Exercise 19a

Question 1.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx}=\left(1+x^2\right)\left(1+y^2\right)$$

Answer:

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

Rearranging the terms,we get:

$$\Rightarrow \frac{dy}{1 + y^2} = (1 + x^2)dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{1 + y^2} = \int (1 + x^2)dx + c$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + c \ldots (\int \frac{dy}{1 + y^2} = \tan^{-1} y, \int x^n = \frac{x^{n+1}}{n+1})$$

Ans: $\tan^{-1} y = x + \frac{x^3}{3} + c$

Question 2.
Find the general solution of each of the following differential equations:

$$x^4\frac{dy}{dx}=-y^4$$

Answer:

$$x^4\frac{dy}{dx} = -y^4$$

$$\Rightarrow \frac{dy}{-y^4} = \frac{dx}{x^4}$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{-y^4} = \int \frac{dx}{x^4} + c'$$

$$\Rightarrow \frac{-y^{-4+1}}{-4+1} = \frac{x^{-4+1}}{-4+1} + c'$$

$$\Rightarrow \frac{1}{3y^3} = -\frac{1}{3x^3} + c'$$

$$\Rightarrow \frac{1}{y^3} + \frac{1}{x^3} = 3c'$$

$\Rightarrow \frac{1}{x^3} + \frac{1}{y^3} = c$ ...(3c′ = c)

Question 3.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = 1 + x + y + xy$$

Answer:
$$\frac{dy}{dx} = 1 + x + y + xy = 1 + y + x(1 + y)$$

$$\Rightarrow \frac{dy}{dx} = (1 + y)(1 + x)$$

Rearranging the terms we get:

$$\Rightarrow \frac{dy}{1 + y} = (1 + x)dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{1 + y} = \int (1 + x)dx + c$$

$$\Rightarrow \log|1 + y| = x + \frac{x^2}{2} + c \ldots (\int \frac{dy}{1+y} = \log|1 + y|)$$

Ans: $\log|1 + y| = x + \frac{x^2}{2} + c$

Question 4.
Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = 1 - x + y - xy$$

Answer:
$$\Rightarrow \frac{dy}{dx} = 1 - x + y - xy = 1 + y - x(1 + y)$$

$$\Rightarrow \frac{dy}{dx} = (1 + y)(1 - x)$$

Rearranging the terms we get:

$$\Rightarrow \frac{dy}{1 + y} = (1 - x)dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{1 + y} = \int (1 - x)dx + c$$

$$\Rightarrow \log|1 + y| = x - \frac{x^2}{2} + c \ldots (\int \frac{dy}{1+y} = \log|1 + y|)$$

Ans: $\log|1 + y| = x - \frac{x^2}{2} + c$

Question 5.

Find the general solution of each of the following differential equations:

$$(x-1)\frac{dy}{dx} = 2x^3y$$

Answer:

$$(x-1)\frac{dy}{dx} = 2x^3y$$

Separating the variables we get:

$$\Rightarrow \frac{dy}{y} = 2x^3\frac{dx}{(x-1)}$$

$$\Rightarrow \frac{dy}{y} = \frac{2\left((x-1)(x^2+x+1)+1\right)}{(x-1)}dx$$

$$\Rightarrow \frac{dy}{y} = 2\left(x^2+x+1+\frac{1}{x-1}\right)dx$$

Integrating both the sides we get,

$$\Rightarrow \int\frac{dy}{y} = \int 2\left(x^2+x+1+\frac{1}{x-1}\right)dx + c$$

$$\Rightarrow \log|y| = \frac{2x^3}{3} + \frac{2x^2}{2} + 2x + 2\log|x-1| + c$$

$$\Rightarrow \log|y| = \frac{2x^3}{3} + x^2 + 2x + 2\log|x-1| + c$$

Ans: $\log|y| = \frac{2x^3}{3} + x^2 + 2x + 2\log|x-1| + c$

Question 6.

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = e^{x+y}$$

Answer:

$$\frac{dy}{dx} = e^{x}e^{y}$$

Rearringing the terms we get:

$$\Rightarrow \frac{dy}{e^{y}} = e^{x}dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{e^{y}} = \int e^{x}dx + c$$

$$\Rightarrow \frac{e^{-y}}{-1} = e^{x} + c$$

$$\Rightarrow e^{x} + e^{-y} = c$$

Ans: $e^{x} + e^{-y} = c$

Question 7.

Find the general solution of each of the following differential equations:

$$\left(e^{x} + e^{-x}\right)dy - \left(e^{x} - e^{-x}\right)dx = 0$$

Answer:

$(e^{x} + e^{-x})dy - (e^{x} - e^{-x})dx = 0$

$$\Rightarrow dy = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}dx$$

Integrating both the sides we get,

$$\Rightarrow \int dy = \int \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}dx + c$$

$\Rightarrow y = \log|e^{x} + e^{-x}| + c$ ...$\left(\frac{d}{dx}(e^{x} + e^{-x}) = e^{x} - e^{-x}\right)$

Ans:$y = \log|e^{x} + e^{-x}| + c$

Question 8.
Find the general solution of each of the following differential equations:

Answer:
Given:$\frac{dy}{dx} = e^{x}e^{-y} + x^{2}e^{-y}$

$$\Rightarrow \frac{dy}{dx} = e^{-y}(e^{x} + x^{2})$$

$$\Rightarrow \frac{dy}{e^{-y}} = (e^{x} + x^{2})dx$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dy}{e^{-y}} = \int (e^{x} + x^{2})dx + c$$

$$\Rightarrow e^{y} = e^{x} + \frac{x^{3}}{3} + c$$

Ans:$e^{y} = e^{x} + \frac{x^{3}}{3} + c$

Question 9.
Find the general solution of each of the following differential equations:

$$e^{2x-3y}dx + e^{2y-3x}dy = 0$$

Answer:
$e^{2x}e^{-3y}dx + e^{2y}e^{-3x}dy = 0$

Rearringing the terms we get:

$$\Rightarrow \frac{e^{2x}dx}{e^{-3x}} = -\frac{e^{2y}dy}{e^{-3y}}$$

$\Rightarrow e^{2x+3x}dx = -e^{2y+3y}dy$

$\Rightarrow e^{5x}dx = - e^{5y}dy$

Integrating both the sides we get:

$$\Rightarrow \int e^{5x}dx = -\int e^{5y}dy + c'$$

$$\Rightarrow \frac{e^{5x}}{5} = -\frac{e^{5y}}{5} + c'$$

$\Rightarrow e^{5x} + e^{5y} = 5c' = c$

Ans: $e^{5x} + e^{5y} = c$

Question 10.
Find the general solution of each of the following differential equations:

$e^x \tan y\, dx + (1 – e^x) \sec^2 y\, dy = 0$

Answer:
Rearranging all the terms we get:

$$\frac{e^x dx}{1 - e^x} = -\frac{sec^2 y\, dy}{tany}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{e^x dx}{1 - e^x} = -\int \frac{sec^2 y\, dy}{tany} + c$$

$$\Rightarrow \frac{\log|1 - e^x|}{-1} = -\log|tany| + logc$$

$\Rightarrow \log|1 - e^x| = \log|tany| - \log c$

$\Rightarrow \log|1 - e^x| + \log c = \log|tany|$

$\Rightarrow tany = c(1 - e^x)$

Ans: tany = c(1 - e^x)

Question 11.
Find the general solution of each of the following differential equations:

$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

Answer:
Rearranging the terms we get:

$$\frac{sec^2 x\, dx}{tanx} = -\frac{sec^2 y\, dy}{tany}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{sec^2 x\, dx}{tanx} = -\int \frac{sec^2 y\, dy}{tany} + c$$

⇒log|tanx| = - log|tany| + logc

⇒ log|tanx| + log|tany| = logc

⇒tanx.tany = c

Ans: tanx.tany = c

Question 12.
Find the general solution of each of the following differential equations:

cos x(1 + cos y)dx – sin y(1 + sin x)dy = 0

Answer:
Rearranging the terms we get:

$$\frac{\cos x\, dx}{(1 + \sin x)} = \frac{siny\, dy}{(1 + cosy)}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{\cos x \, dx}{(1 + \sin x)} = \int \frac{\sin y \; dy}{(1 + \cos y)} + c$$

$\Rightarrow \log|1 + \sin x| = -\log|1 + \cos y| + \log c$

$\Rightarrow \log|1 + \sin x| + \log|1 + \cos y| = \log c$

$\Rightarrow (1 + \sin x)(1 + \cos y) = c$

Ans: $(1 + \sin x)(1 + \cos y) = c$

Question 13.
For each of the following differential equations, find a particular solution satisfying the given condition :

$.\cos\left(\frac{dy}{dx}\right) = a$, . where $a \in R$ and $y = 2$ when $x = 0$.

Answer:
$$\cos\left(\frac{dy}{dx}\right) = a$$

$$\Rightarrow \frac{dy}{dx} = \cos^{-1} a$$

$\Rightarrow dy = \cos^{-1} a \, dx$

Integrating both the sides we get:

$$\Rightarrow \int dy = \int \cos^{-1} a \; dx + c$$

$\Rightarrow y = x\cos^{-1} a + c$

when $x = 0$, $y = 2$

$\therefore 2 = 0 + c$

$\therefore c = 2$

$\therefore y = x\cos^{-1}a + 2$

$$\Rightarrow \frac{y-2}{x} = \cos^{-1}a$$

$$\Rightarrow \cos\left(\frac{y-2}{x}\right) = a$$

Ans: $\cos\left(\frac{y-2}{x}\right) = a$

Question 14.
For each of the following differential equations, find a particular solution satisfying the given condition :

$\frac{dy}{dx} = -4xy^2$, it being given that y = 1 when x = 0.

Answer:
Rearranging the terms we get:

$$\frac{dy}{y^2} = -4x\,dx$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dy}{y^2} = -\int 4x\,dx + c$$

$$\Rightarrow \frac{y^{-1}}{-1} = -\frac{4x^2}{2} + c$$

$\Rightarrow y^{-1} = 2x^2 + c$

y = 1 when x = 0

$\Rightarrow (1)^{-1} = 2(0)^2 + c$

$\Rightarrow c = 1$

$$\Rightarrow \frac{1}{y} = 2x^2 + 1$$

$$\Rightarrow \frac{1}{2x^2 + 1} = y$$

Ans: $y = \frac{1}{2x^2 + 1}$

Question 15.
For each of the following differential equations, find a particular solution satisfying the given condition :

x dy = $(2x^2 + 1)$ dx ($x \neq 0$), given that
y = 1 when x = 1.

Answer:
Rearranging the terms we get:

$$dy = \frac{2x^2 + 1}{x} dx$$

$$\Rightarrow dy = 2x\, dx + \frac{1}{x}\, dx$$

Integrating both the sides we get:

$$\Rightarrow \int dy = \int 2x\, dx + \int \frac{1}{x}\, dx + c$$

$\Rightarrow y = x^2 + \log|x| + c$

y = 1 when x = 1

$\therefore 1 = 1^2 + \log 1 + c$

$\therefore 1 - 1 = 0 + c$ ...($\log 1 = 0$)

$\Rightarrow c = 0$

$\therefore y = x^2 + \log|x|$

Ans: $y = x^2 + \log|x|$

Question 16.
For each of the following differential equations, find a particular solution satisfying the given condition :

$\frac{dy}{dx} = y \tan x,$ it being given that y = 1 when x = 0.

Answer:
Rearranging the terms we get:

$$\frac{dy}{y} = tanx\ dx$$

$$\Rightarrow \int \frac{dy}{y} = \int tanx\ dx + c$$

$\Rightarrow \log|y| = \log|\sec x| + \log c$

$\Rightarrow \log|y| - \log|\sec x| = \log c$

$\Rightarrow \log|y| + \log|\cos x| = \log c$

$\Rightarrow y\cos x = c$

y = 1 when x = 0

$\therefore 1 \times \cos 0 = c$

$\therefore c = 1$

$\Rightarrow y\cos x = 1$

$\Rightarrow y = 1/\cos x$

$\Rightarrow y = \sec x$

Ans: $y = \sec x$