Exercise 28c

Question 1.

Find the distance of the point $\left(2\hat{i}-\hat{j}-4\hat{k}\right)$ from the plane $\vec{r}\cdot\left(3\hat{i}-4\hat{j}+12\hat{k}\right)=9.$

Answer:

Formula : $Distance\ = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

Plane $r.(3i - 4j\ +\ 12k)\ =\ 9$ can be written in cartesian form as

3x - 4y + 12z = 9

3x - 4y + 12z - 9 = 0

Point = (2i - j - 4k)

Which can be also written as

Point = (2 , - 1 , - 4)

Distance $= \frac{|(2\times3) + (-1\times-4) + (-4\times12) + (-9)|}{\sqrt{(3)^2 + (-4)^2 + 12^2}}$

$= \frac{|6\ +\ 4 - 48 - 9|}{\sqrt{9\ +\ 16\ +\ 144}}$

$= \frac{|-47|}{\sqrt{169}}$

$= \frac{47}{13}\ units$

Question 2.

Find the distance of the point $\left(\hat{i}+2\hat{j}+5\hat{k}\right)$ from the plane $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)+17=0.$

Answer:

Formula : $Distance\ = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

Plane $r.(i\ +\ j\ +\ k)\ +\ 17\ =\ 0$ can be written in cartesian form as

x + y + z + 17 = 0

Point = (i + 2j + 5k)

Which can be also written as

Point = (1 , 2 , 5)

Distance $= \frac{|(1\times1) + (2\times1) + (5\times1) + (17)|}{\sqrt{(1)^2 + (1)^2 + 1^2}}$

$$= \frac{|1\ +\ 2\ +\ 5\ +\ 17|}{\sqrt{1\ +\ 1\ +\ 1}}$$

$$= \frac{|25|}{\sqrt{3}}$$

$$= \frac{25\sqrt{3}}{3}\ units$$

Question 3.

Find the distance of the point (3, 4, 5) from the plane $\vec{r}\cdot\left(2\hat{i}-5\hat{j}+3\hat{k}\right)=13.$

Answer:

Formula : $Distance = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

Plane $r.(2i - 5j + 3k) = 13$ can be written in cartesian form as

2x - 5y + 3z = 13

2x - 5y + 3z - 13 = 0

Point = (3 , 4 , 5)

Distance $= \frac{|(3\times2) + (4\times-5) + (5\times3) - (13)|}{\sqrt{(2)^2 + (-5)^2 + 3^2}}$

$$= \frac{|6 - 20 + 15 - 13|}{\sqrt{4 + 25 + 9}}$$

$$= \frac{|-12|}{\sqrt{38}}$$

$$= \frac{12\sqrt{38}}{38} = \frac{6\sqrt{38}}{19}\ units$$

Question 4.

Find the distance of the point (1, 1, 2) from the plane $\vec{r} \cdot \left(2\hat{i} - 2\hat{j} + 4\hat{k}\right) + 5 = 0.$

Answer:

Formula : $Distance = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

Plane $r.(2i - 2j + 4k) + 5 = 0$ can be written in cartesian form as

2x - 2y + 4z + 5 = 0

Point = (1 , 1 , 2)

Distance $= \frac{|(1\times2)+(1\times-2)+(2\times4)+(5)|}{\sqrt{(2)^2+(-2)^2+(4)^2}}$

$$= \frac{|2 - 2 + 8 + 5|}{\sqrt{4 + 4 + 16}}$$

$$= \frac{|13|}{\sqrt{24}}$$

$$= \frac{13}{2\sqrt{6}} = \frac{13\sqrt{6}}{12}\ units$$

Question 5.
Find the distance of the point (2, 1, 0) from the plane 2x + y + 2z + 5 = 0.

Answer:
Formula : $Distance = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

2x + y + 2z + 5 = 0

Point = (2 , 1 , 0)

Distance $= \frac{|(2\times2)+(1\times1)+(0\times2)+(5)|}{\sqrt{(2)^2+(1)^2+(2)^2}}$

$$= \frac{|4 + 1 + 0 + 5|}{\sqrt{4 + 1 + 4}}$$

$$= \frac{|10|}{\sqrt{9}}$$

$$= \frac{10}{3}\ units$$

Question 6.
Find the distance of the point (2, 1, - 1) from the plane x – 2y + 4z = 9.

Answer:
Formula : $Distance\ = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

x - 2y + 4z = 9

x - 2y + 4z – 9 = 0

Point = (2 , 1 , - 1)

Distance $= \frac{|(2\times1) + (1\times-2) + (-1\times4) - (9)|}{\sqrt{(1)^2 + (-2)^2 + (4)^2}}$

$$= \frac{|2 - 2 - 4 - 9|}{\sqrt{1\ +\ 4\ +\ 16}}$$

$$= \frac{|-13|}{\sqrt{21}}$$

$$= \frac{13}{\sqrt{21}} = \frac{13\sqrt{21}}{21}\ units$$

Question 7.

Show that the point (1, 2, 1) is equidistant from the planes $\vec{r}\cdot\left(\hat{i}+2\hat{j}-2\hat{k}\right)=5$ and $\vec{r}\cdot\left(2\hat{i}-2\hat{j}+\hat{k}\right)+3=0.$

Answer:

Formula : $Distance \;=\; \frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

First Plane $r.(i\;+\;2j-2k)\;=\;5$ can be written in cartesian form as

x + 2y - 2z = 5

x + 2y - 2z – 5 = 0

Point = (1 , 2 , 1)

Distance for first plane $= \frac{|(1\times1)+(2\times2)+(1\times-2)-(5)|}{\sqrt{(1)^2+(2)^2+(-2)^2}}$

$$= \frac{|1\;+\;4-2-5|}{\sqrt{1\;+\;4\;+\;4}}$$

$$= \frac{|-2|}{\sqrt{9}}$$

$$= \frac{2}{3}\;units$$

Second Plane $r.(2i-2j\;+\;k)\;+\;3\;=\;0$ can be written in cartesian form as

2x - 2y + z + 3 = 0

Point = (1 , 2 , 1)

Distance for second plane $= \frac{|(1\times2)+(2\times-2)+(1\times1)+(3)|}{\sqrt{(2)^2+(-2)^2+(1)^2}}$

$= \frac{|2-4\ +\ 1\ +\ 3|}{\sqrt{4\ +\ 4\ +\ 1}}$

$= \frac{|2|}{\sqrt{9}}$

$= \frac{2}{3}\ units$

Hence proved.

Question 8.
Show that the points (- 3, 0, 1) and (1, 1, 1) are equidistant from the plane 3x + 4y – 12z + 13 = 0.

Answer:
Formula : $Distance\ = \frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

Plane = 3x + 4y - 12z + 13 = 0

First Point = (- 3 , 0 , 1)

Distance for first point $= \frac{|(-3\times3)+(0\times4)+(1\times-12)+(13)|}{\sqrt{(3)^2+(4)^2+(-12)^2}}$

$= \frac{|-9\ +\ 0-12\ +\ 13|}{\sqrt{9\ +\ 16\ +\ 144}}$

$= \frac{|-8|}{\sqrt{169}}$

$= \frac{8}{13}\ units$

Plane = 3x + 4y - 12z + 13 = 0

Second Point = (1 , 1 , 1)

Distance for first point $= \frac{|(1\times3)+(1\times4)+(1\times-12)+(13)|}{\sqrt{(3)^2+(4)^2+(-12)^2}}$

$$= \frac{|3\ +\ 4 - 12\ +\ 13|}{\sqrt{9\ +\ 16\ +\ 144}}$$

$$= \frac{|8|}{\sqrt{169}}$$

$$= \frac{8}{13}\ units$$

Hence proved.

Question 9.
Find the distance between the parallel planes 2x + 3y + 4 = 4 and 4x + 6y + 8z = 12.

Answer:
Formula : The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Plane 2:ax + by + cz + d2 = 0 is given by the formula

$$Distance\ = \frac{|d_2 - d_1|}{\sqrt{a^2\ +\ b^2\ +\ c^2}}$$

where (d_1, d_2) are constants of the planes

Therefore ,

First Plane 2x + 3y + 4 = 4

2x + 3y + 4 – 4 = 0 ...... (1)

Second plane 4x + 6y + 8z = 12

4x + 6y + 8z - 12 = 0

2(2x + 3y + 4z – 6) = 0

2x + 3y + 4z – 6 = 0 ...... (2)

Using equation (1) and (2)

Distance between both planes $= \frac{|-6-(-4)|}{\sqrt{(2)^2+(3)^2+(4)^2}}$

$$= \frac{|-6 + 4|}{\sqrt{4 + 9 + 16}}$$

$$= \frac{|-2|}{\sqrt{29}}$$

$$= \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}\ units$$

Question 10.
Find the distance between the parallel planes x + 2y - 2z + 4 = 0 and x + 2y – 2z – 8 = 0.

Answer:
Formula : The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Palne 2:ax + by + cz + d2 = 0 is given by the formula

$$Distance = \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

where (d_1, d_2) are costants of the planes

Therefore ,

First Plane x + 2y - 2z + 4 = 0 ...... (1)

Second plane x + 2y – 2z – 8 = 0 ...... (2)

Using equation (1) and (2)

Distance between both planes = $\frac{|-8-(4)|}{\sqrt{(1)^2+(2)^2+(2)^2}}$

$$= \frac{|-12|}{\sqrt{1 + 4 + 4}}$$

$$= \frac{12}{\sqrt{9}}$$

$$= \frac{12}{3} = 4\,units$$

Question 11.
Find the equation of the planes parallel to the plane x – 2y + 2z – 3 = 0, each one of which is at a unit distance from the point (1, 1, 1).

Answer:
Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same

Therefore ,

Parallel Plane x – 2y + 2z – 3 = 0

Normal vector = (i - 2j + 2k)

∴ Normal vector of required plane = (i - 2j + 2k)

Equation of required planes r . (i - 2j + 2k) = d

In cartesian form x – 2y + 2y = d

It should be at unit distance from point (1,1,1)

Distance $= \frac{|(1\times1)+(1\times-2)+(1\times2)-(d)|}{\sqrt{(1)^2+(-2)^2+(2)^2}}$

$= \frac{|1-2\ +\ 2-d|}{\sqrt{1\ +\ 4\ +\ 4}}$

$= \frac{|1-d|}{\sqrt{9}}$

$1\ =\ \frac{\pm(1-d)}{3}$

$3\ =\ \pm(1-d)$

For + sign = > 3 = 1 - d = > d = - 2

For - sign = > 3 = - 1 + d = > d = 4

Therefore equations of planes are : -

For d = - 2 For d = 4

x – 2y + 2y = d x – 2y + 2y = d

x – 2y + 2y = - 2 x – 2y + 2y = 4

x – 2y + 2y + 2 = 0 x – 2y + 2y – 4 = 0

Required planes = x – 2y + 2y + 2 = 0

x – 2y + 2y – 4 = 0

Question 12.
Find the equation of the plane parallel to the plane 2x – 3y + 5z + 7 = 0 and passing through the point (3, 4, - 1). Also, find the distance between the two planes.

Answer:
Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Palne 2:ax + by + cz + d2 = 0 is given by the formula

$$Distance = \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

If two planes are parallel , then their normal vectors are same

Therefore ,

Parallel Plane 2x – 3y + 5z + 7 = 0

Normal vector = (2i - 3j + 5k)

∴ Normal vector of required plane = (2i - 3j + 5k)

Equation of required plane r . (2i - 3j + 5k) = d

In cartesian form 2x – 3y + 5y = d

Plane passes through point (3,4, - 1) therefore it will satisfy it.

2(3) – 3(4) + 5(- 1) = d

6 – 12 - 5 = d

d = - 11

Equation of required plane 2x – 3y + 5z = - 11

2x – 3y + 5z + 11 = 0

Therefore ,

First Plane 2x – 3y + 5z + 7 = 0 ...... (1)

Second plane 2x – 3y + 5z + 11 = 0 ...... (2)

Using equation (1) and (2)

Distance between both planes $= \frac{|11-(7)|}{\sqrt{(2)^2+(-3)^2+(5)^2}}$

$$= \frac{|4|}{\sqrt{4 + 9 + 25}}$$

$$= \frac{4}{\sqrt{38}}$$

$$= \frac{4\sqrt{38}}{38} = \frac{2\sqrt{38}}{19}\ units$$

Question 13.
Find the equation of the plane mid - parallel to the planes 2x – 3y + 6z + 21 = 0 and 2x – 3y + 6z – 14 = 0

Answer:
Formula : The equation of mid parallel plane is , say

Plane 1:ax + by + cz + d1 = 0 &

Plane 2:ax + by + cz + d2 = 0 is given by the formula

$$Mid\ parallel\ plane\ = ax + by + cy + \frac{(d_1 + d_2)}{2} = 0$$

where (d_1, d_2) are constants of the planes

Therefore ,

First Plane 2x – 3y + 6z + 21 = 0 …… (1)

Second plane 2x – 3y + 6z – 14 = 0 …… (2)

Using equation (1) and (2)

Mid parallel plane = $2x - 3y + 6z + \frac{21-14}{2} = 0$

4x – 6y + 12z + 7 = 0