Exercise 13c

Question 1.
Evaluate the following integrals:

$$\int x e^{x} dx$$

Answer:
Using BY PART METHOD.

Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function and e^{x} is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x.e^{x}dx = x\int e^{x} - \int\frac{dx}{dx}.\int e^{x}dx$$

$$= xe^{x} - \int 1.e^{x}dx$$

$$= xe^{x} - e^{x} + c$$

$$= e^{x}\left(x-1\right)+c$$

Question 2.
Evaluate the following integrals:

$$\int x \cos x \, dx$$

Answer:
Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and cos x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\cos xdx = x\int \cos x - \int\left[\frac{dx}{dx}.\int \cos xdx\right]dx$$

$$= x\sin x - \int 1.\sin xdx$$

$$= x\sin x + \cos x + c$$

Question 3.
Evaluate the following integrals:

$$\int x\,e^{2x}dx$$

Answer:
Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function and e^{2x} is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int xe^{2x}dx = x\int e^{2x}dx - \int\left[\frac{dx}{dx}.\int e^{2x}dx\right]dx$$

$$= x\frac{e^{2x}}{2} - \int 1.\frac{e^{2x}}{2}dx$$

$$= x\frac{e^{2x}}{2} - \frac{e^{2x}}{2\times 2} + c$$

$$= x\frac{e^{2x}}{2} - \frac{e^{2x}}{4} + c$$

Question 4.
Evaluate the following integrals:

$$\int x \sin 3x \, dx$$

Answer:
Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and Sin 3x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\sin 3xdx = x\int \sin 3xdx - \int\left[\frac{dx}{dx}.\int \sin 3xdx\right]dx$$

$$= x\left(\frac{-\cos 3x}{3}\right) - \int 1.\left(\frac{-\cos 3x}{3}\right)dx$$

$$= x\left(\frac{-\cos 3x}{3}\right) + \left(\frac{\sin 3x}{3\times 3}\right) + c$$

$$= x\left(\frac{-\cos 3x}{3}\right) + \left(\frac{\sin 3x}{9}\right) + c$$

Question 5.
Evaluate the following integrals:

$$\int x\cos 2x \, dx$$

Answer:
Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and Cos 2x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\cos 2xdx = x\int \cos 2xdx - \int\left[\frac{dx}{dx}.\int \cos 2xdx\right]dx$$

$$= x\left(\frac{\sin 2x}{2}\right) - \int 1.\left(\frac{\sin 2x}{2}\right)dx$$

$$= x\left(\frac{\sin 2x}{2}\right) + \left(\frac{\cos 2x}{2\times 2}\right) + c$$

$$= x\left(\frac{\sin 2x}{2}\right) + \left(\frac{\cos 2x}{4}\right) + c$$

Question 6.
Evaluate the following integrals:

$$\int x \log 2x \, dx$$

Answer:
Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log 2x is the first function, and x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x \log 2x dx = \log 2x \int x dx - \int \left[\frac{d \log 2x}{dx} . \int x dx \right] dx$$

$$= \log 2x . \frac{x^2}{2} - \int \left[\frac{1 \times 2}{2x} \frac{x^2}{2} \right] dx$$

$$= \frac{x^2}{2} \log 2x - \int \frac{x}{2} dx$$

$$= \frac{x^2}{2} \log 2x - \frac{x^2}{2 \times 2} + c$$

$$= \frac{x^2}{2} \log 2x - \frac{x^2}{4} + c$$

Question 7.

Evaluate the following integrals:

$$\int x \ \operatorname{cosec}^2 x \ dx$$

Answer:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and $\operatorname{cosec}^2 x$ is the second function.

Using Integration by part

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} . \int b dx \right] dx$$

$$\Rightarrow \int x \operatorname{cosec}^2 x dx = x \int \operatorname{cosec}^2 x - \int \left[\frac{dx}{dx} . \int \operatorname{cosec}^2 x dx \right] dx$$

$$= x(-\cot x) - \int 1.(-\cot x) dx$$

$$= -x \cot x + \int \cot x dx$$

$$= -x \cot x + \ln |\sin x| + c$$

Question 8.

Evaluate the following integrals:

$$\int x^2 \cos x \, dx$$

Answer:
Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and cos x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x^2 \cos x dx = x^2 \int \cos x dx - \int\left[\frac{dx^2}{dx}.\int \cos x dx\right]dx$$

$$= x^2 \sin x - \int[2x \times \sin x]dx$$

$$= x^2 \sin x - 2\left[\int x \sin x dx\right]$$

Again applying by the part method in the second half, we get

$$x^2 \sin x - 2\int x \sin x dx$$

$$= x^2 \sin x - 2\left[x\int \sin x dx - \int\left(\frac{dx}{dx}.\int \sin x dx\right)dx\right]$$

$$= x^2 \sin x - 2\left[x(-\cos x) - \int 1.(-\cos x)dx\right]$$

$$= x^2 \sin x - 2[-x\cos x + \sin x] + c$$

$$= x^2 \sin x + 2x\cos x - 2\sin x + c$$

Question 9.
Evaluate the following integrals:

$$\int x \sin^2 x \, dx$$

Answer:
Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

Writing $\mathrm{Sin}^2 x = \dfrac{1+\cos 2x}{2}$

We have

$$\int x\sin^2 xdx = \int x\left(\frac{1-\cos 2x}{2}\right)dx$$

$$= \int\left(\frac{x}{2} - \frac{x\cos 2x}{2}\right)dx$$

$$= \int\frac{x}{2}dx - \int\frac{x\cos 2x}{2}dx$$

$$= \frac{x^2}{2\times 2} - \frac{1}{2}\int x\cos 2xdx$$

Taking X as first function and Cos 2x as the second function.

$$= \frac{x^2}{4} - \frac{1}{2}\left\{x\int\cos 2xdx - \int\left(\frac{dx}{dx}.\int\cos 2xdx\right)dx\right\}$$

$$= \frac{x^2}{4} - \frac{1}{2}\left\{x.\frac{\sin 2x}{2} - \int\left(1.\frac{\sin 2x}{2}\right)dx\right\}$$

$$= \frac{x^2}{4} - \frac{1}{2}\left\{\frac{x\sin 2x}{2} - \left(\frac{-\cos 2x}{2\times 2}\right)\right\} + c$$

$$= \frac{x^2}{4} - \frac{1}{2}\left\{\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right\} + c$$

$$= \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + c$$

Question 10.
Evaluate the following integrals:

$$\int x \tan^2 x \, dx$$

Answer:
Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

Writing $\tan^2 x = \sec^2 x - 1$

We have

$$\int x\tan^2 xdx = \int x\left(\sec^2 x - 1\right)dx$$
$$= \int x\sec^2 xdx - \int xdx$$

Using x as the first function and Sec^2x as the second function

$$\int x\sec^2 xdx - \int xdx$$
$$= \left\{x\int \sec^2 xdx - \int\left(\frac{dx}{dx}.\int \sec^2 xdx\right)dx\right\} - \frac{x^2}{2}$$
$$= \left\{x.\tan x - \int 1.\tan xdx\right\} - \frac{x^2}{2}$$
$$= x\tan x - \ln|\sec x| - \frac{x^2}{2} + c$$
$$= x\tan x - \ln\left|\frac{1}{\cos x}\right| - \frac{x^2}{2} + c$$
$$x\tan x + \ln|\cos x| - \frac{x^2}{2} + c$$

Question 11.
Evaluate the following integrals:

$$\int x^2 e^x dx$$

Answer:
Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and e^x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x^2e^xdx = \left[x^2\int e^xdx - \int\left(\frac{dx^2}{dx}.\int e^xdx\right)dx\right]$$

$$= x^2e^x - \int 2x.e^xdx$$

$$= x^2e^x - 2\int xe^xdx$$

$$= x^2e^x - 2\left[x\int e^xdx - \int\left(\frac{dx}{dx}.\int e^xdx\right)dx\right]$$

$$= x^2e^x - 2\left[xe^x - \int 1.e^xdx\right]$$

$$= x^2e^x - 2\left[xe^x - e^x\right] + c$$

$$= x^2e^x - 2xe^x + 2e^x + c$$

$$= e^x\left(x^2 - 2x + 2\right) + c$$

Question 12.
Evaluate the following integrals:

$$\int x^2 \cos^3 x\ dx$$

Answer:
We know that Cos3x = 4Cos^{3}x - 3Cosx

Cos^{3}x = $\frac{\cos 3x + 3\cos x}{4}$

$$\int x^2 \cos^3 x dx = \int x^2\left(\frac{\cos 3x + 3\cos x}{4}\right)dx$$
$$= \frac{1}{4}\left(\int x^2 \cos 3x dx + 3\int x^2 \cos x dx\right)$$

Taking X^2 as the first function and cos 3x and cos x as the second function and applying By part method.

$$\frac{1}{4}\left(\int x^2 \cos 3x dx + 3\int x^2 \cos x dx\right)$$
$$= \frac{1}{4}\left\{\left(x^2\int \cos 3x dx - \int\left[\frac{dx^2}{dx}.\int \cos 3x dx\right]dx\right) + 3\left(x^2\int \cos x dx - \int\left[\frac{dx^2}{dx}.\int \cos x dx\right]dx\right)\right\}$$
$$= \frac{1}{4}\left\{\left(\frac{x^2.\sin 3x}{3} - \int 2x.\frac{\sin 3x}{3}dx\right) + 3\left(x^2 \sin x - \int 2x.\sin x dx\right)\right\}$$
$$= \frac{1}{4}\left\{\left(\frac{x^2 \sin 3x}{3} - \frac{2}{3}\int x \sin 3x dx\right) + 3\left(x^2 \sin x - 2\int x \sin x dx\right)\right\}$$
$$= \frac{1}{4}\left\{\left(\frac{x^2 \sin 3x}{3} - \frac{2}{3}\left[x\int \sin 3x dx - \int\left(\frac{dx}{dx}.\int \sin 3x dx\right)dx\right]\right) + 3\left(x^2 \sin x - 2\left[x\int \sin x dx - \int\left(\frac{dx}{dx}.\int \sin x dx\right)dx\right]\right)\right\}$$
$$= \frac{1}{4}\left\{\left(\frac{x^2 \sin 3x}{3} - \frac{2}{3}\left[x\frac{-\cos 3x}{3} - \int 1.\frac{-\cos 3x}{3}dx\right]\right) + 3\left(x^2 \sin x - 2\left[-x \cos x - \int -\cos x dx\right]\right)\right\}$$
$$= \frac{1}{4}\left\{\left(\frac{x^2 \sin 3x}{3} - \frac{2}{3}\left[\frac{-x \cos 3x}{3} + \frac{\sin 3x}{9}\right]\right) + 3\left(x^2 \sin x + 2x \cos x - 2 \sin x\right)\right\} + c$$
$$= \frac{1}{4}\left\{\frac{x^2 \sin 3x}{3} + \frac{2x \cos 3x}{9} - \frac{2 \sin 3x}{27} + 3x^2 \sin x + 6x \cos x - 6 \sin x\right\} + c$$
$$= \frac{x^2 \sin 3x}{12} + \frac{x \cos 3x}{18} - \frac{\sin 3x}{54} + \frac{3x^2 \sin x}{4} + \frac{3x \cos x}{2} - \frac{3}{2}\sin x + c$$

Question 13.
Evaluate the following integrals:

$$\int x^2 e^{3x} dx$$

Answer:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and e^{3x} is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x^2 e^{3x}dx = x^2\int e^{3x}dx - \int\left(\frac{dx^2}{dx}.\int e^{3x}dx\right)dx$$

$$= x^2\frac{e^{3x}}{3} - \int 2x.\frac{e^{3x}}{3}dx$$

$$= x^2\frac{e^{3x}}{3} - \frac{2}{3}\int xe^{3x}dx$$

$$= x^2\frac{e^{3x}}{3} - \frac{2}{3}\left(x\int e^{3x}dx - \int\left[\frac{dx}{dx}.\int e^{3x}dx\right]dx\right)$$

$$= x^2\frac{e^{3x}}{3} - \frac{2}{3}\left(x\frac{e^{3x}}{3} - \int\frac{e^{3x}}{3}dx\right)$$

$$= x^2\frac{e^{3x}}{3} - \frac{2}{3}\left(x\frac{e^{3x}}{3} - \frac{e^{3x}}{9}\right) + c$$

$$= x^2\frac{e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2e^{3x}}{27} + c$$

$$= e^{3x}\left(\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27}\right) + c$$

Question 14.

Evaluate the following integrals:

$$\int x^2\sin^2 x\,dx$$

Answer:

We can write $\sin^2 x = \frac{1-\cos 2x}{2}$

We have

$$\int x^2\left(\frac{1-\cos 2x}{2}\right)dx = \int \frac{x^2}{2} - \frac{x^2\cos 2x}{2}dx$$
$$= \int \frac{x^2}{2}dx - \int \frac{x^2\cos 2x}{2}dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and Cos 2x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$= \frac{x^3}{3 \times 2} - \frac{1}{2}\int x^2 \cos 2x dx$$

$$= \frac{x^3}{6} - \frac{1}{2}\left(x^2 \int \cos 2x dx - \int \left[\frac{dx^2}{dx} . \int \cos 2x dx\right] dx\right)$$

$$= \frac{x^3}{6} - \frac{1}{2}\left(x^2 . \frac{\sin 2x}{2} - \int 2x . \frac{\sin 2x}{2} dx\right)$$

$$= \frac{x^3}{6} - \frac{1}{2}\left(x^2 . \frac{\sin 2x}{2} - \int x . \sin 2x dx\right)$$

$$= \frac{x^3}{6} - \frac{1}{2}\left(x^2 . \frac{\sin 2x}{2} - \left[x \int \sin 2x dx - \int \left(\frac{dx}{dx} . \int \sin 2x dx\right) dx\right]\right)$$

$$= \frac{x^3}{6} - \frac{1}{2}\left(x^2 . \frac{\sin 2x}{2} - \left[x \frac{-\cos 2x}{2} - \int 1 . \frac{-\cos 2x}{2} dx\right]\right)$$

$$= \frac{x^3}{6} - \frac{1}{2}\left(x^2 . \frac{\sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{\sin 2x}{4}\right) + c$$

$$= \frac{x^3}{6} - \frac{x^2 \sin 2x}{4} - \frac{x \cos 2x}{4} + \frac{\sin 2x}{8} + c$$

Question 15.
Evaluate the following integrals:

$$\int x^3 \log\ 2x\ dx$$

Answer:
Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log2x is the first function, and x^3 is the second function.

Using Integration by part

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} . \int b dx\right] dx$$

$$\int x^3 \log 2x dx = \log 2x \int x^3 dx - \int\left(\frac{d \log 2x}{dx} . \int x^3 dx\right) dx$$

$$= \log 2x \frac{x^4}{4} - \int \frac{1.2}{2x} . \frac{x^4}{4} dx$$

$$= \log 2x \frac{x^4}{4} - \frac{1}{4} \int x^3 dx$$

$$= \log 2x \frac{x^4}{4} - \frac{1}{4} . \frac{x^4}{4} + c$$

$$= \log 2x \frac{x^4}{4} - \frac{x^4}{16} + c$$

Question 16.

Evaluate the following integrals:

$$\int x \cdot \log(x+1) dx$$

Answer:

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log(x + 1) is first function and x is second function.

$$\int a.b.dx = a \int b dx - \int\left[\frac{da}{dx} . \int b dx\right] dx$$

$$\int x \log(x+1) = \log(x+1) \int x dx - \int\left(\frac{d \log(x+1)}{dx} . \int x dx\right) dx$$

$$= \log(x+1) \frac{x^2}{2} - \int \frac{1}{x+1} \times \frac{x^2}{2} dx$$

$$= \log(x+1) \frac{x^2}{2} - \frac{1}{2} \int \frac{x^2 - 1 + 1}{x+1} dx$$

Adding and subtracting 1 in the numerator,

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int \frac{x^2-1}{x+1} + \frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int \frac{(x+1)(x-1)}{x+1} + \frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int (x-1) + \frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\frac{x^2}{2} - x + \log(x+1)\right] + c$$

$$= \log(x+1)\frac{x^2}{2} - \frac{x^2}{4} + \frac{x}{2} - \frac{\log(x+1)}{2} + c$$

$$= \log(x+1)\frac{x^2-1}{2} - \frac{x^2}{4} + \frac{x}{2} + c$$

Question 17.
Evaluate the following integrals:

$$\int \frac{\log x}{x^n} dx$$

Answer:
We can write it as $\int x^{-n}.\log x dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logx is the first function, and x^{-n} is the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x^{-n}\log x dx = \log x\int x^{-n}dx - \int\left(\frac{d\log x}{dx}.\int x^{-n}dx\right)dx$$

$$= \log x\left(\frac{x^{-n+1}}{-n+1}\right) - \int\frac{1}{x}.\frac{x^{-n+1}}{-n+1}dx$$

$$= \frac{x^{-n+1}\log x}{1-n} + \frac{1}{1-n}\int\frac{x^{-n}.x}{x}dx$$

$$= \frac{x^{-n+1}\log x}{1-n} + \frac{1}{1-n}\times\frac{x^{-n+1}}{-n+1} + c$$

$$= \frac{x^{-n+1}\log x}{1-n} - \frac{x^{-n+1}}{(1-n)^2} + c$$

Question 18.
Evaluate the following integrals:

$$\int 2x^3 e^{x^2}\,dx$$

Answer:
We can write it as $\int 2.x.x^2.e^{x^2}dx$

Let $x^2 = t$

$2xdx = dt$

Using the relation in the above condition, we get

$$\int 2x.x^2.e^{x^2}dx = \int t.e^t dt$$

Integrating with respect to t

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function, and e^t is the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int te^t dt = t\int e^t dt - \int\left(\frac{dt}{dt}.\int e^t dt\right)dt$$

$$= te^t - \int 1.e^t dt$$

$$= te^t - e^t + c$$

Replacing t with x^2,we get

$$x^2 e^{x^2} - e^{x^2} + c$$

$$= e^{x^2}\left(x^2 - 1\right) + c$$

Question 19.
Evaluate the following integrals:

$$\int x\sin^3 x\ dx$$

Answer:
We know that Sin3x = 3Sinx - 4Sin^{3}x

Sin^{3}x = (3Sinx - Sin3x)/4

$$\int x\sin^3 x dx = \int x\left(\frac{3\sin x - \sin 3x}{4}\right)dx$$

$$= \frac{1}{4}\int 3x\sin x - x\sin 3x dx$$

$$= \frac{3}{4}\int x\sin x dx - \frac{1}{4}\int x\sin 3x dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and sinx and sin3x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$= \frac{3}{4}\int x\sin xdx - \frac{1}{4}\int x\sin 3xdx$$

$$= \frac{3}{4}\left(x\int \sin xdx - \int\left[\frac{dx}{dx}.\int \sin xdx\right]dx\right) - \frac{1}{4}\left(s\int \sin 3xdx - \int\left[\frac{dx}{dx}.\int \sin 3xdx\right]dx\right)$$

$$= \frac{3}{4}\left(-x\cos x + \int \cos xdx\right) - \frac{1}{4}\left(\frac{-x\cos 3x}{3} + \int\frac{\cos 3x}{3}dx\right)$$

$$= \frac{3}{4}(-x\cos x + \sin x) - \frac{1}{4}\left(\frac{-x\cos 3x}{3} + \frac{\sin 3x}{9}\right) + c$$

$$= \frac{-3x\cos x}{4} + \frac{3\sin x}{4} + \frac{x\cos 3x}{12} - \frac{\sin 3x}{36} + c$$

Question 20.
Evaluate the following integrals:

$$\int x\ \cos^3 x\ dx$$

Answer:
We can write $\cos^3 x = (\cos 3x + 3\cos x)/4$, we have

$$\int x\cos^3 xdx = \int x\left(\frac{\cos 3x + 3\cos x}{4}\right)dx$$

$$= \frac{1}{4}\int x\cos 3xdx + \frac{3}{4}\int x\cos xdx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and cosx and cos3x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$=\frac{1}{4}\left(x\int\cos 3x dx-\int\left[\frac{dx}{dx}.\int\cos 3x dx\right]dx\right)+\frac{3}{4}\left(x\int\cos x dx-\int\left[\frac{dx}{dx}.\int\cos x dx\right]dx\right)$$

$$=\frac{1}{4}\left(x\frac{\sin 3x}{3}-\int\frac{\sin 3x}{3}dx\right)+\frac{3}{4}\left(x\sin x-\int\sin x dx\right)$$

$$=\frac{1}{4}\left(\frac{x\sin 3x}{3}+\frac{\cos 3x}{9}\right)+\frac{3}{4}(x\sin x+\cos x)+c$$

$$=\frac{x\sin 3x}{12}+\frac{\cos 3x}{36}+\frac{3x\sin x}{4}+\frac{3\cos x}{4}+c$$

Question 21.
Evaluate the following integrals:

$$\int x^3\cos x^2 dx$$

Answer:
We can write it as

$$\int x.x^2\cos x^2 dx$$

Now let $x^2 = t$

$2xdx = dt$

$Xdx = dt/2$

Now

$$\frac{1}{2}\int t\cos t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and cost as the second function.

$$\int a.b.dx = a\int b dx-\int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\frac{1}{2}\int t\cos t dt = \frac{1}{2}\left(t\int \cos t dt - \int\left[\frac{dt}{dt}.\int \cos t dt\right]dt\right)$$

$$= \frac{1}{2}\left(t\sin t - \int \sin t dt\right)$$

$$= \frac{1}{2}\left(t\sin t + \cos t\right) + c$$

Replacing t with x^2

$$= \frac{1}{2}x^2 \sin x^2 + \frac{1}{2}\cos x^2 + c$$

Question 22.
Evaluate the following integrals:

$$\int \sin x \log(\cos x)dx$$

Answer:
Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log(cosx) is the first function and sinx as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \sin x \log(\cos x)dx = \log(\cos x)\int \sin x dx - \int\left(\frac{d\log(\cos x)}{dx}.\int \sin x dx\right)dx$$

$$= -\cos x \log(\cos x) + \int \frac{-\sin x}{\cos x}.\cos x dx$$

$$= -\cos x \log(\cos x) - \int \sin x dx$$

$$= -\cos x \log(\cos x) + \cos x + c$$

Question 23.

Evaluate the following integrals:

$$\int x \sin x \cos x \, dx$$

Answer:
We know that Sin2x = 2Sinxcosx

$$\int x \sin x \cos x dx = \frac{1}{2}\int x \sin 2x dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and sin2x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\frac{1}{2}\int x \sin 2xdx = \frac{1}{2}\left(x\int \sin 2xdx - \int\left[\frac{dx}{dx}.\int \sin 2xdx\right]dx\right)$$

$$= \frac{1}{2}\left(x\frac{-\cos 2x}{2} + \int\frac{\cos 2x}{2}dx\right)$$

$$= \frac{1}{2}\left(\frac{-x\cos 2x}{2} + \frac{\sin 2x}{4}\right) + c$$

$$= \frac{-x\cos 2x}{4} + \frac{\sin 2x}{8} + c$$

Question 24.
Evaluate the following integrals:

$$\int \cos\sqrt{x} \, dx$$

Answer:
Let $\sqrt{x} = t$

$$\frac{1}{2\sqrt{x}}dx = dt$$
$$\Rightarrow dx = 2\sqrt{x}dt$$
$$\Rightarrow dx = 2tdt$$

We can write it as

$$\int \cos\sqrt{x}dx = 2\int t\cos tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is first function and cos t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow 2\int t\cos tdt = 2\left(t\int \cos tdt - \int\left[\frac{dt}{dt}\right]\int \cos tdt\right)dt$$
$$= 2\left(t\sin t - \int \sin tdt\right)$$
$$= 2t\sin t + 2\cos t + c$$

Replacing t with √x

= 2√xsin√x + 2cos√x + c

= 2(cos√x + √xsin√x) + c

Question 25.
Evaluate the following integrals:

$$\int \operatorname{cosec}^3 x\, dx$$

Answer:

We can write it as $\int \operatorname{cosec}^3 dx = \int \operatorname{cosec}x.\operatorname{cosec}^2 x dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here cosecx is first function and cosec^{2}x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \operatorname{cosec}x.\operatorname{cosec}^2 xdx = \operatorname{cosec}x\int .\operatorname{cosec}^2 xdx - \int\left(\frac{d\operatorname{cosec}x}{dx}.\int .\operatorname{cosec}^2 xdx\right)dx$$

$$= \operatorname{cosec}x(-\cot x) - \int(-\operatorname{cosec}x.\cot x)(-\cot x)dx$$

$$= -\operatorname{cosec}x.\cot x - \int \operatorname{cosec}x.\cot^2 xdx$$

We know that Cot^{2}x = Cosec^{2}x - 1

$$-\operatorname{cosec}x.\cot x - \int \operatorname{cosec}x(\operatorname{cosec}^2 x - 1)dx$$

$$= -\operatorname{cosec}x.\cot x - \int \operatorname{cosec}^3 xdx + \int \operatorname{cosec}xdx$$

We can write $\int \operatorname{cosec}^3 xdx = I$

$$\Rightarrow \int \operatorname{cosec}^3 xdx - \operatorname{cosec}x.\cot x - \int \operatorname{cosec}^3 xdx + \int \operatorname{cosec}xdx$$

$$\Rightarrow 2\int \operatorname{cosec}^3 xdx = -\operatorname{cosec}x.\cot x + \int \operatorname{cosec}xdx$$

$$\Rightarrow 2\int \operatorname{cosec}^3 xdx = -\operatorname{cosec}x.\cot x + \ln|\sec x + \tan x| + c_1$$

$$\Rightarrow \int \operatorname{cosec}^3 xdx = \frac{-\operatorname{cosec}x.\cot x + \ln|\sec x + \tan x|}{2} + c$$

Question 26.
Evaluate the following integrals:

$$\int x \sin^3 x \cos x \, dx$$

Answer:
We can write it as $\int x \sin^2 x \sin x \cos x dx$

We also know that 2sinx.cosx = sin2x

$$\int x \sin^2 x \sin x \cos x dx = \frac{1}{2}\int x \sin^2 x \sin 2x dx$$

We also know that $\sin^2 x = \dfrac{1-\cos 2x}{2}$

$$\frac{1}{2}\int x \sin^2 x \sin 2x dx = \frac{1}{2}\int x.\left(\frac{1-\cos 2x}{2}\right)\sin 2x dx$$

$$= \frac{1}{2}\left[\left(\int \frac{x \sin 2x}{2} dx - \int \frac{x \cos 2x \sin 2x}{2} dx\right)\right]$$

Here Sin4x = 2sin2x.cos2x

$$= \frac{1}{2}\left[\left(\int \frac{x \sin 2x}{2} dx - \frac{1}{4}\int x \sin 4x dx\right)\right]$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and Sin2x and sin4x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$=\frac{1}{2}\left[\left(\frac{1}{2}\left\{x\int\sin 2x dx-\int\left(\frac{dx}{dx}.\int\sin 2x dx\right)dx\right\}\right)-\left(\frac{1}{4}\left\{x\int\sin 4x-\int\left(\frac{dx}{dx}.\int\sin 4x dx\right)dx\right\}\right)\right]$$

$$=\frac{1}{2}\left[\left(\frac{1}{2}\left\{-x\frac{\cos 2x}{2}+\int\frac{\cos 2x}{2}dx\right\}\right)-\left(\frac{1}{4}\left\{-x\frac{\cos 4x}{4}+\int\frac{\cos 4x}{4}dx\right\}\right)\right]$$

$$=\frac{1}{2}\left[\left(\frac{1}{2}\left\{-x\frac{\cos 2x}{2}+\frac{\sin 2x}{4}\right\}\right)-\left(\frac{1}{4}\left\{-x\frac{\cos 4x}{4}+\frac{\sin 4x}{16}\right\}\right)\right]+c$$

$$=\frac{-x\cos 2x}{8}+\frac{\sin 2x}{16}+\frac{x\cos 4x}{32}-\frac{\sin 4x}{128}+c$$

Question 27.
Evaluate the following integrals:

$$\int\sin x\log(\cos x)dx$$

Answer:
Let cosx = t

- sinxdx = dt

Now the integral we have is

$$\int\sin x\log(\cos x)dx=-\int\log t dt$$

$$=-\int 1.\log t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logt is first function and 1 as the second function.

$$\int a.b.dx=a\int bdx-\int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$-\int 1.\log t dt = \log t\int 1dt - \int\left(\frac{d\log t}{dt}.\int 1.dt\right)dt$$

$$= -\log t.t + \int \frac{1}{t}.tdt$$

$$= -t\log t + t + c$$

Replacing t with cosx

$$t(-\log t + 1) + c$$

$$= \cos x(1 - \log(\cos x)) + c$$

Question 28.
Evaluate the following integrals:

$$\int \frac{\log(\log x)}{x}dx$$

Answer:
Let logx = t

1/x dx = dt

$$\int \frac{\log(\log x)}{x}dx = \int \log t dt = \int 1.\log t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logt is first function and 1 as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int 1.\log t dt = \log t \int 1 dt - \int \left(\frac{d \log t}{dt} . \int 1.dt \right) dt$$

$$= t.\log t - \int \frac{1}{t} t dt$$

$$= t \log t - t + c$$

Now replacing t with logx

$$\log x.\log(\log x) - \log x + c$$

$$= \log x(\log(\log x) - 1) + c$$

Question 29.
Evaluate the following integrals:

$$\int \log(2 + x^2) dx$$

Answer:
= $\int 1.\log(2 + x^2) dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\log(2 + x^2)$ is the first function and 1 as the second function.

$$\int a.b.dx = a \int b dx - \int \left[\frac{da}{dx} . \int b dx \right] dx$$

$$\int 1.\log\left(2+x^2\right)dx = \log\left(2+x^2\right)\int 1dx - \int\left(\frac{d\log\left(2+x^2\right)}{dx}.\int 1dx\right)dx$$

$$= \log\left(2+x^2\right).x - \int \frac{1.2x}{2+x^2}.xdx$$

$$= x\log\left(2+x^2\right) - \int \frac{2x^2}{2+x^2}dx$$

$$= x\log\left(2+x^2\right) - 2\int \frac{x^2+2-2}{2+x^2}dx$$

$$= x\log\left(2+x^2\right) - 2\left[\left(\int 1dx\right) - \int \frac{2}{2+x^2}dx\right]$$

$$= x\log\left(2+x^2\right) - 2\left[x - \left(2\int \frac{1}{2+x}\right)dx\right]$$

$$= x\log\left(2+x^2\right) - 2\left[x - 2\left(\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}\right)\right] + c$$

$$= x\log\left(2+x^2\right) - 2x + 2\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}} + c$$

Question 30.
Evaluate the following integrals:

$$\int \frac{x}{(1+\sin\ x)}dx$$

Answer:

$$\int \frac{x}{1+\sin x}dx = \int \frac{x(1-\sin x)}{(1+\sin x).(1-\sin x)}dx$$

We can write it as $= \int \frac{x(1-\sin x)}{1-\sin^2 x}dx$

$$= \int \frac{x(1-\sin x)}{\cos^2 x}dx$$

$$= \int x\sec^2 xdx - \int x\tan x\sec xdx$$

Using by part and ILATE

Taking x as first function and $\sec^2 x$ and secxtanx as the second function, we have

$$\int x\sec^2 x dx - \int x\sec x\tan x dx = \left(x\int \sec^2 x dx - \int\left(\frac{dx}{dx}.\int \sec^2 x dx\right)dx\right)$$

$$-\left(x\int \sec x\tan x dx - \int\left(\frac{dx}{dx}.\int \sec x\tan x dx\right)dx\right)$$

$$=\left(x\tan x - \int 1.\tan x dx\right) - \left(x.\sec x - \int 1.\sec x dx\right)$$

$$= x\tan x - \ln|\sec x| - x\sec x + \ln|\sec x + \tan x| + c$$

$$= x(\tan x - \sec x) + \ln\left|\frac{\sec x + \tan x}{\sec x}\right| + c$$

$$= x(\tan x - \sec x) + \ln|1 + \sin x| + c$$

Question 31.
Evaluate the following integrals:

$$\int\left\{\frac{1}{\log x} - \frac{1}{(\log\ x)^2}\right\}dx$$

Answer:
Let us assume logx = t

$X = e^t$

$dx = e^t dt$

Now we have

$$\int\left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right)dx = \int\left(\frac{1}{t} - \frac{1}{t^2}\right)e^t dt$$

Considering f(x) = 1/t ; $f'(x) = -1/t^2$

$$\frac{d}{dt}\left(\frac{1}{t}\right) = -\frac{1}{t^2}$$

By the integral property of $\int \{f(x)+f'(x)\}e^x dx = e^x.f(x)+c$

So the solution of the integral is

$$\int\left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right)dx = e^t \times \frac{1}{t}+c$$

Substituting the value of t as logx

$$= e^{\log x} \times \frac{1}{\log x}+c$$

$$= \frac{x}{\log x}+c$$

Question 32.
Evaluate the following integrals:

$$\int e^{-x} \cos 2x \ \cos 4x \ dx$$

Answer:

$$\cos A.\cos B = \frac{1}{2}\left[\cos(A+B)+\cos(A-B)\right]$$

We know that $\Rightarrow \cos 4x.\cos 2x = \frac{1}{2}\left[\cos(4x+2x)+\cos(4x-2x)\right]$

$$= \frac{1}{2}\left[\cos 6x + \cos 2x\right]$$

Putting in the original equation

$$\int e^{-x} \cos 2x.\cos 4x dx = \int e^{-x}\left(\frac{1}{2}\left[\cos 6x + \cos 2x\right]\right)$$

$$= \frac{1}{2}\left[\left(\int e^{-x} \cos 6x dx\right)+\left(\int e^{-x} \cos 2x dx\right)\right]$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here cos6x and cos2x is first function and e^{-x} as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

Solving both parts individually

$$I = \int e^{-x}\cos 6x dx = \cos 6x\int e^{-x}dx - \int\left(\frac{d\cos 6x}{dx}.\int e^{-x}dx\right)dx$$

$$I = \cos 6x.\left(-e^{-x}\right) - \int\left(-6\sin 6x\right).\left(-e^{-x}\right)dt$$

$$I = -\cos 6x.e^{-x} - 6\int \sin 6x.e^{-x}dx$$

$$I = -e^{-x}\cos 6x - 6\left[\sin 6x\int e^{-x}dx - \int\left(\frac{d\sin 6x}{dx}.\int e^{-x}dx\right)dx\right]$$

$$I = -e^{-x}\cos 6x - 6\left[\sin 6x\left(-e^{-x}\right) - \int\left(6\cos 6x\right).\left(-e^{-x}\right)dt\right]$$

$$I = -e^{-x}\cos 6x - 6\left[-e^{-x}\sin 6x + 6\int e^{-x}\cos 6x dx\right]$$

$$I = -e^{-x}\cos 6x - 6\left[-e^{-x}\sin 6x + 6I\right]$$

$$I = -e^{-x}\cos 6x + 6e^{-x}\sin 6x - 36I$$

$$37I = e^{-x}\left(6\sin 6x - \cos 6x\right)$$

$$I = \frac{e^{-x}\left(6\sin 6x - \cos 6x\right)}{37}$$

Solving the second part,

$$I = \int e^{-x} \cos 2x dx = \cos 2x \int e^{-x} dx - \int \left(\frac{d \cos 2x}{dx} . \int e^{-x} dx \right) dx$$

$$J = \cos 2x. \left(-e^{-x}\right) - \int \left(-2\sin 2x\right).\left(-e^{-x}\right) dt$$

$$J = -\cos 2x.e^{-x} - 2\int \sin 2x.e^{-x} dx$$

$$J = -e^{-x} \cos 2x - 2\left[\sin 2x \int e^{-x} dx - \int \left(\frac{d \sin 2x}{dx} . \int e^{-x} dx \right) dx \right]$$

$$J = -e^{-x} \cos 2x - 2\left[\sin 2x \left(-e^{-x}\right) - \int \left(2\cos 2x\right).\left(-e^{-x}\right) dt \right]$$

$$J = -e^{-x} \cos 2x - 2\left[-e^{-x} \sin 2x + 2\int e^{-x} \cos 2x dx \right]$$

$$J = -e^{-x} \cos 2x - 2\left[-e^{-x} \sin 2x + 2J \right]$$

$$J = -e^{-x} \cos 2x + 2e^{-x} \sin 2x - 4J$$

$$5J = e^{-x} \left(2\sin 2x - \cos 2x\right)$$

$$J = \frac{e^{-x}\left(2\sin 2x - \cos 2x\right)}{5}$$

Putting in the obtained equation

$$= \frac{1}{2}\left[\frac{e^{-x}\left(6\sin 6x - \cos 6x\right)}{37} + \frac{e^{-x}\left(2\sin 2x - \cos 2x\right)}{5} \right] + c$$

$$= \frac{e^{-x}\left(6\sin 6x - \cos 6x\right)}{74} + \frac{e^{-x}\left(2\sin 2x - \cos 2x\right)}{10} + c$$

$$= e^{-x}\left(\frac{\left(6\sin 6x - \cos 6x\right)}{74} + \frac{\left(2\sin 2x - \cos 2x\right)}{10} \right) + c$$

Question 33.
Evaluate the following integrals:

$$\int e^{\sqrt{x}} dx$$

Answer:
Let $\sqrt{x} = t$

$$\frac{1}{2\sqrt{x}}dx = dt$$
$$dx = 2\sqrt{x}dt$$
$$\Rightarrow dx = 2tdt$$

Replacing in the original equation , we get

$$\int e^{\sqrt{x}}dx = \int e^t.2tdt$$
$$= 2\int te^t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and e^t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$2\int te^t dt = 2\left[t\int e^t dt - \int\left(\frac{dt}{dt}.\int e^t dt\right)dt\right]$$
$$= 2\left[te^t - \int 1.e^t dt\right]$$
$$= 2\left[te^t - e^t\right] + c$$
$$= 2e^t(t-1) + c$$

Replacing t with $\sqrt{x}$

$= 2e^{\sqrt{x}}(\sqrt{x} - 1) + c$

Question 34.
Evaluate the following integrals:

$$\int e^{\sin x} \sin 2x \, dx$$

Answer:

We can write Sin2x = 2sinx.cosx

$$\int e^{\sin x} \sin 2x dx = 2\int e^{\sin x}.\sin x \cos x dx$$

Let Sinx = t

Cosxdx = dt

$$2\int e^{\sin x} \sin x \cos x dx = 2\int e^{t}.t.dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and e^t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$2\int e^{t}.tdt = 2\left[t\int e^{t}dt - \int\left(\frac{dt}{dt}.\int e^{t}dt\right)dt\right]$$

$$= 2\left[t.e^{t} - \int 1.e^{t}dt\right]$$

$$= 2\left[t.e^{t} - e^{t}\right] + c$$

$$= 2e^{t}(t-1) + c$$

Replacing t with sin x

$= 2e^{\sin x}(\sin x - 1) + c$

Question 35.
Evaluate the following integrals:

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$$

Answer:

Let $\sin^{-1}x = t$

$X = \sin t$

$$\frac{1}{\sqrt{1-x^2}}dx = dt$$

Putting this in the original equation, we get

$$\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx = \int t.\sin t dt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and sin t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int t.\sin tdt = t\int \sin tdt - \int\left(\frac{dt}{dt}.\int \sin tdt\right)dt$$
$$= t(-\cos t) - \int 1.(-\cos t)dt$$
$$= -t\cos t + \sin t + c$$

We can write $\cos t = \sqrt{1 - \sin^2 t}$

$= -t(\sqrt{1 - \sin^2 t}) + \sin t + c$

Now replacing $\sin^{-1}x = t$

$= -\sin^{-1}x(\sqrt{1 - x^2}) + x + c$

Question 36.

Evaluate the following integrals:

$$\int \frac{x^2 \tan^{-1} x}{\left(1+x^2\right)} dx$$

Answer:
Let $\tan^{-1} x = t$ and $x = \tan t$

Differentiating both sides, we get

$$\frac{1}{1+x^2} dx = dt$$

Now we have

$$\int \frac{x^2 \tan^{-1} x}{\left(1+x^2\right)} dx = \int \tan^2 t.tdt$$

$$\int t.\tan^2 tdt = \int t\left(\sec^2 t - 1\right)dt$$
$$= \int t\sec^2 tdt - \int tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and $\sec^2 t$ as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int t\sec^2 tdt - \int tdt = t\int \sec^2 tdt - \int\left(\frac{dt}{dt}.\int \sec^2 tdt\right)dt - \frac{t^2}{2}$$
$$= t.\tan t - \int \tan tdt - \frac{t^2}{2}$$
$$= t.\tan t - \ln|\sec t| - \frac{t^2}{2} + c$$

We know that sec t = $\sqrt{\tan^2 t + 1}$

$$= \tan^{-1} x.x - \ln|\sqrt{\tan^2 t + 1}| - \frac{\tan^2 x}{2} + c$$

$$= x\tan^{-1} x - \ln|\sqrt{x^2 + 1}| - \frac{\tan^2 x}{2} + c$$

Question 37.
Evaluate the following integrals:

$$\int \frac{\log(x+2)}{(x+2)^2} dx$$

Answer:

We can write it as $\int \log(x+2) . \frac{1}{(x+2)^2} dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log(x + 2) is first function and $(x + 2)^{-2}$ as second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \log(x+2).\frac{1}{(x+2)^2}dx = \log(x+2)$$

$$\int \frac{1}{(x+2)^2}dx - \int\left(\frac{d\log(x+2)}{dx}.\int\frac{1}{(x+2)^2}dx\right)dx$$

$$= \log(x+2).\frac{-1}{(x+2)} - \int \frac{1}{x+2}.\frac{-1}{(x+2)}dx$$

$$= -\log(x+2)\frac{1}{(x+2)} + \int \frac{1}{(x+2)^2}dx$$

$$= -\log(x+2)\frac{1}{(x+2)} - \frac{1}{(x+2)} + c$$

Question 38.

Evaluate the following integrals:

$$\int x \sin^{-1} x \, dx$$

Answer:

Let x = sin t ; t = $\sin^{-1}x$

dx = cos t dt

$$\Rightarrow \int x\sin^{-1}x dx = \int \sin t.\sin^{-1}(\sin t)\cos t dt$$

$$= \int \sin t.t.\cos t dt$$

We know that sin 2t = 2 sint×cost

We have $\int t\cos t\sin t dt = \frac{1}{2}\int t\sin 2t dt$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and sin 2t as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\frac{1}{2}\int t\sin 2tdt = \frac{1}{2}\left(t\int \sin 2tdt - \int\left[\frac{dt}{dt}.\int \sin 2tdt\right]dt\right)$$

$$= \frac{1}{2}\left(t.\frac{-\cos 2t}{2} + \int \frac{\cos 2t}{2}dt\right)$$

$$= \frac{1}{2}\left(\frac{-t\cos 2t}{2} + \frac{\sin 2t}{4}\right) + c$$

$$= \frac{-t\cos 2t}{4} + \frac{\sin 2t}{8} + c$$

We know that cos2t = 1 - 2sin^{2}t , sin2t = 2sint×cost and cos t = $\sqrt{}$1 - sin^{2}t

Replacing in above equation

$$= \frac{-t\left(1-2\sin^2 t\right)}{4} + \frac{2\sin t \times \cos t}{8} + c$$

$$= \frac{-t\left(1-2\sin^2 t\right)}{4} + \frac{\sqrt{1-\sin^2 t}}{4}.\sin t + c$$

$$= \frac{-\sin^{-1}x\left(1-2x^2\right)}{4} + \frac{x\sqrt{1-x^2}}{4} + c$$

$$= \frac{1}{2}x^2\sin^{-1}x - \frac{\sin^{-1}x}{4} + \frac{1}{4}x\sqrt{1-x^2} + c$$

$$= \frac{1}{2}x^2\sin^{-1}x - \frac{\sin^{-1}x}{4} + \frac{1}{4}x\sqrt{1-x^2} + c$$

Question 39.
Evaluate the following integrals:

$$\int x\ \cos^{-1}x\,dx$$

Answer:
Let x = cos t ; t = cos $^{-1}$x

dx = - sin t dt

$$\int x\cos^{-1}x dx = -\int \cos t.\cos^{-1}(\cos t)\sin t dt$$
$$= -\int \cos t.t.\sin t.dt$$

We know that sin 2t = 2 sint×cost

We have $-\int t\cos t\sin t dt = \frac{-1}{2}\int t\sin 2t dt$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking first function to the one which comes first in the list.

Here t is first function and sin 2t as second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\frac{-1}{2}\int t\sin 2t dt = \frac{-1}{2}\left(t\int \sin 2t dt - \int\left[\frac{dt}{dt}.\int \sin 2t dt\right]dt\right)$$
$$= \frac{-1}{2}\left(t.\frac{-\cos 2t}{2} + \int\frac{\cos 2t}{2}dt\right)$$
$$= \frac{-1}{2}\left(\frac{-t\cos 2t}{2} + \frac{\sin 2t}{4}\right) + c$$
$$= \frac{t\cos 2t}{4} - \frac{\sin 2t}{8} + c$$

We know that cos2t = 2cos^{2}t - 1 and sin2t = 2sint×cost and sint = $\sqrt{1 - \cos^2 t}$

Replacing in above equation

$$= \frac{t(2\cos^2 t - 1)}{4} - \frac{2\sin t \times \cos t}{8} + c$$

$$= \frac{t(2\cos^2 t - 1)}{4} - \frac{\sqrt{1-\cos^2 t}}{4}.\cos t + c$$

$$= \frac{\cos^{-1} x(2x^2 - 1)}{4} - \frac{x\sqrt{1-x^2}}{4} + c$$

$$= \frac{1}{2}x^2 \cos^{-1} x - \frac{\cos^{-1} x}{4} - \frac{1}{4}x\sqrt{1-x^2} + c$$

$$= \frac{1}{2}x^2 \cos^{-1} x + \frac{\sin^{-1} x}{4} - \frac{1}{4}x\sqrt{1-x^2} + c$$

Question 40.

Evaluate the following integrals:

$$\int \cot^{-1} x \, dx$$

Answer:

We can write it as $\int \cot^{-1} x.1dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here $\cot^{-1}x$ is first function and 1 as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \cot^{-1} x.1dx = \cot^{-1} x\int 1dx - \int\left(\frac{d\cot^{-1} x}{dx}.\int 1dx\right)dx$$

$$= \cot^{-1} x.x - \int \frac{-1}{1+x^2}.x.dx$$

$$= x\cot^{-1} x + \int \frac{x}{1+x^2}dx$$

Let $1 + x^2 = t$

2xdx = dt

Xdx = dt/2

$$\Rightarrow \int \cot^{-1} x dx = x\cot^{-1} x + \int \frac{dt}{2t}$$

$$= x\cot^{-1} x + \frac{\log t}{2} + c$$

Now replacing t with $1 + x^2$

$= x\cot^{-1}x + \log(1 + x^2)/2 + c$

Question 41.
Evaluate the following integrals:

$$\int x\cot^{-1} x\, dx$$

Answer:
Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot^{-1}x$ and $f_2(x) = x$,

$$\therefore \int x\cot^{-1} x\, dx$$

$$= \cot^{-1} x \int xdx - \int \left\{\frac{d}{dx}(\cot^{-1} x) \int xdx\right\}dx$$

$$= \frac{x^2 \cot^{-1} x}{2} - \int \frac{1}{(1 + x^2)} \times \frac{x^2}{2} dx$$

$$= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{(1 + x^2)} dx$$

$$= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2}\int \frac{1 + x^2 - x^2}{(1 + x^2)} dx$$

$$= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2}\int 1 - \frac{1}{(1 + x^2)} dx$$

$= \frac{x^2 \cot^{-1} x}{2} - \frac{1}{2}[x - \tan^{-1} x] + c$, where c is the integrating constant

Question 42.
Evaluate the following integrals:

$$\int x^2 \cot^{-1} x \; dx$$

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Answer:
Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot^{-1}x$ and $f_2(x) = x^2$,

$$\therefore \int x^2 \cot^{-1} x \, dx$$

$$= \cot^{-1} x \int x^2 dx - \int \left\{ \frac{d}{dx}(\cot^{-1} x) \int x^2 dx \right\} dx$$

$$= \frac{x^3 \cot^{-1} x}{3} - \int \frac{1}{(1 + x^2)} \times \frac{x^3}{3} dx$$

$$= \frac{x^3 \cot^{-1} x}{3} - \frac{1}{3}\int \frac{x^3}{(1 + x^2)} dx$$

Taking $(1+x^2)=a$,

2xdx=da i.e. xdx=da/2

Again, x^2=a-1

$$\therefore \frac{1}{3}\int \frac{x^2 \times xdx}{(1+x^2)}$$

$$=\frac{1}{3}\int \frac{(a-1)da}{2a}$$

$$=\frac{1}{6}\int \left(1-\frac{1}{a}\right)da$$

$$=\frac{1}{6}(a-\ln a)$$

Replacing the value of a, we get,

$$\therefore \frac{1}{6}(a-\ln a)$$

$$=\frac{1}{6}[(1+x^2)-\ln|x^2+1|+c_1$$

$$=\frac{x^2}{6}-\frac{\ln|x^2+1|}{6}+\left(c_1+\frac{1}{6}\right)$$

$$=\frac{x^2}{6}-\frac{\ln|x^2+1|}{6}+c$$

The total integration yields as

$=\frac{x^3\cot^{-1}x}{3}+\frac{x^2}{6}-\frac{\ln|x^2+1|}{6}+c$, where c is the integrating constant

Question 43.
Evaluate the following integrals:

$$\int \sin^{-1}\sqrt{x}\ dx$$

Answer:

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \sin^{-1}\sqrt{x}\,dx$$

$$= \sin^{-1}\sqrt{x} \int dx - \int \left\{\frac{d}{dx}\left(\sin^{-1}\sqrt{x}\right) \int dx\right\} dx$$

$$= x \sin^{-1}\sqrt{x} - \int \frac{1}{2\sqrt{x}\sqrt{1-x}} \times x dx$$

$$= x \sin^{-1}\sqrt{x} - \frac{1}{2} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx$$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$$\therefore \frac{1}{2} \int \frac{\sqrt{x}}{\sqrt{1-x}} dx$$

$$= \frac{1}{2} \int \frac{\sqrt{1-a^2}}{a} (-2ada)$$

$$= -\int \sqrt{1-a^2}\,da$$

$$= -\left[\frac{1}{2} a\sqrt{1-a^2} + \frac{1}{2} \sin^{-1} a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2}+\frac{1}{2}\sin^{-1}a\right]$$

$$= -\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]+c$$

The total integration yields as

$= \mathbf{x\sin^{-1}\sqrt{x}+\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]+c}$, where c is the integrating constant

Question 44.
Evaluate the following integrals:

$$\int \cos^{-1}\sqrt{x}\ dx$$

Answer:
Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \cos^{-1}\sqrt{x}dx$$

$$= \cos^{-1}\sqrt{x}\int dx-\int\left\{\frac{d}{dx}\left(\cos^{-1}\sqrt{x}\right)\int dx\right\}dx$$

$$= x\cos^{-1}\sqrt{x}-\int\frac{-1}{2\sqrt{x}\sqrt{1-x}}\times xdx$$

$$= x\cos^{-1}\sqrt{x}+\frac{1}{2}\int\frac{\sqrt{x}}{\sqrt{1-x}}dx$$

Taking $(1-x)=a^2$,

-dx=2ada i.e. dx=-2ada

Again, x=1-a^{2}

$$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

$$=\frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$$

$$=-\int \sqrt{1-a^2}da$$

$$=-\left[\frac{1}{2}a\sqrt{1-a^2}+\frac{1}{2}\sin^{-1}a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2}+\frac{1}{2}\sin^{-1}a\right]$$

$$=-\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]+c$$

The total integration yields as

$\mathbf{= x\cos^{-1}\sqrt{x}-\left[\frac{1}{2}x\sqrt{1-x}+\frac{1}{2}\sin^{-1}\sqrt{1-x}\right]+c}$, where c is the integrating constant

Question 45.
Evaluate the following integrals:

$$\int \cos^{-1}\left(4x^3-3x\right)dx$$

Answer:
Formula to be used – We know , cos3x = 4cos^{3}x-3cosx

$$\therefore \int \cos^{-1}(4x^3-3x)\,dx$$

Assuming x = cosa, $4\cos^3 a - 3\cos a = \cos 3a$

And, dx = -sinada

Hence, $a = \cos^{-1}x$

Again, $\sin a = \sqrt{(1-x^2)}$

$$\therefore \int \cos^{-1}(4x^3 - 3x)\,dx$$

$$= \int \cos^{-1}(\cos 3a)\{-\sin a da\}$$

$$= -3\int a\sin a da$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = a and $f_2(x)$ = sina,

$$\therefore -3\int a\sin a da$$

$$= -3\left[a\int \sin a da - \int\left\{\frac{d}{dx}a\int \sin a da\right\}da\right]$$

$$= 3a\cos a - \int \cos a da$$

$$= 3a\cos a - \sin a + c$$

Replacing the value of a we get,

$$\therefore 3a\cos a - \sin a + c$$

$= \mathbf{3x\cos^{-1}x - \sqrt{1-x^2} + c}$, where c is the integrating constant

Question 46.
Evaluate the following integrals:

$$\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$$

Answer:
Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and $f_2(x) = 1$,

$$\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) dx$$

$$= \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \int dx - \int \left[\frac{d}{dx}\left\{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\} \int dx\right] dx$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \int \left[\frac{\frac{(1+x^2)(-2x) - (1-x^2)(2x)}{(1+x^2)^2}}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}}\right] dx$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \int \frac{-4x^2dx}{(1+x^2)^2 \times \frac{1}{1+x^2} \times 2x}$$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) - \int \frac{2xdx}{1+x^2}$$

Now,

$$\int \frac{2xdx}{1+x^2}$$

$$= \int \frac{d(1+x^2)}{1+x^2}$$

$$= \ln(1+x^2)+c$$

Again, we know,

$$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$$

$$\Rightarrow 2x = \cos^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)$$

Replacing x by tanx, it is obtained that,

$$2\tan x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

So, the final integral yielded is

$\mathbf{2x\tan x - \ln(1+x^2)+c}$, where c is the integrating constant

Question 47.
Evaluate the following integrals:

$$\int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

Answer:
Formula to be used – We know, $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$$\therefore \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

Assuming x = tana,

$$\frac{2\tan a}{1-\tan^2 a} = \tan 2a$$

And, $dx = \sec^2 a\,da$

Hence, $a=\tan^{-1}x$

Now, $\sec^2 a-\tan^2 a=1$, so, $\sec a=\sqrt{(1+x^2)}$

$$\therefore \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

$$= \int \tan^{-1}(\tan 2a)\{\sec^2 a\,da\}$$

$$= 2\int a\sec^2 a\,da$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = a$ and $f_2(x) = \sec^2 a$,

$$\therefore 2\int a\sec^2 a\,da$$

$$= 2\left[a\int \sec^2 a\,da - \int\left\{\frac{d}{dx}a\int \sec^2 a\,da\right\}da\right]$$

$$= 2a\tan a - \int \tan a\,da$$

$$= 2a\tan a - \ln|\sec a| + c$$

Replacing the value of a we get,

$$\therefore 2a\tan a - \ln|\sec a| + c$$

$= \mathbf{2x \tan^{-1} x - \ln\sqrt{1 + x^2} + c}$, where c is the integrating constant

Question 48.
Evaluate the following integrals:

$$\int \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)dx$$

Answer:
Formula to be used – We know, $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

$$\therefore \int \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)dx$$

Assuming x = tana,

$$\frac{3\tan a - \tan^3 a}{1 - 3\tan^2 a} = \tan 3a$$

And, dx = $\sec^2 a da$

Hence, $a = \tan^{-1} x$

Now, $\sec^2 a - \tan^2 a = 1$, so, $\sec a = \sqrt{(1+x^2)}$

$$\therefore \int \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)dx$$

$$= \int \tan^{-1}(\tan 3a)\{\sec^2 a da\}$$

$$= 3\int a\sec^2 a da$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = a$ and $f_2(x) = \sec^2 a$,

$$\therefore 3\int a\sec^2 a\,da$$

$$= 3\left[a\int \sec^2 a\,da - \int\left\{\frac{d}{dx}a\int \sec^2 a\,da\right\}da\right]$$

$$= 3a\tan a - \frac{3}{2}\int \tan a\,da$$

$$= 3a\tan a - \frac{3}{2}\ln|\sec a| + c$$

Replacing the value of a we get,

$$\therefore 3a\tan a - \frac{3}{2}\ln|\sec a| + c$$

$\mathbf{= 3x\tan^{-1}x - \frac{3}{2}\ln\sqrt{1+x^2} + c}$, where c is the integrating constant

Question 49.
Evaluate the following integrals:

$$\int \frac{\sin^{-1}x}{x^2}dx$$

Answer:
Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}x$ and $f_2(x) = 1/x^2$,

$$\therefore \int \frac{\sin^{-1} x}{x^2} dx$$

$$= \sin^{-1} x \int \frac{1}{x^2} dx - \int \left\{ \frac{d}{dx} (\sin^{-1} x) \int \frac{1}{x^2} dx \right\} dx$$

$$= \frac{-\sin^{-1} x}{x} - \int \frac{1}{\sqrt{1 - x^2}} \times \left(-\frac{1}{x}\right) dx$$

$$= \frac{-\sin^{-1} x}{x} + \int \frac{1}{x\sqrt{1 - x^2}} dx$$

Taking x= sina, dx = cosada

Hence, coseca=1/x

Now, $\text{cosec}^2 a - \cot^2 a = 1$ so cota=$\sqrt{}(1-x^2)/x$

$$\therefore \int \frac{1}{x\sqrt{1 - x^2}} dx$$

$$= \int \frac{1}{\text{sinacosa}} (\text{cosada})$$

$$= \int \text{cosecada}$$

$$= \ln|\text{coseca} - \text{cota}| + c$$

Replacing the value of a, we get,

$$\therefore \ln|\text{coseca} - \text{cota}| + c$$

$$= \ln\left|\frac{1}{x} - \frac{\sqrt{1 - x^2}}{x}\right| + c$$

The total integration yields as

$$= \frac{-\sin^{-1} x}{x} + \ln\left|\frac{1}{x} - \frac{\sqrt{1-x^2}}{x}\right| + c$$, where c is the integrating constant

Question 50.
Evaluate the following integrals:

$$\int \frac{\tan x \sec^2 x}{\left(1-\tan^2 x\right)} dx$$

Answer:
Say, tanx = a

Hence, $\sec^2 x dx = da$

$$\therefore \int \frac{\tan x \sec^2 x}{1-\tan^2 x} dx$$

$$= \int \frac{ada}{1-a^2}$$

Now, taking $1-a^2 = k$, $-2ada=dk$ i.e. $ada=-dk/2$

$$\therefore \int \frac{ada}{1-a^2}$$

$$= \int \frac{-dk}{2k}$$

$$= -\frac{1}{2}\ln|k| + c$$

Replacing the value of k,

$$-\frac{1}{2}\ln|k| + c$$

$$= -\frac{1}{2}\ln|1-a^2| + c$$

Replacing the value of a,

$$-\frac{1}{2}\ln|1-a^2|+c$$

$= -\frac{1}{2}\ln\left|1-\tan^2 x\right| + c$, where c is the integrating constant

Question 51.
Evaluate the following integrals:

$$\int e^{3x} \sin 4x \; dx$$

Answer:
Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = sin4x and $f_2(x) = e^{3x}$,

$$\therefore \int e^{3x}\sin 4x dx$$

$$= \sin 4x \int e^{3x}dx - \int\left\{\frac{d}{dx}(\sin 4x)\int e^{3x}dx\right\}dx$$

$$= \frac{e^{3x}\sin 4x}{3} - \int 4\cos 4x \times \frac{e^{3x}}{3}dx$$

$$= \frac{e^{3x}\sin 4x}{3} - \frac{4}{3}\int e^{3x}\cos 4x dx$$

$$= \frac{e^{3x}\sin 4x}{3} - \frac{4}{3}\left[\cos 4x \int e^{3x}dx - \int\left\{\frac{d}{dx}(\cos 4x)\int e^{3x}dx\right\}dx\right]$$

$$= \frac{e^{3x}\sin 4x}{3} - \frac{4e^{3x}\cos 4x}{9} - \frac{4}{3}\int 4\sin 4x \times \frac{e^{3x}}{3}dx$$

$$= \frac{e^{3x}\sin 4x}{3} - \frac{4e^{3x}\cos 4x}{9} - \frac{16}{9}\int e^{3x}\sin 4x dx$$

$$\therefore \left(1 + \frac{16}{9}\right)\int e^{3x}\sin 4x dx = \frac{e^{3x}\sin 4x}{3} - \frac{4e^{3x}\cos 4x}{9} + c_1$$

$$\Rightarrow \frac{25}{9}\int e^{3x}\sin 4x dx = \frac{3e^{3x}\sin 4x - 4e^{3x}\cos 4x}{9} + c_1$$

$\Rightarrow \int e^{3x}\sin 4x dx = \frac{e^{3x}}{25}(3\sin 4x - 4\cos 4x) + c$, where c is the integrating constant

Question 52.
Evaluate the following integrals:

$$\int e^{2x} \sin x \, dx$$

Answer:
Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin x$ and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x}\sin x dx$$

$$= \sin x\int e^{2x}dx - \int\left\{\frac{d}{dx}(\sin x)\int e^{2x}dx\right\}dx$$

$$= \frac{e^{2x}\sin x}{2} - \int \cos x \times \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\sin x}{2} - \frac{1}{2}\int e^{2x}\cos x dx$$

$$= \frac{e^{2x}\sin x}{2} - \frac{1}{2}\left[\cos x \int e^{2x}dx - \int \left\{\frac{d}{dx}(\cos x) \int e^{2x}dx\right\}dx\right]$$

$$= \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{2}\int \sin x \times \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}\int e^{2x}\sin x dx$$

$$\therefore \left(1 + \frac{1}{4}\right)\int e^{2x}\sin x dx = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} + c_1$$

$$\Rightarrow \frac{5}{4}\int e^{2x}\sin x dx = \frac{2e^{2x}\sin x - e^{2x}\cos x}{4} + c_1$$

$\Rightarrow \int e^{2x}\sin x dx = \frac{e^{2x}}{5}(\mathbf{2\sin x - \cos x}) + \mathbf{c}$, where c is the integrating constant

Question 53.
Evaluate the following integrals:

$$\int e^{2x} \sin x \cos x \, dx$$

Answer:

$$\int e^{2x}\sin x\cos x dx$$

$$= \frac{1}{2}\int e^{2x} \times 2\sin x\cos x dx$$

$$= \frac{1}{2}\int e^{2x}\sin 2x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin 2x$ and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x}\sin 2x dx$$

$$= \sin 2x \int e^{2x}dx - \int \left\{\frac{d}{dx}(\sin 2x)\int e^{2x}dx\right\}dx$$

$$= \frac{e^{2x}\sin 2x}{2} - \int 2\cos 2x \times \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\sin 2x}{2} - \int e^{2x}\cos 2x dx$$

$$= \frac{e^{2x}\sin 2x}{2} - \left[\cos 2x \int e^{2x}dx - \int \left\{\frac{d}{dx}(\cos 2x)\int e^{2x}dx\right\}dx\right]$$

$$= \frac{e^{2x}\sin 2x}{2} - \frac{e^{2x}\cos 2x}{2} - \int 2\sin 2x \times \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\sin 2x}{2} - \frac{e^{2x}\cos 2x}{2} - \int e^{2x}\sin x dx$$

$$\therefore (1+1)\int e^{2x}\sin 2x dx = \frac{e^{2x}\sin 2x}{2} - \frac{e^{2x}\cos 2x}{2} + c_1$$

$$\Rightarrow 2\int e^{2x}\sin 2x dx = \frac{e^{2x}\sin 2x - e^{2x}\cos 2x}{2} + c_1$$

$$\Rightarrow \int e^{2x}\sin 2x dx = \frac{e^{2x}}{4}(\sin 2x - \cos 2x) + c'$$

$$\therefore \frac{1}{2}\int e^{2x}\sin 2x dx$$

$$= \frac{1}{2} \times \left[\frac{e^{2x}}{4}(\sin 2x - \cos 2x) + c'\right]$$

$= \boldsymbol{\frac{e^{2x}}{8}(\sin 2x - \cos 2x) + c}$, where c is the integrating constant

Question 54.
Evaluate the following integrals:

$$\int e^{2x}\cos(3x+4)dx$$

Answer:
Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cos(3x+4)$ and $f_2(x) = e^{2x}$,

$$\therefore \int e^{2x}\cos(3x+4)\ dx$$

$$= \cos(3x+4)\int e^{2x}dx - \int\left\{\frac{d}{dx}\cos(3x+4)\int e^{2x}dx\right\}dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \int 3\sin(3x+4) \times \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3}{2}\int e^{2x}\sin(3x+4)dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3}{2}\left[\sin(3x+4)\int e^{2x}dx - \int\left\{\frac{d}{dx}\sin(3x+4)\int e^{2x}dx\right\}dx\right]$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} - \frac{3}{2}\int 3\cos(3x+4) \times \frac{e^{2x}}{2}dx$$

$$= \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} - \frac{9}{4}\int e^{2x}\cos(3x+4)dx$$

$$\therefore \left(1+\frac{9}{4}\right)\int e^{2x}\cos(3x+4)dx = \frac{e^{2x}\cos(3x+4)}{2} + \frac{3e^{2x}\sin(3x+4)}{4} + c_1$$

$$\Rightarrow \frac{13}{4}\int e^{2x}\cos(3x+4)\,dx = \frac{2e^{2x}\cos(3x+4)+3e^{2x}\sin(3x+4)}{4} + c_1$$

$\Rightarrow \int e^{2x}\cos(3x+4)\,dx = \frac{e^{2x}}{13}\left(2\cos(3x+4)+3\sin(3x+4)\right)+c$, where c is the integrating constant

Question 55.
Evaluate the following integrals:

$$\int e^{-x}\cos x\,dx$$

Answer:
Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = cosx and $f_2(x) = e^{-x}$,

$$\therefore \int e^{-x}\cos x\,dx$$

$$= \cos x\int e^{-x}dx - \int\left\{\frac{d}{dx}\cos x\int e^{-x}dx\right\}dx$$

$$= -e^{-x}\cos x - \int e^{-x}\sin x\,dx$$

$$= -e^{-x}\cos x - \left[\sin x\int e^{-x}dx - \int\left\{\frac{d}{dx}\sin x\int e^{-x}dx\right\}dx\right]$$

$$= -e^{-x}\cos x - \left[-e^{-x}\sin x + \int e^{-x}\cos x dx\right]$$

$$= -e^{-x}\cos x + e^{-x}\sin x - \int e^{-x}\cos x dx$$

$$\therefore (1+1)\int e^{-x}\cos x dx = -e^{-x}\cos x + e^{-x}\sin x + c_1$$

$$\Rightarrow 2\int e^{-x}\cos x dx = -e^{-x}\cos x + e^{-x}\sin x + c_1$$

$\Rightarrow \int e^{-x}\cos x dx = \frac{e^{-x}}{2}(\mathbf{\sin x - \cos x}) + \mathbf{c}$, where c is the integrating constant

Question 56.
Evaluate the following integrals:

$$\int e^x (\sin x + \cos x) dx$$

Answer:

$$\int e^x(\sin x + \cos x)dx$$

$$= \int e^x \sin x dx + \int e^x \cos x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \sin x dx + \int e^x \cos x dx$$

$$= \sin x \int e^x dx - \int \left[\frac{d}{dx}(\sin x)\int e^x dx\right] dx + \int e^x \cos x dx$$

$$= e^x \sin x - \int e^x \cos x dx + \int e^x \cos x dx + c$$

$= \mathbf{e^x \sin x + c}$, where c is the integrating constant

Question 57.
Evaluate the following integrals:

$$\int e^{x}\left(\cot x-\operatorname{cosec}^{2}x\right)dx$$

Answer:

$$\int e^{x}(\cot x-\operatorname{cosec}^{2}x)dx$$

$$=\int e^{x}\cot x dx+\int e^{x}\operatorname{cosec}^{2}x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^{x}\cot x dx+\int e^{x}\operatorname{cosec}^{2}x dx$$

$$=\cot x\int e^{x}dx-\int\left[\frac{d}{dx}(\cot x)\int e^{x}dx\right]dx+\int e^{x}\operatorname{cosec}^{2}x dx$$

$$=e^{x}\cot x-\int e^{x}\operatorname{cosec}^{2}x dx+\int e^{x}\operatorname{cosec}^{2}x dx+c$$

$=\mathbf{e^{x}\cot x+c}$, where c is the integrating constant

Question 58.
Evaluate the following integrals:

$$\int e^{x}\sec x\left(1+\tan x\right)dx$$

Answer:

$$\int e^x \sec x(1+\tan x)dx$$

$$= \int e^x \sec x dx + \int e^x \sec x \tan x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sec x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \sec x dx + \int e^x \sec x \tan x dx$$

$$= \sec x \int e^x dx - \int\left[\frac{d}{dx}(\sec x)\int e^x dx\right]dx + \int e^x \sec x \tan x dx$$

$$= e^x \sec x - \int e^x \sec x \tan x dx + \int e^x \sec x \tan x dx + c$$

$= \mathbf{e^x \sec x + c}$, where c is the integrating constant

Question 59.
Evaluate the following integrals:

$$\int e^x\left(\tan^{-1}x + \frac{1}{1+x^2}\right)dx$$

Answer:
$$\int e^x\left(\tan^{-1}x + \frac{1}{1+x^2}\right)dx$$

$$= \int e^x \tan^{-1}x\,dx + \int \frac{e^x}{1+x^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan^{-1}x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \tan^{-1} x\, dx + \int \frac{e^x}{1+x^2}dx$$

$$= \tan^{-1} x \int e^x dx - \int\left[\frac{d}{dx}(\tan^{-1} x)\int e^x dx\right]dx + \int \frac{e^x}{1+x^2}dx$$

$$= e^x \tan^{-1} x - \int \frac{e^x}{1+x^2}dx + \int \frac{e^x}{1+x^2}dx + c$$

$\mathbf{= e^x \tan^{-1} x + c}$, where c is the integrating constant

Question 60.
Evaluate the following integrals:

$$\int e^x(\cot x + \log \sin x)dx$$

Answer:

$$\int e^x(\cot x + \log \sin x)dx$$

$$= \int e^x \cot x\, dx + \int e^x \log \sin x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \log \sin x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \cot x\, dx + \int e^x \log \sin x dx$$

$$= \int e^x \cot x \, dx + \text{logsinx} \int e^x dx - \int \left[\frac{d}{dx}(\text{logsinx}) \int e^x dx\right]$$

$$= \int e^x \cot x \, dx + e^x \text{logsinx} - \int e^x \cot x \, dx + c$$

$= \mathbf{e^x \log|\sin x| + c}$, where c is the integrating constant

Question 61.
Evaluate the following integrals:

$$\int e^x (\tan x - \log \cos x) dx$$

Answer:

$$\int e^x (\tan x + \text{logcosx}) dx$$

$$= \int e^x \tan x \, dx + \int e^x \text{logcosxdx}$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logcosx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \tan x \, dx - \int e^x \text{logcosxdx}$$

$$= \int e^x \tan x \, dx - \text{logcosx} \int e^x dx + \int \left[\frac{d}{dx}(\text{logcosx}) \int e^x dx\right]$$

$$= \int e^x \tan x \, dx - e^x \text{logcosx} - \int e^x \tan x \, dx + c$$

$= \mathbf{e^x \log|\sec x| + c}$, where c is the integrating constant

Question 62.
Evaluate the following integrals:

$$\int e^x\left[\sec x+\log(\sec x+\tan x)\right]dx$$

Answer:

$$\int e^x[\sec x+\log(\sec x+\tan x)]dx$$

$$=\int e^x\sec x\,dx+\int e^x\log(\sec x+\tan x)dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logcosx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x\sec x\,dx+\int e^x\log(\sec x+\tan x)dx$$

$$=\int e^x\sec x dx+\log(\sec x+\tan x)\int e^x dx-\int\left[\frac{d}{dx}(\log(\sec x+\tan x))\int e^x dx\right]$$

$$=\int e^x\sec x\,dx+e^x\log(\sec x+\tan x)-\int\frac{e^x\tan x\times(\sec^2 x+\sec x\tan x)\,dx}{\sec x+\tan x}+c$$

$$=\int e^x\sec x\,dx+e^x\log(\sec x+\tan x)-\int e^x\sec x\,dx+c$$

$= \mathbf{e^x\log|\sec x+\tan x|+c}$, where c is the integrating constant

Question 63.

Evaluate the following integrals:

$$\int e^{x}\left(\frac{1+\sin x\cos x}{\cos^{2}x}\right)dx$$

Answer:

$$\int e^{x}\left(\frac{1+\sin x\cos x}{\cos^{2}x}\right)dx$$

$$=\int e^{x}(\sec^{2}x+\tan x)dx$$

$$=\int e^{x}\sec^{2}x\,dx+\int e^{x}\tan x\,dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^{x}\sec^{2}x\,dx+\int e^{x}\tan x\,dx$$

$$=\int e^{x}\sec^{2}x\,dx+\tan x\int e^{x}dx-\int\left[\frac{d}{dx}(\tan x)\int e^{x}dx\right]$$

$$=\int e^{x}\sec^{2}x\,dx+e^{x}\tan x-\int e^{x}\sec^{2}x\,dx+c$$

$=\mathbf{e^{x}\tan x+c}$, where c is the integrating constant

Question 64.
Evaluate the following integrals:

$$\int e^{x}\left(\frac{\sin x\cos x-1}{\sin^{2}x}\right)dx$$

Answer:

$$\int e^x\left(\frac{\sin x\cos x - 1}{\sin^2 x}\right)dx$$

$$= \int e^x(\cot x - \operatorname{cosec}^2 x)dx$$

$$= \int e^x \cot x dx - \int e^x \operatorname{cosec}^2 x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \cot x dx - \int e^x \operatorname{cosec}^2 x dx$$

$$= \cot x \int e^x dx - \int \left\{\frac{d}{dx}(\cot x) \int e^x dx\right\}dx - \int e^x \operatorname{cosec}^2 x dx$$

$$= e^x \cot x + \int e^x \operatorname{cosec}^2 x dx - \int e^x \operatorname{cosec}^2 x dx + c$$

$= \mathbf{e^x \cot x + c}$, where c is the integrating constant

Question 65.
Evaluate the following integrals:

$$\int e^x\left(\frac{\cos x + \sin x}{\cos^2 x}\right)dx$$

Answer:

$$\int e^x\left(\frac{\cos x + \sin x}{\cos^2 x}\right)dx$$

$$= \int e^x(\sec x + \sec x \tan x)dx$$

$$= \int e^x \sec x dx + \int e^x \sec x \tan x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sec x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \sec x dx + \int e^x \sec x \tan x dx$$

$$= \sec x \int e^x dx - \int \left[\frac{d}{dx}(\sec x) \int e^x dx\right] dx + \int e^x \sec x \tan x dx$$

$$= e^x \sec x - \int e^x \sec x \tan x dx + \int e^x \sec x \tan x dx + c$$

$= \mathbf{e^x \sec x + c}$, where c is the integrating constant

Question 66.
Evaluate the following integrals:

$$\int e^x \left(\frac{2 - \sin 2x}{1 - \cos 2x}\right) dx$$

Answer:

$$\int e^x \left(\frac{2 - \sin 2x}{1 - \cos 2x}\right) dx$$

$$= \int e^x \left(\frac{1 - \sin x \cos x}{\sin^2 x}\right) dx$$

$$= \int e^x (\operatorname{cosec}^2 x - \cot x) dx$$

$$= \int e^x \operatorname{cosec}^2 x dx - \int e^x \cot x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int e^x \operatorname{cosec}^2 x dx - \int e^x \cot x dx$$

$$= \int e^x \operatorname{cosec}^2 x dx - \cot x \int e^x dx + \int \left\{\frac{d}{dx}(\cot x) \int e^x dx\right\} dx$$

$$= \int e^x \operatorname{cosec}^2 x dx - e^x \cot x - \int e^x \operatorname{cosec}^2 x dx$$

$= \mathbf{-e^x \cot x + c}$, where c is the integrating constant

Question 67.

Evaluate the following integrals:

$$\int e^x \left(\frac{1 + \sin x}{1 + \cos x}\right) dx$$

Answer:

$$\left(\frac{1 + \sin x}{1 + \cos x}\right)$$

$$= \left(\frac{1 + \frac{2\tan {}^x/_2}{1 + \tan^2({}^x/_2)}}{1 + \frac{1 - \tan^2({}^x/_2)}{1 + \tan^2({}^x/_2)}}\right)$$

$$= \frac{\left(1 + \tan {}^{x}/_{2}\right)^{2}}{2}$$

$$\therefore \int e^{x}\left(\frac{1 + \sin x}{1 + \cos x}\right)dx$$

$$= \int e^{x} \times \frac{\left(1 + \tan {}^{x}/_{2}\right)^{2}}{2}$$

$$= \int \frac{e^{x}\left(1 + \tan^{2} {}^{x}/_{2} + 2\tan {}^{x}/_{2}\right)}{2}dx$$

$$= \int \frac{e^{x}\left(\sec^{2} {}^{x}/_{2} + 2\tan {}^{x}/_{2}\right)}{2}dx$$

$$= \int \frac{e^{x}\sec^{2} {}^{x}/_{2}\,dx}{2} + \int e^{x}\tan {}^{x}/_{2}\,dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan(x/2)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^{x}\sec^{2} {}^{x}/_{2}\,dx}{2} + \int e^{x}\tan {}^{x}/_{2}\,dx$$

$$= \int \frac{e^{x}\sec^{2} {}^{x}/_{2}\,dx}{2} + \tan {}^{x}/_{2}\int e^{x}dx - \int\left[\frac{d}{dx}\left(\tan {}^{x}/_{2}\right)\int e^{x}dx\right]dx$$

$$= \int \frac{e^{x}\sec^{2} {}^{x}/_{2}\,dx}{2} + e^{x}\tan {}^{x}/_{2} - \int \frac{e^{x}\sec^{2} {}^{x}/_{2}\,dx}{2} + c$$

$= \mathbf{e^{x}\tan {}^{x}/_{2} + c}$, where c is the integrating constant

Question 68.

Evaluate the following integrals:

$$\int e^x\left(\frac{\sin\ 4x-4}{1-\cos\ 4x}\right)dx$$

Answer:

$$\int e^x\left(\frac{\sin 4x-1}{1-\cos 4x}\right)dx$$

$$=\int e^x\left(\frac{2\sin 2x\cos 2x-4}{2\sin^2 2x}\right)dx$$

$$=\int e^x(\cot 2x-2\operatorname{cosec}^2 2x)dx$$

$$=\int e^x\cot 2x dx-\int 2e^x\operatorname{cosec}^2 2x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot 2x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x\cot 2x dx-\int 2e^x\operatorname{cosec}^2 2x dx$$

$$=\cot 2x\int e^x dx-\int\left\{\frac{d}{dx}(\cot 2x)\int e^x dx\right\}dx-\int 2e^x\operatorname{cosec}^2 2x dx$$

$$=e^x\cot 2x+\int 2e^x\operatorname{cosec}^2 2x dx-\int 2e^x\operatorname{cosec}^2 2x dx+c$$

$=\mathbf{e^x\cot 2x+c}$, where c is the integrating constant

Question 69.

Evaluate the following integrals:

$$\int \frac{e^x\left[\sqrt{1-x^2}\sin^{-1}x+1\right]}{\sqrt{1-x^2}}dx$$

Answer:

$$\int \frac{e^x[\sqrt{1-x^2}\sin^{-1}x+1]}{\sqrt{1-x^2}}dx$$

$$=\int e^x\left(\sin^{-1}x+\frac{1}{\sqrt{1-x^2}}\right)dx$$

$$=\int e^x\sin^{-1}x\,dx+\int\frac{e^x}{\sqrt{1-x^2}}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x\sin^{-1}x\,dx+\int\frac{e^x}{\sqrt{1-x^2}}dx$$

$$=\sin^{-1}x\int e^x dx-\int\left\{\frac{d}{dx}(\sin^{-1}x)\int e^x dx\right\}dx+\int\frac{e^x}{\sqrt{1-x^2}}dx$$

$$=e^x\sin^{-1}x-\int\frac{e^x}{\sqrt{1-x^2}}dx+\int\frac{e^x}{\sqrt{1-x^2}}dx+c$$

$\mathbf{= e^x\sin^{-1}x+c}$, where c is the integrating constant

Question 70.

Evaluate the following integrals:

$$\int e^x\left(\frac{1+x\log x}{x}\right)dx$$

Answer:

$$\int e^x\left(\frac{1+x\log x}{x}\right)dx$$

$$=\int e^x\left(\frac{1}{x}+\log x\right)dx$$

$$=\int \frac{e^x}{x}dx+\int e^x\log x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \log x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x}{x}dx+\int e^x\log x dx$$

$$=\int \frac{e^x}{x}dx+\log x\int e^x dx-\int\left[\frac{d}{dx}(\log x)\int e^x dx\right]dx$$

$$=\int \frac{e^x}{x}dx+e^x\log x-\int \frac{e^x}{x}dx+c$$

$\mathbf{= e^x\log x + c}$, where c is the integrating constant

Question 71.
Evaluate the following integrals:

$$\int e^x\cdot\frac{x}{(1+x)^2}dx$$

Answer:

$$\frac{x}{(1+x)^2} = \frac{A}{(1+x)} + \frac{B}{(1+x)^2}$$

$$\Rightarrow x = A(1+x) + B$$

For x=-1, equation: -1 = B i.e. B = -1

For x=0, equation: 0 = A-1 i.e. A = 1

$$\therefore \frac{x}{(1+x)^2}$$

$$= \frac{1}{(1+x)} - \frac{1}{(1+x)^2}$$

The given equation becomes

$$\int e^x \left[\frac{1}{(1+x)} - \frac{1}{(1+x)^2}\right] dx$$

$$= \int e^x \times \frac{1}{(1+x)} dx - \int e^x \times \frac{1}{(1+x)^2} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+x)$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int \frac{e^x}{(1+x)} dx - \int \frac{e^x}{(1+x)^2} dx$$

$$= \frac{1}{(1+x)} \int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{1+x}\right) \int e^x dx\right] dx - \int \frac{e^x}{(1+x)^2} dx$$

$$= \frac{e^x}{(1+x)} + \int \frac{e^x}{(1+x)^2}dx - \int \frac{e^x}{(1+x)^2}dx + c$$

$= \frac{\mathbf{e^x}}{\mathbf{(1+x)}} + \mathbf{c}$, where c is the integrating constant

Question 72.
Evaluate the following integrals:

$$\int e^x \frac{(x-1)}{(x+1)^3}dx$$

Answer:

$$\frac{x-1}{(x+1)^3} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

$\Rightarrow x - 1 = A(x+1)^2 + B(x+1) + C$

For x=-1, equation: -2 = C i.e. C = -2

For x=0, equation: -1 = A+B-2 i.e. A+B = 1

For x=1, equation: 0 = 4A+2B-2

i.e. 2(A+B+A) = 2

⇨1+A = 1

⇨A = 0

And, B = 1

$$\therefore \frac{x-1}{(x+1)^3}$$

$$= \frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}$$

The given equation becomes

$$\int e^x\left[\frac{1}{(x+1)^2}-\frac{2}{(x+1)^3}\right]dx$$

$$=\int e^x \times \frac{1}{(x+1)^2}dx-\int e^x \times \frac{2}{(x+1)^3}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+x)^2$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int \frac{e^x}{(x+1)^2}dx-\int \frac{2e^x}{(x+1)^3}dx$$

$$=\frac{1}{(x+1)^2}\int e^x dx-\int\left[\frac{d}{dx}\left(\frac{1}{(x+1)^2}\right)\int e^x dx\right]dx-\int \frac{2e^x}{(x+1)^3}dx$$

$$=\frac{e^x}{(x+1)^2}+\int \frac{2e^x}{(x+1)^3}dx-\int \frac{2e^x}{(x+1)^3}dx+c$$

$=\frac{e^x}{(x+1)^2}+c$, where c is the integrating constant

Question 73.
Evaluate the following integrals:

$$\int e^x \frac{(2-x)}{(1-x)^2}dx$$

Answer:
$$\frac{2-x}{(1-x)^2}=\frac{A}{(1-x)}+\frac{B}{(1-x)^2}$$

$$\Rightarrow 2-x=A(1-x)+B$$

For x=1, equation: 1 = B i.e. B = 1

For x=2, equation: 0 = -A+1 i.e. A = 1

$$\therefore \frac{2-x}{(1-x)^2}$$

$$= \frac{1}{(1-x)} + \frac{1}{(1-x)^2}$$

The given equation becomes

$$\int e^x\left[\frac{1}{(1-x)} + \frac{1}{(1-x)^2}\right]dx$$

$$= \int e^x \times \frac{1}{(1-x)^2}dx + \int e^x \times \frac{1}{1-x}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1-x)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x}{(1-x)^2}dx + \int \frac{e^x}{1-x}dx$$

$$= \int \frac{e^x}{(1-x)^2}dx + \frac{1}{1-x}\int e^x dx - \int\left[\frac{d}{dx}\left(\frac{1}{1-x}\right)\int e^x dx\right]dx$$

$$= \int \frac{e^x}{(1-x)^2}dx + \frac{e^x}{1-x} - \int \frac{e^x}{(1-x)^2}dx + c$$

$= \frac{e^x}{1-x} + c$, where c is the integrating constant

Question 74.

Evaluate the following integrals:

$$\int e^x \cdot \frac{(x-3)}{(x-1)^3} dx$$

Answer:

$$\frac{x-3}{(x-1)^3} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$$

$$\Rightarrow x - 3 = A(x-1)^2 + B(x-1) + C$$

For x=1, equation: -2 = C i.e. C = -2

For x=0, equation: -3 = A-B-2 i.e. B = A+1

For x=3, equation: 0 = 4A+2B-2

i.e. 2(A+B+A) = 2

⇨1+3A = 1

⇨A = 0

And, B = 1

$$\therefore \frac{x-3}{(x-1)^3}$$

$$= \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}$$

The given equation becomes

$$\int e^x \left[\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right] dx$$

$$= \int e^x \times \frac{1}{(x-1)^2} dx - \int e^x \times \frac{2}{(x-1)^3} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1-x)^2$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int \frac{e^x}{(x-1)^2} dx - \int \frac{2e^x}{(x-1)^3} dx$$

$$= \frac{1}{(x-1)^2} \int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{(x-1)^2}\right) \int e^x dx\right] dx - \int \frac{2e^x}{(x-1)^3} dx$$

$$= \frac{e^x}{(x-1)^2} + \int \frac{2e^x}{(x-1)^3} dx - \int \frac{2e^x}{(x-1)^3} dx + c$$

$= \frac{e^x}{(x-1)^2} + c$, where c is the integrating constant

Question 75.
Evaluate the following integrals:

$$\int e^{3x} \left(\frac{3x-1}{9x^2}\right) dx$$

Answer:
$$\int e^{3x} \left(\frac{3x-1}{9x^2}\right) dx$$

$$= \int \frac{e^{3x}}{3x} dx - \int \frac{e^{3x}}{9x^2} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/3x$and $f_2(x) = e^{3x}$ in the first integral and keeping the second integral intact,

$$\int \frac{e^{3x}}{3x} dx - \int \frac{e^{3x}}{9x^2} dx$$

$$= \frac{1}{3x} \int e^{3x} dx - \int \left[\frac{d}{dx}\left(\frac{1}{3x}\right) \int e^{3x} dx\right] dx - \int \frac{e^{3x}}{9x^2} dx$$

$$= \frac{e^{3x}}{9x} + \int \frac{e^{3x}}{9x^2} dx - \int \frac{e^{3x}}{9x^2} dx + c$$

$= \mathbf{\frac{e^{3x}}{9x} + c}$, where c is the integrating constant

Question 76.
Evaluate the following integrals:

$$\int \frac{(x+1)}{(x+2)^2} e^x dx$$

Answer:
$$\frac{x+1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$$

$$\Rightarrow x + 1 = A(x+2) + B$$

For x=-2, equation: -1 = B i.e. B = -1

For x=-1, equation: 0 = A-1 i.e. A = 1

$$\therefore \frac{x+1}{(x+2)^2}$$

$$= \frac{1}{(x+2)} - \frac{1}{(x+2)^2}$$

The given equation becomes

$$\int e^x \left[\frac{1}{(x+2)} - \frac{1}{(x+2)^2}\right] dx$$

$$= \int e^x \times \frac{1}{x+2} dx - \int e^x \times \frac{1}{(x+2)^2} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(x+2)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x}{x+2} dx - \int \frac{e^x}{(x+2)^2} dx$$

$$= \frac{1}{x+2} \int e^x dx - \int \left[\frac{d}{dx} \left(\frac{1}{x+2} \right) \int e^x dx \right] dx - \int \frac{e^x}{(x+2)^2} dx$$

$$= \frac{e^x}{x+2} + \int \frac{e^x}{(x+2)^2} dx - \int \frac{e^x}{(x+2)^2} dx + c$$

$= \frac{\mathbf{e^x}}{\mathbf{x+2}} + \mathbf{c}$, where c is the integrating constant

Question 77.
Evaluate the following integrals:

$$\int \frac{x e^{2x}}{(1+2x)^2} dx$$

Answer:
$$\frac{x}{(1+2x)^2} = \frac{A}{(1+2x)} + \frac{B}{(1+2x)^2}$$

$\Rightarrow x = A(1+2x) + B$

For x=-1/2, equation: -1/2 = B i.e. B = -1/2

For x=0, equation: 0 = A-1/2 i.e. A = 1/2

$\therefore \frac{x}{(1+2x)^2}$

$= \frac{1}{2(1+2x)} - \frac{1}{2(1+2x)^2}$

The given equation becomes

$$\int e^{2x}\left[\frac{1}{2(1+2x)} - \frac{1}{2(1+2x)^2}\right]dx$$

$$= \int e^{2x} \times \frac{1}{2(1+2x)}\,dx - \int e^{2x} \times \frac{1}{2(1+2x)^2}\,dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+2x)$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$$\int e^{2x} \times \frac{1}{2(1+2x)}\,dx - \int e^{2x} \times \frac{1}{2(1+2x)^2}\,dx$$

$$= \frac{1}{2}\left[\frac{1}{1+2x}\int e^{2x}dx - \int\left[\frac{d}{dx}\left(\frac{1}{1+2x}\right)\int e^{2x}dx\right]dx - \int \frac{e^{2x}}{(1+2x)^2}dx\right]$$

$$= \frac{1}{2}\left[\frac{e^{2x}}{2(2x+1)} + \int \frac{e^{2x}}{(2x+1)^2}dx - \int \frac{e^{2x}}{(2x+1)^2}dx\right]$$

$= \frac{e^{2x}}{4(2x+1)} + c$, where c is the integrating constant

Question 78.
Evaluate the following integrals:

$$\int e^{2x}\left(\frac{2x-1}{4x^2}\right)dx$$

Answer:

$$\int e^{2x}\left(\frac{2x-1}{4x^2}\right)dx$$

$$=\int \frac{e^{2x}}{2x}dx-\int \frac{e^{2x}}{4x^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/2x$ and $f_2(x) = e^{2x}$ in the first integral and keeping the second integral intact,

$$\int \frac{e^{2x}}{2x}dx-\int \frac{e^{2x}}{4x^2}dx$$

$$=\frac{1}{2x}\int e^{2x}dx-\int\left[\frac{d}{dx}\left(\frac{1}{2x}\right)\int e^{2x}dx\right]dx-\int \frac{e^{2x}}{4x^2}dx$$

$$=\frac{e^{2x}}{4x}+\int \frac{e^{2x}}{4x^2}dx-\int \frac{e^{2x}}{4x^2}dx+c$$

$=\frac{e^{2x}}{4x}+c$, where c is the integrating constant

Question 79.
Evaluate the following integrals:

$$\int e^x\left(\log x+\frac{1}{x^2}\right)dx$$

Answer:

$$\int e^x\left(\log x+\frac{1}{x^2}\right)dx$$

$$= \int e^x \log x dx - \int \frac{e^x}{x^2} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \log x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \log x dx - \int \frac{e^x}{x^2} dx$$

$$= \log x \int e^x dx - \int \left[\frac{d}{dx}(\log x) \int e^x dx\right] dx - \int \frac{e^x}{x^2} dx$$

$$= e^x \log x - \int \frac{e^x}{x} dx - \int \frac{e^x}{x^2} dx$$

$$= e^x \log x - \left[\frac{1}{x} \int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{x}\right) \int e^x dx\right] dx\right] - \int \frac{e^x}{x^2} dx$$

$$= e^x \log x - \frac{e^x}{x} + \int \frac{e^x}{x^2} dx - \int \frac{e^x}{x^2} dx + c$$

$= \mathbf{e^x \left(\log x - \frac{1}{x}\right) + c}$, where c is the integrating constant

Question 80.
Evaluate the following integrals:

$$\int \frac{\log x}{(1 + \log x)^2} dx$$

Answer:
$$\frac{\log x}{(1 + \log x)^2} = \frac{A}{(1 + \log x)} + \frac{B}{(1 + \log x)^2}$$

$$\Rightarrow \log x = A(1 + \log x) + B$$

For x=1, equation: 0 = A+B

For x=1/e, equation: -1 = B i.e. B = -1

So, A = 1

$$\therefore \frac{\log x}{(1+\log x)^2}$$

$$= \frac{1}{(1+\log x)} - \frac{1}{(1+\log x)^2}$$

The given equation becomes

$$\int \left[\frac{1}{(1+\log x)} - \frac{1}{(1+\log x)^2}\right] dx$$

$$= \int \frac{1}{(1+\log x)} dx - \int \frac{1}{(1+\log x)^2} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+\log x)$ and $f_2(x) = 1$in the second integral and keeping the first integral intact,

$$\int \frac{1}{(1+\log x)} dx - \int \frac{1}{(1+\log x)^2} dx$$

$$= \frac{1}{(1+\log x)} \int dx - \int \left[\frac{d}{dx}\left(\frac{1}{(1+\log x)}\right) \int dx\right] dx - \int \frac{1}{(1+\log x)^2} dx$$

$$= \frac{x}{(1+\log x)} + \int \frac{1}{(1+\log x)^2} dx - \int \frac{1}{(1+\log x)^2} dx + c$$

$= \frac{\mathbf{x}}{\mathbf{(1+\log x)}} + \mathbf{c}$, where c is the integrating constant

Question 81.
Evaluate the following integrals:

$$\int\{\sin(\log x)+\cos(\log x)\}dx$$

Answer:
Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = sin(logx) and $f_2(x)$ = 1in the first integral and keeping the second integral intact,

$$\int \sin(\log x)dx + \int \cos(\log x)dx$$

$$= \sin(\log x)\int dx - \int\left[\frac{d}{dx}(\sin(\log x))\int dx\right]dx + \int \cos(\log x)dx$$

$$= x\sin(\log x) - \int \cos(\log x)\,dx + \int \cos(\log x)dx + c$$

$\mathbf{= e^{\log x}\sin(\log x) + c}$, where c is the integrating constant

Question 82.
Evaluate the following integrals:

$$\int\left\{\frac{1}{\log x}-\frac{1}{(\log x)^2}\right\}dx$$

Answer:
Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(\log x)$ and $f_2(x) = 1$in the first integral and keeping the second integral intact,

$$\int \frac{1}{\log x} dx - \int \frac{1}{(\log x)^2} dx$$

$$= \frac{1}{\log x}\int dx - \int \left[\frac{d}{dx}\left(\frac{1}{\log x}\right)\int dx\right] dx - \int \frac{1}{(\log x)^2} dx$$

$$= \frac{x}{\log x} + \int \frac{1}{(\log x)^2} dx - \int \frac{1}{(\log x)^2} dx + c$$

$= \frac{x}{\log x} + c$, where c is the integrating constant

Question 83.
Evaluate the following integrals:

$$\int \left\{ \log(\log x) + \frac{1}{(\log x)^2} \right\} dx$$

Answer:
Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \log(\log x)$ and $f_2(x) = 1$in the first integral and keeping the second integral intact,

$$\int \log(\log x) dx + \int \frac{1}{(\log x)^2} dx$$

$$= \log(\log x)\int dx - \int \left[\frac{d}{dx}(\log(\log x))\int dx\right] dx + \int \frac{1}{(\log x)^2} dx$$

$$= x\log(\log x) - \int \frac{1}{\log x} dx + \int \frac{1}{(\log x)^2} dx$$

$$= x\log(\log x) - \left[\frac{1}{\log x}\int dx - \int\left[\frac{d}{dx}\left(\frac{1}{\log x}\right)\int dx\right]dx\right] + \int\frac{1}{(\log x)^2}dx$$

$$= x\log(\log x) - \frac{x}{\log x} - \int\frac{1}{(\log x)^2}dx + \int\frac{1}{(\log x)^2}dx + c$$

$= \mathbf{x\left[\log(\log x) - \frac{1}{\log x}\right] + c}$, where c is the integrating constant

Question 84.
Evaluate the following integrals:

$$\int\left(\frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\right)dx$$

Answer:
It is know that $\sin^{-1}x + \cos^{-1}x = \pi/2$

$$\therefore \left(\frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\right)$$

$$= \frac{2}{\pi}\left(\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}\right)$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Now, for the first term,

Taking $f_1(x) = \sin^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \sin^{-1}\sqrt{x}\,dx$$

$$= \sin^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}\left(\sin^{-1}\sqrt{x}\right)\int dx\right\}dx$$

$$= x\sin^{-1}\sqrt{x} - \int \frac{1}{2\sqrt{x}\sqrt{1-x}} \times x dx$$

$$= x\sin^{-1}\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}} dx$$

Taking (1-x)=a^2,

-dx=2ada i.e. dx=-2ada

Again, x=1-a^2

$$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}} dx$$

$$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$$

$$= -\int \sqrt{1-a^2}\, da$$

$$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

$$= -\left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c$$

The total integration yields as

$= x\sin^{-1}\sqrt{x} + \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c'$, where c' is the integrating constant

For the second term,

Taking $f_1(x) = \cos^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \cos^{-1}\sqrt{x}\,dx$$

$$= \cos^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}\left(\cos^{-1}\sqrt{x}\right)\int dx\right\}dx$$

$$= x\cos^{-1}\sqrt{x} - \int \frac{-1}{2\sqrt{x}\sqrt{1-x}} \times x dx$$

$$= x\cos^{-1}\sqrt{x} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

Taking (1-x)=a^2,

-dx=2ada i.e. dx=-2ada

Again, x=1-a^2

$$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx$$

$$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$$

$$= -\int \sqrt{1-a^2}\,da$$

$$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1}a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1}a\right]$$

$$= -\left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c$$

The total integration yields as

$= x\cos^{-1}\sqrt{x} - \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c''$, where c'' is the integrating constant

$$\therefore \int \left(\frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\right) dx$$

$$= \frac{2}{\pi}\int (\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x})dx$$

$$= \frac{2}{\pi}\left[x\sin^{-1}\sqrt{x} + \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] - x\cos^{-1}\sqrt{x} + \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right]\right] + c$$

$= \frac{2}{\pi}\left[\sqrt{x-x^2} + x(\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}) + \sin^{-1}\sqrt{1-x}\right] + c$ where c is the integrating constant

Question 85.
Evaluate the following integrals:

$$\int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x \, dx$$

Answer:
Tip – 5^x is to be replaced by a

$$\therefore 5^x = a$$

$$\Rightarrow 5^x \log 5 dx = da$$

$$\Rightarrow 5^x dx = \frac{da}{\log 5}$$

The equation becomes as follows:

$$\int 5^{5^a} \times 5^a \times \frac{1}{\log 5} da$$

Tip – 5^a is to be replaced by k

$\therefore 5^{a} = k$

$\Rightarrow 5^{a}\log 5 da = dk$

$\Rightarrow 5^{a} da = \dfrac{dk}{\log 5}$

The equation becomes as follows:

$$\int 5^{k} \times \frac{1}{(\log 5)^{2}} dk$$

$$= \frac{1}{(\log 5)^{2}} \int 5^{k} dk$$

$$= \frac{5^{k}}{(\log 5)^{3}} + c$$

Re-replacing the value of k,

$$\frac{5^{5^{a}}}{(\log 5)^{3}} + c$$

Re-replacing the value of a,

$\dfrac{5^{5^{5^{x}}}}{(\log 5)^{3}} + c$, where c is the integrating constant

Question 86.
Evaluate the following integrals:

$$\int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx$$

Answer:
$$\left(\frac{1+\sin 2x}{1+\cos 2x}\right)$$

$$= \left(\frac{1 + \frac{2\tan x}{1+\tan^2 x}}{1 + \frac{1-\tan^2 x}{1+\tan^2 x}} \right)$$

$$= \frac{(1+\tan x)^2}{2}$$

$$\therefore \int e^{2x} \left(\frac{1+\sin 2x}{1+\cos 2x} \right) dx$$

$$= \int e^{2x} \times \frac{(1+\tan x)^2}{2}$$

$$= \int \frac{e^{2x}(1+\tan^2 x + 2\tan x)}{2} dx$$

$$= \int \frac{e^{2x}(\sec^2 x + 2\tan x)}{2} dx$$

$$= \int \frac{e^{2x}\sec^2 x dx}{2} + \int e^{2x} \tan x\, dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{ \frac{d}{dx} f_1(x) \int f_2(x) dx \right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan x$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$$\int \frac{e^{2x}\sec^2 x dx}{2} + \int e^{2x} \tan x dx$$

$$= \int \frac{e^{2x}\sec^2 x dx}{2} + \tan x \int e^{2x} dx - \int \left[\frac{d}{dx}(\tan x) \int e^{2x} dx \right] dx$$

$$= \int \frac{e^{2x}\sec^2 x dx}{2} + \frac{1}{2} e^{2x} \tan x - \int \frac{e^{2x}\sec^2 x dx}{2} + c$$

$= \frac{1}{2} e^x \tan {}^x/_2 + c$, where c is the integrating constant

Question 87.
Evaluate the following integrals:

$$\int e^{2x}\left(\frac{1-\sin\ 2x}{1-\cos\ 2x}\right)dx$$

Answer:
$\left(\frac{1-\sin 2x}{1-\cos 2x}\right)$

$$=\left(\frac{1-\frac{2\tan x}{1+\tan^2 x}}{1-\frac{1-\tan^2 x}{1+\tan^2 x}}\right)$$

$$=\frac{(1-\tan x)^2}{2}$$

$$\therefore \int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$$

$$=\int e^{2x}\times\frac{(1-\tan x)^2}{2}$$

$$=\int\frac{e^{2x}(1+\tan^2 x-2\tan x)}{2}dx$$

$$=\int\frac{e^{2x}(\sec^2 x-2\tan x)}{2}dx$$

$$=\int\frac{e^{2x}\sec^2 x dx}{2}-\int e^{2x}\tan x\, dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = tanx and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$$\int \frac{e^{2x}\sec^2 x dx}{2} - \int e^{2x}\tan x dx$$

$$= \int \frac{e^{2x}\sec^2 x dx}{2} - \tan x\int e^{2x}dx + \int\left[\frac{d}{dx}(\tan x)\int e^{2x}dx\right]dx$$

$$= \int \frac{e^{2x}\sec^2 x dx}{2} - \frac{1}{2}e^{2x}\tan x + \int \frac{e^{2x}\sec^2 x dx}{2} + c$$

$= -\frac{1}{2}\mathbf{e^x \tan {}^{x}/_{2} + c}$, where c is the integrating constant