Exercise 29a

Question 1.
Let A and B be the events such that

$$P(A)=\frac{7}{13}, P(B)=\frac{9}{13} \text{ and } P(A\cap B)=\frac{4}{13}.$$

Find

(i) P(A / B)

(ii) P(B / A)

(iii) P(A ∪ B)

(iv) $P\left(\overline{B}/\overline{A}\right)$

Answer:
Given - A and B be the events such that $P(A)=\frac{7}{13}$, $P(B)=\frac{9}{13}$ and

$$P(A\cap B)=\frac{4}{13}$$

To find – (i) $P(A/B)$ (ii) $P(B/A)$ (iii) $P(A\cup B)$ (iv) $P(\overline{B}/\overline{A})$

Formula to be used – By conditional probability, $P(A/B)=\frac{P(A\cap B)}{P(B)}$ where $P(A/B)$ is the probability of occurrence of the event A given that B has already occurred.

(i)$P(A/B)$

$$=\frac{P(A\cap B)}{P(B)}$$

$$=\frac{4}{13}\div\frac{9}{13}$$

$$=\frac{4}{9}$$

(ii) $P(B/A)$

$$=\frac{P(A\cap B)}{P(A)}$$

$$=\frac{4}{13}\div\frac{7}{13}$$

$$=\frac{4}{7}$$

(iii) $P(A\cup B)$

$$=P(A)+P(B)-P(A\cap B)$$

$$=\frac{7}{13}+\frac{9}{13}-\frac{4}{13}$$

$$=\frac{12}{13}$$

(iv) $P(\overline{B}/\overline{A})=\frac{P(\overline{A}\cap\overline{B})}{P(\overline{A})}$

Now, by De-Morgan's Law, $(A\cup B)^{C}=A^{C}\cap B^{C}$

$$\therefore P(\overline{A}\cap\overline{B})=P\overline{(A\cup B)}$$

$$\therefore \frac{P(\overline{A}\cap\overline{B})}{P(\overline{A})}$$

$$=\frac{P\overline{(A\cup B)}}{P(\overline{A})}$$

$$=\frac{1-P(A\cup B)}{1-P(A)}$$

$$=\frac{1-\frac{12}{13}}{1-\frac{7}{13}}$$

$$= \frac{1}{6}$$

Question 2.
Let A and B be the events such that

$P(A) = \frac{5}{11}, P(B) = \frac{6}{11}$ and $P(A \cup B) = \frac{7}{11}$.

Find

(i) P(A ∩ B)

(ii) P(A / B)

(iii) P(B / A)

(iv) $P(\overline{A} / \overline{B})$

Answer:
Given - A and B be the events such that $P(A) = \frac{5}{11}$, $P(B) = \frac{6}{11}$ and

$P(A \cup B) = \frac{7}{11}$

To find – (i) $P(A \cap B)$ (ii) $P(A/B)$ (iii)$P(B/A)$ (iv) $P(\bar{A}/\bar{B})$

Formula to be used – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ where $P(A/B)$ is the probability of occurrence of the event A given that B has already occurred.

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

⇨ $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$= \frac{5}{11} + \frac{6}{11} - \frac{7}{11}$$

$$= \frac{4}{11}$$

(ii) $P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{4}{11} \div \frac{6}{11}$

$= \frac{4}{6}$

$= \frac{2}{3}$

(iii) $P(B/A)$

$= \frac{P(A \cap B)}{P(A)}$

$= \frac{4}{11} \div \frac{5}{11}$

$= \frac{4}{5}$

(iv) $P(\bar{A}/\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$

Now, by De-Morgan's Law, $(A \cup B)^C = A^C \cap B^C$

$\therefore P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$\therefore \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$

$= \frac{P(\overline{A \cup B})}{P(\bar{A})}$

$= \frac{1 - P(A \cup B)}{1 - P(B)}$

$$=\frac{1-\frac{7}{11}}{1-\frac{6}{11}}$$

$$=\frac{4}{5}$$

Question 3.
Let A and B be the events such that

$$P(A)=\frac{3}{10}, P(B)=\frac{1}{2} \text{ and } P(B/A)=\frac{2}{5}.$$

Find

(i) P(A ∩ B)

(ii) P(A ∪ B)

(iii) P(A / B)

Answer:
Given - A and B be the events such that $P(A)=\frac{3}{10}$, $P(B)=\frac{1}{2}$ and

$$P(B/A)=\frac{2}{5}$$

To find – (i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A/B)$

Formula to be used – By conditional probability, $P(A/B)=\frac{P(A\cap B)}{P(B)}$ where $P(A/B)$ is the probability of occurrence of the event A given that B has already occurred.

(i)$P(B/A)=\frac{P(A\cap B)}{P(A)}$

⇨ $P(A \cap B)=P(A)P(B/A)$

$$=\frac{3}{10}\times\frac{2}{5}$$

$$= \frac{3}{25}$$

(ii) $P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{10} + \frac{1}{2} - \frac{3}{25}$$

$$= \frac{15 + 25 - 6}{50}$$

$$= \frac{34}{50}$$

$$= \frac{17}{50}$$

(iii) $P(A/B)$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{3}{25} \div \frac{1}{2}$$

$$= \frac{6}{25}$$

Question 4.
Let A and B be the events such that

$2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$.

Find

(i) $P(A \cap B)$

(ii) $P(A \cup B)$.

Answer:

Given - A and B be the events such that $2P(A) = P(B) = \frac{5}{13}$ and

$$P(A/B) = \frac{2}{5}$$

To find – (i) $P(A \cap B)$ (ii) $P(A \cup B)$

Formula to be used – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ where $P(A/B)$ is the probability of occurrence of the event A given that B has already occurred.

(i)$P(A/B) = \frac{P(A \cap B)}{P(B)}$

⇨ $P(A \cap B) = P(B)P(A/B)$

$$= \frac{5}{13} \times \frac{2}{5}$$

$$= \frac{2}{13}$$

(ii) $P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$$

$$= \frac{5 + 10 - 4}{26}$$

$$= \frac{11}{26}$$

Question 5.

A die is rolled. If the outcome is an even number, what is the probability that it is a number greater than 2?

Answer:
A die has 6 faces and its sample space S={1,2,3,4,5,6}.

The total number of outcomes = 6.

Let P(A) be the probability of getting an even number.

The sample space of A = {2,4,6}

$$\therefore \mathrm{P(A)} = \frac{3}{6} = \frac{1}{2}$$

Let P(B) be the probability of getting a number whose value is greater than 2.

The sample space of B = {3,4,5,6}

$$\therefore (\mathrm{A \cap B}) = \{4{,}6\}$$

$$\therefore \mathrm{P(A \cap B)} = \frac{2}{6} = \frac{1}{3}$$

Tip – By conditional probability, $P(A/B) = \frac{P(A\cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability of getting a number greater than 2 given that the outcome is even is given by:

$$\mathrm{P(B/A)}$$

$$= \frac{\mathrm{P(A \cap B)}}{\mathrm{P(A)}}$$

$$= \frac{1/3}{1/2}$$

$$= \frac{2}{3}$$

Question 6.

A coin is tossed twice. If the outcome is at most one tail, what is the probability that both head and tail have appeared?

Answer:
A coin has 2 sides and its sample space S={H,T}

The total number of outcomes = 2.

A coin is tossed twice.

Let P(A) be the probability of getting at most 1 tail.

The sample space of A = {(H,H),(H,T),(T,H)}

Let P(B) be the probability of getting a head.

The sample space of B = {H}

$$\therefore \mathrm{P(B)} = \frac{1}{2}$$

The probability of getting at most one tail and a head

$$\mathrm{i.e.\,(A \cap B) = \{(H,H)\}}$$

$$\therefore \mathrm{P(A \cap B)} = \frac{1}{3}$$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that both head and tail have appeared:

$$\mathrm{P(A/B)}$$

$$= \frac{\mathrm{P(A \cap B)}}{\mathrm{P(B)}}$$

$$= \frac{1/3}{1/2}$$

$= \frac{2}{3}$

Question 7.
Three coins are tossed simultaneously. Find the probability that all coins show heads if at least one of the coins shows a head.

Answer:
When three coins are tossed simultaneously, the total number of outcomes = 2^3 = 8, and the sample space is given by S= {(H,H,H),(H,H,T),(H,T,T),(H,T,H),(T,H,T),(T,T,H),(T,H,H),(T,T,T)}

Let P(A) be the probability of getting 3 heads.

The sample space of A = {(H,H,H)}

$\therefore P(A) = \frac{1}{8}$

Let P(B) be the probability of getting at least head.

Probability of one head = 1 – probability of no heads = 1-1/8 = 7/8

$\therefore P(B) = \frac{7}{8}$

The probability that the throw is either all heads or at least one head i.e. $P(A \cup B) = \frac{7}{8}$

Now,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cap B)$

$= P(A) + P(B) - P(A \cup B)$

$= \frac{1}{8} + \frac{7}{8} - \frac{7}{8}$

$= \frac{1}{8}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that all coins show heads if at least one of the coins

showed a head:

$$P(A/B)$$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/8}{7/8}$$

$$= \frac{1}{7}$$

Question 8.
Two unbiased dice are thrown. Find the probability that the sum of the numbers appearing is 8 or greater, if 4 appears on the first die.

Answer:
Two die having 6 faces each when tossed simultaneously will have a total outcome of 6^2=36

Let P(A) be the probability of getting a sum greater than 8.

Let P(B) be the probability of getting 4 on the first die.

The sample space of B = {(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)}

$$\therefore P(B) = \frac{6}{36} = \frac{1}{6}$$

Let $P(A \cap B)$ be the probability of getting 4 on the first die and the sum greater than or equal to 8

The sample space of $(A \cap B) = \{(4,4),(4,5),(4,6)\}$

$$\therefore P(A \cap B) = \frac{3}{36} = \frac{1}{12}$$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that sum of the numbers is greater than or equal to 8 given that 4 was thrown first:

$$P(A/B)$$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/12}{1/6}$$

$$= \frac{1}{2}$$

Question 9.
A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared at least once?

Answer:
A die thrown twice will have a total outcome of 6^2=36.

Let P(A) be the probability of getting the number 5 at least once.

Let P(B) be the probability of getting sum = 8.

The sample space of B = {(2,6),(3,5),(4,4),(5,3),(6,2)}

$$\therefore P(B) = \frac{5}{36}$$

Let $P(A \cap B)$ be the probability of getting the number 5 at least once and the sum equal to 8

The sample space of $(A \cap B) = \{(3,5),(5,3)\}$

$$\therefore P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that the number 5 have appeared at least once given that the sum = 8:

$$P(A/B)$$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/18}{5/36}$$

$$= \frac{2}{5}$$

Question 10.
Two dice were thrown and it is known that the numbers which come up were different. Find the probability that the sum of the two numbers was 5.

Answer:
Two die having 6 faces each when tossed simultaneously will have a total outcome of 6^2=36

Let P(A) be the probability of getting a sum equal to 5.

Let P(B) be the probability of getting 2 different numbers.

Probability of getting 2 different numbers

= 1 – probability of getting same numbers

= 1 – 1/6

=5/6

$\therefore \mathrm{P(B)} = \frac{5}{6}$

Let $\mathrm{P(A \cap B)}$ be the probability of getting a sum = 5 and two different numbers at the same time.

The sample space of $(\mathrm{A \cap B}) = \{(1,4),(2,3),(3,2),(4,1)\}$

$\therefore \mathrm{P(A \cap B)} = \frac{4}{36} = \frac{1}{9}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that the sum = 5 given that two different numbers were thrown:

$\mathrm{P(A/B)}$

$= \frac{\mathrm{P(A \cap B)}}{\mathrm{P(B)}}$

$= \frac{1/9}{5/6}$

$= \frac{2}{15}$

Question 11.
A coin is tossed and then a die is thrown. Find the probability of obtaining a 6, given that a head came up.

Answer:
A coin is tossed and a die thrown.

A coin having two sides have a total outcome of 2 viz. {H,T}

A die has 6 faces and will have a total outcome of 6 i.e. {1, 2,3,4,5,6}

Let P(A) be the probability of getting the number 6.

$\therefore P(A) = \frac{1}{6}$

Let P(B) be the probability of getting a head.

The sample space of B ={H}

$\therefore P(B) = \frac{1}{2}$

Let $P(A \cap B)$ be the probability of getting the number 6 and a head.

$\therefore P(A \cap B) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that 6 came up given that head came up:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/12}{1/2}$

$= \frac{1}{6}$

Question 12.
A couple has 2 children. Find the probability that both are boys if it is known that (i) one of the children is a boy, and (ii) the elder child is a boy.

Answer:
A couple has two children.

The sample space S = {(B,B),(B,G),(G,B),(G,G)}

Let P(A) be the probability of both being boys.

(i) Let P(B) be the probability of one of them being a boy.

The sample space of B ={(B,B),(B,G),(G,B)}

$\therefore \mathrm{P}(\mathrm{B})=\frac{3}{4}$

Let $\mathrm{P}(\mathrm{A} \cap \mathrm{B})$ be the probability of one of them being a boy and both being boys.

$\therefore (\mathrm{A} \cap \mathrm{B})=\{(\mathrm{B}, \mathrm{B})\}$

$\therefore \mathrm{P}(\mathrm{A} \cap \mathrm{B})=\frac{1}{4}$

Tip – By conditional probability, $P(A/B)=\frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that both are boys given that one of them is a boy:

$\mathrm{P}(\mathrm{A}/\mathrm{B})$

$=\frac{\mathrm{P}(\mathrm{A} \cap \mathrm{B})}{\mathrm{P}(\mathrm{B})}$

$=\frac{1/4}{3/4}$

$=\frac{1}{3}$

(ii) Let P(B) be the probability of the elder being a boy.

The sample space of B ={(B,B),(B,G)}

$\therefore \mathrm{P}(\mathrm{B})=\frac{1}{2}$

Let $P(A \cap B)$ be the probability of the elder being a boy and both being boys.

$\therefore (A \cap B) = \{(B, B)\}$

$\therefore P(A \cap B) = \frac{1}{4}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that both are boys given that the elder is a boy:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/4}{1/2}$

$= \frac{1}{2}$

Question 13.
In a class, 40% students study mathematics; 25% study biology and 15% study both mathematics and biology. One student is selected at random. Find the probability that

(i) he studies mathematics if it is known that he studies biology

(ii) he studies biology if it is known that he studies mathematics.

Answer:
Let P(A) be the probability of students studying mathematics.

$\therefore P(A) = 0.40$

Let P(B) be the probability of students studying biology.

$\therefore P(B) = 0.25$

Let $\mathrm{P}(\mathrm{A} \cap \mathrm{B})$ be the probability of students studying both mathematics and biology.

∴P(A∩B)=0.15

One student is selected at random.

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

(i)The probability that he studies mathematics given that he studies biology:

$\mathrm{P}(\mathrm{A}/\mathrm{B})$

$= \frac{\mathrm{P}(\mathrm{A} \cap \mathrm{B})}{\mathrm{P}(\mathrm{B})}$

$= \frac{0.15}{0.25}$

$= \frac{3}{5}$

(ii)The probability that he studies biology given that he studies mathematics:

$\mathrm{P}(\mathrm{A}/\mathrm{B})$

$= \frac{\mathrm{P}(\mathrm{A} \cap \mathrm{B})}{\mathrm{P}(\mathrm{B})}$

$= \frac{0.15}{0.40}$

$= \frac{3}{8}$

Question 14.

The probability that a student selected at random from a class will pass in Hindi is $\frac{4}{5}$ and the probability that he passes in Hindi and English is $\frac{1}{2}$. What is the probability that he will pass in English if it is known that he has passed in Hindi?

Answer:
One student is selected at random.

Let P(A) be the probability of students passing in English.

Let P(B) be the probability of students passing in Hindi.

$$\therefore \mathrm{P(B)} = \frac{4}{5}$$

Let $\mathrm{P(A \cap B)}$ be the probability of students passing in both English and Hindi.

$$\therefore \mathrm{P(A \cap B)} = \frac{1}{2}$$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that he will pass in English given that he passes in Hindi:

$$\mathrm{P(A/B)}$$

$$= \frac{\mathrm{P(A \cap B)}}{\mathrm{P(B)}}$$

$$= \frac{1/2}{4/5}$$

$$= \frac{5}{8}$$

Question 15.

The probability that a certain person will buy a shirt is 0.2, the probability that he will buy a coat is 0.3 and the probability that he will buy a shirt given that he buys a coat is 0.4. Find the probability that he will buy both a shirt and a coat.

Answer:
Let P(A) be the probability of a certain person buying a shirt.

$\therefore P(A)=0.2$

Let P(B) be the probability of him buying a coat.

$\therefore P(B)=0.3$

Let $P(A \cap B)$ be the probability that he buys both a shirt and a coat.

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that he will buy a shirt given that he buys a coat:

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = 0.4$$

$\Rightarrow P(A \cap B)=P(B)P(A/B)$

$=0.3 \times 0.4$

$= 0.12$

Question 16.
In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(i) Find the probability that he reads neither Hindi nor English news paper.

(ii) If he reads Hindi newspaper, what is the probability that he reads English newspaper?

(iii) If he reads English newspaper, what is the probability that he reads Hindi newspaper?

Answer:
Let P(A) be the probability of students reading Hindi newspaper.

$\therefore$P(A)=0.60

Let P(B) be the probability of them reading English newspaper.

$\therefore$P(B)=0.40

Let P(A∩B) be the probability them reading both.

$\therefore \mathrm{P}(\mathrm{A} \cap \mathrm{B}) = 0.20$

Let $\mathrm{P}(\mathrm{A} \cup \mathrm{B})$ be the probability them reading either one of them.

$\therefore$P(A∪B)

=P(A)+P(B)-P(A∩B)

=0.60+0.40-0.20

=0.80

(i)The probability that none of them reads either of them

= 1 – 0.8

= 0.2

=*1/5*

Tip – By conditional probability,$P(A/B) = \frac{P(A \cap B)}{P(B)}$*where*$P(A/B)$*is the probability of occurrence of the event A given that B has already occurred.*

(ii)The probability that he reads the English one given that he reads the Hindi one:

$\mathrm{P}(\mathrm{A}/\mathrm{B})$

$= \frac{\mathrm{P}(\mathrm{A} \cap \mathrm{B})}{\mathrm{P}(\mathrm{A})}$

$$= \frac{0.20}{0.60}$$

$$= \frac{1}{3}$$

(iii)The probability that he reads the Hindi one given that he reads the English one:

$$P(A/B)$$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.20}{0.40}$$

$$= \frac{1}{2}$$

Question 17.
Two integers are selected at random from integers 1 through 11. If the sum is even, find the probability that both the numbers selected are odd.

Answer:
Two integers are selected at random.

The first choice has 11 options from the 11 integers, and the second choice has 10 options from the remaining 10 integers.

Let P(A) be the probability of choosing both numbers odd.

Let P(B) be the probability of choosing the numbers to yield an even number.

Sample space of B = {(1,3),(1,5),(1,7),(1,9),(1,11),(3,5),(3,7),(3,9),(3,11),(5,7),(5,9),(5,11),(7,9),(7,11),(9,11),(2,4),(2,6),(2,8),(2,10),(4,6),(4,8),(4,10),(6,8),(6,10),(8,10)}

$$\therefore P(B) = \frac{25}{11 \times 10} = \frac{25}{110}$$

Let $P(A \cap B)$ be the probability of getting both odd numbers giving an even sum.

$\therefore (A \cap B) = \{(1,3),(1,5),(1,7),(1,9),(1,11),(3,5),(3,7),(3,9),(3,11),(5,7),$

$(5,9),(5,11),(7,9),(7,11),(9,11),\}$

$$\therefore P(A \cap B) = \frac{15}{110}$$

The probability of getting both numbers odd given that sum is even:

$$P(A/B)$$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{15/_{110}}{25/_{110}}$$

$$= \frac{15}{25}$$

$$= \frac{3}{5}$$