

Exercise 28g

Question 1.

Find the angle between the line $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and the plane

$$\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4.$$

Answer:

Given $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4$

To find – The angle between the line and the plane

Direction ratios of the line = (1, -1, 1)

Direction ratios of the normal of the plane = (2, -1, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by

$$\sin^{-1} \left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \sqrt{a'^2 + b'^2 + c'^2}} \right)$$

The angle between the line and the plane

$$= \sin^{-1} \left(\frac{1 \times 2 + (-1) \times (-1) + 1 \times 1}{\sqrt{1^2 + 1^2 + 1^2} \sqrt{2^2 + 1^2 + 1^2}} \right)$$

$$= \sin^{-1} \left(\frac{2 + 1 + 1}{\sqrt{3} \sqrt{6}} \right)$$

$$= \sin^{-1} \left(\frac{4}{3\sqrt{2}} \right)$$

$$= \sin^{-1} \left(\frac{2\sqrt{2}}{3} \right)$$

Question 2.

Find the angle between the line $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(3\hat{i} - \hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 3$.

Answer:

Given $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(3\hat{i} - \hat{j} + 2\hat{k})$ and $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 3$

To find – The angle between the line and the plane

Direction ratios of the line = (3, -1, 2)

Direction ratios of the normal of the plane = (1, 1, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by

$$\sin^{-1} \left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \sqrt{a'^2 + b'^2 + c'^2}} \right)$$

The angle between the line and the plane

$$= \sin^{-1} \left(\frac{3 \times 1 + (-1) \times 1 + 2 \times 1}{\sqrt{3^2 + 1^2 + 2^2} \sqrt{1^2 + 1^2 + 1^2}} \right)$$

$$= \sin^{-1} \left(\frac{3 - 1 + 2}{\sqrt{14} \sqrt{3}} \right)$$

$$= \sin^{-1} \left(\frac{4}{\sqrt{42}} \right)$$

Question 3.

Find the angle between the line $\vec{r} = (3\hat{i} + \hat{k}) + \lambda(\hat{j} + \hat{k})$ and the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 1$.

Answer:

Given $\vec{r} = (3\hat{i} + \hat{k}) + \lambda(\hat{j} + \hat{k})$ and $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 1$

To find – The angle between the line and the plane

Direction ratios of the line = (0, 1, 1)

Direction ratios of the normal of the plane = (2, -1, 2)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by

$$\sin^{-1} \left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \sqrt{a'^2 + b'^2 + c'^2}} \right)$$

The angle between the line and the plane

$$= \sin^{-1} \left(\frac{0 \times 2 + 1 \times (-1) + 1 \times 2}{\sqrt{0^2 + 1^2 + 1^2} \sqrt{2^2 + 1^2 + 2^2}} \right)$$

$$= \sin^{-1} \left(\frac{-1 + 2}{3\sqrt{2}} \right)$$

$$= \sin^{-1} \left(\frac{1}{3\sqrt{2}} \right)$$

Question 4.

Find the angle between the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{2}$ and the plane $3x + 4y + z + 5 = 0$.

Answer:

Given $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{2}$ and $3x + 4y + z + 5 = 0$

To find – The angle between the line and the plane

Direction ratios of the line = (3, -1, 2)

Direction ratios of the normal of the plane = (3, 4, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by

$$\sin^{-1} \left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \sqrt{a'^2 + b'^2 + c'^2}} \right)$$

The angle between the line and the plane

$$= \sin^{-1} \left(\frac{3 \times 3 + (-1) \times 4 + 2 \times 1}{\sqrt{3^2 + 1^2 + 2^2} \sqrt{3^2 + 4^2 + 1^2}} \right)$$

$$= \sin^{-1} \left(\frac{9 - 4 + 2}{\sqrt{14} \sqrt{26}} \right)$$

$$= \sin^{-1} \left(\frac{7}{\sqrt{2} \sqrt{7} \times \sqrt{2} \times \sqrt{13}} \right)$$

$$= \sin^{-1} \left(\frac{7}{2\sqrt{91}} \right)$$

Question 5.

Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

Answer:

Given $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and $10x + 2y - 11z = 3$

To find – The angle between the line and the plane

Direction ratios of the line = (2, 3, 6)

Direction ratios of the normal of the plane = (10, 2, -11)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by

$$\sin^{-1} \left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \sqrt{a'^2 + b'^2 + c'^2}} \right)$$

The angle between the line and the plane

$$= \sin^{-1} \left(\frac{2 \times 10 + 3 \times 2 + 6 \times (-11)}{\sqrt{2^2 + 3^2 + 6^2} \sqrt{10^2 + 2^2 + 11^2}} \right)$$

$$= \sin^{-1} \left(\frac{20 + 6 - 66}{7 \times 15} \right)$$

$$= \sin^{-1}\left(\frac{-40}{7 \times 15}\right)$$

$$= \sin^{-1}\left(-\frac{8}{21}\right)$$

Question 6.

Find the angle between the line joining the points A(3, - 4, - 2) and B(12, 2, 0) and the plane $3x - y + z = 1$.

Answer:

Given - A = (3, - 4, - 2) , B = (12, 2, 0) and $3x - y + z = 1$

To find – The angle between the line joining the points A and B and the plane

Tip – If P = (a, b, c) and Q = (a', b', c'), then the direction ratios of the line PQ is given by ((a' - a), (b' - b), (c' - c))

The direction ratios of the line AB can be given by

$$((12 - 3), (2 + 4), (0 + 2))$$

$$= (9, 6, 2)$$

Direction ratios of the normal of the plane = (3, - 1, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by

$$\sin^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \sqrt{a'^2 + b'^2 + c'^2}}\right)$$

The angle between the line and the plane

$$= \sin^{-1}\left(\frac{9 \times 3 + 6 \times (-1) + 2 \times 1}{\sqrt{9^2 + 6^2 + 2^2} \sqrt{3^2 + 1^2 + 1^2}}\right)$$

$$= \sin^{-1}\left(\frac{27 - 6 + 2}{11 \times \sqrt{11}}\right)$$

$$= \sin^{-1} \left(\frac{23}{11\sqrt{11}} \right)$$

Question 7.

If the plane $2x - 3y - 6z = 13$ makes an angle $\sin^{-1}(\lambda)$ with the x - axis, then find the value of λ .

Answer:

Given $-y = z = 0$ and $2x - 3y - 6z = 13$

To find – The angle between the line and the plane

Direction ratios of the line = (1, 0, 0)

Direction ratios of the normal of the plane = (2, -3, -6)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by

$$\sin^{-1} \left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \sqrt{a'^2 + b'^2 + c'^2}} \right)$$

The angle between the line and the plane

$$= \sin^{-1} \left(\frac{1 \times 2 + 0 \times (-3) + 0 \times (-6)}{\sqrt{1^2 + 0^2 + 0^2} \sqrt{2^2 + 3^2 + 6^2}} \right)$$

$$= \sin^{-1} \left(\frac{2}{7} \right)$$

Question 8.

Show that the line $\vec{r} = (2\hat{i} + 5\hat{j} + 7\hat{k}) + \lambda(\hat{i} + 3\hat{j} + 4\hat{k})$ is parallel to the plane

$\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 7$. Also, find the distance between them.

Answer:

Given $\vec{r} = (2\hat{i} + 5\hat{j} + 7\hat{k}) + \lambda(\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 7$

To prove – The line and the plane are parallel &

To find – The distance between them

Direction ratios of the line = (1, 3, 4)

Direction ratios of the normal of the plane = (1, 1, -1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by

$$\sin^{-1} \left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \sqrt{a'^2 + b'^2 + c'^2}} \right)$$

The angle between the line and the plane

$$= \sin^{-1} \left(\frac{1 \times 1 + 3 \times 1 + 4 \times (-1)}{\sqrt{1^2 + 3^2 + 4^2} \sqrt{1^2 + 1^2 + 1^2}} \right)$$

$$= \sin^{-1} \left(\frac{1 + 3 - 4}{\sqrt{26} \sqrt{3}} \right)$$

$$= \sin^{-1}(0)$$

$$= 0$$

Hence, **the line and the plane are parallel.**

Now, the equation of the plane may be written as $x + y - z = 7$.

Tip – If $ax + by + c + d = 0$ be a plane and $\vec{r} = (a'\hat{i} + b'\hat{j} + c'\hat{k}) + \lambda(a''\hat{i} + b''\hat{j} + c''\hat{k})$ be a line vector, then the distance between them is given by $\left| \frac{a \times a' + b \times b' + c \times c' + d}{\sqrt{a^2 + b^2 + c^2}} \right|$

The distance between the plane and the line

$$= \left| \frac{1 \times 2 + 1 \times 5 - 1 \times 7 - 7}{\sqrt{1^2 + 1^2 + 1^2}} \right|$$

$$= \left| \frac{2 + 5 - 7 - 7}{\sqrt{3}} \right|$$

$$= \frac{7}{\sqrt{3}} \text{units}$$

Question 9.

Find the value of m for which the line $\vec{r} = (\hat{i} + 2\hat{k}) + \lambda(2\hat{i} - m\hat{j} - 3\hat{k})$ is parallel to the plane $\vec{r} \cdot (m\hat{i} + 3\hat{j} + \hat{k}) = 4$.

Answer:

Given $\vec{r} = (\hat{i} + 2\hat{k}) + \lambda(2\hat{i} - m\hat{j} - 3\hat{k})$ and $\vec{r} \cdot (m\hat{i} + 3\hat{j} + \hat{k}) = 4$ and they are parallel

To find – The value of m

Direction ratios of the line = (2, -m, -3)

Direction ratios of the normal of the plane = (m, 3, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by

$$\sin^{-1} \left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \sqrt{a'^2 + b'^2 + c'^2}} \right)$$

$$\therefore \sin^{-1} \left(\frac{2 \times m + (-m) \times 3 + (-3) \times 1}{\sqrt{2^2 + m^2 + 3^2} \sqrt{m^2 + 3^2 + 1^2}} \right) = 0$$

$$\Rightarrow \sin^{-1} \left(\frac{2m - 3m - 3}{\sqrt{13 + m^2} \sqrt{10 + m^2}} \right) = 0$$

$$\Rightarrow \frac{-m - 3}{\sqrt{13 + m^2} \sqrt{10 + m^2}} = 0$$

$$\Rightarrow m = -3$$

Question 10.

Find the vector equation of a line passing through the origin and perpendicular to the plane

$$\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 3.$$

Answer:

Given $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 3$

To find – The vector equation of the line passing through the origin and perpendicular to the given plane

Tip – The equation of a plane can be given by $\vec{r} \cdot \hat{n} = d$ where \hat{n} is the normal of the plane

A line parallel to the given plane will be in the direction of the normal and will have the direction ratios same as that of the normal.

Formula to be used – If a line passes through the point (a, b, c) and has the direction ratios as (a', b', c'), then its vector equation is given by $\vec{r} = (a\hat{i} + b\hat{j} + c\hat{k}) + \lambda(a'\hat{i} + b'\hat{j} + c'\hat{k})$ where λ is any scalar constant

The required equation will be $\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$= \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ for some scalar λ

Question 11.

Find the vector equation of the line passing through the point with position vector $(\hat{i} - 2\hat{j} + 5\hat{k})$ and perpendicular to the plane $\vec{r} \cdot (2\hat{i} - 3\hat{j} - \hat{k}) = 0$.

Answer:

Given $\vec{r} \cdot (2\hat{i} - 3\hat{j} - \hat{k}) = 0$ and the vector has position vector $(\hat{i} - 2\hat{j} + 5\hat{k})$

To find – The vector equation of the line passing through (1, -2, 5) and perpendicular to the given plane

Tip – The equation of a plane can be given by $\vec{r} \cdot \hat{n} = d$ where \hat{n} is the normal of the plane

A line parallel to the given plane will be in the direction of the normal and will have the direction ratios same as that of the normal.

Formula to be used – If a line passes through the point (a, b, c) and has the direction ratios as (a', b', c'), then its vector equation is given by $\vec{r} = (a\hat{i} + b\hat{j} + c\hat{k}) + \lambda(a'\hat{i} + b'\hat{j} + c'\hat{k})$ where λ is any scalar constant

The required equation will be $\vec{r} = (\hat{i} - 2\hat{j} + 5\hat{k}) + \lambda(2\hat{i} - 3\hat{j} - \hat{k})$ for some scalar λ

Question 12.

Show that the equation $ax + by + d = 0$ represents a plane parallel to the z - axis. Hence, find the equation of a plane which is parallel to the z - axis and passes through the points $A(2, -3, 1)$ and $B(-4, 7, 6)$.

Answer:

Given – The equation of the plane is given by $ax + by + d = 0$

To prove – The plane is parallel to z - axis

Tip – If $ax + by + cz + d$ is the equation of the plane then its angle with the z - axis will be given by $\sin^{-1}\left(\frac{c}{\sqrt{a^2 + b^2 + c^2}}\right)$

Considering the equation, the direction ratios of its normal is given by $(a, b, 0)$

The angle the plane makes with the z - axis $= \sin^{-1}[0/\sqrt{a^2 + b^2}] = 0$

Hence, **the plane is parallel to the z - axis**

To find – Equation of the plane parallel to z - axis and passing through points $A = (2, -3, 1)$ and $B = (-4, 7, 6)$

The given equation $ax + by + d = 0$ passes through $(2, -3, 1)$ & $(-4, 7, 6)$

$$\therefore 2a - 3b + d = 0 \dots\dots(i)$$

$$\therefore -4a + 7b + d = 0 \dots\dots(ii)$$

Solving (i) and (ii),

$$\therefore \frac{a}{\begin{vmatrix} -3 & 1 \\ 7 & 1 \end{vmatrix}} = \frac{b}{\begin{vmatrix} 1 & 2 \\ 1 & -4 \end{vmatrix}} = \alpha \text{ } [\alpha \rightarrow \text{arbitrary constant}]$$

$$\therefore a = -10\alpha$$

$$\therefore b = -6\alpha$$

Substituting the values of a and b in eqn (i), we get,

$$-2 \times 10\alpha + 3 \times 6\alpha + d = 0 \text{ i.e. } d = -2\alpha$$

Putting the value of a, b and d in the equation $ax + by + d = 0$,

$$(-10a)x + (-6a)y + (-2a) = 0$$

i.e. $5x + 3y + 1 = 0$

Question 13.

Find the equation of the plane passing through the points (1, 2, 3) and (0, -1, 0) and parallel to

the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$.

Answer:

Given – A plane passes through points (1, 2, 3) and (0, -1, 0) and is parallel to the line

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$$

To find – Equation of the plane

Tip – If a plane passes through points (a', b', c'), then its equation may be given as $a(x - a') + b(y - b') + c(z - c') = 0$

Taking points (1, 2, 3):

$$a(x - 1) + b(y - 2) + c(z - 3) = 0 \dots\dots\dots (i)$$

The plane passes through (0, -1, 0):

$$a(0 - 1) + b(-1 - 2) + c(0 - 3) = 0$$

i.e. $a + 3b + 3c = 0 \dots\dots\dots (ii)$

The plane is parallel to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$

Tip – The normal of the plane will be normal to the given line since both the line and plane are parallel.

Direction ratios of the line is (2, 3, -3)

Direction ratios of the normal of the plane is (a, b, c)

So, $2a + 3b - 3c = 0 \dots\dots\dots (iii)$

Solving equations (ii) and (iii),

$$\therefore \frac{a}{\begin{vmatrix} 3 & 3 \\ 3 & -3 \end{vmatrix}} = -\frac{b}{\begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix}} = \alpha \quad [\alpha \rightarrow \text{arbitrary constant}]$$

$$\therefore a = -18\alpha$$

$$\therefore b = 9\alpha$$

$$\therefore c = -3\alpha$$

Putting these values in equation (i) we get,

$$-18\alpha(x-1) + 9\alpha(y-2) - 3\alpha(z-3) = 0$$

$$\Rightarrow 18(x-1) - 9(y-2) + 3(z-3) = 0$$

$$\Rightarrow 6(x-1) - 3(y-2) + (z-3) = 0$$

$$\Rightarrow 6x - 3y + z - 3 = 0$$

$$\Rightarrow \mathbf{6x - 3y + z = 3}$$

Question 14.

Find the equation of a plane passing through the point (2, -1, 5), perpendicular to the plane $x + 2y - 3z = 7$ and parallel to the line $\frac{x+5}{3} = \frac{y+1}{-1} = \frac{z-2}{1}$.

$$\frac{x+5}{3} = \frac{y+1}{-1} = \frac{z-2}{1}$$

Answer:

Given – A plane passes through (2, -1, 5), perpendicular to the plane $x + 2y - 3z = 7$ and parallel to the line $\frac{x+5}{3} = \frac{y+1}{-1} = \frac{z-2}{1}$

To find – The equation of the plane

Let the equation of the required plane be $ax + by + cz + d = 0$(a)

The plane passes through (2, -1, 5)

So, $2a - b + 5c + d = 0$(i)

The direction ratios of the normal of the plane is given by (a, b, c)

Now, this plane is perpendicular to the plane $x + 2y - 3z = 7$ having direction ratios (1, 2, - 3)

So, $a + 2b - 3c = 0$(ii)

This plane is also parallel to the line having direction ratios (3, - 1, 1)

So, the direction of the normal of the required plane is also at right angles to the given line.

So, $3a - b + c = 0$(iii)

Solving equations (ii) and (iii),

$$\therefore \frac{a}{\begin{vmatrix} 2 & -3 \\ -1 & 1 \end{vmatrix}} = -\frac{b}{\begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}} = \alpha \quad [\alpha \rightarrow \text{arbitrary constant}]$$

$$\therefore a = -\alpha$$

$$\therefore b = -10\alpha$$

$$\therefore c = -7\alpha$$

Putting these values in equation (i) we get,

$$2x(-\alpha) - (-10\alpha) + 5(-7\alpha) + d = 0 \text{ i.e. } d = 27\alpha$$

Substituting all the values of a, b, c and d in equation (a) we get,

$$-\alpha x - 10\alpha y - 7\alpha z + 27\alpha = 0$$

$$\Rightarrow x + 10y + 7z + 27 = 0$$

Question 15.

Find the equation of the plane passing through the intersection of the planes $5x - y + z = 10$ and $x + y - z = 4$ and parallel to the line with direction ratios 2, 1, 1. Find also the perpendicular distance of (1, 1, 1) from this plane.

Answer:

Given – A plane passes through the intersection of $5x - y + z = 10$ and $x + y - z = 4$ and parallel to the line with direction ratios $(2, 1, 1)$

To find – Equation of the plane

Tip – If $ax + by + cz + d = 0$ and $a'x + b'y + c'z + d' = 0$ be two planes, then the equation of the plane passing through their intersection will be given by

$(ax + by + cz + d) + \lambda(a'x + b'y + c'z + d') = 0$, where λ is any scalar constant

So, the equation of the plane maybe written as

$$(5x - y + z - 10) + \lambda(x + y - z - 4) = 0$$

$$\Rightarrow (5 + \lambda)x + (-1 + \lambda)y + (1 - \lambda)z + (-10 - 4\lambda) = 0$$

This is plane parallel to a line with direction ratios $(2, 1, 1)$

So, the normal of this line with direction ratios $((5 + \lambda), (-1 + \lambda), (1 - \lambda))$ will be perpendicular to the given line.

Hence,

$$2(5 + \lambda) + (-1 + \lambda) + (1 - \lambda) = 0$$

$$\Rightarrow \lambda = -5$$

The equation of the plane will be

$$(5 + (-5))x + (-1 + (-5))y + (1 - (-5))z + (-10 - 4(-5)) = 0$$

$$\Rightarrow -6y + 6z + 10 = 0$$

$$\Rightarrow 3y - 3z = 5$$

To find – Perpendicular distance of point $(1, 1, 1)$ from the plane

Formula to be used - If $ax + by + c + d = 0$ be a plane and (a', b', c') be the point, then the distance between them is given by $\left| \frac{a \times a' + b \times b' + c \times c' + d}{\sqrt{a^2 + b^2 + c^2}} \right|$

The distance between the plane and the line

$$= \left| \frac{0 \times 2 + 3 \times 1 - 3 \times 1 - 5}{\sqrt{0^2 + 3^2 + 3^2}} \right|$$

$$= \left| \frac{3 - 3 - 5}{2\sqrt{3}} \right|$$

$$= \frac{5}{2\sqrt{3}} \text{units}$$