Exercise 11h

Question 1.
Find the slope of the tangent to the curve

i. $y = \left(x^3 - x\right)$ at $x = 2$

ii. $y = \left(2x^2 + 3\sin x\right)$ at $x = 0$

iii. $y = \left(\sin 2x + \cot x + 2\right)^2$ at $x = \frac{\pi}{2}$

Answer:
i. $\frac{dy}{dx} = 3x^2 - 1$

$$\frac{dy}{dx} \text{ at } (x = 2) = 11$$

ii. $\frac{dy}{dx} = 4x + 3\cos x$

$$\frac{dy}{dx} \text{ at } (x = 0) = 3$$

iii. $\frac{dy}{dx} = 2(\sin 2x + \cot x + 2)(2\cos 2x - \operatorname{cosec}^2 x)$

$$\frac{dy}{dx} \text{ at } \left(x = \frac{\pi}{2}\right) = 2(0 + 0 + 2)(-2 - 1) = -12$$

Question 2.
Find the equations of the tangent and the normal to the given curve at the indicated point for

$$y = x^3 - 2x + 7 \text{ at } (1, 6)$$

Answer:
$m: \frac{dy}{dx} = 3x^2 - 2$

m at (1, 6) = 1

Tangent : $y - b = m(x - a)$

$y - 6 = 1(x - 1)$

$x - y + 5 = 0$

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - 6 = -1(x - 1)$

$x + y - 7 = 0$

Question 3.
Find the equations of the tangent and the normal to the given curve at the indicated point for

$$y^2 = 4ax \text{ at } \left(\frac{a}{m^2}, \frac{2a}{m}\right)$$

Answer:
$m: \; 2y\frac{dy}{dx} = 4a$

$m \text{ at } \left(\frac{a}{m^2}, \frac{2a}{m}\right) = m$

Tangent : $y - b = m(x - a)$

$$y - \frac{2a}{m} = m\left(x - \frac{a}{m^2}\right)$$

$m^2x - my + a = 0$

Normal : $y - b = \frac{-1}{m}(x - a)$

$$y - \frac{2a}{m} = \frac{-1}{m}\left(x - \frac{a}{m^2}\right)$$

$m^2x + m^3y - 2am^2 - a = 0$

Question 4.
Find the equations of the tangent and the normal to the given curve at the indicated point for

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ at } (a\cos\theta, b\sin\theta)$$

Answer:
$m : \frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$

$$m \text{ at } (a\cos\theta, b\sin\theta) = \frac{-b\cos\theta}{a\sin\theta}$$

Tangent : y – b = m(x – a)

$$y - b\sin\theta = \frac{-b\cos\theta}{a\sin\theta}(x - a\cos\theta)$$

$$bx\cos\theta + ay\sin\theta = ab$$

Normal : $y - b = \frac{-1}{m}(x-a)$

$$y - b\sin\theta = \frac{a\sin\theta}{b\cos\theta}(x - a\cos\theta)$$

ax sec θ – by cosec θ = $a^2 - b^2$

Question 5.
Find the equations of the tangent and the normal to the given curve at the indicated point for

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ at } (a\sec\theta, b\tan\theta)$$

Answer:
$m : \frac{2x}{a^2}-\frac{2y}{b^2}\frac{dy}{dx}=0$

$$m \text{ at } (a\sec\theta, b\tan\theta) = \frac{b\sec\theta}{a\tan\theta}$$

Tangent : y – b = m(x – a)

$$y - b\tan\theta = \frac{b\sec\theta}{a\tan\theta}(x - a\sec\theta)$$

$$bx\sec\theta - ay\tan\theta = ab$$

Normal : $y - b = \frac{-1}{m}(x - a)$

$$y - b\sin\theta = \frac{-a\sin\theta}{b\cos\theta}(x - a\cos\theta)$$

$$by\,\mathrm{cosec}\,\theta + ax\sec\theta = (a^2 + b^2)$$

Question 6.
Find the equations of the tangent and the normal to the given curve at the indicated point for

$$y = x^3 \text{ at } P(1,1)$$

Answer:
$m : \frac{dy}{dx} = 3x^2$

m at (1, 1) = 3

Tangent : y – b = m(x – a)

y – 1 = 3(x – 1)

y = 3x – 2

Normal : $y - b = \frac{-1}{m}(x - a)$

$$y - 1 = \frac{-1}{3}(x - 1)$$

x + 3y = 4

Question 7.

Find the equations of the tangent and the normal to the given curve at the indicated point for

$$y^2 = 4ax \text{ at } \left(at^2, 2at\right)$$

Answer:

$m: 2y\frac{dy}{dx} = 4a$

m at $(at^2, 2at) = 1/t$

Tangent : $y - b = m(x - a)$

$$y - 2at = \frac{1}{t}(x - at^2)$$

$x - ty + at^2 = 0$

Normal : $y - b = \frac{-1}{m}(x - a)$

$y - 2at = -t(x - at^2)$

$tx + y = at^3 + 2at$

Question 8.
Find the equations of the tangent and the normal to the given curve at the indicated point for

$$y = \cot^2 x - 2\cot x + 2 \text{ at } x = \frac{\pi}{4}$$

Answer:

$m: \frac{dy}{dx} = 2\cot x(-\text{cosec}^2 x) + 2\,\text{cosec}^2 x$

m at $(x = \pi/4) = 2(-2) + 2(2) = 0$

Tangent : $y - b = m(x - a)$

$y - 1 = 0(x - \pi/4)$

$y = 1$

Normal : $y - b = \frac{-1}{m}(x - a)$

$$y - 1 = \frac{-1}{0}\left(x - \frac{\pi}{4}\right)$$

x = π/4

Question 9.
Find the equations of the tangent and the normal to the given curve at the indicated point for

$16x^2 + 9y^2 = 144$ at $(2, y_1)$, where $y_1 > 0$

Answer:
$m : 32x + 18y\frac{dy}{dx} = 0$

$$m \text{ at } (2,y_1) = \frac{-32}{9y_1}$$

$16(2)^2 + 9(y_1)^2 = 144$

$$y_1 = \frac{4\sqrt{5}}{3}$$

Tangent : y – b = m(x – a)

$$y - \frac{4\sqrt{5}}{3} = \frac{-32}{9\frac{4\sqrt{5}}{3}}(x - 2)$$

$$8x + 3\sqrt{5}y - 36 = 0$$

Normal : $y - b = \frac{-1}{m}(x - a)$

$$y - \frac{4\sqrt{5}}{3} = \frac{9\frac{4\sqrt{5}}{3}}{32}(x - 2)$$

$$9\sqrt{5x} - 24y + 14\sqrt{5} = 0$$

Question 10.
Find the equations of the tangent and the normal to the given curve at the indicated point for

$y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at the point where x = 1

Answer:
$m : \frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$

m at (x = 1) = 2

y at (x = 1) = $(1)^4 - 6(1)^3 + 13(1)^2 - 10(1) + 5 = 3$

Tangent : y – b = m(x – a)

y – 3 = 2(x – 1)

2x – y + 1 = 0

Normal : $y - b = \frac{-1}{m}(x - a)$

$$y - 3 = \frac{-1}{2}(x - 1)$$

x + 2y – 7 = 0

Question 11.

Find the equation of the tangent to the curve $\sqrt{x} + \sqrt{y} = a$ at $\left(\frac{a^2}{4}, \frac{a^2}{4}\right)$

Answer:
$m : \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$

$$\text{m at} \left(\frac{a^2}{4}, \frac{a^2}{4}\right) = -1$$

y – b = m(x – a)

$$y - \frac{a^2}{4} = -1\left(x - \frac{a^2}{4}\right)$$

$2(x + y) = a^2$

Question 12.

Show that the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$.

Answer:

$$m \text{ at } (x_1, y_1) = \frac{b^2 x_1}{a^2 y_1}$$

At (x_1, y_1) : $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1 \Rightarrow b^2x_1^2 - a^2y_1^2 = a^2b^2$

$y - b = m(x - a)$

$$y - y_1 = \frac{b^2 x_1}{a^2 y_1}(x - x_1)$$

$a^2y_1y - a^2y_1^2 = b^2x_1x - b^2x_1^2$

$b^2x_1x - a^2y_1y = a^2b^2$

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

Question 13.

Find the equation of the tangent to the curve $y = \left(\sec^4 x - \tan^4 x\right)$ at $x = \frac{\pi}{3}$.

Answer:

$$m : \frac{dy}{dx} = 4\sec^3 x(\tan x \sec x) - 4\tan^3 x(\sec^2 x)$$

$$m \text{ at } \left(x = \frac{\pi}{3}\right) = 4(2)^3(\sqrt{3} \times 2) - 4\left(\sqrt{3}\right)^3(2)^2 = 16\sqrt{3}$$

At x = π/3 , y = 7

y – b = m(x – a)

$$y - 7 = 16\sqrt{3}\left(x - \frac{\pi}{3}\right)$$

$$3y - 48\sqrt{3}x + 16\sqrt{3}\pi - 21 = 0$$

Question 14.

Find the equation of the normal to the curve $y = (\sin 2x + \cot x + 2)^2$ at $x = \dfrac{\pi}{2}$

Answer:

$$m : \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)(2\cos 2x - \operatorname{cosec}^2 x)$$

$$\frac{dy}{dx} \text{ at } \left(x = \frac{\pi}{2}\right) = 2(0 + 0 + 2)(-2 - 1) = -12$$

At x = π/2, y = 4

$$y - b = \frac{-1}{m}(x - a)$$

$$y - 4 = \frac{1}{12}\left(x - \frac{\pi}{2}\right)$$

24y – 2x + π – 96 = 0

Question 15.

Show that the tangents to the curve $y = 2x^3 - 4$ at the point x = 2 and x = –2 are parallel.

Answer:

$$m : \frac{dy}{dx} = 6x^2$$

m at (x = 2) = 24

m at (x = -2) = 24

We know that if the slope of curve at two different point is

equal then straight lines are parallel at that points.

Question 16.
Find the equation of the tangent to the curve $x^2 + 3y = 3$, where is parallel to the line $y - 4x + 5 = 0$.

Answer:
We know that if two straight lines are parallel then their slope

are equal. So, slope of required tangent is also equal to 4.

$$m : \frac{dy}{dx} = \frac{-2x}{3} = 4$$

x = -6 and y = -11

y – b = m(x – a)

y – (-11) = 4(x – (-6))

4x – y + 13 = 0

Question 17.
At what point on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$, is the tangent parallel to the y-axis?

Answer:
If the tangent is parallel to y-axis it means that it's slope is

not defined or 1/0.

$$m : 2x + 2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-(2x-2)}{(2y-4)} = \frac{1}{0}$$

2y – 4 = 0 ⇒ y = 2

$x^2 + (2)^2 - 2x - 4(2) + 1 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x = 3$ and $x = -1$

So, the requied points are (-1, 2) and (3, 2).

Question 18.
Find the point on the curve $x^2 + y^2 - 2x - 3 = 0$ where the tangent is parallel to the x-axis.

Answer:
If the tangent is parallel to x-axis it means that it's slope is 0

$$m:\ 2x + 2y\frac{dy}{dx} - 2 = 0$$

$2x + 2y(0) - 2 = 0$

$x = 1$

$(1)^2 + y^2 - 2(1) - 3 = 0$

$\Rightarrow y^2 = 4 \Rightarrow y = 2$ and $y = -2$

So, the requied points are (1, 2) and (1, -2).

Question 19.
Prove the tangent to the curve $y = x^2 - 5x + 6$ at the point (2, 0) and (3, 0) are at right angles.

Answer:
We know that if the slope of two tangent of a curve are satisfies a relation $m_1m_2 = -1$, then tangents are at right angles

$$m:\ \frac{dy}{dx} = 2x - 5$$

m_1 at (2, 0) = -1

m_2 at (3, 0) = 1

$m_1m_2 = (-1)(1) = -1$

So, we can say that tangent at (2, 0) and (3, 0) are at right angles.

Question 20.
Find the point on the curve $y = x^2 + 3x + 4$ at which the tangent passes through the origin.

Answer:
If tangent is pass through origin it means that equation of tangent is y = mx

Let us suppose that tangent is made at point (x_1, y_1)

$y_1 = x_1^2 + 3x_1 + 4$ ...(1)

$$m : \frac{dy}{dx} = 2x + 3$$

m at $(x_1, y_1) = 2x_1 + 3$

Equation of tangent : $y_1 = (2x_1 + 3)x_1$ ...(2)

On compairing eq(1) and eq(2)

$x_1^2 + 3x_1 + 4 = (2x_1 + 3)x_1$

$x_1^2 - 4 = 0 \Rightarrow x_1 = 2$ and -2

At $x_1 = 2$, $y_1 = 14$

At $x_1 = -2$, $y_1 = 2$

So, required points are (2, 14) and (-2, 2)

Question 21.
Find the point on the curve $y = x^3 - 11x + 5$ at which the equation of tangent is $y = x - 11$.

Answer:
Slope of y = x – 11 is equal to 1

$$m: \frac{dy}{dx} = 3x^2 - 11$$

$3x^2 - 11 = 1 \Rightarrow x = 2$ and -2

At x = 2

From the equation of curve, $y = (2)^3 - 11(2) + 5 = -9$

From the equation of tangent, y = 2 – 11 = -9

At x = -2

From the equation of curve, $y = (-2)^3 - 11(-2) + 5 = 19$

From the equation of tangent, y = -2 – 11 = -13

So, the final answer is (2, -9) because at x = -2, y is come different from the equation of curve and tangent which is not possible.

Question 22.
Find the equation of the tangents to the curve $2x^2 + 3y^2 = 14$, parallel to the line $x = 3y = 4$.

Answer:
If tangent is parallel to the line x + 3y = 4 then it's slope is -1/3.

$$m: 4x + 6y\frac{dy}{dx} = 0$$

$$m = \frac{-2x}{3y} = \frac{-2x}{3\sqrt{\frac{14 - 2x^2}{3}}} = \frac{-1}{3}$$

$$2x = \sqrt{\frac{14 - 2x^2}{3}}$$

$$4x^2 = \frac{14 - 2x^2}{3}$$

x = 1 and -1

At x = 1, y = 2 and y = -2 (not possible)

At x = -1, y = -2 and y = 2 (not possible)

y – b = m(x – a)

At (1, 2)

$$y - 2 = \frac{-1}{3}(x - 1)$$

3y + x = 7

At (-1, -2)

$$y - (-2) = \frac{-1}{3}(x - (-1))$$

3y + x = -7

Question 23.
Find the equation of the tangent to the curve $x^2 + 2y = 8$, which is perpendicular to the line $x - 2y + 1 = 0$.

Answer:
.: If tangent is perpendicular to the line x – 2y + 1 = 0 then it's -1/m is -2.

$$m:\ 2x + 2\frac{dy}{dx} = 0$$

m = -x = 1/2

x = -1/2

At x = -1/2, y = 31/8

$$y - b = \frac{-1}{m}(x - a)$$

At (-1/2, 31/8)

$$y - \frac{31}{8} = \frac{-1}{\frac{1}{2}}\left(x - \left(-\frac{1}{2}\right)\right)$$

16x + 8y – 23 = 0

Question 24.
Find the point on the curve $y = 2x^2 - 6x - 4$ at which the tangent is parallel to the x-axis.

Answer:
We know that if tangent is parallel to x-axis then it's slope is equal to 0.

$$m : \frac{dy}{dx} = 4x - 6$$

4x – 6 = 0 ⇒ x = 3/2

At x = 3/2 , y = -17/2

So, the required points are $\left(\frac{3}{2}, \frac{-17}{2}\right)$.

Question 25.
Find the point on the parabola $y = (x - 3)^2$, where the tangent is parallel to the chord joining the point (3, 0) and (4, 1).

Answer:
If the tangent is parallel to chord joining the points (3, 0) and (4, 1) then slope of tangent is equal to slope of chord.

$$m = \frac{1 - 0}{4 - 3} = 1$$

$$m : \frac{dy}{dx} = 2(x - 3)$$

2(x – 3) = 1 ⇒ x = 7/2

At x = 7/2, y = 1/4

So, the required points are $\left(\frac{7}{2}, \frac{1}{4}\right)$.

Question 26.

Show that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

Answer:

If curves cut at right angle if $8k^2 = 1$ then vice versa also true. So, we have to prove that $8k^2 = 1$ if curve cut at right angles.

If curve cut at right angle then the slope of tangent at their intersecting point satisfies the relation $m_1m_2 = -1$

We have to find intersecting point of two curves.

$x = y^2$ and $xy = k$ then $y = k^{\frac{1}{3}}$ and $x = k^{\frac{2}{3}}$

$$m_1 : \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$m_1 \text{ at } \left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right) = \frac{1}{2k^{\frac{1}{3}}}$$

$$m_2 : \frac{dy}{dx} = \frac{-k}{x^2}$$

$$m_2 \text{ at } \left(k^{\frac{2}{3}}, k^{\frac{1}{3}}\right) = \frac{-k}{k^{\frac{4}{3}}} = -\frac{1}{k^{\frac{1}{3}}}$$

$$m_1 m_2 = -1$$

$$\left(\frac{1}{2k^{\frac{1}{3}}}\right)\left(-\frac{1}{k^{\frac{1}{3}}}\right) = -1$$

$$k^{\frac{2}{3}} = \frac{1}{2} \Rightarrow k^2 = \frac{1}{8} \Rightarrow 8k^2 = 1$$

Question 27.

Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

Answer:
If the two curve touch each other then the tangent at their intersecting point formed a angle of 0.

We have to find the intersecting point of these two curves.

$xy = a^2$ and $x^2 + y^2 = 2a^2$

$\Rightarrow x^2 + \left(\frac{a^2}{x}\right)^2 = 2a^2$

$\Rightarrow x^4 - 2a^2x^2 + a^4 = 0$

$\Rightarrow (x^2 - a^2) = 0$

$\Rightarrow x = +a$ and $-a$

At $x = a$, $y = a$

At $x = -a$, $y = -a$

$m_1 : \frac{dy}{dx} = \frac{-a^2}{x^2}$

m_1 at $(a, a) = -1$

m_1 at $(-a, -a) = -1$

$m_2 : 2x + 2y\frac{dy}{dx} = 0$

m_2 at $(a, a) = -1$

m_2 at $(-a, -a) = -1$

At (a, a)

$\tan\theta = \frac{m_1 - m_2}{1 + m_1m_2}$

$\tan\theta = \frac{(-1)-(-1)}{1+(-1)(-1)} = 0 \Rightarrow \theta = 0$

At (-a, -a)

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\tan\theta = \frac{(-1)-(-1)}{1+(-1)(-1)} = 0 \Rightarrow \theta = 0$$

So, we can say that two curves touch each other because the angle between two tangent at their intersecting point is equal to 0.

Question 28.

Show that the curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ cut orthogonally.

Answer:

If the two curve cut orthogonally then angle between their tangent at intersecting point is equal to 90º.

We have to find their intersecting point.

$x^3 - 3xy^2 + 2 = 0$ ...(1) and $3x^2y - y^3 - 2 = 0$ ...(2)

On adding eq (1) and eq (2)

$x^3 - 3xy^2 + 2 + 3x^2y - y^3 - 2 = 0$

$x^3 - y^3 - 3xy^2 + 3x^2y = 0$

$(x - y)^3 = 0 \Rightarrow x = y$

Put x = y in eq (1)

$y^3 - 3y^3 + 2 = 0 \Rightarrow y = 1$

At y = 1, x = 1

$$m_1 : 3x^2 - 3\left(x \times 2y\frac{dy}{dx} + y^2\right) = 0$$

m_1 at (1, 1) = 0

$$m_2 : 3\left(x^2\frac{dy}{dx} + 2xy\right) - 3y^2\frac{dy}{dx} = 0$$

m_2 at (1, 1) = -2/0

At (1, 1)

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\tan\theta = \frac{m_2\left(1 - \frac{m_1}{m_2}\right)}{m_2\left(\frac{1}{m_2} + m_1\right)}$$

$$\tan\theta = \frac{(1-0)}{(0+0)} = \text{not defined} \Rightarrow \theta = \frac{\pi}{2}$$

So, we can say that two curve cut each other orthogonally because angle between two tangent at their intersecting point is equal to 90º.

Question 29.

Find the equation of tangent to the curve $x = (\theta + \sin\theta), y = (1 + \cos\theta)$ at $\theta = \frac{\pi}{4}$.

Answer:

$$m : \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-\sin\theta}{1+\cos\theta}$$

$$m \text{ at } \left(\theta = \frac{\pi}{4}\right) = \frac{-1}{1 + \sqrt{2}} = 1 - \sqrt{2}$$

At $\theta = \frac{\pi}{4}, x = \left(\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right)$ and $y = \left(1 + \frac{1}{\sqrt{2}}\right)$

y – b = m(x – a)

$$y - \left(1 + \frac{1}{\sqrt{2}}\right) = (1 - \sqrt{2})\left(x - \left(\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right)\right)$$

$$y = (1-\sqrt{2})x + \frac{(\sqrt{2}-1)\pi}{4} + 2$$

Question 30.

Find the equation of the tangent at $t = \frac{\pi}{4}$ for the curve $x = \sin 3t, y = \cos 2t$.

Answer:

$$m : \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t}{3\cos 3t}$$

$$m \text{ at } \left(t = \frac{\pi}{4}\right) = \frac{2\sqrt{2}}{3}$$

At $t = \frac{\pi}{4}, x = \frac{1}{\sqrt{2}}$ and $y = 0$

y – b = m(x – a)

$$y - 0 = \frac{2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right)$$

$$4x - 3\sqrt{2}y - 2\sqrt{2} = 0$$

Objective Questions

Question 1.
Mark (√) against the correct answer in the following:

If $y = 2^x$ then $\frac{dy}{dx} = ?$

A. $x\left(2^{x-1}\right)$

B. $\frac{2^x}{(\log 2)}$

C. $2^x(\log 2)$

D. none of these

Answer:
Given that $y=2^x$

Taking log both sides, we get

$\log_e y = x\log_e 2$ (Since $\log_a b^c = c\log_a b$)

Differentiating with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \log_e 2 \text{ or } \frac{dy}{dx} = \log_e 2 \times y$$

Hence $\frac{dy}{dx} = 2^x \log_e 2$

Question 2.
Mark (√) against the correct answer in the following:

If $y = \log_{10} x$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{x}$

B. $\frac{1}{x}(\log 10)$

C. $\frac{1}{x(\log 10)}$

D. none of these

Answer:
Given that $y = \log_{10} x$

Using the property that $\log_a b = \frac{\log_e b}{\log_e a}$, we get

$$y = \frac{\log_e x}{\log_e 10}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{x\log_e 10}$$

Question 3.
Mark (√) against the correct answer in the following:

If $y = e^{1/x}$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{x} \cdot e^{(1/x-1)}$

B. $\frac{-e^{1/x}}{x^2}$

C. $e^{1/x} \log x$

D. none of these

Answer:
Given that $y = e^{\frac{1}{x}}$

Taking log both sides, we get

$$\log_e y = \frac{1}{x} \text{ (Since } \log_a b^c = c\log_a b)$$

Differentiating with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{x^2} \text{or} \frac{dy}{dx} = -\frac{1}{x^2} \times y$$

Hence $\frac{dy}{dx} = -\frac{1}{x^2} \times e^{\frac{1}{x}}$

Question 4.
Mark (√) against the correct answer in the following:

If $y = x^x$ then $\frac{dy}{dx} = ?$

A. $x^x \log x$

B. $x^x(1 + \log x)$

C. $x(1 + \log x)$

D. none of these

Answer:
Let y=f(x)=x^x

Taking log both sides, we get

$\log_e y = x \times \log_e x$-(1) (Since $\log_a b^c = c \log_a b$)

Differentiating (1) with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x} + \log_e x \times 1$$

$$\Rightarrow \frac{dy}{dx} = y \times (1 + \log_e x)$$

$$\Rightarrow \frac{dy}{dx} = f'(x) = x^x(1 + \log_e x)$$

Question 5.
Mark (√) against the correct answer in the following:

If $y = x^{\sin x}$ then $\frac{dy}{dx} = ?$

A. $(\sin\ x) \cdot x^{(\sin x - 1)}$

B. $(\sin\ x \cos x) \cdot x^{(\sin x - 1)}$

C. $x^{\sin x}\left\{\dfrac{\sin x + x\log x \cdot \cos x}{x}\right\}$

D. none of these

Answer:
Let y=f(x)=$x^{\sin x}$

Taking log both sides, we get

$\log_e y = \sin x \times \log_e x$-(1) (Since $\log_a b^c = c\log_a b$)

Differentiating (1) with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \sin x \times \frac{1}{x} + \log_e x \times \cos x$$

$$\Rightarrow \frac{dy}{dx} = y \times \left(\frac{\sin x}{x} + \log_e x \cos x\right)$$

$$\Rightarrow \frac{dy}{dx} = f'(x) = x^x\left(\frac{\sin x + x\log_e x\cos x}{x}\right)$$

Question 6.
Mark (√) against the correct answer in the following:

f $y = x^{\sqrt{x}}$ then $\dfrac{dy}{dx} = ?$

A. $\sqrt{x} \cdot x^{\left(\sqrt{x}-1\right)}$

B. $\dfrac{x^{\sqrt{x}}\log x}{2\sqrt{x}}$

C. $x^{\sqrt{x}}\left\{\dfrac{2+\log x}{2\sqrt{x}}\right\}$

D. none of these

Answer:

Let $y = f(x) = x^{\sqrt{x}}$

Taking log both sides, we get

$\log_e y = \sqrt{x} \times \log_e x$ -(1)

(Since $\log_a b^c = c \log_a b$)

Differentiating (1) with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \sqrt{x} \times \frac{1}{x} + \log_e x \times \frac{1}{2\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = y \times \left(\frac{2 + \log_e x}{2\sqrt{x}}\right)$$

$$\Rightarrow \frac{dy}{dx} = f'(x)$$

$$= x^{\sqrt{x}}\left(\frac{2 + \log_e x}{2\sqrt{x}}\right)$$

Question 7.

Mark (√) against the correct answer in the following:

If $y = e^{\sin\sqrt{x}}$ then $\dfrac{dy}{dx}$ =?

A. $e^{\sin\sqrt{x}} \cdot \cos\sqrt{x}$

B. $\dfrac{e^{\sin\sqrt{x}} \cos\sqrt{x}}{2\sqrt{x}}$

C. $\dfrac{e^{\sin\sqrt{x}}}{2\sqrt{x}}$

D. none of these

Answer:
Given that $y = e^{\sin\sqrt{x}}$

Taking log both sides, we get

$$\log_e y = \sin\sqrt{x}$$

$$(\text{Since } \log_a b^c = c\log_a b)$$

Differentiating with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \cos\sqrt{x} \times \frac{1}{2\sqrt{x}}$$

Or

$$\frac{dy}{dx} = \cos\sqrt{x} \times \frac{1}{2\sqrt{x}} \times y$$

Hence $\frac{dy}{dx} = \frac{e^{\sin\sqrt{x}}\cos\sqrt{x}}{2\sqrt{x}}$

Question 8.
Mark (√) against the correct answer in the following:

If $y = (\tan x)^{\cot x}$ then $\dfrac{dy}{dx} = ?$

A. $\cot x \cdot (\tan x)^{\cot x - 1} \cdot \sec^2 x$

B. $-(\tan x)^{\cot x} \cdot \operatorname{cosec}^2 x$

C. $(\tan x)^{\cot x} \cdot \operatorname{cosec}^2 x (1 - \log \tan x)$

D. none of these

Answer:
Given that $y = (\tan x)^{\cot x}$

Taking log both sides, we get

$\log_e y = \cot x \times \log_e \tan x$ (Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \cot x \times \frac{1}{\tan x} \times \sec^2 x - \log_e \tan x \times \operatorname{cosec}^2 x = \operatorname{cosec}^2 x(1 - \log_e \tan x)$$

Hence, $\frac{dy}{dx} = \operatorname{cosec}^2 x(1 - \log_e \tan x \times y = \operatorname{cosec}^2 x(1 - \log_e \tan x)(\tan x)^{\cot x}$

Question 9.
Mark (√) against the correct answer in the following:

If $y = (\sin x)^{\log x}$ then $\frac{dy}{dx} = ?$

A. $(\log x) \cdot (\sin x)^{(\log x - 1)} \cdot \cos x$

B. $(\sin x)^{\log x} \cdot \left\{\frac{x \log x + \log \sin x}{x}\right\}$

C. $(\sin x)^{\log x} \cdot \left\{\frac{(x \log x)\cot x + \log \sin x}{x}\right\}$

D. none of these

Answer:
Given that $y = (\sin x)^{\log_e x}$

Taking log both sides, we get

$\log_e y = \log_e x \times \log_e \sin x$ (Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \log_e x \times \frac{1}{\sin x} \times \cos x + \log_e \sin x \times \frac{1}{x}$$

$$= \frac{x\cot x \log_e x + \log_e \sin x}{x}$$

Hence, $\frac{dy}{dx} = \frac{x\cot x \log_e x + \log_e \sin x}{x} \times y$

$$= \frac{x\cot x \log_e x + \log_e \sin x}{x} (\sin x)^{\log_e x}$$

Question 10.
Mark (√) against the correct answer in the following:

If $y = \sin\left(x^x\right)$ then $\frac{dy}{dx} = ?$

A. $x^x \cos\left(x^x\right)$

B. $x^x \cos x^x (1 + \log x)$

C. $x^x \cos x^x \log x$

D. none of these

Answer:
Given that $y=\sin(x^x)$

Let $x^x=u$, then $y=\sin u$

Differentiating with respect to x, we get

$\frac{dy}{dx} = \cos u \times \frac{du}{dx} = \cos(x^x)\frac{du}{dx}$ −(1)

Also, $u=x^x$

Taking log both sides, we get

$\log_e u = x \times \log_e x$

$(\text{Since } \log_a b^c = c \log_a b)$

Differentiating with respect to x, we get

$$\frac{1}{u}\frac{du}{dx} = x \times \frac{1}{x} + \log_e x \times 1$$

$$\Rightarrow \frac{du}{dx} = u \times (1 + \log_e x)$$

$$\Rightarrow \frac{du}{dx} = x^x(1 + \log_e x) \text{ -(2)}$$

From (1) and (2), we get

$$\frac{dy}{dx} = \cos(x^x)\, x^x(1 + \log_e x)$$

Question 11.
Mark (√) against the correct answer in the following:

If $y = \sqrt{x \sin x}$ then $\frac{dy}{dx} = ?$

A. $\frac{(x \cos x + \sin x)}{2\sqrt{x \sin x}}$

B. $\frac{1}{2}(x \cos x + \sin x) \cdot \sqrt{x \sin x}$

C. $\frac{1}{2\sqrt{x \sin x}}$

D. none of these

Answer:
Given that $y = \sqrt{x\sin x}$

Squaring both sides, we get

$$y^2 = x\sin x$$

Differentiating with respect to x, we get

$2y\frac{dy}{dx} = x\cos x + \sin x$ or $\frac{dy}{dx} = \frac{x\cos x+\sin x}{2y}$

Hence, $\frac{dy}{dx} = \frac{x\cos x+\sin x}{2\sqrt{x\sin x}}$

Question 12.
Mark (√) against the correct answer in the following:

If $e^{x+y} = xy$ then $\frac{dy}{dx}$ =?

A. $\frac{x(1-y)}{y(x-1)}$

B. $\frac{y(1-x)}{x(y-1)}$

C. $\frac{(x-xy)}{(xy-y)}$

D. none of these

Answer:
Given that $xy=e^{x+y}$

Taking log both sides, we get

$\log_e xy = x + y$ (Since $\log_a b^c = c\log_a b$)

Since $\log_a bc = \log_a b + \log_a c$, we get

$\log_e x + \log_e y = x + y$

Differentiating with respect to x, we get

$$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

Or

$$\frac{dy}{dx}\left(\frac{y-1}{y}\right)=\frac{1-x}{x}$$

Hence, $\frac{dy}{dx}=\frac{y(1-x)}{x(y-1)}$

Question 13.
Mark (√) against the correct answer in the following:

If $(x+y)=\sin(x+y)$ then $\frac{dy}{dx}=?$

A. -1

B. 1

C. $\frac{1-\cos(x+y)}{\cos^2(x+y)}$

D. none of these

Answer:
Given that x+y=sin(x+y)

Differentiating with respect to x, we get

$1+\frac{dy}{dx}=\cos(x+y)\left(1+\frac{dy}{dx}\right)$ or $(\cos(x+y)-1)\left(1+\frac{dy}{dx}\right)=0$

Hence, cos(x+y)=1 or $\frac{dy}{dx}=-1$

If cos(x+y)=1 then, x+y=2nπ, n∈$\mathbb{Z}$

Hence x+y=sin(2nπ)=0 or y=-x

Differentiating with respect to x, we get

$$\frac{dy}{dx}=-1$$

Hence, $\frac{dy}{dx}=-1$

Question 14.
Mark (√) against the correct answer in the following:

If $\sqrt{x}+\sqrt{y}=\sqrt{a}$ then $\frac{dy}{dx}$=?

A. $\frac{-\sqrt{x}}{\sqrt{y}}$

B. $-\frac{1}{2}\cdot\frac{\sqrt{y}}{\sqrt{x}}$

C. $\frac{-\sqrt{y}}{\sqrt{x}}$

D. None of these

Answer:
Given that $\sqrt{x}+\sqrt{y}=\sqrt{a}$

Differentiating with respect to x, we get

$$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$$

Or

$$\frac{dy}{dx}=-\sqrt{\frac{y}{x}}$$

Question 15.
Mark (√) against the correct answer in the following:

If $x^{y}=y^{x}$ then $\frac{dy}{dx}=?$

A. $\frac{(y-x\log y)}{(x-y\log x)}$

B. $\dfrac{y(y-x\log y)}{x(x-y\log x)}$

C. $\dfrac{y(y+x\log y)}{x(x+y\log x)}$

D. none of these

Answer:
Given that $x^y=y^x$

Taking log both sides, we get

$$y\log_e x = x\log_e y$$

$$(\text{Since } \log_a b^c = c\log_a b)$$

Differentiating with respect to x, we get

$$\frac{y}{x}+\log_e x\frac{dy}{dx}=\frac{x}{y}\frac{dy}{dx}+\log_e y$$

$$\Rightarrow \frac{x-y\log_e x}{y}\frac{dy}{dx}=\frac{y-x\log_e y}{x}$$

Hence $\frac{dy}{dx}=\frac{y(y-x\log_e y)}{x(x-y\log_e x)}$

Question 16.
Mark (√) against the correct answer in the following:

If $x^p y^q = (x+y)^{(p+q)}$ then $\dfrac{dy}{dx}=?$

A. $\dfrac{x}{y}$

B. $\dfrac{y}{x}$

C. $\dfrac{x^{p-1}}{y^{q-1}}$

D. none of these

Answer:
Given that $x^p y^q = (x+y)^{p+q}$

Taking log both sides, we get

$$\log_e x^p y^q = (p+q)\log_e (x+y)$$

$$(\text{Since } \log_a b^c = c \log_a b)$$

Since $\log_a bc = \log_a b + \log_a c$, we get

$$\log_e x^p + \log_e y^q = (p+q)\log_e (x+y)$$

$$p\log_e x + q\log_e y = (p+q)\log_e (x+y)$$

Differentiating with respect to x, we get

$$\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p+q}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{xq - yp}{y(x+y)}\right) = \frac{xq - yp}{x(x+y)}$$

Hence, $\frac{dy}{dx} = \frac{y}{x}$

Question 17.
Mark (√) against the correct answer in the following:

If $y = x^2 \sin\dfrac{1}{x}$ then $\dfrac{dy}{dx} = ?$

A. $x\sin\dfrac{1}{x} - \cos\dfrac{1}{x}$

B. $-\cos\frac{1}{x}+2x\sin\frac{1}{x}$

C. $-x\sin\frac{1}{x}+\cos\frac{1}{x}$

D. None of these

Answer:
Given that $y = x^2 \sin\frac{1}{x}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = x^2 \cos\frac{1}{x} \times -\frac{1}{x^2} + 2x\sin\frac{1}{x} = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$$

Question 18.
Mark (√) against the correct answer in the following:

If $y = \cos^2 x^3$ then $\frac{dy}{dx}$ =?

A. $-3x^2 \sin$

B. $-3x^2 \sin^2 x^3$

C. $-3x^2 \cos^2\left(2x^3\right)$

D. none of these

Answer:
$y=\cos^2 x^3=(\cos(x^3))^2$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = 2\cos(x^3) \times -\sin(x^3) \times 3x^2$$

Using 2sinAcosA=sin2A

$$\frac{dy}{dx} = -3x^2 \sin(2x^3)$$

Question 19.
Mark (√) against the correct answer in the following:

If $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{2\left(x + \sqrt{x^2 + a^2}\right)}$

B. $\frac{-1}{\sqrt{x^2 + a^2}}$

C. $\frac{1}{\sqrt{x^2 + a^2}}$

D. none of these

Answer:
Given that $y = \log_e\left(x + \sqrt{x^2 + a^2}\right)$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + a^2}}\left(1 + \frac{1}{2\sqrt{x^2 + a^2}} \times 2x\right)$$

Hence, $\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}} \times \frac{x+\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}} = \frac{1}{\sqrt{x^2+a^2}}$

Question 20.
Mark (√) against the correct answer in the following:

If $y = \log\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{\sqrt{x}\,(1-x)}$

B. $\frac{-1}{x\left(1-\sqrt{x}\right)^2}$

C. $\frac{-\sqrt{x}}{2\left(1-\sqrt{x}\right)}$

D. none of these

Answer:

Given that $y = \log_e \frac{1+\sqrt{x}}{1-\sqrt{x}}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1-\sqrt{x}}{1+\sqrt{x}} \times \frac{(1-\sqrt{x}) \times \frac{1}{2\sqrt{x}} - (1+\sqrt{x}) \times -\frac{1}{2\sqrt{x}}}{(1-\sqrt{x})^2} = \frac{1}{(1-x)\sqrt{x}}$$

Question 21.
Mark (√) against the correct answer in the following:

If $y = \log\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{2}{\sqrt{1+x^2}}$

B. $\frac{2\sqrt{1+x^2}}{x^2}$

C. $\frac{-2}{\sqrt{1+x^2}}$

D. none of these

Answer:

Given that $y = \log_e\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$

Differentiating with respect to x, we get

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1+x^2}-x}{\sqrt{1+x^2}+x} \times \frac{\left(\sqrt{1+x^2}-x\right)\times\left(\frac{1}{2\sqrt{1+x^2}}\times 2x+1\right)-\left(\sqrt{1+x^2}+x\right)\times\left(\frac{1}{2\sqrt{1+x^2}}\times 2x-1\right)}{\left(\sqrt{1+x^2}-x\right)^2}$$

Hence, $\frac{dy}{dx} = \frac{2}{\sqrt{1+x^2}}$

Question 22.
Mark (√) against the correct answer in the following:

If $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$ then $\frac{dy}{dx}$ =?

A. $\frac{1}{2}\sec^2\left(\frac{\pi}{4}-\frac{\pi}{2}\right)$

B. $\frac{1}{2}\operatorname{cosec}^2\left(\frac{\pi}{4}-\frac{\pi}{2}\right)$

C. $\frac{1}{2}\operatorname{cosec}^2\left(\frac{\pi}{4}-\frac{\pi}{2}\right)\cot\left(\frac{\pi}{4}-\frac{\pi}{2}\right)$

D. none of these

Answer:

Given that $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$

Using, $\cos^2\theta + \sin^2\theta = 1$ and $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$$y = \sqrt{\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}$$

$$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$$

Dividing by $\sin\frac{x}{2}$ in numerator and denominator, we get

$$y = \frac{\cot\frac{x}{2} + 1}{\cot\frac{x}{2} - 1} = \cot\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\left(\text{Using } \cot\left(\frac{\pi}{4} - A\right) = \frac{\cot A + 1}{\cot A - 1}\right)$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = -\mathrm{cosec}^2\left(\frac{\pi}{4} - \frac{x}{2}\right) \times -\frac{1}{2}$$

Hence, $\frac{dy}{dx} = \frac{1}{2}\mathrm{cosec}^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

Question 23.
Mark (√) against the correct answer in the following:

If $y = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$ then $\frac{dy}{dx}$ =?

A. $\sec^2 x$

B. $\frac{1}{2}\sec^2\frac{x}{2}$

C. $\frac{-1}{2}\mathrm{cosec}^2\frac{x}{2}$

D. none of these

Answer:

Given that $y = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$

Multiplying by cos x in numerator and denominator, we get

$$y = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

Using $1 - \cos x = 2\sin^2\frac{x}{2}$ and $1 + \cos x = 2\cos^2\frac{x}{2}$, we get

$$y = \sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}$$

$$= \tan\left(\frac{x}{2}\right)$$

Differentiating with respect to x, we get

$$y = \sec^2\frac{x}{2} \times \frac{1}{2}$$

$$= \frac{1}{2}\sec^2\frac{x}{2}$$

Question 24.
Mark (√) against the correct answer in the following:

If $y = \sqrt{\frac{1 + \tan x}{1 - \tan x}}$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{2}\sec^2 x \cdot \tan\left(x + \frac{\pi}{4}\right)$

B. $\dfrac{\sec^2\left(x + \frac{\pi}{4}\right)}{2\sqrt{\tan\left(x + \frac{\pi}{4}\right)}}$

C. $\dfrac{\sec^2\left(\frac{x}{4}\right)}{\sqrt{\tan\left(x + \frac{\pi}{4}\right)}}$

D. none of these

Answer:

Given that $y = \sqrt{\frac{1+\tan x}{1-\tan x}}$

Using $\tan\left(\frac{\pi}{4} + x\right) = \frac{1+\tan x}{1-\tan x}$, we get

$$y = \sqrt{\tan\left(\frac{\pi}{4} + x\right)}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\tan\left(\frac{\pi}{4} + x\right)}} \times \sec^2\left(\frac{\pi}{4} + x\right) \times 1$$

Hence, $\frac{dy}{dx} = \frac{\sec^2\left(\frac{\pi}{4}+x\right)}{2\sqrt{\tan\left(\frac{\pi}{4}+x\right)}}$

Question 25.
Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ then $\frac{dy}{dx} = ?$

A. 1

B. -1

C. $\frac{1}{2}$

D. $\frac{-1}{2}$

Answer:

Given that $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$

Using $1 - \cos x = 2\sin^2\frac{x}{2}$ and Using $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$, we get

$$y = \tan^{-1}\left(\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right) \text{ or } y = \tan^{-1}\tan\frac{x}{2}$$

$$y = \frac{x}{2}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{2}$$

Question 26.
Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\left\{\frac{\cos x + \sin x}{\cos x - \sin x}\right\}$ then $\frac{dy}{dx} = ?$

A. 1

B. -1

C. $\frac{1}{2}$

D. $\frac{-1}{2}$

Answer:
Given that $y = \tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)$

Dividing numerator and denominator with cosx, we get

$$y = \tan^{-1}\left(\frac{1 + \tan x}{1 - \tan x}\right)$$

Using $\tan\left(\frac{\pi}{4} + x\right) = \frac{1 + \tan x}{1 - \tan x}$, we get

$$y = \tan^{-1}\tan\left(\frac{\pi}{4}+x\right) = \frac{\pi}{4}+x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = 1$$

Question 27.
Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\left\{\frac{\cos x}{1+\sin x}\right\}$ then $\frac{dy}{dx}$ =?

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. 1

D. -1

Answer:
Given that $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

Using $\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$, $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$ and $\cos^2\theta + \sin^2\theta = 1$

Hence, $y = \tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}\right) = \tan^{-1}\left(\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}\right)$

$$\Rightarrow\ y = \tan^{-1}\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}$$

Dividing by $\cos\frac{x}{2}$ in numerator and denominator, we get

$$y = \tan^{-1}\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}$$

Using $\tan\left(\frac{\pi}{4} - x\right) = \frac{1-\tan x}{1+\tan x}$, we get

$$y = \tan^{-1}\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$= \frac{\pi}{4} - \frac{x}{2}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = -\frac{1}{2}$$

Question 28.
Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ then $\frac{dy}{dx} =$

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. $\frac{1}{(1+x^2)}$

D. none of these

Answer:
Given that $y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

Using $1 - \cos x = 2\sin^2\frac{x}{2}$ and $1 + \cos x = 2\cos^2\frac{x}{2}$, we get

$$y = \tan^{-1}\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}} = \tan^{-1}\tan\left(\frac{x}{2}\right) = \frac{x}{2}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{2}$$

Question 29.
Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$ then $\frac{dy}{dx}$ =?

A. $\frac{a}{b}$

B. $\frac{-b}{a}$

C. 1

D. -1

Answer:
Given that $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

Dividing by bcosx in numerator and denominator, we get

$$y = \tan^{-1}\left(\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}\right)$$

Let $\frac{a}{b} = \tan\alpha \Rightarrow \alpha = \tan^{-1}\frac{a}{b}$

Then $y = \tan^{-1}\left(\frac{\tan\alpha - \tan x}{1 + \tan\alpha\tan x}\right)$

Using $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$, we get

$$y = \tan^{-1}\tan(\alpha - x) = \alpha - x = \tan^{-1}\frac{a}{b} - x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = -1$$

Question 30.
Mark (√) against the correct answer in the following:

If $y = \sin^{-1}\left(3x - 4x^3\right)$ then $\frac{dy}{dx}$ =?

A. $\frac{3}{\sqrt{1-x^2}}$

B. $\frac{-4}{\sqrt{1-x^2}}$

C. $\frac{3}{\sqrt{1+x^2}}$

D. none of these

Answer:
Given that $y=\sin^{-1}(3x-4x^3)$

Let $x=\sin\theta$

$\Rightarrow \theta=\sin^{-1}x$

Then, $y= \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

Using $\sin3\theta=3\sin\theta - 4\sin^3\theta$, we get

$y=\sin^{-1}(\sin3\theta)=3\theta=3\sin^{-1}x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$$

Question 31.
Mark (√) against the correct answer in the following:

If $y = \cos^{-1}\left(4x^3 - 3x\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{3}{\sqrt{1-x^2}}$

B. $\frac{-3}{\sqrt{1-x^2}}$

C. $\frac{4}{\sqrt{1-x^2}}$

D. $\frac{4}{\left(3x^2 - 1\right)}$

Answer:
Given that $y=\cos^{-1}(4x^3-3x)$

Let $x=\cos\theta$

$\Rightarrow \theta=\cos^{-1}x$

Then, $y=\cos^{-1}(4\cos^3\theta-3\cos\theta)$

Using $\cos3\theta=4\cos^3\theta-3\cos\theta$, we get

$y=\cos^{-1}(\cos3\theta)=3=3\cos^{-1}x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{-3}{\sqrt{1-x^2}}$$

Question 32.

Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\left(\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{(1+x)}$

B. $\frac{1}{\sqrt{x}(1+x)}$

C. $\frac{2}{\sqrt{x}(1+x)}$

D. $\frac{1}{2\sqrt{x}(1+x)}$

Answer:

Given that $y = \tan^{-1}\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}$

Let $\sqrt{a} = \tan A$ and $\sqrt{x} = \tan B$, then $A = \tan^{-1}\sqrt{a}$ and $B = \tan^{-1}\sqrt{x}$

Hence, $y = \tan^{-1}\frac{\tan A+\tan B}{1-\tan A\tan B}$

Using $\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$, we get

$y=\tan^{-1}\tan(A+B)=A+B$

$= \tan^{-1}\sqrt{a} + \tan^{-1}\sqrt{x}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = 0 + \frac{1}{1+(\sqrt{x})^2} \times \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}(1+x)}$$

Question 33.

Mark (√) against the correct answer in the following:

If $y = \cos^{-1}\left(\frac{x^2 - 1}{x^2 + 1}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{2}{(1+x^2)}$

B. $\frac{-2}{(1+x^2)}$

C. $\frac{2x}{(1+x^2)}$

D. none of these

Answer:

Given that $y = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

$\Rightarrow \cos y = \frac{x^2-1}{x^2+1}$ or $\sec y = \frac{x^2+1}{x^2-1}$

Since $\tan^2 x = \sec^2 x - 1$, therefore

$$\tan^2 y = \left(\frac{x^2 + 1}{x^2 - 1}\right)^2 - 1$$

$$= \frac{4x^2}{(x^2 - 1)^2}$$

Hence, $\tan y = -\frac{2x}{1-x^2}$ or $y = \tan^{-1}\left(-\frac{2x}{1-x^2}\right)$

Let $x = \tan\theta$

$\Rightarrow \theta = \tan^{-1} x$

Hence, $y = \tan^{-1}\left(-\frac{2\tan\theta}{1-\tan^2\theta}\right)$

Using $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, we get

$$y = \tan^{-1}(-\tan 2\theta)$$

Using -tan x = tan(-x), we get

$$y = \tan^{-1}(\tan(-2\theta))$$

=-2θ

=-2 $\tan^{-1} x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{-2}{1+x^2}$$

Question 34.
Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\left(\frac{1+x^2}{1-x^2}\right)$ then $\frac{dy}{dx}$ =?

A. $\frac{2x}{\left(1+x^4\right)}$

B. $\frac{-2x}{\left(1+x^4\right)}$

C. $\frac{x}{\left(1+x^4\right)}$

D. none of these

Answer:
Given that $y = \tan^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

Let $x^2=\tan\theta$

$\Rightarrow \theta=\tan^{-1}x^2$

Hence, $y=\tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$

Using $\tan\left(\frac{\pi}{4}+x\right)=\frac{1+\tan x}{1-\tan x}$, we get

$$y=\tan^{-1}\tan\left(\frac{\pi}{4}+\theta\right)=\frac{\pi}{4}+\theta=\frac{\pi}{4}+\tan^{-1}(x^2)$$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{1}{1+x^4}\times 2x=\frac{2x}{1+x^4}$$

Question 35.
Mark (√) against the correct answer in the following:

If $y=\tan^{-1}\left(-\sqrt{x}\right)$

A. $\dfrac{-1}{(1+x)}$

B. $\dfrac{2}{\sqrt{(1+x)}}$

C. $\dfrac{-1}{2\sqrt{x}(1+x)}$

D. none of these

Answer:
Given that $y=\tan^{-1}(-\sqrt{x})$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{1}{1+\left(-\sqrt{x}\right)^2}\times\frac{-1}{2\sqrt{x}}=\frac{-1}{2\sqrt{x}(1+x)}$$

Question 36.
Mark (√) against the correct answer in the following:

If $y=\cos^{-1}x^3$ then $\frac{dy}{dx}=?$

A. $\frac{-1}{\sqrt{1-x^6}}$

B. $\frac{-3x^2}{\sqrt{1-x^6}}$

C. $\frac{-3}{x^2\sqrt{1-x^6}}$

D. none of these

Answer:
Given that $y=\cos^{-1}x^3$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{-1}{\sqrt{1-(x^3)^2}}\times 3x^2=\frac{-3x^2}{\sqrt{1-x^6}}$$

Question 37.
Mark (√) against the correct answer in the following:

If $y=\tan^{-1}\left(\sec x+\tan x\right)$ then $\frac{dy}{dx}=?$

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. 1

D. none of these

Answer:
Given that y=tan^{-1}(sec x + tan x)

Hence, $y = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

Using $\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$, $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$ and $\cos^2\theta + \sin^2\theta = 1$

Hence, $y = \tan^{-1}\left(\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right) = \tan^{-1}\left(\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right)$

Dividing by $\cos\frac{x}{2}$ in numerator and denominator, we get

$$y = \tan^{-1}\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$$

Using $\tan\left(\frac{\pi}{4}+x\right) = \frac{1+\tan x}{1-\tan x}$, we get

$$y = \tan^{-1}\tan\left(\frac{\pi}{4}+\frac{x}{2}\right) = \frac{\pi}{4}+\frac{x}{2}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{2}$$

Question 38.
Mark (√) against the correct answer in the following:

If $y = \cot^{-1}\left(\frac{1-x}{1+x}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{-1}{\left(1+x^2\right)}$

B. $\frac{1}{\left(1+x^2\right)}$

C. $\frac{-1}{\left(1+x^2\right)^{3/2}}$

D. none of these

Answer:

Given that $y = \cot^{-1}\left(\frac{1-x}{1+x}\right)$

Let x=tanθ ⇒ θ=tan^{-1}x and using $\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$

Hence, $y = \frac{\pi}{2} - \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$

Using $\tan\left(\frac{\pi}{4} - x\right) = \frac{1-\tan x}{1+\tan x}$, we get

$$y = \frac{\pi}{2} - \tan^{-1}\tan\left(\frac{\pi}{4} - \theta\right) = \frac{\pi}{2} - \left(\frac{\pi}{4} - \theta\right) = \frac{\pi}{4} + \theta = \frac{\pi}{4} + \tan^{-1}x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

Question 39.

Mark (√) against the correct answer in the following:

If $y = \sqrt{\frac{1+x}{1-x}}$ then $\frac{dy}{dx} = ?$

A. $\frac{2}{(1-x)^2}$

B. $\dfrac{x}{(1-x)^{3/2}}$

C. $\dfrac{1}{(1-x)^{3/2}\cdot(1+x)^{1/2}}$

D. none of these

Answer:

Given that $y=\sqrt{\frac{1+x}{1-x}}$

Let x=-cosθ ⇒ θ=cos^{-1}(-x).

Using $1-\cos\theta=2\sin^2\frac{\theta}{2}$ and $1+\cos\theta=2\cos^2\frac{\theta}{2}$, we get

$$y=\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}=\tan\left(\frac{\theta}{2}\right)$$

Differentiating with respect to x, we get

$\frac{dy}{dx}=\sec^2\left(\frac{\theta}{2}\right)\times\frac{1}{2}\frac{d\theta}{dx}$ −(1)

Since, x=-cosθ ⇒ $2\cos^2\frac{\theta}{2}=1+\cos\theta=1-x$ or $\sec^2\left(\frac{\theta}{2}\right)=\frac{2}{1-x}$ -(2)

Also, since θ=cos^{-1}(-x), therefore $\frac{d\theta}{dx}=\frac{1}{\sqrt{1-x^2}}$ −(3)e

Substituting (2) and (3) in (1), we get

$$\frac{dy}{dx}=\frac{2}{1-x}\times\frac{1}{2}\times\frac{1}{\sqrt{1-x^2}}=\frac{1}{(1-x)\sqrt{1-x^2}}=\frac{1}{(1-x)^{\frac{3}{2}}(1+x)^{\frac{1}{2}}}$$

Question 40.

Mark (√) against the correct answer in the following:

If $y = \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{-2}{\left(1+x^2\right)}$

B. $\frac{2}{\left(1+x^2\right)}$

C. $\frac{-1}{\left(1+x^2\right)}$

D. none of these

Answer:

Given that $y = \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$

$$\Rightarrow \sec y = \frac{x^2+1}{x^2-1}$$

Since $\tan^2 x = \sec^2 x - 1$, therefore

$$\tan^2 y = \left(\frac{x^2+1}{x^2-1}\right)^2 - 1 = \frac{4x^2}{(x^2-1)^2}$$

Hence, $\tan y = -\frac{2x}{1-x^2}$ or $y = \tan^{-1}\left(-\frac{2x}{1-x^2}\right)$

Let $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

Hence, $y = \tan^{-1}\left(-\frac{2\tan\theta}{1-\tan^2\theta}\right)$

Using $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, we get

$$y = \tan^{-1}(-\tan 2\theta)$$

Using -tan x=tan(-x), we get

$$y = \tan^{-1}(\tan(-2\theta)) = -2\theta = -2\tan^{-1}x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{-2}{1+x^2}$$

Question 41.
Mark (√) against the correct answer in the following:

If $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$ then $\frac{dy}{dx}$ =?

A. $\frac{-2}{(1+x^2)}$

B. $\frac{-2}{(1-x^2)}$

C. $\frac{-2}{\sqrt{1+x^2}}$

D. none of these

Answer:
$\Rightarrow\ y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$

$\Rightarrow\ \sec y\ = \frac{1}{2x^2-1}$

$\Rightarrow$ cos y=2x^2 -1

$\Rightarrow$ y=$\cos^{-1}$ (2x^2 -1)

Put x = cos θ

$\Rightarrow$ y = $\cos^{-1}$(2 $\cos^2 \theta$ – 1)

$\Rightarrow$ y = $\cos^{-1}$(cos 2 θ)

$\Rightarrow$ y = 2θ

But θ = $\cos^{-1}$x.

$$\Rightarrow \frac{dy}{dx} = \frac{d(\cos^{-1}x)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 2.\frac{d(\cos^{-1}x)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 2.\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$$

Question 42.
Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{(1+x^2)}$

B. $\frac{2}{(1+x^2)}$

C. $\frac{1}{2(1+x^2)}$

D. none of these

Answer:

Put x = tan θ

$$\Rightarrow\ y = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$\Rightarrow\ y = \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$\Rightarrow\ y = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$\Rightarrow\ y = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}\right)$$

$$\Rightarrow\ y = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$\Rightarrow\ y = \frac{\theta}{2}$$

$\theta = \tan^{-1}x$

$$\Rightarrow\ y = \frac{\tan^{-1}x}{2}$$

$$\Rightarrow\ \frac{dy}{dx} = \frac{1}{2(1+x^2)}$$

Question 43.
Mark (√) against the correct answer in the following:

If $y = \sin^{-1}\left\{\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right\}$ then $\frac{dy}{dx} = ?$

A. $\frac{-1}{2\sqrt{1-x^2}}$

B. $\frac{1}{2\sqrt{1-x^2}}$

C. $\dfrac{1}{2\sqrt{1+x^2}}$

D. none of these

Answer:
Put x = cos2θ

$$\Rightarrow\ y=\sin^{-1}\left(\frac{\sqrt{1+\cos 2\theta}}{2}+\frac{\sqrt{1-\cos 2\theta}}{2}\right)$$

$$\Rightarrow\ y=\sin^{-1}\left(\frac{\sqrt{2\cos^2 2\theta}}{2}+\frac{\sqrt{2.\sin^2\theta}}{2}\right)$$

$$\Rightarrow\ y=\sin^{-1}\left(\frac{\cos 2\theta}{\sqrt{2}}+\frac{\sin 2\theta}{\sqrt{2}}\right)$$

$$\Rightarrow\ y=\sin^{-1}(\sin\left(\frac{\pi}{4}+2\theta\right)$$

$$\Rightarrow\ y=\frac{\pi}{4}+2\theta.$$

$$\Rightarrow\ \frac{dy}{d\theta}=2$$

Put $\theta=\frac{\cos^{-1}x}{2}$

$$\Rightarrow\ \frac{d\theta}{dx}=\frac{-1}{4\sqrt{1-x^2}}$$

$$\therefore\ \frac{dy}{dx}=\frac{-1}{2\sqrt{1-x^2}}$$

Question 44.

Mark (√) against the correct answer in the following:

If $x = at^2, y = 2at$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{t}$

B. $\frac{-1}{t^2}$

C. $\frac{-2}{t}$

D. none of these

Answer:
$x = at^2$

$\therefore \frac{dx}{dt} = 2at$

$\therefore \frac{dt}{dx} = \frac{1}{2at}$

Y = 2at

$\therefore \frac{dy}{dt} = 2a$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = 2a \times \frac{1}{2at}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{t}$

Question 45.

Mark (√) against the correct answer in the following:

If $x = a\sec\theta, y = b\tan\theta$ then $\frac{dy}{dx} = ?$

A. $\frac{b}{a}\sec\theta$

B. $\frac{b}{a}\operatorname{cosec}\theta$

C. $\frac{b}{a}\cot\theta$

D. none of these

Answer:
x = a sec θ

$$\therefore \frac{dx}{d\theta} = a\sec\theta . \tan\theta$$

$$\therefore \frac{d\theta}{dx} = \frac{1}{a\sec\theta . \tan\theta}$$

y = b tan θ

$$\therefore \frac{dy}{d\theta} = b.\sec^2\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$\Rightarrow \frac{dy}{dx} = b.\sec^2\theta \times \frac{1}{a\sec\theta . \tan\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b\sec\theta}{a\tan\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b.\frac{1}{\cos\theta}}{a.\frac{\sin\theta}{\cos\theta}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b}{a}\csc\theta$$

Question 46.
Mark (√) against the correct answer in the following:

If $x = a\cos^2\theta, y = b\sin^2\theta$ then $\frac{dy}{dx} = ?$

A. $\frac{-a}{b}$

B. $\frac{-a}{b}\cot\theta$

C. $\frac{-b}{a}$

D. none of these

Answer:
x = a.cos^{2}θ

$\therefore \frac{dx}{d\theta} = -2\,a\cos\theta\,.\sin\theta$

$$\Rightarrow \frac{d\theta}{dx} = \frac{-1}{2\,a.\cos\theta\,.\sin\theta}$$

y = b.sin^{2}θ

$\therefore \frac{dy}{d\theta} = 2b\sin\theta\,.\cos\theta$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 2b\sin\theta.\cos\theta \times \frac{-1}{2\,a\cos\theta.\sin\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-b}{a}$$

Question 47.
Mark (√) against the correct answer in the following:

If $x = \theta(\cos\theta + \sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$ then $\frac{dy}{dx} = ?$

A. cot θ

B. tan θ

C. a cot θ

D. a tan θ

Answer:
x = a(cos θ + θ sin θ)

$$\therefore \frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$$

$$\Rightarrow \frac{d\theta}{dx} = \frac{1}{a\theta\cos\theta}$$

y = a(sin θ – θ cos θ)

$$\therefore \frac{dy}{d\theta} = a(\cos\theta - (\cos\theta + \theta(-\sin\theta))$$

$$\Rightarrow \frac{dy}{d\theta} = a\cos\theta - a\cos\theta + \theta a\sin\theta$$

$$\Rightarrow \frac{dy}{d\theta} = a\theta\sin\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$\Rightarrow \frac{dy}{dx} = a\theta\sin\theta \times \frac{1}{a\theta\cos\theta}$$

$$\Rightarrow \frac{dy}{dx} = \tan\theta$$

Question 48.
Mark (√) against the correct answer in the following:

If $y = x^{x^{x\ldots\infty}}$ then $\frac{dy}{dx} = ?$

A. $\frac{y}{x(1-\log x)}$

B. $\frac{y^2}{x(1-\log x)}$

C. $\frac{y}{x(1-y\log x)}$

D. none of these

Answer:
Given:

$\Rightarrow y = x^{x^{x^{x\ldots\infty}}}$

We can write it as

$\Rightarrow y=x^y$

Taking log of both sides we get

log y = y log x

Differentiating

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log x + y.\frac{1}{x}$$

$$\Rightarrow \left(\frac{1}{y} - \log x\right)\frac{dy}{dx} = \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{y}{1 - \log x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1 - \log x)}$$

Question 49.
Mark (√) against the correct answer in the following:

If $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\infty}}}$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{(2y - 1)}$

B. $\frac{1}{(y^2 - 1)}$

C. $\frac{2y}{(y^2 - 1)}$

D. none of these

Answer:
Given:

$$\Rightarrow y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\infty}}}$$

We can write it as

$$\Rightarrow y = \sqrt{x + y}$$

Squaring we get

$\Rightarrow y^2 = x + y$

Differentiating

$$\Rightarrow \ 2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \ \frac{dy}{dx} = \frac{1}{(2y-1)}$$

Question 50.
Mark (√) against the correct answer in the following:

If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots\infty}}}$ then $\frac{dy}{dx} = ?$

A. $\frac{\sin x}{(2y-1)}$

B. $\frac{\cos x}{(y-1)}$

C. $\frac{\cos x}{(2y-1)}$

D. none of these

Answer:
Given:

$$\Rightarrow \ y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots\infty}}}$$

We can write it as

$$\Rightarrow \ y = \sqrt{\sin x + y}$$

Squaring we get

$\Rightarrow y^2 = \sin x + y$

Differentiating

$$\Rightarrow\ 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$$

$$\Rightarrow\ \frac{dy}{dx} = \frac{\cos x}{(2y-1)}$$

Question 51.
Mark (√) against the correct answer in the following:

If $y = e^x + e^{x+\ldots\infty}$ then $\frac{dy}{dx}$ =?

A. $\frac{1}{(1-y)}$

B. $\frac{y}{(1-y)}$

C. $\frac{y}{(y-1)}$

D. none of these

Answer:
We can write it as

$\Rightarrow y = e^{x+y}$

logy = (x + y) log e

Differentiating

$$\Rightarrow\ \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \left(\frac{1}{y} - 1\right)\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = 1\left(\frac{y}{1-y}\right)$$

Question 52.
Mark (√) against the correct answer in the following:

The value of k for which $f(x) = \begin{cases} \frac{\sin 5x}{3x}, \text{if } x \neq 0 \\ k, \text{if } x = 0 \end{cases}$ is continuous at x = 0 is

A. $\frac{1}{3}$

B. 0

C. $\frac{3}{5}$

D. $\frac{5}{3}$

Answer:
Since f(x) is continuous on 0.

$$\Rightarrow \lim_{x\to 0} \frac{\sin 5x}{3x} = f(0)$$

$$\Rightarrow \lim_{x\to 0} \frac{\sin 5x}{3x} \times \frac{5x}{5x} = f(0)$$

$$\Rightarrow \lim_{x\to 0} \frac{\sin 5x}{5x} \times \frac{5x}{3x} = f(0)$$

$$\Rightarrow f(0) = \frac{5}{3}$$

$$\Rightarrow k = \frac{5}{3}$$

Question 53.
Mark (√) against the correct answer in the following:

Let $f(x)=\begin{cases} x\sin\frac{1}{x}, \text{ if } x\neq 0 \\ 0, \text{ when } x=0. \end{cases}$

Then, which of the following is the true statement?

A. $f(x)$ is not defined at x = 0

B. $\lim\limits_{x\to 0} f(x)$ does not exist

C. $f(x)$ is continuous at x = 0

D. $f(x)$ is discontinuous at x = 0

Answer:
Left hand limit =

$$\Rightarrow \lim_{x\to 0^-} f(x)$$

$$\Rightarrow \lim_{h\to 0} f(0-h)$$

$$\Rightarrow \lim_{h\to 0} h.\sin\left(\frac{-1}{h}\right)$$

$$\Rightarrow \lim_{h\to 0} -h.\frac{\sin\left(\frac{-1}{h}\right)}{-\frac{1}{h}}\times\frac{-1}{h} = 1$$

Right hand limit =

$$\Rightarrow \lim_{x\to 0^+} f(x)$$

$$\Rightarrow \lim_{h\to 0} f(0+h)$$

$$\Rightarrow \lim_{h\to 0} h.\sin\left(\frac{1}{h}\right)$$

$$\Rightarrow \lim_{h\to 0} h.\frac{\sin\left(\frac{1}{h}\right)}{\frac{1}{h}} \times \frac{1}{h}$$

= 1

As L.H.L = R.H.L

F(x) is continuous.

Question 54.
Mark (√) against the correct answer in the following:

The value of k for which $f(x) = \begin{cases} \frac{3x+4\tan x}{2}, & \text{when } x \neq 0 \\ k, & \text{when } x = 0 \end{cases}$ is continuous at x = 0, is

A. 7

B. 4

C. 3

D. none of these

Answer:
$\Rightarrow f(x) = \frac{3x+4\tan x}{x}$ is continuous at x = 0.

$$\Rightarrow f(x) = \lim_{x\to 0}\frac{3x+4\tan x}{x}$$

$$\Rightarrow f(x) = \lim_{x\to 0}\frac{3x}{x} + \frac{4\tan x}{x}$$

$$\Rightarrow f(x) = 3 + 4\lim_{x\to 0}\frac{\tan x}{x}$$

⇒ f(x) = 3+4

∴ K = 7.

Question 55.
Mark (√) against the correct answer in the following:

Let $f(x) = x^{3/2}$. Then, $f'(0) = ?$

A. $\frac{3}{2}$

B. $\frac{1}{2}$

C. does not exist

D. none of these

Answer:
$f(x) = x^{3/2}$

$$\Rightarrow f'(x) = \frac{3}{2\sqrt{x}}$$

As $x \to 0$, $f'(x) \to \infty$

∴ f′(x) does not exist.

Question 56.
Mark (√) against the correct answer in the following:

The function $f(x) = |x| \forall x \in R$ is

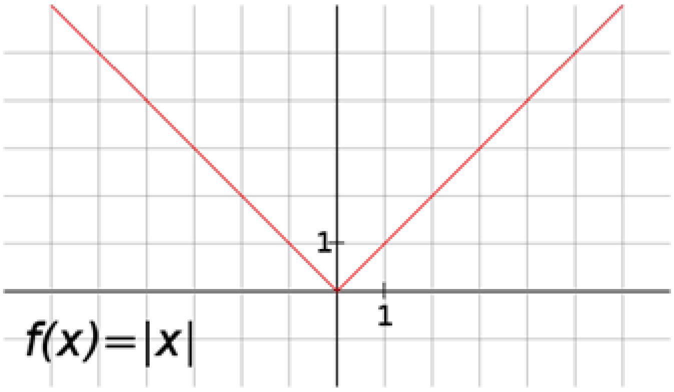
A. continuous but not differentiable at x = 0

B. differentiable but not continuous at x = 0

C. neither continuous nor differentiable at x = 0

D. none of these

Answer:



(Sometimes it's easier to get the answer by graphs)

Now in the above graph

We can see f(x) is Continuous on 0.

But it has sharp curve on x = 0 which implies it is not differentiable.

Question 57.
Mark (√) against the correct answer in the following:

The function $f(x)=\begin{cases}1+x, & \text{when } x \le 2\\ 5-x, & \text{when } x > 2\end{cases}$ is

A. continuous as well as differentiable at x = 2

B. continuous but not differentiable at x = 2

C. differentiable but not continuous at x = 2

D. none of these

Answer:
For continuity left hand limit must be equal to right hand limit and value at the point.

Continuity at x =2.

For continuity at x=2,

L.H.L = $\lim_{x\to 2^-}(1+x) = 3$

R.H.L = $\lim_{x\to 2^+}(5-x) = 3$

f(2) = 1+2 = 3

∴ f(x) is continuous at x = 2

Now for differentiability.

$$\Rightarrow\ f'(2^-) = \lim_{x\to 2^-}\frac{f(x)-f(2)}{x-2}$$

$$\Rightarrow\ f'(2^-) = \lim_{h\to 0}\frac{f(2-h)-f(2)}{2-h-2}$$

$$\Rightarrow\ f'(2^-) = \lim_{h\to 0}\frac{1+2-h-3}{2-h-2} = \lim_{h\to 0}\frac{-h}{-h} = 1.$$

$$\Rightarrow\ f'(2^+) = \lim_{x\to 2^-}\frac{f(x)-f(2)}{x-2}$$

$$\Rightarrow\ f'(2^+) = \lim_{h\to 0}\frac{f(2+h)-f(2)}{2+h-2}$$

$$\Rightarrow\ f'(2^-) = \lim_{h\to 0}\frac{5-(2-h)-3}{2+h-2}$$

$$= \lim_{h\to 0}\frac{h}{-h}$$

=-1

As, $f'(2^-)$ is not equal to $f(2^+)$

∴ f(x) is not differentiable.

Question 58.
Mark (√) against the correct answer in the following:

If $f(x) = \begin{cases} kx+5, \text{ when } x \le 2 \\ x+1, \text{ when } x > 2 \end{cases}$ is continuous at x = 2 then k = ?

A. 2

B. -2

C. 3

D. -3

Answer:
For continuity left hand limit must be equal to right hand limit and value at the point.

Continuous at x =2.

L.H.L = $\lim_{x\to 2^-}(kx+5)$

$\Rightarrow \lim_{h\to 0}(k(2-h)+5)$

⇒ k(2-0)+5 = 2k+5

R.H.L $= \lim_{x\to 2^+}(x+1)$

$\Rightarrow \lim_{h\to 0}(2+h+1)$

⇒ 2+0+1

=3

As f(x) is continuous

∴ 2k+5 = 3

K = -1.

Question 59.
Mark (√) against the correct answer in the following:

If the function $f(x)=\begin{cases}\dfrac{1-\cos 4x}{8x^2}, x\neq 0\\ k, x=0\end{cases}$ is continuous at x = 0 and then k = ?

A. 1

B. 2

C. $\frac{1}{2}$

D. $\frac{-1}{2}$

Answer:
Given:

$\Rightarrow f(x) = \frac{1-\cos 4x}{8x^2}$ is continuous at x = 0.

$\Rightarrow$ 1-cos4x = 2sin^{2}2x

$$\Rightarrow\ f(x) = \lim_{x\to 0} \frac{2\sin^2 2x}{8x^2}$$

$$\Rightarrow\ f(x) = \lim_{x\to 0} \frac{2\sin^2 2x}{2\times 4x^2}$$

$$\Rightarrow\ f(x) = \lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right)^2$$

$\Rightarrow$ f(x) = 1

$\therefore$ Kk= 1

Question 60.
Mark (√) against the correct answer in the following:

If the function $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, \text{ when } x \neq 0 \\ k, \text{ when } x = 0 \end{cases}$ is continuous at x = 0 then k = ?

A. a

B. a^2

C. -2

D. -4

Answer:
F(x) is continuous at x = 0.

$$\Rightarrow f(x) = \lim_{x \to 0} \frac{\sin^2 ax}{x^2}$$

$$\Rightarrow f(x) = \lim_{x \to 0} \frac{\sin^2 ax}{x^2} \times \frac{a^2}{a^2}$$

$$\Rightarrow f(x) = \lim_{x \to 0} \left(\frac{\sin ax}{ax}\right)^2 \times a^2$$

$\Rightarrow f(x) = a^2$

$\therefore k = a^2$

Question 61.
Mark (√) against the correct answer in the following:

If the function $f(x) = \begin{cases} \frac{k\cos x}{(\pi - 2x)}, & \text{when } x \neq \frac{\pi}{2} \\ 3, & \text{when } x = \frac{\pi}{2} \end{cases}$ be continuous at $x = \frac{\pi}{2}$, then the value of k is

A. 3

B. -3

C. -5

D. 6

Answer:
Given: f(x) is continuous at x= π/2.

$$\therefore \text{L.H.L} = \lim_{x \to \frac{\pi}{2}^-} f(x)$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{k \cos x}{\pi - 2x}$$

Putting $x = \frac{\pi}{2} - h;$

As $x \to \frac{\pi}{2}^-$ then $h \to 0.$

$$\therefore \lim_{x \to \frac{\pi}{2}} \frac{k \cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} = k. \lim_{h \to 0} \frac{\sin h}{h}$$

∴ L.H.L = k

As it is continuous which implies right hand limit equals left hand limit equals the value at that point.

∴ k=3.

Question 62.
Mark (√) against the correct answer in the following:

At $x = 2,\ f(x) = |x|$ is

A. continuous but not differentiable

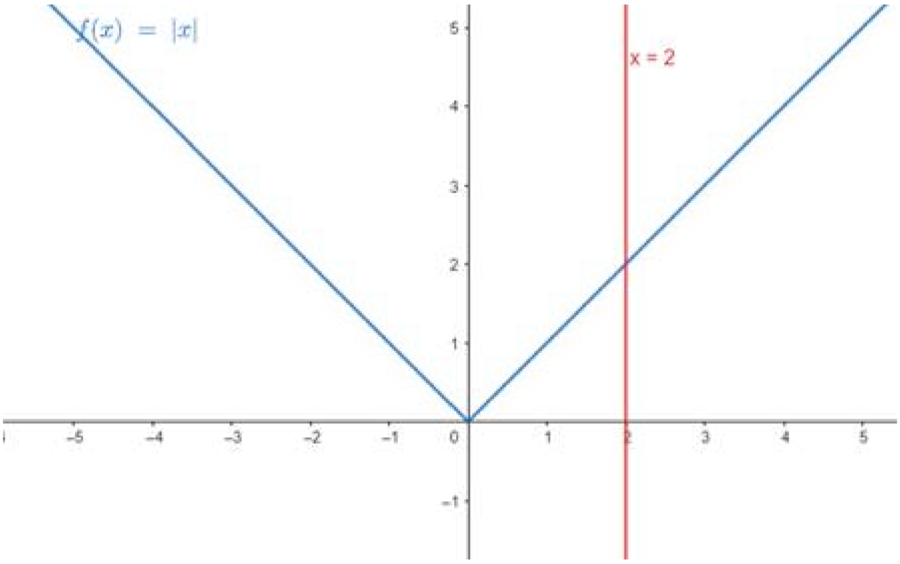
B. differentiable but not continuous

C. continuous as well as differentiable

D. none of these

Answer:
Given:

Let us see that graph of the modulus function.



We can see that f(x) = |x| is neither continuous and nor differentiable at x = 2. Hence, D is the correct answer.

Question 63.
Mark (√) against the correct answer in the following:

Let $f(x) = \begin{cases} \dfrac{x^2 - 2x - 3}{x + 1^2}, \text{ when } x \neq -1 \\ k, \text{ when } x = -1 \end{cases}$

If $f(x)$ is continuous at x = -1 then k = ?

A. 4

B. -4

C. -3

D. 2

Answer:
$\Rightarrow$ $f(x) = \frac{x^2 - 2x - 3}{x + 1}$ is continuous at x = 0.

$$\Rightarrow f(x) = \lim_{x \to -1} \frac{(x+1)(x-3)}{x+1}$$

$$\Rightarrow f(x) = \lim_{x \to -1} x - 3$$

⇒ f(x) = -4

∴ K = 1.

Question 64.
Mark (√) against the correct answer in the following:

The function $f(x) = x^3 + 6x^2 + 15x - 12$ is

A. strictly decreasing on R

B. strictly increasing on R

C. increasing in $(-\infty, 2)$ and decreasing in $(2, \infty)$

D. none of these

Answer:
Given:

$f(x) = x^3+6x^2+15x-12.$

$f'(x) = 3x^2+12x+15$

$f'(x) = 3x^2+12x+12+3$

$f'(x) = 3(x^2+4x+4)+3$

$f'(x) = 3(x+2)^2+3$

As square is a positive number

∴ f′(x) will be always positive for every real number

Hence $f'(x) > 0$ for all x ϵ R

∴ f(x) is strictly increasing.

Question 65.
Mark (√) against the correct answer in the following:

The function $f(x) = 4 - 3x + 3x^2 - x^3$ is

A. decreasing on R

B. increasing on R

C. strictly decreasing on R

D. strictly increasing on R

Answer:
$f(x) = -x^3+3x^2-3x+4.$

$f'(x) = -3x^2+6x-3$

$f'(x) = -3(x^2-2x+1)$

$f'(x) = -3(x-1)^2$

As f′(x) has –ve sign before 3

⇒ f′(x) is decreasing over R.

Question 66.
Mark (√) against the correct answer in the following:

The function $f(x) = 3x + \cos 3x$ is

A. increasing on R

B. decreasing on R

C. strictly increasing on R

D. strictly decreasing on R

Answer:
Given:

$f(x) = 3x+\cos 3x$

$f'(x) = 3-3\sin 3x$

$f'(x) = 3(1-\sin 3x)$

sin3x varies from[-1,1]

when sin3x is 1 f′(x) = 0 and sin3x is -1 f′(x) = 6

As the function is increasing in 0 to 6.

∴ The function is increasing on R.

Question 67.
Mark (√) against the correct answer in the following:

The function $f(x) = x^3 + 6x^2 + 9x + 3$ is decreasing for

A. $1 < x < 3$

B. $x > 1$

C. $x < 1$

D. $x < 1$ or $x > 3$

Answer:
Given:

$f(x) = x^3+6x^2+9x+3.$

$f'(x) = 3x^2+12x+9 = 0$

$f'(x) = 3(x^2+4x+3) =0$

$f'(x) = 3(x+1)(x+3) = 0$

$x = -1$ or $x =-3$

for $x>-1$ f(x) is increasing

for $x<-3$ f(x) is increasing

But for $-1<x<-3$ it is decreasing.

Question 68.
Mark (√) against the correct answer in the following:

The function $f(x) = x^3 - 27x + 8$ is increasing when

A. $|x| < 3$

B. $|x| > 3$

C. -3 < x < 3

D. none of these

Answer:
Given:

$f(x) = x^3 - 27x + 8.$

$f'(x) = 3x^2 - 27x = 0$

$f'(x) = 3(x^2 - 9) = 0$

$f'(x) = 3(x-3)(x+3) = 0$

$x = 3$ or $x = -3$

for $x>3$ $f(x)$ is increasing

for $x<-3$ $f(x)$ is increasing

$\therefore$ for $|x|>3$ $f(x)$ is increasing.

Question 69.
Mark (√) against the correct answer in the following:

$f(x) = \sin x$ is increasing in

A. $\left(\frac{\pi}{2}, \pi\right)$

B. $\left(\pi, \frac{3\pi}{2}\right)$

C. $(0, \pi)$

D. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Answer:
Given: f(x) is sin x

∴ $f'(x) = \cos x$

⇒ $f'(x) = \cos x$

= 0

$$\Rightarrow \text{ for } x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

f′(x) is increasing

$$\therefore \text{ f(x) is increasing in } \left(\frac{-\pi}{2}, \frac{\pi}{2}\right).$$

Question 70.
Mark (√) against the correct answer in the following:

$f(x) = \frac{2x}{\log x}$ is increasing in

A. (0, 1)

B. (1, e)

C. (e, ∞)

D. (-∞, e)

Answer:

$$\Rightarrow f(x) = \frac{2x}{\log x}$$

$$\Rightarrow f'(x) = \frac{2.\log x - 2}{\log^2 x}$$

Put f′(x) = 0

We get

$$\Rightarrow \frac{2.\log x - 2}{\log^2 x} = 0$$

⇒ 2.logx = 2

log x = 1

⇒ x= e

We only have one critical point

So, we can directly say x>e f(x) would be increasing

∴ f(x) will be increasing in (e, ∞)

Question 71.
Mark (√) against the correct answer in the following:

$f(x) = (\sin x - \cos x)$ is decreasing in

A. $\left(0, \frac{3\pi}{4}\right)$

B. $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

C. $\left(\frac{7\pi}{4}, 2\pi\right)$

D. none of these

Answer:
Given:

f(x) = sin x – cos x

$f'(x) = \cos x + \sin x$

Multiply and divide by $\sqrt{2}$.

$$\Rightarrow \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)$$

$$\Rightarrow \sqrt{2}\left(\sin\frac{\pi}{4}.\cos x + \cos\frac{\pi}{4}.\sin x\right)$$

$$\Rightarrow \sqrt{2}(\sin\left(\frac{\pi}{4} + x\right))$$

$$\Rightarrow f'(x) = \sqrt{2}\sin\left(\frac{\pi}{4} + x\right)$$

For f(x) to be decreasing .f′(x)<0

$$\Rightarrow f'(x) = \sqrt{2}\sin\left(\frac{\pi}{4} + x\right) < 0$$

$$\Rightarrow \pi < x + \frac{\pi}{4} < 2\pi$$

($\because$ $\sin\theta < 0$ for $\pi < \theta < 2\pi$)

$$\Rightarrow \pi - \frac{\pi}{4} < x < 2\pi - \frac{\pi}{4}$$

$$\Rightarrow \frac{3\pi}{4} < x < \frac{7\pi}{4}$$

$\therefore$ f(x) decreases in the interval.

$$\Rightarrow \left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$$

Question 72.
Mark (√) against the correct answer in the following:

$f(x) = \dfrac{x}{\sin x}$ is

A. increasing in (0, 1)

B. decreasing in (0, 1)

C. increasing in $\left(0, \frac{1}{2}\right)$ and decreasing in $\left(\frac{1}{2}, 1\right)$

D. none of these

Answer:

$\Rightarrow\ f(x) = \frac{x}{\sin x}$

$\Rightarrow\ f'(x) = \frac{\sin x - x\cos x}{\sin x}$

Now see

In (0,1) sin x is increasing and cos x is decreasing

sin x – x cos x will be increasing

∴ f(x) is increasing in (0,1)

Question 73.
Mark (√) against the correct answer in the following:

$f(x) = x^x$ is decreasing in the interval

A. (0, e)

B. $\left(0, \frac{1}{e}\right)$

C. (0,1)

D. none of these

Answer:
Given: $f(x) = x^x$.

$\Rightarrow f'(x)=(\log x+1)\ x^x$

⇒ keeping f′(x) = 0

We get

⇒ $x = 0$ or $x = \frac{1}{e}$

Now

When x>1/e the function is increasing

x<0 function is increasing.

But in the interval (0,1/e) the function is decreasing.

Question 74.
Mark (√) against the correct answer in the following:

$f(x) = x^2e^{-x}$ is increasing in

A. (-2, 0)

B. (0, 2)

C. (2, ∞)

D. (-∞, ∞)

Answer:
Given $f(x) = x^2.e^{-x}$

⇒ $f'(x) = 2x.\ e^{-x} - x^2\ e^{-x}$

⇒ Put f′(x) = 0

⇒ $-(x^2 - 2x)e^{-x} = 0$

⇒ x = 0 or x =2.

Now as there is a -ve sign before f′(x)

When x>2 the function is decreasing

x<0 function is decreasing

But in the interval (0,2) the function is increasing.

Question 75.
Mark (√) against the correct answer in the following:

$f(x) = \sin x - kx$ is decreasing for all $x \notin R$, when

A. k < 1

B. k ≤ 1

C. k > 1

D. k ≤ 1

Answer:
f(x) =sin x - kx

f′(x) = cos x –k

∴ f decreases, if f′(x)≤ 0

⇒ cos x – k ≤ 0

⇒ cos x≤ k

So, for decreasing k≥ 1.

Question 76.
Mark (√) against the correct answer in the following:

$f(x) = (x+1)^3 (x-3)^3$ is increasing in

A. (-∞, 1)

B. (-1, 3)

C. (3, ∞)

D. (1, ∞)

Answer:
Given:

$\Rightarrow$ $f(x) = (x+1)^3.(x-3)^3$

$\Rightarrow$ $f'(x) = 3(x+1)^2(x-3)^3 + 3(x-3)^3\ (x+1)^3$

Put $f'(x) = 0$

$\Rightarrow$ $3(x+1)^2(x-3)^3 = -3(x-3)^2(x+1)^3$

$\Rightarrow$ $x-3 = -(x+1)$

$\Rightarrow$ $2x = 2$

$\Rightarrow$ $x =1$

When $x>1$ the function is increasing.

$x<1$ function is decreasing.

So, f(x) is increasing in $(1, \infty)$.

Question 77.
Mark (√) against the correct answer in the following:

$f(x) = [x(x-3)]^2$ is increasing in

A. $(0, \infty)$

B. $(-\infty, 0)$

C. $(1, 3)$

D. $\left(0, \frac{3}{2}\right) \cup (3, \infty)$

Answer:
$\Rightarrow$ $f(x)=[x(x-3)]^2$

$\Rightarrow$ f' (x)=2[x(x-3)] =0

$$\Rightarrow x = 3 \text{ and } x = \frac{3}{2}$$

When x> 3/2 the function is increasing

X<3 function is increasing.

$\Rightarrow \left(0, \frac{3}{2}\right) \cup (3, \infty)$ Function is increasing.

Question 78.
Mark (√) against the correct answer in the following:

If $f(x) = kx^3 - 9x^2 + 9x + 3$ is increasing for every real number x, then

A. k > 3

B. k ≥ 3

C. k < 3

D. k ≤ 3

Answer:
Given f(x) = kx^3 -$9x^2$+ 9x+3

$\Rightarrow$ f′(x) = $3kx^2$-18x+9

$\Rightarrow$ f′(x) = 3(kx^2 – 6x + 3)>0

$\Rightarrow$ kx^2 – 6x + 3 >0

For quadratic equation to be greater than 0. a>0 and D<0.

$\Rightarrow$ k>0 and $(-6)^2$- 4(k)(3)<0

$\Rightarrow$ 36 – 12k<0

$\Rightarrow$ 12k>36

⇒ k>3

∴ k>3.

Question 79.
Mark (√) against the correct answer in the following:

$f(x) = \frac{x}{(x^2 - 1)}$ is increasing in

A. (-1, 1)

B. (-1, ∞)

C. $(-\infty, -1) \cup (1, \infty)$

D. none of these

Answer:
$$\Rightarrow\ f(x) = \frac{x}{x^2 + 1}$$

$$\Rightarrow\ f'(x) = \frac{x^2 - 2x^2 + 1}{x^2 + 1}$$

$$\Rightarrow\ f'(x) = -\frac{x^2 - 1}{x^2 + 1}$$

⇒ For critical points f′(x) = 0

When f′(x) = 0

We get x = 1 or x = -1

When we plot them on number line as f′(x) is multiplied by –ve sign we get

For x>1 function is decreasing

For x<-1 function is decreasing

But between -1 to 1 function is increasing.

∴ Function is increasing in(-1,1).

Question 80.
Mark (√) against the correct answer in the following:

The least value of k for which $f(x) = x^2 + kx + 1$ is increasing on (1, 2), is

A. -2

B. -1

C. 1

D. 2

Answer:
f(x) =x^2+kx+1

For increasing

f′(x) = 2x+k

k≥ -2x

thus,

k≥ -2.

Least value of -2.

Question 81.
Mark (√) against the correct answer in the following:

$f(x) = |x|$ has

A. minimum at x = 0

B. maximum x = 0

C. neither a maximum nor a minimum at x = 0

D. none f these

Answer:

f(x) = |x|

Now to check the maxima and minima at x =0.

It can be easily seen through the option.

See |x| is x for x>0 and –x for x<0

That is no matter if you put a number greater than zero or number less than zero you will get positive answer.

∴ for x = 0 we will get minima.

Question 82.
Mark (√) against the correct answer in the following:

When x is positive, the minimum value of x^x is

A. e^e

B. $e^{1/e}$

C. $e^{-1/e}$

D. $\left(1/e\right)$

Answer:
Given: f(x) = x^x.

⇒ f' (x)=(log x+1) x^x

⇒ keeping f′(x) = 0

We get

$$\Rightarrow\ x = 0 \text{ or } x = \frac{1}{e}$$

$$\Rightarrow\ f''(x) = x^x(1 + \log x)\left[1 + \log x + \frac{1}{x(1 + \log x)}\right]$$

When x is greater than zero,

We get a maximum value as the function will be negative.

Therefore,

$F(x) = x^x$

$F(e) = \left(\frac{1}{e}\right)^{1/e} = e^{-\frac{1}{e}}$

Hence, C is the correct answer.

Question 83.
Mark (√) against the correct answer in the following:

The maximum value of $\left(\frac{\log x}{x}\right)$ is

A. $\left(\frac{1}{e}\right)$

B. $\frac{2}{e}$

C. e

D. 1

Answer:
$\Rightarrow f(x) = \frac{\log x}{x}$

$\therefore f'(x) = \frac{\log x - x.\frac{1}{x}}{x^2}$

$\Rightarrow f'(x) = \log x - 1$

$\Rightarrow$ Put $f'(x) = 0$

We get x =e

$F''(x) = 1/x$

Put x =e in $f''(X)$

1/e is point of maxima

∴ The max value is 1/e.

Question 84.
Mark (√) against the correct answer in the following:

$f(x) = \text{cosec } x$ in (-π, 0) has a maxima at

A. x = 0

B. $x = \frac{-\pi}{4}$

C. $x = \frac{-\pi}{3}$

D. $x = \frac{-\pi}{2}$

Answer:
We can go through options for this question

Option a is wrong because 0 is not included in (-π,0)

At x = -π/4 value of f(x) is -√2 = -1.41

At x = -π/3 value of f(x) is -2.

At x = -π/2 value of f(x) is -1.

∴ f(x) has max value at x =-π/2.

Which is -1.

Question 85.

Mark (√) against the correct answer in the following:

If x > 0 and xy = 1, the minimum value of (x + y) is

A. -2

B. 1

C. 2

D. none of these

Answer:
Given: x>0 and xy = 1

We need to find the minimum value of (x + y).

$$\Rightarrow\ y = \frac{1}{x}$$

$$\Rightarrow\ f(x) = x + \frac{1}{x}$$

$$\Rightarrow\ f(x) = \frac{x^2 + 1}{x}$$

$$\Rightarrow\ f'(x) = \frac{x.2x - (x^2 + 1).1}{x^2}$$

$$\Rightarrow\ f'(x) = \frac{x^2 - 1}{x^2}$$

$$\Rightarrow\ f''(x) = \frac{x^2(2x) - (x^2 - 1).2x}{x^4}$$

$$\Rightarrow\ f''(x) = \frac{2x}{x^4}$$

$$\Rightarrow\ f''(x) = \frac{2}{x^3}$$

For maximum or minimum value f′(x) = 0.

$\therefore \frac{x^2 - 1}{x^2} = 0$

∴ x = 1 or x =-1

f''(x) at x = 1.

∴ f''(x) = 2.

F''(x)>0 it is decreasing and has minimum value at x = 1

At x = -1

f''(x) = -2

f''(x)<0 it is increasing and has maximum value at x = -1.

∴ Substituting x = 1 in f(x) we get

f(x) = 2.

∴ The minimum value of given function is 2.

Question 86.
Mark (√) against the correct answer in the following:

The minimum value of $\left(x^2 + \frac{250}{x}\right)$ is

A. 0

B. 25

C. 50

D. 75

Answer:
$\Rightarrow f(x) = x^2 + \frac{250}{x}$

$$\Rightarrow f'(x) = 2x - \frac{250}{x^2} = 0$$

$\Rightarrow 2x^3 = 250$

$\Rightarrow x^3 = 125$

$\Rightarrow x = 5$

Substituting x = 5 in f(x) we get

f(x) = 25+50

f(x) = 75.

Question 87.
Mark (√) against the correct answer in the following:

The minimum value of $f(x) = 3x^4 - 8x^3 - 48x + 25$ on [0, 3] is

A. 16

B. 25

C. -39

D. none of these

Answer:
Given:

$f(x) = 3x^4-8x^3-48x+25.$

$F'(x) = 12x^3-24x^2-48 = 0$

$F'(x) = 12(x^3-2x^2-4) = 0$

Differentiating again, we get,

$F''(x) = 3x^2 – 4x = 0$

$x(3x – 4) = 0$

x = 0 or x = 4/3

Putting the value in equation, we get,

f(x) = -39

Hence, C is the correct answer.

Question 88.
Mark (√) against the correct answer in the following:

The maximum value of $f(x) = (x-2)(x-3)^2$ is

A. $\frac{7}{3}$

B. 3

C. $\frac{4}{27}$

D. 0

Answer:
$f(x) = (x-2)(x-3)^2$

$f(x) = (x-2)(x^2-6x+9)$

$f(x) = x^3-8x^2+21x-18.$

$f'(x) = 3x^2-16x+21$

$f''(x) = 6x-16$

For maximum or minimum value f′(x) = 0.

$\therefore 3x^2-9x-7x+21 = 0$

$\Rightarrow 3x(x-3)-7(x-3)=0$

⇒ x = 3 or x =7/3.

f′′(x) at x = 3.

∴ f′′(x) = 2

f′′(x)>0 it is decreasing and has minimum value at x = 3

At x = 7/3

F′′(x) = -2

F′′(x)<0 it is increasing and has maximum value at x = 7/3.

Substituting x = 7/3 in f(x) we get

$\Rightarrow \left(\frac{7}{3}-2\right)\left(\frac{7}{3}-3\right)^2$

$\Rightarrow \left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)^2$

$\Rightarrow \frac{4}{27}$

Question 89.
Mark (√) against the correct answer in the following:

The least value of $f(x) = (e^x + e^{-x})$ is

A. -2

B. 0

C. 2

D. none of these

Answer:
f(x) = $e^x + e^{-x}$

$\Rightarrow\ f(x) = e^x + \frac{1}{e^x}$

$$\Rightarrow f(x) = \frac{e^{2x} + 1}{e^x}$$

f(x) is always increasing at x = 0it has the least value

$$\Rightarrow f(x) = \frac{1 + 1}{1} = 2$$

∴ The least value is 2.