

# Congruence of Triangles



## 7.1 INTRODUCTION

You are now ready to learn a very important geometrical idea, **Congruence**. In particular, you will study a lot about congruence of triangles.

To understand what congruence is, we turn to some activities.

### Do This

Take two stamps (Fig 7.1) of same denomination. Place one stamp over the other. What do you observe?



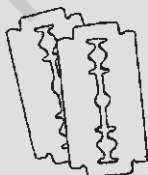
Fig 7.1



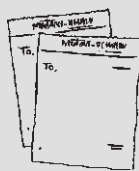
One stamp covers the other completely and exactly. This means that the two stamps are of the same shape and same size. Such objects are said to be congruent. The two stamps used by you are congruent to one another. Congruent objects are exact copies of one another.

Can you, now, say if the following objects are congruent or not?

1. Shaving blades of the same company [Fig 7.2 (i)].
2. Sheets of the same letter-pad [Fig 7.2 (ii)].
3. Biscuits in the same packet [Fig 7.2 (iii)].
4. Toys made of the same mould. [Fig 7.2(iv)]



(i)



(ii)



(iii)



(iv)

Fig 7.2

The relation of two objects being congruent is called **congruence**. For the present, we will deal with plane figures only, although congruence is a general idea applicable to three-dimensional shapes also. We will try to learn a precise meaning of the congruence of plane figures already known.

## 7.2 CONGRUENCE OF PLANE FIGURES

Look at the two figures given here (Fig 7.3). Are they congruent?

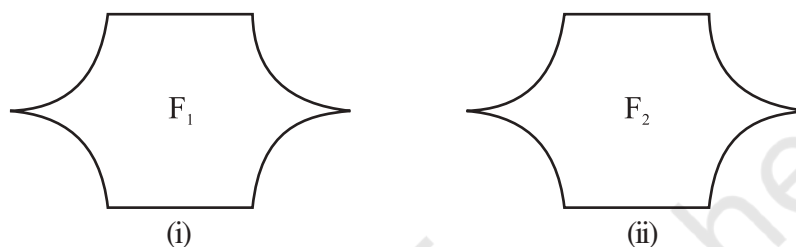


Fig 7.3

You can use the method of superposition. Take a trace-copy of one of them and place it over the other. If the figures cover each other completely, they are congruent. Alternatively, you may cut out one of them and place it over the other. Beware! You are not allowed to bend, twist or stretch the figure that is cut out (or traced out).

In Fig 7.3, if figure  $F_1$  is congruent to figure  $F_2$ , we write  $F_1 \cong F_2$ .

## 7.3 CONGRUENCE AMONG LINE SEGMENTS

When are two line segments congruent? Observe the two pairs of line segments given here (Fig 7.4).



Fig 7.4

Use the 'trace-copy' superposition method for the pair of line segments in [Fig 7.4(i)]. Copy  $\overline{CD}$  and place it on  $\overline{AB}$ . You find that  $\overline{CD}$  covers  $\overline{AB}$ , with C on A and D on B. Hence, the line segments are congruent. We write  $\overline{AB} \cong \overline{CD}$ .

Repeat this activity for the pair of line segments in [Fig 7.4(ii)]. What do you find? They are not congruent. How do you know it? It is because the line segments do not coincide when placed one over other.

You should have by now noticed that the pair of line segments in [Fig 7.4(i)] matched with each other because they had same length; and this was not the case in [Fig 7.4(ii)].

*If two line segments have the same (i.e., equal) length, they are congruent. Also, if two line segments are congruent, they have the same length.*

In view of the above fact, when two line segments are congruent, we sometimes just say that the line segments are equal; and we also write  $AB = CD$ . (What we actually mean is  $\overline{AB} \cong \overline{CD}$ ).

## 7.4 CONGRUENCE OF ANGLES

Look at the four angles given here (Fig 7.5).

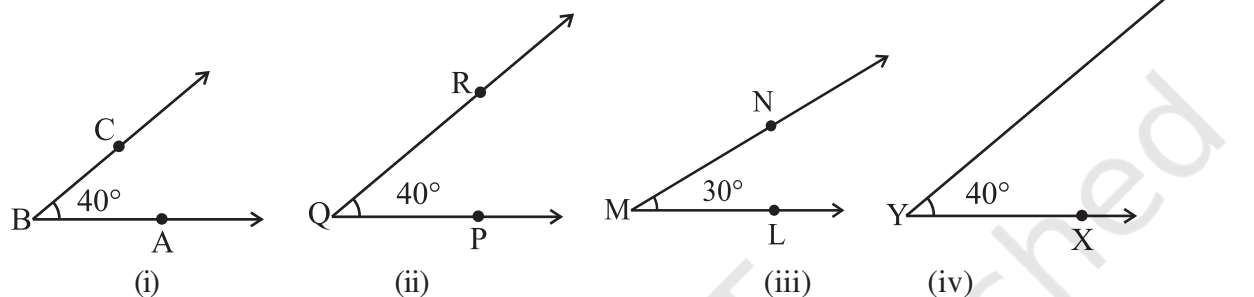


Fig 7.5

Make a trace-copy of  $\angle PQR$ . Try to superpose it on  $\angle ABC$ . For this, first place Q on B and QP along  $\overline{BA}$ . Where does  $\overline{QR}$  fall? It falls on  $\overline{BC}$ .

Thus,  $\angle PQR$  matches exactly with  $\angle ABC$ .

That is,  $\angle ABC$  and  $\angle PQR$  are congruent.

(Note that the measurement of these two congruent angles are same).

We write  $\angle ABC \cong \angle PQR$  (i)

or  $m\angle ABC = m\angle PQR$  (In this case, measure is  $40^\circ$ ).

Now, you take a trace-copy of  $\angle LMN$ . Try to superpose it on  $\angle ABC$ . Place M on B and  $\overline{ML}$  along  $\overline{BA}$ . Does  $\overline{MN}$  fall on  $\overline{BC}$ ? No, in this case it does not happen. You find that  $\angle ABC$  and  $\angle LMN$  do not cover each other exactly. So, they are not congruent.

(Note that, in this case, the measures of  $\angle ABC$  and  $\angle LMN$  are not equal).

What about angles  $\angle XYZ$  and  $\angle ABC$ ? The rays  $\overline{YX}$  and  $\overline{YZ}$ , respectively appear [in Fig 7.5 (iv)] to be longer than  $\overline{BA}$  and  $\overline{BC}$ . You may, hence, think that  $\angle ABC$  is 'smaller' than  $\angle XYZ$ . But remember that the rays in the figure only indicate the direction and not any length. On superposition, you will find that these two angles are also congruent.

We write  $\angle ABC \cong \angle XYZ$  (ii)

or  $m\angle ABC = m\angle XYZ$

In view of (i) and (ii), we may even write

$$\angle ABC \cong \angle PQR \cong \angle XYZ$$

*If two angles have the same measure, they are congruent. Also, if two angles are congruent, their measures are same.*

As in the case of line segments, congruency of angles entirely depends on the equality of their measures. So, to say that two angles are congruent, we sometimes just say that the angles are equal; and we write

$$\angle ABC = \angle PQR \text{ (to mean } \angle ABC \cong \angle PQR \text{)}.$$

## 7.5 CONGRUENCE OF TRIANGLES

We saw that two line segments are congruent where one of them, is just a copy of the other. Similarly, two angles are congruent if one of them is a copy of the other. We extend this idea to triangles.

Two triangles are congruent if they are copies of each other and when superposed, they cover each other exactly.

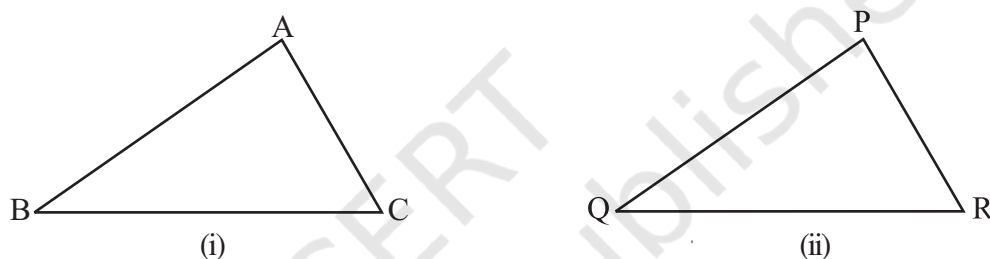


Fig 7.6

$\triangle ABC$  and  $\triangle PQR$  have the same size and shape. They are congruent. So, we would express this as

$$\triangle ABC \cong \triangle PQR$$

This means that, when you place  $\triangle PQR$  on  $\triangle ABC$ , P falls on A, Q falls on B and R falls on C, also  $\overline{PQ}$  falls along  $\overline{AB}$ ,  $\overline{QR}$  falls along  $\overline{BC}$  and  $\overline{PR}$  falls along  $\overline{AC}$ . If, under a given correspondence, two triangles are congruent, then their corresponding parts (i.e., angles and sides) that match one another are equal. Thus, in these two congruent triangles, we have:

Corresponding vertices : A and P, B and Q, C and R.

Corresponding sides :  $\overline{AB}$  and  $\overline{PQ}$ ,  $\overline{BC}$  and  $\overline{QR}$ ,  $\overline{AC}$  and  $\overline{PR}$ .

Corresponding angles :  $\angle A$  and  $\angle P$ ,  $\angle B$  and  $\angle Q$ ,  $\angle C$  and  $\angle R$ .

If you place  $\triangle PQR$  on  $\triangle ABC$  such that P falls on B, then, should the other vertices also correspond suitably? *It need not happen!* Take trace, copies of the triangles and try to find out.

This shows that while talking about congruence of triangles, not only the measures of angles and lengths of sides matter, but also the matching of vertices. In the above case, the correspondence is

$$A \leftrightarrow P, B \leftrightarrow Q, C \leftrightarrow R$$

We may write this as  $ABC \leftrightarrow PQR$

**EXAMPLE 1**  $\triangle ABC$  and  $\triangle PQR$  are congruent under the correspondence:

$$ABC \leftrightarrow RQP$$

Write the parts of  $\triangle ABC$  that correspond to

- (i)  $\overline{PQ}$                       (ii)  $\angle Q$                       (iii)  $\overline{RP}$

**SOLUTION** For better understanding of the correspondence, let us use a diagram (Fig 7.7).

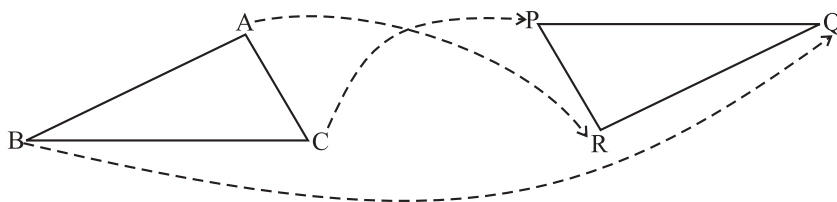


Fig 7.7

The correspondence is  $ABC \leftrightarrow RQP$ . This means

$$A \leftrightarrow R ; \quad B \leftrightarrow Q; \text{ and } C \leftrightarrow P.$$

So, (i)  $\overline{PQ} \leftrightarrow \overline{CB}$     (ii)  $\angle Q \leftrightarrow \angle B$  and (iii)  $\overline{RP} \leftrightarrow \overline{AC}$

### THINK, DISCUSS AND WRITE

When two triangles, say  $ABC$  and  $PQR$  are given, there are, in all, six possible matchings or correspondences. Two of them are

- (i)  $ABC \leftrightarrow PQR$     and    (ii)  $ABC \leftrightarrow QRP$ .

Find the other four correspondences by using two cutouts of triangles. Will all these correspondences lead to congruence? Think about it.



### EXERCISE 7.1

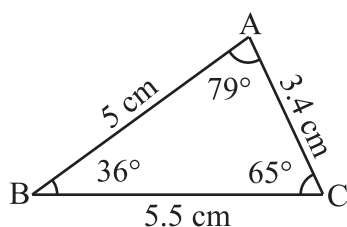
- Complete the following statements:
  - Two line segments are congruent if \_\_\_\_\_.
  - Among two congruent angles, one has a measure of  $70^\circ$ ; the measure of the other angle is \_\_\_\_\_.
  - When we write  $\angle A = \angle B$ , we actually mean \_\_\_\_\_.
- Give any two real-life examples for congruent shapes.
- If  $\triangle ABC \cong \triangle FED$  under the correspondence  $ABC \leftrightarrow FED$ , write all the corresponding congruent parts of the triangles.
- If  $\triangle DEF \cong \triangle BCA$ , write the part(s) of  $\triangle BCA$  that correspond to
  - $\angle E$
  - $\overline{EF}$
  - $\angle F$
  - $\overline{DF}$



## 7.6 CRITERIA FOR CONGRUENCE OF TRIANGLES

We make use of triangular structures and patterns frequently in day-to-day life. So, it is rewarding to find out when two triangular shapes will be congruent. If you have two triangles drawn in your notebook and want to verify if they are congruent, you cannot everytime cut out one of them and use method of superposition. Instead, if we can judge congruency in terms of appropriate measures, it would be quite useful. Let us try to do this.

### A Game



**Fig 7.8**

Triangle drawn by  
Appu

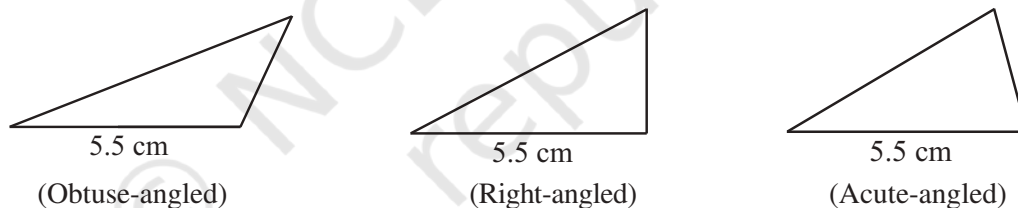
Appu and Tippu play a game. Appu has drawn a triangle ABC (Fig 7.8) and has noted the length of each of its sides and measure of each of its angles. Tippu has not seen it. Appu challenges Tippu if he can draw a copy of his  $\triangle ABC$  based on bits of information that Appu would give. Tippu attempts to draw a triangle congruent to  $\triangle ABC$ , using the information provided by Appu.

The game starts. Carefully observe their conversation and their games.

### SSS Game

**Appu :** One side of  $\triangle ABC$  is 5.5 cm.

**Tippu :** With this information, I can draw any number of triangles (Fig 7.9) but they need not be copies of  $\triangle ABC$ . The triangle I draw may be obtuse-angled or right-angled or acute-angled. For example, here are a few.



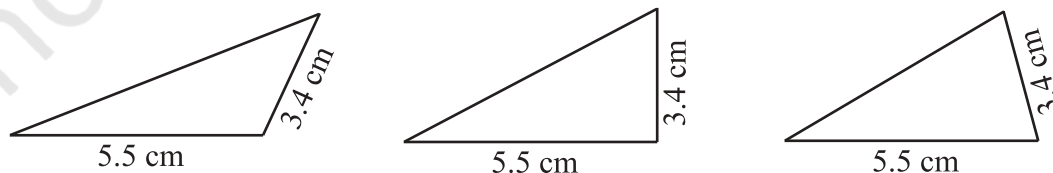
**Fig 7.9**

I have used some arbitrary lengths for other sides. This gives me many triangles with length of base 5.5 cm.

So, giving only one side-length will not help me to produce a copy of  $\triangle ABC$ .

**Appu :** Okay. I will give you the length of one more side. Take two sides of  $\triangle ABC$  to be of lengths 5.5 cm and 3.4 cm.

**Tippu :** Even this will not be sufficient for the purpose. I can draw several triangles (Fig 7.10) with the given information which may not be copies of  $\triangle ABC$ . Here are a few to support my argument:



**Fig 7.10**

One cannot draw an exact copy of your triangle, if only the lengths of two sides are given.

**Appu :** Alright. Let me give the lengths of all the three sides. In  $\triangle ABC$ , I have  $AB = 5\text{ cm}$ ,  $BC = 5.5\text{ cm}$  and  $AC = 3.4\text{ cm}$ .

**Tippu :** I think it should be possible. Let me try now.

First I draw a rough figure so that I can remember the lengths easily.

I draw  $\overline{BC}$  with length  $5.5\text{ cm}$ .

With B as centre, I draw an arc of radius  $5\text{ cm}$ . The point A has to be somewhere on this arc. With C as centre, I draw an arc of radius  $3.4\text{ cm}$ . The point A has to be on this arc also.

So, A lies on both the arcs drawn. This means A is the point of intersection of the arcs.

I know now the positions of points A, B and C. Aha! I can join them and get  $\triangle ABC$  (Fig 7.11).

**Appu :** Excellent. So, to draw a copy of a given  $\triangle ABC$  (i.e., to draw a triangle congruent to  $\triangle ABC$ ), we need the lengths of three sides. Shall we call this condition as side-side-side criterion?

**Tippu :** Why not we call it SSS criterion, to be short?

#### SSS Congruence criterion:

If under a given correspondence, the three sides of one triangle are equal to the three corresponding sides of another triangle, then the triangles are congruent.

**EXAMPLE 2** In triangles ABC and PQR,  $AB = 3.5\text{ cm}$ ,  $BC = 7.1\text{ cm}$ ,  $AC = 5\text{ cm}$ ,  $PQ = 7.1\text{ cm}$ ,  $QR = 5\text{ cm}$  and  $PR = 3.5\text{ cm}$ . Examine whether the two triangles are congruent or not. If yes, write the congruence relation in symbolic form.

**SOLUTION** Here,  $AB = PR (= 3.5\text{ cm})$ ,  
 $BC = PQ (= 7.1\text{ cm})$   
 and  $AC = QR (= 5\text{ cm})$

This shows that the three sides of one triangle are equal to the three sides of the other triangle. So, by SSS congruence rule, the two triangles are congruent. From the above three equality relations, it can be easily seen that  $A \leftrightarrow R$ ,  $B \leftrightarrow P$  and  $C \leftrightarrow Q$ .

So, we have  $\triangle ABC \cong \triangle RPQ$

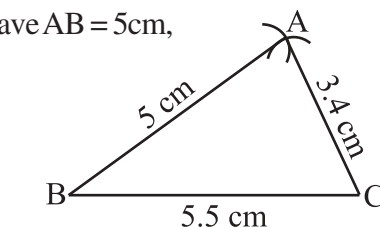


Fig 7.11

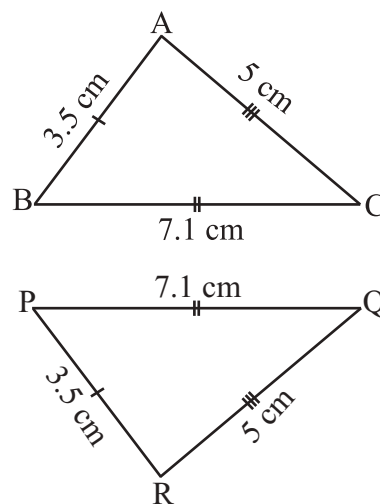


Fig 7.12

**Important note:** The order of the letters in the names of congruent triangles displays the corresponding relationships. Thus, when you write  $\triangle ABC \cong \triangle RPQ$ , you would know that A lies on R, B on P, C on Q,  $\overline{AB}$  along  $\overline{RP}$ ,  $\overline{BC}$  along  $\overline{PQ}$  and  $\overline{AC}$  along  $\overline{RQ}$ .



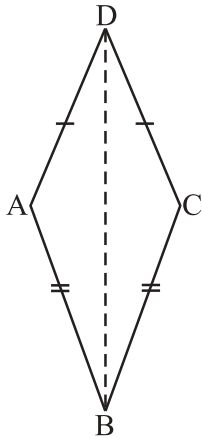


Fig 7.13

**EXAMPLE 3** In Fig 7.13,  $AD = CD$  and  $AB = CB$ .

- State the three pairs of equal parts in  $\triangle ABD$  and  $\triangle CBD$ .
- Is  $\triangle ABD \cong \triangle CBD$ ? Why or why not?
- Does  $BD$  bisect  $\angle ABC$ ? Give reasons.

**SOLUTION**

- In  $\triangle ABD$  and  $\triangle CBD$ , the three pairs of equal parts are as given below:  
 $AB = CB$  (Given)  
 $AD = CD$  (Given)  
 and  $BD = BD$  (Common in both)
- From (i) above,  $\triangle ABD \cong \triangle CBD$  (By SSS congruence rule)
- $\angle ABD = \angle CBD$  (Corresponding parts of congruent triangles)  
 So,  $BD$  bisects  $\angle ABC$ .

**TRY THESE**



- In Fig 7.14, lengths of the sides of the triangles are indicated. By applying the SSS congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form:

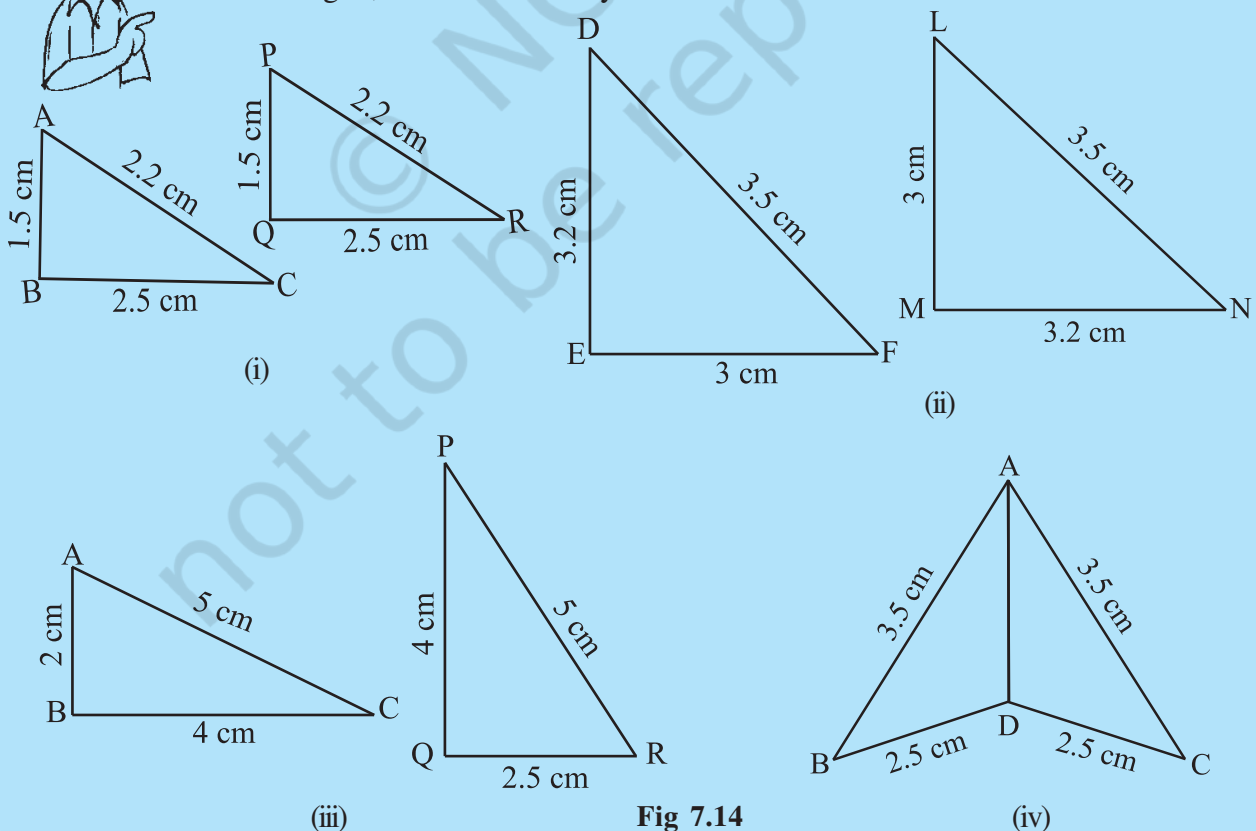


Fig 7.14



2. In Fig 7.15,  $AB = AC$  and  $D$  is the mid-point of  $\overline{BC}$ .

- State the three pairs of equal parts in  $\triangle ADB$  and  $\triangle ADC$ .
- Is  $\triangle ADB \cong \triangle ADC$ ? Give reasons.
- Is  $\angle B = \angle C$ ? Why?

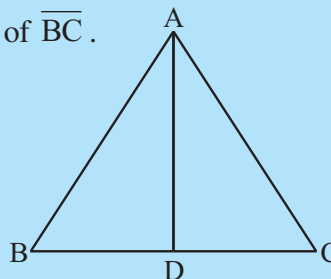


Fig 7.15

3. In Fig 7.16,  $AC = BD$  and  $AD = BC$ . Which of the following statements is meaningfully written?

- $\triangle ABC \cong \triangle ABD$
- $\triangle ABC \cong \triangle BAD$ .

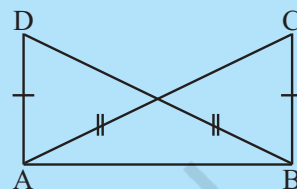


Fig 7.16

### THINK, DISCUSS AND WRITE

$ABC$  is an isosceles triangle with  $AB = AC$  (Fig 7.17).

Take a trace-copy of  $\triangle ABC$  and also name it as  $\triangle ACB$ .

- State the three pairs of equal parts in  $\triangle ABC$  and  $\triangle ACB$ .
- Is  $\triangle ABC \cong \triangle ACB$ ? Why or why not?
- Is  $\angle B = \angle C$ ? Why or why not?

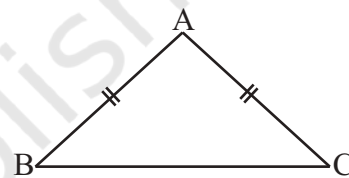


Fig 7.17

Appu and Tippu now turn to playing the game with a slight modification.

### SAS Game

**Appu** : Let me now change the rules of the triangle-copying game.

**Tippu** : Right, go ahead.

**Appu** : You have already found that giving the length of only one side is useless.

**Tippu** : Of course, yes.

**Appu** : In that case, let me tell that in  $\triangle ABC$ , one side is 5.5 cm and one angle is  $65^\circ$ .

**Tippu** : This again is not sufficient for the job. I can find many triangles satisfying your information, but are not copies of  $\triangle ABC$ . For example, I have given here some of them (Fig 7.18):

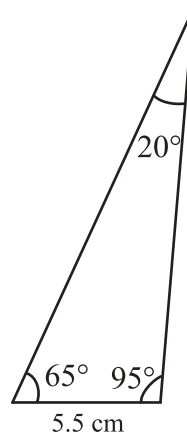
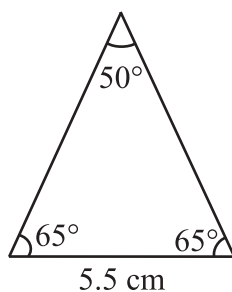
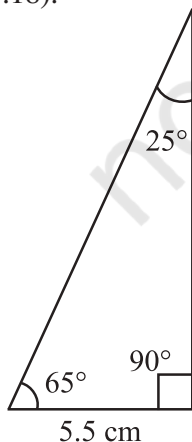


Fig 7.18

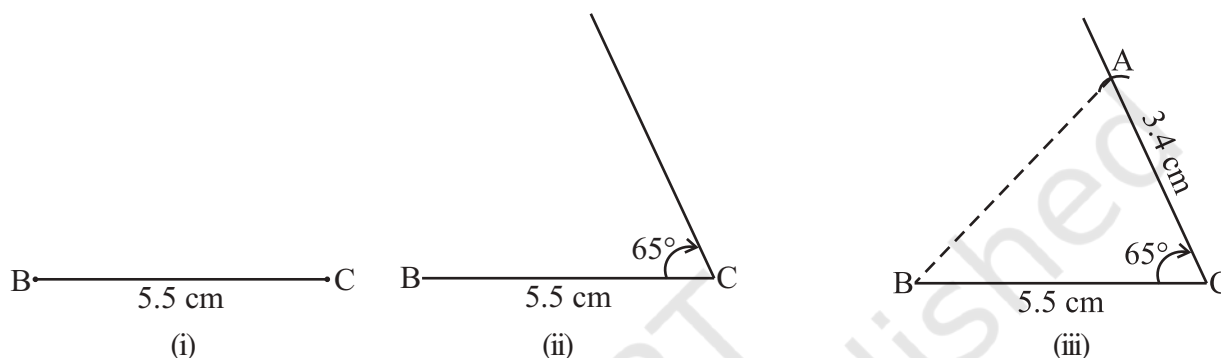


**Appu** : So, what shall we do?

**Tippu** : More information is needed.

**Appu** : Then, let me modify my earlier statement. In  $\triangle ABC$ , the length of two sides are 5.5 cm and 3.4 cm, and the angle between these two sides is  $65^\circ$ .

**Tippu** : This information should help me. Let me try. I draw first  $\overline{BC}$  of length 5.5 cm [Fig 7.19 (i)]. Now I make  $65^\circ$  at C [Fig 7.19 (ii)].



**Fig 7.19**

Yes, I got it, A must be 3.4 cm away from C along this angular line through C.

I draw an arc of 3.4 cm with C as centre. It cuts the  $65^\circ$  line at A.

Now, I join AB and get  $\triangle ABC$  [Fig 7.19(iii)].

**Appu** : You have used side-angle-side, where the angle is ‘included’ between the sides!

**Tippu** : Yes. How shall we name this criterion?

**Appu** : It is SAS criterion. Do you follow it?

**Tippu** : Yes, of course.

#### **SAS Congruence criterion:**

If under a correspondence, two sides and the angle included between them of a triangle are equal to two corresponding sides and the angle included between them of another triangle, then the triangles are congruent.

**EXAMPLE 4** Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, by using SAS congruence rule. If the triangles are congruent, write them in symbolic form.

**$\triangle ABC$**

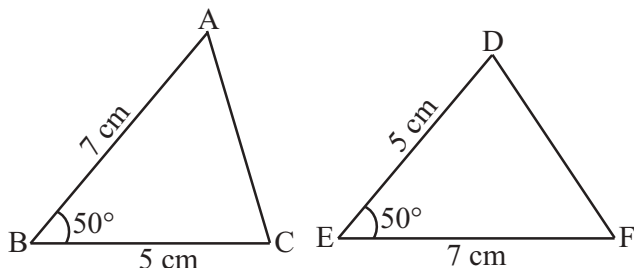
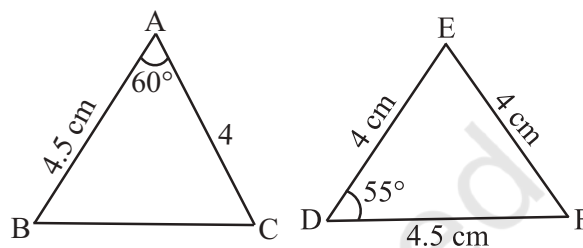
**$\triangle DEF$**

- |   |   |
|---|---|
| (a) $AB = 7$ cm, $BC = 5$ cm, $\angle B = 50^\circ$   | $DE = 5$ cm, $EF = 7$ cm, $\angle E = 50^\circ$   |
| (b) $AB = 4.5$ cm, $AC = 4$ cm, $\angle A = 60^\circ$ | $DE = 4$ cm, $FD = 4.5$ cm, $\angle D = 55^\circ$ |
| (c) $BC = 6$ cm, $AC = 4$ cm, $\angle B = 35^\circ$   | $DF = 4$ cm, $EF = 6$ cm, $\angle E = 35^\circ$   |

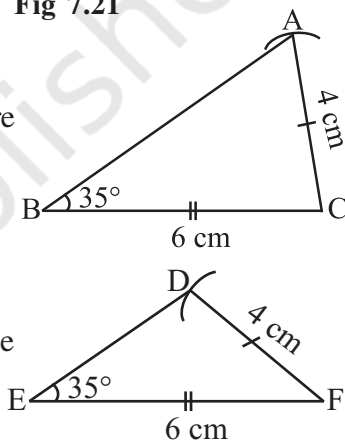
(It will be always helpful to draw a rough figure, mark the measurements and then probe the question).

**SOLUTION**

- (a) Here,  $AB = EF$  ( $= 7$  cm),  $BC = DE$  ( $= 5$  cm) and included  $\angle B =$  included  $\angle E$  ( $= 50^\circ$ ). Also,  $A \leftrightarrow F$ ,  $B \leftrightarrow E$  and  $C \leftrightarrow D$ .  
Therefore,  $\triangle ABC \cong \triangle FED$  (By SAS congruence rule) (Fig 7.20)

**Fig 7.20****Fig 7.21**

- (b) Here,  $AB = FD$  and  $AC = DE$  (Fig 7.21).  
But included  $\angle A \neq$  included  $\angle D$ . So, we cannot say that the triangles are congruent.
- (c) Here,  $BC = EF$ ,  $AC = DF$  and  $\angle B = \angle E$ .  
But  $\angle B$  is not the included angle between the sides  $AC$  and  $BC$ .  
Similarly,  $\angle E$  is not the included angle between the sides  $EF$  and  $DF$ .  
So, SAS congruence rule cannot be applied and we cannot conclude that the two triangles are congruent.

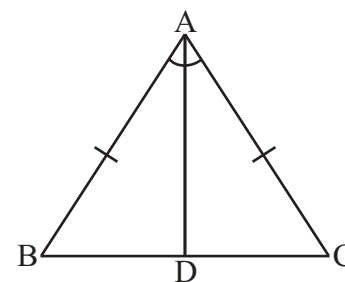
**Fig 7.22**

**EXAMPLE 5** In Fig 7.23,  $AB = AC$  and  $AD$  is the bisector of  $\angle BAC$ .

- State three pairs of equal parts in triangles  $ADB$  and  $ADC$ .
- Is  $\triangle ADB \cong \triangle ADC$ ? Give reasons.
- Is  $\angle B = \angle C$ ? Give reasons.

**SOLUTION**

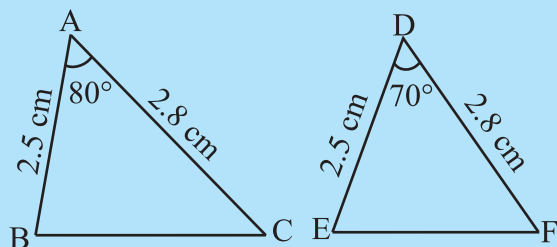
- The three pairs of equal parts are as follows:  
 $AB = AC$  (Given)  
 $\angle BAD = \angle CAD$  ( $AD$  bisects  $\angle BAC$ ) and  $AD = AD$  (common)
- Yes,  $\triangle ADB \cong \triangle ADC$  (By SAS congruence rule)
- $\angle B = \angle C$  (Corresponding parts of congruent triangles)

**Fig 7.23****TRY THESE**

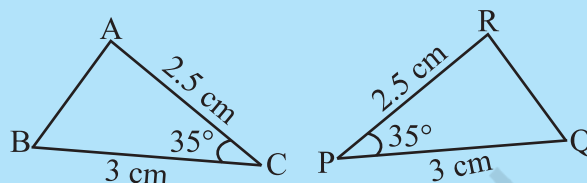
- Which angle is included between the sides  $\overline{DE}$  and  $\overline{EF}$  of  $\triangle DEF$ ?
- By applying SAS congruence rule, you want to establish that  $\triangle PQR \cong \triangle FED$ . It is given that  $PQ = FE$  and  $RP = DF$ . What additional information is needed to establish the congruence?



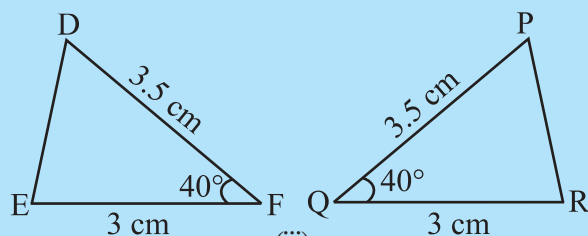
3. In Fig 7.24, measures of some parts of the triangles are indicated. By applying SAS congruence rule, state the pairs of congruent triangles, if any, in each case. In case of congruent triangles, write them in symbolic form.



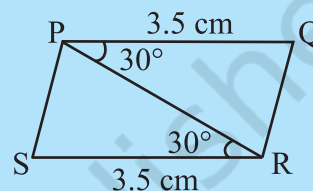
(i)



(ii)



(iii)



(iv)

Fig 7.24

4. In Fig 7.25,  $\overline{AB}$  and  $\overline{CD}$  bisect each other at O.

- State the three pairs of equal parts in two triangles AOC and BOD.
- Which of the following statements are true?
  - $\triangle AOC \cong \triangle DOB$
  - $\triangle AOC \cong \triangle BOD$

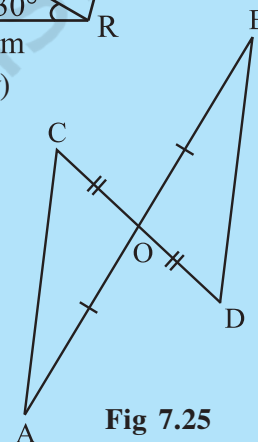


Fig 7.25

### ASA Game

Can you draw Appu's triangle, if you know

- only one of its angles?
- only two of its angles?
- two angles and any one side?
- two angles and the side included between them?

Attempts to solve the above questions lead us to the following criterion:

#### ASA Congruence criterion:

If under a correspondence, two angles and the included side of a triangle are equal to two corresponding angles and the included side of another triangle, then the triangles are congruent.

**EXAMPLE 6** By applying ASA congruence rule, it is to be established that  $\triangle ABC \cong \triangle QRP$  and it is given that  $BC = RP$ . What additional information is needed to establish the congruence?

**SOLUTION** For ASA congruence rule, we need the two angles between which the two sides BC and RP are included. So, the additional information is as follows:

$$\angle B = \angle R$$

and

$$\angle C = \angle P$$

**EXAMPLE 7** In Fig 7.26, can you use ASA congruence rule and conclude that  $\triangle AOC \cong \triangle BOD$ ?

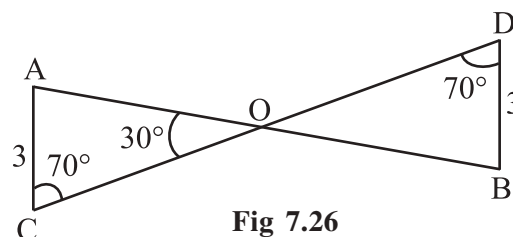


Fig 7.26

**SOLUTION** In the two triangles AOC and BOD,  $\angle C = \angle D$  (each  $70^\circ$ )

Also,  $\angle AOC = \angle BOD = 30^\circ$  (vertically opposite angles)

So,  $\angle A$  of  $\triangle AOC = 180^\circ - (70^\circ + 30^\circ) = 80^\circ$

(using angle sum property of a triangle)

Similarly,  $\angle B$  of  $\triangle BOD = 180^\circ - (70^\circ + 30^\circ) = 80^\circ$

Thus, we have  $\angle A = \angle B$ ,  $AC = BD$  and  $\angle C = \angle D$

Now, side AC is between  $\angle A$  and  $\angle C$  and side BD is between  $\angle B$  and  $\angle D$ .

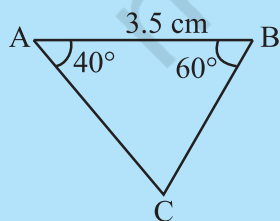
So, by ASA congruence rule,  $\triangle AOC \cong \triangle BOD$ .

### Remark

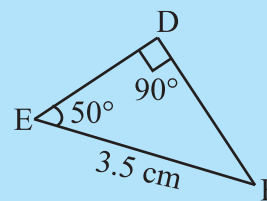
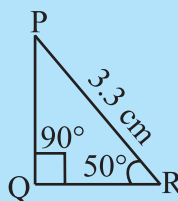
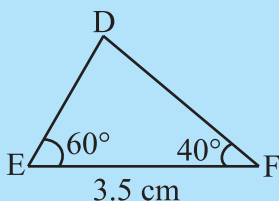
Given two angles of a triangle, you can always find the third angle of the triangle. So, whenever, two angles and one side of one triangle are equal to the corresponding two angles and one side of another triangle, you may convert it into 'two angles and the included side' form of congruence and then apply the ASA congruence rule.

### TRY THESE

1. What is the side included between the angles M and N of  $\triangle MNP$ ?
2. You want to establish  $\triangle DEF \cong \triangle MNP$ , using the ASA congruence rule. You are given that  $\angle D = \angle M$  and  $\angle F = \angle P$ . What information is needed to establish the congruence? (Draw a rough figure and then try!)
3. In Fig 7.27, measures of some parts are indicated. By applying ASA congruence rule, state which pairs of triangles are congruent. In case of congruence, write the result in symbolic form.



(i)



(ii)

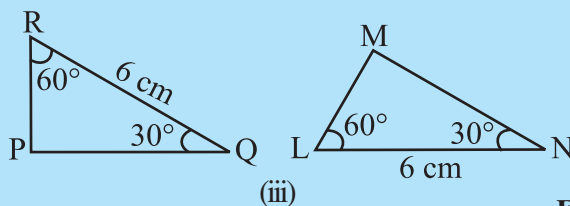
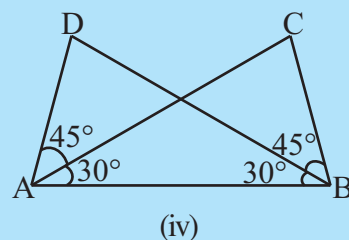


Fig 7.27



4. Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, by ASA congruence rule. In case of congruence, write it in symbolic form.

 **$\triangle DEF$** 

- (i)  $\angle D = 60^\circ$ ,  $\angle F = 80^\circ$ ,  $DF = 5$  cm  
 (ii)  $\angle D = 60^\circ$ ,  $\angle F = 80^\circ$ ,  $DF = 6$  cm  
 (iii)  $\angle E = 80^\circ$ ,  $\angle F = 30^\circ$ ,  $EF = 5$  cm

 **$\triangle PQR$** 

- $\angle Q = 60^\circ$ ,  $\angle R = 80^\circ$ ,  $QR = 5$  cm  
 $\angle Q = 60^\circ$ ,  $\angle R = 80^\circ$ ,  $QP = 6$  cm  
 $\angle P = 80^\circ$ ,  $PQ = 5$  cm,  $\angle R = 30^\circ$

5. In Fig 7.28, ray AZ bisects  $\angle DAB$  as well as  $\angle DCB$ .

- (i) State the three pairs of equal parts in triangles BAC and DAC.  
 (ii) Is  $\triangle BAC \cong \triangle DAC$ ? Give reasons.  
 (iii) Is  $AB = AD$ ? Justify your answer.  
 (iv) Is  $CD = CB$ ? Give reasons.

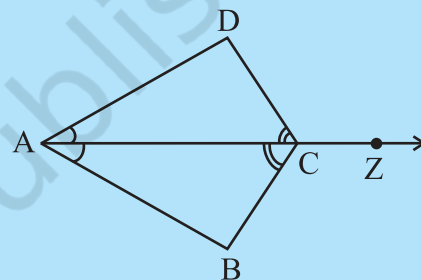


Fig 7.28

## 7.7 CONGRUENCE AMONG RIGHT-ANGLED TRIANGLES

Congruence in the case of two right triangles deserves special attention. In such triangles, obviously, the right angles are equal. So, the congruence criterion becomes easy.

Can you draw  $\triangle ABC$  (shown in Fig 7.29) with  $\angle B = 90^\circ$ , if

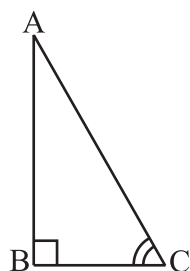


Fig 7.29

- (i) only BC is known?                      (ii) only  $\angle C$  is known?  
 (iii)  $\angle A$  and  $\angle C$  are known?              (iv) AB and BC are known?  
 (v) AC and one of AB or BC are known?

Try these with rough sketches. You will find that (iv) and (v) help you to draw the triangle. But case (iv) is simply the SAS condition. Case (v) is something new. This leads to the following criterion:

### RHS Congruence criterion:

If under a correspondence, the hypotenuse and one side of a right-angled triangle are respectively equal to the hypotenuse and one side of another right-angled triangle, then the triangles are congruent.

Why do we call this 'RHS' congruence? Think about it.

**EXAMPLE 8** Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, using RHS congruence rule. In case of congruent triangles, write the result in symbolic form:

$\triangle ABC$

$\triangle PQR$

(i)  $\angle B = 90^\circ$ ,  $AC = 8$  cm,  $AB = 3$  cm

$\angle P = 90^\circ$ ,  $PR = 3$  cm,  $QR = 8$  cm

(ii)  $\angle A = 90^\circ$ ,  $AC = 5$  cm,  $BC = 9$  cm

$\angle Q = 90^\circ$ ,  $PR = 8$  cm,  $PQ = 5$  cm

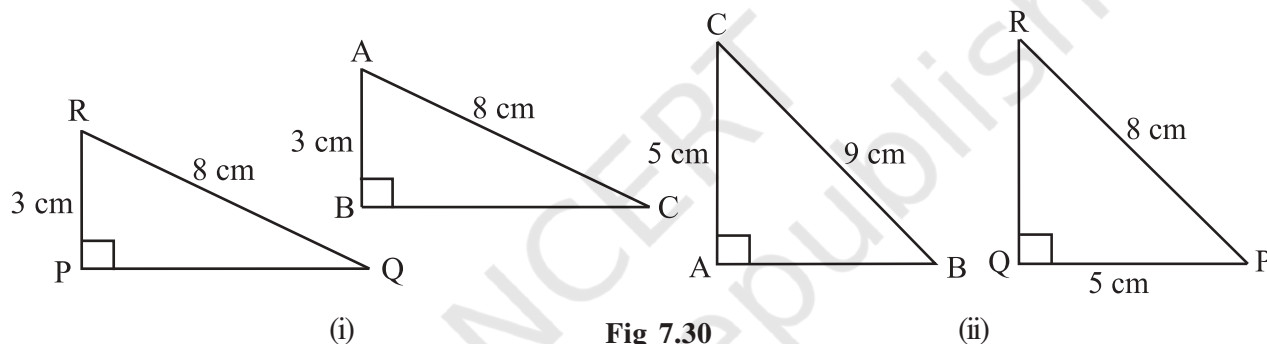
**SOLUTION**

(i) Here,  $\angle B = \angle P = 90^\circ$ ,

hypotenuse,  $AC = \text{hypotenuse, } RQ (= 8 \text{ cm})$  and

side  $AB = \text{side } RP (= 3 \text{ cm})$

So,  $\triangle ABC \cong \triangle RPQ$  (By RHS Congruence rule). [Fig 7.30(i)]



**Fig 7.30**

(ii) Here,  $\angle A = \angle Q (= 90^\circ)$  and  
side  $AC = \text{side } PQ (= 5 \text{ cm})$ .

But hypotenuse  $BC \neq \text{hypotenuse } PR$  [Fig 7.30(ii)]

So, the triangles are not congruent.

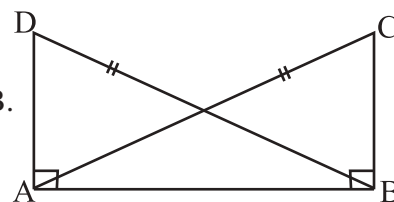
**EXAMPLE 9** In Fig 7.31,  $DA \perp AB$ ,  $CB \perp AB$  and  $AC = BD$ .

State the three pairs of equal parts in  $\triangle ABC$  and  $\triangle DAB$ .

Which of the following statements is meaningful?

(i)  $\triangle ABC \cong \triangle BAD$

(ii)  $\triangle ABC \cong \triangle ABD$



**Fig 7.31**

**SOLUTION** The three pairs of equal parts are:

$$\angle ABC = \angle BAD (= 90^\circ)$$

$$AC = BD \text{ (Given)}$$

$$AB = BA \text{ (Common side)}$$

From the above,

$$\triangle ABC \cong \triangle BAD \text{ (By RHS congruence rule).}$$

So, statement (i) is true

Statement (ii) is not meaningful, in the sense that the correspondence among the vertices is not satisfied.



## TRY THESE

1. In Fig 7.32, measures of some parts of triangles are given. By applying RHS congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form.

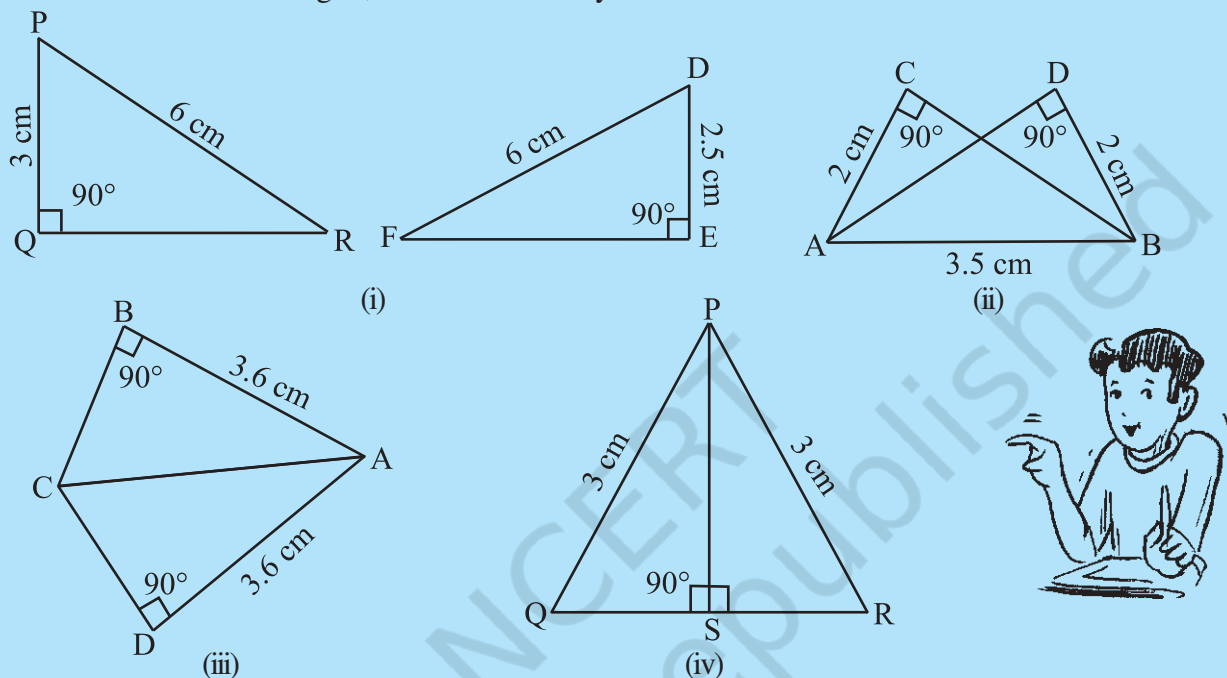


Fig 7.32

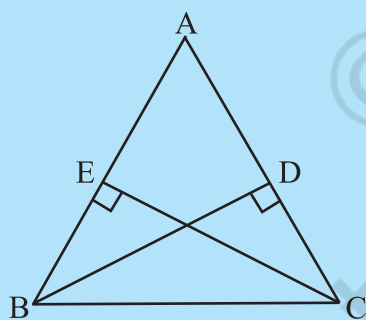


Fig 7.33

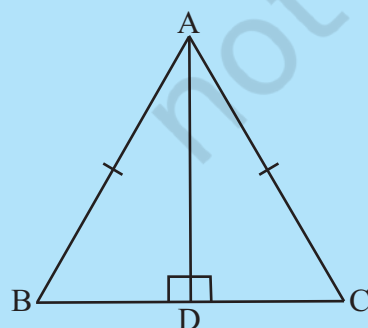


Fig 7.34

2. It is to be established by RHS congruence rule that  $\triangle ABC \cong \triangle RPQ$ . What additional information is needed, if it is given that  $\angle B = \angle P = 90^\circ$  and  $AB = RP$ ?
3. In Fig 7.33, BD and CE are altitudes of  $\triangle ABC$  such that  $BD = CE$ .
- State the three pairs of equal parts in  $\triangle CBD$  and  $\triangle BCE$ .
  - Is  $\triangle CBD \cong \triangle BCE$ ? Why or why not?
  - Is  $\angle DCB = \angle ECB$ ? Why or why not?
4. ABC is an isosceles triangle with  $AB = AC$  and AD is one of its altitudes (Fig 7.34).
- State the three pairs of equal parts in  $\triangle ADB$  and  $\triangle ADC$ .
  - Is  $\triangle ADB \cong \triangle ADC$ ? Why or why not?
  - Is  $\angle B = \angle C$ ? Why or why not?
  - Is  $BD = CD$ ? Why or why not?

We now turn to examples and problems based on the criteria seen so far.

### EXERCISE 7.2



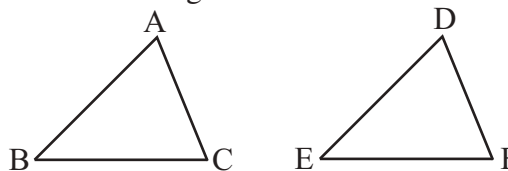
1. Which congruence criterion do you use in the following?

(a) **Given:**  $AC = DF$

$$AB = DE$$

$$BC = EF$$

$$\text{So, } \triangle ABC \cong \triangle DEF$$

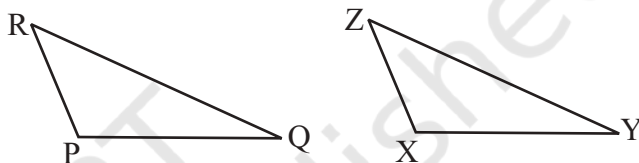


(b) **Given:**  $ZX = RP$

$$RQ = ZY$$

$$\angle PRQ = \angle XZY$$

$$\text{So, } \triangle PQR \cong \triangle XYZ$$

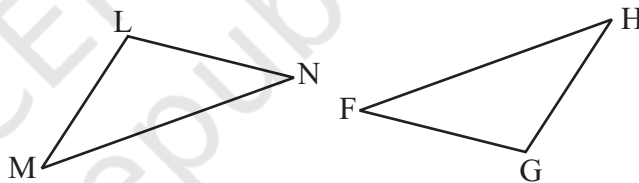


(c) **Given:**  $\angle MLN = \angle FGH$

$$\angle NML = \angle GFH$$

$$ML = FG$$

$$\text{So, } \triangle LMN \cong \triangle GFH$$

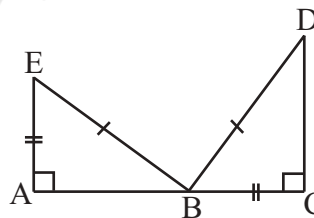


(d) **Given:**  $EB = DB$

$$AE = BC$$

$$\angle A = \angle C = 90^\circ$$

$$\text{So, } \triangle ABE \cong \triangle CDB$$



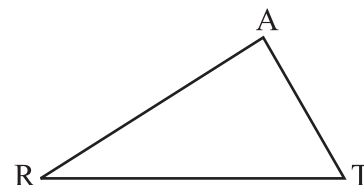
2. You want to show that  $\triangle ART \cong \triangle PEN$ ,

(a) If you have to use SSS criterion, then you need to show

(i)  $AR =$

(ii)  $RT =$

(iii)  $AT =$

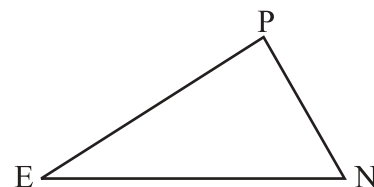


(b) If it is given that  $\angle T = \angle N$  and you are to use SAS criterion, you need to have

(i)  $RT =$

and

(ii)  $PN =$



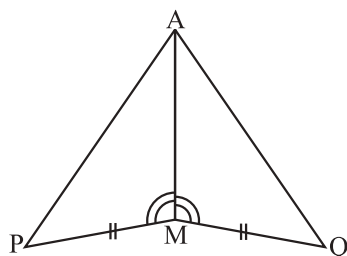
(c) If it is given that  $AT = PN$  and you are to use ASA criterion, you need to have

(i) ?

(ii) ?

3. You have to show that  $\triangle AMP \cong \triangle AMQ$ .

In the following proof, supply the missing reasons.



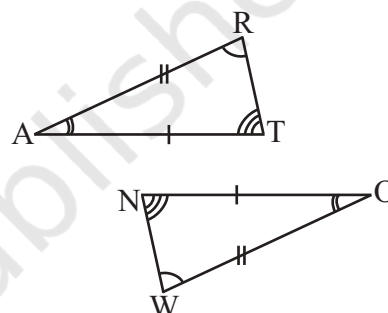
Steps	Reasons
(i) $PM = MQ$	(i) ...
(ii) $\angle PMA = \angle QMA$	(ii) ...
(iii) $AM = AM$	(iii) ...
(iv) $\triangle AMP \cong \triangle AMQ$	(iv) ...

4. In  $\triangle ABC$ ,  $\angle A = 30^\circ$ ,  $\angle B = 40^\circ$  and  $\angle C = 110^\circ$

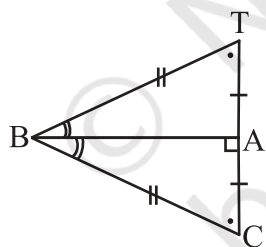
In  $\triangle PQR$ ,  $\angle P = 30^\circ$ ,  $\angle Q = 40^\circ$  and  $\angle R = 110^\circ$

A student says that  $\triangle ABC \cong \triangle PQR$  by AAA congruence criterion. Is he justified? Why or why not?

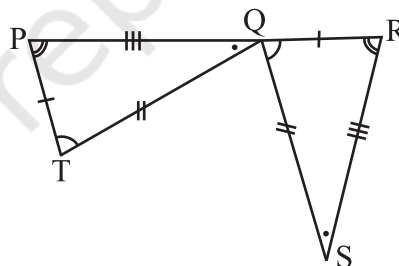
5. In the figure, the two triangles are congruent. The corresponding parts are marked. We can write  $\triangle RAT \cong ?$



6. Complete the congruence statement:



$\triangle BCA \cong ?$



$\triangle QRS \cong ?$

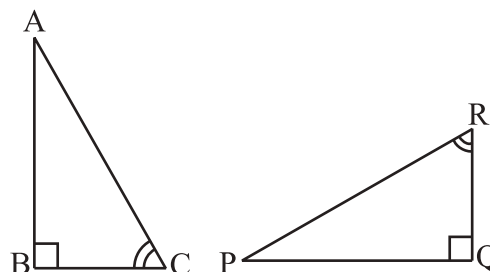
7. In a squared sheet, draw two triangles of equal areas such that

- the triangles are congruent.
- the triangles are not congruent.

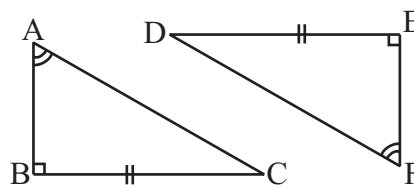
What can you say about their perimeters?

8. Draw a rough sketch of two triangles such that they have five pairs of congruent parts but still the triangles are not congruent.

9. If  $\triangle ABC$  and  $\triangle PQR$  are to be congruent, name one additional pair of corresponding parts. What criterion did you use?



10. Explain, why  
 $\triangle ABC \cong \triangle FED$ .



### Enrichment activity

We saw that superposition is a useful method to test congruence of plane figures. We discussed conditions for congruence of line segments, angles and triangles. You can now try to extend this idea to other plane figures as well.

1. Consider cut-outs of different sizes of squares. Use the method of superposition to find out the condition for congruence of squares. How does the idea of 'corresponding parts' under congruence apply? Are there corresponding sides? Are there corresponding diagonals?
2. What happens if you take circles? What is the condition for congruence of two circles? Again, you can use the method of superposition. Investigate.
3. Try to extend this idea to other plane figures like regular hexagons, etc.
4. Take two congruent copies of a triangle. By paper folding, investigate if they have equal altitudes. Do they have equal medians? What can you say about their perimeters and areas?

### WHAT HAVE WE DISCUSSED?

1. Congruent objects are exact copies of one another.
2. The method of superposition examines the congruence of plane figures.
3. Two plane figures, say,  $F_1$  and  $F_2$  are congruent if the trace-copy of  $F_1$  fits exactly on that of  $F_2$ . We write this as  $F_1 \cong F_2$ .
4. Two line segments, say,  $\overline{AB}$  and  $\overline{CD}$ , are congruent if they have equal lengths. We write this as  $\overline{AB} \cong \overline{CD}$ . However, it is common to write it as  $\overline{AB} = \overline{CD}$ .
5. Two angles, say,  $\angle ABC$  and  $\angle PQR$ , are congruent if their measures are equal. We write this as  $\angle ABC \cong \angle PQR$  or as  $m\angle ABC = m\angle PQR$ . However, in practice, it is common to write it as  $\angle ABC = \angle PQR$ .
6. SSS Congruence of two triangles:  
 Under a given correspondence, two triangles are congruent if the three sides of the one are equal to the three corresponding sides of the other.
7. SAS Congruence of two triangles:  
 Under a given correspondence, two triangles are congruent if two sides and the angle included between them in one of the triangles are equal to the corresponding sides and the angle included between them of the other triangle.

**8. ASA Congruence of two triangles:**

Under a given correspondence, two triangles are congruent if two angles and the side included between them in one of the triangles are equal to the corresponding angles and the side included between them of the other triangle.

**9. RHS Congruence of two right-angled triangles:**

Under a given correspondence, two right-angled triangles are congruent if the hypotenuse and a leg of one of the triangles are equal to the hypotenuse and the corresponding leg of the other triangle.

**10. There is no such thing as AAA Congruence of two triangles:**

Two triangles with equal corresponding angles need not be congruent. In such a correspondence, one of them can be an enlarged copy of the other. (They would be congruent only if they are exact copies of one another).

