Exercise 28j

Question 1.
Find the direction ratios of the normal to the plane x + 2y - 3z = 5.

Answer:
Given :

Equation of plane : x + 2y – 3z = 5

To Find : direction ratios of normal

Answer :

Given equation of plane : x + 2y – 3z = 5

It can be written as

$$\left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(\hat{\imath} + 2\hat{\jmath} - 3\hat{k}\right) = 5$$

Comparing with $\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Therefore, normal vector is $\bar{n} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

Hence, direction ratios of normal are (1, 2, -3).

Question 2.
Find the direction cosines of the normal to the plane 2x + 3y - z = 4.

Answer:
Given :

Equation of plane : 2x + 3y – z = 4

To Find : Direction cosines of the normal i.e. $l, m \,\&\, n$

Formula :

1) Direction cosines :

If a, b & c are direction ratios of the vector then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Answer :

For the given equation of plane

2x + 3y – z = 4

Direction ratios of normal vector are (2, 3, -1)

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{2^2 + 3^2 + (-1)^2}$$

$$= \sqrt{4 + 9 + 1}$$

$$= \sqrt{14}$$

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{2}{\sqrt{14}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{3}{\sqrt{14}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{-1}{\sqrt{14}}$$

$$(l, m, n) = \left(\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}\right)$$

Question 3.
Find the direction cosines of the normal to the plane y = 3.

Answer:
Given :

Equation of plane : y = 3

To Find : Direction cosines of the normal i.e. $l, m \,\&\, n$

Formula :

1) Direction cosines :

If a, b & c are direction ratios of the vector then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Answer :

For the given equation of plane

y = 3

Direction ratios of normal vector are (0, 1, 0)

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{0^2 + 1^2 + 0^2}$$

$$= \sqrt{0 + 1 + 0}$$

$$= \sqrt{1}$$

= 1

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{0}{1} = 0$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{1}{1} = 1$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{0}{1} = 0$$

$(l, m, n) = (0,1,0)$

Question 4.
Find the direction cosines of the normal to the plane 3x + 4 = 0.

Answer:
Given :

Equation of plane : 3x + 4 = 0

To Find : Direction cosines of the normal i.e. $l, m \,\&\, n$

Formula :

1) Direction cosines :

If a, b & c are direction ratios of the vector then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Answer :

For the given equation of plane

-3x = 4

Direction ratios of normal vector are (-3, 0, 0)

$$\sqrt{a^2+b^2+c^2} = \sqrt{(-3)^2+0^2+0^2}$$

$$= \sqrt{9+0+0}$$

$$= \sqrt{9}$$

= 3

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{-3}{3} = -1$$

$$m = \frac{b}{\sqrt{a^2+b^2+c^2}} = \frac{0}{3} = 0$$

$$n = \frac{c}{\sqrt{a^2+b^2+c^2}} = \frac{0}{3} = 0$$

$(l, m, n) = (-1,0,0)$

Question 5.
Write the equation of the plane parallel to XY-plane and passing through the point (4, -2, 3).

Answer:
Given :

Point : (4, -2, 3)

To Find : equation of plane

Formula :

1) Equation of plane :

Equation of plane passing through point A with position vector $\bar{a}$ and perpendicular to vector $\bar{n}$ is given by,

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Answer :

Position vector for given point A ≡ (4, -2, 3) is

$\bar{a} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

As required plane is parallel to XY plane, therefore Z-axis is perpendicular to the plane.

$\therefore \bar{n} = \hat{k}$

Therefore, equation of plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(\hat{k}\right) = \left(4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}\right).\left(\hat{k}\right)$

$\therefore (x \times 0) + (y \times 0) + (z \times 1) = (4 \times 0) + (-2 \times 0) + (3 \times 1)$

$\therefore z = 3$

This is required equation of plane.

Question 6.
Write the equation of the plane parallel to YZ-plane and passing through the point (-3, 2, 0).

Answer:

Given :

Point : (-3, 2, 0)

To Find : equation of plane

Formula :

1) Equation of plane :

Equation of plane passing through point A with position vector $\bar{a}$ and perpendicular to vector $\bar{n}$ is given by,

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Answer :

Position vector for given point A ≡ (-3, 2, 0) is

$\bar{a} = -3\hat{\imath} + 2\hat{\jmath} + 0\hat{k}$

As required plane is parallel to YZ plane, therefore X-axis is perpendicular to the plane.

$\therefore \bar{n} = \hat{\imath}$

Therefore, equation of plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).(\hat{\imath}) = \left(-3\hat{\imath} + 2\hat{\jmath} + 0\hat{k}\right).(\hat{\imath})$

$\therefore (x \times 1) + (y \times 0) + (z \times 0) = (-3 \times 1) + (2 \times 0) + (0 \times 0)$

$\therefore x = -3$

This is required equation of plane.

Question 7.
Write the general equation of a plane parallel to the x-axis.

Answer:
Let, normal vector of plane be

$$\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$$

Equation of plane is given by,

$$\bar{r}.\bar{n} = d$$

$$\therefore \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(a\hat{\imath} + b\hat{\jmath} + c\hat{k}\right) = d$$

$$\therefore ax + by + cz = d$$

As the required plane is parallel to the given plane, hence normal vector of plane is perpendicular to x-axis.

$$\therefore \bar{n}.\hat{\imath} = 0$$

$$\therefore \left(a\hat{\imath} + b\hat{\jmath} + c\hat{k}\right).\hat{\imath} = 0$$

a = 0

Therefore, equation of plane is

by + cz = d

Question 8.
Write the intercept cut off by the plane 2x + y - z = 5 on the x-axis.

Answer:
Given :

Equation of plane : 2x + y – z = 5

To Find : Intercept made by the plane with the X-axis.

Formula :

If $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

is the equation of plane in intercept form then intercept made by it with co-ordinate axes are

X-intercept = a

Y-intercept = b

Z-intercept = c

Answer :

Given equation of plane:

2x + y − z = 5

Dividing above equation throughout by 5

$$\therefore \frac{2x}{5}+\frac{y}{5}+\frac{-z}{5}=1$$

$$\therefore \frac{x}{5/2}+\frac{y}{5}+\frac{z}{-5}=1$$

Comparing above equation with

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

We get,

a = 5/2

Therefore, intercepts made by plane with X-axis are

X-intercept = 5/2

Question 9.
Write the intercepts made by the plane
4x - 3y + 2z = 12 on the coordinate axes.

Answer:
Given :

Equation of plane : 4x – 3y + 2z = 12

To Find :

1) Equation of plane in intercept form

2) Intercepts made by the plane with the co-ordinate axes.

Formula :

If $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

is the equation of plane in intercept form then intercept made by it with co-ordinate axes are

X-intercept = a

Y-intercept = b

Z-intercept = c

Answer :

Given equation of plane:

4x – 3y + 2z = 12

Dividing above equation throughout by 12

$\therefore \frac{4x}{12}+\frac{-3y}{12}+\frac{2z}{12}=1$

$$\therefore \frac{x}{3}+\frac{y}{-4}+\frac{z}{6}=1$$

This is the equation of plane in intercept form.

Comparing above equation with

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

We get,

a = 3

b = -4

c = 6

Therefore, intercepts made by plane with co-ordinate axes are

X-intercept = 3

Y-intercept = -4

Z-intercept = 6

Question 10.
Reduce the equation 2x – 3y + 5z + 4 = 0 to intercept form and find the intercepts made by it on the coordinate axes.

Answer:
Given :

Equation of plane : 2x – 3y + 5z + 4 = 0

To Find :

1) Equation of plane in intercept form

2) Intercepts made by the plane with the co-ordinate axes.

Formula :

If $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

is the equation of plane in intercept form then intercept made by it with co-ordinate axes are

X-intercept = a

Y-intercept = b

Z-intercept = c

Answer :

Given equation of plane:

2x – 3y + 5z = -4

Dividing above equation throughout by -4

$\therefore \frac{2x}{-4} + \frac{-3y}{-4} + \frac{5z}{-4} = 1$

$\therefore \frac{x}{-2} + \frac{y}{4/3} + \frac{z}{-4/5} = 1$

This is the equation of plane in intercept form.

Comparing above equation with

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

We get,

a = -2

$b = \frac{4}{3}$

$$c = \frac{-4}{5}$$

Therefore, intercepts made by plane with co-ordinate axes are

X-intercept = -2

$$Y - intercept = \frac{4}{3}$$

$$Z - intercept = -\frac{4}{5}$$

Question 11.
Find the equation of a plane passing through the points A(a, 0, 0), B(0, b, 0) and C(0, 0, c).

Answer:
Given : Plane is passing through points

A ≡ (a, 0, 0)

B ≡ (0, b, 0)

C ≡ (0, 0, c)

To Find : Equation of plane

Formulae :

Equation of plane making intercepts (a, b, c) on X, Y & Z axes respectively is given by,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Answer : As plane is passing through points A ≡ (a, 0, 0),

B ≡ (0, b, 0) & C ≡ (0, 0, c)

Therefore, intercepts made by it on X, Y & Z axes respectively are a, b & c.

hence, equation of plane is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Question 12.
Write the value of k for which the planes 2x – 5y + kz = 4 and x + 2y – z = 6 are perpendicular to each other.

Answer:
Given : equations of perpendicular planes-

2x – 5y + kz = 4

x + 2y – z = 6

To Find : k

Formulae :

Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$$\bar{n}=a\hat{\imath}+b\hat{\jmath}+c\hat{k}$$

Answer :

For given planes –

2x – 5y + kz = 4

x + 2y – z = 6

normal vectors are

$\overline{n_1} = 2\hat{\imath} - 5\hat{\jmath} + k\hat{k}$

$\overline{n_2} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

As given vectors are perpendicular, hence their normal vectors are also perpendicular to each other.

$\therefore \overline{n_1}.\overline{n_2} = 0$

$\therefore \left(2\hat{\imath} - 5\hat{\jmath} + k\hat{k}\right).\left(\hat{\imath} + 2\hat{\jmath} - \hat{k}\right) = 0$

(2×1) + (-5×2) + (k×(-1)) = 0

2 – 10 – k = 0

- 8 – k = 0

k = -8

Question 13.
Find the angle between the planes 2x + y – 2z = 5 and 3x – 6y – 2z = 7.

Answer:
Given : equations of planes-

2x + y – 2z = 5

3x – 6y – 2z = 7

To Find : angle between two planes

Formulae :

1) Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

2) Angle between two planes :

The angle Θ between the planes $\bar{r}.\overline{n_1} = p_1$ and $\bar{r}.\overline{n_2} = p_2$ is given by

$$\cos\theta = \frac{\overline{n_1}\,.\overline{n_2}}{|\overline{n_1}|\,.|\overline{n_2}|}$$

Answer :

For given planes

2x + y – 2z = 5

3x – 6y – 2z = 7

Normal vectors are

$\overline{n_1} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and

$\overline{n_2} = 3\hat{\imath} - 6\hat{\jmath} - 2\hat{k}$

$\therefore |\overline{n_1}| = \sqrt{2^2 + 1^2 + (-2)^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$

$\therefore |\overline{n_2}| = \sqrt{3^2 + (-6)^2 + (-2)^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$

Therefore, angle between two planes is

$$\cos\theta = \frac{\overline{n_1}\,.\overline{n_2}}{|\overline{n_1}|\,.|\overline{n_2}|}$$

$$\therefore \cos\theta = \frac{\left(2\hat{\imath} + \hat{\jmath} - 2\hat{k}\right).\left(3\hat{\imath} - 6\hat{\jmath} - 2\hat{k}\right)}{3 \times 7}$$

$$\therefore \cos\theta = \frac{(2 \times 3) + (1 \times (-6)) + ((-2) \times (-2))}{21}$$

$$\therefore \cos\theta = \frac{6-6+4}{21}$$

$$\therefore \cos\theta = \frac{4}{21}$$

$$\therefore \theta = cos^{-1}\left(\frac{4}{21}\right)$$

Question 14.

Find the angle between the planes $\bar{r}\cdot(\hat{i}+\hat{j})=1$ and $\bar{r}\cdot(\hat{i}+\hat{k})=3$.

Answer:

Given : equations of planes-

$$\bar{r}.(\hat{\imath}+\hat{\jmath}) = 1$$

$$\bar{r}.(\hat{\jmath}+\hat{k}) = 3$$

To Find : angle between two planes

Formulae :

Angle between two planes :

The angle Ѳ between the planes $\bar{r}.\overline{n_1} = p_1$ and $\bar{r}.\overline{n_2} = p_2$ is given by

$$\cos\theta = \frac{\overline{n_1}\,.\overline{n_2}}{|\overline{n_1}|\,.|\overline{n_2}|}$$

Answer :

For given planes

$$\bar{r}.(\hat{\imath}+\hat{\jmath}) = 1$$

$$\bar{r}.(\hat{\jmath}+\hat{k}) = 3$$

Normal vectors are

$\overline{n_1} = \hat{\imath} + \hat{\jmath}$ and

$\overline{n_2} = \hat{\jmath} + \hat{k}$

$\therefore |\overline{n_1}| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{1 + 1 + 0} = \sqrt{2}$

$\therefore |\overline{n_2}| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{0 + 1 + 1} = \sqrt{2}$

Therefore, angle between two planes is

$$\cos\theta = \frac{\overline{n_1} . \overline{n_2}}{|\overline{n_1}| . |\overline{n_2}|}$$

$$\therefore \cos\theta = \frac{(\hat{\imath} + \hat{\jmath}) . (\hat{\jmath} + \hat{k})}{\sqrt{2} \times \sqrt{2}}$$

$$\therefore \cos\theta = \frac{(1 \times 0) + (1 \times 1) + (0 \times 1)}{2}$$

$$\therefore \cos\theta = \frac{0 + 1 + 0}{2}$$

$$\therefore \cos\theta = \frac{1}{2}$$

$$\therefore \theta = cos^{-1}\left(\frac{1}{2}\right)$$

$$\therefore \theta = \frac{\pi}{3}$$

Question 15.
Find the angle between the planes $\bar{r} \cdot (3\hat{i} - 4\hat{j} + 5\hat{k}) = 0$ and $\bar{r} \cdot (2\hat{i} - \hat{j} - 2\hat{k}) = 7$.

Answer:
Given : equations of planes-

$$\bar{r}.(3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}) = 0$$

$\bar{r}.\left(2\hat{\imath} - \hat{\jmath} - 2\hat{k}\right) = 7$

To Find : angle between two planes

Formulae :

Angle between two planes :

The angle Θ between the planes $\bar{r}.\overline{n_1} = p_1$ and $\bar{r}.\overline{n_2} = p_2$ is given by

$$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$$

Answer :

For given planes

$\bar{r}.\left(3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}\right) = 0$

$\bar{r}.\left(2\hat{\imath} - \hat{\jmath} - 2\hat{k}\right) = 7$

Normal vectors are

$\overline{n_1} = 3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and

$\overline{n_2} = 2\hat{\imath} - \hat{\jmath} - 2\hat{k}$

$\therefore |\overline{n_1}| = \sqrt{3^2 + (-4)^2 + 5^2} = \sqrt{9 + 16 + 25} = \sqrt{50} = 5\sqrt{2}$

$\therefore |\overline{n_2}| = \sqrt{2^2 + (-1)^2 + (-2)^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$

Therefore, angle between two planes is

$$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$$

$$\therefore \cos\theta = \frac{(3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}).(2\hat{\imath} - \hat{\jmath} - 2\hat{k})}{5\sqrt{2} \times 3}$$

$$\therefore \cos\theta = \frac{(3 \times 2) + ((-4) \times (-1)) + (5 \times (-2))}{15\sqrt{2}}$$

$$\therefore \cos\theta = \frac{6 + 4 - 10}{2}$$

$$\therefore \cos\theta = 0$$

$$\therefore \theta = cos^{-1}(0)$$

$$\therefore \theta = \frac{\pi}{2}$$

Question 16.
Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the planes 10x + 2y – 11z = 3.

Answer:
Given :

Equation of line : $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$

Equation of plane : 10x + 2y – 11z = 3

To Find : angle between line and plane

Formulae :

1) Parallel vector to the line :

If equation of the line is $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ then,

Vector parallel to the line is given by,

$$\bar{b} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}$$

2) Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

3) Angle between a line and a plane :

If Θ is a angle between the line $\bar{r} = \bar{a} + \lambda\bar{b}$ and the plane $\bar{r}.\bar{n} = p$, then

$$\sin\theta = \frac{\bar{b}\,.\,\bar{n}}{|\bar{b}|\,.\,|\bar{n}|}$$

Where, $\bar{b}$ is vector parallel to the line and

$\bar{n}$ is the vector normal to the plane.

Answer :

For given equation of line,

$$\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$$

Parallel vector to the line is

$\bar{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$

$\therefore |\bar{b}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$

For given equation of plane,

10x + 2y – 11z = 3

normal vector to the plane is

$\bar{n} = 10\hat{\imath} + 2\hat{\jmath} - 11\hat{k}$

$\therefore |\bar{n}| = \sqrt{10^2 + 2^2 + (-11)^2} = \sqrt{100 + 4 + 121} = \sqrt{225} = 15$

Therefore, angle between given line and plane is

$$\sin\theta = \frac{\bar{b}\,.\,\bar{n}}{|\bar{b}|\,.\,|\bar{n}|}$$

$$\therefore \sin\theta = \frac{\left(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}\right).\left(10\hat{\imath} + 2\hat{\jmath} - 11\hat{k}\right)}{7 \times 15}$$

$$\therefore \sin\theta = \frac{(2 \times 10) + (3 \times 2) + (6 \times (-11))}{105}$$

$$\therefore \sin\theta = \frac{20 + 6 - 66}{105}$$

$$\therefore \sin\theta = \frac{-40}{105}$$

$$\therefore \sin\theta = \frac{-8}{21}$$

$$\therefore \theta = sin^{-1}\left(\frac{-8}{21}\right)$$

Question 17.
Find the angle between the line $\bar{r} = \left(\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and the plane $\bar{r} \cdot \left(2\hat{i} - \hat{j} + \hat{k}\right) = 4$.

Answer:
Given :

Equation of line : $\bar{r} = \left(\hat{\imath} + \hat{\jmath} - 2\hat{k}\right) + \lambda\left(\hat{\imath} - \hat{\jmath} + \hat{k}\right)$

Equation of plane : $\bar{r}.\left(2\hat{\imath} - \hat{\jmath} + \hat{k}\right) = 4$

To Find : angle between line and plane

Formulae :

1) Angle between a line and a plane :

If Θ is a angle between the line $\bar{r} = \bar{a} + \lambda\bar{b}$ and the plane $\bar{r}.\bar{n} = p$, then

$$\sin\theta = \frac{\bar{b}\,.\,\bar{n}}{|\bar{b}|\,.\,|\bar{n}|}$$

Where, $\bar{b}$ is vector parallel to the line and

$\bar{n}$ is the vector normal to the plane.

Answer :

For given equation of line,

$$\bar{r} = \left(\hat{\imath} + \hat{\jmath} - 2\hat{k}\right) + \lambda\left(\hat{\imath} - \hat{\jmath} + \hat{k}\right)$$

Parallel vector to the line is

$$\bar{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$$

$$\therefore\ |\bar{b}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{1 + 1 + 1} = \sqrt{3}$$

For given equation of plane,

$$\bar{r}.\left(2\hat{\imath} - \hat{\jmath} + \hat{k}\right) = 4$$

normal vector to the plane is

$$\bar{n} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$$

$$\therefore\ |\bar{n}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4 + 1 + 1} = \sqrt{6}$$

Therefore, angle between given line and plane is

$$\sin\theta = \frac{\bar{b}\,.\,\bar{n}}{|\bar{b}|\,.\,|\bar{n}|}$$

$$\therefore \sin\theta = \frac{(\hat{\imath} - \hat{\jmath} + \hat{k})\,.\,(2\hat{\imath} - \hat{\jmath} + \hat{k})}{\sqrt{3} \times \sqrt{6}}$$

$$\therefore \sin\theta = \frac{(1 \times 2) + ((-1) \times (-1)) + (1 \times 1)}{\sqrt{18}}$$

$$\therefore \sin\theta = \frac{2 + 1 + 1}{3\sqrt{2}}$$

$$\therefore \sin\theta = \frac{4}{3\sqrt{2}}$$

$$\therefore \sin\theta = \frac{2 \times 2}{3\sqrt{2}}$$

$$\therefore \sin\theta = \frac{2\sqrt{2}}{3}$$

$$\therefore \theta = sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

Question 18.

Find the value of λ such that the line $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{4}$ is perpendicular to the plane 3x - y – 2z = 7.

Answer:

Given :

Equation of line : $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{4}$

Equation of plane : 3x – y – 2z = 7

To Find : λ

Formulae :

1) Parallel vector to the line :

If equation of the line is $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ then,

Vector parallel to the line is given by,

$$\bar{b} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}$$

2) Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$$\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Answer :

For given equation of line,

$$\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{4}$$

Parallel vector to the line is

$\bar{b} = 6\hat{\imath} + \lambda\hat{\jmath} + 4\hat{k}$

For given equation of plane,

3x – y – 2z = 7

normal vector to the plane is

$\bar{n} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$

As given line and plane are perpendicular to each other.

$\therefore \bar{b} \times \bar{n} = 0$

$$\therefore \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 6 & \lambda & 4 \\ 3 & -1 & -2 \end{vmatrix} = 0$$

$\therefore \hat{\imath}(-2\lambda + 4) - \hat{\jmath}(-12 - 12) + \hat{k}(-6 - 3\lambda) = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

Comparing coefficients of $\hat{k}$ on both sides

$\therefore -6 - 3\lambda = 0$

$3\lambda = -6$

$\lambda = -2$

Question 19.
Write the equation of the plane passing through the point (a, b, c) and parallel to the plane $\bar{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 2.$

Answer:
Given :

A ≡ (a, b, c)

Equation of plane parallel to required plane

$\therefore \bar{r}.\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right)=2$

To Find : Equation of plane

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

$\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

then,

$\bar{a}.\bar{b}=(a_1\times b_1)+(a_2\times b_2)+(a_3\times b_3)$

3) Equation of plane :

If a plane is passing through point A, then equation of plane is

$\bar{r}.\bar{n}=\bar{a}.\bar{n}$

Where, $\bar{a}=position\ vector\ of\ A$

$\bar{n}=vector\ perpendicular\ to\ the\ plane$

$\bar{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$

Answer :

For point A ≡ (a, b, c), position vector is

$$\bar{a} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$$

As plane $\bar{r}.\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right) = 2$ is parallel to the required plane, the vector normal to required plane is

$$\bar{n} = \hat{\imath} + \hat{\jmath} + \hat{k}$$

Now, $\bar{a}.\bar{n} = (a \times 1) + (b \times 1) + (c \times 1)$

= a + b + c

Equation of the plane passing through point A and perpendicular to vector $\bar{n}$ is

$$\bar{r}.\bar{n} = \bar{a}.\bar{n}$$

$$\therefore \bar{r}.\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right) = a + b + c$$

Question 20.
Find the length of perpendicular drawn from the origin to the plane 2x – 3y + 6z + 21 = 0.

Answer:
Given :

Equation of plane : 2x – 3y + 6z + 21 = 0

To Find :

Length of perpendicular drawn from origin to the plane = d

Formulae :

1) Distance of the plane from the origin :

Distance of the plane from the origin is given by,

$d = \frac{p}{|\bar{n}|}$

Answer :

For the given equation of plane

2x – 3y + 6z = -21

Direction ratios of normal vector are (2, -3, 6)

Therefore, equation of normal vector is

$\bar{n} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$

$\therefore |\bar{n}| = \sqrt{2^2 + (-3)^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

= 7

From given equation of plane,

p = -21

Now, distance of the plane from the origin is

$d = \frac{p}{|\bar{n}|}$

$\therefore d = \frac{-21}{7}$

d = 3 units

Question 21.
Find the direction cosines of the perpendicular from the origin to the plane $\bar{r} \cdot \left(6\hat{i} - 3\hat{j} - 2\hat{k}\right) + 1 = 0$.

Answer:
Given :

Equation of plane : $\bar{r}.\left(6\hat{\imath}-3\hat{\jmath}-2\hat{k}\right)+1=0$

To Find :

Direction cosines of the normal i.e. l, m & n

Formulae :

1) Direction cosines :

If a, b & c are direction ratios of the vector then its direction cosines are given by

$$l=\frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$m=\frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$n=\frac{c}{\sqrt{a^2+b^2+c^2}}$$

Answer :

For the given equation of plane

$$\bar{r}.\left(6\hat{\imath}-3\hat{\jmath}-2\hat{k}\right)+1=0$$

Equation of normal vector is

$$\bar{n}=6\hat{\imath}-3\hat{\jmath}-2\hat{k}$$

$$\sqrt{a^2+b^2+c^2}=\sqrt{6^2+(-3)^2+(-2)^2}$$

$$=\sqrt{36+9+4}$$

$= \sqrt{49}$

= 7

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{6}{7}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{-3}{7}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{-2}{7}$$

$$(l, m, n) = \left(\frac{6}{7}, \frac{-3}{7}, \frac{-2}{7}\right)$$

Question 22.
Show that the line $\bar{r} = \left(4\hat{i} - 7\hat{k}\right) + \lambda\left(4\hat{i} - 2\hat{j} + 3\hat{k}\right)$ is parallel to the plane $\bar{r} \cdot \left(5\hat{i} + 4\hat{j} - 4\hat{k}\right) = 7$.

Answer:
Given :

Equation of plane : : $\bar{r}.\left(5\hat{\imath} + 4\hat{\jmath} - 4\hat{k}\right) = 7$

Equation of line :

$$\bar{r} = \left(4\hat{\imath} - 7\hat{k}\right) + \lambda\left(4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}\right)$$

To Prove : Given line is parallel to the given plane.

Answer :

Comparing given plane i.e.

$$\bar{r}.\left(5\hat{\imath} + 4\hat{\jmath} - 4\hat{k}\right) = 7$$

with $\bar{r}.\bar{n} = \bar{a}.\bar{n}$, we get,

$$\bar{n} = 5\hat{\imath} + 4\hat{\jmath} - 4\hat{k}$$

This is the vector perpendicular to the given plane.

Now, comparing given equation of line i.e.

$$\bar{r} = \left(4\hat{\imath} - 7\hat{k}\right) + \lambda\left(4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}\right)$$

with $\bar{r} = \bar{a} + \lambda\bar{b}$, we get,

$$\bar{b} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$$

Now,

$$\bar{n}.\bar{b} = \left(5\hat{\imath} + 4\hat{\jmath} - 4\hat{k}\right).\left(4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}\right)$$

$$= (5 \times 4) + (4 \times (-2)) + ((-4) \times 3)$$

= 20 – 8 – 12

= 0

$$\therefore \bar{n}.\bar{b} = 0$$

Therefore, vector normal to the plane is perpendicular to the vector parallel to the line.

Hence, the given line is parallel to the given plane.

Question 23.
Find the length of perpendicular from the origin to the plane $\bar{r} \cdot \left(2\hat{i} - 3\hat{j} + 6\hat{k}\right) + 14 = 0$.

Answer:
Given :

Equation of plane : $\bar{r}.\left(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}\right) + 14 = 0$

To Find : Length of perpendicular = d

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then unit vector of $\bar{a}$ is

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$$

Where, $|\bar{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$

2) Length of perpendicular :

The length of the perpendicular from the origin to the plane

$\bar{r}.\bar{n} = p$ is given by,

$$d = \frac{p}{|\bar{n}|}$$

Answer :

Given equation of the plane is

$$\bar{r}.\left(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}\right) + 14 = 0$$

$$\therefore \bar{r}.\left(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}\right) = -14$$

$$\therefore \bar{r}.\left(-2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}\right) = 14$$

Comparing above equation with

$$\bar{r}.\bar{n} = p$$

We get,

$\bar{n} = -2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ & p = 14

Therefore,

$|\bar{n}| = \sqrt{(-2)^2 + 3^2 + (-6)^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

= 7

The length of the perpendicular from the origin to the given plane is

$d = \frac{p}{|\bar{n}|}$

$\therefore d = \frac{14}{7}$

$\therefore d = 2\ units$

Question 24.
Find the value of λ for which the line

$\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{\lambda}$ is parallel to the plane $\bar{r}.\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right) = 4$

Answer:
Given :

Equation of line : $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{\lambda}$

Equation of plane : $\bar{r}.\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right) = 4$

To Find : λ

Formulae :

1) Parallel vector to the line :

If equation of the line is $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ then,

Vector parallel to the line is given by,

$$\bar{b} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}$$

2) Angle between a line and a plane :

If Θ is a angle between the line $\bar{r} = \bar{a} + \lambda\bar{b}$ and the plane $\bar{r}.\bar{n} = p$, then

$$\sin\theta = \frac{\bar{b}\,.\,\bar{n}}{|\bar{b}|\,.\,|\bar{n}|}$$

Where, $\bar{b}$ is vector parallel to the line and

$\bar{n}$ is the vector normal to the plane.

Answer :

For given equation of line,

$$\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{\lambda}$$

Parallel vector to the line is

$$\bar{b} = 2\hat{\imath} + 3\hat{\jmath} + \lambda\hat{k}$$

For given equation of plane,

$$\bar{r}.\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right) = 4$$

normal vector to the plane is

$\bar{n} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$

Therefore, angle between given line and plane is

$$\sin\theta = \frac{\bar{b}\,.\,\bar{n}}{|\bar{b}|\,.\,|\bar{n}|}$$

As given line is parallel too the given plane, angle between them is 0.

$\therefore \theta = 0$

$\therefore \sin\theta = 0$

$\therefore \bar{b}\,.\,\bar{n} = 0$

$\therefore \left(2\hat{\imath} + 3\hat{\jmath} + \lambda\hat{k}\right).\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right) = 0$

$\therefore (2 \times 2) + (3 \times 3) + (\lambda \times 4) = 0$

4 + 9 + 4 λ = 0

13 + 4λ = 0

4λ = -13

$\therefore \lambda = -\frac{13}{4}$

$\lambda = -\frac{13}{4}$

Question 25.
Write the angle between the line

$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{-2}$ and the plane x + y + 4 = 0.

Answer:
Given :

Equation of line : $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{-2}$

Equation of plane : x + y + 4 = 0

To Find : angle between line and plane

Formulae :

1) Parallel vector to the line :

If equation of the line is $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ then,

Vector parallel to the line is given by,

$$\bar{b} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}$$

2) Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$$\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$$

3) Angle between a line and a plane :

If Θ is a angle between the line $\bar{r} = \bar{a} + \lambda\bar{b}$ and the plane $\bar{r}.\bar{n} = p$, then

$$\sin\theta = \frac{\bar{b}\,.\,\bar{n}}{|\bar{b}|\,.\,|\bar{n}|}$$

Where, $\bar{b}$ is vector parallel to the line and

$\bar{n}$ is the vector normal to the plane.

Answer :

For given equation of line,

$$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+3}{-2}$$

Parallel vector to the line is

$$\bar{b}=2\hat{\imath}+\hat{\jmath}-2\hat{k}$$

$$\therefore \left|\bar{b}\right| = \sqrt{2^2+1^2+(-2)^2} = \sqrt{4+1+4} = \sqrt{9}=3$$

For given equation of plane,

x + y + 4 = 0

normal vector to the plane is

$$\bar{n}=\hat{\imath}+\hat{\jmath}+0\hat{k}$$

$$\therefore |\bar{n}| = \sqrt{1^2+1^2+0^2} = \sqrt{1+1+0} = \sqrt{2}$$

Therefore, angle between given line and plane is

$$\sin\theta=\frac{\bar{b}\,.\,\bar{n}}{\left|\bar{b}\right|.\,|\bar{n}|}$$

$$\therefore \sin\theta=\frac{\left(2\hat{\imath}+\hat{\jmath}-2\hat{k}\right).\left(\hat{\imath}+\hat{\jmath}+0\hat{k}\right)}{3\times\sqrt{2}}$$

$$\therefore \sin\theta=\frac{(2\times1)+(1\times1)+((-2)\times0)}{3\sqrt{2}}$$

$$\therefore \sin\theta=\frac{2+1-0}{3\sqrt{2}}$$

$$\therefore \sin\theta=\frac{3}{3\sqrt{2}}$$

$\therefore \sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\theta = \frac{\pi}{4}$

Question 26.
Write the equation of a plane passing through the point (2, -1, 1) and parallel to the plane 3x + 2y - z = 7.

Answer:
Given :

A ≡ (2, -1, 1)

Plane parallel to the required plane : 3x + 2y – z = 7

To Find : Equation of plane

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

If a plane is passing through point A, then equation of plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{a} = position\ vector\ of\ A$

$\bar{n} = vector\ perpendicular\ to\ the\ plane$

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Answer :

For point A ≡ (2, -1, 1), position vector is

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

As required plane is parallel to 3x + 2y – z = 7.

Therefore, normal vector of given plane is also perpendicular to required plane

$\bar{n} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$

Now, $\bar{a}.\bar{n} = (2 \times 3) + ((-1) \times 2) + (1 \times (-1))$

= 6 – 2 – 1

= 3

Equation of the plane passing through point A and perpendicular to vector $\bar{n}$ is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore \bar{r}.\left(3\hat{\imath} + 2\hat{\jmath} - \hat{k}\right) = 3$

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore \bar{r}.\left(3\hat{\imath} + 2\hat{\jmath} - \hat{k}\right) = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(3\hat{\imath} + 2\hat{\jmath} - \hat{k}\right)$

= 3x + 2y – z

Therefore, equation of the plane is

3x + 2y – z = 3

3x + 2y – z - 3 = 0