

MATHEMATICS

Textbook for Class XII

PART I



12079

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PART I



राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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Foreword

The National Curriculum Framework, 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that, given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

NCERT appreciates the hard work done by the textbook development committee responsible for this book. We wish to thank the Chairperson of the advisory group in Science and Mathematics, Professor J.V. Narlikar and the Chief Advisor for this book, Professor P.K. Jain for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

New Delhi
20 December 2005

Director
National Council of Educational
Research and Training

Preface

The National Council of Educational Research and Training (NCERT) had constituted 21 Focus Groups on Teaching of various subjects related to School Education, to review the National Curriculum Framework for School Education - 2000 (NCFSE - 2000) in face of new emerging challenges and transformations occurring in the fields of content and pedagogy under the contexts of National and International spectrum of school education. These Focus Groups made general and specific comments in their respective areas. Consequently, based on these reports of Focus Groups, National Curriculum Framework (NCF)-2005 was developed.

NCERT designed the new syllabi and constituted Textbook Development Teams for Classes XI and XII to prepare textbooks in mathematics under the new guidelines and new syllabi. The textbook for Class XI is already in use, which was brought in 2005.

The first draft of the present book (Class XII) was prepared by the team consisting of NCERT faculty, experts and practicing teachers. The draft was refined by the development team in different meetings. This draft of the book was exposed to a group of practicing teachers teaching mathematics at higher secondary stage in different parts of the country, in a review workshop organised by the NCERT at Delhi. The teachers made useful comments and suggestions which were incorporated in the draft textbook. The draft textbook was finalised by an editorial board constituted out of the development team. Finally, the Advisory Group in Science and Mathematics and the Monitoring Committee constituted by the HRD Ministry, Government of India have approved the draft of the textbook.

In the fitness of things, let us cite some of the essential features dominating the textbook. These characteristics have reflections in almost all the chapters. The existing textbook contain 13 main chapters and two appendices. Each Chapter contain the followings:

- Introduction: Highlighting the importance of the topic; connection with earlier studied topics; brief mention about the new concepts to be discussed in the chapter.
- Organisation of chapter into sections comprising one or more concepts/sub concepts.
- Motivating and introducing the concepts/sub concepts. Illustrations have been provided wherever possible.

- Proofs/problem solving involving deductive or inductive reasoning, multiplicity of approaches wherever possible have been inducted.
- Geometric viewing / visualisation of concepts have been emphasised whenever needed.
- Applications of mathematical concepts have also been integrated with allied subjects like science and social sciences.
- Adequate and variety of examples/exercises have been given in each section.
- For refocusing and strengthening the understanding and skill of problem solving and applicabilities, miscellaneous types of examples/exercises have been provided involving two or more sub concepts at a time at the end of the chapter. The scope of challenging problems to talented minority have been reflected conducive to the recommendation as reflected in NCF-2005.
- For more motivational purpose, brief historical background of topics have been provided at the end of the chapter and at the beginning of each chapter relevant quotation and photograph of eminent mathematician who have contributed significantly in the development of the topic undertaken, are also provided.
- Lastly, for direct recapitulation of main concepts, formulas and results, brief summary of the chapter has also been provided.

I am thankful to Professor Krishan Kumar, Director, NCERT who constituted the team and invited me to join this national endeavor for the improvement of mathematics education. He has provided us with an enlightened perspective and a very conducive environment. This made the task of preparing the book much more enjoyable and rewarding. I express my gratitude to Professor J.V. Narlikar, Chairperson of the Advisory Group in Science and Mathematics, for his specific suggestions and advice towards the improvement of the book from time to time. I, also, thank Prof. G. Ravindra, Joint Director, NCERT for his help from time to time.

I express my sincere thanks to Professor Hukum Singh, Chief Coordinator and Head DESM, Dr. V. P. Singh, Coordinator and Professor S. K. Singh Gautam who have been helping for the success of this project academically as well as administratively. Also, I would like to place on records my appreciation and thanks to all the members of the team and the teachers who have been associated with this noble cause in one or the other form.

PAWAN K. JAIN
Chief Advisor

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THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a ¹**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the ²[unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

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Constitution of India

Part IV A (Article 51 A)

Fundamental Duties

It shall be the duty of every citizen of India —

- (a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;
- (b) to cherish and follow the noble ideals which inspired our national struggle for freedom;
- (c) to uphold and protect the sovereignty, unity and integrity of India;
- (d) to defend the country and render national service when called upon to do so;
- (e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;
- (f) to value and preserve the rich heritage of our composite culture;
- (g) to protect and improve the natural environment including forests, lakes, rivers, wildlife and to have compassion for living creatures;
- (h) to develop the scientific temper, humanism and the spirit of inquiry and reform;
- (i) to safeguard public property and to abjure violence;
- (j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;
- * (k) who is a parent or guardian, to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

Note: The Article 51A containing Fundamental Duties was inserted by the Constitution (42nd Amendment) Act, 1976 (with effect from 3 January 1977).

* (k) was inserted by the Constitution (86th Amendment) Act, 2002 (with effect from 1 April 2010).

Contents

PART I

<i>Foreword</i>	<i>v</i>
<i>Preface</i>	<i>vii</i>
1. Relations and Functions	1
1.1 Introduction	1
1.2 Types of Relations	2
1.3 Types of Functions	7
1.4 Composition of Functions and Invertible Function	12
1.5 Binary Operations	19
2. Inverse Trigonometric Functions	33
2.1 Introduction	33
2.2 Basic Concepts	33
2.3 Properties of Inverse Trigonometric Functions	42
3. Matrices	56
3.1 Introduction	56
3.2 Matrix	56
3.3 Types of Matrices	61
3.4 Operations on Matrices	65
3.5 Transpose of a Matrix	83
3.6 Symmetric and Skew Symmetric Matrices	85
3.7 Elementary Operation (Transformation) of a Matrix	90
3.8 Invertible Matrices	91
4. Determinants	103
4.1 Introduction	103
4.2 Determinant	103
4.3 Properties of Determinants	109
4.4 Area of a Triangle	121
4.5 Minors and Cofactors	123
4.6 Adjoint and Inverse of a Matrix	126
4.7 Applications of Determinants and Matrices	133

5. Continuity and Differentiability	147
5.1 Introduction	147
5.2 Continuity	147
5.3 Differentiability	161
5.4 Exponential and Logarithmic Functions	170
5.5 Logarithmic Differentiation	174
5.6 Derivatives of Functions in Parametric Forms	179
5.7 Second Order Derivative	181
5.8 Mean Value Theorem	184
6. Application of Derivatives	194
6.1 Introduction	194
6.2 Rate of Change of Quantities	194
6.3 Increasing and Decreasing Functions	199
6.4 Tangents and Normals	206
6.5 Approximations	213
6.6 Maxima and Minima	216
Appendix 1: Proofs in Mathematics	247
A.1.1 Introduction	247
A.1.2 What is a Proof?	247
Appendix 2: Mathematical Modelling	256
A.2.1 Introduction	256
A.2.2 Why Mathematical Modelling?	256
A.2.3 Principles of Mathematical Modelling	257
Answers	268
Supplementary Material	286