Chapter 15 Waves

Wave Motion

- The motion of a disturbance from one point to another by the vibrations of the particles of the medium about their mean position is known as wave motion.
- It is a mode of transfer of energy from one point to another.
- The waves are mainly of three types: (a) mechanical waves, (b) electromagnetic waves and (c) matter waves.

Mechanical waves

- Exist only within a material medium, such as water, air, and rock
- examples : – water waves, sound waves, seismic waves, etc
- two types : –

1) transverse waves

2) longitudinal waves

Electromagnetic waves

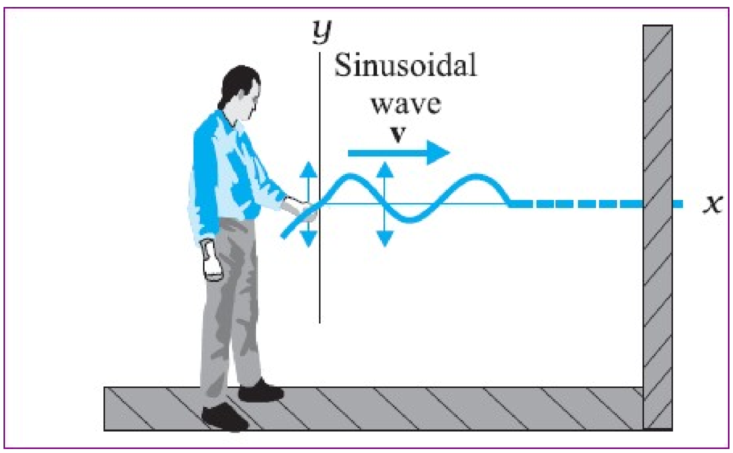
- The electromagnetic waves do not require any medium for their propagation.
- All electromagnetic waves travel through vacuum at the same speed c, given by c = 299, 792,458 m s–1
- Examples of electromagnetic waves are visible and ultraviolet light, radio waves, microwaves, x-rays etc.

Matter Waves

- Matter waves are associated with moving electrons, protons, neutrons and other fundamental particles, and even atoms and molecules
- Matter waves associated with electrons are employed in electron microscopes

Transverse Waves

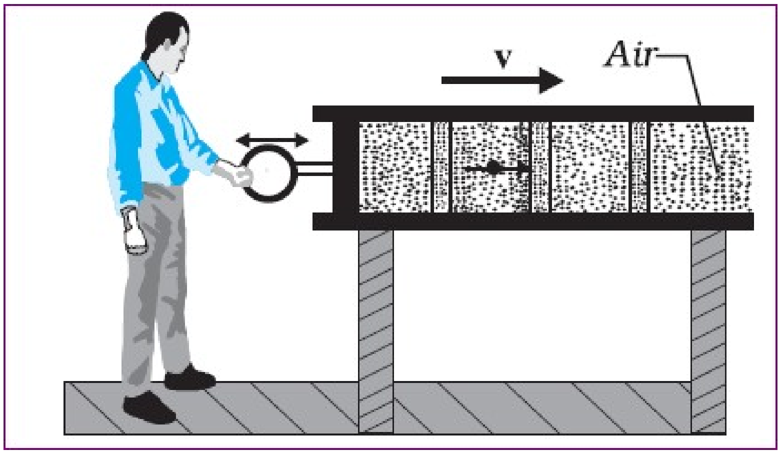
- In transverse waves, the constituents of the medium oscillate perpendicular to the direction of wave propagation.
- A point of maximum positive displacement in a wave is called crest, and a point of maximum negative displacement is called trough.



- Transverse waves can be propagated only through solids and strings, and not in fluids.

Longitudinal Waves

- In longitudinal waves the constituents of the medium oscillate along the direction of wave propagation.



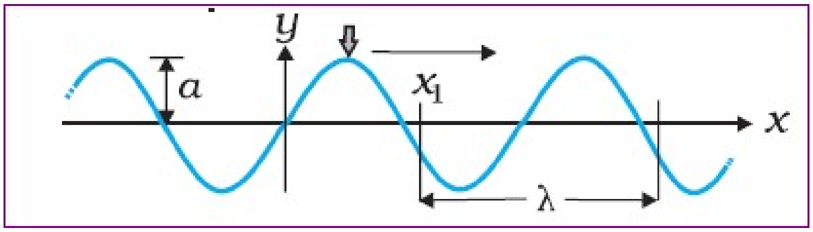
- Longitudinal sound waves propagates as compressions(high pressure region) and rarefactions(low pressure regions)
- longitudinal waves can propagate in all elastic media (solids and fluids)
- transverse and longitudinal waves travel with different speeds in the same medium.

The waves on the surface of water

- The waves on the surface of water are of two kinds: capillary waves and gravity waves.
- Capillary waves are ripples of short wavelength.
- The restoring force that produces capillary waves is the surface tension of water.
- Gravity waves have wavelengths typically ranging from several metres to several hundred metres.
- The restoring force that produces gravity waves is the pull of gravity, which tends to keep the water surface at its lowest level.
- The waves in an ocean are a combination of both longitudinal and transverse waves.

Travelling or progressive wave

- A wave which travels from one point of the medium to another is called a travelling wave.

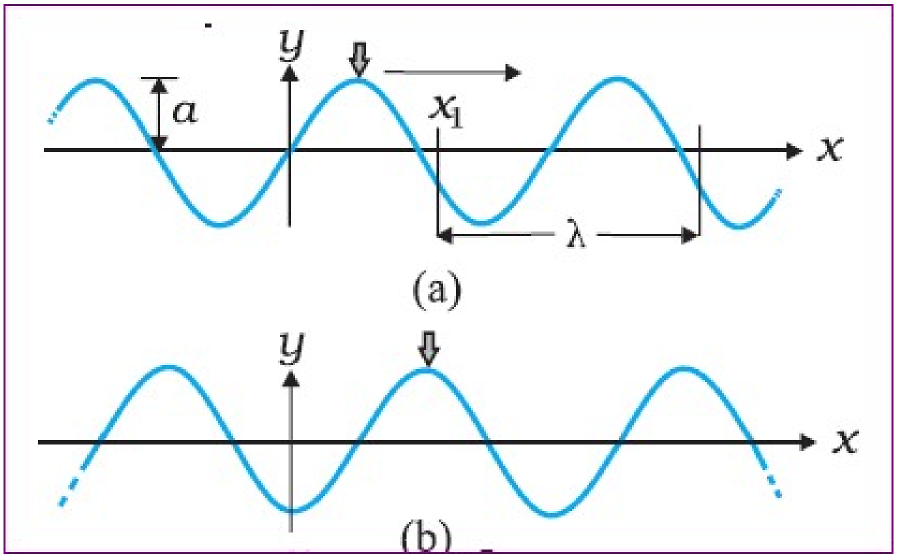


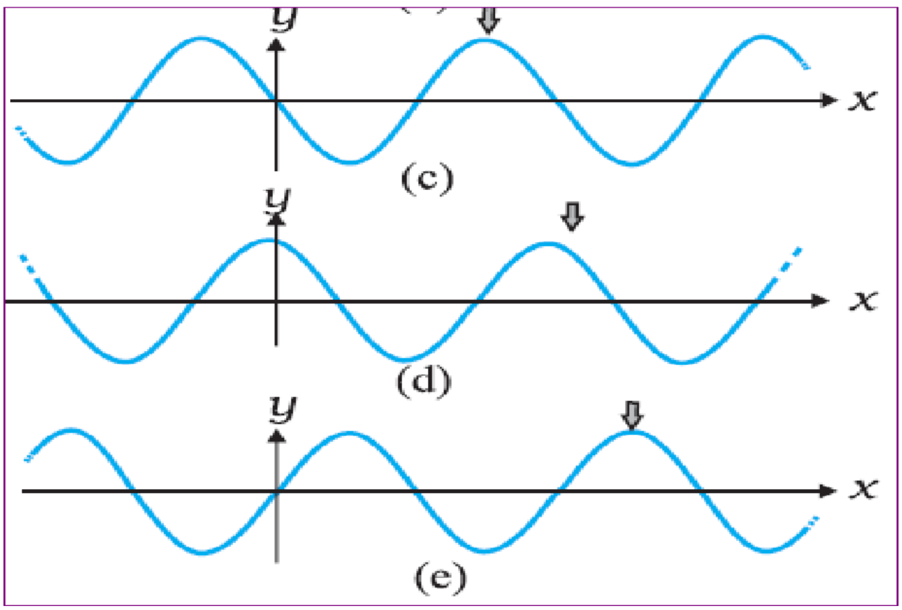
Displacement Relation in a Progressive Wave

- At any time t, the displacement of a wave travelling in positive x-axis is given by

$$y(x, t) = a \sin(kx - \omega t + \phi)$$

- Where , a- amplitude , k- angular wave number or propagation constant , ω- angular frequency , φ- initial phase angle and (kx- ωt+ φ) – phase Plots for a wave travelling in the positive direction of an x-axis at different values of timet.





- A wave travelling in the negative direction of x-axis can be represented by

$$y(x, t) = a \sin(kx + \omega t + \phi)$$

Amplitude

- The amplitude a of a wave is the magnitude of the maximum displacement of the elements from their equilibrium positions as the wave passes through them.
- It is a positive quantity, even if the displacement is negative.

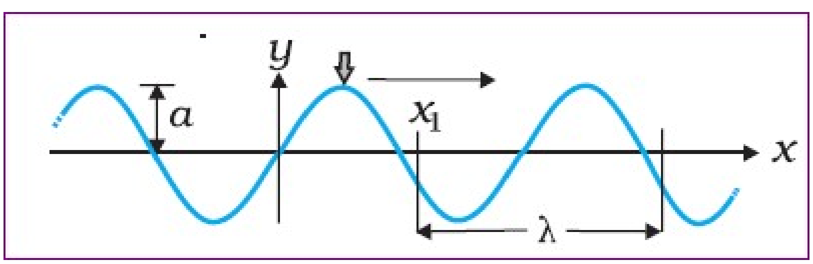
Phase

- It describes the state of motion as the wave sweeps through a string element at a particular position x
- The constant φ is called the initial phase angle.

The value of φ is determined by the initial (t = 0) displacement and velocity of the element (say, at x = 0).

Wavelength (λ)

- It is the minimum distance between two consecutive troughs or crests or two consecutive points in the same phase of wave motion.



Propagation constant or the angular wave number (k)

- For t = 0 and φ = 0

$$\boxed{y(x, 0) = a \sin kx}$$

- By definition, the displacement y is same at both ends of this wavelength, that is at x = x1 and at x = x1 + λ.
- Thus

$$\begin{aligned} a \sin k x_1 &= a \sin k (x_1 + \lambda) \\ &= a \sin (k x_1 + k \lambda) \end{aligned}$$

- This condition can be satisfied only when,

$$\boxed{k \lambda = 2\pi n}$$

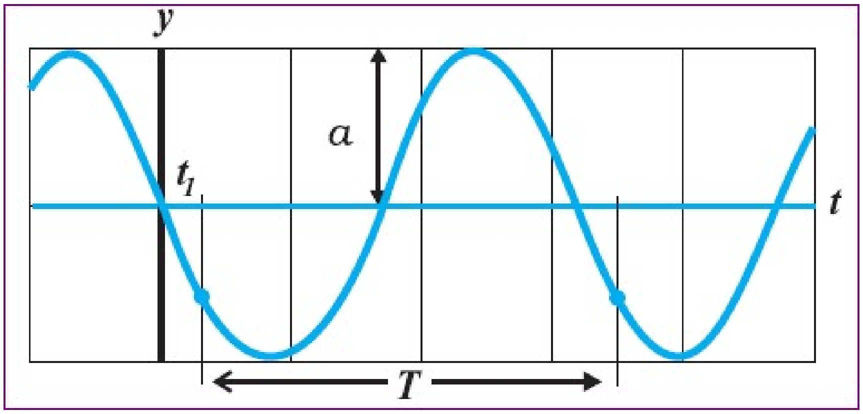
- where n = 1, 2, 3… Since λ is defined as the least distance between points with the same phase, n =1 and therefore

$$k = \frac{2\pi}{\lambda}$$

- k is called the propagation constant or the angular wave number ; its SI unit is radian per metre or rad m–1

Period

- The period of oscillation T of a wave is the time any string element takes to move through one complete oscillation.



Angular Frequency

- The angular frequency of the wave is given by

$$\omega = 2\pi / T$$

- Its SI unit is rad s-1

Frequency

- It is the number of oscillations per unit time made by a string element as the wave passes through it
- The frequency v of a wave is defined as 1/T and is related to the angular frequency ω by

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

- It is usually measured in hertz

Displacement relation of a longitudinal wave

- In a longitudinal wave, the displacement of an element of the medium is parallel to the direction of propagation of the wave.
- The displacement function for a longitudinal wave is written as,

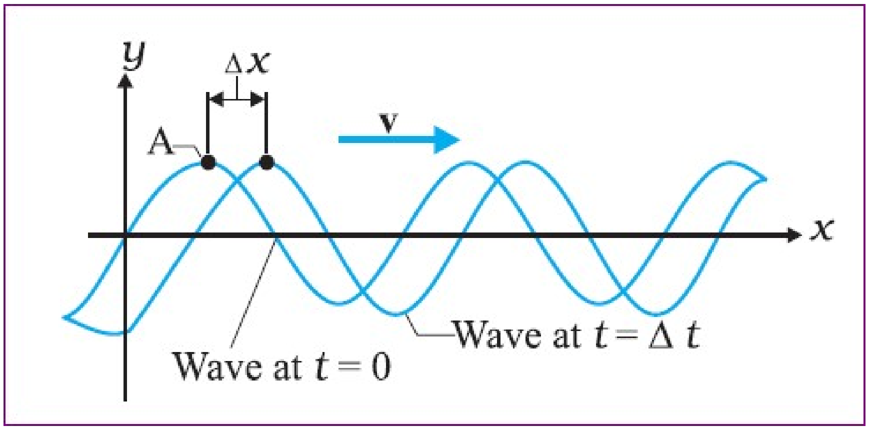
$$s(x, t) = a \sin(kx - \omega t + \phi)$$

- where s(x, t) is the displacement of an element of the medium in the direction of propagation of the wave at position x and time t.

The Speed of a Travelling Wave

- The speed of a wave is related to its wavelength and frequency by the relation

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda \nu$$



- The speed is determined by the properties of the medium.

Speed of a Transverse Wave on Stretched String

- The speed of transverse waves on a string is determined by two factors,

(i) the linear mass density or mass per unit length, μ, and (ii) (ii) the tension T.

- The linear mass density, μ, of a string is the mass m of the string divided by its length l. therefore its dimension is $[ML^{-1}]$.
- The tension T has the dimension of force $[M L T^{-2}]$.
- Let the speed $v = C \mu^a T^b$, where c is a dimensionless constant.
- Taking dimensions on both sides $[M^0L^1T^{-1}] = [M^1L^{-1}]^a[M L T^{-2}]^b =[M^{a+b}L^{-a+b}T^{-2b}]$
- Equating the dimensions on both sides we get a+b = 0 , therefore a=-b, -a+b = 1, therefore 2b=1 or b= ½ and a= – ½
- Thus

$$v = C \mu^{-\frac{1}{2}} T^{\frac{1}{2}},$$

or

$$\boxed{v = C\sqrt{\frac{T}{\mu}}}$$

- It can be shown that C=1, therefore the speed of transverse waves on a stretched string is

$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

- The speed of a wave along a stretched ideal string depends only on the tension and the linear mass density of the string and does not depend on the frequency of the wave.

Speed of a Longitudinal Wave – Speed of Sound

- In a longitudinal wave the constituents of the medium oscillate forward and backward in the direction of propagation of the wave.
- The sound waves travel in the form of compressions and rarefactions of small volume elements of air.
- The speed of sound waves depends on

1. Bulk modulus , B and
2. Density of the medium, ρ

- Using dimensional analysis we may write

v = C Ba ρ b

- Taking dimensions [M0L1T-1] = [ML-1T-2]a [M L-3] b =[Ma+b L-a-3 bT-2a]

- Equating the dimensions on both sides we get

a+b = 0 , therefore a=-b, -2a=-1, a=1/2 , therefore b=-1/2

- Therefore

$$\boxed{v = C\sqrt{\frac{B}{\rho}}}$$

- where C is a dimensionless constant and can be shown to be unity.
- Thus, the speed of longitudinal waves in a medium is given by,

$$\boxed{v = \sqrt{\frac{B}{\rho}}}$$

- The speed of propagation of a longitudinal wave in a fluid therefore depends only on the bulk modulus and the density of the medium.
- The bulk modulus is given by

$$\boxed{B = -\frac{\Delta P}{\Delta V/V}}$$

- Here ΔV/V is the fractional change in volume produced by a change in pressure ΔP.

Speed of sound wave in a material of a bar

- The speed of a longitudinal wave in the bar is given by,

$$\boxed{v = \sqrt{\frac{Y}{\rho}}}$$

- where Y is the Young's modulus of the material of the bar.

Speed of sound in different media

Medium	Speed (m s^{-1})
Gases	
Air (0 °C)	331
Air (20 °C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0 °C)	1402
Water (20 °C)	1482
Seawater	1522
Solids	
Aluminium	6420
Copper	3560
Steel	5941
Granite	6000
Vulcanised Rubber	54

Newton's Formula

- In the case of an ideal gas, the relation between pressure P and volume V is given by

$$\boxed{PV = Nk_BT}$$

- Therefore, for an isothermal change it follows that

$$V\Delta P + P\Delta V = 0$$

$$-\frac{\Delta P}{\Delta V / V} = P$$

- Thus B=P
- Therefore, the speed of a longitudinal wave in an ideal gas is given by,

$$v = \sqrt{\frac{P}{\rho}}$$

- This relation was first given by Newton and is known as Newton's formula. Laplace correction According to Newton's formula for the speed of sound in a medium, we get for the speed of sound in air at STP,

$$v = \left[\frac{1.01 \times 10^5 \text{ N m}^{-2}}{1.29 \text{ kg m}^{-3}}\right]^{1/2} = 280 \text{ m s}^{-1}$$

- This is about 15% smaller as compared to the experimental value of 331 m s–1
- Laplace pointed out that the pressure variations in the propagation of sound waves are adiabatic and not isothermal.
- For adiabatic processes the ideal gas satisfies the relation,

$$PV^{\gamma} = \text{constant}$$

$$\text{i.e.} \quad \Delta(PV^{\gamma}) = 0$$

$$P\gamma V^{\gamma-1} \Delta V + V^{\gamma} \Delta P = 0$$

- Thus for an ideal gas the adiabatic bulk modulus is given by,

$$B_{ad} = -\frac{\Delta P}{\Delta V / V} = \gamma P$$

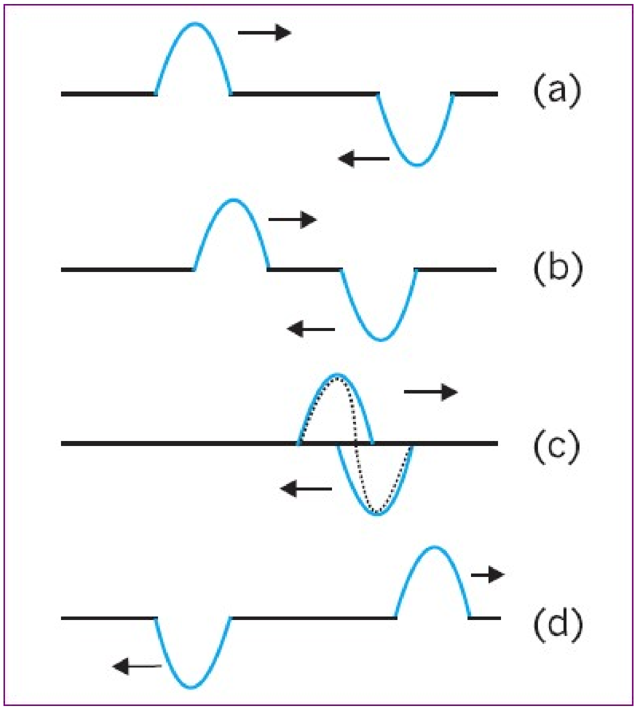
- where γ is the ratio of two specific heats, Cp/Cv.
- The speed of sound is, therefore, given by,

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

- This modification of Newton's formula is referred to as the Laplace correction.
- For air γ = 7/5,therefore the speed of sound in air at STP, we get a value 331.3 m s–1, which agrees with the measured speed.

The Principle of Superposition of Waves

- The principle of super position of waves states that the net displacement at a given time of a number of waves is the algebraic sum of the displacements due to each wave.



- Let y1(x, t) and y2(x, t) be the displacements that any element of the string would experience if each wave travelled alone.
- The displacement y (x,t) of an element of the string when the waves overlap is then given by,

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

- Let a wave travelling along a stretched string be given by,

$$y_1(x, t) = a \sin(kx - \omega t)$$

- And another wave, shifted from the first by a phase φ,

$$y_2(x, t) = a \sin(kx - \omega t + \phi)$$

- Both the waves have the same angular frequency, same angular wave number k (same wavelength) and the same amplitude a.
- Applying the superposition principle

$$y(x, t) = a \sin(kx - \omega t) + a \sin(kx - \omega t + \phi)$$

- Using the trigonometric relation

$$\omega = \sqrt{\frac{mgL}{I}}$$

$$y(x, t) = [2a \cos \frac{1}{2}\phi] \sin(kx - \omega t + \frac{1}{2}\phi)$$

- Thus, the resultant wave is also a sinusoidal wave, travelling in the positive direction of x-axis.
- The resultant wave differs from the constituent waves in two respects:

i) its phase angle is (½)φ and

II) its amplitude is the quantity given by

$$\boxed{A(\phi) = 2a\cos(½)\phi}$$

- If φ = 0,the amplitude of the resultant wave is 2a, which is the largest possible value of A(φ).
- If φ = π, the two waves are completely out of phase, the amplitude of the resultant reduces to zero.

REFLECTION OF WAVES

- When a pulse or a travelling wave encounters a rigid boundary it gets reflected.
- If the boundary is not completely rigid or is an interface between two different elastic media, a part of the wave is reflected and a part is transmitted into the second medium.
- The incident and refracted waves obey Snell's law of refraction, and the incident and reflected waves obey the laws of reflection.
- A travelling wave, at a rigid boundary or a closed end, is reflected with a phase reversal.
- A travelling wave ,at an open boundary is reflected without any phase change.
- Let the incident wave be represented by

$$\boxed{y_i(x, t) = a\sin(kx - \omega t)}$$

- then, for reflection at a rigid boundary the reflected wave is represented by,

$$\boxed{\begin{aligned} y_r(x, t) &= a \sin(kx + \omega t + \pi) \\ &= -a \sin(kx + \omega t) \end{aligned}}$$

- For reflection at an open boundary, the reflected wave is represented by

$$\boxed{y_r(x, t) = a \sin(kx + \omega t)}$$

Standing Waves and Normal Modes

- The waveform or the disturbance does not move to either side is known as stationary wave or standing wave.
- Let the wave travelling in the positive direction of x-axis be

$$\boxed{y_1(x, t) = a \sin(kx - \omega t)}$$

- And the wave travelling in the negative direction of x-axis

$$\boxed{y_2(x, t) = a \sin(kx + \omega t)}$$

- The principle of superposition gives, for the combined wave

$$\boxed{\begin{aligned} y(x, t) &= y_1(x, t) + y_2(x, t) \\ &= a \sin(kx - \omega t) + a \sin(kx + \omega t) \\ &= (2a \sin kx) \cos \omega t \end{aligned}}$$

- The amplitude is zero for values of kx that give sin kx = 0 . Those values are given by

$$\boxed{kx = n\pi, \text{ for } n = 0, 1, 2, 3, \ldots}$$

- Substituting k = 2π/λ in this equation, we get

$$\boxed{x = n\frac{\lambda}{2}, \text{ for } n = 0, 1, 2, 3, \ldots}$$

Nodes

- The positions of zero amplitude in a standing wave are called nodes.
- A distance of λ/2 or half a wavelength separates two consecutive nodes.
- The amplitude has a maximum value of 2a, which occurs for the values of kx that give | sin k x | = 1.
- The values are

$$\boxed{kx = (n + ½)\pi \text{ for } n = 0, 1, 2, 3, \ldots}$$

- Substituting k = 2π/λ in this equation, we get

$$\boxed{x = (n + ½)\frac{\lambda}{2} \text{ for } n = 0, 1, 2, 3, \ldots}$$

Antinodes

♦ The positions of maximum amplitude are called antinodes.

♦ The antinodes are separated by λ/2 and are located half way between pairs of nodes.

Standing waves of Stretched Rings

- For a stretched string of length L, fixed at both ends, the two ends of the string have to be nodes.
- If one of the ends is chosen as position x = 0, then the other end is x = L. In order that this end is a node; the length L must satisfy the condition

$$\boxed{L = n\,\frac{\lambda}{2}, \text{ for } n = 1, 2, 3, \ldots}$$

The standing waves on a string of length L have restricted wavelength given by

$$\boxed{\lambda = \frac{2L}{n}, \text{ for } n = 1, 2, 3, \ldots \text{ etc.}}$$

- The frequencies corresponding to these wavelengths is given by

$$\boxed{\nu = n\frac{v}{2L}, \text{ for } n = 1, 2, 3, \dots \text{ etc.}}$$

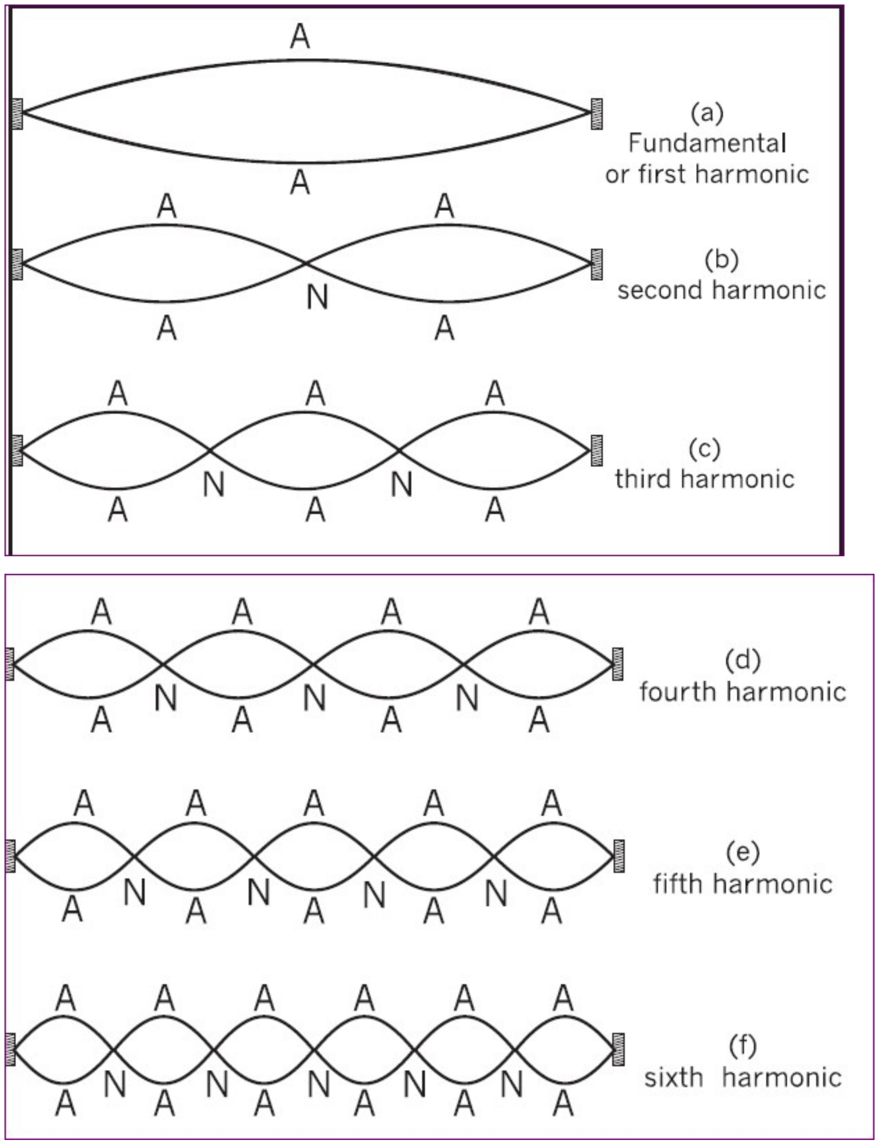
- where v is the speed of travelling waves on the string.
- The set of frequencies possible in a standing wave are called the natural frequencies or modes of oscillation of the system.
- The frequency corresponding to n=1 is

$$\boxed{\nu = \frac{v}{2L}}$$

- The oscillation mode with this lowest frequency (n=1) is called the fundamental mode or the first harmonic.
- The second harmonic is the oscillation mode with n = 2. The third harmonic corresponds to n = 3 and so on.

The frequencies associated with these modes are often labelled as v1, v2, v3 and so on.

- The collection of all possible modes is called the harmonic series and n is called the harmonic number.



Modes of vibration of a pipe closed at one end

- In a closed pipe standing waves are formed such that a node at the closed end and antinode at open end.
- Now if the length of the air column is L, then the open end, x = L, is an antinode and therefore,

$$L = (n + \frac{1}{2})\frac{\lambda}{2}$$

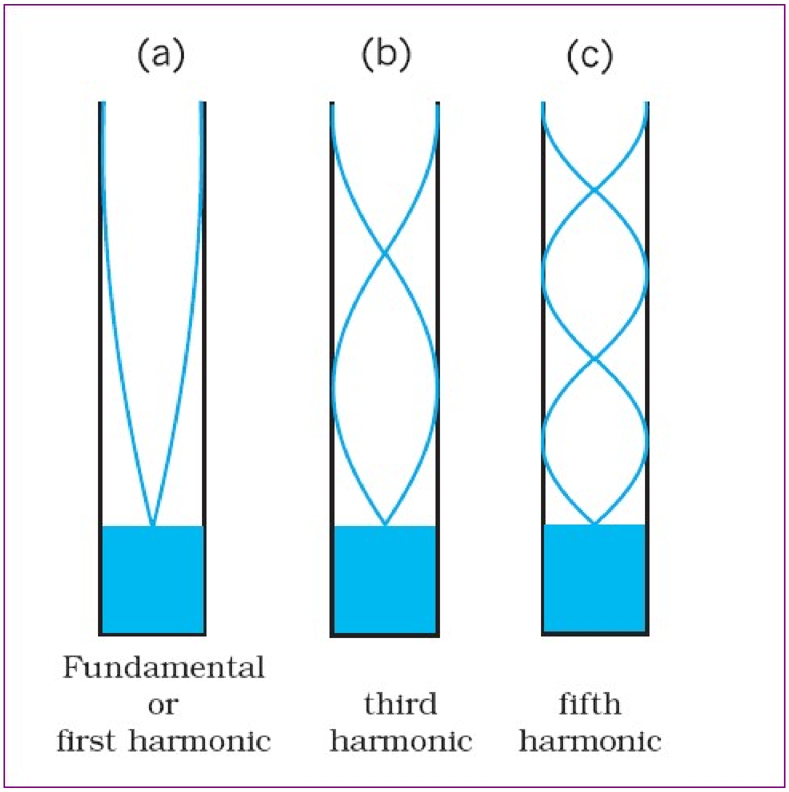
- Where n=0,1,2,3….
- The modes, which satisfy the condition

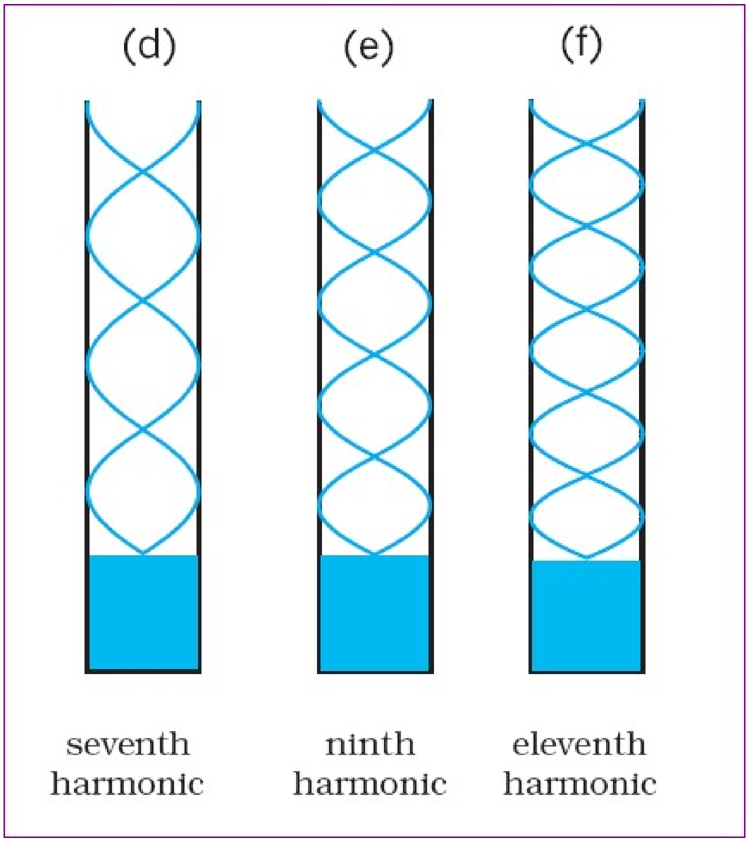
$$\boxed{\lambda = \frac{2L}{(n + 1/2)}, \text{ for } n = 0, 1, 2, 3, ...}$$

- The corresponding frequencies of various modes of such an air column are given by,

$$\boxed{\nu = (n + \frac{1}{2})\frac{v}{2L}, \text{ for } n = 0, 1, 2, 3, ...}$$

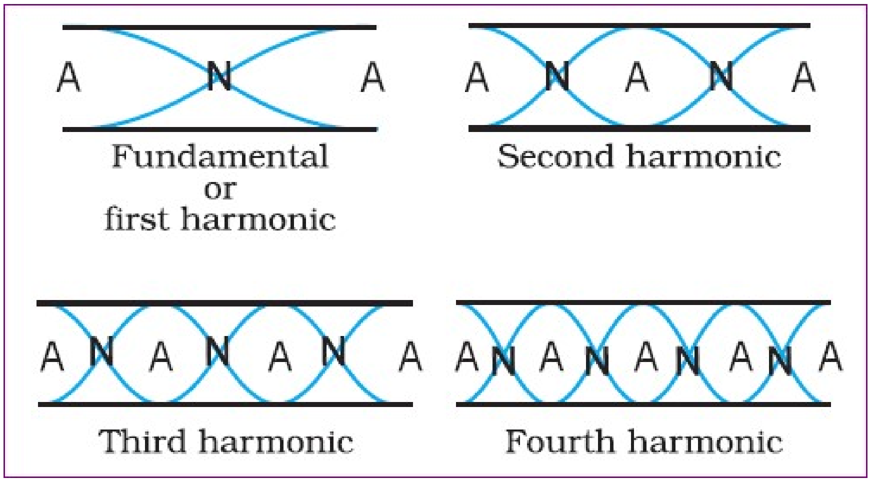
- The fundamental frequency is v/4L and the higher frequencies are odd harmonics of the fundamental frequency, i.e. 3 v/4L,5 v/4L,…


(a)
(b)
(c)
Fundamental
or
first harmonic
third
harmonic
fifth
harmonic



Pipe open at both ends

- In the case of a pipe open at both ends, there will be antinodes at both ends, and all harmonics will be generated.

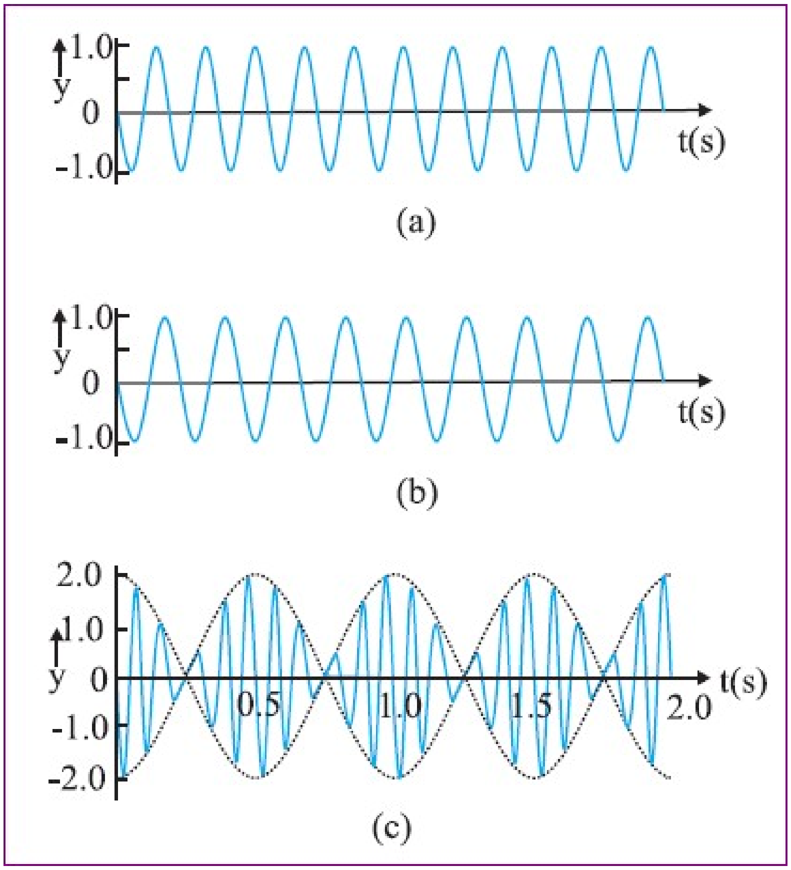


BEATS

- The phenomenon of wavering of sound intensity when two waves of nearly same frequencies and amplitudes travelling in the same direction, are superimposed on each other is called beats.
- The beat frequency, is given by

$$\boxed{\nu_{beat} = \nu_1 - \nu_2}$$

The time-displacement graphs of two waves of frequency 11 Hz and 9 Hz



- Musicians use the beat phenomenon in tuning their instruments.
- If an instrument is sounded against a standard frequency and tuned until the beat disappears, then the instrument is in tune with that standard.

Doppler Effect

- ***The apparent change in the pitch or the frequency of sound produced by a source due to relative motion of the source, listener or the medium is called Doppler effect.***

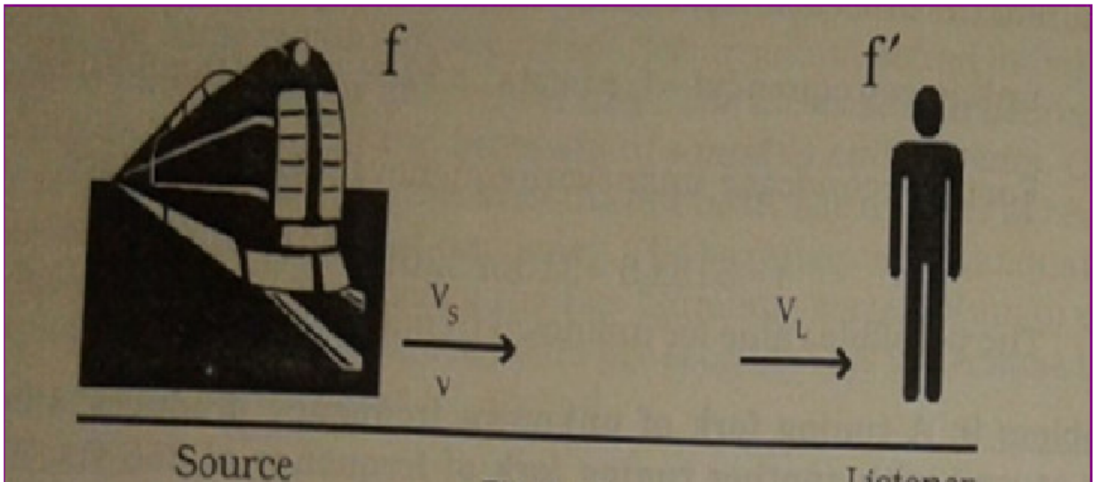


Fig 11.17

- It was proposed by Christian Doppler and tested experimentally by Buys Ballot
- All types of waves shows Doppler effect.
- S- source
- f – frequency of sound from source
- V – velocity of sound
- λ- wavelength

When Source and Listener at Rest

- When the source and the listener are at rest , the frequency of sound heard by the listener

$$f = \frac{V}{\lambda} \quad \text{or} \quad \lambda = \frac{V}{f}$$

When source and listener moving in the direction of sound

- The relative velocity of sound wave with respect to source is V – Vs

- Vs – velocity of source
- Thus, apparent wavelength is

$$\lambda' = \frac{V - V_S}{f}$$

- The relative velocity of sound with respect to listener is *V ‘ = −V VL*
- The apparent frequency of sound heard by the listener is

$$f' = \frac{V'}{\lambda'}$$

- Thus

$$f' = \frac{V - V_L}{\frac{V - V_S}{f}} = f\,\frac{(V - V_L)}{(V - V_S)}$$

Special cases

Source moving and listener stationary

a) Source moves towards the listener

- Now VS = +ve , VL= 0

- Thus $$f' = f\left(\frac{V}{V - V_S}\right)$$

b) Source moves away from the listener

- Now VS = – ve , VL= 0

- Thus $f' = f\left(\frac{V}{V + V_S}\right)$

Source stationary, listener moving

a) Listener moves towards the source

- Now VL = – ve , VS= 0

- Thus $f' = f\left(\frac{V + V_L}{V}\right)$

b) Listener moves away from the source

- Now VL = + ve , VS= 0

- Thus $f' = f\left(\frac{V - V_L}{V}\right)$

Both source and listener moving

a) Source and listener move towards each other

- Now VS = +ve, VL = -ve

- Thus $f' = f\left(\dfrac{V - V_L}{V - V_S}\right)$

b) Source and listener move away from each other

  - Now VS = -ve, VL = +ve

- **Thus** $f' = f\left(\dfrac{V - V_L}{V + V_S}\right)$

c) Source moves towards the listener and listener moves away

  - Now VS = +ve, VL = +ve

- Thus $f' = f\left(\dfrac{V - V_L}{V - V_S}\right)$

1. Source moves away from the listener and listener moves towards the source
   - Now VS = -ve , VL = -ve

- **Thus** $f' = f\left(\frac{V+V_L}{V+V_S}\right)$

Effect of motion of the medium

- When the wind blows the air medium will moves with a velocity w

When wind moves towards the listener the velocity of sound is V+ w

- Thus, the apparent frequency

$$f' = f\left(\frac{(V+w)-V_L}{(V+w)-V_S}\right)$$

- If the wind is blowing from listener to the source , velocity of sound is V – w

- Thus $f' = f\left(\frac{(V-w)-V_L}{(V-w)-V_S}\right)$

Uses of Doppler Effect

- To estimate the speed of submarine , aero plane, automobile , etc
- To track artificial satellite
- To estimate velocity and rotation of star
- Doctors use it to study heart beats and blood flow in different part of

the body. Here they use ultrasonic waves, and in common practice, it is called sonography.

- In the case of the heart, the picture generated is called echocardiogram.