Question 1.
Find the principal value of :

(i) $\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

(ii) $\sin^{-1}\left(\dfrac{1}{2}\right)$

(iii) $\cos^{-1}\left(\dfrac{1}{2}\right)$

(iv) $\tan^{-1}(1)$

(v) $\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

(vi) $\sec^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$

(vii) $\operatorname{cosec}^{-1}\left(\sqrt{2}\right)$

Answer:
NOTE:

Trigonometric Table

	0° (0)	30° ($\frac{\pi}{6}$)	45° ($\frac{\pi}{4}$)	60° ($\frac{\pi}{3}$)	90° ($\frac{\pi}{2}$)
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
cosec	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Undefined
cot	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

(i) Let $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$

$\Rightarrow \frac{\sqrt{3}}{2} = \sin x$ [We know which value of x when placed in sin gives us this answer]

$\therefore X = \frac{\pi}{3}$

(ii) Let $\sin^{-1}\left(\frac{1}{2}\right) = x$

$\Rightarrow \frac{1}{2} = \sin x$ [We know which value of x when put in this expression will give us this result]

$\Rightarrow X = \frac{\pi}{6}$

(iii) Let $\cos^{-1}\left(\frac{1}{2}\right) = x$

$\Rightarrow \frac{1}{2} = \cos x$ [We know which value of x when put in this expression will give us this result]

$\therefore X = \frac{\pi}{3}$

(iv) Let $\tan^{-1}(1) = x$

$\Rightarrow 1 = \tan x$ [We know which value of x when put in this expression will give us this result]

$$\therefore X = \frac{\pi}{4}$$

(v) Let $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = x$

$\Rightarrow \frac{1}{\sqrt{3}} = \tan x$ [We know which value of x when put in this expression will give us this result]

$$\therefore X = \frac{\pi}{6}$$

(vi) Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = x$

$\Rightarrow \frac{2}{\sqrt{3}} = \sec x$ [We know which value of x when put in this expression will give us this result]

$$\therefore X = \frac{\pi}{6}$$

(vii) Let $\csc^{-1}\left(\sqrt{2}\right) = x$

$\Rightarrow \sqrt{2} = \csc x$

[We know which value of x when put in this expression will give us this result]

$$\therefore X = \frac{\pi}{4}$$

Question 2.
Find the principal value of :

(i) $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

(ii) $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

(iii) $\tan^{-1}\left(-\sqrt{3}\right)$

(iv) $\sec^{-1}\left(-2\right)$

(v) $\cosec^{-1}\left(-\sqrt{2}\right)$

(vi) $\cot^{-1}\left(\dfrac{-1}{\sqrt{3}}\right)$

Answer:

(i). Let $\sin^{-1}\left(\dfrac{-1}{\sqrt{2}}\right) = x$

$\Rightarrow -\sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = x$ [Formula: $\sin^{-1}(-x) = -\sin^{-1} x$]

$\Rightarrow \dfrac{1}{\sqrt{2}} = -\sin x$ [We know which value of x when put in this expression will give us this result]

$\therefore X = -\dfrac{\pi}{4}$

(ii)$\cos^{-1}\left(\dfrac{-\sqrt{3}}{2}\right) = \pi - \cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ [Formula: $\cos^{-1}(-x) = \pi - \cos^{-1} x$]

Let $\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = x$

$\Rightarrow \left(\dfrac{\sqrt{3}}{2}\right) = \cos x$ [We know which value of x when put in this expression will give us this result]

$\therefore X = \dfrac{\pi}{6}$

Putting this value back in the equation

$\pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$

(iii). Let $\tan^{-1}\left(-\sqrt{3}\right) = x$

$\Rightarrow -\tan^{-1}\left(\sqrt{3}\right) = x$ [Formula: $\tan^{-1}(-x) = -\tan^{-1}(x)$]

$\Rightarrow \sqrt{3} = -\tan x$ [We know which value of x when put in this expression will give us this result]

$$\therefore X = \frac{-\pi}{3}$$

(iv).$\sec^{-1}(-2) = \pi - \sec^{-1}(2)$ ...(i) [Formula:$\sec^{-1}(-x) = \pi - \sec^{-1}(x)$]

Let $\sec^{-1}(2) = x$

$\Rightarrow 2 = \sec x$ [We know which value of x when put in this expression will give us this result]

$$\therefore X = \frac{\pi}{3}$$

Putting the value in (i)

$$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

(v). Let $\csc^{-1}\left(-\sqrt{2}\right) = x$

$\Rightarrow -\csc^{-1}(\sqrt{2}) = x$ [Formula: $\csc^{-1}(-x) = -\csc^{-1}(x)$]

$\Rightarrow \sqrt{2} = -\csc x$

$$\therefore X = -\frac{\pi}{4}$$

(vi).$\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \pi - \cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$ ... (i)

Let $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = x$

$\Rightarrow \frac{1}{\sqrt{3}} = \cot^{-1} x$ [We know which value of x when put in this expression will give us this result]

$$\Rightarrow X = \frac{\pi}{3}$$

Putting in (i)

$$\pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

Question 3.

Evaluate $\cos\left\{\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right\}$.

Answer:

$\cos\left\{\pi - \frac{\pi}{6} + \frac{\pi}{6}\right\}$ [Refer to question 2(ii)]

$= \cos\{\pi\}$

$= \cos\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$

$= -1$

Question 4.

Evaluate $\sin\left\{\frac{\pi}{2} - \left(\frac{-\pi}{3}\right)\right\}$

Answer:

$\sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$

$= \sin\left(\frac{5\pi}{6}\right)$

$= \sin\left(\pi - \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{6}$

$= \frac{1}{2}$