CBSE SAMPLE PAPER-03
Class – XI
PHYSICS (Theory)

Time allowed: 3 hours, Maximum Marks: 70

General Instructions:

1. All the questions are compulsory.
2. There are **26** questions in total.
3. Questions **1** to **5** are very short answer type questions and carry **one** mark each.
4. Questions **6** to **10** carry **two** marks each.
5. Questions **11** to **22** carry **three** marks each.
6. Questions **23** is value based questions carry **four** marks.
7. Questions **24** to **26** carry **five** marks each.
8. There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all three questions in five marks each. You have to attempt only one of the choices in such questions.
9. Use of calculators is **not** permitted. However, you may use log tables if necessary.
10. You may use the following values of physical constants wherever necessary:
    $c = 3 \times 10^8 m/s,\ \ h = 6.63 \times 10^{-34} Js,\ \ e = 1.6 \times 10^{-19} C,$
    $\mu_o = 4\pi \times 10^{-7} TmA^{-1},\ \ \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2C^{-2},\ \ m_e = 9.1 \times 10^{-31} kg$

1. When real gases obey the ideal gas equation pV = RT?

Ans. a) Low pressure

b) High temperature

2. Give reason: “Though earth received solar energy, it is not warmed up continuously”.

Ans. This is because

a) The energy received is less due to large distance.

b) Loss of energy takes place due to radiation, absorption and convection currents.

3. Find the internal energy possessed at a temperature T, if there are ‘f’ degrees freedom with ‘n’ moles of a gas.

Ans. For 1 mole with ‘f’ degrees of freedom,

Internal energy U = 1 x C_V x T = $\frac{f}{2}RT$

For ‘n’ moles,

U = n x C_V x T = $\frac{nf}{2}RT$

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4. Give reason: “Curved roads are banked”.

Ans. Curved roads are generally banked so as to help in providing centripetal force needed to balance the centrifugal force arising due to the circular motion of the curved road.

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5. Give reason: “A spring balance show wrong measure after long use”.

Ans. This is because the spring loses its elastic character after long use.

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6. Give the limitations of the first law of Thermodynamics?

Or

If work required blowing a soap bubble of radius r is W, then what additional work is required to be done below it to a radius 3r?

Ans. (i) It does not tell us about the direction of flow of heat

(ii) It fails to explain why heat cannot be spontaneously converted into work.

Or

Increase in surface area = $2[4\pi (3r)^2 - 4\pi r^2]$

Increase in surface energy = $\sigma \times 2 \times 4\pi \times 8r^2 = 8W$

Additional work done = 8W

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7. The measured quantities a, b, c and x is calculated by using the relation $x = \frac{ab^2}{c^3}$. If the percentage errors in measurements of a, b, and c are ± 1%, ± 2% and ± 1.5 %, then calculate the maximum percentage error in value of x obtained.

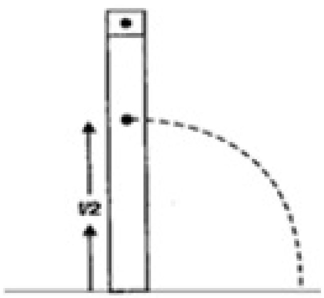
Ans. $x = \frac{ab^2}{c^3}$

$$\left(\frac{\Delta x}{x}\right)_{max} = \frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c}$$

$$\frac{\Delta a}{a} = \pm 1\%,$$

$\frac{\Delta b}{b} = \pm 2\%$,
$\frac{\Delta c}{c} = \pm 3\%$
$\left(\frac{\Delta x}{x}\right)_{\max}$ = 1%+2 x 2%+3 x 1.5%
= (1 + 4 + 4.5)%
= 9.5%

8. What would be the velocity of the top end at the time of touching the ground if a rod of length l and mass M held vertically is let go down, without slipping at the point of contact?
Ans. Loss in potential energy = Gain in rotational Kinetic energy
$\mathrm{mg}\frac{1}{2} = \frac{1}{2}\frac{\mathrm{Ml}^2}{3}.\omega^2$



$\omega = \sqrt{\frac{3gl}{l^2}} - \sqrt{\frac{3g}{l}}$
v =lm
= $\sqrt{3gl}$

9. State the laws of limiting friction.
Ans. The laws of limiting friction are as follows:
(a) The value of limiting friction depends on the nature of the two surfaces in contact and on the state of their smoothness.
(b) The force of friction acts tangential to the surfaces in contact in a direction opposite to the direction of relative motion.
(c) The value of limiting friction is directly proportional to the normal reaction between the two given surfaces.
(d) For any two given surface and for a given value of normal reaction the force of limiting friction is independent of the shape and surface area of surfaces in contact. Coefficient of

limiting friction for two given surfaces in contact is defined as the ratio of the force of normal reaction N.

$\mu_r = \frac{f_1}{N}$

10. If 1A^0 = 10^{-10} m and the size of a hydrogen atom is about 0.5 A^0, then what is the total atomic volume in m^3 of a mole of hydrogen atoms?

Ans. Volume of the one hydrogen atom = $\frac{4}{3}$ πr^3

$= \frac{4}{3}$ x 3.14 x (0.5×10^{-10}) m^3

$= 5.23 \times 10^{-31} m^3$

According to Avogadro's hypothesis, one mole of hydrogen contains 6.023×10^{23} atoms.

Atomic volume of 1 mole of hydrogen atom = $6.023 \times 10^{23} \times 5.23 \times 10^{-31}$

$= 3.15 \times 10^{-7} m^3$.

11. What is the average speed during the whole journey, if a body covers half of its journey with a speed of 40 m/s and other half with a speed of 60 m/s.

Ans. Average speed = $\frac{\text{Total Distance}}{\text{Timetaken}}$

Let x be the distance to be covered,

Average speed = $\frac{x}{\frac{x}{2v_1} + \frac{x}{2v_2}}$

Where $\frac{x}{2v_1}$ = time taken to cover first half of the distance, $\frac{x}{2v_2}$ = time taken to cover the second half of the distance,

Average speed = $\frac{x}{\frac{x(v_1+v_2)}{2v_1v_2}}$

$= \frac{\text{x }(x)\text{ 2v}_1\text{v}_2}{\text{x(v}_1\text{v}_2)} = \frac{2\text{v}_1\text{v}_2}{\text{v}_1\text{v}_2}$

$V_{av} = \frac{\text{2 x 40m/s x 60m/s}}{\text{100m/s}}$ = 48 ms^{-1}

12. Find the height to which it rises above the earth's surface if a particle is projected vertically upwards from the surface of earth of radius R with kinetic energy equal to half of the minimum value needed for it to escape.

Ans. Escape velocity from the surface of earth is

$v_{es} = \sqrt{\frac{2GM}{R}}$

K.E of a body $K_{es}=1/2\ mv^2_{es} = \frac{GMm}{R}$

The body is projected from the surface of earth with a K.E half of that needed to escape from earth surface hence

Initial K.E. of body $K = \frac{K_{es}}{2} = \frac{GMm}{2R}$

And its potential energy $\mu = -\frac{GMm}{R}$

Total initial energy of body $-\frac{GMm}{2R}$

The body goes up to a maximum height h from surface of earth, where the final K.E. =0 and P.E = $-\frac{GMm}{(R+h)}$

Total energy = $-\frac{GMm}{(R+h)}$

From conservation law of mechanical energy,

$-\frac{GMm}{2R} = -\frac{GMm}{(R+h)}$

On simplifying, h =R.

13. If the density of hydrogen at S.T.P is 0.09 kg m^{-3}, then calculate

i. RMS velocity

ii. Mean kinetic energy E of one gram molecule of hydrogen at S.T.P.

Ans. Hence ρ = 0.09 kg m^{-3}

S.T.P pressure P = 101 x 10^5 Pa.

According to K.E of gases

$P=\frac{1}{3}\rho C^2$ or $C=\sqrt{\frac{3P}{\rho}}$

$\sqrt{\frac{3x1.01x10^5}{0.09}}$

= 1837.5 ms^{-1}

Volume occupied by one mole of hydrogen at S.T.P = 22.4 litres = 22.4 x 10^{-3}m^3

Mass of hydrogen M = volume x density

= 22.4 x 10^{-3} x 0.09

= 2.016 x 10^{-3} kg

Average K.E / mole = ½ MC^2

= ½ x (2.016 x 10^{-3}) x (1837.5)^2

= 3403.4 J

14. The planet Mars has two moons, A and B

(i) How would you calculate the mass of Mars, A has a period 7 hours, 39 minutes and an orbital radius of 9.4 x 10^3 km

(ii) Assuming that Earth and Mars move in circular orbits around the sun with the Martian orbit being 1.52 times the orbital radius of the earth, then what is the length of the Martian year in days?

Ans. (i) The Sun's Mass replaced by the Martian mass M_m

$$T^2 = \frac{4\pi^2}{GMm} R^3$$

$$Mm = \frac{4\pi^2}{G} \text{x} \frac{R^3}{T^2}$$

$$Mm = \frac{4\text{x}(3.14)^2\text{x}(9.4)^3\text{x}10^{18}}{6.67\text{x}10^{-11}\text{x}(459\text{x}60)^2}$$

$$= \frac{4\text{x}(3.14)^2\text{x}(9.4)^3\text{x}10^{18}}{6.67\text{x}(4.59\text{x}6)^2\text{x}10^{-5}}$$

= 6.48 x 10^{23}kg

(ii) Using Kepler's third law

$$\frac{T_M^2}{T_E^2} = \frac{R_{MS}^3}{R_{ES}^3}$$

Where RMS is the Mars (Earth) – Sun distance

$$T_M = \left(\frac{R_{MS}}{R_{ES}}\right)^{3/2} - T_E^2$$

= $(1.52)^{3/2}$ x 365 = 684 days

15. Deduce the height at which the value of g is the same as at a depth of $\frac{R}{2}$?

Ans. At depth = $\frac{R}{2}$, value of acceleration due to gravity

$$g' = g\left(1 - \frac{R}{2R}\right) = \frac{g}{2}$$

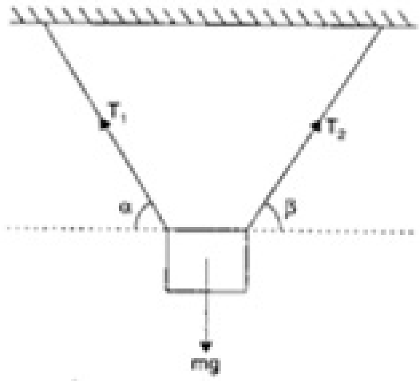
At height x,

$$g' = g\left(1 - \frac{2x}{R}\right)$$

$$g\left(1 - \frac{2x}{R}\right) = \frac{g}{2}$$

$$\frac{1}{2} = \frac{2x}{R} \Rightarrow x = \frac{R}{4}$$

16. A body of mass m is suspended by two strings making angles α and β with the horizontal. Calculate the tensions in the two strings.



Ans. Consider components of tensions T_1 and T_2 along the horizontal and vertical directions

$- T_1\cos\alpha + T_2\cos\beta = 0$ ---------------------- (i)

$T_1\cos\alpha = T_2\cos\beta$ ------------------------- (ii)

$T_1 \sin\alpha + T_2 \sin\beta = mg$

From (i)

$T_2 = \frac{T_1\cos\alpha}{\cos\beta}$ And substituting if in (ii) we get

$$T_1\sin\alpha + \left(\frac{T_1\cos\alpha}{\cos\beta}\right)\sin\beta = mg$$

$$T_1\frac{\sin(\alpha+\beta)}{\cos\beta} = mg$$

$$T_1 = \frac{mg\cos\beta}{\sin(\alpha+\beta)}$$

Hence, $T_2 = \frac{T_1\cos\alpha}{\cos\beta} = \frac{mg\cos\beta}{\sin(\alpha+\beta)} \cdot \frac{\cos\alpha}{\cos\beta}$

$$T_2 = \frac{mg\cos\alpha}{\sin(\alpha+\beta)}$$

17. i. Calculate the angular momentum and rotational kinetic energy of earth about its own axis.

ii. How long could this amount of energy supply one KW power to each of the 3.5 x 10^9 persons on earth?

Ans. Assume that earth to be a solid sphere. We know that the moment of inertia of a solid sphere about its axis is

$I = \frac{2}{5} MR^2$

$= \frac{2}{5}$ x (6.0 x 10^{24}kg) x (6.4 x 10^6m)^2

= 9.8 x 10^{37} kg m^2

In one day the earth completes one revolution. Hence the angular velocity is given by

$\omega = \frac{2\pi}{24 \text{ x } 60 \text{ x } 60} = 7.27 \text{ x } 10^{-5} \text{rad/sec}$

Angular momentum $I\omega = (98 \text{ x } 10^{37} \text{ kg m}^2)(7.27 \text{ x } 10^{-5} \text{ s}^{-1})$

$= 7.1 \text{ x } 10^{33} \text{ kg m}^2 / \text{sec.}$

The rotational energy

$\frac{1}{2} I\omega^2 = \frac{1}{2}(9.8 \text{ x } 10^{37} \text{ kg m}^2)(7.27 \text{ x } 10^{-5} \text{ s}^{-1})^2$

$= 2.6 \text{ x } 10^{29}$ J

Power supplied by this energy

$P = \frac{\text{Energy}}{\text{Time}} = \frac{2.6 \text{ x} 10^{29}}{t} \text{watt}$

$= \frac{2.6 \text{ x } 10^{29}}{10^{39} t} \text{KW}$

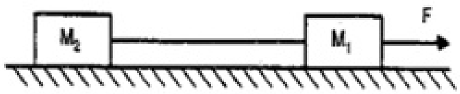
Power required by $3.5 \text{ x } 10^9$ persons = $3.5 \text{ x } 10^9$ x 1 kilowatt

$\frac{2.6 \text{ x } 10^{29}}{10^3 t} = 3.5 \text{ x } 10^9$

$t = \frac{2.6 \text{ x } 10^{29}}{10^3 \text{ x } 3.5 \text{ x } 10^9} \text{sec}$

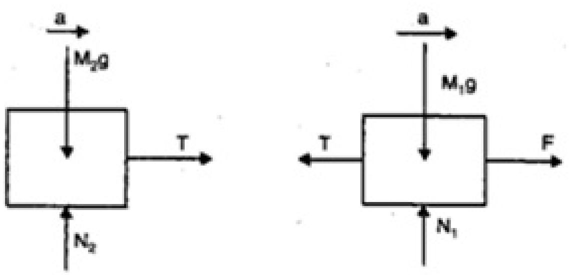
$= 2.35 \text{ x } 10^9$ years

18. A diagram below is a light inextensible string. If a force F as shown acts upon M_1, find the acceleration of the system and tension in string.



Ans. As the string is inextensible acceleration of two blocks will be same. Also string is massless so tension throughout the string will be same. Contact force will be normal force only.

Acceleration of each block is a , tension in string is T and connect force between M_1 and surface is N_1 and contact force between M_2 and surface is N_2



Applying Newton’s second law for the blocks:

$F - T = M_1a$--------------------------(i)

$M_1g - N_1 = 0$--------------------------(ii)

$T = M_2a$--------------------------(iii)

$M_2g - N_2 = 0$--------------------------(iv)

Solving equation (i) and (iii)

$$a = \frac{F}{M_1+M_2}$$

$$T = \frac{M_2F}{M_1+M_2}$$

19. i. State ideal gas equation.

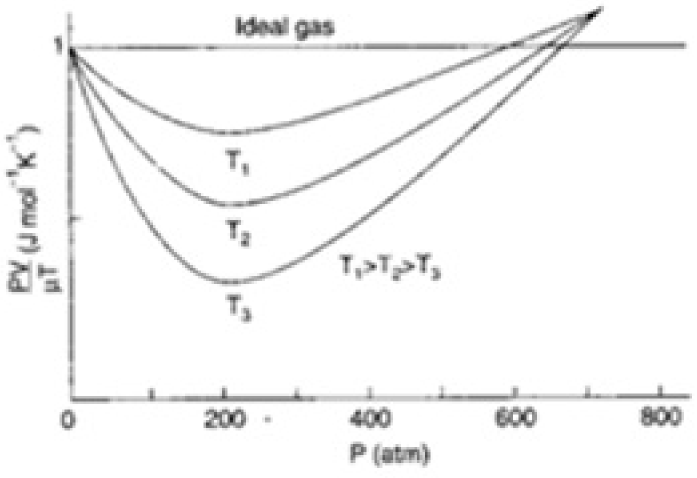
ii. Draw graph to check whether a real gas obeys this equation.

iii. What is the conclusion drawn?

Ans. According to the ideal gas equation PV = μ RT

According to this equation

$\frac{PV}{\mu T}$ = R



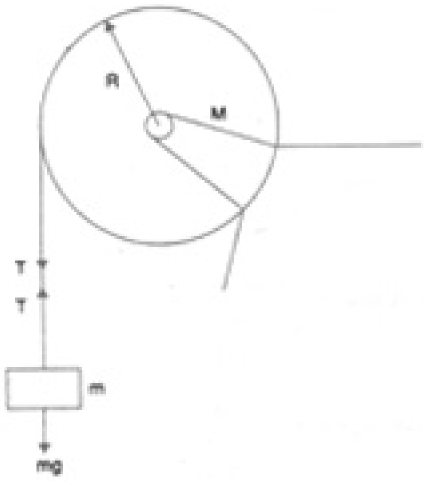
Experimentally value of $\frac{PV}{\mu T}$ for real gas was calculated by altering the pressure of gas at different temperatures. The graphs obtained have been shown in the diagram. Here for the purpose of comparison, graph for an ideal gas has also been drawn, which is a straight line parallel to pressure axis.

From the graph it is clear that behavior of real gases is differ from an ideal gas. However at high temperatures and low pressures behaviors is nearly same as that of an ideal gas.

20. A uniform disc of radius R and mass M is mounted on an axis supported in fixed frictionless bearing. A light chord is wrapped around the rim of the wheel and supposes that we hang a body of mass m from the chord. Find the angular of the disc and tangential acceleration of point on the rim.

Ans. Let T be the tension in the chord

mg – T = ma,---------------------------(i)



Where a is the tangential acceleration of appoint on the rim of the disc.

$\tau = I\alpha$

But the resultant torque on the disc = TR and the rotational inertia

I = ½ MR^2

$TR = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$

2TR = Ma

$a = \frac{2T}{M}$--------------------------- (ii)

From the equation (i) and (ii)

$\text{mg - }\left(\frac{\text{Ma}}{2}\right) = \text{ma}$

$\text{a} = \left(\frac{\text{2m}}{\text{M + 2M}}\right)\text{g}$

$\text{mg - T} = \text{m x}\left(\frac{\text{2T}}{\text{M}}\right)$

$\text{T} = \left(\frac{\text{mM}}{\text{M+2m}}\right)\text{g}$

21. The following equation represents standing wave set up in medium

$\text{y} = 4\cos\frac{\pi x}{5}\sin 40\pi\text{t}$**, where x and y are in cm and t in seconds. Find out the amplitude and the velocity of the two component waves and calculate the distance between**

adjacent nodes. What is the velocity of a medium particle at x = 3cm at time $\frac{1}{8}$ second?

Ans. The equation of stationary wave is

$$y = 4 \cos \frac{\pi x}{3} \sin 40\pi t$$

$$y = 2 \text{ x } 2 \cos \frac{2\pi x}{6} \sin \frac{2x(120)t}{6}$$

We know that

$$y = 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2xvt}{\lambda}$$

By comparing tow equations we get,

a= 2cm, λ=6cm and v=220 cm/sec.

The component waves are

$$y_1 = a \sin \frac{2\pi}{\lambda}(vt\text{-}x)$$

$$y_2 = a \sin \frac{2\pi}{\lambda}(vt+x)$$

Distance between two adjacent nodes

$$= \frac{\lambda}{2} = \frac{6}{2} = 3\text{cm}$$

Particle velocity

$$\frac{dy}{dt} = 4 \cos \frac{\pi x}{3} \cos (40\pi t).\ 40\pi$$

$$= 160 \cos \frac{\pi x}{3} \cos 40\pi t$$

$$= 160\pi \cos \frac{\pi x}{3} \cos \left(40\pi x\ \frac{1}{8}\right)$$

=160 π

Particle velocity = 160 cm/sec.

22. Uniform spring whose upstretched length is l has a force constant k. the spring is cut into two pieces of upstretched lengths, l_1 and l_2, where $l_1 = n\, l_2$ and n is an integer. What are the corresponding force constant K_1 and K_2 in terms of n and k?

Or

If a steel rod has a radius of 10 mm and a length of 1.0 m. A 100 kN force stretches it along its length, then calculate [Given Y = 2.0 x 10^{11}N/m^2]

a) Stress

b) Elongation

c) Strain of the rod

Ans.

$l = l_1 + l_2$-- (i)

$l_1 = n\, l_2$--- (ii)

$k = \frac{Mg}{l}$ ------------------------------------- (iii)

$k_1 = \frac{Mg}{l_1}$ ------------------------------ (iv)

$k_2 = \frac{Mg}{l_2}$ -------------------------------- (v)

Dividing the equation (iv) by (iii),

$\frac{k_1}{k} = \frac{l}{l_2} = 2\frac{l_1+l_2}{l_1} = 1 + \frac{l_2}{l_1}$

From the equation (ii) we find $\frac{l_1}{l_2}$ = n

$\frac{k_1}{k} = 1 + \frac{1}{n}$

From equation (v) and (iii)

$\frac{k_2}{k} = \frac{l}{l_2} = \frac{l_1+l_2}{l_2} = \frac{l_1}{l_2} + 1$

From equation (ii) we have

$\frac{l_1}{l_2}$ = n

$\frac{k_2}{k} = (n+1)$

k_2 = k(n+1)

Or

a) Stress = $\frac{F}{A} = \frac{F}{\pi r^2}$

= 3.18 x 10^8 N/m^2

b) Elongation ΔL = $\frac{\left(\frac{F}{A}\right)L}{Y}$

= 1.59 mm

g) Strain = $\frac{\Delta L}{L}$

= 0.16%

23. Radha was 70 kg at the age of 16 years. She then decided to lose weight. She started walking daily for 30 minutes and started taking only milk and fruits. Her parents advised her not to cut off her meal so drastically. She showed the result of losing 30 kg but she was feeling weak. So her parents took her to doctor who advised her to take proper balanced diet and exercise regularly.

a) Is taking crash diet advisable? Why?

b) Give the relation between SI and CGS unit of heat.

c) What would be her rise in temperature, if Radha weighing 40 kg now was advised to take 4000 kcal diet in a day and this energy was to be used in heating her without any

losses? [Given: Sp. Heat of human body = 0.83 cal $g^{-1 0}C^{-1}$

Ans. a) Crash diet should not be taken as it makes body weak and less immune to diseases. It makes the body deficient in certain nutrients which is harmful for body.

b) The SI unit of heat = Joule and the CGS unit of heat = Calorie

1 calorie = 4.18 J

c) Substituting the values for $\Delta Q = cm\Delta T$, we get

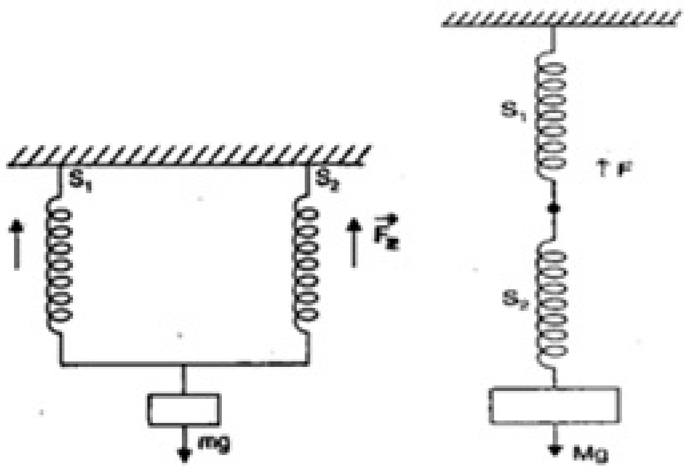
$\Delta T = 40.16^0C$

24. Find the expression for time period of motion of a body suspended by two springs connected in parallel and series.

Or

Calculate the frequency of the first and last fork if a set of 24 tuning forks is arranged in series of increasing frequencies. If each fork gives with the preceding one and the last fork is octave of the first.

Ans.



Consider the body of mass M suspended by two springs connected in parallel. Let K_1 and K_2 be the spring constants of two springs.

Let the body be pulled down so that each spring is stretched through a distance y. Restoring F_1 and F_2 will be developed in the springs S_1 and S_2.

According to Hooke's law $F_1 = -K_1y$ and $F_2 = -K_2y$

Since both the forces acting in the same direction, total restoring force acting on the body is given by

$F = F_1 + F_2 = - K_1 y – K_2 y = - (K_1 + K_2)y$

Acceleration produced in the body is given by

$a = \frac{F}{M} = - \frac{(K_1+K_2)Y}{M}$ --------------- (i)

Since $\frac{(K_1+ K_2)}{M}$ is constant a = - y

Hence motion of the body is SHM

The time period of body is given by

$T = 2\pi\sqrt{\frac{y}{|a|}} = 2\pi\sqrt{\frac{M}{K_1+ K_2}}$----------------- (ii)

$K_1 = K_2 = K$

$T = 2\pi\sqrt{\frac{M}{2K}}$

For series:

Consider the body of mass M suspended by two springs S_1 and S_2 which are connected in series. Let K_1 and K_2 be the spring constants of spring S1 and S_2.

At any instant the displacement of the body from equilibrium position is y in the downward direction. If y_1 and y_2 be the extension produced in the springs S_1 and S_2.

$y = y_1 + y_2$ --------------------------- (i)

Restoring the forces developed in S_1 and S_2 are given by,

$F_1 = -K_1 + y_1$ ---------------------------------- (ii)

$F_2 = -K_2 + y_2$---------------------------------- (iii)

Multiplying the equation (ii) by K_2 and equation (iii) by K_1 and adding we get,

$K_2F_1+k_1F_2 = -k_1k_2(y_1+y_2) = -k_1k_2 y$

Since both the springs are connected in series.

$F_1 = F_2 =F$

$F (K_1+ K_2) =-k_1k_2y$

$F = \frac{k_1k_2}{(k_1+ k_2)y}$

If ‘a’ be the acceleration produced in the body of mass ‘M’ then,

$a = \frac{F}{M} = \frac{k_1k_2y}{(k_1+ k_2)M}$ -------------------------- (iv)

Time period of the body is given by,

$T = 2\pi\sqrt{\frac{y}{|a|}} = 2\pi\sqrt{\frac{(k_1+k_2)M}{k_1k_2}}$

$$T = 2\pi\sqrt{\left(\frac{1}{k_1}+\frac{1}{k_2}\right)M}$$

Or

Let the frequency of the first tuning fork be n. as last fork is octave of the first therefore frequency of the last fork = 2n

As each fork gives 4 beats / seconds with the preceding one,

Therefore, frequency of 2^{nd} fork = n + 4

Frequency of the 3^{rd} fork = n +8 = n +4(2) = n + 4(3-1)

Proceeding in this way, frequency of 24^{th} fork = n + 4(24 -1) = n +92

2n = n + 92 or n = 92 Hz.

Frequency of the last fork = 2n = 2 x 92 = 184 Hz.

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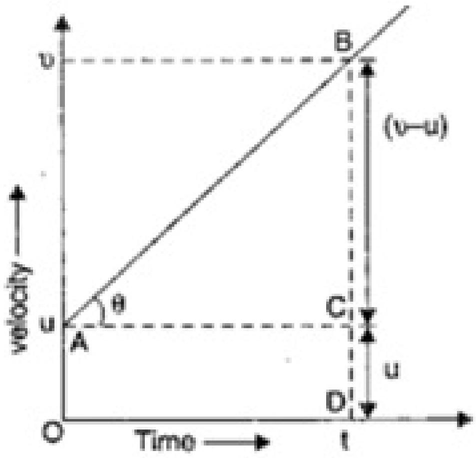
25. Explain the kinematic equation for uniformly accelerated motion.

Or

A particle is thrown over a triangle from one end of a horizontal base that grazing h vertex fall on the other end of the base. If α and β be the same angles and θ the angle of projection, prove that tan θ = tan α + tan β.

Ans. Uniformly accelerated motion, we can drive some simple equation that relate displacements (x), time taken (f), initial velocity (u), final velocity (v) and acceleration (a).

(i) Velocity attends after time t: the velocity-time graph for positive constant acceleration of a particle.



Let u be the initial velocity of the particle at l =0 and v is the final velocity of the particle after time t. consider two points A and B on the curve corresponding to t=0 and t =t.

Draw ZBD perpendicular to time axis. Also draw AC perpendicular to BD.

OA = CD = u

BC = (v – u) and OD = t

Now,

Slope of v – t graph = acceleration (a)

a = slope of v – t graph

$\tan\theta = \frac{BC}{AC} = \frac{BC}{OD}$

$a = \frac{\text{v - u}}{\text{t}}$

v –u = at

v = u + at

(ii) Distance travelled in time t will be,

x_0 = position of the particle at t =0 from the origin

x = position of the particle at t =t from the origin

$(x - x_0)$ = S = distance travelled by the particle in the time interval (t -0) =t

Distance travelled by a particle in the given time

Interval = area under velocity-time graph

$(x - x_0)$ = area OABD

= area of Trapezium OABD

= ½ [Sum of parallel sides x perpendicular distance between parallel sides]

= ½ (OA + BZD) x AC

= ½ (u + v) x t

v =u +at

$(x - x_0)$ = ½ (u + u + at) x t

= ½ (2u + at) x t

= ut + ½ at^2

$x - x_0 = S$

S = ut + ½ at^2

(iii) Velocity attained after travelling a distance S:

Distance travelled by a particle in time t is equal to the area under velocity-time graph. The distance (s) travelled by a particle during time interval t is given by

S = area under v – t graph

S = area of Trapezium OABD

= ½ (sum of parallel sides) x perpendicular distance between these parallel sides

S = ½ (OA + OD) x AC ---------------------------- (i)

Acceleration a = slope of v – t graph

$$A = \frac{BC}{AC} = \frac{BD - CD}{AC} = \frac{v - u}{AC}$$

$$\mathrm{AC} = \left(\frac{\text{v - u}}{\text{a}}\right)$$ ---------------------------- (ii)

OA = u and BD = v---------------------------- (iii)

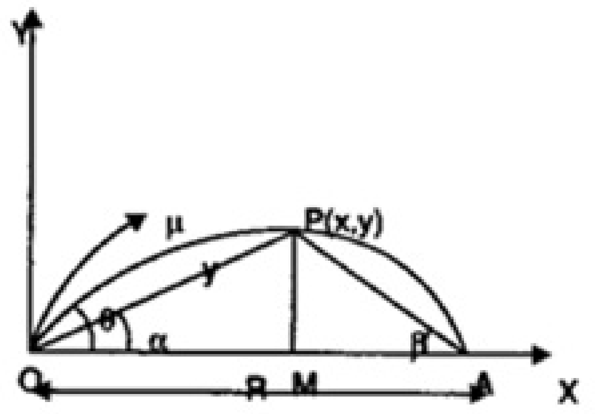
From equation (i), (ii) and (iii) we get

$$\mathrm{S} = \frac{1}{2}(\mathrm{v}+\mathrm{u})\,\frac{(\text{v-u})}{\text{a}}$$

$$\mathrm{S} = \frac{\mathrm{v}^2 - \mathrm{u}^2}{2\mathrm{a}}$$

$v^2 - u^2 = 2aS$

Or



From the

$$\tan\alpha = \frac{\mathrm{y}}{\mathrm{x}}$$

$$\tan\beta = \frac{\mathrm{y}}{\mathrm{MA}} = \frac{\mathrm{y}}{\text{R - x}}$$

where R is horizontal range.

$$\tan\alpha + \tan\beta = \frac{\mathrm{y}}{\mathrm{x}} + \frac{\mathrm{y}}{\text{R - x}}$$

$$= \frac{(\text{R - x} + \mathrm{x})\mathrm{y}}{\mathrm{x}(\text{R - x})} = \frac{\mathrm{yR}}{\mathrm{x}(\text{R - x})}$$

$$\tan\alpha + \tan\beta = \frac{\mathrm{yR}}{\mathrm{x}(\text{R - x})}$$ -------------------------------- (i)

$$\mathrm{x} = (\mathrm{u}\cos\theta)\,\mathrm{t}$$ -------------------------------- (ii)

$$\mathrm{y} = (\mathrm{u}\sin\theta)\,\mathrm{t} - \frac{1}{2}\mathrm{gt}^2$$ -------------------- (iii)

From equation (ii) and (iii),

$$\mathrm{y} = \mathrm{x}\tan\theta\left[1 - \frac{\mathrm{xg}}{2\mathrm{u}^2\cos^2\theta\tan\theta}\right]$$

Substituting,

$$R = \frac{2u^2 \sin\theta \cos\theta}{g}$$

$$y = x\ \tan\theta \left[1 - \frac{xg}{2u^2 \cos\theta \sin\theta}\right]$$

$$y = x\ \tan\theta \left[1 - \frac{x}{R}\right]$$

$$\frac{y}{x} = \tan\theta \left(\frac{R - x}{R}\right) \text{--------------------- (iv)}$$

Putting (iv) in (i) we get,

$$\tan\alpha + \ \tan\beta = \frac{yR}{x(R - X)} = \tan\theta$$

$$\tan\alpha + \tan\beta = \tan\theta$$

26. If a stone is dropped from the top of a mountain and n second later another stone is thrown vertically downwards with a velocity of u m/s, then how far below the top of the mountain will be the second stone overtake the first?

Or

A particle is projected horizontally with a speed u from top of a plane inclined at an angle θ with the horizontal direction. How far from the point of projection will the particle strike the plane?

Ans. The second stone will catch up with the first stone when the distance covered by it in (t – n) second will equal the distance covered by the first stone in t second.

The distance covered by the first stone in t second = ½ gt^2 and distance covered by the second stone in (1 –n) second.

u (t –n) + ½ g $(t - n)^2$

½ gt^2 = u (t – n) + ½ g $(t - n)^2$

½ g [t_2 – $(t-n)^2$] = u (t –n)

½ g [(2t –n) n] = u (t –n)

gnt – ½ gn^2 = ut –un

t (gn – u) = (½ gn –u)n

$$t = \frac{n\left(\frac{1}{2}gn - u\right)}{(gn - u)}$$

The distance covered by the first stone in this time

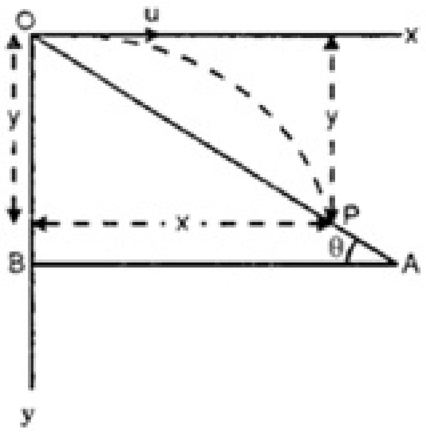
$$h = \frac{1}{2}gt^2 = \frac{1}{2}g\left[\frac{n\left(\frac{1}{2}gn - u\right)}{(gn - u)}\right]$$

Thus the second stone will overtake the first at distance

$\frac{1}{2}g\left[\frac{n\left(\frac{gn}{2} - u\right)}{(gn - u)}\right]^2$

Or

Let the particle projected from O strike the inclined plane OA at P after time t and coordinates of P be (x,y).



Taking motion of projectile from O to P along x-axis we have

$x_0 = 0$, x = x, $u_x = u$, ax =0, t=t

Using the relation $x = x_0 + u_x t + ½ a_x t^2$

We get x = ut or i =x/u

Taking motion of projectile along y – axis

$y_0 = y$, y = y, u = 0, $a_y = g$, t = t

Using the relation $y = y_0 + u_y t + ½ a_y t^2$

$$y = 0 + 0 + \frac{1}{2}\ gt^2 = \frac{1}{2}gt^2 = \frac{1}{2}g\frac{x^2}{u^2}$$

y = x tan θ, so $gx^2 / 2u^2 = x \tan\theta$

x = $\frac{2u^2\tan\theta}{g}$

And y = x tanθ = $\frac{2u^2\tan^2\theta}{g}$

Distance OP = $\sqrt{x^2 + y^2}$

= $\frac{2u^2\ \tan\theta}{g}\sqrt{1 + \tan^2\theta}$

= $\frac{2u^2\ \tan\theta\sec\theta}{g}$