# Exercise 4b

**Question 1.**
Find the principal value of each of the following :

$$\sin^{-1}\left(\frac{-1}{2}\right)$$

**Answer:**
$\sin^{-1}\left(\frac{-1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right)$ [Formula: $\sin^{-1}(-x) = \sin^{-1}(x)$ ]

$= -\frac{\pi}{6}$

**Question 2.**
Find the principal value of each of the following :

$$\cos^{-1}\left(\frac{-1}{2}\right)$$

**Answer:**
$\cos^{-1}\left(\frac{-1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right)$ [ Formula: $\cos^{-1}(-x) = -\cos^{-1}(x)$ ]

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

**Question 3.**
Find the principal value of each of the following :

$$\tan^{-1}(-1)$$

**Answer:**
$\tan(-1) = -\tan(1)$ [ Formula: $\tan^{-1}(-x) = -\tan^{-1}(x)$ ]

[ We know that $\tan\frac{\pi}{4} = 1$, thus $\tan^{-1}\frac{\pi}{4} = 1$ ]

$= -\frac{\pi}{4}$

**Question 4.**
Find the principal value of each of the following :

$\sec^{-1}(-2)$

**Answer:**
$\sec^{-1}(-2) = \pi - \sec^{-1}(2)$ [ Formula: $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$ ]

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

**Question 5.**
Find the principal value of each of the following :

$\text{cosec}^{-1}\left(-\sqrt{2}\right)$

**Answer:**
$\text{cosec}^{-1}\left(-\sqrt{2}\right) = -\text{cosec}^{-1}\left(\sqrt{2}\right)$ [Formula: $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}(x)$ ]

$= -\frac{\pi}{4}$

This can also be solved as

$\text{cosec}^{-1}\left(-\sqrt{2}\right)$

Since cosec is negative in the third quadrant, the angle we are looking for will be in the third quadrant.

$= \pi + \frac{\pi}{4}$

$=\frac{5\pi}{4}$

**Question 6.**
Find the principal value of each of the following :

$$\cot^{1}(-1)$$

**Answer:**
$\cot^{-1}(-1) = \pi - \cot^{-1}(1)$ [Formula: $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$ ]

$= \pi - \frac{\pi}{4}$

$=\frac{3\pi}{4}$

**Question 7.**
Find the principal value of each of the following :

$$\tan^{-1}\left(-\sqrt{3}\right)$$

**Answer:**
$\tan^{-1}\left(-\sqrt{3}\right) = -\tan^{-1}\left(\sqrt{3}\right)$ [Formula: $\tan^{-1}(-x) = -\tan^{-1}(x)$ ]

$= -\frac{\pi}{3}$

**Question 8.**
Find the principal value of each of the following :

$$\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$$

**Answer:**
$\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right) = \pi - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ [ Formula: $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$ ]

$= \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

**Question 9.**
Find the principal value of each of the following :

$\text{cosec}^{-1}(2)$

**Answer:**
$\text{cosec}^{-1}(2)$

Putting the value directly

$= \frac{\pi}{6}$

**Question 10.**
Find the principal value of each of the following :

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

**Answer:**
$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right)$

[ Formula: $\sin(\pi - x) = \sin x$ )

$= \sin^{-1}\left(\sin\frac{\pi}{3}\right)$

[ Formula: $\sin^{-1}(\sin x) = x$ ]

$= \frac{\pi}{3}$

**Question 11.**

Find the principal value of each of the following :

$$\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$$

**Answer:**

$$\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{4}\right)\right)$$

[Formula: tan(π – x) = -tan (x) , as tan is negative in the second quadrant. ]

$$= \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$$

[Formula: $\tan^{-1}(\tan x) = x$ ]

$$= -\frac{\pi}{4}$$

**Question 12.**

Find the principal value of each of the following :

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$$

**Answer:**

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{5\pi}{6}\right)\right)$$

[Formula: cos(2π – x) = cos (x), as cos has a positive vaule in the fourth quadrant. ]

$= \cos^{-1}\left(\cos\frac{5\pi}{6}\right)$ [Formula: $\cos^{-1}(\cos x) = x$

$$= \frac{5\pi}{6}$$

**Question 13.**

Find the principal value of each of the following :

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$$

**Answer:**

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi+\frac{\pi}{6}\right)\right)$$

[ Formula: cos (2π + x) = cos x , cos is positive in the first quadrant. ]

$= \cos^{-1}\left(\cos\frac{\pi}{6}\right)$ [Formula: $\cos^{-1}(\cos x) = x$]

$= \frac{\pi}{6}$

**Question 14.**
Find the principal value of each of the following :

$$\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$$

**Answer:**

$$\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{-1}\left(\tan\left(\pi+\frac{\pi}{6}\right)\right)$$

[ Formula: tan( π + x) = tan x, as tan is positive in the third quadrant.]

$=\tan^{-1}\left(\tan\frac{\pi}{6}\right)$ [Formula: $\tan^{-1}(\tan x) = x$ ]

$= \frac{\pi}{6}$

**Question 15.**
Find the principal value of each of the following :

$$\tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right)$$ 3

**Answer:**

$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

Putting the value of $\tan^{-1}\sqrt{3}$ and using the formula

$\cot^{-1}(-x) = \pi - \cot^{-1}x$

$= \frac{\pi}{3} - (\pi - \cot^{-1}(\sqrt{3}))$

Putting the value of $\cot^{-1}(\sqrt{3})$

$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right)$

$= \frac{\pi}{3} - \frac{5\pi}{6}$

$= -\frac{3\pi}{6}$

$= -\frac{\pi}{2}$

**Question 16.**

Find the principal value of each of the following :

$$\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right\}$$

**Answer:**

$\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right\}$ [Formula: $\sin^{-1}(-x) = -\sin^{-1}x$ ]

$= \sin\left\{\frac{\pi}{3} - \left(-\sin^{-1}\frac{1}{2}\right)\right\}$

$= \sin\left\{\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right\}$

Putting value of $\sin^{-1}\left(\frac{1}{2}\right)$

$= \sin\left\{\frac{\pi}{3} + \frac{\pi}{6}\right\}$

$= \sin\frac{3\pi}{6}$

$= \sin\frac{\pi}{2}$

= 1

**Question 17.**
Find the principal value of each of the following :

$$\cot\left(\tan^{-1}x + \cot^{-1}x\right)$$

**Answer:**
$\cot(\tan^{-1}x + \cot^{-1}x) = \cot\left(\frac{\pi}{2}\right)$ [Formula: $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$]

Putting value of $\cot\left(\frac{\pi}{2}\right)$

= 0

**Question 18.**
Find the principal value of each of the following :

$$\text{cosec}\left(\sin^{-1}x + \cos^{-1}x\right)$$

**Answer:**
$\text{cosec}\,(\sin^{-1}x + \cos^{-1}x) = \text{cosec}\,\frac{\pi}{2}$ [Formula: $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$]

Putting the value of $\text{cosec}\,\frac{\pi}{2}$

= 1

**Question 19.**

Find the principal value of each of the following :

$$\sin\left(\sec^{-1}x+\operatorname{cosec}^{-1}x\right)$$

**Answer:**

$\sin(\sec^{-1}x+\ \operatorname{cosec}^{-1}x)=\sin\left(\frac{\pi}{2}\right)$ [Formula: $\sec^{-1}x+\ \operatorname{cosec}^{-1}x=\ \frac{\pi}{2}$ ]

Putting the value of $\sin\left(\frac{\pi}{2}\right)$

=1

**Question 20.**

Find the principal value of each of the following :

$$\cos^{-1}\frac{1}{2}+2\sin^{-1}\frac{1}{2}$$

**Answer:**

Putting the values of the inverse trigonometric terms

$$\frac{\pi}{3}+\ 2\times\frac{\pi}{6}$$

$=\frac{\pi}{3}+\ \frac{\pi}{3}$

$=\frac{2\pi}{3}$

**Question 21.**

Find the principal value of each of the following :

$$\tan^{-1}1+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$$

**Answer:**

[Formula: $\cos^{-1}(-x)=\pi-\cos(x)$ and $\sin^{-1}(-x)=-\sin(x)$ ]

$$\tan^{-1} 1 + \left(\pi - \cos^{-1}\left(\frac{1}{2}\right)\right) + \left(-\sin^{-1}\left(\frac{1}{2}\right)\right)$$

Putting the values for each of the inverse trigonometric terms

$$= \frac{\pi}{4} + \left(\pi - \frac{\pi}{3}\right) - \frac{\pi}{6}$$

$$= \frac{\pi}{12} + \frac{2\pi}{3}$$

$$= \frac{9\pi}{12}$$

$$= \frac{3\pi}{4}$$

**Question 22.**
Find the principal value of each of the following :

$$\sin^{-1}\left\{\sin\frac{3\pi}{5}\right\}$$

**Answer:**

$$\sin^{-1}\left\{\sin\left(\frac{3\pi}{5}\right)\right\}$$

$$= \sin^{-1}\left\{\sin\left(\pi - \frac{2\pi}{5}\right)\right\}$$

[Formula: sin(π – x) = sin x, as sin is positive in the second quadrant.]

$$= \sin^{-1}\left\{\sin\frac{2\pi}{5}\right\}$$ [Formula: $\sin^{-1}(\sin x)=x$ ]

$$= \frac{2\pi}{5}$$