Exercise 2d

Question 1.
Let A = {2, 3, 4, 5} and B = {7, 9, 11, 13}, and

let f = {(2, 7), (3, 9), (4, 11), (5, 13)}.

Show that f is invertible and find f^{-1}.

Answer:
To Show: that f is invertible

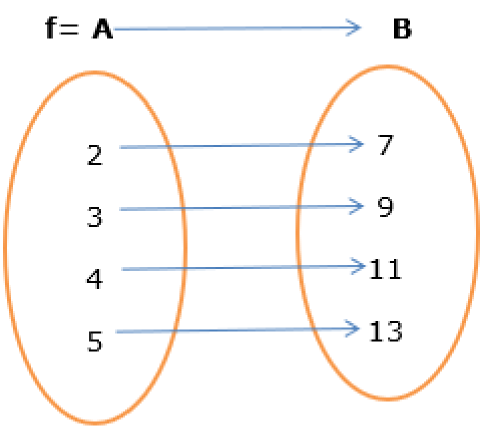
To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for x_l, $x_2 \in$ A & $f(x_1)$, $f(x_2) \in$ B, f(x1) = f(x2) $\leftrightarrow$ $x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow$ f(x1) $\neq$ f(x2)

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.



As we see that inthe above figure (2 is mapped with 7), (3 is mapped with 9), (4 is mapped with 11),

(5 is mapped with 13)

So it is one-one functions.

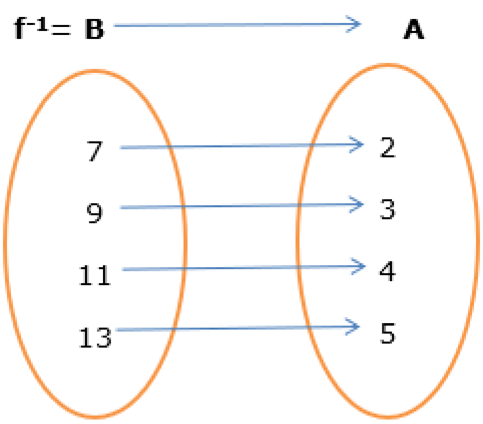
Now elements of B are known as co-domain. Also, a range of a function is also the elements of B(by definition)

So it is onto functions.

Hence Proved that f is invertible.

Now, We know that if $f : A \to B$ then $f^{-1} : B \to A$ (if it is invertible)

So,



So $f^{-1} = \{(7, 2), (9, 3), (11, 4), (13, 5)\}$

Question 2.
Show that the function $f : R \to R : f(x) = 2x + 3$ is invertible and find f^{-1}.

Answer:
To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function $f : A \to B$ is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let x_1, $x_2 \in R$ and $f(x) = 2x+3$.So $f(x_1) = f(x_2) \rightarrow 2x_1+3 = 2x_2+3 \rightarrow x_1=x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1= x_2$, f(x) is one-one

Given co-domain of f(x) is R.

Let y = f(x) = 2x+3 , So $x = \frac{y-3}{2}$ [Range of f(x) = Domain of y]

So Domain of y is R(real no.) = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = R

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{y-3}{2}$

Question 3.
Let f : Q → Q : f(x) = 3x —4. Show that f is invertible and find f^{-1}.

Answer:
To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for x_1, $x_2 \in A$ & $f(x_1)$, $f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1= x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let $x_1, x_2 \in Q$ and $f(x) = 3x-4$.So $f(x_1) = f(x_2) \rightarrow 3x_1 - 4 = 3x_2 - 4 \rightarrow x_1=x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1= x_2$, f(x) is one-one

Given co-domain of f(x) is Q.

Let $y = f(x) = 3x- 4$, So $x = \frac{y+4}{3}$ [Range of f(x) = Domain of y]

So Domain of y is Q = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = Q

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{y+4}{3}$

Question 4.

Let $f : R \rightarrow R : f(x) = \frac{1}{2}(3x+1)$.. Show that f is invertible and find f^{-1}.

Answer:

To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function $f : A \rightarrow B$ is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1= x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let $x_1, x_2 \in Q$ and $f(x) = \frac{(3x+1)}{2}$.So $f(x_1) = f(x_2) \rightarrow \frac{(3x_1+1)}{2} = \frac{(3x_2+1)}{2} \rightarrow x_1=x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1= x_2$, f(x) is one-one

Given co-domain of f(x) is R.

Let $y = f(x) = \frac{(3x+1)}{2}$, So $x = \frac{2y-1}{3}$ [Range of f(x) = Domain of y]

So Domain of y is R = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = R

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{2y-1}{3}$

Question 5.

If $f(x) = \frac{(4x+3)}{(6x-4)}, x \neq \frac{2}{3}$, show that (f o f) (x) = x for all $x \neq \frac{2}{3}$.

Hence, find f^{-1}.

Answer:

To Show: that f o f (x) = x

Finding (f o f) (x) = $\frac{(4\frac{(4x+3)}{(6x-4)}+3)}{(6\frac{(4x+3)}{(6x-4)}-4)} = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{16x+12+18x-12}{24x+18-24x+16} = \frac{35x}{35} = x$.

Question 6.

Show that the function f on $A = R - \left\{\frac{2}{3}\right\}$, defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence, find f^{-1}.

Answer:
To Show: that f is one-one and onto

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function $f : A \rightarrow B$ is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let $x_1, x_2 \in Q$ and $f(x) = \frac{(4x+3)}{(6x-4)}$ .So $f(x_1) = f(x_2) \rightarrow \frac{(4x_1+3)}{(6x_1-4)} = \frac{(4x_2+3)}{(6x_2-4)} \rightarrow$ on solving we get $x_1 = x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is R except 3x-2=0.

Let $y = f(x) = \frac{(4x+3)}{(6x-4)}$ So $x = \frac{4y+3}{6y-4}$ [Range of f(x) = Domain of y]

So Domain of y is R (except 3x-2=0) = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = R except 3x-2=0

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{4y+3}{6y-4}$.

Question 7.

Show that the function f on $A = R - \left\{ \frac{-4}{3} \right\}$ into itself, defined by $f(x) = \frac{4x}{(3x+4)}$ is one-one and onto. Hence, find f^{-1}.

Answer:
To Show: that f is one-one and onto

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let $x_1, x_2 \in Q$ and $f(x) = \frac{4x}{(3x+4)}$ .So $f(x_1) = f(x_2) \rightarrow \frac{(4x_1)}{(3x_1+4)} = \frac{(4x_2)}{(3x_2+4)} \rightarrow$ on solving we get $x_1 = x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is R except 3x+4=0.

Let $y = f(x) = \frac{(4x)}{(3x+4)}$ So $x = \frac{4y}{4-3y}$ [Range of f(x) = Domain of y]

So Domain of y is R = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = R except 3x+4=0

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{4y}{4-3y}$.

Question 8.
Let R_+ be the set of all positive real numbers. show that the function f : $R_+ \rightarrow [-5, \infty]$: $f(x) = (9x^2 + 6x - 5)$ is invertible. Find f^{-1}.

Answer:
To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let $x_1, x_2 \in R$ and $f(x) = (9x^2 + 6x - 5)$.So $f(x_1) = f(x_2) \rightarrow (9x_1^2 + 6x_1 - 5) = (9x_2^2 + 6x_2 - 5)$ on solving we get→ $x_1=x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is $[-5, \infty]$

Let $y = f(x) = (9x^2 + 6x - 5)$, So $x = \frac{-1+\sqrt{y+6}}{3}$ [Range of f(x) = Domain of y]

So Domain of y = Range of f(x) = $[-5, \infty]$

Hence, Range of f(x) = co-domain of f(x) =$[-5, \infty]$

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{-1+\sqrt{y+6}}{3}$.

Question 9.
Let f : N → R : $f(x) = 4x^2 + 12x + 15$. Show that f: N → range (f) is invertible. Find f^{-1}.

Answer:
To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let $x_1, x_2 \in R$ and $f(x) = 4x^2 + 12x + 15$ So $f(x_1) = f(x_2) \rightarrow (4x_1^2 + 12x_1 + 15) = (4x_2^2 + 12x_2 + 15)$, on solving we get→$x_1 = x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is Range(f).

Let $y = f(x) = 4x^2 + 12x + 15$, So $x = \frac{-3+\sqrt{y-6}}{2}$ [Range of f(x) = Domain of y]

So Domain of y = Range of f(x) = [6, ∞]

Hence, Range of f(x) = co-domain of f(x) = [6, ∞]

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{-3+\sqrt{y-6}}{2}$.

Question 10.

Let A = R – {2} and B = R – {1}. If $f : A \to B : f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto. Hence, find f^{-1}.

Answer:
To Show: that f is one-one and onto

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let $x_1, x_2 \in Q$ and $f(x) = \frac{x-1}{x-2}$.So $f(x_1) = f(x_2) \to \frac{x_1-1}{x_1-2} = \frac{(x_2-1)}{x_2-2}$, on solving we get $\to x_1 = x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is R – {1}

Let $y = f(x) = \frac{x-1}{x-2}$, So $x = \frac{2y-1}{y-1}$ [Range of f(x) = Domain of y]

So Domain of y = Range of f(x) = R – {1}

Hence, Range of f(x) = co-domain of f(x) = R – {1}.

So, f(x) is onto function

As it is a bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{2y-1}{y-1}$

Question 11.
Let f and g be two functions from R into R, defined by f(x) = |x| + x and g(x) = |x| - x for all x ∈ R. Find f o g and g o f.

Answer:
To Find: Inverse of f o g and g o f.

Given: f(x) = |x| + x and g(x) = |x| - x for all x ∈ R

f o g (x) = f(g(x)) = |g(x)| + g(x) = ||x| - x | + |x| - x

Case 1) when x $\geq$ 0

f(g(x)) = 0 (i.e. |x| - x)

Case 2) when x$<$ 0

f(g(x)) = -4x

g o f (x) = g(f(x)) = |f(x)| - f(x) = ||x| + x | - |x| - x

Case 1) when x $\geq$ 0

g(f(x)) = 0 (i.e. |x| - x)

Case 2) when x$<$ 0

g(f(x)) = 0