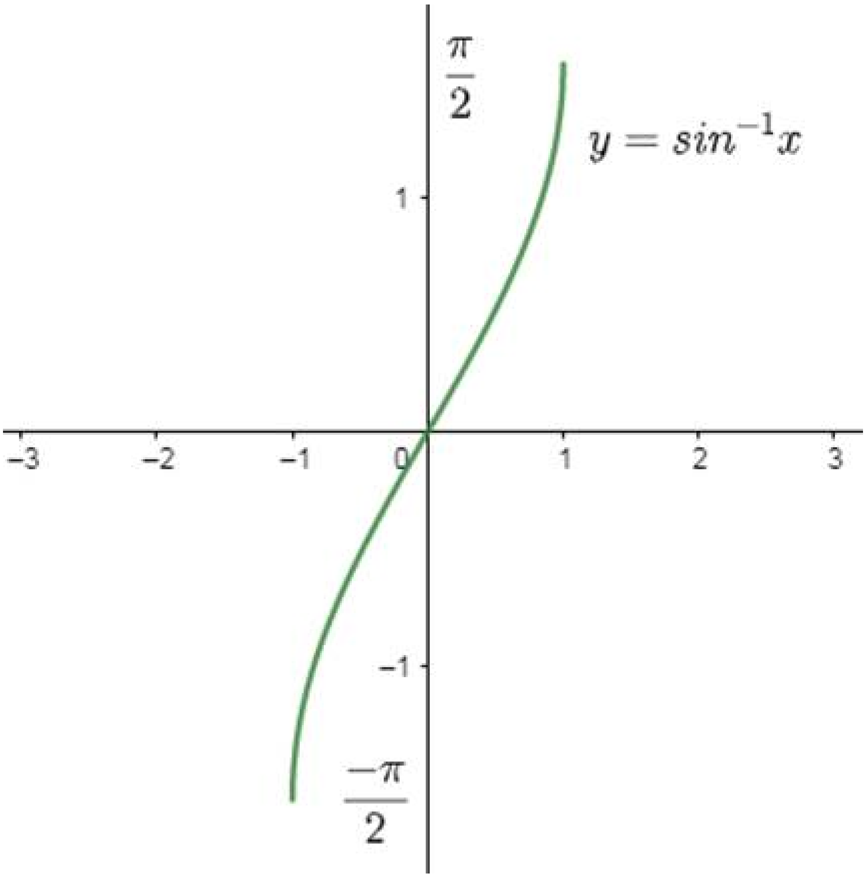
Exercise 4d

Question 1.
Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\sin^{-1} x$

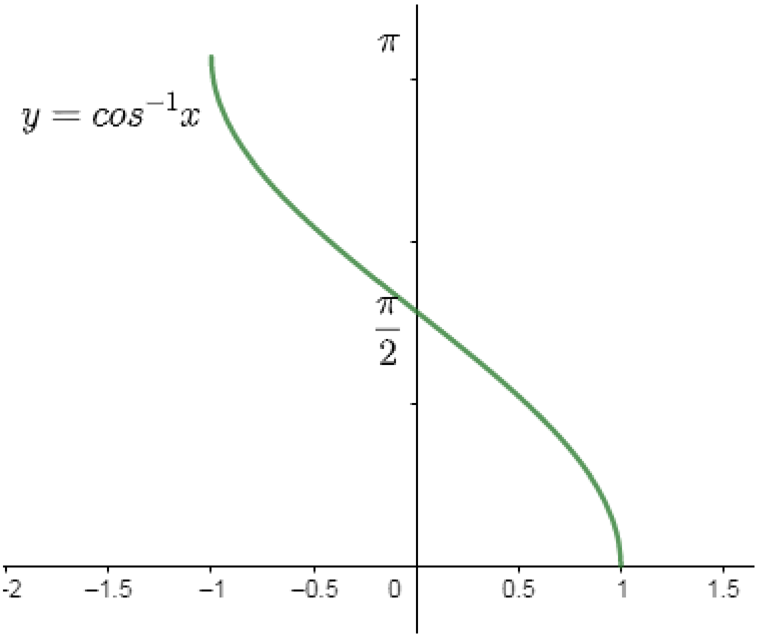
Answer:
Principal value branch of $\sin^{-1} x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$



Question 2.
Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\cos^{-1} x$

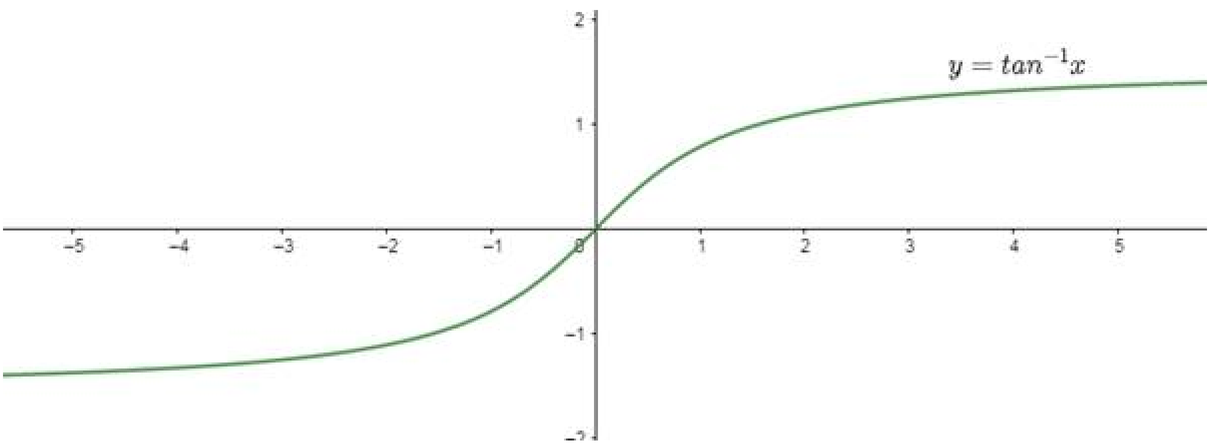
Answer:
Principal value branch of $\cos^{-1} x$ is $[0, \pi]$



Question 3.
Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\tan^{-1} x$

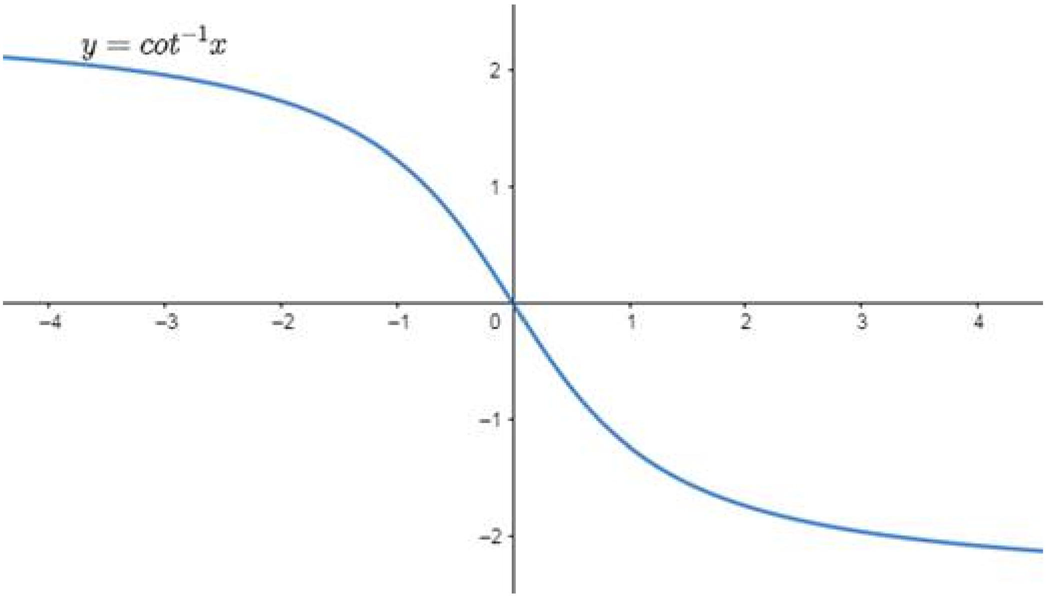
Answer:
Principal value branch of $\tan^{-1} x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$



Question 4.
Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\cot^{-1} x$

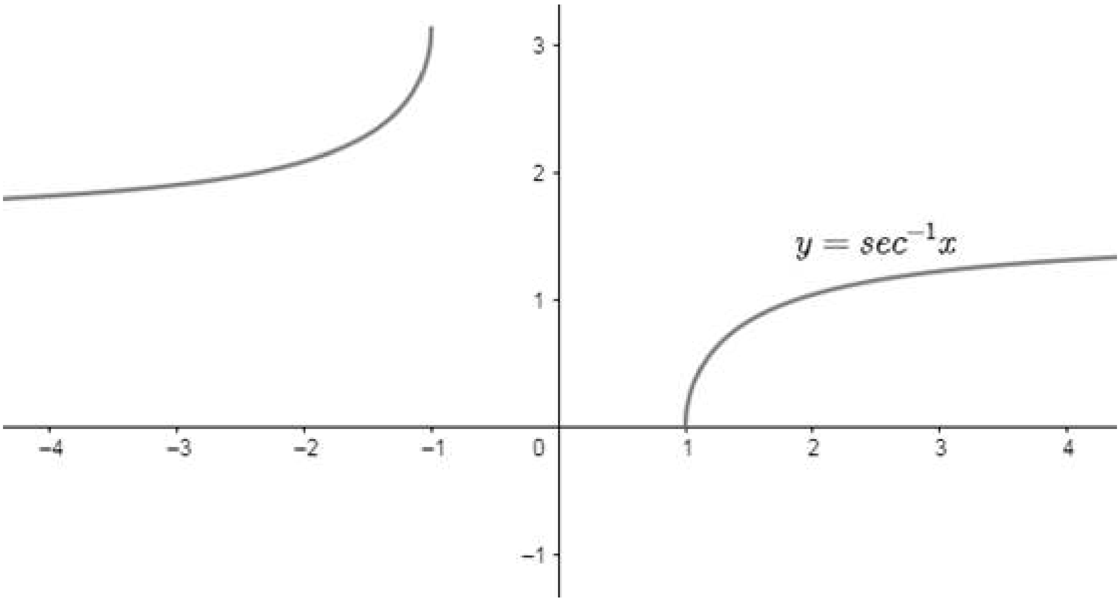
Answer:
Principal value branch of $\cot^{-1} x$ is $(0, \pi)$



Question 5.
Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\sec^{-1} x$

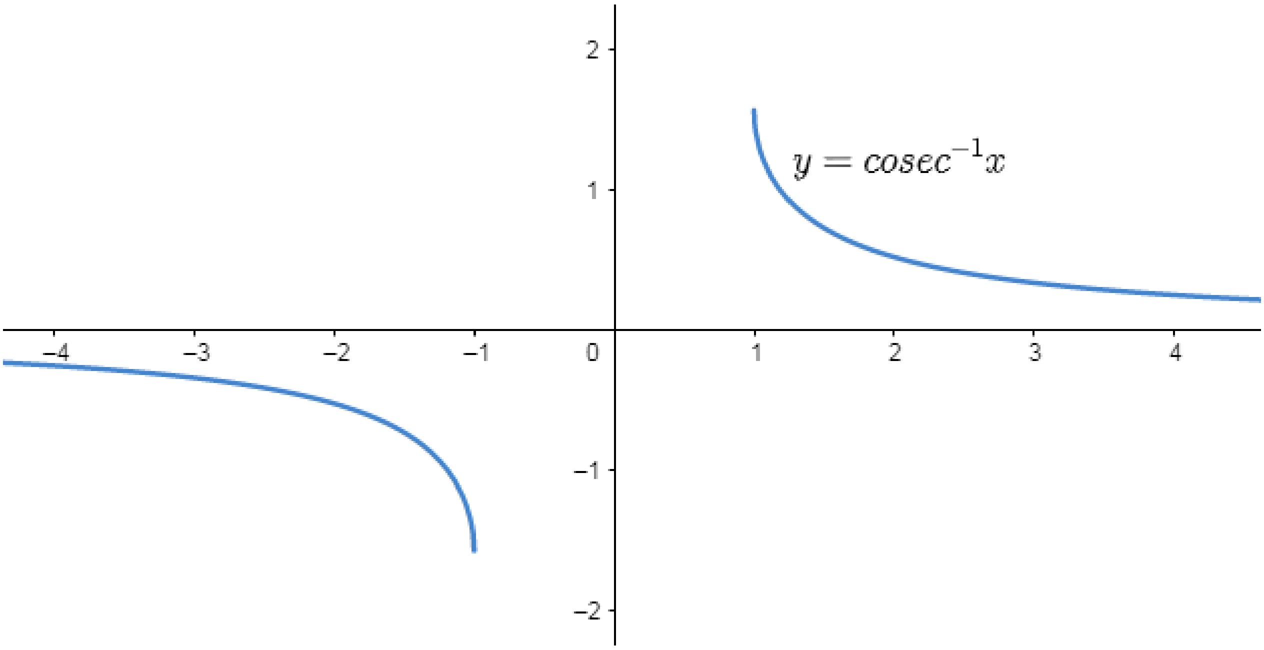
Answer:
Principal value branch of $\sec^{-1}$ x is $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



Question 6.
Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$cosec^{-1} x$

Answer:
Principal value branch of $cosec^{-1}$ x is $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$



Objective Questions

Question 1.
Mark the tick against the correct answer in the following:

The principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is

A. $\frac{\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{7\pi}{6}$

D. none of these

Answer:

To Find:The Principle value of $\cos^{-1}(\frac{\sqrt{3}}{2})$

Let the principle value be given by x

Now, let x = $\cos^{-1}(\frac{\sqrt{3}}{2})$

$\Rightarrow$ cos x=$\frac{\sqrt{3}}{2}$

$\Rightarrow$ cos x=cos($\frac{\pi}{6}$) ($\because \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$)

$\Rightarrow$ x = $\frac{\pi}{6}$

Question 2.

Mark the tick against the correct answer in the following:

The principal value of cosec^{-1}(2) is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

Answer:

To Find: The Principle value of $\operatorname{cosec}^{-1}(2)$

Let the principle value be given by x

Now, let x = $\operatorname{cosec}^{-1}(2)$

⇒ cosec x =2

⇒ cosec x=cosec($\frac{\pi}{6}$) (∵ $\cos\left(\frac{\pi}{6}\right) = 2$)

⇒ x = $\frac{\pi}{6}$

Question 3.
Mark the tick against the correct answer in the following:

The principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ is

A. $\frac{-\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{4}$

D. $\frac{5\pi}{4}$

Answer:
To Find: The Principle value of $\cos^{-1}(\frac{-1}{\sqrt{2}})$

Let the principle value be given by x

Now, let x = $\cos^{-1}(\frac{-1}{\sqrt{2}})$

⇒ cos x =$\frac{-1}{\sqrt{2}}$

⇒ cos x= - cos($\frac{\pi}{4}$) (∵ $\cos(\frac{\pi}{4})$=$\frac{1}{\sqrt{2}}$)

⇒ cos x=cos($\pi - \frac{\pi}{4}$) (∵ $-\cos(\theta) = \cos(\pi - \theta$))

$\Rightarrow$ x = $\frac{3\pi}{4}$

Question 4.
Mark the tick against the correct answer in the following:

The principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is

A. $\frac{-\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{7\pi}{6}$

D. none of these

Answer:
To Find: The Principle value of $\sin^{-1}(\frac{-1}{2})$

Let the principle value be given by x

Now, let x = $\sin^{-1}(\frac{-1}{2})$

$\Rightarrow$ sin x =$\frac{-1}{2}$

$\Rightarrow$ sin x= - sin($\frac{\pi}{6}$) ($\because \sin(\frac{\pi}{6})=\frac{1}{2}$)

$\Rightarrow$ sin x=sin($-\frac{\pi}{6}$) ($\because -\sin(\theta) = \sin(-\theta)$))

$\Rightarrow$ x = $-\frac{\pi}{4}$

Question 5.

Mark the tick against the correct answer in the following:

The principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is

A. $\frac{-\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{4\pi}{3}$

D. $\frac{\pi}{3}$

Answer:
To Find: The Principle value of $\cos^{-1}(\frac{-1}{2})$

Let the principle value be given by x

Now, let x = $\cos^{-1}(\frac{-1}{2})$

⇒ cos x =$\frac{-1}{2}$

⇒ cos x= - cos($\frac{\pi}{3}$) (∵ $\cos(\frac{\pi}{3})$=$\frac{1}{2}$)

⇒ cos x=cos($\pi - \frac{\pi}{3}$) (∵ $-\cos(\theta) = \cos(\pi - \theta$))

⇒ x = $\frac{2\pi}{3}$

Question 6.
Mark the tick against the correct answer in the following:

The principal value of $\tan^{-1}\left(-\sqrt{3}\right)$ is

A. $\frac{2\pi}{3}$

B. $\frac{4\pi}{3}$

C. $\frac{-\pi}{3}$

D. none of these

Answer:
To Find: The Principle value of $\tan^{-1}(-\sqrt{3})$

Let the principle value be given by x

Now, let x = $\tan^{-1}(-\sqrt{3})$

⇒ tan x =$-\sqrt{3}$

⇒ tan x= - tan($\frac{\pi}{3}$) ($\because \tan(\frac{\pi}{3})= -\sqrt{3}$)

⇒ $\tan x = \tan(-\frac{\pi}{3})$ ($\because -\tan(\theta) = \tan(-\theta)$)

⇒ x = $-\frac{\pi}{3}$

Question 7.
Mark the tick against the correct answer in the following:

The principal value of $\cot^{-1}$ (-1) is

A. $\frac{-\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{5\pi}{4}$

D. $\frac{3\pi}{4}$

Answer:
To Find: The Principle value of $\cot^{-1}(-1)$

Let the principle value be given by x

Now, let x = $\cot^{-1}(-1)$

⇒ cot x =-1

⇒ cot x= - cot($\frac{\pi}{4}$) (∵ $\cot(\frac{\pi}{4})= 1$)

⇒ cot x=cot($\pi - \frac{\pi}{4}$) (∵ $-\cot(\theta) = \cot(\pi - \theta)$)

⇒ x = $\frac{3\pi}{4}$

Question 8.
Mark the tick against the correct answer in the following:

The principal value of $\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is

A. $\frac{\pi}{6}$

B. $\frac{-\pi}{6}$

C. $\frac{5\pi}{6}$

D. $\frac{7\pi}{6}$

Answer:

To Find: The Principle value of $\sec^{-1}(\frac{-2}{\sqrt{3}})$

Let the principle value be given by x

Now, let x = $\sec^{-1}(\frac{-2}{\sqrt{3}})$

⇒ sec x =$\frac{-2}{\sqrt{3}}$

⇒ sec x= - sec($\frac{\pi}{6}$) ($\because \sec(\frac{\pi}{6})$=$\frac{2}{\sqrt{3}}$)

⇒ sec x=sec($\pi - \frac{\pi}{6}$) ($\because -\sec(\theta) = \sec(\pi - \theta$))

⇒ x = $\frac{5\pi}{6}$

Question 9.
Mark the tick against the correct answer in the following:

The principal value of $\text{cosec}^{-1}\left(-\sqrt{2}\right)$ is

A. $\frac{-\pi}{4}$

B. $\frac{3\pi}{4}$

C. $\frac{5\pi}{4}$

D. none of these

Answer:
To Find: The Principle value of $\text{cosec}^{-1}(-\sqrt{2})$

Let the principle value be given by x

Now, let x = $\text{cosec}^{-1}(-\sqrt{2})$

$\Rightarrow$ cosec x $= -\sqrt{2}$

$\Rightarrow$ cosec x= - cosec($\frac{\pi}{4}$) ($\because$ $cosec(\frac{\pi}{4}) = \sqrt{2}$)

$\Rightarrow$ cosec x=cosec($-\frac{\pi}{4}$) ($\because$ $-cosec(\theta) = \text{cosec}(-\theta\,)$)

$\Rightarrow$ x = $-\frac{\pi}{4}$

Question 10.
Mark the tick against the correct answer in the following:

The principal value of $\cot^{-1}\left(-\sqrt{3}\right)$ is

A. $\frac{2\pi}{6}$

B. $\frac{\pi}{6}$

C. $\frac{7\pi}{6}$

D. $\frac{5\pi}{6}$

Answer:
To Find: The Principle value of $\cot^{-1}(-\sqrt{3})$

Let the principle value be given by x

Now, let x = $\cot^{-1}(-\sqrt{3})$

$\Rightarrow$ cot x $= -\sqrt{3}$

$\Rightarrow$ cot x= - cot($\frac{\pi}{6}$) ($\because$ $cot(\frac{\pi}{6}) = \sqrt{3}$)

$\Rightarrow$ cot x=cot$(\pi - \frac{\pi}{6})$ $(\because -cot(\theta) = \cot(\pi - \theta\ ))$

$\Rightarrow$ x = $\frac{5\pi}{6}$

Question 11.
Mark the tick against the correct answer in the following:

The value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is

A. $\frac{2\pi}{3}$

B. $\frac{5\pi}{3}$

C. $\frac{\pi}{3}$

D. none of these

Answer:
To Find: The value of $\sin^{-1}(\sin(\frac{2\pi}{3}))$

Now, let x =$\sin^{-1}(\sin(\frac{2\pi}{3}))$

$\Rightarrow$ sin x =sin $(\frac{2\pi}{3})$

Here range of principle value of sine is $[-\frac{\pi}{2},\frac{\pi}{2}]$

$\Rightarrow$ x = $\frac{2\pi}{3}$ $\notin [-\frac{\pi}{2},\frac{\pi}{2}]$

Hence for all values of x in range $[-\frac{\pi}{2},\frac{\pi}{2}]$,the value of

$\sin^{-1}(\sin(\frac{2\pi}{3}))$ is

$\Rightarrow$ sin x =sin ($\pi - \frac{\pi}{3}$) ($\because$sin ($\frac{2\pi}{3}$)= sin ($\pi - \frac{\pi}{3}$))

$\Rightarrow$ sin x =sin ($\frac{\pi}{3}$) ($\because$sin ($\pi - \theta$)= sin θas here θ $lies\ in\ II\ quadrant\ and\ sine\ is\ positive$)

$\Rightarrow$ x = $\frac{\pi}{3}$

Question 12.
Mark the tick against the correct answer in the following:

The value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ is

A. $\frac{13\pi}{6}$

B.

C. $\frac{5\pi}{6}$

D. $\frac{\pi}{6}$ $\frac{7\pi}{6}$

Answer:
To Find: The value of $\cos^{-1}(\cos(\frac{13\pi}{6}))$

Now, let x =$\cos^{-1}(\cos(\frac{13\pi}{6}))$

$\Rightarrow$ cos x =cos ($\frac{13\pi}{6}$)

Here ,range of principle value of cos is [0,π]

$\Rightarrow$ x = $\frac{13\pi}{6}$ $\notin$ [0,π]

Hence for all values of x in range [0,π] ,the value of

$\cos^{-1}(\cos(\frac{13\pi}{6}))$ is

$\Rightarrow$ cos x =cos $(2\pi - \frac{\pi}{6})$ ($\because$cos $(\frac{13\pi}{6})$= cos $(2\pi - \frac{\pi}{6})$)

$\Rightarrow$ cos x =cos $(\frac{\pi}{6})$ ($\because$cos $(2\pi - \theta)$= cos θ)

$\Rightarrow$ x = $\frac{\pi}{6}$

Question 13.
Mark the tick against the correct answer in the following:

The value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ is

A. $\frac{7\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{\pi}{6}$

D. none of these

Answer:
To Find: The value of $\tan^{-1}(\tan(\frac{7\pi}{6}))$

Now, let x =$\tan^{-1}(\tan(\frac{7\pi}{6}))$

$\Rightarrow$ tan x =tan $(\frac{7\pi}{6})$

Here range of principle value of tan is $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\Rightarrow$ x = $\frac{7\pi}{6} \notin [-\frac{\pi}{2}, \frac{\pi}{2}]$

Hence for all values of x in range $[-\frac{\pi}{2}, \frac{\pi}{2}]$,the value of

$\tan^{-1}(\tan(\frac{13\pi}{6}))$ is

$\Rightarrow$ tan x =tan $(\pi + \frac{\pi}{6})$ ($\because$tan $(\frac{7\pi}{6})$= tan $(\pi + \frac{\pi}{6})$)

$\Rightarrow$ tan x =tan $(\frac{\pi}{6})$ ($\because$tan $(\pi + \theta)$= tan θ)

$\Rightarrow$ x = $\frac{\pi}{6}$

Question 14.
Mark the tick against the correct answer in the following:

The value of $\cot^{-1}\left(\cot\frac{5\pi}{4}\right)$ is

A. $\frac{\pi}{4}$

B. $\frac{-\pi}{4}$

C. $\frac{3\pi}{4}$

D. none of these

Answer:
To Find: The value of $\cot^{-1}(\cot(\frac{5\pi}{4}))$

Now, let x =$\cot^{-1}(\cot(\frac{5\pi}{4}))$

$\Rightarrow$ cot x =cot $(\frac{5\pi}{4})$

Here range of principle value of cot is $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\Rightarrow$ x = $\frac{5\pi}{4}$ $\notin [-\frac{\pi}{2}, \frac{\pi}{2}]$

Hence for all values of x in range $[-\frac{\pi}{2}, \frac{\pi}{2}]$,the value of

$\cot^{-1}(\cot(\frac{5\pi}{4}))$ is

⇒ cot x =cot $(\pi + \frac{\pi}{4})$ (∵cot $(\frac{5\pi}{4})$= cot $(\pi + \frac{\pi}{4})$)

⇒ cot x =cot $(\frac{\pi}{4})$ (∵cot $(\pi + \theta)$= cot θ)

⇒ x = $\frac{\pi}{4}$

Question 15.
Mark the tick against the correct answer in the following:

The value of $\sec^{-1}\left(\sec\frac{8\pi}{5}\right)$ is

A. $\frac{2\pi}{5}$

B. $\frac{3\pi}{5}$

C. $\frac{8\pi}{5}$

D. none of these

Answer:
To Find: The value of $\sec^{-1}(\sec(\frac{8\pi}{5}))$

Now, let x =$\sec^{-1}(\sec(\frac{8\pi}{5}))$

⇒ sec x =sec $(\frac{8\pi}{5})$

Here range of principle value of sec is $[0,\pi]$

$\Rightarrow$ x = $\frac{8\pi}{5}$ $\notin[0,\pi]$

Hence for all values of x in range $[0,\pi]$,the value of

$\sec^{-1}(\sec(\frac{8\pi}{5}))$ is

$\Rightarrow$ sec x =sec $(2\pi - \frac{2\pi}{5})$ ($\because$sec $(\frac{8\pi}{5})$= sec $(2\pi - \frac{2\pi}{5})$)

$\Rightarrow$ sec x =sec $(\frac{2\pi}{5})$ ($\because$sec $(2\pi - \theta)$= sec θ)

$\Rightarrow$ x = $\frac{2\pi}{5}$

Question 16.
Mark the tick against the correct answer in the following:

The value of $\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{4\pi}{3}\right)$ is

A. $\frac{\pi}{3}$

B. $\frac{-\pi}{3}$

C. $\frac{2\pi}{3}$

D. none of these

Answer:
To Find: The value of $\operatorname{cosec}^{-1}(\operatorname{cosec}(\frac{4\pi}{3}))$

Now, let x =$\operatorname{cosec}^{-1}(\operatorname{cosec}(\frac{4\pi}{3}))$

$\Rightarrow$ cosec x =cosec $(\frac{4\pi}{3})$

Here range of principle value of cosec is [-$\frac{\pi}{2}$,$\frac{\pi}{2}$]

⇒ x = $\frac{4\pi}{3}$ $\notin$ [-$\frac{\pi}{2}$,$\frac{\pi}{2}$]

Hence for all values of x in range [-$\frac{\pi}{2}$,$\frac{\pi}{2}$] ,the value of

$\operatorname{cosec}^{-1}(\operatorname{cosec}(\frac{4\pi}{3}))$ is

⇒ cosec x =cosec $(\pi + \frac{\pi}{3})$ (∵cosec $(\frac{4\pi}{3})$= cosec $(\pi + \frac{\pi}{3})$)

⇒ cosec x =cosec $(-\frac{\pi}{3})$ (∵cosec $(\pi + \theta)$= cosec$(-\theta)$)

⇒ x = - $\frac{\pi}{3}$

Question 17.
Mark the tick against the correct answer in the following:

The value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is

A. $\frac{3\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{-\pi}{4}$

D. none of these

Answer:
To Find: The value of $\tan^{-1}(\tan(\frac{3\pi}{4}))$

Now, let x =$\tan^{-1}(\tan(\frac{3\pi}{4}))$

$\Rightarrow$ tan x =tan $(\frac{3\pi}{4})$

Here range of principle value of tan is $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\Rightarrow$ x = $\frac{3\pi}{4}$ $\notin [-\frac{\pi}{2}, \frac{\pi}{2}]$

Hence for all values of x in range $[-\frac{\pi}{2}, \frac{\pi}{2}]$,the value of

$\tan^{-1}(\tan(\frac{3\pi}{4}))$ is

$\Rightarrow$ tan x =tan $(\pi - \frac{\pi}{4})$ ($\because$tan $(\frac{3\pi}{4})$= tan $(\pi - \frac{\pi}{4})$)

$\Rightarrow$ tan x =tan $(-\frac{\pi}{4})$ ($\because$tan $(\pi - \theta)$= tan$(-\theta)$)

$\Rightarrow$ x = $-\frac{\pi}{4}$

Question 18.
Mark the tick against the correct answer in the following:

$$\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right) = ?$$

A. 0

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{2}$

D. π

Answer:
To Find: The value of $\frac{\pi}{3} - \sin^{-1}(\frac{-1}{2})$

Now, let x $=\frac{\pi}{3} - \sin^{-1}(\frac{-1}{2})$

$\Rightarrow$ x $=\frac{\pi}{3} - (-\sin^{-1}(\frac{1}{2}))$ ($\because \sin(-\theta) = -\sin(\theta)$)

$\Rightarrow$ x $= \frac{\pi}{3} - (-\frac{\pi}{6})$ ($\because \sin\frac{\pi}{6} = \frac{1}{2}$)

$\Rightarrow$ x $= \frac{\pi}{3} + \frac{\pi}{6}$

$\Rightarrow$ x $= \frac{3\pi}{6} = \frac{\pi}{2}$

Question 19.
Mark the tick against the correct answer in the following:

The value of $\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}\right) = ?$

A. 0

B. 1

C. -1

D. none of these

Answer:
To Find: The value of $\sin(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2})$

Now, let x = $\sin(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2})$

$\Rightarrow$ x = $\sin(\frac{\pi}{2})$ ($\because \sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$)

$\Rightarrow$ x = 1 ($\because \sin\left(\frac{\pi}{2}\right) = 1$)

Question 20.
Mark the tick against the correct answer in the following:

If $x \neq 0$ then $\cos(\tan^{-1} x + \cot^{-1} x) = ?$

A. -1

B. 1

C. 0

D. none of these

Answer:
Given: $x \neq 0$

To Find: The value of $\cos(\tan^{-1} x + \cot^{-1} x)$

Now, let $x = \cos(\tan^{-1} x + \cot^{-1} x)$

$\Rightarrow x = \cos\left(\frac{\pi}{2}\right)$ $(\because \tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2})$

$\Rightarrow x = 0$ $\left(\because \cos\left(\frac{\pi}{2}\right) = 0\right)$

Question 21.
Mark the tick against the correct answer in the following:

The value of $\sin\left(\cos^{-1}\frac{3}{5}\right)$ is

A. $\frac{2}{5}$

B. $\frac{4}{5}$

C. $\frac{-2}{5}$

D. none of these

Answer:
To Find: The value of $\sin(\cos^{-1}\frac{3}{5})$

Now, let $x = \cos^{-1}\frac{3}{5}$

$\Rightarrow$ cos x = $\frac{3}{5}$

Now ,sin x = $\sqrt{1-\cos^2 x}$

= $\sqrt{1-(\frac{3}{5})^2}$

= $\frac{4}{5}$

$\Rightarrow$ x = $\sin^{-1}\frac{4}{5}$ = $\cos^{-1}\frac{3}{5}$

Therefore,

sin($\cos^{-1}\frac{3}{5}$) = sin($\sin^{-1}\frac{4}{5}$)

Let , Y= sin($\sin^{-1}\frac{4}{5}$)

$\Rightarrow$ $\sin^{-1}$Y = $\sin^{-1}\frac{4}{5}$

$\Rightarrow$ Y = $\frac{4}{5}$

Question 22.
Mark the tick against the correct answer in the following:

$$\cos^{-1}\left(\cos\frac{2\pi}{3}\right)+\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=?$$

A. $\frac{4\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{5\pi}{3}$

D. π

Answer:
To Find: The value of $\cos^{-1}(\cos(\frac{2\pi}{3})) + \sin^{-1}(\sin(\frac{2\pi}{3}))$

Here,consider $\cos^{-1}(\cos(\frac{2\pi}{3}))$ (
$\because$ *the principle value of* $\cos$ *lies in the range* $[0, \pi]$*and since* $\frac{2\pi}{3} \in [0, \pi]$)

$\Rightarrow \cos^{-1}(\cos(\frac{2\pi}{3})) = \frac{2\pi}{3}$

Now,consider $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

Since here the principle value of sine lies in range $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and since $\frac{2\pi}{3} \notin [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\Rightarrow \sin^{-1}(\sin(\frac{2\pi}{3})) = \sin^{-1}(\sin(\pi - \frac{\pi}{3}))$

$=\sin^{-1}(\sin(\frac{\pi}{3}))$

$= \frac{\pi}{3}$

Therefore,

$\cos^{-1}(\cos(\frac{2\pi}{3})) + \sin^{-1}(\sin(\frac{2\pi}{3})) = \frac{2\pi}{3} + \frac{\pi}{3}$

$=\frac{3\pi}{3}$

$=\pi$

Question 23.
Mark the tick against the correct answer in the following:

$\tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}\left(-2\right) = ?$

A. $\frac{\pi}{3}$

B. $\frac{-\pi}{3}$

C. $\frac{5\pi}{3}$

D. none of these

Answer:
To Find: The value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

Let , x = $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

⇒x = $\frac{\pi}{3}$ – [π- $\sec^{-1}(2)$] (∵ $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}\ and\ \sec^{-1}(-\theta) = \pi -\ \sec^{-1}(\theta)$)

⇒x = $\frac{\pi}{3}$ – [π - $\frac{\pi}{3}$]

⇒x = $\frac{\pi}{3}$ – [$\frac{2\pi}{3}$]

⇒x = - $\frac{\pi}{3}$

Question 24.
Mark the tick against the correct answer in the following:

$$\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2} = ?$$

A. $\frac{2\pi}{3}$

B. $\frac{3\pi}{2}$

C. 2π

D. none of these

Answer:

To Find: The value of $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$

Now, let x = $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$

$\Rightarrow$x= $\frac{\pi}{3}$ +2($\frac{\pi}{6}$) ($\because$cos ($\frac{\pi}{3}$)= $\frac{1}{2}$ and sin ($\frac{\pi}{6}$)= $\frac{1}{2}$)

$\Rightarrow$x= $\frac{\pi}{3}$ +$\frac{\pi}{3}$

$\Rightarrow$x= $\frac{2\pi}{3}$

Question 25.

Mark the tick against the correct answer in the following:

$$\tan^{-1}1 + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right) = ?$$

A. π

B. $\frac{2\pi}{3}$

C. $\frac{3\pi}{4}$

D. $\frac{\pi}{2}$

Answer:

To Find: The value of $\tan^{-1} 1 + \cos^{-1}(\frac{-1}{2}) + \sin^{-1}(\frac{-1}{2})$

Now, let x = $\tan^{-1} 1 + \cos^{-1}(\frac{-1}{2}) + \sin^{-1}(\frac{-1}{2})$

$\Rightarrow$ x = $\frac{\pi}{4}$ + [π- $\cos^{-1}(\frac{1}{2})$] + [-$\sin^{-1}\frac{1}{2}$] (

$\because \tan\left(\frac{\pi}{4}\right) = 1$ *and* $\cos^{-1}(-\theta) = [\pi - \cos^{-1}\theta]$ *and* $\sin^{-1}(-\theta) = -\sin^{-1}\theta$)

$\Rightarrow$ x = $\frac{\pi}{4}$ + [π- $\frac{\pi}{3}$] + [- $\frac{\pi}{6}$]

$\Rightarrow$ x = $\frac{\pi}{4}$ + $\frac{2\pi}{3}$ - $\frac{\pi}{6}$

$\Rightarrow$ x = $\frac{3\pi}{4}$

Question 26.
Mark the tick against the correct answer in the following:

$$\tan\left[2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right]=?$$

A. $\frac{7}{17}$

B. $\frac{-7}{17}$

C. $\frac{7}{12}$

D. $\frac{-7}{12}$

Answer:
To Find: The value of tan(2 $\tan^{-1}\frac{1}{5}$ - $\frac{\pi}{4}$)

Consider , tan(2 $\tan^{-1}\frac{1}{5}$ - $\frac{\pi}{4}$) =tan($\tan^{-1}$($\frac{2(\frac{1}{5})}{1-(\frac{1}{5})^2}$) - $\frac{\pi}{4}$)

($\because$ $2\tan^{-1}x = \tan^{-1}(\frac{2x}{1-x^2})$)

= tan($\tan^{-1}$($\frac{\frac{2}{5}}{1-\frac{1}{25}}$) - $\frac{\pi}{4}$)

= tan($\tan^{-1}$($\frac{5}{12}$) - $\frac{\pi}{4}$)

$= \tan(\tan^{-1}(\frac{5}{12}) - \tan^{-1}(1))$ ($\because \tan(\frac{\pi}{4})=1$)

$= \tan(\tan^{-1}(\frac{\frac{5}{12}-1}{1+\frac{5}{12}}))$

$(\tan^{-1} x - \tan^{-1} y = \tan^{-1}(\frac{x-y}{1+xy})$

$= \tan(\tan^{-1}(\frac{-7}{17}))$

$\tan(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}) = \frac{-7}{17}$

Question 27.

Mark the tick against the correct answer in the following:

$$\tan\frac{1}{2}\left(\cos^{-1}\frac{\sqrt{5}}{3}\right) = ?$$

A. $\frac{(3-\sqrt{5})}{2}$

B. $\frac{(3+\sqrt{5})}{2}$

C. $\frac{(5-\sqrt{3})}{2}$

D. $\frac{(5+\sqrt{3})}{2}$

Answer:

To Find: The value of $\tan\frac{1}{2}(\cos^{-1}\frac{\sqrt{5}}{3})$

Let , $x = \cos^{-1}\frac{\sqrt{5}}{3}$

$\Rightarrow \cos x = \frac{\sqrt{5}}{3}$

Now, $\tan \frac{1}{2}(\cos^{-1}\frac{\sqrt{5}}{3})$ becomes

$\tan \frac{1}{2}(\cos^{-1}\frac{\sqrt{5}}{3}) = \tan \frac{1}{2}(x) = \tan \frac{x}{2}$

$= \sqrt{\frac{1-\cos x}{1+\cos x}}$

$= \sqrt{\frac{1-(\frac{\sqrt{5}}{3})}{1+\frac{\sqrt{5}}{3}}}$

$= \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}$

$= \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}} \times \sqrt{\frac{3-\sqrt{5}}{3-\sqrt{5}}}$

$\tan \frac{1}{2}(\cos^{-1}\frac{\sqrt{5}}{3}) = \frac{3-\sqrt{5}}{2}$

Question 28.
Mark the tick against the correct answer in the following:

$$\sin\left(\cos^{-1}\frac{3}{5}\right) = ?$$

A. $\frac{3}{4}$

B. $\frac{4}{5}$

C. $\frac{3}{5}$

D. none of these

Answer:

To Find: The value of sin($\cos^{-1}\frac{3}{5}$)

Let, x = $\cos^{-1}\frac{3}{5}$

⇒cos x = $\frac{3}{5}$

Now , sin($\cos^{-1}\frac{3}{5}$) becomes sin (x)

Since we know that sin x = $\sqrt{1-\cos^2 x}$

= $\sqrt{1-(\frac{3}{5})^2}$

sin($\cos^{-1}\frac{3}{5}$) = Sin x = $\frac{4}{5}$

Question 29.

Mark the tick against the correct answer in the following:

$$\cos\left(\tan^{-1}\frac{3}{4}\right) = ?$$

A. $\frac{3}{5}$

B. $\frac{4}{5}$

C. $\frac{4}{9}$

D. none of these

Answer:

To Find: The value of cos($\tan^{-1}\frac{3}{4}$)

Let x = $\tan^{-1}\frac{3}{4}$

⇒tan x = $\frac{3}{4}$

⇒tan x = $\frac{3}{4} = \frac{opposite\,side}{adjacent\,side}$

We know that by pythagorus theorem ,

(Hypotenuse)^2 = (opposite side)^2 + (adjacent side)^2

Therefore, Hypotenuse = 5

⇒cos x = $\frac{adjacent\,side}{hypotenuse} = \frac{4}{5}$

Since here x = $\tan^{-1}\frac{3}{4}$ hence cos($\tan^{-1}\frac{3}{4}$) becomes cos x

Hence , cos($\tan^{-1}\frac{3}{4}$) = cos x = $\frac{4}{5}$

Question 30.
Mark the tick against the correct answer in the following:

$$\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right\} = ?$$

A. 1

B. 0

C. $\frac{-1}{2}$

D. none of these

Answer:
To Find: The value of of sin $\{\frac{\pi}{3} - \sin^{-1}(\frac{-1}{2})\}$

Let, x = sin $\{\frac{\pi}{3} - \sin^{-1}(\frac{-1}{2})\}$

$\Rightarrow$ x = sin $\{\frac{\pi}{3} - \ (-\sin^{-1}\frac{1}{2})\}$ ($\because \sin^{-1}(-\theta) = -\sin\theta$)

$\Rightarrow$ x = sin $(\frac{\pi}{3} + \frac{\pi}{6})$

$\Rightarrow$ x = sin $(\frac{3\pi}{6})$ = sin $(\frac{\pi}{2})$ = 1

Question 31.
Mark the tick against the correct answer in the following:

$$\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = ?$$

A. $\frac{1}{\sqrt{5}}$

B. $\frac{2}{\sqrt{5}}$

C. $\frac{1}{\sqrt{10}}$

D. $\frac{2}{\sqrt{10}}$

Answer:
To Find: The value of sin$(\frac{1}{2}\cos^{-1}\frac{4}{5})$

Let x = $\cos^{-1}\frac{4}{5}$

$\Rightarrow$cos x = $\frac{4}{5}$

Therefore sin$(\frac{1}{2}\cos^{-1}\frac{4}{5})$ becomes sin$(\frac{1}{2}x)$,i.e sin $(\frac{x}{2})$

We know that sin $(\frac{x}{2})$ = $\sqrt{\frac{1-\cos x}{2}}$

$= \sqrt{\frac{1-\frac{4}{5}}{2}}$

$= \sqrt{\frac{\frac{1}{5}}{2}}$

sin $(\frac{x}{2})$ $= \frac{1}{\sqrt{10}}$

Question 32.
Mark the tick against the correct answer in the following:

$$\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\} = ?$$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{4}$

D. $\frac{2\pi}{3}$

Answer:
To Find: The value of $\tan^{-1}\{2\cos(2\sin^{-1}\frac{1}{2})\}$

Let , x = $\tan^{-1}\{2\cos(2\sin^{-1}\frac{1}{2})\}$

$\Rightarrow$x = $\tan^{-1}\{2\cos(2(\frac{\pi}{6}))\}$ ($\because$sin $(\frac{\pi}{6})$= $\frac{1}{2}$)

$\Rightarrow$x = $\tan^{-1}(2\cos\frac{\pi}{3})$

$\Rightarrow$x = $\tan^{-1}(2(\frac{1}{2}))$ = $\tan^{-1} 1$ = $\frac{\pi}{4}$ ($\because$cos $(\frac{\pi}{3})$= $\frac{1}{2}$ $and \tan\left(\frac{\pi}{4}\right) = 1$)

Question 33.
Mark the tick against the correct answer in the following:

If $\cot^{-1}\left(\frac{-1}{5}\right) = x$ then sin x = ?

A. $\frac{1}{\sqrt{26}}$

B. $\frac{5}{\sqrt{26}}$

C. $\frac{1}{\sqrt{24}}$

D. none of these

Answer:
Given: $\cot^{-1}\frac{-1}{5}$ = x

To Find: The value of sin x

Since , x = $\cot^{-1}\frac{-1}{5}$

⇒cot x = $\frac{-1}{5} = \frac{adjacent\ side}{opposite\ side}$

By pythagorus theroem ,

(Hypotenuse)^2 = (opposite side)^2 + (adjacent side)^2

Therefore, Hypotenuse = $\sqrt{26}$

⇒sin x = $\frac{opposite\ side}{hypotenuse} = \frac{5}{\sqrt{26}}$

Question 34.

Mark the tick against the correct answer in the following:

$$\sin^{-1}\left(\frac{-1}{2}\right)+2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)=?$$

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{2}$

D. none of these

Answer:

To Find: The value of $\sin^{-1}(\frac{-1}{2})$ + $2\cos^{-1}(\frac{-\sqrt{3}}{2})$

Let , x = $\sin^{-1}(\frac{-1}{2})$ + $2\cos^{-1}(\frac{-\sqrt{3}}{2})$

⇒ x = - $\sin^{-1}(\frac{1}{2})$ + 2 $[\pi - \cos^{-1}(\frac{\sqrt{3}}{2})]$ (
$\because \sin^{-1}(-\theta) = -\sin^{-1}(\theta)\ and\ \cos^{-1}(-\theta) = \pi - \cos^{-1}(\theta)$)

⇒ x = - $(\frac{\pi}{6})$ + 2 $[\pi - \frac{\pi}{6}]$

⇒ x = - $(\frac{\pi}{6})$ + 2 $[\frac{5\pi}{6}]$

⇒ x = - $\frac{\pi}{6} + \frac{5\pi}{3}$

⇒ x = $\frac{3\pi}{2}$

Tag:

Question 35.

Mark the tick against the correct answer in the following:

$$\tan^{-1}(-1) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = ?$$

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{2}$

D. $\frac{2\pi}{3}$

Answer:
To Find: The value of $\tan^{-1}(-1) + \cos^{-1}(\frac{-1}{\sqrt{2}})$

Let , x = $\tan^{-1}(-1) + \cos^{-1}(\frac{-1}{\sqrt{2}})$

⇒x = - $\tan^{-1}(1)$ + $(\pi - \cos^{-1}(\frac{1}{\sqrt{2}}))$

(∵ $\tan^{-1}(-\theta) = -\tan^{-1}(\theta)$ *and* $\cos^{-1}(-\theta) = \pi - \cos^{-1}(\theta)$)

⇒x = - $\frac{\pi}{4}$ + $(\pi - \frac{\pi}{4})$

⇒x = - $\frac{\pi}{4}$ + $\frac{3\pi}{4}$

⇒x = $\frac{\pi}{2}$

Question 36.
Mark the tick against the correct answer in the following:

$$\cot\left(\tan^{-1} x + \cot^{-1} x\right) = ?$$

A. 1

B. $\frac{1}{2}$

C. 0

D. none of these

Answer:
To Find: The value of cot $(\tan^{-1} x + \cot^{-1} x)$

Let , x = cot $(\tan^{-1} x + \cot^{-1} x)$

⇒x = cot $(\frac{\pi}{2})$ $(\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2})$

⇒x = 0

Question 37.
Mark the tick against the correct answer in the following:

$$\tan^{-1} 1 + \tan^{-1} \frac{1}{3} = ?$$

A. $\tan^{-1} \frac{4}{3}$

B. $\tan^{-1} \frac{2}{3}$

C. $\tan^{-1} 2$

D. $\tan^{-1} 3$

Answer:
To Find: The value of $\tan^{-1} 1 + \tan^{-1} \frac{1}{3}$

Let , x = $\tan^{-1} 1 + \tan^{-1} \frac{1}{3}$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow \tan^{-1} 1 + \tan^{-1} \frac{1}{3} = \tan^{-1}(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}) = \tan^{-1} 2$

Question 38.
Mark the tick against the correct answer in the following:

$$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = ?$$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer:
To Find: The value of $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$

Let , x = $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow \tan^{-1} 1 + \tan^{-1} \frac{1}{3} = \tan^{-1}(\frac{\frac{1}{2}+\frac{1}{3}}{1-(\frac{1}{3}\times\frac{1}{2})}) = \tan^{-1} 1 = \frac{\pi}{4}$

Question 39.
Mark the tick against the correct answer in the following:

$$2\tan^{-1}\frac{1}{3} = ?$$

A. $\tan^{-1}\frac{3}{2}$

B. $\tan^{-1}\frac{3}{4}$

C. $\tan^{-1}\frac{4}{3}$

D. none of these

Answer:
To Find: The value of 2 $\tan^{-1}\frac{1}{3}$ i.e, $\tan^{-1}\frac{1}{3}$+ $\tan^{-1}\frac{1}{3}$

Let , x = $\tan^{-1}\frac{1}{3}$+ $\tan^{-1}\frac{1}{3}$

Since we know that $\tan^{-1}x$+ $\tan^{-1}y$ = $\tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow$ $\tan^{-1}1$+ $\tan^{-1}\frac{1}{3}$ = $\tan^{-1}(\frac{\frac{1}{3}+\frac{1}{3}}{1-(\frac{1}{3}\times\frac{1}{3})})$ = $\tan^{-1}\frac{3}{4}$

Question 40.
Mark the tick against the correct answer in the following:

$$\cos\left(2\tan^{-1}\frac{1}{2}\right)=?$$

A. $\frac{3}{5}$

B. $\frac{4}{5}$

C. $\frac{7}{8}$

D. none of these

Answer:
To Find: The value of cos (2 $\tan^{-1}\frac{1}{2}$)

Let , x = cos $(2\tan^{-1}\frac{1}{2})$

⇒x = cos $(\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{2})$

Since we know that $\tan^{-1}x+\tan^{-1}y=\tan^{-1}(\frac{x+y}{1-xy})$

⇒ $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{2}=\tan^{-1}(\frac{\frac{1}{2}+\frac{1}{2}}{1-(\frac{1}{2}\times\frac{1}{2})})=\tan^{-1}\frac{4}{3}$

⇒x = cos $(\tan^{-1}\frac{4}{3})$

Now , let y = $\tan^{-1}\frac{4}{3}$

⇒ tan y = $\frac{4}{3}=\frac{opposite\ side}{adjacent\ side}$

By pythagorus theroem ,

(Hypotenuse $)^2$ = (opposite side $)^2$ + (adjacent side $)^2$

Therefore, Hypotenuse = 5

⇒ cos $(\tan^{-1}\frac{4}{3})$=cos y = $\frac{3}{5}$

Question 41.
Mark the tick against the correct answer in the following:

$$\sin\left[2\tan^{-1}\frac{5}{8}\right]$$

A. $\frac{25}{64}$

B. $\frac{80}{89}$

C. $\frac{75}{128}$

D. none of these

Answer:

To Find: The value of sin (2 $\tan^{-1}\frac{5}{8}$)

Let , x = sin(2 $\tan^{-1}\frac{5}{8}$)

We know that $2\tan^{-1}x = \sin^{-1}(\frac{2x}{1+x^2})$

$\Rightarrow$ x = sin($\sin^{-1}(\frac{2(\frac{5}{8})}{1+(\frac{5}{8})^2})$) = sin($\sin^{-1}(\frac{80}{89})$) = $\frac{80}{89}$

Question 42.

Mark the tick against the correct answer in the following:

$$\sin\left[2\sin^{-1}\frac{4}{5}\right]$$

A. $\frac{12}{25}$

B. $\frac{16}{25}$

C. $\frac{24}{25}$

D. None of these

Answer:

To Find: The value of sin (2 $\sin^{-1}\frac{4}{5}$)

Let , x = $\sin^{-1}\frac{4}{5}$

$\Rightarrow$sin x = $\frac{4}{5}$

We know that ,cos x = $\sqrt{1 - sin^2 x}$

= $\sqrt{1-(\frac{4}{5})^2}$

=$\frac{3}{5}$

Now since, x = $\sin^{-1}\frac{4}{5}$,hence sin (2 $\sin^{-1}\frac{4}{5}$) becomes sin(2x)

Here, sin(2x)= 2 sin x cos x

=2× $\frac{4}{5}$ × $\frac{3}{5}$

=$\frac{24}{25}$

Question 43.
Mark the tick against the correct answer in the following:

If $\tan^{-1}x = \frac{\pi}{4} - \tan^{-1}\frac{1}{3}$ then x = ?

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{1}{6}$

D. None of these

Answer:
To Find: The value of $\tan^{-1}x$ = $\frac{\pi}{4}$ - $\tan^{-1}\frac{1}{3}$

Now , $\tan^{-1}x$ = $\tan^{-1}1$ - $\tan^{-1}\frac{1}{3}$ ($\because$ $\tan\frac{\pi}{4} = 1$)

Since we know that $\tan^{-1}x$- $\tan^{-1}y$ = $\tan^{-1}(\frac{x-y}{1+xy})$

$\Rightarrow$ $\tan^{-1}1$+ $\tan^{-1}\frac{1}{3}$ = $\tan^{-1}(\frac{1-\frac{1}{3}}{1+\frac{1}{3}})$ = $\tan^{-1}\frac{1}{2}$

$\Rightarrow$ $\tan^{-1}x$ = $\tan^{-1}\frac{1}{2}$

$\Rightarrow$x = $\frac{1}{2}$

Question 44.
Mark the tick against the correct answer in the following:

If $\tan^{-1}(1+x)+\tan^{-1}(1-x)=\frac{\pi}{2}$ then x = ?

A. 1

B. -1

C. 0

D. $\frac{1}{2}$

Answer:
To Find: The value of $\tan^{-1}(1+x)$ + $\tan^{-1}(1-x)$=$\frac{\pi}{2}$

Since we know that $\tan^{-1}x$+ $\tan^{-1}y$ = $\tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow$ $\tan^{-1}(1+x)$ + $\tan^{-1}(1-x)$ = $\tan^{-1}(\frac{(1+x)+(1-x)}{1-(1+x)(1-x)})$

= $\tan^{-1}(\frac{2}{1-(1-x^2)})$

=$\tan^{-1}(\frac{2}{x^2})$

Here since $\tan^{-1}(1+x)$ + $\tan^{-1}(1-x)$=$\frac{\pi}{2}$

$\Rightarrow \tan^{-1}(\frac{2}{x^2}) = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}(\frac{2}{x^2}) = \tan^{-1}(\infty)$ ($\because \tan\frac{\pi}{2} = \infty$)

$\Rightarrow \frac{2}{x^2} = \infty$

$\Rightarrow x^2 = \frac{2}{\infty}$

$\Rightarrow$ x = 0

Question 45.
Mark the tick against the correct answer in the following:

If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$ then $\left(\cos^{-1} x + \cos^{-1} y\right) = ?$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. π

D. $\frac{2\pi}{3}$

Answer:
Given: $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$

To Find: The value of $\cos^{-1} x + \cos^{-1} y$

Since we know that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$\Rightarrow \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$

Similarly $\cos^{-1} y = \frac{\pi}{2}$ - $\sin^{-1} y$

Now consider $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2}$ - $\sin^{-1} x + \frac{\pi}{2}$ - $\sin^{-1} y$

$= \frac{2\pi}{2} - [\sin^{-1} x + \sin^{-1} y]$

$= \pi$ - $\frac{2\pi}{3}$

$= \frac{\pi}{3}$

Question 46.
Mark the tick against the correct answer in the following:

$(\tan^{-1} 2 + \tan^{-1} 3) = ?$

A. $\frac{-\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{4}$

D. π

Answer:
To Find: The value of $\tan^{-1} 2 + \tan^{-1} 3$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow \tan^{-1} 2 + \tan^{-1} 3 = \tan^{-1}(\frac{2+3}{1-(2\times3)})$

$= \tan^{-1}(\frac{5}{-5})$

$= \tan^{-1}(-1)$

Since the principle value of tan lies in the range $[0,\pi]$

$\Rightarrow\ \tan^{-1}(-1) = \frac{3\pi}{4}$

Question 47.
Mark the tick against the correct answer in the following:

If $\tan^{-1} x + \tan^{-1} 3 = \tan^{-1} 8$ then x = ?

A. $\frac{1}{3}$

B. $\frac{1}{5}$

C. 3

D. 5

Answer:
Given: $\tan^{-1} x + \tan^{-1} 3 = \tan^{-1} 8$

To Find: The value of x

Here $\tan^{-1} x + \tan^{-1} 3 = \tan^{-1} 8$ can be written as

$\tan^{-1} x = \tan^{-1} 8 - \tan^{-1} 3$

Since we know that $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

$\tan^{-1} x = \tan^{-1} 8 - \tan^{-1} 3 = \tan^{-1}\left(\frac{8-3}{1+(8\times3)}\right)$

$= \tan^{-1}\left(\frac{5}{25}\right)$

$= \tan^{-1}\left(\frac{1}{5}\right)$

$\Rightarrow x = \frac{1}{5}$

Question 48.
Mark the tick against the correct answer in the following:

If $\tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$ then x = ?

A. $\frac{1}{2}$ or -2

B. $\frac{1}{3}$ or -3

C. $\frac{1}{4}$ or -2

D. $\frac{1}{6}$ or -1

Answer:
Given: $\tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$

To Find: The value of x

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow \tan^{-1} 3x + \tan^{-1} 2x = \tan^{-1}(\frac{3x+2x}{1-(3x\times 2x)})$

$= \tan^{-1}(\frac{5x}{1-6x^2})$

Now since $\tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$

$\tan^{-1} 3x + \tan^{-1} 2x = \tan^{-1} 1$ ($\because \tan\frac{\pi}{4} = 1$)

$\Rightarrow \tan^{-1}(\frac{5x}{1-6x^2}) = \tan^{-1} 1$

$\Rightarrow \frac{5x}{1-6x^2} = 1$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow x = \frac{1}{6}$ or $x = -1$

Question 49.
Mark the tick against the correct answer in the following:

$$\tan\left\{\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right\} = ?$$

A. $\frac{13}{6}$

B. $\frac{17}{6}$

C. $\frac{19}{6}$

D. $\frac{23}{6}$

Answer:
To Find: The value of tan $\{\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\}$

Let x = $\cos^{-1}\frac{4}{5}$

$\Rightarrow \cos x = \frac{4}{5} = \frac{adjacent\ side}{hypotenuse}$

By pythagorus theroem ,

$(\text{Hypotenuse})^2 = (\text{opposite side})^2 + (\text{adjacent side})^2$

Therefore , opposite side = 3

$\Rightarrow \tan x = \frac{opposite\ side}{adjacent\ side} = \frac{3}{4}$

$\Rightarrow$x $=\tan^{-1}\frac{3}{4}$

Now tan $\{\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\}$ = tan $\{\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\}$

Since we know that $\tan^{-1}x+\tan^{-1}y=\tan^{-1}(\frac{x+y}{1-xy})$

tan $\{\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\}$ = tan $(\tan^{-1}(\frac{\frac{3}{4}+\frac{2}{3}}{1-(\frac{3}{4}\times\frac{2}{3})}))$

= tan $(\tan^{-1}(\frac{17}{6}))$

= $\frac{17}{6}$

Question 50.
Mark the tick against the correct answer in the following:

$$\cos^{-1}9+\operatorname{cosec}^{-1}\frac{\sqrt{41}}{4}=?$$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{3\pi}{4}$

Answer:

To Find: The value of $\cot^{-1}9+\operatorname{cosec}^{-1}\frac{\sqrt{41}}{4}$

Now $\cot^{-1}9+\operatorname{cosec}^{-1}\frac{\sqrt{41}}{4}$ can be written in terms of tan inverse as

$\cot^{-1} 9 + \operatorname{cosec}^{-1}\frac{\sqrt{41}}{4} = \tan^{-1}\frac{1}{9} + \tan^{-1}\frac{4}{5}$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}(\frac{x+y}{1-xy})$

$\Rightarrow \tan^{-1}\frac{1}{9} + \tan^{-1}\frac{4}{5} = \tan^{-1}(\frac{\frac{1}{9}+\frac{4}{5}}{1-(\frac{1}{9}\times\frac{4}{5})})$

$= \tan^{-1}(\frac{41}{41})$

$= \tan^{-1}(1) = \frac{\pi}{4}$

Question 51.
Mark the tick against the correct answer in the following:

Range of $\sin^{-1} x$ is

A. $\left[0, \frac{\pi}{2}\right]$

B. $[0, \pi]$

C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

D. None of these

Answer:
To Find: The range of $\sin^{-1} x$

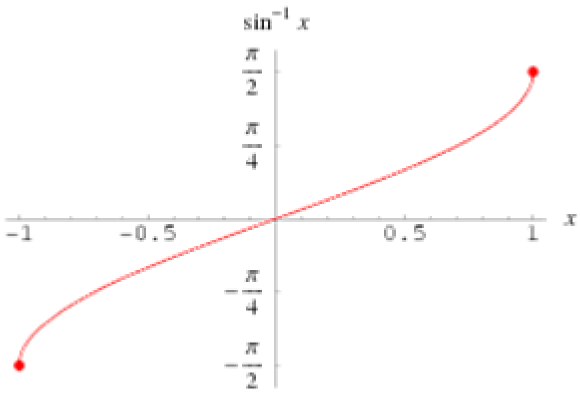
Here,the inverse function is given by y = $f^{-1}(x)$

The graph of the function y = $\sin^{-1}(x)$ can be obtained from the graph of

Y = sin x by interchanging x and y axes.i.e, if (a,b) is a point on Y = sin x then (b,a) is

The point on the function y = $\sin^{-1}(x)$

Below is the Graph of range of $\sin^{-1}(x)$



From the graph, it is clear that the range of $\sin^{-1}(x)$ is restricted to the interval

$[-\frac{\pi}{2}, \frac{\pi}{2}]$

Question 52.
Mark the tick against the correct answer in the following:

Range of $\cos^{-1}$ x is

A. [0, π]

B. $\left[0, \frac{\pi}{2}\right]$

C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

D. None of these

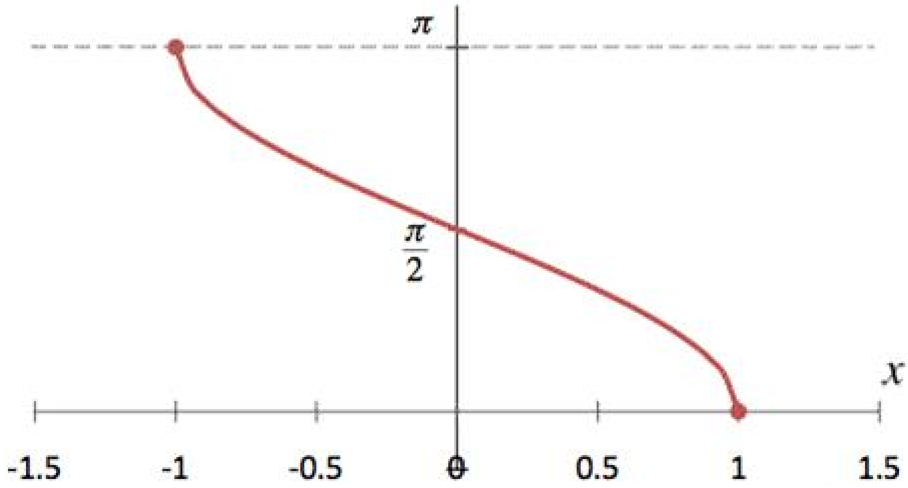
Answer:
To Find: The range of $\cos^{-1} x$

Here,the inverse function is given by y = $f^{-1}(x)$

The graph of the function y = $\cos^{-1}(x)$ can be obtained from the graph of

Y = cos x by interchanging x and y axes.i.e, if (a,b) is a point on Y = cos x then (b,a) is the point on the function y = $\cos^{-1}(x)$

Below is the Graph of the range of $\cos^{-1}(x)$



From the graph, it is clear that the range of $\cos^{-1}(x)$ is restricted to the interval

$[0, \pi]$

Question 53.
Mark the tick against the correct answer in the following:

Range of $\tan^{-1} x$ is

A. $\left(0, \frac{\pi}{2}\right)$

B. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

D. None of these

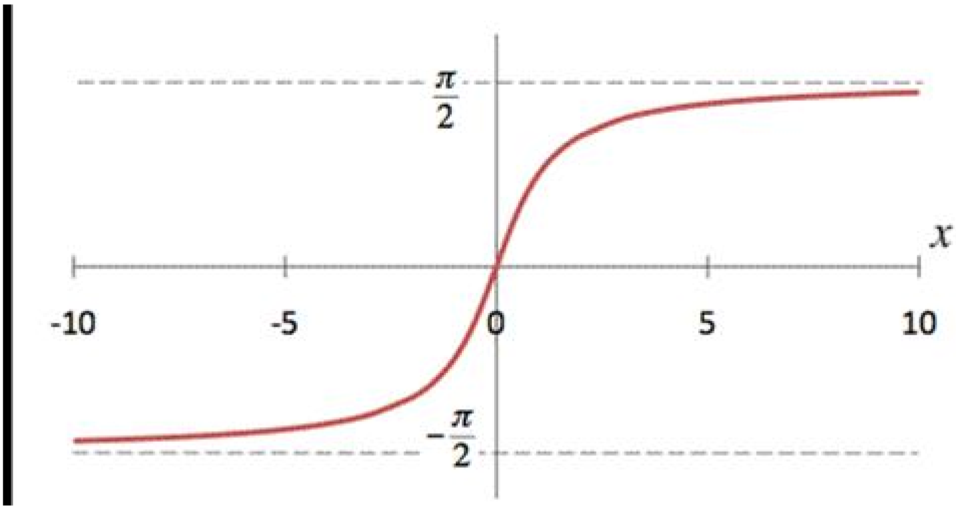
Answer:
To Find: The range of $\tan^{-1} x$

Here,the inverse function is given by y = $f^{-1}(x)$

The graph of the function y = $\tan^{-1}(x)$ can be obtained from the graph of

Y = tan x by interchanging x and y axes.i.e, if (a,b) is a point on Y = tan x then (b,a) is the point on the function y = $\tan^{-1}(x)$

Below is the Graph of the range of $\tan^{-1}(x)$



From the graph, it is clear that the range of $\tan^{-1}(x)$ is restricted to any of the intervals like $[-\frac{3\pi}{2}, -\frac{\pi}{2}]$, $[-\frac{\pi}{2}, \frac{\pi}{2}]$, $[\frac{\pi}{2}, \frac{3\pi}{2}]$ and so on. Hence the range is given by

$(-\frac{\pi}{2}, \frac{\pi}{2})$.

Question 54.
Mark the tick against the correct answer in the following:

Range of $\sec^{-1} x$ is

A. $\left[0, \frac{\pi}{2}\right]$

B. $[0, \pi]$

C. $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

D. None of these

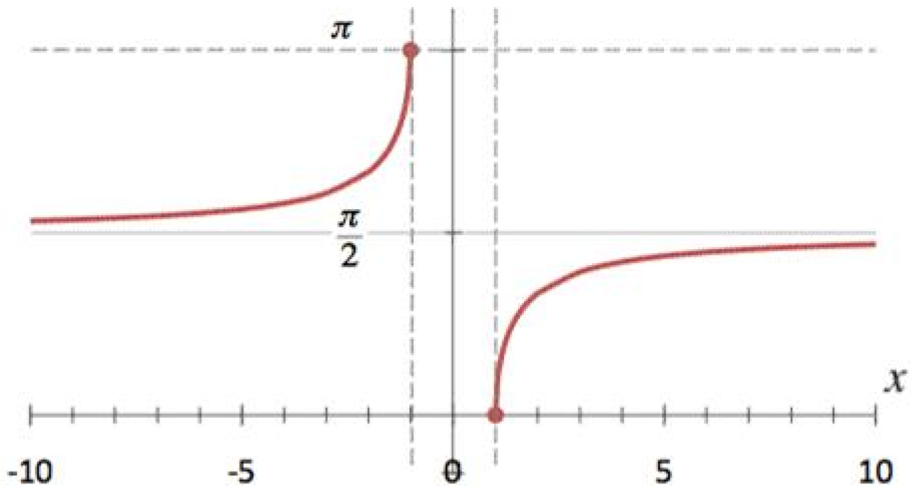
Answer:
To Find:The range of $\sec^{-1}(x)$

Here,the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \sec^{-1}(x)$ can be obtained from the graph of

Y = sec x by interchanging x and y axes.i.e, if (a,b) is a point on Y = sec x then (b,a) is the point on the function y = $\sec^{-1}(x)$

Below is the Graph of the range of $\sec^{-1}(x)$



From the graph, it is clear that the range of $\sec^{-1}(x)$ is restricted to interval

$[0,\pi] - \{\frac{\pi}{2}\}$

Question 55.
Mark the tick against the correct answer in the following:

Range of coses^{-1} x is

A. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

B. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

D. None of these

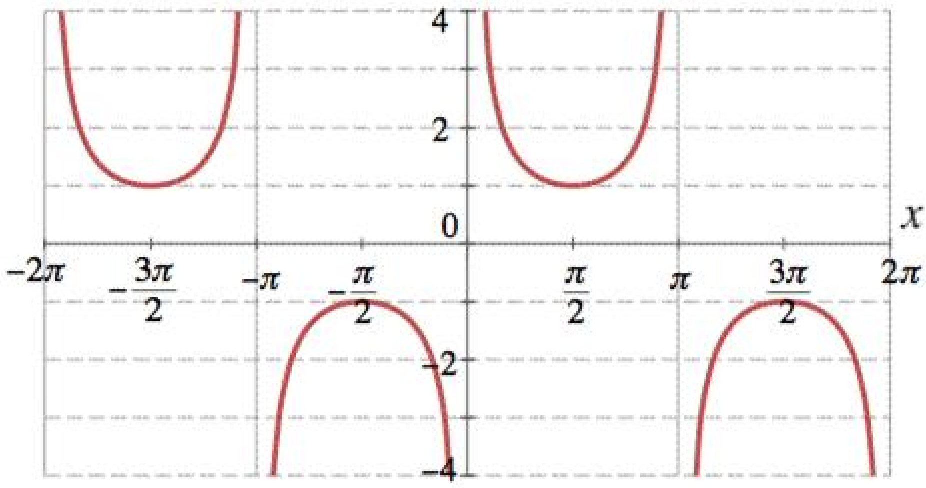
Answer:
To Find: The range of $\operatorname{cosec}^{-1}(x)$

Here,the inverse function is given by y = $f^{-1}(x)$

The graph of the function y = $\text{cosec}^{-1}(x)$ can be obtained from the graph of

Y = cosec x by interchanging x and y axes.i.e, if (a,b) is a point on Y = cosec x then (b,a) is the point on the function y = $\text{cosec}^{-1}(x)$

Below is the Graph of the range of $\text{cosec}^{-1}(x)$



From the graph it is clear that the range of $\text{cosec}^{-1}(x)$ is restricted to interval

$[-\frac{\pi}{2},\frac{\pi}{2}] - \{0\}$

Question 56.
Mark the tick against the correct answer in the following:

Domain of cos-1 x is

A. [0, 1]

B. [-1, 1]

C. [-1, 0]

D. None of these

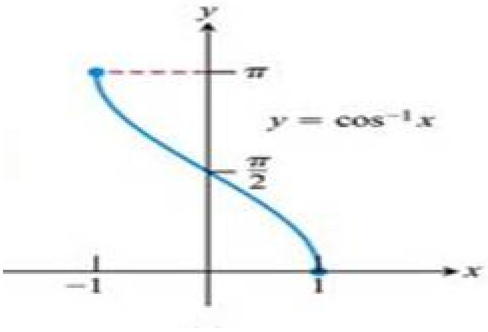
Answer:
To Find: The Domain of $\cos^{-1}(x)$

Here,the inverse function of cos is given by y = $f^{-1}(x)$

The graph of the function y = $\cos^{-1}(x)$ can be obtained from the graph of

Y = cos x by interchanging x and y axes.i.e, if (a,b) is a point on Y = cos x then (b,a) is the point on the function y = $\cos^{-1}(x)$

Below is the Graph of the domain of $\cos^{-1}(x)$



From the graph, it is clear that the domain of $\cos^{-1}(x)$ is [-1,1]

Question 57.
Mark the tick against the correct answer in the following:

Domain of sec^{-1} x is

A. [-1, 1]

B. R – {0}

C. R – [-1, 1]

D. R – {-1, 1}

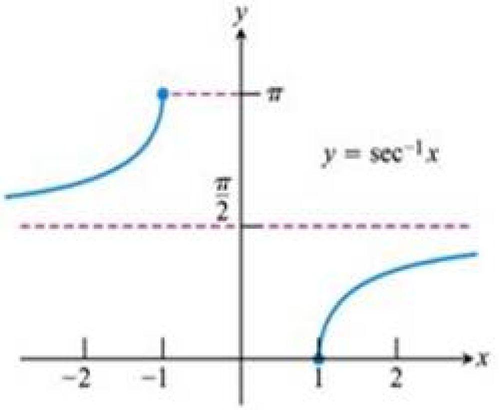
Answer:
To Find: The Domain of $\sec^{-1}(x)$

Here,the inverse function is given by y = $f^{-1}(x)$

The graph of the function y = $\sec^{-1}(x)$ can be obtained from the graph of

Y = sec x by interchanging x and y axes.i.e, if (a,b) is a point on Y = sec x then (b,a) is the point on the function y = $\sec^{-1}(x)$

Below is the Graph of the domain of $\sec^{-1}(x)$



From the graph, it is clear that the domain of $\sec^{-1}(x)$ is a set of all real numbers excluding -1 and 1 i.e, R – [-1,1]