Exercise 16a

Question 1.
Evaluate:

$\int_1^3 x^4 dx$

Answer:
$\frac{242}{5}$

Evaluation:

$$\int_1^3 x^4 dx = \left[\frac{x^5}{5}\right]$$

$$= \frac{3^5}{5} - \frac{1}{5}$$

$$= \frac{243 - 1}{5}$$

$$= \frac{242}{5}$$

Question 2.
Evaluate:

$\int_1^4 \sqrt{x} dx$

Answer:
$\frac{14}{3}$

Evaluation:

$$\int_1^4 \sqrt{x} dx = \left[\frac{2}{3} x^{\frac{3}{2}}\right]$$

$$= \frac{2}{3}\left[4^{\frac{3}{2}} - 1\right]$$

$$= \frac{14}{3}$$

Question 3.
Evaluate:

$\int_1^2 x^{-5} dx$

Answer:
$\frac{15}{64}$

Evaluation:

$$\int_1^2 x^{-5} dx = \left[\frac{x^{-4}}{-4}\right]$$

$$= \frac{2^{-4}}{-4} - \frac{1}{-4}$$

$$= \frac{16-1}{64}$$

$$= \frac{15}{64}$$

Question 4.
Evaluate:

$\int_0^{16} x^{\frac{3}{4}} dx$

Answer:
$\frac{512}{7}$

Evaluation:

$$\int_0^{16} x^{\frac{3}{4}} dx = \left[\frac{4}{7} x^{\frac{7}{4}}\right]$$

$$=\frac{4}{7}\left[16^{\frac{7}{4}}-1\right]$$

$$=\frac{512}{7}$$

Question 5.
Evaluate:

$\int_{-4}^{-1}\frac{\mathrm{dx}}{\mathrm{x}}$

Answer:
$-log4$

Evaluation:

$$\int_{-4}^{-1}\frac{\mathrm{dx}}{\mathrm{x}}=-[logx]$$

=[log(-1)-log(-4)]

=-[log(-4)-log(-1)]

$$=-\left[log\left(\frac{-4}{-1}\right)\right]$$

=-log 4

Question 6.
Evaluate:

$\int_{1}^{4}\frac{\mathrm{dx}}{\sqrt{\mathrm{x}}}$

Answer:
2

Evaluation:

$$\int_{1}^{4}\frac{\mathrm{dx}}{\sqrt{\mathrm{x}}}=[2\sqrt{\mathrm{x}}]$$

$=[2\sqrt{4}-2]$

$=[4-2]$

$=2$

Question 7.
Evaluate:

$$\int_0^1 \frac{dx}{\sqrt[3]{x}}$$

Answer:
$\frac{3}{2}$

Evaluation:

$$\int_0^1 \frac{dx}{\sqrt[3]{x}} = \left[\frac{3}{2}x^{\frac{2}{3}}\right]$$

$$= \left[\frac{3}{2}1^{\frac{4}{3}} - 0\right]$$

$$= \frac{3}{2}$$

Question 8.
Evaluate:

$$\int_1^8 \frac{dx}{x^{\frac{2}{3}}}$$

Answer:
3

Evaluation:

$$\int_1^8 \frac{dx}{x^{\frac{2}{3}}} = \left[\frac{3}{1}x^{\frac{1}{3}}\right]$$

$$= \left[3(8)^{\frac{1}{3}} - 3(1)^{\frac{1}{3}}\right]$$

=[6-3]

=3

Question 9.
Evaluate:

$$\int_{2}^{4} 3\,dx$$

Answer:
6

Evaluation:

$$\int_{2}^{4} 3dx = 3[x]$$

=3[4-2]

=6

Question 10.
Evaluate:

$$\int_{0}^{1} \frac{dx}{\left(1+x^2\right)}$$

Answer:
$\frac{\pi}{4}$

Evaluation:

$$\int_0^1 \frac{dx}{1+x^2} = [\tan^{-1}x]$$

$=[\tan^{-1} 1-\tan^{-1} 0]$

$=\pi/4$

Question 11.
Evaluate:

$$\int_0^{\infty} \frac{dx}{\left(1+x^2\right)}$$

Answer:
$\frac{\pi}{2}$

Evaluation:

$$\int_0^{\infty} \frac{dx}{1+x^2} = [\tan^{-1}x]$$

$=[\tan^{-1} \infty-\tan^{-1} 0]$

$=\pi/2$

Question 12.
Evaluate:

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

Answer:
$\frac{\pi}{2}$

Evaluation:

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}} = [\sin^{-1}x]$$

$=[\sin^{-1} 1-\sin^{-1} 0]$

$$= \frac{\pi}{2}$$

Question 13.
Evaluate:

$$\int_0^{\pi/6} \sec^2 x\, dx$$

Answer:
$$\frac{1}{\sqrt{3}}$$

Evaluation:

$$\int_0^{\frac{\pi}{6}} \sec^2 x\, dx = [\tan x]$$

$$= \left[\tan\left(\frac{\pi}{6}\right) - \tan 0\right]$$

$$= \frac{1}{\sqrt{3}}$$

Question 14.
Evaluate:

$$\int_{-\pi/4}^{\pi/4} \operatorname{cosec}^2 x\, dx$$

Answer:
-2

Evaluation:

$$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \operatorname{cosec}^2 x dx = [-\cot x]$$

$$= \left[-\cot\left(\frac{\pi}{4}\right) + \cot\left(-\frac{\pi}{4}\right)\right]$$

$$= \left[-\cot\left(\frac{\pi}{4}\right) - \cot\left(\frac{\pi}{4}\right)\right]$$

=-2

Question 15.
Evaluate:

$$\int_{\pi/4}^{\pi/2} \cot^2 x \, dx$$

Answer:

$$\left(1 - \frac{\pi}{4}\right)$$

Evaluation:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^2 x dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\operatorname{cosec}^2 x - 1) dx$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\operatorname{cosec}^2 x - 1) dx = [-\cot x - x]$$

$$= \left[-\cot\left(\frac{\pi}{2}\right) - \frac{\pi}{2} + \cot\left(\frac{\pi}{4}\right) + \frac{\pi}{4}\right]$$

$$= \left[0 - \frac{\pi}{4} + 1\right]$$

$$= \left[1 - \frac{\pi}{4}\right]$$

Question 16.
Evaluate:

$$\int_0^{\pi/4} \tan^2 x \, dx$$

Answer:

$$\left(1 - \frac{\pi}{4}\right)$$

Evaluation:

$$\int_0^{\frac{\pi}{4}} \tan^2 x dx = \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) dx$$

$$\int_0^{\frac{\pi}{4}} (\sec^2 x - 1) dx = [\tan x - x]$$

$$= \left[\tan\left(\frac{\pi}{4}\right) - \frac{\pi}{4} - \tan(0) - 0\right]$$

$$= \left[1 - \frac{\pi}{4}\right]$$

Question 17.
Evaluate:

$$\int_0^{\pi/2} \sin^2 x \, dx$$

Answer:

$$\frac{\pi}{4}$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} \sin^2 x dx = \int_0^{\frac{\pi}{2}} \frac{1}{2}(1 - \cos 2x) dx$$

$$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - \frac{\sin\pi}{2} - 0 + \frac{\sin 0}{2}\right]$$

$$= \frac{\pi}{4}$$

Question 18.
Evaluate:

$$\int_0^{\pi/4} \cos^2 x \, dx$$

Answer:
$$\left(\frac{\pi}{8} + \frac{1}{4}\right)$$

Evaluation:

$$\int_0^{\frac{\pi}{4}} \cos^2 x dx = \int_0^{\frac{\pi}{2}} \frac{1}{2}(1 + \cos 2x) dx$$

$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{4} + \frac{\sin(\frac{\pi}{2})}{2} - 0 - \frac{\sin 0}{2}\right]$$

$$= \frac{\pi}{8} + \frac{1}{4}$$

Question 19.

Evaluate:

$$\int_{0}^{\pi/3} \tan x \, dx$$

Answer:
log 2

Evaluation:

$$\int_{0}^{\frac{\pi}{3}} tanxdx = log|secx|$$

$$= log\left|sec\left(\frac{\pi}{3}\right)\right| - ln|cos0|$$

=log|2|-log|1|

=log2

Question 20.
Evaluate:

$$\int_{\pi/6}^{\pi/4} \text{cosec } x \, dx$$

Answer:
$$\log\left(\sqrt{2}-1\right)+\log\left(2+\sqrt{3}\right)$$

Evaluation:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} cosecxdx = -log|cosecx + cotx|$$

$$= -log\left|cosec\left(\frac{\pi}{4}\right) + cot\left(\frac{\pi}{4}\right)\right| + log|cosec(\frac{\pi}{6}) + cot(\frac{\pi}{6})|$$

=-log|√2+1|+log|2+√3|

Question 21.
Evaluate:

$$\int_{0}^{\pi/3} \cos^3 x\, dx$$

Answer:
$$\frac{3\sqrt{3}}{8}$$

Evaluation:

$$\int_0^{\frac{\pi}{3}} cos^3 x\, dx = \frac{1}{4}\int_0^{\frac{\pi}{3}} (3cosx + cos3x)dx$$

$$\frac{1}{4}\int_0^{\frac{\pi}{3}} (3cosx - cos3x)dx = \frac{1}{4}\left[3sinx + \frac{sin3x}{3}\right]$$

$$= \frac{1}{4}\left[3sin\left(\frac{\pi}{3}\right) + \frac{sin\pi}{3}\right] - \frac{1}{4}\left[3sin0 + \frac{sin0}{3}\right]$$

$$= \frac{1}{4}\left[\frac{3\sqrt{3}}{2}\right]$$

$$= \frac{3\sqrt{3}}{8}$$

Question 22.
Evaluate:

$$\int_{0}^{\pi/2} \sin^3 x\, dx$$

Answer:
$$\frac{2}{3}$$

Evaluation:

Evaluation:

$$\int_0^{\frac{\pi}{2}} sin^3 x\ dx = \frac{1}{4}\int_0^{\frac{\pi}{2}}(3sinx - sin3x)dx$$

$$\frac{1}{4}\int_0^{\frac{\pi}{2}}(3sinx - sin3x)dx = \frac{1}{4}\left[-3cosx + \frac{cos3x}{3}\right]$$

$$= \frac{1}{4}\left[-3cos(\frac{\pi}{2}) + \frac{cos(\frac{3\pi}{2})}{3}\right] - \frac{1}{4}\left[-3cos0 + \frac{cos0}{3}\right]$$

$$= \frac{1}{4}\left[\frac{9-1}{3}\right]$$

$$= \frac{2}{3}$$

Question 23.
Evaluate:

$$\int_{\pi/4}^{\pi/2} \frac{(1-3\cos x)}{\sin^2 x} dx$$

Answer:
$\left(4-3\sqrt{2}\right)$

Evaluation:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(1-3cosx)}{sin^2 x} dx \ = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(cosec^2(x) - 3cosec(x)cot(x))dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(cosec^2(x) - 3cosec(x)cot(x))dx$$

Question 24.

Evaluate:

$$\int_{0}^{\pi/4} \sqrt{1+\cos\ 2x}\, dx$$

Answer:
1

Evaluation:

$$\int_{0}^{\frac{\pi}{4}} \sqrt{1+\cos 2x}\, dx = \int_{0}^{\frac{\pi}{4}} \sqrt{2\cos^2 x} dx$$

$$= \sqrt{2}[\sin x]$$

$$= \sqrt{2}\left[\sin\left(\frac{\pi}{4}\right) - \sin 0\right]$$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}}\right]$$

=1

Question 25.
Evaluate:

$$\int_{0}^{\pi/4} \sqrt{1-\sin\ 2x}\, dx$$

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Answer:
$\left(\sqrt{2}-1\right)$

Evaluation:

$$\int_{0}^{\frac{\pi}{4}} \sqrt{1-\sin 2x}\ \ dx = \int_{0}^{\frac{\pi}{4}} \sqrt{\sin^2 x + \cos^2 x - 2\sin x\cos x}\, dx$$

$$= \int_{0}^{\frac{\pi}{4}} (cosx - sinx)\, dx$$

=[sin x + cos x]

$$= \left[cos\left(\frac{\pi}{4}\right) + sin\left(\frac{\pi}{4}\right) - cos0 - sin0\right]$$

$$= \left[+\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right]$$

=[√2-1]

Question 26.
Evaluate:

$$\int_{-\pi/4}^{\pi/4} \frac{dx}{(1+\sin\ x)}$$

Answer:
2

Evaluation:

$$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1 + sinx} = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{sec^2\left(\frac{x}{2}\right)}{\left(tan^2\left(\frac{x}{2}\right) + 1\right)^2} dx$$

Let $u = \left(tan\left(\frac{x}{2}\right) + 1\right)$

$$dx = \frac{2}{sec^2\left(\frac{x}{2}\right)} du$$

$$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1 + sinx} = 2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{u^2} du$$

$$= -\frac{2}{u}$$

$$= -\frac{2}{tan\left(\frac{x}{2}\right) + 1}$$

=2

Question 27.
Evaluate:

$$\int_{0}^{\pi/4} \frac{dx}{(1 + \cos 2x)}$$

Answer:

$$\frac{1}{2}$$

Evaluation:

$$\int_0^{\frac{\pi}{4}} \frac{dx}{1 + cos2x} = \int_0^{\frac{\pi}{4}} \frac{dx}{2cos^2x}$$

$$\int_0^{\frac{\pi}{4}} \frac{dx}{2cos^2x} = \int_0^{\frac{\pi}{4}} \frac{1}{2} sec^2xdx$$

$$\int_0^{\frac{\pi}{4}} \frac{1}{2} sec^2xdx = \frac{1}{2}[tanx]$$

$$= \frac{1}{2}\left[tan\left(\frac{\pi}{4}\right) - tan0\right]$$

$$= \frac{1}{2}[1]$$

$$= \frac{1}{2}$$

Question 28.
Evaluate:

$$\int_{\pi/4}^{\pi/2} \frac{dx}{1-\cos 2x}$$

Answer:

$$\frac{1}{2}$$

Evaluation:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{1-cos2x} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{2sin^2 x}$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{2sin^2 x} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} cosec^2 xdx$$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} cosec^2 xdx = \frac{1}{2}[cotx]$$

$$= \frac{1}{2}\left[cot(\frac{\pi}{4}) - cot0\right]$$

$$= \frac{1}{2}[1]$$

$$= \frac{1}{2}$$

Question 29.
Evaluate:

$$\int_{0}^{\pi/4} \sin 2x \sin 3x \, dx$$

Answer:

$$\frac{3}{5\sqrt{2}}$$

Evaluation:

$$\int_0^{\frac{\pi}{4}} sin2xsin3xdx = \frac{1}{2}\int_0^{\frac{\pi}{4}}(cosx - cos5x)dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}}(cosx - cos5x)dx$$

$$= \frac{1}{2}\left[sinx - \frac{sin5x}{5}\right]$$

$$= \frac{1}{2}\left[sin(\frac{\pi}{4}) - \frac{sin\left(\frac{5\pi}{4}\right)}{5}\right] - \frac{1}{2}\left[sin(0) - \frac{sin(0)}{5}\right]$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2}} + \frac{1}{5\sqrt{2}}\right]$$

$$= \frac{3}{5\sqrt{2}}$$

Question 30.
Evaluate:

$$\int_0^{\pi/6} \cos x \cos 2x \, dx$$

Answer:

$$\frac{5}{12}$$

Evaluation:

$$\int_0^{\frac{\pi}{6}} cosxcos2xdx = \frac{1}{2}\int_0^{\frac{\pi}{6}}(cos3x + cosx)dx$$

$$= \frac{1}{2}\left[\frac{sin3x}{3} + sinx\right]$$

$$= \frac{1}{2}\left[\frac{sin\left(\frac{\pi}{2}\right)}{3} + sin\left(\frac{\pi}{6}\right)\right] - 0$$

$$= \frac{1}{2}\left[\frac{1}{3} + \frac{1}{2}\right]$$

$$= \frac{5}{12}$$

Question 31.
Evaluate:

$$\int_{0}^{\pi} \sin 2x \cos 3x \, dx$$

Answer:

$$\frac{-4}{5}$$

Evaluation:

$$\int_0^\pi sin2xcos3xdx = \frac{1}{2}\int_0^\pi (sin5x - sinx)dx$$

$$= \frac{1}{2}\left[\frac{-cos5x}{5} + cosx\right]$$

$$= \frac{1}{2}\left[-\frac{cos(5\pi)}{5} + cos(\pi)\right] - \frac{1}{2}\left[-\frac{cos(0)}{5} + cos(0)\right]$$

$$= \frac{1}{2}\left[\frac{-(-1)}{5} - 1\right] - \frac{1}{2}\left[-\frac{1}{5} + 1\right]$$

$$= \frac{1}{2}\left[\frac{-4}{5} - \frac{4}{5}\right]$$

$= \frac{1}{2} 2 \left(-\frac{4}{5}\right)$

$= -\frac{4}{5}$

Question 32.
Evaluate:

$$\int_0^{\pi/2} \sqrt{1+\sin x}\ dx$$

Answer:
2

Explanation:

$$\int_0^{\frac{\pi}{2}} \sqrt{1+sin(x)}dx = \int_0^{\frac{\pi}{2}} \sqrt{2}cos\left(\frac{2x-\pi}{4}\right) dx$$

$$= 2^{\frac{3}{2}} sin\left(\frac{2x-\pi}{4}\right)$$

$$= 2^{\frac{3}{2}} \left(0 - sin(-\frac{\pi}{4}\right)$$

$$= \frac{2\sqrt{2}}{\sqrt{2}}$$

=2

Question 33.
Evaluate:

$$\int_0^{\pi/2} \sqrt{1+\cos x}\, dx$$

Answer:

2

Explanation:

$$\int_0^{\frac{\pi}{2}} \sqrt{1+cos(x)}\,dx = \int_0^{\frac{\pi}{2}} \sqrt{2}cos\left(\frac{x}{2}\right)dx$$

$$= 2^{\frac{3}{2}} sin\left(\frac{x}{2}\right)$$

$$= 2^{\frac{3}{2}}\left(sin\left(\frac{\pi}{4}\right) - 0\right)$$

$$= \frac{2\sqrt{2}}{\sqrt{2}}$$

=2

Question 34.
Evaluate:

$$\int_0^2 \frac{\left(x^4+1\right)}{\left(x^2+1\right)}dx$$

Answer:

$$\left(\frac{2}{3} + 2\tan^{-1} 2\right)$$

Explanation:

$$\int_0^2 \left\{\frac{(x^4+1)}{x^2+1}\right\} dx = \int_0^2 \frac{x^4+2-1}{x^2+1}\,dx$$

$$= \int_0^2 \frac{x^4-1}{x^2+1}dx + \int_0^2 \frac{2}{x^2+1}dx$$

$$= \int_0^2 \frac{(x^2-1)(x^2+1)}{x^2+1}dx + \int_0^2 \frac{2}{x^2+1}dx$$

$$= \int_0^2 (x^2 - 1)dx + 2tan^{-1}x$$

$$= \left[\frac{x^3}{3} - x + 2tan^{-1}x\right]_0^2$$

$$= \frac{2}{3} + 2tan^{-1}2$$

Question 35.
Evaluate:

$$\int_1^2 \frac{dx}{(x+1)(x+2)}$$

Answer:
(2 log 3 – 3 log 2)

Explanation:

$$\int_1^2 \frac{dx}{(x+1)(x+2)} = \int_1^2 \frac{(x+2)-(x+1))}{(x+1)(x+2)} dx$$

$$= \int_1^2 \frac{1}{(x+1)} dx - \int_1^2 \frac{1}{(x+2)} dx$$

$$= [log(x+1) - log(x+2)]_1^2$$

=2log3-3log2

Question 36.
Evaluate:

$$\int_1^2 \frac{(x+3)}{x(x+2)} dx$$

Answer:

$$\frac{1}{2}(\log 2+\log 3)$$

Explanation:

$$\int_1^2 \frac{x+3}{x(x+2)}dx = \int_1^2 \frac{3}{2x}dx - \int_1^2 \frac{1}{x+2}dx$$

$$= \frac{3}{2} log\ x - log(x+2)$$

$$= \frac{1}{2}(log 2 + log 3)$$

Question 37.
Evaluate:

$$\int_3^4 \frac{dx}{(x^2-4)}$$

Answer:

$$\frac{1}{4}(\log 5-\log 3)$$

Evaluation:

$$\int_3^4 \frac{dx}{x^2-4} = \int_3^4 \frac{1}{(x-2)(x+2)}dx$$

$$= \int_3^4 \frac{1}{4(x-2)}dx - \int_3^4 \frac{1}{4(x+2)}dx$$

$$= \frac{1}{4}\log(x-2) - \frac{1}{4}\log(x+2)$$

$$= \frac{1}{4} log 3 - \frac{1}{4} log 1 - \frac{1}{4}\log 6 + \frac{1}{4} log 5$$

$$= \frac{1}{4}\left(log 5 - \log\left(\frac{6}{2}\right)\right)$$

$$= \frac{1}{4}(log5 - log3)$$

Question 38.
Evaluate:

$$\int_0^4 \frac{dx}{\sqrt{x^2 + 2x + 3}}$$

Answer:

$$\log\left(\frac{5 + 3\sqrt{3}}{1 + \sqrt{3}}\right)$$

Evaluation:

$$\int \frac{dx}{\sqrt{x^2 + 2x + 3}} = \int \frac{dx}{\sqrt{(x+1)^2 + 2}}$$

Substitute:

$$\frac{x+1}{\sqrt{2}} = u$$

$$\therefore\ dx = \sqrt{2}du$$

$$= \int \frac{\sqrt{2}du}{\sqrt{2u^2 + 2}}$$

$$= log(\sqrt{u^2 + 1} + u)$$

Undo substitution: $u = \frac{x+1}{\sqrt{2}}$

$$\therefore \int_0^4 \frac{dx}{\sqrt{x^2 + 4x + 3}} = log(\sqrt{(x+1)^2 + 2} + x + 1)$$

$$= log\left(\sqrt{(4+1)^2 + 2} + 4 + 1\right) - log(\sqrt{(0+1)^2 + 2} + 0 + 1)$$

$$= log\left(5+3\sqrt{3}\right) - log(1+\sqrt{3})$$

$$= \log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$$

Question 39.
Evaluate:

$$\int_{1}^{2} \frac{dx}{\sqrt{x^2+4x+3}}$$

Answer:

$$\log\left(4+\sqrt{15}\right) - \log\left(3+\sqrt{8}\right)$$

Evaluation:

$$\int \frac{dx}{\sqrt{x^2+4x+3}} = \int \frac{dx}{\sqrt{(x+2)^2-1}}$$

Substitute:

x+2=u

∴ dx=du

$$= \int \frac{du}{\sqrt{u^2-1}}$$

$$= log(\sqrt{u^2-1}+u)$$

Undo substitution: $u = x + 2$

$$\therefore \int_1^2 \frac{dx}{\sqrt{x^2+4x+3}} = log(\sqrt{(x+2)^2-1}+x+2)$$

$$= log\left(\sqrt{(2+2)^2-1}+2+2\right) - log(\sqrt{(1+2)^2-1}+1+2)$$

=log(4+√15)-log(3+√8)

Question 40.
Evaluate:

$$\int_0^1 \frac{dx}{\left(1+x+2x^2\right)}$$

Answer:

$$\frac{2}{\sqrt{7}}\left\{\text{ran}^{-1}\frac{5}{\sqrt{7}}-\tan^{-1}\frac{1}{\sqrt{7}}\right\}$$

Evaluation:

$$\int_0^1 \frac{1}{2x^2+x+1}dx = \int_0^1 \frac{1}{\left(\left(\sqrt{2x}+\frac{1}{2^{\frac{3}{2}}}\right)2+\frac{7}{8}\right)dx}$$

Substitute 4x+1√7=u

$$\therefore\ dx = \frac{\sqrt{7}}{4}du$$

Now solving:

$$\int\left(\frac{1}{u^2}+\ 1\right)du = \tan^{-1}u$$

$$\frac{2}{\sqrt{7}}\int\frac{1}{u^2+1}du = \frac{2}{\sqrt{7}}\tan^{-1}u$$

$$\therefore\ \int_0^1 \frac{1}{2x^2+x+1}dx = \frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right)$$

$$= \frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{4+1}{\sqrt{7}}\right)-\frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)$$

$$= \frac{2}{\sqrt{7}}\{\tan^{-1}\left(\frac{5}{\sqrt{7}}\right) - \tan^{-1}\left(\frac{1}{\sqrt{7}}\right)\}$$

Question 41.
Evaluate:

$$\int_{0}^{\pi/2} \left(a\cos^2 x + b\sin^2 x\right)dx$$

Answer:

$$\frac{\pi}{4}(a+b)$$

Evaluation:

$$\int_{0}^{\frac{\pi}{2}} (acos^2 x + bsin^2 x)dx = \int_{0}^{\frac{\pi}{2}} \left[\frac{a}{2}(cos2x+1) + \frac{b}{2}(1-cos2x)\right]dx$$

$$= \left[\frac{a}{2}\left(\frac{sin2x}{2} + x\right) + \frac{b}{2}\left(x - \frac{sin2x}{2}\right)\right]$$

$$= \left[\frac{a}{2}\left(\frac{sin\pi}{2} + \frac{\pi}{2}\right) + \frac{b}{2}\left(\frac{\pi}{2} - \frac{sin\pi}{2}\right) - \frac{a}{2}\left(\frac{sin0}{2} + 0\right) - \frac{b}{2}\left(0 - \frac{sin0}{2}\right)\right]$$

$$= \left[\frac{a}{2}\left(0 + \frac{\pi}{2}\right) + \frac{b}{2}\left(\frac{\pi}{2} - 0\right) - \frac{a}{2}(0+0) - \frac{b}{2}(0-0)\right]$$

$$= \frac{\pi}{4}(a+b)$$

Question 42.
Evaluate:

$$\int_{\pi/3}^{\pi/4} (\tan x + \cot x)^2 dx$$

Answer:

$$\frac{-2}{\sqrt{3}}$$

Evaluation:

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}(tanx + cotx)^2 dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\left(\frac{tan^2 x + 1}{tanx}\right)^2 dx$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\left(\frac{tan^2 x + 1}{tanx}\right)^2 dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\frac{sec^2 x(tan^2 x + 1)}{tan^2 x}dx$$

Substitute:

tan(x)=u

$$\therefore dx = \frac{1}{sec^2(x)}du$$

$$\therefore = \int \frac{(u^2 + 1)}{u^2}\, du$$

$$\therefore = u - \frac{1}{u}$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\left(\frac{tan^2 x + 1}{tanx}\right)^2 dx = [tan(x) - cot(x)]$$

$$= \left[tan\left(\frac{\pi}{4}\right) - cot\left(\frac{\pi}{4}\right) - tan\left(\frac{\pi}{3}\right) + cot(\frac{\pi}{3})\right]$$

$$= \left[1 - 1 - \sqrt{3} + \frac{1}{\sqrt{3}}\right]$$

$$= -\frac{2}{\sqrt{3}}$$

Question 43.

Evaluate:

$$\int_0^{\pi/2} \cos^4 x \, dx$$

Answer:

$$\frac{3\pi}{16}$$

Evaluation:

By reduction formula:

$$\int_0^{\frac{\pi}{2}} cos^4 x dx = \frac{cos^3(x) sin(x)}{4} + \frac{3}{4}\int cos^2 x dx$$

We know that,

$$\int cos^2 x \, dx = \frac{1}{2}\left[\frac{sin2x}{2} + x\right]$$

$$\int_0^{\frac{\pi}{2}} cos^4 x dx = \frac{cos^3(x) sin(x)}{4} + \frac{3}{8}\left[\frac{sin2x}{2} + x\right]$$

$$= \frac{cos^3\left(\frac{\pi}{2}\right) sin\left(\frac{\pi}{2}\right)}{4} + \frac{3}{8}\left[\frac{sin\pi}{2} + \frac{\pi}{2}\right] - \frac{cos^3(0) sin(0)}{4} - \frac{3}{8}\left[\frac{sin0}{2} + 0\right]$$

$$= 0 + \frac{3}{8}\left[0 + \frac{\pi}{2}\right] - 0 - \frac{3}{8}[0 + 0]$$

$$= \frac{3\pi}{16}$$

Question 44.
Evaluate:

$$\int_0^{a} \frac{dx}{\left(ax + a^2 - x^2\right)}$$

Answer:

$$\frac{1}{\sqrt{5}a}\log\left\{\frac{7+3\sqrt{5}}{2}\right\}$$

Evaluation:

Assume that a≠0.

$$\int_0^2 \frac{1}{-x^2+ax+a^2}\ dx = -\int_0^2 \frac{1}{x^2-ax-a^2}\ dx$$

$$= \int_0^2 \frac{4}{(2x+(-\sqrt{5}-1)a)(2x+(\sqrt{5}-1)a)}dx$$

$$= \int_0^2 \left(\frac{2}{\sqrt{5}a(2x+(-\sqrt{5}-1)a)} - \frac{2}{\sqrt{5}a(2x+(\sqrt{5}-1)a)}\right)dx$$

Now,

$$\int \frac{1}{2x+(-\sqrt{5}-1)a}dx$$

Substitute:

u=2x+(-√5-1)a

$$\therefore dx = \frac{1}{2}du$$

$$= \frac{1}{2}\int \frac{1}{u}\ du$$

$$= \frac{1}{2}logu$$

Undo substitution:

$$u = 2x + (-\sqrt{5}-1)a$$

$$\therefore \int \frac{1}{2x+\left(-\sqrt{5}-1\right)a}\,dx = \frac{1}{2}log\left(2x+\left(-\sqrt{5}-1\right)a\right)$$

Now,

$$\int \frac{1}{2x+\left(\sqrt{5}-1\right)a}\,dx$$

Substitute:

$$u = 2x + (\sqrt{5}-1)a$$

$$\therefore dx = \frac{1}{2}du$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}logu$$

Undo substitution:

$$u = 2x + (\sqrt{5}-1)a$$

$$\therefore \int \frac{1}{2x+\left(\sqrt{5}-1\right)a}\,dx = \frac{1}{2}log\left(2x+\left(\sqrt{5}-1\right)a\right)$$

$$\frac{2}{\sqrt{5}a}\int_0^2 \frac{1}{\left(2x+\left(-\sqrt{5}-1\right)a\right)}\,dx - \frac{2}{\sqrt{5}a}\int_0^2 \frac{1}{2x+\left(\sqrt{5}-1\right)a}\,dx$$

$$= \frac{log\left(2x+\left(-\sqrt{5}-1\right)a\right)}{\sqrt{5}a} - \frac{log\left(2x+\left(\sqrt{5}-1\right)a\right)}{\sqrt{5}a}$$

$$-\int_0^2 \frac{1}{x^2-ax-a^2}\,dx = \frac{log\left(2x+\left(\sqrt{5}-1\right)a\right)}{\sqrt{5}a} - \frac{log\left(2x+\left(-\sqrt{5}-1\right)a\right)}{\sqrt{5}a}$$

$$= \frac{log(4 + (\sqrt{5} - 1)a)}{\sqrt{5}a} - \frac{log(4 + (-\sqrt{5} - 1)a)}{\sqrt{5}a} - \frac{log(0 + (\sqrt{5} - 1)a)}{\sqrt{5}a} + \frac{log(0 + (-\sqrt{5} - 1)a)}{\sqrt{5}a}$$

$$= \frac{1}{\sqrt{5}a} log\left(\frac{7 + 3\sqrt{5}}{2}\right)$$

Question 45.
Evaluate:

$$\int_{1/4}^{1/2} \frac{dx}{\sqrt{x - x^2}}$$

Answer:

$$\frac{\pi}{6}$$

Evaluation:

$$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{dx}{\sqrt{x - x^2}} = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{\sqrt{\frac{1}{4} - \left(x - \frac{1}{2}\right)^2}}$$

Substitute:

2x-1=u

$$\therefore dx = \frac{1}{2} du$$

$$\int \frac{1}{\sqrt{1 - u^2}} du = sin^{-1}(u)$$

Undo Substitution:

u=2x-1

$\therefore = \sin^{-1}(2x-1)$

$$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{dx}{\sqrt{x-x^2}} = sin^{-1}(2x-1)$$

$$= sin^{-1}(1-1) - sin^{-1}(\frac{1}{2}-1)$$

$$= \frac{\pi}{6}$$

Question 46.
Evaluate:

$$\int_0^1 \sqrt{x(1-x)}dx$$

Answer:
$$\frac{\pi}{8}$$

Evaluation:

$$\int_0^1 \sqrt{x-x^2}dx = \int_0^1 \sqrt{\frac{1}{4}-\left(x-\frac{1}{2}\right)^2}\ dx$$

$$= \frac{1}{2}\int_0^1 \sqrt{1-(2x-1)^2}\ dx$$

Substitute:

2x-1=u

$$\therefore dx = \frac{1}{2}du$$

$$\therefore \frac{1}{2}\int \sqrt{1-u^2}du$$

Substitute:

u=sin(v)

∴$\sin^{-1}$ (u)=v

∴du=cos(v)dv

$$= \int cos(v)\sqrt{a - sin^2(v)}dv$$

$$= \int cos^2(v)dv$$

We know that,

$$\int cos^2(v)\ dv = \frac{1}{2}\left[\frac{sin(2v)}{2} + v\right]$$

Undo Substitution:

v=$\sin^{-1}$ (u)
sin($\sin^{-1}$ (u))=u$cos(sin^{-1}(u)) = \sqrt{1 - u^2}$

$$= \frac{sin^{-1}(u)}{2} + \frac{u\sqrt{1 - u^2}}{2}$$

Undo Substitution:

u=2x-1

$$\therefore = \frac{sin^{-1}(2x - 1)}{4} + \frac{(2x - 1)\sqrt{1 - (2x - 1)^2}}{4}$$

$$\frac{1}{2}\int_0^1 \sqrt{1 - (2x - 1)^2}\ dx\ = \frac{sin^{-1}(2x - 1)}{8} + \frac{(2x - 1)\sqrt{1 - (2x - 1)^2}}{8}$$

$$= \frac{sin^{-1}(2 - 1)}{8} + \frac{(2 - 1)\sqrt{1 - (2 - 1)^2}}{8} - \frac{sin^{-1}(0 - 1)}{8} - \frac{(0 - 1)\sqrt{1 - (0 - 1)^2}}{8}$$

$$= \frac{\pi}{16} + 0 - \frac{\pi}{8} - 0$$

$$= \frac{\pi}{8}$$

Question 47.
Evaluate:

$$\int_1^3 \frac{dx}{x^2(x+1)}$$

Answer:

$$\log 2 - \log 3 + \frac{2}{3}$$

Evaluation:

$$\int_1^3 \frac{1}{x^2(x+1)} dx$$

Perform partial fraction decomposition:

$$\int_1^3 \frac{1}{x^2(x+1)} dx = \int_1^3 \left(\frac{1}{x+1} - \frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$= \left[log(x+1) - log(x) - \frac{1}{x}\right]$$

$$= \left[log(4) - log(3) - \frac{1}{3} - log(2) + log(1) + \frac{1}{1}\right]$$

$$= log(2) - log(3) + \frac{2}{3}$$

Question 48.
Evaluate:

$$\int_1^2 \frac{dx}{x(1+2x)^2}$$

Answer:

$$\log\, 6 - \log\, 5 - \frac{2}{15}$$

Evaluation:

$$\int_1^2 \frac{1}{x(2x+1)^2} dx \;= \int_1^2 \left(-\frac{2}{2x+1} - \frac{2}{(2x+1)^2} + \frac{1}{x}\right) dx$$

$$= -2\int_1^2 \frac{1}{2x+1} dx - 2\int_1^2 \frac{1}{(2x+1)^2} dx + \int_1^2 \frac{1}{x} dx$$

$$= -2\left[\frac{1}{2} log(2x+1)\right] - 2\left[\frac{-1}{2(2x+1)}\right] + [log(x)]$$

$$= -[log(5)] + \left[\frac{1}{(5)}\right] + [log(2)] + [log(3)] - \left[\frac{1}{(3)}\right] + [log(1)]$$

$$= log(6) - log(5) - \frac{2}{15}$$

Question 49.
Evaluate:

$$\int_0^1 x\, e^x\, dx$$

Answer:
1

Evaluation:

$$\int_0^1 xe^x dx = \int_0^1 (x-1+1)e^x dx$$

=[(x-1)e^x]

=[(1-1) e^1-(0-1) e^0]

=1

Question 50.
Evaluate:

$$\int_0^{\pi/2} x^2 \cos x \, dx$$

Answer:

$$\left(\frac{\pi^2}{4} - 2\right)$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} x^2 cos(x)dx = x^2 sin(x) - \int 2xsin(x)dx$$

$$\int_0^{\frac{\pi}{2}} x^2 cos(x)dx = [x^2 sin(x) - 2sin(x) - 2xcos(x)]$$

$$= [\left(\frac{\pi}{2}\right)^2 sin\left(\frac{\pi}{2}\right) - 2sin\left(\frac{\pi}{2}\right) - \pi cos\left(\frac{\pi}{2}\right) - (0)^2 sin(0) + 2sin(0) + 0]$$

$$= \left[\frac{\pi^2}{4} - 2 - 0 - 0 + 0 + 0\right]$$

$$= \left(\frac{\pi^2}{4} - 2\right)$$

Question 51.
Evaluate:

$$\int_0^{\pi/4} x^2 \sin x \, dx$$

Answer:

$$\left(\sqrt{2}+\frac{\pi}{2\sqrt{2}}-\frac{\pi^2}{16\sqrt{2}}-2\right)$$

Evaluation:

From integrate by parts:

$$\int_0^{\frac{\pi}{4}} x^2 sin(x)dx = -x^2 cos(x) - \int -2xcos(x)dx$$

From integrate by parts:

$$\int_0^{\frac{\pi}{4}} x^2 cos(x)dx = [-x^2 cos(x) + 2xsin(x) + 2cos(x)]$$

$$= [2xsin(x) + (2-x^2)cos(x)]$$

$$= [\frac{\pi}{2} sin\left(\frac{\pi}{4}\right) + \left(2-\frac{\pi^2}{16}\right) cos\left(\frac{\pi}{4}\right) - 2(0)sin(0) - (2-0)cos(0)]$$

$$= \left[\frac{\pi}{2\sqrt{2}} + \frac{2}{\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} + 0 - 0 - 2\right]$$

$$= \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$$

Question 52.
Evaluate:

$$\int_0^{\pi/2} x^2 \cos 2x \, dx$$

Answer:

$$\frac{-\pi}{4}$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} x^2 cos(2x)dx = \frac{x^2 sin(2x)}{2} - \int xsin(x)dx$$

$$\int_0^{\frac{\pi}{2}} x^2 cos(x)dx = \left[\frac{x^2 sin(2x)}{2} - \frac{sin(2x)}{4} + \frac{xcos(2x)}{2}\right]$$

$$= \left[\frac{(\frac{\pi}{2})^2 sin(\pi)}{2} - \frac{sin(\pi)}{4} + \frac{(\frac{\pi}{2})cos(\pi)}{2} - \frac{(0)^2 sin(0)}{2} + \frac{sin(0)}{4} - \frac{(0)cos(0)}{2}\right]$$

$$= \left[0 - 0 - \frac{\pi}{4} - 0 + 0 - 0\right]$$

$$= -\frac{\pi}{4}$$

Question 53.
Evaluate:

$$\int_0^{\pi/2} x^3 \sin 3x \, dx$$

Answer:

$$\left(\frac{2}{27} - \frac{\pi^2}{12}\right)$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} x^3 sin(3x)dx = -\frac{x^3 cos(3x)}{3} - \int -x^2 cos(3x)dx$$

$$= -\frac{x^3 cos(3x)}{3} + \frac{x^2 sin(3x)}{3} - \int \frac{2xsin(3x)}{3} dx$$

$$= -\frac{x^3 cos(3x)}{3} + \frac{x^2 sin(3x)}{3} + \frac{2xcos(3x)}{9} + \frac{2}{3}\int -\frac{cos(3x)}{3} dx$$

$$= -\frac{x^3 cos(3x)}{3} + \frac{x^2 sin(3x)}{3} + \frac{2xcos(3x)}{9} - \frac{2sin(3x)}{27}$$

$$= -0 + \frac{\left(\frac{\pi}{2}\right)^2 sin\left(\frac{3\pi}{2}\right)}{3} + 0 - \frac{2sin\left(\frac{3\pi}{2}\right)}{27} + 0 - 0 - 0 + 0$$

$$= \left(\frac{2}{27} - \frac{\pi^2}{12}\right)$$

Question 54.
Evaluate:

$$\int_0^{\pi/2} x^2 \cos^2 x \, dx$$

Answer:

$$\left(\frac{\pi^3}{48} - \frac{\pi}{8}\right)$$

Evaluation:

$$\int_0^{\frac{\pi}{2}} x^2 cos^2 x dx = \int_0^{\frac{\pi}{2}} \frac{x^2}{2}(cos(2x) + 1)dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{x^2}{2} cos(2x) + \frac{x^2}{2}\right) dx$$

$$\int_0^{\frac{\pi}{2}} \left(\frac{x^2}{2} cos(2x) + \frac{x^2}{2}\right) dx = \frac{x^2 sin(2x)}{2} - \int xsin(2x)dx + \frac{x^3}{6}$$

$$= \frac{x^2 sin(2x)}{2} + \frac{xcos(2x)}{4} + \int -\frac{cos(2x)}{2} dx + \frac{x^3}{6}$$

$$= \frac{x^2 sin(2x)}{2} + \frac{xcos(2x)}{4} - \frac{sin(2x)}{4} + \frac{x^3}{6}$$

$$= \frac{x^2 sin(2x)}{2} + \frac{xcos(2x)}{4} - \frac{sin(2x)}{4} + \frac{x^3}{6}$$

$$= 0 + \frac{\frac{\pi}{2} cos(\pi)}{4} - 0 + \frac{\left(\frac{\pi}{2}\right)^3}{6} - 0 - 0 + 0 - 0$$

$$= \left(\frac{\pi^3}{48} - \frac{\pi}{8}\right)$$

Question 55.
Evaluate:

$$\int_1^2 \log x \ dx$$

Answer:
(2 log 2 – 1)

Evaluation:

$$\int_1^2 log(x)dx = xlog(x) - (x)$$

$$= 2log(2) - (2) - 1log(1) + (1)$$

$$= 2log(2) - 1$$

Question 56.
Evaluate:

$$\int_1^3 \frac{\log x}{(1+x)^2} dx$$

Answer:
$\frac{3}{4}\log 3 - \log 2$

Evaluation:

$$\int_1^3 \frac{log(x)}{(1+x)^2}dx = -\frac{log(x)}{1+x} - \int\left(-\frac{1}{x(1+x)}\right)dx$$

Now,

$$\int\left(-\frac{1}{x(1+x)}\right)dx = -\int\left(\frac{1}{x^2\left(\frac{1}{x}+1\right)}\right)dx$$

Let,

$$\frac{1}{x}+1 = u$$

$\therefore$dx=-x^2 du

$$\therefore -\int\left(\frac{1}{x^2\left(\frac{1}{x}+1\right)}\right)dx = \int\frac{1}{u}du$$

$$= log(u)$$

Undo substitution:

$$u = \frac{1}{x}+1$$

$$\int_1^3 \frac{log(x)}{(1+x)^2}dx = -\frac{log(x)}{1+x} + log\left(\frac{1}{x}+1\right)$$

$$= -\frac{log(3)}{4} + log\left(\frac{4}{3}\right) + \frac{log(1)}{2} - log(2)$$

$$= -\frac{log(3)}{4} + log(4) + log(3) - log2$$

$$= \frac{3}{4}log3 - log2$$

Question 57.
Evaluate:

$$\int_0^{e^2}\left\{\frac{1}{(\log x)}-\frac{1}{(\log x)^2}\right\}dx$$

Answer:

$$\left(\frac{e^2}{2}-e\right)$$

Correct answer is $\frac{e^2}{2}$

Evaluation:

Let,

log(x)=u

→x=e^u

→dx=e^u du

$$\int\left\{\frac{1}{u}-\frac{1}{u^2}\right\}e^u du=\frac{e^u}{u}$$

Undo substitution:

$$u=log(x)$$

$$\int_0^{e^2}\left\{\frac{1}{log(x)}-\frac{1}{log(x)^2}\right\}dx=\frac{x}{log(x)}$$

$$=\frac{e^2}{log(e^2)}-0$$

$$=\frac{e^2}{2}$$

Question 58.
Evaluate:

$$\int_1^e e^x\left(\frac{1+x\log x}{x}\right)dx$$

Answer:
e^e

Evaluation:

$$\int_1^e e^x\left(\frac{(1+xlog(x))}{x}\right)dx=\int_1^e e^x\left(\frac{1}{x}+log(x)\right)dx$$

$=\log(x)\ e^x$

$=\log(e)\ e^e-\log(1)\ e^1$

$=e^e$

Question 59.
Evaluate:

$$\int_0^1 \frac{x\ e^x}{(1+x)^2}dx$$

Answer:
$\left(\frac{e}{2}-1\right)$

Evaluation:

$$\int_0^1 \frac{xe^x}{(1+x)^2}dx$$

From Integrates by parts:

$$= -\frac{xe^x}{x+1} - \int \frac{-xe^x - e^x}{x+1}\, dx$$

$$\therefore \int \frac{-xe^x - e^x}{x+1}\, dx = \int -e^x dx$$

=-e^x

$$\int_0^1 \frac{xe^x}{(1+x)^2} dx = \left[-\frac{xe^x}{x+1} - e^x\right]$$

$$= \left[-\frac{1e^1}{1+1} - e^1 - \frac{0}{1+0} + e^0\right]$$

$$= \left[-\frac{e}{2} + e + 0 - 1\right]$$

$$= \left[\frac{e}{2} - 1\right]$$

Question 60.
Evaluate:

$$\int_0^{\pi/2} 2\tan^3 x\, dx$$

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Answer:
(1 – log 2)

Evaluation:

$$\int_0^{\frac{\pi}{2}} 2tan^3 x dx = 2\int_0^{\frac{\pi}{2}} tan^2 x tanx dx$$

$$= 2\int_0^{\frac{\pi}{2}} tan^2 x tanx dx$$

$$= 2\int_0^{\frac{\pi}{2}} (sec^2 x - 1) tanx dx$$

Substitute:

$$sec(x) = u$$

$$\therefore \mathrm{dx} = \frac{1}{\mathrm{sec(x)tan(x)}} \mathrm{du}$$

$$= 2\int_0^{\frac{\pi}{2}} \frac{(u^2 - 1)}{u} du$$

$$= 2\int_0^{\frac{\pi}{2}} \left(u - \frac{1}{u}\right) du$$

$$= 2\int_0^{\frac{\pi}{2}} \left(u - \frac{1}{u}\right) du$$

$$= 2\left[\frac{\mathrm{u}^2}{2} - \mathrm{logu}\right]$$

Undo substitution:

$$u = sec(x)$$

$$\therefore \int_0^{\frac{\pi}{2}} 2tan^3 x dx = 2\left[\frac{\mathrm{sec}^2\mathrm{x}}{2} - \mathrm{log(secx)}\right]$$

$$= 2\left[\frac{\sec^2\left(\frac{\pi}{2}\right)}{2} - \log\left(\sec\left(\frac{\pi}{2}\right)\right) - \frac{\sec^2(0)}{2} + \log(\sec(0))\right]$$

$$= 2\left[\frac{1}{2} - \log(1)\right]$$

=1-log2

Question 61.
Evaluate:

$$\int_{1}^{2} \frac{5x^2}{\left(x^2+4x+3\right)} dx$$

Answer:

$$5-\frac{5}{2}\left(9\log\frac{5}{4}-\log\frac{3}{2}\right)$$

Explanation:

$$\int_1^2 \frac{5x^2}{(x^2+4x+3)}dx = 5[\int_1^2 \frac{x^2}{(x+3)(x+1)}dx]$$

$$= 5[\int_1^2 \left(1-\frac{9}{2(x+3)}+\frac{1}{2(x+1)}\right)dx]$$

$$= 5\left[x-\frac{9}{2}log(x+3)+\frac{1}{2}log(x+1)\right]_1^2$$

$$= 5[2-\frac{9}{2}log5+\frac{1}{2}log3-1+\frac{9}{2}log4-\frac{1}{2}log2]$$

$$= 5[1-\frac{9}{2}\log\left(\frac{5}{4}\right)+\frac{1}{2}\log\left(\frac{3}{2}\right)$$

$$= 5-\frac{5}{2}\left(9\log\left(\frac{5}{4}\right)-\log\left(\frac{3}{2}\right)\right)$$