Exercise 5f

Question 1.
Construct a 3 × 2 matrix whose elements are given by

$$a_{ij} = \frac{1}{2}(i - 2j)^2$$

Answer:
Here, i is the subscript for a row, and j is the subscript for column

And the given matrix is 3×2, so 1≤ i ≤ 3 and 1≤j ≤2

Hence for i=1, j=1, $a_{11} = \frac{1}{2}(1-(2\times 1))^2 = \frac{1}{2}$

For i=1, j=2, $a_{12} = \frac{1}{2}(1-(2\times 2))^2 = \frac{9}{2}$

For i=2, j=1 $a_{21} = \frac{1}{2}(2-(2\times 1))^2 = 0$

For i=2, j=2 $a_{22} = \frac{1}{2}(2-(2\times 2))^2 = 2$

For i=3, j=1 $a_{31} = \frac{1}{2}(3-(2\times 1))^2 = \frac{1}{2}$

For i=3, j=2 $a_{32} = \frac{1}{2}(3-(2\times 2))^2 = \frac{1}{2}$

Hence the required matrix is :- $\begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

Question 2.
Construct a 2 × 3 matrix whose elements are given by

$$a_{ij} = \frac{1}{2}|-3i + j|.$$

Answer:
The elements of the matrix are given by, $a_{ij} = \frac{1}{2}|-3j + j|$

Matrix is 2× 3 hence, $1 \leq i \leq 2, 1 \leq j \leq 3$

Here, i is the subscript for a row, and j is the subscript for column

For i=1, j=1, $a_{11} = \frac{1}{2}|-3(1) + 1| = 1$

For i=1, j=2, $a_{12} = \frac{1}{2}|-3(1) + 2| = \frac{1}{2}$

For i=1, j=3, $a_{13} = \frac{1}{2}|-3(1) + 3| = 0$

For i=2, j=1, $a_{21} = \frac{1}{2}|-3(2) + 1| = \frac{5}{2}$

For i=2, j=2, $a_{22} = \frac{1}{2}|-3(2) + 2| = 2$

For i=2, j=3, $a_{23} = \frac{1}{2}|-3(2) + 3| = \frac{3}{2}$

Hence the required matrix is :-

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 \\ \frac{5}{2} & 2 & \frac{3}{2} \end{bmatrix}$$

Question 3.
If $\begin{bmatrix} x + 2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$, find the values of x and y.

Answer:
On comparing L.H.S. and R. H.S we get,

$$\begin{bmatrix} x+2y & -y \\ 3x & 4 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$$

On comparing each term we get,

$x + 2y = -4$ ....(i)

$-y = 3$ ...(ii)

$3x = 6$ .....(iii)

From (i), (ii) and (iii), we get,

$y = -3 \; and \; x = 2$

Question 4.
Find the values of x and y, if

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}.$$

Answer:
Given,

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Using the property of matrix multiplication such that h is scalar, $h\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ah & bh \\ ch & dh \end{bmatrix}$

Using the matrix property of matrix addition, when two matrices are of the same order then, each element gets added to the corresponding element,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix}$$

$$\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Comparing each element we get,

2+y=5, ⇒ y=3

2x+2=8, ⇒x=3

Question 5.
If $x.\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y.\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the values of x and y.

Answer:
Given, $x. \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y.\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$$\begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix}$$

And we have,

$$\begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

Solving the linear equations, we get,

$x = 3, y = -4$

Question 6.
If $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$, find the values of x, y, z, ω.

Answer:
Given,

$$\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$$

On comparing each element of the two matrices we get,

x=3,

3x-y=2

y=7

2x+z=4,

z=-2,

3y-w=7,

w=14

Question 7.

If $\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} = 3\begin{bmatrix} x & y \\ z & w \end{bmatrix}$, find the values of x, y, z, ω.

Answer:
Given,

$$\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} = 3\begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

First applying matrix addition then, comparing each element of the matrix with the corresponding element we get,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

$$\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix} = \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix}$$

$$\begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix} = \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix}$$

We now have, $x+4=3x$, ...(i)

x=2

$2w+3=3w$, ...(ii)

$w=3$

6+x+y=3y, substituting x from (i) we get,

y = 4,

And -1+z+w=3z, substituting w from (ii), we get,

z=1

Question 8.
If A = diag (3 -2, 5) and B = diag (1 3 -4), find (A + B).

Answer:
We are given two diagonal matrices A and B,

On adding the two diagonal matrices of order (3×3) we get an diagonal matrix of order (3×3)

Each of the elements get added to the corresponding element hence, we get after adding,

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, we get A+B = diag(4 1 1)

Question 9.

Show that

$$\cos\theta \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \cdot$$
$$\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} = I$$

Answer:
We have to show that

$$\cos\theta \,.\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + sin\theta .\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Multiplying the scalars with we get,

$$\begin{bmatrix} \cos\theta \times \cos\theta & \cos\theta \times \sin\theta \\ \cos\theta \times (-\sin\theta) & \cos\theta \times \cos\theta \end{bmatrix} + \begin{bmatrix} \sin\theta \times \sin\theta & \sin\theta \times (-\cos\theta) \\ \sin\theta \times \cos\theta & \sin\theta \times \sin\theta \end{bmatrix}$$

$$\begin{bmatrix} cos^2\,\theta + sin^2\theta & 0 \\ 0 & cos^2\,\theta + sin^2\theta \end{bmatrix}$$

And we know that $cos^2\,\theta + sin^2\theta = 1$

$$\begin{bmatrix} cos^2\,\theta + sin^2\theta & 0 \\ 0 & cos^2\,\theta + sin^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence, proved.

Question 10.
If $A = \begin{bmatrix} 1 & -5 \\ -3 & 2 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 2 & -1 \\ -2 & 3 \end{bmatrix}$, find the matrix C such that A + B + C is a zero matrix

Answer:
Given, A+B+C = zero matrix

We know that zero matrix is a matrix whose all elements are zero, so we have,

$$A = \begin{bmatrix} 1 & -5 \\ -3 & 2 \\ 4 & -2 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 2 & -1 \\ -2 & 3 \end{bmatrix}$$

WE have A+B+C=0,

So C = -A+B,

$$-C = \begin{bmatrix} 1 & -5 \\ -3 & 2 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 2 & -1 \\ -2 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} -4 & 4 \\ 1 & -1 \\ -2 & -1 \end{bmatrix}$$

Question 11.

If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ then find the least value of α for which A + A′ = I.

Answer:

Given, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Here, A′ i.e. A transpose is $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

We are given that A+A′=I

So, $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

After doing addition of matrices, we get,

$$\begin{bmatrix} \cos\alpha + \cos\alpha & \sin\alpha - \sin\alpha \\ \sin\alpha - \sin\alpha & \cos\alpha + \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing the elements we get,

$2\cos\alpha = 1$

This implies, $\cos\alpha = \frac{1}{2}$

For α belongs 0 to π, $\alpha = \frac{\pi}{3}$

Question 12.
Find the value of x and y for which

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Answer:
Given,

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Applying matrix multiplication we get,

$$\begin{bmatrix} 2x - 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

On comparing the elements we get, 2x-3y = 1,

x+y = 3,

On solving the equations we get, x=2, y=1

Question 13.
Find the value of x and y for which

$$\begin{bmatrix} x & y \\ 3y & x \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}.$$

Answer:
Given,

$$\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Applying matrix multiplication we have, $\begin{bmatrix} x + 2y \\ 3y + 2x \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

On comparing the elements with each other we get,

The linear equations, x+2y=3, 3y+2x=5

On solving these equations we get x = 1, y = 1

Question 14.

If $A = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$, show that (A + A′) is symmetric

Answer:

Given, $A = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix} and\ A' = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$

Then, (A +A′) will be, $\begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$

The matrix $\begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$ is a symmetrical matrix.

Question 15.

If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, and show that (A – A′) is skew-symmetric

Answer:

Given,

$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, and

$A' = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$

(A - A′) = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

The matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is skew-symmetric.

Question 16.

If $\mathrm{A} = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$ and $\mathrm{B} = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$, find a matrix X such that A + 2B + X = O.

Answer:

Given, $A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$

We need to a matrix X such that, A +2B + X = 0

We have, X = -(A + 2B),

$$X = - \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} + 2\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$$

$$X = -\begin{bmatrix} 2+(-2) & -3+(2\times 2) \\ 4+0 & 5+(2\times 3) \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & -1 \\ -4 & -11 \end{bmatrix}$$

Question 17.

If $\mathrm{A} = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $\mathrm{B} = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$, find a matrix X such that

3 A – 2B + X = O.

Answer:

Given, $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$

We have 3A – 2B + X = 0

So X = -(3A – 2B)

Thus,

$$X = -\ 3\begin{bmatrix}4 & 2\\1 & 3\end{bmatrix} - 2\begin{bmatrix}-2 & 1\\3 & 2\end{bmatrix}$$

$$X = -\begin{bmatrix}3\times 4 + 2\times 2 & 3\times 2 - 2\times 1\\3\times 1 - 2\times 3 & 3\times 3 - 2\times 2\end{bmatrix}$$

$$X = \begin{bmatrix}-16 & -4\\3 & -5\end{bmatrix}$$

Question 18.

If $A = \begin{bmatrix}\cos\alpha & \sin\alpha\\-\sin\alpha & \cos\alpha\end{bmatrix}$, show that A′ A = I.

Answer:

Given, $A = \begin{bmatrix}\cos\alpha & \sin\alpha\\-\sin\alpha & \cos\alpha\end{bmatrix}$

$$A' = \begin{bmatrix}\cos\alpha & -\sin\alpha\\\sin\alpha & \cos\alpha\end{bmatrix}$$

Then , $AA' = \begin{bmatrix}\cos\alpha & \sin\alpha\\-\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\\sin\alpha & \cos\alpha\end{bmatrix}$

Applying matrix multiplication we get,

$$AA' = \begin{bmatrix}\cos\alpha\times\cos\alpha + \sin\alpha\times\sin\alpha & \cos\alpha\times(-\sin\alpha) + \sin\alpha\times\cos\alpha\\(-\sin\alpha)\times\cos\alpha + \cos\alpha\times\sin\alpha & (-\sin\alpha)\times(-\sin\alpha) + \cos\alpha\times\cos\alpha\end{bmatrix}$$

$$AA' = \begin{bmatrix}cos^2\alpha + sin^2\alpha & 0\\0 & cos^2\alpha + sin^2\alpha\end{bmatrix}$$

Hence, $AA' = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I$

As we know that $cos^2\alpha + sin^2\alpha = 1$

Question 19.

If A and B are symmetric matrices of the same order, show that (AB – BA) is a skew symmetric matrix.

Answer:
We are given that A and B are symmetric matrices of the same order then, we need to show that (AB – BA) is a skew symmetric matrix.

Let us consider P is a matrix of the same order as A and B

And let P = (AB – BA),

we have A = A′ and B = B′

then, P′ = (AB – BA)′

P′ = ((AB)′ – (BA)′) …….using reversal law we have (CD)′=D′C′

P′ = (B′A′ – A′B′)

P′ = (BA – AB)

P′ = -P

Hence, P is a skew symmetric matrix.

Question 20.
If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and f(x) = x^2 – 4x + 1, find f(A).

Answer:
Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

f(x) = x^2 – 4x + 1,

f(A) = A^2 – 4A + I,

f(A) = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$f(A) = \begin{bmatrix} 4+3-8+1 & 6+6-12+0 \\ 2+2-4+0 & 3+4-8+1 \end{bmatrix}$$

$$f(A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Question 21.
If the matrix A is both symmetric and skew-symmetric, show that A is a zero matrix.

Answer:
Given that matrix A is both symmetric and skew symmetric, then,

We have A = A′ ……(i)

And A = -A′ ……(ii)

From (i) and (ii) we get,

A′ = -A′,

2A′ = 0

A′ = 0

Then, A = 0

Hence proved.