Exercise 6a

Question 1.
If A is a 2 × 2 matrix such that $|A| \neq 0$ and $|A| = 5$, write the value of $|4A|$.

Answer:
Theorem: If A be k × k matrix then $|pA|=p^k|A|$.

Given, p=4,k=2 and |A|=5.

$|4A|=4^2 \times 5$

$=16 \times 5$

$=80$

Question 2.
If A is a 3 × 3 matrix such that $|A| \neq 0$ and $|3A| = k|A|$ then write the value of k.

Answer:
Theorem: If Let A be k × k matrix then $|pA|=p^k|A|$.

Given: k=3 and p=3.

$|3A|=3^3 \times |A|$

$=27|A|$.

Comparing above with k|A| gives k=27.

Question 3.
Let A be a square matrix of order 3, write the value of $|2A|$, where $|A| = 4$.

Answer:
Theorem: If A be k × k matrix then $|pA|=p^k|A|$.

Given: p=2, k=3 and|A|=4

$|2A|=2^3 \times |A|$

$=8 \times 4$

= 32

Question 4.

If A_{ij} is the cofactor of the element a_{ij} of $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ then write the value of $(a_{32}A_{32})$.

Answer:
Theorem: A_{ij} is found by deleting i^{th} rowand j^{th} column, the determinant of left matrix is called cofactor with multiplied by $(-1)^{(i+j)}$.

Given: i=3 and j=2.

$A_{32}=(-1)^{(3+2)}(2 \times 4-6 \times 5)$

$=-1 \times (-22)$

$=22$

$a_{32}=5$

$a_{32}A_{32}= 5 \times 22$

$=110$

Question 5.

Evaluate $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$.

Answer:
Theorem: This evaluation can be done in two different ways either by taking out the common things and then calculating the determinants or simply take determinant.

I will prefer first method because with that chances of silly mistakes reduces.

Take out x+1 from second row.

$(x+1) \times \begin{vmatrix} x^2-x+1 & x-1 \\ 1 & 1 \end{vmatrix}$

$\Rightarrow (x+1) \times (x^2-x+1-(x-1))$

$\Rightarrow (x+1) \times (x^2-2x+2)$

$\Rightarrow x^3-2x^2+2x+x^2-2x+2$

$\Rightarrow x^3-x^2+2$.

Question 6.
Evaluate $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$.

Answer:
This we can very simply go through directly.

$((a+ib)(a-ib))-((-c+id)(c+id))$.

$\Rightarrow (a^2+b^2)-(-c^2 -d^2)$.

$\Rightarrow a^2+b^2 + c^2+d^2$

$\because i \times i=-1$

Question 7.
If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, write the value of x.

Answer:
Here the determinant is compared so we need to take determinant both sides then find x.

$12x+14=32-42$

$\Rightarrow 12x=-10-14$

⇒ 12x=-24

⇒ x=-2

Question 8.

If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, write the value of x.

Answer:

this question is having the same logic as above.

$2x^2-40=18+14$

⇒ $2x^2=72$

⇒ $x^2=36$

⇒ $x=\pm 6$.

Question 9.

If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, write the value of x.

Answer:

Simply by equating both sides we can get the value of x.

$2x^2+2x-2(x^2+4x+3)=-12$

⇒ $-6x-6=-12$

⇒ $-6x=-6$

⇒ $x = 1$

Question 10.

If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, find the value of 3|A|.

Answer:
Find the determinant of A and then multiply it by 3

|A|=2

3|A|=3 × 2

=6

Question 11.
Evaluate $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$.

Answer:
It is determinant multiplied by a scalar number 2, just find determinant of matrix and multiply it by 2.

2 × (35-20)

2 × 15= 30

Question 12.
Evaluate $\begin{vmatrix} \sqrt{6} & \sqrt{5} \\ \sqrt{20} & \sqrt{24} \end{vmatrix}$.

Answer:
Find determinant

$\sqrt{6}$ × √24-√20 × √5

√144-√100.

=12-10

=2.

Question 13.

Evaluate $\begin{vmatrix} 2\cos\theta & -2\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$.

Answer:
After finding determinant we will get a trigonometric identity.

$2\cos^2\theta + 2\sin^2\theta$

$=2$

$\because \sin^2\theta + \cos^2\theta = 1$

Question 14.

Evaluate $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$.

Answer:
After finding determinant we will get a trigonometric identity.

$\cos^2\alpha + \sin^2\alpha$

$=1$

$\because \sin^2\theta + \cos^2\theta = 1$

Question 15.

Evaluate $\begin{vmatrix} \sin 60^\circ & \cos 60^\circ \\ -\sin 30^\circ & \cos 30^\circ \end{vmatrix}$.

Answer:
After finding determinant we will get,

$\text{Sin}60^\circ = \frac{\sqrt{3}}{2} = \cos 30^\circ$

$\text{Cos}60° = \frac{1}{2} = \sin 30°$

$\sin 60° \times \cos 30° + \sin 30° \times \cos 60°$

$$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$= \frac{3}{4} + \frac{1}{4}$

= 1.

Question 16.

Evaluate $\begin{vmatrix} \cos 65^{\circ} & \sin 65^{\circ} \\ \sin 25^{\circ} & \cos 25^{\circ} \end{vmatrix}$.

Answer:

By directly opening this determinant

$\cos 65° \times \cos 25° - \sin 25° \times \sin 65°$

$= \cos(65°+25°)$ $\because \cos A \cos B - \sin A \sin B = \cos(A+B)$

$= \cos 90°$

= 0

$\because \cos A \cos B - \sin A \sin B = \cos(A+B)$

Question 17.

Evaluate $\begin{vmatrix} \cos 15^{\circ} & \sin 15^{\circ} \\ \sin 75^{\circ} & \cos 75^{\circ} \end{vmatrix}$.

Answer:

$\cos 15° \cos 75° - \sin 75° \sin 15°$

$= \cos(15°+75°)$ $\because \cos A \cos B - \sin A \sin B = \cos(A+B)$

= cos90°

= 0

∵ cosAcosB-sinAsinB=cos(A+B)

Question 18.

Evaluate $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$.

Answer:
We know that expansion of determinant with respect to first row is $a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}$.

0(3 × 6-5 × 4)-2(2 × 6-4 × 4)+0(2 × 5-4 × 3)

= 8.

Question 19.

Without expanding the determinant, prove that $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix} = 0.$

SINGULAR MATRIX A square matrix A is said to be singular if |A| = 0.

Also, A is called non singular if |A| ≠ 0.

Answer:
We know that $C_1 \Rightarrow C_1 - C_2$, would not change anything for the determinant.

Applying the same in above determinant, we get

$\begin{bmatrix} 40 & 1 & 5 \\ 72 & 7 & 9 \\ 24 & 5 & 3 \end{bmatrix}$ Now it can clearly be seen that $C_1 = 8 \times C_3$

Applying above equation we get,

$$\begin{bmatrix} 0 & 1 & 5 \\ 0 & 7 & 9 \\ 0 & 3 & 3 \end{bmatrix}$$

We know that if a row or column of a determinant is 0. Then it is singular determinant.

Question 20.

For what value of x, the given matrix $A = \begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ is a singular matrix?

Answer:
For A to be singular matrix its determinant should be equal to 0.

0= (3-2x) × 4-(x+1) × 2

0= 12-8x-2x-2

0=10-10x

X=1.

Question 21.

Evaluate $\begin{vmatrix} 14 & 9 \\ -8 & -7 \end{vmatrix}$.

Answer:
$\begin{vmatrix} 14 & 9 \\ -8 & -7 \end{vmatrix}$.=14 × (-7)-9 × (-8)

= -26

Question 22.

Evaluate $\begin{vmatrix} \sqrt{3} & \sqrt{5} \\ -\sqrt{5} & 3\sqrt{3} \end{vmatrix}$.

Answer:

$\begin{vmatrix} \sqrt{3} & \sqrt{5} \\ -\sqrt{5} & 3\sqrt{3} \end{vmatrix}$. = 3√3 × √3 – (-√5 × √5)

= 14.