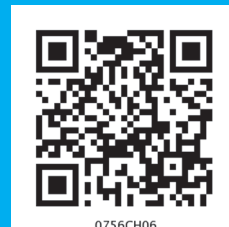


# The Triangle and its Properties



## 6.1 INTRODUCTION

A triangle, you have seen, is a simple closed curve made of three line segments. It has three vertices, three sides and three angles.

Here is  $\triangle ABC$  (Fig 6.1). It has

**Sides:**  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CA}$

**Angles:**  $\angle BAC$ ,  $\angle ABC$ ,  $\angle BCA$

**Vertices:** A, B, C

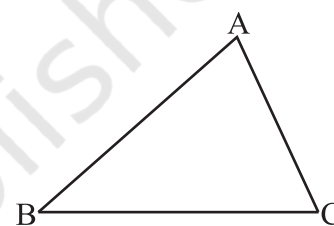


Fig 6.1

The side opposite to the vertex A is BC. Can you name the angle opposite to the side AB?

You know how to classify triangles based on the (i) sides (ii) angles.

- (i) Based on Sides: Scalene, Isosceles and Equilateral triangles.
- (ii) Based on Angles: Acute-angled, Obtuse-angled and Right-angled triangles.

Make paper-cut models of the above triangular shapes. Compare your models with those of your friends and discuss about them.

### TRY THESE

- Write the six elements (i.e., the 3 sides and the 3 angles) of  $\triangle ABC$ .
- Write the:
  - Side opposite to the vertex Q of  $\triangle PQR$
  - Angle opposite to the side LM of  $\triangle LMN$
  - Vertex opposite to the side RT of  $\triangle RST$
- Look at Fig 6.2 and classify each of the triangles according to its
  - Sides
  - Angles



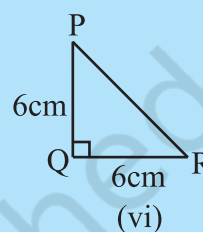
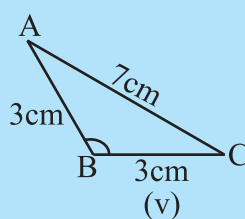
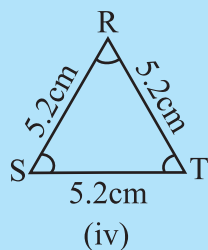
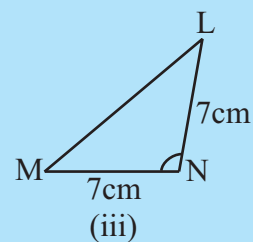
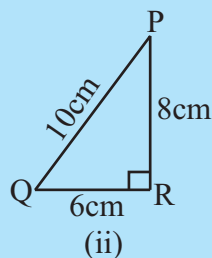
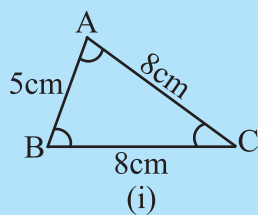


Fig 6.2

Now, let us try to explore something more about triangles.

## 6.2 MEDIANS OF A TRIANGLE

Given a line segment, you know how to find its perpendicular bisector by paper folding. Cut out a triangle ABC from a piece of paper (Fig 6.3). Consider any one of its sides, say,  $\overline{BC}$ . By paper-folding, locate the perpendicular bisector of  $\overline{BC}$ . The folded crease meets  $\overline{BC}$  at D, its mid-point. Join AD.

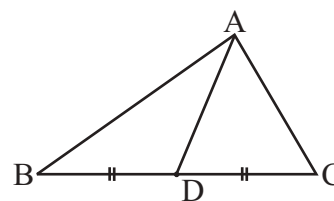
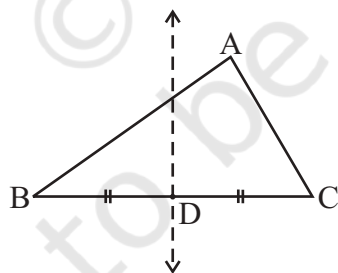


Fig 6.3

The line segment AD, joining the mid-point of  $\overline{BC}$  to its opposite vertex A is called a **median** of the triangle.

Consider the sides  $\overline{AB}$  and  $\overline{CA}$  and find two more medians of the triangle. A median connects a vertex of a triangle to the mid-point of the opposite side.



### THINK, DISCUSS AND WRITE

1. How many medians can a triangle have?
2. Does a median lie wholly in the interior of the triangle? (If you think that this is not true, draw a figure to show such a case).

### 6.3 ALTITUDES OF A TRIANGLE

Make a triangular shaped cardboard ABC. Place it upright on a table. How 'tall' is the triangle? The **height** is the distance from vertex A (in the Fig 6.4) to the base  $\overline{BC}$ .

From A to  $\overline{BC}$ , you can think of many line segments (see the next Fig 6.5). Which among them will represent its height?

The **height** is given by the line segment that starts from A, comes straight down to  $\overline{BC}$ , and is perpendicular to  $\overline{BC}$ .

This line segment  $\overline{AL}$  is an **altitude** of the triangle.

An altitude has one end point at a vertex of the triangle and the other on the line containing the opposite side. Through each vertex, an altitude can be drawn.

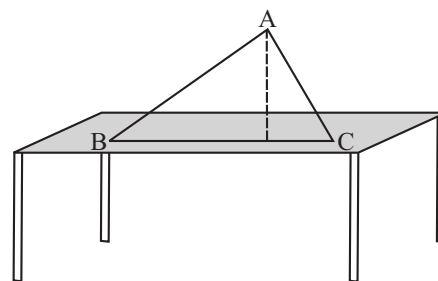


Fig 6.4

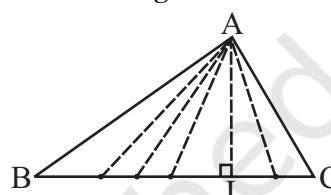


Fig 6.5

#### THINK, DISCUSS AND WRITE

1. How many altitudes can a triangle have?
2. Draw rough sketches of altitudes from A to  $\overline{BC}$  for the following triangles (Fig 6.6):

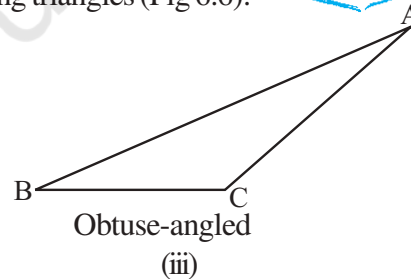
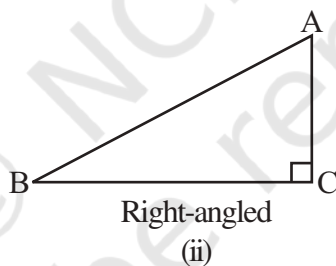
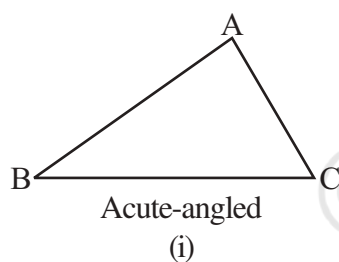


Fig 6.6

3. Will an altitude always lie in the interior of a triangle? If you think that this need not be true, draw a rough sketch to show such a case.
4. Can you think of a triangle in which two altitudes of the triangle are two of its sides?
5. Can the altitude and median be same for a triangle?  
(Hint: For Q.No. 4 and 5, investigate by drawing the altitudes for every type of triangle).

#### Do This

Take several cut-outs of

- (i) an equilateral triangle
- (ii) an isosceles triangle and
- (iii) a scalene triangle.

Find their altitudes and medians. Do you find anything special about them? Discuss it with your friends.



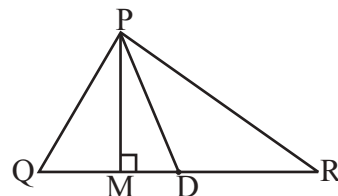
## EXERCISE 6.1

1. In  $\triangle PQR$ ,  $D$  is the mid-point of  $\overline{QR}$ .

$\overline{PM}$  is \_\_\_\_\_.

$\overline{PD}$  is \_\_\_\_\_.

Is  $QM = MR$ ?



2. Draw rough sketches for the following:

- In  $\triangle ABC$ ,  $BE$  is a median.
- In  $\triangle PQR$ ,  $PQ$  and  $PR$  are altitudes of the triangle.
- In  $\triangle XYZ$ ,  $YL$  is an altitude in the exterior of the triangle.

3. Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.

## 6.4 EXTERIOR ANGLE OF A TRIANGLE AND ITS PROPERTY

## Do This

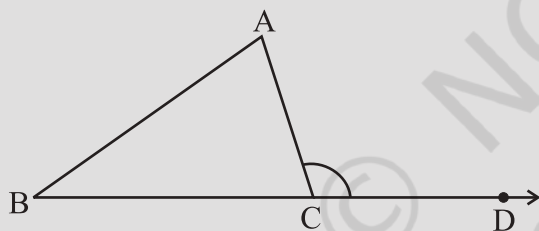


Fig 6.7



1. Draw a triangle  $ABC$  and produce one of its sides, say  $BC$  as shown in Fig 6.7. Observe the angle  $ACD$  formed at the point  $C$ . This angle lies in the exterior of  $\triangle ABC$ . We call it an **exterior angle** of the  $\triangle ABC$  formed at vertex  $C$ .

Clearly  $\angle BCA$  is an adjacent angle to  $\angle ACD$ . The remaining two angles of the triangle namely  $\angle A$  and  $\angle B$  are called the two **interior opposite angles** or the two remote

interior angles of  $\angle ACD$ . Now cut out (or make trace copies of)  $\angle A$  and  $\angle B$  and place them adjacent to each other as shown in Fig 6.8.

Do these two pieces together entirely cover  $\angle ACD$ ?

Can you say that

$$m \angle ACD = m \angle A + m \angle B?$$

2. As done earlier, draw a triangle  $ABC$  and form an exterior angle  $ACD$ . Now take a protractor and measure  $\angle ACD$ ,  $\angle A$  and  $\angle B$ .

Find the sum  $\angle A + \angle B$  and compare it with the measure of  $\angle ACD$ . Do you observe that  $\angle ACD$  is equal (or nearly equal, if there is an error in measurement) to  $\angle A + \angle B$ ?

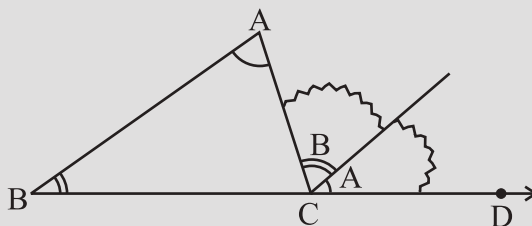


Fig 6.8

You may repeat the two activities as mentioned by drawing some more triangles along with their exterior angles. Every time, you will find that the exterior angle of a triangle is equal to the sum of its two interior opposite angles.

A logical step-by-step argument can further confirm this fact.

**An exterior angle of a triangle is equal to the sum of its interior opposite angles.**

**Given:** Consider  $\triangle ABC$ .

$\angle ACD$  is an exterior angle.

**To Show:**  $m\angle ACD = m\angle A + m\angle B$

Through C draw  $\overline{CE}$ , parallel to  $\overline{BA}$ .

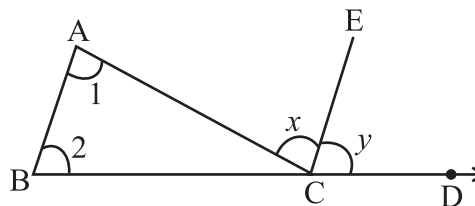


Fig 6.9

### Justification

#### Steps

(a)  $\angle 1 = \angle x$

(b)  $\angle 2 = \angle y$

(c)  $\angle 1 + \angle 2 = \angle x + \angle y$

(d) Now,  $\angle x + \angle y = m\angle ACD$  From Fig 6.9

Hence,  $\angle 1 + \angle 2 = \angle ACD$

#### Reasons

$\overline{BA} \parallel \overline{CE}$  and  $\overline{AC}$  is a transversal.

Therefore, alternate angles should be equal.

$\overline{BA} \parallel \overline{CE}$  and  $\overline{BD}$  is a transversal.

Therefore, corresponding angles should be equal.

The above relation between an exterior angle and its two interior opposite angles is referred to as the **Exterior Angle Property of a triangle**.

### THINK, DISCUSS AND WRITE

- Exterior angles can be formed for a triangle in many ways. Three of them are shown here (Fig 6.10)

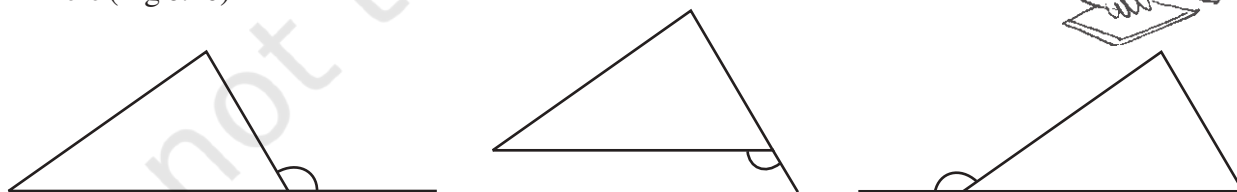


Fig 6.10

There are three more ways of getting exterior angles. Try to produce those rough sketches.

- Are the exterior angles formed at each vertex of a triangle equal?
- What can you say about the sum of an exterior angle of a triangle and its adjacent interior angle?

**EXAMPLE 1** Find angle  $x$  in Fig 6.11.

**SOLUTION** Sum of interior opposite angles = Exterior angle

or  $50^\circ + x = 110^\circ$

or  $x = 60^\circ$

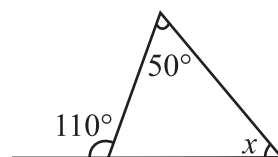


Fig 6.11



### THINK, DISCUSS AND WRITE

- What can you say about each of the interior opposite angles, when the exterior angle is
  - a right angle?
  - an obtuse angle?
  - an acute angle?
- Can the exterior angle of a triangle be a straight angle?

### TRY THESE

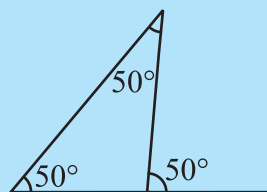


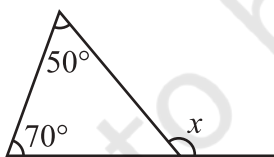
Fig 6.12

- An exterior angle of a triangle is of measure  $70^\circ$  and one of its interior opposite angles is of measure  $25^\circ$ . Find the measure of the other interior opposite angle.
- The two interior opposite angles of an exterior angle of a triangle are  $60^\circ$  and  $80^\circ$ . Find the measure of the exterior angle.
- Is something wrong in this diagram (Fig 6.12)? Comment.

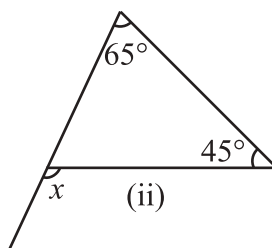
### EXERCISE 6.2



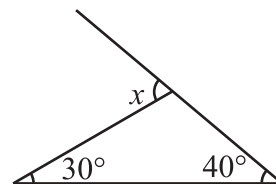
- Find the value of the unknown exterior angle  $x$  in the following diagrams:



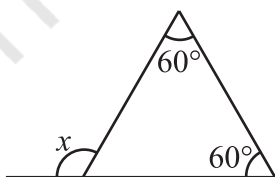
(i)



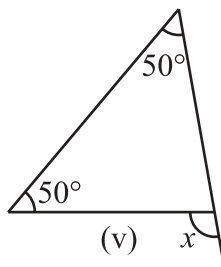
(ii)



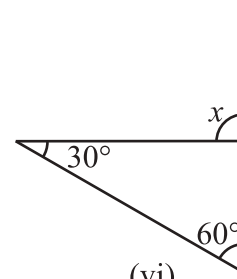
(iii)



(iv)

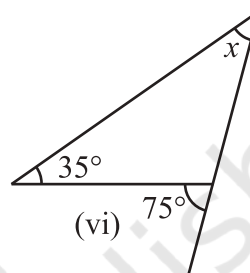
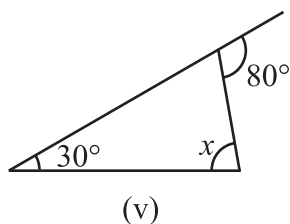
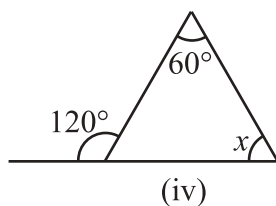
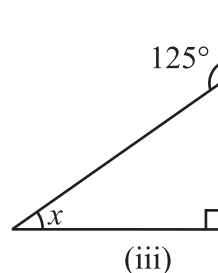
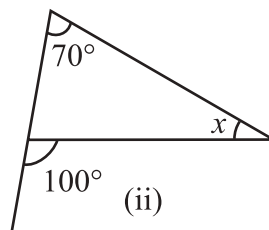
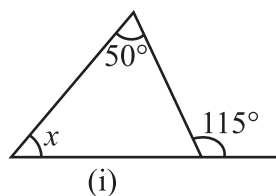


(v)



(vi)

2. Find the value of the unknown interior angle  $x$  in the following figures:



## 6.5 ANGLE SUM PROPERTY OF A TRIANGLE

There is a remarkable property connecting the three angles of a triangle. You are going to see this through the following four activities.

1. Draw a triangle. Cut on the three angles. Rearrange them as shown in Fig 6.13 (i), (ii). The three angles now constitute one angle. This angle is a straight angle and so has measure  $180^\circ$ .

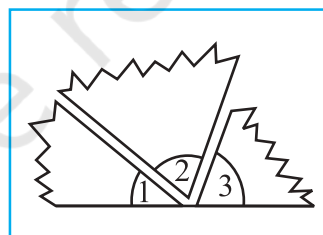
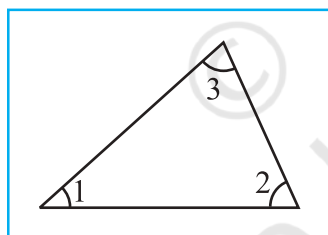


Fig 6.13

Thus, the sum of the measures of the three angles of a triangle is  $180^\circ$ .

2. The same fact you can observe in a different way also. Take three copies of any triangle, say  $\triangle ABC$  (Fig 6.14).

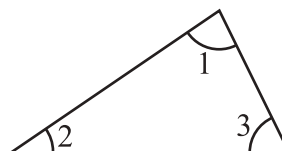
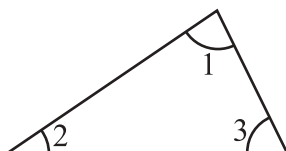
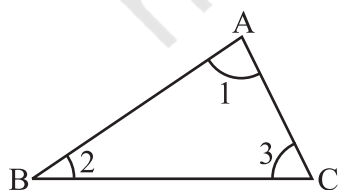


Fig 6.14

Arrange them as in Fig 6.15.

What do you observe about  $\angle 1 + \angle 2 + \angle 3$ ?

(Do you also see the 'exterior angle property'?)

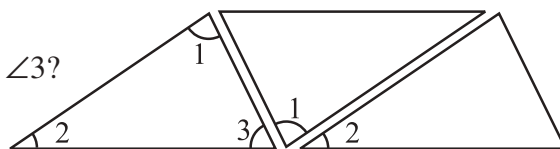
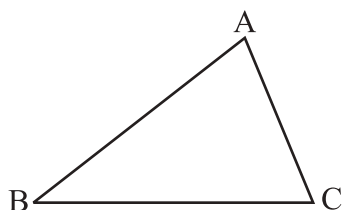


Fig 6.15

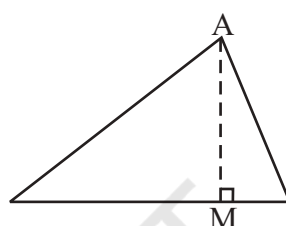
3. Take a piece of paper and cut out a triangle, say,  $\triangle ABC$  (Fig 6.16).

Make the altitude  $AM$  by folding  $\triangle ABC$  such that it passes through  $A$ .

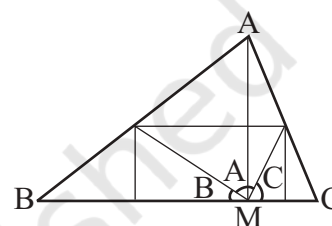
Fold now the three corners such that all the three vertices  $A$ ,  $B$  and  $C$  touch at  $M$ .



(i)



(ii)



(iii)

Fig 6.16

You find that all the three angles form together a straight angle. This again shows that the sum of the measures of the three angles of a triangle is  $180^\circ$ .

4. Draw any three triangles, say  $\triangle ABC$ ,  $\triangle PQR$  and  $\triangle XYZ$  in your notebook.

Use your protractor and measure each of the angles of these triangles.

Tabulate your results

Name of $\Delta$	Measures of Angles	Sum of the Measures of the three Angles
$\triangle ABC$	$m\angle A =$ $m\angle B =$ $m\angle C =$	$m\angle A + m\angle B + m\angle C =$
$\triangle PQR$	$m\angle P =$ $m\angle Q =$ $m\angle R =$	$m\angle P + m\angle Q + m\angle R =$
$\triangle XYZ$	$m\angle X =$ $m\angle Y =$ $m\angle Z =$	$m\angle X + m\angle Y + m\angle Z =$

Allowing marginal errors in measurement, you will find that the last column always gives  $180^\circ$  (or nearly  $180^\circ$ ).

When perfect precision is possible, this will also show that the sum of the measures of the three angles of a triangle is  $180^\circ$ .

You are now ready to give a formal justification of your assertion through logical argument.

**Statement** The total measure of the three angles of a triangle is  $180^\circ$ .

To justify this let us use the exterior angle property of a triangle.

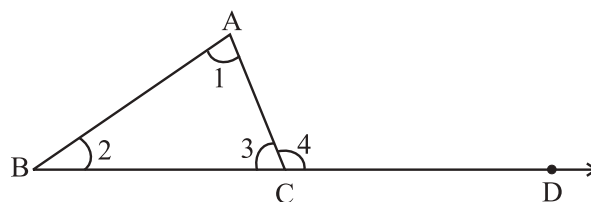


Fig 6.17



**Given**  $\angle 1, \angle 2, \angle 3$  are angles of  $\triangle ABC$  (Fig 6.17).  
 $\angle 4$  is the exterior angle when  $BC$  is extended to  $D$ .

**Justification**  $\angle 1 + \angle 2 = \angle 4$  (by exterior angle property)  
 $\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 3$  (adding  $\angle 3$  to both the sides)

But  $\angle 4$  and  $\angle 3$  form a linear pair so it is  $180^\circ$ . Therefore,  $\angle 1 + \angle 2 + \angle 3 = 180^\circ$ .

Let us see how we can use this property in a number of ways.

**EXAMPLE 2** In the given figure (Fig 6.18) find  $m\angle P$ .

**SOLUTION** By angle sum property of a triangle,

$$m\angle P + 47^\circ + 52^\circ = 180^\circ$$

Therefore

$$\begin{aligned} m\angle P &= 180^\circ - 47^\circ - 52^\circ \\ &= 180^\circ - 99^\circ = 81^\circ \end{aligned}$$

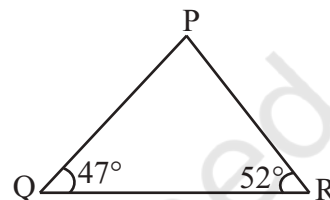
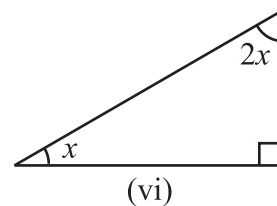
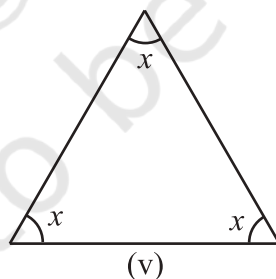
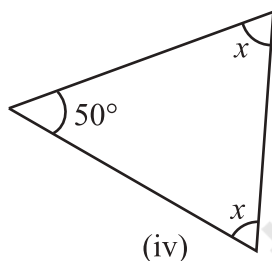
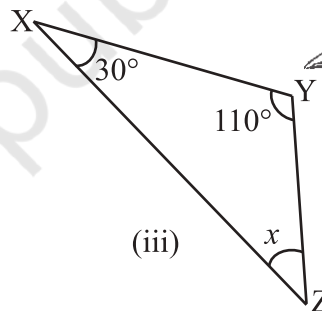
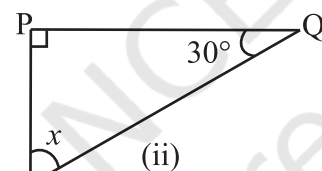
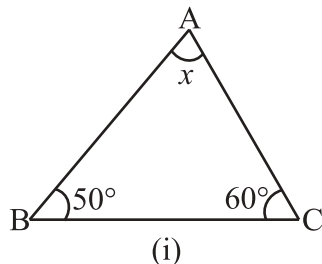


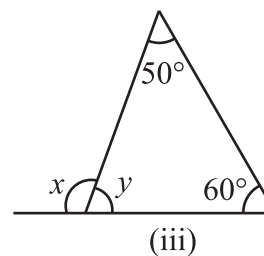
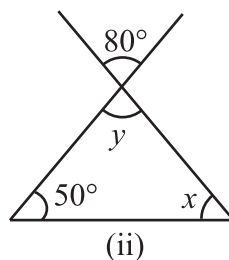
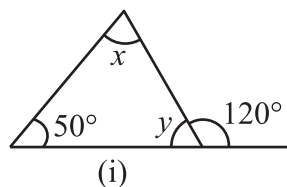
Fig 6.18

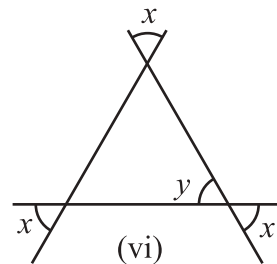
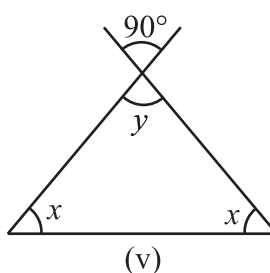
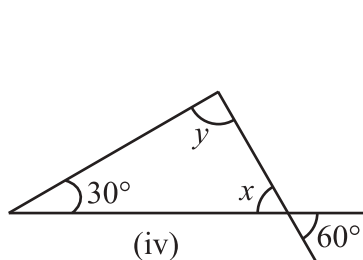
### EXERCISE 6.3

1. Find the value of the unknown  $x$  in the following diagrams:



2. Find the values of the unknowns  $x$  and  $y$  in the following diagrams:





### TRY THESE



- Two angles of a triangle are  $30^\circ$  and  $80^\circ$ . Find the third angle.
- One of the angles of a triangle is  $80^\circ$  and the other two angles are equal. Find the measure of each of the equal angles.
- The three angles of a triangle are in the ratio 1:2:1. Find all the angles of the triangle. Classify the triangle in two different ways.

### THINK, DISCUSS AND WRITE



- Can you have a triangle with two right angles?
- Can you have a triangle with two obtuse angles?
- Can you have a triangle with two acute angles?
- Can you have a triangle with all the three angles greater than  $60^\circ$ ?
- Can you have a triangle with all the three angles equal to  $60^\circ$ ?
- Can you have a triangle with all the three angles less than  $60^\circ$ ?

## 6.6 TWO SPECIAL TRIANGLES : EQUILATERAL AND ISOSCELES

A triangle in which all the three sides are of equal lengths is called an equilateral triangle.

Take two copies of an equilateral triangle ABC (Fig 6.19). Keep one of them fixed. Place the second triangle on it. It fits exactly into the first. Turn it round in any way and still they fit with one another exactly. Are you able to see that when the three sides of a triangle have equal lengths then the three angles are also of the same size?

We conclude that in an equilateral triangle:

- all sides have same length.
- each angle has measure  $60^\circ$ .

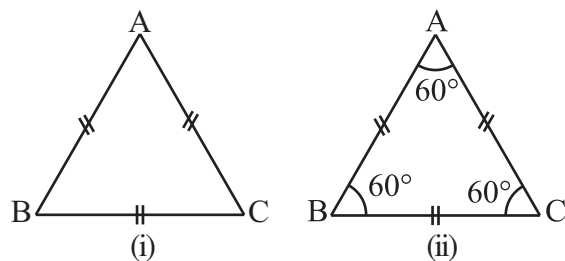


Fig 6.19

A triangle in which two sides are of equal lengths is called an isosceles triangle.

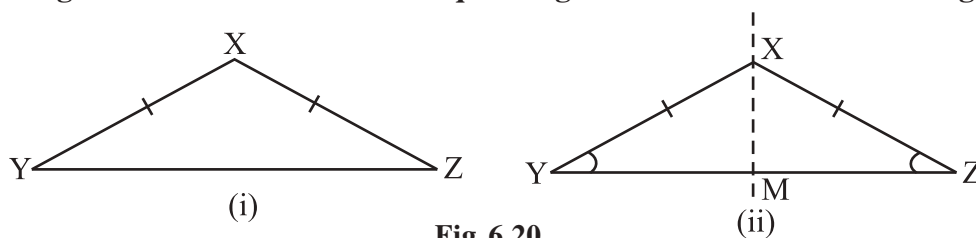


Fig 6.20

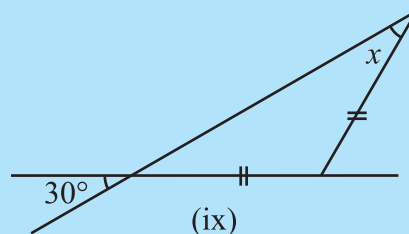
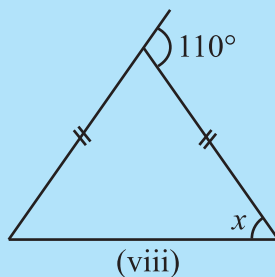
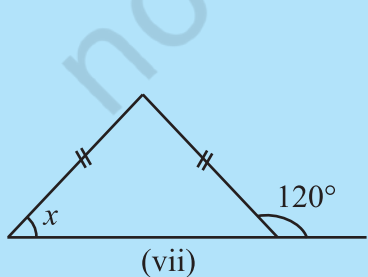
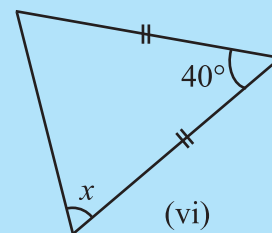
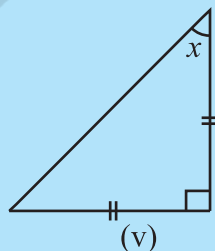
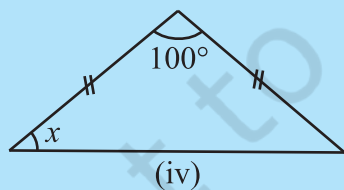
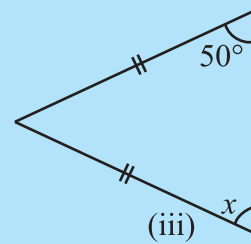
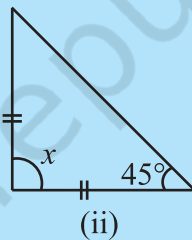
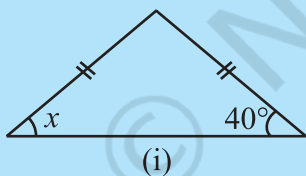
From a piece of paper cut out an isosceles triangle  $XYZ$ , with  $XY = XZ$  (Fig 6.20). Fold it such that  $Z$  lies on  $Y$ . The line  $XM$  through  $X$  is now the axis of symmetry (which you will read in Chapter 14). You find that  $\angle Y$  and  $\angle Z$  fit on each other exactly.  $XY$  and  $XZ$  are called equal sides;  $YZ$  is called the base;  $\angle Y$  and  $\angle Z$  are called base angles and these are also equal.

Thus, in an isosceles triangle:

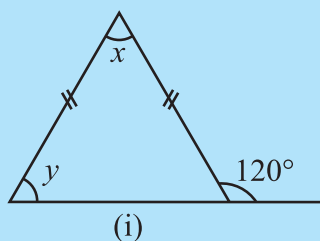
- (i) two sides have same length.
- (ii) base angles opposite to the equal sides are equal.

### TRY THESE

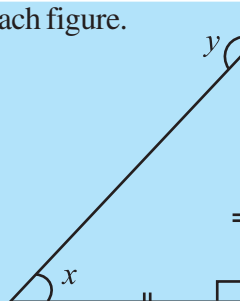
1. Find angle  $x$  in each figure:



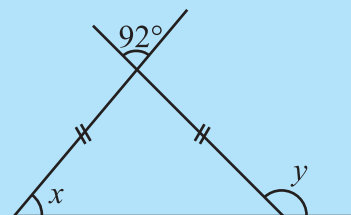
2. Find angles  $x$  and  $y$  in each figure.



(i)



(ii)



(iii)

## 6.7 SUM OF THE LENGTHS OF TWO SIDES OF A TRIANGLE

1. Mark three non-collinear spots A, B and C in your playground. Using lime powder mark the paths AB, BC and AC.

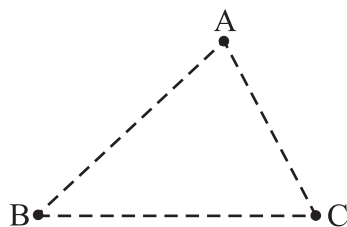


Fig 6.21

Ask your friend to start from A and reach C, walking along one or more of these paths. She can, for example, walk first along  $\overline{AB}$  and then along  $\overline{BC}$  to reach C; or she can walk straight along  $\overline{AC}$ . She will naturally prefer the direct path AC. If she takes the other path ( $\overline{AB}$  and then  $\overline{BC}$ ), she will have to walk more. In other words,

$$AB + BC > AC \quad (i)$$

Similarly, if one were to start from B and go to A, he or she will not take the route  $\overline{BC}$  and  $\overline{CA}$  but will prefer  $\overline{BA}$ . This is because

$$BC + CA > AB \quad (ii)$$

By a similar argument, you find that

$$CA + AB > BC \quad (iii)$$

These observations suggest that **the sum of the lengths of any two sides of a triangle is greater than the third side.**

2. Collect fifteen small sticks (or strips) of different lengths, say, 6 cm, 7 cm, 8 cm, 9 cm, ..., 20 cm.

Take any three of these sticks and try to form a triangle. Repeat this by choosing different combinations of three sticks.

Suppose you first choose two sticks of length 6 cm and 12 cm. Your third stick has to be of length more than  $12 - 6 = 6$  cm and less than  $12 + 6 = 18$  cm. Try this and find out why it is so.

To form a triangle you will need any three sticks such that the sum of the lengths of any two of them will always be greater than the length of the third stick.

This also suggests that the sum of the lengths of any two sides of a triangle is greater than the third side.

3. Draw any three triangles, say  $\triangle ABC$ ,  $\triangle PQR$  and  $\triangle XYZ$  in your notebook (Fig 6.22).

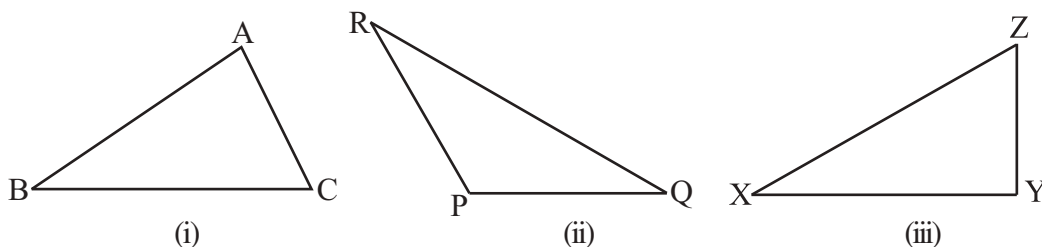


Fig 6.22

Use your ruler to find the lengths of their sides and then tabulate your results as follows:

Name of $\Delta$	Lengths of Sides	Is this True?	
$\Delta ABC$	AB ____	$AB - BC < CA$	(Yes/No)
		____ + ____ > ____	
	BC ____	$BC - CA < AB$	(Yes/No)
		____ + ____ > ____	
	CA ____	$CA - AB < BC$	(Yes/No)
		____ + ____ > ____	
$\Delta PQR$	PQ ____	$PQ - QR < RP$	(Yes/No)
		____ + ____ > ____	
	QR ____	$QR - RP < PQ$	(Yes/No)
		____ + ____ > ____	
	RP ____	$RP - PQ < QR$	(Yes/No)
		____ + ____ > ____	
$\Delta XYZ$	XY ____	$XY - YZ < ZX$	(Yes/No)
		____ + ____ > ____	
	YZ ____	$YZ - ZX < XY$	(Yes/No)
		____ + ____ > ____	
	ZX ____	$ZX - XY < YZ$	(Yes/No)
		____ + ____ > ____	

This also strengthens our earlier guess. Therefore, we conclude that **sum of the lengths of any two sides of a triangle is greater than the length of the third side.**

We also find that the difference between the length of any two sides of a triangle is smaller than the length of the third side.

**EXAMPLE 3** Is there a triangle whose sides have lengths 10.2 cm, 5.8 cm and 4.5 cm?

**SOLUTION** Suppose such a triangle is possible. Then the sum of the lengths of any two sides would be greater than the length of the third side. Let us check this.

$$\text{Is } 4.5 + 5.8 > 10.2? \quad \text{Yes}$$

$$\text{Is } 5.8 + 10.2 > 4.5? \quad \text{Yes}$$

$$\text{Is } 10.2 + 4.5 > 5.8? \quad \text{Yes}$$

Therefore, the triangle is possible.

**EXAMPLE 4** The lengths of two sides of a triangle are 6 cm and 8 cm. Between which two numbers can length of the third side fall?

**SOLUTION** We know that the sum of two sides of a triangle is always greater than the third.

Therefore, third side has to be less than the sum of the two sides. The third side is thus, less than  $8 + 6 = 14$  cm.

The side cannot be less than the difference of the two sides. Thus, the third side has to be more than  $8 - 6 = 2$  cm.

The length of the third side could be any length greater than 2 and less than 14 cm.

### EXERCISE 6.4



1. Is it possible to have a triangle with the following sides?

(i) 2 cm, 3 cm, 5 cm

(ii) 3 cm, 6 cm, 7 cm

(iii) 6 cm, 3 cm, 2 cm

2. Take any point O in the interior of a triangle PQR. Is

(i)  $OP + OQ > PQ$ ?

(ii)  $OQ + OR > QR$ ?

(iii)  $OR + OP > RP$ ?

3. AM is a median of a triangle ABC.

Is  $AB + BC + CA > 2 AM$ ?

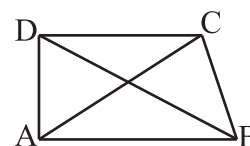
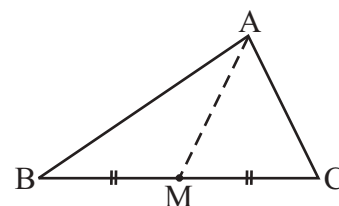
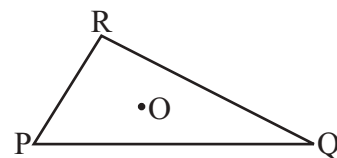
(Consider the sides of triangles  $\triangle ABM$  and  $\triangle AMC$ .)

4. ABCD is a quadrilateral.

Is  $AB + BC + CD + DA > AC + BD$ ?

5. ABCD is quadrilateral. Is

$AB + BC + CD + DA < 2 (AC + BD)$ ?



6. The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

### THINK, DISCUSS AND WRITE

1. Is the sum of any two angles of a triangle always greater than the third angle?

## 6.8 RIGHT-ANGLED TRIANGLES AND PYTHAGORAS PROPERTY

Pythagoras, a Greek philosopher of sixth century B.C. is said to have found a very important and useful property of right-angled triangles given in this section. The property is, hence, named after him. In fact, this property was known to people of many other countries too. The Indian mathematician Baudhayan has also given an equivalent form of this property. We now try to explain the Pythagoras property.

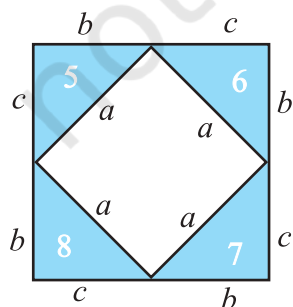
In a right-angled triangle, the sides have some special names. The side opposite to the right angle is called the **hypotenuse**; the other two sides are known as the **legs** of the right-angled triangle.

In  $\triangle ABC$  (Fig 6.23), the right-angle is at B. So, AC is the hypotenuse.  $\overline{AB}$  and  $\overline{BC}$  are the legs of  $\triangle ABC$ .

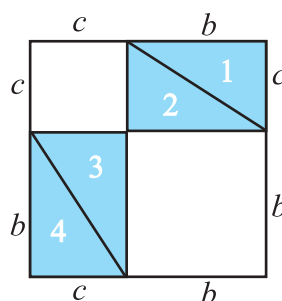
Make eight identical copies of right angled triangle of any size you prefer. For example, you make a right-angled triangle whose hypotenuse is  $a$  units long and the legs are of lengths  $b$  units and  $c$  units (Fig 6.24).

Draw two identical squares on a sheet with sides of lengths  $b + c$ .

You are to place four triangles in one square and the remaining four triangles in the other square, as shown in the following diagram (Fig 6.25).



Square A



Square B

Fig 6.25

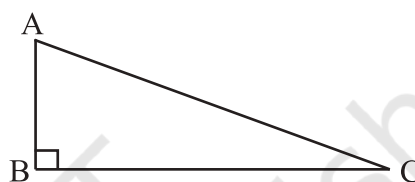


Fig 6.23

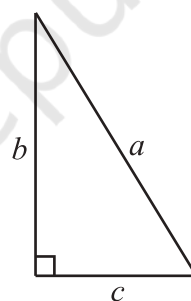


Fig 6.24



The squares are identical; the eight triangles inserted are also identical.

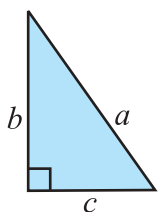
Hence the uncovered area of square A = Uncovered area of square B.

i.e., Area of inner square of square A = The total area of two uncovered squares in square B.

$$a^2 = b^2 + c^2$$

This is Pythagoras property. It may be stated as follows:

In a right-angled triangle,  
the square on the hypotenuse = sum of the squares on the legs.



Pythagoras property is a very useful tool in mathematics. It is formally proved as a theorem in later classes. You should be clear about its meaning.

It says that for any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the legs.

Draw a right triangle, preferably on a square sheet, construct squares on its sides, compute the area of these squares and verify the theorem practically (Fig 6.26).

If you have a right-angled triangle, the Pythagoras property holds. If the Pythagoras property holds for some triangle, will the triangle be right-angled? (Such problems are known as converse problems). We will try to answer this. Now, we will show that, if there is a triangle such that sum of the squares on two of its sides is equal to the square of the third side, it must be a right-angled triangle.

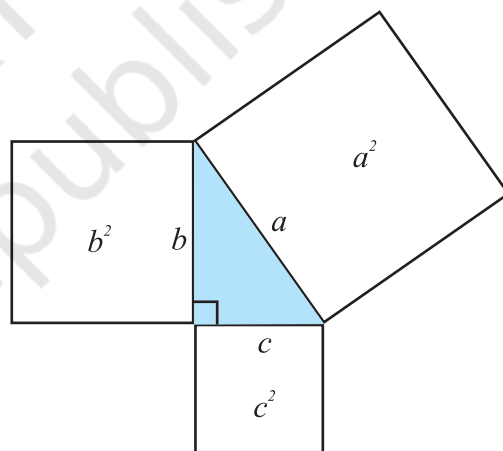


Fig 6.26

## Do This



1. Have cut-outs of squares with sides 4 cm, 5 cm, 6 cm long. Arrange to get a triangular shape by placing the corners of the squares suitably as shown in the figure (Fig 6.27). Trace out the triangle formed. Measure each angle of the triangle. You find that there is no right angle at all.

In fact, in this case each angle will be acute! Note that  $4^2 + 5^2 \neq 6^2$ ,  $5^2 + 6^2 \neq 4^2$  and  $6^2 + 4^2 \neq 5^2$ .

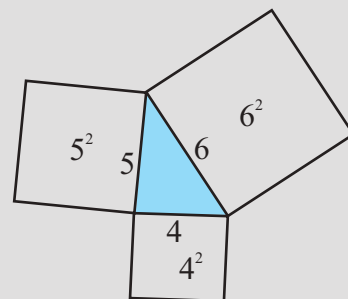


Fig 6.27



2. Repeat the above activity with squares whose sides have lengths 4 cm, 5 cm and 7 cm. You get an obtuse-angled triangle! Note that

$$4^2 + 5^2 \neq 7^2 \text{ etc.}$$

This shows that Pythagoras property holds if and only if the triangle is right-angled. Hence we get this fact:

If the Pythagoras property holds, the triangle must be right-angled.

**EXAMPLE 5** Determine whether the triangle whose lengths of sides are 3 cm, 4 cm, 5 cm is a right-angled triangle.

**SOLUTION**  $3^2 = 3 \times 3 = 9$ ;  $4^2 = 4 \times 4 = 16$ ;  $5^2 = 5 \times 5 = 25$

We find  $3^2 + 4^2 = 5^2$ .

Therefore, the triangle is right-angled.

**Note:** In any right-angled triangle, the hypotenuse happens to be the longest side. In this example, the side with length 5 cm is the hypotenuse.

**EXAMPLE 6**  $\triangle ABC$  is right-angled at C. If AC = 5 cm and BC = 12 cm find the length of AB.

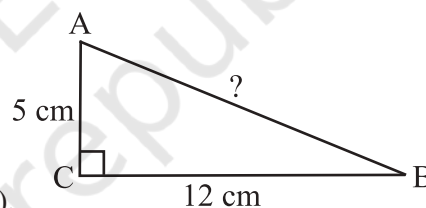


Fig 6.28

**SOLUTION** A rough figure will help us (Fig 6.28).

By Pythagoras property,

$$\begin{aligned} AB^2 &= AC^2 + BC^2 \\ &= 5^2 + 12^2 = 25 + 144 = 169 = 13^2 \end{aligned}$$

or

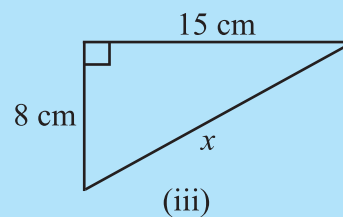
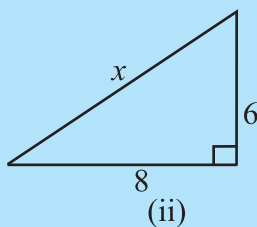
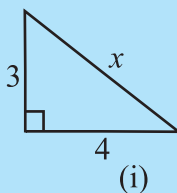
$$AB^2 = 13^2. \text{ So, } AB = 13$$

or the length of AB is 13 cm.

**Note:** To identify perfect squares, you may use prime factorisation technique.

## TRY THESE

Find the unknown length  $x$  in the following figures (Fig 6.29):



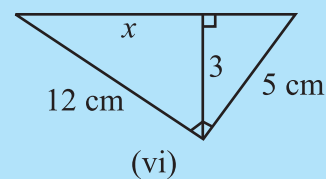
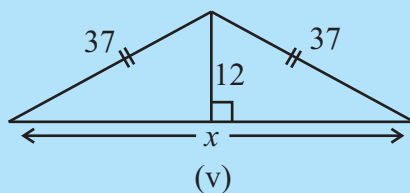
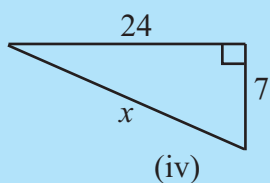


Fig 6.29

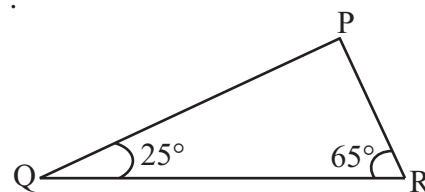
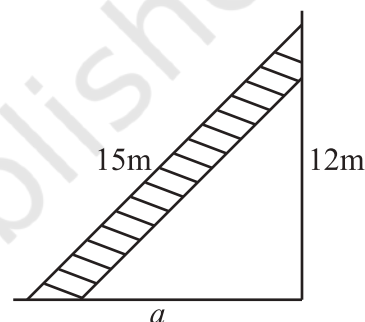
## EXERCISE 6.5



1. PQR is a triangle, right-angled at P. If  $PQ = 10$  cm and  $PR = 24$  cm, find QR.
2. ABC is a triangle, right-angled at C. If  $AB = 25$  cm and  $AC = 7$  cm, find BC.
3. A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance  $a$ . Find the distance of the foot of the ladder from the wall.
4. Which of the following can be the sides of a right triangle?
  - (i) 2.5 cm, 6.5 cm, 6 cm.
  - (ii) 2 cm, 2 cm, 5 cm.
  - (iii) 1.5 cm, 2 cm, 2.5 cm.

In the case of right-angled triangles, identify the right angles.

5. A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.
6. Angles Q and R of a  $\triangle PQR$  are  $25^\circ$  and  $65^\circ$ . Write which of the following is true:
  - (i)  $PQ^2 + QR^2 = RP^2$
  - (ii)  $PQ^2 + RP^2 = QR^2$
  - (iii)  $RP^2 + QR^2 = PQ^2$



7. Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.
8. The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.

**THINK, DISCUSS AND WRITE**

1. Which is the longest side in the triangle PQR, right-angled at P?
2. Which is the longest side in the triangle ABC, right-angled at B?
3. Which is the longest side of a right triangle?
4. 'The diagonal of a rectangle produce by itself the same area as produced by its length and breadth' – This is Baudhayan Theorem. Compare it with the Pythagoras property.

**Do This****Enrichment activity**

There are many proofs for Pythagoras theorem, using 'dissection' and 'rearrangement' procedure. Try to collect a few of them and draw charts explaining them.

**WHAT HAVE WE DISCUSSED?**

1. The **six elements** of a triangle are its **three angles** and the **three sides**.
2. The line segment joining a vertex of a triangle to the mid point of its opposite side is called a **median** of the triangle. A triangle has 3 medians.
3. The perpendicular line segment from a vertex of a triangle to its opposite side is called an **altitude** of the triangle. A triangle has 3 altitudes.
4. An **exterior angle** of a triangle is formed, when a side of a triangle is produced. At each vertex, you have two ways of forming an exterior angle.
5. A property of exterior angles:  
The measure of any exterior angle of a triangle is equal to the sum of the measures of its interior opposite angles.
6. The angle sum property of a triangle:  
The total measure of the three angles of a triangle is  $180^\circ$ .
7. A triangle is said to be **equilateral**, if each one of its sides has the same length.  
In an equilateral triangle, each angle has measure  $60^\circ$ .
8. A triangle is said to be **isosceles**, if atleast any two of its sides are of same length.  
The non-equal side of an isosceles triangle is called its **base**; the base angles of an isosceles triangle have equal measure.
9. Property of the lengths of sides of a triangle:  
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.  
The difference between the lengths of any two sides is smaller than the length of the third side.

This property is useful to know if it is possible to draw a triangle when the lengths of the three sides are known.

10. In a right angled triangle, the side opposite to the right angle is called the **hypotenuse** and the other two sides are called its **legs**.
11. **Pythagoras property:**

In a right-angled triangle,

the square on the hypotenuse = the sum of the squares on its legs.

If a triangle is not right-angled, this property does not hold good. This property is useful to decide whether a given triangle is right-angled or not.

