Exercise 11g

Question 1.
Show that the function $f(x)=5x-2$ is a strictly increasing function on R.

Answer:
Domain of the function is R

Finding derivative f′(x)=5

Which is greater than 0

Mean strictly increasing in its domain i.e R

Question 2.
Show the function $f(x)=-2x+7$ is a strictly decreasing function on R.

Answer:
Domain of the function is R

Finding derivative f′(x)=-2

Which is less than 0

Means strictly decreasing in its domain i.e R

Question 3.
Prove that $f(x)=ax+b$, where a and b are constants and a > 0, is a strictly increasing function on R.

Answer:
Domain of the function is R

Finding derivative i.e f′(x)=a

As given in question it is given that a>0

Derivative>0

Means strictly increasing in its domain i.e R

Question 4.
Prove that the function $f(x) = e^{2x}$ is strictly increasing on R.

Answer:
Domain of the function is R

finding derivative i.e $f'(x)=2e^{x}$

As we know e^{x} is strictly increasing its domain

$f'(x)>0$

hence f(x) is strictly increasing in its domain

Question 5.
Show that the function $f(x) = x^2$ is

a. strictly increasing on [0, ∞[

b. strictly decreasing on [0, ∞[

c. neither strictly increasing nor strictly decreasing on R

Answer:
Domain of function is **R.**

$f'(x)=2x$

for $x>0$ $f'(x)>0$ i.e. increasing

for $x<0$ $f'(x)<0$ i.e. decreasing

hence it is neither increasing nor decreasing in R

Question 6.
Show that the function $f(x) = |x|$ is

a. strictly increasing on]0, ∞[

b. strictly decreasing on] – ∞, 0[

Answer:
For x>0

Modulus will open with + sign

f(x)=+x

⇒ f′(x)=+1 which is <0

for x<0

Modulus will open with -ve sign

f(x)=-x => f′(x)=-1 which is >0

hence f(x) is increasing in x>0 and decreasing in x<0

Question 7.
Prove that the function $f(x)=\log_e x$ is strictly increasing on]0, ∞[.

Answer:
f(x)=ln(x)

$$f'(x) = \frac{1}{x}$$

for x<0

f′(x)=-ve →increasing

for x>0

f′(x)=+ve →decreasing

f(x) in increasing when x>0 i.e x∈(0,∞)

Question 8.
Prove that the function $f(x)=\log_a x$ is strictly increasing on]0, ∞[when a > 1 and strictly decreasing on]0, ∞[when 0 < a < 1.

Answer:
Consider $f(x)=\log_a x$

domain of f(x) is x>0

$$f'(x)=\frac{1}{x}\ln(a)$$

⇒ for a>1, ln(a)>0,

hence f′(x) >0 which means strictly increasing.

⇒ for 0<a<1, ln(a)<0,

hence f′(x)<0 which means strictly decreasing.

Question 9.
Prove that $f(x)=3^x$ is strictly increasing on R.

Answer:
Consider $f(x)=3^x$

The domain of f(x) is R.

$f'(x)=3^x\ln(3)$

3^x is always greater than 0 and ln(3) is also + ve.

Overall f′(x) is >0 means strictly increasing in its domain i.e. R.

Question 10.
Show that $f(x)=x^3-15x^2+75x-50$ is increasing on R.

Answer:
Consider $f(x)=x^3-15x^2+75x-50$

Domain of the function is R.

$f'(x)=3x^2-30x+75$

$=3(x^2-10+25)$

$=3(x-5)\ (x-5)$

$=3(x-5)^2$

$f'(x)=0$ for $x=5$

for $x<5$

$f'(x)>5$

and

for $x>5$

$f'(x)>5$

we can see throughout R the derivative is +ve but at x=5 it is 0 so it is increasing.

Question 11.

Show that $f(x)=\left(x-\frac{1}{x}\right)$ is increasing all $x \in R$, where $x \neq 0$.

Answer:

$f(x)=\left(x-\frac{1}{x}\right)$

domain of function is R-{0}

$f'(x)=1+\frac{1}{x^2}$

$f'(x)\ \forall\ x \in R$ is greater than 0.

Question 12.

Show that $f(x)=\left(\frac{1}{x}+5\right)$ is decreasing for all $x \in R$, where $x \neq 0$.

Answer:
$f(x)=\frac{1}{x}+5$

domain of function is R-{0}

$$f'(x)=-\frac{1}{x^2}$$

for all x, f′(x)<0

Hence function is decreasing.

Question 13.

Show that $f(x)=\frac{1}{\left(1+x^2\right)}$ is decreasing for all $x \geq 0$

Answer:
Consider $f(x)=\frac{1}{(1+x^2)}$,

$$f'(x)=-\frac{2x}{(1+x^2)^2}$$

for x ≥ 0 ,

f′(x) is -ve.

hence function is decreasing for x ≤ 0

Question 14.

Show that $f(x)=\left(x^3+\frac{1}{x^3}\right)$ is decreasing on $]-1,1[$.

Answer:
$f(x)=x^3+x^{-3}$

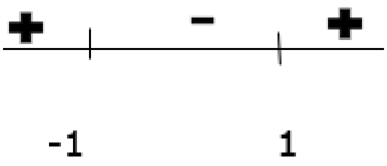
$f'(x)=3x^2-3x^{-4}$

$=3(x^2-1/x^4)$

$$= 3\left(\frac{x^3-1}{x^2} \cdot \frac{x^3+1}{x^2}\right)$$

$$= \frac{3(x-1)(x^2+x+1)(x+1)(x^2-x+1)}{x^4}$$

Root of $f'(x)$=1and-1



Here we can clearly see that $f'(x)$ is decreasing in [-1,1]

So, f(x) is decreasing in interval [-1,1]

Question 15.

Show that $f(x)=\frac{x}{\sin x}$ is increasing on $\left]0,\frac{\pi}{2}\right[$.

Answer:

Consider $f(x)=\frac{x}{\sin x}$,

$$f'(x)=\frac{\sin x - x.\cos x}{\sin^2 x}$$

$$f'(x)=\frac{\cos x(\tan x - x)}{\sin^2 x}$$

in $\left]0,\frac{\pi}{2}\right[$ cos>0,

tan x-x>0,

$\sin^2 x > 0$

hence $f'(x) > 0$,

so, function is increasing in the given interval.

Question 16.

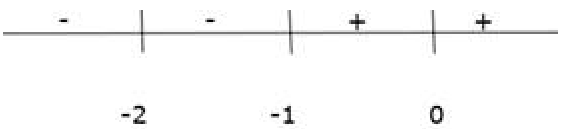
Prove that the function $f(x) = \log(1+x) - \frac{2x}{(x+2)}$ is increasing for all $x > -1$.

Answer:
Consider $f(x) = \log(1+x) - \frac{2x}{(x+2)}$,

$$f'(x) = \frac{1}{1+x} - \frac{4}{(x+2)^2}$$

$$= \frac{(x+2)^2 - 4(x+1)}{(x+1)(x+2)^2}$$

$$= \frac{x^2}{(x+1)(x+2)^2}$$



Clearly we can see that $f'(x) > o$ for $x > -1$.

Hence function is increasing for all $x > -1$

Question 17.

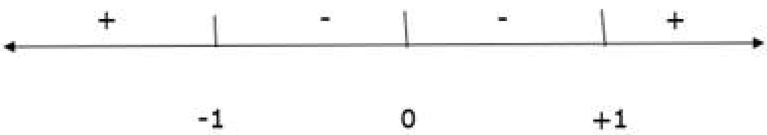
Let I be an interval disjoint from $]-1, 1[$. Prove that the function $f(x) = \left(x + \frac{1}{x}\right)$ is strictly increasing on I.

Answer:
Consider $f(x) = \left(x + \frac{1}{x}\right)$

$$f'(x) = 1 - \frac{1}{x^2}$$

$$f'(x) = \frac{x^2 - 1}{x^2}$$

$$= \frac{x - 1 \,.\, x + 1}{x^2}$$



We can see f′(x) <0 in [-1,1]

i.e. f(x) is decreasing in this interval.

We can see f′(x) >0 in (-∞, -1) ∪(1, ∞)

i.e. f(x) is increasing in this interval.

Question 18.

Show that $f(x) = \frac{(x-2)}{(x+1)}$ is increasing for all $x \in R$, except at $x = -1$.

Answer:

Consider $f(x) = \frac{(x-2)}{(x+1)}$,

$$f'(x) = \frac{3}{(x+1)^2}$$

f′(x) at x=-1 is not defined

and for all x ∈ R- {-1}

f′(x)>0

hence f(x) is increasing.

Question 19.

Find the intervals on which the function $f(x)=(2x^2-3x)$ is

(a) strictly increasing

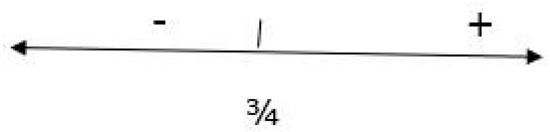
(b) strictly decreasing.

Answer:

$f(x)=(2x^2-3x)$

$f'(x)=4x-3$

$f'(x)=0$ at $x=3/4$



Clearly we can see that function is increasing for $x\in[3/4, \infty)$ and is decreasing for $x\in(-\infty,3/4)$

Question 20.

Find the intervals on which the function $f(x)=2x^3-3x^2-36x+7$ is

(a) strictly increasing (b) strictly decreasing.

Answer:

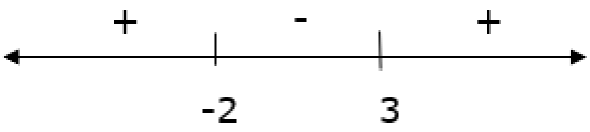
$f(x)=2x^3-3x^2-36x+7$

$f'(x)=6x^2-6x-36$

$f'(x)=6(x^2-x-6)$

$f'(x)=6(x-3)(x+2)$

$f'(x)$ is 0 at $x=3$ and $x=-2$



$F'(x)>0$ for $x \in (-\infty, -2] \cup [3, \infty)$

hence in this interval function is increasing.

$F'(x)<0$ for $x \in (-2, 3)$

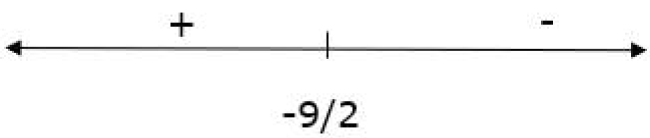
hence in this interval function is decreasing.

Question 21.
Find the intervals on which the function $f(x) = 6 - 9x - x^2$ is

(a) strictly increasing (b) strictly decreasing.

Answer:
$f(x)=6-9x-x^2$

$f'(x)=-(2x+9)$



We can see that f(x) is increasing for $x \in \left(-\infty, -\frac{9}{2}\right]$ and decreasing in $x \in \left(-\frac{9}{2}, \infty\right)$

Question 22.
Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

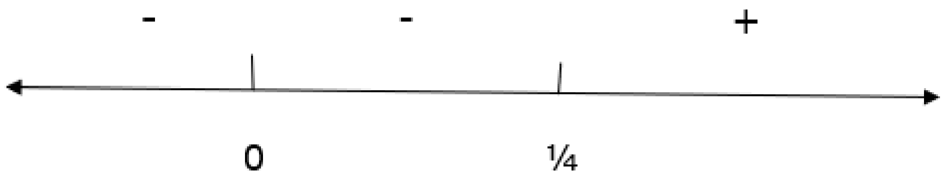
$$f(x) = \left(x^4 - \frac{x^3}{3}\right)$$

Answer:
Consider $f(x) = \left(x^4 - \frac{x^3}{3}\right)$

$f'(x)=4x^3-x^2$

$=x^2(4x-1)$

$F'(x)=0$ for $x=0$ and $x=1/4$



Function f(x) is decreasing for $x \in (-\infty,1/4]$ and increasing in $x \in (1/4 , \infty)$

Question 23.
Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

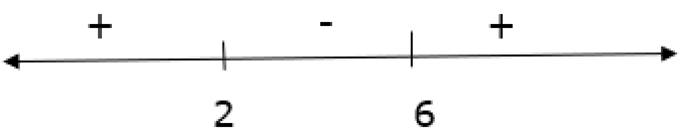
$$f(x)=x^3-12x^2+36x+17$$

Answer:
$f(x)=x^3-12x^2+36x+17$

$f'(x)=3x^2-24x+36$

$f'(x)=3(x^2-8x+12)$

$=3(x-6)(x-2)$



Function f(x) is decreasing for $x \in [2,6]$ and increasing in $x \in (-\infty,2) \cup (6, \infty)$

Question 24.
Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

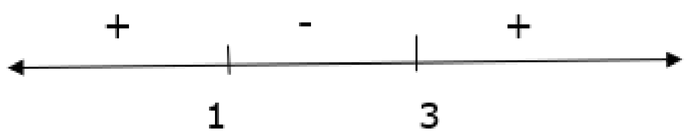
$$f(x)=\left(x^3-6x^2+9x+10\right)$$

Answer:
$f(x)=x^3-6x^2+9x+10$

$f'(x)=3x^2-12x+9$

$f'(x)=3(x^2-4x+3)$

$=3(x-3)(x-1)$



Function f(x) is decreasing for $x \in [1,3]$ and increasing in $x \in (-\infty,1) \cup (3, \infty)$

Question 25.
Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

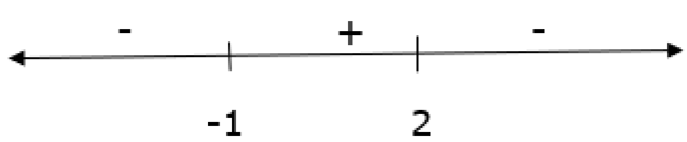
$$f(x)=\left(6+12x+3x^2-2x^3\right)$$

Answer:
$f(x)=-2x^3+3x^2+12x+6$

$f'(x)=-6x^2+6x+12$

$f'(x)=-6(x^2-x-2)$

$=-6(x-2)(x+1)$



Function f(x) is increasing for $x \in [-1,2]$ and decreasing in $x \in (-\infty,-1) \cup (2, \infty)$

Question 26.

Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

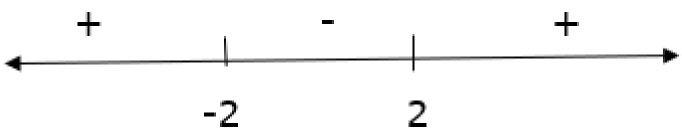
$$f(x) = 2x^3 - 24x + 5$$

Answer:
$f(x)=2x^3-24x+5$

$f'(x)=6x^2-24$

$f'(x)=6(x^2-4)$

$=6(x-2)(x+2)$



Function f(x) is decreasing for $x \in [-2,2]$ and increasing in $x \in (-\infty,-2) \cup (2, \infty)$

Question 27.
Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

$$f(x) = (x-1)(x-2)^2$$

Answer:
$f(x)=(x-1)(x-2)^2$=x2-4x+4 * x-1= x3-4x2+4x-x2+4x-4

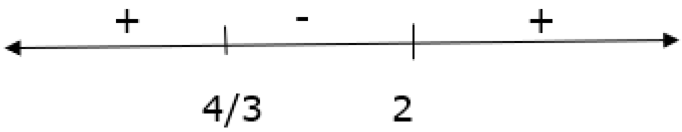
$f(x)=x^3-5x^2+8x-4$

$f'(x)=3x^2-10x+8$

$f'(x)=3x^2-6x-4x+8$

$=3x(x-2)-4(x-2)$

$=(3x-4)(x-2)$



Function f(x) is decreasing for x ∈ [4/3,2] and increasing in x ∈ (-∞,4/3) ∪ (2, ∞)

Question 28.
Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

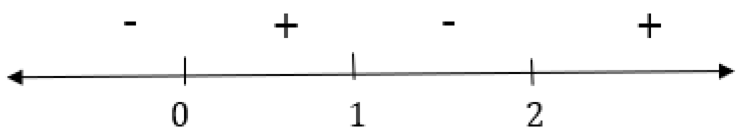
$$f(x)=\left(x^4-4x^3+4x^2+15\right)$$

Answer:
$f(x)=x^4-4x^3+4x^2+15$

$f'(x)=4x^3-12x^2+8x$

$= 4x(x^2-3x+2)$

$=4x(x-1)(x-2)$



Function f(x) is decreasing for x ∈ (-∞,0] ∪ [1, 2] and increasing in x ∈ (0,1) ∪ (2, ∞)

Question 29.
Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

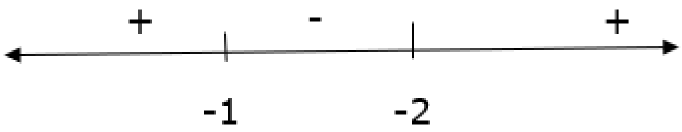
$$f(x)=2x^3+9x^2+12x+15$$

Answer:
$f(x)=2x^3+9x^2+12x+15$

$f'(x)=6x^2+18x+12$

$f'(x)=6(x^2+3x+2)$

=6(x+2)(x+1)



Function f(x) is decreasing for x ∈ [-1,-2] and increasing in x ∈ (-∞,-1) ∪ (-2, ∞)

Question 30.
Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

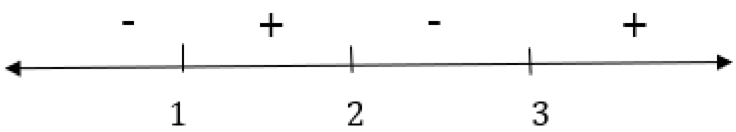
$$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$$

Answer:
$f(x)=x^4-8x^3+22x^2-24x+21$

$f'(x)=4x^3-24x^2+44x-24$

$= 4(x^3 -6x^2+11x-6)$

$=4(x-3)(x-1)(x-2)$



Function f(x) is decreasing for x ∈ (-∞,1] ∪ [2, 3] and increasing in x ∈ (1,2) ∪ (3, ∞)

Question 31.
Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$$

Answer:
$f(x)=3x^4-4x^3-12x^2+5$

$f'(x)=12x^3-12x^2-24x$

$=12x(x^2 - x - 2)$

$=12(x)(x+1)(x-2)$

Function f(x) is decreasing for $x \in (-\infty,-1] \cup [0, 2]$ and increasing in $x \in (-1,0) \cup (2, \infty)$

Question 32.
Find the intervals on which each of the following functions is (a) increasing (b) decreasing.

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

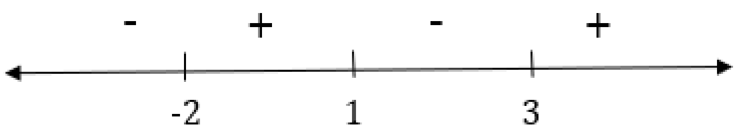
Answer:

$$f'(x) = \frac{12x^3}{10} - \frac{12x^2}{5} - 6x + \frac{36}{5}$$

$f'(x)=(12x^3-24x^2-60x+72)/10$

$=1.2(x^3-2x^2-5x+6)$

$=1.2(x-1)(x-3)(x+2)$



Function f(x) is decreasing for $x \in (-\infty,-2] \cup [1, 3]$ and increasing in $x \in (-2,1) \cup (3, \infty)$