Exercise 1a

Question 1.
Find the domain and range of the relation

R = {(-1, 1), (1, 1), (-2, 4), (2, 4)}.

Answer:
dom (R) = {-1, 1, -2, 2} and range (R) = {1, 4}

Question 2.
Let R = {(a, a^3) : a is a prime number less than 5}.

Find the range of R.

Answer:
range (R) = {8 27}

Question 3.
Let R = {(a, a^3) : a is a prime number less than 10}.

Find (i) R (ii) dom (R) (iii) range (R).

Answer:
(i) R = {(2, 8),(3, 27),(5, 125), (7, 343)}

(ii) dom (R) = {2, 3, 5, 7}

(iii) range (R) = {8, 27, 125, 343}

Question 4.
Let R = (x, y) : x + 2y = be are relation on N.

Write the range of R.

Answer:
{3, 2, 1}

Question 5.

Let R ={(a, b): a, b $\in$ N and a + 3b = 12}.

Find the domain and range of R.

Answer:
dom (R) = {3, 6, 9} and range (R) = {3, 2, 1}

Question 6.
Let R = {(a, b) : b = |a – 1|, a $\in$ Z and |a| < 3}.

Find the domain and range of R.

Answer:
dom (R) = {-2, -1, 0, 1, 2} and range (R) = {3, 2, 1, 0}

Question 7.
Let $R = \left\{\left(a, \frac{1}{a}\right) : a \in N \text{ and } 1 < a < 5\right\}$.

Find the domain and range of R.

Answer:
dom (R) = {2, 3, 4} and range $(R) = \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$

Question 8.
Let R = {(a, b) : a, b $\in$ N and b = a + 5, a < 4}.

Find the domain and range of R.

Answer:
dom (R) = {1, 2, 3} and range (R) = {6, 7, 8}

Question 9.
Let S be the set of all sets and let R = {(A, B) : A $\subset$ B)}, i.e., A is a proper subset of B. Show that R is (i) transitive (ii) not reflexive (iii) not symmetric.

Answer:

Let R = {(A, B) : A $\subset$ B)}, i.e., A is a proper subset of B, be a relation defined on S.

Now,

Any set is a subset of itself, but not a proper subset.

$\Rightarrow$ (A,A) $\notin$ R $\forall$ A $\in$ S

$\Rightarrow$ R is not reflexive.

Let (A,B) $\in$ R $\forall$ A, B $\in$ S

$\Rightarrow$ A is a proper subset of B

$\Rightarrow$ all elements of A are in B, but B contains at least one element that is not in A.

$\Rightarrow$ B cannot be a proper subset of A

$\Rightarrow$ (B,A) $\notin$ R

For e.g. , if B = {1,2,5} then A = {1,5} is a proper subset of B . we observe that B is not a proper subset of A.

$\Rightarrow$ R is not symmetric

Let (A,B) $\in$ R and (B,C) $\in$ R $\forall$ A, B,C $\in$ S

$\Rightarrow$ A is a proper subset of B and B is a proper subset of C

$\Rightarrow$ A is a proper subset of C

$\Rightarrow$ (A,C) $\in$ R

For e.g. , if B = {1,2,5} then A = {1,5} is a proper subset of B .

And if C = {1,2,5,7} then B = {1,2,5} is a proper subset of C.

We observe that A = {1,5} is a proper subset of C also.

$\Rightarrow$ R is transitive.

Thus, R is transitive but not reflexive and not symmetric.

Question 10.
Let A be the set of all points in a plane and let O be the origin. Show that the relation R = {(P, Q) : P, Q $\in$ A and OP = OQ) is an equivalence relation.

Answer:
In order to show R is an equivalence relation, we need to show R is Reflexive, Symmetric and Transitive.

Given that, A be the set of all points in a plane and O be the origin. Then, R = {(P, Q) : P, Q $\in$ A and OP = OQ)}

Now,

R is Reflexive if $(P,P) \in R \forall P \in A$

$\forall$ P $\in$ A , we have

OP=OP

$\Rightarrow (P,P) \in R$

Thus, R is reflexive.

R is Symmetric if $(P,Q) \in R \Rightarrow (Q,P) \in R \forall P, Q \in A$

Let P, Q $\in$ A such that,

$(P,Q) \in R$

$\Rightarrow OP = OQ$

$\Rightarrow OQ = OP$

$\Rightarrow (Q,P) \in R$

Thus, R is symmetric.

R is Transitive if (P,Q)∈R and (Q,S)∈R⇒(P,S)∈R∀P, Q, S∈A

Let (P,Q) ∈ R and (Q,S) ∈ R ∀ P, Q, S ∈ A

⇒ OP = OQ and OQ = OS

⇒ OP = OS

⇒ (P,S) ∈ R

Thus, R is transitive.

Since R is reflexive, symmetric and transitive it is an equivalence relation on A.

Question 11.
On the set S of all real numbers, define a relation R = {(a, b) : a ≤ b}.

Show that R is (i) reflexive (ii) transitive (iii) not symmetric.

Answer:
Let R = {(a, b) : a ≤ b} be a relation defined on S.

Now,

We observe that any element x ∈ S is less than or equal to itself.

⇒ (x,x) ∈ R ∀ x ∈ S

⇒ R is reflexive.

Let (x,y) ∈ R ∀ x, y ∈ S

⇒ x is less than or equal to y

But y cannot be less than or equal to x if x is less than or equal to y.

$\Rightarrow (y,x) \notin R$

For e.g. , we observe that $(2,5) \in R$ i.e. $2 < 5$ but 5 is not less than or equal to $2 \Rightarrow (5,2) \notin R$

$\Rightarrow$ R is not symmetric

Let $(x,y) \in R$ and $(y,z) \in R\ \forall\ x, y, z \in S$

$\Rightarrow x \le y$ and $y \le z$

$\Rightarrow x \le z$

$\Rightarrow (x,z) \in R$

For e.g. , we observe that

$(4,5) \in R \Rightarrow 4 \le 5$ and $(5,6) \in R \Rightarrow 5 \le 6$

And we know that $4 \le 6 \therefore (4,6) \in R$

$\Rightarrow$ R is transitive.

Thus, R is reflexive and transitive but not symmetric.

Question 12.
Let A = {1, 2, 3, 4, 5, 6) and let R = {(a, b) : a, b ∈ A and b = a + 1}.

Show that R is (i) not reflexive, (ii) not symmetric and (iii) not transitive.

Answer:
Given that,

A = {1, 2, 3, 4, 5, 6) and R = {(a, b) : a, b ∈ A and b = a + 1}.

$\therefore R = \{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

Now,

R is Reflexive if $(a,a) \in R \ \forall\ a \in A$

Since, $(1,1),(2,2),(3,3),(4,4),(5,5),(6,6) \notin R$

Thus, R is not reflexive .

R is Symmetric if $(a,b) \in R \Rightarrow (b,a) \in R \ \forall\ a,b \in A$

We observe that $(1,2) \in R$ but $(2,1) \notin R$.

Thus, R is not symmetric .

R is Transitive if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R \ \forall\ a,b,c \in A$

We observe that $(1,2) \in R$ and $(2,3) \in R$ but $(1,3) \notin R$

Thus, R is not transitive.