Exercise 27b

Question 1.
Show that the points A(2, 1, 3), B(5, 0, 5) and C(-4, 3, -1) are collinear.

Answer:
Given -

A = (2,1,3)

B = (5,0,5)

C = (-4,3,-1)

To prove – A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((5-2),(0-1),(5-3))

=(3,-1,-2)

Similarly, the direction ratios of the line BC can be given by

((-4-5),(3-0),(-1-5))

=(-9,3,-6)

Tip – If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(3,-1,-2)

=(-1/3)X(-9,3,-6)

=(-1/3)Xd.r. of BC

Hence, **A, B and C are collinear**

Question 2.
Show that the points A(2, 3, -4), B(1, -2, 3) and C(3, 8, -11) are collinear.

Answer:
Given -

A = (2,3,-4)

B = (1,-2,3)

C = (3,8,-11)

To prove – A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((1-2),(-2-3),(3+4))

=(-1,-5,7)

Similarly, the direction ratios of the line BC can be given by

((3-1),(8+2),(-11-3))

=(2,10,-14)

Tip – If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(-1,-5,7)

$=(-1/2)X(2,10,-14)$

$=(-1/2)X$d.r. of BC

Hence, **A, B and C are collinear**

Question 3.
Find the value of λ for which the points A(2, 5, 1), B(1, 2, -1) and C(3, λ, 3) are collinear.

Answer:
Given -

A = (2,5,1)

B = (1,2,-1)

C = (3,λ,3)

To find – The value of λ so that A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((1-2),(2-5),(-1-1))

=(-1,-3,-2)

Similarly, the direction ratios of the line BC can be given by

((3-1),(λ-2),(3+1))

=(2,λ-2,4)

Tip – If it is shown that direction ratios of AB=α times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(-1,-3,-2)

=(-1/2)X(2,λ-2,4)

=(-1/2)Xd.r. of BC

Since, A, B and C are collinear,

$$\therefore -\frac{1}{2}(\lambda-2) = -3$$

$\Rightarrow \lambda - 2 = 6$

$\Rightarrow \boldsymbol{\lambda = 8}$

Question 4.
Find the values of λ and μ so that the points A(3, 2, -4), B(9, 8, -10) and C(λ, μ -6) are collinear.

Answer:
Given -

A = (3,2,-4)

B = (9,8,-10)

C = (λ,μ,-6)

To find – The value of λ and μ so that A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((9-3),(8-2),(-10+4))

=(6,6,-6)

Similarly, the direction ratios of the line BC can be given by

((λ-9),(μ-8),(-6+10))

=(λ-9,μ-8,4)

Tip – If it is shown that direction ratios of AB=α times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(6,6,-6)

=(-6/4)X(-4,-4,4)

=(-3/2)Xd.r. of BC

Since, A, B and C are collinear,

$\therefore -\frac{3}{2}(\lambda - 9) = 6$

⇨ $\lambda - 9 = -4$

⇨ $\boldsymbol{\lambda = 5}$

And,

$\therefore -\frac{3}{2}(\mu - 8) = 6$

⇨ $\mu - 8 = -4$

⇨ $\boldsymbol{\lambda = 4}$

Question 5.
Find the values of λ and μ so that the points A(-1, 4, -2), B(λ, μ 1) and C(0, 2, -1) are collinear.

Answer:
Given -

A = (-1,4,-2)

B = (λ,μ,1)

C = (0,2,-1)

To find – The value of λ and μ so that A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((λ+1),(μ-4),(1+2))

=(λ+1,μ-4,3)

Similarly, the direction ratios of the line BC can be given by

((0-λ),(2-μ),(-1-1))

=(-λ,2-μ,-2)

Tip – If it is shown that direction ratios of AB=α times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(λ+1,μ-4,3)

Say, α be an arbitrary constant such that d.r. of AB = α X d.r. of BC

So, 3 = α X (-2)

i.e. α = -3/2

Since, A, B and C are collinear,

$$\therefore -\frac{3}{2}(-\lambda) = \lambda + 1$$

⇨ $3\lambda = 2\lambda + 2$

$\Rightarrow \boldsymbol{\lambda = 2}$

And,

$\therefore -\frac{3}{2}(2-\mu) = \mu - 4$

$\Rightarrow -6 + 3\mu = 2\mu - 8$

$\Rightarrow \boldsymbol{\mu = -2}$

Question 6.

The position vectors of three points A, B and C are $\left(-4\hat{i}+2\hat{j}-3\hat{k}\right), \left(\hat{i}+3\hat{j}-2\hat{k}\right)$ and $\left(-9\hat{i}+\hat{j}-4\hat{k}\right)$respectively. show that the points A, B and C are collinear.

Answer:
Given -

$\vec{A} = -4\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{B} = \hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{C} = -9\hat{i} + \hat{j} - 4\hat{k}$

It can thus be written as:

A = (-4,2,-3)

B = (1,3,-2)

C = (-9,1,-4)

To prove – A, B and C are collinear

Formula to be used – If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((1+4),(3-2),(-2+3))

=(5,1,1)

Similarly, the direction ratios of the line BC can be given by

((-9-1),(1-3),(-4+2))

=(-10,-2,-2)

Tip – If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(5,1,1)

=(-1/2)X(-10,-2,-2)

=(-1/2)Xd.r. of BC

Hence, **A, B and C are collinear**