Exercise 6c

Question 1.
Find the area of the triangle whose vertices are:

A(3, 8), B(-4, 2) and C(5, -1)

Answer:

Area of a triangle = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

Expanding with C_3

$$= \frac{1}{2}[\,(4-10)-(-3-40)+(6+32)]$$

$$= \frac{1}{2}[\,-6+43+38]$$

$$= \frac{75}{2}$$

= 37.5 sq. units

Question 2.
Find the area of the triangle whose vertices are:

A(-2, 4), B(2, -6) and C(5, 4)

Answer:

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

Expanding with C_3

$$= \frac{1}{2}[\,(8+30)-(-8-20)+(12-8)]$$

$$= \frac{1}{2}[\,38+28+4\,]$$

$$= \frac{68}{2}$$

= 34 sq. units

Question 3.
Find the area of the triangle whose vertices are:

A(-8, -2), B(-4, -6) and C(-1, 5)

Answer:

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -8 & -2 & 1 \\ -4 & -6 & 1 \\ -1 & 5 & 1 \end{vmatrix}$$

Expanding with R_3

$$= \frac{1}{2}[\,(-20-6)-(-40-2)+(48-8)]$$

$$= \frac{1}{2}[-26+42+40]$$

$$= \frac{56}{2}$$

=28 sq. units

Question 4.
Find the area of the triangle whose vertices are:

P(0, 0), Q(6, 0) and R(4, 3)

Answer:

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

Expanding with R_1

$$= \frac{1}{2}[\,18\,]$$

= 9 sq. units

Question 5.
Find the area of the triangle whose vertices are:

P(1, 1), Q(2, 7) and R(10, 8)

Answer:

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 7 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

Operating $R_1 \rightarrow R_1 - R_3$, $R_2 \rightarrow R_2 - R_3$

$$= \frac{1}{2}\begin{vmatrix} -9 & -7 & 0 \\ -8 & -1 & 0 \\ 10 & 8 & 1 \end{vmatrix}$$

Expanding with C_3

$$= \frac{1}{2}[\,9 - 56\,]$$

$$= \frac{1}{2}[-47]$$

$$= \frac{-47}{2}$$

= -23.5 sq. units = 23.5 sq units

Question 6.
Use determinants to show that the following points are collinear.

A(2, 3), B(-1, -2) and C(5, 8)

Answer:

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$$

Expanding with C_3

$$= \frac{1}{2}[\,(-8 + 10) - (16 - 15) + (-4 + 3)]$$
$$= \frac{1}{2}\,[\,2 - 1 - 1]$$

=0

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

Question 7.

Use determinants to show that the following points are collinear.

A(3, 8), B(-4, 2) and C(10, 14)

Answer:

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 10 & 14 & 1 \end{vmatrix}$$

Expanding with C_3

$$= \frac{1}{2}\left[(-56-20)-(42-80)+(6+32)\right]$$

$$= \frac{1}{2}\left[-76+38+38\right]$$

=0

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

Question 8.
Use determinants to show that the following points are collinear.

P(-2, 5), Q(-6, -7) and R(-5, -4)

Answer:

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -2 & 5 & 1 \\ -6 & -7 & 1 \\ -5 & -4 & 1 \end{vmatrix}$$

Expanding with C_3

$= \frac{1}{2}[(24-35)-(8+25)+(14+30)]$

$= \frac{1}{2}[-11-33+44]$

=0

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

Question 9.
Find the value of k for which the
points A(3, -2), B(k, 2) and C(8, 8) are collinear.

Answer:

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since they are collinear, the area will be 0

$$\rightarrow 0 = \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ k & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix}$$

Expanding with C_3

$$\rightarrow 0 = \frac{1}{2}[(8k-16)-(24+16)+(6+2k)]$$

$$\rightarrow 0 = \frac{1}{2}[10k-50]$$

→ 10k – 50=0

→ 10k=50

∴ k=5

Question 10.
Find the value of k for which the
points P(5, 5), Q(k, 1) and R(11, 7) are collinear.

Answer:

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since they are collinear, the area will be 0

$$\rightarrow 0 = \frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ k & 1 & 1 \\ 11 & 7 & 1 \end{vmatrix}$$

Expanding with C_3

→ 0 = (7k-11)-(35-55)+(5-5k)

→ 0= 2k-14

→ 2k=14

∴ k=7

Question 11.
Find the value of k for which the
points A(1, -1), B(2, k) and C(4, 5) are collinear.

Answer:

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since they are collinear, the area will be 0

$$\rightarrow 0 = \frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & k & 1 \\ 4 & 5 & 1 \end{vmatrix}$$

Expanding with C_3

→ 0 = (10-4k)-(5+4)+(k+2)

→ 0=-3k+3

→ 3k=3

∴ k= 1

Question 12.
Find the value of k for which the area of aABC having vertices A(2, -6), B(5, 4) and C(k, 4) is 35 sq units.

Answer:

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$35 = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

Expanding with C_3

→ 70 = (20-4k)-(8+6k)+(8+30)

→ 70= -10k+50

→ 20=-2k

→ k=-2

Question 13.
If A(-2, 0), B(0, 4) and C(0, k) be three points such that area of a ABC is 4 sq units, find the value of k.

Answer:

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$4 = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

Expanding with C_1

→ 8=-2(4-k)

→ -4=4-k

→ k=8

Question 14.

If the points A(a, 0), B(0, b) and C(1, 1) are collinear, prove that $\frac{1}{a}+\frac{1}{b}=1.$

Answer:

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since the points are collinear, the area they enclose is 0

$$0 = \frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

Expanding with C_1

→ 0 = a(b-1)+(-b)

→ 0= ab-a-b

→ a+b=ab

→ $\frac{a+b}{ab} = 1$

→ $\frac{1}{a}+\frac{1}{b} = 1$

Hence proved