Exercise 10e

Question 1.
Find , when:

$x^2 + y^2 = 4$

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to the chain rule of differentiation

$$d(y^2)/_{dx} = \frac{d(y^2)}{dy} \times \frac{dy}{dx} = 2y \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(x^2)}{dx} + \frac{d(y^2)}{dx} = \frac{d(4)}{dx}$$

$$2x + 2y \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

Question 2.
Find , when:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to the chain rule of differentiation

$$\frac{d(^{y^2}/_{b^2})}{dx} = \frac{d(^{y^2}/_{b^2})}{dy} \times \frac{dy}{dx} = \frac{2y}{b^2} \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(^{x^2}/_{a^2})}{dx} + \frac{d(^{y^2}/_{b^2})}{dx} = \frac{d(1)}{dx}$$

$$\frac{2x}{a^2} + \frac{2y}{b^2} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-\frac{2x}{a^2}}{\frac{2y}{b^2}}$$

$$\frac{dy}{dx} = \frac{-b^2x}{a^2y}$$

Question 3.
Find , when:

$$\sqrt{x} + \sqrt{y} = \sqrt{a}$$

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to the chain rule of differentiation

$$\frac{d(\sqrt{y})}{dx} = \frac{d(\sqrt{y})}{dy} \times \frac{dy}{dx} = \frac{1}{2\sqrt{y}} \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(\sqrt{x})}{dx} + \frac{d(\sqrt{y})}{dx} = \frac{d(\sqrt{a})}{dx}$$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-\frac{1}{2\sqrt{x}}}{\frac{1}{2\sqrt{y}}}$$

$$\frac{dy}{dx} = \frac{-2\sqrt{y}}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{-\sqrt{y}}{\sqrt{x}}$$

Question 4.
Find , when:

$$x^{2/3} + y^{2/3} = a^{2/3}$$

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to the chain rule of differentiation

$$\frac{d(y^{2/3})}{dx} = \frac{d(y^{2/3})}{dy} \times \frac{dy}{dx} = \frac{2}{3y^{1/3}} \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(x^{2/3})}{dx} + \frac{d(y^{2/3})}{dx} = \frac{d(a^{2/3})}{dx}$$

$$\frac{2}{3\,x^{1/3}} + \frac{2}{3y^{1/3}} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-\frac{2}{3x^{1/3}}}{\frac{2}{3y^{1/3}}}$$

$$\frac{dy}{dx} = \frac{-y^{1/3}}{x^{1/3}}$$

Question 5.
Find , when:

xy = c^2

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xd(y)}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(xy)}{dx} = \frac{d(c^2)}{dx}$$

$$x \times \frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

$$\frac{dy}{dx} = \frac{-xy}{x^2}$$

$$\frac{dy}{dx} = \frac{-c^2}{x^2}$$

Question 6.
Find , when:

$x^2 + y^2 — 3xy = 1$

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx} = \frac{d(y^2)}{dy} \times \frac{dy}{dx} = 2y \times \frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xd(y)}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(x^2)}{dx} + \frac{d(y^2)}{dx} - 3\frac{d(xy)}{dx} = \frac{d(1)}{dx}$$

$$2x + 2y \times \frac{dy}{dx} - 3(x \times \frac{dy}{dx} + y) = 0$$

$$(2y-3x)\frac{dy}{dx}+2x-3y=0$$

$$\frac{dy}{dx}=\frac{-(2x-3y)}{2y-3x}$$

$$\frac{dy}{dx}=\frac{2x-3y}{3x-2y}$$

Question 7.
Find , when:

$xy^2 - x^2y - 5 = 0$

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx}=n\times x^{(n-1)}$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx}=\frac{d(y^2)}{dy}\times\frac{dy}{dx}=2y\times\frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx}=\frac{xd(y)}{dx}+\frac{yd(x)}{dx}=x\times\frac{dy}{dx}+y$$

Therefore ,

$$\frac{d(xy^2)}{dx}-\frac{d(x^2y)}{dx}=\frac{d(5)}{dx}$$

$$x\times\frac{d(y^2)}{dx}+y^2-[x^2\times\frac{d(y)}{dx}+y\times 2x]=0$$

$$x \times (2y \times \frac{dy}{dx}) + y^2 - [x^2 \times \frac{d(y)}{dx} + y \times 2x] = 0$$

$$2xy\frac{dy}{dx} - x^2\frac{dy}{dx} + y^2 - 2xy = 0$$

$$\frac{dy}{dx} = \frac{2xy - y^2}{2xy - x^2}$$
$$\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$$

Question 8.
Find , when:

$(x^2 + y^2)^2 = xy$

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx} = \frac{d(y^2)}{dy} \times \frac{dy}{dx} = 2y \times \frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xd(y)}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d((x^2 + y^2)^2)}{dx} = \frac{d(xy)}{dx}$$

$$2(x^2 + y^2) \times \frac{d(x^2 + y^2)}{dx} = [x \times \frac{d(y)}{dx} + y]$$

$$2(x^2+y^2)\times[2x+2y\times\frac{dy}{dx}]=[x\times\frac{d(y)}{dx}+y]$$

$$4x(x^2+y^2)+4y(x^2+y^2)\frac{dy}{dx}=x\frac{dy}{dx}+y$$

$$\frac{dy}{dx}[4y(x^2+y^2)-x]=y-4x(x^2+y^2)$$

$$\frac{dy}{dx}=\frac{y-4x(x^2+y^2)}{[4y(x^2+y^2)-x]}$$

$$\frac{dy}{dx}=\frac{y-4x^3-4xy^2}{4y^3+4x^2y-x}$$

Question 9.
Find , when:

$x^2 + y^2 = \log(xy)$

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx}=n\times x^{(n-1)}$, $\frac{d(\log x)}{dx}=\frac{1}{x}$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx}=\frac{d(y^2)}{dy}\times\frac{dy}{dx}=2y\times\frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx}=\frac{xd(y)}{dx}+\frac{yd(x)}{dx}=x\times\frac{dy}{dx}+y$$

Therefore ,

$$\frac{d(x^2)}{dx} + \frac{d(y^2)}{dx} = \frac{d(\log xy)}{dx}$$

$$2x + 2y\frac{d(y)}{dx} = [\frac{1}{xy}\frac{d(xy)}{dx}]$$

$$2x + 2y\frac{d(y)}{dx} = \frac{1}{xy}(x\frac{dy}{dx} + y)$$

$$2x + 2y\frac{dy}{dx} = \frac{1}{y}\frac{dy}{dx} + \frac{1}{x}$$

$$\frac{dy}{dx}\left[2y - \frac{1}{y}\right] = \frac{1}{x} - 2x$$

$$\frac{dy}{dx}(\frac{2y^2 - 1}{y}) = \frac{1 - 2x^2}{x}$$

$$\frac{dy}{dx} = \frac{y(1 - 2x^2)}{x(2y^2 - 1)}$$

Question 10.
Find , when:

$x^n + y^n = a^n$

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to the chain rule of differentiation

$$\frac{d(y^n)}{dx} = \frac{d(y^n)}{dy} \times \frac{dy}{dx} = ny^{n-1} \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(x^n)}{dx} + \frac{d(y^n)}{dx} = \frac{d(a^n)}{dx}$$

$$nx^{n-1} + ny^{n-1} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-nx^{n-1}}{ny^{n-1}}$$

$$\frac{dy}{dx} = \frac{-x^{n-1}}{y^{n-1}}$$

Question 11.
Find , when:

x sin 2y = y cos 2x

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\sin x)}{dx} = \cos x \, , \frac{d(\cos x)}{dx} = -\sin x$

According to the chain rule of differentiation

$$\frac{d(\sin 2y)}{dx} = \frac{d(\sin 2y)}{dy} \times \frac{dy}{dx} = 2\cos 2y \times \frac{dy}{dx}$$

According to the product rule of differentiation

$$\frac{d(x\sin 2y)}{dx} = \frac{xd(\sin 2y)}{dx} + \frac{\sin 2y\, d(x)}{dx} = x \times \frac{d(\sin 2y)}{dx} + \sin 2y$$

Therefore ,

$$\frac{d(x\sin 2y)}{dx} = \frac{d(y\cos 2x)}{dx}$$

$$x \times \frac{d(\sin 2y)}{dx} + \sin 2y = \cos 2x \times \frac{d(y)}{dx} + y(-2\sin 2x)$$

$$x \times 2\cos 2y \times \frac{dy}{dx} + \sin 2y = \cos 2x \times \frac{d(y)}{dx} + y(-2\sin 2x)$$

$$\frac{dy}{dx}[2x\, cos\ 2y - \cos 2x] = -2y\sin 2x - \sin 2y$$

$$\frac{dy}{dx} = \frac{-(2y\sin 2x + \sin 2y)}{2x\, cos\ 2y - \cos 2x}$$

$$\frac{dy}{dx} = \frac{(2y\sin 2x + \sin 2y)}{\cos 2x - 2x\, cos\ 2y}$$

Question 12.
Find , when:

$\sin^2 x + 2\cos y + xy$

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(sin\ x)}{dx} = cos\ x\ , \frac{d(cos\ x)}{dx} = -sin\ x$

According to chain rule of differentiation

$$\frac{d(\cos y)}{dx} = \frac{d(\cos y)}{dy} \times \frac{dy}{dx} = -\sin y \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(sin^2 x)}{dx} + \frac{d(2\cos y)}{dx} + \frac{d(xy)}{dx} = 0$$

$$2\sin x \times \frac{d(\sin x)}{dx} + 2\left(-\sin y \times \frac{dy}{dx}\right) + x \times \frac{dy}{dx} + y = 0$$

$$2\sin x \times \cos x + y = 2\left(\sin y \times \frac{dy}{dx}\right) - x \times \frac{dy}{dx}$$

$$\frac{dy}{dx}[2\sin y - x] = \sin 2x + y$$

$$\frac{dy}{dx} = \frac{\sin 2x + y}{2x\,cos\,2y - \cos 2x}$$

$$\frac{dy}{dx} = \frac{\sin 2x + y}{2\sin y - x}$$

Question 13.
Find , when:

y sec x + tan x + x^2y = 0

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\sec x)}{dx} = \sec x \tan x$, $\frac{d(\tan x)}{dx} = sec^2 x$

According to product rule of differentiation

$$\frac{d(x^2y)}{dx} = \frac{x^2\mathrm{dy}}{dx} + \frac{yd(x^2)}{dx} = x^2\frac{dy}{dx} + 2xy$$

Therefore ,

$$\frac{d(y\sec x)}{dx} + \frac{d(\tan x)}{dx} + \frac{d(x^2y)}{dx} = 0$$

$$\sec x \times \frac{d(y)}{dx} + y\sec x \tan x + \sec^2\mathrm{x} + x^2\frac{dy}{dx} + 2xy = 0$$

$$\frac{dy}{dx}[x^2 + \sec x] = -(y\sec x \tan x + \sec^2\mathrm{x} + 2\mathrm{xy})$$

$$\frac{dy}{dx} = \frac{-(y\sec x\tan x + \sec^2 x + 2xy)}{x^2 + \sec x}$$

Question 14.
Find , when:

cot (xy) + xy = y

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\cot x)}{dx} = -cosec^2 x$

According to chain rule of differentiation

$$\frac{d(\cot xy)}{dx} = -\text{cosec}^2\,\text{xy} \times \frac{d(xy)}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xdy}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(\cot xy)}{dx} + \frac{d(xy)}{dx} = \frac{dy}{dx}$$

$$-\text{cosec}^2\,\text{xy} \times \frac{d(xy)}{dx} + \frac{d(xy)}{dx} = \frac{dy}{dx}$$

$$\frac{d(xy)}{dx}[-\text{cosec}^2\,\text{xy} + 1] = \frac{dy}{dx}$$

$[x\frac{dy}{dx} + y][-\cot^2 \text{xy}] = \frac{dy}{dx}$ (Since, $1 - \text{cosec}^2\,\text{xy} = -\cot^2\,\text{xy}$)

$$x\frac{dy}{dx}(-\cot^2 \text{xy}) - y\cot^2 \text{xy} = \frac{dy}{dx}$$

$$\frac{dy}{dx}[-\mathrm{xcot}^2\,\mathrm{xy}-1]=y\mathrm{cot}^2\,\mathrm{xy}$$

$$\frac{dy}{dx}=\frac{-\,y\cot^2\mathrm{xy}}{\mathrm{xcot}^2\,\mathrm{xy}+1}$$

Question 15.
Find , when:

y tan x — y^2 cos x + 2x = 0

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\tan x)}{dx}=sec^2x\,,\frac{d(\cos x)}{dx}=-sin\;x$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx}=\frac{d(y^2)}{dy}\times\frac{dy}{dx}=2y\times\frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(y\tan x)}{dx}=\mathrm{ysec}^2\,\mathrm{x}+\tan x\times\frac{dy}{dx}$$

Therefore ,

$$\frac{d(y\tan x)}{dx}-\frac{d(y^2\cos x)}{dx}+\frac{d(2x)}{dx}=0$$

$$\mathrm{ysec}^2\,\mathrm{x}+\tan x\times\frac{dy}{dx}-\cos x\frac{d(y^2)}{dx}-y^2(-\sin x)+2=0$$

$$\mathrm{ysec}^2\,\mathrm{x}+\tan x\times\frac{dy}{dx}-\cos x\left(2y\frac{dy}{dx}\right)+y^2(\sin x)+2=0$$

$$y\sec^2 x + \frac{dy}{dx}[\tan x - 2y\cos x] + y^2(\sin x) + 2 = 0$$

$$y\sec^2 x + y^2(\sin x) + 2 = \frac{dy}{dx}[2y\cos x - \tan x]$$

$$\frac{dy}{dx} = \frac{y\sec^2 x + y^2 \sin x + 2}{2y\cos x - \tan x}$$

Question 16.
Find , when:

$e^x \log y = \sin^{-1} x + \sin^{-1} y$

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$, $\frac{d(\log x)}{dx} = \frac{1}{x}$

According to chain rule of differentiation

$$\frac{d(\sin^{-1} y)}{dx} = \frac{d(\sin^{-1} y)}{dy} \times \frac{dy}{dx} = \frac{1}{\sqrt{1-y^2}} \times \frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(e^x \log y)}{dx} = e^x \log y + e^x \times \frac{d(\log y)}{dx} = e^x \log y + e^x \times \frac{1}{y} \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(e^x \log y)}{dx} = \frac{d(\sin^{-1} x)}{dx} + \frac{d(\sin^{-1} y)}{dx}$$

$$e^x \log y + e^x \frac{1}{y}\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}} \times \frac{dy}{dx}$$

$$\frac{dy}{dx}[e^x\frac{1}{y}-\frac{1}{\sqrt{1-y^2}}]=\frac{1}{\sqrt{1-x^2}}-e^x\log y$$

$$\frac{dy}{dx}[\frac{e^x\sqrt{1-y^2}-y}{y\sqrt{1-y^2}}]=\frac{1-(e^x\log y\sqrt{1-x^2})}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx}=\frac{y\sqrt{1-y^2}}{e^x\sqrt{1-y^2}-y}\times\frac{1-(e^x\log y\sqrt{1-x^2})}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx}=y\times\sqrt{\frac{1-y^2}{1-x^2}}\times\frac{1-(e^x\log y\sqrt{1-x^2})}{(e^x\sqrt{1-y^2})-y}$$

Question 17.
Find , when:

xy log (x + y) = 1

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx}=n\times x^{(n-1)}$, $\frac{d(\log x)}{dx}=\frac{1}{x}$

According to product rule of differentiation

$$\frac{d(xy)}{dx}=\frac{xd(y)}{dx}+\frac{yd(x)}{dx}=x\times\frac{dy}{dx}+y$$

Therefore ,

$$\frac{d(xy\times\log x+y)}{dx}=\frac{d(1)}{dx}$$

$$\log x+y\times\frac{d(xy)}{dx}+xy\times\frac{d(\log x+y)}{dx}=\frac{d(1)}{dx}$$

$$\log x+y\left[x\frac{dy}{dx}+y\right]+xy[\frac{1}{x+y}\times\left(1+\frac{dy}{dx}\right)]=0$$

$$\frac{dy}{dx}[x \times \log x + y] + y \times \log(x+y) + \frac{xy}{x+y}\left(1 + \frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx}\left(x\log(x+y) + \frac{xy}{x+y}\right) = -\left(y\log(x+y) + \frac{xy}{x+y}\right)$$

$$\frac{dy}{dx}[(x^2+xy)\log(x+y) + xy] = -[(y^2+xy)\log(x+y) + xy]$$

$\frac{dy}{dx} = \frac{-y^2\log(x+y) - xy\log(x+y) - xy}{x[(x+y)\log(x+y)+y]} \times \frac{x}{x}$ (Multiply and divide by x)

$$\frac{dy}{dx} = \frac{-y\,xy\log(x+y) - x\,xy\log(x+y) - x^2y}{x^2[(x+y)\log(x+y) + y]}$$

$$\frac{dy}{dx} = \frac{-y\,(1) - x\,(1) - x^2y}{x^2[(x+y)\log(x+y) + y]}$$

$$\frac{dy}{dx} = \frac{-\left(x + y + x^2y\right)}{x^2\{y + (x+y)\log(x+y)\}}$$

Question 18.
Find , when:

tan (x + y) + tan (x — y) = 1

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\tan x)}{dx} = sec^2 x$

Therefore ,

$$\frac{d(\tan(x+y))}{dx} + \frac{d(\tan(x-y))}{dx} = \frac{d(1)}{dx}$$

$$\sec^2(x+y)[1+\frac{dy}{dx}]+\sec^2(x-y)[1-\frac{dy}{dx}]=0$$

$$\sec^2(x+y)+\sec^2(x+y)\frac{dy}{dx}+\sec^2(x-y)-\sec^2(x-y)\frac{dy}{dx}=0$$

$$\sec^2(x+y)+\sec^2(x-y)=\frac{dy}{dx}[\sec^2(x-y)-\sec^2(x+y)]$$

$$\frac{dy}{dx}=\frac{\sec^2(x+y)+\sec^2(x-y)}{\sec^2(x+y)-\sec^2(x-y)}$$

Question 19.
Find , when:

$$\log\sqrt{x^2+y^2}=\tan^{-1}\frac{y}{x}$$

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\tan^{-1}x)}{dx}=\frac{1}{1+x^2}$, $\frac{d(\log x)}{dx}=\frac{1}{x}$

According to quotient rule of differentiation

$$\frac{d(^y/_x)}{dx}=\frac{\frac{xd(y)}{dx}-\frac{yd(x)}{dx}}{x^2}=\frac{x\frac{dy}{dx}-y}{x^2}$$

Therefore ,

$$\frac{d(\log\sqrt{x^2+y^2})}{dx}=\frac{d(\tan^{-1}\frac{y}{x})}{dx}$$

$$\frac{1}{\sqrt{x^2+y^2}}\times\frac{d(\sqrt{x^2+y^2})}{dx}=\frac{1}{1+(\frac{y}{x})^2}\times\frac{d(\frac{y}{x})}{dx}$$

$$\frac{1}{\sqrt{x^2+y^2}} \times \frac{1}{2\sqrt{x^2+y^2}} \times [2x + 2y\frac{d(y)}{dx}] = \frac{1}{1+(\frac{y}{x})^2} \times \frac{x\frac{dy}{dx} - y}{x^2}$$

$$\frac{1}{x^2+y^2} \times [x + y\frac{dy}{dx}] = \frac{x^2}{x^2+y^2} \times \frac{x\frac{dy}{dx} - y}{x^2}$$

$$x + y\frac{dy}{dx} = x\frac{dy}{dx} - y$$

$$\frac{dy}{dx}[x-y] = x + y$$

$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

Question 20.
Find , when:

If y = x sin y , prove that $\left(x.\frac{dy}{dx}\right) = \frac{y}{(1 - x\cos y)}$.

There is correction in question .... Prove that should be $\frac{dy}{dx} = \frac{\sin y}{1 - x\cos y}$ instead of $\left(x.\frac{dy}{dx}\right) = \frac{y}{(1 - x\cos y)}$ to get the required answer.

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\sin x)}{dx} = \cos x$

According to chain rule of differentiation

$$\frac{d(\sin y)}{dx} = \frac{d(\sin y)}{dy} \times \frac{dy}{dx} = \cos y \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(y)}{dx} = \frac{d(x \sin y)}{dx}$$

$$\frac{dy}{dx} = x\frac{d(\sin y)}{dx} + \sin y$$

$$\frac{dy}{dx} = xcos\, y\frac{dy}{dx} + \sin y$$

$$\frac{dy}{dx}[1 - xcos\, y] = \sin y$$

$$\frac{dy}{dx} = \frac{\sin y}{1 - x\cos y}$$

Question 21.
Find , when:

If xy = tan (xy), show that $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{-\mathrm{y}}{\mathrm{x}}$.

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\tan x)}{dx} = sec^2 x$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xdy}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(xy)}{dx} = \frac{d(\tan xy)}{dx}$$

$$x\frac{dy}{dx} + y = sec^2(xy) \times \frac{d(xy)}{dx}$$

$$x\frac{dy}{dx} + y = sec^2(xy) \times [x\frac{dy}{dx} + y]$$

$$\frac{dy}{dx}[x - xsec^2(xy)] = ysec^2(xy) - y$$

$$x\frac{dy}{dx}(1 - \sec^2 \mathrm{xy}) = y(sec^2(xy) - 1)$$

$$\frac{dy}{dx} = \frac{-y(1 - sec^2(xy))}{x(1 - \sec^2 \mathrm{xy})}$$

$$\frac{dy}{dx} = \frac{-\mathrm{y}}{\mathrm{x}}$$

Question 22.
Find , when:

If y log x = (x — y), prove that $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\log \mathrm{x}}{(1 + \log \mathrm{x})^2}$.

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$, $\frac{d(log\ x)}{dx} = \frac{1}{x}$

According to product rule of differentiation

$$\frac{d(y \log x)}{dx} = \frac{\log x\, d(y)}{dx} + \frac{y\, d(\log x)}{dx} = \log x \times \frac{dy}{dx} + \frac{y}{x}$$

Therefore ,

$$\frac{d(y \times \log x)}{dx} = \frac{d(x - y)}{dx}$$

$$\log x \times \frac{d(y)}{dx} + \frac{y}{x} = 1 - \frac{d(y)}{dx}$$

$$\frac{dy}{dx}[\log x \ + 1] = 1 - \frac{y}{x}$$

$$\frac{dy}{dx}[(1+\log x)^2] = 1 - \frac{y}{x}(1+\log x)$$

(Multiply by 1+log x on both sides)

$$\frac{dy}{dx}[(1+\log x)^2] = 1 + \log x - \frac{y}{x} - \frac{y}{x}\log x$$

$\frac{dy}{dx}[(1+\log x)^2] = 1 + \log x - \frac{y}{x} - \frac{(x-y)}{x}$ (y log x = x - y)

$$\frac{dy}{dx}[(1+\log x)^2] = 1 + \log x - \frac{y}{x} - 1 + \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$$

Question 23.
Find , when:

If cos y = x cos (y + a), prove that $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\cos^2(\mathrm{y}+\mathrm{a})}{\sin \mathrm{a}}$.

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\cos x)}{dx} = -\sin x$

According to chain rule of differentiation

$$\frac{d(\cos y)}{dx} = \frac{d(\cos y)}{dy} \times \frac{dy}{dx} = -\sin y \times \frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(x\, cos\,(y+a))}{dx} = x\frac{d(\cos y + a)}{dx} + \cos(y+a)$$

Therefore ,

$$\frac{d(\cos y)}{dx} = \frac{d(x\cos(y+a))}{dx}$$

$$-\sin y\frac{dy}{dx} = x\frac{d(\cos(y+a))}{dx} + \cos(y+a)$$

$$-\sin y\frac{dy}{dx} = x(-\sin(y+a)\frac{dy}{dx}) + \cos(y+a)$$

$$\frac{dy}{dx}[-\sin y + x sin\,(y+a)] = \cos(y+a)$$

$$\frac{dy}{dx} = \frac{\cos(y+a)}{x\sin(y+a) - \sin y}$$

$$\frac{dy}{dx} = \frac{cos^2(y+a)}{x\cos(y+a)\sin(y+a) - \cos(y+a)\sin y}$$

(Multiply and divide by cos (y+a))

$\frac{dy}{dx} = \frac{cos^2(y+a)}{\sin(y+a)\cos y - \cos(y+a)\sin y}$ (Since cos y = x cos (y + a))

$\frac{dy}{dx} = \frac{cos^2(y+a)}{\sin(y+a-y)}$ (Formula sin(a-b)=sin a cos b – cos a sin b)

$$\frac{dy}{dx} = \frac{cos^2(y+a)}{\sin a}$$

Question 24.

Find , when:

If $\cos^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = \tan^{-1} a$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

Answer:
Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\cos^{-1} x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$

According to the chain rule of differentiation

$$\frac{d(y^2)}{dx} = \frac{d(y^2)}{dy} \times \frac{dy}{dx} = 2y \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(\cos^{-1}\frac{x^2-y^2}{x^2+y^2})}{dx} = \frac{d(\tan^{-1} a)}{dx}$$

$$-\frac{1}{\sqrt{1-(\frac{x^2-y^2}{x^2+y^2})^2}} \times \frac{d(\frac{x^2-y^2}{x^2+y^2})}{dx} = 0$$

$$\frac{d(\frac{x^2-y^2}{x^2+y^2})}{dx} = 0$$

$$\frac{x^2+y^2\left[\frac{d(x^2-y^2)}{dx}\right] - (x^2-y^2)[\frac{d(x^2+y^2)}{dx}]}{(x^2+y^2)^2} = 0$$

$$x^2+y^2\left[\frac{d(x^2-y^2)}{dx}\right] - (x^2-y^2)[\frac{d(x^2+y^2)}{dx}] = 0$$

$$(x^2+y^2)\left(2x-2y\frac{dy}{dx}\right)-(x^2-y^2)(2x+2y\frac{dy}{dx})=0$$

$$(x^2+y^2)\left(x-y\frac{dy}{dx}\right)=(x^2-y^2)(x+y\frac{dy}{dx})$$

$$\frac{dy}{dx}[-x^2y-y^3-x^2y+y^3]=x^3-xy^2-x^3-xy^2$$

$$\frac{dy}{dx}[-2x^2y]=-2xy^2$$

$$\frac{dy}{dx}=\frac{-2xy^2}{-2yx^2}$$

$$\frac{dy}{dx}=\frac{y}{x}$$

Exercise 10f

Question 1.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y}=\mathrm{x}^{1/\mathrm{x}}$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$lny=\frac{lnx}{x}\ \{\ln(x^m)=m(lnx)\}$$

Now differentiating both sides by x we get,

$$\frac{1}{y}\times\frac{dy}{dx}=\frac{x\left(\frac{1}{x}\right)-lnx(1)}{x^2}$$

$$\frac{dy}{dx} = \frac{1 - lnx}{x^2} \times y \left\{ divide\, rule\ \frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \right\}$$

$$\frac{dy}{dx} = \frac{1 - lnx}{x^2} \times x^{\frac{1}{x}} \{y = x^{\frac{1}{x}}\}$$

Question 2.

Find $\frac{dy}{dx}$, when:

$$y = x^{\sqrt{x}}$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = \sqrt{x}\, lnx$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \sqrt{x}\left(\frac{1}{x}\right) + lnx\left(\frac{1}{2\sqrt{x}}\right) \left\{product\, rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x}}\left(1 + \frac{lnx}{2}\right) \times y$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x}} \times \left(1 + \frac{lnx}{2}\right) \times \left(x^{\sqrt{x}}\right)$$

Question 3.

Find $\frac{dy}{dx}$, when:

$$y = (\log x)^x$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = x\ln(lnx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = x\left(\frac{1}{lnx} \times \frac{1}{x}\right) + \ln(lnx)\ \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{1}{lnx} + \ln(lnx)\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{1}{lnx} + \ln(lnx\,)\right) \times (lnx)^{x}$$

Question 4.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y} = \mathrm{x}^{\sin \mathrm{x}}$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = sinx\ lnx$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = sinx \times \frac{1}{x} + \ lnx \times cosx\ \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(sinx \times \frac{1}{x} + lnx \times cosx\ \right) \times y$$

$$\frac{dy}{dx} = \left(\frac{sinx}{x} + cosx(lnx\,)\right) \times x^{sinx}$$

Question 5.

Find $\frac{dy}{dx}$, when:

$$y = x^{(\cos^{-1} x)}$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$lny = \cos^{-1} x \, lnx$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \cos^{-1} x \times \left(\frac{1}{x}\right) + lnx \times \left(-\frac{1}{\sqrt{1-x^2}}\right) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \cos^{-1} x \times \left(\frac{1}{x}\right) + lnx \times \left(-\frac{1}{\sqrt{1-x^2}}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{\cos^{-1} x}{x} - \frac{lnx}{\sqrt{1-x^2}}\right) \times x^{(\cos^{-1} x)}$$

Question 6.
Find $\frac{dy}{dx}$, when:

$$y = (\tan x)^{1/x}$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$lny = \left(\frac{1}{x}\right) \ln(tanx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{1}{x}\right) \times \left(\frac{1}{tanx} \times \sec^2 x\right) + \ln(tanx) \times \left(-\frac{1}{x^2}\right) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{\sec^2 x}{x \times tanx} - \frac{\ln(tanx)}{x^2}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{\sec^2 x}{x \times tanx} - \frac{\ln(tanx)}{x^2}\right) \times tanx^{\frac{1}{x}}$$

Question 7.

Find $\frac{dy}{dx}$, when:

$$y = (\sin x)^{\cos x}$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (cosx)\ln(sinx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (cosx) \times \left(\frac{1}{sinx} \times cosx\right) + \ln(sinx) \times (-sinx) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{\cos^2 x}{sinx} - sinx(\ln(sinx))\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{\cos^2 x}{sinx} - sinx(\ln(sinx))\right) \times sinx^{cosx}$$

Question 8.

Find $\frac{dy}{dx}$, when:

$$y = (\log x)^{\sin x}$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$lny = (sinx)\ln(lnx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (sinx) \times \left(\frac{1}{lnx} \times \frac{1}{x}\right)$$
$$+ \ln(lnx) \times (cosx)\left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{sin\ x}{x \times lnx} - cosx(\ln(lnx))\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{sin\ x}{x \times lnx} - cosx(\ln(lnx))\right) \times lnx^{sinx}$$

Question 9.
Find $\frac{dy}{dx}$, when:

$$y = (\cos x)^{\log x}$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$lny = (lnx)\ln(cosx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (lnx) \times \left(\frac{1}{cosx} \times (-sinx)\right)$$
$$+ \ln(cosx) \times \left(\frac{1}{x}\right) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(-\frac{\sin x \times lnx}{cosx} + \frac{(\ln cosx)}{x}\right) \times y$$

$$\frac{dy}{dx} = \left(-\frac{\sin x \times lnx}{cosx} + \frac{(\ln cosx)}{x}\right) \times cosx^{lnx}$$

Question 10.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y} = (\tan \mathrm{x})^{\sin \mathrm{x}}$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (sinx)\ln(tanx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (sinx) \times \left(\frac{1}{tanx} \times (\sec^2 x)\right)$$
$$+ \ln(tanx) \times (cosx) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{\sin x \times \sec^2 x}{tanx} + \ln(tanx)\, cosx\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{\sin x \times \sec^2 x}{tanx} + \ln(tanx)\, cosx\right) \times tanx^{sinx}$$

Question 11.

Find $\frac{dy}{dx}$, when:

$$y = (\cos x)^{\cos x}$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$lny = (cosx)\ln(cosx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (cosx) \times \left(\frac{1}{cosx} \times (-sinx)\right) + \ln(cosx) \times (-sinx) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = (-sinx - \ln(cosx)\, sinx) \times y$$

$$\frac{dy}{dx} = (-sinx - \ln(cosx)\, sinx) \times cosx^{cosx}$$

Question 12.
Find $\frac{dy}{dx}$, when:

$$y = (\tan x)^{\cot x}$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$lny = (\cot x)\ln(\tan x)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (cotx) \times \left(\frac{1}{tanx} \times (-\sec^2 x)\right)$$
$$+ \ln(tanx) \times (-cosec^2 x) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = (-cosec^2 x - \ln(tanx)cosec^2 x) \times y$$

$$\frac{dy}{dx} = -cosec^2 x \times (1 + \ln(cosx)) \times tanx^{\cot x}$$

Question 13.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y} = \mathrm{x}^{\sin 2\mathrm{x}}$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (sin2x)\ln(x)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (sin2x) \times \left(\frac{1}{x}\right)$$
$$+ \ln(x) \times (cos2x \times 2) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{sin2x}{x} + 2\cos 2x \times \ln x\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{sin2x}{x} + 2\cos 2x \times \ln x\right) \times x^{\sin 2x}$$

Question 14.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$y = (\sin^{-1} x)^x$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$lny = (x)\ln(\sin^{-1} x)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (x) \times \left(\frac{1}{\sin^{-1} x} \times \frac{1}{\sqrt{1-x^2}}\right) + \ln(\sin^{-1} x) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{x}{\sin^{-1} x \times \sqrt{1-x^2}} \times \ln \sin^{-1} x\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{x}{\sin^{-1} x \times \sqrt{1-x^2}} \times \ln \sin^{-1} x\right) \times \sin^{-1} x^x$$

Question 15.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$y = \sin(x^x)$$

Answer:
Here, the argument of the sinusoidal function has exponent as x itself.

For that, we will consider $x^x = u$ for simplicity.

$$y = \sin u$$

Differentiating both the sides,

$\frac{dy}{dx} = \cos u \times \frac{du}{dx}$. ....(1)

Now we have to find $\frac{du}{dx}$, where $u = x^x$

take log both the sides

$$\ln u = x \ln x$$

Now differentiating both sides by x, we get,

$$\frac{1}{u} \times \frac{du}{dx} = x\left(\frac{1}{x}\right) + \ln x$$

$$\frac{du}{dx} = (1 + \ln x) \times u$$

$$\frac{du}{dx} = (1 + \ln x) \times x^x$$

Substituting the value in equation 1,

$$\frac{dy}{dx} = cosx\ (1 + \ln x\ ) \times x^x$$

Question 16.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y} = (3\mathrm{x} + 5)^{(2\mathrm{x} - 3)}$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (2x - 3)\ln(3x - 5)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y}\times\frac{dy}{dx}=(2x-3)\times\left(\frac{1}{3x-5}\times 3\right)$$

$$+\ln(3x-5)\times 2\left\{product\,rule,\frac{d(uv)}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx}=\left(\frac{2x-3}{3x-5}\times 3+\ln(3x-5)\times 2\right)\times y$$

$$\frac{dy}{dx}=\left(\frac{2x-3}{3x-5}\times 3+2\times\ln 3x-5\right)\times(3x-5)^{2x-3}$$

Question 17.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y}=(\mathrm{x}+1)^3(\mathrm{x}+2)^4(\mathrm{x}+3)^5$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$lny=3\ln(x+1)+\ 4\ln(x+2)+5\ln(x+3)\ \{\ln(mn)=\ln n+\ln m\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y}\times\frac{dy}{dx}=\frac{3}{x+1}+\frac{4}{x+2}+\frac{5}{x+3}$$

$$\frac{dy}{dx}=\left(\frac{3}{x+1}+\frac{4}{x+2}+\frac{5}{x+3}\right)\times y$$

$$\frac{dy}{dx}=\left(\frac{3}{x+1}+\frac{4}{x+2}+\frac{5}{x+3}\right)\times(x+1)^3(x+2)^4(x+3)^5$$

Question 18.

Find $\frac{dy}{dx}$, when:

$$y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = \frac{1}{2}(\ln(x-1) + \ln(x-2) - \ln(x-3) - \ln(x-4) - \ln(x-5))$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right) \times y$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right) \times \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

Question 19.
Find $\frac{dy}{dx}$, when:

$$y = (2-x)^3(3+2x)^5$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 3\ln(2-x) + 5\ln(3+2x)$$

$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = 3\left(\frac{-1}{2-x}\right) + 5\left(\frac{1}{3+2x} \times 2\right)$$

$$\frac{dy}{dx} = \left(\frac{3}{x-2} + \frac{10}{3+2x}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{3}{x-2} + \frac{10}{3+2x}\right) \times (2-x)^3(3+2x)^5$$

Question 20.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y} = \cos \mathrm{x} \cos 2\mathrm{x} \cos 3\mathrm{x}$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = \ln(cos x) + \ln(\cos 2x) + \ln \cos 3x$$

$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{1}{\cos x} \times (-\sin x) + \frac{1}{\cos 2x} \times (-2\sin 2x) + \frac{1}{\cos 3x}(-3\sin 3x)$$

$$\frac{dy}{dx} = \left(\frac{-\sin x}{\cos x} - \frac{2\sin 2x}{\cos 2x} - \frac{3\sin 3x}{\cos 3x}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{-\sin x}{\cos x} - \frac{2\sin 2x}{\cos 2x} - \frac{3\sin 3x}{\cos 3x}\right) \times \cos x \cos 2x \cos 3x$$

$$\frac{dy}{dx} = (-\tan x - 2\tan 2x - 3\tan 3x)\ \times \cos x \cos 2x \cos 3x$$

Question 21.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$y = \frac{x^5\sqrt{x+4}}{(2x+3)^2}$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 5\ln(x) + \frac{1}{2}\ln(x+4) - 2\ln(2x+3)$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{5}{x} + \frac{1}{2(x+4)} - \frac{4}{2x+3}$$

$$\frac{dy}{dx} = \left(\frac{5}{x} + \frac{1}{2(x+4)} - \frac{4}{2x+3}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{5}{x} + \frac{1}{2(x+4)} - \frac{4}{2x+3}\right) \times \frac{x^5\sqrt{x+4}}{(2x+3)^2}$$

Question 22.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$y = \frac{(x+1)^2\sqrt{x-1}}{(x+4)^3 . e^x}$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 2\ln(x+1) + \frac{1}{2}\ln(x-1) - 3\ln(x+4) - x$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{2}{x+1} + \frac{1}{2(x-1)} - \frac{3}{x+4} - 1$$

$$\frac{dy}{dx} = \left(\frac{2}{x+1} + \frac{1}{2(x-1)} - \frac{3}{x+4} - 1\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{2}{x+1} + \frac{1}{2(x-1)} - \frac{3}{x+4} - 1\right) \times \frac{(x+1)^2\sqrt{x-1}}{(x+4)^3.e^x}$$

Question 23.

Find $\frac{dy}{dx}$, when:

$$y = \frac{\sqrt{x}(3x+5)^2}{\sqrt{x+1}}$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 2\ln(3x+5) + \frac{1}{2}\ln(x) - \frac{1}{2}\ln(x+1)$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y}\times\frac{dy}{dx}=\frac{2}{3x+5}+\frac{1}{2x}-\frac{1}{2(x+1)}$$

$$\frac{dy}{dx}=\left(\frac{2}{3x+5}+\frac{1}{2x}-\frac{1}{2(x+1)}\right)\times y$$

$$\frac{dy}{dx}=\left(\frac{2}{3x+5}+\frac{1}{2x}-\frac{1}{2(x+1)}\right)\times\frac{(3x+5)^2\sqrt{x}}{\sqrt{x+1}}$$

Question 24.

Find $\frac{dy}{dx}$, when:

$$y=\frac{x^2\sqrt{1+x}}{(1+x^2)^{3/2}}$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y=\ 2\ln(x)+\frac{1}{2}\ln(x+1)-\frac{3}{2}\ln(x^2+1)$$

$$\{\ln(mn)\ =\ln n+\ln m\}\ \{\ln\left(\frac{m}{n}\right)\ =\ln m-\ln n\}\ \{\ln e\ =\ 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y}\times\frac{dy}{dx}=\frac{2}{x}+\frac{1}{2(x+1)}-\frac{3}{2(x^2+1)}\times 2x$$

$$\frac{dy}{dx}=\left(\frac{2}{x}+\frac{1}{2(x+1)}-\frac{6x}{2(x^2+1)}\right)\times y$$

$$\frac{dy}{dx}=\left(\frac{2}{x}+\frac{1}{2(x+1)}-\frac{6x}{2(x^2+1)}\right)\times\frac{(x)^2\sqrt{x+1}}{(1+x^2)^{\frac{3}{2}}}$$

Question 25.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$y=\sqrt{(x-2)(2x-3)(3x-4)}$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$\ln y=\frac{1}{2}(\ln(x-2)+\ln(2x-3)+\ln(3x-4))$$

$$\{\ln(mn)=\ln n+\ln m\}\ \{\ln\left(\frac{m}{n}\right)=\ln m-\ln n\}\ \{\ln e=1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y}\times\frac{dy}{dx}=\frac{1}{2}\left(\frac{1}{x-2}+\frac{2}{2x-3}+\frac{3}{3x-4}\right)$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{1}{x-2}+\frac{2}{2x-3}+\frac{3}{3x-4}\right)\times y$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{1}{x-2}+\frac{2}{2x-3}+\frac{3}{3x-4}\right)\times\sqrt{(x-2)(2x-3)(3x-4)}$$

Question 26.
Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$y=\sin 2x\sin 3x\sin 4x$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$\ln y=\ (\ln(\sin 2x)+\ln(\sin 3x)+\ln(\sin 4x))$$

$$\{\ln(mn)=\ln n+\ln m\}\ \{\ln\left(\frac{m}{n}\right)=\ln m-\ln n\}\ \{\ln e=1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{2}{\sin 2x} + \frac{3}{\sin 3x} + \frac{4}{\sin 4x}\right)$$

$$\frac{dy}{dx} = \left(\frac{2}{\sin 2x} + \frac{3}{\sin 3x} + \frac{4}{\sin 4x}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{2}{\sin 2x} + \frac{3}{\sin 3x} + \frac{4}{\sin 4x}\right) \times \sin 2x \sin 3x \sin 4x$$

Question 27.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y} = \frac{\mathrm{x}^3 \sin \mathrm{x}}{\mathrm{e}^{\mathrm{x}}}$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 3 \ln x + \ln \sin x - x$$

$$\{\ln(mn) = \ln n + \ln m\} \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\} \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{1}{\sin x} \times \cos x + \frac{3}{x} - 1\right)$$

$$\frac{dy}{dx} = \left(\frac{\cos x}{\sin x} + \frac{3}{x} - 1\right) \times y$$

$$\frac{dy}{dx} = \left(\cot x + \frac{3}{x} - 1\right) \times \frac{x^3 \sin x}{e^x}$$

Question 28.

Find $\frac{dy}{dx}$, when:

$$y = \frac{e^x \log x}{x^2}$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = x + \ln(\ln x) - 2\ln x$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(1 + \frac{1}{x\ln x} - \frac{2}{x}\right)$$

$$\frac{dy}{dx} = \left(1 + \frac{1}{x\ln x} - \frac{2}{x}\right) \times y$$

$$\frac{dy}{dx} = \left(1 + \frac{1}{x\ln x} - \frac{2}{x}\right) \times \frac{e^x \log x}{x^2}$$

Question 29.
Find $\frac{dy}{dx}$, when:

$$y = \frac{x\cos^{-1}x}{\sqrt{1-x^2}}$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = \ln\cos^{-1}x + \ln(x) - \frac{1}{2}\ln(1 - x^2)$$

$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(-\frac{1}{\sqrt{1-x^2}} + \frac{1}{x} + \frac{2x}{2(1-x^2)}\right)$$

$$\frac{dy}{dx} = \left(-\frac{1}{\sqrt{1-x^2}} + \frac{1}{x} + \frac{2x}{2(1-x^2)}\right) \times y$$

$$\frac{dy}{dx} = \left(-\frac{1}{\sqrt{1-x^2}} + \frac{1}{x} + \frac{x}{(1-x^2)}\right) \times \frac{x\cos^{-1}x}{\sqrt{1-x^2}}$$

Question 30.

Find $\frac{dy}{dx}$, when:

$$y = (1+x)(1+x^2)(1+x^4)(1+x^6)$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = \ln(1+x) + \ln(1+x^2) + \ln(1+x^4) + \ln(1+x^6)$$

$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{6x^5}{1+x^6}\right)$$

$$\frac{dy}{dx} = \left(\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{6x^5}{1+x^6}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{6x^5}{1+x^6}\right) \times (1+x)(1+x^2)(1+x^4)(1+x^6)$$

Question 31.

Find $\frac{dy}{dx}$, when:

$$y = x^x - 2^{\sin x}$$

Answer:

simply taking log both sides would not help more.

For that let us assume $u = x^x$ and $v = 2^{\sin x}$

$$\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

$$u = x^x$$

Take log both sides

$$\ln u = x \ln x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = x\left(\frac{1}{x}\right) + \ln x$$

$$\frac{du}{dx} = (1 + \ln x) \times u$$

$\frac{du}{dx} = (1 + \ln x) \times x^x$ ....(1)

$$v = 2^{\sin x}$$

Take log both sides,

$$\ln v = \sin x . \ln 2$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \sin x(0) + \ln 2.\cos x$$

$$\frac{dv}{dx} = \ln 2.\cos x \times v$$

$\frac{dv}{dx} = \ln 2.\cos x \times 2^{\sin x}$ ......(2)

$$\frac{dy}{dx} = (1 + \ln x) \times x^x - \ln 2.\cos x \times 2^{\sin x}$$

Question 32.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y} = (\log \mathrm{x})^{\mathrm{x}} + \mathrm{x}^{\log \mathrm{x}}$$

Answer:

simply taking log both sides would not help more.

For that let us assume $u = (\ln x)^x$ and $v = x^{\ln x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (\ln x)^x$$

Take log both sides

$$\ln u = x \ln(\ln x)$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = x\left(\frac{1}{x} \times \frac{1}{\ln x}\right) + \ln(\ln x)$$

$$\frac{du}{dx} = \left(\frac{1}{\ln x} + \ln(\ln x)\right) \times u$$

$\frac{du}{dx} = \left(\frac{1}{\ln x} + \ln(\ln x)\right) \times (\ln x)^x$ ....(1)

$v = x^{\ln x}$

Take log both sides,

$\ln v = \ln x . \ln x$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = 2. \ln x \times \frac{1}{x}$$

$$\frac{dv}{dx} = \frac{2. \ln x}{x} \times v$$

$\frac{dv}{dx} = \frac{2. \ln x}{x} \times x^{\ln x}$ ......(2)

$$\frac{dy}{dx} = \left(\frac{1}{\ln x} + \ln(\ln x)\right) \times (\ln x)^x + \frac{2. \ln x}{x} \times x^{\ln x}$$

Question 33.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y} = \mathrm{x}^{\sin \mathrm{x}} + (\sin \mathrm{x})^{\cos \mathrm{x}}$$

Answer:

simply taking log both sides would not help more.

For that let us assume $u = x^{\sin x}$ and $v = \sin x^{\cos x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$u = (x)^{\sin x}$

Take log both sides

$$\ln u = \sin x \ln(x)$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \sin x \left(\frac{1}{x}\right) + \ln(x) \times \cos x$$

$$\frac{du}{dx} = \left(\frac{\sin x}{x} + \ln(x) \times \cos x\right) \times u$$

$$\frac{du}{dx} = \left(\frac{\sin x}{x} + \ln(x) \times \cos x\right) \times (x^{\sin x} + (\sin x)^{\cos x}) \text{ ....(1)}$$

$$v = (\sin x)^{\cos x}$$

Take log both sides,

$$\ln v = \cos\ x \ln(\sin x)$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \cos x(\frac{1}{\sin x} \times \cos x)$$

$$\frac{dv}{dx} = \frac{\cos^2 x}{\sin x} \times v$$

$$\frac{dv}{dx} = \frac{\cos^2 x}{\sin x} \times (\sin x)^{\cos x} \text{ ......(2)}$$

$$\frac{dy}{dx} = \frac{\cos^2 x}{\sin x} \times (\sin x)^{\cos x} + \left(\frac{\sin x}{x} + \ln(x) \times \cos x\right) \times (x^{\sin x} + (\sin x)^{\cos x})$$

Question 34.

Find $\frac{dy}{dx}$, when:

$$y = (x\cos x)^x + (x\sin x)^{1/x}$$

Answer:
simply taking log both sides would not help more.

For that let us assume $u = (x.\cos x)^x$ and $v = (x.\sin x)^{\frac{1}{x}}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (x.\cos x)^x$$

Take log both sides

$$\ln u = x\left(\ln(x) + \ln\cos x\right)$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = x\left(\frac{1}{x} - \frac{\sin x}{\cos x}\right) + \ (\ln(x) + \ln\cos x)$$

$$\frac{du}{dx} = \left(x\left(\frac{1}{x} - \frac{\sin x}{\cos x}\right) + \ (\ln(x) + \ln\cos x)\ \right) \times u$$

$$\frac{du}{dx} = \left(x\left(\frac{1}{x} - \frac{\sin x}{\cos x}\right) + \ (\ln(x) + \ln\cos x)\ \right) \times ((x.\cos x)^x) \text{ ....(1)}$$

$$v = (x.\sin x)^{\frac{1}{x}}$$

Take log both sides,

$$\ln v = \frac{1}{x} \times (\ln x + \ln\sin x)$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \frac{1}{x}\left(\frac{1}{x} + \frac{\cos x}{\sin x}\right) - \frac{1}{x^2} \times (\ln x + \ln \sin x)$$

$$\frac{dv}{dx} = \left(\frac{1}{x}\left(\frac{1}{x} + \frac{\cos x}{\sin x}\right) - \frac{1}{x^2} \times (\ln x + \ln \sin x)\right) \times v$$

$$\frac{dv}{dx} = \left(\frac{1}{x}\left(\frac{1}{x} + \frac{\cos x}{\sin x}\right) - \frac{1}{x^2} \times (\ln x + \ln \sin x)\right) \times (x.\sin x)^{\frac{1}{x}} \text{......(2)}$$

$$\frac{dy}{dx} = \left(\frac{1}{x}\left(\frac{1}{x} + \frac{\cos x}{\sin x}\right) - \frac{1}{x^2} \times (\ln x + \ln \sin x)\right) \times (x.\sin x)^{\frac{1}{x}} + \left(x\left(\frac{1}{x} - \frac{\sin x}{\cos x}\right) + (\ln(x) + \ln \cos x)\right) \times ((x.\cos x)^x)$$

Question 35.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y} = (\sin \mathrm{x})^{\mathrm{x}} + \sin^{-1}\sqrt{\mathrm{x}}$$

Answer:

simply taking log both sides would not help more.

For that let us assume $u = (\sin x)^x$ and $v = \sin^{-1}\sqrt{x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (\sin x)^x$$

Take log both sides

$$\ln u = x.\ln \sin x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = x\left(\frac{\cos x}{\sin x}\right) + \ln \sin x$$

$$\frac{du}{dx} = \left(x\left(\frac{\cos x}{\sin x}\right) + \ln \sin x\right) \times u$$

$\frac{du}{dx} = \left(x\left(\frac{\cos x}{\sin x}\right) + \ln \sin x\right) \times ((\sin x)^x)$ ....(1)

for v we do not have to take log just simply differentiate it,

$\frac{dv}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \times \frac{1}{2\sqrt{x}}$ ....(2)

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \times \frac{1}{2\sqrt{x}} + \left(x\left(\frac{\cos x}{\sin x}\right) + \ln \sin x\right) \times ((\sin x)^x)$$

Question 36.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$y = x^{x \cos x} + \left(\frac{x^2+1}{x^2-1}\right)$$

Answer:

simply taking log both sides would not help more.

For that let us assume $u = (x)^{x.\cos x}$ and $v = \frac{x^2+1}{x^2-1}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (x)^{x.\cos x}$$

Take log both sides

$$\ln u = x\,.\cos x\,.\ln x$$

Here there are three terms to differentiate for this; we can take two term as one and then apply product rule, I am taking $x.\ln x$ as a single term

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cos x \left(x \left(\frac{1}{x} \right) + \ln x \right) + x.\ln x(-\sin x)$$

$$\frac{du}{dx} = (\cos x(1 + \ln x) - x.\ln x.\sin x) \times u$$

$\frac{du}{dx} = (\cos x(1 + \ln x) - x.\ln x.\sin x) \times (x.\cos x.\ln x)$..........(1)

for v we do not have to take log just simply differentiate it,

$$\frac{dv}{dx} = \frac{(x^2 - 1)(2x) - (x^2 + 1)(2x)}{(x^2 - 1)^2}$$

$\frac{dy}{dx} = \frac{2x(-2)}{(x^2-1)^2}$........(2)

$$\frac{dy}{dx} = (\cos x(1 + \ln x) - x.\ln x.\sin x) \times (x.\cos x.\ln x) + \frac{2x(-2)}{(x^2 - 1)^2}$$

Question 37.

Find $\frac{dy}{dx}$, when:

$$y = e^x \sin^3 x \cos^4 x$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = x + 3.\ln \sin x + 4 \ln \cos x$$

$$\{\ln(mn) = \ln n + \ln m\} \{\ln \left(\frac{m}{n} \right) = \ln m - \ln n\} \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{3.\cos x}{\sin x} - \frac{4\sin x}{\cos x} + 1\right)$$

$$\frac{dy}{dx} = \left(\frac{3.\cos x}{\sin x} - \frac{4\sin x}{\cos x} + 1\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{3.\cos x}{\sin x} - \frac{4\sin x}{\cos x} + 1\right) \times e^{x}.\sin^{3} x.\cos^{4} x$$

Question 38.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$y = 2^{x}.e^{3x}\sin 4x$$

Answer:

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = x.\ln 2 + 3x + \ln\sin 4x$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \ln 2 + 3 + \frac{\cos 4x}{\sin 4x} \times 4$$

$$\frac{dy}{dx} = \left(\ln 2 + 3 + \frac{\cos 4x}{\sin 4x} \times 4\right) \times y$$

$$\frac{dy}{dx} = \left(\ln 2 + 3 + \frac{\cos 4x}{\sin 4x} \times 4\right) \times e^{3x}.\sin 4x.2^{x}$$

Question 39.

Find $\frac{dy}{dx}$, when:

$$y = x^x . e^{(2x+5)}$$

Answer:
Here we need to take log both the sides to get that differentiation simple.

$$\ln y = x . \ln x + 2x + 5$$

$$\{\ln(mn) = \ln n + \ln m\} \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\} \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = 1 + \ln x + 2$$

$$\frac{dy}{dx} = (\ln x + 3) \times y$$

$$\frac{dy}{dx} = (\ln x + 3) \times x^x . e^{2x+5}$$

Question 40.
Find $\frac{dy}{dx}$, when:

$$y = (2x+3)^5 (3x-5)^7 (5x-1)^3$$

Answer:
Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 5 . \ln(2x + 5) + 7 . \ln(3x - 5) + 3 . \ln(5x - 1)$$

$$\{\ln(mn) = \ln n + \ln m\} \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\} \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{5 \times 2}{2x + 5} + \frac{7 \times 3}{3x - 5} + \frac{3 \times 5}{5x - 1}$$

$$\frac{dy}{dx} = \left(\frac{10}{2x + 5} + \frac{21}{3x - 5} + \frac{15}{5x - 1}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{10}{2x + 5} + \frac{21}{3x - 5} + \frac{15}{5x - 1}\right) \times (2x + 5)^5 (3x - 5)^7 (5x - 1)^3$$

Question 41.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$(\cos \mathrm{x})^{\mathrm{y}} = (\cos \mathrm{y})^{\mathrm{x}}$$

Answer:

. So the equation given is implicit, we will just take log both sides

$$y.\ln(\cos x) = x.\ln(\cos y)$$

Now differentiate it with respect to x and consider $\frac{dy}{dx} = y'$

$$y\left(\frac{-\sin x}{\cos x}\right) + \ln \cos x . y' = x\left(\frac{-\sin y}{\cos y} \times y'\right) + \ln \cos y$$

Taking y′ one side, we get

$$y'(\ln \cos x + x.\tan x) = \ln \cos y + y.\tan x$$

$$y' = \frac{\ln \cos y + y.\tan x}{\ln \cos x + x.\tan x}$$

Question 42.

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$(\tan \mathrm{x})^{\mathrm{y}} = (\tan \mathrm{y})^{\mathrm{x}}$$

Answer:
. So the equation given is implicit, we will just take log both sides

$$y.\ln(\tan x) = x.\ln(\tan y)$$

Now differentiate it with respect to x and consider $\frac{dy}{dx} = y'$

$$y\left(\frac{\sec^2 x}{\tan x}\right) + \ln\tan x.\, y' = x\left(\frac{\sec^2 y}{\tan y} \times y'\right) + \ln\tan y$$

Taking y′ one side, we get

$$y'\left(\ln\tan x + \frac{x}{\sin y.\cos y}\right) = \ln\tan y + \frac{y}{\sin x.\cos x}$$

$$y' = \frac{\sin 2\text{x}.\ln\tan y + 2y}{\sin 2\text{y}.\ln\tan x + 2x}$$

Question 43.
Find $\frac{\text{dy}}{\text{dx}}$, when:

$$\text{y} = ((\log \text{x})^{\text{x}} + (\text{x})^{\log \text{x}}$$

Answer:
we can write this equation as,

$$y = e^{x ln(\ln x)} + e^{\ln x.\ln x}$$

Differentiate

$$y' = (\ln x)^x\left(x\left(\frac{1}{\ln x} \times \frac{1}{x}\right) + \ln(ln\, x)\right) + x^{\ln x}\left(2.\frac{\ln x}{x}\right)$$

$$y' = (\ln x)^x\left(\frac{1}{\ln x} + \ln(ln\, x)\right) + x^{\ln x}\left(\frac{2\ln x}{x}\right)$$

Question 44.

If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$, prove that $(1-x^2)\frac{dy}{dx}=(xy+1)$.

Answer:
differentiate the given y to get the result,

$$\frac{dy}{dx}=\frac{\sqrt{1-x^2}\left(\frac{1}{\sqrt{1-x^2}}\right)-\sin^{-1}x\left(\frac{-x}{\sqrt{1-x^2}}\right)}{\left(\sqrt{1-x^2}\right)^2}$$

$$\frac{dy}{dx}=\frac{1+\frac{x.\sin^{-1}x}{\sqrt{1-x^2}}}{1-x^2}$$

$$\frac{dy}{dx}(1-x^2)=\ 1+xy$$

Question 45.

If $y=\sqrt{x+y}$, prove that $\frac{dy}{dx}=\frac{1}{(2y-1)}$.

Answer:
differentiate the given y to get the result,

$$\frac{dy}{dx}=\frac{1+\frac{dy}{dx}}{2\sqrt{x+y}}$$

$$let\,,\frac{dy}{dx}=y'$$

$y'=\frac{1+y\prime}{2\sqrt{x+y}}$ {taking y′ one side}

$$y'\left(2\sqrt{x+y}-1\right)=1$$

$$\frac{dy}{dx}=\frac{1}{2y-1}$$

Question 46.

If $x^a y^b = (x+y)^{(a+b)}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

Answer:
taking log both sides,

$$a \ln x + b \ln y = (a+b).\ln(x+y)$$

differentiating both sides,

$$\frac{a}{x} + \frac{b}{y} \times y' = \frac{a+b}{x+y} \times (1+y')$$

Take y′ one side,

$$y'\left(\frac{b}{y} - \frac{a+b}{x+y}\right) = \frac{a+b}{x+y} - \frac{a}{x}$$

$$y' = \frac{ax+bx-(ax+ay)}{x.(x+y)} \times \frac{y.(x+y)}{bx+by-(ay+by)}$$

$$y' = \frac{bx-ay}{x} \times \frac{y}{bx-ay}$$

$$y' = \frac{y}{x}$$

Question 47.

If $(x^x + y^x) = 1$, show that $\frac{dy}{dx} = -\left\{\frac{x^x(1+\log x) + y^x(\log y)}{xy^{x-1}}\right\}$

Answer:
differentiate both sides,

$$x^x(1+\ln x) + y^x\left(\frac{x}{y} \times y' + \ln y\right) = 0$$

Taking y′ one side,

$$y' = \left(\frac{x^x(1+\ln x)}{y^x} - \ln y\right) \times \frac{y}{x}$$

$$y' = \frac{x^x(1+\ln x) - y^x.\ln y}{x.y^{x-1}}$$

Question 48.

If $y = e^{\sin x} + (\tan x)^x$, prove that $\frac{dy}{dx} = e^{\sin x}\cos x + (\tan x)^x\,[2x\cos ec2x + \log\tan x]$.

Answer:
differentiate both sides,

$$y' = e^{\sin x}(\cos x) + (\tan x)^x\left(x\left(\frac{\sec^2 x}{\tan x}\right) + \ln\tan x\right)$$

$$y' = e^{\sin x}(\cos x) + (\tan x)^x(2x.cosec2x + \ln\tan x)$$

Question 49.

If $y = \log(x + \sqrt{1+x^2})$, prove that $\frac{dy}{dx} = \frac{1}{\log(x+\sqrt{1+x^2})}.\frac{1}{\sqrt{1+x^2}}$.

Answer:
differentiate both sides,

$$y' = \frac{1}{x+\sqrt{1+x^2}} \times \left(1 + \frac{x}{\sqrt{1+x^2}}\right)$$

$$y' = \frac{1}{x+\sqrt{1+x^2}} \times \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}$$

$$y' = \frac{1}{\sqrt{1+x^2}}$$

Question 50.

If $y = \log \sin \sqrt{x^2+1}$, prove that $\frac{dy}{dx} = \frac{x \cot \sqrt{x^2+1}}{\sqrt{x^2+1}}$.

Answer:
differentiate both sides,

$$y' = \frac{1}{\sin(\sqrt{1+x^2})} \times \cos\left(\sqrt{1+x^2}\right) \times \frac{x}{\sqrt{1+x^2}}$$

$$y' = \frac{\left(\cot(\sqrt{1+x^2}).x\right)}{\sqrt{1+x^2}}$$

Question 51.
If $y = \log \sqrt{\frac{1-\cos x}{1+\cos x}}$, show that $\frac{dy}{dx} = \operatorname{cosec} x$.

Answer:
differentiate both sides,

$$y' = \sqrt{\frac{1+\cos x}{1-\cos x}} \times \frac{(1+\cos x)(\sin x) - (1-\cos x)(-\sin x)}{(1+\cos x)^2}$$

$$y' = \sqrt{\frac{1+\cos x}{1-\cos x}} \times \frac{(\sin x + \sin x.\cos x) + (\sin x - \sin x.\cos x)}{(1+\cos x)^2}$$

$$y' = \sqrt{\frac{1+\cos x}{1-\cos x}} \times \frac{2\sin x}{(1+\cos x)^2}$$

$$y' = \sqrt{\frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}}} \times \frac{4\sin\frac{x}{2}.\cos\frac{x}{2}}{(1+\cos x)^2}$$

$$y' = 4\cos\frac{x}{2} \times \frac{\cos\frac{x}{2}}{4\cos^4\frac{x}{2}}$$

$$y' = \frac{1}{\cos\frac{x}{2}}$$

$$y' = \sec^2\frac{x}{2}$$

Question 52.

If $y = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, show that $\frac{dy}{dx} = \sec x$.

Answer:
differentiate both sides,

$$y' = \frac{1}{\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)} \times \sec^2\left(\frac{\pi}{4} + \frac{x}{2}\right) \times \frac{1}{2}$$

$$y' = \frac{1}{2 \times \sin\left(\frac{\pi}{4} + \frac{x}{2}\right).\cos\left(\frac{\pi}{4} + \frac{x}{2}\right)}$$

$$y' = \frac{1}{\sin\left(\frac{\pi}{2} + x\right)}$$

$$y' = \sec x$$

Question 53.

If $y = \sqrt{\frac{1 - \sin 2x}{1 + \sin 2x}}$, show that $\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} - x\right) = 0$.

Answer:
differentiate both sides,

$$y' = \frac{1}{2} \times \sqrt{\frac{1 + \sin 2x}{1 - \sin 2x}} \times \frac{(1 + \sin 2x)(-2\cos 2x) - (1 - \sin 2x)(2\cos 2x)}{(1 + \sin 2x)^2}$$

$$y' = \frac{1}{2} \times \sqrt{\frac{1}{1 - \sin^2 2x}} \times \frac{2\cos 2x(-1 - \sin 2x - 1 + \sin 2x)}{1 + \sin 2x}$$

$$y' = \frac{1}{2} \times \frac{-4}{1 + \sin 2x}$$

$$y' = \frac{-2}{(\cos x + \sin x)^2}$$

$$y' = \frac{-1}{\left(\frac{\cos x}{\sqrt{2}} + \frac{\sin x}{\sqrt{2}}\right)^2}$$

$$y' = \frac{-1}{\cos^2\left(\frac{\pi}{4} + x\right)}$$

$$\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} + x\right) = 0$$

Question 54.

If $y = \log\sqrt{\frac{1+\cos^2 x}{1-e^{2x}}}$, show that $\frac{dy}{dx} = \frac{e^{2x}}{(1-e^{2x})} - \frac{\sin x \cos x}{(1+\cos^2 x)}$.

Answer:
differentiate both sides,

$$y' = \sqrt{\frac{1-e^{2x}}{1+\cos^2 x}} \times \frac{(1-e^{2x})(-2\cos x.\sin x) - (1+\cos^2 x)(-2e^{2x})}{(1-e^{2x})^2} \times \frac{1}{2} \times \sqrt{\frac{1-e^{2x}}{1+\cos^2 x}}$$

$$y' = \sqrt{\frac{1-e^{2x}}{\cos 2x}} \times \frac{(e^{2x}-1)(\sin 2x) + 2e^{2x}(\cos 2x)}{(1-e^{2x})^2} \times \frac{1}{2} \times \sqrt{\frac{1-e^{2x}}{\cos 2x}}$$

$$y' = \frac{(e^{2x}-1)\tan 2x + 2e^{2x}}{2.(1-e^{2x})}$$

$$y' = \frac{e^{2x}}{(1-e^{2x})} - \frac{\sin x.\cos x}{(1+\cos^2 x)}$$

Question 55.

If $y = (x)^{\cos x} + (\sin x)^{\tan x}$, prove that $\frac{dy}{dx} = x^{\cos x}\left\{\frac{\cos x}{x} - (\sin x)\log x\right\} + (\sin x)^{\tan x}$. $\left\{1 + (\log \sin x)\sec^2 x\right\}$.

Answer:
simply taking log both sides would not help more.

For that let us assume $u = (x)^{\cos x}$ and $v = (\sin x)^{\tan x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (x)^{\cos x}$$

Take log both sides

$$\ln u = \cos x.\ln x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cos x\left(\frac{1}{x}\right) + \ln x(-\sin x)$$

$$\frac{du}{dx} = \left(\frac{\cos x}{x} - \ln x.\sin x\right) \times u$$

$\frac{du}{dx} = \left(\frac{\cos x}{x} - \ln x.\sin x\right) \times (x^{\cos x})$ ....(1)

$$v = (\sin x)^{\tan x}$$

Take log both sides,

$$\ln v = \tan x \times (\ln \sin x)$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \tan x \left(\frac{\cos x}{\sin x}\right) + \ln \sin x (\sec^2 x)$$

$$\frac{dv}{dx} = \left(\tan x \left(\frac{\cos x}{\sin x}\right) + \ln \sin x (\sec^2 x)\right) \times v$$

$\frac{dv}{dx} = (1 + \ln \sin x (\sec^2 x)) \times (\sin x)^{\tan x}$ ......(2)

$$\frac{dy}{dx} = (1 + \ln \sin x (\sec^2 x)) \times (\sin x)^{\tan x} + \left(\frac{\cos x}{x} - \ln x . \sin x\right) \times (x^{\cos x})$$

Question 56.

If $y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$, prove tha $\frac{dy}{dx} = (\sin x)^{\cos x} . [\cot x \cos x - \sin x (\log \sin x)] + (\cos x)^{\sin x} . [\cos x (\log \cos x) - \sin x \tan x]$.

Answer:
simply taking log both sides would not help more.

For that let us assume $u = (\sin x)^{\cos x}$ and $v = (\cos x)^{\sin x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (\sin x)^{\cos x}$$

Take log both sides

$$\ln u = \cos x . \ln \sin x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cos x \left(\frac{\cos x}{\sin x}\right) + \ln(\sin x)(-\sin x)$$

$$\frac{du}{dx} = \left(\frac{\cos^2 x}{\sin x} - \ln \sin x \,.\, \sin x\right) \times u$$

$$\frac{du}{dx} = \left(\frac{\cos^2 x}{\sin x} - \ln \sin x \,.\, \sin x\right) \times (\sin x^{\cos x}) \text{ ....(1)}$$

$$v = (\cos x)^{\sin x}$$

Take log both sides,

$$\ln v = \sin x \times (\ln \cos x)$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \sin x \left(\frac{-\sin x}{\cos x}\right) + \ln \cos x(\cos x)$$

$$\frac{dv}{dx} = \left(\sin x \left(\frac{-\sin x}{\cos x}\right) + \ln \cos x(\cos x)\right) \times v$$

$$\frac{dv}{dx} = \left(\sin x \left(\frac{-\sin x}{\cos x}\right) + \ln \cos x(\cos x)\right) \times (\cos x)^{\sin x} \text{ ......(2)}$$

$$\frac{dy}{dx} = \left(\sin x \left(\frac{-\sin x}{\cos x}\right) + \ln \cos x(\cos x)\right) \times (\cos x)^{\sin x} + \left(\frac{\cos^2 x}{\sin x} - \ln \sin x \,.\, \sin x\right) \times (\sin x^{\cos x})$$

Question 57.

If $y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$, $\frac{dy}{dx} = (\tan x)^{\cot x} \cdot \operatorname{cosec}^2 x(1 - \log \tan x)$ $+ (\cot x)^{\tan x} . \sec^2 x[\log(\cot x) - 1]$.

Answer:
simply taking log both sides would not help more.

For that let us assume $u = (\tan x)^{\cot x}$ and $v = (\cot x)^{\tan x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$u = (\tan x)^{\cot x}$

Take log both sides

$\ln u = \cot x . \ln \tan x$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cot x \left(\frac{\sec^2 x}{\tan x} \right) + \ln(\tan x)(-cosec^2 x)$$

$$\frac{du}{dx} = \left(\cot x \left(\frac{\sec^2 x}{\tan x} \right) - \ln(\tan x)(cosec^2 x) \right) \times u$$

$\frac{du}{dx} = (cosec^2 x(1 - \ln(\tan x))) \times (\tan x)^{\cot x}$ ....(1)

$v = (\cot x)^{\tan x}$

Take log both sides,

$\ln v = \tan x \times (\ln \cot x)$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \tan x \left(\frac{-cosec^2 x}{\cot x} \right) + \ln \cot x(\sec^2 x)$$

$$\frac{dv}{dx} = (\sec^2 x(\ln \cot x - 1)) \times v$$

$\frac{dv}{dx} = (\sec^2 x(\ln \cot x - 1)) \times (\cot x)^{\tan x}$ ......(2)

$$\frac{dy}{dx} = (\sec^2 x(\ln \cot x - 1)) \times (\cot x)^{\tan x} + (cosec^2 x(1 - \ln(\tan x))) \times (\tan x)^{\cot x}$$

Question 58.

If $y = x^{\cos x} + (\cos x)^{x}$, prove that $\frac{dy}{dx} = x^{\cos x}.\left\{\frac{\cos x}{x} - (\sin x)\log x\right\} + (\cos x)^{x}$ $[(\log \cos x) - x \tan x]$.

Answer:
simply taking log both sides would not help more.

For that let us assume $u = (x)^{\cos x}$ and $v = (\cos x)^{x}$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (x)^{\cos x}$$

Take log both sides

$$\ln u = \cos x . \ln x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = \cos x\left(\frac{1}{x}\right) + \ln(x)(-\sin x)$$

$$\frac{du}{dx} = \left(\cos x\left(\frac{1}{x}\right) + \ln(x)(-\sin x)\right) \times u$$

$\frac{du}{dx} = \left(\cos x\left(\frac{1}{x}\right) + \ln(x)(-\sin x)\right) \times (x)^{\cos x}$ ....(1)

$$v = (\cos x)^{x}$$

Take log both sides,

$$\ln v = x \times (\ln \cos x)$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \quad x\left(\frac{-\sin x}{\cos x}\right) + \ln\cos x.1$$

$$\frac{dv}{dx} = \left(x\left(\frac{-\sin x}{\cos x}\right) + \ln\cos x.1\right) \times v$$

$\frac{dv}{dx} = \left(x\left(\frac{-\sin x}{\cos x}\right) + \ln\cos x.1\right) \times (\cos x)^x$ ......(2)

$$\frac{dy}{dx} = \left(x\left(\frac{-\sin x}{\cos x}\right) + \ln\cos x.1\right) \times (\cos x)^x + \left(\cos x\left(\frac{1}{x}\right) + \ln(x)(-\sin x)\right) \times (x)^{\cos x}$$

Question 59.

If $y = x^{\log x} + (\log x)^x$, prove that $\frac{dy}{dx} = x^{(\log x)}\left\{\frac{2\log x}{x}\right\} + (\log x)^x.$ $\left\{\frac{1}{\log x} + \log(\log x)\right\}.$

Answer:
simply taking log both sides would not help more.

For that let us assume $u = (x)^{\ln x}$ and $v = (\ln x)^x$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (x)^{\ln x}$$

Take log both sides

$$\ln u = \ln x.\ln x$$

Differentiate

$$\frac{1}{u} \times \frac{du}{dx} = 2\ln x\left(\frac{1}{x}\right)$$

$$\frac{du}{dx} = \left(2\ln x\left(\frac{1}{x}\right)\right) \times u$$

$\frac{du}{dx} = \left(2\ln x\left(\frac{1}{x}\right)\right) \times (x)^{\ln x}$ ....(1)

$$v = (\ln x)^x$$

Take log both sides,

$$\ln v = x \times (\ln x)$$

Differentiate ,

$$\frac{1}{v} \times \frac{dv}{dx} = \mathrm{x}\left(\frac{1}{x}\right) + \ln x$$

$$\frac{dv}{dx} = (1 + \ln x) \times v$$

$\frac{dv}{dx} = (1 + \ln x) \times (\ln x)^x$ ......(2)

$$\frac{dy}{dx} = (1 + \ln x) \times (\ln x)^x + \left(2\ln x\left(\frac{1}{x}\right)\right) \times (x)^{\ln x}$$

Question 60.

If $\mathrm{y} - \mathrm{x}^{\left(\mathrm{x}^2 - 3\right)} + (\mathrm{x} - 3)^{\mathrm{x}^2}$, find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

Answer:

equality is not given but we may assume that it is equal to 0.

We can also write this equation as

$$y - e^{(x^2-3).\ln x} + e^{x^2.\ln(x-3)} = 0$$

Now differentiating it,

$$y' - x^{x^2-3}\left(\frac{x^2-3}{x} + \ln x . 2x\right) + (x-3)^{x^2} . \left(\frac{x^2}{x-3} + \ln(x-3) . 2x\right) = 0$$

$$y' = x^{x^2-3}\left(\frac{x^2-3}{x} + \ln x . 2x\right) - (x-3)^{x^2} . \left(\frac{x^2}{x-3} + \ln(x-3) . 2x\right)$$

Question 61.

If $f(x) = \left(\frac{3+x}{1+x}\right)^{2+3x}$, find $f'(0)$.

Answer:

take log both the side,

$$\ln f(x) = (2+3x) . \ln\left(\frac{3+x}{1+x}\right)$$

Now differentiate it,

$$\frac{1}{f(x)} \times f'(x) = (2+3x)\left(\frac{1+x}{3+x}\right)\left(\frac{1+x-(3+x)}{(1+x)^2}\right) + \ln\left(\frac{3+x}{1+x}\right) . 3$$

$$f'(x) = \left(\frac{(2+3x)(-2)}{(3+x)(1+x)} + 3 . \ln\left(\frac{3+x}{1+x}\right)\right) \times f(x)$$

To get f′(0) we need to find f(0),

Putting x=0 in f

$$f(0) = \left(\frac{3}{1}\right)^2$$

$$f(0) = 9$$

Now put x=0 in f′(x),

$$f'(0) = \left(\left(\frac{2 \times (-2)}{3}\right) + 3\ln 3\right) \times 9$$

$$f'(0) = 9\left(3\ln 3 - \frac{4}{3}\right)$$

Question 62.

If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$, find $\frac{dy}{dx}$.

Answer:
we can write this equation as,

$$y = e^{x\, ln(\sin x)} + \sin^{-1}\sqrt{x}$$

Differentiate it,

$$y' = (\sin x)^x\left(\frac{x \times \cos x}{\sin x} + \ln(\sin x)\right) + \frac{1}{\sqrt{1 - \sqrt{x}^2}} \times \frac{1}{2\sqrt{x}}$$

$$y' = (\sin x)^x(x.\cot x + \ln\sin x) + \frac{1}{2\sqrt{x}.\sqrt{1-x}}$$

Question 63.

If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$.

Answer:
simply differentiate both sides,

$$2(x^2 + y^2)(2x + 2y.y') = x.y' + y$$

Take y′ one side

$$4x^3 + 4x^2.y.y' + 4y^2.x + 4y^3.y' = x.y' + y$$

$$y'(4x^2.y + 4y^3 - x) = y - 4x^3 - 4y^2x$$

$$y' = \frac{y - 4x^3 - 4y^2x}{4x^2.y + 4y^3 - x}$$

Question 64.

$$y = x^{\cot x} + \frac{2x^2 - 3}{x^2 + x + 2}$$, find $\frac{dy}{dx}$.

Answer:

we can write this as,

$$y = e^{\cot x . \ln x} + \frac{2x^2 - 3}{x^2 + x + 2}$$

Differentiate ,

$$y' = x^{\cot x}\left(\frac{\cot x}{x} + \ln x(-cosec^2 x\,)\right) + \frac{(x^2 + x + 2)(4x) - (2x^2 - 3)(2x + 1)}{(x^2 + x + 2)^2}$$

$$y' = x^{\cot x}\left(\frac{\cot x}{x} + \ln x(-cosec^2 x\,)\right) + \frac{2x^2 + 14x + 3}{(x^2 + x + 2)^2}$$

Question 65.

Find , when:

If $y = \tan^{-1}\frac{a}{x} + \log\sqrt{\frac{x - a}{x + a}}$, prove that $\frac{dy}{dx} = \frac{2a^3}{(x^4 - a^4)}$.

Answer:

Differentiate it,

$$y' = \frac{1}{1 + \frac{a^2}{x^2}} \times \left(-\frac{a}{x^2}\right) + \sqrt{\frac{x + a}{x - a}} \times \frac{1}{2} \times \sqrt{\frac{x + a}{x - a}} \times \frac{(x + a) - (x - a)}{(x + a)^2}$$

$$y' = \frac{-a}{x^2 + a^2} + \frac{x - a}{2(x + a)} \times \frac{2a}{(x - a)^2}$$

$$y' = -\frac{a}{(x^2 + a^2)} + \frac{a}{x^2 - a^2}$$

$$y' = \frac{ax^2 + a^3 - ax^2 + a^3}{x^4 - a^4}$$

$$y' = \frac{2a^3}{x^4 - a^4}$$

Question 66.

If $x^m y^n = (x + y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

Answer:

taking log both sides,

$$m \ln x + n \ln y = (m + n).\ln(x + y)$$

differentiating both sides,

$$\frac{m}{x} + \frac{n}{y} \times y' = \frac{m + n}{x + y} \times (1 + y')$$

Take y′ one side,

$$y'\left(\frac{n}{y} - \frac{m + n}{x + y}\right) = \frac{m + n}{x + y} - \frac{m}{x}$$

$$y' = \frac{mx + nx - (mx + my)}{x.(x + y)} \times \frac{y.(x + y)}{nx + ny - (my + ny)}$$

$$y' = \frac{nx - my}{x} \times \frac{y}{nx - my}$$

$$y' = \frac{y}{x}$$