Objective Questions

Question 1.
Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} = e^{x+y}$ is

A. $e^x + e^y = C$

B. $e^x - e^{-y} = C$

C. $e^x + e^{-y} = C$

D. None of these

Answer:
Given, $\frac{dy}{dx} = e^{x+y}$

$$\frac{dy}{dx} = e^x e^y$$

$$e^{-y} dy = e^x dx$$

On integrating on both sides, we get

$$-e^{-y} + c_1 = e^x + c_2$$

$$e^{-y} + e^x = c$$

Conclusion: Therefore, $e^{-y} + e^x = c$ is the solution of $\frac{dy}{dx} = e^{x+y}$

Question 2.
Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} = 2^{x+y}$ is

A. $2^x + 2^y = C$

B. $2^x + 2^{-y} = C$

C. $2^x - 2^{-y} = C$

D. None of these

Answer:
Given, $\frac{dy}{dx} = 2^{x+y}$

$$\frac{dy}{dx} = 2^x 2^y$$

$$2^{-y} dy = 2^x dx$$

On integrating on both sides, we get

$$-\frac{2^{-y}}{\log 2} + c_2 = \frac{2^x}{\log 2} + c_2$$

$$2^x + 2^{-y} = c_3 \log 2$$

$$2^x + 2^{-y} = c$$

Conclusion: Therefore, $2^x + 2^{-y} = c$ is the solution of $\frac{dy}{dx} = 2^{x+y}$

Question 3.
Mark (√) against the correct answer in the following:

The solution of the $\mathrm{DE}(e^x + 1)y\,dy = (y + 1)e^x dx$ is

A. $e^y = C(e^x + 1)(y + 1)$

B. $e^y = e^x + y + 1$

C. $y = (e^x + 1)(y + 1)$

D. None of these

Answer:
$(e^x + 1)y\,dy = (y + 1)e^x dx$

$$\frac{y\,dy}{y+1} = \frac{e^x\,dx}{(e^x + 1)}$$

Let, $e^x + 1 = t$

On differentiating on both sides we get $e^x dx = dt$

Now we can write this equation as $\frac{y\,dy}{y+1} = \frac{e^x\,dx}{(e^x+1)}$

$$\frac{((y+1) - 1)\,dy}{y+1} = \frac{e^x\,dx}{(e^x + 1)}$$

$$\left(1 - \frac{1}{y+1}\right) dy = \frac{e^x\,dx}{(e^x + 1)}$$

$$\left(1 - \frac{1}{y+1}\right) dy = \frac{dt}{t}$$

On integrating on both sides, we get

$$y - \log(y+1) = \log(e^x + 1) + \log c$$

$$y = \log(y+1) + \log(e^x + 1) + \log c$$

$$y = \log(y+1)\,(e^x + 1)c$$

$$e^y = c(y+1)(e^x + 1)$$

Conclusion: Therefore, $e^y = c(y+1)(e^x + 1)$ is the solution of $(e^x + 1)y\,dy = (y+1)e^x dx$

Question 4.
Mark (√) against the correct answer in the following:

The solution of the $\mathrm{DE} x dy + y dx = 0$ is

A. $x + y = C$

B. $xy = C$

C. $\log(x + y) = C$

D. None of these

Answer:
Given xdy + ydx = 0

xdy = -ydx

$$-\frac{dy}{y} = \frac{dx}{x}$$

On integrating on both sides we get,

-log y = log x + c

log x + log y = c

log xy = c

xy = C

Conclusion: Therefore xy = c is the solution of xdy + ydx = 0

Question 5.
Mark (√) against the correct answer in the following:

The solution of the $x\frac{dy}{dx} = \cot y$ is

A. $x \cos y = C$

B. $x \tan y = C$

C. $x \sec y = C$

D. None of these

Answer:

Given: $x\frac{dy}{dx} = \cot y$

Separating the variables, we get,

$$\frac{dy}{\cot y} = \frac{dx}{x}$$

$$\tan y\, dy = \frac{dx}{x}$$

Integrating both sides, we get,

$$\int \tan y\, dy = \int \frac{dx}{x}$$

log sec y = log x + log c

xcosy = c

Hence, A is the correct answer.

Question 6.
Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} = \frac{(1+y^2)}{(1+x^2)}$ is.

A. ($\mathscr{Y}$ +$\mathscr{X}$)=C(1-$\mathscr{Y}\mathscr{X}$)

B. ($\mathscr{Y}$ − $\mathscr{X}$) = C(1+$\mathscr{Y}\mathscr{X}$)

C. $\mathscr{Y}$ = (1+$\mathscr{X}$)C

D. None of these

Answer:
Given $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

On integrating on both sides, we get

$$\tan^{-1} y = \tan^{-1} x + c$$

$$\tan^{-1} y - \tan^{-1} x = c$$

$\frac{y-x}{1+yx} = c$ (since $\tan^{-1} y - \tan^{-1} x = \frac{y-x}{1+yx}$)

y-x = C(1+yx)

Conclusion: Therefore, y-x = C(1+yx) is the solution of $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

Question 7.
Mark (√) against the correct answer in the following:

The solution of the $\mathrm{DE}\,\frac{dy}{dx}$ =1 - $\mathscr{X}$ + $\mathscr{Y}$ – $\mathscr{X}\mathscr{Y}$ is

A. Log (1 + $\mathscr{Y}$) = $\mathscr{X}$ - $\frac{x^2}{2}$ +C

B. $e^{(1+y)} = x - \frac{x^2}{2} + C$

C. $e^{y} = x - \frac{x^2}{2} + C$

D. None of these

Answer:
$$\frac{dy}{dx} = 1 - x + y - xy$$

$$\frac{dy}{dx} = 1 - x + y(1 - x)$$

$$\frac{dy}{dx} = (1 + y)(1 - x)$$

$$\frac{dy}{1+y} = (1-x)dx$$

On integrating on both sides, we get

$$\log(1+y) = x - \frac{x^2}{2} + c$$

Conclusion: Therefore, $\log(1+y) = x - \frac{x^2}{2} + c$ is the

solution of $\frac{dy}{dx} = 1 - x + y - xy$

Question 8.
Mark (√) against the correct answer in the following:

The solution of the $\mathrm{DE}\,\frac{dy}{dx} = e^{x+y} + x^2.e^y$ is

A. $e^{x-y} + \frac{x^3}{3} + C$

B. $e^x + e^{-y} + \frac{x^3}{3} + C'$

C. $e^x - e^{-y} + \frac{x^3}{3} + C$

D. None of these

Answer:
Given $\frac{dy}{dx} = e^{x+y} + x^2 e^y$

$$(e^{-y})dy = (e^x + x^2)dx$$

On integrating on both sides, we get

$$-e^{-y} = e^x + \frac{x^3}{3} + C$$

$$e^{-y} + e^{x} + \frac{x^3}{3} = C$$

Conclusion: Therefore, $e^{-y} + e^{x} + \frac{x^3}{3} = C$ is the

solution of $\frac{dy}{dx} = e^{x+y} + x^2 e^{y}$

Question 9.
Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is

A. $y + \sin^{-1} y = \sin^{-1} x + C$

B. $\sin^{-1} y - \sin^{-1} x = C$

C. $\sin^{-1} y + \sin^{-1} x = C$

D. None of these

Answer:
Given $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

$$-\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$$

On integrating on both sides, we get

$-\sin^{-1} y = \sin^{-1} x + C$ (As $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$)

$$\sin^{-1} y + \sin^{-1} x = C$$

Conclusion: Therefore, $\sin^{-1} y + \sin^{-1} x = C$ is the

solution of $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

Question 10.
Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$ is

A. $y = 2\tan\frac{x}{2} - x + C$

B. $y = \tan\frac{x}{2} - 2x + C$

C. $y = \tan x - x + C$

D. None of these

Answer:
Given $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

$$\frac{dy}{dx} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$\frac{dy}{dx} = \tan^2\frac{x}{2}$$

$$dy = dx(\tan^2\frac{x}{2})$$

On integrating on both sides, we get

$$y = 2\tan\frac{x}{2} - x + C$$

Conclusion: Therefore, $y = 2\tan\frac{x}{2} - x + C$ is the solution

of $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

Question 11.

Mark (√) against the correct answer in the following:

The solution of the $\mathrm{DE}\, \frac{dy}{dx} = \frac{-2xy}{(x^2+1)}$ is

A. $\mathscr{Y}^2\ (\mathscr{X} + 1) = C$

B. $\mathscr{Y}\ (\mathscr{X}^2 + 1) = C$

C. $\mathscr{X}^2\ (\mathscr{Y} + 1) = C$

D. None of these

Answer:

Given $\frac{dy}{dx} = \frac{-2xy}{(x^2+1)}$

$$\frac{dy}{y} = \frac{-2xdx}{(x^2+1)}$$

Let $x^2 + 1 = t$

On differentiating on both sides we get 2xdx = dt

$$\frac{dy}{y} = \frac{-dt}{t}$$

On integrating on both sides, we get

$$\log y = -\log t + C$$

$$\log y + \log t = C$$

$$\log yt = C$$

yt = C

As $t = x^2 + 1$

$$y(x^2+1) = C$$

Conclusion: Therefore, $y(x^2+1) = C$ is the solution of $\frac{dy}{dx} = \frac{-2xy}{(x^2+1)}$

Question 12.
Mark (√) against the correct answer in the following:

The solution of the DE cos $\mathscr{X}$ (1 + cos $\mathscr{Y}$) $d\mathscr{X}$ – sin $\mathscr{Y}$ (1 + sin $\mathscr{X}$) $d\mathscr{Y}$ = 0 is

A. 1 + sin $\mathscr{X}$ cos $\mathscr{Y}$ = C

B. (1 + sin $\mathscr{X}$) (1 + cos $\mathscr{Y}$) = C

C. sin $\mathscr{X}$ cos $\mathscr{Y}$ + cos $\mathscr{X}$ = C

D. none of these

Answer:
Given cos x (1+cos y) dx – sin y (1+sin x) dy = 0

Let 1+cos y = t and 1+sin x = u

On differentiating both equations, we get

-sin y dy = dt and cos x dx = du

Substitute this in the first equation

t du + u dt = 0

$$-\frac{du}{u} = \frac{dt}{t}$$

-log u = log t + C

log u + log t = C

log ut = C

ut = C

(1+sin x)(1+cos y) = C

Conclusion: Therefore, (1+sin x)(1+cos y) = C is the solution of cos x (1+cos y) dx – sin y (1+sin x) dy = 0

Question 13.
Mark (√) against the correct answer in the following:

the solution of the DE $\mathscr{X}$ cos $\mathscr{Y}$ $d\mathscr{Y}$ = ($\mathscr{X}$e$^{\mathscr{X}}$ log $\mathscr{X}$ + e$^{\mathscr{X}}$) $d\mathscr{X}$ is

A. sin $\mathscr{Y}$ = e$^{\mathscr{X}}$ log $\mathscr{X}$ +C

B. sin $\mathscr{Y}$ - e$^{\mathscr{X}}$ log $\mathscr{X}$ = C

C. sin $\mathscr{Y}$ = e$^{\mathscr{X}}$ (log $\mathscr{X}$) +C

D. none of these

Answer:
Given $x\cos y\ dy = (xe^{x}\log x +\ e^{x})dx$

$$\cos y\ dy = \frac{(xe^{x}\log x +\ e^{x})}{x}dx$$

On integrating on both sides we get

$$\sin y = \log x\int e^{x}dx - \int \frac{1}{x}\left(\int e^{x}\right)dx + \int \frac{e^{x}}{x}dx$$

$$\sin y = \log x\,(e^{x}) - \int \frac{e^{x}}{x}dx + \int \frac{e^{x}}{x}dx + C$$

$$\sin y =\ e^{x}\log x + C$$

Conclusion: Therefore, $\sin y =\ e^{x}\log x + C$ the solution of

$$x\cos y\ dy = (xe^{x}\log x +\ e^{x})dx$$

Question 14.
Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx}$+ $\mathscr{Y}$ log $\mathscr{Y}$ cot $\mathscr{X}$ = 0 is

A. cos $\mathscr{X}$ log $\mathscr{Y}$ = C

B. sin $\mathscr{X}$ log $\mathscr{Y}$ = C

C. log $\mathscr{Y}$ = C sin $\mathscr{X}$

D. none of these

Answer:
Given $\frac{dy}{dx} + y \log y \cot x = 0$

$$\frac{dy}{y \log y} = -\cot x \ dx$$

Let log y = t

On differentiating we get

$$\frac{1}{y} \ dy = dt$$

$$\frac{dt}{t} = -\cot x \ dx$$

log t = -log (sin x) + C

log t + log(sin x) = C

log(tsin x) = C

tsin x = C

(log y)(sin x) = C

Conclusion: Therefore, (log y)(sin x) = C is the solution of $\frac{dy}{dx} + y \log y \cot x = 0$

Question 15.
Mark (√) against the correct answer in the following:

the general solution of the DE (1 + $\mathscr{X}^2$) $d\mathscr{Y}$ – $\mathscr{X}\mathscr{Y}$ $d\mathscr{X}$ = 0 is

A. $y = C(1 + x^2)$

B. $y^2 = C(1 + x^2)$

C. $y\sqrt{1+x^2} = C$

D. None of these

Answer:
Given $(1 + x^2)dy - xy\,dx = 0$

$$\frac{dy}{y} = \frac{x}{1+x^2}dx$$

Let $1 + x^2 = t$

2x dx = dt

$$\frac{dy}{y} = \frac{dt}{2t}$$

On integrating on both sides we get

$$\log y = \frac{\log t}{2} + C$$

2 log y = log t + C

$$\log y^2 = \log t + C$$

$$y^2 = (1 + x^2)c$$

Conclusion: Therefore, $y^2 = (1 + x^2)c$ is the solution of

$$(1 + x^2)dy - xy\,dx = 0$$

Question 16.

Mark (√) against the correct answer in the following:

The general solution of the DE $x\sqrt{1+y^2}\,dx + y\sqrt{1+x^2}\,dy = 0$ is

A. $\sin^{-1}x + \sin^{-1}y = C$

B. $\sqrt{1+x^2} + \sqrt{1+y^2} = C$

C. $\tan^{-1}x + \tan^{-1}y = C$

D. None of these

Answer:

Given $x\sqrt{1+y^2}\,dx + y\sqrt{1+x^2}\,dy = 0$

$$\frac{ydy}{\sqrt{1+y^2}} = -\frac{xdx}{\sqrt{1+x^2}}$$

Let $1+y^2 = t$ and $1+x^2 = u$

2y dy = dt and 2x dx = du

$$\frac{dt}{\sqrt{t}} = -\frac{du}{\sqrt{u}}$$

On integrating on both sides we get

$$\sqrt{t} = -\sqrt{u} + C$$

$$\sqrt{1+y^2} + \sqrt{1+x^2} = C$$

Conclusion: Therefore, $\sqrt{1+y^2} + \sqrt{1+x^2} = C$ is the

solution of $x\sqrt{1+y^2}\,dx + y\sqrt{1+x^2}\,dy = 0$

Question 17.

Mark (√) against the correct answer in the following:

The general solution of the DE $\log\left(\frac{dy}{dx}\right) = (ax + by)$ is

A. $\frac{-e^{-by}}{b} = \frac{e^{ax}}{a} + C$

B. $e^{ax} - e^{-by} = C$

C. $be^{ax} + ae^{by} = C$

D. None of these

Answer:
Given $\log(\frac{dy}{dx}) = (ax + by)$

$$\frac{dy}{dx} = e^{ax+by}$$

$$\frac{dy}{e^{by}} = e^{ax}dx$$

On integrating on both sides we get

$$-\frac{e^{-by}}{b} = \frac{e^{ax}}{a} + C$$

Conclusion: Therefore, $-\frac{e^{-by}}{b} = \frac{e^{ax}}{a} + C$ is the solution of

$$\log(\frac{dy}{dx}) = (ax + by)$$

Question 18.
Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} = \left(\sqrt{1-x^2}\right)\left(\sqrt{1-y^2}\right)$ is

A. $\sin^{-1} y - \sin^{-1} x = x\sqrt{1-x^2} + C$

B. $2\sin^{-1}y - \sin^{-1}x = x\sqrt{1-x^2} + C$

C. $2\sin^{-1}y - \sin^{-1}x = C$

D. None of these

Answer:

Given $\frac{dy}{dx} = (\sqrt{1-x^2})(\sqrt{1-y^2})$

$$\frac{dy}{\sqrt{1-y^2}} = \sqrt{1-x^2}\,dx$$

Let x = sin t

dx = cos t dt

We know $\cos t = \sqrt{1-x^2}$

On integrating on both sides we get

$$\sin^{-1}y = \frac{t}{2} + \frac{\sin 2t}{4}$$

Sin 2t = 2 sin t cost

= $2x\sqrt{1-x^2}$

$$\sin^{-1}y = \frac{\sin^{-1}x}{2} + \frac{x\sqrt{1-x^2}}{2} + C$$

$$2\sin^{-1}y - \sin^{-1}x = x\sqrt{1-x^2} + C$$

Conclusion: Therefore, $2\sin^{-1}y - \sin^{-1}x = x\sqrt{1-x^2} + C$ is the solution of $\frac{dy}{dx} = (\sqrt{1-x^2})(\sqrt{1-y^2})$

Question 19.

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ is

A. $x^2 - y^2 = C_1 x$

B. $x^2 + y^2 = C_1 y$

C. $x^2 + y^2 = C_1 x$

D. None of these

Answer:

Given $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\frac{x^2v^2 - x^2}{2vx^2} = v + x\frac{dv}{dx}$$

$$\frac{v^2 - 1}{2v} - v = x\frac{dv}{dx}$$

$$\frac{-v^2 - 1}{2v} = x\frac{dv}{dx}$$

$$\frac{dx}{x} + \frac{2vdv}{v^2 + 1} = 0$$

On integrating on both sides, we get

$$\log x + \log(v^2 + 1) = c$$

$$\log(x(v^2 + 1)) = c$$

$$x\left(\frac{y^2}{x^2}+1\right)=C$$

$$y^2+x^2=Cx$$

Conclusion: Therefore, $y^2+x^2=Cx$ is the solution of

$$\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$$

Question 20.
Mark (√) against the correct answer in the following:

The general solution of the DE $x^2\frac{dy}{dx}=x^2+xy+y^2$is.

A. $\tan^{-1}\frac{y}{x}=\log x+C$

B. $\tan^{-1}\frac{x}{y}=\log x+C$

C. $\tan^{-1}\frac{y}{x}=\log y+C$

D. None of these

Answer:
Given $x^2\frac{dy}{dx}=x^2+xy+y^2$

$$\frac{dy}{dx}=1+\frac{y}{x}+\frac{y^2}{x^2}$$

Let y = vx

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$1+v+v^2=v+x\frac{dv}{dx}$$

$$1 + v^2 = x\frac{dv}{dx}$$

$$\frac{dx}{x} = \frac{dv}{v^2 + 1}$$

On integrating on both sides, we get

$$\log x = \tan^{-1} v + C$$

$$\tan^{-1}\frac{y}{x} = \log x + C$$

Conclusion: Therefore, $\tan^{-1}\frac{y}{x} = \log x + C$ is the solution of

$$x^2\frac{dy}{dx} = x^2 + xy + y^2$$

Question 21.
Mark (√) against the correct answer in the following:

The general solution od the DE $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$ is

A. $\sin\left(\frac{y}{x}\right) = C$

B. $\sin\left(\frac{y}{x}\right) = Cx$

C. $\sin\left(\frac{y}{x}\right) = Cy$

D. None of these

Answer: Given DE: $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$

Now,
Dividing both sides by x, we get,

$$\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$$

Let y = vx
Differentiating both sides,
dy/dx = v + xdv/dx
Now, our differential equation becomes,

$$v + x\frac{dv}{dx} = v + \tan v$$

On separating the variables, we get,

$$\frac{dv}{\tan v} = \frac{dx}{x}$$

Integrating both sides, we get,

sinv = Cx

Putting the value of v we get,

$$\sin\left(\frac{y}{x}\right) = Cx$$

Hence, B is the correct answer.

Question 22.

Mark (√) against the correct answer in the following:

The general solution of the DE $2xy\ dy + (x^2 - y^2)\ dx = 0$ is

A. $x^2 + y^2 = Cx$

B. $x^2 + y^2 = Cy$

C. $x^2 + y^2 = C$

D. None of these

Answer:

Given $2xy\,dy + (x^2 - y^2)dx = 0$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

Let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\frac{x^2v^2 - x^2}{2vx^2} = v + x\frac{dv}{dx}$$

$$\frac{v^2-1}{2v} - v = x\frac{dv}{dx}$$

$$\frac{-v^2-1}{2v} = x\frac{dv}{dx}$$

$$\frac{dx}{x} + \frac{2vdv}{v^2+1} = 0$$

On integrating on both sides, we get

$$\log x + \log(v^2+1) = c$$

$$\log(x(v^2+1)) = c$$

$$x\left(\frac{y^2}{x^2}+1\right) = C$$

$$y^2 + x^2 = Cx$$

Conclusion: Therefore, $y^2 + x^2 = Cx$ is the solution of

$$2xy\,dy + (x^2 - y^2)dx = 0$$

Question 23.
Mark (√) against the correct answer in the following:

The general solution of the DE ($\mathscr{X}$ - $\mathscr{Y}$) $d\mathscr{Y}$ + ($\mathscr{X}$ + $\mathscr{Y}$) $d\mathscr{X}$ is

A. $\tan^{-1}\frac{y}{x} = C\sqrt{x^2+y^2}$

B. $\tan^{-1(y-x)} = C\sqrt{x^2+y^2}$

C. $\tan^{-1}\left(\frac{y}{x}\right) = x^2+y^2+C$

D. None of these

Answer:

Given (x-y)dy + (x+y) dx = 0

$$\frac{dy}{dx} = \frac{x+y}{y-x}$$

Let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx+x}{vx-x}$$

$$v + x\frac{dv}{dx} = \frac{v+1}{v-1}$$

$$x\frac{dv}{dx} = \frac{v+1-v^2+v}{v-1}$$

$$x\frac{dv}{dx} = \frac{2v+1-v^2}{v-1}$$

Question is wrong. I think subtraction should be there instead of addition in LHS(left hand side)

Question 24.
Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$ is

A. $\tan\frac{y}{2x} = Cx$

B. $\tan\frac{y}{x}Cx$

C. $\tan\frac{y}{2x} = C$

D. None of these

Answer:

Given $\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$

Let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = v + \sin v$$

$$x\frac{dv}{dx} = \sin v$$

$$\frac{dv}{\sin v} = \frac{dx}{x}$$

$$\log\tan\frac{v}{2} = \log x + C$$

$$\tan\frac{v}{2} = Cx$$

$$\tan\frac{y}{2x} = Cx$$

Conclusion: Therefore, $\tan\frac{y}{2x} = Cx$ is the solution of $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Question 25.

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} + y\tan x = \sec x$ is

A. $y = \sin x - C\cos x$

B. $y = \sin x + C\cos x$

C. $y = \cos x - C\sin x$

D. None of these

Answer:

Given $\frac{dy}{dx} + y\tan x = \sec x$

It is in the form $\frac{dy}{dx} + py = Qx$

Integrating factor $= e^{\int \tan x dx} = e^{\log \sec x} = \sec x$

General solution $y \sec x = \int (\sec x)(\sec x)\, dx + C$

$$y \sec x = \int \sec^2 x\, dx + C$$

y sec x = tan x + C

y = sin x + C cos x

Conclusion: Therefore, y = sin x + C cos x is the solution of $\frac{dy}{dx} + y\tan x = \sec x$

Question 26.

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} + y\cot x = 2\cos x$ is

A. ($\mathscr{Y}$ + sin $\mathscr{X}$)sin $\mathscr{X}$ = C

B. ($\mathscr{Y}$ + cos $\mathscr{X}$) sin $\mathscr{X}$ = C

C. ($\mathscr{Y}$ – sin $\mathscr{X}$) sin $\mathscr{X}$ = C

D. None of these

Answer:

Given $\frac{dy}{dx} + y\cot x = 2\cos x$

It is in the form $\frac{dy}{dx} + py = Qx$

Integrating factor $= e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

General solution is $y \sin x = \int 2 \cos x \sin x \, dx + C$

$$y \sin x = \int \sin 2x \, dx + C$$

$$y \sin x = -\frac{\cos 2x}{2} + C$$

$$y \sin x = \sin^2 x + C$$

(y-sin x)sin x = C

Conclusion: Therefore, (y-sin x)sin x = C is the solution of $\frac{dy}{dx} + y \cot x = 2 \cos x$

Question 27.
Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} + \frac{y}{x} = x^2$ is

A. $\mathscr{X}\mathscr{Y} = \mathscr{X}^4 + C$

B. $4\mathscr{X}\mathscr{Y} = \mathscr{X}^4 + C$

C. $3\mathscr{X}\mathscr{Y} = \mathscr{X}^3 + C$

D. None of these

Answer:
Given $\frac{dy}{dx} + \frac{y}{x} = x^2$

It is in the form $\frac{dy}{dx} + py = Qx$

Integrating factor $= e^{\int \frac{1}{x} dx} = e^{\log x} = x$

General solution is $yx = \int x^2 . x dx + C$

$$yx = \frac{x^4}{4} + C$$

Conclusion: Therefore, $yx = \frac{x^4}{4} + C$ is the solution of $\frac{dy}{dx} + \frac{y}{x} = x^2$