Exercise 28i

Question 1.
Show that the lines $\bar{r} = (2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\bar{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$ are coplanar.

Also find the equation of the plane containing these lines.

Answer:
Given : Equations of lines -

$$\bar{r_1} = (2\hat{j} - 3\hat{k}) + \lambda(\hat{\imath} + 2\hat{j} + 3\hat{k})$$

$$\bar{r_2} = (2\hat{\imath} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{\imath} + 3\hat{j} + 4\hat{k})$$

To Prove : $\bar{r_1}$ & $\bar{r_2}$ are coplanar.

To Find : Equation of plane.

Formulae :

1) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Coplanarity of two lines :

If two lines $\overline{r_1} = \bar{a} + \lambda\bar{b}$ & $\overline{r_2} = \bar{c} + \mu\bar{d}$ are coplanar then

$\bar{a}.\left(\bar{b} \times \bar{d}\right) = \bar{c}.\left(\bar{b} \times \bar{d}\right)$

4) Equation of plane :

If two lines $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$\bar{r}.\left(\overline{b_1} \times \overline{b_2}\right) = \overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right)$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Answer :

Given equations of lines are

$\overline{r_1} = \left(2\hat{\jmath} - 3\hat{k}\right) + \lambda\left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right)$

$\overline{r_2} = \left(2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right)$

Let, $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$

Where,

$\overline{a_1} = 2\hat{j} - 3\hat{k}$

$\overline{b_1} = \hat{\imath} + 2\hat{j} + 3\hat{k}$

$\overline{a_2} = 2\hat{\imath} + 6\hat{j} + 3\hat{k}$

$\overline{b_2} = 2\hat{\imath} + 3\hat{j} + 4\hat{k}$

Now,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$= \hat{\imath}(8 - 9) - \hat{j}(4 - 6) + \hat{k}(3 - 4)$

$\therefore \left(\overline{b_1} \times \overline{b_2}\right) = -\hat{\imath} + 2\hat{j} - \hat{k}$

Therefore,

$\overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = (0 \times (-1)) + (2 \times 2) + ((-3) \times (-1))$

= 0 + 4 + 3

= 7

$\therefore \overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = 7$ ……… eq(1)

And

$\overline{a_2}.\left(\overline{b_1} \times \overline{b_2}\right) = (2 \times (-1)) + (6 \times 2) + (3 \times (-1))$

= - 2 + 12 – 3

= 7

$\therefore \overline{a_2}.\left(\overline{b_1} \times \overline{b_2}\right) = 7$ ……… eq(2)

From eq(1) and eq(2)

$$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = \overline{a_2}.(\overline{b_1} \times \overline{b_2})$$

Hence lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar.

Equation of plane containing lines $\overline{r_1}$ & $\overline{r_2}$ is

$$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$$

Now,

$$\overline{b_1} \times \overline{b_2} = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$$

From eq(1)

$$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = 7$$

Therefore, equation of required plane is

$$\bar{r}.(-\hat{\imath} + 2\hat{\jmath} - \hat{k}) = 7$$

$$\therefore \bar{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k}) = -7$$

$$\therefore \bar{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k}) + 7 = 0$$

$$\bar{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k}) + 7 = 0$$

Question 2.
Find the vector and Cartesian forms of the equations of the plane containing the two lines $\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $.\bar{r} = (9\hat{i} + 5\hat{j} - \hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$..

Answer:
Given : Equations of lines -

$$\overline{r_1} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$$

$\overline{r_2} = (9\hat{\imath} + 5\hat{\jmath} - \hat{k}) + \mu(-2\hat{\imath} + 3\hat{\jmath} + 8\hat{k})$

To Find : Equation of plane.

Formulae :

1) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

If two lines $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Where,

Given equations of lines are

$\overline{r_1} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

$\overline{r_2} = (9\hat{\imath} + 5\hat{\jmath} - \hat{k}) + \mu(-2\hat{\imath} + 3\hat{\jmath} + 8\hat{k})$

Let, $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$

Where,

$\overline{a_1} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k}$

$\overline{b_1} \; = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$

$\overline{a_2} = 9\hat{\imath} + 5\hat{\jmath} - \hat{k}$

$\overline{b_2} = -2\hat{\imath} + 3\hat{\jmath} + 8\hat{k}$

Now,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix}$$

$= \hat{\imath}(24 - 18) - \hat{\jmath}(16 + 12) + \hat{k}(6 + 6)$

$\therefore (\overline{b_1} \times \overline{b_2}) = 6\hat{\imath} - 28\hat{\jmath} + 12\hat{k}$

Therefore,

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = (1 \times 6) + (2 \times (-28)) + ((-4) \times 12)$

= 6 – 56 – 48

= - 98

$\therefore \overline{a_1}.(\overline{b_1} \times \overline{b_2}) = -98$ ....eq(1)

Equation of plane containing lines $\overline{r_1}$ & $\overline{r_2}$ is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Now,

$\overline{b_1} \times \overline{b_2} = 6\hat{\imath} - 28\hat{\jmath} + 12\hat{k}$

From eq(1)

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = -98$

Therefore, equation of required plane is

$\bar{r}.(6\hat{\imath} - 28\hat{\jmath} + 12\hat{k}) = -98$

$\therefore \bar{r}.(6\hat{\imath} - 28\hat{\jmath} + 12\hat{k}) + 98 = 0$

This vector equation of plane.

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore \bar{r}.(\overline{b_1} \times \overline{b_2}) = (x \times 6) + (y \times (-28)) + (z \times 12)$

= 6x – 28y + 12z

Therefore, equation of plane is

6x – 28y + 12z = -98

6x – 28y + 12z + 98 = 0

This Cartesian equation of plane.

Question 3.

Find the vector and Cartesian equations of a plane containing the two lines $\bar{r} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 5\hat{k}\right)$ and $\bar{r} = \left(3\hat{i} + 3\hat{j} + 2\hat{k}\right) + \mu\left(3\hat{i} - 2\hat{j} + 5\hat{k}\right)$. Also show that the lines $\bar{r} = \left(2\hat{i} + 5\hat{j} + 2\hat{k}\right) + p\left(3\hat{i} - 2\hat{j} + 5\hat{k}\right)$ lies in the plane.

Answer:
Given : Equations of lines -

$$\overline{r_1} = \left(2\hat{\imath} + \hat{\jmath} - 3\hat{k}\right) + \lambda\left(\hat{\imath} + 2\hat{\jmath} + 5\hat{k}\right)$$

$$\overline{r_2} = \left(3\hat{\imath} + 3\hat{\jmath} + 2\hat{k}\right) + \mu\left(3\hat{\imath} - 2\hat{\jmath} + 5\hat{k}\right)$$

To Prove : $\overline{r_1}$ & $\overline{r_2}$ are coplanar.

To Find : Equation of plane.

Formulae :

1) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

3) Coplanarity of two lines :

If two lines $\overline{r_1} = \bar{a} + \lambda\bar{b}$ & $\overline{r_2} = \bar{c} + \mu\bar{d}$ are coplanar then

$$\bar{a}.\left(\bar{b} \times \bar{d}\right) = \bar{c}.\left(\bar{b} \times \bar{d}\right)$$

4) Equation of plane :

If two lines $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$$\bar{r}.\left(\overline{b_1} \times \overline{b_2}\right) = \overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right)$$

Where,

$$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

Answer :

Given equations of lines are

$$\overline{r_1} = \left(2\hat{\imath} + \hat{\jmath} - 3\hat{k}\right) + \lambda\left(\hat{\imath} + 2\hat{\jmath} + 5\hat{k}\right)$$

$$\overline{r_2} = \left(3\hat{\imath} + 3\hat{\jmath} + 2\hat{k}\right) + \mu\left(3\hat{\imath} - 2\hat{\jmath} + 5\hat{k}\right)$$

Let, $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$

Where,

$$\overline{a_1} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$$

$$\overline{b_1} = \hat{\imath} + 2\hat{\jmath} + 5\hat{k}$$

$\overline{a_2} = 3\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

$\overline{b_2} = 3\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$

Now,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix}$$

$= \hat{\imath}(10 + 10) - \hat{\jmath}(5 - 15) + \hat{k}(-2 - 6)$

$\therefore \left(\overline{b_1} \times \overline{b_2}\right) = 20\hat{\imath} + 10\hat{\jmath} - 8\hat{k}$

Therefore,

$\overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = (2 \times 20) + (1 \times 10) + ((-3) \times (-8))$

= 40 + 10 + 24

= 74

$\therefore \overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = 74$ ……… eq(1)

And

$\overline{a_2}.\left(\overline{b_1} \times \overline{b_2}\right) = (3 \times 20) + (3 \times 10) + (2 \times (-8))$

= 60 + 30 – 16

= 74

$\therefore \overline{a_2}.\left(\overline{b_1} \times \overline{b_2}\right) = 74$ ……… eq(2)

From eq(1) and eq(2)

$\overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = \overline{a_2}.\left(\overline{b_1} \times \overline{b_2}\right)$

Hence lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar.

Equation of plane containing lines $\overline{r_1}$ & $\overline{r_2}$ is

$$\bar{r}.\left(\overline{b_1} \times \overline{b_2}\right) = \overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right)$$

Now,

$$\overline{b_1} \times \overline{b_2} = 20\hat{\imath} + 10\hat{\jmath} - 8\hat{k}$$

From eq(1)

$$\overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = 74$$

Therefore, equation of required plane is

$$\bar{r}.\left(20\hat{\imath} + 10\hat{\jmath} - 8\hat{k}\right) = 74$$

$$\therefore \bar{r}.\left(10\hat{\imath} + 5\hat{\jmath} - 4\hat{k}\right) = 37$$

$$\therefore \bar{r}.\left(10\hat{\imath} + 5\hat{\jmath} - 4\hat{k}\right) - 37 = 0$$

This vector equation of plane.

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$$\therefore \bar{r}.\left(\overline{b_1} \times \overline{b_2}\right) = (x \times 20) + (y \times 10) + (z \times (-8))$$

= 20x + 10y – 8z

Therefore, equation of plane is

20x + 10y – 8z = 74

20x + 10y – 8z – 74 = 0

10x + 5y – 4z – 37 = 0

This Cartesian equation of plane.

Question 4.

Prove that the lines $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ are coplanar. Also find the equation of the plane containing these lines.

Answer:

Given : Equations of lines –

Line 1 : $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$

Line 2 : $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$, then these lines are coplanar, if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Answer :

Given lines –

Line 1 : $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$

Line 2 : $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$

Here, $x_1 = 0$, $y_1 = 2$, $z_1 = -3$, $a_1 = 1$, $b_1 = 2$, $c_1 = 3$

$x_2 = 2$, $y_2 = 6$, $z_2 = 3$, $a_2 = 2$, $b_2 = 3$, $c_2 = 4$

Now,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 2-0 & 6-2 & 3+3 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= 2(8-9) - 4(4-6) + 6(3-4)$$

$$= 2(-1) - 4(-2) + 6(-1)$$

= - 2 + 8 – 6

= 0

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Hence, given two lines are coplanar.

Equation of plane passing through line1 and line 2 is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x-0 & y-2 & z+3 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$$\therefore (x-0)\times(8-9)-(y-2)\times(4-6)+(z+3)\times(3-4)=0$$

$$\therefore -1(x)-(y-2)(-2)+(z+3)(-1)=0$$

-x + 2y – 4 – z – 3 = 0

- x + 2y – z – 7 = 0

x – 2y + z + 7 = 0

Therefore, equation of plane is

x – 2y + z + 7 = 0

Question 5.

Prove that the lines $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$ and $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ are coplanar. Also find the equation of the plane containing these lines.

Answer:

Given : Equations of lines –

Line 1 : $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$

Line 2 : $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$, then these lines are coplanar, if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by,

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Answer :

Given lines –

Line 1 : $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$

Line 2 : $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$

Here, $x_1 = 2$, $y_1 = 4$, $z_1 = 6$, $a_1 = 1$, $b_1 = 4$, $c_1 = 7$

$x_2 = -1$, $y_2 = -3$, $z_2 = -5$, $a_2 = 3$, $b_2 = 5$, $c_2 = 7$

Now,

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} -1-2 & -3-4 & -5-6 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & -7 & -11 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix}$$

$$= -3(28-35) - (-7)(7-21) - 11(5-12)$$

$$= -3(-7) + 7(-14) - 11(-7)$$

= 21 - 98 + 77

= 0

$$\therefore \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Hence, given two lines are coplanar.

Equation of plane passing through line 1 and line 2 is given by,

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x-2 & y-4 & z-6 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix} = 0$$

$$\therefore (x-2) \times (28-35) - (y-4) \times (7-21) + (z-6) \times (5-12) = 0$$

$$\therefore -7(x-2) - (y-4)(-14) + (z-6)(-7) = 0$$

-7x + 14 + 14y - 56 – 7z + 42 = 0

- 7x + 14y – 7z = 0

x – 2y + z = 0

Therefore, equation of plane is

x – 2y + z = 0

Question 6.

Show that the lines $\frac{5-x}{-4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{2y-8}{2}=\frac{z-5}{3}$ are coplanar. Find the equation of the plane containing these lines.

Answer:

Given : Equations of lines –

Line 1 : $\frac{5-x}{-4}=\frac{y-7}{4}=\frac{z+3}{-5}$ or $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$

Line 2 : $\frac{x-8}{7}=\frac{2y-8}{2}=\frac{z-5}{3}$ or $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$, then these lines are coplanar, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Answer :

Given lines –

Line 1 : $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$

Line 2 : $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$

Here, $x_1 = 5$, $y_1 = 7$, $z_1 = -3$, $a_1 = 4$, $b_1 = 4$, $c_1 = -5$

$x_2 = 8$, $y_2 = 4$, $z_2 = 5$, $a_2 = 7$, $b_2 = 1$, $c_2 = 3$

Now,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 8-5 & 4-7 & 5+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & -3 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$$= 3(12+5) - (-3)(12+35) + 8(4-28)$$

$$= 3(17) + 3(47) + 8(-24)$$

= 51 + 141 – 192

= 0

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Hence, given two lines are coplanar.

Equation of plane passing through line1 and line 2 is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x-5 & y-7 & z+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix} = 0$$

$$\therefore (x-5)\times(12+5)-(y-7)\times(12+35)+(z+3)\times(4-28)=0$$

$$\therefore 17(x-5)-47(y-7)+(z+3)(-24)=0$$

17x – 85 – 47y + 329 – 24z – 72 = 0

17x - 47y – 24z + 172 = 0

Therefore, equation of plane is

17x - 47y – 24z + 172 = 0

Question 7.

Show that the lines $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$ are coplanar. Find the equation of the plane containing these lines.

Answer:

Given : Equations of lines –

Line 1 : $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$

Line 2 : $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$, then these lines are coplanar, if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by,

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Answer :

Given lines –

Line 1 : $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$

Line 2 : $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$

Here, $x_1 = -1$, $y_1 = 3$, $z_1 = -2$, $a_1 = -3$, $b_1 = 2$, $c_1 = 1$

$x_2 = 0$, $y_2 = 7$, $z_2 = -7$, $a_2 = 1$, $b_2 = -3$, $c_2 = 2$

Now,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 0+1 & 7-3 & -7+2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 4 & -5 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= 1(4+3) - 4(-6-1) - 5(9-2)$$

$$= 1(7) - 4(-7) - 5(7)$$

= 7 + 28 – 35

= 0

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Hence, given two lines are coplanar.

Equation of plane passing through line1 and line 2 is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x+1 & y-3 & z+2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix} = 0$$

$$\therefore (x+1) \times (4+3) - (y-3) \times (-6-1) + (z+2) \times (9-2) = 0$$

$$\therefore 7(x+1) - (y-3)(-7) + (z+2)(7) = 0$$

7x + 7 + 7y – 21 + 7z + 14 = 0

7x + 7y + 7z = 0

x + y + z = 0

Therefore, equation of plane is

Question 8.
Show that the lines $\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z}{-1}$ and $\frac{x-4}{3}=\frac{y-1}{-2}=\frac{z-1}{-1}$ are coplanar. Also find the equation of the plane containing these lines.

Answer:
Given : Equations of lines –

Line 1 : $\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z}{-1}$

Line 2 : $\frac{x-4}{3}=\frac{y-1}{-2}=\frac{z-1}{-1}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$, then these lines are coplanar, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Answer :

Given lines –

Line 1 : $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z}{-1}$

Line 2 : $\frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{-1}$

Here, $x_1 = 1$, $y_1 = 3$, $z_1 = 0$, $a_1 = 2$, $b_1 = -1$, $c_1 = -1$

$x_2 = 4$, $y_2 = 1$, $z_2 = 1$, $a_2 = 3$, $b_2 = -2$, $c_2 = -1$

Now,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 4-1 & 1-3 & 1-0 \\ 2 & -1 & -1 \\ 3 & -2 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & -2 & 1 \\ 2 & -1 & -1 \\ 3 & -2 & -1 \end{vmatrix}$$

$$= 3(1-2) - (-2)(-2+3) + 1(-4+3)$$

$$= 3(-1) + 2(1) + 1(-1)$$

= - 2

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} \neq 0$$

Hence, given two lines are not coplanar.

Question 9.

Find the equation of the plane which contains two parallel lines given by $\frac{x-3}{1}=\frac{y+2}{-4}=\frac{z}{5}$ and $\frac{x-4}{1}=\frac{y-3}{-4}=\frac{z-2}{5}$.

Answer:
Given : Equations of lines –

Line 1 : $\frac{x-3}{1}=\frac{y+2}{-4}=\frac{z}{5}$

Line 2 : $\frac{x-4}{1}=\frac{y-3}{-4}=\frac{z-2}{5}$

To Find : Equation of plane.

Formulae :

Equation of plane :

The equation of plane containing two parallel lines $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

& $\frac{x-x_2}{a}=\frac{y-y_2}{b}=\frac{z-z_2}{c}$ is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a & b & c \end{vmatrix}=0$$

Answer :

Given lines –

Line 1 : $\frac{x-3}{1}=\frac{y+2}{-4}=\frac{z}{5}$

Line 2 : $\frac{x-4}{1}=\frac{y-3}{-4}=\frac{z-2}{5}$

Here, $x_1 = 3$, $y_1 = -2$, $z_1 = 0$, a= 1 , b = -4 , c = 5

$x_2 = 4$, $y_2 = 3$, $z_2 = 2$

Therefore, equation of plane containing line 1 & line 2 is given by,

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a & b & c \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x - 3 & y + 2 & z - 0 \\ 4 - 3 & 3 + 2 & 2 - 0 \\ 1 & -4 & 5 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x - 3 & y + 2 & z \\ 1 & 5 & 2 \\ 1 & -4 & 5 \end{vmatrix} = 0$$

$$\therefore (x - 3) \times (25 + 8) - (y + 2) \times (5 - 2) + (z) \times (-4 - 5) = 0$$

$$\therefore 33(x - 3) - (y + 2)(3) + (z)(-9) = 0$$

33x – 99 – 3y – 6 – 9z = 0

33x – 3y – 9z – 105 = 0

11x – y – 3z = 35