Exercise 28e

Question 1.
Find the equation of the plane through the line of intersection of the planes x + y + z = 6 and 2x + 2y + 4z + 5 = 0, and passing through the point (1, 1, 1).

Answer:
Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x + B_1y + C_1z + D_1 + \lambda(A_2x + B_2y + C_2z + D_2) = 0 \quad (1)$$

For the standard equation of planes,

$$A_1x + B_1y + C_1z + D_1 \text{ and } A_2x + B_2y + C_2z + D_2$$

So, putting in equation (1), we have

$x + y + z - 6 + \lambda(2x + 2y + 4z + 5) = 0$

$(1 + 2\lambda)x + (1 + 2\lambda)y + (1 + 4\lambda)z - 6 + 5\lambda = 0$ (2)

Now plane passes through (1,1,1) then it must satisfy the plane equation,

$(1 + 2\lambda).1 + (1 + 2\lambda).1 + (1 + 4\lambda).1 - 6 + 5\lambda = 0$

$1 + 2\lambda + 1 + 2\lambda + 1 + 4\lambda - 6 + 5\lambda = 0$

$3 + 8\lambda - 6 + 5\lambda = 0$

$13\lambda = 3$

$$\lambda = \frac{3}{13}$$

Putting in equation (2)

$$\left(1 + 2.\frac{3}{13}\right)x + \left(1 + 2.\frac{3}{13}\right)y + \left(1 + 4.\frac{3}{13}\right)z - 6 + 5.\frac{3}{13} = 0$$

$$\left(\frac{13+6}{13}\right)x+\left(\frac{13+6}{13}\right)y+\left(\frac{13+12}{13}\right)z+\frac{-78+15}{13}=0$$

19x + 19y + 25z-63=0

So, the required equation of plane is 19x + 19y + 25z=63.

Question 2.
Find the equation of the plane through the line of intersection of the planes x - 3y + z + 6 = 0 and x + 2y + 3z + 5 = 0, and passing through the origin.

Answer:
Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda\left(A_2x+B_2y+C_2z+D_2\right)=0 \quad (1)$$

For the standard equation of planes,

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation (1), we have

x-3y + z + 6 + λ(x + 2y + 3z + 5)=0

(1 + λ)x + (-3 + 2λ)y + (1 + 3λ)z + 6 + 5λ=0 (2)

Now plane passes through (0,0,0) then it must satisfy the plane equation,

(1 + λ).0 + (-3 + 2λ).0 + (1 + 3λ).0 + 6 + 5λ=0

5λ=-6

$$\lambda=\frac{-6}{5}$$

Putting in equation (2)

$$\left(1+\frac{-6}{5}\right)x+\left(-3+2.\frac{-6}{5}\right)y+\left(1+3.\frac{-6}{5}\right)z+6+5.\frac{-6}{5}=0$$

$$\left(\frac{5+(-6)}{5}\right)x+\left(\frac{-15-12}{5}\right)y+\left(\frac{5+(-18)}{5}\right)z+\frac{30+(-30)}{5}=0$$

-x-27y-13z=0

x + 27y + 13z=0

So, required equation of plane is x + 27y + 13z=0.

Question 3.
Find the equation of the plane passing through the intersection of the planes 2x + 3y – z + 1 = 0 and x + y – 2z + 3 = 0, and perpendicular to the plane 3x - y -2z -4 = 0.

Answer:
Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0$$ (1)

For the standard equation of planes,

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation (1), we have

2x + 3y–z + 1 + λ(x + y–2z + 3)=0

(2 + λ)x + (3 + λ)y + (-1-2λ)z + 1 + 3λ=0 (2)

Now as the plane 3x-y-2z-4=0 is perpendicular the given plane,

For θ=90°, cos90°=0

$$A_1A_2+B_1B_2+C_1C_2=0$$ (3)

On comparing with standard equations in Cartesian form,

$$A_1 = 2+\lambda, B_1 = 3+\lambda, C_1 = -1-2\lambda \text{ and } A_2 = 3, B_2 = -1, C_2 = -2$$

Putting values in equation (3), we have

(2 + λ).3 + (3 + λ).(-1) + (-1-2λ).(-2)=0

6 + 3λ-3-λ + 2 + 4λ=0

5 + 6λ=0

$$\lambda = \frac{-5}{6}$$

Putting in equation(2)

$$\left(2+\frac{-5}{6}\right)x+\left(3+\frac{-5}{6}\right)y+\left(-1-2.\frac{-5}{6}\right)z+1+3.\frac{-5}{6}=0$$

$$\left(\frac{12-5}{6}\right)x+\left(\frac{18-5}{6}\right)y+\left(\frac{-6+10}{6}\right)z+\frac{6-15}{6}=0$$

7x + 13y + 4z-9=0

7x + 13y + 4z=9

So, required equation of plane is 7x + 13y + 4z=9.

Question 4.
Find the equation of the plane passing through the line of intersection of the planes 2x - y = 0 and 3z - y = 0, and perpendicular to the plane 4x + 5y - 3z = 9.

Answer:
Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda\left(A_2x+B_2y+C_2z+D_2\right)=0 \text{ (1)}$$

For the standard equation of planes,

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation (1), we have

2x-y + λ(3z-y)=0

2x + (-1-λ)y + 3λz=0 (2)

Now as the plane is perpendicular the given plane,

For θ=90°, cos90°=0

$A_1A_2+B_1B_2+C_1C_2=0$ (3)

On comparing with standard equations in Cartesian form,

$$A_1=2, B_1=-1-\lambda, C_1=3\lambda \text{ and } A_2=4, B_2=5, C_2=-3$$

Putting values in equation(3),

2.4 + (-1-λ).5 + 3λ.-3=0

8-5-5λ-9λ=0

-14λ=-3

$$\lambda=\frac{3}{14}$$

Putting in equation(2)

$$2x+\left(-1-\frac{3}{14}\right)y+3\left(\frac{3}{14}\right)z=0$$

$$2x+\left(\frac{-14-3}{14}\right)y+\frac{9}{14}.z=0$$

28x-17y + 9z=0

So, required equation of plane is 28x-17y + 9z=0.

Question 5.
Find the equation of the plane passing through the intersection of the planes x - 2y + z = 1 and 2x + y + z = 8, and parallel to the line with direction ratios 1, 2, 1. Also, find the perpendicular distance of (1, 1, 1) from the plane.

Answer:
Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda\left(A_2x+B_2y+C_2z+D_2\right)=0 \quad (1)$$

For the standard equation of planes,

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation (1), we have

x-2y + z-1 + λ(2x + y + z-8)=0

(1 + 2λ)x + (-2 + λ)y + (1 + λ)z-1-8λ=0 (2)

For plane the normal is perpendicular to line given parallel to this i.e.

$$A_1A_2+B_1B_2+C_1C_2=0$$

Where A_1, B_1, C_1 are direction ratios of plane and A_2, B_2, C_2 are of line.

(1 + 2λ).1 + (-2 + λ).2 + (1 + λ).1=0

1 + 2λ-4 + 2λ + 1 + λ=0

-2 + 5λ=0

$$\lambda=\frac{2}{5}$$

Putting the value of λ in equation (2)

$$\left(1+2.\left(\frac{2}{5}\right)\right).x+\left(-2+\frac{2}{5}\right).y+\left(1+\frac{2}{5}\right).z-1-8.\left(\frac{2}{5}\right)=0$$

$$\left(\frac{5+4}{5}\right)x+\left(\frac{-10+2}{5}\right)y+\left(\frac{5+2}{5}\right)z+\frac{-5-16}{5}=0$$

9x-8y + 7z-21=0

9x-8y + 7z=21

For the equation of plane Ax + By + Cz=D and point (x1,y1,z1), a distance of a point from a plane can be calculated as

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

$$\left|\frac{9.1-8.1+7.1-21}{\sqrt{(9)^2+(-8)^2+(7)^2}}\right| \Rightarrow \left|\frac{9-8+7-21}{\sqrt{81+64+49}}\right| = \left|\frac{13}{\sqrt{194}}\right|$$

So, the required equation of the plane is 9x-8y + 7z=21, and distance of the plane from (1,1,1) is

$$d=\frac{13}{\sqrt{194}}$$

Question 6.
Find the equation of the plane passing through the line of intersection of the planes x + 2y + 3z – 5 = 0 and 3x - 2y –z + 1 = 0 and cutting off equal intercepts on the x-axis and z-axis.

Answer:
Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0 \quad (1)$$

For the standard equation of planes in Cartesian form

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation 1 we have

x + 2y + 3 z-5 + λ(3x -2y -z + 1)=0

(1 + 3λ)x + (2-2λ)y + (3-λ)z-5 + λ=0

Now equation of plane in intercept form

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

As given equal intercept means a=c

First, we transform equation of a plane in intercept form

$$\frac{x}{\frac{1}{(1+3\lambda)}}+\frac{y}{\frac{1}{(2-2\lambda)}}+\frac{z}{\frac{1}{(3-\lambda)}}=5-\lambda$$

$$\frac{x}{\frac{5-\lambda}{(1+3\lambda)}}+\frac{y}{\frac{5-\lambda}{(2-2\lambda)}}+\frac{z}{\frac{5-\lambda}{(3-\lambda)}}=1$$

On comparing with the standard equation of a plane in intercept form

$$a=\frac{5-\lambda}{(1+3\lambda)},c=\frac{5-\lambda}{(3-\lambda)}$$

Now as a=b=c

$$\frac{5-\lambda}{(1+3\lambda)}=\frac{5-\lambda}{(3-\lambda)} \Rightarrow 3-\lambda=1+3\lambda$$

$$4\lambda = 2 \Rightarrow \lambda = \frac{1}{2}$$

Putting in equation (2), we have

$$\left(1+3.\frac{1}{2}\right)x+\left(2-2.\frac{1}{2}\right)y+\left(3-\frac{1}{2}\right)z-5+\frac{1}{2}=0$$

$$\left(\frac{2+3}{2}\right)x+\left(\frac{4-2}{2}\right)y+\left(\frac{6-1}{2}\right)z+\frac{-10+1}{2}=0$$

5x + 2y + 5z-9=0

5x + 2y + 5z=9

So, required equation of plane is 5x + 2y + 5z=9.

Question 7.
Find the equation of the plane through the intersection of the planes 3x – 4y + 5z =10 and 2x + 2y - 3z = 4 and parallel to the line x = 2y = 3z.

Answer:
Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda\left(A_2x+B_2y+C_2z+D_2\right)=0 \quad (1)$$

For the standard equation of planes in Cartesian form

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation (1), we have

3x – 4y + 5z -10 + λ(2x + 2y - 3z - 4)=0

(3 + 2λ)x + (-4 + 2λ)y + (5-3λ)z-10-4λ=0

Given line is parallel to plane then the normal of plane is perpendicular to line,

$$A_1A_2 + B_1B_2 + C_1C_2 = 0$$

Where A_1, B_1, C_1 are direction ratios of plane and A_2, B_2, C_2 are of line.

(3 + 2λ).6 + (-4 + 2λ).3 + (5-3λ).2=0

18 + 12λ-12 + 6λ + 10-6λ=0

16 + 12λ=0

$$\lambda = \frac{-16}{12} \Rightarrow \frac{-4}{3}$$

Putting the value of λ in equation (2)

$$\left(3+2.\left(\frac{-4}{3}\right)\right)x+\left(-4+2.\left(\frac{-4}{3}\right)\right)y+\left(5-3\left(\frac{-4}{3}\right)\right)z-10-4.\left(\frac{-4}{3}\right)=0$$

$$\left(\frac{9-8}{3}\right)x+\left(\frac{-12-8}{3}\right)y+\left(\frac{15+12}{3}\right)z+\frac{-30+16}{3}=0$$

x-20y + 27z-14=0

So, required equation of plane is x-20y + 27z-14=0.

Question 8.

Find the vector equation of the plane through the intersection of the planes $\vec{r}.\left(\hat{i}+3\hat{j}-\hat{k}\right)=0$ and $\vec{r}.\left(\hat{j}+2\hat{k}\right)=0,$ and passing through the point (2, 1, -1).

Answer:

Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2 \text{ where } \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1 = d_1$ and $\vec{r}.\vec{n}_2 = d_2$

Putting values in equation(1)

$$\vec{r}(\hat{i}+3\hat{j}-\hat{k}+\lambda\left(\hat{j}+2\hat{k}\right)=0+\lambda.0$$

$$\vec{r}\left(\hat{i}+(3+\lambda)\hat{j}+(-1+2\lambda)\hat{k}\right)=0 \quad (2)$$

Now as the plane passes through (2,1,-1)

$$\vec{r} = 2\hat{i}+\hat{j}-\hat{k}$$

Putting in equation (2)

$$\left(2\hat{i}+\hat{j}-\hat{k}\right)(\left(\hat{i}+(3+\lambda)\hat{j}+(-1+2\lambda)\hat{k}\right)=0$$

2.1 + 1.(3 + λ) + (-1)(-1 + 2λ)=0

2 + 3 + λ + 1-2λ=0

λ=6

Putting the value of λ in equation (2)

$$\vec{r}\left(\hat{i}+(3+6)\hat{j}+(-1+2(6))\hat{k}\right)=0$$

$$\vec{r}\left(\hat{i}+9\hat{j}+11\hat{k}\right)=0$$

So, required equation of plane is $\vec{r}\left(\hat{i}+9\hat{j}+11\hat{k}\right)=0$.

Question 9.

Find the vector equation of the plane through the point (1, 1, 1), and passing through the intersection of the planes $\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)+1=0$ and $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)-5=0.$

Answer:
Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation(1)

$\vec{r}(\hat{i}-\hat{j}+3\hat{k}+\lambda\left(2\hat{i}+\hat{j}-\hat{k}\right)=-1+\lambda.5$

$\vec{r}\left((1+2\lambda)\hat{i}+(-1+\lambda)\hat{j}+(3-\lambda)\hat{k}\right)=-1+5\lambda$ (2)

Now as the plane passes through (1,1,1)

$\vec{r}=\hat{i}+\hat{j}+\hat{k}$

Putting in equation (2)

$\left(\hat{i}+\hat{j}+\hat{k}\right)(\left((1+2\lambda)\hat{i}+(-1+\lambda)\hat{j}+(3-\lambda)\hat{k}\right)=-1+5\lambda$

1.(1 + 2λ) + 1.(-1 + λ) + 1.(3-λ)=-1 + 5λ

1 + 2λ-1 + λ + 3- λ + 1-5λ=0

-3λ + 4=0

$\lambda=\frac{4}{3}$

Putting the value of λ in equation (2)

$$\vec{r}\left(\left(1+2.\frac{4}{3}\right)\hat{i}+\left(-1+\frac{4}{3}\right)\hat{j}+\left(3-\frac{4}{3}\right)\hat{k}\right)=-1+5.\frac{4}{3}$$

$$\vec{r}\left(\left(\frac{3+8}{3}\right)\hat{i}+\left(\frac{-3+4}{3}\right)\hat{j}+\left(\frac{9-4}{3}\right)\hat{k}\right)=\frac{-3+20}{3}$$

$$\vec{r}\left(11\hat{i}+\hat{j}+5\hat{k}\right)=17$$

So, required equation of plane is $\vec{r}\left(11\hat{i}+\hat{j}+5\hat{k}\right)=17.$

Question 10.
Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.\left(2\hat{i}-7\hat{j}+4\hat{k}\right)=3$ and $\vec{r}.\left(3\hat{i}-5\hat{j}+4\hat{k}\right)+11=0,$ and passing through the point (-2, 1, 3).

Answer:
Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation(1)

$$\vec{r}(2\hat{i}-7\hat{j}+4\hat{k}+\lambda\left(3\hat{i}-5\hat{j}+4\hat{k}\right)=3-\lambda.11$$

$\vec{r}\left((2+3\lambda)\hat{i}+(-7-5\lambda)\hat{j}+(4+4\lambda)\hat{k}\right)=3-11\lambda$ (2)

Now as the plane passes through (-2,1,3)

$$\vec{r}=-2\hat{i}+\hat{j}+3\hat{k}$$

Putting in equation (2)

$$\left(-2\hat{i}+\hat{j}+3\hat{k}\right)(\left(\left(2+3\lambda\right)\hat{i}+\left(-7-5\lambda\right)\hat{j}+\left(4+4\lambda\right)\hat{k}\right)=3-11\lambda$$

-2.(2 + 3λ) + 1.(-7-5λ) + 3.(4 + 4λ)=3-11λ

-4-6λ-7-5λ + 12 + 12λ-3 + 11λ=0

-14 + 12 + 12λ=0

$$\lambda=\frac{1}{6}$$

Putting the value of λ in equation (2)

$$\vec{r}\left(\left(2+3.\frac{1}{6}\right)\hat{i}+\left(-7-5.\frac{1}{6}\right)\hat{j}+\left(4+4\frac{1}{6}\right)\hat{k}\right)=3-11.\frac{1}{6}$$

$$\vec{r}\left(\left(\frac{12+3}{6}\right)\hat{i}+\left(\frac{-42-5}{6}\right)\hat{j}+\left(\frac{24+4}{6}\right)\hat{k}\right)=\frac{18-11}{6}$$

$$\vec{r}\left(15\hat{i}-47\hat{j}+28\hat{k}\right)=7$$

So, required equation of plane is $\vec{r}\left(15\hat{i}-47\hat{j}+28\hat{k}\right)=7.$

Question 11.
Find the equation of the plane through the line of intersection of the planes $\vec{r}.\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=1$ and $\vec{r}.\left(\hat{i}-\hat{j}\right)+4=0$ and perpendicular to the plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)+8=0.$

Answer:
Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation (1), we have

$$\vec{r}(2\hat{i}-3\hat{j}+4\hat{k}+\lambda(\hat{i}-\hat{j})=1-\lambda.4$$

$$\vec{r}\left((2+\lambda)\hat{i}+(-3-\lambda)\hat{j}+4\hat{k}\right)=1-4\lambda \quad (2)$$

Given a plane perpendicular to this plane, So if n1 and n2 are normal

vectors of planes

$$\vec{n}_1.\vec{n}_2=0$$

$$(2\hat{i}-\hat{j}+\hat{k}).\left((2+\lambda)\hat{i}+(-3-\lambda)\hat{j}+4\hat{k}\right)=0$$

$2.(2+\lambda)+(-1).(-3-\lambda)+1.4=0$

$4+2\lambda+3+\lambda+4=0$

$11+3\lambda=0$

$$\lambda=\frac{-11}{3}$$

Putting the value of λ in equation (2)

$$\vec{r}\left(\left(2+\frac{-11}{3}\right)\hat{i}+\left(-3-\frac{-11}{3}\right)\hat{j}+4\hat{k}\right)=1-4.\frac{-11}{3}$$

$$\vec{r}\left(\left(\frac{6-11}{3}\right)\hat{i}+\left(\frac{-9+11}{3}\right)\hat{j}+4\hat{k}\right)=\frac{3+44}{3}$$

$$\vec{r}\left(-5\hat{i}-2\hat{j}+12\hat{k}\right)=47$$

So required equation of plane is $\vec{r}\left(-5\hat{i}-2\hat{j}+12\hat{k}\right)=47.$

Question 12.
Find the Cartesian and vector equations of the planes through the line of intersection of the planes $\vec{r}.\left(\hat{i}-\hat{j}\right)+6=0$ and $\vec{r}.\left(3\hat{i}+3\hat{j}-4\hat{k}\right)=0,$ which are at a unit distance from the origin.

Answer:
Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation (1)

$$\vec{r}(\hat{i}-\hat{j}+\lambda\left(3\hat{i}+3\hat{j}-4\hat{k}\right)=6+\lambda.0$$

$\vec{r}\left((1+3\lambda)\hat{i}+(-1+3\lambda)\hat{j}+(-4\lambda)\hat{k}\right)=6$ (2)

For the equation of plane Ax + By + Cz=D and point (x1,y1,z1), a distance of a point from a plane can be calculated as

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

$$\left|\frac{(1+3\lambda)0+(-1+3\lambda).0+(-4\lambda).0-6}{\sqrt{(1+3\lambda)^2+(-1+3\lambda)^2+(-4\lambda)^2)}}\right|=1$$

$$\left|\frac{-6}{\sqrt{1}+9\lambda^2+6\lambda+1+9\lambda^2-6\lambda+16\lambda^2}\right|=1$$

$$\sqrt{2+34\lambda^2}=-6$$

$$2+34\lambda^2=(-6)^2$$

$$34\lambda^2=36-2$$

$$34\lambda^2=34$$

$$\lambda^2=1\Rightarrow\lambda=1,-1$$

Putting value of λ in equation (2)

λ=1

$$\vec{r}\left((1+3.1)\hat{i}+(-1+3.1)\hat{j}+(-4.1)\hat{k}\right)=6$$

$$\vec{r}\left(4\hat{i}+2\hat{j}-4\hat{k}\right)=6\Rightarrow\vec{r}.\left(2\hat{i}+\hat{j}-2\hat{k}\right)=3$$

λ=-1

$$\vec{r}\left((1+3.(-1))\hat{i}+(-1+3(-1))\hat{j}+(-4(-1))\hat{k}\right)=6$$

$$\vec{r}\left(-2\hat{i}-4\hat{j}+4\hat{k}\right)=6\Rightarrow\vec{r}.\left(\hat{i}+2\hat{j}-2\hat{k}\right)=-3$$

For equations in Cartesian form put

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

For λ=1

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(2\hat{i} + \hat{j} - 2\hat{k} - 3\right) = 0$$

x.2 + y.1 + z.(-2)-3=0

2x + y-2z-3=0

For λ=-1

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(\hat{i} + 2\hat{j} - 2\hat{k} + 3\right) = 0$$

x.1 + y.2 + z.(-2) + 3=0

x + 2y-2z + 3=0

So, required equation of plane

in vector form are $\vec{r}.\left(2\hat{i} + \hat{j} - 2\hat{k}\right) = 3$ for $\lambda = 1$

$$\vec{r}.\left(\hat{i} + 2\hat{j} - 2\hat{k}\right) = -3 \text{ for } \lambda = -1$$

In Cartesian form are 2x + y-2z-3=0 & x + 2y-2z + 3=0