Exercise 28a

Question 1.
Find the equation of the plane passing through each group of points:

(i) A(2, 2, -1), B(3, 4, 2) and C(7, 0, 6)

(ii) A(0, -1, -1), B(4, 5, 1) and C(3, 9, 4)

(iii) A(-2, 6, -6), B(-3, 10, 9) and

Answer:
(i) A(2, 2, -1), B(3, 4, 2) and C(7, 0, 6)

Given Points :

A = (2, 2, -1)

B = (3, 4, 2)

C = (7, 0, 6)

To Find : Equation of plane passing through points A, B & C

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

2) Vector :

If A and B be two points with position vectors $\bar{a}$ & $\bar{b}$ respectively, where

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\overline{AB} = \bar{b} - \bar{a}$$

$$= (b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

5) Equation of Plane :

If A = (a_1, a_2, a_3), B = (b_1, b_2, b_3), C = (c_1, c_2, c_3) are three non-collinear points,

Then, the vector equation of the plane passing through these points is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given points,

A = (2, 2, -1)

B = (3, 4, 2)

C = (7, 0, 6)

Position vectors are given by,

$\bar{a} = 2\hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\bar{b} = 3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$

$\bar{c} = 7\hat{\imath} + 6\hat{k}$

Now, vectors $\overline{AB}$ & $\overline{AC}$ are

$\overline{AB} = \bar{b} - \bar{a}$

$= (3-2)\hat{\imath} + (4-2)\hat{\jmath} + (2+1)\hat{k}$

$\therefore \overline{AB} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\overline{AC} = \bar{c} - \bar{a}$

$= (7-2)\hat{\imath} + (0-2)\hat{\jmath} + (6+1)\hat{k}$

$\therefore \overline{AC} = 5\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$

Therefore,

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix}$$

$$= \hat{\imath}(2 \times 7 - (-2) \times 3) - \hat{\jmath}(1 \times 7 - 5 \times 3) + \hat{k}(1 \times (-2) - 5 \times 2)$$

$$= 20\hat{\imath} + 8\hat{\jmath} - 12\hat{k}$$

Now,

$$\bar{a}.(\overline{AB} \times \overline{AC}) = (2 \times 20) + (2 \times 8) + ((-1) \times (-12))$$

= 40 + 16 + 12

= 68

$\therefore \bar{a}.(\overline{AB} \times \overline{AC}) = 68$ ....eq(1)

And

$$\bar{r}.(\overline{AB} \times \overline{AC}) = (x \times 20) + (y \times 8) + (z \times (-12))$$

= 20x + 8y – 12z

$\therefore \bar{r}.(\overline{AB} \times \overline{AC}) = 20x + 8y - 12z$ ....eq(2)

Vector equation of the plane passing through points A, B & C is

$$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$$

From eq(1) and eq(2)

20x + 8y – 12z = 68

This is 5x + 2y – 3z = 17 vector equation of required plane.

(ii) Given Points :

A = (0, -1, -1)

B = (4, 5, 1)

C = (3, 9, 4)

To Find : Equation of plane passing through points A, B & C

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

2) Vector :

If A and B be two points with position vectors $\bar{a}$ & $\bar{b}$ respectively, where

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\overline{AB} = \bar{b} - \bar{a}$$

$$= (b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

5) Equation of Plane :

If A = (a_1, a_2, a_3), B = (b_1, b_2, b_3), C = (c_1, c_2, c_3) are three non-collinear points,

Then, vector equation of the plane passing through these points is

$$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$$

Where,

$$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

For given points,

A = (0, -1, -1)

B = (4, 5, 1)

C = (3, 9, 4)

Position vectors are given by,

$\bar{a} = -\hat{j} - \hat{k}$

$\bar{b} = 4\hat{\imath} + 5\hat{j} + \hat{k}$

$\bar{c} = 3\hat{\imath} + 9\hat{j} + 4\hat{k}$

Now, vectors $\overline{AB}$ & $\overline{AC}$ are

$\overline{AB} = \bar{b} - \bar{a}$

$= (4-0)\hat{\imath} + (5+1)\hat{j} + (1+1)\hat{k}$

$\therefore \overline{AB} = 4\hat{\imath} + 6\hat{j} + 2\hat{k}$

$\overline{AC} = \bar{c} - \bar{a}$

$= (3-0)\hat{\imath} + (9+1)\hat{j} + (4+1)\hat{k}$

$\therefore \overline{AC} = 3\hat{\imath} + 10\hat{j} + 5\hat{k}$

Therefore,

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{\imath} & \hat{j} & \hat{k} \\ 4 & 6 & 2 \\ 3 & 10 & 5 \end{vmatrix}$$

$= \hat{\imath}(6 \times 5 - 10 \times 2) - \hat{j}(4 \times 5 - 2 \times 3) + \hat{k}(4 \times 10 - 3 \times 6)$

$= 10\hat{\imath} - 14\hat{j} + 22\hat{k}$

Now,

$\bar{a}.(\overline{AB} \times \overline{AC}) = (0 \times 10) + ((-1) \times (-14)) + ((-1) \times 22)$

= 0 + 14 – 22

= - 8

$\therefore \bar{a}.(\overline{AB} \times \overline{AC}) = -8$ ....eq(1)

And

$\bar{r}.(\overline{AB} \times \overline{AC}) = (x \times 10) + (y \times (-14)) + (z \times 22)$

= 10x - 14y + 22z

$\therefore \bar{r}.(\overline{AB} \times \overline{AC}) = 10x - 14y + 22z$ ....eq(2)

Vector equation of plane passing through points A, B & C is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

From eq(1) and eq(2)

10x - 14y + 22z = - 8

This is 5x - 7y + 11z = - 4 vector equation of required plane

(iii) Given Points :

A = (-2, 6, -6)

B = (-3, 10, 9)

C = (-5, 0, -6)

To Find : Equation of plane passing through points A, B & C

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Vector :

If A and B be two points with position vectors $\bar{a}$ & $\bar{b}$ respectively, where

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\overline{AB} = \bar{b} - \bar{a}$$

$$= (b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

5) Equation of Plane :

If A = (a_1, a_2, a_3), B = (b_1, b_2, b_3), C = (c_1, c_2, c_3) are three non-collinear points,

Then, vector equation of the plane passing through these points is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given points,

A = (-2, 6, -6)

B = (-3, 10, 9)

C = (-5, 0, -6)

Position vectors are given by,

$\bar{a} = -2\hat{\imath} + 6\hat{\jmath} - 6\hat{k}$

$\bar{b} = -3\hat{\imath} + 10\hat{\jmath} + 9\hat{k}$

$\bar{c} = -5\hat{\imath} - 6\hat{k}$

Now, vectors $\overline{AB}$ & $\overline{AC}$ are

$\overline{AB} = \bar{b} - \bar{a}$

$= (-3 + 2)\hat{\imath} + (10 - 6)\hat{\jmath} + (9 + 6)\hat{k}$

$\therefore \overline{AB} = -\hat{\imath} + 4\hat{\jmath} + 15\hat{k}$

$\overline{AC} = \bar{c} - \bar{a}$

$= (-5+2)\hat{\imath} + (0-6)\hat{\jmath} + (-6+6)\hat{k}$

$\therefore \overline{AC} = -3\hat{\imath} - 6\hat{\jmath} + 0\hat{k}$

Therefore,

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 4 & 15 \\ -3 & -6 & 0 \end{vmatrix}$$

$$= \hat{\imath}(4 \times 0 - (-6) \times 15) - \hat{\jmath}\big((-1) \times 0 - (-3) \times 15\big) + \hat{k}((-1) \times (-6) - (-3) \times 4)$$

$= 90\hat{\imath} - 45\hat{\jmath} + 18\hat{k}$

Now,

$\bar{a}.(\overline{AB} \times \overline{AC}) = ((-2) \times 90) + (6 \times (-45)) + ((-6) \times 18)$

= - 180 - 270 – 108

= - 558

$\therefore \bar{a}.(\overline{AB} \times \overline{AC}) = -558$ ....eq(1)

And

$\bar{r}.(\overline{AB} \times \overline{AC}) = (x \times 90) + (y \times (-45)) + (z \times 18)$

= 90x - 45y + 18z

$\therefore \bar{r}.(\overline{AB} \times \overline{AC}) = 90x - 45y + 18z$ ....eq(2)

Vector equation of plane passing through points A, B & C is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

From eq(1) and eq(2)

90x - 45y + 18z = - 558

This is 10x - 5y + 2z = - 62 vector equation of required plane

Question 2.
Show that the four points A(3, 2, -5),
B(-1, 4, -3), C(-3, 8, -5) and D(-3, 2, 1) are coplanar. Find the equation of the plane containing them.

Answer:
Given Points :

A = (3, 2, -5)

B = (-1, 4, -3)

C = (-3, 8, -5)

D = (-3, 2, 1)

To Prove : Points A, B, C & D are coplanar.

To Find : Equation of plane passing through points A, B, C & D.

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

2) Equation of line

If A and B are two points having position vectors $\bar{a}$ & $\bar{b}$ then equation of line passing through two points is given by,

$\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

5) Coplanarity of two lines :

If two lines $\bar{r_1} = \bar{a} + \lambda\bar{b}$ & $\bar{r_2} = \bar{c} + \mu\bar{d}$ are coplanar then

$\bar{a}.(\bar{b} \times \bar{d}) = \bar{c}.(\bar{b} \times \bar{d})$

6) Equation of plane :

If two lines $\bar{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\bar{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given points,

A = (3, 2, -5)

B = (-1, 4, -3)

C = (-3, 8, -5)

D = (-3, 2, 1)

Position vectors are given by,

$\bar{a} = 3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$

$\bar{b} = -1\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$

$\bar{c} = -3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}$

$\bar{d} = -3\hat{\imath} + 2\hat{\jmath} + \hat{k}$

Equation of line passing through points A & B is

$\overline{r_1} = \bar{a} + \lambda(\bar{b} - \bar{a})$

$\bar{b} - \bar{a} = (-1 - 3)\hat{\imath} + (4 - 2)\hat{\jmath} + (-3 + 5)\hat{k}$

$= -4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

$\therefore \overline{r_1} = (3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}) + \lambda(-4\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

Let, $\overline{r_1} = \overline{a_1} + \lambda b_1$

Where,

$\overline{a_1} = 3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ & $b_1 = -4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

And the equation of the line passing through points C & D is

$\overline{r_2} = \bar{c} + \mu(\bar{d} - \bar{c})$

$\bar{d} - \bar{c} = (-3+3)\hat{\imath} + (2-8)\hat{\jmath} + (1+5)\hat{k}$

$= -6\hat{\jmath} + 6\hat{k}$

$\therefore \overline{r_1} = (-3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}) + \lambda(-6\hat{\jmath} + 6\hat{k})$

Let, $\overline{r_2} = \overline{a_2} + \lambda b_2$

Where,

$\overline{a_2} = -3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}$ & $b_2 = -6\hat{\jmath} + 6\hat{k}$

Now,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -4 & 2 & 2 \\ 0 & -6 & 6 \end{vmatrix}$$

$= \hat{\imath}(12+12) - \hat{\jmath}(-24-0) + \hat{k}(24+0)$

$\therefore (\overline{b_1} \times \overline{b_2}) = 24\hat{\imath} + 24\hat{\jmath} + 24\hat{k}$

Therefore,

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = (3 \times 24) + (2 \times 24) + ((-5) \times 24)$

= 72 + 48 – 120

= 0

$\therefore \overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = 0$ ……… eq(1)

And

$\overline{a_2}.\left(\overline{b_1} \times \overline{b_2}\right) = ((-3) \times 24) + (8 \times 24) + ((-5) \times 24)$

= - 72 + 192 – 120

= 0

$\therefore \overline{a_2}.\left(\overline{b_1} \times \overline{b_2}\right) = 0$ ……… eq(2)

From eq(1) and eq(2)

$\overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = \overline{a_2}.\left(\overline{b_1} \times \overline{b_2}\right)$

Hence lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar

Therefore, points A, B, C & D are also coplanar.

As lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar therefore equation of the plane passing through two lines containing four given points is

$\bar{r}.\left(\overline{b_1} \times \overline{b_2}\right) = \overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right)$

Now,

$\bar{r}.\left(\overline{b_1} \times \overline{b_2}\right) = (x \times 24) + (y \times 24) + (z \times 24)$

= 24x + 24y + 24z

From eq(1)

$\overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = 0$

Therefore, equation of required plane is

24x + 24y + 24z = 0

x + y + z = 0

Question 3.
Show that the four points A(0, -1, 0),
B(2, 1, -1), C(1, 1, 1) and D(3, 3, 0) are coplanar. Find the equation of the plane containing them.

Answer:
Given Points :

A = (0, -1, 0)

B = (2, 1, -1)

C = (1, 1, 1)

D = (3, 3, 0)

To Prove : Points A, B, C & D are coplanar.

To Find : Equation of plane passing through points A, B, C & D.

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

2) Equation of line

If A and B are two points having position vectors $\bar{a}$ & $\bar{b}$ then equation of line passing through two points is given by,

$$\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

5) Coplanarity of two lines :

If two lines $\bar{r_1} = \bar{a} + \lambda\bar{b}$ & $\bar{r_2} = \bar{c} + \mu\bar{d}$ are coplanar then

$$\bar{a}.(\bar{b} \times \bar{d}) = \bar{c}.(\bar{b} \times \bar{d})$$

6) Equation of plane :

If two lines $\bar{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\bar{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given points,

A = (0, -1, 0)

B = (2, 1, -1)

C = (1, 1, 1)

D = (3, 3, 0)

Position vectors are given by,

$\bar{a} = -\hat{\jmath}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\bar{d} = 3\hat{\imath} + 3\hat{\jmath}$

Equation of line passing through points A & D is

$\overline{r_1} = \bar{a} + \lambda(\bar{d} - \bar{a})$

$\bar{d} - \bar{a} = (3 - 0)\hat{\imath} + (3 + 1)\hat{\jmath} + (0 - 0)\hat{k}$

$= 3\hat{\imath} + 4\hat{\jmath}$

$\therefore \overline{r_1} = (-\hat{\jmath}) + \lambda(3\hat{\imath} + 4\hat{\jmath})$

Let, $\overline{r_1} = \overline{a_1} + \lambda b_1$

Where,

$\overline{a_1} = -\hat{\jmath}$ & $b_1 = 3\hat{\imath} + 4\hat{\jmath}$

And equation of line passing through points B & C is

$\overline{r_2} = \bar{b} + \mu(\bar{c} - \bar{b})$

$\bar{c} - \bar{b} = (1-2)\hat{\imath} + (1-1)\hat{\jmath} + (1+1)\hat{k}$

$= -\hat{\imath} + 0\hat{\jmath} + 2\hat{k}$

$\therefore \overline{r_1} = (2\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(-\hat{\imath} + 2\hat{k})$

Let, $\overline{r_2} = \overline{a_2} + \lambda b_2$

Where,

$\overline{a_2} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ & $b_2 = -\hat{\imath} + 2\hat{k}$

Now,

$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 4 & 0 \\ -1 & 0 & 2 \end{vmatrix}$

$= \hat{\imath}(8-0) - \hat{\jmath}(6-0) + \hat{k}(0+4)$

$\therefore (\overline{b_1} \times \overline{b_2}) = 8\hat{\imath} - 6\hat{\jmath} + 4\hat{k}$

Therefore,

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = (0 \times 8) + ((-1) \times (-6)) + (0 \times 4)$

= 0 + 6 + 0

= 6

$\therefore \overline{a_1}.(\overline{b_1} \times \overline{b_2}) = 6$ ……… eq(1)

And

$\overline{a_2}.(\overline{b_1} \times \overline{b_2}) = (2 \times 8) + (1 \times (-6)) + ((-1) \times 4)$

= 16 – 6 – 4

= 6

$\therefore \overline{a_2}.(\bar{b} \times \bar{d}) = 6$ ……… eq(2)

From eq(1) and eq(2)

$$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = \overline{a_2}.(\overline{b_1} \times \overline{b_2})$$

Hence lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar

Therefore, points A, B, C & D are also coplanar.

As lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar therefore equation of the plane passing through two lines containing four given points is

$$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$$

Now,

$$\bar{r}.(\overline{b_1} \times \overline{b_2}) = (x \times 8) + (y \times (-6)) + (z \times 4)$$

= 8x - 6y + 4z

From eq(1)

$$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = 6$$

Therefore, equation of required plane is

8x - 6y + 4z = 6

4x - 3y + 2z = 3

Question 4.
Write the equation of the plane whose intercepts on the coordinate axes are 2, -4 and 5 respectively.

Answer:
Given :

X – intercept, a = 2

Y – intercept, b = - 4

Z – intercept, c = 5

To Find : Equation of plane

Formula :

If a, b & c are the intercepts made by plane on X, Y & Z axes respectively, then equation of the plane is given by,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

$$\therefore \frac{x}{2}+\frac{y}{-4}+\frac{z}{5}=1$$

Multiplying above equation throughout by 40,

$$\therefore \frac{40x}{2}+\frac{40y}{-4}+\frac{40z}{5}=40$$

20x – 10y + 8z = 40

10x – 5y + 4z = 20

This the equation of the required plane.

Question 5.
Reduce the equation of the plane 4x – 3y + 2z = 12 to the intercept form, and hence find the intercepts made by the plane with the coordinate axes.

Answer:
Given :

Equation of plane : 4x – 3y + 2z = 12

To Find :

1) Equation of plane in intercept form

2) Intercepts made by the plane with the co-ordinate axes.

Formula :

If $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

is the equation of a plane in intercept form then intercept made by it with co-ordinate axes are

X-intercept = a

Y-intercept = b

Z-intercept = c

Given the equation of plane:

4x – 3y + 2z = 12

Dividing the above equation throughout by 12

$$\therefore \frac{4x}{12}+\frac{-3y}{12}+\frac{2z}{12}=1$$

$$\therefore \frac{x}{3}+\frac{y}{-4}+\frac{z}{6}=1$$

This is the equation of a plane in intercept form.

Comparing the above equation with

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

We get,

a = 3

b = -4

c = 6

Therefore, intercepts made by plane with co-ordinate axes are

X-intercept = 3

Y-intercept = -4

Z-intercept = 6

Question 6.
Find the equation of the plane which passes through the point (2, -3,7) and makes equal intercepts on the coordinate axes.

Answer:
Equation of the plane making a, b & c intercepts with X, Y & Z axes respectively is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

But, the plane makes equal intercepts on the co-ordinate axes

Therefore, a = b = c

Therefore the equation of the plane is

$$\frac{x}{a}+\frac{y}{a}+\frac{z}{a}=1$$

x + y + z = a

As plane passes through the point (2, -3, 7),

Substituting x = 2, y = -3 & z = 7

2 – 3 + 7 = a

Therefore, a = 6

Hence, required equation of plane is

x + y + z = 6

Question 7.
A plane meets the coordinate axes at A, B and C respectively such that the centroid of ΔABC is (1, -2, 3). Find the equation of the plane.

Answer:
Given :

X-intercept = A

Y-intercept = B

Z-intercept = C

Centroid of ΔABC = (1, -2, 3)

To Find : Equation of a plane

Formulae :

1) Centroid Formula :

For ΔABC if co-ordinates of A, B & C are

A = (x_1, x_2, x_3)

B = (y_1, y_2, y_3)

C = (z_1, z_2, z_3)

Then co-ordinates of the centroid of ΔABC are

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

2) Equation of plane :

Equation of the plane making a, b & c intercepts with X, Y & Z axes respectively is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

As the plane makes intercepts at points A, B & C on X, Y & Z axes respectively, let co-ordinates of A, B, C be

A = (a, 0, 0)

B = (0, b, 0)

C = (0, 0, c)

By centroid formula,

The centroid of ΔABC is given by

$$G=\left(\frac{a+0+0}{3},\frac{0+b+0}{3},\frac{0+0+c}{3}\right)$$

$$G=\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$$

But, Centroid of ΔABC = (1, -2, 3) ...... given

$$\therefore \frac{a}{3}=1,\frac{b}{3}=-2,\frac{c}{3}=3$$

Therefore, a = 3, b = - 6, c = 9

Therefore,

X-intercept = a = 3

Y-intercept = b = - 6

Z-intercept = c = 9

Therefore, equation of required plane is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

$$\therefore \frac{x}{3}+\frac{y}{-6}+\frac{z}{9}=1$$

Question 8.
Find the Cartesian and vector equations of a plane passing through the point (1, 2, 3) and perpendicular to a line with direction ratios 2, 3, -4.

Answer:
Given :

A = (1, 2, 3)

Direction ratios of perpendicular vector = (2, 3, -4)

To Find : Equation of a plane

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

If a plane is passing through point A, then the equation of a plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{a} = position\ vector\ of\ A$

$\bar{n} = vector\ perpendicular\ to\ the\ plane$

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For point A = (1, 2, 3), position vector is

$\bar{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Vector perpendicular to the plane with direction ratios (2, 3, -4) is

$\bar{n} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

Now, $\bar{a}.\bar{n} = (1 \times 2) + (2 \times 3) + (3 \times (-4))$

= 2 + 6 – 12

= - 4

Equation of the plane passing through point A and perpendicular to vector $\bar{n}$ is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore \bar{r}.\left(2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}\right) = -4$

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore \bar{r}.\left(2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}\right) = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}\right)$

= 2x + 3y – 4z

Therefore, equation of the plane is

2x + 3y – 4z = -4

Or

2x + 3y – 4z + 4 = 0

Question 9.
If O is the origin and P(1, 2, -3) be a given point, then find the equation of the plane passing through P and perpendicular to OP.

Answer:
Given :

P = (1, 2, -3)

O = (0, 0, 0)

$\bar{n} = \overline{OP}$

To Find : Equation of a plane

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Vector :

If A and B be two points with position vectors $\bar{a}$ & $\bar{b}$ respectively, where

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\overline{AB} = \bar{b} - \bar{a}$

$= (b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

3) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

4) Equation of plane :

If a plane is passing through point A, then the equation of a plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{a} = position\ vector\ of\ A$

$\bar{n} = vector\ perpendicular\ to\ the\ plane$

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For points,

P = (1, 2, -3)

O = (0, 0, 0)

Position vectors are

$\bar{p} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{o} = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

Vector

$\overline{OP} = \bar{p} - \bar{o}$

$= (1-0)\hat{\imath} + (2-0)\hat{\jmath} + (3-0)\hat{k}$

$\therefore \overline{OP} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Now,

$\bar{p}.\overline{OP} = (1 \times 1) + (2 \times 2) + (3 \times 3)$

= 1 + 4 + 9

= 14

And

$\bar{r}.\overline{OP} = (x \times 1) + (y \times 2) + (z \times 3)$

= x + 2y + 3z

Equation of the plane passing through point A and perpendicular to the vector $\bar{n}$ is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

But,
$\bar{n} = \overline{OP}$

Therefore, the equation of the plane is

$\bar{r}.\overline{OP} = \bar{p}.\overline{OP}$

$$x + 2y + 3z = 14$$

$$x + 2y + 3z - 14 = 0$$