# Exercise 10c

**Question 1.**
Differentiate each of the following w.r.t. x:

$\cos^{-1} 2x$

**Answer:**
Formulae :

i) $\frac{d}{dx}\left(\cos^{-1}x\right) = \frac{-1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}(kx) = k$

Answer :

Let,

$y = \cos^{-1}2x$

and u = 2x

therefore, $y = \cos^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ ............ By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}\left(\cos^{-1}u\right) \cdot \frac{d}{dx}(2x)$$

$$= \frac{-1}{\sqrt{1-u^2}} \cdot 2$$

............ $\left(\because \frac{d}{dx}\left(\cos^{-1}x\right) = \frac{-1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx}(kx) = k\right)$

$$= \frac{-2}{\sqrt{1-(2x)^2}}$$

$$= \frac{-2}{\sqrt{1-4x^2}}$$

$$\therefore \frac{dy}{dx} = \frac{-2}{\sqrt{1-4x^2}}$$

**Question 2.**
Differentiate each of the following w.r.t. x:

$$\tan^{-1} x^2$$

**Answer:**
Formulae :

i) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}(x^n) = n.x^{n-1}$

Answer :

Let,

$y = \tan^{-1}x^2$

and u = $x^2$

therefore, $y = \tan^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ ............ By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\tan^{-1}u) \cdot \frac{d}{dx}(x^2)$$

$$= \frac{1}{1+u^2} \cdot 2x$$

$$\ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \ \& \ \frac{d}{dx}(x^n) = n.x^{n-1}\right)$$

$$= \frac{2x}{1+(x^2)^2}$$

$$= \frac{2x}{1+x^4}$$

$$\therefore \frac{dy}{dx} = \frac{2x}{1+x^4}$$

**Question 3.**
Differentiate each of the following w.r.t. x:

$$\sec^{-1}\sqrt{x}$$

**Answer:**
Formulae :

i) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

ii) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Answer :

Let,

$$y = \sec^{-1}\sqrt{x}$$

and $u = \sqrt{x}$

therefore, $y = \sec^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ ............ By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\sec^{-1}u) . \frac{d}{dx}(\sqrt{x})$$

$$= \frac{1}{u\sqrt{u^2 - 1}} . \frac{1}{2\sqrt{x}}$$

............ $\left(\because \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}} \ \& \ \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}\right)$

$$= \frac{1}{\sqrt{x}\sqrt{\left(\sqrt{x}\right)^2 - 1}} . \left(\frac{1}{2\sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{x}.\sqrt{x}\sqrt{x-1}}$$

$$= \frac{1}{2x\sqrt{x-1}}$$

$\therefore \frac{dy}{dx} = \frac{1}{2x\sqrt{x-1}}$

**Question 4.**
Differentiate each of the following w.r.t. x:

$$\sin^{-1}\frac{x}{a}$$

**Answer:**
Formulae :

i) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}(kx) = k$

<u>Answer</u> :

Let,

$y = \sin^{-1}\left(\frac{x}{a}\right)$

and $u = \frac{x}{a}$

therefore, $y = \sin^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ ............ By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\sin^{-1}u) \cdot \frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{1}{\sqrt{1-u^2}} \cdot \frac{1}{a}$$

............ $\left(\because \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx}(kx) = k\right)$

$$= \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}} \cdot \frac{1}{a}$$

$$= \frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}} \cdot \frac{1}{a}$$

$$= \frac{a}{\sqrt{a^2 - x^2}} . \frac{1}{a}$$

$$= \frac{1}{\sqrt{a^2 - x^2}}$$

$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{a^2 - x^2}}$

**Question 5.**
Differentiate each of the following w.r.t. x:

$\tan^{-1}(\log x)$

**Answer:**
Formulae :

i) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}(\log x) = \frac{1}{x}$

Answer :

Let,

$y = \tan^{-1}(\log x)$

and u = log x

therefore, $y = \tan^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ ............ By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\tan^{-1}u) . \frac{d}{dx}(\log x)$$

$$= \frac{1}{1+u^2} . \frac{1}{x}$$

............. $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \ \& \ \frac{d}{dx}(\log x) = \frac{1}{x}\right)$

$$= \frac{1}{1+(\log x)^2} . \frac{1}{x}$$

$$= \frac{1}{x\{1+(\log x)^2\}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{x\{1+(\log x)^2\}}$$

**Question 6.**
Differentiate each of the following w.r.t. x:

$$\cot^{-1}(e^x)$$

**Answer:**
Formulae :

i) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

ii) $\frac{d}{dx}(e^x) = e^x$

Answer :

Let,

$y = \cot^{-1}(e^x)$

and $u = e^x$

therefore, $y = \cot^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ ............ By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}\left(\cot^{-1}u\right) . \frac{d}{dx}\left(e^{x}\right)$$

$$= \frac{-1}{1+u^{2}} . e^{x}$$

............ $\left(\because \frac{d}{dx}\left(\cot^{-1}x\right) = \frac{-1}{1+x^{2}} \;\&\; \frac{d}{dx}\left(e^{x}\right) = e^{x}\right)$

$$= \frac{-1}{1+\left(e^{x}\right)^{2}} . e^{x}$$

$$= \frac{-e^{x}}{1+e^{2x}}$$

$\therefore \frac{dy}{dx} = \frac{-e^{x}}{1+e^{2x}}$

**Question 7.**
Differentiate each of the following w.r.t. x:

$\log(\tan^{-1}x)$

**Answer:**
Formulae :

i) $\frac{d}{dx}(\log x) = \frac{1}{x}$

ii) $\frac{d}{dx}\left(\tan^{-1}x\right) = \frac{1}{1+x^{2}}$

Answer :

Let,

$y = \log(\tan^{-1})$

and u = $\tan^{-1}x$

therefore, $y = \log u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ ............ By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\log u) . \frac{d}{dx}(\tan^{-1}x)$$

$$= \frac{1}{u} . \frac{1}{1+x^2}$$

............ $\left(\because \frac{d}{dx}(\log x) = \frac{1}{x} \;\&\; \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}\right)$

$$= \frac{1}{\tan^{-1}x} . \frac{1}{1+x^2}$$

$$= \frac{1}{(1+x^2).\tan^{-1}x}$$

$$\therefore \frac{dy}{dx} = \frac{1}{(1+x^2).tan^{-1}x}$$

**Question 8.**
Differentiate each of the following w.r.t. x:

$\cot^{-1}x^3$

**Answer:**

Formulae :

i) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

ii) $\frac{d}{dx}(x^n) = n.x^{n-1}$

Answer :

Let,

$y = \cot^{-1}(x^3)$

and u = $x^3$

therefore, $y = \cot^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ ............ By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\cot^{-1}u).\frac{d}{dx}(x^3)$$

$$= \frac{-1}{1+u^2}.3x^2$$

............ $\left(\because \frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2} \ \& \ \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$$= \frac{-1}{1+(x^3)^2}.3x^2$$

$$= \frac{-3x^2}{1+x^6}$$

$$\therefore \frac{dy}{dx} = \frac{-3x^2}{1+x^6}$$

**Question 9.**
Differentiate each of the following w.r.t. x:

$\sin^{-1}(\cos x)$

**Answer:**
Formulae :

i) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}(\cos x) = -\sin x$

iii) $\sin^2 x + \cos^2 x = 1$

Answer :

Let,

$y = \sin^{-1}(\cos x)$

and $u = \cos x$

therefore, $y = \sin^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ ............ By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\sin^{-1}u).\frac{d}{dx}(\cos x)$$

$$= \frac{1}{\sqrt{1-u^2}}.(-\sin x)$$

............ $\left(\because \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}\ \&\ \frac{d}{dx}(\cos x) = -\sin x\right)$

$$= \frac{1}{\sqrt{1-(\cos x)^2}}.(-\sin x)$$

$= \frac{1}{\sqrt{\sin^2 x}}.(-\sin x)$ .........$(\because \sin^2 x + \cos^2 x = 1)$

$$= \frac{1}{\sin x}.(-\sin x)$$

$$= -1$$

$$\therefore \frac{dy}{dx} = -1$$

**Question 10.**
Differentiate each of the following w.r.t. x:

$$(1+x^2)\tan^{-1}x$$

**Answer:**
Formulae :

i) $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$

ii) $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

iii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

iv) $\frac{d}{dx}(k) = 0$

v) $\frac{d}{dx}(x^n) = n.x^{n-1}$

Answer :

Let,

$$y = (1 + x^2)\tan^{-1}x$$

Let, u = $(1+x^2)$ and v=$\tan^{-1}x$

therefore, y=u.v

$$\therefore \frac{dy}{dx} = (1 + x^2).\frac{d}{dx}(\tan^{-1}x) + (\tan^{-1}u).\frac{d}{dx}(1 + x^2)$$

$$.........\left(\because \frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\ \frac{du}{dx}\right)$$

$$= (1 + x^2)\ .\frac{1}{1 + x^2} + (\tan^{-1}x)\left\{\frac{d}{dx}(1) + \frac{d}{dx}(x^2)\right\}$$

$$.............\left(\because \frac{d}{dx}\ (\tan^{-1}x) = \frac{1}{1+x^2}\ \&\ \frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}\right)$$

$$= 1 + (\tan^{-1}x)(0 + 2x)$$

$$.............\left(\because \frac{d}{dx}\ (k) = 0\ \&\ \frac{d}{dx}(x^n) = n.x^{n-1}\right)$$

$$= 1 + 2x\tan^{-1}x$$

$$\therefore \frac{dy}{dx} = 1 + 2x\,tan^{-1}x$$

**Question 11.**
Differentiate each of the following w.r.t. x:

$$\tan^{-1}(\cot x)$$

**Answer:**
Formulae :

i) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

iii) $1 + \cot^2 x = \operatorname{cosec}^2 x$

Answer :

Let,

$y = \tan^{-1}(\cot x)$

and u = cot x

therefore, $y = \tan^{-1} u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ ............ By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\tan^{-1}u) \cdot \frac{d}{dx}(\cot x)$$

$$= \frac{1}{1+u^2} \cdot (-\operatorname{cosec}^2 x)$$

............ $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \ \& \ \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x\right)$

$$= \frac{-\operatorname{cosec}^2 x}{1 + (\cot x)^2}$$

$= \frac{-\operatorname{cosec}^2 x}{\operatorname{cosec}^2 x}$ .........$(\because 1 + \cot^2 x = \operatorname{cosec}^2 x)$

= -1

$\therefore \frac{dy}{dx} = -1$

**Question 12.**

Differentiate each of the following w.r.t. x:

$\log(\sin^{-1} x^4)$

**Answer:**

Formulae :

i) $\frac{d}{dx}(\log x) = \frac{1}{x}$

ii) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

iii) $\frac{d}{dx}(x^n) = n.x^{n-1}$

Answer :

Let,

$y = \log(\sin^{-1}x^4)$

and u = $x^4$

therefore, $y = \log(\sin^{-1}u)$

let, $v = \sin^{-1}u$

therefore, y= log v

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ ........... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(\log v).\frac{d}{du}(\sin^{-1}u).\frac{d}{dx}(x^4)$$

$$= \frac{1}{v}.\left(\frac{1}{\sqrt{1-u^2}}\right).4x^3$$

............ $\left(\because \frac{d}{dx}(\log x) = \frac{1}{x}, \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$$= \frac{1}{\sin^{-1}u}.\left(\frac{1}{\sqrt{1-(x^4)^2}}\right).4x^3$$

$$= \frac{1}{\sin^{-1}x^4}.\left(\frac{1}{\sqrt{1-x^8}}\right).4x^3$$

$$= \frac{4x^3}{\sin^{-1}x^4 .\sqrt{1-x^8}}$$

$$\therefore \frac{dy}{dx} = \frac{4x^3}{\sin^{-1}x^4 .\sqrt{1-x^8}}$$

**Question 13.**
Differentiate each of the following w.r.t. x:

$$(\cot^{-1}x^2)^3$$

**Answer:**
Formulae :

i) $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

ii) $\frac{d}{dx}(x^n) = n.x^{n-1}$

Answer :

Let,

$y = (\cot^{-1}x^2)^3$

and u = $x^2$

therefore, $y = (\cot^{-1}u)^3$

let, $v = \cot^{-1}u$

therefore, y= $v^3$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ ........... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^3).\frac{d}{du}(\cot^{-1}u).\frac{d}{dx}(x^2)$$

$$= 3v^2.\left(\frac{-1}{1+u^2}\right).2x$$

............ $\left(\because \frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2} \;\&\; \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$$= 3(\cot^{-1}u)^2.\left(\frac{-1}{1+(x^2)^2}\right).2x$$

$$= \left(\cot^{-1}(x^2)\right)^2.\frac{-6x}{1+(x^2)^2}$$

$$= \frac{-6x\left(\cot^{-1}(x^2)\right)^2}{1+x^4}$$

$$\therefore \frac{dy}{dx} = \frac{-6x\left(cot^{-1}(x^2)\right)^2}{1+x^4}$$

**Question 14.**

Differentiate each of the following w.r.t. x:

$\tan^{-1}(\cos\sqrt{x})$

**Answer:**
Formulae :

i) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$

iii) $\frac{d}{dx}(\cos x) = -\sin x$

Answer :

Let,

$y = \tan^{-1}\left(\cos\sqrt{x}\right)$

and $u = \sqrt{x}$

therefore, $y = \tan^{-1}(\cos u)$

let, $v = \cos u$

therefore, $y = \tan^{-1}v$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ ............ By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(\tan^{-1}v).\frac{d}{du}(\cos u).\frac{d}{dx}\left(\sqrt{x}\right)$$

$$= \frac{1}{1+v^2} . (-\sin u) . \frac{1}{2\sqrt{x}}$$

$\ldots\ldots\ldots \left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}, \frac{d}{dx}(\cos x) = -\sin x \ \& \ \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}\right)$

$$= \frac{1}{1+(\cos u)^2} . (-\sin\sqrt{x}) . \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{1+\left(\cos\sqrt{x}\right)^2} . (-\sin\sqrt{x}) . \frac{1}{2\sqrt{x}}$$

$$= \frac{-\sin\sqrt{x}}{2\sqrt{x}\left(1+\left(\cos\sqrt{x}\right)^2\right)}$$

$$\therefore \frac{dy}{dx} = \frac{-\sin\sqrt{x}}{2\sqrt{x}\left(1+\left(\cos\sqrt{x}\right)^2\right)}$$

**Question 15.**
Differentiate each of the following w.r.t. x:

$$\tan(\sin^{-1} x)$$

**Answer:**
Formulae :

i) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}(\tan x) = \sec^2 x$

Answer :

Let,

$$y = \tan(\sin^{-1}x)$$

and u = $\sin^{-1}x$

therefore, $y = \tan u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ ............ By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\tan u) \cdot \frac{d}{dx}(\sin^{-1}x)$$

$$= \sec^2 u \cdot \frac{1}{\sqrt{1-x^2}}$$

............ $\left(\because \frac{d}{dx}(\tan x) = \sec^2 x \;\&\; \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}\right)$

$$= \sec^2(\sin^{-1}x) \cdot \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{\sec^2(\sin^{-1}x)}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy}{dx} = \frac{\sec^2(\sin^{-1}x)}{\sqrt{1-x^2}}$$

**Question 16.**
Differentiate each of the following w.r.t. x:

$$e^{\tan^{-1}\sqrt{x}}$$

**Answer:**
<u>Formulae</u> :

i) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

ii) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

iii) $\frac{d}{dx}(e^x) = e^x$

Answer :

Let,

$y = e^{\tan^{-1}\sqrt{x}}$

and $u = \sqrt{x}$

therefore, $y = e^{\tan^{-1}u}$

let, $v = \tan^{-1}u$

therefore, $y = e^v$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ ............ By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(e^v).\frac{d}{du}(\tan^{-1}u).\frac{d}{dx}(\sqrt{x})$$

$$= e^v \cdot \left(\frac{1}{1+u^2}\right) \cdot \frac{1}{2\sqrt{x}}$$

......... $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}, \frac{d}{dx}(e^x) = e^x \;\&\; \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}\right)$

$$= e^{\tan^{-1}u} \cdot \left(\frac{1}{1+(\sqrt{x})^2}\right) \cdot \frac{1}{2\sqrt{x}}$$

$$= e^{\tan^{-1}\sqrt{x}} \cdot \left(\frac{1}{1+x}\right) \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{e^{\tan^{-1}\sqrt{x}}}{2\sqrt{x}\,(1+x)}$$

$$\therefore \frac{dy}{dx} = \frac{e^{\tan^{-1}\sqrt{x}}}{2\sqrt{x}\,(1+x)}$$

**Question 17.**
Differentiate each of the following w.r.t. x:

$$\sqrt{\sin^{-1} x^2}$$

**Answer:**
Formulae :

i) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

iii) $\frac{d}{dx}(x^n) = n.x^{n-1}$

Answer :

Let,

$y = \sqrt{\sin^{-1}x^2}$

and $u = x^2$

therefore, $y = \sqrt{\sin^{-1}u}$

let, $v = \sin^{-1}u$

therefore, $y = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$ ………… By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(\sqrt{v}) . \frac{d}{du}(\sin^{-1}u) . \frac{d}{dx}(x^2)$$

$$= \frac{1}{2\sqrt{v}} . \left(\frac{1}{\sqrt{1-u^2}}\right) . 2x$$

……… $\left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}, \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$$= \frac{1}{2\sqrt{\sin^{-1}u}} . \left(\frac{1}{\sqrt{1-(x^2)^2}}\right) . 2x$$

$$= \frac{1}{\sqrt{\sin^{-1}(x^2)}} . \left(\frac{1}{\sqrt{1-x^4}}\right) . x$$

$$= \frac{x}{\sqrt{\sin^{-1}(x^2)}\left(\sqrt{1-x^4}\right)}$$

$$\therefore \frac{dy}{dx} = \frac{x}{\sqrt{sin^{-1}(x^2)}\left(\sqrt{1-x^4}\right)}$$

**Question 18.**

If $y = \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$, prove that $\frac{dy}{dx} = -2$.

**Answer:**

Given : $y = \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$

To Prove : $\frac{dy}{dx} = -2$

Formulae :

i) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

ii) $\frac{d}{dx}\left(\cos^{-1}x\right) = \frac{-1}{\sqrt{1-x^2}}$

iii) $\frac{d}{dx}\left(\cos x\right) = -\sin x$

iv) $\frac{d}{dx}\left(\sin x\right) = \cos x$

v) $\sin^2 x + \cos^2 x = 1$

vi) $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Answer :

Given equation,

$y = \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$

Let $s = \sin^{-1}(\cos x)$ & $t = \cos^{-1}(\sin x)$

Therefore, y = s + t .........eq(1)

I. For $\sin^{-1}(\cos x)$

let $u = \cos x$

therefore, $s = \sin^{-1}u$

Differentiating above equation w.r.t. x,

$\therefore \frac{ds}{dx} = \frac{ds}{du} \cdot \frac{du}{dx}$ ............ By chain rule

$\therefore \frac{ds}{dx} = \frac{d}{du}\left(\sin^{-1}u\right) \cdot \frac{d}{dx}(\cos x)$

$= \frac{1}{\sqrt{1-u^2}} \cdot (-\sin x)$

............ $\left(\because \frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx}(\cos x) = -\sin x\right)$

$= \frac{1}{\sqrt{1-(\cos x)^2}} \cdot (-\sin x)$

$= \frac{1}{\sqrt{\sin^2 x}} \cdot (-\sin x)$ .........$(\because \sin^2 x + \cos^2 x = 1)$

$= \frac{1}{\sin x} \cdot (-\sin x)$

$= -1$

$\therefore \frac{ds}{dx} = -1$ .........eq(2)

II. For $\cos^{-1}(\sin x)$

let $u = \sin x$

therefore, $t = \cos^{-1} u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dt}{dx} = \frac{dt}{du} \cdot \frac{du}{dx}$ ............ By chain rule

$\therefore \frac{dt}{dx} = \frac{d}{du}(\cos^{-1}u) \cdot \frac{d}{dx}(\sin x)$

$= \frac{-1}{\sqrt{1-u^2}} \cdot (\cos x)$

............ $\left(\because \frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}} \;\&\; \frac{d}{dx}(\sin x) = \cos x\right)$

$$= \frac{-1}{\sqrt{1-(\sin x)^2}}.(\cos x)$$

$$= \frac{-1}{\sqrt{\cos^2 x}}.(\cos x) \text{ .........} (\because \sin^2 x + \cos^2 x = 1)$$

$$= \frac{-1}{\cos x}.(\cos x)$$

$$= -1$$

$\therefore \frac{dt}{dx} = -1$ .........eq(2)

Differentiating eq(1) w.r.t. x,

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(s+t)$$

$$= \frac{ds}{dx} + \frac{dt}{dx} \text{ .........} \left(\because \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$$

= -1 -1 .........from eq(2) and eq(3)

= -2

$$\therefore \frac{dy}{dx} = -2$$

Hence proved !!!

**Question 19.**

Prove that $\frac{d}{dx}\left\{2x \tan^{-1}x - \log\left(1+x^2\right)\right\} = 2 \tan^{-1}x$.

**Answer:**

To Prove: $\frac{d}{dx}\{2x \tan^{-1}x - \log(1+x^2)\} = 2 \tan^{-1}x$

Formulae :

i) $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$

ii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

iii) $\frac{d}{dx}(kx) = k$

iv) $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

v) $\frac{d}{dx}(kx) = 0$

vi) $\frac{d}{dx}(x^n) = n.x^{n-1}$

vii) $\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$

Answer :

Let,

$y = 2x\tan^{-1}x - \log(1+x^2)$

Let $s = 2x\tan^{-1}x$ & $t = \log(1+x^2)$

Therefore, y = s - t .........eq(1)

I. For $2x\tan^{-1}x$

let $u = 2x$ & $v = \tan^{-1}x$

therefore, $s = u.v$

Differentiating above equation w.r.t. x,

$\therefore \frac{ds}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ .........$\left(\because \frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}\right)$

$\therefore \frac{ds}{dx} = 2x\frac{d}{dx}(\tan^{-1}x) + \tan^{-1}x\frac{d}{dx}(2x)$

$= 2x.\frac{1}{1+x^2} + \tan^{-1}x.2$

............ $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}\ \&\ \frac{d}{dx}(kx) = k\right)$

$= \frac{2x}{1+x^2} + 2\tan^{-1}x$

$\therefore \frac{ds}{dx} = \frac{2x}{1+x^2} + 2\tan^{-1}x$ .........eq(2)

II. For $\log(1+x^2)$

let $u = (1+x^2)$

therefore, $t = \log u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dt}{dx} = \frac{dt}{du}.\frac{du}{dx}$ ............ By chain rule

$\therefore \frac{dt}{dx} = \frac{d}{du}(\log u).\frac{d}{dx}(1+x^2)$

$= \frac{1}{u}.\left(\frac{d}{dx}(1) + \frac{d}{dx}(x^2)\right)$ ......... $\left(\because \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= \frac{1}{(1+x^2)}.(0+2x)$

......... $\left(\because \frac{d}{dx}(k) = 0\ \&\ \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$$= \frac{2x}{1 + x^2}$$

$\therefore \frac{dt}{dx} = \frac{2x}{1+x^2}$ .........eq(3)

Differentiating eq(1) w.r.t. x,

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(s - t)$$

$= \frac{ds}{dx} - \frac{dt}{dx}$ .........$\left(\because \frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}\right)$

$= \frac{2x}{1+x^2} + 2\tan^{-1}x - \frac{2x}{1+x^2}$ .........from eq(2) and eq(3)

$= 2 \tan^{-1}x$

$\therefore \frac{dy}{dx} = 2\tan^{-1}x$

Hence proved !!!