Exercise 15b

Question 1.
Evaluate:

$$\int x^{-6} dx$$

Answer:

$$\int x^{-6} dx = \frac{x^{-6+1}}{-6+1} + c$$

$$\because \left\{ \int x^n = \frac{x^{n+1}}{n+1} + c \right\}$$

$$= \frac{x^{-5}}{-5} + c$$

$$\int x^{-6} dx = -\frac{1}{5x^5} + c$$

Question 2.
Evaluate:

$$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$$

Answer:

$$\int (\sqrt{x} + 1/\sqrt{x}) dx = \int \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}} \right) dx$$

$$\left\{ \int x^n = \frac{x^{n+1}}{n+1} + c \right\}$$

$$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx = \int \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} dx$$

$$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx = \int \frac{2}{3} x^{\frac{3}{2}} + 2\sqrt{x} + c$$

Question 3.
Evaluate:

$$\int \sin\ 3x\ dx$$

Answer:

$$\int sin\,3x\,dx = \frac{-1}{3} cos\,3x + c$$

$$\left\{ \int \sin ax\,dx = \frac{-1}{a} \cos ax \right\}$$

Question 4.
Evaluate:

$$\int \frac{x^2}{\left(1+x^3\right)} dx$$

Answer:
Let $x^3 + 1 = t$

$$3x^2 dx = dt$$

$$\frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} ln\, t + c$$

$$\int \frac{x^2}{1+x^3} dx = \frac{1}{3} ln(x^3 + 1) + c$$

Question 5.
Evaluate:

$$\int \frac{2\cos x}{3\sin^2 x} dx$$

Answer:
Let sin x=t

cos x dx=dt

$$\int \frac{2\cos x}{\quad} dx = \int \frac{2}{\quad} dt = -\frac{2}{\quad} + c$$

$$\int \frac{2\cos x}{\quad} dx = \frac{-2}{\quad} cs\,c\,x + c$$

Question 6.
Evaluate:

$$\int \frac{(3\sin\phi - 2)\cos\phi}{\left(5 - \cos^2\phi - 4\sin\phi\right)} d\phi$$

Answer:

$$\frac{(3\sin\emptyset - 6 + 4)\cos\emptyset}{(4+1-cos^2\emptyset - 4\sin\emptyset)} = \frac{3(\sin\emptyset - 2)\cos\emptyset + 4\cos\emptyset}{(\sin\emptyset - 2)^2}$$

$$= \frac{3\cos\emptyset}{(\sin\emptyset - 2)} + \frac{4\cos\emptyset}{(\sin\emptyset - 2)^2}$$

$$\int \left(\frac{3\cos\emptyset}{(\sin\emptyset - 2)} + \frac{4\cos\emptyset}{(\sin\emptyset - 2)^2}\right) d\emptyset$$

Let (sin ⌀-2)= t

cos ⌀ d⌀=dt

$$\int \frac{3dt}{t} + \frac{4dt}{t^2} = 3\ln t - \frac{4}{t} + c$$

$$\int \frac{(3sin\phi - 2)cos\phi}{(5 - cos^2\phi - 4sin\phi)} d\phi = 3ln|sin\phi - 2| - \frac{4}{(sin\phi - 2)} + c$$

Question 7.
Evaluate:

$$\int \sin^2 x\, dx$$

Answer:

$$\int sin^2 x\, dx = \int \frac{1}{2} - \frac{\cos 2x}{2}\, dx$$

{1-cos 2x=2 $\sin^2$ x}

$$\int sin^2 x\, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$\left\{\int \cos ax\, dx = \frac{1}{a}\sin ax\right\}$$

Question 8.
Evaluate:

$$\int \frac{(\log x)^2}{x}\, dx$$

Answer:
Let log x = t

$$\frac{1}{x}dx = dt$$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$\int \frac{(log\, x)^2}{x}dx = \frac{(log\, x)^3}{3} + c$$

Question 9.
Evaluate:

$$\int \frac{(x+1)(x+\log x)^2}{x}\, dx$$

Answer:

$$\int \frac{(x+1)(x+logx)]^2}{x} = \int \left(1+\frac{1}{x}\right)(x+logx)^2 dx$$

Let $x + logx = t$

$$\left(1+\frac{1}{x}\right)dx = dt$$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$\int \frac{(x+1)(x+logx)^2}{x} = \frac{(x+logx)^3}{3} + c$$

Question 10.
Evaluate:

$$\int \frac{\sin x}{(1+\cos x)} dx$$

Answer:
Let 1+cosx=t

-sin x dx=dt

$$\int \frac{-dt}{t} = -ln\, t + c$$

$$\int \frac{sin\, x}{(1+cos\, x)} dx = -ln|1+cosx| + c$$

Question 11.
Evaluate:

$$\int \frac{(1+\tan x)}{(1-\tan x)} dx$$

Answer:

$$\frac{1+tanx}{1-tanx} = \frac{cos\, x + sin\, x}{cos\, x - sin\, x}$$

$$\int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Let cos x-sin x=t

-(sin x + cos x)dx=dt

$$\int \frac{-dt}{t} = -\ln t + c$$

$$\int \frac{1+tanx}{1-tanx} dx = -ln|cos\, x - sin\, x| + c$$

Question 12.
Evaluate:

$$\int \frac{(1-\cot x)}{(1+\cot x)} \mathrm{dx}$$

Answer:

$$\frac{1-\cot x}{1+\cot x} = \frac{\sin x - \cos x}{\sin x + \cos x}$$

$$\int \frac{\sin x - \cos x}{\sin x + \cos x} \mathrm{dx}$$

Let sin x + cos x=t

(cos x-sin x)dx=dt

$$\int \frac{sin\, x - cos\, x}{sin\, x + cos\, x} dx = \int \frac{-dt}{t} = -ln|sin\, x + cos\, x| + c$$

$$\int \frac{1-cot\, x}{1+cot\, x} dx = -ln|sin\, x + cos\, x| + c$$

Question 13.
Evaluate:

$$\int \frac{(1+\cot x)}{(x+\log \sin x)} \mathrm{dx}$$

Answer:
Let (x + log (sin x))=t

(1+cot x) dx=dt

$$\int \frac{dt}{t} = ln\, t + c$$

$$\int \frac{(1+\cot x)}{(x+\log \sin x)} = \ln|x + \log(\sin x)| + c$$

Question 14.
Evaluate:

$$\int \frac{(1-\sin\ 2x)}{\left(x+\cos^2 x\right)} dx$$

Answer:
Let $(x + \cos^2 x)=t$

(1-sin 2x) dx=dt

$$\int \frac{dt}{t} = \ln t + c$$

$$\int \frac{1 - sin\, 2x}{x + \ cos^2 x} = l\, n(|x + cos^2 x|) + c$$

Question 15.
Evaluate:

$$\int \frac{\sec^2 (\log x)}{x} dx$$

Answer:
Let $\log x = t$

$$\frac{1}{x} dx = dt$$

$$\int sec^2 t dt = tan\, t + c$$

$$\int \frac{sec^2\,(\log x)}{x} dx = tan(\log x) + c$$

Question 16.
Evaluate:

$$\int \frac{\sin\left(2\tan^{-1} x\right)}{\left(1+x^2\right)} dx$$

Answer:
Let $\tan^{-1} x = t$

$$\frac{1}{1+x^2} dx = dt$$

$$\int \sin 2t = -\frac{\cos 2t}{2} + c$$

$$\int \frac{sin(2\,tan^{-1} x)}{(1+x^2)} dx = \frac{-1}{2}\cos(2\tan^{-1} x) + c$$

Question 17.
Evaluate:

$$\int \frac{\tan x \sec^2 x}{\left(1-\tan^2 x\right)} dx$$

Answer:
Let $1-\tan^2 x=t$

$-2 \tan x. \sec^2 x\, dx=dt$

$$\frac{-1}{2}\int \frac{dt}{t} = \frac{-1}{2} log\, t + c$$

$$\int \frac{\tan x \sec^2 x}{} dx = \frac{-1}{} log\,|1-tan^2 x| + c$$

Question 18.
Evaluate:

$$\int \frac{\left(x^4+1\right)}{\left(x^2+1\right)} dx$$

Answer:

$$\frac{x^4+1}{x^2+1} = \frac{x^4-1+2}{x^2+1}$$

$$= x^2 - 1 + \frac{2}{x^2+1}$$

$$\int \left(x^2 - 1 + \frac{2}{x^2+1}\right) dx = \frac{x^3}{3} - x + 2tan^{-1} x + c$$

Question 19.
Evaluate:

$$\int \tan^{-1} \sqrt{\frac{1-\sin x}{1+\sin x}} dx$$

Answer:

$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$

$$tan^{-1} \sqrt{\frac{1-cos\left(\frac{\pi}{2}-x\right)}{1+cos\left(\frac{\pi}{2}-x\right)}} = tan^{-1} \sqrt{\frac{2sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}}$$

$$= tan^{-1}\left(\left(tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right)$$

$$\int \left(\frac{\pi}{4}-\frac{x}{2}\right)dx = \frac{\pi}{4}x-\frac{x^2}{4}+c$$

Question 20.
Evaluate:

$$\int \log\left(1+x^2\right)dx$$

Answer:
Using Integration by Parts

$$\int u_{II}v_I dx = u\int vdx - \int u' \int vdx\, dx + c$$

Here 1 is the first function and $\log(x^2+1)$ is second function

$$\int log(1+x^2)dx = (log(1+x^2))x - \int \frac{2x}{1+x^2}xdx$$

$$= (log(1+x^2))x - 2\int \frac{x^2+1-1}{x^2+1}dx$$

$$= (log(1+x\text{^}2\,)\,)x - 2x + 2\ \ tan\ \text{^}(-1)\,x + c$$

Question 21.
Evaluate:

$$\int \cos x \cos 3x\, dx$$

Answer:

$$\frac{1}{2}\int 2\cos x \cos 3x\, dx$$

{2 cos A cos B=cos(A+B)+cos(A-B)}

$$\frac{1}{2}\int (\cos 4x+\cos 2x)\, dx = \frac{\sin 4x}{8}+\frac{\sin 2x}{4}+c$$

Question 22.

Evaluate:Evaluate $\int \sin\ 3x\ \sin\ x\ dx$

Answer:

$\frac{1}{2}\int 2\sin 3x \sin x\, dx$

{ 2 sin A sin B=cos(A-B)-cos(A+B) }

$$\frac{1}{2}\int (cos\ 2x - cos\ 4x)dx\ = \frac{sin\ 2x}{4} - \frac{sin\ 4x}{8} + c$$

Question 23.
Evaluate:

$$\int \frac{xe^x}{(x+1)^2} dx$$

Answer:

$$\frac{e^x(x+1-1)}{(x+1)^2} = e^x\left(\frac{1}{x+1} - \frac{1}{(x+1)^2}\right)$$

$$\{\int \left(e^x(f(x)+f'(x)\right)dx = e^x f(x) + c\}$$

$$\int \frac{xe^x}{(x+1)^2}dx\ = \frac{e^x}{x+1} + c$$

Question 24.
Evaluate:

$$\int e^x \{\tan x - \log \cos x\} dx$$

Answer:

$$\int \left(e^x(f(x)+f'(x)\right)dx = e^x f(x) + c$$

Here f(x)=-log cos x

$\int e^x$ (tan x- log cos x)dx=-e^x (log cos x)+c

Question 25.
Evaluate:

$$\int \frac{dx}{(1-\sin x)}$$

Answer:
Multiplying Num^r and Den^r with (1+sinx)

$$\int \frac{1+sin\,x}{cos^2 x} dx = \int \sec^2 x + \sec x \tan x \; dx$$

= tan x + sec x + c

Question 26.
Evaluate:

$$\int c \cos x^2 dx$$

Answer:
Let x^2 =t

2xdx=dt

$$\frac{1}{2}\int \cos t \, dt = \frac{1}{2}\sin t + c$$

$$\int x \cos x^2 \; dx = \frac{1}{2}\sin x^2 + c$$

Question 27.
Evaluate:

$$\int \frac{\cot x}{\sqrt{\sin x}} dx$$

Answer:
$\frac{\cot x}{\sqrt{\sin x}} = \frac{\cos x}{(\sin x)^{3/2}}$

Let sin x=t

cos x dx=dt

$$\int \frac{dt}{t^{3/2}} = \frac{-2}{\sqrt{t}} + c$$

$$\int \frac{\cot x}{\sqrt{\sin x}} dx = \frac{-2}{\sqrt{\sin x}} + c$$

Question 28.
Evaluate:

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$$

Answer:

$$\frac{sec^2 x}{cosec^2 x} = tan^2 x$$

$$\int tan^2 x \, dx = \int (sec^2 x - 1) \, dx$$

$$\int \frac{sec^2 x}{cosec^2 x} dx = \tan x - x + c$$

Question 29.
Evaluate:

$$\int \sin^{-1}(\cos x) dx$$

Answer:

$$\int sin^{-1}(cos\, x)\, dx = \int \left(\frac{\pi}{2} - \cos^{-1}(cos x)\right) dx$$

$$\int (\frac{\pi}{2} - x)\, dx = \frac{\pi}{2} x - \frac{x^2}{2} + c$$

Question 30.

Evaluate:

$$\int \frac{dx}{\left(\sqrt{x+2}+\sqrt{x+1}\right)}$$

Answer:
On rationalizing

$$\int \frac{dx}{(\sqrt{x+2}+\sqrt{x+1})} = \int \frac{\sqrt{x+2}-\sqrt{x+1}}{(\sqrt{x+2}+\sqrt{x+1})\sqrt{x+2}-\sqrt{x+1}}dx$$

$$= \int \frac{\sqrt{x+2}-\sqrt{x+1}}{(x+2-x-1)}dx$$

$$\int \frac{\sqrt{x+2}-\sqrt{x+1}}{1}dx = \frac{2}{3}(x+2)^{3/2} - \frac{2}{3}(x+1)^{\frac{3}{2}} + c$$

Question 31.
Evaluate:

$$\int 2^x \, dx$$

Answer:
We know that,

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int 2^x dx = \frac{2^x}{\ln 2} + c$$

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Question 32.
Evaluate:

$$\int \frac{(1+\tan x)}{(x+\log \sec x)} dx$$

Answer:
Let (x + log (sec x))=t

(1+tan x) dx=dt

$$\int \frac{dt}{t} = ln\, t + c$$

$$\int \frac{(1 + tan\, x)}{(x + log\, sec\, x)} = ln|x + log(sec\, x)| + c$$

Question 33.
Evaluate:

$$\int \frac{\sec^2(\log x)}{x} dx$$

Answer:
Let $\log x = t$

$$\frac{1}{x} dx = dt$$

$$\int sec^2 t dt = tan\, t + c$$

$$\int \frac{sec^2(\log x)}{x} dx = \tan(\log x) + c$$

Question 34.
Evaluate:

$$\int (2x + 1)\left(\sqrt{x^2 + x + 1}\right) dx$$

Answer:
Let x^2+x+1=t

(2x+1)dx=dt

$$\int \sqrt{t}dt = \frac{2}{3}t^{3/2} + c = \frac{2}{3}(x^2 + x + 1)^{3/2} + c$$

Question 35.
Evaluate:

$$\int \frac{dx}{\sqrt{9x^2 + 16}}$$

Answer:
We know that,

$$\int \frac{dx}{\sqrt{(ax)^2 + b^2}} = \frac{1}{a}\log\left|ax + \sqrt{(ax)^2 + b^2}\right| + c$$

$$\int \frac{dx}{\sqrt{(3x)^2 + 4^2}} = \frac{1}{3}\log\left|3x + \sqrt{9x^2 + 16}\right| + c$$

Question 36.
Evaluate:

$$\int \frac{dx}{\sqrt{4 - 9x^2}}$$

Answer:
We know that,

$$\int \frac{dx}{\sqrt{b^2 - (ax)^2}} = \frac{1}{a}\sin^{-1}\frac{ax}{b} + c$$

$$\int \frac{dx}{\sqrt{2^2 - (3x)^2}} = \frac{1}{3}\sin^{-1}\frac{3x}{2} + c$$

Question 37.
Evaluate:

$$\int \frac{dx}{\sqrt{4x^2 - 25}}$$

Answer:
We know that,

$$\int \frac{dx}{\sqrt{(ax)^2 - b^2}} = \frac{1}{a}\log\left|ax + \sqrt{(ax)^2 - b^2}\right| + c$$

$$\int \frac{dx}{\sqrt{(2x)^2 - 5^2}} = \frac{1}{2}\log\left|2x + \sqrt{4x^2 - 25}\right| + c$$

Question 38.
Evaluate:

$$\int \sqrt{4 - x^2}\, dx$$

Answer:
We know that,

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$$

$$\int \sqrt{2^2 - x^2}\, dx = \frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2} + c$$

Question 39.
Evaluate:

$$\int \sqrt{9 + x^2}\, dx$$

Answer:
We know that,

$$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log\left|x + \sqrt{a^2 + x^2}\right| + c$$

$$\int \sqrt{3^2 + x^2}\, dx = \frac{x}{2}\sqrt{9 + x^2} + \frac{9}{2}\log\left|x + \sqrt{9 + x^2}\right| + c$$

Question 40.

Evaluate:

$$\int \sqrt{x^2 - 16}\, dx$$

Answer:
We know that,

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$$

$$\int \sqrt{x^2 - 4^2}\, dx = \frac{x}{2}\sqrt{x^2 - 16} - 8\log\left|x + \sqrt{x^2 - 16}\right| + c$$