Exercise 2b

Question 1.
Let A = {1, 2, 3, 4}. Let f : A → A and g : A → A,

defined by f = {(1, 4), (2, 1), (3,3),(4, 2)} and g = {(1, 3), (2, 1), (3, 2), (4, 4)}.

Find (i) g of (ii) f o g (iii) f o f.

Answer:
(i) g o f

To find: g o f

Formula used: g o f = g(f(x))

Given: f = {(1, 4), (2, 1), (3, 3), (4, 2)} and g = {(1, 3), (2, 1),

(3, 2), (4, 4)}

Solution: We have,

gof(1) = g(f(1)) = g(4) = 4

gof(2) = g(f(2)) = g(1) = 3

gof(3) = g(f(3)) = g(3) = 2

gof(4) = g(f(4)) = g(2) = 1

Ans) g o f = {(1, 4), (2, 3), (3, 2), (4, 1)}

(ii) f o g

To find: f o g

Formula used: f o g = f(g(x))

Given: f = {(1, 4), (2, 1), (3, 3), (4, 2)} and g = {(1, 3), (2, 1), (3, 2), (4, 4)}

Solution: We have,

fog(1) = f(g(1)) = f(3) = 3

fog(2) = f(g(2)) = f(1) = 4

fog(3) = f(g(3)) = f(2) = 1

fog(4) = f(g(4)) = f(4) = 2

Ans) f o g = {(1, 3), (2, 4), (3, 1), (4, 2)}

(iii) f o f

To find: f o f

Formula used: f o f = f(f(x))

Given: f = {(1, 4), (2, 1), (3, 3), (4, 2)}

Solution: We have,

fof(1) = f(f(1)) = f(4) = 2

fof(2) = f(f(2)) = f(1) = 4

fof(3) = f(f(3)) = f(3) = 3

fof(4) = f(f(4)) = f(2) = 1

Ans) f o f = {(1, 2), (2, 4), (3, 3), (4, 1)}

Question 2.
Let f : {3, 9, 12} → {1, 3, 4} and g : {1, 3, 4, 5} → {3, 9} be

defined as f = {(3, 1), (9, 3), (12, 4)} and

g = {(1, 3), (3, 3), (4, 9), (5, 9)}.

Find (i) (g o f) (ii) (f o g).

Answer:
(i) g o f

To find: g o f

Formula used: g o f = g(f(x))

Given: f = {(3, 1), (9, 3), (12, 4)} and g = {(1, 3), (3, 3),(4, 9), (5, 9)}

Solution: We have,

gof(3) = g(f(3)) = g(1) = 3

gof(9) = g(f(9)) = g(3) = 3

gof(12) = g(f(12)) = g(4) = 9

Ans) g o f = {(3, 3), (9, 3), (12, 9)}

(ii) f o g

To find: f o g

Formula used: f o g = f(g(x))

Given: f = {(3, 1), (9, 3), (12, 4)} and g = {(1, 3), (3, 3),(4, 9), (5, 9)}

Solution: We have,

fog(1) = f(g(1)) = f(3) = 1

fog(3) = f(g(3)) = f(3) = 1

fog(4) = f(g(4)) = f(9) = 3

fog(5) = f(g(5)) = f(9) = 3

Ans) f o g = {(1, 1), (3, 1), (4, 3), (5, 3)}

Question 3.
Let f : R → R : $f(x) = x^2$ and g : R → R : $g(x) = (x + 1)$.

Show that (g o f) ≠ (f o g).

Answer:
To prove: (g o f) ≠ (f o g)

Formula used: (i) g o f = g(f(x))

(ii) f o g = f(g(x))

Given: (i) f : R → R : $f(x) = x^2$

(ii) g : R → R : $g(x) = (x + 1)$

Proof: We have,

g o f = $g(f(x)) = g(x^2) = (x^2 + 1)$

f o g = $f(g(x)) = g(x+1) = [(x+1)^2 + 1] = x^2 + 2x + 2$

From the above two equation we can say that (g o f) ≠ (f o g)

Hence Proved

Question 4.
Let f : R → R : $f(x) = (2x + 1)$ and g : R → R : $g(x) = (x^2 - 2)$.

Write down the formulae for

(i) (g o f) (ii) (f o g)

(iii) (f o f) (iv) (g o g)

Answer:
(i) g o f

To find: g o f

Formula used: g o f = g(f(x))

Given: (i) f : R → R : f(x) = (2x + 1)

(ii) g : R → R : g(x) = (x^2 - 2)

Solution: We have,

g o f = g(f(x)) = g(2x + 1) = [$(2x + 1)^2$ – 2]

⇒ $4x^2 + 4x + 1 – 2$

⇒ $4x^2 + 4x – 1$

Ans). g o f (x) = $4x^2 + 4x – 1$

(ii) f o g

To find: f o g

Formula used: f o g = f(g(x))

Given: (i) f : R → R : f(x) = (2x + 1)

(ii) g : R → R : g(x) = (x^2 - 2)

Solution: We have,

f o g = f(g(x)) = f(x^2 - 2) = [2(x^2 - 2) + 1]

⇒ $2x^2 - 4 + 1$

⇒ $2x^2 – 3$

Ans). f o g (x) = $2x^2 – 3$

(iii) f o f

To find: f o f

Formula used: f o f = f(f(x))

Given: (i) f : R → R : f(x) = (2x + 1)

Solution: We have,

f o f = f(f(x)) = f(2x + 1) = [2(2x + 1) + 1]

⇒ 4x + 2 + 1

⇒ 4x + 3

Ans). f o f (x) = 4x+ 3

(iv) g o g

To find: g o g

Formula used: g o g = g(g(x))

Given: (i) g : R → R : g(x) = (x^2 - 2)

Solution: We have,

g o g = g(g(x)) = g(x^2 - 2) = [$(x^2 - 2)^2$ – 2]

⇒ x^4 -4x^2 + 4 - 2

⇒ x^4 -4x^2 + 2

Ans). g o g (x) = x^4 -4x^2 + 2

Question 5.
Let f : R → R : f(x) = (x^2 + 3x + 1) and g: R → R : g(x) = (2x - 3). Write down the formulae for

(i) g o f

(ii) f o g

(iii) g o g

Answer:
(i) g o f

To find: g o f

Formula used: g o f = g(f(x))

Given: (i) f : R → R : f(x) = $(x^2 + 3x + 1)$

(ii) g: R → R : g(x) = (2x - 3)

Solution: We have,

g o f = g(f(x)) = $g(x^2 + 3x + 1) = [\ 2(x^2 + 3x + 1) - 3\ ]$

$\Rightarrow 2x^2 + 6x + 2 - 3$

$\Rightarrow 2x^2 + 6x - 1$

Ans). g o f (x) = $2x^2 + 6x - 1$

(ii) f o g

To find: f o g

Formula used: f o g = f(g(x))

Given: (i) f : R → R : f(x) = $(x^2 + 3x + 1)$

(ii) g: R → R : g(x) = (2x - 3)

Solution: We have,

f o g = f(g(x)) = f(2x - 3) = $[\ (2x - 3)^2 + 3(2x - 3) + 1\ ]$

$\Rightarrow 4x^2 - 12x + 9 + 6x - 9 + 1$

$\Rightarrow 4x^2 - 6x + 1$

Ans). f o g (x) = $4x^2 - 6x + 1$

(iii) g o g

To find: g o g

Formula used: g o g = g(g(x))

Given: (i) g: R → R : g(x) = (2x - 3)

Solution: We have,

g o g = g(g(x)) = g(2x - 3) = [2(2x – 3) - 3]

$\Rightarrow 4x - 6 - 3$

$\Rightarrow 4x - 9$

Ans). g o g (x) = 4x – 9

Question 6.
Let f : R → R : f(x) = |x|, prove that f o f = f.

Answer:
To prove: f o f = f

Formula used: f o f = f(f(x))

Given: (i) f : R → R : f(x) = |x|

Solution: We have,

f o f = f(f(x)) = f(|x|) = $||x||$ = |x| = f(x)

Clearly f o f = f.

Hence Proved.

Question 7.

Let f : R → R : $f(x) = x^2$, g : R → R : g(x) = tan x

and h : R → R : h(x) = log x.

Find a formula for h o (g o f).

Show that [h o (g o f)] $\sqrt{\frac{\pi}{4}} = 0$.

Answer:

|

To find: formula for h o (g o f)

To prove: Show that [h o (g o f)] $\sqrt{\frac{\pi}{4}} = 0$

Formula used: f o f = f(f(x))

Given: (i) f : R → R : $f(x) = x^2$

(ii) g : R → R : g(x) = tan x

(iii) h : R → R : h(x) = log x

Solution: We have,

h o (g o f) = h o g(f(x)) = h o $g(x^2)$

= $h(g(x^2))$ = h (tan x^2)

= log (tan x^2)

h o (g o f) = log (tan x^2)

For, [h o (g o f)] $\sqrt{\frac{\pi}{4}}$

$= \log\left[\tan\left(\sqrt{\frac{\pi}{4}}\right)^2\right]$

$= \log\left[\tan\frac{\pi}{4}\right]$

$= \log 1$

= 0

Hence Proved.

Question 8.

Let f : R → R : f(x) (2x - 3) and $g : R \rightarrow R : g(x) = \frac{1}{2}(x + 3)$.

Show that (f o g) = I_R = (g o f).

Answer:
To prove: (f o g) = I_R = (g o f).

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) f : R → R : f(x) = (2x - 3)

(ii) $g : R \rightarrow R : g(x) = \frac{1}{2}(x+3)$

Solution: We have,

f o g = f(g(x))

$= f\left(\frac{1}{2}(x+3)\right)$

$= \left[2\left(\frac{1}{2}(x+3)\right) - 3\right]$

$= x + 3 - 3$

$= x$

$= I_R$

g o f = g(f(x))

$= g(2x - 3)$

$= \frac{1}{2}(2x-3+3)$

$= \frac{1}{2}(2x)$

$= x$

$= I_R$

Clearly we can see that (f o g) = I_R = (g o f) = x

Hence Proved.

Question 9.
Let f : Z → Z : f(x) = 2x. Find g : Z → Z : g o f = I_Z.

Answer:
To find: g : Z → Z : g o f = I_Z

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) g : Z → Z : g o f = I_Z

Solution: We have,

f(x) = 2x

Let f(x) = y

⇒ y = 2x

$\Rightarrow x = \frac{y}{2}$

$\Rightarrow x = \frac{y}{2}$

Let $g(y) = \frac{y}{2}$

Where g: Z → Z

For g o f,

⇒ g(f(x))

⇒ g(2x)

$\Rightarrow \frac{2x}{2}$

⇒ x = I_Z

Clearly we can see that (g o f) = x = I_Z

The required function is $g(x) = \frac{x}{2}$

Question 10.
Let f : N → N : f(x) = 2x, g : N → N : g(y) = 3y + 4 and h : N → N : h(z) = sin z. Show that h o (g o f) = (h o g) o f.

Answer:
To show: h o (g o f) = (h o g) o f

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) $f : N \to N : f(x) = 2x$

(ii) $g : N \to N : g(y) = 3y + 4$

(iii) $h : N \to N : h(z) = \sin z$

Solution: We have,

LHS = h o (g o f)

$\Rightarrow$ h o (g(f(x))

$\Rightarrow h(g(2x))$

$\Rightarrow h(3(2x) + 4)$

$\Rightarrow h(6x +4)$

$\Rightarrow \sin(6x + 4)$

RHS = (h o g) o f

$\Rightarrow$ (h(g(x))) o f

$\Rightarrow$ (h(3x + 4)) o f

$\Rightarrow$ sin(3x+4) o f

Now let sin(3x+4) be a function u

RHS = u o f

$\Rightarrow u(f(x))$

$\Rightarrow u(2x)$

$\Rightarrow \sin(3(2x) + 4)$

$\Rightarrow \sin(6x + 4) = \text{LHS}$

Hence Proved.

Question 11.
If f be a greatest integer function and g be an absolute value function, find the value of $(f \circ g)\left(\frac{-3}{2}\right) + (g \circ f)\left(\frac{4}{3}\right)$.

Answer:
To find: $(fog)\left(\frac{-3}{2}\right) + (gof)\left(\frac{4}{3}\right)$

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) f is a greatest integer function

(ii) g is an absolute value function

f(x) = [x] (greatest integer function)

g(x) = $|x|$ (absolute value function)

$f\left(\frac{4}{3}\right) = \left[\frac{4}{3}\right] = 1$ ... (i)

$g\left(\frac{-3}{2}\right) = \left|\frac{-3}{2}\right| = 1.5$ ... (ii)

Now, for $(fog)\left(\frac{-3}{2}\right) + (gof)\left(\frac{4}{3}\right)$

$$\Rightarrow f\left(g\left(\frac{-3}{2}\right)\right)+g\left(f\left(\frac{4}{3}\right)\right)$$

Substituting values from (i) and (ii)

$$\Rightarrow f(1.5) + g(1)$$

$\Rightarrow$ [1.5] + $|1|$

$\Rightarrow 1 + 1 = 2$

Ans) 2

Question 12.

Let f : R → R : f(x) = x^2 + 2 and $g : R \to R : g(x) = \frac{x}{x-1}, x \neq 1.$ find f o g and g o f and hence find (f o g) (2) and (g o f) (-3).

Answer:
To find: f o g, g o f ,(f o g) (2) and (g o f) (-3)

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) f : R → R : f(x) = x^2 + 2

$$\text{(ii) } g : R \to R : g(x) = \frac{x}{x-1}, x \neq 1$$

f o g = f(g(x))

$$\Rightarrow f\left(\frac{x}{x-1}\right)$$

$$\Rightarrow \left(\frac{x}{x-1}\right)^2 + 2$$

Ans) $\Rightarrow \dfrac{(x)^2}{(x-1)^2} + 2$

$f \circ g\,(2) = \dfrac{(2)^2}{(2-1)^2} + 2$

$= \dfrac{4}{1} + 2$

Ans) = 6

g o f = g(f(x))

$\Rightarrow g(x^2+2)$

$\Rightarrow \dfrac{x^2+2}{x^2+2-1}$

Ans) $\Rightarrow \dfrac{x^2+2}{x^2+1}$

$(g \circ f)\,(-3) = \dfrac{-3^2+2}{-3^2+1}$

$= \dfrac{9+2}{9+1}$

Ans) $= \dfrac{11}{10}$