Exercise 25b

Question 1.

If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors the x + y + z = ?

Answer:

Two vectors are equal if and only if their corresponding components are equal.

Thus, the given vectors $\vec{a}$ and $\vec{b}$ are equal if and only if

x = 3, - y = 2, - z = 1

x = 3, y = - 2, z = - 1

x + y + z = 3 + (- 2) + (- 1) = 3 - 3 = 0

Question 2.

If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors the x + y + z = ?

Answer:

$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{j} + \hat{k}$

$\vec{a} = x\hat{\imath} + 2\hat{\jmath} - z\hat{k}$

$\vec{b} = 3\hat{\imath} - y\hat{\jmath} + \hat{k}$

Since, these two vectors are equal, therefore comparing these two vectors we get,

x = 3 , - y = 2 , - z = 1

⇒x = 3,y = - 2,z = - 1

∴x + y + z = 3 + (- 2) + (- 1) = 3 - 2 - 1 = 0

Ans:x + y + z = 0

Question 3.

Write a unit vector in the direction of the sum of the vectors $\vec{a} = \left(2\hat{i} + 2\hat{j} - 5\hat{k}\right)$ and $\vec{b} = \left(2\hat{i} + \hat{j} - 7\hat{k}\right)$.

Answer:
The sum of vectors is

$$\vec{a} + \vec{b} = 2\hat{i} + 2\hat{j} - 5\hat{k} + 2\hat{i} + \hat{j} - 7\hat{k}$$

$$= 4\hat{i} + 3\hat{j} - 12\hat{k}$$

Let the unit vector in the direction of $\vec{a} + \vec{b}$ be $\hat{c}$, then

$$\hat{c} = \frac{\left(\vec{a} + \vec{b}\right)}{\left|\vec{a} + \vec{b}\right|} \Rightarrow \frac{\left(4\hat{i} + 3\hat{j} - 12\hat{k}\right)}{\left|4\hat{i} + 3\hat{j} - 12\hat{k}\right|} = \frac{4\hat{i} + 3\hat{j} - 12\hat{k}}{\sqrt{\left(4^2 + 3^2 + (-12)^2\right)}}$$

$$\Rightarrow \frac{4\hat{i} + 3\hat{j} - 12\hat{k}}{\sqrt{169}} = \frac{1}{13}\left(4\hat{i} + 3\hat{j} - 12\hat{k}\right)$$

Question 4.

Write a unit vector in the direction of the sum of the vectors $\vec{a} = \left(2\hat{i} + 2\hat{j} - 5\hat{k}\right)$ and $\vec{b} = \left(2\hat{i} + \hat{j} - 7\hat{k}\right)$.

Answer:
Let $\vec{s}$ be the sum of the vectors $\vec{a}$ *and* $\vec{b}$

$$\Rightarrow \vec{s} = \vec{a} + \vec{b}$$

$$\Rightarrow \vec{s} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k} + 2\hat{\imath} + \hat{\jmath} - 7\hat{k}$$

$$\Rightarrow \vec{s} = 4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}$$

$|\vec{s}| = (4^2 + 3^2 + (-12)^2)^{1/2}$

$\Rightarrow |\vec{s}| = (16 + 9 + 144)^{1/2} = (169)^{1/2} = 13$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{s} = \frac{\vec{s}}{|\vec{s}|} = \frac{4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}}{13}$$

Ans: $\hat{s} = \frac{4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}}{13}$

Question 5.

Write the value of λ so that the vectors $\vec{a} = \left(2\hat{i} + \lambda\hat{j} + \hat{k}\right)$ and $\vec{b} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$ are perpendicular to each other.

Answer:

If the scalar product (dot product) is zero, two non - zero vectors are perpendicular.

$$\vec{a}.\vec{b} \Rightarrow \left(2\hat{i} + \lambda\hat{j} + \hat{k}\right).\left(\hat{i} - 2\hat{j} + 3\hat{k}\right) = 0$$

$$\Rightarrow 2.1 + \lambda.(-2) + 1.3 = 0\left(\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1\right)$$

2 - 2λ + 3 = 0

- 2λ = 5

$$\lambda = \frac{5}{2}$$

Question 6.

Write the value of λ so that the vectors $\vec{a} = \left(2\hat{i} + \lambda\hat{j} + \hat{k}\right)$ and $\vec{b} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$ are perpendicular to each other.

Answer:

$$\vec{a} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$$

$$\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$$

Since these two vectors are perpendicular the dot product of these two vectors is zero.

i.e.: $\vec{a}.\vec{b}$ = 0

$\Rightarrow(2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}).(\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) = 0$

⇒2 + λ×(- 2) + 3 = 0

⇒5 = 2 λ

⇒ λ = 5/2

Ans: λ = 5/2

Question 7.

Find the value of p for which the vectors $\vec{a} = \left(3\hat{i} + 2\hat{j} + 9\hat{k}\right)$ and $\vec{b} = \left(\hat{i} - 2p\hat{j} + 3\hat{k}\right)$ are parallel.

Answer:

$\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$

$\vec{b} = \hat{\imath} - 2p\hat{\jmath} + 3\hat{k}$

Since these two vectors are parallel

$\therefore \frac{3}{1} = \frac{2}{-2p} = \frac{9}{3}$

$\Rightarrow \frac{3}{1} = \frac{1}{-p}$

$\Rightarrow p = \frac{-1}{3}$

Ans: $p = \frac{-1}{3}$

Question 8.

Find the value of p for which the vectors $\vec{a} = \left(3\hat{i} + 2\hat{j} + 9\hat{k}\right)$ and $\vec{b} = \left(\hat{i} - 2p\hat{j} + 3\hat{k}\right)$ are parallel.

Answer:
Two nonzero vectors are parallel if their vector product (cross product) is zero,

$$\vec{a} \times \vec{b} \Rightarrow \begin{vmatrix} i & j & k \\ 3 & 2 & 9 \\ 1 & -2p & 3 \end{vmatrix} = 0$$

$$\Rightarrow \left(2.3 - (-2p).9\right)\hat{i} - \left(3.3 - 9.1\right)\hat{j} + \left(3.(-2p) - 1.2\right)\hat{k} = 0$$

$$\Rightarrow \left(6 + 18p\right)\hat{i} - \left(9 - 9\right)\hat{j} + \left(-6p - 2\right)\hat{k} = 0$$

$$\Rightarrow \left(6 + 18p\right)\hat{i} - 0.\hat{j} + \left(-6p - 2\right)\hat{k} = 0 \Rightarrow 0.\hat{i} - 0.\hat{j} + 0.\hat{k}$$

On comparing with right hand side,

6 + 18p = 0

$p = \frac{-6}{18} \Rightarrow -\frac{1}{3}$ (You can solve using - 6p - 2)

Question 9.

Find the value of λ when the projection of $\vec{a} = \left(\lambda\hat{i} + \hat{j} + 4\hat{k}\right)$ on $\vec{b} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right)$ is 4 units.

Answer:

Projection of vector $\vec{a}$ on vector $\vec{b} = \frac{1}{\left|\vec{b}\right|}\left(\vec{a}.\vec{b}\right)$

So we first calculate the magnitude of vector b and the scalar product of a vector $\vec{a}$ and $\vec{b}$.

$$\left|\vec{b}\right| = \sqrt{\left(2^2 + 6^2 + 3^2\right)} \Rightarrow \sqrt{\left(4 + 36 + 9\right)} = \sqrt{49} \Rightarrow 7$$

$\vec{a}.\vec{b} = (\lambda\hat{i} + \hat{j} + 4\hat{k}).(2\hat{i} + 6\hat{j} + 3\hat{k}) \Rightarrow \lambda.2 + 1.6 + 4.3 = 2\lambda + 6 + 12 = 2\lambda + 18$

Projection of vector $\vec{a}$ on vector $\vec{b} = \frac{1}{|\vec{b}|}(\vec{a}.\vec{b})$ = 4 (1)

Putting the values from above in equation (1),

$\frac{2\lambda + 18}{7} = 4 \Rightarrow 2\lambda = 28 - 18$

2λ = 10

λ = 5

Question 10.

Find the value of λ when the projection of $\vec{a} = (\lambda\hat{i} + \hat{j} + 4\hat{k})$ on $\vec{b} = (2\hat{i} + 6\hat{j} + 3\hat{k})$ is 4 units.

Answer:

$\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$

$\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$

projection of a on b is given by: $\vec{a}.\hat{b}$

$|\vec{b}| = (2^2 + 6^2 + 3^2)^{1/2}$

$\Rightarrow |\vec{b}| = (4 + 36 + 9)^{1/2} = (49)^{1/2} = 7$

a unit vector in the direction of the sum of the vectors is given by:

$\hat{b} = \frac{b}{|\vec{b}|} = \frac{2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}}{7}$

Now it is given that: $\vec{a}.\hat{b} = 4$

$\Rightarrow (\lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}).\left(\frac{2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}}{7}\right) = 4$

⇒2 λ + 6 + (3×4) = 28

⇒ λ = (28 - 12 - 6)/2

⇒ λ = 10/2 = 5

Ans: λ = 5

Question 11.
If $\vec{a}$ and $\vec{b}$ are perpendicular vectors such that $|\vec{a} + \vec{b}| = 13$ and $|\vec{a}| = 5,$ find the value of $|\vec{b}|$.

Answer:
As vector $\vec{a}$ and $\vec{b}$ is perpendicular, $\vec{a}.\vec{b} = 0$. So, using $\left(\vec{a} + \vec{b}\right)^2$

$$\left(\vec{a} + \vec{b}\right)^2 \Rightarrow \left(\vec{a} + \vec{b}\right).\left(\vec{a} + \vec{b}\right) = \left|\vec{a} + \vec{b}\right|^2 \left(\text{using } \vec{a}.\vec{a} = |\vec{a}|^2\right)$$

$$\Rightarrow \vec{a}.\vec{a} + \vec{b}.\vec{a} + \vec{a}.\vec{b} + \vec{b}.\vec{b} = 13^2$$

$$\Rightarrow |\vec{a}|^2 + 2.\vec{a}.\vec{b} + \left|\vec{b}\right|^2 = 169$$

$$\Rightarrow 5^2 + 2.0 + \left|\vec{b}\right|^2 = 169$$

$$\Rightarrow \left|\vec{b}\right|^2 = 169 - 25$$

$$\Rightarrow \left|\vec{b}\right|^2 = 144$$

$\left|\vec{b}\right| = \sqrt{144} \Rightarrow 12$ (Negative value not considered as magnitude is positive).

Question 12.

If $\vec{a}$ and $\vec{b}$ are perpendicular vectors such that $\left|\vec{a}+\vec{b}\right|=13$ and $\left|\vec{a}\right|=5,$ find the value of $\left|\vec{b}\right|$.

Answer:

Since a and b vectors are perpendicular .

$\Rightarrow\theta = \frac{\pi}{2}$

Now,

$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 13^2 = 5^2 + |\vec{b}|^2 + 0$ ...($\cos\theta = cos\frac{\pi}{2} = 0$)

$\Rightarrow|\vec{b}|^2 = 169 - 25 = 144$

$\Rightarrow|\vec{b}| = 12$

Ans:$|\vec{b}| = 12$

Question 13.

If $\vec{a}$ is a unit vector such that $\left(\vec{x}-\vec{a}\right)\cdot\left(\vec{x}+\vec{a}\right)=15,$ find $\left|\vec{x}\right|$.

Answer:

$(\vec{x} - \vec{a})(\vec{x} + \vec{a}) = 15$

$\Rightarrow|\vec{x}|^2 - |\vec{a}|^2 = 15$

$\Rightarrow|\vec{x}|^2 = |\vec{a}|^2 + 15$

Now , a is a unit vector,

$\Rightarrow|\vec{a}| = 1$

$\Rightarrow|\vec{x}|^2 = 1^2 + 15$

$\Rightarrow |\vec{x}|^2 = 16$

$\Rightarrow |\vec{x}| = 4$

Ans: $|\vec{x}| = 4$

Question 14.
If $\vec{a}$ is a unit vector such that $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a}) = 15$, find $|\vec{x}|$.

Answer:
$(\vec{x}-\vec{a}).(\vec{x}+\vec{a}) \Rightarrow \vec{x}.\vec{x} - \vec{a}.\vec{x} + \vec{a}.\vec{x} - \vec{a}.\vec{a} = 15$

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 15$ (Using $\vec{x}.\vec{a} = \vec{a}.\vec{x}$, commutative law)

$\Rightarrow |\vec{x}|^2 - 1^2 = 15$ (As magnitude of unit vector is 1)

$\Rightarrow |\vec{x}|^2 = 15 + 1$

$\Rightarrow |\vec{x}| = \sqrt{16} \Rightarrow 4$

Question 15.
Find the sum of the vectors $\vec{a} = (\hat{i} - 3\hat{k}), \vec{b} = (2\hat{j} - \hat{k})$ and $\vec{c} = (2\hat{i} - 3\hat{j} + 2\hat{k})$.

Answer:
$\vec{a} + \vec{b} + \vec{c} = \hat{i} - 3\hat{k} + 2\hat{j} - \hat{k} + 2\hat{i} - 3\hat{j} + 2\hat{k}$

$= (1+2)\hat{i} + (2-3)\hat{j} + (-3-1+2)\hat{k}$

$= 3\hat{i} - \hat{j} - 2\hat{k}$

Question 16.
Find the sum of the vectors $\vec{a} = (\hat{i} - 3\hat{k}), \vec{b} = (2\hat{j} - \hat{k})$ and $\vec{c} = (2\hat{i} - 3\hat{j} + 2\hat{k})$.

Answer:
$\vec{a} = \hat{\imath} - 3\hat{k}$

$\vec{b} = 2\hat{\jmath} - \hat{k}$

$\vec{c} = 2\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

Now ,

$\vec{a} + \vec{b} + \vec{c} = \hat{\imath} - 3\hat{\jmath} + 2\hat{\jmath} - \hat{k} + 2\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$

Ans: $\vec{a} + \vec{b} + \vec{c} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$

Question 17.

Find the sum of the vectors $\vec{a} = (\hat{i} - 2\hat{j}), \vec{b} = (2\hat{i} - 3\hat{j})$ and $\vec{c} = (2\hat{i} + 3\hat{k})$.

Answer:
$\vec{a} + \vec{b} + \vec{c} = \hat{i} - 2\hat{j} + 2\hat{i} - 3\hat{j} + 2\hat{i} + 3\hat{k}$

$= (1 + 2 + 2)\hat{i} + (-2 - 3)\hat{j} + 3\hat{k}$

$= 5\hat{i} - 5\hat{j} + 3\hat{k}$

Question 18.

Find the sum of the vectors $\vec{a} = (\hat{i} - 2\hat{j}), \vec{b} = (2\hat{i} - 3\hat{j})$ and $\vec{c} = (2\hat{i} + 3\hat{k})$.

Answer:
$\vec{a} = \hat{\imath} - 2\hat{\jmath}$

$\vec{b} = 2\hat{\imath} - 3\hat{\jmath}$

$\vec{c} = 2\hat{\imath} + 3\hat{k}$

Now ,

$\vec{a} + \vec{b} + \vec{c} = \hat{\imath} - 2\hat{\jmath} + 2\hat{\imath} - 3\hat{\jmath} + 2\hat{\imath} + 3\hat{k}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 5\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$

Ans: $\vec{a} + \vec{b} + \vec{c} = 5\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$

Question 19.
Write the projection of the vector $\left(\hat{i} + \hat{j} + \hat{k}\right)$ along the vector $\hat{j}$.

Answer:
projection of a on b is given by: $\vec{a}.\hat{b}$

$\therefore$ the projection of the vector $\left(\hat{i} + \hat{j} + \hat{k}\right)$ along the vector $\hat{j}$.is :

$(\hat{\imath} + \hat{\jmath} + \hat{k}).\hat{\jmath} = 0 + 1 + 0 = 1$

Ans: the projection of the vector $\left(\hat{i} + \hat{j} + \hat{k}\right)$ along the vector $\hat{j}$.is:1

Question 20.
Write the projection of the vector $\left(\hat{i} + \hat{j} + \hat{k}\right)$ along the vector $\hat{j}$.

Answer:

Projection of vector $\vec{a}$ on vector $\vec{b} = \frac{1}{\left|\vec{b}\right|}\left(\vec{a}.\vec{b}\right)$.

$= \frac{1}{1}\left(\left(\hat{i} + \hat{j} + \hat{k}\right).\hat{j}\right)$

$= 0 + 1 + 0\left(\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0\right)$ = 1

Question 21.

Write the projection of the vector $\left(7\hat{i}+\hat{j}-4\hat{k}\right)$ on the vector $\left(2\hat{i}+6\hat{j}+3\hat{k}\right)$.

Answer:

$\vec{a} = 7\hat{\imath} + \hat{\jmath} - 4\hat{k}$

$\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$

projection of a on b is given by: $\vec{a}.\hat{b}$

$|\vec{b}| = (2^2 + 6^2 + 3^2)^{1/2}$

$\Rightarrow|\vec{b}| = (4 + 36 + 9)^{1/2} = (49)^{1/2} = 7$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}}{7}$$

$$\vec{a}.\hat{b} = (7\hat{\imath} + \hat{\jmath} - 4\hat{k}).\left(\frac{2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}}{7}\right) = \frac{(7\times 2) + (1\times 6) - (4\times 3)}{7}$$
$$= \frac{14 + 6 - 12}{7} = \frac{8}{7}$$

Ans: the projection of the vector $\left(7\hat{i}+\hat{j}-4\hat{k}\right)$ on the vector $\left(2\hat{i}+6\hat{j}+3\hat{k}\right)$.

Question 22.

Write the projection of the vector $\left(7\hat{i}+\hat{j}-4\hat{k}\right)$ on the vector $\left(2\hat{i}+6\hat{j}+3\hat{k}\right)$.

Answer:

Projection of vector $\vec{a}$ on vector $\vec{b} = \frac{1}{|\vec{b}|}\left(\vec{a}.\vec{b}\right)$

$$= \frac{1}{\sqrt{\left(2^2 + 6^2 + 3^2\right)}}\left(\left(7\hat{i} + \hat{j} - 4\hat{k}\right).(2\hat{i} + 6\hat{j} + 3\hat{k}\right)$$

$$= \frac{1}{\sqrt{(4+36+9)}}(7.2+1.6+(-4).3)$$

$$= \frac{1}{\sqrt{49}}(14+6-12)$$

$$= \frac{1}{7}(20-12)$$

$$= \frac{8}{7}$$

Question 23.
Find $\vec{a}\cdot(\vec{b}\times\vec{c})$ when $\vec{a}=(2\hat{i}+\hat{j}+3\hat{k}), \vec{b}=(-\hat{i}+2\hat{j}+\hat{k})$ and $\vec{c}=(3\hat{i}+\hat{j}+2\hat{k})$.

Answer:
We will first find vector product of $\vec{b}$ and $\vec{c}$ then scalar product of that with $\vec{a}$.

$$\vec{b}\times\vec{c} \Rightarrow \begin{vmatrix} i & j & k \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = (2.2-1.1)\hat{i}-((-1).2-3.1)\hat{j}+((-1).1-3.2)\hat{k}$$

$$= (4-1)\hat{i}-(-2-3)\hat{j}+(-1-6)\hat{k}$$

$$= 3\hat{i}+5\hat{j}-7\hat{k}$$

$$\vec{a}.(\vec{b}\times\vec{c}) = (2\hat{i}+\hat{j}+3\hat{k}).(3\hat{i}+5\hat{j}-7\hat{k})$$

= 2.3 + 1.(5) + 3.(- 7)

= 6 + 5 - 21

= - 10

Question 24.

Find $\vec{a} \cdot \left(\vec{b} \times \vec{c}\right)$ when $\vec{a} = \left(2\hat{i} + \hat{j} + 3\hat{k}\right)$, $\vec{b} = \left(-\hat{i} + 2\hat{j} + \hat{k}\right)$ and $\vec{c} = \left(3\hat{i} + \hat{j} + 2\hat{k}\right)$.

Answer:

$\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

$\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\vec{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

$$\vec{b} \times \vec{c} = \left(-\hat{\imath} + 2\hat{\jmath} + \hat{k}\right) \times \left(3\hat{\imath} + \hat{\jmath} + 2\hat{k}\right) = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} = \hat{\imath}(4-1) - \hat{\jmath}(-2-3) + \hat{k}(-1-6) = 3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}$$

$\therefore \vec{b} \times \vec{c} = 3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}$

$\therefore \vec{a}.\left(\vec{b} \times \vec{c}\right) = \left(2\hat{\imath} + \hat{\jmath} + 3\hat{k}\right).\left(3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}\right)$ = (2×3) + (1×5) + (3× - 7)

= 6 + 5 - 21 = - 10

Ans: - 10

Question 25.

Find a vector in the direction of $\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$ which has magnitude 21 units.

Answer:

$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$

$|\vec{a}| = (2^2 + (-3)^2 + 6^2)^{1/2}$

$\Rightarrow |\vec{a}| = (4 + 9 + 36)^{1/2} = (49)^{1/2} = 7$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$$

a vector in the direction of $\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$ which has magnitude 21 units.

= $21\hat{a} = 21 \times \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7} = 3\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right) = 6\hat{i} - 9\hat{j} + 18\hat{k}$

Ans: $6\hat{i} - 9\hat{j} + 18\hat{k}$

Question 26.

Find a vector in the direction of $\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$ which has magnitude 21 units.

Answer:

First, we find the unit vector in the direction of a given vector,

$$\hat{a} \Rightarrow \frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{\left(2^2 + (-3)^2 + 6^2\right)}}$$

$$\Rightarrow \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{(4 + 9 + 36)}} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{49}}$$

$$= \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$$

Now vector in the direction of the given vector and with magnitude 21 is

$$= 21.\frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7} \Rightarrow 3\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$$

$$= 6\hat{i} - 9\hat{j} + 18\hat{k}$$

Question 27.

If $\vec{a} = \left(2\hat{i} + 2\hat{j} + 3\hat{k}\right), \vec{b} = \left(-\hat{i} + 2\hat{j} + \hat{k}\right)$ and $\vec{c} = \left(3\hat{i} + \hat{j}\right)$ are such that $\left(\vec{a} + \lambda\vec{b}\right)$ is perpendicular to then find the value of λ.

Answer:

For perpendicular vectors scalar product is zero.

$$\left(\vec{a} + \lambda\vec{b}\right).\vec{c} = 0$$

$$\left(2\hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(-\hat{i} + 2\hat{j} + \hat{k}\right)\right).\left(3\hat{i} + \hat{j}\right) = 0$$

$$\left((2-\lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}\right).\left(3\hat{i} + \hat{j}\right) = 0$$

(2 - λ).3 + (2 + 2λ).1 + (3 + λ).0 = 0

6 - 3λ + 2 + 2λ = 0

8 - λ = 0

λ = 8

Question 28.

If $\vec{a} = \left(2\hat{i} + 2\hat{j} + 3\hat{k}\right), \vec{b} = \left(-\hat{i} + 2\hat{j} + \hat{k}\right)$ and $\vec{c} = \left(3\hat{i} + \hat{j}\right)$ are such that $\left(\vec{a} + \lambda\vec{b}\right)$ is perpendicular to $\vec{c}$ then find the value of λ.

Answer:

$$\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

$$\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$$

$$\vec{c} = 3\hat{\imath} + \hat{\jmath}$$

$$\vec{a} + \lambda\vec{b} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda\left(-\hat{\imath} + 2\hat{\jmath} + \hat{k}\right)$$

$$\Rightarrow \vec{a} + \lambda\vec{b} = (2 - \lambda)\hat{\imath} + (2 + 2\lambda)\hat{\jmath} + (3 + \lambda)\hat{k}$$

Since $\vec{a} + \lambda\vec{b}\ is\ perpendicular\ to\ \vec{c}$

$\Rightarrow \overrightarrow{(a + \lambda\vec{b})}.\vec{c} = 0$

$\Rightarrow \left((2-\lambda)\hat{\imath} + (2 + 2\lambda)\hat{\jmath} + (3 + \lambda)\hat{k}\right).(3\hat{\imath} + \hat{\jmath}) = 0$

$\Rightarrow (2 - \lambda)\times 3 + (2 + 2\lambda)\times 1 = 0$

$\Rightarrow 6 + 2 - 3\lambda + 2\lambda = 0$

$\Rightarrow \lambda = 8$

Ans: $\lambda = 8$

Question 29.

Write the vector of magnitude 15 units in the direction of vector $\left(\hat{\mathrm{i}} - 2\hat{\mathrm{j}} + 2\hat{\mathrm{k}}\right)$.

Answer:

$\vec{a} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$|\vec{a}| = (1^2 + (-2)^2 + 2^2)^{1/2}$

$\Rightarrow |\vec{a}| = (1 + 4 + 4)^{1/2} = (9)^{1/2} = 3$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3}$$

a vector in the direction of $\left(\hat{\mathrm{i}} - 2\hat{\mathrm{j}} + 2\hat{\mathrm{k}}\right)$. which has magnitude 15 units.

$= 15\hat{a} = 15 \times \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3} = 5\left(\hat{\imath} - 2\hat{\jmath} + 2\hat{k}\right) = 5\hat{\imath} - 10\hat{\jmath} + 10\hat{k}.$

Ans: $5\hat{\imath} - 10\hat{\jmath} + 10\hat{k}.$

Question 30.

Write the vector of magnitude 15 units in the direction of the vector $\left(\hat{i}-2\hat{j}+2\hat{k}\right)$.

Answer:
First, we find the unit vector in the direction of a given vector,

$$\hat{a} \Rightarrow \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{\left(1^2+(-2)^2+2^2\right)}}$$

$$\Rightarrow \frac{\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{(1+4+4)}} = \frac{\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{9}}$$

$$= \frac{\hat{i}-2\hat{j}+2\hat{k}}{3}$$

Now vector in the direction of the given vector and with magnitude 15 is

$$= 15.\frac{\hat{i}-2\hat{j}+2\hat{k}}{3} \Rightarrow 5\left(\hat{i}-2\hat{j}+2\hat{k}\right)$$

$$= 5\hat{i}-10\hat{j}+10\hat{k}$$

Question 31.
If $\vec{a}=\left(\hat{i}+\hat{j}+\hat{k}\right), \vec{b}=\left(4\hat{i}-2\hat{j}+3\hat{k}\right)$ and $\vec{c}=\left(\hat{i}-2\hat{j}+\hat{k}\right)$, find a vector of magnitude 6 units which is parallel to the vector $\left(2\vec{a}-\vec{b}+3\vec{c}\right)$.

Answer:
First, we will find vector, then we will find a unit vector in the given direction,

$$2\vec{a}-\vec{b}+3\vec{c} = 2\left(\hat{i}+\hat{j}+\hat{k}\right)-\left(4\hat{i}-2\hat{j}+3\hat{k}\right)+3\left(\hat{i}-2\hat{j}+\hat{k}\right)$$

$$= (2-4+3)\hat{i}+(2+2-6)\hat{j}+(2-3+3)\hat{k}$$

$= \hat{i} - 2\hat{j} + 2\hat{k}$

$|2\vec{a} - \vec{b} + 3\vec{c}| = |\hat{i} - 2\hat{j} + 2\hat{k}| \Rightarrow \sqrt{\left(1^2 + (-2)^2 + 2^2\right)} = \sqrt{(1 + 4 + 4)}$

$\Rightarrow \sqrt{9} = 3$

$\hat{a} \Rightarrow \frac{\vec{a}}{|\vec{a}|} = \frac{2\vec{a} - \vec{b} + 3\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|} \Rightarrow \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$

Vector with magnitude 6 in the direction of the vector is

$\vec{m} = 6.\left(\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}\right) \Rightarrow 2\left(\hat{i} - 2\hat{j} + 2\hat{k}\right) = 2\hat{i} - 4\hat{j} + 4\hat{k}$

Question 32.

If $\vec{a} = \left(\hat{i} + \hat{j} + \hat{k}\right), \vec{b} = \left(4\hat{i} - 2\hat{j} + 3\hat{k}\right)$ and $\vec{c} = \left(\hat{i} - 2\hat{j} + \hat{k}\right)$, find a vector of magnitude 6 units which is parallel to the vector $\left(2\vec{a} - \vec{b} + 3\vec{c}\right)$.

Answer:

$\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\vec{b} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\therefore (2\vec{a} - \vec{b} + 3\vec{c}) = 2(\hat{\imath} + \hat{\jmath} + \hat{k}) - (4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + 3(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

$\Rightarrow (2\vec{a} - \vec{b} + 3\vec{c}) = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

LET, $(2\vec{a} - \vec{b} + 3\vec{c}) = \vec{s}$

$\vec{s} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$|\vec{s}| = (1^2 + (-2)^2 + 2^2)^{1/2}$

$\Rightarrow |\vec{s}| = (1 + 4 + 4)^{1/2} = (9)^{1/2} = 3$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{s} = \frac{\vec{s}}{|\vec{s}|} = \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3}$$

a vector of magnitude 6 units which is parallel to the vector $\left(2\vec{a} - \vec{b} + 3\vec{c}\right)$. is:

$$6\hat{s} = 6 \times \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3} = 2(\hat{\imath} - 2\hat{\jmath} + 2\hat{k}) = 2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}.$$

Ans: $2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$

Question 33.

Write the projection of the vector $\left(\hat{i} - \hat{j}\right)$ on the vector $\left(\hat{i} + \hat{j}\right)$.

Answer:

Projection of vector $\vec{a}$ on vector $\vec{b} = \frac{1}{\left|\vec{b}\right|}\left(\vec{a}.\vec{b}\right)$

$$= \frac{1}{\left|\overline{\hat{i} + \hat{j}}\right|}\left(\left(\hat{i} - \hat{j}\right).(\hat{i} + \hat{j}\right))$$

$$= \frac{1}{\sqrt{\left(1^2 + 1^2\right)}}\left(\left(\hat{i} - \hat{j}\right).(\hat{i} + \hat{j}\right)$$

$$= \frac{1}{\sqrt{(1+1)}}\left(1.1 + (-1.1)\right)$$

$= \frac{1}{\sqrt{2}}(1-1)$

= 0

So, projection of vector on other is 0.

Question 34.

Write the projection of the vector $\left(\hat{i}-\hat{j}\right)$ on the vector $\left(\hat{i}+\hat{j}\right)$.

Answer:

$\vec{a} = \hat{\imath} - \hat{\jmath}$

$\vec{b} = \hat{\imath} + \hat{\jmath}$

projection of a on b is given by: $\vec{a}.\hat{b}$

$|\vec{b}| = (1^2 + 1^2 + 0^2)^{1/2}$

$\Rightarrow |\vec{b}| = (1 + 1)^{1/2} = (2)^{1/2}$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}}$$

$$\vec{a}.\hat{b} = (\hat{\imath} - \hat{\jmath}).\left(\frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}}\right) = \frac{(1 \times 1) + (-1 \times 1)}{\sqrt{2}} = \frac{0}{\sqrt{2}} = 0$$

Ans: the projection of the vector $\left(7\hat{i}+\hat{j}-4\hat{k}\right)$ on the vector $\left(2\hat{i}+6\hat{j}+3\hat{k}\right)$.

Question 35.

Write the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a}\cdot\vec{b} = \sqrt{6}$.

Answer:
$|\vec{a}| = \sqrt{3}$

$|\vec{b}| = 2$

Since, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|cos\theta$

Substituting the given values we get:

$\Rightarrow \sqrt{6} = \sqrt{3} \times 2 \times cos\theta$

$\Rightarrow cos\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \cos^{-1}\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 45^\circ = \frac{\pi}{4}$

Ans: $\theta = 45^\circ = \frac{\pi}{4}$

Question 36.
Write the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$.

Answer:
Using scalar product, we can find the angle between two vectors.

$$\cos\theta = \frac{|\vec{a}.\vec{b}|}{|\vec{a}||\vec{b}|} = \frac{\sqrt{6}}{\sqrt{3.2}} = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}$$

Question 37.

If $\vec{a} = \left(\hat{i} - 7\hat{j} + 7\hat{k}\right)$ and $\vec{b} = \left(3\hat{i} - 2\hat{j} + 2\hat{k}\right)$ then find $\left|\vec{a} \times \vec{b}\right|$.

Answer:

$\vec{a} = \hat{\imath} - 7\hat{\jmath} + 7\hat{k}$

$\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$$\vec{a} \times \vec{b} = \left(\hat{\imath} - 7\hat{\jmath} + 7\hat{k}\right) \times \left(3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}\right) = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix} = \hat{\imath}(-14 - (-14)) - \hat{\jmath}(2 - 21) + \hat{k}\left(-2 - (-21)\right)$$
$$= 0\hat{\imath} + 19\hat{\jmath} + 19\hat{k}$$

$\therefore \vec{a} \times \vec{b} = 0\hat{\imath} + 19\hat{\jmath} + 19\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = (0^2 + 19^2 + 19^2)^{1/2} = (2 \times 19^2)^{1/2} = 19\sqrt{2}$

Ans: $\therefore |\vec{a} \times \vec{b}| = 19\sqrt{2}$

Question 38.

If $\vec{a} = \left(\hat{i} - 7\hat{j} + 7\hat{k}\right)$ and $\vec{b} = \left(3\hat{i} - 2\hat{j} + 2\hat{k}\right)$ then find $\left|\vec{a} \times \vec{b}\right|$.

Answer:

$$\vec{a} \times \vec{b} \Rightarrow \begin{vmatrix} i & j & k \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix} = \left((-7).2 - (-2).7\right)\hat{i} + \left(1.2 - 3.7\right)\hat{j} + \left(1.(-2) - 3.(-7)\right)\hat{k}$$

$$= \left(-14 - (-14)\right)\hat{i} + \left(2 - 21\right)\hat{j} + \left(-2 - (-21)\right)\hat{k}$$

$$= 0.\hat{i} - 19\hat{j} + 19\hat{k}$$

$$\left|\vec{a} \times \vec{b}\right| = \sqrt{\left(19^2 + 19^2\right)} = \sqrt{2.19^2} = 19\sqrt{2}$$

Question 39.

Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively, when $\left|\vec{a} \times \vec{b}\right| = \sqrt{3}.$

Answer:

$|\vec{a}| = 1$

$|\vec{b}| = 2$

Since, $\overrightarrow{|a} \times \vec{b}| = |\vec{a}||\overrightarrow{b}|sin\theta$

Substituting the given values we get:

$\Rightarrow\sqrt{3} = 1 \times 2 \times sin\theta$

$\Rightarrow sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow\theta = \sin^{-1}\frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 60^{\circ} = \frac{\pi}{3}$

Ans: $\theta = 60^{\circ} = \frac{\pi}{3}$

Question 40.

Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively, when $\left|\vec{a} \times \vec{b}\right| = \sqrt{3}.$

Answer:

Using vector product, we can calculate the angle between vectors.

$$\sin\theta = \frac{\left|\vec{a} \times \vec{b}\right|}{|\vec{a}|\left|\vec{b}\right|} = \frac{\sqrt{3}}{1.2} = \frac{\sqrt{3}}{2}$$

$\theta = \frac{\pi}{3}$

Question 41.
What conclusion can you draw about vectors $\vec{a}$ and $\vec{b}$ when $\vec{a} \times \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 0$?

Answer:
It is given that:

$\vec{a} \times \vec{b} = \vec{0}$ and $\vec{a}.\vec{b} = \vec{0}$

$\Rightarrow |\vec{a}||\vec{b}| \sin\theta = |\vec{a}||\vec{b}| \cos\theta = \vec{0}$

Since sinθ and cosθ cannot be 0 simultaneously $\therefore |\vec{a}| = |\vec{b}|$ = 0

Conclusion: when $\vec{a} \times \vec{b} = \vec{0}$ and $\vec{a}.\vec{b} = \vec{0}$

Then $|\vec{a}| = |\vec{b}|$ = 0

Question 42.
What conclusion can you draw about vectors $\vec{a}$ and $\vec{b}$ when $\vec{a} \times \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 0$?

Answer:
As $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$ and $\cos\theta = \frac{|\vec{a}.\vec{b}|}{|\vec{a}||\vec{b}|}$, using scalar product and vector product.

Now $\vec{a} \times \vec{b} = 0$ and $\vec{a}.\vec{b} = 0$ also.

As cosθ and sinθ cannot be 0 simultaneously So, then either vector a is o or b is 0.

Question 43.
Find the value of λ when the vectors $\vec{a} = \left(\hat{i} + \lambda\hat{j} + 3\hat{k}\right)$ and $\vec{b} = \left(3\hat{i} + 2\hat{j} + 9\hat{k}\right)$ are parallel.

Answer:
If the vector product is zero, two vectors are parallel.

$$\vec{a} \times \vec{b} \Rightarrow \begin{vmatrix} i & j & k \\ 1 & \lambda & 3 \\ 3 & 2 & 9 \end{vmatrix} = 0$$

$$(9.\lambda - 2.3)\hat{i} - (1.9 - 3.3)\hat{j} + (1.2 - 3.\lambda) = 0$$

$$(9\lambda - 6)\hat{i} - 0.\hat{j} + (2 - 3\lambda)\hat{k} = 0$$

On comparing with the right hand side, we have

$9\lambda - 6 = 0$

$$\lambda = \frac{6}{9} = \frac{2}{3}$$

Question 44.

Find the value of λ when the vectors $\vec{a} = \left(\hat{i} + \lambda\hat{j} + 3\hat{k}\right)$ and $\vec{b} = \left(3\hat{i} + 2\hat{j} + 9\hat{k}\right)$ are parallel.

Answer:

$\vec{a} = \hat{\imath} + \lambda\hat{\jmath} + 3\hat{k}$

$\vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$

It is given that $\vec{a} \parallel \vec{b}$

$\Rightarrow \frac{1}{3} = \frac{\lambda}{2} = \frac{3}{9}$

$\Rightarrow \frac{1}{3} = \frac{\lambda}{2}$

$\Rightarrow \lambda = 2 \times \frac{1}{3} = \frac{2}{3}$

Ans: λ = 2/3

Question 45.
Write the value of

$$\hat{i}\cdot\left(\hat{j}\times\hat{k}\right)+\hat{j}\cdot\left(\hat{i}\times\hat{k}\right)+\hat{k}\cdot\left(\hat{i}\times\hat{j}\right).$$

Answer:
According to the right hand coordinate system,

$$\hat{i}\times\hat{j} = \hat{k}$$

$$\hat{j}\times\hat{k} = \hat{i}$$

$$\hat{k}\times\hat{i} = \hat{j}$$

Then putting values in the equation

$$\hat{i}.\left(\hat{j}\times\hat{k}\right) + \hat{j}\left(\hat{i}\times\hat{k}\right) + \hat{k}\left(\hat{i}\times\hat{j}\right) = \hat{i}.\hat{i} + \hat{j}.\left(-\hat{j}\right) + \hat{k}.\hat{k}$$

= 1 - 1 + 1 = 1

Question 46.
Write the value of

$$\hat{i}\cdot\left(\hat{j}\times\hat{k}\right)+\hat{j}\cdot\left(\hat{i}\times\hat{k}\right)+\hat{k}\cdot\left(\hat{i}\times\hat{j}\right).$$

Answer:
We know that:

$$\hat{\imath}\times\hat{\jmath} = \hat{k}, \hat{\jmath}\times\hat{k} = \hat{\imath}, \hat{k}\times\hat{\imath} = \hat{\jmath},$$

$$\hat{\jmath}\times\hat{\imath} = -\hat{k}, \hat{k}\times\hat{\jmath} = -\hat{\imath}, \hat{\imath}\times\hat{k} = -\hat{\jmath}$$

$$\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$$

$$\therefore \hat{\imath}.\left(\hat{\jmath}\times\hat{k}\right) + \hat{\jmath}.\left(\hat{\imath}\times\hat{k}\right) + \hat{k}.\left(\hat{\imath}\times\hat{\jmath}\right) = \hat{\imath}.\hat{\imath} + \hat{\jmath}.\left(-\hat{\jmath}\right) + \hat{k}.\hat{k} = 1 - 1 + 1 = 1$$

Ans: $\hat{\imath}.(\hat{\jmath} \times \hat{k}) + \hat{\jmath}.(\hat{\imath} \times \hat{k}) + \hat{k}.(\hat{\imath} \times \hat{\jmath})$ = 1

Question 47.
Find the volume of the parallelepiped whose edges are represented by the vectors $\vec{a} = (2\hat{i} - 3\hat{j} + 4\hat{k}), \vec{b} = (\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{c} = (3\hat{i} - 2\hat{j} + 2\hat{k})$.

Answer:
Scalar triple product geometrically represents the volume of the parallelepiped whose three coterminous edges are represented by $\vec{a}, \vec{b}, \vec{c}$.i.e. V = $[\vec{a}\vec{b}\vec{c}]$

$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\vec{c} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

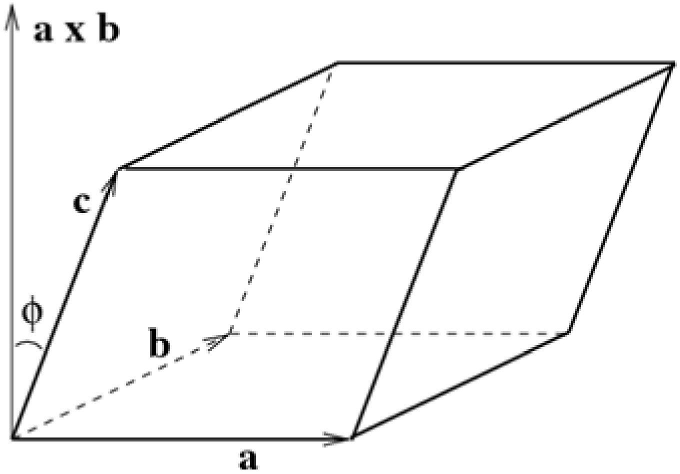
$\therefore$ V = $[\vec{a}\vec{b}\vec{c}] = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -2 & 2 \end{bmatrix}$ = 2(4 - 2) - (- 3)(2 - (- 3)) + 4(- 2 - 6) = 4 + 15 - 32 = | - 13| =

13 cubic units.

Ans:13 cubic units.

Question 48.
Find the volume of the parallelepiped whose edges are represented by the vectors $\vec{a} = (2\hat{i} - 3\hat{j} + 4\hat{k}), \vec{b} = (\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{c} = (3\hat{i} - 2\hat{j} + 2\hat{k})$.

Answer:
The volume of parallelepiped $= \left|(\vec{a} \times \vec{b}).\vec{c}\right|$



$$\left(\vec{a}\times\vec{b}\right).\vec{c}=\begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} 3 & -2 & 2 \\ 2 & -3 & 4 \\ 1 & 2 & -1 \end{vmatrix}$$

= (- 3).(- 1) - 2.4).3 - (2.(- 1) - 1.4).(- 2) + (2.2 - 1.(- 3)).

= (3 - 8).3 + (- 2 - 4).2 + (4 - (- 3)).2

= - 15 - 12 + 14

= - 27 + 14

= - 13

Volume of parallelepiped $=\left|\left(\vec{a}\times\vec{b}\right).\vec{c}\right|=|-13|=13$ cubic unit

Question 49.

If $\vec{a}=\left(-2\hat{i}-2\hat{j}+4\hat{k}\right),\vec{b}=\left(-2\hat{j}+4\hat{j}-2\hat{k}\right)$ and $\vec{c}=\left(4\hat{i}-2\hat{j}-2\hat{k}\right)$ then prove that $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar.

Answer:

If three planes lie in a single plane, then the volume of parallelepiped will be zero. So, planes are coplanar if

The volume of parallelepiped $=\left|\left(\vec{a}\times\vec{b}\right).\vec{c}\right|=\begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$$= \begin{vmatrix} 4 & -2 & -2 \\ -2 & -2 & 4 \\ -2 & 4 & -2 \end{vmatrix}$$

= (- 2. - 2 - 4.4)4 - (- 2. - 2 - 4. - 2) - 2 + (4. - 2 - (- 2). - 2) - 2

= (4 - 16)4 + (4 + 8)2 - (- 8 - 4)2

= - 48 + 24 - (- 24)

= - 48 + 48 = 0

So, planes are coplanar.

Question 50.

If $\vec{a} = \left(-2\hat{i} - 2\hat{j} + 4\hat{k}\right), \vec{b} = \left(-2\hat{j} + 4\hat{j} - 2\hat{k}\right)$ and $\vec{c} = \left(4\hat{i} - 2\hat{j} - 2\hat{k}\right)$ then prove that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar.

Answer:

$\vec{a} = -2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$

$\vec{b} = -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

$\vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$

If $\vec{a}, \vec{b}, \vec{c}$ are coplanar then $[\vec{a}\vec{b}\vec{c}]$ = 0

L.H.S = $\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & -2 \\ 4 & -2 & -2 \end{bmatrix}$ = - 2(- 8 - 4) + 2(4 + 8) + 4(4 - 16) = 24 + 24 - 48 = 0 = R.H.S

∴L.H.S = R.H.S

Hence proved that the vectors $\vec{a} = -2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$

$\vec{b} = -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

$\vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$

Are coplanar.

Question 51.

If $\vec{a} = \left(2\hat{i} + 6\hat{j} + 27\hat{k}\right)$ and $\vec{b} = \left(\hat{i} + \lambda\hat{j} + \mu\hat{k}\right)$ are such that $\vec{a} \times \vec{b} = \vec{0}$ then find the values of λ and μ.

Answer:

Given that the vector product is zero.

$$\vec{a} \times \vec{b} \Rightarrow \begin{vmatrix} i & j & k \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = \vec{0}$$

$$= (6.\mu - 27.\lambda)\hat{i} + (2.\mu - 27.1)\hat{j} + (2.\lambda - 1.6)\hat{k} = \vec{0}$$

On comparing with the right hand side, we have

$6\mu - 27\lambda = 0$

$2\mu - 27 = 0$

$$\mu = \frac{27}{2}$$

$2\lambda - 6 = 0$

$$\lambda = \frac{6}{2} \Rightarrow 3$$

Question 52.

If $\vec{a} = \left(2\hat{i} + 6\hat{j} + 27\hat{k}\right)$ and $\vec{b} = \left(\hat{i} + \lambda\hat{j} + \mu\hat{k}\right)$ are such that $\vec{a} \times \vec{b} = \vec{0}$ then find the values of λ and μ.

Answer:

$\vec{a} = 2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}$

$\vec{b} = \hat{\imath} + \lambda\hat{\jmath} + \mu\hat{k}$

It is given that $\vec{a} \times \vec{b} = \vec{0}$

$\Rightarrow (2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}) \times (\hat{\imath} + \lambda\hat{\jmath} + \mu\hat{k}) = 0$

$\Rightarrow \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{bmatrix} = 0 = \hat{\imath}(6\mu - 27\lambda) - \hat{\jmath}(2\mu - 27) + \hat{k}(2\lambda - 6)$

$\Rightarrow 2\lambda - 6 = 0$

$\Rightarrow \lambda = 6/2 = 3$

$\Rightarrow 2\mu - 27 = 0$

$\Rightarrow \mu = 27/2$

Ans: $\lambda = 3$, $\mu = 27/2$

Question 53.

If θ is the angle between $\vec{a}$ and $\vec{b}$, and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ then what is the value of θ?

Answer:

It is given that:

$|\vec{a} \times \vec{b}| = |\vec{a}.\vec{b}|$

$\Rightarrow |\vec{a}||\vec{b}|sin\theta = |\vec{a}||\vec{b}|cos\theta$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = \tan^{-1} 1 = \frac{\pi}{4}$

Ans: $\theta = \frac{\pi}{4}$

Question 54.
If θ is the angle between $\vec{a}$ and $\vec{b}$, and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ then what is the value of θ?

Answer:
We have

$$|\vec{a}.\vec{b}| = |\vec{a}|.|\vec{b}|.\cos\theta$$

$$|\vec{a} \times \vec{b}| = |\vec{a}|.|\vec{b}|.\sin\theta$$

Equating both

$$|\vec{a} \times \vec{b}| = |\vec{a}.\vec{b}|\ |\vec{a}|.|\vec{b}|.\sin\theta = |\vec{a}|.|\vec{b}|.\cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 1 \Rightarrow \tan\theta = 1$$

$$\theta = \frac{\pi}{4}$$

Question 55.
When does $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$ hold?

Answer:
When the two vectors are parallel or collinear, they can be added in a scalar way because the angle between them is zero degrees,they are I the same or opposite direction.

Therefore when two vectors $\vec{a}\ and\ \vec{b}$ are either parallel or collinear then

$$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$$

Question 56.
When does $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}\right| + \left|\vec{b}\right|$ hold?

Answer:
$$\left(\vec{a}+\vec{b}\right)^2 = \vec{a}^2 + \vec{b}^2 + 2\vec{a}.\vec{b}$$

$$\left|\vec{a}+\vec{b}\right|^2 = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2.\left|\vec{a}\right|.\left|\vec{b}\right|.\cos\theta\left(\text{using } \vec{a}^2 = \left|\vec{a}\right|^2\right)$$

$$\left(\left|\vec{a}\right| + \left|\vec{b}\right|\right)^2 = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2.\left|\vec{a}\right|.\left|\vec{b}\right|.\cos\theta$$

$$\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2.\left|\vec{a}\right|.\left|\vec{b}\right| = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2.\left|\vec{a}\right|.\left|\vec{b}\right|.\cos\theta$$

$$\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2.\left|\vec{a}\right|.\left|\vec{b}\right| - \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 - 2.\left|\vec{a}\right|.\left|\vec{b}\right|.\cos\theta = 0$$

$$2.\left|\vec{a}\right|.\left|\vec{b}\right|\left(1-\cos\theta\right) = 0$$

As, magnitude of a vector cannot be zero (leaving zero vector)

1 - cosθ = 0

Cosθ = 1

θ = 0˚

So, vectors are either parallel or collinear.

Question 57.
Find the direction cosines of a vector which is equally inclined to the x - axis, y - axis and z - axis.

Answer:
Direction cosines of a vector l, m, n are related to each other as

$$l^2 + m^2 + n^2 = 1$$

Now given that equally inclined to three axes with an angle of θ. Then direction cosines l, m, n are

l = m = n = cosθ

Putting values of direction cosines in equation,

$\cos^2\theta + \cos^2\theta + \cos^2\theta = 1$

$3\cos^2\theta = 1$

$$\cos^2\theta = \frac{1}{3} \Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$$

$$l = m = n = \cos\theta = \frac{1}{\sqrt{3}}$$

Question 58.
Find the direction cosines of a vector which is equally inclined to the x - axis, y - axis and z - axis.

Answer:
Let the inclination with:

x - axis = α

y - axis = β

z - axis = γ

The vector is equally inclined to the three axes.

$\Rightarrow \alpha = \beta = \gamma$

Direction cosines: $cos\alpha, cos\beta, cos\gamma$

We know that: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$ ...($\alpha = \beta = \gamma$)

$\Rightarrow 3\cos^2 \alpha = 1$

$$\cos\alpha = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$$

$$\cos\beta = \frac{1}{\sqrt{3}}$$

$$\cos\gamma = \frac{1}{\sqrt{3}}$$

Ans: $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Question 59.
If P(1, 5, 4) and Q(4, 1, - 2) be the position vectors of two points P and Q, find the direction ratios of $\overrightarrow{PQ}$.

Answer:
$$\overrightarrow{PQ} \Rightarrow \vec{P} - \vec{Q} = (4-1)\hat{i} + (1-5)\hat{j} + (-2-4)\hat{k}$$

$$= 3\hat{i} - 4\hat{j} - 6\hat{k}$$

So direction ratios are 3, - 4, - 6.

Question 60.
If P(1, 5, 4) and Q(4, 1, - 2) be the position vectors of two points P and Q, find the direction ratios of $\overrightarrow{PQ}$.

Answer:
Let P(x_1,y_1,z_1) and Q(x_2,y_2,z_2) be the two points then Direction ratios of line joining P and Q i.e. PQ are x_2 - x_1,y_2 - y_1,z_2- z

Here, P is(1, 5, 4) and Q is (4, 1, - 2)

Direction ratios of PQ are:(4 - 1),(1 - 5),(- 2 - 4) = 3, - 4, - 6

Ans: the direction ratios of $\overrightarrow{PQ}$. are: 3, - 4, - 6

Question 61.

Find the direction cosines of the vector $\vec{a} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$.

Answer:

$\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Let the inclination with:

x - axis = α

y - axis = β

z - axis = γ

Direction cosines: $cos\alpha, cos\beta, cos\gamma = l, m, n$

For a vector $\vec{a} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, l = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

$$\therefore l = \frac{1}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{1}{\sqrt{1 + 4 + 9}} = \frac{1}{\sqrt{14}}.$$

$$\therefore m = \frac{2}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{2}{\sqrt{1 + 4 + 9}} = \frac{2}{\sqrt{14}}$$

$$\therefore n = \frac{3}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{3}{\sqrt{1 + 4 + 9}} = \frac{3}{\sqrt{14}}$$

Ans: $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

Question 62.

Find the direction cosines of the vector $\vec{a} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$.

Answer:
The direction cosines and direction ratios are related as

$l = \frac{a}{r},\ m = \frac{b}{r}, n = \frac{c}{r}$, where a, b, c are direction ratios and r is magnitude.

Now direction ratios are 1, 2, 3 respectively and magnitude of vector is
$r = \sqrt{\left(1^2 + 2^2 + 3^2\right)}\sqrt{(1 + 4 + 9)} = \sqrt{14}$

Putting the values

$$l = \frac{1}{\sqrt{14}},\ m = \frac{2}{\sqrt{14}}, n = \frac{3}{\sqrt{14}}$$

Question 63.

If $\hat{a}$ and $\hat{b}$ are unit vectors such that $\left(\hat{a} + \hat{b}\right)$ is a unit vector, what is the angle between $\hat{a}$ and $\hat{b}$?

Answer:
It is given that $\hat{a}\ and\ \hat{b}$ are unit vectors ,as well as $(\hat{a}\ +\ \hat{b})$ is also a unit vector

$\Rightarrow |\hat{a}| = |\hat{b}| = |\hat{a}\ +\ \hat{b}| = 1$

Since the modulus of a unit vector is unity.

Now,

$|\hat{a}\ +\ \hat{b}|^2 = |\hat{a}|^2 + |\hat{b}|^2 + 2|\hat{a}||\hat{b}|\cos\theta$

$\Rightarrow 1^2 = 1^2 + 1^2 + 2\times1\times1\times \cos\theta$

$\Rightarrow \cos\theta = (1 - 1 - 1)/2$

$\Rightarrow \cos\theta\ = \frac{-1}{2}$

$\Rightarrow \theta = \cos^{-1}\frac{-1}{2} = \frac{2\pi}{3}$

Ans: $\frac{2\pi}{3}$

Question 64.

If $\hat{a}$ and $\hat{b}$ are unit vectors such that $\left(\hat{a}+\hat{b}\right)$ is a unit vector, what is the angle between $\hat{a}$ and $\hat{b}$?

Answer:

$$\left(\hat{a} + \hat{b}\right)^2 = \hat{a}^2 + b^2 + 2\hat{a}.\hat{b}$$

$$\left|\hat{a} + \hat{b}\right|^2 = |\hat{a}|^2 + \left|\hat{b}\right|^2 + 2.|\hat{a}|.\left|\hat{b}\right|.\cos\theta$$

$$1^2 = 1^2 + 1^2 + 2.1.1.\cos\theta$$

1 - 1 - 1 = 2cosθ

- 1 = 2cosθ

$$\cos\theta = -\frac{1}{2} \Rightarrow \theta = \pi - \frac{\pi}{3} \Rightarrow \frac{2\pi}{3}$$