Exercise 14a

Question 1.
Evaluate:

$$\int \frac{dx}{(1-9x)^2}$$

Answer:
To find: $\int \frac{dx}{(1-9x)^2}$

Formula Used: $\int x^n = \frac{x^{n+1}}{n+1} + C$

Let y = (1 – 9x) ... (1)

Differentiating with respect to x,

$$\frac{dy}{dx} = -9$$

i.e., dy = -9 dx

Substituting in the equation to evaluate,

$$\Rightarrow \int \frac{\frac{dy}{-9}}{y^2}$$

$$\Rightarrow \frac{-1}{9} \int \frac{dy}{y^2}$$

$$\Rightarrow \frac{-1}{9} \times \int y^{-2}\ dy$$

$$\Rightarrow \frac{-1}{9} \times \frac{y^{-2+1}}{-2+1} + C$$

Simplifying and substituting the value of y from (1),

$$\Rightarrow \frac{-1}{9} \times \frac{-1}{(1-9x)} + C$$

$$\Rightarrow \frac{1}{9(1-9x)} + C$$

Therefore,

$$\int \frac{dx}{(1-9x)^2} = \frac{1}{9(1-9x)} + C$$

Question 2.
Evaluate:

$$\int \frac{dx}{\left(25-4x^2\right)}$$

Answer:
To find: $\int \frac{dx}{(25-4x^2)}$

Formula Used: $\frac{dx}{(a^2-x^2)} = \frac{1}{2a} \times \log\left|\frac{a+x}{a-x}\right| + C$

Given equation = $\int \frac{dx}{4(\frac{25}{4}-x^2)}$

$\Rightarrow \frac{1}{4}\int \frac{dx}{\left(\left(\frac{5}{2}\right)^2-x^2\right)}$ ... (1)

Here $a = \frac{5}{2}$

Therefore, (1) becomes

$$\Rightarrow \frac{1}{4} \times \frac{1}{5} \times \log \left| \frac{\frac{5}{2} + x}{\frac{5}{2} - x} \right| + C$$

$$\Rightarrow \frac{1}{20} \times \log \left| \frac{5 + 2x}{5 - 2x} \right| + C$$

Therefore,

$$\int \frac{dx}{(25 - 4x^2)} = \frac{1}{20} \times \log \left| \frac{5 + 2x}{5 - 2x} \right| + C$$

Question 3.
Evaluate:

$$\int \frac{\mathrm{dx}}{\left(\mathrm{x}^2 + 16\right)}$$

Answer:
To find: $\int \frac{dx}{(x^2+16)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{4^2 + x^2}$$

Here a = 4

$$\Rightarrow \frac{1}{4} \times \tan^{-1}\left(\frac{x}{4}\right) + C$$

Therefore,

$$\int \frac{dx}{(x^2 + 16)} = \frac{1}{4} \times \tan^{-1}\left(\frac{x}{4}\right) + C$$

Question 4.
Evaluate:

$$\int \frac{dx}{\left(4+9x^2\right)}$$

Answer:
To find: $\int \frac{dx}{(4+9x^2)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{9}\int \frac{dx}{\left(\frac{4}{9}\right) + x^2}$$

$$\Rightarrow \frac{1}{9}\int \frac{dx}{\left(\frac{2}{3}\right)^2 + x^2}$$

Here $a = \frac{2}{3}$

$$\Rightarrow \frac{1}{9} \times \frac{3}{2} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

$$\Rightarrow \frac{1}{6} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

Therefore,

$$\int \frac{dx}{(4+9x^2)} = \frac{1}{6} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

Question 5.

Evaluate:

$$\int \frac{dx}{\left(50+2x^2\right)}$$

Answer:

To find: $\int \frac{dx}{(50+2x^2)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{2}\int \frac{dx}{25+x^2}$$

$$\Rightarrow \frac{1}{2}\int \frac{dx}{5^2+x^2}$$

Here a = 5

$$\Rightarrow \frac{1}{10} \times \tan^{-1}\left(\frac{x}{5}\right) + C$$

Therefore,

$$\int \frac{dx}{(x^2+16)} = \frac{1}{10} \times \tan^{-1}\left(\frac{x}{5}\right) + C$$

Question 6.

Evaluate:

$$\int \frac{dx}{\left(16x^2-25\right)}$$

Answer:

To find: $\int \frac{dx}{(16x^2-25)}$

Formula Used: $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{16} \int \frac{dx}{x^2 - \left(\frac{25}{16}\right)}$$

$$\Rightarrow \frac{1}{16} \int \frac{dx}{x^2 - \left(\frac{5}{4}\right)^2}$$

Here $a = \frac{5}{4}$

$$\Rightarrow \frac{1}{16} \times \frac{2}{5} \times \ln\left|\frac{x - \frac{5}{4}}{x + \frac{5}{4}}\right| + C$$

$$\Rightarrow \frac{1}{40} \times ln\left|\frac{4x - 5}{4x + 5}\right| + C$$

Therefore,

$$\int \frac{dx}{(16x^2 - 25)} = \frac{1}{40} \times \log\left|\frac{4x - 5}{4x + 5}\right| + C$$

Question 7.
Evaluate:

$$\int \frac{(x^2 - 1)}{(x^2 + 4)} dx$$

Answer:
To find: $\int \frac{(x^2 - 1)}{(x^2 + 4)}\, dx$

Formula Used: $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Given equation can be rewritten as the following:

$$\Rightarrow \int \frac{(x^2+4-5)}{(x^2+4)}\, dx$$

$$\Rightarrow \int \frac{(x^2+4)}{(x^2+4)} dx \; - \int \frac{5}{(x^2+4)}\, dx$$

$$\Rightarrow \int dx \, - \, 5 \int \frac{1}{(x^2+2^2)}\, dx$$

Here a = 2,

$$\Rightarrow x - \frac{5}{2}\, tan^{-1} \frac{x}{2} + C$$

Therefore,

$$\int \frac{(x^2-1)}{(x^2+4)}\, dx = \; x - \frac{5}{2}\, tan^{-1} \frac{x}{2} + C$$

Question 8.
Evaluate:

$$\int \frac{x^2}{(9+4x^2)} dx$$

Answer:
To find: $\int \frac{x^2}{(9+4x^2)}\, dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Given equation can be rewritten as the following:

$$\Rightarrow \frac{1}{4}\int \frac{x^2}{(x^2+\frac{9}{4})}\,dx$$

$$\Rightarrow \frac{1}{4}\int \frac{x^2+\frac{9}{4}-\frac{9}{4}}{(x^2+\frac{9}{4})}\,dx$$

$$\Rightarrow \frac{1}{4}\int dx\ -\frac{9}{16}\int \frac{1}{\left(x^2+\left(\frac{3}{2}\right)^2\right)}\,dx$$

Here $a = \frac{3}{2}$,

$$\Rightarrow \frac{x}{4}-\left(\frac{9}{16}\times\frac{2}{3}\,tan^{-1}\frac{2x}{3}\right)+C$$

$$\Rightarrow \frac{x}{4}-\frac{3}{8}\,tan^{-1}\left(\frac{2x}{3}\right)+C$$

Therefore,

$$\int \frac{x^2}{(9+4x^2)}\,dx = \frac{x}{4}-\frac{3}{8}\,tan^{-1}\left(\frac{2x}{3}\right)+C$$

Question 9.
Evaluate:

$$\int \frac{e^x}{(e^{2x}+1)}dx$$

Answer:
To find: $\int \frac{e^x}{(e^{2x}+1)}\,dx$

Formula Used: $\int \frac{dx}{1+x^2} = \tan^{-1} x$

Let y = e^x ... (1)

Differentiating both sides, we get

dy = e^x dx

Substituting in given equation,

$$\Rightarrow \int \frac{dy}{y^2 + 1}$$

$\Rightarrow \tan^{-1} y$

From (1),

$\Rightarrow \tan^{-1} (e^x)$

Therefore,

$$\int \frac{e^x}{(e^{2x} + 1)}\, dx = \tan^{-1}(e^x) + C$$

Question 10.
Evaluate:

$$\int \frac{\sin x}{(1 + \cos^2 x)} dx$$

Answer:
To find: $\int \frac{\sin x}{(1+\cos^2 x)}\, dx$

Formula Used: $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Let y = cos x ... (1)

Differentiating both sides, we get

dy = –sin x dx

Substituting in given equation,

$$\Rightarrow \int \frac{-dy}{1+y^2}$$

⇒ – $\tan^{-1} y$

From (1),

⇒ $-\tan^{-1} (\cos x)$

Therefore,

$$\int \frac{\sin x}{(1+\cos^2 x)}\, dx = -\tan^{-1}(\cos x) + C$$

Question 11.
Evaluate:

$$\int \frac{\cos x}{(1+\sin^2 x)} dx$$

Answer:
To find: $\int \frac{\cos x}{(1+\sin^2 x)}\, dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Let y = sin x ... (1)

Differentiating both sides, we get

dy = cos x dx

Substituting in given equation,

$$\Rightarrow \int \frac{dy}{1+y^2}$$

$\Rightarrow \tan^{-1} y$

From (1),

$\Rightarrow \tan^{-1} (\sin x)$

Therefore,

$$\int \frac{\cos x}{(1+\sin^2 x)}\, dx = \tan^{-1}(\sin x) + C$$

Question 12.
Evaluate:

$$\int \frac{3x^5}{(1+x^{12})} dx$$

Answer:
To find: $\int \frac{3x^5}{(1+x^{12})}\, dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Let $y = x^6$ ... (1)

Differentiating both sides, we get

$dy = 6x^5\, dx$

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2}\, dy}{1+y^2}$$

$$\Rightarrow \frac{1}{2}\tan^{-1} y + C$$

From (1),

$$\Rightarrow \frac{1}{2}\tan^{-1}(x^6) + C$$

Therefore,

$$\int \frac{3x^5}{(1+x^{12})}\,dx = \frac{1}{2}\tan^{-1}(x^6) + C$$

Question 13.
Evaluate:

$$\int \frac{2x^3}{(4+x^8)}dx$$

Answer:
To find: $\int \frac{2x^3}{(4+x^8)}\,dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Let y = x^4 ... (1)

Differentiating both sides, we get

dy = $4x^3$ dx

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2}\,dy}{4+y^2}$$

$$\Rightarrow \frac{1}{2}\int \frac{1}{2^2+y^2}\,dy$$

$$\Rightarrow \frac{1}{4}\tan^{-1}\left(\frac{y}{2}\right) + C$$

From (1),

$$\Rightarrow \frac{1}{4}\tan^{-1}\left(\frac{x^4}{2}\right)+C$$

Therefore,

$$\int \frac{2x^3}{(4+x^8)}\,dx = \frac{1}{4}\tan^{-1}\left(\frac{x^4}{2}\right)+C$$

Question 14.
Evaluate:

$$\int \frac{\mathrm{dx}}{(\mathrm{e}^{\mathrm{x}}+\mathrm{e}^{-\mathrm{x}})}$$

Answer:
To find: $\int \frac{dx}{(e^x+e^{-x})}$

Formula Used: $\int \frac{dx}{1+x^2} = \tan^{-1} x$

Given equation is:

$\int \frac{dx}{(e^x+e^{-x})} = \int \frac{e^x\,dx}{(e^{2x}+1)}$ ... (1)

Let y = e^x ... (1)

Differentiating both sides, we get

dy = e^x dx

Substituting in (1),

$$\Rightarrow \int \frac{dy}{y^2+1}$$

$\Rightarrow \tan^{-1} y$

From (1),

$\Rightarrow \tan^{-1} (e^x)$

Therefore,

$$\int \frac{dx}{(e^x + e^{-x})} = \tan^{-1}(e^x) + C$$

Question 15.
Evaluate:

$$\int \frac{x}{(1-x^4)} dx$$

Answer:
To find: $\int \frac{x\,dx}{(1-x^4)}$

Formula Used: $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$

Let $y = x^2$ ... (1)

Differentiating both sides, we get

$dy = 2x\,dx$

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2}\,dy}{1 - y^2}$$

Here $a = 1$,

$$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \log \left|\frac{1+y}{1-y}\right| + C$$

$$\Rightarrow \frac{1}{4} \log \left|\frac{1+y}{1-y}\right| + C$$

From (1),

$$\Rightarrow \frac{1}{4} \log \left|\frac{1+x^2}{1-x^2}\right| + C$$

Therefore,

$$\int \frac{x\,dx}{(1-x^4)} = \frac{1}{4} \log \left|\frac{1+x^2}{1-x^2}\right| + C$$

Question 16.
Evaluate:

$$\int \frac{x^2}{(a^6 - x^6)} dx$$

Answer:
To find: $\int \frac{x^2\,dx}{(a^6 - x^6)}$

Formula Used: $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left|\frac{a+x}{a-x}\right| + C$

Let y = x^3 ... (1)

Differentiating both sides, we get

dy = $3x^2$ dx

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{3}\,dy}{a^6 - y^2}$$

$$\Rightarrow \frac{1}{3}\int \frac{1}{(a^3)^2 - y^2}\,dy$$

$$\Rightarrow \frac{1}{3} \times \frac{1}{2a^3} \times \log\left|\frac{a^3 + y}{a^3 - y}\right| + C$$

$$\Rightarrow \frac{1}{6a^3}\log\left|\frac{a^3 + y}{a^3 - y}\right| + C$$

From (1),

$$\Rightarrow \frac{1}{6a^3}\log\left|\frac{a^3 + x^3}{a^3 - x^3}\right| + C$$

Therefore,

$$\int \frac{x^2\,dx}{(a^6 - x^6)} = \frac{1}{6a^3}\log\left|\frac{a^3 + x^3}{a^3 - x^3}\right| + C$$

Question 17.
Evaluate:

$$\int \frac{\mathrm{dx}}{(\mathrm{x}^2 + 4\mathrm{x} + 8)}$$

Answer:
To find: $\int \frac{dx}{(x^2+4x+8)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{((x+2)^2+4)}$$

$$\Rightarrow \int \frac{dx}{((x+2)^2+2^2)} \dots (1)$$

Let y = x + 2 ... (2)

Differentiating both sides,

dy = dx

Substituting in (1),

$$\Rightarrow \int \frac{dy}{(y^2+2^2)}$$

Here a = 2,

$$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{y}{2}\right)+C$$

From (2),

$$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right)+C$$

Therefore,

$$\int \frac{dx}{(x^2+4x+8)} = \frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right)+C$$

Question 18.
Evaluate:

$$\int \frac{dx}{(4x^2-4x+3)}$$

Answer:

To find: $\int \frac{dx}{(4x^2-4x+3)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$\Rightarrow \int \frac{dx}{((2x-1)^2+2)}$ ... (1)

Let y = 2x – 1 ... (2)

Differentiating both sides,

dy = 2dx

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{2}dy}{\left(y^2+\left(\sqrt{2}\right)^2\right)}$$

Here a = √2,

$$\Rightarrow \frac{1}{2} \times \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{y}{\sqrt{2}}\right) + C$$

From (2),

$$\Rightarrow \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$$

Therefore,

$$\int \frac{dx}{(4x^2-4x+3)} = \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$$

Question 19.

Evaluate:

$$\int \frac{dx}{(2x^2+x+3)}$$

Answer:

To find: $\int \frac{dx}{(2x^2+x+3)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x + \frac{1}{2\sqrt{2}}\right)^2 + 3 - \frac{1}{8}\right)}$$

$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x+\frac{1}{2\sqrt{2}}\right)^2+\frac{23}{8}\right)}$... (1)

Let $y = \sqrt{2}x + \frac{1}{2\sqrt{2}}$... (2)

Differentiating both sides,

dy = $\sqrt{2}$ dx

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{\sqrt{2}}dy}{\left(y^2 + \left(\frac{\sqrt{23}}{2\sqrt{2}}\right)^2\right)}$$

Here $a = \frac{\sqrt{23}}{2\sqrt{2}}$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{23}} \tan^{-1}\left(\frac{y \times 2\sqrt{2}}{\sqrt{23}}\right) + C$$

From (2),

$$\Rightarrow \frac{2}{\sqrt{23}}\tan^{-1}\left(\frac{4x+1}{\sqrt{23}}\right)+C$$

Therefore,

$$\int \frac{dx}{(2x^2+x+3)} = \frac{2}{\sqrt{23}}\tan^{-1}\left(\frac{4x+1}{\sqrt{23}}\right)+C$$

Question 20.
Evaluate:

$$\int \frac{dx}{(2x^2-x-1)}$$

Answer:
To find: $\int \frac{dx}{(2x^2-x-1)}$

Formula Used: $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x-\frac{1}{2\sqrt{2}}\right)^2-1-\left(\frac{1}{2\sqrt{2}}\right)^2\right)}$$

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x-\frac{1}{2\sqrt{2}}\right)^2-1-\frac{1}{8}\right)}$$

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x-\frac{1}{2\sqrt{2}}\right)^2-\frac{9}{8}\right)}$$

$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x - \frac{1}{2\sqrt{2}}\right)^2 - \left(\frac{3}{2\sqrt{2}}\right)^2\right)}$ ... (1)

Let $y = \sqrt{2}x - \frac{1}{2\sqrt{2}}$ ... (2)

Differentiating both sides,

dy = √2 dx

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{\sqrt{2}}dy}{\left(y^2 - \left(\frac{3}{2\sqrt{2}}\right)^2\right)}$$

Here $a = \frac{3}{2\sqrt{2}}$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{3} \times \log\left|\frac{\frac{3}{2\sqrt{2}} + y}{\frac{3}{2\sqrt{2}} - y}\right| + C$$

$$\Rightarrow \frac{1}{3} \times \log\left|\frac{3 + 2\sqrt{2}y}{3 - 2\sqrt{2}y}\right| + C$$

From (2),

$$\Rightarrow \frac{1}{3} \times \log\left|\frac{3 + 4x - 1}{3 - 4x + 1}\right| + C$$

$$\Rightarrow \frac{1}{3}\log\left|\frac{1 + 2x}{2(1 - x)}\right| + C$$

$$\Rightarrow \frac{1}{3}\log\left|\frac{2(x - 1)}{2x + 1}\right| + C$$

Therefore,

$$\int \frac{dx}{(2x^2 - x - 1)} = \frac{1}{3}\log\left|\frac{2(x-1)}{2x+1}\right| + C$$

Question 21.
Evaluate:

$$\int \frac{dx}{(3 - 2x - x^2)}$$

Answer:
To find: $\int \frac{dx}{(3-2x-x^2)}$

Formula Used: $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{-dx}{(x^2 + 2x - 3)}$$

$$\Rightarrow \int \frac{-dx}{(x+1)^2 - 4}$$

$\Rightarrow \int \frac{-dx}{(x+1)^2-2^2}$ ... (1)

Let y = x + 1 ... (2)

Differentiating both sides wrt x,

dy = dx

Substituting in (1),

$$\Rightarrow \int \frac{-dy}{y^2 - 2^2}$$

$$\Rightarrow \int \frac{dy}{2^2 - y^2}$$

Here a = 2,

$$\Rightarrow \frac{1}{4}\log\left|\frac{2+y}{2-y}\right| + C$$

From (2),

$$\Rightarrow \frac{1}{4}\log\left|\frac{x+3}{1-x}\right| + C$$

Therefore,

$$\int \frac{dx}{(3-2x-x^2)} = \frac{1}{4}\log\left|\frac{x+3}{1-x}\right| + C$$

Question 22.
Evaluate:

$$\int \frac{x}{(x^2+3x+2)} dx$$

Answer:
To find: $\int \frac{x\,dx}{(x^2+3x+2)}$

Formula Used:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$

Using partial fractions,

$$x = A\left(\frac{d}{dx}(x^2 + 3x + 2)\right) + B$$

x = A (2x + 3) + B

Equating the coefficients of x,

1 = 2A

$$A = \frac{1}{2}$$

Also, 0 = 3A + B

$$B = \frac{-3}{2}$$

Therefore, the given equation becomes,

$$\Rightarrow \int \frac{\frac{1}{2}(2x+3) - \frac{3}{2}}{(x^2 + 3x + 2)}\, dx$$

$$\Rightarrow \frac{1}{2}\log|x^2 + 3x + 2| - \frac{3}{2}\int \frac{1}{\left(\left(x+\frac{3}{2}\right)^2 + 2 - \left(\frac{3}{2}\right)^2\right)} dx$$

$$\Rightarrow \frac{1}{2}\log|x^2 + 3x + 2| - \frac{3}{2}\int \frac{1}{\left(\left(x+\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right)} dx$$

$$\Rightarrow \frac{1}{2}\log|x^2 + 3x + 2| - \frac{3}{2} \times \log\left|\frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}}\right| + C$$

$$\Rightarrow \frac{1}{2}\log|x^2 + 3x + 2| - \frac{3}{2}\log\left|\frac{x+1}{x+2}\right| + C$$

Therefore,

$$\int \frac{x\,dx}{(x^2+3x+2)} = \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\log\left|\frac{x+1}{x+2}\right| + C$$

Question 23.
Evaluate:

$$\int \frac{(x-3)}{(x^2+2x-4)}dx$$

Answer:
To find: $\int \frac{(x-3)\,dx}{(x^2+2x-4)}$

Formula Used:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$

Using partial fractions,

$$(x-3) = A\left(\frac{d}{dx}(x^2+2x-4)\right) + B$$

x – 3 = A (2x + 2) + B

Equating the coefficients of x,

1 = 2A

$$\Rightarrow A = \frac{1}{2}$$

Also, -3 = 2A + B

⇒ B = -4

Substituting in the given equation,

$$\Rightarrow \int \frac{\frac{1}{2}(2x+2)-4}{(x^2+2x-4)}dx$$

$$\Rightarrow \frac{1}{2}\log|x^2+2x-4| - 4\int \frac{1}{(x+1)^2-(\sqrt{5})^2}\,dx$$

$$\Rightarrow \frac{1}{2}\log|x^2+2x-4| - \left(4 \times \frac{1}{2\sqrt{5}} \times \log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right|\right) + C$$

$$\Rightarrow \frac{1}{2}\log|x^2+2x-4| - \frac{2}{\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right| + C$$

Therefore,

$$\int \frac{(x-3)\,dx}{(x^2+2x\ -\ 4)} = \frac{1}{2}\log|x^2+2x-4| - \frac{2}{\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right| + C$$

Question 24.
Evaluate:

$$\int \frac{(2x-3)}{(x^2+3x-18)}dx$$

Answer:
To find: $\int \frac{(2x-3)}{(x^2+3x-18)}dx$

Formula Used:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$

Using partial fractions,

$$(2x-3)=A\left(\frac{d}{dx}(x^2+3x-18)\right)+B$$

2x – 3 = A(2x + 3) + B

Equating the coefficients of x,

2 = 2A

A = 1

Also, -3 = 3A + B

⇒ B = -6

Substituting in the given equation,

$$\Rightarrow \int \frac{(2x+3)-6}{(x^2+3x-18)}dx$$

$$\Rightarrow \log|x^2+3x-18|+C_1-6\int \frac{1}{\left(x+\frac{3}{2}\right)^2-18-\left(\frac{3}{2}\right)^2}dx$$ ... (1)

Let I $= \ 6\int \frac{1}{\left(x+\frac{3}{2}\right)^2-18-\left(\frac{3}{2}\right)^2}dx$

$$\Rightarrow \ 6\int \frac{1}{\left(x+\frac{3}{2}\right)^2-\left(\frac{9}{2}\right)^2}dx$$

Here a $= \ \frac{9}{2}$

$$\Rightarrow \frac{6}{9}\times\log\left|\frac{x+\frac{3}{2}-\frac{9}{2}}{x+\frac{3}{2}+\frac{9}{2}}\right|+C_2$$

$\Rightarrow \frac{2}{3} \times \log\left|\frac{x-3}{x+6}\right| + C_2$ ... (2)

Substituting (2) in (1),

$$\Rightarrow \log|x^2 + 3x - 18| - \frac{2}{3}\log\left|\frac{x-3}{x+6}\right| + C$$

Therefore,

$$\int \frac{(2x-3)}{(x^2+3x-18)}dx = \log|x^2+3x-18| - \frac{2}{3}\log\left|\frac{x-3}{x+6}\right| + C$$

Question 25.
Evaluate:

$$\int \frac{x^2}{(x^2+6x-3)}dx$$

Answer:
To find: $\int \frac{x^2}{(x^2+6x-3)}dx$

Formula Used:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$

Given equation can be rewritten as following:

$$\Rightarrow \int \frac{x^2+(6x-3)-(6x-3)}{(x^2+6x-3)}dx$$

$$\Rightarrow \int \frac{(x^2+6x-3)-(6x-3)}{(x^2+6x-3)}\,dx$$

$$\Rightarrow x - \int \frac{6x-3}{x^2+6x-3}\,dx$$

Let I $= \int \frac{6x-3}{x^2+6x-3}\,dx$ ... (2)

Using partial fractions,

$$(6x-3) = A\left(\frac{d}{dx}(x^2+6x-3)\right) + B$$

6x – 3 = A (2x + 6) + B

Equating the coefficients of x,

6 = 2A

A = 3

Also, -3 = 6A + B

⇒ B = -21

Substituting in (1),

$$\Rightarrow \int \frac{3(2x+6)-21}{(x^2+6x-3)}dx$$

$$\Rightarrow 3 \times \log|x^2+6x-3| + C_1 - 21\int \frac{1}{(x+3)^2-\left(\sqrt{12}\right)^2}\,dx$$

$$\Rightarrow 3 \times \log|x^2+6x-3| + C_1 - 21 \times \frac{1}{2\sqrt{12}} \times \log\left|\frac{x+3-\sqrt{12}}{x+3+\sqrt{12}}\right| + C_2$$

I $= 3\log|x^2+6x-3| - \frac{7\sqrt{3}}{4} \times \log\left|\frac{x+3-2\sqrt{3}}{x+3+2\sqrt{3}}\right| + C$

Therefore,

$$\int \frac{x^2}{(x^2+6x\ -\ 3)}dx = x -\ 3\log|x^2+6x-3| + \frac{7\sqrt{3}}{4} \times \log\left|\frac{x+3-2\sqrt{3}}{x+3+2\sqrt{3}}\right| + C$$

Question 26.
Evaluate:

$$\int \frac{(2x-1)}{(2x^2+2x+1)}dx$$

Answer:
To find: $\int \frac{2x-1}{(2x^2+2x+1)}dx$

Formula Used:

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

2. $\int \frac{f'(x)}{f(x)}\ dx = \log|f(x)| + C$

Using partial fractions,

$$(2x-1) = A\left(\frac{d}{dx}(2x^2+2x+1)\right) + B$$

2x – 1 = A (4x + 2) + B

Equating the coefficients of x,

2 = 4A

A = $\frac{1}{2}$

Also, -1 = 2A + B

⇒ B = -2

Substituting in the given equation,

$$\Rightarrow \int \frac{\frac{1}{2}(4x+2)-2}{(2x^2+2x+1)}dx$$

$$\Rightarrow \frac{1}{2}\log|2x^2+2x+1| - 2\int \frac{1}{2\left(x^2+x+\frac{1}{2}\right)}dx$$

Let I $= 2\int \frac{1}{2\left(x^2+x+\frac{1}{2}\right)}dx$ ... (1)

$$\Rightarrow \int \frac{1}{\left(x^2+x+\frac{1}{2}\right)}dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{1}{2}\right)^2+\frac{1}{2}-\left(\frac{1}{2}\right)^2\right)}dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{1}{2}\right)^2+\frac{1}{2}-\frac{1}{4}\right)}dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2\right)}dx$$

Here $a = \frac{1}{2}$

$$\Rightarrow 2\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{1}{2}}\right)+C$$

$\Rightarrow 2\tan^{-1}(2x + 1) + C$

Substituting in (1) and combining with original equation,

$$\Rightarrow \frac{1}{2}\log|2x^2 + 2x + 1| - 2\tan^{-1}(2x + 1) + C$$

Therefore,

$$\int \frac{2x - 1}{(2x^2 + 2x + 1)}dx = \frac{1}{2}\log|2x^2 + 2x + 1| - 2\tan^{-1}(2x + 1) + C$$

Question 27.
Evaluate:

$$\int \frac{(1-3x)}{(3x^2+4x+2)}dx$$

Answer:
To find: $\int \frac{1-3x}{(3x^2+4x+2)}dx$

Formula Used:

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

2. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$

Rewriting the given equation,

$$\Rightarrow -\int \frac{3x - 1}{(3x^2 + 4x + 2)}\,dx$$

Using partial fractions,

$$(3x - 1) = A\left(\frac{d}{dx}(3x^2 + 4x + 2)\right) + B$$

3x – 1 = A (6x + 4) + B

Equating the coefficients of x,

3 = 6A

A = $\frac{1}{2}$

Also, -1 = 4A + B

⇒ B = -3

Substituting in the original equation,

$$\Rightarrow -\int \frac{\frac{1}{2}(6x+4)-3}{(3x^2+4x+2)}\,dx$$

$$\Rightarrow -\frac{1}{2}\log|3x^2+4x+2| + 3\int \frac{1}{3\left(x^2+\frac{4}{3}x+\frac{2}{3}\right)}\,dx$$

Let I $= 3\int \frac{1}{3\left(x^2+\frac{4}{3}x+\frac{2}{3}\right)}\,dx$

$$\Rightarrow \int \frac{1}{\left(x^2+\frac{4}{3}x+\frac{2}{3}\right)}\,dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{2}{3}\right)^2+\frac{2}{3}-\frac{4}{9}\right)}dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{2}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right)}dx$$

Here $a = \frac{\sqrt{2}}{3}$

$$\Rightarrow \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{x+\frac{2}{3}}{\frac{\sqrt{2}}{3}}\right)+C$$

$$\Rightarrow \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)+C$$

Substituting in (1) and combining with original equation,

$$\Rightarrow -\frac{1}{2}\log|3x^2+4x+2|+\frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)+C$$

Therefore,

$$\int\frac{1-3x}{(3x^2+4x+2)}dx = -\frac{1}{2}\log|3x^2+4x+2|+\frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)+C$$

Question 28.
Evaluate:

$$\int\frac{2x}{(2+x-x^2)}dx$$

Answer:
To find: $\int\frac{2x}{(2+x-x^2)}dx$

Formula Used:

1. $\int\frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+C$

2. $\int\frac{f'(x)}{f(x)}\,dx = \log|f(x)|+C$

Rewriting the given equation,

$$\Rightarrow -2\int\frac{x}{(x^2-x-2)}\,dx$$

Using partial fractions,

$x = A\left(\frac{d}{dx}(x^2 - x - 2)\right) + B$

x = A (2x – 1) + B

Equating the coefficients of x,

1 = 2A

A = $\frac{1}{2}$

Also, 0 = -A + B

B = $\frac{1}{2}$

Substituting in the original equation,

$\Rightarrow -2\int \frac{\frac{1}{2}(2x-1) + \frac{1}{2}}{(x^2 - x - 2)}\, dx$

$\Rightarrow -\log|x^2 - x - 2| - \int \frac{1}{(x^2 - x - 2)}\, dx$

Let I = $\int \frac{1}{(x^2 - x - 2)} dx$

$\Rightarrow \int \frac{1}{\left(\left(x - \frac{1}{2}\right)^2 - 2 - \frac{1}{4}\right)} dx$

$\Rightarrow \int \frac{1}{\left(\left(x - \frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right)} dx$

Here $a = \frac{3}{2}$

$$\Rightarrow \frac{1}{3}\log\left|\frac{x-\frac{1}{2}-\frac{3}{2}}{x-\frac{1}{2}+\frac{3}{2}}\right|+C$$

$$\Rightarrow \frac{1}{3}\log\left|\frac{x-2}{x+1}\right|+C$$

Substituting for I and combining with the original equation,

$$\Rightarrow -\log|x^2-x-2|+\frac{1}{3}\log\left|\frac{x-2}{x+1}\right|+C$$

Therefore,

$$\int\frac{2x}{(2+x-x^2)}dx = -\log|x^2-x-2|+\frac{1}{3}\log\left|\frac{x-2}{x+1}\right|+C$$

or

$$\int\frac{2x}{(2+x-x^2)}dx = -\log|2+x-\ x^2|+\frac{1}{3}\log\left|\frac{1+x}{2-x}\right|+C$$

Question 29.
Evaluate:

$$\int\frac{\mathrm{dx}}{(1+\cos^2\mathrm{x})}$$

Answer:
To find: $\int\frac{1}{(1+\cos^2x)}dx$

Formula Used:

1. $\int\frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+C$

2. $\sec^2 x = 1 + \tan^2 x$

Dividing the given equation by $\cos^2 x$ in the numerator and denominator gives us,

$\Rightarrow \int \frac{\sec^2 x\, dx}{1+\sec^2 x}$ ... (1)

Let y = tan x

dy = $\sec^2$ x dx ... (2)

Also, $y^2 = \tan^2 x$

i.e., $y^2 = \sec^2 x - 1$

$\sec^2 x = y^2 + 1$ ... (3)

Substituting (2) and (3) in (1),

$$\Rightarrow \int \frac{dy}{1+y^2+1}$$

$$\Rightarrow \int \frac{dy}{y^2+\left(\sqrt{2}\right)^2}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{y}{\sqrt{2}}\right)+C$$

Since y = tan x,

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)+C$$

Therefore,

$$\int \frac{1}{(1+\cos^2 x)}dx = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)+C$$

Question 30.

Evaluate:

$$\int \frac{dx}{(2+\sin^2 x)}$$

Answer:

To find: $\int \frac{1}{(2+\sin^2 x)} dx$

Formula Used:

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

2. $\sec^2 x = 1 + \tan^2 x$

Dividing the given equation by $\cos^2 x$ in the numerator and denominator gives us,

$\Rightarrow \int \frac{\sec^2 x \, dx}{2\sec^2 x + \tan^2 x}$ ... (1)

Let y = tan x

$dy = \sec^2 x \, dx$ ... (2)

Also, $y^2 = \tan^2 x$

i.e., $y^2 = \sec^2 x - 1$

$\sec^2 x = y^2 + 1$ ... (3)

Substituting (2) and (3) in (1),

$$\Rightarrow \int \frac{dy}{2y^2 + 2 + y^2}$$

$$\Rightarrow \int \frac{dy}{3y^2 + 2}$$

$$\Rightarrow \frac{1}{3}\int \frac{dy}{y^2 + \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2}$$

$$\Rightarrow \frac{1}{3} \times \frac{\sqrt{3}}{\sqrt{2}} \tan^{-1}\left(\frac{y\sqrt{3}}{\sqrt{2}}\right) + C$$

Since y = tan x,

$$\Rightarrow \frac{1}{\sqrt{6}} \tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{2}}\right) + C$$

Therefore,

$$\int \frac{1}{(2+\sin^2 x)} dx = \frac{1}{\sqrt{6}} \tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{2}}\right) + C$$

Question 31.
Evaluate:

$$\int \frac{dx}{(a^2\cos^2 x + b^2 \sin^2 x)}$$

Answer:
To find: $\int \frac{dx}{(a^2\cos^2 x + b^2\sin^2 x)}$

Formula Used:

1. $Sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \ dx}{a^2 + b^2 \tan^2 x}$$

Let y = tan x

dy = sec^2 x dx

Therefore,

$$\Rightarrow \int \frac{dy}{a^2 + b^2 y^2}$$

$$\Rightarrow \frac{1}{b^2} \int \frac{dy}{\left(\frac{a}{b}\right)^2 + y^2}$$

$$\Rightarrow \frac{1}{b^2} \times \frac{b}{a} \tan^{-1} \frac{yb}{a} + C$$

Since y = tan x,

$$\Rightarrow \frac{1}{ab} \tan^{-1} \left(\frac{b \tan x}{a}\right) + C$$

Therefore,

$$\int \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)} = \frac{1}{a^2} \tan^{-1} \left(\frac{b}{a} \tan x\right) + C$$

Question 32.
Evaluate:

$$\int \frac{\mathrm{dx}}{(\cos^2 \mathrm{x} - 3\sin^2 \mathrm{x})}$$

Answer:
To find: $\int \frac{dx}{(\cos^2 x - 3\sin^2 x)}$

Formula Used:

1. sec^2 x = 1 + tan^2 x

2. $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \ dx}{1-3\tan^2 x}$$

Let y = tan x

dy = $\sec^2$ x dx

Therefore,

$$\Rightarrow \int \frac{dy}{1-3y^2}$$

$$\Rightarrow \frac{1}{3}\int \frac{dy}{\left(\frac{1}{\sqrt{3}}\right)^2 - y^2}$$

$$\Rightarrow \frac{1}{3} \times \frac{\sqrt{3}}{2}\log\left|\frac{\frac{1}{\sqrt{3}}+y}{\frac{1}{\sqrt{3}}-y}\right| + C$$

$$\Rightarrow \frac{1}{2\sqrt{3}}\log\left|\frac{1+y\sqrt{3}}{1-y\sqrt{3}}\right| + C$$

Since y = tan x,

$$\Rightarrow \frac{1}{2\sqrt{3}}\log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right| + C$$

Therefore,

$$\int \frac{dx}{(\cos^2 x \ - \ 3\sin^2 x)} = \frac{1}{2\sqrt{3}}\log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right| + C$$

Question 33.
Evaluate:

$$\int \frac{dx}{(\sin^2 x - 4\cos^2 x)}$$

Answer:
To find: $\int \frac{dx}{(\sin^2 x - 4\cos^2 x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{x^2-a^2}\, dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x\ dx}{\tan^2 x - 4}$$

Let y = tan x

dy = $\sec^2$ x dx

Therefore,

$$\Rightarrow \int \frac{dy}{y^2 - 2^2}$$

$$\Rightarrow \frac{1}{4}\log\left|\frac{y-2}{y+2}\right| + C$$

Since y = tan x,

$$\Rightarrow \frac{1}{4}\log\left|\frac{\tan x - 2}{\tan x + 2}\right| + C$$

Therefore,

$$\int \frac{dx}{(\sin^2 x - 4\cos^2 x)} = \frac{1}{4}\log\left|\frac{\tan x - 2}{\tan x + 2}\right| + C$$

Question 34.
Evaluate:

$$\int \frac{\mathrm{dx}}{(\sin x \cos x + 2\cos^2 x)}$$

Answer:
To find: $\int \frac{dx}{(\sin x \cos x + 2\cos^2 x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{x} dx = \log x + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \; dx}{\tan x + 2}$$

Let y = tan x

$dy = \sec^2 x \, dx$

Therefore,

$$\Rightarrow \int \frac{dy}{y + 2}$$

⇒ log |y + 2| + C

Since y = tan x,

⇒ log |tan x + 2| + C

Therefore,

$$\int \frac{dx}{(\sin x \cos x + 2\cos^2 x)} = \log|\tan x + 2| + C$$

Question 35.
Evaluate:

$$\int \frac{\sin 2x}{(\sin^4 x + \cos^4 x} dx$$

Answer:
To find: $\int \frac{\sin 2x \, dx}{(\sin^4 x + \cos^4 x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

3. sin 2x = 2 sin x cos x

Rewriting the given equation,

$$\Rightarrow \int \frac{2\sin x \cos x}{\sin^4 x + \cos^4 x} \, dx$$

Dividing by $\cos^4$ x in the numerator and denominator,

$$\Rightarrow \int \frac{2\tan x \sec^2 x \; dx}{\tan^4 x + 1}$$

Let y = tan x

dy = $\sec^2$ x dx

Therefore,

$$\Rightarrow \int \frac{2y}{y^4+1}\, dy$$

Let z = y^2

dz = 2y dy

$$\Rightarrow \int \frac{dz}{1+z^2}$$

⇒ $\tan^{-1} z + C$

Since z = y^2,

⇒ $\tan^{-1}(y^2) + C$

Since y = tan x,

⇒ $\tan^{-1}(\tan^2 x) + C$

Therefore,

$$\int \frac{\sin 2x\ dx}{(\sin^4 x + \cos^4 x)} = \tan^{-1}(\tan^2 x) + C$$

Question 36.
Evaluate:

$$\int \frac{(2\sin 2\phi - \cos\phi)}{(6 - \cos^2\phi - 4\sin\phi)} d\phi$$

Answer:
To find: $\int \frac{(2\sin 2\phi - cos\phi)}{(6-\cos^2\phi - 4\sin\phi)}\ d\phi$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

3. $\sin 2x = 2 \sin x \cos x$

Rewriting the given equation,

$$\Rightarrow \int \frac{4\sin\phi\cos\phi - \cos\phi}{6 - \cos^2\phi - 4\sin\phi}\, d\phi$$

$$\Rightarrow \int \frac{\cos\phi\ (4\sin\phi - 1)}{6 - (1 - \sin^2\phi) - 4\sin\phi}\, d\phi$$

$$\Rightarrow \int \frac{\cos\phi\ (4\sin\phi - 1)}{5 + \sin^2\phi - 4\sin\phi}\, d\phi$$

Let $y = \sin \phi$

$dy = \cos \phi \, d\phi$

Substituting in the original equation,

$\Rightarrow \int \frac{4y-1}{y^2-4y+5}\, dy$ ... (1)

Using partial fraction,

$$4y - 1 = A\left(\frac{d}{dy}(y^2 - 4y + 5)\right) + B$$

$4y - 1 = A\ (2y - 4) + B$

Equating the coefficients of y,

$4 = 2A$

$A = 2$

Also, -1 = -4A + B

B = 7

Substituting in (1),

$$\Rightarrow \int \frac{2(2y-4)+7}{y^2-4y+5}\,dy$$

$$\Rightarrow 2\log|y^2-4y+5| + 7\int \frac{1}{((y-2)^2+1)}\,dy$$

⇒ 2 log $|y^2 - 4y + 5|$ + 7 $\tan^{-1}(y - 2)$ + C

But y = sin ϕ

⇒ 2 log $|\sin^2\phi - 4\sin\phi + 5|$ + 7 $\tan^{-1}(\sin\phi - 2)$ + C

Therefore,

$$\int \frac{(2\sin 2\phi - cos\phi)}{(6-\cos^2\phi - 4\sin\phi)}\,d\phi = 2\log|\sin^2\phi - 4\sin\phi + 5| + 7\tan^{-1}(\sin\phi - 2) + C$$

Question 37.
Evaluate:

$$\int \frac{\mathrm{dx}}{(\sin x - 2\cos x)(2\sin x + \cos x)}$$

Answer:
To find: $\int \frac{dx}{(\sin x - 2\cos x)(2\sin x + \cos x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{x} dx = \log x + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \ dx}{(\tan x - 2)(2\tan x + 1)}$$

Let y = tan x

dy = $\sec^2$ x dx

Therefore,

$\Rightarrow \int \frac{dy}{(y-2)(2y+1)}$ ... (1)

Let

$$\frac{1}{(y-2)(2y+1)} = \frac{A}{(y-2)} + \frac{B}{(2y+1)}$$

1 = A (2y + 1) + B(y – 2)

When y = 0,

1 = A – 2B ... (2)

When y = 1,

1 = 3A – B ⇒ 2 = 6A – 2B ... (3)

Solving (2) and (3),

1 = 5A

A = $\frac{1}{5}$

So, B = $\frac{-2}{5}$

(1) becomes,

$$\Rightarrow \int \frac{\frac{1}{5}}{(y-2)} + \frac{\frac{-2}{5}}{(2y+1)}$$

$$\Rightarrow \frac{1}{5}\log|y-2| - \frac{2}{5}\log|2y+1| \times \frac{1}{2} + C$$

Since y = tan x,

$$\Rightarrow \frac{1}{5}\log|\tan x - 2| - \frac{1}{5}\log|2\tan x + 1| + C$$

$$\Rightarrow \frac{1}{5}\log\left|\frac{\tan x - 2}{2\tan x + 1}\right| + C$$

Therefore,

$$\int \frac{dx}{(\sin x - 2\cos x)(2\sin x + \cos x)} = \frac{1}{5}\log\left|\frac{\tan x - 2}{2\tan x + 1}\right| + C$$

Question 38.
Evaluate:

$$\int \frac{\left(1-x^2\right)}{\left(1+x^4\right)} dx$$

Answer:
To find: $\int \frac{(1-x^2)}{(1+x^4)}\, dx$

Formula used: $\int \frac{1}{x^2-a^2}\, dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

On dividing by x^2 in the numerator and denominator of the given equation,

$$\Rightarrow \int \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} + x^2} dx$$

$$\Rightarrow \int \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} + x^2 + 2 - 2} dx$$

$$\Rightarrow \int \frac{-\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 - 2} dx$$

Let $y = x + \frac{1}{x}$

Differentiating wrt x,

$$dy = \left(1 - \frac{1}{x^2}\right) dx$$

Substituting in the original equation,

$$\Rightarrow \int \frac{-dy}{y^2 - \left(\sqrt{2}\right)^2}$$

$$\Rightarrow \frac{-1}{2\sqrt{2}} \log \left| \frac{y - \sqrt{2}}{y + \sqrt{2}} \right| + C$$

Substituting for $y = x + \frac{1}{x}$ and taking reciprocal of the value within logarithm, we get

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \left| \frac{x + \frac{1}{x} + \sqrt{2}}{x + \frac{1}{x} - \sqrt{2}} \right| + C$$

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2}x + x^2 + 1}{\sqrt{2}x - x^2 + 1} \right| + C$$

Therefore,

$$\int \frac{(1-x^2)}{(1+x^4)}\,dx = \frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}x+x^2+1}{\sqrt{2}x-x^2+1}\right| + C$$

Question 39.
Evaluate:

$$\int \frac{\left(x^2+1\right)}{\left(x^4+x^2+1\right)}dx$$

Answer:
To find: $\int \frac{(x^2+1)}{(x^4+x^2+1)}\,dx$

Formula used: $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

On dividing by x^2 in the numerator and denominator of the given equation,

$$\Rightarrow \int \frac{1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$$

$$\Rightarrow \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+3}dx$$

Let $y = x - \frac{1}{x}$

Differentiating wrt x,

$$dy = \left(1+\frac{1}{x^2}\right)dx$$

Substituting in the original equation,

$$\Rightarrow \int \frac{dy}{y^2 + \left(\sqrt{3}\right)^2}$$

$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \frac{y}{\sqrt{3}} + C$$

Substituting for $y = x - \frac{1}{x}$

$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x - \frac{1}{x}}{\sqrt{3}} \right) + C$$

$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x} \right) + C$$

Therefore,

$$\int \frac{(x^2 + 1)}{(x^4 + x^2 + 1)}\, dx = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x} \right) + C$$

Question 40.
Evaluate:

$$\int \frac{\mathrm{dx}}{\left(\sin^4 \mathrm{x} + \cos^4 \mathrm{x}\right)}$$

Answer:
To find: $\int \frac{dx}{(\sin^4 x + \cos^4 x)}$

Formula used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

Dividing by $\cos^4 x$ in the numerator and denominator of the given equation,

$$\Rightarrow \int \frac{\sec^4 x}{(\tan^4 x + 1)} dx$$

$$\Rightarrow \int \frac{\sec^2 x\,(1 + \tan^2 x)}{(1 + \tan^4 x)} dx$$

Let y = tan x

dy = $\sec^2$ x dx

Substituting in the original equation,

$$\Rightarrow \int \frac{1 + y^2}{1 + y^4} dy$$

Dividing by y^2 in the numerator and denominator,

$$\Rightarrow \int \frac{y^{-2} + 1}{y^{-2} + y^2} dy$$

$$\Rightarrow \int \frac{1 + y^{-2}}{y^2 + y^{-2} - 2 + 2} dy$$

$$\Rightarrow \int \frac{1 + y^{-2}}{\left(y - \frac{1}{y}\right)^2 + 2} dy$$

Let $z = y - \frac{1}{y}$

$$dz = \left(1 + \frac{1}{y^2}\right) dy$$

Therefore,

$$\Rightarrow \int \frac{dz}{z^2 + \left(\sqrt{2}\right)^2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{z}{\sqrt{2}}\right) + C$$

Substituting for z,

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{y - \frac{1}{y}}{\sqrt{2}}\right) + C$$

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{y^2 - 1}{y\sqrt{2}}\right) + C$$

Substituting for y = tan x,

$$\Rightarrow \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\tan^2 x - 1}{\sqrt{2}\tan x}\right) + C$$

Therefore,

$$\int \frac{dx}{(\sin^4 x + \cos^4 x)} = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\tan^2 x - 1}{\sqrt{2}\tan x}\right) + C$$