Exercise 5a

Question 1.

If
$$A = \begin{bmatrix} 5 & -2 & 6 & 1 \\ 7 & 0 & 8 & -3 \\ \sqrt{2} & \frac{3}{5} & 4 & 3 \end{bmatrix}$$
 then write

i. the number of rows in A,

ii. the number of columns in A,

iii. the order of the matrix A,

iv. the number of all entries in A,

v. the elements a₂₃, a₃₁, a₁₄, a₃₃, a₂₂ of A.

Answer:

(i) Number of rows = 3

(ii) Number of columns = 4

(iii) Order of matrix = Number of rows x Number of columns = (3×4)

(iv) Number of entries = (Number of rows) x (Number of columns)

 $= 3 \times 4$

= 12

(V) $a_{ij} = element \ of \ i^{th} \ row \ and \ j^{th} \ column$

 $a_{23} = 8$

 $a_{31} = \sqrt{2}$

 $a_{14} = 1$

$$a_{33} = 4$$

$$a_{22} = 0$$

Question 2.

Write the order of each of the following matrices:

i. A =
$$\begin{bmatrix} 3 & 5 & 4 & -2 \\ 0 & \sqrt{3} & -1 & \frac{4}{9} \end{bmatrix}$$

ii.
$$B = \begin{bmatrix} 6 & -5 \\ \frac{1}{2} & \frac{3}{4} \\ -2 & -1 \end{bmatrix}$$

iii.
$$C = \begin{bmatrix} 7 - \sqrt{2} & 5 & 0 \end{bmatrix}$$

iv.
$$D = [8 - 3]$$

v.
$$E = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$$

$$vi, F = [6]$$

Answer:

i. A =
$$\begin{bmatrix} 3 & 5 & 4 & -2 \\ 0 & \sqrt{3} & -1 & \frac{4}{9} \end{bmatrix}$$

Order of matrix = Number of rows x Number of columns

$$= (2 \times 4)$$

ii.
$$B = \begin{bmatrix} 6 & -5 \\ \frac{1}{2} & \frac{3}{4} \\ -2 & -1 \end{bmatrix}$$

Order of matrix = Number of rows x Number of columns

$$= (4 \times 2)$$

iii.
$$C = \begin{bmatrix} 7 - \sqrt{2} & 5 & 0 \end{bmatrix}$$

Order of matrix = Number of rows x Number of columns

$$= (1 \times 4)$$

iv.
$$D = [8 - 3]$$

Order of matrix = Number of rows x Number of columns

$$= (1 \times 2)$$

v.
$$E = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$$

Order of matrix = Number of rows x Number of columns

$$= (3 \times 1)$$

$$vi, F = [6]$$

Order of matrix = Number of rows x Number of columns

$$= (1 \times 1)$$

Question 3.

If a matrix has 18 elements, what are the possible orders it can have?

Answer:

Number of entries = (Number of rows) x (Number of columns) = 18

If order is $(a \times b)$ then, Number of entries = $a \times b$

So now a x b = 18 (in this case)

Possible cases are (1 x 18), (2 x 9), (3 x 6), (6 x 3), (9 x 2), (18 x 1)

Conclusion: If a matrix has 18 elements, then possible orders are (1×18) , (2×9) , (3×6) , (6×3) , (9×2) , (18×1)

Question 4.

Find all possible orders of matrices having 7 elements.

Answer:

Number of entries = (Number of rows) x (Number of columns) = 7

If order is $(a \times b)$ then, Number of entries = $a \times b$

So now a x b = 7 (in this case)

Possible cases are (1 x 7), (7 x 1)

Conclusion: If a matrix has 18 elements, then possible orders are (1×7) , (7×1)

Question 5.

Construct a 3×2 matrix whose elements are given by $a_{ij} = (2i - j)$.

Answer:

Given: $a_{ii} = (2i - j)$

Now,
$$a_{11} = (2 \times 1 - 1) = 2 - 1 = 1$$

$$a_{12} = 2 \times 1 - 2 = 2 - 2 = 0$$

$$a_{21} = 2 \times 2 - 1 = 4 - 1 = 3$$

$$a_{22} = 2 \times 2 - 2 = 4 - 2 = 2$$

$$a_{31} = 2 \times 3 - 1 = 6 - 1 = 5$$

$$a_{32} = 2 \times 3 - 2 = 6 - 2 = 4$$

Therefore,

$$A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$$

Question 6.

Construct a 4 × 3 matrix whose elements are given by $a_{ij} = \frac{i}{i}$.

Answer:

It is (4 x 3) matrix. So it has 4 rows and 3 columns

Given
$$a_{ij} = \frac{i}{j}$$
.

So,
$$a_{11} = 1$$
, $a_{12} = \frac{1}{2}$, $a_{13} = \frac{1}{3}$

$$a_{21} = 2 \cdot a_{22} = 1 \cdot a_{23} = \frac{2}{3}$$

$$a_{31} = 3, a_{32} = \frac{3}{2}, a_{33} = 1$$

$$a_{41} = 4$$
, $a_{42} = 2$, $a_{43} = \frac{4}{3}$

So, the matrix =
$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \\ 4 & 2 & \frac{4}{3} \end{bmatrix}$$

Conclusion: Therefore, Matrix is
$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \\ 4 & 2 & \frac{4}{3} \end{bmatrix}$$

Question 7.

Construct a 2 × 2 matrix whose elements are $a_{ij} = \frac{\left(i+2j\right)^2}{2}$.

Answer:

It is a (2 x 2) matrix. So, it has 2 rows and 2 columns.

Given
$$a_{ij} = \frac{(i+2j)^2}{2}$$

So,
$$a_{11} = \frac{9}{2}$$
, $a_{12} = \frac{25}{2}$,

$$a_{21} = 8, a_{22} = 18$$

So, the matrix =
$$\begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$$

Conclusion: Therefore, Matrix is =
$$\begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$$

Question 8.

Construct a 2 × 3 matrix whose elements are $a_{ij} = \frac{\left(i-2j\right)^2}{2}$.

Answer:

It is a (2 x 3) matrix. So, it has 2 rows and 3 columns.

Given
$$a_{ij} = \frac{(i-2j)^2}{2}$$

So,
$$a_{11} = \frac{1}{2}$$
, $a_{12} = \frac{9}{2}$, $a_{13} = \frac{25}{3}$

$$a_{21} = 0, a_{22} = 2, a_{23} = 8$$

So, the matrix =
$$\begin{bmatrix} \frac{1}{2} & \frac{9}{2} & \frac{25}{2} \\ 0 & 2 & 8 \end{bmatrix}$$

Conclusion: Therefore, Matrix is
$$\begin{bmatrix} \frac{1}{2} & \frac{9}{2} & \frac{25}{2} \\ 0 & 2 & 8 \end{bmatrix}$$

Question 9.

Construct a 3 × 4 matrix whose elements are given by $a_{ij} = \frac{1}{2} \left| -3i + j \right|$.

Answer:

It is a (3 x 4) matrix. So, it has 3 rows and 4 columns.

Given
$$a_{ij} = \frac{|-3i+j|}{2}$$

So,
$$a_{11} = 1$$
, $a_{12} = \frac{1}{2}$, $a_{13} = 0$, $a_{13} = \frac{1}{2}$,

$$a_{21} = \frac{5}{2}$$
, $a_{22} = 2$, $a_{23} = \frac{3}{2}$, $a_{13} = 1$,

$$a_{31} = 4$$
, $a_{32} = \frac{7}{2}$, $a_{33} = 3$, $a_{13} = \frac{5}{2}$

So, the matrix =
$$\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

Conclusion: Therefore, Matrix is
$$\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$