Chapter 5 - Measures Of Central Tendency

Question 1:

Which average would be suitable in the following cases?

(i) Average size of readymade garments.

(ii) Average intelligence of students in a class.

(iii) Average production in a factory per shift.

(iv) Average wage in an industrial concern.

(v) When the sum of absolute deviations from average is least.

(vi) When quantities of the variable are in ratios.

(vii) In case of open-ended frequency distribution.

Answer:

(i) The demand for the average size of any readymade garment is the maximum. As, the modal value represents the value with the highest frequency, so the number of the average size to be produced is given by the Modal value.

(ii) Median will be the best measure for calculating the average intelligence of students in a class. It is the value that divides the series into two equal parts. So, number of students below and above the average intelligence can easily be estimated by median.

(iii) It is advisable to use mean for calculating the average production in a factory per shift. The average production is best calculated by arithmetic mean.

(iv) Mean will be the most suitable measure. It is calculated by dividing the sum of wages of all the labour by the total number of labours in the industry.

(v) When the sum of absolute deviations from average is the least, then mean could be used to calculate the average. This is an important mathematical property of arithmetic mean. The algebraic sum of the deviations of a set of n values from A.M. is 0.

(vi) Median will be the most suitable measure in case the variables are in ratios. It is least affected by the extreme values.

(vii) In case of open ended frequency distribution, Median is the most suitable measure as it can be easily computed. Moreover, the median value can be estimated even in case of incomplete statistical series.

Question 2(i):

The most suitable average for qualitative measurement is

(a) arithmetic mean

(b) median

(c) mode

(d) geometric mean

(e) none of the above

Answer:

Median is the most suitable average for qualitative measurement. This is because Median divides a series in two equal parts.

Question 2(ii):

Which average is affected most by the presence of extreme items?

(a) median

(b) mode

(c) arithmetic mean

(d) geometric mean

(e) harmonic mean

Answer:

Arithmetic mean is the most affected by the presence of extreme items. It is one of the prime demerits of the arithmetic mean. It is easily distorted by the extreme values, and also the value of arithmetic mean may not figure out at all in the series.

Question 2(iii):

The algebraic sum of deviation of a set of n values from A.M. is

(a) n

(b) 0

(c) 1

(d) none of the above

Answer:

The algebraic sum of deviation of a set of n values from A.M. is zero. This is one of the mathematical properties of arithmetic mean.

Question 3:

Comment whether the following statements are true or false.

(i) The sum of deviation of items from median is zero.

(ii) An average alone is not enough to compare series.

(iii) Arithmetic mean is a positional value.

(iv) Upper quartile is the lowest value of top 25% of items.

(v) Median is unduly affected by extreme observations.

Answer:

(i) The sum of deviation of items from median is zero. **False**

The statement is false. This mathematical property applies to the arithmetic mean that states that the sum of the deviation of all items from the mean is zero.

(ii) An average alone is not enough to compare series. **True**

An average indicates only the behaviour of a particular series. Therefore, in order to measure the extent of divergence of different items from the central tendency is measured by dispersion. So, average is not enough to compare the series.

(iii) Arithmetic mean is a positional value. **False**

This statement is false as mean is not a positional average, rather the statement holds true for median and mode. The calculation of median and modal values is based on the position of the items in the series, i.e. why these are also termed as positional averages.

(iv) Upper quartile is the lowest value of top 25% of items. **True**

The value that divides a statistical series into four equal parts, the end value of each part is called quartile. The third quartile or the upper quartile has 75 % of the items below it and 25 % of items above it,

(v) Median is unduly affected by extreme observations. **False**

This statement is true for Arithmetic mean. Arithmetic mean is most affected by the presence of extreme items. It is one of the prime demerits of the arithmetic mean. It is easily distorted by the extreme values, and also the value of arithmetic mean may not figure out at all in the series.

Question 4:

If the arithmetic mean of the data given below is 28, find (a) the missing frequency, and (b) the median of the series:

Profit per retail shop (in Rs)	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of retail shops	12	18	27	–	17	6

Answer:

(i) Let the missing frequency be f_1
Arithmetic Mean = 28

Profit per Retail Shop (in Rs) **Class Interval**	**No of Retail Shops** **(*f*)**	**Mid Value** **(*m*)**	***fm***
0 – 10	12	5	60
10 – 20	18	15	270
20 – 30	27	25	675
30 – 40	f_1	35	$35f_1$
40 – 50	17	45	765
50 – 60	6	55	330
	$\sum f = 80 + f_1$		$\sum fm = 2100 + 35f_1$

$$\bar{X} = \frac{\sum fm}{\sum f}$$

$$\text{or, } 28 = \frac{2100 + 35f_1}{80 + f_1}$$

or, $2240 + 28f_1 = 2100 + 35f_1$
or, $2240 - 2100 = 35f_1 - 28f_1$
or, $140 = 7f_1$
$f_1 = 20$

(ii)

Class Interval	**Frequency** ***(f)***	**Cumulative Frequency** ***(CF)***
0 – 10	12	12
10 – 20	18	30
20 – 30	27	57
30 – 40	20	77
40 – 50	17	94
50 – 60	6	100

Total	$\sum f = 100$	

$\sum f = N = 100$

So, the Median class = Size of $\left(\frac{N}{2}\right)^{th}$ item

= 50^{th} item

50^{th} item lies in the 57^{th} cumulative frequency and the corresponding class interval is 20 – 30.

$$\text{Median} = L + \frac{\frac{N}{2} - CF}{f} \times i$$

$$= 20 + \frac{\frac{100}{2} - 30}{27} \times 10$$

$$= 20 + \frac{50 - 30}{27} \times 10$$

$$= 20 + \frac{20}{27} \times 10$$

$$= 27.40$$

Question 5:

The following table gives the daily income of ten workers in a factory. Find the arithmetic mean.

Workers	A	B	C	D	E	F	G	H	I	J
Daily Income (in Rs)	120	150	180	200	250	300	220	350	370	260

Answer:

Workers	**Daily Income (in Rs)** **(*X*)**
A	120
B	150
C	180
D	200
E	250
F	300
G	220

H	350
I	370
J	260
Total	$\sum X = 2400$

$N = 10$

$$\bar{X} = \frac{\sum X}{N}$$

$$= \frac{2400}{10} = 240$$

Arithmetic mean = Rs 240

Question 6:

Following information pertains to the daily income of 150 families. Calculate the arithmetic mean.

Income (in Rs)	*Number of families*
More than 75	150
More than 85	140
More than 95	115
More than 105	95
More than 115	70
More than 125	60
More than 135	40
More than 145	25

Answer:

Income Class Interval	**No. of families (*CF*)**	**Frequency (*f*)**	**Mid Value (*m*)**	***fm***
75 – 85	150	150 – 140 = 10	80	800
85 – 95	140	140 – 115 = 25	90	2250

95 – 105	115	115 – 95 = 20	100	2000
105 – 115	95	95 – 70 = 25	110	2750
115 – 125	70	70 – 60 = 10	120	1200
125 – 135	60	60 – 40 = 20	130	2600
135 – 145	40	40 – 25 = 15	140	2100
145 – 155	25	25	150	3750
Total		$\sum f = 150$		$\sum fm = 17450$

$$\text{Mean} = \frac{\sum fm}{\sum f} = \frac{17450}{150}$$

= Rs 116.33

Question 7:

The size of land holdings of 380 families in a village is given below. Find the median size of land holdings.

Size of Land Holdings (in acres)	Less than 100	100 – 200	200 – 300	300 – 400	400 and above
Number of families	40	89	148	64	39

Answer:

Size of Land Holdings **Class Interval**	**No. of Families** **(*f*)**	**Cumulative Frequency** **(*CF*)**
0 – 100	40	40
100 – 200	89	129
200 – 300	148	277
300 – 400	64	341
400 – 500	39	380
Total	$\sum f = 380$	

$$\sum f = N = 380$$

So, the Median class = Size of $\left(\frac{N}{2}\right)^{th}$ item = 190^{th} item

190^{th} item lies in the 129^{th} cumulative frequency and the corresponding class interval is 200 – 300.

$$\text{So, Median} = L + \frac{\frac{N}{2} - CF}{f} \times i$$

$$= 200 + \frac{190 - 129}{148} \times 100$$

$$= 200 + \frac{61}{148} \times 100$$

$$= 200 + 41.22$$

$$= 241.22$$

Median size of land holdings = 241.22 acres

Question 8:

The following series relates to the daily income of workers employed in a firm. Compute (a) highest income of lowest 50% workers (b) minimum income earned by the top 25% workers and (c) maximum income earned by lowest 25% workers.

Daily Income (in Rs)	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34	35 – 39
Number of workers	5	10	15	20	10	5

(Hint: Compute median, lower quartile and upper quartile)

Answer:

Daily Income (in Rs) Class Interval	**No. of Workers (*f*)**	**Cumulative frequency *(CF)***
9.5 – 14.5	5	5
14.5 – 19.5	10	15
19.5 – 24.5	15	30
24.5 – 29.5	20	50
29.5 – 34.5	10	60
34.5 – 39.5	5	65
	$\sum f = 65$	

(a) Highest income of lowest 50% workers

$$\sum f = N = 65$$

$$\text{Median class} = \text{Size of } \left(\frac{N}{2}\right)^{\text{th}} \text{ item}$$

$$= \text{Size of } \left(\frac{65}{2}\right)^{\text{th}} \text{ item} = 32.5^{\text{th}} \text{ item}$$

32.5^{th} item lies in the 50^{th} cumulative frequency and the corresponding class interval is 24.5 – 29.5.

$$\text{Median} = L + \frac{\frac{N}{2} - CF}{f} \times i$$

$$= 24.5 + \frac{32.5 - 30}{20} \times 5$$

$$= 24.5 + \frac{2.5}{20} \times 5$$

$$= \text{Rs } 25.125$$

(b) Minimum income earned by top 25% workers

In order to calculate the minimum income earned by top 25% workers, we need to ascertain Q_3.

$$\text{Class interval of } Q_3 = 3\left(\frac{N}{4}\right)^{\text{th}} \text{ item}$$

$$= 3\left(\frac{65}{4}\right)^{\text{th}} \text{ item}$$

$$= 3 \times 16.25^{\text{th}} \text{ item}$$

$$= 48.75^{\text{th}} \text{ item}$$

48.75^{th} item lies in 50^{th} item and the corresponding class interval is 24.5 – 29.5.

$$Q_3 = L + \frac{\frac{3N}{4} - CF}{f} \times i$$

$$= 24.5 + \frac{\frac{3 \times 65}{4} - 30}{20} \times 5$$

$$= 24.5 + \frac{\frac{195}{4} - 30}{20} \times 5$$

$$= 24.5 + \frac{48.75 - 30}{20} \times 5$$

$$= \text{Rs } 29.187$$

(c) Maximum income earned by lowest 25% workers

In order to calculate the maximum income earned by lowest 25% workers, we need to ascertain Q_1.

$$\text{Class interval of } Q_1 = \left(\frac{N}{4}\right)^{th} \text{ items}$$

$$= \left(\frac{65}{4}\right)^{th} \text{ item} = 16.25^{th} \text{item}$$

16.25^{th} item lies in the 30^{th} cumulative frequency and the corresponding class interval is 19.5 – 24.5

$$Q_1 = L + \frac{\frac{N}{4} - CF}{f} \times i$$

$$= 19.5 + \frac{16.25 - 15}{15} \times 5$$

$$= 19.5 + \frac{1.25}{15} \times 5$$

$$= \text{Rs } 19.92$$

Question 9:

The following table gives production yield in kg. per hectare of wheat of 150 farms in a village. Calculate the mean, median and mode values.

Production yield (kg. per hectare)	50 – 53	53 – 56	56 – 59	59 – 62	62 – 65	65 – 68	68 – 71	71 – 74	74 – 77
Number of farms	3	8	14	30	36	28	16	10	5

Answer:

(i) Mean

Production Yield	**No. of farms**	**Mid value**	***A* = 63.5**	$d' = \frac{X - A}{c}$	fd'
50 – 53	3	51.5	–12	–4	–12
53 – 56	8	54.5	–9	–3	–24
56 – 59	14	57.5	–6	–2	–28
59 – 62	30	60.5	–3	–1	–30
62 – 65	36	63.5	0	0	0
65 – 68	28	66.5	+3	+1	28
68 – 71	16	69.5	+6	+2	32
71 – 74	10	72.5	+9	+3	30
74 – 77	5	75.5	+12	+4	20

Total	$\sum f = 150$				$\sum fd' = 16$

$$\bar{X} = A + \frac{\sum fd'}{\sum f} \times c$$

$$= 63.5 + \frac{16}{150} \times 3$$

= 63.5 + 0.32

= 63.82 kg per hectare

(ii) Median

Class Interval	**Frequency (*f*)**	***CF***
50 – 53	3	3
53 – 56	8	11
56 – 59	14	25
59 – 62	30	55
62 – 65	36	91
65 – 68	28	119
68 – 71	16	135
71 – 74	10	145
74 – 77	5	150
Total	$\sum f = 150$	

$$\text{Median class} = \text{Size of } \left(\frac{N}{2}\right)^{\text{th}} \text{ item}$$

$$= \text{Size of } \left(\frac{150}{2}\right)^{\text{th}} \text{ item} = 75^{\text{th}} \text{ item}$$

75^{th} item lies in the 91^{st} cumulative frequency and the corresponding class interval is 62 – 65.

$$\text{Median} = L + \frac{\frac{N}{2} - CF}{f} \times i$$

$$= 62 + \frac{75 - 55}{36} \times 3$$

$$= 62 + \frac{20}{36} \times 3$$

$$= 63.67 \text{ kg per hectare}$$

(iii) Mode

Grouping Table

Class Interval	I	II	III	IV	V	VI
50 – 53	3					
		11	22	25	52	80
53 – 56	8					
56 – 59	14					
		44	66		94	
59 – 62	30					
62 – 65	36					
		64	44		80	54
65 – 68	28					
68 – 71	16					
		26	15	31		
71 – 74	10					
74 – 77	5					

Analysis Table

Column	50 – 53	53 – 56	56 – 59	59 – 62	62 – 65	65 – 68	68 – 71	71 – 74	74 – 77
I					√				
II					√	√			
III				√	√				
IV				√	√	√			
V					√	√	√		
VI			√	√	√				

Total	–	–	1	3	6	3	1	–	–

Modal class = 62 – 65

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$$= 62 + \frac{36 - 30}{2 \times 36 - 30 - 28} \times 3$$

$$= 62 + \frac{6}{72 - 30 - 28} \times 3$$

$$= 62 + \frac{6}{14} \times 3$$

$$= 62 + \frac{18}{14}$$

$$= 63.28 \text{ kg per hectare}$$