Exercise 14b

Question 1.
Evaluate:

$$\int \frac{dx}{\sqrt{16 - x^2}}$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{16 - x^2}}$$

$$= \int \frac{dx}{\sqrt{4^2 - x^2}}$$

$= \sin^{-1}\frac{x}{4} + c$, c being the integrating constant

Question 2.
Evaluate:

$$\int \frac{dx}{\sqrt{1 - 9x^2}}$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1 - 9x^2}}$$

$$= \int \frac{dx}{\sqrt{9\left\{\left(\frac{1}{9}\right) - x^2\right\}}}$$

$$= \frac{1}{3}\int \frac{dx}{\sqrt{1^2 - \left(\frac{x}{3}\right)^2}}$$

$$= \frac{1}{3}\sin^{-1}\frac{x}{\frac{1}{3}} + c$$

$= \frac{1}{3}\sin^{-1} 3x + c$, c being the integrating constant

Question 3.
Evaluate:

$$\int \frac{dx}{\sqrt{15 - 8x^2}}$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{15 - 8x^2}}$$

$$= \int \frac{dx}{\sqrt{15\left\{1 - \left(\frac{\sqrt{8}}{\sqrt{15}}x\right)^2\right\}}}$$

$$= \frac{1}{\sqrt{15}}\int \frac{dx}{\sqrt{1^2 - \left(\frac{\sqrt{8}}{\sqrt{15}}x\right)^2}}$$

$$= \frac{1}{\sqrt{15}}\sin^{-1}\frac{x}{\left(\frac{\sqrt{15}}{\sqrt{8}}\right)} + c$$

$= \frac{1}{\sqrt{15}}\sin^{-1}\frac{\sqrt{8}}{\sqrt{15}}x + c$, c being the integrating constant

Question 4.
Evaluate:

$$\int \frac{dx}{\sqrt{x^2 - 4}}$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 - 4}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 2^2}}$$

$= \log\left|x + \sqrt{x^2 - 4}\right| + c$, c being the integrating constant

Question 5.
Evaluate:

$$\int \frac{dx}{\sqrt{4x^2 - 1}}$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{4x^2 - 1}}$$

$$= \int \frac{dx}{\sqrt{(2x)^2 - 1^2}}$$

$= \frac{1}{2}\log\left|2x + \sqrt{4x^2 - 1}\right| + c$, c being the integrating constant

Question 6.
Evaluate:

$$\int \frac{dx}{\sqrt{9x^2 - 7}}$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{9x^2 - 7}}$$

$$= \int \frac{dx}{\sqrt{(3x)^2 - \sqrt{7}^2}}$$

$= \log|3x + \sqrt{9x^2 - 7}| + c$, c being the integrating constant

Question 7.
Evaluate:

$$\int \frac{dx}{\sqrt{x^2 - 9}}$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 - 9}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 3^2}}$$

$= \log|x + \sqrt{x^2 - 9}| + c$, c being the integrating constant

Question 8.
Evaluate:

$$\int \frac{dx}{\sqrt{1+4x^2}}$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1 + 4x^2}}$$

$$= \int \frac{dx}{\sqrt{(2x)^2 + 1^2}}$$

$= \frac{1}{2} \log|2x + \sqrt{4x^2 + 1}| + c$, c being the integrating constant

Question 9.
Evaluate:

$$\int \frac{dx}{\sqrt{9+4x^2}}$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{9 + 4x^2}}$$

$$= \int \frac{dx}{\sqrt{(2x)^2 + 3^2}}$$

$= \frac{1}{2} \log|2x + \sqrt{4x^2 + 9}| + c$, c being the integrating constant

Question 10.
Evaluate:

$$\int \frac{x}{\sqrt{9-x^4}} dx$$

Answer:
Tip – $d(x^2) = 2xdx$ i.e. $xdx = (1/2)\times d(x^2)$

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{xdx}{\sqrt{9-x^4}}$$

$$= \frac{1}{2}\int \frac{d(x^2)}{\sqrt{3^2-(x^2)^2}}$$

$= \frac{1}{2}\sin^{-1}\frac{x^2}{3} + c$, c being the integrating constant

Question 11.
Evaluate:

$$\int \frac{3x^2}{\sqrt{9-16x^6}} dx$$

Answer:
Tip – $d(x^3) = 3x^2dx$ so, $d(4x^3) = 4\times 3x^2dx$ i.e $3x^2dx = (1/4)d(2x^3)$

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{3x^2dx}{\sqrt{9-16x^6}}$$

$$= \frac{1}{4}\int \frac{d(2x^3)}{\sqrt{3^2 - (4x^3)^2}}$$

$= \frac{1}{4}\sin^{-1}\frac{4x^3}{3} + c$, c being the integrating constant

Question 12.
Evaluate:

$$\int \frac{\sec^2 x}{\sqrt{16 + \tan^2 x}} dx$$

Answer:
Tip – d(tanx) = $\sec^2$xdx

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{\sec^2 x dx}{\sqrt{16 + \tan^2 x}}$$

$$= \int \frac{d(\tan x)}{\sqrt{4^2 + (\tan x)^2}}$$

$= \log|\tan x + \sqrt{16 + \tan^2 x}| + c$, c being the integrating constant

Question 13.
Evaluate:

$$\int \frac{\sin x}{\sqrt{4 + \cos^2 x}} dx$$

Answer:
Tip – d(cosx) = - sinxdx i.e. sinxdx = - d(cosx)

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{\text{sinxdx}}{\sqrt{4 + \cos^2 x}}$$

$$= \int \frac{-d(\text{cosx})}{\sqrt{(\text{cosx})^2 + 2^2}}$$

$= -\log|\text{cosx} + \sqrt{4 + \cos^2 x}| + c$, c being the integrating constant

Question 14.
Evaluate:

$$\int \frac{\cos x}{\sqrt{9\sin^2 x - 1}} dx$$

Answer:
Tip – d(sinx) = cosxdx so, d(3sinx) = 3cosxdx i.e. cosxdx = (1/3)d(3sinx)

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{\text{cosxdx}}{\sqrt{9\sin^2 x - 1}}$$

$$= \frac{1}{3} \int \frac{d(3\text{sinx})}{\sqrt{(3\text{sinx})^2 - 1^2}}$$

$= \frac{1}{3} \log|\text{cosx} + \sqrt{4 + \cos^2 x}| + c$, c being the integrating constant

Question 15.
Evaluate:

$$\int \frac{e^x}{\sqrt{4 + e^{2x}}} dx$$

Answer:
Tip – $d(e^x) = e^x dx$

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{e^x dx}{\sqrt{4 + e^{2x}}}$$

$$= \int \frac{d(e^x)}{\sqrt{2^2 + (e^x)^2}}$$

$= \log|e^x + \sqrt{4 + e^{2x}}| + c$, c being the integrating constant

Question 16.
Evaluate:

$$\int \frac{2e^x}{\sqrt{4 - e^{2x}}} dx$$

Answer:
Tip – $d(e^x) = e^x dx$

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{2e^x dx}{\sqrt{4 - e^{2x}}}$$

$$= 2\int \frac{d(e^x)}{\sqrt{2^2 - (e^x)^2}}$$

$= 2\sin^{-1}\left(\frac{e^x}{2}\right) + c$, c being the integrating constant

Question 17.
Evaluate:

$$\int \frac{dx}{\sqrt{1 - e^x}}$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1-e^x}}$$

$$= \int \frac{dx}{\sqrt{e^x(e^{-x}-1)}}$$

$$= \int \frac{e^{-\frac{x}{2}}\,dx}{\sqrt{e^{-x}-1}}$$

$$= \int \frac{e^{-\frac{x}{2}}\,dx}{\sqrt{\left(e^{-\frac{x}{2}}\right)^2 - 1^2}}$$

Tip – Assuming $e^{-(x/2)} = a$, $-(1/2)\,e^{-(x/2)}dx = da$ i.e. $e^{-(x/2)}dx = -2da$

$$\therefore \int \frac{e^{-\frac{x}{2}}\,dx}{\sqrt{\left(e^{-\frac{x}{2}}\right)^2 - 1^2}}$$

$$= \int \frac{-2da}{\sqrt{a^2 - 1^2}}$$

$$= -2\log|a + \sqrt{a^2-1}| + c$$

$= -2\log|e^{-\frac{x}{2}} + \sqrt{e^{-x}-1}| + c$, c being the integrating constant

Question 18.
Evaluate:

$$\int \sqrt{\frac{a-x}{a+x}}\,dx$$

Answer:

Tip – Taking $x = a\cos 2\theta$,

$dx = -2a\sin 2\theta\, d\theta$ and $\theta = \frac{1}{2}\cos^{-1}\frac{x}{a}$

$x = a\cos 2\theta$ i.e $\cos 2\theta = \frac{x}{a}$

$$\therefore \sin 2\theta = \sqrt{1 - \frac{x^2}{a^2}}$$

$$\therefore \int \sqrt{\frac{a - x}{a + x}} dx$$

$$= \int \sqrt{\frac{a - a\cos 2\theta}{a + a\cos 2\theta}} \times (-2a\sin 2\theta\, d\theta)$$

$$= \int \sqrt{\frac{a(1 - \cos 2\theta)}{a(1 + \cos 2\theta)}} \times (-2a\sin 2\theta\, d\theta)$$

Formula to be used - $\cos 2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$\therefore \int \sqrt{\frac{1 - \cos 2\theta}{1 + \cos 2\theta}} \times (-2a\sin 2\theta\, d\theta)$$

$$= \int \sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}} \times (-2a\sin 2\theta\, d\theta)$$

$$= \int \frac{\sin\theta}{\cos\theta} \times (-2a \times 2\sin\theta\cos\theta\, d\theta)$$

$$= -2a\int 2\sin^2\theta d\theta$$

$$= -2a\int 1-\cos 2\theta\, d\theta$$

$$= -2a\left[\theta - \frac{\sin 2\theta}{2}\right]$$

$$= -2a\left[\theta - \frac{\sin 2\theta}{2}\right] + c$$

$$= -2a\left[\frac{1}{2}\cos^{-1}\frac{x}{a} - \frac{\sqrt{1-\frac{x^2}{a^2}}}{2}\right] + c$$

$$= -a\cos^{-1}\frac{x}{a} + a\sqrt{1-\frac{x^2}{a^2}} + c$$

$= a\sin^{-1}\frac{x}{a} + \sqrt{a^2-x^2} + c$, c being the integrating constant

Question 19.
Evaluate:

$$\int \frac{dx}{\sqrt{x^2+6x+5}}$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 + 6x + 5}}$$

$$= \int \frac{dx}{\sqrt{(x^2 + 2\times x\times 3 + 3^2) + 5 - 3^2}}$$

$$= \int \frac{dx}{\sqrt{(x + 3)^2 - 2^2}}$$

$= \log|(x + 3) + \sqrt{x^2 + 6x + 5}| + c$, c being the integrating constant

Question 20.
Evaluate:

$$\int \frac{dx}{\sqrt{(2-x)^2 + 1}}$$

Answer:
Tip – d(2 - x) = - dx i.e. dx = - d(2 - x)

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{(2 - x)^2 + 1}}$$

$$= \int \frac{-d(2 - x)}{\sqrt{(2 - x)^2 + 1}}$$

$$= -\log|(2 - x) + \sqrt{(2 - x)^2 + 1}| + c$$

$= -\log|(2 - x) + \sqrt{x^2 - 4x + 5}| + c$, c being the integrating constant

Question 21.
Evaluate:

$$\int \frac{dx}{\sqrt{(x-3)^2 + 1}}$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{(x-3)^2 + 1}}$$

$$= \log|(x-3) + \sqrt{(x-3)^2 + 1}| + c$$

$= \log|(x-3) + \sqrt{x^2 - 6x + 10}| + c$, c being the integrating constant

Question 22.
Evaluate:

$$\int \frac{dx}{\sqrt{x^2 - 6x + 10}}$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 - 6x + 10}}$$

$$= \int \frac{dx}{\sqrt{(x-3)^2 + 1}}$$

$$= \log|(x-3) + \sqrt{(x-3)^2 + 1}| + c$$

$= \log|(x-3) + \sqrt{x^2 - 6x + 10}| + c$, c being the integrating constant

Question 23.
Evaluate:

$$\int \frac{dx}{\sqrt{2 + 2x - x^2}}$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{2 + 2x - x^2}}$$

$$= \int \frac{dx}{\sqrt{3 - (x^2 - 2x + 1)}}$$

$$= \int \frac{dx}{\sqrt{\left(\sqrt{3}\right)^2 - (x - 1)^2}}$$

$= \sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right) + c$, c being the integrating constant

Question 24.

Evaluate:

$$\int \frac{dx}{\sqrt{8 - 4x - 2x^2}}$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{8 - 4x - 2x^2}}$$

$$= \int \frac{dx}{\sqrt{10 - 2(x^2 + 2x + 1)}}$$

$$= \int \frac{dx}{\sqrt{\left(\sqrt{10}\right)^2 - 2(x + 1)^2}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(\sqrt{5}\right)^2 - (x + 1)^2}}$$

$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{x+1}{\sqrt{5}}\right) + c$, c being the integrating constant

Question 25.
Evaluate:

$$\int \frac{dx}{\sqrt{16 - 6x - x^2}}$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{16 - 6x - x^2}}$$

$$= \int \frac{dx}{\sqrt{25 - (x^2 + 6x + 9)}}$$

$$= \int \frac{dx}{\sqrt{(5)^2 - (x + 3)^2}}$$

$= \sin^{-1}\left(\frac{x+3}{5}\right) + c$, c being the integrating constant

Question 26.
Evaluate:

$$\int \frac{dx}{\sqrt{7 - 6x - x^2}}$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{7 - 6x - x^2}}$$

$$= \int \frac{dx}{\sqrt{16 - (x^2 + 6x + 9)}}$$

$$= \int \frac{dx}{\sqrt{(4)^2 - (x + 3)^2}}$$

$= \sin^{-1}\left(\frac{x+3}{4}\right) + c$, c being the integrating constant

Question 27.
Evaluate:

$$\int \frac{dx}{\sqrt{x - x^2}}$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x - x^2}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - (x^2 - 2 \times x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2)}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - (x - \frac{1}{2})^2}}$$

$$= \sin^{-1}\left(\frac{x - \frac{1}{2}}{\frac{1}{2}}\right) + c$$

$= \sin^{-1}(2x - 1) + c$, c being the integrating constant

Question 28.
Evaluate:

$$\int \frac{dx}{\sqrt{8 + 2x - x^2}}$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{8 + 2x - x^2}}$$

$$= \int \frac{dx}{\sqrt{9 - (x^2 - 2x + 1)}}$$

$$= \int \frac{dx}{\sqrt{(3)^2 - (x - 1)^2}}$$

$= \sin^{-1}\left(\frac{x-1}{3}\right) + c$, c being the integrating constant

Question 29.
Evaluate:

$$\int \frac{dx}{\sqrt{x^2 - 3x + 2}}$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 - 3x + 2}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 2 \times x \times \frac{3}{2} + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2}}$$

$$= \int \frac{dx}{\sqrt{(x - \frac{3}{2})^2 - \frac{1}{4}}}$$

$= \log|(x - \frac{3}{2}) + \sqrt{x^2 - 3x + 2}| + c$, c being the integrating constant

Question 30.
Evaluate:

$$\int \frac{dx}{\sqrt{2x^2 + 3x - 2}}$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{2x^2 + 3x - 2}}$$

$$= \int \frac{dx}{\sqrt{2\left(x^2 + 2 \times x \times \frac{3}{4} + \left(\frac{3}{4}\right)^2\right) - \frac{7}{8}}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x + \frac{3}{4})^2 - \left(\frac{\sqrt{7}}{4}\right)^2}}$$

$= \frac{1}{\sqrt{2}} \log|(x + \frac{3}{4}) + \sqrt{2x^2 + 3x - 2}| + c$, c being the integrating constant

Question 31.

Evaluate:

$$\int \frac{dx}{\sqrt{2x^2 + 4x + 6}}$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{2x^2 + 4x + 6}}$$

$$= \int \frac{dx}{\sqrt{2(x^2 + 2x + 1) + 4}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x + 1)^2 + (\sqrt{2})^2}}$$

$= \frac{1}{\sqrt{2}} \log|(x + 1) + \sqrt{2x^2 + 4x + 6}| + c$, c being the integrating constant

Question 32.

Evaluate:

$$\int \frac{dx}{\sqrt{1 + 2x - 3x^2}}$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1 + 2x - 3x^2}}$$

$$= \int \frac{dx}{\sqrt{\left(1 - \frac{1}{3}\right) - 3\left(x^2 - 2 \times x \times \frac{1}{3} + \left(\frac{1}{3}\right)^2\right)}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 - 3\left(x - \frac{1}{3}\right)^2}}$$

$$= \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{\left(\frac{\sqrt{2}}{3}\right)^2 - \left(x - \frac{1}{3}\right)^2}}$$

$$= \frac{1}{\sqrt{3}} \sin^{-1}\left(\frac{x - \frac{1}{3}}{\frac{\sqrt{2}}{3}}\right) + c$$

$= \frac{1}{\sqrt{3}} \sin^{-1}\left(\frac{3x-1}{\sqrt{2}}\right) + c$, c being the integrating constant

Question 33.
Evaluate:

$$\int \frac{dx}{\sqrt{x}\sqrt{5-x}}$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{5x - x^2}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{5}{2}\right)^2 - \left(x^2 - 2 \times x \times \frac{5}{2} + \left(\frac{5}{2}\right)^2\right)}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{5}{2}\right)^2 - \left(x - \frac{5}{2}\right)^2}}$$

$$= \sin^{-1}\left(\frac{x-\frac{5}{2}}{\frac{5}{2}}\right) + c$$

$= \sin^{-1}\left(\frac{2x-5}{5}\right) + c$, c being the integrating constant

Question 34.
Evaluate:

$$\int \frac{dx}{\sqrt{3+4x-2x^2}}$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{3 + 4x - 2x^2}}$$

$$= \int \frac{dx}{\sqrt{5 - 2(x^2 - 2x + 1)}}$$

$$= \int \frac{dx}{\sqrt{\left(\sqrt{5}\right)^2 - 2(x-1)^2}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(\sqrt{\frac{5}{2}}\right)^2 - (x-1)^2}}$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{x-1}{\sqrt{\frac{5}{2}}}\right) + c$$

$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{\sqrt{2}(x-5)}{\sqrt{5}}\right) + c$, c being the integrating constant

Question 35.
Evaluate:

$$\int \frac{x^2}{\sqrt{x^6 + 2x^3 + 3}} dx$$

Answer:
Tip – $d(x^3) = 3x^2dx$ i.e. $x^2dx = (1/3)d(x^3)$

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{x^2 dx}{\sqrt{x^6 + 2x^3 + 3}}$$

$$= \int \frac{\frac{1}{3} d(x^3)}{\sqrt{(x^3)^2 + 2x^3 + 3}}$$

$$= \frac{1}{3} \int \frac{d(x^3)}{\sqrt{(x^3 + 1)^2 + (\sqrt{2})^2}}$$

$= \frac{1}{3} \log|(x^3 + 1) + \sqrt{x^6 + 2x^3 + 3}| + c$, c being the integrating constant

Question 36.
Evaluate:

$$\int \frac{(2x + 3)}{\sqrt{x^2 + x + 1}} dx$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{(2x + 3)}{\sqrt{x^2 + x + 1}} dx$$

$$= \int \frac{(2x + 1) + 2}{\sqrt{x^2 + x + 1}} dx$$

$$= \int \frac{(2x + 1)}{\sqrt{x^2 + x + 1}} dx + \int \frac{2}{\sqrt{x^2 + x + 1}} dx$$

Tip – Assuming $x^2 + x + 1 = a^2$, $(2x + 1)dx = 2ada$

$$\therefore \int \frac{(2x + 1)}{\sqrt{x^2 + x + 1}} dx$$

$$= \int \frac{2ada}{a}$$

$$= \int 2da$$

$$= 2a + c_1$$

$$= 2\sqrt{x^2 + x + 1} + c_1$$

$$\therefore \int \frac{2}{\sqrt{x^2 + x + 1}} dx$$

$$= 2\int \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}}$$

$$= 2\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + 1}\right| + c_2$$

$$\therefore \int \frac{(2x + 1)}{\sqrt{x^2 + x + 1}} dx + \int \frac{2}{\sqrt{x^2 + x + 1}} dx$$

$= 2\sqrt{x^2 + x + 1} + 2\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + 1}\right| + c$, c is the integrating constant

Question 37.
Evaluate:

$$\int \frac{(5x+3)}{\sqrt{x^2+4x+10}}dx$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{(5x + 3)}{\sqrt{x^2 + 4x + 10}}dx$$

$$= \int \frac{\frac{5}{2} \times (2x + 4) - 7}{\sqrt{x^2 + 4x + 10}}dx$$

$$= \frac{5}{2}\int \frac{(2x + 4)}{\sqrt{x^2 + 4x + 10}}dx - \int \frac{7}{\sqrt{x^2 + 4x + 10}}dx$$

Tip – Assuming $x^2 + 4x + 10 = a^2$, $(2x + 4)dx = 2ada$

$$\therefore \frac{5}{2}\int \frac{(2x + 4)}{\sqrt{x^2 + 4x + 10}}dx$$

$$= \frac{5}{2}\int \frac{2ada}{a}$$

$$= \frac{5}{2}\int 2da$$

$$= 5a + c_1$$

$$= 5\sqrt{x^2 + 4x + 10} + c_1$$

$$\therefore \int \frac{7}{\sqrt{x^2 + 4x + 10}}dx$$

$$= 7\int \frac{dx}{\sqrt{(x+2)^2 + \left(\sqrt{6}\right)^2}}$$

$$= 7\log\left|(x+2) + \sqrt{x^2 + 4x + 10}\right| + c_2$$

$$\therefore \frac{5}{2}\int \frac{(2x+4)}{\sqrt{x^2 + 4x + 10}}dx - \int \frac{7}{\sqrt{x^2 + 4x + 10}}dx$$

$= 5\sqrt{x^2 + 4x + 10} - 7\log\left|(x+2) + \sqrt{x^2 + 4x + 10}\right| + c$, c is the integrating constant

Question 38.
Evaluate:

$$\int \frac{(4x+3)}{\sqrt{2x^2 + 2x - 3}}$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{(4x+3)}{\sqrt{2x^2 + 2x - 3}}dx$$

$$= \int \frac{(4x+2)+1}{\sqrt{2x^2 + 2x - 3}}dx$$

$$= \int \frac{(4x+2)}{\sqrt{2x^2 + 2x - 3}}dx + \int \frac{1}{\sqrt{2x^2 + 2x - 3}}dx$$

Tip – Assuming $2x^2 + 2x - 3 = a^2$, $(4x + 2)dx = 2ada$

$$\therefore \int \frac{(4x+2)}{\sqrt{2x^2 + 2x - 3}}dx$$

$$= \int \frac{2ada}{a}$$

$$= \int 2da$$

$$= 2a + c_1$$

$$= 2\sqrt{2x^2 + 2x - 3} + c_1$$

$$\therefore \int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= \int \frac{dx}{\sqrt{2\left(x + \frac{1}{2}\right)^2 - \left(\sqrt{\frac{7}{2}}\right)^2}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 - \left(\frac{\sqrt{7}}{2}\right)^2}}$$

$$= \frac{1}{\sqrt{2}} \log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x - \frac{3}{2}}\right| + c_2$$

$$\therefore \int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}} dx + \int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$$

$= 2\sqrt{2x^2 + 2x - 3} + \frac{1}{\sqrt{2}} \log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x - \frac{3}{2}}\right| + c$, c is the integrating constant

Question 39.
Evaluate:

$$\int \frac{(3 - 2x)}{\sqrt{2 + x - x^2}} dx$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{(3-2x)}{\sqrt{2+x-x^2}}dx$$

$$= \int \frac{(1-2x)+2}{\sqrt{2+x-x^2}}dx$$

$$= \int \frac{(1-2x)}{\sqrt{2+x-x^2}}dx + \int \frac{2}{\sqrt{2+x-x^2}}dx$$

Tip – Assuming $2 + x - x^2 = a^2$, $(1 - 2x)dx = 2ada$

$$\therefore \int \frac{(1-2x)}{\sqrt{2+x-x^2}}dx$$

$$= \int \frac{2ada}{a}$$

$$= 2a + c_1$$

$$= 2\sqrt{2+x-x^2} + c_1$$

$$\therefore \int \frac{2}{\sqrt{2+x-x^2}}dx$$

$$= 2\int \frac{dx}{\sqrt{\left(\frac{3}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2}}$$

$$= 2\sin^{-1}\frac{\left(x-\frac{1}{2}\right)}{\left(\frac{3}{2}\right)} + c_2$$

$$= 2\sin^{-1}\left(\frac{2x-1}{3}\right) + c_2$$

$$\therefore \int \frac{(1-2x)}{\sqrt{2+x-x^2}}dx + \int \frac{2}{\sqrt{2+x-x^2}}dx$$

$= 2\sqrt{2+x-x^2} + 2\sin^{-1}\left(\frac{2x-1}{3}\right) + c$, c is the integrating constant

Question 40.
Evaluate:

$$\int \frac{(x+2)}{\sqrt{2x^2+2x-3}}dx$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{(x+2)}{\sqrt{2x^2+2x-3}}dx$$

$$= \int \frac{\frac{1}{4} \times (4x+2) + \frac{3}{2}}{\sqrt{2x^2+2x-3}}dx$$

$$= \frac{1}{4}\int \frac{(4x+2)}{\sqrt{2x^2+2x-3}}dx + \frac{3}{2}\int \frac{1}{\sqrt{2x^2+2x-3}}dx$$

Tip – Assuming $2x^2 + 2x - 3 = a^2$, $(4x + 2)dx = 2ada$

$$\therefore \frac{1}{4}\int \frac{(4x+2)}{\sqrt{2x^2+2x-3}}dx$$

$$= \frac{1}{4}\int \frac{2ada}{a}$$

$$= \frac{1}{2}\int da$$

$$= \frac{a}{2} + c_1$$

$$= \frac{\sqrt{2x^2 + 2x - 3}}{2} + c_1$$

$$\therefore \frac{3}{2}\int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= \frac{3}{2}\int \frac{dx}{\sqrt{2\left(x + \frac{1}{2}\right)^2 - \left(\sqrt{\frac{7}{2}}\right)^2}}$$

$$= \frac{3}{2\sqrt{2}}\int \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 - \left(\frac{\sqrt{7}}{2}\right)^2}}$$

$$= \frac{3}{2\sqrt{2}}\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x - \frac{3}{2}}\right| + c_2$$

$$\therefore \frac{1}{4}\int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}} dx + \frac{3}{2}\int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$$

$= \frac{\sqrt{2x^2 + 2x - 3}}{2} + \frac{3}{2\sqrt{2}}\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x - \frac{3}{2}}\right| + c$, c is the integrating constant

Question 41.
Evaluate:

$$\int \frac{(3x + 1)}{\sqrt{5 - 2x - x^2}} dx$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{(3x + 1)}{\sqrt{5 - 2x - x^2}} dx$$

$$= \int \frac{3(x + 1) - 2}{\sqrt{5 - 2x - x^2}} dx$$

$$= \int \frac{3(x + 1)}{\sqrt{5 - 2x - x^2}} dx - \int \frac{2}{\sqrt{5 - 2x - x^2}} dx$$

Tip – Assuming $5 - 2x - x^2 = a^2$, $(- 2 - 2x)dx = 2ada$ i.e. $(x + 1)dx = - ada$

$$\therefore \int \frac{3(x + 1)}{\sqrt{5 - 2x - x^2}} dx$$

$$= -3 \int \frac{ada}{a}$$

$$= -3a + c_1$$

$$= -3\sqrt{5 - 2x - x^2} + c_1$$

$$\therefore \int \frac{2}{\sqrt{5 - 2x - x^2}} dx$$

$$= 2 \int \frac{dx}{\sqrt{\left(\sqrt{6}\right)^2 - (x + 1)^2}}$$

$$= 2 \sin^{-1} \frac{(x + 1)}{\sqrt{6}} + c_2$$

$$\therefore \int \frac{3(x + 1)}{\sqrt{5 - 2x - x^2}} dx - \int \frac{2}{\sqrt{5 - 2x - x^2}} dx$$

$= -3\sqrt{5 - 2x - x^2} - 2 \sin^{-1} \left(\frac{x + 1}{\sqrt{6}}\right) + c$, c is the integrating constant

Question 42.

Evaluate:

$$\int \frac{(6x+5)}{\sqrt{6+x-2x^2}}dx$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{(6x + 5)}{\sqrt{6 + x - 2x^2}}dx$$

$$= \int \frac{\frac{6}{4}(4x-1) + \frac{13}{2}}{\sqrt{6 + x - 2x^2}}dx$$

$$= \frac{3}{2}\int \frac{(4x-1)}{\sqrt{6 + x - 2x^2}}dx + \frac{13}{2}\int \frac{1}{\sqrt{6 + x - 2x^2}}dx$$

Tip – Assuming 6 + x - 2x^2 = a^2, (1 - 4x)dx = 2ada i.e. (4x - 1)dx = - 2ada

$$\therefore \frac{3}{2}\int \frac{(4x-1)}{\sqrt{6 + x - 2x^2}}dx$$

$$= -\frac{3}{2}\int \frac{2ada}{a}$$

$$= -3a + c_1$$

$$= -3\sqrt{6 + x - 2x^2} + c_1$$

$$\therefore \frac{13}{2}\int \frac{1}{\sqrt{6 + x - 2x^2}}dx$$

$$= \frac{13}{2}\int \frac{dx}{\sqrt{\left(\frac{7}{2\sqrt{2}}\right)^2 - 2\left(x-\frac{1}{4}\right)^2}}$$

$$= \frac{13}{2\sqrt{2}} \int \frac{dx}{\sqrt{\left(\frac{7}{4}\right)^2 - \left(x - \frac{1}{4}\right)^2}}$$

$$= \frac{13}{2\sqrt{2}} \sin^{-1} \frac{\left(x - \frac{1}{4}\right)}{\left(\frac{7}{4}\right)} + c_2$$

$$= \frac{13}{2\sqrt{2}} \sin^{-1} \left(\frac{4x - 1}{7}\right) + c_2$$

$$\therefore \frac{3}{2} \int \frac{(4x - 1)}{\sqrt{6 + x - 2x^2}} dx + \frac{13}{2} \int \frac{1}{\sqrt{6 + x - 2x^2}} dx$$

$= -3\sqrt{6 + x - 2x^2} + \frac{13}{2\sqrt{2}} \sin^{-1}\left(\frac{4x-1}{7}\right) + c$, c is the integrating constant

Question 43.
Evaluate:

$$\int \sqrt{\frac{1+x}{x}} dx$$

Answer:
Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\int \sqrt{\frac{1 + x}{x}} dx$$

$$= \int \sqrt{\frac{(1 + x)^2}{x(1 + x)}} dx$$

$$= \int \frac{1 + x}{\sqrt{x^2 + x}} dx$$

$$= \int \frac{\frac{1}{2}(2x + 1) + \frac{1}{2}}{\sqrt{x^2 + x}} dx$$

$$= \frac{1}{2}\int \frac{2x + 1}{\sqrt{x^2 + x}} dx + \frac{1}{2}\int \frac{dx}{\sqrt{x^2 + x}}$$

Tip – Taking $x^2 + x = a^2$, $(2x + 1)dx = 2ada$

$$\therefore \frac{1}{2}\int \frac{2x + 1}{\sqrt{x^2 + x}} dx$$

$$= \frac{1}{2}\int \frac{2ada}{a}$$

$$= a + c_1$$

$$= \sqrt{x^2 + x} + c_1$$

$$\therefore \frac{1}{2}\int \frac{1}{\sqrt{x^2 + x}} dx$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

$$= \frac{1}{2}\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x}\right| + c_2$$

$$\therefore \frac{1}{2}\int \frac{2x + 1}{\sqrt{x^2 + x}} dx + \frac{1}{2}\int \frac{dx}{\sqrt{x^2 + x}}$$

$= \sqrt{x^2 + x} + \frac{1}{2}\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x}\right| + c$, c is the integrating constant

Question 44.

Evaluate:

$$\int \frac{(x+2)}{\sqrt{x^2+5x+6}} dx$$

Answer:

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\int \frac{(x + 2)}{\sqrt{x^2 + 5x + 6}} dx$$

$$= \int \frac{\frac{1}{2}(2x + 5) - \frac{1}{2}}{\sqrt{x^2 + 5x + 6}} dx$$

$$= \frac{1}{2}\int \frac{2x + 5}{\sqrt{x^2 + 5x + 6}} dx - \frac{1}{2}\int \frac{dx}{\sqrt{x^2 + 5x + 6}}$$

Tip – Taking $x^2 + 5x + 6 = a^2$, $(2x + 5)dx = 2ada$

$$\therefore \frac{1}{2}\int \frac{2x + 5}{\sqrt{x^2 + 5x + 6}} dx$$

$$= \frac{1}{2}\int \frac{2ada}{a}$$

$$= a + c_1$$

$$= \sqrt{x^2 + 5x + 6} + c_1$$

$$\therefore -\frac{1}{2}\int \frac{1}{\sqrt{x^2 + 5x + 6}} dx$$

$$= -\frac{1}{2}\int \frac{dx}{\sqrt{\left(x + \frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

$$= -\frac{1}{2}\log\left|\left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 6}\right| + c_2$$

$$\therefore \frac{1}{2}\int \frac{2x + 5}{\sqrt{x^2 + 5x + 6}}dx - \frac{1}{2}\int \frac{dx}{\sqrt{x^2 + 5x + 6}}$$

$= \sqrt{x^2 + 5x + 6} - \frac{1}{2}\log\left|\left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 6}\right| + c$, c is the integrating constant