Exercise 18a

Question 1.
Write order and degree (if defined)of each of the following differential equations:

$$\frac{d^4y}{dx^4} - \cos\left(\frac{d^3y}{dx^3}\right) = 0$$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 4 as we have $\frac{d^4y}{dx^4}$ and the degree is the highest power to which a derivative is raised. But when we open the Cos x series, we get $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + - - - - -$. This leads to an undefined power on the highest derivative.
Therefore the deg9ee of this function becomes undefined.

So the answer is 4, not defined.

Question 2.
Write order and degree (if defined)of each of the following differential equations:

$$\left(\frac{dy}{dx}\right)^4 + 3y\left(\frac{d^2y}{dx^2}\right) = 0$$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 2 as we have $\frac{d^2y}{dx^2}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 1.

So the answer is 2, 1.

Question 3.

Write order and degree (if defined)of each of the following differential equations:

$$x^3\left(\frac{d^2y}{dx^2}\right)^2+x\left(\frac{dy}{dx}\right)^4=0$$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 2 as we have $\frac{d^2y}{dx^2}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 2.

So the answer is 2, 2.

Question 4.
Write order and degree (if defined)of each of the following differential equations:

$$\left(\frac{d^2s}{dt^2}\right)^2+\left(\frac{ds}{dt}\right)^3+4=0$$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 2 as we have $\frac{d^2s}{dt^2}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 2.

So the answer is 2, 2.

Question 5.
Write order and degree (if defined)of each of the following differential equations:

$$\left(\frac{d^3y}{dx^3}\right)^2+\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^4+y^5=0$$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 3 as we have $\frac{d^3y}{dx^3}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 2.

So the answer is 3, 2.

Question 6.
Write order and degree (if defined)of each of the following differential equations:

$$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+2y=0$$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 2 as we have $\frac{d^2y}{dx^2}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 1.

So the answer is 2, 1.

Question 7.
Write order and degree (if defined)of each of the following differential equations:

$$\frac{dy}{dx}+y=e^x$$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 1 as we have $\frac{dy}{dx}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 1. Also, the equation has to be a polynomial, but here the exponential function does not take any derivative with this. Hence it is a polynomial.

So the answer is 1, 1.

Question 8.
Write order and degree (if defined)of each of the following differential equations:

$$\frac{d^2y}{dx^2}+y^2+e^{(dy/dx)}=0$$

Answer:

The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 2 as we have $\frac{d^2y}{dx^2}$ and the degree is the highest power to which a derivative is raised. But here when we open the series of e^x as $1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+--------$. Also, the equation has to be polynomial. Therefore the degree is not defined. Also, the equation has to be a polynomial, but opening the exponential function will give undefined power to the highest derivative, so the degree of this function is not defined.

So the answer is 2, not defined .

Question 9.
Write order and degree (if defined)of each of the following differential equations:

$$\frac{dy}{dx}+\sin\left(\frac{dy}{dx}\right)=0$$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 1 as we have $\frac{dy}{dx}$ and the degree is the highest power to which a derivative is raised. But when we open Sin x as $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+----$. Also, the equation has to be polynomial, and opening thus, Sin function will lead to an undefined power of the highest derivative. Therefore the degree is not defined.

So the answer is 1, not defined.

Question 10.
Write order and degree (if defined)of each of the following differential equations:

$$\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2-6y=\log x$$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 2 as we have $\frac{d^2y}{dx^2}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 1. Because the logarithm function is not at any

derivative, so it doesn't destroy the polynomial.Hence degree is 1

So the answer is 2, 1.

Question 11.
Write order and degree (if defined)of each of the following differential equations:

$$\left(\frac{dy}{dx}\right)^3 - 4\left(\frac{dy}{dx}\right)^2 + 7y = \sin x$$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 1 as we have $\frac{dy}{dx}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 3. Because the Sine function is not at any derivative, so it doesn't destroy the polynomial.Hence the degree is 3.

So the answer is 1, 3.

Question 12.
Write order and degree (if defined)of each of the following differential equations:

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation. So the order comes out to be 3 as we have $\frac{d^3y}{dx^3}$ and the degree is the highest power to which a derivative is raised. So the power at this order is 1.

So the answer is 3, 1.

Question 13.
Write order and degree (if defined)of each of the following differential equations:

$$x\left(\frac{dy}{dx}\right) + \frac{2}{\left(\frac{dy}{dx}\right)} + 9 = y^2$$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 1 as we have $x\frac{dy}{dx}+\frac{2}{\left(\frac{dy}{dx}\right)}+9=y^2$

$$x\left(\frac{dy}{dx}\right)^2+2+9\frac{dy}{dx}=y^2\frac{dy}{dx}$$

and the degree is the highest power to which a derivative is raised. So the power at this order is 2.

So the answer is 1, 2.

Question 14.
Write order and degree (if defined)of each of the following differential equations:

$$\sqrt{1-\left(\frac{dy}{dx}\right)^2}=\left(a\frac{d^2y}{dx^2}\right)^{1/3}$$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 2/3 as we have $\sqrt{1-\left(\frac{dy}{dx}\right)^2}=\left(a\frac{d^2y}{dx^2}\right)^{1/3}$

and the degree is the highest power to which a derivative is raised. So the power at this order is 2.

So the answer is 2/3, 2.

Question 15.
Write order and degree (if defined)of each of the following differential equations:

$$\sqrt{1-y^2}dx+\sqrt{1-x^2}dy=0$$

Answer:

The order of a differential equation is the order of the highest derivative involved in the equation.
So, the order comes out to be 1 as we have $\sqrt{1-y^2}dx+\sqrt{1-x^2}dy=0$

$$\frac{dy}{dx}=-\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

and the degree is the highest power to which a derivative is raised. So the power at this order is 1.

So the answer is 1, 1.

Question 16.
Write order and degree (if defined)of each of the following differential equations:

$$(y'')^3+(y')^2+\sin y'+1=0$$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation.
So, the order comes out to be 3 as we have $(y'')^3+(y')^2+\sin y'+1=0$

and the degree is the highest power to which a derivative is raised. So the power at this order is 2.

So the answer is 3, 2.

Question 17.
Write order and degree (if defined)of each of the following differential equations:

$(3x + 5y)dy - 4x^2 dx = 0$

Answer:
The order of a differential equation is the order of the highest derivative involved in the equation.
So, the order comes out to be 1 as we have $(3x + 5y)dy - 4x^2 dx = 0$

and the degree is the highest power to which a derivative is raised. So the power at this order is 1.

So the answer is 1, 1.

Question 18.
Write order and degree (if defined)of each of the following differential equations:

$$y = \frac{dy}{dx} + \frac{5}{\left(\frac{dy}{dx}\right)}$$

Answer:

Given: $y = \frac{dy}{dx} + \frac{5}{\left(\frac{dy}{dx}\right)}$

Solving, we get,

$$y \times \frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2 + 5$$

Now,

The order of a differential equation is the order of the highest derivative involved in the equation. So, the order comes out to be 2 as we have, $y \times \frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2 + 5$

and the degree is the highest power to which a derivative is raised. So the power at this order is 1.

So the answer is 2, 1.