Exercise 29b

Question 1.
A bag contains 17 tickets, numbered from 1 to 17. A ticket is drawn, and then another ticket is drawn without replacing the first one. Find the probability that both the tickets may show even numbers.

Answer:
Given: A bag contains 17 tickets , numbered 1 to 17, and each trial is independent of the other.

Hence the sample space is given by S = {1,2,3,......,17}

To find: the probability that both the tickets are drawn show even numbers.

Let , success : ticket drawn is even.i.e $\frac{8}{17}$

Now , the Probability of success in the first trial is

P_1(success) = $\frac{8}{17}$

Probability of success in the second trial without replacement of the first draw is given by

P_2(success) = $\frac{7}{16}$

Hence , the probability that both the tickets show even numbers with each trial being independent is given by

$P_1 \times P_2 = \frac{8}{17} \times \frac{7}{16} = \frac{7}{34}$

Question 2.
Two marbles are drawn successively from a box containing 3 black and 4 white marbles. Find the probability that both the marbles are black if the first marble is not replaced before the second draw.

Answer:
Given: A box containing 3 black and 4 white marbles .Each trail is independent of the other trial.

Hence the sample space is given by S = {1B,2B,3B,1W,2W,3W,4W}

To find: the probability that both the marbles are drawn are black.

Let , success : marble drawn is black.i.e $\frac{3}{7}$

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{3}{7}$

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{2}{6}$

Hence , the probability that both the marbles are drawn are black ,with each trial being independent is given by

$P_1 \times P_2 = \frac{3}{7} \times \frac{2}{6} = \frac{1}{7}$

Question 3.
A card is drawn from a well-shuffled deck of 52 cards and without replacing this card, a second card is drawn. Find the probability that the first card is a club and the second card is a spade.

Answer:
Given: a well shuffled deck of 52 cards. Each draw is independent of the other.

To find: the probability that the first card is drawn is a club and the second card is a spade.

Let , success for the first trail be getting a club.

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{13}{52}$

let , success for the second trail be getting a spade.

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{13}{51}$

Hence , the probability that the first card is drawn is a club and the second card is a spade ,with each trial being independent is given by

$P_1 \times P_2 = \frac{13}{52} \times \frac{13}{51} = \frac{13}{204}$

Question 4.
There is a box containing 30 bulbs, of which 5 are defective. If two bulbs are chosen at random from the box in succession without replacing the first, what is the probability that both the bulbs are chosen are defective?

Answer:
Given: A box containing 30 bulbs of which 5 are defective.Each trail is independent of the other trial.

To find: the probability that both the bulbs are chosen are defective.

Let , success :bulb chosen is defective .i.e $\frac{5}{30}$

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{5}{30}$

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{4}{29}$

Hence , the probability that both the bulbs are chosen are defective,with each trial being independent is given by

$P_1 \times P_2 = \frac{5}{30} \times \frac{4}{29} = \frac{2}{87}$

Question 5.
A bag contains 1.0 white and 15 black balls. Two balls are drawn in succession without replacement. What is the probability that the first ball is white and the second is black?

Answer:

Given: A bag containing 10 white and 15 black balls .Each trial is independent of the other trial.

To find: the probability that the first ball is drawn is white and the second ball drawn is black.

Let , success in the first draw be getting a white ball.

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{10}{25}$

Let success in the second draw be getting a black ball.

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{15}{24}$

Hence , the probability that the first ball is drawn is white and the second ball drawn is black,with each trial being independent is given by

$P_1 \times P_2 = \frac{10}{25} \times \frac{15}{24} = \frac{1}{4}$

Question 6.
An urn contains 5 white and 8 black balls. Two successive drawings of 3 balls at a time are made such that the balls drawn in the first draw are not replaced before the second draw. Find the probability that the first draw gives 3 white balls and the second draw gives 3 black balls.

Answer:
Given: An urn containing 5 white and 8 black balls .Each trial is independent of the other trial.

To find: the probability that the first draws gives 3 white and the second draw gives 3 black balls.

Let , success in the first draw be getting 3 white balls.

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{^5C_3}{^{13}C_3} = \frac{10}{286} = \frac{5}{143}$

Let success in the second draw be getting 3 black balls.

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{^{8}C_{3}}{^{10}C_{3}} = \frac{56}{120} = \frac{7}{15}$

Hence , the probability that the first draws gives 3 white and the second draw gives 3 black balls,with each trial being independent is given by

$P_1 \times P_2 = \frac{5}{143} \times \frac{7}{15} = \frac{7}{429}$

Question 7.

Let E_1 and E_2 be the events such that $P(E_1) = \frac{1}{3}$ and $P(E_2) = \frac{3}{5}$.

Find:

(i)$P(E_1 \cup E_2)$, when E_1 and E_2 are mutually exclusive.

(ii)$P(E_1 \cap E_2)$, when E_1 and E_2 are independent

Answer:

Given: E_1 and E_2 are two events such that $P(E_1) = \frac{1}{3}$ and $P(E_2) = \frac{3}{5}$

To Find: i)$P(E_1 \cup E_2)$ when E_1 and E_2 are mutually exclusive.

We know that,

When two events are mutually exclusive $P(E_1 \cap E_2) = 0$

Hence, $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$= \frac{1}{3} + \frac{3}{5}$

$= \frac{14}{15}$

Therefore , $P(E_1 \cup E_2) = \frac{14}{15}$ when E_1 and E_2 are mutually exclusive.

ii) $P(E_1 \cap E_2)$ when E_1 and E_2 are independent.

We know that when E_1 and E_2 are independent ,

$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

$= \frac{1}{3} \times \frac{3}{5}$

$= \frac{1}{5}$

Therefore, $P(E_1 \cap E_2) = \frac{1}{5}$ when E_1 and E_2 are independent.

Question 8.

If E_1 and E_2 are the two events such that $P(E_1) = \frac{1}{4}, P(E_2) = \frac{1}{3}$ and $P(E_1 \cup E_2) = \frac{1}{2}$, show that E_1 and E_2 are independent events.

Answer:

Given: E_1 and E_2 are two events such that $P(E_1) = \frac{1}{4}$ and $P(E_2) = \frac{1}{3}$ and

$P(E_1 \cup E_2) = \frac{1}{2}$

To show: E_1 and E_2 are independent events.

We know that,

Hence, $P(E_1 \cap E_2) = P(E_1) + P(E_2) - P(E_1 \cup E_2)$

$= \frac{1}{4} + \frac{1}{3} - \frac{1}{2}$

$= \frac{1}{12}$ Equation 1

Since The condition for two events to be independent is

$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

$= \frac{1}{4} \times \frac{1}{3}$

$= \frac{1}{12}$ Equation 2

Since, Equation 1 = Equation 2

$\Rightarrow$ E_1 and E_2 are independent events.

Hence proved.

Question 9.
If E_1 and E_2 are independent events such that $P(E_1) = 0.3$ and $P(E_2) = 0.4$, find

(i) $P(E_1 \cap E_2)$

(ii) $P(E_1 \cap E_2)$

(iii) $P\left(\overline{E}_1 \cap \overline{E}_2\right)$

(iv) $P\left(\overline{E}_1 \cap E_2\right)$

Answer:
Given: E_1 and E_2 are two independent events such that $P(E_1) = 0.3$ and $P(E_2) = 0.4$

To Find: i)$P(E_1 \cap E_2)$

We know that,

when E_1 and E_2 are independent ,

$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

$= 0.3 \times 0.4$

$= 0.12$

Therefore, $P(E_1 \cap E_2) = 0.12$ when E_1 and E_2 are independent.

ii) $P(E_1 \cup E_2)$ when E_1 and E_2 are independent.

We know that,

Hence, $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= 0.3 + 0.4 - (0.3 \times 0.4)$

$= 0.58$

Therefore , $P(E_1 \cup E_2) = 0.58$ when E_1 and E_2 are Independent.

iii) $P(\overline{E_1} \cap \overline{E_2}) = P(\overline{E_1}) \times P(\overline{E_2})$

since , $P(E_1) = 0.3$ and $P(E_2) = 0.4$

$\Rightarrow P(\overline{E_1}) = 1 - P(E_1) = 0.7$ and $P(\overline{E_2}) = 1 - P(E_2) = 0.6$

Since, E_1 and E_2 are two independent events

$\Rightarrow$ $\overline{E_1}$ and $\overline{E_2}$ are also independent events.

Therefore, $P(\overline{E_1} \cap \overline{E_2}) = 0.7 \times 0.6 = 0.42$

iv) $P(\overline{E_1} \cap E_2) = P(\overline{E_1}) \times P(E_2)$

$= 0.7 \times 0.4$

$= 0.28$

Therefore , $P(\overline{E_1} \cap E_2) = 0.28$

Question 10.

Let A and B be the events such that $P(A)=\frac{1}{2}, P(B)=\frac{7}{12}$ and P(not A or not B) $=\frac{1}{4}$.

State whether A and B are

(i) mutually exclusive

(ii) independent

Answer:

Given: A and B are the events such that P(A) $=\frac{1}{2}$ and P(B) = $\frac{7}{12}$ and

P(not A or not B) = $\frac{1}{4}$

To Find: i)If A and B are mutually exclusive

Since P(not A or not B) = $\frac{1}{4}$ i.e., P($\bar{A} \cup \bar{B}$) = $\frac{1}{4}$

we know that , P($\bar{A} \cup \bar{B}$) = P(A $\cap$ B)′= 1- P(A $\cap$ B) = $\frac{1}{4}$

$\Rightarrow$ P(A $\cap$ B) = 1- $\frac{1}{4}$ = $\frac{3}{4}$ Equation 1

Since for two mutually exclusive events P(A $\cap$ B) = 0

But here P(A $\cap$ B) $\neq$ 0

Therefore , A and B are not mutually exclusive.

ii)If A and B are independent

The condition for two events to be independent is given by

$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

$=\frac{1}{2} \times \frac{7}{12}$

$= \frac{7}{24}$ Equation 2

Since Equation 1 $\neq$ Equation 2

$\Rightarrow$ A and B are not independent

Question 11.
Kamal and Vimal appeared for an interview for two vacancies. The probability of Kamal's selection is 1/3, and that of Vimal's selection is 3. Find the probability that only one of them will be selected.

Answer:
Given: let A denote the event 'kamal is selected' and let B denote the event 'vimal is selected'.

Therefore , P(A) = $\frac{1}{3}$ and P(B) = $\frac{1}{5}$

Also, A and B are independent .A and not B are independent, not A and B are independent.

To Find:The probability that only one of them will be selected.

Now ,

P(only one of them is selected) = P(A and not B or B and not A)

= P(A and not B) + (B and not A)

= P(A $\cap$ $\bar{B}$) + P(B $\cap$ $\bar{A}$)

= P(A) $\times$P($\bar{B}$) + P(B) $\times$ P($\bar{A}$)

= P(A) $\times$[1-P(B)]+ P(B) $\times$ [1-P(A)]

$= \frac{1}{3}\left[1-\frac{1}{5}\right] + \frac{1}{5}\left[1-\frac{1}{3}\right]$

$= \frac{4}{15} + \frac{2}{15}$

$= \frac{2}{5}$

Therefore , The probability that only one of them will be selected is $\frac{2}{5}$

Question 12.
Arun and Ved appeared for an interview for two vacancies. The probability of Arun's selection is 1/4, and that of Ved's rejection is 2/3. Find the probability that at least one of them will be selected.

Answer:
Given : let A denote the event 'Arun is selected' and let B denote the event 'ved is selected'.

Therefore , P(A) = $\frac{1}{4}$ and P($\bar{B}$) = $\frac{2}{3}$ $\Rightarrow$P(B) = $\frac{1}{3}$ and P($\bar{A}$) = $\frac{3}{4}$

Also, A and B are independent .A and not B are independent, not A and B are independent.

To Find: The probability that atleast one of them will get selected.

Now,

P(atleast one of them getting selected) = P(selecting only Arun) + P(selecting only ved) + P(selecting both)

= P(A and not B) +P (B and not A) +P (A and B)

= P(A $\cap$ $\bar{B}$) + P(B $\cap$ $\bar{A}$) + P(A $\cap$ B)

= P(A) $\times$P($\bar{B}$) + P(B) $\times$ P($\bar{A}$) + P(A) $\times$P(B)

$= (\frac{1}{4} \times \frac{2}{3}) + (\frac{1}{3} \times \frac{3}{4}) + (\frac{1}{4} \times \frac{1}{3})$

$= \frac{2}{12} + \frac{3}{12} + \frac{1}{12}$

$= \frac{1}{2}$

Therefore , The probability that atleast one of them will get selected is $\frac{1}{2}$

Question 13.
A and B appear for an interview for two vacancies in the same post. The probability of A's selection is 1/6 and that of B's selection is 1/4. Find the probability that

(i) both of them are selected

(ii) only one of them is selected

(iii) none is selected

(iv) at least one of them is selected.

Answer:
Given : A and B appear for an interview ,then P(A) = $\frac{1}{6}$ and P(B) = $\frac{1}{4}$ $\Rightarrow$ P($\bar{A}$) = $\frac{5}{6}$ and P($\bar{B}$) = $\frac{3}{4}$

Also, A and B are independent .A and not B are independent, not A and B are independent.

To Find: i) The probability that both of them are selected.

We know that, P(both of them are selected) = P(A $\cap$ B) = P(A) $\times$P(B)

= $\frac{1}{6} \times \frac{1}{4}$

= $\frac{1}{24}$

Therefore , The probability that both of them are selected is $\frac{1}{24}$

ii) P(only one of them is selected) = P(A and not B or B and not A)

= P(A and not B) + (B and not A)

= P(A $\cap$ $\bar{B}$) + P(B $\cap$ $\bar{A}$)

= P(A) $\times$P($\bar{B}$) + P(B) $\times$ P($\bar{A}$)

= $(\frac{1}{6} \times \frac{3}{4}) + (\frac{1}{4} \times \frac{5}{6})$

$= \frac{3}{24} + \frac{5}{24}$

$= \frac{1}{3}$

Therefore, the probability that only one of them Is selected is $\frac{1}{3}$

iii)none is selected

we know that P(none is selected) = P($\bar{A} \cap \bar{B}$)

= P($\bar{A}$) $\times$P($\bar{B}$)

$= \frac{5}{6} \times \frac{3}{4}$

$= \frac{5}{8}$

Therefore , the probability that none is selected is $\frac{5}{8}$

iv) atleast one of them is selected

Now, P(atleast one of them is selected) = P(selecting only A) + P(selecting only B) + P(selecting both)

= P(A and not B) +P (B and not A) +P (A and B)

= P(A $\cap$ $\bar{B}$) + P(B $\cap$ $\bar{A}$) + P(A $\cap$ B)

= P(A) $\times$P($\bar{B}$) + P(B) $\times$ P($\bar{A}$) + P(A) $\times$P(B)

$= (\frac{1}{6} \times \frac{3}{4}) + (\frac{1}{4} \times \frac{5}{6}) + (\frac{1}{6} \times \frac{1}{4})$

$= \frac{3}{24} + \frac{5}{24} + \frac{1}{24}$

$= \frac{3}{8}$

Therefore, the probability that atleast one of them is selected is $\frac{3}{8}$

Question 14.
Given the probability that A can solve a problem is 2/3, and the probability that B can solve the same problem is %, find the probability that

(i)at least one of A and B will solve the problem

(ii)none of the two will solve the problem

Answer:
Given : Here probability of A and B that can solve the same problem is given , i.e., P(A) = $\frac{2}{3}$ and P(B) = $\frac{3}{5}$ $\Rightarrow$ P($\bar{A}$) = $\frac{1}{3}$and P($\bar{B}$) = $\frac{2}{5}$

Also, A and B are independent . not A and not B are independent.

To Find: i) atleast one of A and B will solve the problem

Now , P(atleast one of them will solve the problem) = 1 – P(both are unable to solve)

= 1 - P($\bar{A} \cap \bar{B}$)

= 1- P($\bar{A}$) $\times$P($\bar{B}$)

= 1- ($\frac{1}{3} \times \frac{2}{5}$)

= $\frac{13}{15}$

Therefore , atleast one of A and B will solve the problem is $\frac{13}{15}$

ii) none of the two will solve the problem

Now, P(none of the two will solve the problem) = P($\bar{A} \cap \bar{B}$)

= P($\bar{A}$) $\times$P($\bar{B}$)

$= \frac{1}{3} \times \frac{2}{5}$

$= \frac{2}{15}$

Therefore , none of the two will solve the problem is $\frac{2}{15}$

Question 15.
A problem is given to three students whose chances of solving it are 1/4, 1/5 and 1/6, respectively. Find the probability that the problem is solved.

Answer:
Given : let A , B and C be three students whose chances of solving a problem is given i.e , P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{5}$ and P(C) = $\frac{1}{6}$.

$\Rightarrow$ P($\bar{A}$) = $\frac{3}{4}$, P($\bar{B}$) = $\frac{4}{5}$ and P($\bar{C}$)= $\frac{5}{6}$

To Find: The probability that the problem is solved .

Here , P(the problem is solved) = 1 - P(the problem is not solved)

$= 1- P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= 1 – [P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})]$

$= 1- [\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}]$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

Therefore , The probability that the problem is solved is $\frac{1}{2}$.

Question 16.

The probabilities of A, B, C solving a problem are 1/3, 1/4 and 1/6, respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.

Answer:
Given : let A , B and C be three students whose chances of solving a problem is given i.e , P(A) = $\frac{1}{3}$, P(B) = $\frac{1}{4}$ and P(C) = $\frac{1}{6}$.

$\Rightarrow$ P($\bar{A}$) = $\frac{2}{3}$, P($\bar{B}$) = $\frac{3}{4}$ and P($\bar{C}$)= $\frac{5}{6}$

To Find: The probability that excatly one of them will solve it .

Now, P(excatly one of them will solve it) = P(A and not B and not c) +P (B and not A and not C) +P (C and not A and not B)

= P(A ∩ $\bar{B}$ ∩ $\bar{C}$) + P(B ∩ $\bar{A}$ ∩ $\bar{C}$) + P(C ∩ $\bar{A}$ ∩ $\bar{B}$)

= P(A) ×P($\bar{B}$) × P($\bar{C}$) + P(B) × P($\bar{A}$) × P($\bar{C}$) + P(C) ×P($\bar{B}$) × P($\bar{A}$)

= $[\frac{1}{3} \times \frac{3}{4} \times \frac{5}{6}] + [\frac{1}{4} \times \frac{2}{3} \times \frac{5}{6}] + [\frac{1}{6} \times \frac{3}{4} \times \frac{2}{3}]$

= $\frac{15}{72} + \frac{10}{72} + \frac{6}{72}$

= $\frac{31}{72}$

Therefore , The probability that excatly one of them will solve the problem is $\frac{31}{72}$

Question 17.
A can hit a target 4 times in 5 shots, B can hit 3 times in 4 shots, and C can hit 2 times in 3 shots. Calculate the probability that

(i) A, B and C all hit the target

(ii) B and C hit and A does not hit the target.

Answer:
Given : let A , B and C chances of hitting a target is given i.e , P(A) = $\frac{4}{5}$, P(B) = $\frac{3}{4}$ and P(C) = $\frac{2}{3}$.

$\Rightarrow$ P($\bar{A}$) = $\frac{1}{5}$, P($\bar{B}$) = $\frac{1}{4}$ and P($\bar{C}$)= $\frac{1}{3}$

To Find: i)The probability that A , B and C all hit the target.

Now ,P(all hitting the target) = P($A \cap B \cap C$)

= P(A) $\times$P(B) $\times$ P(C)

= $\frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$

= $\frac{2}{5}$

Hence , The probability that A , B and C all hit the target is $\frac{2}{5}$

ii) B and C hit and A does not hit the target

Here, P(B and C hit and not A) = P(B $\cap$ C $\cap$ $\bar{A}$)

= P(B) $\times$P(C) $\times$ P($\bar{A}$)

= $\frac{3}{4} \times \frac{2}{3} \times \frac{1}{5}$

= $\frac{1}{10}$

Hence , the probability that B and C hit and A does not hit the target is $\frac{1}{10}$

Question 18.
Neelam has offered physics, chemistry and mathematics in Class XII. She estimates that her probabilities of receiving a grade A in these courses are 0.2, 0.3 and 0.9 respectively. Find the probabilities that Neelam receives

(i) all A grades

(ii) no A grade

(iii) exactly 2 A grades.

Answer:
Given : let A , B and C represent the subjects physics,chemistry and mathematics respectively ,the probability of neelam getting A grade in these three subjects is given i.e , P(A) = 0.2 , P(B) = 0.3 and P(C) =0.9

⇒ P($\bar{A}$) = 0.8 , P($\bar{B}$) = 0.7 and P($\bar{C}$)= 0.1

To Find: i)The probability that neelam gets all A grades

Here, P(getting all A grades) = P(A ∩ B ∩ C)

= P(A) $\times$P(B) $\times$ P(C)

=0.2 $\times$ 0.3 $\times$ 0.9

= 0.054

Therefore, The probability that neelam gets all A grades is 0.054.

ii)no A grade

Here , P(getting no A grade) = P($\bar{A}$ ∩ $\bar{B}$ ∩ $\bar{C}$)

= P($\bar{A}$) $\times$ P($\bar{B}$) $\times$ P($\bar{C}$)

= 0.8 $\times$ 0.7 $\times$0.1

= 0.056

Therefore, The probability that neelam gets no A grade is 0.056.

iii)excatly 2 a grades

P(getting excatly 2 A grades) = P(A and B and not c) +P (B and C and not A) +P (C and A and not B)

= P(A ∩ B ∩ $\bar{C}$) + P(B ∩ C ∩ $\bar{A}$) + P(C ∩ A ∩ $\bar{B}$)

= P(A) $\times$P(B) $\times$ P($\bar{C}$) + P(B) $\times$ P(C) $\times$ P($\bar{A}$) + P(C) $\times$P(A) $\times$ P($\bar{B}$)

= [0.2 $\times$ 0.3 $\times$0.1] + [0.3 $\times$ 0.9 $\times$ 0.8] + [0.9 $\times$ 0.2 $\times$0.7]

= 0.006 + 0.216 +0.126

= 0.348

Therefore, The probability that neelam gets excatly 2 A grades is 0.348.

Question 19.
An article manufactured by a company consists of two parts X and Y. In the process of manufacture of part X. 8 out of 100 parts may be defective. Similarly, 5 out of 100 parts of Y may be defective. Calculate the probability that the assembled product will not be defective.

Answer:
Given: X and Y are the two parts of a company that manufactures an article.

Here the probability of the parts being defective is given i.e, P(X) = $\frac{8}{100}$ and P(Y) = $\frac{5}{100}$ $\Rightarrow$ P($\bar{X}$) = $\frac{92}{100}$ and P($\bar{Y}$) = $\frac{95}{100}$

To Find: the probability that the assembled product will not be defective.

Here,

P(product assembled will not be defective) = 1 – P(product assembled to be defective)

=1 – [P(X and not Y) + P(Y and not X) +P(both)]

= 1- [P(X $\cap$ $\bar{Y}$) + P(Y $\cap$ $\bar{X}$) + P(X $\cap$ Y)]

= 1- [P(X) $\times$P($\bar{Y}$) + P(Y) $\times$ P($\bar{X}$) + P(X) $\times$P(Y)]

= 1-[($\frac{8}{100}$ $\times$ $\frac{95}{100}$) + ($\frac{5}{100}$ $\times$ $\frac{92}{100}$)+ ($\frac{8}{100}$ $\times$ $\frac{5}{100}$)]

=1- [$\frac{760}{10000}$ + $\frac{460}{10000}$ + $\frac{40}{10000}$]

$= \frac{437}{500}$

Therefore, The probability that the assembled product will not be defective is $\frac{437}{500}$.

Question 20.
A town has two fire-extinguishing engines, functioning independently. The probability of availability of each engine when needed is 0.95. What is the probability that

(i) neither of them is available when needed?

(ii) an engine is available when needed?

Answer:
Given: Let A and B be two fire extinguishing engines . The probability of availability of each of the two fire extinguishing engines is given i.e., P(A) = 0.95 and P(B) = 0.95 ⇒ P($\bar{A}$) = 0.05 and P($\bar{B}$) = 0.05

To Find: i) The probability that neither of them is available when needed

Here, P(not A and not B) = P($\bar{A} \cap \bar{B}$)

= P($\bar{A}$) $\times$P($\bar{B}$)

=0.05 $\times$ 0.05

= 0.0025 = $\frac{1}{400}$

Therefore, The probability that neither of them is available when needed is $\frac{1}{400}$

ii)an engine is available when needed

Here, P(A and not B or B and not A) = P(A $\cap$ $\bar{B}$) + P(B $\cap$ $\bar{A}$)

= P(A) $\times$P($\bar{B}$) + P(B) $\times$ P($\bar{A}$)

= (0.95 $\times$0.05) + (0.95 $\times$ 0.05)

= 0.0475 + 0.0475

= 0.095

= $\frac{19}{200}$

Therefore, The probability that an engine is available when needed is $\frac{19}{200}$

Question 21.
A machine operates only when all of its three components function. The probabilities of the failures of the first, second and third components are 0.14, 0.10 and 0.05, respectively. What is the probability that the machine will fail?

Answer:
Given: let A ,B and C be the three components of a machine which works only if all its three compenents function.the probabilities of the failures of A,B

and C respectively is given i.e, P(A) = 0.14 ,P(B) = 0.10 and P(C) = 0.05

⇒ P($\bar{A}$) = 0.86 and P($\bar{B}$) = 0.90 and P($\bar{C}$) = 0.95

To Find: The probability that the machine will fail.

Here, P(the machine will fail) = 1 – P(the machine will function)

= 1 – P(all three components are working)

= 1- P($\bar{A} \cap \bar{B} \cap \bar{C}$)

= 1 – [P($\bar{A}$) × P($\bar{B}$) × P($\bar{C}$)]

= 1- [0.86× 0.90 × 0.95]

= 1 – 0.7353

= 0.2647

Therefore, The probability that the machine will fail is 0.2647.

Question 22.

An anti-aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shots are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that at least one shot hits the plane?

Answer:

Given:Let A ,B , C and Dbe first second third and fourth shots whose probability of hitting the plane is given i.e, P(A) = 0.4 ,P(B) = 0.3 , P(C) = 0.2 and P(D) = 0.1 respectively

⇒ P($\bar{A}$) = 0.6 and P($\bar{B}$) = 0.7 and P($\bar{C}$) = 0.8 and P($\bar{D}$) = 0.9

To Find: The probability that atleast one shot hits the plane .

Here , P(atleast one shot hits the plane) = 1 – P(none of the shots hit the plane)

= 1- P($\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}$)

= 1 – [P($\bar{A}$) × P($\bar{B}$) × P($\bar{C}$) × P($\bar{D}$)]

= 1- [0.6× 0.7 × 0.8× 0.9]

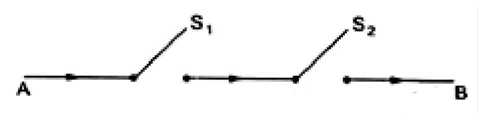
= 1 – 0.3024

= 0.6976

Therefore, The probability that atleast one shot hits the plane is 0.6976.

Question 23.

Let S_1 and S_2 be the two switches and let their probabilities of working be given by $P(S_1) = 4/5$ and $P(S_2) = 9/10$. Find the probability that the current flows from the terminal A to terminal B when S_1 and S_2 are installed in series, shown as follows:



Answer:

Given: S_1 and S_2 are two swiches whose probabilities of working be given by

$P(S_1) = \frac{4}{5}$ and $P(S_2) = \frac{9}{10}$

To Find: the probability that the current flows from terminal A to terminal B when

S_1 and S_2 are connected in series.

Now, since the current in series flows from end to end

⇒ the flow of current from terminal A to terminal B is given by

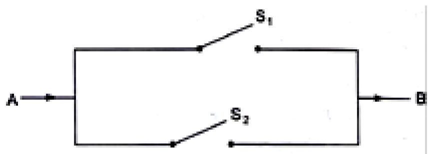
$P(S_1 \cap S_2) = P(S_1) \times P(S_2)$

$= \frac{4}{5} \times \frac{9}{10}$

$= \frac{18}{25}$

Therefore, The probability that the current flows from terminal A to terminal B when S_1 and S_2 are connected in series is $\frac{18}{25}$

Question 24.
Let S_1 and S_2 be two the switches and let their probabilities of working be given by $P(S_1) = 2/3$ and $P(S_2) = 3/4$. Find the probability that the current flows from terminal A to terminal B, when S_1 and S_2 are installed in parallel, as shown below:



Answer:
Given: S_1 and S_2 are two swiches whose probabilities of working be given by

$P(S_1) = \frac{2}{3}$ and $P(S_2) = \frac{3}{4}$

To Find: the probability that the current flows from terminal A to terminal B when

S_1 and S_2 are connected in parallel.

Now, since current in parallel flows in two or more paths and hence the sum of currents through each path is equal to total current that flows from the source.

⇒ the flow of current from terminal A to terminal B in a parallel circuit is given by

$P(S_1 \cup S_2) = P(S_1) + P(S_2) - P(S_1 \cap S_2)$

$= P(S_1) + P(S_2) - [P(S_1) \times P(S_2)]$

$= \frac{2}{3} + \frac{3}{4} - \frac{1}{2}$

$= \frac{11}{12}$

Therefore, The probability that the current flows from terminal A to terminal B when S_1 and S_2 are connected in parallel is $\frac{11}{12}$

Question 25.
A coin is tossed. If a head comes up, a die is thrown, but if a tail comes up, the coin is tossed again. Find the probability of obtaining

(i) two tails

(ii) a head and the number 6

(iii) a head and an even number.

Answer:
Given : let H be head, and T be tails where as 1,2,3,4,5,6 be the numbers on the dice which are thrown when a head comes up or else coin is tossed again if its tail.

According to the question ,sample space S = {(TH),(TT) ,(H1),(H2),(H3),(H4),(H5),(H6)}

To Find: i)the probability of obtaining two tails

From sample space, it is clear that the probability of obtaining two tails is $\frac{1}{8}$

i.e., {TT} with total no of elements in sample space as 8.

ii) the probability of obtaining a head and the number 6

From sample space, it is clear that the probability of obtaining a head and the number 6 is $\frac{1}{8}$

i.e., {H6} with total no of elements in sample space as 8.

iii) the probability of obtaining a head and an even number

From sample space, it is clear that the probability of obtaining a head and an even number is $\frac{3}{8}$

i.e, {H2,H4,H6} with total no of elements in sample space as 8.