

## Objective Questions

### Question 1.

Mark (✓) against the correct answer in each of the following:

$$\int x^6 dx = ?$$

A.  $7x^7 + C$

B.  $\frac{x^7}{7} + C$

C.  $6x^5 + C$

D.  $6x^7 + C$

### Answer:

Given:

$$\int x^6 dx,$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^6 dx = \frac{x^{6+1}}{6+1} + c$$

$$= \frac{x^7}{7} + c$$

### Question 2.

Mark (✓) against the correct answer in each of the following:

$$\int x^{5/3} dx = ?$$

A.  $\frac{3}{5} x^{2/3} + C$

B.  $\frac{8}{3} x^{8/3} + C$

C.  $\frac{3}{8}x^{\frac{8}{3}} + C$

D.  $\frac{5}{3}x^{\frac{8}{3}} + C$

**Answer:**

Given:

$$\int x^{\frac{5}{3}} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\frac{5}{3}} dx = \frac{x^{\frac{5}{3}+1}}{\frac{5}{3}+1} + c$$

$$= \frac{x^{\frac{8}{3}}}{\frac{8}{3}} + c$$

$$= \frac{3}{8}x^{\frac{8}{3}} + c$$

**Question 3.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{x^3} dx = ?$$

A.  $\frac{-3}{x^2} + C$

B.  $\frac{-1}{2x^2} + C$

C.  $\frac{-1}{3x^2} + C$

D.  $\frac{x^{-2}}{2} + C$

**Answer:**

Given:

$$\int \frac{1}{x^3} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{1}{x^3} dx = \frac{x^{-3+1}}{-3+1} + c$$

$$= -\frac{x^{-2}}{2} + c$$

$$= -\frac{1}{2x^2} + c$$

**Question 4.**

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt[3]{x} dx = ?$$

A.  $\frac{3}{4}x^{3/4} + C$

B.  $\frac{4}{3}x^{3/4} + C$

C.  $\frac{3}{4}x^{4/3} + C$

D.  $\frac{4}{3}x^{4/3} + C$

**Answer:**

Given:

$$\int \sqrt[3]{x} \, dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \sqrt[3]{x} \, dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + c$$

$$= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + c$$

$$= \frac{3}{4} x^{\frac{4}{3}} + c$$

**Question 5.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{\sqrt[3]{x}} \, dx = ?$$

A.  $\frac{3}{2} x^{2/3} + C$

B.  $\frac{3}{2x^{2/3}} + C$

C.  $\frac{2}{3x^{2/3}} + C$

D.  $\frac{2}{3} x^{3/2} + C$

**Answer:**

Given:

$$\int \frac{1}{\sqrt[3]{x}} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{1}{\sqrt[3]{x}} dx = \frac{x^{\frac{-1}{3}+1}}{\frac{-1}{3}+1} + c$$

$$= \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c$$

$$= \frac{3}{2} x^{\frac{2}{3}} + c$$

**Question 6.**

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt[3]{x^2} dx = ?$$

A.  $\frac{5}{3} x^{\frac{5}{3}} + C$

B.  $\frac{3}{5} x^{\frac{5}{3}} + C$

C.  $\frac{5}{3} x^{\frac{3}{5}} + C$

D.  $\frac{3}{5} x^{\frac{3}{5}} + C$

**Answer:**

Given:

$$\int \sqrt[3]{x^2} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \sqrt[3]{x^2} dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + c$$

$$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + c$$

$$= \frac{3}{5} x^{\frac{5}{3}} + c$$

### Question 7.

Mark (✓) against the correct answer in each of the following:

$$\int 3^x dx = ?$$

A.  $3^x (\log 3) + C$

B.  $3^x + C$

C.  $\frac{3^x}{\log 3} + C$

D.  $\frac{\log 3}{3^x} + C$

**Answer:**

Given:

$$\int 3^x dx$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int 3^x dx = \frac{3^x}{\ln 3} + c$$

**Question 8.**

Mark (✓) against the correct answer in each of the following:

$$\int 2^{\log x} dx = ?$$

A.  $\frac{2^{\log x + 1}}{(\log x + 1)} + C$

B.  $\frac{x^{(\log 2 + 1)}}{(\log 2 + 1)} + C$

C.  $\frac{2^{\log x}}{\log 2} + C$  2adc

D.  $\frac{2^{\log x}}{2} + C$

**Answer:**

Given:

$$\int 2^{\log x} dx$$

As  $2^{\log x} = x^{\log 2}$

$$I = \int x^{\log 2} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\log 2} dx = \frac{x^{\log 2 + 1}}{\log 2 + 1} + c$$

**Question 9.**

Mark (✓) against the correct answer in each of the following:

$$\int \operatorname{cosec} x (\operatorname{cosec} x + \cot x) dx = ?$$

- A.  $\cot x - \operatorname{cosec} x + C$
- B.  $-\cot x + \operatorname{cosec} x + C$
- C.  $\cot x + \operatorname{cosec} x + C$
- D.  $-\cot x - \operatorname{cosec} x + C$

**Answer:**

Given:

$$\int \operatorname{cosec} x (\operatorname{cosec} x + \cot x) dx = \int (\csc x)^2 + \cot x \csc x dx$$

$$= -\cot x - \csc x + C$$

**Question 10.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sec x}{(\sec x + \tan x)} dx = ?$$

- A.  $\tan x + \sec x + C$
- B.  $\tan x - \sec x + C$
- C.  $-\tan x + \sec x + C$
- D.  $-\tan x - \sec x + C$

**Answer:**

Given:

$$\int \frac{\sec x}{(\sec x + \tan x)} dx$$

Multiply and divide by  $(\sec x - \tan x)$

$$= \int \frac{(\sec x)^2 - \tan x \sec x}{(\sec x)^2 - (\tan x)^2} dx$$

$$= \int (\sec x)^2 - \tan x \sec x dx$$

$$= \tan x - \sec x + C$$



**Question 11.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(1 - \cos 2x)}{(1 + \cos 2x)} dx = ?$$

- A.  $\tan x + x + C$
- B.  $\tan x - x + C$
- C.  $-\tan x + x + C$
- D.  $-\tan x - x + C$

**Answer:**

Given:

$$\int \frac{(1 - \cos 2x)}{(1 + \cos 2x)} dx = \int \frac{(2(\sin x)^2)}{(2(\cos x)^2)} dx$$

$$= \int (\tan x)^2$$

$$= \int ((\sec x)^2 - 1) dx$$

$$= \tan x - x + C$$

**Question 12.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{\sin^2 x \cos^2 x} dx = ?$$

- A.  $\tan x + \cot x + C$
- B.  $-\tan x + \cot x + C$
- C.  $\tan x - \cot x + C$
- D. none of these

**Answer:**

Given:

$$\int \frac{1}{\sin^2 x \cos^2 x} dx$$

As we know  $\sin^2 x + \cos^2 x = 1$

$$\int \frac{1}{\sin^2 x \cos^2 x} dx = \int \frac{(\sin x)^2 + (\cos x)^2}{(\sin x)^2 (\cos x)^2} dx$$

$$= \int \frac{1}{(\cos x)^2} + \frac{1}{(\sec x)^2} dx$$

$$= \int (\sec x)^2 + (\csc x)^2 dx$$

$$= \tan x - \cot x + c$$

### Question 13.

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = ?$$

A.  $-\cot x - \tan x + C$

B.  $-\cot x + \tan x + C$

C.  $\cot x - \tan x + C$

D.  $\cot x + \tan x + C$

**Answer:**

Given:

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = \int \frac{(\cos x)^2 - (\sin x)^2}{(\sin x)^2 (\cos x)^2} dx$$

$$= \int \frac{1}{(\sin x)^2} - \frac{1}{(\cos x)^2} dx$$

$$= \int (\csc x)^2 - (\sec x)^2 dx$$

$$= -\cot x - \tan x + c$$

**Question 14.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx = ?$$

- A.  $2 \sin x + 2x \cos \alpha + C$
- B.  $2 \sin x - 2x \cos \alpha + C$
- C.  $-2 \sin x + 2x \cos \alpha + C$
- D.  $-2 \sin x - 2x \cos \alpha + C$

**Answer:**

Given:

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx = \int \frac{2 \sin(x + \alpha) \sin(x - \alpha)}{2 \sin\left(\frac{x + \alpha}{2}\right) \sin\left(\frac{x - \alpha}{2}\right)} dx$$

$$= \int 4 \cos\left(\frac{x + \alpha}{2}\right) \cos\left(\frac{x - \alpha}{2}\right) dx$$

$$= 2 \int \cos(x) + \cos(\alpha) dx$$

$$= 2 \sin x + 2x \cos \alpha + C$$

**Question 15.**

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{1 + \cos 2x} \, dx = ?$$

- A.  $\sqrt{2} \cos x + C$
- B.  $\sqrt{2} \sin x + C$
- C.  $-\sqrt{2} \cos x + C$
- D.  $-\sqrt{2} \sin x + C$

**Answer:**

Given:

$$\int \sqrt{1 + \cos 2x} dx = \int \sqrt{2(\cos x)^2} dx$$

$$= \sqrt{2} \int \cos x dx$$

$$= \sqrt{2} \sin x + c$$

**Question 16.**

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{1 + \sin 2x} dx = ?$$

- A.  $\sin x + \cos x + C$
- B.  $-\sin x + \cos x + C$
- C.  $\sin x - \cos x + C$
- D.  $-\sin x - \cos x + C$

**Answer:**

Given:

$$\int \sqrt{1 + \sin 2x} dx = \int \sqrt{1 + \frac{2 \tan x}{1 + (\tan x)^2}} dx$$

$$= \int \sqrt{\frac{(1 + \tan x)^2}{(\sec x)^2}} dx$$

$$= \int \frac{1 + \tan x}{\sec x} dx$$

$$= \int \cos x + \sin x dx$$

$$= \sin x - \cos x + c$$

**Question 17.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx = ?$$

- A.  $\cot x + \tan x + C$
- B.  $-\cot x + \tan x + C$
- C.  $\cot x - \tan x + C$
- D.  $-\cot x - \tan x + C$

**Answer:**

Given:

$$\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx = \int \frac{(\cos x)^2 - (\sin x)^2}{(\sin x)^2 (\cos x)^2} dx$$

$$= \int \frac{1}{(\sin x)^2} - \frac{1}{(\cos x)^2} dx$$

$$= \int (\csc x)^2 - (\sec x)^2 dx$$

$$= -\cot x - \tan x + C$$

**Question 18.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(1 - \cos 2x)} = ?$$

- A.  $\frac{1}{2} \cot x + C$
- B.  $2 \cot x + C$
- C.  $-\frac{1}{2} \cot x + C$
- D.  $-2 \cot x + C$

**Answer:**

Given:

$$\int \frac{dx}{(1 - \cos 2x)} = \int \frac{1}{2(\sin x)^2} dx$$

$$= \frac{1}{2} \int (\csc x)^2 dx$$

$$= -\frac{1}{2} \cot x + c$$

**Question 19.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin 2x}{\sin x} dx = ?$$

A.  $2 \sin x + C$

B.  $\frac{1}{2} \sin x + C$

C.  $2 \cos x + C$

D.  $\frac{1}{2} \cos x + C$

**Answer:**

Given:

$$\int \frac{\sin 2x}{\sin x} dx = \int \frac{2 \sin x \cos x}{\sin x} dx$$

$$= 2 \int \cos x dx$$

$$= 2 \sin x + c$$

**Question 20.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(1 - \sin x)}{\cos^2 x} dx = ?$$

A.  $\tan x + \sec x + C$

- B.  $\tan x - \sec x + C$
- C.  $-\tan x + \sec x + C$
- D.  $-\tan x - \sec x + C$

**Answer:**

Given:

$$\int \frac{(1 - \sin x)}{\cos^2 x} dx = \int \frac{1}{(\cos x)^2} - \frac{\sin x}{(\cos x)^2} dx$$

$$= \int (\sec x)^2 - \tan x \sec x dx$$

$$= \tan x - \sec x + C$$

**Question 21.**

Mark (✓) against the correct answer in each of the following:

$$\int \cot^2 x dx = ?$$

- A.  $-\cot x - x + C$
- B.  $\cot x - x + C$
- C.  $-\cot x + x + C$
- D.  $\cot x + x + C$

**Answer:**

Given:

$$\int \cot^2 x dx = \int ((\csc x)^2 - 1) dx$$

$$= -\cot x - x + C$$

**Question 22.**

Mark (✓) against the correct answer in each of the following:

$$\int \sec x (\sec x + \tan x) dx = ?$$

- A.  $\tan x - \sec x + C$
- B.  $-\tan x + \sec x + C$

C.  $\tan x + \sec x + C$

D.  $-\tan x - \sec x + C$

**Answer:**

Given:

$$\int \sec x (\sec x + \tan x) dx = \int (\sec x)^2 + \sec x \tan x dx$$

$$= \tan x + \sec x + C$$

**Question 23.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx = ?$$

A.  $\tan x + x + C$

B.  $\tan x - x + C$

C.  $-\tan x + x + C$

D.  $-\tan x - x + C$

**Answer:**

Given:

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx = \int \frac{(\sin x)^2}{(\cos x)^2} dx$$

$$= \int (\tan x)^2 dx$$

$$= \int (\sec x)^2 - 1 dx$$

$$= \tan x - x + C$$

**Question 24.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin^2 x}{(1 + \cos x)} dx = ?$$



- A.  $x + \sin x + C$
- B.  $x - \sin x + C$
- C.  $\sin x - x + C$
- D.  $-\sin x - x + C$

**Answer:**

Given:

$$\int \frac{\sin^2 x}{(1 + \cos x)} dx = \int \frac{1 - (\cos x)^2}{(1 + \cos x)} dx$$

$$= \int \frac{(1 + \cos x)(1 - \cos x)}{(1 + \cos x)} dx$$

$$= \int (1 - \cos x) dx$$

$$= x - \sin x + C$$

**Question 25.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\cot x}{(\operatorname{cosec} x - \cot x)} dx = ?$$

- A.  $-\operatorname{cosec} x - \cot x - x + C$
- B.  $\operatorname{cosec} x - \cot x - x + C$
- C.  $-\operatorname{cosec} x + \cot x - x + C$
- D.  $\operatorname{cosec} x + \cot x - x + C$

**Answer:**

Given:

$$\int \frac{\cot x}{(\operatorname{cosec} x - \cot x)} dx = \int \frac{\cot x (\csc x + \cot x)}{((\csc x)^2 - (\cot x)^2)} dx$$

$$= \int \cot x \csc x + (\csc x)^2 dx$$

$$= -\csc x - \cot x + C$$

**Question 26.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin x}{(1 + \sin x)} dx = ?$$

- A.  $\sec x + \tan x + x + C$
- B.  $\sec x - \tan x + x + C$
- C.  $-\sec x + \tan x + x + C$
- D. None of these

**Answer:**

Given:

$$\int \frac{\sin x}{(1 + \sin x)} dx$$

Multiply and divide by  $(1 - \sin x)$

$$= \int \frac{\sin x - (\sin x)^2}{1 - (\sin x)^2} dx$$

$$= \int \frac{\sin x - (\sin x)^2}{(\cos x)^2} dx$$

$$= \int (\tan x \sec x - (\tan x)^2) dx$$

$$= \int (\tan x \sec x - (\sec x)^2 + 1) dx$$

$$= \sec x - \tan x + x + C$$

**Question 27.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(1 + \sin x)}{(1 - \sin x)} dx = ?$$

- A.  $2 \tan x + 2 \sec x + x + C$
- B.  $2 \tan x + 2 \sec x - x + C$

C.  $\tan x + \sec x - x + C$

D. None of these

**Answer:**

Given:

$$\int \frac{(1 + \sin x)}{(1 - \sin x)} dx$$

Multiply and divide with  $(1 + \sin x)$  to get,

$$\int \frac{(1 + \sin x)}{(1 - \sin x)} dx$$

$$= \int \frac{1 + (\sin x)^2 + 2 \sin x}{1 - (\sin x)^2} dx$$

$$= \int \frac{1 + (\sin x)^2 + 2 \sin x}{(\cos x)^2} dx$$

$$= \int (\sec x)^2 + (\tan x)^2 + 2 \tan x \sec x dx$$

$$= \int 2(\sec x)^2 - 1 + 2 \tan x \sec x dx$$

$$= 2 \tan x - x + 2 \sec x + C$$

**Question 28.**

Mark ( $\checkmark$ ) against the correct answer in each of the following:

$$\int \frac{1}{(1 + \cos x)} dx = ?$$

A.  $-\cot x + \operatorname{cosec} x + C$

B.  $\cot x - \operatorname{cosec} x + C$

C.  $\cot x + \operatorname{cosec} x + C$

D. None of these

**Answer:**

Given:

$$\int \frac{1}{(1 + \cos x)} dx$$

Multiply and divide by (1-cos x)

$$\int \frac{1}{(1 + \cos x)} dx = \int \frac{1 - \cos x}{1 - (\cos x)^2} dx$$

$$= \int \frac{1 - \cos x}{(\sin x)^2} dx$$

$$= \int (\csc x)^2 - \cot x \csc x dx$$

$$= -\cot x + \csc x + C$$

**Question 29.**

Mark (✓) against the correct answer in each of the following:

$$\int \sin^{-1}(\cos x) dx = ?$$

A.  $\operatorname{cosec} x + C$

B.  $\frac{\pi x}{2} + \frac{x^2}{2} + C$

C.  $\frac{\pi x}{2} - \frac{x^2}{2} + C$

D.  $\frac{x^2}{2} - \frac{\pi x}{2} + C$

**Answer:**

Given:

$$\int \sin^{-1}(\cos x) dx$$

$$\sin^{-1}(\cos x) + \cos^{-1}(\cos x) = \frac{\pi}{2}$$

$$= \int \frac{\pi}{2} - \cos^{-1}(\cos x) dx$$

$$= \int \frac{\pi}{2} - x dx$$

$$= \frac{\pi}{2}x - \frac{x^2}{2} + c$$

**Question 30.**

Mark (✓) against the correct answer in each of the following:

$$\int \tan^{-1} \left\{ \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} \right\} dx = ?$$

A.  $\frac{-1}{(1+x^2)} + C$

B.  $\frac{1}{\sqrt{1+x^2}} + C$

C.  $\frac{1}{\sqrt{1-x^2}} + C$

D.  $\frac{x^2}{2} + C$

**Answer:**

Given:

$$\int \tan^{-1} \left\{ \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} \right\} dx = \int \tan^{-1} \sqrt{\frac{2(\sin x)^2}{2(\cos x)^2}} dx$$

$$= \int \tan^{-1}(\tan x) dx$$

$$= \int x \, dx$$

$$= \frac{x^2}{2} + c$$

**Question 31.**

Mark (✓) against the correct answer in each of the following:

$$\int \cot^{-1} \left( \frac{\sin 2x}{1 - \cos 2x} \right) dx = ?$$

A.  $\frac{-1}{(1+x^2)} + C$

B.  $\frac{-1}{(1-x^2)} + C$

C.  $\frac{x^2}{2} + C$

D.  $2x^2 + C$

**Answer:**

Given:

$$\int \cot^{-1} \left( \frac{\sin 2x}{1 - \cos 2x} \right) dx = \int \cot^{-1} \left( \frac{2 \sin x \cos x}{1 - 1 + 2(\sin x)^2} \right) dx$$

$$= \int \cot^{-1}(\cot x) \, dx$$

$$= \int x \, dx$$

$$= \frac{x^2}{2} + c$$

**Question 32.**

Mark (✓) against the correct answer in each of the following:

$$\int \sin^{-1} \left( \frac{2 \tan x}{1 + \tan^2 x} \right) dx = ?$$

A.  $-x^2 + C$

B.  $x^2 + C$

C.  $\frac{x^2}{2} + C$

D.  $2x^2 + C$

**Answer:**

Given:

$$\int \sin^{-1} \left( \frac{2 \tan x}{1 + \tan^2 x} \right) dx = \int \sin^{-1}(\sin 2x) dx$$

$$= \int 2x dx$$

$$= x^2 + C$$

**Question 33.**

Mark (✓) against the correct answer in each of the following:

$$\int \cos^{-1} \left( \frac{1 - \tan^2 x}{1 + \tan^2 x} \right) dx = ?$$

A.  $x^2 + C$

B.  $-x^2 + C$

C.  $\frac{1}{\sqrt{1+x^2}} + C$

D.  $\frac{1}{\sqrt{1-x^2}} + C$

**Answer:**

Given:

$$\int \cos^{-1} \left( \frac{1 - \tan^2 x}{1 + \tan^2 x} \right) dx = \int \cos^{-1}(\cos 2x) dx$$

$$= \int 2x dx$$

$$= x^2 + C$$

**Question 34.**

Mark (✓) against the correct answer in each of the following:

$$\int \tan^{-1}(\operatorname{cosec} x - \cot x) dx = ?$$

A.  $\frac{x^2}{4} + C$

B.  $\frac{-x^2}{4} + C$

C.  $\frac{x^2}{2} + C$

D.  $\frac{-x^2}{2} + C$

**Answer:**

Given:

$$\int \tan^{-1}(\operatorname{cosec} x - \cot x) dx = \int \tan^{-1} \left( \frac{1 - \cos x}{\sin x} \right) dx$$

$$= \int \tan^{-1} \left( \frac{2 \sin \frac{x}{2} \sin \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} \right) dx$$

$$= \int \tan^{-1} \left( \tan \frac{x}{2} \right) dx$$

$$= \int \frac{x}{2} dx$$



$$= \frac{x^2}{4} + c$$

**Question 35.**

Mark (✓) against the correct answer in each of the following:

$$\int \left( \frac{(x^4 + 1)}{(x^2 + 1)} \right) dx = ?$$

A.  $\frac{x^3}{3} + x - \tan^{-1} x + C$

B.  $\frac{x^3}{3} - x - 2 \tan^{-1} x + C$

C.  $\frac{x^3}{3} + x - 2 \tan^{-1} x + C$

D. None of these

**Answer:**

Given:

$$\int \left( \frac{(x^4 + 1)}{(x^2 + 1)} \right) dx = \int \frac{(x^4 + 2x^2 + 1)}{(x^2 + 1)} - \frac{2x^2}{(x^2 + 1)} dx$$

$$= \int \frac{(x^2 + 1)^2}{(x^2 + 1)} dx - 2 \left\{ \int \frac{(x^2 + 1)}{(x^2 + 1)} - \frac{1}{(x^2 + 1)} dx \right\}$$

$$= \int (x^2 + 1) dx - 2\{x - \tan^{-1} x\} + c$$

$$= \frac{x^3}{3} - x - 2 \tan^{-1} x + c$$

**Question 36.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(ax + b)}{(cx + d)} dx = ?$$

A.  $\frac{ax}{c} + \log|cx + d| + C$

B.  $\frac{a}{c} + \log|cx + d| + C$

C.  $\frac{ax}{c} + \frac{(bc - ad)}{c^2} \log|cx + d| + C$

D. None of these

**Answer:**

Given:

$$\int \frac{(ax + b)}{(cx + d)} dx = \int \frac{ax}{cx + d} + \frac{b}{cx + d} dx$$

$$= a \int \frac{x}{cx + d} \times \frac{c}{c} dx + b \int \frac{1}{cx + d} dx$$

$$= \frac{a}{c} \left( \int \frac{cx + d}{cx + d} dx - \frac{d}{cx + d} \right) + b \ln|cx + d| + c$$

$$= \frac{a}{c} \left( x - \frac{d}{c} \ln|cx + d| \right) + \frac{b}{c} \ln|cx + d| + c$$

$$= \frac{a}{c} x + \frac{(bc - ad)}{c^2} \ln|cx + d| + c$$

**Question 37.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(\sin^3 x + \cos^3 x)}{\sin^2 x \cos^2 x} dx = ?$$

A.  $\sin x - \cos x + C$

B.  $\tan x - \cos x + C$

C.  $\sec x - \operatorname{cosec} x + C$

D. None of these

**Answer:**

Given:

$$\int \frac{(\sin^3 x + \cos^3 x)}{\sin^2 x \cos^2 x} dx = \int \frac{(\sin x)^3}{(\sin x)^2 (\cos x)^2} + \frac{(\sin x)^3}{(\sin x)^2 (\cos x)^2} dx$$

$$= \int (\tan x \sec x + \csc x \cot x) dx$$

$$= \sec x - \csc x + C$$

**Question 38.**

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin x}{\sin(x - \alpha)} dx = ?$$

A.  $x \cos \alpha + (\sin \alpha) \log |\sin(x - \alpha)| + C$

B.  $x \sin \alpha + (\sin \alpha) \log |\sin(x - \alpha)| + C$

C.  $x \cos \alpha - (\sin \alpha) \log |\sin(x - \alpha)| + C$

D.  $x \sin \alpha - (\sin \alpha) \log |\sin(x - \alpha)| + C$

**Answer:**

Given:

$$\int \frac{\sin x}{\sin(x - \alpha)} dx$$

Let  $x - \alpha = t$

$dx = dt$

$$I = \int \frac{\sin(t + \alpha)}{\sin t} dx$$

$$= \int \frac{\sin t \cos \alpha + \cos t \sin \alpha}{\sin t} dt$$

$$= \int \cos \alpha + \sin \alpha \cot t \, dt$$

$$= t \cos \alpha + \sin \alpha \ln |\sin t| + c$$

$$= (x - \alpha) \cos \alpha + (\sin \alpha) \ln |\sin(x - \alpha)| + c$$

$$= x \cos \alpha + (\sin \alpha) \ln |\sin(x - \alpha)| + c$$

**Question 39.**

Mark (✓) against the correct answer in each of the following:

$$\int \sin 3x \sin 2x \, dx = ?$$

A.  $-\frac{1}{5} \cos 5x + C$

B.  $\frac{1}{2} \sin x + \frac{1}{10} \sin 5x - C$

C.  $\frac{1}{2} \sin x - \frac{1}{10} \sin 5x - C$

D.  $-\frac{1}{3} \cos 3x - \frac{1}{2} \sin 2x + C$

**Answer:**

Given:

$$\int \sin 3x \sin 2x \, dx = \frac{1}{2} \int 2 \sin 3x \sin 2x \, dx$$

$$= \frac{1}{2} \int \cos x - \cos 5x \, dx$$

$$= \frac{1}{2} \left\{ \frac{\sin x}{1} - \frac{\sin 5x}{5} \right\} + c$$

$$= \frac{\sin x}{2} - \frac{\sin 5x}{10} + c$$

**Question 40.**

Mark (✓) against the correct answer in each of the following:

$$\int \cos 3x \sin 2x \, dx = ?$$

A.  $\frac{1}{2} \cos x - \frac{1}{10} \cos 5x + C$

B.  $-\frac{1}{2} \sin x + \frac{1}{10} \sin 5x + C$

C.  $-\frac{1}{2} \cos x + \frac{1}{10} \cos 5x + C$

D. None of these

**Answer:**

Given:

$$\int \cos 3x \sin 2x \, dx = \frac{1}{2} \int 2 \cos 3x \sin 2x \, dx$$

$$= \frac{1}{2} \int \sin 5x + \cos x \, dx$$

$$= \frac{1}{2} \left\{ \frac{-\cos 5x}{5} + \frac{\sin x}{1} \right\} + c$$

$$= -\frac{\cos 5x}{10} + \frac{\sin x}{2} + c$$

**Question 41.**

Mark (✓) against the correct answer in each of the following:

$$\int \cos 4x \cos x \, dx = ?$$

A.  $\frac{1}{5} \sin 5x + \frac{1}{3} \sin 3x + C$

B.  $\frac{1}{5}\cos 5x - \frac{1}{3}\cos 3x + C$

C.  $\frac{1}{10}\sin 5x + \frac{1}{6}\sin 3x + C$

D. None of these

**Answer:**

Given:

$$\int \cos 4x \cos x \, dx = \frac{1}{2} \int 2 \cos 4x \cos x \, dx$$

$$= \frac{1}{2} \int \cos 5x + \cos 3x \, dx$$

$$= \frac{1}{2} \left\{ \frac{\sin 5x}{5} + \frac{\sin 3x}{3} \right\} + c$$

$$= \frac{\sin 5x}{10} + \frac{\sin 3x}{6} + c$$