Exercise 12

Question 1.
Evaluate :

Evaluate

i. $\int x^7 dx$

ii. $\int x^{-7} dx$

iii. $\int x^{-1} dx$

iv. $\int x^{5/3} dx$

v. $\int x^{-5/4} dx$

vi. $\int 2^x dx$

vii. $\int \sqrt[3]{x^2}\, dx$

viii. $\int \frac{1}{\sqrt[4]{x^3}} dx$

ix. $\int \frac{2}{x^2} dx$

Answer:
i Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^7 dx = \frac{x^{7+1}}{7+1} + c$$

$$= \frac{x^8}{8} + c$$

ii. Given:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{-7}\,dx = \frac{x^{-7+1}}{-7+1} + c$$

$$= \frac{x^{-6}}{-6} + c$$

iii. Given:

$$\int \frac{1}{x}\,dx = \ln|x| + c$$

iv. Given:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\frac{5}{3}}\,dx = \frac{x^{\frac{5}{3}+1}}{\frac{5}{3}+1} + c$$

$$= \frac{3x^{\frac{8}{3}}}{8} + c$$

v. Given:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{-\frac{5}{4}}\,dx = \frac{x^{-\frac{5}{4}+1}}{-\frac{5}{4}+1} + c$$

$$= -4x^{-\frac{1}{4}} + c$$

vi. Given:

$$\int a^x\,dx = \frac{a^x}{\ln a} + c$$

$$\int 2^x\,dx = \frac{2^x}{\ln 2} + c$$

vii. Given:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\frac{2}{3}}\,dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + c$$

$$= \frac{3x^{\frac{5}{3}}}{5} + c$$

viii. Given:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{-\frac{3}{4}}\,dx = \frac{x^{-\frac{3}{4}+1}}{-\frac{3}{4}+1} + c$$

$$= 4x^{\frac{1}{4}} + c$$

ix. Given:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int 2x^{-2}\,dx = 2\frac{x^{-2+1}}{-2+1} + c$$

$$= \frac{-2}{x} + c$$

Question 2.
Evaluate :

i. $\int \left(6x^5 - \frac{2}{x^4} - 7x + \frac{3}{x} - 5 + 4e^x + 7^x \right) dx$

ii. $\int \left(8 - x + 2x^3 - \frac{6}{x^3} + 2x^{-5} + 5x^{-1} \right) dx$

iii. $\int \left(\frac{x}{a} + \frac{a}{x} + x^a + a^x + ax \right) dx$

Answer:
i Given:

$$\int \left(6x^5 - \frac{2}{x^4} - 7x + \frac{3}{x} - 5 + 4e^x + 7^x\right) dx$$
$$= 6\frac{x^{5+1}}{5+1} - 2\frac{x^{-4+1}}{-4+1} - 7\frac{x^2}{2} + 3\ln|x| - 5x + 4e^x + \frac{7^x}{\ln 7} + c$$

$$= 6\frac{x^6}{6} - 2\frac{x^{-3}}{-3} - 7\frac{x^2}{2} + 3\ln|x| - 5x + 4e^x + \frac{7^x}{\ln 7} + c$$

$$= x^6 + \frac{2}{3}x^{-3} - \frac{7}{2}x^2 + 3\ln|x| - 5x + 4e^x + \frac{7^x}{\ln 7} + c$$

ii. Given:

$$\int \left(8 - x + 2x^3 - \frac{6}{x^3} + 2x^{-5} + 5x^{-1}\right) dx$$
$$= 8x - \frac{x^2}{2} + 2\frac{x^{3+1}}{3+1} - 6\frac{x^{-3+1}}{-3+1} + 2\frac{x^{-5+1}}{-5+1} + 5\ln|x| + c$$

$$= 8x - \frac{x^2}{2} + \frac{2}{4}x^4 + \frac{6}{2}x^2 - \frac{2}{4}x^{-4} + 5\ln|x| + c$$

$$= 8x - \frac{x^2}{2} + \frac{1}{2}x^4 + 3x^2 - \frac{1}{2}x^{-4} + 5\ln|x| + c$$

iii. Given:

$$\int \left(\frac{x}{a} + \frac{a}{x} + x^a + a^x + ax\right)dx = \frac{1}{a}\frac{x^2}{2} + a\ln|x| + \frac{x^{a+1}}{a+1} + \frac{a^x}{\ln a} + a\frac{x^2}{2} + c$$

Question 3.
Evaluate :

i. $\int (2-5x)(3+2x)(1-x)dx$

ii. $\int \sqrt{x}\left(ax^2 + bx + c\right)dx$

iii. $\int \left(\sqrt{x} - \sqrt[3]{x^4} + \frac{7}{\sqrt[3]{x^2}} - 6x^x + 1\right)dx$

Answer:
i. Given:

$\int(2-5x)(3+2x)(1-x)dx$

$=\int(6-11x-10x^2)(1-x)\,dx$

$=\int(10\,x^3+x^2-17x+6)dx$

$$= \frac{10x^4}{4} + \frac{x^3}{3} - \frac{17x^2}{2} + 6x + c$$

$$= \frac{5x^4}{2} + \frac{x^3}{3} - \frac{17x^2}{2} + 6x + c$$

ii. Given:

$$= \int \left(ax^{\frac{5}{2}} + bx^{\frac{3}{2}} + cx^{\frac{1}{2}}\right)dx$$

$$= a\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} + b\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + c\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{2a}{7}x^{\frac{7}{2}} + \frac{2b}{5}x^{\frac{5}{2}} + \frac{2c}{3}x^{\frac{3}{2}} + C$$

iii. Given:

$$\int \left(x^{\frac{1}{2}} - x^{\frac{4}{3}} + 7x^{\frac{-2}{3}} - 6e^{\ln x^x} + 1\right)dx$$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1} + 7\frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} - 6e^{\ln x^x} + x + c$$

$$= \frac{2x^{\frac{3}{2}}}{3} - \frac{3x^{\frac{7}{3}}}{7} - 21x^{-\frac{1}{3}} - 6x^x + x + c$$

Question 4.
Evaluate :

i. $\int\left(x^2 - \frac{1}{x^2}\right)^3 dx$

ii. $\int\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)dx$

iii. $\int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$

iv. $\int\frac{(1+2x)^3}{x^4}dx$

v. $\int\frac{(1+x)^3}{\sqrt{x}}dx$

vi. $\int\frac{2x^2 + x - 2}{(x-2)}dx$

Answer:
i. Given:

$=\int x^6 + x^{-6} - 3x^2 - 3x^{-2}\, dx$

$$= \frac{x^7}{7} + \frac{x^{-5}}{5} - x^3 + 3x^{-1} + c$$

ii. Given:

$$= \int x^{\frac{1}{2}} - x^{\frac{-1}{2}} dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + c$$

iii. Given:

$$= \int \left(x + \frac{1}{x} + 2\right) dx$$

$$= \frac{x^2}{2} + \ln|x| + 2x + c$$

iv. Given:

$$= \int \frac{1 + 8x^3 + 6x + 12x^2}{x^4} dx$$

$$= \int x^{-4} + \frac{8}{x} + 6x^{-3} + 12x^{-2} dx$$

$$= -\frac{x^{-3}}{3} + 8\ln|x| - 3x^{-2} - 12x^{-1} + c$$

v. Given:

$$= \int \frac{1 + x^3 + 3x + 3x^2}{\sqrt{x}} dx$$

$$= \int x^{\frac{-1}{2}} + x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 3x^{\frac{3}{2}}\,dx$$

$$= 2x^{\frac{1}{2}} + \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + \frac{6}{5}x^{\frac{5}{2}} + c$$

vi. Given:

$$= \int \left(\frac{2x^2}{x-2} + \frac{x-2}{x-2}\right) dx$$

$$= 2\int \left(\frac{x^2 - 4x + 4}{x-2} + \frac{4x}{x-2} - \frac{4}{x-2}\right) dx + \int dx$$

$$= 2\left[\int \frac{(x-2)^2}{x-2} dx + 4\int \frac{x-2+2}{x-2} dx - 4\int \frac{1}{x-2} dx\right] + x + c$$

$$= 2\left[\int (x-2) dx + 4\left(\int dx + 2\int \frac{1}{x-2} dx\right) - 4\ln|x-2|\right] + x + c$$

$$= 2\left[\frac{x^2}{2} - 2x + 4x + 8\ln|x-2| - 4\ln|x-2|\right] + x + c$$

$=x^2- 4x+8x+8 \ln|x-2|+x + c$

$=x^2 +5x+8 \ln|x-2|+c$

Question 5.
Evaluate :

$$\int \left[1 + \frac{1}{\left(1+x^2\right)} - \frac{2}{\sqrt{1-x^2}} + \frac{5}{x\sqrt{x^2-1}} + a^x\right] dx$$

Answer:
Given:

Since, $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$;

$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c;$

$\int a^x \, dx = \frac{a^x}{\ln a} + c$ &

$\int \frac{1}{|x|\sqrt{(x^2-1)}} dx = \sec^{-1} x + c$

So,

$= x + \tan^{-1} x - 2\sin^{-1} x + 5\sec^{-1} x + \frac{a^x}{\ln a} + c$

Question 6.
Evaluate :

i. $\int \left(\frac{x^2 - 1}{x^2 + 1} \right) dx$

ii. $\int \left(\frac{x^6 - 1}{x^2 + 1} \right) dx$

iii. $\int \left(\frac{x^4}{1 + x^2} \right) dx$

iv. $\int \left(\frac{x^2}{1 + x^2} \right) dx$

Answer:
i. Given:

$= \int \frac{x^2 + 1 - 2}{x^2 + 1} dx$

$= \int \frac{x^2 + 1}{x^2 + 1} - \frac{2}{x^2 + 1} dx$

=x-2 $\tan^{-1} x + c$

ii. Given:

$$= \int \left[\frac{x^6}{x^2+1} - \frac{1}{x^2+1}\right] dx$$

$$= \int \left[\frac{x^6+3x^2+3x^4+1-3x^2-3x^4-1}{x^2+1} - \frac{1}{x^2+1}\right] dx$$

$$= \int \left[\frac{(x^2+1)^3}{x^2+1} - 3\frac{x^2}{x^2+1} - 3\frac{x^4}{x^2+1} - \frac{1}{x^2+1} - \frac{1}{x^2+1}\right] dx$$

$$= \int (x^2+1)^2\,dx - 3\int \left[\frac{x^2+1-1}{x^2+1}\right] dx - 3\left[\int \frac{x^4+2x^2+1}{x^2+1} + \frac{-2x^2-1}{x^2+1} dx\right]$$
$$- 2\int \frac{1}{x^2+1} dx$$

$$= \int (x^4+2x^2+1)dx - 3\left[\int dx - \int \frac{1}{x^2+1} dx\right]$$
$$- 3\left[\int \frac{(x^2+1)^2}{x^2+1} dx - 2\int \frac{x^2}{x^2+1} dx - \int \frac{1}{x^2+1} dx\right] - 2\int \frac{1}{x^2+1} dx$$

$$= \int (x^4+2x^2+1)dx - 3\left[\int dx - \int \frac{1}{x^2+1} dx\right]$$
$$- 3\left[\int (x^2+1)dx - 2\int \frac{x^2+1-1}{x^2+1} dx - \int \frac{1}{x^2+1} dx\right]$$
$$- 2\int \frac{x^2+1}{x^2+1} dx$$

$$= \frac{x^5}{5} + \frac{2}{3}x^3 + x - 3x + 3\tan^{-1} x - x^3 - 3x + 6x - 3\tan^{-1} x - 2\tan^{-1} x + c$$

$$= \frac{x^5}{5} + \frac{1}{3}x^3 + x - 2\tan^{-1} x + c$$

iii. Given:

$$= \int \frac{x^4+2x^2+1}{x^2+1} + \frac{-2x^2-1}{x^2+1} dx$$

$$= \int \frac{(x^2+1)^2}{x^2+1} dx - 2\int \frac{x^2}{x^2+1} dx - \int \frac{1}{x^2+1} dx$$

$$= \int (x^2+1) dx - 2\int \frac{x^2+1-1}{x^2+1} dx - \int \frac{1}{x^2+1} dx$$

$$= \int (x^2+1) dx - 2\int dx + 2\int \frac{1}{x^2+1} dx - \int \frac{1}{x^2+1} dx$$

$$= \int (x^2+1) dx - 2\int dx + \int \frac{1}{x^2+1} dx$$

$$= \frac{1}{3}x^3 - x + \tan^{-1} x + c$$

iv. Given:

$$= \int \left[\frac{x^2+1-1}{x^2+1}\right] dx$$

$$= \int dx - \int \frac{1}{x^2+1} dx$$

$=x - \tan^{-1} x + c$

Question 7.
Evaluate :

$$\int \left(9\sin x - 7\cos x - \frac{6}{\cos^2 x} + \frac{2}{\sin^2 x} + \cot^2 x \right) dx$$

Answer:
Given:

$$= \int (9\sin x - 7\cos x - 6(\sec x)^2 + 2(\csc x)^2 + (\csc x)^2 - 1)\, dx$$

$=-9 \cos x-7 \sin x-6 \tan x-3 \cot x – x + c$

Question 8.
Evaluate :

$$\int\left(\frac{\cot x}{\sin x}-\tan^2 x-\frac{\tan x}{\cos x}+\frac{2}{\cos^2 x}\right)dx$$

Ans. - cosec x + tan x + x – sec x + C

Answer:
Given:

$$=\int(\cot x\csc x-(\sec x)^2+1-\tan x\sec x+2(\sec x)^2)$$

=-csc x-tan x + x-sec x+2 tan x + c

=-csc x + tan x + x -sec x + c

Question 9.
Evaluate :

i. $\int\sec x(\sec x+\tan x)dx$

ii. $\int\operatorname{cosec} x(\operatorname{cosec} x-\cot x)dx$

Answer:
i. Given:

$$=\int(\sec x)^2+\sec x\tan x\,dx$$

=tan x+sec x+c

ii. Given:

$$=\int(\csc x)^2-\cot x\csc x\,dx$$

=-cot x + csc x + c

Question 10.
Evaluate :

i. $\int (\tan x + \cot x)^2 \, dx$

ii. $\int \left(\frac{1 + 2\sin x}{\cos^2 x} \right) dx$

iii. $\int \left(\frac{3\cos x + 4}{\sin^2 x} \right) dx$

Answer:
i. Given:

$$= \int ((\tan x)^2 + (\cot x)^2 + 2) dx$$

$$= \int ((\sec x)^2 - 1 + (\csc x)^2 - 1 + 2) dx$$

$$= \int ((\sec x)^2 + (\csc x)^2) dx$$

=tan x-cot x+c

ii. Given:

$$= \int \left(\frac{1}{(\cos x)^2} + 2\frac{\sin x}{(\cos x)^2} \right) dx$$

$$= \int ((\sec x)^2 + 2\tan x \sec x) dx$$

=tan x+2 sec x + c

iii. Given:

$$= \int (2\cot x \csc x + 4(\csc x)^2)$$

=-2 csc x-4 cot x + c

Question 11.
Evaluate :

i. $\int \frac{1}{(1-\cos x)} dx$

ii. $\int \frac{1}{(1-\sin x)} dx$

Answer:
i. Given:

Multiply and divide by $(1+\cos x)$

$$= \int \frac{1+\cos x}{1-(\cos x)^2} dx$$

$$= \int \frac{1+\cos x}{(\sin x)^2} dx$$

$$= \int ((\csc x)^2 + \csc x \cot x) dx$$

=-cot x-csc x + c

ii. Given:

Multiply and divide by $(1+\sin x)$

$$= \int \frac{1+\sin x}{1-(\sin x)^2} dx$$

$$= \int \frac{1+\sin x}{(\cos x)^2} dx$$

$$= \int ((\sec x)^2 + \sec x \tan x) dx$$

=tan x + sec x + c

Question 12.
Evaluate :

i. $\int \frac{\tan x}{(\sec x + \tan x)} dx$

ii. $\int \frac{\text{cosec } x}{(\text{cosec } x - \cot x)} dx$

Answer:
i. Given:

Multiply and divide by $(\sec x - \tan x)$

$$= \int \frac{\tan x \sec x - (\tan x)^2}{(\sec x)^2 - (\tan x)^2} dx$$

$$= \int \tan x \sec x - (\tan x)^2 dx$$

$$= \int (\tan x \sec x - (\sec x)^2 + 1) dx$$

=sec x-tan x + x + c

ii. Given:

Multiply and divide by $(\csc x + \cot x)$

$$= \int \frac{(\csc x)^2 + \csc x \cot x}{(\csc x)^2 - (\cot x)^2} dx$$

$$= \int (\csc x)^2 + \csc x \cot x \, dx$$

=-cot x-csc x + c

Question 13.
Evaluate :

i. $\int \frac{\cos x}{1+\cos x} dx$

ii. $\int \frac{\sin x}{(1-\sin x)} dx$

Answer:
i. Given:

Multiply and divide by $(1-\cos x)$

$$= \int \frac{\cos x - (\cos x)^2}{1-(\cos x)^2} dx$$

$$= \int \frac{\cos x - (\cos x)^2}{(\sin x)^2} dx$$

$$= \int (\cot x \csc x - (\cot x)^2) dx$$

$$= \int (\cot x \csc x - (\csc x)^2 + 1) dx$$

=-csc x + cot x + x + c

ii. Given:

Multiply and divide by $(1+\sin x)$

$$= \int \frac{\sin x + (\sin x)^2}{1-(\sin x)^2} dx$$

$$= \int \frac{\sin x - (\sin x)^2}{(\cos x)^2} dx$$

$$= \int (\tan x \sec x + (\tan x)^2) dx$$

$$= \int (\tan x \sec x + (\sec x)^2 - 1) dx$$

=sec x + tan x-x + c

Question 14.
Evaluate :

i. $\int \sqrt{1 + \cos 2x}\ dx$

ii. $\int \sqrt{1 - \cos 2x}\ dx$

Answer:
i. Given:

$$= \int \sqrt{2(\cos x)^2}\, dx$$

$$= \sqrt{2} \int \cos x\, dx$$

$$= \sqrt{2} \sin x + c$$

ii. Given:

$$= \int \sqrt{2(\sin x)^2}\, dx$$

$$= \sqrt{2} \int \sin x\, dx$$

=-√2 cos x + c

Question 15.
Evaluate :

i. $\int \frac{1}{(1+\cos 2x)} dx$

ii. $\int \frac{1}{(1-\cos 2x)} dx$

Answer:
i. Given:

$$= \int \frac{1}{2(\cos x)^2} dx$$

$$= \frac{1}{2} \int (\sec x)^2 \, dx$$

$$= \frac{1}{2} \tan x + c$$

ii. Given:

$$= \int \frac{1}{2(\sin x)^2} dx$$

$$= \frac{1}{2} \int (\csc x)^2 dx$$

$$= -\frac{1}{2} \cot x + c$$

Question 16.
Evaluate :

$$\int \sqrt{1+\sin 2x} \, dx$$

Answer:

Given:

$$= \int \sqrt{\left(1 + \frac{2\tan x}{1 + (\tan x)^2}\right)} dx$$

$$= \int \sqrt{\left(\frac{(1 + \tan x)^2}{(\sec x)^2}\right)} dx$$

$$= \int \left(\frac{1 + \tan x}{\sec x}\right) dx$$

=∫(cos x + sin x) d

=sin x-cos x+c

Question 17.
Evaluate :

$$\int \left(\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} \right) dx$$

Ans. sec x – cosec x + C

Answer:
Given:

$$= \int \frac{(\sin x)^3}{(\sin x)^2 (\cos x)^2} + \frac{(\sin x)^3}{(\sin x)^2 (\cos x)^2} dx$$

=∫(tan x sec x + csc x cot x)dx

=sec x- csc x + c

Question 18.

Evaluate :

$$\int \tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right)dx$$

Ans. $\frac{x^2}{2}+C$

Answer:
Given:

$$=\int \tan^{-1}\left(\frac{2\sin x\cos x}{2(\cos x)^2}\right)dx$$

$=\int \tan^{-1}(\tan x)dx$

$=\int x\,dx$

$=\frac{x^2}{2}+c$

Question 19.
Evaluate :

$$\int \cos^{1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)dx$$

Ans. x^2+C

Answer:
Given:

$=\int \cos^{-1}(\cos 2x)dx$

$=\int 2x\,dx$

$=x^2+c$

Question 20.
Evaluate :

$$\int \cos^{-1}(\sin x)\,dx$$

Ans. $\left(\frac{\pi x}{2} - \frac{x^2}{2} + C\right)$

Answer:
Given:

$$\sin^{-1}(\sin x) + \cos^{-1}(\sin x) = \frac{\pi}{2}$$

$$= \int \left[\frac{\pi}{2} - \sin^{-1}(\sin x)\right] dx$$

$$= \int \left[\frac{\pi}{2} - x\right] dx$$

$$= \frac{\pi}{2}x - \frac{x^2}{2} + c$$

Question 21.
Evaluate :

$$\int \tan^{-1} \sqrt{\frac{1-\sin x}{1+\sin x}}\,dx$$

Ans. $\frac{\pi x}{4} - \frac{x^2}{4} + C$

Answer:
Given:

$$= \int \tan^{-1} \sqrt{\left(\frac{(1-\sin x)^2}{1-(\sin x)^2}\right)} dx$$

$$= \int \tan^{-1}\left(\frac{1-\sin x}{\cos x}\right) dx$$

$$= \int \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}\right) dx$$

$$= \int \tan^{-1}\left(\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right) dx$$

$$= \int \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right) dx$$

$$= \int \left(\frac{\pi}{4}-\frac{x}{2}\right) dx$$

$$= \frac{\pi}{4}x - \frac{x^2}{4} + c$$

Question 22.
Evaluate :

$$\int (3\cot x - 2\tan x)^2 \, dx$$

Ans. 4 tan x – 9 cos x – 25x + C

Answer:
Given:

$$= \int (9(\cot x)^2 + 4(\tan x)^2 - 12) dx$$

$$= \int (9(\csc x)^2 + 4(\sec x)^2 - 25)\, dx$$

=-9 cot x+4 tan x-25x+c

Question 23.

Evaluate :

$$\int (3 \sin x + 4 \operatorname{cosec} x)^2 \, dx$$

Ans. $\frac{57}{2}x - \frac{9}{4}\sin 2x - 16 \cot x + C$

Answer:
Given:

$$= \int (9(\sin x)^2 + 16(\csc x)^2 + 24) dx$$

$$= \int \left(\frac{9}{2}(1 - \cos 2x) + 16(\csc x)^2 + 24\right) dx$$

$$= \frac{9}{2}x - \frac{9}{4}\sin 2x - 16 \cot x + 24x + c$$

$$= \frac{57}{2}x - \frac{9}{4}\sin 2x - 16 \cot x + c$$

Question 24.
Evaluate :

$$\int \frac{dx}{\left(\sqrt{x+1} + \sqrt{x+2}\right)}$$

Ans. $\frac{2}{3}(x+2)^{3/2} - \frac{2}{3}(x+1)^{3/2} + C$

Answer:
Given:

Multiply and divide by $\sqrt{(x+1)} - \sqrt{(x+2)}$

$$= \int \frac{\left(\sqrt{(x+1)} - \sqrt{(x+2)}\right)}{x+1-x-2} dx$$

$$= -\int \sqrt{(x+3)} + \sqrt{(x+2)}\, dx$$

$$= \frac{-2}{3}(x+1)^{\frac{3}{2}} + \frac{2}{3}(x+2)^{\frac{3}{2}} + c$$

Question 25.
Evaluate :

$$\int \frac{dx}{\left(\sqrt{x+3} - \sqrt{x+2}\right)}$$

Ans. $\frac{2}{3}(x+3)^{3/2} + \frac{2}{3}(x+2)^{3/2} + C$

Answer:
Given:

Multiply and divide by $\sqrt{(x+3)} + \sqrt{(x+2)}$

$$= \int \frac{\left(\sqrt{(x+3)} + \sqrt{(x+2)}\right)}{x+3-x-2} dx$$

$$= \int \sqrt{(x+3)} + \sqrt{(x+2)}\, dx$$

$$= \frac{2}{3}(x+3)^{\frac{3}{2}} + \frac{2}{3}(x+2)^{\frac{3}{2}} + c$$

Question 26.
Evaluate :

$$\int \left(\frac{1+\cos x}{1-\cos x}\right) dx$$

Ans. $-2\cot\frac{x}{2} - x + C$

Answer:
Given:

Multiply and divide by $(1+\cos x)$

$$= \int \left(\frac{(1+\cos x)^2}{1-(\cos x)^2}\right) dx$$

$$= \int \frac{1+(\cos x)^2+2\cos x}{(\sin x)^2} dx$$

$$= \int (\csc x)^2 + (\cot x)^2 + 2\cot x \csc x \ dx$$

$$= \int (\csc x)^2 + (\csc x)^2 - 1 + 2\cot x \csc x \ dx$$

$$= -2\cot x - 2\csc x - x + c$$

$$= -2(\csc x + \cot x) - x + c$$

$$= -2\left(\frac{1+\cos x}{\sin x}\right) - x + c$$

$$= -2\left(\frac{2\cos\frac{x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right) - x + c$$

$$= -2\cot\frac{x}{2} - x + c$$

Question 27.
Evaluate :

$$\int \frac{(1+\tan x)}{(1-\tan x)} dx$$

Ans. -log |cos x – sin x| + C

Answer:
Given:

$$= \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}} dx$$

$$= \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Let cos x-sin x=t

(-sin x-cos x) dx=dt

-(sin x + cos x)dx=dt

dx=-dt

So, I= $-\int \frac{dt}{t}$

=-ln|t|+c

=-ln|cos x-sin x |+c

Question 28.
Evaluate :

$$\int \frac{\cos(x+a)}{\sin(x+b)} dx$$

Ans. cos(a – b)log |sin (x + b)| -x sin(a –b) + C

Answer:
Given:

$$= \int \frac{\cos(x+b+a-b)}{\sin(x+b)} dx$$

$$= \int \frac{\cos(x+b)\cos(a-b) - \sin(x+b)\sin(a-b)}{\sin(x+b)}\,dx$$

=cos(a-b)∫cot(x + b) dx-∫sin(a-b)dx

=cos (a-b)ln|sin(x + b)|-x sin(a-b)+c

Question 29.
Evaluate :

$$\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)}dx$$

Ans. x cos 2α - sin 2α . log |sin(x + α)| + C

Answer:
Given:

$$= \int \frac{\sin(x-\alpha+\alpha-\alpha)}{\sin(x+\alpha)}\,dx$$

$$= \int \frac{\sin(x+\alpha)\cos(2\alpha) - \sin(2\alpha)\cos(x+\alpha)}{\sin(x+\alpha)}\,dx$$

=∫cos 2α dx-sin2α ∫cot(x+α)dx

=x cos 2α-sin 2α×ln|sin(x+α) |+c

Question 30.
Evaluate :

$$\int (1-x)\sqrt{x}\ dx$$

Ans. $\frac{2}{15}x\sqrt{x}(5-3x)+C$

Answer:
Given:

$$= \int \left(x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + c$$

$$= \frac{2}{15}x^{\frac{3}{2}}(5 - 3x) + c$$

Question 31.
Evaluate :

$$\int \frac{\sec^2 x}{\text{cosec}^2 x} dx$$

Ans. tan x – x + C

Answer:
Given:

$=\int (\tan x)^2 \, dx$

$=\int ((\sec x)^2 - 1) dx$

=tan x – x + c

Question 32.
Evaluate :

$$\int \left\{ \frac{2 - 3\sin x}{\cos^2 x} \right\} dx$$

Ans. 2 tan x – 3 sec x + C

Answer:
Given:

$$= \int \frac{2}{(\cos x)^2} dx - \int \frac{3\sin x}{(\cos x)^2} dx$$

$$= 2\int (\sec x)^2\, dx - 3\int \tan x \sec x\, dx$$

=2 tan x-3 sec x+c