Exercise 18b

Question 1.

Verify that $x^2 = 2y^2 \log y$ is a solution of the differential equation $(x^2 + y^2)\frac{dy}{dx} - xy = 0$.

Answer:

Given $x^2 = 2y^2 \log y$

On differentiating both sides with respect to x, we get

$$2x = 2(2y)\log y\left(\frac{dy}{dx}\right) + 2y^2\left(\frac{1}{y}\right)\frac{dy}{dx}$$

$$x = (2y)\log y\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right)$$

$$x = \left(\frac{dy}{dx}\right)((2y)\log y + y)$$

Multiply both sides with y

$$xy = (2y^2 \log y + y^2)\frac{dy}{dx}$$

We know, $x^2 = 2y^2 \log y$. So replace $2y^2 \log y$ with x^2 in the above equation.

$$xy = (x^2 + y^2)\frac{dy}{dx}$$

$$(x^2 + y^2)\frac{dy}{dx} - xy = 0$$

Conclusion: Therefore $x^2 = 2y^2 \log y$ is the solution of $(x^2 + y^2)\frac{dy}{dx} - xy = 0$

Question 2.

Verify that $x^2 = 2y^2 \log y$ is a solution of the differential equation $(x^2 + y^2)\frac{dy}{dx} - xy = 0$.

Answer:
Given $x^2 = 2y^2 \log y$

On differentiating both sides with respect to x, we get

$$2x = 2(2y) \log y \left(\frac{dy}{dx}\right) + 2y^2 \left(\frac{1}{y}\right) \frac{dy}{dx}$$

$$x = (2y) \log y \left(\frac{dy}{dx}\right) + 2y \left(\frac{dy}{dx}\right)$$

$$x = \left(\frac{dy}{dx}\right)((2y) \log y + y)$$

Multiply both sides with y

$$xy = (2y^2 \log y + y^2)\frac{dy}{dx}$$

We know, $x^2 = 2y^2 \log y$. So replace $2y^2 \log y$ with x^2 in the above equation.

$$xy = (x^2 + y^2)\frac{dy}{dx}$$

$$(x^2 + y^2)\frac{dy}{dx} - xy = 0$$

Conclusion: Therefore $x^2 = 2y^2 \log y$ is the solution of $(x^2 + y^2)\frac{dy}{dx} - xy = 0$

Question 3.

Verify that y = e^x cos bx is a solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$

Answer:
Given $y = e^x \cos bx$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x \cos bx + e^x(-b \sin bx)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x \cos bx + e^x(-b \sin bx) + e^x(-b^2 \cos bx) + e^x(-b \sin bx)$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$\begin{aligned}\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y \\ &= e^x \cos bx + e^x(-b \sin bx) + e^x(-b^2 \cos bx) + e^x(-b \sin bx) \\ &- 2e^x \cos bx - 2e^x(-b \sin bx) + 2e^x \cos bx\end{aligned}$$

$$= e^x \cos bx - e^x(b^2 \cos bx)$$

This is not a solution

Conclusion: Therefore, y = e^x cos bx is not the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

Question 4.

Verify that y = e^x cos bx is a solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$

Answer:
Given $y = e^x \cos bx$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x \cos bx + e^x(-b \sin bx)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x \cos bx + e^x(-b \sin bx) + e^x(-b^2 \cos bx) + e^x(-b \sin bx)$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$$
$$= e^x \cos bx + e^x(-b\sin bx) + e^x(-b^2\cos bx) + e^x(-b\sin bx) - 2e^x\cos bx - 2e^x(-b\sin bx) + 2e^x\cos bx$$

$$= e^x\cos bx - e^x(b^2\cos bx)$$

This is not a solution

Conclusion: Therefore, y = e^x cos bx is not the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

Question 5.

Verify that y = $e^{m\cos^{-1}x}$ is a solution of the differential equation (1 - x^2)

$$\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0.$$

Answer:

Given $y = e^{(m)\cos^{-1}x}$

On differentiating with x, we get

$$\frac{dy}{dx} = e^{(m)\cos^{-1}x}(m)\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{-ym}{\sqrt{1-x^2}}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = \frac{ym^2}{1-x^2} - \frac{mx}{(\sqrt{1-x^2})(1-x^2)}$$

We want to find $(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$

$$= ym^2 - \frac{mxy}{\sqrt{1-x^2}} + \frac{ymx}{\sqrt{1-x^2}} - m^2y$$

= 0

Therefore, $y = e^{(m)\cos^{-1}x}$ is the solution of $(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$

Conclusion: Therefore, $y = e^{(m)\cos^{-1}x}$ is the solution of

$$(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$$

Question 6.

Verify that y = $e^{m\cos^{-1}x}$ is a solution of the differential equation (1 - x^2)

$$\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0.$$

Answer:

Given $y = e^{(m)\cos^{-1}x}$

On differentiating with x, we get

$$\frac{dy}{dx} = e^{(m)\cos^{-1}x}(m)\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{-ym}{\sqrt{1-x^2}}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = \frac{ym^2}{1-x^2} - \frac{mx}{(\sqrt{1-x^2})(1-x^2)}$$

We want to find $(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$

$$= ym^2 - \frac{mxy}{\sqrt{1-x^2}} + \frac{ymx}{\sqrt{1-x^2}} - m^2y$$

= 0

Therefore, $y = e^{(m)\cos^{-1}x}$ is the solution of $(1-x^2)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - m^2y$

Conclusion: Therefore, $y = e^{(m)\cos^{-1}x}$ is the solution of

$$(1-x^2)\frac{d^2y}{dx^2}-2\frac{dy}{dx}-m^2y$$

Question 7.
Verify that y = (a + bx) e^{2x} is the general solution of the differential equation

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=0.$$

Answer:
Given $y = (a + bx)e^{2x}$

On differentiating with x, we get

$$\frac{dy}{dx} = be^{2x} + 2(a + bx)e^{2x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 2be^{2x} + 2be^{2x} + 4(a + bx)e^{2x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2}-4\frac{dy}{dx} + 4y$

$$= 2be^{2x} + 2be^{2x} + 4(a + bx)e^{2x}-4be^{2x}-8(a + bx)e^{2x} + 4(a + bx)e^{2x}$$

= 0

Conclusion: Therefore, $y = (a + bx)e^{2x}$ is the solution of $\frac{d^2y}{dx^2}-4\frac{dy}{dx} + 4y = 0$

Question 8.
Verify that y = (a + bx) e^{2x} is the general solution of the differential equation

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=0.$$

Answer:
Given $y = (a + bx)e^{2x}$

On differentiating with x, we get

$$\frac{dy}{dx} = be^{2x} + 2(a + bx)e^{2x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 2be^{2x} + 2be^{2x} + 4(a + bx)e^{2x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y$

$$= 2be^{2x} + 2be^{2x} + 4(a + bx)e^{2x} - 4be^{2x} - 8(a + bx)e^{2x} + 4(a + bx)e^{2x}$$

= 0

Conclusion: Therefore, $y = (a + bx)e^{2x}$ is the solution of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

Question 9.
Verify that y = e^x(A cos x + B sin x) is the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$$

Answer:
Given $y = e^x(A\cos x + B\sin x)$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\sin x + B\cos x)$$

$$+ e^x(-A\cos x - B\sin x)$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$= e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) - 2e^x(A\cos x + B\sin x) - 2e^x(-A\sin x + B\cos x) + 2e^x(A\cos x + B\sin x)$$

= 0

Conclusion: Therefore, $y = e^x(A\cos x + B\sin x)$ is the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

Question 10.
Verify that y = e^x(A cos x + B sin x) is the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$$

Answer:
Given $y = e^x(A\cos x + B\sin x)$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\sin x + B\cos x)$$

$$+ e^x(-A\cos x - B\sin x)$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$= e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) - 2e^x(A\cos x + B\sin x) - 2e^x(-A\sin x + B\cos x) + 2e^x(A\cos x + B\sin x)$$

= 0

Conclusion: Therefore, $y = e^x(A\cos x + B\sin x)$ is the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

Question 11.

Verify that y = A cos 2x - B sin 2x is the general solution of the differential equation $\frac{d^2y}{dx^2} + 4y =$ 0.

Answer:
Given $y = A\cos 2x - B\sin 2x$

On differentiating with x, we get

$$\frac{dy}{dx} = -2A\sin 2x - 2B\cos 2x$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = -4A\cos 2x + 4B\sin 2x$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + 4y$

$$= -4A\cos 2x + 4B\sin 2x + 4\cos 2x - 4B\sin 2x$$

= 0

Conclusion: Therefore, $y = A\cos 2x - B\sin 2x$ is the solution of $\frac{d^2y}{dx^2} + 4y = 0$

Question 12.

Verify that y = A cos 2x - B sin 2x is the general solution of the differential equation $\frac{d^2y}{dx^2} + 4y =$ 0.

Answer:
Given $y = A\cos 2x - B\sin 2x$

On differentiating with x, we get

$$\frac{dy}{dx} = -2A\sin 2x - 2B\cos 2x$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = -4A\cos 2x + 4B\sin 2x$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + 4y$

$$= -4A\cos 2x + 4B\sin 2x + 4\cos 2x - 4B\sin 2x$$

= 0

Conclusion: Therefore, $y = A\cos 2x - B\sin 2x$ is the solution of $\frac{d^2y}{dx^2} + 4y = 0$

Question 13.
Verify that y = ae^{2x} + be^{-x} is the general solution of the differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0.$$

Answer:
Given $y = ae^{2x} + be^{2x}$

On differentiating with x, we get

$$\frac{dy}{dx} = 2ae^{2x} + 2be^{2x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 4ae^{2x} + 4be^{2x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$

$$= 4ae^{2x} + 4be^{2x} - 2ae^{2x} - 2be^{2x} - 2ae^{2x} - 2be^{2x}$$

= 0

Conclusion : Therefore, $y = ae^{2x} + be^{2x}$ is the solution of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

Question 14.
Verify that y = ae^{2x} + be^{-x} is the general solution of the differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0.$$

Answer:
Given $y = ae^{2x} + be^{2x}$

On differentiating with x, we get

$$\frac{dy}{dx} = 2ae^{2x} + 2be^{2x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 4ae^{2x} + 4be^{2x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$

$$= 4ae^{2x} + 4be^{2x} - 2ae^{2x} - 2be^{2x} - 2ae^{2x} - 2be^{2x}$$

= 0

Conclusion : Therefore, $y = ae^{2x} + be^{2x}$ is the solution of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

Question 15.
Show that y = e^{x}(A cos x + B sin x) is the solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$$

Answer:
Given $y = e^{x}(A\cos x + B\sin x)$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x)$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$= e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x) - 2e^x(A\cos x + B\sin x) - 2e^x(-A\sin x + B\cos x) + 2e^x(A\cos x + B\sin x)$$

= 0

Conclusion: Therefore, $y = e^x(A\cos x + B\sin x)$ is the solution of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Question 16.
Show that y = e^x(A cos x + B sin x) is the solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$$

Answer:
Given $y = e^x(A\cos x + B\sin x)$

On differentiating with x, we get

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x)$$

Now let’s see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$$= e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x) - 2e^x(A\cos x + B\sin x) - 2e^x(-A\sin x + B\cos x) + 2e^x(A\cos x + B\sin x)$$

= 0

Conclusion: Therefore, $y = e^x(A\cos x + B\sin x)$ is the solution of

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Question 17.

Verify that $y^2 = 4a(x + a)$ is a solution of the differential equation $y\left\{1-\left(\frac{dy}{dx}\right)^2\right\} = 2x\frac{dy}{dx}$

Answer:
Given, $y^2 = 4a(x + a)$

On differentiating with x, we get

$$2y\frac{dy}{dx} = 4a$$

Now let’s see what is the value of $y(1 - (\frac{dy}{dx})^2) - 2x\frac{dy}{dx}$

$$= y\left(1 - \left(\frac{2a}{y}\right)^2\right) - 4\frac{ax}{y}$$

$$= y - \frac{4a^2}{y} - 4\frac{ax}{y}$$

$$= \frac{y^2 - 4a(a + x)}{y}$$

$$= \frac{4a(a + x) - 4a(a + x)}{y}$$

= 0

Conclusion: Therefore, $y^2 = 4a(x + a)$ is the solution of $y(1 - (\frac{dy}{dx})^2)) = 2x\frac{dy}{dx}$

Question 18.

Verify that y^{2} = 4a(x + a) is a solution of the differential equation $y\left\{1-\left(\frac{dy}{dx}\right)^2\right\}=2x\frac{dy}{dx}$

Answer:
Given, $y^2 = 4a(x + a)$

On differentiating with x, we get

$$2y\frac{dy}{dx} = 4a$$

Now let's see what is the value of $y(1 - (\frac{dy}{dx})^2)) - 2x\frac{dy}{dx}$

$$= y\left(1 - \left(\frac{2a}{y}\right)^2\right) - 4\frac{ax}{y}$$

$$= y - \frac{4a^2}{y} - 4\frac{ax}{y}$$

$$= \frac{y^2 - 4a(a + x)}{y}$$

$$= \frac{4a(a + x) - 4a(a + x)}{y}$$

= 0

Conclusion: Therefore, $y^2 = 4a(x + a)$ is the solution of $y(1 - (\frac{dy}{dx})^2)) = 2x\frac{dy}{dx}$

Question 19.

Verify that y = $ce^{\tan^{-1}x}$ is a solution of the differential equation $(1 + x^2)\frac{d^2y}{dx^2} + (2x - 1)\frac{dy}{dx} = 0$

Answer:
Given $y = c\,e^{\tan^{-1}x}$

On differentiating with x, we get

$$\frac{dy}{dx} = c\tan^{-1}x\left(\frac{1}{1+x^2}\right)e^{\tan^{-1}x} = y\tan^{-1}x\left(\frac{1}{1+x^2}\right)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = c(\frac{1}{1+x^2})^2e^{\tan^{-1}x} + c\ \tan^{-1}x\left(\frac{-2x}{(1+x^2)^2}\right)e^{\tan^{-1}x} \\ + c\,(\tan^{-1}x)^2\left(\frac{1}{(1+x^2)^2}\right)e^{\tan^{-1}x}$$

$$= y(\frac{1}{1+x^2})^2 + y\tan^{-1}x\left(\frac{-2x}{(1+x^2)^2}\right) + y\,(\tan^{-1}x)^2\left(\frac{1}{(1+x^2)^2}\right)$$

Now let’s see what is the value of $(1 + x^2)\frac{d^2y}{dx^2} + (2x - 1)\frac{dy}{dx}$

$$= y\left(\frac{1}{1+x^2}\right) + y\tan^{-1}x\left(\frac{-2x}{1+x^2}\right) + y(\tan^{-1}x)^2\left(\frac{1}{1 + x^2}\right) + \left(\frac{2xy}{1 + x^2}\right)\tan^{-1}x \\ - \tan^{-1}x\,(\frac{y}{1 + x^2})$$

$$= \left(\frac{1}{1+x^2}\right)y + y(\tan^{-1}x)^2\left(\frac{1}{1+x^2}\right) - \tan^{-1}x\left(\frac{y}{1+x^2}\right)$$

Conclusion: Therefore, $y = c\,e^{\tan^{-1}x}$ is not the solution of

$$(1 + x^2)\frac{d^2y}{dx^2} + (2x - 1)\frac{dy}{dx}$$

Question 20.

Verify that y = $ce^{\tan^{-1}x}$ is a solution of the differential equation $(1 + x^2)\frac{d^2y}{dx^2} + (2x - 1)\frac{dy}{dx} = 0$

Answer:
Given $y = c\,e^{\tan^{-1}x}$

On differentiating with x, we get

$$\frac{dy}{dx} = c\tan^{-1}x\left(\frac{1}{1+x^2}\right)e^{\tan^{-1}x} = y\tan^{-1}x\left(\frac{1}{1+x^2}\right)$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = c(\frac{1}{1+x^2})^2e^{\tan^{-1}x} + c\tan^{-1}x\left(\frac{-2x}{(1+x^2)^2}\right)e^{\tan^{-1}x} + c\,(\tan^{-1}x)^2\left(\frac{1}{(1+x^2)^2}\right)e^{\tan^{-1}x}$$

$$= y(\frac{1}{1+x^2})^2 + y\tan^{-1}x\left(\frac{-2x}{(1+x^2)^2}\right) + y\,(\tan^{-1}x)^2\left(\frac{1}{(1+x^2)^2}\right)$$

Now let's see what is the value of $(1 + x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx}$

$$= y\left(\frac{1}{1+x^2}\right) + y\tan^{-1}x\left(\frac{-2x}{1+x^2}\right) + y(\tan^{-1}x)^2\left(\frac{1}{1+x^2}\right) + \left(\frac{2xy}{1+x^2}\right)\tan^{-1}x - \tan^{-1}x\,(\frac{y}{1+x^2})$$

$$= \left(\frac{1}{1+x^2}\right)y + y(\tan^{-1}x)^2\left(\frac{1}{1+x^2}\right) - \tan^{-1}x\left(\frac{y}{1+x^2}\right)$$

Conclusion: Therefore, $y = c\,e^{\tan^{-1}x}$ is not the solution of

$$(1 + x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx}$$

Question 21.

Verify that y = ae^{bx} is a solution of the differential equation $\frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2$

Answer:
Given $y = a\,e^{bx}$

On differentiating with x, we get

$$\frac{dy}{dx} = ab\,e^{bx}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = ab^2\,e^{bx}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right)^2$

$$= ab^2\,e^{bx} - \left(\frac{1}{y}\right)(abe^{bx})^2$$

$$= ab^2\,e^{bx} - ab^2\,e^{bx}$$

= 0

Conclusion: Therefore, $y = a\,e^{bx}$ is the solution of $\frac{d^2y}{dx^2} = \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right)^2$

Question 22.

Verify that y = ae^{bx} is a solution of the differential equation $\frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2$

Answer:
Given $y = a\,e^{bx}$

On differentiating with x, we get

$$\frac{dy}{dx} = ab\, e^{bx}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = ab^2\, e^{bx}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} - \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right)^2$

$$= ab^2\, e^{bx} - \left(\frac{1}{y}\right)(abe^{bx})^2$$

$$= ab^2\, e^{bx} - ab^2\, e^{bx}$$

= 0

Conclusion: Therefore, $y = a\, e^{bx}$ is the solution of $\frac{d^2y}{dx^2} = \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right)^2$

Question 23.

Verify that y = $\frac{a}{x} + b$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{2}{x}\left(\frac{dy}{dx}\right) = 0$

Answer:

Given $y = \frac{a}{x} + b$

On differentiating with x, we get

$$\frac{dy}{dx} = -\frac{a}{x^2}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = \frac{2a}{x^3}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + \left(\frac{2}{x}\right)\left(\frac{dy}{dx}\right)$

$$= \frac{2a}{x^3} - \frac{2a}{x^3}$$

= 0

Conclusion: Therefore, $y = \frac{a}{x} + b$ is the solution of $\frac{d^2y}{dx^2} + \left(\frac{2}{x}\right)\left(\frac{dy}{dx}\right) = 0$

Question 24.

Verify that y = $\frac{a}{x} + b$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{2}{x}\left(\frac{dy}{dx}\right) = 0$

Answer:

Given $y = \frac{a}{x} + b$

On differentiating with x, we get

$$\frac{dy}{dx} = -\frac{a}{x^2}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = \frac{2a}{x^3}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + \left(\frac{2}{x}\right)\left(\frac{dy}{dx}\right)$

$$= \frac{2a}{x^3} - \frac{2a}{x^3}$$

= 0

Conclusion: Therefore, $y = \frac{a}{x} + b$ is the solution of $\frac{d^2y}{dx^2} + \left(\frac{2}{x}\right)\left(\frac{dy}{dx}\right) = 0$

Question 25.

Verify that y = e^{-x} + Ax + B is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right)=1$

Answer:
Given $y = e^{-x} + Ax + B$

On differentiating with x, we get

$$\frac{dy}{dx} = -e^{-x} + A$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^{-x}$$

Now let's see what is the value of $e^x\left(\frac{d^2y}{dx^2}\right)$

$= e^x(e^{-x})$

= 1

Conclusion: Therefore, $y = e^{-x} + Ax + B$ is the solution of $e^x\left(\frac{d^2y}{dx^2}\right) = 1$

Question 26.

Verify that y = e^{-x} + Ax + B is a solution of the differential equation $e^x\left(\frac{d^2y}{dx^2}\right)=1$

Answer:
Given $y = e^{-x} + Ax + B$

On differentiating with x, we get

$$\frac{dy}{dx} = -e^{-x} + A$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = e^{-x}$$

Now let's see what is the value of $e^x\left(\frac{d^2y}{dx^2}\right)$

$$= e^x(e^{-x})$$

= 1

Conclusion: Therefore, $y = e^{-x} + Ax + B$ is the solution of $e^x\left(\frac{d^2y}{dx^2}\right) = 1$

Question 27.

Verify that $Ax^2 + By^2 = 1$ is a solution of the differential equation $x\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$

Answer:
Given $Ax^2 + By^2 = 1$

On differentiating with x, we get

$$2Ax + 2By\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{Ax}{By}$$

On differentiating again with x, we get

$$2A + 2B\left(\frac{dy}{dx}\right)^2 + 2By\left(\frac{d^2y}{dx^2}\right) = 0$$

$$y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = -\frac{A}{B}$$

Now let's see what is the value of $x\left(y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2\right) - y\frac{dy}{dx}$

$$= x\left(-\frac{A}{B}\right) - y\left(-\frac{Ax}{By}\right)$$

$$= \left(-\frac{Ax}{B}\right) + \left(\frac{Ax}{B}\right)$$

= 0

Conclusion: Therefore, $Ax^2 + By^2 = 1$ is the solution of

$$x\left(y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2\right) = y\frac{dy}{dx}$$

Question 28.

Verify that $Ax^2 + By^2 = 1$ is a solution of the differential equation $x\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$

Answer:

Given $Ax^2 + By^2 = 1$

On differentiating with x, we get

$$2Ax + 2By\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{Ax}{By}$$

On differentiating again with x, we get

$$2A + 2B\left(\frac{dy}{dx}\right)^2 + 2By\left(\frac{d^2y}{dx^2}\right) = 0$$

$$y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = -\frac{A}{B}$$

Now let's see what is the value of $x\left(y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2\right) - y\frac{dy}{dx}$

$$= x\left(-\frac{A}{B}\right) - y\left(-\frac{Ax}{By}\right)$$

$$= \left(-\frac{Ax}{B}\right) + \left(\frac{Ax}{B}\right)$$

= 0

Conclusion: Therefore, $Ax^2 + By^2 = 1$ is the solution of

$$x\left(y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2\right) = y\frac{dy}{dx}$$

Question 29.

Verify that y = $\frac{c-x}{1+cx}$ is a solution of the differential equation (1 + x^2) $\frac{dy}{dx}$ + (1 + y^2) = 0.

Answer:

Given $y = \frac{c-x}{1+cx}$

On differentiating with x, we get

$$\frac{dy}{dx} = \frac{-1-c^2}{(1+cx)^2}$$

Now let's see what is the value of $(1 + x^2)\frac{dy}{dx} + (1 + y^2)$

$$= -\frac{(1+x^2)(1+c^2)}{(1+cx)^2} + (1 + \left(\frac{c-x}{1+cx}\right)^2)$$

$$= \frac{(-1-c^2-x^2-x^2c^2) + (1 + c^2x^2 + 2cx + c^2 + x^2 - 2cx)}{(1+cx)^2}$$

= 0

Conclusion: Therefore, $y = \frac{c-x}{1+cx}$ is the solution of $(1 + x^2)\frac{dy}{dx} + (1 + y^2) = 0$

Question 30.

Verify that y = $\frac{c-x}{1+cx}$ is a solution of the differential equation (1 + x^2) $\frac{dy}{dx}$ + (1 + y^2) = 0.

Answer:

Given $y = \frac{c-x}{1+cx}$

On differentiating with x, we get

$$\frac{dy}{dx} = \frac{-1-c^2}{(1+cx)^2}$$

Now let's see what is the value of $(1 + x^2)\frac{dy}{dx} + (1 + y^2)$

$$= -\frac{(1+x^2)(1+c^2)}{(1+cx)^2} + \left(1 + \left(\frac{c-x}{1+cx}\right)^2\right)$$

$$= \frac{(-1-c^2-x^2-x^2c^2) + (1 + c^2x^2 + 2cx + c^2 + x^2 - 2cx)}{(1+cx)^2}$$

= 0

Conclusion: Therefore, $y = \frac{c-x}{1+cx}$ is the solution of $(1 + x^2)\frac{dy}{dx} + (1 + y^2) = 0$

Question 31.

Verify that y = log (x + $\sqrt{x^2 + a^2}$) satisfies the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

Answer:

Given $y = \log\left(x + \sqrt{x^2 + a^2}\right)$

On differentiating with x, we get

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}}(1 + \frac{x}{\sqrt{x^2 + a^2}})$$

$$= \frac{1}{\sqrt{x^2 + a^2}}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = -\frac{x}{(x^2 + a^2)^{\frac{3}{2}}}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

$$= -\frac{x}{(x^2 + a^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{x^2 + a^2}}$$

Conclusion: Therefore, $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ is not the solution of

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$$

Question 32.

Verify that y = log (x + $\sqrt{x^2 + a^2}$) satisfies the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

Answer:
Given $y = \log\left(x + \sqrt{x^2 + a^2}\right)$

On differentiating with x, we get

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}}(1 + \frac{x}{\sqrt{x^2 + a^2}})$$

$$= \frac{1}{\sqrt{x^2 + a^2}}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = -\frac{x}{(x^2 + a^2)^{\frac{3}{2}}}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + x\frac{dy}{dx}$

$$= -\frac{x}{(x^2 + a^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{x^2 + a^2}}$$

Conclusion: Therefore, $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ is not the solution of

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$$

Question 33.

Verify that y = e^{-3x} is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

Answer:
Given, $y = e^{-3x}$

On differentiating with x, we get

$$\frac{dy}{dx} = -3e^{-3x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 9\,e^{-3x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y$

$$= 9\,e^{-3x} - 3e^{-3x} - 6e^{-3x}$$

= 0

Conclusion: Therefore, $y = e^{-3x}$ is the solution of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

Question 34.

Verify that y = e $^{-3x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

Answer:
Given, $y = e^{-3x}$

On differentiating with x, we get

$$\frac{dy}{dx} = -3e^{-3x}$$

On differentiating again with x, we get

$$\frac{d^2y}{dx^2} = 9\,e^{-3x}$$

Now let's see what is the value of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y$

$$= 9\,e^{-3x} - 3e^{-3x} - 6e^{-3x}$$

= 0

Conclusion: Therefore, $y = e^{-3x}$ is the solution of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$