Exercise 10g

Question 1.

If $y=(\sin x)^{(\sin x)^{(\sin x)\ldots\infty}}$, prove that $\frac{dy}{dx}=\frac{y^2\cot x}{(1-y\log\sin x)}$.

Answer:

Given : $y = (\sin x)^{(\sin x)^{(\sin x)^{(\sin x)\ldots\ldots\infty}}}$

To prove : $\frac{dy}{dx} = \frac{y^2 \cot x}{1-y\log \sin x}$

Formula used : $\log a = \log b^m$

$\log a = m \log b$

$$\frac{d(\log y)}{dx} = \frac{1}{y}\frac{dy}{dx}$$

$\frac{d(\sin x)}{dx}$ = cos x

If u and v are functions of x,then $\frac{d(uv)}{dx}$ = u$\frac{dv}{dx}$ + v$\frac{du}{dx}$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

y = $(\sin x)^y$

taking log on both sides

log y = log $(\sin x)^y$

log y = y log $(\sin x)$

Differentiating both sides with respect to x

$$\frac{d(\log y)}{dx} = \frac{d[y\log(\sin x)]}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log(\sin x) + y\,\frac{d\log(\sin x)}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log(\sin x) + y\,\frac{1}{\sin x} \times \frac{d(\sin x)}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log(\sin x) + y\,\frac{\cos x}{\sin x}$$

$$\left(\frac{1}{y} - \log \sin x\right)\frac{dy}{dx} = y\cot x$$

$$\frac{1 - y\log \sin x}{y}\frac{dy}{dx} = y\cot x$$

$$\frac{dy}{dx} = \frac{y^2 \cot x}{1 - y\log \sin x}$$

$$\frac{dy}{dx} = \frac{y^2 \cot x}{1 - y\log \sin x}$$

Question 2.

If $y = (\cos x)^{(\cos x)^{(\cos x)\ldots\infty}}$, prove that $\frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y\log \cos x)}$.

Answer:

Given : $y = (\cos x)^{(\cos x)^{(\cos x)^{(\cos x)\ldots\ldots\ldots\infty}}}$

To prove : $\frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y\log \cos x}$

Formula used : $\log a = \log b^m$

$\log a = m \log b$

$$\frac{d(\log y)}{dx} = \frac{1}{y}\frac{dy}{dx}$$

$\frac{d(\cos x)}{dx}$ = - sinx

If u and v are functions of x,then $\frac{d(uv)}{dx}$ = u$\frac{dv}{dx}$ + v$\frac{du}{dx}$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

Given that y = $(\cos x)^y$

taking log on both sides

log y = log $(\cos x)^y$

log y = y log $(\cos x)$

Differentiating both sides with respect to x

$\frac{d(\log y)}{dx} = \frac{d[y \log(\cos x)]}{dx}$

$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}$ log$(\cos x)$ + y $\frac{d \log(\cos x)}{dx}$

$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}$ log$(\cos x)$ + y $\frac{1}{\cos x} \times \frac{d(\cos x)}{dx}$

$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}$ log$(\cos x)$ + y $\frac{-\sin x}{\cos x}$

$(\frac{1}{y} -$log$\cos x)\frac{dy}{dx}$ = - y$\tan x$

$$\frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y\log \cos x}$$

$$\frac{dy}{dx} = \frac{-y^2 \tan x}{1 - y\log \cos x}$$

Question 3.

If $y=\sqrt{x+\sqrt{x+\sqrt{x+\ldots\infty}}}$, prove that $\frac{dy}{dx}=\frac{1}{(2y-1)}$.

Answer:

Given : $y=\sqrt{x+\sqrt{x+\sqrt{x+\ldots\ldots\ldots\infty}}}$

To prove : $\frac{dy}{dx}=\frac{1}{2y-1}$

Formula used : $\log a=\log b^{m}$

$\log a=m\log b$

$$\frac{d(\log y)}{dx}=\frac{1}{y}\frac{dy}{dx}$$

$\frac{dx}{dx}=1$

If u and v are functions of x,then $\frac{d(uv)}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

$$y=\sqrt{x+\sqrt{x+\sqrt{x+\ldots\ldots\ldots\infty}}}.$$

$y=\sqrt{x+y}$

squaring on both sides

$y^{2}=x+y$

Differentiating with respect to x

$2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$(2y - 1)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y-1}$

$\frac{dy}{dx} = \frac{1}{2y-1}$

Question 4.

If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \ldots\infty}}}$, prove that $\frac{dy}{dx} = \frac{\sin x}{(1-2y)}$.

Answer:

Given : $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \ldots\ldots\ldots.\infty}}}$

To prove : $\frac{dy}{dx} = \frac{\sin x}{2y-1}$

Formula used : $\log a = \log b^{m}$

$\log a = m \log b$

$\frac{d(\log y)}{dx} = \frac{1}{y}\frac{dy}{dx}$

$\frac{d(\cos x)}{dx}$ = - sinx

If u and v are functions of x,then $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

y = $\sqrt{\cos x + y}$

squaring on both sides

$y^2 = \cos x + y$

Differentiating with respect to x

$2y\frac{dy}{dx} = -\sin x + \frac{dy}{dx}$

$(2y - 1)\frac{dy}{dx} = -\sin x$

$\frac{dy}{dx} = \frac{-\sin x}{2y-1} = \frac{\sin x}{1-2y}$

$\frac{dy}{dx} = \frac{\sin x}{1-2y}$

$\frac{dy}{dx} = \frac{\sin x}{1-2y}$

Question 5.

If $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \ldots\infty}}}$, prove that $\frac{dy}{dx} = \frac{\sec^2 x}{(2y-1)}$.

Answer:

Given : $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \ldots\ldots\ldots.\infty}}}$

To prove : $\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$

Formula used : $\log a = \log b^m$

$\log a = m \log b$

$$\frac{d(\log y)}{dx} = \frac{1}{y}\frac{dy}{dx}$$

$\frac{d(\tan x)}{dx} = \sec^2 x$

If u and v are functions of x,then $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

y = $\sqrt{\tan x + y}$

squaring on both sides

$y^2 = \tan x + y$

Differentiating with respect to x

$2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$

$(2y - 1)\frac{dy}{dx} = \sec^2 x$

$\frac{dy}{dx} = \frac{\sec^2 x}{2y-1} = \frac{\sec^2 x}{2y-1}$

$\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$

$\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$

Question 6.

If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots\infty}}}$, show that $(2y - 1).\frac{dy}{dx} = \frac{1}{x}$.

Answer:

Given : $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots \ldots \ldots \infty}}}$

To show : $(2y-1).\frac{dy}{dx} = \frac{1}{x}$

Formula used : $\log a = \log b^{m}$

$\log a = m \log b$

$$\frac{d(\log y)}{dx} = \frac{1}{y}\frac{dy}{dx}$$

If u and v are functions of x,then $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

y = $\sqrt{\log x + y}$

squaring on both sides

$y^2 = \log x + y$

Differentiating with respect to x

$2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$

$(2y-1)\frac{dy}{dx} = \frac{1}{x}$

$(2y-1)\frac{dy}{dx} = \frac{1}{x}$

Question 7.

If $y = a^{x^{a^{x \ldots \infty}}}$, prove that $\frac{dy}{dx} = \frac{y^2(\log y)}{x[1-y(\log x)(\log y)]}$.

Answer:

Given : $y = a^{x^{a^{x \ldots\ldots\ldots \infty}}}$

To show : $\frac{dy}{dx} = \frac{y^2(\log y)}{x[1-y(\log x)(\log y)]}$

Formula used : $\log a = \log b^m$

$\log a = \mathrm{m} \log b$

$\frac{d(\log y)}{dx} = \frac{1}{y}\frac{dy}{dx}$

If u and v are functions of x,then $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

$y = a^{x^y}$

taking log on both sides

$\log y = \log a^{x^y}$

$\log y = x^y . \log a$

taking log on both sides

$\log(\log y) = \log(x^y . \log a)$

$\log(\log y) = y.\log x + \log(\log a)$

Differentiating both sides with respect to x

$\frac{d(\log[\log y])}{dx} = \frac{d(y.\log x)}{dx} + 0$ (as differentiation of $\log(\log a)$ [constant] is zero)

$\frac{1}{\log y}\frac{d\log y}{dx} = \log x\frac{dy}{dx} + y.\frac{d\log x}{dx}$

$\frac{1}{\log y}.\frac{1}{y}\frac{dy}{dx} = \log x\frac{dy}{dx} + y.\frac{1}{x}$

$(\frac{1}{\log y} \cdot \frac{1}{y} - \log x)\frac{dy}{dx} = \frac{y}{x}$

$(\frac{1 - y(\log x)(\log y)}{y(\log y)})\frac{dy}{dx} = \frac{y}{x}$

$\frac{dy}{dx} = \frac{y^2(\log y)}{x[1 - y(\log x)(\log y)]}.$

$$\frac{dy}{dx} = \frac{y^2(\log y)}{x[1 - y(\log x)(\log y)]}$$

Question 8.

If $y = x + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x_{\,...\infty,}}}}$ prove that $\frac{dy}{dx} = \frac{y}{(2y - x)}$.

Answer:

Given : $y = x + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x_{\,...\infty,}}}}$

To show : $\frac{dy}{dx} = \frac{y}{(2y - x)}$

Formula used : $\log a = \log b^m$

$\log a = m \log b$

$$\frac{d(\log y)}{dx} = \frac{1}{y}\frac{dy}{dx}$$

If u and v are functions of x,then $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

The CHAIN RULE states that the derivative of f(g(x)) is f′(g(x)).g′(x)

$y = x + \frac{1}{y}$

$y^2 = xy + 1$

Differentiating with respect to x

$\frac{d(y^2)}{dx} = \frac{d(xy)}{dx} + 0$ (as differentiation of constant is zero)

$2y.\frac{dy}{dx} = x.\frac{dy}{dx} + y$

$(2y - x)\frac{dy}{dx} = y$

$\frac{dy}{dx} = \frac{y}{(2y-x)}$

$\frac{dy}{dx} = \frac{y}{(2y-x)}$