frac{\theta}{2}$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\theta}{2}\right)}{dx}$$

$$\Rightarrow \frac{1}{2}\frac{d(\theta)}{dx}$$

$$\Rightarrow \frac{1}{2}\frac{d\sin^{-1}x}{dx}$$

$$\Rightarrow \frac{1}{2\sqrt{1-x^2}}$$

Ans) $\frac{1}{2\sqrt{1-x^2}}$

Question 23.
Differentiate each of the following w.r.t x:

$$\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

Answer:

To find: Value of $\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

⇒ Putting x = sinθ

θ = $\sin^{-1}x$ ... (i)

Putting x = sinθ in the equation

$$\Rightarrow \cot^{-1}\left(\frac{\sqrt{1-(\sin\theta)^2}}{\sin\theta}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\sqrt{\cos^2\theta}}{\sin\theta}\right)$$

$\Rightarrow \cot^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$

$\Rightarrow \cot^{-1}(\cot\theta)$

$\Rightarrow \theta$

Now, we can see that $\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) = \theta$

Now Differentiating

$\Rightarrow \frac{d(\theta)}{dx}$

$\Rightarrow \frac{d(\sin^{-1}x)}{dx}$

$\Rightarrow \frac{1}{\sqrt{1-x^2}}$

Ans) $\frac{1}{\sqrt{1-x^2}}$

Question 24.
Differentiate each of the following w.r.t x:

$$\sec^{-1}\left(\frac{1}{1-2x^2}\right)$$

Answer:
To find: Value of $\sec^{-1}\left(\frac{1}{1-2x^2}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sec^{-1}\left(\frac{1}{1-2x^2}\right)$

$\Rightarrow$ Putting x = sinθ

$\theta = \sin^{-1}x$ ... (i)

Putting x = sinθ in the equation

$\Rightarrow \sec^{-1}\left(\frac{1}{1-2(\sin\theta)^2}\right)$

$\Rightarrow \sec^{-1}\left(\frac{1}{1-2\sin^2\theta}\right)$

$\Rightarrow \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

$\Rightarrow \sec^{-1}(\sec 2\theta)$

$\Rightarrow 2\theta$

Now, we can see that $\sec^{-1}\left(\frac{1}{1-2x^2}\right) = 2\theta$

Now Differentiating

$\Rightarrow \frac{d(2\theta)}{dx}$

$\Rightarrow 2\frac{d(\sin^{-1}x)}{dx}$

$\Rightarrow \frac{2}{\sqrt{1-x^2}}$

Ans) $\frac{2}{\sqrt{1-x^2}}$

Question 25.
Differentiate each of the following w.r.t x:

$$\sin^{-1}\left\{\frac{1}{\sqrt{1+x^2}}\right\}$$

Answer:
To find: Value of $\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

⇒ Putting x = cotθ

θ = $\cot^{-1}x$ ... (i)

Putting x = cotθ in the equation

$$\Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{1+(\cot\theta)^2}}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{1+\cot^2\theta}}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{\mathrm{cosec}^2\theta}}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{1}{\mathrm{cosec}\theta}\right)$$

$$\Rightarrow \sin^{-1}(\sin\theta)$$

$\Rightarrow \theta$

Now, we can see that $\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \theta$

Now Differentiating

$$\Rightarrow \frac{d(\theta)}{dx}$$

$$\Rightarrow \frac{d(\cot^{-1}x)}{dx}$$

$$\Rightarrow -\frac{1}{1+x^2}$$

Ans) $-\frac{1}{1+x^2}$

Question 26.
Differentiate each of the following w.r.t x:

$$\tan^{-1}\left(\frac{1+x}{1-x}\right)$$

Answer:
To find: Value of $\tan^{-1}\left(\frac{1+x}{1-x}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{1+x}{1-x}\right)$

$\Rightarrow$ Putting x = $\tan\theta$

$\theta = \tan^{-1}x$ ... (i)

Putting x = tanθ in the equation

$$\Rightarrow \tan^{-1}\left(\frac{1 + \tan\theta}{1 - \tan\theta}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4}\tan\theta}\right)$$

$$\Rightarrow \tan^{-1}\left(\tan\frac{\pi}{4}+\theta\right)$$

$$\Rightarrow \frac{\pi}{4} + \theta$$

Now, we can see that $\tan^{-1}\left(\frac{1 + x}{1 - x}\right) = \frac{\pi}{4} + \theta$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\pi}{4} + \theta\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\pi}{4}\right)}{dx} + \frac{d(\theta)}{dx}$$

$$\Rightarrow 0 + \frac{d(\theta)}{dx}$$

$$\Rightarrow \frac{d(\tan^{-1}x)}{dx}$$

$$\Rightarrow \frac{1}{1+x^2}$$

Ans) $\frac{1}{1+x^2}$

Question 27.

Differentiate each of the following w.r.t x:

$$\cot^{-1}\left(\frac{1+x}{1-x}\right)$$

Answer:

To find: Value of $\cot^{-1}\left(\frac{1+x}{1-x}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\cot^{-1}\left(\frac{1+x}{1-x}\right)$

⇒ Putting x = tanθ

θ = $\tan^{-1}x$ ... (i)

Putting x = tanθ in the equation

$$\Rightarrow \cot^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}\right)$$

$$\Rightarrow \cot^{-1}\left(\tan\frac{\pi}{4}+\theta\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\left(\frac{\pi}{2}-\left(\frac{\pi}{4}+\theta\right)\right)\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\left(\frac{\pi}{2}-\left(\frac{\pi}{4}+\theta\right)\right)\right)$$

$$\Rightarrow \cot^{-1}\left(\cot\left(\frac{\pi}{4}-\theta\right)\right)$$

$\Rightarrow \frac{\pi}{4} - \theta$

Now, we can see that $\cot^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4} - \theta$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\pi}{4} - \theta\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\pi}{4}\right)}{dx} - \frac{d(\theta)}{dx}$$

$$\Rightarrow 0 - \frac{d(\theta)}{dx}$$

$$\Rightarrow -\frac{d(\tan^{-1}x)}{dx}$$

$$\Rightarrow -\frac{1}{1+x^2}$$

Ans) $-\frac{1}{1+x^2}$

Question 28.
Differentiate each of the following w.r.t x:

$$\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$$

Answer:
To find: Value of $\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

⇒ Putting x = tanθ

θ = $\tan^{-1}x$ ... (i)

Putting x = tanθ in the equation

$\Rightarrow \tan^{-1}\left(\frac{3\tan\theta - (\tan\theta)^3}{1 - 3(\tan\theta)^2}\right)$

$\Rightarrow \tan^{-1}(\tan 3\theta)$

$\Rightarrow \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$

$\Rightarrow \tan^{-1}(\tan 3\theta)$

$\Rightarrow 3\theta$

Now, we can see that $\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) = 3\theta$

Now Differentiating

$\Rightarrow \frac{d(3\theta)}{dx}$

$\Rightarrow 3\frac{d(\tan^{-1}x)}{dx}$

$\Rightarrow \frac{3}{1+x^2}$

Ans) $\frac{3}{1+x^2}$

Question 29.
Differentiate each of the following w.r.t x:

$$\operatorname{cosec}^{-1}\left(\frac{1+x^2}{2x}\right)$$

Answer:
To find: Value of $\operatorname{cosec}^{-1}\left(\frac{1+x^2}{2x}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\operatorname{cosec}^{-1}\left(\frac{1+x^2}{2x}\right)$

⇒ Putting $x = \tan\theta$

$\theta = \tan^{-1}x$ ... (i)

Putting $x = \tan\theta$ in the equation

$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{1+(\tan\theta)^2}{2\tan\theta}\right)$

$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{1+\tan^2\theta}{2\tan\theta}\right)$

$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{1}{\sin 2\theta}\right)$

$\Rightarrow \operatorname{cosec}^{-1}(\operatorname{cosec}2\theta)$

$\Rightarrow 2\theta$

Now, we can see that $\operatorname{cosec}^{-1}\left(\frac{1+x^2}{2x}\right) = 2\theta$

Now Differentiating

$$\Rightarrow \frac{d(2\theta)}{dx}$$

$$\Rightarrow 2\frac{d(\tan^{-1}x)}{dx}$$

$$\Rightarrow \frac{2}{1+x^2}$$

Ans) $\frac{2}{1+x^2}$

Question 30.
Differentiate each of the following w.r.t x:

$$\sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$$

Answer:
To find: Value of $\sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

⇒ Putting $x = \tan\theta$

$\theta = \tan^{-1}x$ ... (i)

Putting $x = \tan\theta$ in the equation

$\Rightarrow \sec^{-1}\left(\frac{1+(\tan\theta)^2}{1-(\tan\theta)^2}\right)$

$\Rightarrow \sec^{-1}\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)$

$\Rightarrow \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

$\Rightarrow \sec^{-1}(\sec 2\theta)$

$\Rightarrow 2\theta$

Now, we can see that $\sec^{-1}\left(\frac{1+x^2}{1-x^2}\right) = 2\theta$

Now Differentiating

$$\Rightarrow \frac{d(2\theta)}{dx}$$

$$\Rightarrow 2\frac{d(\tan^{-1}x)}{dx}$$

$$\Rightarrow \frac{2}{1+x^2}$$

Ans) $\frac{2}{1+x^2}$

Question 31.
Differentiate each of the following w.r.t x:

$$\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$$

Answer:

To find: Value of $\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

⇒ Putting $x = \tan\theta$

$\theta = \tan^{-1}x$ ... (i)

Putting $x = \tan\theta$ in the equation

$$\Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{1+(\tan\theta)^2}}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{1+\tan^2\theta}}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{1}{\sqrt{\sec^2\theta}}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{1}{\sec\theta}\right)$$

$$\Rightarrow \sin^{-1}(\cos\theta)$$

$$\Rightarrow \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \theta\right)\right)$$

$$\Rightarrow \frac{\pi}{2} - \theta$$

Now, we can see that $\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \frac{\pi}{2} - \theta$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\pi}{2}-\theta\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\pi}{2}\right)}{dx}-\frac{d(\theta)}{dx}$$

$$\Rightarrow 0-\frac{d(\tan^{-1}x)}{dx}$$

$$\Rightarrow -\frac{1}{1+x^2}$$

Ans) $-\frac{1}{1+x^2}$

Question 32.
Differentiate each of the following w.r.t x:

$$\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$$

Answer:
To find: Value of $\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$

⇒ Putting $x = \tan\theta$

$\theta = \tan^{-1}x$ ... (i)

Putting x = tanθ in the equation

$$\Rightarrow \sec^{-1}\left(\frac{(\tan\theta)^2 + 1}{(\tan\theta)^2 - 1}\right)$$

$$\Rightarrow \sec^{-1}\left(\frac{\tan^2\theta + 1}{\tan^2\theta - 1}\right)$$

$$\Rightarrow \sec^{-1}\left[-\left(\frac{1 + \tan^2\theta}{1 - \tan^2\theta}\right)\right]$$

$$\Rightarrow \pi - \sec^{-1}\left(\frac{1 + \tan^2\theta}{1 - \tan^2\theta}\right)$$

$$\Rightarrow \pi - \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$$

$$\Rightarrow \pi - \sec^{-1}(\sec 2\theta)$$

$$\Rightarrow \pi - 2\theta$$

$$\Rightarrow \pi - 2\tan^{-1}x$$

Now, we can see that $\sec^{-1}\left(\frac{x^2 + 1}{x^2 - 1}\right) = \pi - 2\tan^{-1}x$

Now Differentiating

$$\Rightarrow \frac{d(\pi - 2\tan^{-1}x)}{dx}$$

$$\Rightarrow \frac{d(\pi)}{dx} - \frac{d(2\tan^{-1}x)}{dx}$$

$$\Rightarrow 0-2\frac{d(\tan^{-1}x)}{dx}$$

$$\Rightarrow -\frac{2}{1+x^2}$$

Ans) $-\frac{1}{1+x^2}$

Question 33.
Differentiate each of the following w.r.t x:

$$\cos^{-1}\left(\frac{1-x^{2n}}{1+x^{2n}}\right)$$

Answer:
To find: Value of $\cos^{-1}\left(\frac{1-x^{2n}}{1+x^{2n}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\cos^{-1}\left(\frac{1-x^{2n}}{1+x^{2n}}\right)$

$\Rightarrow \cos^{-1}\left(\frac{1-(x^n)^2}{1+(x^n)^2}\right)$

$\Rightarrow$ Putting $x^n = \tan\theta$

$\theta = \tan^{-1}(x^n)$ ... (i)

Putting $x^n = \tan\theta$ in the equation

$\Rightarrow \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\Rightarrow \cos^{-1}(\cos 2\theta)$

$\Rightarrow 2\theta$

$\Rightarrow 2\tan^{-1}(x^n)$

Now, we can see that $\cos^{-1}\left(\frac{1 - x^{2n}}{1 + x^{2n}}\right) = 2\tan^{-1}(x^n)$

Now Differentiating

$$\Rightarrow \frac{d\left(2\tan^{-1}(x^n)\right)}{dx}$$

$$\Rightarrow 2\frac{d\left(\tan^{-1}(x^n)\right)}{dx^n}\frac{dx^n}{dx}$$

$$\Rightarrow 2\frac{1}{1+(x^n)^2}nx^{n-1}$$

$$\Rightarrow \frac{2nx^{n-1}}{1+x^{2n}}$$

Ans) $\frac{2nx^{n-1}}{1+x^{2n}}$

Question 34.
Differentiate each of the following w.r.t x:

$$\tan^{-1}\left\{\frac{x}{\sqrt{a^2 - x^2}}\right\}$$

Answer:
To find: Value of $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$

$\Rightarrow$ Putting x = asinθ

$\sin\theta = \frac{x}{a}$

$\theta = \sin^{-1}\left(\frac{x}{a}\right)$ ... (i)

Putting x = asinθ in the equation

$$\Rightarrow \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-(a\sin\theta)^2}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2\left(1-\sin^2\theta\right)}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$$

$$\Rightarrow \tan^{-1}(\tan\theta)$$

$$\Rightarrow \theta$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{a}\right)$$

Now, we can see that $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right) = \sin^{-1}\left(\frac{x}{a}\right)$

Now Differentiating

$$\Rightarrow \frac{d\left(\sin^{-1}\left(\frac{x}{a}\right)\right)}{dx}$$

$$\Rightarrow \frac{d\left(\sin^{-1}\left(\frac{x}{a}\right)\right)}{d\left(\frac{x}{a}\right)}\frac{d\left(\frac{x}{a}\right)}{dx}$$

$$\Rightarrow \left(\frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\right)\frac{1}{a}$$

$$\Rightarrow \left(\frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\right)\frac{1}{a}$$

$$\Rightarrow \left(\frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}}\right)\frac{1}{a}$$

$$\Rightarrow \left(\frac{a}{\sqrt{a^2-x^2}}\right)\frac{1}{a}$$

$$\Rightarrow \frac{1}{\sqrt{a^2-x^2}}$$

Ans) $\frac{1}{\sqrt{a^2-x^2}}$

Question 35.
Differentiate each of the following w.r.t x:

$$\sin^{-1}\left\{2ax\sqrt{1-a^2x^2}\right\}$$

Answer:

To find: Value of $\sin^{-1}\left\{2ax\sqrt{1-a^2x^2}\right\}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\left\{2ax\sqrt{1-a^2x^2}\right\}$

⇒ Putting ax = sinθ

$\theta = \sin^{-1}(ax)$ ... (i)

Putting ax = sinθ in the equation

$\Rightarrow \sin^{-1}\left\{2\sin\theta\sqrt{1-(\sin\theta)^2}\right\}$

$\Rightarrow \sin^{-1}\left\{2\sin\theta\sqrt{1-\sin^2\theta}\right\}$

$\Rightarrow \sin^{-1}\{2\sin\theta\cos\theta\}$

$\Rightarrow \sin^{-1}\{\sin 2\theta\}$

$\Rightarrow 2\theta$

$\Rightarrow 2\sin^{-1}(ax)$

Now, we can see that $\sin^{-1}\left\{2ax\sqrt{1-a^2x^2}\right\} = 2\sin^{-1}(ax)$

Now Differentiating

$$\Rightarrow \frac{d(2\sin^{-1}(ax))}{dx}$$

$$\Rightarrow 2\frac{d(\sin^{-1}(ax))}{dax}\frac{dax}{dx}$$

$$\Rightarrow \left(2\frac{1}{\sqrt{1-(ax)^2}}\right)a$$

$$\Rightarrow \left(\frac{2a}{\sqrt{1-a^2x^2}}\right)$$

Ans) $\frac{2a}{\sqrt{1-a^2x^2}}$

Question 36.
Differentiate each of the following w.r.t x:

$$\tan^{-1}\left\{\frac{\sqrt{1+a^2x^2}-1}{ax}\right\}$$

Answer:

To find: Value of $\tan^{-1}\left\{\frac{\sqrt{1+a^2x^2}-1}{ax}\right\}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left\{\frac{\sqrt{1+a^2x^2}-1}{ax}\right\}$

⇒ Putting $ax = \tan\theta$

$\theta = \tan^{-1}(ax)$ ... (i)

Putting ax = tanθ in the equation

$$\Rightarrow \tan^{-1}\left\{\frac{\sqrt{1+(\tan\theta)^2-1}}{\tan\theta}\right\}$$

$$\Rightarrow \tan^{-1}\left\{\frac{\sqrt{1+\tan^2\theta-1}}{\tan\theta}\right\}$$

$$\Rightarrow \tan^{-1}\left\{\frac{\sec\theta-1}{\tan\theta}\right\}$$

$$\Rightarrow \tan^{-1}\left\{\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right\}$$

$$\Rightarrow \tan^{-1}\left\{\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right\}$$

$$\Rightarrow \tan^{-1}\left\{\frac{1-\cos\theta}{\sin\theta}\right\}$$

$$\Rightarrow \tan^{-1}\left\{\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right\}$$

$$\Rightarrow \tan^{-1}\left\{\tan\frac{\theta}{2}\right\}$$

$$\Rightarrow \frac{\theta}{2}$$

$$\Rightarrow \frac{\tan^{-1}(ax)}{2}$$

Now, we can see that $\tan^{-1}\left\{\frac{\sqrt{1+a^2x^2-1}}{ax}\right\} = \frac{\tan^{-1}(ax)}{2}$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\tan^{-1}(ax)}{2}\right)}{dx}$$

$$\Rightarrow \frac{1}{2}\frac{d(\tan^{-1}(ax))}{dax}\frac{dax}{dx}$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{1+(ax)^2}\right)a$$

$$\Rightarrow \frac{a}{2(1+a^2x^2)}$$

Ans) $\frac{a}{2(1+a^2x^2)}$

Question 37.
Differentiate each of the following w.r.t x:

$$\sin^{-1}\left\{\frac{x^2}{\sqrt{x^4+a^4}}\right\}$$

Answer:

To find: Value of $\sin^{-1}\left\{\frac{x^2}{\sqrt{x^4+a^4}}\right\}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\left\{\frac{x^2}{\sqrt{x^4+a^4}}\right\}$

$\Rightarrow$ Putting $x^2 = a^2 \cot\theta$

$\theta = \cot^{-1}\left(\frac{x^2}{a^2}\right)$ ... (i)

Putting $x^2 = a^2 \cot\theta$ in the equation

$$\Rightarrow \sin^{-1}\left\{\frac{a^2 \cot\theta}{\sqrt{(a^2 \cot\theta)^2 + a^4}}\right\}$$

$$\Rightarrow \sin^{-1}\left\{\frac{a^2 \cot\theta}{\sqrt{a^4 \cot^2\theta + a^4}}\right\}$$

$$\Rightarrow \sin^{-1}\left\{\frac{a^2 \cot\theta}{\sqrt{a^4(\cot^2\theta + 1)}}\right\}$$

$$\Rightarrow \sin^{-1}\left\{\frac{a^2 \cot\theta}{a^2 cosec\theta}\right\}$$

$$\Rightarrow \sin^{-1}\{\cos\theta\}$$

$$\Rightarrow \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - \theta\right)\right\}$$

$$\Rightarrow \frac{\pi}{2} - \theta$$

$$\Rightarrow \frac{\pi}{2} - \cot^{-1}\left(\frac{x^2}{a^2}\right)$$

Now, we can see that $\sin^{-1}\left\{\frac{x^2}{\sqrt{x^4 + a^4}}\right\} = \frac{\pi}{2} - \cot^{-1}\left(\frac{x^2}{a^2}\right)$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\pi}{2} - \cot^{-1}\left(\frac{x^2}{a^2}\right)\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\pi}{2}\right)}{dx} - \frac{d\left(\cot^{-1}\left(\frac{x^2}{a^2}\right)\right)}{dx}$$

$$\Rightarrow 0-\frac{d\left(\cot^{-1}\left(\frac{x^2}{a^2}\right)\right)}{d\frac{x^2}{a^2}}\frac{d\frac{x^2}{a^2}}{dx}$$

$$\Rightarrow \left(\frac{1}{1+\left(\frac{x^2}{a^2}\right)^2}\right)\frac{1}{a^2}2x$$

$$\Rightarrow \left(\frac{a^4}{a^4+x^4}\right)\frac{1}{a^2}2x$$

$$\Rightarrow \left(\frac{2a^2x}{a^4+x^4}\right)$$

Ans) $\frac{2a^2x}{a^4+x^4}$

Question 38.
Differentiate each of the following w.r.t x:

$$\tan^{-1}\left\{\frac{e^{2x}+1}{e^{2x}-1}\right\}$$

Answer:
To find: Value of $\tan^{-1}\left\{\frac{e^{2x}+1}{e^{2x}-1}\right\}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left\{\frac{e^{2x}+1}{e^{2x}-1}\right\}$

$$\Rightarrow \tan^{-1}\left\{\frac{1+e^{2x}}{-(1-e^{2x})}\right\}$$

$$-\tan^{-1}\left\{\frac{1+e^{2x}}{1-e^{2x}}\right\}$$

Putting $e^{2x} = \tan\theta$

$\theta = \tan^{-1}(e^{2x})$ ... (i)

Putting $e^{2x} = \tan\theta$ in the equation

$$\Rightarrow -\tan^{-1}\left\{\frac{1+\tan\theta}{1-\tan\theta}\right\}$$

$$\Rightarrow -\tan^{-1}\left\{\frac{tan\frac{\pi}{4}+\tan\theta}{1-tan\frac{\pi}{4}\tan\theta}\right\}$$

$$\Rightarrow -\tan^{-1}\left\{\tan\left(\frac{\pi}{4}+\theta\right)\right\}$$

$$\Rightarrow -\left(\frac{\pi}{4}+\theta\right)$$

$$\Rightarrow -\frac{\pi}{4}-\theta$$

$$\Rightarrow -\frac{\pi}{4}-\tan^{-1}(e^{2x})$$

Now, we can see that $\tan^{-1}\left\{\frac{e^{2x}+1}{e^{2x}-1}\right\} = -\frac{\pi}{4}-\tan^{-1}(e^{2x})$

Now Differentiating

$$\Rightarrow \frac{d\left(-\frac{\pi}{4}-\tan^{-1}(e^{2x})\right)}{dx}$$

$$\Rightarrow \frac{d\left(-\frac{\pi}{4}\right)}{dx}-\frac{d(\tan^{-1}(e^{2x}))}{dx}$$

$$\Rightarrow 0 - \frac{d(\tan^{-1}(e^{2x}))}{de^{2x}}\frac{de^{2x}}{d2x}\frac{d2x}{dx}$$

$$\Rightarrow -\left(\frac{1}{1+(e^{2x})^2}\right)e^{2x}2$$

$$\Rightarrow -\left(\frac{2e^{2x}}{1+e^{4x}}\right)$$

$$\Rightarrow \frac{-2e^{2x}}{1+e^{4x}}$$

Ans) $\frac{-2e^{2x}}{1+e^{4x}}$

Question 39.
Differentiate each of the following w.r.t x:

$$\cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2}$$

Answer:
To find: Value of $\cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2}$

The formula used: (i) $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\cos^{-1}x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$

We have, $\cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2}$

Putting 2x = cosθ

θ = $\cos^{-1}(2x)$ ... (i)

Putting e^{2x} = tanθ in the equation

$$\Rightarrow \cos^{-1}(\cos\theta) + 2\cos^{-1}\sqrt{1-(\cos\theta)^2}$$

$\Rightarrow \cos^{-1}(\cos\theta) + 2\cos^{-1}\sqrt{1-\cos^2\theta}$

$\Rightarrow \theta + 2\cos^{-1}\sqrt{\sin^2\theta}$

$\Rightarrow \theta + 2\cos^{-1}(\sin\theta)$

$\Rightarrow \theta + 2\cos^{-1}\left(\cos\left(\frac{\pi}{2} - \theta\right)\right)$

$\Rightarrow \theta + 2\left(\frac{\pi}{2} - \theta\right)$

$\Rightarrow \pi - \theta$

$\Rightarrow \pi - \cos^{-1}(2x)$

Now, we can see that $\cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2} = \pi - \cos^{-1}(2x)$

Now Differentiating

$$\Rightarrow \frac{d(\pi - \cos^{-1}(2x))}{dx}$$

$$\Rightarrow \frac{d(\pi)}{dx} - \frac{d(\cos^{-1}(2x))}{dx}$$

$$\Rightarrow 0 - \frac{d(\cos^{-1}(2x))}{d2x}\frac{d2x}{dx}$$

$$\Rightarrow \left(\frac{1}{\sqrt{1-(2x)^2}}\right)2$$

$$\Rightarrow \left(\frac{2}{\sqrt{1-4x^2}}\right)$$

Ans) $\frac{2}{\sqrt{1-4x^2}}$

Question 40.
Differentiate each of the following w.r.t x:

$$\tan^{-1}\left(\frac{a-x}{1+ax}\right)$$

Answer:
To find: Value of $\tan^{-1}\left\{\frac{a-x}{1+ax}\right\}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$

We have, $\tan^{-1}\left\{\frac{a-x}{1+ax}\right\}$

$\Rightarrow \tan^{-1}a - \tan^{-1}x$

Now Differentiating

$$\Rightarrow \frac{d(\tan^{-1}a - \tan^{-1}x)}{dx}$$

$$\Rightarrow \frac{d(\tan^{-1}a)}{dx} - \frac{d(\tan^{-1}x)}{dx}$$

$$\Rightarrow 0 - \frac{1}{1+x^2}$$

Ans) $-\frac{1}{1+x^2}$

Question 41.

Differentiate each of the following w.r.t x:

$$\tan^{-1}\left\{\frac{\sqrt{x}-x}{1+x^{3/2}}\right\}$$

Answer:

To find: Value of $\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{\frac{3}{2}}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{\frac{3}{2}}}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{\sqrt{x}-x}{1+x\sqrt{x}}\right)$$

$$\Rightarrow \tan^{-1}\sqrt{x} - \tan^{-1}x$$

Now Differentiating

$$\Rightarrow \frac{d\left(\tan^{-1}\sqrt{x} - \tan^{-1}x\right)}{dx}$$

$$\Rightarrow \frac{d\left(\tan^{-1}\sqrt{x}\right)}{dx} - \frac{d(\tan^{-1}x)}{dx}$$

$$\Rightarrow \frac{d\left(\tan^{-1}\sqrt{x}\right)}{d\sqrt{x}}\frac{d\sqrt{x}}{x} - \frac{d(\tan^{-1}x)}{dx}$$

$$\Rightarrow \frac{1}{1+\left(\sqrt{x}\right)^2}\frac{1}{2\sqrt{x}} - \frac{1}{1+x^2}$$

$$\Rightarrow \frac{1}{2\sqrt{x}(1+x)} - \frac{1}{1+x^2}$$

Ans) $\frac{1}{2\sqrt{x}(1+x)} - \frac{1}{1+x^2}$

Question 42.
Differentiate each of the following w.r.t x:

$$\tan^{-1}\left(\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}\right)$$

Answer:
To find: Value of $\tan^{-1}\left(\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{x}\sqrt{a}}\right)$$

$$\Rightarrow \tan^{-1}\sqrt{a} + \tan^{-1}\sqrt{x}$$

Now Differentiating

$$\Rightarrow \frac{d\left(\tan^{-1}\sqrt{a} + \tan^{-1}\sqrt{x}\right)}{dx}$$

$$\Rightarrow \frac{d\left(\tan^{-1}\sqrt{a}\right)}{dx} - \frac{d\left(\tan^{-1}\sqrt{x}\right)}{dx}$$

$$\Rightarrow 0 - \frac{d(\tan^{-1}\sqrt{x})}{d\sqrt{x}}\frac{d\sqrt{x}}{x}$$

$$\Rightarrow \ -\frac{1}{1+(\sqrt{x})^2}\frac{1}{2\sqrt{x}}$$

$$\Rightarrow \ -\frac{1}{2\sqrt{x}(1+x)}$$

Ans) $-\frac{1}{2\sqrt{x}(1+x)}$

Question 43.
Differentiate each of the following w.r.t x:

$$\tan^{-1}\left(\frac{3-2x}{1+6x}\right)$$

Answer:
Given: Value of $\tan^{-1}\left(\frac{3-2x}{1+6x}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{3-2x}{1+6x}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{3-2x}{1+3\times 2x}\right)$$

$$\Rightarrow \ \tan^{-1}3 - \tan^{-1}2x$$

Now Differentiating

$$\Rightarrow \frac{d(\tan^{-1}3 - \tan^{-1}2x)}{dx}$$

$$\Rightarrow 0 - \frac{d(\tan^{-1}2x)}{d2x}\frac{d2x}{dx}$$

$$\Rightarrow -\frac{1}{1+(2x)^2}2$$

$$\Rightarrow -\frac{2}{1+4x^2}$$

Ans) $-\frac{2}{1+4x^2}$

Question 44.
Differentiate each of the following w.r.t x:

$$\tan^{-1}\left(\frac{5x}{1-6x^2}\right)$$

Answer:
Given: Value of $\tan^{-1}\left(\frac{5x}{1-6x^2}\right)$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{5x}{1-6x^2}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{3x+2x}{1-3x\times 2x}\right)$$

$$\Rightarrow \tan^{-1}3x + \tan^{-1}2x$$

Now Differentiating

$$\Rightarrow \frac{d(\tan^{-1} 3x \mp \tan^{-1} 2x)}{dx}$$

$$\Rightarrow \frac{d(\tan^{-1} 3x)}{d3x}\frac{d3x}{dx} + \frac{d(\tan^{-1} 2x)}{d2x}\frac{d2x}{dx}$$

$$\Rightarrow \frac{1}{1+(3x)^2}3 + \frac{1}{1+(2x)^2}2$$

$$\Rightarrow \frac{3}{1+9x^2} + \frac{2}{1+4x^2}$$

Ans) $\frac{3}{1+9x^2} + \frac{2}{1+4x^2}$

Question 45.
Differentiate each of the following w.r.t x:

$$\tan^{-1}\left(\frac{2x}{1+15x^2}\right)$$

Answer:
Given: Value of $\tan^{-1}\left(\frac{2x}{1+15x^2}\right)$

The formula used: (i) $\cos \theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{2x}{1+15x^2}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{5x-3x}{1+5x \times 3x}\right)$$

$$\Rightarrow \tan^{-1} 5x - \tan^{-1} 3x$$

Now Differentiating

$$\Rightarrow \frac{d(\tan^{-1}5x - \tan^{-1}3x)}{dx}$$

$$\Rightarrow \frac{d(tan^{-1}5x)}{d5x}\frac{d5x}{dx} - \frac{d(tan^{-1}3x)}{d3x}\frac{d3x}{dx}$$

$$\Rightarrow \frac{1}{1+(5x)^2}5 + \frac{1}{1+(3x)^2}3$$

$$\Rightarrow \frac{5}{1+25x^2} + \frac{3}{1+9x^2}$$

Ans) $\frac{5}{1+25x^2} + \frac{3}{1+9x^2}$

Question 46.

Differentiate each of the following w.r.t x:

If $t = \tan^{-1}\left(\frac{ax-b}{bx+a}\right)$, prove that $\frac{dy}{dx} = \frac{1}{(1+x^2)}$.

Answer:

Given: Value of $\tan^{-1}\left(\frac{ax-b}{bx+a}\right)$

To Prove: $\frac{dy}{dx} = \frac{1}{1+x^2}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\tan^{-1}\left(\frac{ax-b}{bx+a}\right)$

Dividing numerator and denominator with a

$$\Rightarrow \tan^{-1}\left(\frac{\frac{ax-b}{a}}{\frac{bx+a}{a}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{x-\frac{b}{a}}{1+\frac{b}{a}x}\right)$$

$$\Rightarrow \tan^{-1}x - \tan^{-1}\left(\frac{b}{a}\right)$$

Now Differentiating

$$\Rightarrow \frac{d\left(\tan^{-1}x - \tan^{-1}\left(\frac{b}{a}\right)\right)}{dx}$$

$$\Rightarrow \frac{d(tan^{-1}x)}{dx} - \frac{d\left(tan^{-1}\left(\frac{b}{a}\right)\right)}{dx}$$

$$\Rightarrow \frac{1}{1+x^2} + 0$$

Ans) $\frac{1}{1+x^2}$

Question 47.
Differentiate each of the following w.r.t x:

If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$, show that $\frac{dy}{dx} = \frac{4}{(1+x^2)}$.

Answer:
Given: Value of $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

To Prove: $\frac{dy}{dx} = \frac{4}{(1+x^2)}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

Putting x = tanθ

$\theta = \tan^{-1}x$

Dividing numerator and denominator with a

$$\Rightarrow \sin^{-1}\left(\frac{2\tan\theta}{1+(\tan\theta)^2}\right) + \sec^{-1}\left(\frac{1+(\tan\theta)^2}{1-(\tan\theta)^2}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + \sec^{-1}\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)$$

$$\Rightarrow \sin^{-1}(\sin 2\theta) + \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$$

$$\Rightarrow \sin^{-1}(\sin 2\theta) + \sec^{-1}(\sec 2\theta)$$

$$\Rightarrow 2\theta + 2\theta$$

$$\Rightarrow 4\theta$$

$\Rightarrow 4\tan^{-1}x$

Now Differentiating

$$\Rightarrow \frac{d(4\tan^{-1}x)}{dx}$$

$$\Rightarrow 4\frac{1}{1+x^2}$$

Ans) $\frac{4}{1+x^2}$

Question 48.
Differentiate each of the following w.r.t x:

If $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$, show that $\frac{dy}{dx} = 0.$

Answer:
Given: Value of $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

To Prove: $\frac{dy}{dx} = 0$

Formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$\Rightarrow \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$\Rightarrow \frac{\pi}{2}$

Now Differentiating

$\Rightarrow \frac{d\left(\frac{\pi}{2}\right)}{dx}$

$\Rightarrow 0$

Ans) $\frac{4}{1+x^2}$

Question 49.
Differentiate each of the following w.r.t x:

If $y = \sin\left\{2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right\}$, show that $\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}$.

Answer:

Given: Value of $y = \sin\left\{2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right\}$

To Prove: $\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}$

Formula used: (i) $\frac{d(\cos^{-1}x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$

Let x = cosθ

$\theta = \cos^{-1}x$

Putting x = cosθ in equation

$$\Rightarrow \sin\left\{2\tan^{-1}\left(\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)\right\}$$

$$\Rightarrow \sin\left\{2\tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\right)\right\}$$

$$\Rightarrow \sin\left\{2\tan^{-1}\left(\sqrt{\tan^2\frac{\theta}{2}}\right)\right\}$$

$$\Rightarrow \sin\left\{2\tan^{-1}\left(\tan\frac{\theta}{2}\right)\right\}$$

$\Rightarrow \sin\left\{2\frac{\theta}{2}\right\}$

$\Rightarrow \sin\theta$

$\Rightarrow \sin(\cos^{-1}x)$

Now Differentiating

$$\Rightarrow \frac{d(\sin(\cos^{-1}x))}{dx}$$

$$\Rightarrow \frac{d(\sin(\cos^{-1}x))}{d\cos^{-1}x}\frac{d\cos^{-1}x}{dx}$$

$$\Rightarrow -\cos(\cos^{-1}x)\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow -\frac{x}{\sqrt{1-x^2}}$$

Ans) $\frac{4}{1+x^2}$

Question 50.
Differentiate each of the following w.r.t x:

If $y = \tan^{-1}\left\{\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right\}$. Prove that $\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$.

Answer:
Given: Value of $y = \tan^{-1}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

To Prove: $\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\cos^{-1}x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$

Let $x = \cos 2\theta$

$2\theta = \cos^{-1}x$

$\theta = \frac{1}{2}\cos^{-1}x$

Putting $x = \cos 2\theta$

$$y = \tan^{-1}\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}$$

$$y = \tan^{-1}\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}$$

$$y = \tan^{-1}\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}$$

$$y = \tan^{-1}\frac{\sqrt{2}(\cos\theta-\sin\theta)}{\sqrt{2}(\cos\theta+\sin\theta)}$$

Dividing by $\cos\theta$ in the numerator and denominator

$$y = \tan^{-1}\frac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\cos\theta+\sin\theta}{\cos\theta}}$$

$$y = \tan^{-1}\frac{1-\tan\theta}{1+\tan\theta}$$

$$y = \tan^{-1}\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}$$

$$y = \tan^{-1}\tan\left(\frac{\pi}{4} - \theta\right)$$

$$y = \frac{\pi}{4} - \theta$$

$$y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}x\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\pi}{4}\right)}{dx} - \frac{1}{2}\frac{d\cos^{-1}x}{dx}$$

$$\Rightarrow \frac{1}{2}\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{1}{2\sqrt{1-x^2}}$$

Ans) $\frac{1}{2\sqrt{1-x^2}}$

Question 51.
Differentiate each of the following w.r.t x:

Differentiate $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ w. r. t. x

Answer:
Given: Value of $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

To find: $\frac{dy}{dx}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d\left(\tan^{-1}x\right)}{dx} = \frac{1}{1+x^2}$

$$y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$$

$$y = \sin^{-1}\left(\frac{2^x.2}{1+\left(2^2\right)^x}\right)$$

$$y = \sin^{-1}\left(\frac{2^x.2}{1+\left(2^x\right)^2}\right)$$

Let $2^x = \tan\theta$

$\theta = \tan^{-1}(2^x)$

Putting $2^x = \tan\theta$

$$y = \sin^{-1}\left(\frac{\tan\theta.2}{1+(\tan\theta)^2}\right)$$

$$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$y = \sin^{-1}(\sin 2\theta)$$

$$y = 2\theta$$

$$y = 2\tan^{-1}(2^x)$$

Now Differentiating

$$\Rightarrow \frac{d(2\tan^{-1}(2^x))}{dx}$$

$$\Rightarrow 2\frac{d(\tan^{-1}(2^x))}{d2^x}\frac{d2^x}{dx}$$

$$\Rightarrow 2\frac{1}{1+(2^x)^2}.\,2^x\log 2$$

$$\Rightarrow \frac{2^{1+x}\log 2}{1+4^x}.$$

Ans) $\frac{2^{1+x}\log 2}{1+4^x}$