Exercise 16d

Question 1.
Evaluate each of the following integrals as the limit of sums:

$$\int_0^2 (x+4)dx$$

Answer:
f(x) is continuous in [0,2]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a+rh), where\, h = (b-a)/n$$

here h=2/n

$$\int_0^2 (x+4)dx = \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} f(2r/n)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} \left(\frac{2r}{n}\right) + 4$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \left(\frac{(n-1)(n)}{n} + 4(n-1)\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \frac{n^2 - n + 4n^2 - 4n}{n}$$

$$= \lim_{n\to\infty} \frac{2}{n} \frac{5n^2 - 5n}{n}$$

$$= \lim_{n\to\infty} \frac{10n^2 - 10n}{n^2}$$

$$= \lim_{n\to\infty} 10 - (10/n)$$

=10

Question 2.
Evaluate each of the following integrals as the limit of sums:

$$\int_1^2 (3x - 2)dx$$

Answer:
f(x) is continuous in [1,2]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a + rh), where\ h = (b - a)/n$$

here h=1/n

$$\int_1^2 (3x - 2)dx = \lim_{n\to\infty} \left(\frac{1}{n}\right) \sum_{r=0}^{n-1} f\left(1 + \left(\frac{r}{n}\right)\right)$$

$$= \lim_{n\to\infty} \left(\frac{1}{n}\right) \sum_{r=0}^{n-1} (3 + 3\frac{r}{n} - 2)$$

$$\lim_{n\to\infty} \left(\frac{1}{n}\right)\left(n + \frac{3(n-1)(n)}{2n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{1}{n}\right)\left(\frac{2n^2 + 3n^2 - 3n}{2n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{5n^2 - 3n}{2n^2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{5}{2}\right) - \left(\frac{3}{2n}\right)$$

=5/2

Question 3.
Evaluate each of the following integrals as the limit of sums:

$$\int_1^3 x^2 dx$$

Answer:
f(x) is continuous in [1,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a + rh), where\ h = (b - a)/n$$

here h=2/n

$$\int_1^3 (x^2)dx = \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} f\left(1 + \left(\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} \left(1 + \left(\frac{2r}{n}\right)\right)^2$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} (\frac{4r^2}{n^2} + 1 + \frac{4r}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{4(n-1)(n)(2n-1)}{6n^2} + n + \frac{4(n-1)(n)}{2n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{4(2n^3 - 2n^2 - n^2 + n)}{6n^2} + n + \frac{2(n^2 - n)}{n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{(8n^3 - 12n^2 + 4n) + (6n^3) + (12n^3 - 12n^2)}{6n^2}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{26n^3 - 24n^2 + 4n}{6n^2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{52n^3 - 48n^2 + 8n}{6n^3}\right)$$

$$= \lim_{n\to\infty} \left(\frac{52}{6}\right) - \left(\frac{26}{6n}\right) + \left(\frac{8}{6n^2}\right)$$

=26/3

Question 4.
Evaluate each of the following integrals as the limit of sums:

$$\int_0^3 \left(x^2 + 1\right) dx$$

Answer:
f(x) is continuous in [0,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a + rh), where\ h = (b - a)/n$$

here h=3/n

$$\int_0^3 (x^2+1)dx = \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\left(\frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(\left(\frac{3r}{n}\right)^2+1\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(\frac{9r^2}{n^2}+1\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{9(n-1)(n)(2n-1)}{6n^2}+n\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{9(n^2-n)(2n-1)}{6n^2}+n\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{9(2n^3-2n^2-n^2+n)}{6n^2}+n)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{(18n^3-27n^2+9n)+(6n^3)}{6n^2}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{24n^3-27n^2+9n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{72n^3-81n^2+27n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{72}{6}\right)-\left(\frac{81}{6n}\right)+\left(\frac{27}{6n^2}\right)$$

=12

Question 5.
Evaluate each of the following integrals as the limit of sums:

$$\int_2^5 \left(3x^2 - 5\right)dx$$

Answer:
f(x) is continuous in [2,5]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=3/n

$$\int_2^5 (3x^2 - 5)dx = \lim_{n\to\infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} f\left(\left(2 + \frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} (3\left(2 + \frac{3r}{n}\right)^2 - 5)$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right) \sum_{r=0}^{n-1} 3\left(\frac{9r^2}{n^2} + 4 + \frac{12r}{n}\right) - 5$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{27(n-1)(n)(2n-1)}{6n^2} + 12n + \frac{18n(n-1)}{n} - 5n\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{27(n^2-n)(2n-1)}{6n^2} + 12n + \frac{18n(n-1)}{n} - 5n\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{27(2n^3 - 2n^2 - n^2 + n)}{6n^2} + 12n + \frac{18n(n-1)}{n} - 5n\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{(54n^3 - 81n^2 + 27n) + (42n^3) + (108n^3 - 108n^2)}{6n^2}\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{204n^3 - 189n^2 + 27n}{6n^2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{612n^3 - 567n^2 + 27n}{6n^3}\right)$$

$$= \lim_{n\to\infty} \left(\frac{612}{6}\right) - \left(\frac{567}{6n}\right) + \left(\frac{27}{6n^2}\right)$$

=102

Question 6.
Evaluate each of the following integrals as the limit of sums:

$$\int_0^3 \left(x^2 + 2x\right)dx$$

Answer:
f(x) is continuous in [2,5]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a + rh), where\ h = (b - a)/n$$

here h=3/n

$$\int_0^3 (x^2+2x)\,dx = \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\left(\frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}(\left(\frac{3r}{n}\right)^2 + \frac{6r}{n})$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(\frac{9r^2}{n^2} + \frac{6r}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{9(n-1)(n)(2n-1)}{6n^2} + \frac{3n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{9(n^2-n)(2n-1)}{6n^2} + \frac{3n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{9(2n^3-2n^2-n^2+n)}{6n^2} + \frac{3n(n-1)}{n})$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{(18n^3-27n^2+9n)+(18n^3-18n^2)}{6n^2}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{36n^3-45n^2+9n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{108n^3-135n^2+27n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{108}{6}\right)-\left(\frac{135}{6n}\right)+\left(\frac{27}{6n^2}\right)$$

=18

Question 7.
Evaluate each of the following integrals as the limit of sums:

$$\int_1^4 \left(3x^2 + 2x\right)dx$$

Answer:
f(x) is continuous in [1,4]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=3/n

$$\int_1^4 (3x^2+2x)dx = \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\left(1+\frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}\left(3\left(1+\frac{3r}{n}\right)^2 + 2\left(1+\frac{3r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} 3\left(\frac{9r^2}{n^2}+1+\frac{6r}{n}\right) + 2\left(1+\frac{3r}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{27(n-1)(n)(2n-1)}{6n^2} + 3n + \frac{9n(n-1)}{n} + 2n + \frac{3n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{3}{n}\left(\frac{27(n^2-n)(2n-1)}{6n^2} + 5n + \frac{12n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{27(2n^3 - 2n^2 - n^2 + n)}{6n^2} + 5n + \frac{12n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{(54n^3 - 81n^2 + 27n) + (30n^3) + (72n^3 - 72n^2)}{6n^2}\right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{156n^3 - 153n^2 + 27n}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{468n^3 - 459n^2 + 81n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{468}{6}\right) - \left(\frac{459}{6n}\right) + \left(\frac{81}{6n^2}\right)$$

=78

Question 8.
Evaluate each of the following integrals as the limit of sums:

$$\int_1^3 \left(x^2 + 5x\right) dx$$

Answer:
f(x) is continuous in [1,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=3/n

$$\int_1^3 (x^2+5x)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(\left(1+\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} (\left(1 + \frac{2r}{n}\right)^2 + 5\left(1 + \frac{2r}{n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} (1 + \frac{4r^2}{n^2} + \frac{4r}{n} + 5 + \frac{10r}{n})$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} (1 + \frac{4r^2}{n^2} + \frac{4r}{n} + 5 + \frac{10r}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{4(n-1)(n)(2n-1)}{6n^2} + 6n + \frac{7n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{4(n^2-n)(2n-1)}{6n^2} + 6n + \frac{7n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{4(2n^3-2n^2-n^2+n)}{6n^2} + 6n + \frac{7n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{(8n^3-12n^2+4n)+(42n^3-42n^2)+(36n^3)}{6n^2})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{86n^3-54n^2+4n}{6n^2})$$

$$= \lim_{n\to\infty} (\frac{172n^3-108n^2+8n}{6n^3})$$

$$= \lim_{n\to\infty} \left(\frac{172}{6}\right) - \left(\frac{108}{6n}\right) + \left(\frac{8}{6n^2}\right)$$

=86/3

Question 9.
Evaluate each of the following integrals as the limit of sums:

$$\int_1^3 \left(2x^2 + 5x\right) dx$$

Answer:
f(x) is continuous in [1,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=2/n

$$\int_1^3 (2x^2+5x)dx = \lim_{n\to\infty} (\frac{2}{n}) \sum_{r=0}^{n-1} f\left((1+\frac{2r}{n})\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} (2\left(1+\frac{2r}{n}\right)^2 + 5\left(1+\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} (2+\frac{8r^2}{n^2}+\frac{8r}{n}+5+\frac{10r}{n})$$

$$= \lim_{n\to\infty} (\frac{2}{n}) \sum_{r=0}^{n-1} (7+\frac{8r^2}{n^2}+\frac{18r}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{8(n-1)(n)(2n-1)}{6n^2} + 7n + \frac{9n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} (\frac{8(n^2-n)(2n-1)}{6n^2} + 7n + \frac{9n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{8(2n^3 - 2n^2 - n^2 + n)}{6n^2} + 7n + \frac{9n(n-1)}{n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{(16n^3 - 24n^2 + 8n) + (54n^3 - 54n^2) + (42n^3)}{6n^2}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{112n^3 - 78n^2 + 8n}{6n^2}\right)$$

$$= \lim_{n\to\infty} \left(\frac{224n^3 - 156n^2 + 8n}{6n^3}\right)$$

$$= \lim_{n\to\infty} \left(\frac{224}{6}\right) - \left(\frac{156}{6n}\right) + \left(\frac{8}{6n^2}\right)$$

=112/3

Question 10.
Evaluate each of the following integrals as the limit of sums:

$$\int_0^2 x^3 \, dx$$

Answer:
f(x) is continuous in [0,2]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a + rh), where\ h = (b-a)/n$$

here h=2/n

$$\int_0^2 (x^3)dx = \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} f\left(\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{2r}{n}\right)^3$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{8r^3}{n^3}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{8(n-1)^2(n)^2}{4n^3}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{8(n^2-2n+1)(n^2)}{4n^3}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n}\left(\frac{8(n^4-2n^3+n^2)}{4n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{16n^4-32n^3+16n^2}{4n^4}\right)$$

$$= \lim_{n\to\infty}\left(\frac{16}{4}\right)-\left(\frac{32}{4n}\right)+\left(\frac{16}{4n^2}\right)$$

=4

Question 11.
Evaluate each of the following integrals as the limit of sums:

$$\int_2^4\left(x^2-3x+2\right)dx$$

Answer:
f(x) is continuous in [2,4]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=3/n

$$\int_2^4 (x^2-3x+2)dx = \lim_{n\to\infty} (\frac{2}{n})\sum_{r=0}^{n-1} f\left((2+\frac{2r}{n})\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right)\sum_{r=0}^{n-1} (\left(2+\frac{2r}{n}\right)^2 - 3\left(2+\frac{2r}{n}\right)+2)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right)\sum_{r=0}^{n-1} (\frac{4r^2}{n^2}+\frac{8r}{n}+4-6-\frac{6r}{n}+2)$$

$$= \lim_{n\to\infty} \frac{2}{n}(\frac{4(n-1)(n)(2n-1)}{6n^2}+\frac{n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n}(\frac{4(n^2-n)(2n-1)}{6n^2}+\frac{n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n}(\frac{4(2n^3-2n^2-n^2+n)}{6n^2}+\frac{n(n-1)}{n})$$

$$= \lim_{n\to\infty} \frac{2}{n}(\frac{(8n^3-12n^2+4n)+(6n^3-6n^2)}{6n^2})$$

$$= \lim_{n\to\infty} \frac{2}{n}(\frac{14n^3-18n^2+4n}{6n^2})$$

$$= \lim_{n\to\infty}\left(\frac{28n^3 - 36n^2 + 8n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{28}{6}\right) - \left(\frac{36}{6n}\right) + \left(\frac{8}{6n^2}\right)$$

=14/3

Question 12.
Evaluate each of the following integrals as the limit of sums:

$$\int_0^2 \left(x^2 + x\right) dx$$

Answer:
f(x) is continuous in [0,2]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a + rh), where\ h = (b-a)/n$$

here h=2/n

$$\int_0^2 (x^2 + x)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\left(\frac{2r}{n}\right)^2 + \left(\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1}\left(\frac{4r^2}{n^2} + \frac{2r}{n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{4(n-1)(n)(2n-1)}{6n^2} + \frac{n(n-1)}{n} \right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{4(n^2-n)(2n-1)}{6n^2} + \frac{n(n-1)}{n} \right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{4(2n^3-2n^2-n^2+n)}{6n^2} + \frac{n(n-1)}{n} \right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{(8n^3-12n^2+4n)+(6n^3-6n^2)}{6n^2} \right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{14n^3-18n^2+4n}{6n^2} \right)$$

$$= \lim_{n\to\infty} \left(\frac{28n^3-36n^2+8n}{6n^3} \right)$$

$$= \lim_{n\to\infty} \left(\frac{28}{6}\right) - \left(\frac{36}{6n}\right) + \left(\frac{8}{6n^2}\right)$$

=14/3

Question 13.
Evaluate each of the following integrals as the limit of sums:

$$\int_0^3 \left(2x^2+3x+5\right)dx$$

Answer:
f(x) is continuous in [0,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=3/n

$$\int_0^3 (2x^2+3x+5)dx = \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1} f\left(\frac{3r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}(2\left(\frac{3r}{n}\right)^2+3\left(\frac{3r}{n}\right)+5)$$

$$= \lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{r=0}^{n-1}(\frac{18r^2}{n^2}+\frac{9r}{n}+5)$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{18(n-1)(n)(2n-1)}{6n^2}+\frac{9n(n-1)}{2n}+5n)$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{18(n^2-n)(2n-1)}{6n^2}+\frac{9n(n-1)}{2n}+5n)$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{18(2n^3-2n^2-n^2+n)}{6n^2}+\frac{9n(n-1)}{2n}+5n)$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{(36n^3-54n^2+18n)+(27n^3-27n^2)+30n^3}{6n^2})$$

$$= \lim_{n\to\infty}\frac{3}{n}(\frac{93n^3-81n^2+18n}{6n^2})$$

$$= \lim_{n\to\infty}\left(\frac{279n^3 - 243n^2 + 54n}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{279}{6}\right) - \left(\frac{243}{6n}\right) + \left(\frac{54}{6n^2}\right)$$

=93/2

Question 14.
Evaluate each of the following integrals as the limit of sums:

$$\int_0^1 |3x - 1| dx$$

Answer:
Since it is modulus function so we need to break the function and then solve it

$$f(x) = \int_0^{\frac{1}{3}} (1 - 3x)dx + \int_{\frac{1}{3}}^{1} (3x - 1)dx$$

it is continuous in [0,1]

let $g(x) = \int_0^{\frac{1}{3}}(1 - 3x)dx$ and $h(x) = \int_{\frac{1}{3}}^{1}(3x - 1)dx$

$$g(x) = \int_0^{\frac{1}{3}} (1 - 3x)dx$$

here h=1/3n

$$\int_0^{\frac{1}{3}} (1 - 3x)dx = \lim_{n\to\infty}\left(\frac{1}{3n}\right)\sum_{r=0}^{n-1} f(r/3n)$$

$$= \lim_{n\to\infty} \left(\frac{1}{3n}\right) \sum_{r=0}^{n-1} \left(1 - 3\left(\frac{r}{3n}\right)\right)$$

$$= \lim_{n\to\infty} \left(\frac{1}{3n}\right) \left(n - \frac{3(n-1)(n)}{6n}\right)$$

$$= \lim_{n\to\infty} \frac{1}{3n} \frac{6n^2 - 3n^2 + 3n}{3n}$$

$$= \lim_{n\to\infty} \frac{1}{3n} \frac{3n^2 + 3n}{3n}$$

$$= \lim_{n\to\infty} \frac{3n^2 + 3n}{9n^2}$$

$$= \lim_{n\to\infty} \frac{1}{3} + \left(\frac{3}{9n}\right)$$

=1/3

$$h(x) = \int_{\frac{1}{3}}^{1} (3x - 1)dx$$

here h=2/3n

$$\int_{\frac{1}{3}}^{1} (3x - 1)dx = \lim_{n\to\infty} \left(\frac{2}{3n}\right) \sum_{r=0}^{n-1} f\left(\left(\frac{1}{3}\right) + \left(\frac{2r}{3n}\right)\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{3n}\right) \sum_{r=0}^{n-1} (3\left(\frac{1}{3} + \frac{2r}{3n}\right) - 1)$$

$$= \lim_{n\to\infty} \left(\frac{2}{3n}\right)\left(\frac{(n-1)(n)}{n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{3n} \cdot \frac{n^2 - n}{n}$$

$$= \lim_{n\to\infty} \frac{2}{3n} \cdot \frac{n^2 - n}{n}$$

$$= \lim_{n\to\infty} \frac{2n^2 - 2n}{3n^2}$$

$$= \lim_{n\to\infty} \frac{2}{3} - \left(\frac{2}{3n}\right)$$

=2/3

f(x)=g(x)+h(x)

=(1/3)+(2/3)

=3/3

=1

Question 15.
Evaluate each of the following integrals as the limit of sums:

$$\int_0^2 e^x dx$$

Answer:
f(x) is continuous in [0,2]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h = (b-a)/n$$

here h=2/n

$$\int_0^2 (e^x)dx = \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(\frac{2r}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} e^{\frac{2r}{n}}$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)(e^0 + e^h + e^{2h} + \cdots \ldots\ldots\ldots\ldots. + e^{nh}$$

$$sum\ of e^0 + e^h + e^{2h} + \cdots \ldots\ldots\ldots\ldots. + e^{nh}$$

Which is g.p with common ratio $e^{1/n}$

Whose sum is $= \frac{e^h(1-e^{nh})}{1-e^h}$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\left(\frac{e^h(1-e^{nh})}{1-e^h}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right)\left(\frac{e^h(1-e^{nh})}{\frac{1-e^h.h}{h}}\right)$$

$$\lim_{h\to 0}\frac{1-e^h}{h} = -1$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\right).\frac{e^h(1-e^{nh})}{-h}$$

As h=2/n

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right).\frac{e^{\left(\frac{2}{n}\right)}(1 - e^{n*(2/n)})}{-2/n}$$

$=e^2-1$

Question 16.
Evaluate each of the following integrals as the limit of sums:

$$\int_1^3 e^{-x} dx$$

Answer:
f(x) is continuous in [1,3]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h \sum_{r=0}^{n-1} f(a + rh), where\ h = (b - a)/n$$

here h=2/n

$$\int_1^3 (e^{-x})dx = \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} f\left(1 + \left(\frac{2r}{n}\right)\right)$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} e^{-(1+\frac{2r}{n})}$$

$$= \lim_{n\to\infty} \left(\frac{2}{n}\right) \sum_{r=0}^{n-1} e^{-1}.e^{-\frac{2r}{n}}$$

Common ratio is $h = -2/n$

$$sum = e^{-1}(e^0 + e^h + e^{2h} + \cdots \ldots \ldots \ldots \ldots . + e^{nh})$$

$$= \lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)(e^0 + e^h + e^{2h} + \cdots \ldots\ldots\ldots\ldots. + e^{nh}$$

$$sum\ of = e^0 + e^h + e^{2h} + \cdots \ldots\ldots\ldots\ldots. + e^{nh}$$

Which is g.p. with common ratio $e^{1/n}$

Whose sum is $= \frac{e^h(1-e^{nh})}{1-e^h}$

$$= \lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^h(1-e^{nh})}{1-e^h}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^h(1-e^{nh})}{\frac{1-e^h.h}{h}}\right.$$

$$\lim_{h\to 0}\frac{1-e^h}{h} = -1$$

$$= \lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^h(1-e^{nh})}{-h}\right.$$

As h=-2/n

$$= \lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^{(-\frac{2}{n})}(1-e^{n*(-2/n)})}{2/n}\right.$$

$$= \frac{(1-e^{-2})}{e}$$

$$= \frac{(e^2-1)}{e^3}$$

Question 17.
Evaluate each of the following integrals as the limit of sums:

$$\int_a^b \cos x\, dx$$

Answer:
f(x) is continuous in [a,b]

$$\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh), where\ h=(b-a)/n$$

here h=(b-a)/n

$$\int_a^b (cosx)dx = \lim_{n\to\infty}\left(\frac{b-a}{n}\right)\sum_{r=0}^{n-1} f(a+rh)$$

$$=\lim_{n\to\infty}\left(\frac{b-a}{n}\right)\sum_{r=0}^{n-1}\cos(a+rh)$$

S=cos(a)+ cos(a+h)+ cos(a+2h)+ cos(a+3h)+......................+ cos(a+(n-1)h)= $\frac{\sin\left(\frac{nh}{2}\right)\cos\left(a+\frac{(n-1)h}{2}\right)}{\sin\left(\frac{h}{2}\right)}$

Putting h=(b-a)/n

$$=\lim_{n\to\infty}\left(\frac{b-a}{n}\right)\frac{\sin\left(\frac{n(b-a)}{2n}\right)\cos\left(a+\frac{(n-1)(b-a)}{2n}\right)}{\frac{\sin\left(\frac{b-a}{2n}\right)}{\frac{b-a}{2n}}\cdot\frac{b-a}{2n}}$$

As we know

$$\lim_{h\to 0}\left(\frac{sinh}{h}\right)=1$$

$$=\lim_{n\to\infty} 2\sin\left(\frac{(b-a)}{2}\right)\cos(a+\left(\frac{1}{2}-\frac{1}{2n}\right)(b-a)$$

$$=2\sin\left(\frac{b-a}{2}\right)\cos\left(\frac{b+a}{2}\right)$$

Which is trigonometry formula of sin(b)-sin(a)

Final answer is sin(b)-sin(a)