

## Objective Questions

### Question 1.

Mark against the correct answer in each of the following:

The direction cosines of the perpendicular from the origin to the plane

$$\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) + 1 = 0 \text{ are}$$

A.  $\frac{6}{7}, \frac{3}{7}, \frac{-2}{7}$

B.  $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$

C.  $\frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$

D. None of these

### Answer:

Given: Equation of plane is  $\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) + 1 = 0$

Formula Used: Equation of a plane is  $\hat{u} \cdot \vec{r} = p$  where  $\hat{u}$  is the unit vector normal to the plane,  $\vec{r}$  represents a point on the plane and  $p$  is the distance of the plane from the origin.

Explanation:

The equation of the given plane is  $\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = -1 \dots (1)$

$$\text{Now, } |6\hat{i} - 3\hat{j} + 2\hat{k}| = \sqrt{36 + 9 + 4}$$

$$= 7$$

$$\therefore \frac{6}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k} \text{ is a unit vector.}$$

(1) can be rewritten as

$$\vec{r} \cdot \left( \frac{6}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k} \right) = -\frac{1}{7}$$

$$\Rightarrow \vec{r} \cdot \left( -\frac{6}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{2}{7}\hat{k} \right) = \frac{1}{7}$$

which is of the form  $\hat{u} \cdot \vec{r} = p$

Perpendicular vector from the origin to the plane is

$$\hat{u} = -\frac{6}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{2}{7}\hat{k}$$

So, direction cosines of the vector perpendicular from the origin to the plane is  $\left( \frac{-6}{7}, \frac{3}{7}, \frac{-2}{7} \right)$

### Question 2.

Mark against the correct answer in each of the following:

The direction cosines of the normal to the plane  $5y + 4 = 0$  are

A.  $0, \frac{-4}{5}, 0$

B.  $0, 1, 0$

C.  $0, -1, 0$

D. None of these

### Answer:

Given: Equation of plane is  $5y + 4 = 0$

Formula Used: Equation of a plane is  $lx + my + nz = p$  where  $(l, m, n)$  are the direction cosines of the normal to the plane and  $(x, y, z)$  is a point on the plane and  $p$  is the distance of plane from origin.

Explanation:

Given equation is  $5y = -4$

Dividing by  $-5$ ,

$$-y = \frac{4}{5}$$

which is of the form  $lx + my + nz = p$  where  $l = 0$ ,  $m = -1$ ,  $n = 0$

Therefore, direction cosines of the normal to the plane is  $(0, -1, 0)$

### Question 3.

Mark against the correct answer in each of the following:

The length of perpendicular from the origin to the plane  $\vec{r} \cdot (3\hat{i} - 4\hat{j} - 12\hat{k}) + 39 = 0$  is

A. 3 units

B.  $\frac{13}{5}$  units

C.  $\frac{5}{3}$  units

D. None of these

### Answer:

Given: Equation of plane is  $\vec{r} \cdot (3\hat{i} - 4\hat{j} - 12\hat{k}) + 39 = 0$

Formula Used: Equation of a plane is  $\hat{u} \cdot \vec{r} = p$  where  $\hat{u}$  is the unit vector normal to the plane,  $\vec{r}$  represents a point on the plane and  $p$  is the distance of the plane from the origin.

Explanation:

Given equation is  $\vec{r} \cdot (3\hat{i} - 4\hat{j} - 12\hat{k}) = -39 \dots (1)$

Now,  $|3\hat{i} - 4\hat{j} - 12\hat{k}| = \sqrt{9 + 16 + 144} = \sqrt{169}$

$= 13$

Dividing (1) by 13 and multiplying by -1,

$$\vec{r} \cdot \left( \frac{-3}{13}\hat{i} + \frac{4}{13}\hat{j} + \frac{12}{13}\hat{k} \right) = 3$$

which is of the form  $\hat{u} \cdot \vec{r} = p$

Therefore, length of perpendicular from origin to plane is 3 units.

**Question 4.**

Mark against the correct answer in each of the following:

The equation of a plane passing through the point A(2, -3, 7) and making equal intercepts on the axes, is

A.  $x + y + z = 3$

B.  $x + y + z = 6$

C.  $x + y + z = 9$

D.  $x + y + z = 4$

**Answer:**

Given: A(2, -3, 7) is a point on the plane making equal intercepts on the axes.

Formula Used: Equation of plane is  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$  where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Explanation:

Let the equation of the plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \dots (1)$$

Here  $a = b = c = p$  (let's say)

Since (2, -3, 7) is a point on the plane,

(1) becomes

$$\frac{2 - 3 + 7}{p} = 1$$

$$p = 6$$

Therefore equation of the plane is

$$x + y + z = 6$$

**Question 5.**

Mark against the correct answer in each of the following:

A plane cuts off intercepts 3, -4, 6 on the coordinate axes. The length of perpendicular from the origin to this plane is

A.  $\frac{5}{\sqrt{29}}$  units

B.  $\frac{8}{\sqrt{29}}$  units

C.  $\frac{6}{\sqrt{29}}$  units

D.  $\frac{12}{\sqrt{29}}$  units

**Answer:**

Given: Plane makes intercepts 3, -4 and 6 with the coordinate axes.

Formula Used: Equation of plane is  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$  where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Normal Form of a plane  $\Rightarrow lx + my + nz = p$  where (l, m, n) is the direction cosines and p is the distance of perpendicular to the plane from the origin.

Explanation:

Equation of the given plane is

$$\frac{x}{3} + \frac{y}{-4} + \frac{z}{6} = 1$$

i.e.,  $4x - 3y + 2z = 12 \dots (1)$

which is of the form  $ax + by + cz = d$

Direction ratios are (4, -3, 12)

$$\text{So, } \sqrt{4^2 + (-3)^2 + 2^2} = \sqrt{16 + 9 + 4}$$

$$= \sqrt{29}$$

Dividing (1) by 13,

$$\frac{4}{\sqrt{29}}x - \frac{3}{\sqrt{29}} + \frac{2}{\sqrt{29}} = \frac{12}{\sqrt{29}}$$

which is in the normal form

Therefore length of perpendicular from the origin is  $\frac{12}{\sqrt{29}}$  units

### Question 6.

Mark against the correct answer in each of the following:

If the line  $\frac{x+1}{3} = \frac{y-2}{4} = \frac{z+6}{5}$  is parallel to the plane  $2x - 3y + kz = 0$ , then the value of k is

A.  $\frac{5}{6}$

B.  $\frac{6}{5}$

C.  $\frac{3}{4}$

D.  $\frac{4}{5}$

**Answer:**

Given:

1. Equation of line is  $\frac{x+1}{3} = \frac{y-2}{4} = \frac{z+6}{5}$

2. Equation of plane is  $2x - 3y + kz = 0$

Formula Used: If two direction ratios are perpendicular, then

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

Explanation:

Direction ratios of given line is (3, 4, 5)

Direction ratios of given plane is (2, -3, k)

Since the given line is parallel to the plane, the normal to the plane is perpendicular to the line.

So direction ratio of line is perpendicular to direction ratios of plane.

$$\Rightarrow 3 \times 2 + 4 \times -3 + 5 \times k = 0$$

$$\Rightarrow 6 - 12 + 5k = 0$$

$$\Rightarrow k = \frac{6}{5}$$

Therefore,  $k = \frac{6}{5}$

### Question 7.

Mark against the correct answer in each of the following:

If O is the origin and P(1, 2, -3) is a given point, then the equation of the plane through P and perpendicular to OP is

A.  $x + 2y - 3z = 14$

B.  $x - 2y + 3z = 12$

C.  $x - 2y - 3z = 14$

D. None of these

### Answer:

Given: P(1, 2, -3) is a point on the plane. OP is perpendicular to the plane.

Explanation:

Let equation of plane be  $ax + by + cz = d \dots (1)$

Substituting point P,

$$\Rightarrow a + 2b - 3c = d \dots (2)$$

$$\overrightarrow{OP} = \hat{i} + 2\hat{j} - 3\hat{k}$$

Since OP is perpendicular to the plane, direction ratio of the normal is (1, 2, -3)

Substituting in (2)

$$1 + 4 + 9 = d$$

$$d = 14$$

Substituting the direction ratios and value of 'd' in (1), we get

$$x + 2y - 3z = 14$$

Therefore equation of plane is  $x + 2y - 3z = 14$

### Question 8.

Mark against the correct answer in each of the following:

If the line  $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$  lies in the plane  $2x - 4y + z = 7$ , then the value of k is

A. -7

B. 7

C. 4

D. -4

### Answer:

Given: Equation of plane is  $2x - 4y + z = 7$

Line  $\frac{(x-4)}{1} = \frac{(y-2)}{1} = \frac{(z-k)}{2}$  lies on the given plane.



Formula Used: Equation of a line is

$$\frac{(x - x_1)}{b_1} = \frac{(y - y_1)}{b_2} = \frac{(z - z_1)}{b_3} = \lambda$$

Where  $(x_1, y_1, z_1)$  is a point on the line and  $b_1, b_2, b_3$  : direction ratios of line.

Explanation:

$$\text{Let } \frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2} = \lambda$$

So the given line passes through the point  $(4, 2, k)$

Since the line lies on the given plane,  $(4, 2, k)$  is a point on the plane.

Therefore, substituting the point on the equation for the plane,

$$\Rightarrow 8 - 8 + k = 7$$

$$\Rightarrow k = 7$$

### Question 9.

Mark against the correct answer in each of the following:

The plane  $2x + 3y + 4z = 12$  meets the coordinate axes in A, B and C. The centroid of  $\Delta ABC$  is

A.  $(2, 3, 4)$

B.  $(6, 4, 3)$

C.  $\left(2, \frac{4}{3}, 1\right)$

D. None of these

### Answer:

Given: The plane  $2x + 3y + 4z = 12$  meets coordinate axes at A, B and C.

To find: Centroid of  $\Delta ABC$

Formula Used: Equation of plane is  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$  where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

$$\text{Centroid of a triangle} = \left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$$

Explanation:

Equation of given plane is  $2x + 3y + 4z = 12$

Dividing by 12,

$$\frac{x}{6} + \frac{y}{4} + \frac{z}{3} = 1$$

Therefore the intercepts on x, y and z-axis are 6, 6 and 3 respectively.

So, the vertices of  $\Delta ABC$  are (6, 0, 0), (0, 4, 0) and (0, 0, 3)

$$\text{Centroid} = \left( \frac{6+0+0}{3}, \frac{0+4+0}{3}, \frac{0+0+3}{3} \right)$$

$$= (2, 4/3, 1)$$

Therefore, the centroid of  $\Delta ABC$  is (2, 4/3, 1)

### Question 10.

Mark against the correct answer in each of the following:

If a plane meets the coordinate axes in A, B and C such that the centroid of  $\Delta ABC$  is (1, 2, 4), then the equation of the plane is

- A.  $x + 2y + 4z = 6$
- B.  $4x + 2y + z = 12$
- C.  $x + 2y + 4z = 7$
- D.  $4x + 2y + z = 7$

### Answer:

Given: Centroid of  $\Delta ABC$  is (1, 2, 4)

To find: Equation of plane.

Formula Used: Equation of plane is  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$  where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

$$\text{Centroid of a triangle} = \left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$$

Explanation:

Let the equation of plane be

$$\frac{x}{A} + \frac{y}{B} + \frac{z}{C} = 1 \dots (1)$$

Therefore, A = 3a, B = 3b, C = 3c where (a, b, c) is the centroid of the triangle with vertices (A, 0, 0), (0, B, 0) and (0, 0, C)

Substituting in (1),

$$\Rightarrow \frac{x}{3a} + \frac{y}{3b} + \frac{z}{3c} = 1$$

Here a = 1, b = 2 and c = 4

$$\Rightarrow \frac{x}{3} + \frac{y}{6} + \frac{z}{12} = 1$$

Multiplying by 12,

$$4x + 2y + z = 12$$

Therefore equation of required plane is  $4x + 2y + z = 12$

### Question 11.

Mark against the correct answer in each of the following:

The equation of a plane through the point A(1, 0, -1) and perpendicular to the line

$$\frac{x+1}{2} = \frac{y+3}{4} = \frac{z+7}{-3} \text{ is}$$

A.  $2x + 4y - 3z = 3$

B.  $2x - 4y + 3z = 5$

C.  $2x + 4y - 3z = 5$

D.  $x + 3y + 7z = -6$

**Answer:**

Given: Plane passes through the point A(1, 0, -1).

Plane is perpendicular to the line

$$\frac{x+1}{2} = \frac{y+3}{4} = \frac{z+7}{-3}$$

To find: Equation of the plane.

Formula Used: Equation of a plane is  $ax + by + cz = d$  where (a, b, c) are the direction ratios of the normal to the plane.

Explanation:

Let the equation of the plane be

$$ax + by + cz = d \dots (1)$$

Substituting point A,

$$a - z = d$$

Since the given line is perpendicular to the plane, it is the normal.

Direction ratios of line is 2, 4, -3

$$\text{Therefore, } 2 + 3 = d$$

$$d = 5$$

So the direction ratios of perpendicular to plane is 2, 4, -3 and  $d = 5$

Substituting in (1),

$$2x + 4y - 3z = 5$$

Therefore, equation of plane is  $2x + 4y - 3z = 5$

**Question 12.**

Mark against the correct answer in each of the following:

The line  $\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{-3}$  meets the plane  $2x + 3y - z = 14$  in the point

A. (2, 5, 7)

B. (3, 5, 7)

C. (5, 7, 3)

D. (6, 5, 3)

**Answer:**

Given: Line  $\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{-3}$  meets plane  $2x + 3y - z = 14$

To find: Point of intersection of line and plane.

Explanation:

Let the equation of the line be

$$\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{-3} = \lambda$$

Therefore, any point on the line is  $(2\lambda + 1, 4\lambda + 2, -3\lambda + 3)$

Since this point also lies on the plane,

$$2(2\lambda + 1) + 3(4\lambda + 2) - (-3\lambda + 3) = 14$$

$$4\lambda + 2 + 12\lambda + 6 + 3\lambda - 3 = 14$$

$$19\lambda + 5 = 14$$

$$\lambda = \frac{19}{19} = 1$$

Therefore the required point is (3, 5, 7).

**Question 13.**

Mark against the correct answer in each of the following:

The equation of the plane passing through the points A(2, 2, 1) and B(9, 3, 6) and perpendicular to the plane  $2x + 6y + 6z = 1$ , is

A.  $x + 2y - 3z + 5 = 0$

B.  $2x - 3y + 4z - 6 = 0$

C.  $4x + 5y - 6z + 3 = 0$

D.  $3x + 4y - 5z - 9 = 0$

**Answer:**

Given: Plane passes through A(2, 2, 1) and B(9, 3, 6). Plane is perpendicular to  $2x + 6y + 6z = 1$

To find: Equation of the plane

Formula Used: Equation of a plane is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

where  $a:b:c$  is the direction ratios of the normal to the plane.

$(x_1, y_1, z_1)$  is a point on the plane.

Explanation:

Let the equation of plane be  $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Since (2, 2, 1) is a point in the plane,

$$a(x - 2) + b(y - 2) + c(z - 1) = 0 \dots (1)$$

Since B(9, 3, 6) is another point on the plane,

$$a(9 - 2) + b(3 - 2) + c(6 - 1) = 0$$

$$7a + b + 5c = 0 \dots (1)$$

Since this plane is perpendicular to the plane  $2x + 6y + 6z = 1$ , the direction ratios of the normal to the plane will also be perpendicular.

$$\text{So, } 2a + 6b + 6c = 0 \Rightarrow a + 3b + 3c = 0 \dots (2)$$

Solving (1) and (2),

$$\frac{a}{\begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 7 & 5 \\ 1 & 3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 7 & 1 \\ 1 & 3 \end{vmatrix}}$$

$$\frac{a}{-12} = \frac{b}{-16} = \frac{c}{20}$$

$$\frac{a}{3} = \frac{b}{4} = \frac{c}{-5}$$

$$a : b : c = 3 : 4 : -5$$

Substituting in (1),

$$3x - 6 + 4y - 8 - 5z + 5 = 0$$

$$3x + 4y - 5z - 9 = 0$$

Therefore the equation of the plane is  $3x + 4y - 5z - 9 = 0$

#### Question 14.

Mark against the correct answer in each of the following:

The equation of the plane passing through the intersection of the planes  $3x - y + 2z - 4 = 0$  and  $x + y + z - 2 = 0$  and passing through the point  $A(2, 2, 1)$  is given by

A.  $7x + 5y - 4z - 8 = 0$

B.  $7x - 5y + 4z - 8 = 0$

C.  $5x - 7y + 4z - 8 = 0$

D.  $5x + 7y - 4z + 8 = 0$

#### Answer:

Given: Plane passes through the intersection of planes  $3x - y + 2z - 4 = 0$  and  $x + y + z - 2 = 0$ .  
Point  $A(2, 2, 1)$  lies on the plane.

To find: Equation of the plane.

Formula Used: Equation of plane passing through the intersection of 2 planes  $P_1$  and  $P_2$  is given by  $P_1 + \lambda P_2 = 0$

Explanation:

Equation of plane is

$$3x - y + 2z - 4 + \lambda (x + y + z - 2) = 0 \dots (1)$$

Since A(2, 2, 1) lies on the plane,

$$6 - 2 + 2 - 4 + \lambda (2 + 2 + 1 - 2) = 0$$

$$2 + 3\lambda = 0$$

$$\lambda = \frac{-2}{3}$$

Substituting in (1) and multiplying by 3,

$$9x - 3y + 6z - 12 - 2(x + y + z - 2) = 0$$

$$9x - 3y + 6z - 12 - 2x - 2y - 2z + 4 = 0$$

$$7x - 5y + 4z - 8 = 0$$

Therefore the equation of the plane is  $7x - 5y + 4z - 8 = 0$

### Question 15.

Mark against the correct answer in each of the following:

The equation of the plane passing through the points A(0, -1, 0), B(2, 1, -1) and C(1, 1, 1) is given by

A.  $4x + 3y - 2z - 3 = 0$

B.  $4x - 3y + 2z + 3 = 0$

C.  $4x - 3y + 2z - 3 = 0$

D. None of these

**Answer:**



Given: Plane passes through A(0, -1, 0), B(2, 1, -1) and C(1, 1, 1)

To find: Equation of the plane

Formula Used: Equation of a plane is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

where a:b:c is the direction ratios of the normal to the plane.

( $x_1, y_1, z_1$ ) is a point on the plane.

Explanation:

Let the equation of plane be  $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Substituting point A,

$$ax + b(y + 1) + cz = 0 \dots (1)$$

Substituting points B and C,

$$2a + 2b - c = 0 \text{ and } a + 2b + c = 0$$

Solving,

$$\frac{a}{\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix}}$$

$$\frac{a}{4} = \frac{b}{-3} = \frac{c}{2}$$

Therefore,  $a : b : c = 4 : -3 : 2$

Substituting in (1),

$$4x - 3(y + 1) + 2z = 0$$

$$4x - 3y + 2z - 3 = 0$$

Therefore equation of plane is  $4x - 3y + 2z - 3 = 0$

**Question 16.**

Mark against the correct answer in each of the following:

If the plane  $2x - y + z = 0$  is parallel to the line  $\frac{2x-1}{2} = \frac{2-y}{2} = \frac{z+1}{a}$ , then the value of a is

A. -4

B. -2

C. 4

D. 2

**Answer:**

Given: Plane  $2x - y + z = 0$  is parallel to the line

$$\frac{2x-1}{2} = \frac{2-y}{2} = \frac{z+1}{a},$$

To find: value of a

Formula Used: If two lines with direction ratios  $a_1:a_2:a_3$  and  $b_1:b_2:b_3$  are perpendicular, then

$$a_1b_1 + a_2b_2 + a_3b_3 = 0$$

Explanation:

Since the plane is parallel to the line, the normal to the plane will be perpendicular to the line.

Equation of the line can be rewritten as

$$\frac{x - \frac{1}{2}}{1} = \frac{y - 2}{-2} = \frac{z - (-1)}{a}$$

Direction ratio of the normal to the plane is  $2 : -1 : 1$

Direction ratio of line is  $1 : -2 : a$

Therefore,

$$2 + 2 + a = 0$$

$$a = -4$$

Therefore,  $a = -4$

**Question 17.**

Mark against the correct answer in each of the following:

The angle between the line  $\frac{x+1}{1} = \frac{y}{2} = \frac{z-1}{1}$  and a normal to the plane  $x - y + z = 0$  is

A.  $0^\circ$

B.  $30^\circ$

C.  $45^\circ$

D.  $90^\circ$

**Answer:**

Given: Equation of line is  $\frac{x+1}{1} = \frac{y}{2} = \frac{z-1}{1}$

Equation of plane is  $x - y + z = 0$

To find: Angle between a line and the normal to a plane.

Formula Used: If  $\theta$  is the angle between two lines with direction ratios  $b_1:b_2:b_3$  and  $c_1:c_2:c_3$ , then

$$\cos \theta = \frac{b_1 c_1 + b_2 c_2 + b_3 c_3}{\sqrt{b_1^2 + b_2^2 + b_3^2} \times \sqrt{c_1^2 + c_2^2 + c_3^2}}$$

Explanation:

Direction ratios of given line is  $1 : 2 : 1$

Direction ratios of the normal to the plane is  $1 : -1 : 1$

Therefore,

$$\cos \theta = \frac{1 - 2 + 1}{\sqrt{1 + 4 + 1} \times \sqrt{1 + 1 + 1}}$$

$$\cos \theta = 0$$

$$\theta = 90^\circ$$

Therefore angle between them is  $90^\circ$

### Question 18.

Mark against the correct answer in each of the following:

The point of intersection of the line  $\frac{x-1}{3} = \frac{y+2}{4} = \frac{z-3}{-2}$  and the plane  $2x - y + 3z - 1 = 0$ , is

- A. (-10, 10, 3)
- B. (10, 10, -3)
- C. (10, -10, 3)
- D. (10, -10, -3)

### Answer:

Given: Line  $\frac{x-1}{3} = \frac{y+2}{4} = \frac{z-3}{-2}$  meets plane  $2x - y + 3z - 1 = 0$

To find: Point of intersection of line and plane.

Explanation:

Let the equation of the line be

$$\frac{x-1}{3} = \frac{y+2}{4} = \frac{z-3}{-2} = \lambda$$

Therefore, any point on the line is  $(3\lambda + 1, 4\lambda - 2, -2\lambda + 3)$

Since this point also lies on the plane,

$$2(3\lambda + 1) - (4\lambda - 2) + 3(-2\lambda + 3) = 1$$

$$6\lambda + 2 - 4\lambda + 2 - 6\lambda + 9 = 1$$

$$-4\lambda = -12$$

$$\lambda = 3$$

Therefore required point is (10, 10, -3)

**Question 19.**

Mark against the correct answer in each of the following:

The equation of a plane passing through the points A(a, 0, 0), B(0, b, 0) and C(0, 0, c) is given by

A.  $ax + by + cz = 0$

B.  $ax + by + cz = 1$

C.  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$

D.  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

**Answer:**

Given: Plane passes through the points A(a, 0, 0), B(0, b, 0) and C(0, 0, c)

To find: Equation of plane.

Explanation:

The given points lie on the co-ordinate axes.

Therefore, the plane makes intercepts of a, b and c on the x, y and z-axis respectively.

Equation of the plane is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

**Question 20.**

Mark against the correct answer in each of the following:

If  $\theta$  is the angle between the planes  $2x - y + 2z = 3$  and  $6x - 2y + 3z = 5$ , then  $\cos \theta = ?$

A.  $\frac{11}{20}$

B.  $\frac{12}{23}$

C.  $\frac{17}{25}$

D.  $\frac{20}{21}$

**Answer:**

Given: Equation of two planes are  $2x - y + 2z = 3$  and  $6x - 2y + 3z = 5$

To find:  $\cos \theta$  where  $\theta$ : angle between the planes

Formula Used: Angle between two planes  $a_1x + b_1y + c_1z = 0$  and  $a_2x + b_2y + c_2z = 0$  is

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \times \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

where  $\theta$  : angle between the planes,

Explanation:

Here  $a_1 = 2$ ,  $b_1 = -1$ ,  $c_1 = 2$

$a_2 = 6$ ,  $b_2 = -2$ ,  $c_2 = 3$

$$\Rightarrow \cos \theta = \frac{12 + 2 + 6}{\sqrt{4 + 1 + 4} \times \sqrt{36 + 4 + 9}}$$

$$\Rightarrow \cos \theta = \frac{20}{3 \times 7}$$

$$\Rightarrow \cos \theta = \frac{20}{21}$$

Therefore,  $\cos \theta = \frac{20}{21}$

**Question 21.**

Mark against the correct answer in each of the following:

The angle between the planes  $2x - y + z = 6$  and  $x + y + 2z = 7$ , is

A.  $\frac{\pi}{6}$

B.  $\frac{\pi}{4}$

C.  $\frac{\pi}{3}$

D.  $\frac{\pi}{2}$

**Answer:**

Given: Equation of two planes are  $2x - y + z = 6$  and  $x + y + 2z = 7$

To find: Angle between the two planes

Formula Used: Angle between two planes  $a_1x + b_1y + c_1z = 0$  and  $a_2x + b_2y + c_2z = 0$  is

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \times \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

where  $\theta$  : angle between the planes,

Explanation:

Here  $a_1 = 2$ ,  $b_1 = -1$ ,  $c_1 = 1$

$a_2 = 1$ ,  $b_2 = 1$ ,  $c_2 = 2$

$$\Rightarrow \cos \theta = \frac{2 - 1 + 2}{\sqrt{4 + 1 + 1} \times \sqrt{1 + 1 + 4}}$$

$$\Rightarrow \cos \theta = \frac{3}{\sqrt{6} \times \sqrt{6}}$$

$$\Rightarrow \cos \theta = \frac{3}{6}$$

$$\Rightarrow \cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

Therefore angle between the planes is  $\frac{\pi}{3}$

### Question 22.

Mark against the correct answer in each of the following:

The angle between the planes  $\vec{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 4$  and  $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 3$ , is

A.  $\cos^{-1}\left(\frac{16}{21}\right)$

B.  $\cos^{-1}\left(\frac{4}{21}\right)$

C.  $\cos^{-1}\left(\frac{3}{4}\right)$

D.  $\cos^{-1}\left(\frac{1}{4}\right)$

### Answer:

Given: Equation of two planes are  $\vec{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 4$  and  $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 3$

To find: Angle between the two planes

Formula Used: Angle between two planes  $a_1x + b_1y + c_1z = 0$  and  $a_2x + b_2y + c_2z = 0$  is



$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \times \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

where  $\theta$  : angle between the planes,

Explanation:

Since  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ , the given equation of planes can rewritten as:

$$3x - 6y + 2z = 4 \text{ and } 2x - y + 2z = 3$$

$$\text{Here } a_1 = 3, b_1 = -6, c_1 = 2$$

$$a_2 = 2, b_2 = -1, c_2 = 2$$

$$\Rightarrow \cos \theta = \frac{6 + 6 + 4}{\sqrt{9 + 36 + 4} \times \sqrt{4 + 1 + 4}}$$

$$\Rightarrow \cos \theta = \frac{16}{7 \times 3}$$

$$\Rightarrow \cos \theta = \frac{16}{21}$$

$$\Rightarrow \theta = \cos^{-1} \frac{16}{21}$$

Therefore angle between the planes is  $\cos^{-1} \frac{16}{21}$

### Question 23.

Mark against the correct answer in each of the following:

The equation of the plane through the points A(2, 3, 1) and B(4, -5, 3), parallel to the x-axis, is

A.  $x + y - 3z = 2$

B.  $y + 4z = 7$

C.  $y + 3z = 6$

D.  $x + 5y - 3z = 4$

**Answer:**

Given: Plane passes through the points A(2, 3, 1) and B(4, -5, 3) and is parallel to x-axis

To find: Equation of plane

Formula Used: Equation of a plane parallel to x-axis is

$$b(y - y_1) + c(z - z_1) = 0$$

Explanation:

Let the equation of the plane be

$$b(y - y_1) + c(z - z_1) = 0$$

Since A(2, 3, 1) lies on the plane,

$$b(y - 3) + c(z - 1) = 0 \dots (1)$$

Since B(4, -5, 3) lies on the plane,

$$b(-5 - 3) + c(3 - 1) = 0$$

$$-8b + 2c = 0 \text{ or } -4b + c = 0$$

$$b : c = 1 : 4$$

Substituting in (1),

$$y - 3 + 4z - 4 = 0$$

$$y + 4z = 7$$

The equation of the plane is  $y + 4z = 7$

**Question 24.**

Mark against the correct answer in each of the following:

A variable plane moves so that the sum of the reciprocals of its intercepts on the coordinate axes is  $(1/2)$ . Then, the plane passes through the point

A. (0, 0, 0)

B. (1, 1, 1)

C.  $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

D. (2, 2, 2)

**Answer:**

Given: Variable plane moves so that the sum of the reciprocals of its intercepts on the coordinate axes is  $(1/2)$

Formula Used: Equation of a plane is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Explanation:

Let the intercepts made by the plane on the co-ordinate axes be a, b and c.

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2}$$

Let the equation of the plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

On solving for each of the given options,

$$(0, 0, 0) \Rightarrow LHS \neq RHS$$

$$(1, 1, 1) \Rightarrow LHS \neq RHS$$

$$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \Rightarrow LHS = \frac{1}{2a} + \frac{1}{2b} + \frac{1}{2c} = \frac{1}{2} \times \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = \frac{1}{4} \neq RHS$$

$$(2, 2, 2) \Rightarrow LHS = \frac{2}{a} + \frac{2}{b} + \frac{2}{c} = 2 \times \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = 1 = RHS$$

Therefore, plane passes through the point (2, 2, 2)

**Question 25.**

Mark against the correct answer in each of the following:

The equation of a plane which is perpendicular to  $(2\hat{i} - 3\hat{j} + \hat{k})$  and at a distance of 5 units from the origin is

A.  $2x - 3y + z = 5$

B.  $2x - 3y + z = 5\sqrt{14}$

C.  $\frac{x}{2} - \frac{y}{3} + \frac{z}{1} = 5$

D.  $\frac{x}{2} - \frac{y}{3} + \frac{z}{1} = \frac{5}{\sqrt{14}}$

**Answer:**

Given: Plane is perpendicular to  $(2\hat{i} - 3\hat{j} + \hat{k})$  and is at a distance of 5 units from origin.

To find: Equation of plane

Formula Used: Equation of a plane is  $lx + my + nz = p$  where  $p$  is the distance from the origin and  $l$ ,  $m$  and  $n$  are the direction cosines of the normal to the plane

Explanation:

Direction ratio of normal to the plane is 2:-3:1

$$|2\hat{i} - 3\hat{j} + \hat{k}| = \sqrt{4 + 9 + 1} = \sqrt{14}$$

Therefore, direction cosines of the normal to the plane is

$$l = \frac{2}{\sqrt{14}}, m = \frac{-3}{\sqrt{14}}, n = \frac{1}{\sqrt{14}}$$

Since the equation of a plane is  $lx + my + nz = p$  where  $p$  is the distance from the origin,

$$2x - 3y + z = 5\sqrt{14}$$

Therefore, equation of the plane is  $2x - 3y + z = 5\sqrt{14}$

**Question 26.**

Mark against the correct answer in each of the following:

The equation of the plane passing through the point A(2, 3,4) and parallel to the plane  $5x - 6y + 7z = 3$ , is

A.  $5x - 6y + 7z = 20$

B.  $7x - 6y + 5z = 72$

C.  $20x - 18y + 14z = 11$

D.  $10x - 18y + 28z = 13$

**Answer:**

Given: Point A(2, 3, 4) lies on a plane which is parallel to  $5x - 6y + 7z = 3$

To find: Equation of the plane

Formula Used: Equation of a plane is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

where a:b:c is the direction ratios of the normal to the plane

$(x_1, y_1, z_1)$  is a point on the plane.

Explanation:

Since the plane (say  $P_1$ ) is parallel to the plane  $5x - 6y + 7z = 3$  (say  $P_2$ ), the direction ratios of the normal to  $P_1$  is same as the direction ratios of the normal to  $P_2$ .

i.e., direction ratios of  $P_1$  is 5 : -6 : 7

Let the equation of the required plane be

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

Here  $a = 5$ ,  $b = -6$  and  $c = 7$

Since (2, 3, 4) lies on the plane,

$$5(x - 2) - 6(y - 3) + 7(z - 4) = 0$$

$$5x - 6y + 7z - 10 + 18 - 28 = 0$$

$$5x - 6y + 7z = 20$$

The equation of the plane is  $5x - 6y + 7z = 20$

**Question 27.**

Mark against the correct answer in each of the following:

The foot of the perpendicular from the point A(7, 14, 5) to the plane  $2x + 4y - z = 2$  is

A. (3, 1, 8)

B. (1, 2, 8)

C. (3, -3, 5)

D. (5, -3, -4)

**Answer:**

Given: Perpendicular dropped from A(7, 14, 5) on to the plane  $2x + 4y - z = 2$

To find: co-ordinates of the foot of perpendicular

Formula Used: Equation of a line is

$$\frac{x - x_1}{b_1} = \frac{y - y_2}{b_2} = \frac{z - z_3}{b_3} = \lambda$$

Where  $b_1:b_2:b_3$  is the direction ratio and  $(x_1, x_2, x_3)$  is a point on the line.

Explanation:

Let the foot of the perpendicular be (a, b, c)

Since this point lies on the plane,

$$2a + 4b - c = 2 \dots (1)$$

Direction ratio of the normal to the plane is 2 : 4 : -1

Direction ratio perpendicular = direction ratio of normal to the plane

So, equation of the perpendicular is

$$\frac{x - x_1}{2} = \frac{y - y_1}{4} = \frac{z - z_1}{-1} = \lambda$$

Since (a, b, c) is a point on the perpendicular,

$$\frac{x - a}{2} = \frac{y - b}{4} = \frac{z - c}{-1} = \lambda$$

(7, 14, 5) is a point on the perpendicular.

$$\frac{7 - a}{2} = \frac{14 - b}{4} = \frac{5 - c}{-1} = \lambda$$

So,  $a = 7 - 2\lambda$ ,  $b = 14 - 4\lambda$ ,  $c = 5 + \lambda$

Substituting in (1),

$$14 - 4\lambda + 56 - 16\lambda - 5 - \lambda = 2$$

$$21\lambda = 70 - 7 = 63$$

$$\lambda = 3$$

Therefore, foot of the perpendicular is (1, 2, 8)

### Question 28.

Mark against the correct answer in each of the following:

The equation of the plane which makes with the coordinate axes, a triangle with centroid ( $\alpha$ ,  $\beta$ ,  $\gamma$ ) is given by

A.  $\alpha x + \beta y + \gamma z = 1$

B.  $\alpha x + \beta y + \gamma z = 3$

C.  $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$

$$D. \frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$$

**Answer:**

Given: Centroid of triangle is  $(\alpha, \beta, \gamma)$

To find: Equation of plane.

Formula Used: Equation of plane is  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$  where  $(x, y, z)$  is a point on the plane and  $a, b, c$  are intercepts on x-axis, y-axis and z-axis respectively.

$$\text{Centroid of a triangle} = \left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$$

Explanation:

Let the equation of plane be

$$\frac{x}{A} + \frac{y}{B} + \frac{z}{C} = 1 \dots (1)$$

Therefore,  $A = 3\alpha$ ,  $B = 3\beta$ ,  $C = 3\gamma$  where  $(a, b, c)$  is the centroid of the triangle with vertices  $(A, 0, 0)$ ,  $(0, B, 0)$  and  $(0, 0, C)$

Substituting in (1),

$$\Rightarrow \frac{x}{3a} + \frac{y}{3b} + \frac{z}{3c} = 1$$

Here  $a = \alpha$ ,  $b = \beta$  and  $c = \gamma$

$$\Rightarrow \frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$$

Therefore equation of required plane is  $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$

**Question 29.**



Mark against the correct answer in each of the following:

The intercepts made by the plane  $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 12$  are

- A. 2, -3, 4
- B. 2, -3, -6
- C. 6, -4, 3
- D. -6, 4, 3

**Answer:**

Given: Equation of plane is  $\vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 12$

To find: Intercepts made by the plane.

Formula Used: Equation of plane is  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$  where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Explanation:

The equation of the plane can be written as

$$2x - 3y + 4z = 12$$

Dividing by 12,

$$\frac{x}{6} + \frac{y}{-4} + \frac{z}{3} = 1 \text{ which is of the form } \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Therefore the intercepts made by the plane are 6, -4, 3

**Question 30.**

Mark against the correct answer in each of the following:

The angle between the line  $\frac{x-2}{1} = \frac{y+3}{-2} = \frac{z+4}{-3}$  and the plane  $2x - 3y + z = 5$  is

A.  $\cos^{-1}\left(\frac{5}{14}\right)$

B.  $\sin^{-1}\left(\frac{5}{14}\right)$

C.  $\cos^{-1}\left(\frac{3}{7}\right)$

D.  $\sin^{-1}\left(\frac{3}{7}\right)$

**Answer:**

Given: Equation of line is  $\frac{x-2}{1} = \frac{y+3}{-2} = \frac{z+4}{-3}$

Equation of the plane is  $2x - 3y + z = 5$

To find: angle between line and plane

Formula Used: If  $\theta$  is the angle between a line with direction ratio  $b_1:b_2:b_3$  and a plane with direction ratio of normal  $n_1:n_2:n_3$ , then

$$\sin \theta = \frac{n_1 b_1 + n_2 b_2 + n_3 b_3}{\sqrt{n_1^2 + n_2^2 + n_3^2} \times \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

Explanation:

Here direction ratio of the line is  $1 : -2 : -3$

Direction ratio of normal to the plane is  $2 : -3 : 1$

Therefore,

$$\sin \theta = \frac{2 + 6 - 3}{\sqrt{1 + 4 + 9} \times \sqrt{4 + 9 + 1}}$$

$$\sin \theta = \frac{5}{\sqrt{14} \times \sqrt{14}}$$

$$\theta = \sin^{-1} \frac{5}{14}$$

Therefore, angle between the line and plane is  $\sin^{-1} \frac{5}{14}$

**Question 31.**

Mark against the correct answer in each of the following:

The angle between the line  $\vec{r} \cdot (\hat{i} + \hat{j} - 3\hat{k}) + \lambda(2\hat{i} + 2\hat{j} + \hat{k})$  and the plane  $\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 5$ , is

A.  $\cos^{-1} \left( \frac{8}{21} \right)$

B.  $\cos^{-1} \left( \frac{5}{21} \right)$

C.  $\sin^{-1} \left( \frac{5}{21} \right)$

D.  $\sin^{-1} \left( \frac{8}{21} \right)$

**Answer:**

Given: Equation of line is  $\vec{r} \cdot (\hat{i} + \hat{j} - 3\hat{k}) + \lambda(2\hat{i} + 2\hat{j} + \hat{k})$

Equation of plane is  $\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 5$

To find: angle between line and plane

Formula Used: If  $\theta$  is the angle between a line with direction ratio  $b_1:b_2:b_3$  and a plane with direction ratio of normal  $n_1:n_2:n_3$ , then

$$\sin \theta = \frac{n_1 b_1 + n_2 b_2 + n_3 b_3}{\sqrt{n_1^2 + n_2^2 + n_3^2} \times \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

Explanation:

Here direction ratio of the line is 2 : 2 : 1

Direction ratio of normal to the plane is 6 : -3 : 2

Therefore,

$$\sin \theta = \frac{12 - 6 + 2}{\sqrt{4 + 4 + 1} \times \sqrt{36 + 9 + 4}}$$

$$\sin \theta = \frac{8}{3 \times 7}$$

$$\theta = \sin^{-1} \frac{8}{21}$$

Therefore, angle between the line and plane is  $\sin^{-1} \frac{8}{21}$

**Question 32.**

Mark against the correct answer in each of the following:

The distance of the point  $(\hat{i} + 2\hat{j} + 5\hat{k})$  from the plane  $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) + 17 = 0$ , is

A.  $\frac{25}{\sqrt{2}}$  units

B.  $\frac{25}{\sqrt{3}}$  units

C.  $25\sqrt{2}$  units

D.  $25\sqrt{3}$  units

**Answer:**

Given: Point is at  $(\hat{i} + 2\hat{j} + 5\hat{k})$  and equation of plane is  $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) + 17 = 0$

To find: distance of point from plane

Formula Used: Perpendicular distance from  $(x_1, y_1, z_1)$  to the plane  $ax + by + cz + d = 0$  is

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Explanation:

The point is at (1, 2, 5) and equation of plane is  $x + y + z + 17 = 0$

$$\text{Distance} = \frac{1+2+5+17}{\sqrt{1+1+1}}$$

$$= \frac{25}{\sqrt{3}}$$

Therefore, distance =  $\frac{25}{\sqrt{3}}$  units

**Question 33.**

Mark against the correct answer in each of the following:

The distance between the parallel planes  $2x - 3y + 6z = 5$  and  $6x - 9y + 18z + 20 = 0$ , is

A.  $\frac{5}{3}$  units

B.  $5\sqrt{3}$  units

C.  $\frac{8}{5}$  units

D.  $8\sqrt{5}$  units

**Answer:**

Given: The equations of the parallel planes are  $2x - 3y + 6z = 5$  and  $6x - 9y + 18z + 20 = 0$

To find: distance between the planes

Formula Used: Distance between two parallel planes  $ax + by + cz + d_1 = 0$  and  $ax_1 + by_1 + cz_1 + d_1 = 0$  is

$$\left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$

Explanation:

The equations of the parallel planes are:

$$2x - 3y + 6z - 5 = 0$$

$$2x - 3y + 6z + \frac{20}{3} = 0$$

Therefore distance between them is

$$= \left| \frac{\frac{20}{3} + 5}{\sqrt{4 + 9 + 36}} \right|$$

$$= \left| \frac{35}{3 \times \sqrt{49}} \right|$$

$$= \frac{5}{3}$$

Therefore distance between the planes is  $\frac{5}{3}$  units

**Question 34.**

Mark against the correct answer in each of the following:

The distance between the planes  $x + 2y - 2z + 1 = 0$  and  $2x + 4y - 4z - 4z + 5 = 0$ , is

A. 4 units

B. 2 units

C.  $\frac{1}{2}$  units

D.  $\frac{1}{4}$  units

**Answer:**

Given: The equations of the planes are  $x + 2y - 2z + 1 = 0$  and  $2x + 4y - 4z - 4z + 5 = 0$

To find: distance between the planes

Formula Used: Distance between two parallel planes  $ax + by + cz + d_1 = 0$  and  $ax_1 + by_1 + cz_1 + d_1 = 0$  is

$$\left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$

Explanation:

The equations of the planes are:

$$x + 2y - 2z + 1 = 0 \text{ and } 2x + 4y - 4z - 4z + 5 = 0$$

Multiplying the equation of first plane by 2,

$$2x + 4y - 4z + 2 = 0$$

Therefore distance between them is

$$= \left| \frac{\frac{20}{3} + 5}{\sqrt{4 + 9 + 36}} \right|$$

$$= \left| \frac{35}{3 \times \sqrt{49}} \right|$$

$$= \frac{5}{3}$$

Therefore distance between the planes is  $\frac{5}{3}$  units

### Question 35.

Mark against the correct answer in each of the following:

The image of the point P(1, 3, 4) in the plane  $2x - y + z + 3 = 0$ , is

A. (3, -5, 2)

B. (3, 5, -2)

C. (3, 5, 2)

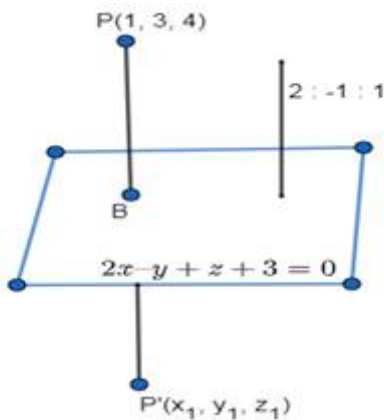
D. (-3, 5, 2)

### Answer:

Given: Equation of plane is  $2x - y + z + 3 = 0$ . P is at (1, 3, 4)

To find: image of P

Explanation:



From the figure, P' is the image of P and B is the midpoint of PP'

If B is (a, b, c), then

$$a = \frac{1 + x_1}{2}$$

$$b = \frac{3 + y_1}{2}$$

$$c = \frac{4 + z_1}{2}$$

B lies on the plane.

So,  $2a - b + c + 3 = 0 \dots (1)$

Also, equation of line PB is

$$\frac{x - 1}{2} = \frac{y - 3}{-1} = \frac{z - 4}{1} = \lambda$$

So any point on the line PB will be of the form

$$x = 2\lambda + 1, y = -\lambda + 3, z = \lambda + 4$$



Since  $(a, b, c)$  will also be of this form we can substitute these values in (1)

$$\Rightarrow 2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) + 3 = 0$$

$$\Rightarrow 4\lambda + 2 + \lambda - 3 + \lambda + 4 + 3 = 0$$

$$\Rightarrow 6\lambda = -6$$

$$\Rightarrow \lambda = -1$$

$$\text{So } (a, b, c) = (-1, 4, 3)$$

Substituting these values in the equations of  $a$ ,  $b$  and  $c$ ,

$$-1 = \frac{1 + x_1}{2}$$

$$4 = \frac{3 + y_1}{2}$$

$$3 = \frac{4 + z_1}{2}$$

Therefore,

$$x_1 = -3, y_1 = 5, z_1 = 2$$

Therefore, the image is  $(-3, 5, 2)$