<u>Exercise 11c</u>

Question 1.
Verify Rolle's theorem for each of the following functions:

$$f(x) = x^2 \text{ on } [-1,1]$$

Answer:
Condition (1):

Since, $f(x)=x^2$ is a polynomial and we know every polynomial function is continuous for all xϵR.

$\Rightarrow$ $f(x)=x^2$ is continuous on [-1,1].

Condition (2):

Here, $f'(x)=2x$ which exist in [-1,1].

So, $f(x)=x^2$ is differentiable on (-1,1).

Condition (3):

Here, $f(-1)=(-1)^2=1$

And $f(1)=1^1=1$

i.e. f(-1)=f(1)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cϵ(-1,1) such that f'(c)=0

i.e. 2c=0

i.e. c=0

Value of c=0ϵ(-1,1)

Thus, Rolle's theorem is satisfied.

Question 2.
Verify Rolle's theorem for each of the following functions:

$$f(x) = x^2 - x - 12 \text{ in } [-3,4]$$

Answer:
Condition (1):

Since, $f(x)=x^2-x-12$ is a polynomial and we know every polynomial function is continuous for all $x \epsilon R$.

⇒ $f(x)= x^2-x-12$ is continuous on [-3,4].

Condition (2):

Here, $f'(x)=2x-1$ which exist in [-3,4].

So, $f(x)= x^2-x-12$ is differentiable on (-3,4).

Condition (3):

Here, $f(-3)=(-3)^2-3-12=0$

And $f(4)=4^2-4-12=0$

i.e. $f(-3)=f(4)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c\epsilon(-3,4)$ such that $f'(c)=0$

i.e. $2c-1=0$

i.e. $c = \frac{1}{2}$

Value of $c = \frac{1}{2} \epsilon(-3,4)$

Thus, Rolle’s theorem is satisfied.

Question 3.
Verify Rolle’s theorem for each of the following functions:

$$f(x) = \cos x \text{ in } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Answer:
Condition (1):

Since, f(x)=cosx is a trigonometric function and we know every trigonometric function is continuous.

⇒ f(x)=cosx is continuous on $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Condition (2):

Here, f′(x)=-sinx which exist in $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

So, f(x)=cosx is differentiable on $(-\frac{\pi}{2}, \frac{\pi}{2})$.

Condition (3):

Here, $f\left(-\frac{\pi}{2}\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

And $f\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

i.e. $f\left(-\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right)$

Conditions of Rolle’s theorem are satisfied.

Hence, there exist at least one $c \epsilon (-\frac{\pi}{2}, \frac{\pi}{2})$ such that f′(c)=0

i.e. -sinc=0

i.e. c=0

Value of $c = 0 \epsilon (-\frac{\pi}{2}, \frac{\pi}{2})$

Thus, Rolle's theorem is satisfied.

Question 4.
Verify Rolle's theorem for each of the following functions:

$$f(x) = x^2 - 5x + 6 \text{ in } [2,3]$$

Answer:
Condition (1):

Since, $f(x)=x^2-5x+6$ is a polynomial and we know every polynomial function is continuous for all xϵR.

⇒ $f(x)= x^2-5x+6$ is continuous on [2,3].

Condition (2):

Here, f′(x)=2x-5 which exist in [2,3].

So, $f(x)= x^2-5x+6$ is differentiable on (2,3).

Condition (3):

Here, $f(2)=2^2-5\times2+6=0$

And $f(3)= 3^2-5\times3+6=0$

i.e. f(2)=f(3)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cϵ(2,3) such that f′(c)=0

i.e. 2c-5=0

i.e. $c = \frac{5}{2}$

Value of $c = \frac{5}{2} \epsilon (2,3)$

Thus, Rolle's theorem is satisfied.

Question 5.
Verify Rolle's theorem for each of the following functions:

$$f(x) = x^2 - 5x + 6 \text{ in } [-3, 6]$$

Answer:
Condition (1):

Since, $f(x)= x^2-5x+6$ is a polynomial and we know every polynomial function is continuous for all $x \epsilon R$.

⇒ $f(x)= x^2-5x+6$ is continuous on $[-3,6]$.

Condition (2):

Here, $f'(x)=2x-5$ which exist in $[-3,6]$.

So, $f(x)= x^2-5x+6$ is differentiable on $(-3,6)$.

Condition (3):

Here, $f(-3)=(-3)^2-5\times(-3)+6=30$

And $f(6)= 6^2-5\times6+6=12$

i.e. $f(-3)\neq f(6)$

Conditions (3) of Rolle's theorem is not satisfied.

So, Rolle's theorem is not applicable.

Question 6.
Verify Rolle's theorem for each of the following functions:

$$f(x)=x^2-4x+3 \text{ in } [1,3]$$

Answer:
Condition (1):

Since, $f(x)=x^2-4x+3$ is a polynomial and we know every polynomial function is continuous for all $x \in R$.

⇒ $f(x)=x^2-4x+3$ is continuous on [1,3].

Condition (2):

Here, $f'(x)=2x-4$ which exist in [1,3].

So, $f(x)=x^2-4x+3$ is differentiable on (1,3).

Condition (3):

Here, $f(1)=(1)^2-4(1)+3=0$

And $f(3)= (3)^2-4(3)+3=0$

i.e. $f(1)=f(3)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c \in (1,3)$ such that $f'(c)=0$

i.e. $2c-4=0$

i.e. $c=2$

Value of $c=2 \in (1,3)$

Thus, Rolle's theorem is satisfied.

Question 7.
Verify Rolle's theorem for each of the following functions:

$$f(x) = x(x-4)^2 \text{ in } [0,4]$$

Answer:
Condition (1):

Since, $f(x)=x(x-4)^2$ is a polynomial and we know every polynomial function is continuous for all $x \epsilon R$.

⇒ $f(x)= x(x-4)^2$ is continuous on [0,4].

Condition (2):

Here, $f'(x)= (x-4)^2+2x(x-4)$ which exist in [0,4].

So, $f(x)= x(x-4)^2$ is differentiable on (0,4).

Condition (3):

Here, $f(0)=0(0-4)^2=0$

And $f(4)= 4(4-4)^2=0$

i.e. f(0)=f(4)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c \epsilon (0,4)$ such that $f'(c)=0$

i.e. $(c-4)^2+2c(c-4)=0$

i.e. $(c-4)(3c-4)=0$

i.e. $c=4$ or $c=3 \div 4$

Value of $c = \frac{3}{4} \epsilon (0,4)$

Thus, Rolle's theorem is satisfied.

Question 8.
Verify Rolle's theorem for each of the following functions:

$$f(x) = x^3 - 7x^2 + 16x - 12 \text{ in } [2,3]$$

Answer:
Condition (1):

Since, $f(x)=x^3- 7x^2+16x-12$ is a polynomial and we know every polynomial function is continuous for all xϵR.

⇒ $f(x)= x^3- 7x^2+16x-12$ is continuous on [2,3].

Condition (2):

Here, $f'(x)=3x^2-14x+16$ which exist in [2,3].

So, $f(x)= x^3- 7x^2+16x-12$ is differentiable on (2,3).

Condition (3):

Here, $f(2)= 2^3- 7(2)^2+16(2)-12=0$

And $f(3)= 3^3- 7(3)^2+16(3)-12=0$

i.e. f(2)=f(3)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cϵ(2,3) such that f′(c)=0

i.e. $3c^2-14c+16=0$

i.e. (c-2)(3c-7)=0

i.e. c=2 or c=7÷3

Value of $c = \frac{7}{3} \epsilon (2,3)$

Thus, Rolle's theorem is satisfied.

Question 9.
Verify Rolle's theorem for each of the following functions:

$$f(x) = x^3 + 3x^2 - 24x - 80 \text{ in } [-4, 5]$$

Answer:
Condition (1):

Since, $f(x)= x^3+3x^2-24x-80$ is a polynomial and we know every polynomial function is continuous for all $x \epsilon R$.

⇒ $f(x)= x^3+3x^2-24x-80$ is continuous on [-4,5].

Condition (2):

Here, $f'(x)= 3x^2+6x-24$ which exist in [-4,5].

So, $f(x)= x^3+3x^2-24x-80$ is differentiable on (-4,5).

Condition (3):

Here, $f(-4)= (-4)^3+3(-4)^2-24(-4)-80=0$

And $f(5)= (5)^3+3(5)^2-24(5)-80=0$

i.e. $f(-4)=f(5)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c \epsilon (-4,5)$ such that $f'(c)=0$

i.e. $3c^2+6c-24=0$

i.e. $c=-4$ or $c=2$

Value of c=2 ϵ(-4,5)

Thus, Rolle's theorem is satisfied.

Question 10.
Verify Rolle's theorem for each of the following functions:

$$f(x)=(x-1)(x-2)(x-3) \text{ in } [1,3]$$

Answer:
Condition (1):

Since, f(x)=(x-1)(x-2)(x-3) is a polynomial and we know every polynomial function is continuous for all xϵR.

⇒ f(x)= (x-1)(x-2)(x-3) is continuous on [1,3].

Condition (2):

Here, f'(x)= (x-2)(x-3)+ (x-1)(x-3)+ (x-1)(x-2) which exist in [1,3].

So, f(x)= (x-1)(x-2)(x-3) is differentiable on (1,3).

Condition (3):

Here, f(1)= (1-1)(1-2)(1-3) =0

And f(3)= (3-1)(3-2)(3-3) =0

i.e. f(1)=f(3)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cϵ(1,3) such that f'(c)=0

i.e. (c-2)(c-3)+ (c-1)(c-3)+ (c-1)(c-2)=0

i.e. (c-3)(2c-3)+(c-1)(c-2)=0

i.e. $(2c^2-9c+9)+(c^2-3c+2)=0$

i.e. $3c^2-12c+11=0$

i.e. $c=\frac{12\pm\sqrt{12}}{6}$

i.e. c=2.58 or c=1.42

Value of $c=1.42\epsilon(1,3)$ and $c=2.58\epsilon(1,3)$

Thus, Rolle's theorem is satisfied.

Question 11.
Verify Rolle's theorem for each of the following functions:

$$f(x)=(x-1)(x-2)^2 \text{ in } [1,2]$$

Answer:
Condition (1):

Since, $f(x)=(x-1)(x-2)^2$ is a polynomial and we know every polynomial function is continuous for all $x\epsilon R$.

⇒ $f(x)= (x-1)(x-2)^2$ is continuous on [1,2].

Condition (2):

Here, $f'(x)= (x-2)^2+2(x-1)(x-2)$ which exist in [1,2].

So, $f(x)= (x-1)(x-2)^2$ is differentiable on (1,2).

Condition (3):

Here, $f(1)= (1-1)(1-2)^2=0$

And $f(2)= (2-1)(2-2)^2=0$

i.e. $f(1)=f(2)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cϵ(1,2) such that f′(c)=0

i.e. $(c-2)^2+2(c-1)(c-2)=0$

(3c-4)(c-2)=0

i.e. c=2 or c=4÷3

Value of $c = \frac{4}{3} = 1.33\epsilon(1,2)$

Thus, Rolle's theorem is satisfied.

Question 12.
Verify Rolle's theorem for each of the following functions:

$$f(x) = (x-2)^4 (x-3)^3 \text{ in } [2,3]$$

Answer:
Condition (1):

Since, $f(x)=(x-2)^4(x-3)^3$ is a polynomial and we know every polynomial function is continuous for all xϵR.

⇒ $f(x)= (x-2)^4(x-3)^3$ is continuous on [2,3].

Condition (2):

Here, $f'(x)= 4(x-2)^3(x-3)^3+3(x-2)^4(x-3)^2$ which exist in [2,3].

So, $f(x)= (x-2)^4(x-3)^3$ is differentiable on (2,3).

Condition (3):

Here, $f(2)= (2-2)^4(2-3)^3=0$

And $f(3)= (3-2)^4(3-3)^3=0$

i.e. f(2)=f(3)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cϵ(2,3) such that f′(c)=0

i.e. $4(c-2)^3(c-3)^3+3(c-2)^4(c-3)^2=0$

$(c-2)^3(c-3)^2(7c-18)=0$

i.e. c=2 or c=3 or c=18÷7

Value of $c = \frac{18}{7} = 2.57 \epsilon (2,3)$

Thus, Rolle's theorem is satisfied.

Question 13.
Verify Rolle's theorem for each of the following functions:

$$f(x) = \sqrt{1-x^2} \text{ in } [-1,1]$$

Answer:
Condition (1):

Since, $f(x) = \sqrt{1-x^2}$ is a polynomial and we know every polynomial function is continuous for all xϵR.

⇒ $f(x) = \sqrt{1-x^2}$ is continuous on [-1,1].

Condition (2):

Here, $f'(x) = -\frac{x}{\sqrt{1-x^2}}$ which exist in [-1,1].

So, $f(x) = \sqrt{1-x^2}$ is differentiable on (-1,1).

Condition (3):

Here, $f(-1) = \sqrt{1-(-1)^2} = 0$

And $f(1) = \sqrt{1-1^2} = 0$

i.e. $f(-1) = f(1)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cϵ(-1,1) such that f′(c)=0

i.e. $-\frac{c}{\sqrt{1-c^2}} = 0$

i.e. c=0

Value of c=0ϵ(-1,1)

Thus, Rolle's theorem is satisfied.

Question 14.
Verify Rolle's theorem for each of the following functions:

$f(x) = \cos 2x \text{ in } [0, \pi]$

Answer:
Condition (1):

Since, f(x)=cos2x is a trigonometric function and we know every trigonometric function is continuous.

⇒ f(x)= cos2x is continuous on [0,π].

Condition (2):

Here, f′(x)= -2sin2x which exist in [0,π].

So, f(x)=cos2x is differentiable on (0,π).

Condition (3):

Here, f(0)=cos0=1

And f(π)=cos2π=1

i.e. f(0)=f(π)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cϵ(0,π) such that f′(c)=0

i.e. -2sin2c =0

i.e. 2c=π

i.e. $c = \frac{\pi}{2}$

Value of $c = \frac{\pi}{2} \epsilon (0, \pi)$

Thus, Rolle's theorem is satisfied.

Question 15.
Verify Rolle's theorem for each of the following functions:

$$f(x) = \sin 3x \text{ in } [0, \pi]$$

Answer:
Condition (1):

Since, f(x)=sin3x is a trigonometric function and we know every trigonometric function is continuous.

⇒ f(x)= sin3x is continuous on [0,π].

Condition (2):

Here, f′(x)= 3cos3x which exist in [0,π].

So, f(x)= sin3x is differentiable on (0,π).

Condition (3):

Here, f(0)=sin0=0

And f(π)=sin3π=0

i.e. f(0)=f(π)

Conditions of Rolle’s theorem are satisfied.

Hence, there exist at least one cϵ(0,π) such that f′(c)=0

i.e. 3cos3c =0

i.e. $3c = \frac{\pi}{2}$

i.e. $c = \frac{\pi}{6}$

Value of $c = \frac{\pi}{6} \epsilon (0, \pi)$

Thus, Rolle’s theorem is satisfied.

Question 16.
Verify Rolle’s theorem for each of the following functions:

$$f(x) = \sin x + \cos x \text{ in } \left[0, \frac{\pi}{2}\right]$$

Answer:
Condition (1):

Since, f(x)=sinx+cosx is a trigonometric function and we know every trigonometric function is continuous.

⇒ f(x)= sinx+cosx is continuous on $[0, \frac{\pi}{2}]$.

Condition (2):

Here, f′(x)= cosx-sinx which exist in $[0,\frac{\pi}{2}]$.

So, f(x)= sinx+cosx is differentiable on $(0,\frac{\pi}{2})$

Condition (3):

Here, f(0)=sin0+cos0=1

And $f(\frac{\pi}{2}) = \sin\left(\frac{\pi}{2}\right) + \cos(\frac{\pi}{2}) \ = 1$

i.e. $f(0) = f(\frac{\pi}{2})$

Conditions of Rolle’s theorem are satisfied.

Hence, there exist at least one $c\epsilon(0,\frac{\pi}{2})$ such that f′(c)=0

i.e. cosc-sinc =0

i.e. $c = \frac{\pi}{4}$

Value of $c = \frac{\pi}{4}\epsilon(0,\frac{\pi}{2})$

Thus, Rolle’s theorem is satisfied.

Question 17.
Verify Rolle’s theorem for each of the following functions:

$$f(x) = e^{-x} \sin x \text{ in } [0,\pi]$$

Answer:
Condition (1):

Since, f(x)=e^{-x} sinx is a combination of exponential and trigonometric function which is continuous.

⇒ f(x)= e^{-x} sinx is continuous on [0,π].

Condition (2):

Here, $f'(x) = e^{-x}(\cos x - \sin x)$ which exist in $[0,\pi]$.

So, $f(x) = e^{-x} \sin x$ is differentiable on $(0,\pi)$

Condition (3):

Here, $f(0) = e^{-0} \sin 0 = 0$

And $f(\pi) = e^{-\pi} \sin\pi = 0$

i.e. $f(0) = f(\pi)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c \epsilon (0,\pi)$ such that $f'(c) = 0$

i.e. $e^{-c}(\cos c - \sin c) = 0$

i.e. $\cos c - \sin c = 0$

i.e. $c = \frac{\pi}{4}$

Value of $c = \frac{\pi}{4} \epsilon (0, \pi)$

Thus, Rolle's theorem is satisfied.

Question 18.
Verify Rolle's theorem for each of the following functions:

$$f(x) = e^{-x}(\sin x - \cos x) \text{ in } \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$$

Answer:
Condition (1):

Since, $f(x)=e^{-x}$ (sinx-cosx) is a combination of exponential and trigonometric function which is continuous.

⇒ $f(x)= e^{-x}$ (sinx-cosx) is continuous on $[\frac{\pi}{4}, \frac{5\pi}{4}]$.

Condition (2):

Here, $f'(x)= e^{-x}$ (sinx + cosx) - e^{-x} (sinx – cosx)

$= e^{-x}$ cosx which exist in $[\frac{\pi}{4}, \frac{5\pi}{4}]$.

So, $f(x)= e^{-x}$ (sinx-cosx) is differentiable on $(\frac{\pi}{4}, \frac{5\pi}{4})$

Condition (3):

Here, $f(\frac{\pi}{4}) = e^{-\frac{\pi}{4}}(\sin\frac{\pi}{4} - \cos\frac{\pi}{4}) = 0$

And $f(\frac{5\pi}{4}) = e^{-\frac{5\pi}{4}}(\sin\frac{5\pi}{4} - \cos\frac{5\pi}{4}) = 0$

i.e. $f(\frac{\pi}{4}) = f(\frac{5\pi}{4})$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c\epsilon(\frac{\pi}{4}, \frac{5\pi}{4})$ such that f′(c)=0

i.e. e^{-c} cos c =0

i.e. cos c = 0

i.e. $c = \frac{\pi}{2}$

Value of $c = \frac{\pi}{2}\epsilon(\frac{\pi}{4}, \frac{5\pi}{4})$

Thus, Rolle's theorem is satisfied.

Question 19.
Verify Rolle's theorem for each of the following functions:

$$f(x) = \sin x - \sin 2x \text{ in } [0, 2\pi]$$

Answer:
Condition (1):

Since, f(x) = sinx-sin2x is a trigonometric function and we know every trigonometric function is continuous.

⇒ f(x) = sinx-sin2x is continuous on [0,2π].

Condition (2):

Here, f′(x)= cosx-2cos2x which exist in [0,2π].

So, f(x)= sinx-sin2x is differentiable on (0,2π)

Condition (3):

Here, f(0)= sin0-sin0 = 0

And f(2π)=sin(2π)-sin(4π) =0

i.e. f(0)=f(2π)

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one cϵ(0,2π) such that f′(c)=0

i.e. cosx-2cos2x =0

i.e. cosx-4cos^{2}x+2=0

i.e. 4cos^{2}x-cosx-2=0

i.e. $\cos x = \frac{1 \pm \sqrt{33}}{8}$

i.e. c=32° 32′ or c=126°23′

Value of $c = 32°32' \in (0, 2\pi)$

Thus, Rolle's theorem is satisfied.

Question 20.
Verify Rolle's theorem for each of the following functions:

$$f(x) = x(x+2)e^x \text{ in } [-2, 0]$$

Answer:
Condition (1):

Since, $f(x)=x(x+2)e^x$ is a combination of exponential and polynomial function which is continuous for all $x \in R$.

$\Rightarrow f(x)= x(x+2)e^x$ is continuous on $[-2,0]$.

Condition (2):

Here, $f'(x)=(x^2+4x+2)e^x$ which exist in $[-2,0]$.

So, $f(x)=x(x+2)e^x$ is differentiable on $(-2,0)$.

Condition (3):

Here, $f(-2)= (-2)(-2+2)e^{-2} =0$

And $f(0)= 0(0+2)e^0=0$

i.e. $f(-2)=f(0)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c \in (-2,0)$ such that $f'(c)=0$

i.e. $(c^2+4c+2)e^c =0$

i.e. $(c+\sqrt{2})^2=0$

i.e. $c=-\sqrt{2}$

Value of $c=-\sqrt{2} \in (-2,0)$

Thus, Rolle's theorem is satisfied.

Question 21.
Verify Rolle's theorem for each of the following functions:

Show that $f(x)=x(x-5)^2$ satisfies Rolle's theorem on [0, 5] and that the value of c is (5/3)

Answer:
Condition (1):

Since, $f(x)=x(x-5)^2$ is a polynomial and we know every polynomial function is continuous for all $x \in R$.

$\Rightarrow f(x)= x(x-5)^2$ is continuous on [0,5].

Condition (2):

Here, $f'(x)= (x-5)^2+ 2x(x-5)$ which exist in [0,5].

So, $f(x)= x(x-5)^2$ is differentiable on (0,5).

Condition (3):

Here, $f(0)= 0(0-5)^2=0$

And $f(5)= 5(5-5)^2=0$

i.e. $f(0)=f(5)$

Conditions of Rolle's theorem are satisfied.

Hence, there exist at least one $c \in (0,5)$ such that $f'(c)=0$

i.e. $(c-5)^2+ 2c(c-5)=0$

i.e.(c-5)(3c-5)=0

i.e. $c = \frac{5}{3}$ or c=5

Value of $c = \frac{5}{3} \epsilon (0,5)$

Thus, Rolle’s theorem is satisfied.

Question 22.
Discuss the applicability for Rolle’s theorem, when:

$f(x) = (x-1)(2x-3)$, where $1 \le x \le 3$

Answer:
Condition (1):

Since, f(x)=(x-1)(2x-3) is a polynomial and we know every polynomial function is continuous for all xєR.

⇒ f(x)= (x-1)(2x-3) is continuous on [1,3].

Condition (2):

Here, f′(x)= (2x-3)+ 2(x-1) which exist in [1,3].

So, f(x)= (x-1)(2x-3) is differentiable on (1,3).

Condition (3):

Here, f(1)= (1-1)(2(1)-3)=0

And f(5)= (3-1)(2(3)-3)=6

i.e. f(1)≠f(3)

Condition (3) of Rolle’s theorem is not satisfied.

So, Rolle’s theorem is not applicable.

Question 23.
Discuss the applicability for Rolle's theorem, when:

$$f(x) = x^{1/2} \text{ on } [-1,1]$$

Answer:
Condition (1):

Since, $f(x)=x^{1/2}$ is a polynomial and we know every polynomial function is continuous for all xϵR.

⇒ $f(x)= x^{1/2}$ is continuous on [-1,1].

Condition (2):

Here, $f'(x) = \frac{1}{2x^{\frac{1}{2}}}$ which does not exist at x=0 in [-1,1].

$f(x)=x^{1/2}$ is not differentiable on (-1,1).

Condition (2) of Rolle's theorem is not satisfied.

So,Rolle's theorem is not applicable.

Question 24.
Discuss the applicability for Rolle's theorem, when:

$$f(x) = 2 + (x-1)^{2/3} \text{ on } [0,2]$$

Answer:
Condition (1):

Since, $f(x)=2+(x-1)^{2/3}$ is a polynomial and we know every polynomial function is continuous for all xϵR.

⇒ $f(x)= 2+(x-1)^{2/3}$ is continuous on [0,2].

Condition (2):

Here, $f'(x) = \frac{2}{3(x-1)^{\frac{1}{3}}}$ which does not exist at x=1 in [0,2].

$f(x)= 2+(x-1)^{2/3}$ is not differentiable on (0,2).

Condition (2) of Rolle's theorem is not satisfied.

So,Rolle's theorem is not applicable.

Question 25.
Discuss the applicability for Rolle's theorem, when:

$$f(x) = \cos\frac{1}{x} \text{ on } [-1,1]$$

Answer:
Condition (1):

Since, $f(x) = \cos\frac{1}{x}$ which is discontinuous at x=0

$\Rightarrow$ $f(x) = \cos\frac{1}{x}$ is not continuous on [-1,1].

Condition (1) of Rolle's theorem is not satisfied.

So,Rolle's theorem is not applicable.

Question 26.
Discuss the applicability for Rolle's theorem, when:

$f(x) = [x] \text{ on } [-1,1]$, where [x] denotes the greatest integer not exceeding x

Answer:
Condition (1):

Since, f(x)=[x] which is discontinuous at x=0

⇒ f(x)=[x] is not continuous on [-1,1].

Condition (1) of Rolle’s theorem is not satisfied.

So,Rolle’s theorem is not applicable.

Question 27.
Using Rolle’s theorem, find the point on the curve $y = x(x-4), x \in [0,4]$, where the tangent is parallel to the x-axis.

Answer:
Condition (1):

Since, y=x(x-4) is a polynomial and we know every polynomial function is continuous for all xϵR.

⇒ y= x(x-4) is continuous on [0,4].

Condition (2):

Here, y’= (x-4)+x which exist in [0,4].

So, y= x(x-4) is differentiable on (0,4).

Condition (3):

Here, y(0)=0(0-4)=0

And y(4)= 4(4-4)=0

i.e. y(0)=y(4)

Conditions of Rolle’s theorem are satisfied.

Hence, there exist at least one cϵ(0,4) such that y’(c)=0

i.e. (c-4)+c=0

i.e. 2c-4=0

i.e. c=2

Value of $c=2\epsilon(0,4)$

So, $y(c)=y(2)=2(2-4)=-4$

By geometric interpretation, $(2,-4)$ is a point on a curve $y=x(x-4)$, where tangent is parallel to x-axis.