Objective Questions I

Question 1.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(9+x^2\right)} = ?$$

A. $\tan^{-1}\frac{x}{3}+C$

B. $\frac{1}{3}\tan^{-1}\frac{x}{3}+C$

C. $3\tan^{-1}\frac{x}{3}+C$

D. none of these

Answer:
$$= \int \frac{dx}{x^2+3^2}$$

We know, $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$= \frac{1}{3}\tan^{-1}\frac{x}{3}+c$$

Question 2.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(4+16x^2\right)} = ?$$

A. $\frac{1}{32}\tan^{-1}4x+C$

B. $\frac{1}{16}\tan^{-1}\frac{x}{2}+C$

C. $\frac{1}{8}\tan^{-1} 2x + C$

D. $\frac{1}{4}\tan^{-1}\frac{x}{2} + C$

Answer:

$$= \int \frac{dx}{(4x)^2 + 2^2}$$

4x=t

4dx=dt

$$dx = \frac{dt}{4}$$

$$= \frac{1}{4}\int \frac{dt}{t^2 + 2^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{8}\tan^{-1}\frac{t}{2} + c$$

put t=4x

$$= \frac{1}{8}\tan^{-1}\frac{4x}{2} + c$$

$$= \frac{1}{8}\tan^{-1} 2x + c$$

Question 3.

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(9 + 4x^2\right)} dx = ?$$

A. $\frac{1}{2}\tan^{-1}\frac{2x}{3}+C$

B. $\frac{1}{6}\tan^{-1}\frac{2x}{3}+C$

C. $\frac{1}{6}\tan^{-1}\frac{3x}{2}+C$

D. none of these

Answer:

$$\int \frac{dx}{(2x)^2+3^2}$$

2x=t

2dx=dt

$$dx=\frac{dt}{2}$$

$$=\frac{1}{2}\int \frac{dt}{t^2+3^2}$$

We know, $\int \frac{1}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$=\frac{1}{6}\tan^{-1}\frac{t}{3}+c$$

put t=2x

$$=\frac{1}{6}\tan^{-1}\frac{2x}{3}+c$$

Question 4.

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{\left(1+\cos^2 x\right)}dx=?$$

A. $-\tan^{-1}(\cos x)+C$

B. $\cot^{-1}(\cos x)+C$

C. $-\cot^{-1}(\cos x)+C$

D. $\tan^{-1}(\cos x)+C$

Answer:

$$\int \frac{\sin x}{(\cos x)^2+1^2}\,dx$$

cos x=t

-sin x dx=dt

$$=-\int \frac{dt}{t^2+1^2}$$

We know, $\int \frac{1}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$=-\tan^{-1}\frac{t}{1}+c$$

put t=cos x

$=-\tan^{-1}(\cos x)+c$

Question 5.

Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos x}{\left(1+\sin^2 x\right)}dx=?$$

A. $-\tan^{-1}(\sin x)+C$

B. $\tan^{-1}(\cos x)+C$

C. $\tan^{-1}(\sin x)+C$

D. $-\tan^{-1}(\cos x)+C$

Answer:
$$\int \frac{\cos x}{(\sin x)^2+1^2}\,dx$$

sin x=t

cos x dx=dt

$$=\int \frac{dt}{t^2+1^2}$$

We know, $\int \frac{1}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$=\tan^{-1}\frac{t}{1}+c$$

put t=sin x

= $\tan^{-1}$ (sin x)+c

Question 6.
Mark (√) against the correct answer in each of the following:

$$\int \frac{e^x}{\left(e^{2x}+1\right)}dx=?$$

A. $\cot^{-1}\left(e^x\right)+C$

B. $\tan^{-1}\left(e^x\right)+C$

C. $2\tan^{-1}\left(e^x\right)+C$

D. none of these

Answer:

$$= \int \frac{e^x}{(e^x)^2 + 1^2}\, dx$$

$e^x = t$

$e^x\, dx = dt$

$$= \int \frac{dt}{t^2 + 1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \tan^{-1}\frac{t}{1} + c$$

put $t = e^x$

$\tan^{-1} e^x + c$

Question 7.

Mark (√) against the correct answer in each of the following:

$$\int \frac{3x^5}{\left(1 + x^{12}\right)} dx = ?$$

A. $\tan^{-1} x^6 + C$

B. $\frac{1}{4}\tan^{-1} x^6 + C$

C. $\frac{1}{2}\tan^{-1} x^6 + C$

D. none of these

Answer:

$$= \int \frac{3x^5}{(x^6)^2 + 1^2}\, dx$$

Let $x^6 = t$

$6x^5\,dx = dt$

$$3x^5\,dx = \frac{dt}{2}$$

$$= \frac{1}{2}\int \frac{dt}{t^2 + 1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{2}\tan^{-1}\frac{t}{1} + c$$

put $t = x^6$

$$= \frac{1}{2}\tan^{-1}\frac{x^6}{1} + c$$

$$= \frac{1}{2}\tan^{-1}x^6 + c$$

Question 8.
Mark (√) against the correct answer in each of the following:

$$\int \frac{2x^3}{\left(4 + x^8\right)}dx = ?$$

A. $\frac{1}{2}\tan^{-1}\frac{x^4}{2} + C$

B. $\frac{1}{4}\tan^{-1}\frac{x^4}{2} + C$

C. $\frac{1}{2}\tan^{-1}x^4 + C$

D. none of these

Answer:

$$= \int \frac{2x^3}{(x^4)^2 + 2^2}\, dx$$

Let $x^4 = t$

$4x^3\, dx = dt$

$$2x^3\, dx = \frac{dt}{2}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 + 2^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{4}\tan^{-1}\frac{t}{2} + c$$

put $t = x^4$

$$= \frac{1}{4}\tan^{-1}\frac{x^4}{2} + c$$

Question 9.

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(x^2 + 4x + 8\right)} = ?$$

A. $\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C$

B. $\frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right) + C$

C. $\frac{1}{2}\tan^{-1}(x+2) + C$

D. $\tan^{-1}\left(\frac{x+2}{2}\right)+C$

Answer:

$$=\int \frac{dx}{x^2+4x+8}$$

Completing the square

$x^2 + 4x + 8 = x^2 + 4x + 8\ (+4-4)$

$= x^2 + 4x + 4 + 4$

$= (x+2)^2 + 2^2$

$$=\int \frac{dx}{(x+2)^2+2^2}$$

Let x+2=t

dx=dt

$$=\int \frac{dt}{t^2+2^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$=\frac{1}{2}\tan^{-1}\frac{t}{2}+c$$

put t=x+2

$$=\frac{1}{2}\tan^{-1}\frac{x+2}{2}+c$$

Question 10.

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(2x^2+x+3\right)} = ?$$

A. $\frac{1}{\sqrt{23}}\tan^{-1}\left(\frac{4x+1}{\sqrt{23}}\right)+C$

B. $\frac{1}{\sqrt{23}}\tan^{-1}\left(\frac{x+1}{\sqrt{23}}\right)+C$

C. $\frac{2}{\sqrt{23}}\tan^{-1}\left(\frac{4x+1}{\sqrt{23}}\right)+C$

D. none of these

Answer:

$$= \int \frac{dx}{2x^2+x+3}$$

Completing the square

$$\Rightarrow 2x^2+x+3 \ = 2x^2+\frac{1}{2}x+\frac{3}{2})$$

$$= 2(x^2+\frac{1}{2}x+\frac{3}{2}+\frac{1}{16}-\frac{1}{16})$$

$$= 2(\ (x+\frac{1}{4})^2+\frac{23}{16})$$

$$= \frac{1}{2}\int \frac{dx}{(\ (x+\frac{1}{4})^2+\frac{23}{16})}$$

$$Let\ x+\frac{1}{4}=t$$

dx=dt

$$= \int \frac{dt}{t^2 + \frac{\sqrt{23}^2}{4}}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{4}{2\sqrt{23}}\tan^{-1}\frac{t}{\frac{\sqrt{23}}{4}} + c$$

$$put\ t = x + \frac{1}{4}$$

$$= \frac{2}{\sqrt{23}}\tan^{-1}\frac{x+\frac{1}{4}}{\frac{\sqrt{23}}{4}} + c$$

$$= \frac{2}{\sqrt{23}}\tan^{-1}\frac{4x+1}{\sqrt{23}} + c$$

Question 11.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(e^x + e^{-x}\right)} = ?$$

A. $\tan^{-1}\left(e^x\right) + C$

B. $\tan^{-1}\left(e^{-x}\right) + C$

C. $-\tan^{-1}\left(e^{-x}\right) + C$

D. none of these

Answer:
$$= \int \frac{1}{e^x + e^{-x}}\, dx$$

$$= \int \frac{e^x}{(e^x)^2 + 1^2}\, dx$$

$e^x = t\ e^x$

$e^x\,dx = dt$

$$= \int \frac{dt}{t^2 + 1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \tan^{-1}\frac{t}{1} + c$$

put $t = e^x$

$= \tan^{-1} e^x + c$

Question 12.
Mark (√) against the correct answer in each of the following:

$$\int \frac{x^2}{\left(9+4x^2\right)} = ?$$

A. $\frac{x}{4} - \frac{1}{8}\tan^{-1}\frac{x}{3} + C$

B. $\frac{x}{4} - \frac{3}{8}\tan^{-1}\frac{x}{3} + C$

C. $\frac{x}{4} - \frac{3}{8}\tan^{-1}\frac{2x}{3} + C$

D. none of these

Answer:
$$\int \frac{x^2}{4x^2+9} = \frac{1}{4}\int \frac{4x^2+9-9}{4x^2+9}\,dx$$

$$= \frac{1}{4}\int 1 + \frac{1}{4}\int \frac{-9}{4x^2+9}\,dx$$

$$= \frac{x}{4} - \frac{9}{4} \int \frac{1}{(2x)^2 + 3^2} dx$$

Let 2x=t

2 dx=dt

$$= \frac{x}{4} - \frac{9}{8} \int \frac{1}{(t)^2 + 3^2} dx$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

$$= \frac{x}{4} - \frac{9}{4.2.3} \tan^{-1} \frac{t}{3} + c$$

put t=2x

$$= \frac{x}{4} - \frac{3}{8} \tan^{-1} \frac{2x}{3} + c$$

Question 13.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\left(x^2 - 1\right)}{\left(x^2 + 4\right)} dx = ?$$

A. $x - 5 \tan^{-1} \frac{x}{2} + C$

B. $x - \frac{5}{2} \tan^{-1} \frac{x}{2} + C$

C. $x - \frac{5}{2} \tan^{-1} \frac{5x}{2} + C$

D. none of these

Answer:
$$\int \frac{x^2 - 1}{x^2 + 4} = \int \frac{x^2}{x^2 + 4} - \int \frac{1}{x^2 + 4}$$

$$= \int \frac{x^2}{x^2+4} - \frac{1}{2}\tan^{-1}\frac{x}{2}$$

$$= \int \frac{x^2+4-4}{x^2+4} - \frac{1}{2}\tan^{-1}\frac{x}{2}$$

$$= \int (1 - \frac{4}{x^2+4}) - \frac{1}{2}\tan^{-1}\frac{x}{2}$$

$$= x - 2\tan^{-1}\frac{x}{2} - \frac{1}{2}\tan^{-1}\frac{x}{2} + c$$

$$= x - \frac{5}{2}\tan^{-1}\frac{x}{2} + c$$

Question 14.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(4+9x^2)} = ?$$

A. $\frac{2}{3}\tan^{-1}\frac{3x}{2} + C$

B. $\frac{1}{6}\tan^{-1}3x + C$

C. $\frac{1}{6}\tan^{-1}\frac{3x}{2} + C$

D. none of these

Answer:
Consider $\int \frac{dx}{(3x)^2+2^2}$,

3x=t

3dx=dt

$$dx = \frac{dt}{3}$$

$$= \frac{1}{3}\int \frac{dt}{t^2 + 2^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{6}\tan^{-1}\frac{t}{2} + c$$

put t=3x

$$= \frac{1}{6}\tan^{-1}\frac{3x}{2} + c$$

Question 15.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(4x^2 - 4x + 3\right)} = ?$$

A. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$

B. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$

C. $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$

D. none of these

Answer:
Consider $\int \frac{dx}{4x^2-4x+3}$,

Completing the square

$$4x^2 - 4x + 3 = 4(x^2 - x + \frac{3}{4})$$

$$= 4(x^2 - x + \frac{3}{4} + \frac{1}{4} - \frac{1}{4})$$

$$= 4(\,(x - \frac{1}{2})^2 + \frac{1}{2})$$

$$= \frac{1}{4}\int \frac{dx}{(\,(x-\frac{1}{2})^2 + \frac{1}{2})}$$

$$Let\ x - \frac{1}{2} = t$$

dx=dt

$$= \frac{1}{4}\int \frac{dt}{t^2 + \frac{1}{\sqrt{2}}^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{\sqrt{2}}{4}\tan^{-1}\frac{t}{\frac{1}{\sqrt{2}}} + c$$

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\sqrt{2}t + c$$

put t=x- �

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\frac{2x-1}{\sqrt{2}} + c$$

Question 16.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(\sin^4 x + \cos^4 x\right)} = ?$$

A. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2 x-1}{\sqrt{2}\tan x}\right)+C$

B. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2 x-1}{\tan x}\right)+C$

C. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}\tan x}\right)+C$

D. None of these

Answer:

$$\int \frac{dx}{sin^4 x + cos^4 x} = \int \frac{1}{cos^4 x(tan^4 x + 1)}\ dx$$

$$= \int \frac{sec^4 x}{tan^4 x + 1}\ dx$$

$$= \int \frac{sec^2 x\ sec^2 x}{tan^4 x + 1}\ dx$$

$$= \int \frac{sec^2 x\ (1\ +\ tan^2 x)}{tan^4 x + 1}\ dx$$

tan x=t

$\sec^2$ x dx=dt

$$= \int \frac{1\ +\ t^2}{t^4 + 1}\ dt$$

$$= \int \frac{t^2 + 1}{t^4 + 1}\ dt$$

$$= \int \frac{1 + t^{-2}}{t^2 + t^{-2}} dt$$

$$= \int \frac{1 + t^{-2}}{t^2 + t^{-2} + 2 - 2} dt$$

$$= \int \frac{1 + t^{-2}}{(t - t^{-1})^2 + 2} dt$$

Let t-t^{-1} =u

1+x^{-2} dt=du

$$= \int \frac{du}{(u)^2 + \sqrt{2}^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{u}{\sqrt{2}} + c$$

put u=t-t^{-1}

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{t - t^{-1}}{\sqrt{2}} + c$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{t^2 - 1}{\sqrt{2}t} + c$$

put t=tan x

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{tan^2 x - 1}{\sqrt{2}\tan x} + c$$

Question 17.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\left(x^2 + 1\right)}{\left(x^4 + x^2 + 1\right)} dx = ?$$

A. $\tan^{-1}\frac{\left(x^2-1\right)}{\sqrt{3}}+C$

B. $\frac{1}{\sqrt{3}}\tan^{-1}\frac{\left(x^2-1\right)}{\sqrt{3}}+C$

C. $\frac{1}{\sqrt{3}}\tan^{-1}\frac{\left(x^2-1\right)}{\sqrt{3}x}+C$

D. none of these

Answer:

$$\int \frac{(x^2+1)}{(x^4+x^2+1)}dx = \int \frac{1+x^{-2}}{x^2+1+x^{-2}}dx$$

$$= \int \frac{1+x^{-2}}{x^2+1+x^{-2}+2-2}dx$$

$$= \int \frac{1+x^{-2}}{(x-x^{-1})^2+3}dx$$

Let $x-x^{-1}=t$

$1+x^{-2}\,dx=dt$

$$= \int \frac{dt}{(t)^2+\sqrt{3}^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\frac{t}{\sqrt{3}}+c$$

put $t=x-x^{-1}$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\frac{x-x^{-1}}{\sqrt{3}}+c$$

$$=\frac{1}{\sqrt{3}}\tan^{-1}\frac{x^2-1}{\sqrt{3}x}+c$$

Question 18.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin\ 2x}{\left(\sin^4 x+\cos^4 x\right)}dx=?$$

A. $\tan^{-1}(\tan^2 x) + C$

B. $x^2 + C$

C. - tan-1 ($\tan^2$ x) +C

D. none of these

Answer:
$$\int \frac{\sin 2x}{sin^4 x+cos^4 x}dx=\int \frac{2\sin x\cos x}{cos^4 x(tan^4 x+1)}dx$$

$$=\int \frac{2\tan x\ sec^2 x}{(tan^2 x)^2+1}dx$$

$$=\int \frac{2\tan x\ sec^2 x}{(sec^2 x-1)^2+1}dx$$

Let $\sec^2$ x-1=t

2 sec x sec x tan x dx=dt

$$=\int \frac{dt}{(t)^2+1}$$

We know, $\int \frac{1}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$=\tan^{-1} t +c$

put $t=\sec^2 x-1$

$= \tan^{-1} \sec^2 x - 1 + c$

$= \tan^{-1} \tan^2 x + c$

Question 19.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(1-9x^2\right)} = ?$$

A. $\frac{1}{3}\log\left|\frac{1+3x}{1-3x}\right|+C$

B. $\frac{1}{3}\log\left|\frac{1-3x}{1+3x}\right|+C$

C. $\frac{1}{6}\log\left|\frac{1+3x}{1-3x}\right|+C$

D. $\frac{1}{6}\log\left|\frac{1-3x}{1+3x}\right|+C$

Answer:
Consider $\int \frac{dx}{(1)^2-(3x)^2}$

3x=t

3dx=dt

$$dx = \frac{dt}{3}$$

$$= \frac{1}{3}\int \frac{dt}{1^2-(t)^2}$$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$$= \frac{1}{6}\log\frac{1+t}{1-t} + c$$

put t=3x

$$\frac{1}{6}\tan^{-1}\frac{1+3x}{1-3x}+c$$

Question 20.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(16-4x^2\right)} = ?$$

A. $\frac{1}{8}\log\left|\frac{2-x}{2+x}\right|+C$

B. $\frac{1}{16}\log\left|\frac{2-x}{2+x}\right|+C$

C. $\frac{1}{8}\log\left|\frac{2+x}{2-x}\right|+C$

D. $\frac{1}{16}\log\left|\frac{2+x}{2-x}\right|+C$

Answer:
Consider $\int \frac{dx}{(4)^2-(2x)^2}$

2x=t

2dx=dt

$$dx = \frac{dt}{2}$$

$$= \frac{1}{2}\int \frac{dt}{4^2-(t)^2}$$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x}+c$

$$= \frac{1}{16} \log \frac{4+t}{4-t} + c$$

put t=2x

$$= \frac{1}{16} \tan^{-1} \frac{4+2x}{4-2x} + c$$

$$= \frac{1}{16} \tan^{-1} \frac{2+x}{2-x} + c$$

Question 21.
Mark (√) against the correct answer in each of the following:

$$\int \frac{x^2}{\left(1-x^6\right)} dx = ?$$

A. $\frac{1}{6} \log \left| \frac{1+x^3}{1-x^3} \right| + C$

B. $\frac{1}{6} \log \left| \frac{1-x^3}{1+x^3} \right| + C$

C. $\frac{1}{3} \log \left| \frac{1-x^3}{1+x^3} \right| + C$

D. none of these

Answer:
$$= \int \frac{x^2}{(1)^2 - (x^3)^2}\, dx$$

Let x^3 =t

$3x^2$ dx=dt

$$x^2\, dx = \frac{dt}{3}$$

$$=\frac{1}{3}\int\frac{dt}{1^2-t^2}$$

We know, $\int\frac{1}{a^2-x^2}=\frac{1}{2a}\log\frac{a+x}{a-x}+c$

$$=\frac{1}{6}\log\frac{1+t}{1-t}+c$$

put t=x^3

$$=\frac{1}{6}\log\frac{1+x^3}{1-x^3}+c$$

Question 22.
Mark (√) against the correct answer in each of the following:

$$\int\frac{x}{\left(1-x^4\right)}dx=?$$

A. $\frac{1}{4}\log\left|\frac{1+x^2}{1-x^2}\right|+C$

B. $\frac{1}{4}\log\left|\frac{1-x^2}{1+x^2}\right|+C$

C. $\frac{1}{2}\log\left|\frac{1+x^2}{1-x^2}\right|+C$

D. none of these

Answer:

$$=\int\frac{x}{(1)^2-(x^2)^2}\,dx$$

Let x^2 =t

2x dx=dt

$x\,dx = \frac{dt}{2}$

$= \frac{1}{2}\int \frac{dt}{1^2 - t^2}$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$= \frac{1}{4}\log\frac{1+t}{1-t} + c$

put t=x^2

$= \frac{1}{4}\log\frac{1+x^2}{1-x^2} + c$

Question 23.
Mark (√) against the correct answer in each of the following:

$\int \frac{x^2}{\left(a^6 - x^6\right)} dx = ?$

A. $\frac{1}{3a^3}\log\left|\frac{a^3 + x^3}{a^3 - x^3}\right| + C$

B. $\frac{1}{6a^3}\log\left|\frac{a^3 + x^3}{a^3 - x^3}\right| + C$

C. $\frac{1}{6a^3}\log\left|\frac{a^3 - x^3}{a^3 + x^3}\right| + C$

D. none of these

Answer:
$= \int \frac{x^2}{(a^3)^2 - (x^3)^2}\,dx$

Let x^3 =t

$3x^2\,dx = dt$

$$x^2\,dx = \frac{dt}{3}$$

$$= \frac{1}{3}\int \frac{dt}{(a^3)^2 - t^2}$$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$$= \frac{1}{6a^3}\log\frac{a^3 + t}{a^3 - t} + c$$

put $t = x^3$

$$= \frac{1}{6a^3}\log\frac{a^3 + x^3}{a^3 - x^3} + c$$

Question 24.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(3 - 2x - x^2\right)} = ?$$

A. $\frac{1}{4}\log\left|\frac{3+x}{3-x}\right| + C$

B. $\frac{1}{4}\log\left|\frac{1+x}{1-x}\right| + C$

C. $\frac{1}{4}\log\left|\frac{3+x}{1-x}\right| + C$

D. none of these

Answer:

$$= -\int \frac{dx}{x^2 + 2x - 3}$$

Completing the square

$x^2 + 2x - 3 = x^2 + 2x - 3 + 1 - 1$

$(x+1)^2 - 4$

$$= -\int \frac{dx}{(x+1)^2 - 4}$$

Let x+1=t

dx=dt

$$= -\int \frac{dt}{t^2 - 2^2}$$

$$= -\int \frac{dt}{2^2 - t^2}$$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$$= \frac{1}{4} \log \frac{2+t}{2-t} + c$$

put t=x+1

$$= \frac{1}{4} \log \frac{2+x+1}{2-x-1} + c$$

$$= \frac{1}{4} \log \frac{x+3}{1-x} + c$$

Question 25.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(\cos^2 x - 3\sin^2 x\right)} = ?$$

A. $\frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right|+C$

B. $\frac{1}{\sqrt{3}}\log\left|\frac{1-\sqrt{3}\tan x}{1+\sqrt{3}\tan x}\right|+C$

C. $\frac{1}{2\sqrt{3}}\log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right|+C$

D. none of these

Answer:

$$\int \frac{1}{cos^2 x - 3sin^2 x}dx = \int \frac{1}{cos^2 x(1-3tan^2 x)}dx$$

$$= \int \frac{sec^2 x}{(1-(\sqrt{3}\tan x)^2)}dx$$

Let $\sqrt{3}$ tan x=t

$\sqrt{3}$ sec^2 x dx=dt

$$= \frac{1}{\sqrt{3}}\int \frac{dt}{1^2 - t^2}$$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$$= \frac{1}{2\sqrt{3}}\log\frac{1+t}{1-t} + c$$

put t=$\sqrt{3}$ tan x

$$= \frac{1}{2\sqrt{3}}\log\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x} + c$$

Question 26.

Mark (√) against the correct answer in each of the following:

$$\int \frac{\operatorname{cosec}^2 x}{\left(1-\cot^2 x\right)} dx = ?$$

A. $\frac{1}{2}\log\left|\frac{1+\cot x}{1-\cot x}\right| + C$

B. $-\frac{1}{2}\log\left|\frac{1+\cot x}{1-\cot x}\right| + C$

C. $\frac{1}{2}\log\left|\frac{1-\cot x}{1+\cot x}\right| + C$

D. none of these

Answer:

$$\int \frac{cosec^2 x}{1-cot^2 x} dx$$

Let cot x=t

-cosec^2 x dx=dt

$$= -\int \frac{dt}{1^2 - t^2}$$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$$= \frac{-1}{2}\log\frac{1+t}{1-t} + c$$

put t=cot x

$$= \frac{-1}{2}\log\frac{1+\cot x}{1-\cot x} + c$$

Question 27.

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(4x^2-1\right)} = ?$$

A. $\frac{1}{2}\log\left|\frac{2x-1}{2x+1}\right| + C$

B. $\frac{1}{2}\log\left|\frac{2x+1}{2x-1}\right| + C$

C. $\frac{1}{4}\log\left|\frac{2x-1}{2x+1}\right| + C$

D. none of these

Answer:
Consider

$$\int \frac{dx}{(2x)^2 - 1^2}$$

2x=t

2dx=dt

$$dx = \frac{dt}{2}$$

$$= \frac{1}{2}\int \frac{dt}{t^2 - 1^2}$$

We know, $\int \frac{1}{x^2-a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} + c$

$$= \frac{1}{4}\log\frac{t-1}{t+1} + c$$

put t=2x

$$= \frac{1}{4} \log \frac{2x-1}{2x+1} + c$$

Question 28.

Mark (√) against the correct answer in each of the following:

$$\int \frac{x}{\left(x^4 - 16\right)} dx = ?$$

A. $\frac{1}{4} \log \left| \frac{x^2+4}{x^2-4} \right| + C$

B. $\frac{1}{16} \log \left| \frac{x^2+4}{x^2-4} \right| + C$

C. $\frac{1}{16} \log \left| \frac{x^2-4}{x^2+4} \right| + C$

D. none of these

Answer:

$$= \int \frac{x}{(x^2)^2 - (4)^2} dx$$

Let $x^2 = t$

$2x\, dx = dt$

$$x\, dx = \frac{dt}{2}$$

$$= \frac{1}{2} \int \frac{1}{(t)^2 - (4)^2} dt$$

We know, $\int \frac{1}{x^2-a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + c$

$$= \frac{1}{16} \log \frac{t-4}{t+4} + c$$

put t=x^2

$$= \frac{1}{16} \log \frac{x^2 - 4}{x^2 + 4} + c$$

Question 29.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(\sin^2 x - 4\cos^2 x\right)} = ?$$

A. $\frac{1}{4} \log \left| \frac{\tan x - 2}{\tan x + 2} \right| + C$

B. $\frac{1}{4} \log \left| \frac{\tan x + 2}{\tan x - 2} \right| + C$

C. $\frac{1}{4} \log \left| \frac{1 - \tan x}{1 + \tan x} \right| + C$

D. none of these

Answer:

$$\int \frac{1}{sin^2 x - 4cos^2 x} dx = \int \frac{1}{cos^2 x(tan^2 x - 4)} dx$$

$$= \int \frac{sec^2 x}{((\tan x)^2 - 2^2)} dx$$

Let tan x=t

sec^2 x dx=dt

$$= \int \frac{dt}{t^2 - 2^2}$$

We know, $\int \frac{1}{x^2 - a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + c$

$$=\frac{1}{4}\log\frac{t-2}{t+2}+c$$

put t=tan x

$$=\frac{1}{4}\log\frac{\tan x\ -2}{\tan x\ +2}+c$$

Question 30.
Mark (√) against the correct answer in each of the following:

$$\int\frac{dx}{\left(4\sin^2 x+5\cos^2 x\right)}=?$$

A. $\frac{1}{2}\tan^{-1}\left(\frac{\tan x}{\sqrt{5}}\right)+C$

B. $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{\tan x}{\sqrt{5}}\right)+C$

C. $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right)+C$

D. none of these

Answer:

$$\int\frac{1}{4sin^2x+5cos^2x}dx=\int\frac{1}{cos^2x(4tan^2x+5)}dx$$

$$\int\frac{sec^2x}{((2\tan x)^2+\sqrt{5}^2)}dx$$

Let 2 tan x=t

2 $\sec^2$ x dx=dt

$$=\frac{1}{2}\int\frac{dt}{t^2+\sqrt{5}^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{2\sqrt{5}}\tan^{-1}\frac{t}{\sqrt{5}} + c$$

put t=2 tan x

$$= \frac{1}{2\sqrt{5}}\tan^{-1}\frac{2\tan x}{\sqrt{5}} + c$$

Question 31.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{\sin 3x} dx = ?$$

A. $\frac{1}{2\sqrt{3}} \log \left|\frac{\sqrt{3}+\sin x}{\sqrt{3}-\sin x}\right| + C$

B. $\frac{1}{2\sqrt{3}} \log \left|\frac{\sqrt{3}+\cos x}{\sqrt{3}-\cos x}\right| + C$

C. $\frac{1}{2\sqrt{3}} \log \left|\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right| + C$

D. none of these

Answer:
$$\int \frac{\sin x}{\sin 3x} dx = \int \frac{\sin x}{3\sin x - 4sin^3 x} dx$$

$$= \int \frac{1}{3 - 4sin^2 x} dx$$

$$= \int \frac{1}{cos^2 x(3sec^2 x - 4tan^2 x)} dx$$

$$= \int \frac{sec^2 x}{3(1 + tan^2 x) - 4tan^2 x} dx$$

$$= \int \frac{sec^2 x}{3 - tan^2 x} dx$$

Let tan x=t

sec^2 x dx=dt

$$= \int \frac{dt}{\sqrt{3}^2 - t^2}$$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$$= \frac{1}{2\sqrt{3}} \log \frac{\sqrt{3} + t}{\sqrt{3} - t} + c$$

put t= tan x

$$= \frac{1}{2\sqrt{3}} \log \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} + c$$

Question 32.
Mark (√) against the correct answer in each of the following:

$$\int \frac{(x^2 + 1)}{(x^4 + 1)} dx = ?$$

A. $\frac{1}{2} \tan^{-1} \left(\frac{x^2 + 1}{\sqrt{2}x} \right) + C$

B. $\frac{1}{2} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right) + C$

C. $\frac{1}{\sqrt{2}} \log \left(\frac{x^2 + 1}{x^2 - 1} \right) + C$

D. none of these

Answer:

$$\int \frac{(x^2+1)}{(x^4+1)} dx = \int \frac{1+x^{-2}}{x^2+x^{-2}} dx$$

$$= \int \frac{1+x^{-2}}{x^2+x^{-2}+2-2} dx$$

$$= \int \frac{1+x^{-2}}{(x-x^{-1})^2+2} dx$$

Let $x-x^{-1}=t$

$1+x^{-2}$ dx=dt

$$= \int \frac{dt}{(t)^2+\sqrt{2}^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}}+c$$

put $t=x-x^{-1}$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{x-x^{-1}}{\sqrt{2}}+c$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{x^2-1}{\sqrt{2}x}+c$$

Objective Questions Ii

Question 1.

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{4-9x^2}} = ?$$

A. $\frac{1}{3}\sin^{-1}\frac{x}{3}+C$

B. $\frac{2}{3}\sin^{-1}\left(\frac{2x}{3}\right)+C$

C. $\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)+C$

D. none of these

Answer:

$$\int \frac{dx}{\sqrt{4-9x^2}} = \int \frac{1}{3}\frac{dx}{\sqrt{\frac{4}{9}-x^2}}$$

$$= \int \frac{1}{3}\frac{dx}{\sqrt{\left(\frac{2}{3}\right)^2 - x^2}}$$

$$= \frac{1}{3}\sin^{-1}\frac{x}{\frac{2}{3}} + c$$

$$= \frac{1}{3}\sin^{-1}\frac{3x}{2} + c.$$

Question 2.

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{16-4x^2}} = ?$$

A. $\frac{1}{2}\sin^{-1}\frac{x}{2}+C$

B. $\frac{1}{4}\sin^{-1}\frac{x}{2}+C$

C. $\frac{1}{2}\sin^{-1}\frac{x}{4}+C$

D. none of these

Answer:

$$\int \frac{dx}{\sqrt{16-4x^2}} = \int \frac{1}{2}\frac{dx}{\sqrt{\frac{16}{4}-x^2}}$$

$$= \int \frac{1}{2}\frac{dx}{\sqrt{(2)^2-x^2}}$$

$$= \frac{1}{2}\sin^{-1}\frac{x}{2}+c$$

Question 3.

Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos x}{\sqrt{4-\sin^2 x}} = ?$$

A. $\sin^{-1}\frac{x}{2}+C$

B. $\sin^{-1}\left(\frac{1}{2}\cos x\right)+C$

C. $\sin^{-1}(2\sin x)+C$

D. $\sin^{-1}\left(\frac{1}{2}\sin x\right)+C$

Answer:

Put sin x =t

⇒ cos x dx = dt

∴ The given equation becomes

$$\int \frac{dt}{\sqrt{4-t^2}}$$

$$= \sin^{-1}\frac{t}{2} + c$$

But t = sin x

$$= \sin^{-1}\left(\frac{\sin x}{2}\right) + c$$

Question 4.
Mark (√) against the correct answer in each of the following:

$$\int \frac{2^x}{\sqrt{1-4^x}}\,dx = ?$$

A. $\sin^{-1}(2^x)\log 2 + C$

B. $\dfrac{\sin^{-1}(2^x)}{\log 2} + C$

C. $\sin^{-1}(2^x) + C$

D. none of these

Answer:
⇒ Let $t=2^x$

$dt = \log 2.\ 2^x.dx$

$$\Rightarrow \frac{dt}{\log 2} = 2^x.dx$$

$$= \int \frac{dt}{\log 2\sqrt{1-t^2}}$$

$$= \frac{1}{\log 2} \int \frac{dt}{\sqrt{1 - t^2}}$$

$$= \frac{1}{\log 2} \sin^{-1} t$$

But t $=2^x$

$$= \frac{1}{\log 2} \sin^{-1}(2^x)$$

Question 5.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{2x - x^2}} = ?$$

A. $\sin^{-1}(x + 1) + C$

B. $\sin^{-1}(x - 2) + C$

C. $\sin^{-1}(x - 1) + C$

D. none of these

Answer:

$$\int \frac{dx}{\sqrt{2x - x^2}} = \int \frac{dx}{\sqrt{2x - x^2 + 1 - 1}}$$

$$= \int \frac{dx}{\sqrt{-x^2 + 2x - 1 + 1}}$$

$$= \int \frac{dx}{\sqrt{1 - (x - 1)^2}}$$

$=\sin^{-1}(x-1)+c$

Question 6.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{x(1 - 2x)} = ?$$

A. $\frac{1}{\sqrt{2}}\sin^{-1}(2x-1)+C$

B. $\frac{1}{\sqrt{2}}\sin^{-1}(2x+1)+C$

C. $\frac{1}{\sqrt{2}}\sin^{-1}(4x+1)+C$

D. $\frac{1}{\sqrt{2}}\sin^{-1}(4x-1)+C$

Answer:

$$\int \frac{dx}{\sqrt{x-2x^2}} = \int \frac{dx}{\sqrt{2}\sqrt{-x^2+\frac{1}{2}x}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{-\left(x^2-\frac{1}{2}x\right)}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{-\left(x^2-\frac{1}{2}x\right)+\frac{1}{16}-\frac{1}{16}}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{-\left(x^2-\frac{1}{2}x+\frac{1}{16}\right)+\frac{1}{16}}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{\frac{1}{16}-\left(x-\frac{1}{4}\right)^2}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{\left(\frac{1}{4}\right)^2-\left(\frac{4x-1}{4}\right)^2}}$$

$$= \frac{1}{\sqrt{2}}(\sin^{-1}\left(\frac{\frac{4x-1}{4}}{\frac{1}{4}}\right)$$

$$= \frac{1}{\sqrt{2}} \sin^{-1}(4x - 1)$$

Question 7.
Mark (√) against the correct answer in each of the following:

$$\int \frac{3x^2}{\sqrt{9-16x^6}} dx = ?$$

A. $\frac{1}{4}\sin^{-1}\left(\frac{x^3}{3}\right) + C$

B. $\frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right) + C$

C. $4\sin^{-1}\left(\frac{x^3}{4}\right) + C$

D. none of these

Answer:
$\Rightarrow \int \frac{3x^2 dx}{\sqrt{9-16x^6}}$

Let $x^3 = t$

$\therefore 3x^2 dx = dt$

$\therefore x^6 = t^2$

$\Rightarrow \int \frac{1}{4} \frac{dt}{\sqrt{\frac{9}{16} - t^2}}$

$\Rightarrow \frac{1}{4} \sin^{-1}\left(\frac{4t}{3}\right) + c$

But $t = x^3$

$\Rightarrow \frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right) + c$

Question 8.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{2 + 2x - x^2}} = ?$$

A. $\sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right) + C$

B. $\sin^{-1}\left(\frac{x-1}{\sqrt{2}}\right) + C$

C. $\sin^{-1}\sqrt{3}(x-1) + C$

D. none of these

Answer:
$\Rightarrow \int \frac{dx}{\sqrt{2+2x-x^2}} = \int \frac{dx}{\sqrt{2x-x^2+2+3-3}}$

$\Rightarrow \int \frac{dx}{\sqrt{-((x^2-2x+1)-3)}}$

$\Rightarrow \int \frac{dx}{\sqrt{3-(x-1)^2}}$

$\Rightarrow \sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right) + c.$

Question 9.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{16 - 6x - x^2}} = ?$$

A. $\sin^{-1}\left(\frac{x-3}{5}\right) + C$

B. $\sin^{-1}\left(\frac{x+3}{5}\right)+C$

C. $\frac{1}{5}\sin^{-1}(x+3)+C$

D. none of these

Answer:
$$\int \frac{dx}{\sqrt{16-6x-x^2}} = \int \frac{dx}{\sqrt{-x^2-6x-9+16+9}}$$

$$= \int \frac{dx}{\sqrt{25-(x+3)^2}}$$

$$= \sin^{-1}\left(\frac{x+3}{5}\right) + c.$$

Question 10.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x-x^2}} = ?$$

A. $\sin^{-1}(x - 1) + C$

B. $\sin^{-1}(x + 1) + C$

C. $\sin^{-1}(2x - 1) + C$

D. none of these

Answer:
$$\int \frac{dx}{\sqrt{x-x^2}} = \int \frac{dx}{\sqrt{-x^2+x-}}$$

$$= \int \frac{dx}{\sqrt{-(x^2-x)+\frac{1}{4}-\frac{1}{4}}}$$

$$= \int \frac{dx}{\sqrt{-\left(x^2 - x + \frac{1}{4}\right) + \frac{1}{4}}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}}$$

$$= \sin^{-1}\left(\frac{\frac{2x-1}{2}}{\frac{1}{2}}\right) + c$$

$= \sin^{-1}(2x-1)+c$

Question 11.

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{1+2x-3x^2}} = ?$$

A. $\frac{1}{\sqrt{3}} \sin^{-1}\left(\frac{3x-1}{2}\right) + C$

B. $\frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{2x-1}{3}\right) + C$

C. $\frac{1}{\sqrt{3}} \sin^{-1}\left(\frac{2x-1}{3}\right) + C$

D. none of these

Answer:

$$\int \frac{dx}{\sqrt{1+2x-3x^2}} = \int \frac{dx}{\sqrt{3}\sqrt{-x^2 + \frac{2}{3}x + \frac{1}{3}}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{-\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right)}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{-\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right) + \frac{1}{9} - \frac{1}{9}}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{-\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) + \frac{1}{3} + \frac{1}{9}}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{\frac{4}{9} - \left(x - \frac{1}{3}\right)^2}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{\left(\frac{2}{3}\right)^2 - \left(\frac{3x-1}{3}\right)^2}}$$

$$= \frac{1}{\sqrt{3}}(\sin^{-1}\left(\frac{\frac{3x-1}{3}}{\frac{2}{3}}\right)$$

$$= \frac{1}{\sqrt{3}}\sin^{-1}\left(\frac{3x-1}{2}\right)$$

Question 12.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2 - 16}} = ?$$

A. $\sin^{-1}\left(\frac{x}{4}\right) + C$

B. $\log\left|x + \sqrt{x^2 - 16}\right| + C$

C. $\log\left|x - \sqrt{x^2 - 16}\right| + C$

D. none of these

Answer:

We know

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right|$$

$$\int \frac{dx}{\sqrt{x^2 - 4^2}} = \log\left|x + \sqrt{x^2 - 16}\right|$$

Question 13.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{4x^2 - 9}} = ?$$

A. $\frac{1}{2}\log\left|2x + \sqrt{4x^2 - 9}\right| + C$

B. $\frac{1}{4}\log\left|x + \sqrt{4x^2 - 9}\right| + C$

C. $\log\left|2x + \sqrt{4x^2 - 9}\right| + C$

D. none of these

Answer:

$$\int \frac{dx}{\sqrt{(2x)^2 - (3)^2}}$$

Put t =2x

dt =2 dx

$\Rightarrow dx = \frac{dt}{2}$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t^2 - 9}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$$

$$= \frac{1}{2} \log |t + \sqrt{t^2 - 9}$$

But t = 2x

$$= \frac{1}{2} \log |2x + \sqrt{4x^2 - 9}|$$

Question 14.
Mark (√) against the correct answer in each of the following:

$$\int \frac{x^2}{x^6 - 1} dx = ?$$

A. $\frac{1}{2} \log \left| x^3 + \sqrt{x^6 - 1} \right| + C$

B. $\frac{1}{3} \log \left| x^3 + \sqrt{x^6 - 1} \right| + C$

C. $\frac{1}{3} \log \left| x^3 - \sqrt{x^6 - 1} \right| + C$

D. none of these

Answer:
$\Rightarrow \int \frac{x^2\, dx}{\sqrt{(x^3)^2 - (1)^2}}$

Put t $=x^3$

dt $=3x^2$dx

$$\Rightarrow \; dx = \frac{dt}{3x^2}$$

$$\Rightarrow \; \frac{1}{3} \int \frac{1}{x^2} \frac{x^2 dt}{\sqrt{t^2 - 1}}$$

$$\Rightarrow \; \int \frac{dx}{\sqrt{x^2 - a^2}} = log \left| x + \sqrt{x^2 - a^2} \right|$$

$= \frac{1}{3} \log |t + \sqrt{t^2 - 1}$

But t = x^3

$= \frac{1}{3} \log |x^3 + \sqrt{x^6 - 1}|$

Question 15.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{\sqrt{4\cos^2 x - 1}} = ?$$

A. $-\frac{1}{2} \log \left|2\cos x + \sqrt{4\cos^2 x - 1}\right| + C$

B. $-\frac{1}{3} \log \left|2\cos x + \sqrt{4\cos^2 x - 1}\right| + C$

C. $-\frac{1}{6} \log \left|2\cos x + \sqrt{2\cos^2 x - 1}\right| + C$

D. none of these

Answer:
$\Rightarrow \int \frac{\sin x dx}{\sqrt{(2\cos x)^2 - (1)^2}}$

Put t =2cos x

dt =-2sinxdx

$\Rightarrow dx = -\frac{dt}{2 \sin x}$

$= -\frac{1}{2} \int \frac{dt}{\sqrt{t^2 - 1}}$

$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log \left|x + \sqrt{x^2 - a^2}\right|$

$$= -\frac{1}{2}\log|t + \sqrt{t^2 - 1}$$

But t = 2cosx

$$\Rightarrow -\frac{1}{2}\log|2\cos x + \sqrt{4\cos^2 x - 1}|$$

Question 16.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\sqrt{\tan^2 x - 4}} dx = ?$$

A. $\log\left|\tan x - \sqrt{\tan^2 x - 4}\right| + C$

B. $\log\left|\tan x + \sqrt{\tan^2 x - 4}\right| + C$

C. $\frac{1}{2}\log\left|\tan x + \sqrt{\tan^2 x - 4}\right| + C$

D. none of these

Answer:

$$\int \frac{\sec^2 x\, dx}{\sqrt{(\tan x)^2 - (1)^2}}$$

Put t =tanx

dt = $\sec^2$x

$$\Rightarrow dx = -\frac{dt}{sec^2 x}$$

$$= \int \frac{1}{sec^2 x} \frac{sec^2 x\, dt}{\sqrt{t^2 - 1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$$

$$= \log|t + \sqrt{t^2 - 1}|$$

But t = tanx

$$= \log|\tan x + \sqrt{4\tan^2 x - 1}|$$

Question 17.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(1 - e^{2x}\right)} = ?$$

A. $\log\left|e^{x} + \sqrt{e^{2x} - 1}\right| + C$

B. $\log\left|e^{-x} + \sqrt{e^{-2x} - 1}\right| + C$

C. $-\log\left|e^{-x} + \sqrt{e^{-2x} - 1}\right| + C$

D. none of these

Answer:
Differentiating both side with respect to t

$$-2e^{2x}\frac{dx}{dt} = 1 \Rightarrow dx = -\frac{1}{2}\frac{dt}{1 - t}$$

$$y = -\frac{1}{2}\int \frac{1}{(1 - t)t} dt$$

$$y = -\frac{1}{2}\int \frac{t + (1 - t)}{(1 - t)t} dt$$

$$y = -\frac{1}{2}\int \frac{1}{(1 - t)} + \frac{1}{t} dt$$

$$y = -\frac{1}{2}(-\log(1 - t) + \log t) + c$$

Again put, $t = 1 - e^{2x}$

$$y = -\frac{1}{2}(-\log e^{2x} + \log(1 - e^{2x})) + c$$

$$y = -\log\sqrt{\frac{1 - e^{2x}}{e^{2x}}} + c$$

$$y = -\log\sqrt{e^{-2x} - 1} + c$$

Question 18.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2 - 3x + 2}} = ?$$

A. $\log\left|\left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + 2}\right| + C$

B. $\log\left|x + \sqrt{x^2 - 3x + 2}\right| + C$

C. $\log\left|x - \sqrt{x^2 - 3x + 2}\right| + C$

D. none of these

Answer:

$$\int \frac{dx}{\sqrt{x^2 - 3x + 2}} = \int \frac{dx}{\sqrt{x^2 - 3x + 2 + \frac{9}{4} - \frac{9}{4}}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 3x + \frac{9}{4} - \frac{1}{4}}}$$

$$= \int \frac{dx}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \frac{1}{4}}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$$

$$= \log|\left(x - \tfrac{3}{2}\right) + \sqrt{x^2 - 3x + 2}.$$

Question 19.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}} dx = ?$$

A. $\log\left|\sin x + \sqrt{\sin^2 x - 2\sin x - 3}\right| + C$

B. $\log\left|(\sin x - 1) + \sqrt{\sin^2 x - 2\sin x - 3}\right| + C$

C. $\log\left|(\sin x - 1) - \sqrt{\sin^2 x - 2\sin x - 3}\right| + C$

D. none of these

Answer:
$$\Rightarrow \int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}} dx$$

Let t =sin x

dt =cos x dx

$$\Rightarrow\ dx = \frac{dt}{\cos x}$$

$$= \frac{\cos x\, dt}{\cos x \sqrt{t^2 - 2t - 3 + 2 - 2}}$$

$$= \frac{dt}{\sqrt{(t^2 - 2t + 2) - 5}}$$

$$= \frac{dt}{\sqrt{(t-1)^2 - 5}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$$

$$\Rightarrow \int \frac{dt}{\sqrt{(t-1)^2 - 5}} = log\,|t - 1 + \sqrt{t^2 - 2t - 3}|$$

But t =sin x

$$\therefore\ log\,|\,sin\,x - 1 + \sqrt{sin^2\,x - 2\,sin\,x - 3}|$$

Question 20.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{2 - 4x + x^2}} = ?$$

A. $\log\left|(x-2) + \sqrt{x^2 - 4x + 2}\right| + C$

B. $\log\left|x + \sqrt{x^2 - 4x + 2}\right| + C$

C. $\log\left|x - \sqrt{x^2 - 4x + 2}\right| + C$

D. none of these

Answer:

$$\int \frac{dx}{\sqrt{x^2 - 4x + 2}} = \int \frac{dx}{\sqrt{x^2 - 4x + 2 + 4 - 4}}$$

$$= \int \frac{dx}{\sqrt{(x-2)^2 - 2}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-2)^2-2}} = log\left|x-2+\sqrt{x^2-4x+2}\right|$$

Question 21.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2+6x+5}} = ?$$

A. $\log\left|x+\sqrt{x^2+6x+5}\right|+C$

B. $\log\left|x-\sqrt{x^2+6x+5}\right|+C$

C. $\log\left|(x+3)+\sqrt{x^2+6x+5}\right|+C$

D. none of these

Answer:

$$\int \frac{dx}{\sqrt{x^2+6x+5}} = \int \frac{dx}{\sqrt{x^2+6x+5+9-9}}$$

$$= \int \frac{dx}{\sqrt{(x+3)^2-4}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x+3)^2-4}} = log\left|x+3+\sqrt{x^2+6x+5}\right|$$

Question 22.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{(x-3)^2-1}} = ?$$

A. $\log\left|(x-3)+\sqrt{x^2-6x+8}\right|+C$

B. $\log\left|x+\sqrt{x^2-6x+8}\right|+C$

C. $\log\left|(x-3)-\sqrt{x^2-6x+8}\right|+C$

D. none of these

Answer:

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-3)^2-1}} = log\left|x-3+\sqrt{x^2-6x+9-1}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-3)^2-1}} = log\left|x-3+\sqrt{x^2-6x+8}\right|$$

Question 23.

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2-6x+10}} = ?$$

A. $\log\left|x+\sqrt{x^2-6x+10}\right|+C$

B. $\log\left|(x-3)+\sqrt{x^2-6x+10}\right|+C$

C. $\log\left|x-\sqrt{x^2-6x+10}\right|+C$

D. none of these

Answer:

$$\int \frac{dx}{\sqrt{x^2-6x+10}} = \int \frac{dx}{\sqrt{x^2-6x+10+9-9}}$$

$$= \int \frac{dx}{\sqrt{(x-3)^2+1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-3)^2+1}} = log\left|x+3+\sqrt{x^2-6x+10}\right|$$

Question 24.
Mark (√) against the correct answer in each of the following:

$$\int \frac{x^2 dx}{\sqrt{x^6+a^6}} dx = ?$$

A. $\frac{1}{3}\log\left|x^6+a^6\right|+C$

B. $\frac{1}{3}\tan^{-1}\left(\frac{x^3}{a^3}\right)+C$

C. $\frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right|+C$

D. none of these

Answer:

$$\int \frac{x^2 dx}{\sqrt{(x^3)^2+(a)^6}}$$

Put t $=x^3$

dt $=3x^2$dx

$$\Rightarrow dx = \frac{dt}{3x^2}$$

$$= \frac{1}{3}\int \frac{1}{x^2}\frac{x^2 dt}{\sqrt{t^2+a^6}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$= \frac{1}{3} \log|t+\sqrt{t^2+a^6}$$

But t = x^3

$$= \frac{1}{3} log\,|x^3+\sqrt{x^6+a^6}|+c.$$

Question 25.
Mark (√) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\sqrt{16+\tan^2 x}} dx = ?$$

A. $\log\left|\tan\ x+\sqrt{\tan^2 x+16}\right|+C$

B. $\log\left|x+\sqrt{\tan^2 x+16}\right|+C$

C. $\log\left|\tan\ x-\sqrt{\tan^2 x+16}\right|+C$

D. none of these

Answer:

$$\int \frac{\sec^2 x\, dx}{\sqrt{(\tan x)^2+(4)^2}}$$

Put t =tan x

dt = $\sec^2 x$

$$\Rightarrow\ dx = \frac{dt}{sec^2\, x}$$

$$= \int \frac{1}{\sec^2 x} \frac{\sec^2 x \, dt}{\sqrt{t^2 + 16}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 + a^2}} = log\left|x + \sqrt{x^2 + a^2}\right|$$

$$= \log|t + \sqrt{t^2 + 16}$$

But t = tan x

$$= \log|\tan x + \sqrt{\tan^2 x + 16}|$$

Question 26.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{3x^2 + 6x + 12}} = ?$$

A. $\log\left|(x+1) + \sqrt{x^2 + 2x + 4}\right| + C$

B. $\frac{1}{3}\log\left|(x+1) + \sqrt{x^2 + 2x + 4}\right| + C$

C. $\frac{1}{\sqrt{3}}\log\left|(x+1) + \sqrt{x^2 + 2x + 4}\right| + C$

D. none of these

Answer:

$$\int \frac{dx}{\sqrt{3x^2 + 6x + 12}} = \int \frac{1}{\sqrt{3}} \frac{dx}{\sqrt{x^2 + 2x + 4}}$$

$$= \int \frac{1}{\sqrt{3}} \frac{dx}{\sqrt{x^2 + 2x + 3 + 1}}$$

$$= \int \frac{1}{\sqrt{3}} \frac{dx}{\sqrt{(x+1)^2 + 3}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$\Rightarrow \frac{1}{\sqrt{3}}\int \frac{dx}{\sqrt{(x+1)^2+3}} = log\left|x+1+\sqrt{x^2+2x+4}\right|$$

Question 27.
Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{2x^2+4x+6}} = ?$$

A. $\frac{1}{2}\log\left|(x+1)+\sqrt{x^2+2x+3}\right|+C$

B. $\frac{1}{\sqrt{2}}\log\left|(x+1)+\sqrt{x^2+2x+3}\right|+C$

C. $\frac{1}{\sqrt{2}}\log\left|x+\sqrt{x^2+2x+3}\right|+C$

D. none of these

Answer:

$$\int \frac{dx}{\sqrt{2x^2+4x+6}} = \int \frac{1}{\sqrt{2}}\frac{dx}{\sqrt{x^2+2x+3}}$$

$$= \int \frac{1}{\sqrt{2}}\frac{dx}{\sqrt{x^2+2x+1+2}}$$

$$= \int \frac{1}{\sqrt{2}}\frac{dx}{\sqrt{(x+1)^2+2}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$\Rightarrow \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{(x+1)^2+2}} = log\left|x+1+\sqrt{x^2+2x+3}\right|$$

Question 28.

Mark (√) against the correct answer in each of the following:

$$\int \frac{x^2}{\sqrt{x^6 + 2x^3 + 3}} dx = ?$$

A. $\frac{1}{3}\log\left|\left(x^3 + 1\right) + \sqrt{x^6 + 2x^3 + 3}\right| + C$

B. $\log\left|x^3 + \sqrt{x^6 + 2x^3 + 3}\right| + C$

C. $\frac{1}{3}\log\left|\left(x^3 + 1\right) - \sqrt{x^6 + 2x^3 + 3}\right| + C$

D. none of these

Answer:

$$\int \frac{x^2 dx}{\sqrt{x^6 + 2x^3 + 3}}$$

Let $x^3 = t$

$\Rightarrow 3x^2 dx = dt$

$$\Rightarrow \frac{dt}{3x^2} = dx$$

$$\int \frac{x^2 dt}{3x^2\sqrt{t^2 + 2t + 3}} = \frac{1}{3}\int \frac{dt}{\sqrt{t^2 + 2t + 3}}$$

$$= \int \frac{1}{3}\frac{dx}{\sqrt{t^2 + 2t + 1 + 2}}$$

$$= \int \frac{1}{3}\frac{dx}{\sqrt{(t + 1)^2 + 2}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 + a^2}} = log\left|x + \sqrt{x^2 + a^2}\right|$$

$$\Rightarrow \frac{1}{3}\int \frac{dx}{\sqrt{(t+1)^2+2}} = log\left|t+1+\sqrt{t^2+2t+3}\right|$$

But t =x^3

$$= \log\left|x^3+1+\sqrt{x^6+2x^3+3}\right|$$

Question 29.
Mark (√) against the correct answer in each of the following:

$$\int \sqrt{4-x^2}\,dx = ?$$

A. $\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+C$

B. $x\sqrt{4-x^2}+\sin^{-1}\frac{x}{2}+C$

C. $\frac{1}{2}x\sqrt{4-x^2}-2\sin^{-1}\frac{x}{2}+C$

D. none of these

Answer:
We know

$$\Rightarrow \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\left(\frac{x}{a}\right)+C$$

$$\Rightarrow \int \sqrt{2^2-x^2} = \frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}sin^{-1}\left(\frac{x}{2}\right)+C$$

$$\Rightarrow \int \sqrt{4-x^2} = \frac{x}{2}\sqrt{4-x^2}+2\,sin^{-1}\left(\frac{x}{2}\right)+C$$

Question 30.
Mark (√) against the correct answer in each of the following:

$$\int \sqrt{1-9x^2}\,dx = ?$$

A. $\frac{x}{2}\sqrt{1-9x^2}+\frac{1}{18}\sin^{-1}3x+C$

B. $\frac{3x}{2}\sqrt{1-9x^2}+\frac{1}{6}\sin^{-1}3x+C$

C. $\frac{x}{2}\sqrt{1-9x^2}+\frac{1}{6}\sin^{-1}3x+C$

D. none of these

Answer:
We know

$$\Rightarrow \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\left(\frac{x}{a}\right)+C$$

$$\Rightarrow \sqrt{1^2-(3x)^2} = 3\sqrt{\frac{1}{9}-x^2}$$

$$\Rightarrow 3\sqrt{\frac{1}{9}-x^2} = \frac{3x}{2}\sqrt{\frac{1}{9}-x^2}+\frac{\frac{1}{9}}{2}sin^{-1}\left(\frac{x}{\frac{1}{3}}\right)+C$$

$$\Rightarrow \sqrt{1^2-(3x)^2} = \frac{x}{2}\sqrt{1-9x^2}+\frac{3}{18}sin^{-1}(3x)+C$$

$$\Rightarrow \sqrt{1^2-(3x)^2} = \frac{x}{2}\sqrt{1-9x^2}+\frac{1}{6}sin^{-1}(3x)+C$$

Question 31.
Mark (√) against the correct answer in each of the following:

$$\int\sqrt{9-4x^2}\,dx = ?$$

A. $\frac{x}{2}\sqrt{9-4x^2}+\frac{9}{4}\sin^{-1}\frac{2x}{3}+C$

B. $x\sqrt{9-4x^2}+\frac{9}{2}\sin^{-1}\frac{2x}{3}+C$

C. $\frac{x}{2}\sqrt{9-4x^2}-\frac{9}{4}\sin^{-1}\frac{2x}{3}+C$

D. none of these

Answer:
We know

$$\Rightarrow \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\left(\frac{x}{a}\right)+C$$

$$\Rightarrow \sqrt{3^2-(2x)^2} = 2\sqrt{\frac{9}{4}-x^2}$$

$$\Rightarrow 2\sqrt{\frac{9}{4}-x^2}. = \frac{x}{2}\sqrt{\frac{9}{4}-x^2}+\frac{\frac{9}{4}}{2}sin^{-1}\left(\frac{x}{\frac{3}{2}}\right)+C$$

$$\Rightarrow \sqrt{9-4x^2} = \frac{x}{2}\sqrt{9-4x^2}+\frac{2.9}{8}sin^{-1}(2x)+C$$

$$\Rightarrow \sqrt{9-4x^2} = \frac{x}{2}\sqrt{9-4x^2}+\frac{9}{4}sin^{-1}(2x)+C$$

Question 32.
Mark (√) against the correct answer in each of the following:

$$\int \cos x\sqrt{9-\sin^2 x}\,dx = ?$$

A. $\frac{1}{2}\sin x\sqrt{9-\sin^2 x}+\frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right)+C$

B. $\frac{\sin x}{2}\sqrt{9-\sin^2 x}+\frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right)+C$

C. $\frac{1}{2}\cos x\sqrt{9-\sin^2 x}+\frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right)+C$

D. none of these

Answer:
Given: $\int \cos x \sqrt{9-\sin^2 x}\, dx$

Let sin x =t

cos x dx =dt

$$\Rightarrow \frac{dt}{\cos x} = dx$$

$$= \frac{dt}{\cos x}\sqrt{9-\sin^2 x}\cos x$$

$$= \sqrt{9-t^2}\, dt$$

$$\Rightarrow \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\Rightarrow \int \sqrt{3^2-t^2} = \frac{t}{2}\sqrt{9-t^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + C$$

But t =sin x

$$\Rightarrow \int \cos x\sqrt{9-\sin^2 x} = \frac{\sin x}{2}\sqrt{9-\sin^2 x} + \frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right) + C$$

Question 33.
Mark (√) against the correct answer in each of the following:

$$\int \sqrt{x^2-16}\, dx = ?$$

A. $x\sqrt{x^2-16} - 4\log\left|x+\sqrt{x^2-16}\right| + C$

B. $\frac{x}{2}\sqrt{x^2-16} - 8\log\left|x+\sqrt{x^2-16}\right| + C$

C. $\frac{x}{2}\sqrt{x^2-16} + 8\log\left|x+\sqrt{x^2-16}\right| + C$

D. none of these

Answer:
We know

$$\Rightarrow \int \sqrt{x^2-a^2} = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} log\left|x+\sqrt{x^2-a^2}\right| + C$$

$$\Rightarrow \int \sqrt{x^2-4^2} = \frac{x}{2}\sqrt{x^2-4^2} - \frac{4^2}{2} log\left|x+\sqrt{x^2-4^2}\right| + C$$

$$\Rightarrow \int \sqrt{x^2-16} = \frac{x}{2}\sqrt{x^2-16} - 8\, log\left|x+\sqrt{x^2-16}\right| + C$$

Question 34.
Mark (√) against the correct answer in each of the following:

$$\int \sqrt{x^2-4x+2}\, dx = ?$$

A. $\frac{1}{2}(x-2)\sqrt{x^2-4x+2} + \log\left|(x-2)+\sqrt{x^2-4x+2}\right| + C$

B. $(x-2)\sqrt{x^2-4x+2} + \frac{1}{2}\log\left|(x-2)+\sqrt{x^2-4x+2}\right| + C$

C. $\frac{1}{2}(x-2)\sqrt{x^2-4x+2} - \log\left|(x-2)+\sqrt{x^2-4x+2}\right| + C$

D. none of these

Answer:
$\sqrt{x^2-4x+2dx}$

It can be written as

$$\Rightarrow \sqrt{x^2-4x+2+2-2} = \sqrt{x^2-4x+4-2}$$

$$= \sqrt{(x-2)^2-2}$$

We know

$$\Rightarrow \int \sqrt{x^2 - a^2} = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} log\left|x + \sqrt{x^2 - a^2}\right| + C$$

$$\Rightarrow \int \sqrt{(x-2)^2 - 2} = \frac{(x-2)}{2}\sqrt{(x-2)^2 - 2} - \frac{\left(\sqrt{2}\right)^2}{2} log\left|\sqrt{(x-2)^2 - 2}\right| + C$$

$$\Rightarrow \int \sqrt{x^2 - 4x + 2} = \frac{x-2}{2}\sqrt{x^2 - 4x + 2} - log|x^2 - 4x + 2| + C$$

Question 35.
Mark (√) against the correct answer in each of the following:

$$\int \sqrt{9x^2 + 16}\,dx = ?$$

A. $\frac{x}{2}\sqrt{9x^2 + 16} + \frac{8}{3}\log\left|3x + \sqrt{9x^2 + 16}\right| + C$

B. $\frac{x}{2}\sqrt{9x^2 + 16} - \frac{8}{3}\log\left|3x + \sqrt{9x^2 + 16}\right| + C$

C. $x\sqrt{9x^2 + 16} + 24\log\left|3x + \sqrt{9x^2 + 16}\right| + C$

D. none of these

Answer:

$$\Rightarrow \int \sqrt{x^2 + a^2} = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} log\left|x + \sqrt{x^2 + a^2}\right| + C$$

$$\Rightarrow 3\int \sqrt{x^2 + \left(\frac{4}{3}\right)^2} = 3\left(\frac{x}{2}\sqrt{x^2 + \left(\frac{4}{3}\right)^2} + \frac{\frac{16}{9}}{2} log\left|x + \sqrt{x^2 + \left(\frac{4}{3}\right)^2}\right|\right)$$

$$\Rightarrow \int \sqrt{9x^2 + 16dx} = \frac{x}{2}\sqrt{9x^2 + 16} + \frac{8}{3} log\left|3x + \sqrt{9x^2 + 16}\right|$$

Question 36.

Mark (√) against the correct answer in each of the following:

$$\int e^{x}\sqrt{e^{2x}+4}\,dx = ?$$

A. $\frac{1}{2}e^{x}\sqrt{e^{2x}+4}-2\log\left|e^{x}+\sqrt{e^{2x}+4}\right|+C$

B. $\frac{1}{2}e^{x}\sqrt{e^{2x}+4}+2\log\left|e^{x}+\sqrt{e^{2x}+4}\right|+C$

C. $e^{x}\sqrt{e^{2x}+4}+\frac{1}{2}\log\left|e^{x}+\sqrt{e^{2x}+4}\right|+C$

D. none of these

Answer:

$$\int e^{x}\sqrt{e^{2x}+4}dx$$

Let $e^x = t$

$e^x\,dx = dt$

$$= \int \sqrt{t^2+2^2}dt$$

$$\Rightarrow \int \sqrt{x^2+a^2} = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}log\left|x+\sqrt{x^2+a^2}\right|+C$$

$$\Rightarrow \int \sqrt{t^2+2^2} = \frac{t}{2}\sqrt{t^2+2^2}+\frac{2^2}{2}log\left|t+\sqrt{t^2+2^2}\right|+C$$

But $t = e^x$

$$\Rightarrow \int e^{x}\sqrt{e^{2x}+4}dx = \frac{e^x}{2}\sqrt{e^{2x}+4}+2\,log\left|e^{x}+\sqrt{e^{2x}+4}\right|+C$$

Question 37.

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sqrt{16+(\log x)^2}}{x} dx = ?$$

A. $\frac{1}{2}\log x \cdot \sqrt{16+(\log x)^2} + 8 \log \left|\log x + \sqrt{16+(\log x)^2}\right| + C$

B. $\frac{1}{2}\log x \cdot \sqrt{16+(\log x)^2} + 4 \log \left|\log x + \sqrt{16+(\log x)^2}\right| + C$

C. $\log x \cdot \sqrt{16+(\log x)^2} + 16 \log \left|\log x + \sqrt{16+(\log x)^2}\right| + C$

D. none of these

Answer:

$$\int \frac{\sqrt{16+(\log x)^2}}{x} dx$$

Let log x =t

$$\Rightarrow \frac{1}{x} dx = dt$$

$$= \int \sqrt{t^2 + 4^2}\, dt$$

$$\Rightarrow \int \sqrt{x^2 + a^2} = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} log\left|x + \sqrt{x^2 + a^2}\right| + C$$

$$\Rightarrow \int \sqrt{t^2 + 4^2}\, dt = \frac{t}{2}\sqrt{t^2 + 4^2} + \frac{4^2}{2} log\left|t + \sqrt{t^2 + 4^2}\right| + C$$

But t =log x

$$\Rightarrow \int \frac{\sqrt{16 + (log\, x)^2}}{x} dx = \frac{log\, x}{2}\sqrt{log^2 x + 16} + 8\, log\left|log\, x + \sqrt{log^2 x + 16}\right| + C$$