

Ex 19.1

Indefinite Integrals Ex 19.1 Q1

(i)

$$\int x^4 dx = \frac{x^{4+1}}{4+1} + C$$

$$= \frac{x^5}{5} + C$$

(ii)

$$\int x^{\frac{5}{4}} dx = \frac{x^{\frac{5}{4}+1}}{\frac{5}{4}+1} + C$$

$$= \frac{x^{\frac{5+4}{4}}}{\frac{5+4}{4}} + C$$

$$= \frac{4x^{\frac{9}{4}}}{9} + C$$

(iii)

$$\int \frac{1}{x^5} dx = \int x^{-5} dx$$

$$= \frac{x^{-5+1}}{-5+1} + C$$

$$= \frac{x^{-4}}{-4} + C$$

$$= \frac{-1}{4x^4} + C$$

(iv)

$$\int \frac{1}{x^{\frac{3}{2}}} dx = \int x^{\frac{-3}{2}} dx$$

$$= \int x^{\frac{-3}{2}} dx$$

$$= \frac{x^{\frac{-3+1}{2}}}{\frac{-3+1}{2}} + C$$

$$= \frac{x^{\frac{-1}{2}}}{\frac{-1}{2}} + C$$

$$= -2 \times \frac{1}{\sqrt{x}} + C$$

$$= \frac{-2}{\sqrt{x}} + C$$

(v)

$$\int 3^x dx = \frac{3^x}{\log 3} + C$$

$$\left[\because \int a^x dx = \frac{a^x}{\log a} + C \right]$$

(vi)

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{x^2}} dx &= \int \frac{1}{x^{\frac{2}{3}}} dx \\
&= \int x^{-\frac{2}{3}} dx \\
&= \frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1} + C \\
&= \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + C \\
&= \frac{1}{3}\sqrt[3]{x} + C
\end{aligned}$$

(vii)

$$\begin{aligned}
\int 3^{2\log_3 x} dx &= \int 3^{\log_3 x^2} dx \\
&= \int x^2 dx \quad [\because 3^{\log_a x} = x] \\
&= \frac{x^3}{3} + C
\end{aligned}$$

(viii)

$$\begin{aligned}
\int \log_x x dx &= \int 1 dx \\
&= x + C.
\end{aligned}$$

Indefinite Integrals Ex 19.1 Q2

(i)

$$\begin{aligned}
\int \sqrt{\frac{1+\cos 2x}{2}} dx &= \int \sqrt{\frac{2\cos^2 x}{2}} dx \quad [\because \cos 2x = 2\cos^2 x - 1] \\
&= \int \cos x dx \\
&= \sin x + C
\end{aligned}$$

$$\begin{aligned}
(ii) \quad \int \sqrt{\frac{1-\cos 2x}{2}} dx &= \int \sqrt{\frac{2\sin^2 x}{2}} dx \\
&= \int \sin x dx \\
&= -\cos x + C
\end{aligned}$$

Indefinite Integrals Ex 19.1 Q3

Evaluate the integral as follows

$$\begin{aligned}
\int \frac{e^{\delta \log_e x} - e^{\delta \log_e x}}{e^{4\log_e x} - e^{3\log_e x}} dx &= \int \frac{x^{\delta} - x^{\delta}}{x^4 - x^3} dx \\
&= \int \frac{x^{\delta}(x-1)}{x^3(x-1)} dx \\
&= \int x^{\delta-3} dx \\
&= \frac{x^3}{3} + C
\end{aligned}$$

Indefinite Integrals Ex 19.1 Q4

$$\begin{aligned}
\int \frac{1}{a^x b^x} dx &= \int a^{-x} b^{-x} dx \\
&= \int (ab)^{-x} dx \\
&= \frac{(ab)^{-x}}{\log_e(ab)^{-1}} + C \\
&= \frac{(ab)^{-x}}{-\log_e(ab)} + C \\
&= \frac{a^{-x} b^{-x}}{-\log_e(ab)} + C
\end{aligned}$$

Indefinite Integrals Ex 19.1 Q5

$$\begin{aligned}
 \text{(i)} \quad & \int \frac{\cos 2x + 2 \sin^2 x}{\sin^2 x} dx \\
 &= \int \frac{\cos 2x + 2 \sin^2 x}{\sin^2 x} dx \\
 &= \int \frac{1 - 2 \sin^2 x + 2 \sin^2 x}{\sin^2 x} dx \\
 &= \int \frac{1}{\sin^2 x} dx \\
 &= \int \csc^2 x dx \\
 &= -\cot x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & \int \frac{2 \cos^2 x - \cos 2x}{\cos^2 x} dx \\
 &= \int \frac{2 \cos^2 x - (2 \cos^2 x - 1)}{\cos^2 x} dx \\
 &= \int \frac{2 \cos^2 x - 2 \cos^2 x + 1}{\cos^2 x} dx \\
 &= \int \frac{1}{\cos^2 x} dx \\
 &= \int \sec^2 x dx \\
 &= \tan x + c
 \end{aligned}$$

Indefinite Integrals Ex 19.1 Q6

$$\begin{aligned}
 \int \frac{e^{\log \sqrt{x}}}{x} dx &= \int \frac{\sqrt{x}}{x} dx \\
 &= \int x^{\frac{1}{2}} \times x^{-1} dx \\
 &= \int x^{\frac{1}{2}-1} dx \\
 &= \int x^{\frac{-1}{2}} dx \\
 &= \frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}} + c \\
 &= \frac{x^{\frac{1}{2}}}{\frac{-1}{2}} + c \\
 &= \frac{1}{2} x^{\frac{1}{2}} \\
 &= 2\sqrt{x} + c
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q1

Ex 19.2

$$\begin{aligned}
 & \int (3x\sqrt{5} + 4\sqrt{x} + 5) dx \\
 &= \int 3x\sqrt{5}dx + \int 4\sqrt{x}dx + \int 5dx \\
 &= \int 3x^{\frac{3}{2}}dx + 4\int x^{\frac{1}{2}}dx + 5\int dx \\
 &= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{4x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 5x + c \\
 &= \frac{6}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + 5x + c
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q2

$$\begin{aligned}
 & \int \left(2^x + \frac{5}{x} - \frac{1}{x^{\frac{1}{3}}} \right) dx \\
 &= \int 2^x dx + 5 \int \frac{1}{x} dx - \int \frac{1}{x^{\frac{1}{3}}} dx \\
 &= \frac{2^x}{\log 2} + 5 \log x - \frac{3}{2}x^{\frac{2}{3}} + c
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q3

$$\begin{aligned}
 & \int \left\{ \sqrt{x} (ax^2 + bx + c) \right\} dx \\
 &= \int \sqrt{x} \times ax^2 dx + \int \sqrt{x} \times bx dx + \int c\sqrt{x} dx \\
 &= \int ax^{\frac{5}{2}} dx + \int bx^{\frac{3}{2}} dx + \int cx^{\frac{1}{2}} dx \\
 &= \frac{ax^{\frac{5}{2}+1}}{\frac{5}{2}+1} + \frac{bx^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{cx^{\frac{1}{2}+1}}{\frac{1}{2}+1} + d \\
 &= \frac{2ax^{\frac{7}{2}}}{7} + \frac{2bx^{\frac{5}{2}}}{5} + \frac{2cx^{\frac{3}{2}}}{3} + d
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q4

$$\begin{aligned}
 & \int (2 - 3x)(3 + 2x)(1 - 2x) dx \\
 &= \int (6 + 4x - 9x - 6x^2)(1 - 2x) dx \\
 &= \int (-6x^2 - 5x + 6)(1 - 2x) dx \\
 &= \int (-6x^2 + 12x^3 - 5x + 10x^2 + 6 - 12x) dx \\
 &= \int (4x^2 + 12x^3 - 17x + 6) dx \\
 &= \int (12x^3 + 4x^2 - 17x + 6) dx \\
 &= \frac{12}{4}x^4 + \frac{4}{3}x^3 - \frac{17}{2}x^2 + 6x + c \\
 &= 3x^4 + \frac{4}{3}x^3 - \frac{17}{2}x^2 + 6x + c
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q5

$$\begin{aligned}
 & \int \left(\frac{m}{x} + \frac{x}{m} + m^x + x^m + mx \right) dx \\
 &= m \int \frac{1}{x} dx + \frac{1}{m} \int x dx + \int m^x dx + \int x^m dx + m \int x dx \\
 &= m \log|x| + \frac{x^2}{2m} + \frac{m^x}{\log m} + \frac{x^{m+1}}{m+1} + \frac{mx^2}{2} + c
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q6

$$\begin{aligned}
& \int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 dx \\
&= \int \left(x + \frac{1}{x} - 2 \right) dx \\
&= \int x dx + \int \frac{1}{x} dx - 2 \int 1 dx \\
&= \frac{x^2}{2} + \log|x| - 2x + C
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q7

$$\begin{aligned}
& \int \frac{(1+x)^3}{\sqrt{x}} dx \\
&= \int \frac{1+x^3+3x^2+3x}{5x} dx \\
&= \int \frac{1}{\sqrt{x}} dx + \int \frac{x^3}{\sqrt{x}} dx + \int \frac{3x^2}{\sqrt{x}} dx + \int \frac{3x}{\sqrt{x}} dx \\
&= \int x^{-\frac{1}{2}} dx + \int x^{\frac{5}{2}} dx + 3 \int x^{\frac{3}{2}} dx + 3 \int x^{\frac{1}{2}} dx \\
&= \frac{x^{\frac{-1+1}{2}}}{\frac{-1+1}{2}} + \frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} + \frac{3x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + 3 \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C \\
&= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3x^{\frac{5}{2}}}{\frac{5}{2}} + 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C \\
&= 2x^{\frac{1}{2}} + \frac{2}{7}x^{\frac{7}{2}} + \frac{6}{5}x^{\frac{5}{2}} + \frac{6}{3}x^{\frac{3}{2}} + C \\
&= 2x^{\frac{1}{2}} + \frac{2}{7}x^{\frac{7}{2}} + \frac{6}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C
\end{aligned}$$

$$\therefore \int \frac{(1+x)^3}{\sqrt{x}} dx = 2x^{\frac{1}{2}} + \frac{2}{7}x^{\frac{7}{2}} + \frac{6}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C$$

Indefinite Integrals Ex 19.2 Q8

$$\begin{aligned}
& \int \left\{ x^2 + e^{\log x} + \left(\frac{e}{2}\right)^x \right\} dx \\
&= \int x^2 dx + \int e^{\log x} dx + \int \left(\frac{e}{2}\right)^x dx \\
&= \frac{x^3}{3} + \int x dx + \int \left(\frac{e}{2}\right)^x dx \\
&= \frac{x^3}{3} + \frac{x^2}{2} + \frac{1}{\log\left(\frac{e}{2}\right)} \times \left(\frac{e}{2}\right)^x + C
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q9

$$\begin{aligned}
& \int (x^e + e^x + e^e) dx \\
&= \int x^e dx + \int e^x dx + \int e^e dx \\
&= \frac{x^{e+1}}{e+1} + e^x + e^e x + C \quad [\because e \text{ is constant}] \\
&\therefore \int (x^e + e^x + e^e) dx = \frac{x^{e+1}}{e+1} + e^x + e^e x + C
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q10

$$\begin{aligned}
\int \sqrt{x} \left(x^3 - \frac{2}{x} \right) dx &= \int x^{\frac{7}{2}} dx - 2 \int x^{\frac{1}{2}} dx \\
&= \frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1} - 2 \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C \\
&= \frac{x^{\frac{9}{2}}}{\frac{9}{2}} - \frac{2x^{\frac{3}{2}}}{\frac{-1}{2}} + C \\
&= \frac{2}{9}x^{\frac{9}{2}} - 4x^{\frac{3}{2}} + C
\end{aligned}$$

$\therefore \int \sqrt{x} \left(x^3 - \frac{2}{x} \right) dx = \frac{2}{9}x^{\frac{9}{2}} - 4\sqrt{x} + C$

Indefinite Integrals Ex 19.2 Q11

$$\begin{aligned}
\int \frac{1}{\sqrt{x}} \left(1 + \frac{1}{x} \right) dx &= \int \left(\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{x} \times x} \right) dx \\
&= \int x^{-\frac{1}{2}} dx + \int x^{-\frac{3}{2}} dx \\
&= 2x^{\frac{1}{2}} - 2x^{\frac{-1}{2}} + C \\
&= 2\sqrt{x} - \frac{2}{\sqrt{x}} + C
\end{aligned}$$

$\therefore \int \frac{1}{\sqrt{x}} \left(1 + \frac{1}{x} \right) dx = 2\sqrt{x} - \frac{2}{\sqrt{x}} + C$

Indefinite Integrals Ex 19.2 Q12

$$\begin{aligned}
\int \frac{x^6 + 1}{x^2 + 1} dx &= \int \frac{(x^2)^3 + (1)^3}{x^2 + 1} dx \\
&= \int \frac{(x^2 + 1)(x^4 + 1 - x^2)}{x^2 + 1} dx \\
&= \int (x^4 - x^2 + 1) dx \\
&= \int x^4 dx - \int x^2 dx + \int 1 dx \\
&= \frac{x^5}{5} - \frac{x^3}{3} + x + C
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q13

$$\begin{aligned}
\int \frac{x^{-\frac{1}{3}} + \sqrt{x} + 2}{\sqrt[3]{x}} dx &= \int \frac{x^{-\frac{1}{3}}}{x^{\frac{1}{3}}} dx + \int \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} dx + \int \frac{2}{x^{\frac{1}{3}}} dx \\
&= \int x^{-\frac{2}{3}} dx + \int x^{\frac{1}{6}} dx + 2 \int x^{-\frac{1}{3}} dx \\
&= 3x^{\frac{1}{3}} + \frac{6}{7}x^{\frac{7}{6}} + 3x^{\frac{2}{3}} + C
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q14

$$\begin{aligned}
& \int \frac{(1+\sqrt{x})^2}{\sqrt{x}} dx \\
&= \int \frac{1+x+2\sqrt{x}}{x^{\frac{1}{2}}} dx \\
&= \int x^{\frac{-1}{2}} + \int x^{\frac{1}{2}} dx + 2 \int dx \\
&= 2\sqrt{x} + \frac{2}{3}x^{\frac{3}{2}} + 2x + c \\
\therefore \quad & \int \frac{(1+\sqrt{x})^2}{\sqrt{x}} dx = 2\sqrt{x} + 2x + \frac{2}{3}x^{\frac{3}{2}} + c
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q15

$$\begin{aligned}
& \int \sqrt{x}(3-5x)dx \\
&= 3 \int \sqrt{x} dx - 5 \int x^{\frac{3}{2}} dx \\
&= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - 5 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c \\
&= 2x^{\frac{3}{2}} - 2x^{\frac{5}{2}} + c
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q16

$$\begin{aligned}
& \int \frac{(x+1)(x-2)}{\sqrt{x}} dx \\
&= \int \frac{x^2 - 2x + x - 2}{x^{\frac{1}{2}}} dx \\
&= \int \frac{x^2 - x - 2}{x^{\frac{1}{2}}} dx \\
&= \int \frac{x^2}{x^{\frac{1}{2}}} dx - \int \frac{x}{x^{\frac{1}{2}}} dx - 2 \int \frac{1}{x^{\frac{1}{2}}} dx \\
&= \frac{2x^{\frac{5}{2}}}{5} - \frac{2x^{\frac{3}{2}}}{3} - 4x^{\frac{1}{2}} + c \\
\therefore \quad & \int \frac{(x+1)(x-2)}{\sqrt{x}} dx = \frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + c \\
&= \frac{2}{5}x^{\frac{5}{2}} - \frac{2x^{\frac{3}{2}}}{3} - 4\sqrt{x} + c
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q17

$$\begin{aligned}
& \int \frac{x^5 + x^{-2} + 2}{x^2} dx \\
&= \int \left(\frac{x^5}{x^2} + \frac{x^{-2}}{x^2} + \frac{2}{x^2} \right) dx \\
&= \int x^3 dx + \int x^{-4} dx + 2 \int x^{-2} dx \\
&= \frac{x^4}{4} + \frac{x^{-3}}{-3} + \frac{2x^{-1}}{-1} + c \\
&= \frac{x^4}{4} - \frac{x^{-3}}{3} - \frac{2}{x} + c
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q18

$$\begin{aligned}
& \int (3x+4)^2 dx \\
&= \int (9x^2 + 16 + 24x) dx \\
&= 9 \int x^2 dx + 16 \int dx + 24 \int x dx \\
&= 9 \frac{x^3}{3} + 16x + 24 \frac{x^2}{2} + C \\
&= 3x^3 + 16x + 12x^2 + C \\
\therefore \quad & \int (3x+4)^2 = 3x^3 + 12x^2 + 16x + C
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q19

$$\begin{aligned}
& \int \frac{2x^4 + 7x^3 + 6x^2}{x^2 + 2x} dx \\
&= \int \frac{x(2x^3 + 7x^2 + 6x)}{x(x+2)} dx \\
&= \int \frac{2x^3 + 7x^2 + 6x}{x+2} dx \\
&= \int \frac{2x^3 + 4x^2 + 3x^2 + 6x}{(x+2)} dx \\
&= \int \frac{2x^2(x+2) + 3x(x+2)}{(x+2)} dx \\
&= \int \frac{(x+2)(2x^2 + 3x)}{x+2} dx \\
&= \int (2x^2 + 3x) dx \\
&= \int 2x^2 dx + \int 3x dx \\
&= \frac{2}{3}x^3 + \frac{3}{2}x^2 + C
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q20

$$\begin{aligned}
& \int \frac{5x^4 + 12x^3 + 7x^2}{x^2 + x} dx \\
&= \int \frac{5x^4 + 7x^3 + 5x^3 + 7x^2}{x^2 + x} dx \\
&= \int \frac{5x^3 + 7x^2 + 5x^2 + 7x}{x+1} dx \\
&= \int \frac{5x^2(x+1) + 7x(x+1)}{x+1} dx \\
&= \int (5x^2 + 7x) dx \\
&= \frac{5x^3}{3} + \frac{7x^2}{2} + C
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q21

$$\begin{aligned}
& \int \frac{\sin^2 x}{1 + \cos x} dx \\
&= \int \frac{1 - \cos^2 x}{1 + \cos x} dx \\
&= \int \frac{(1 - \cos x)(1 + \cos x)}{(1 + \cos x)} dx \\
&= \int (1 - \cos x) dx \\
&= x - \sin x + C
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q22

$$\begin{aligned}
& \int (\sec^2 x + \csc^2 x) dx \\
&= \int \sec^2 x dx + \int \csc^2 x dx \\
&= \tan x - \cot x + c \\
\therefore \quad & \int (\sec^2 x + \csc^2 x) dx = \tan x - \cot x + c
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q23

Evaluate the integral as follows

$$\begin{aligned}
\int \frac{\sin^3 x - \cos^3 x}{\sin^2 x \cos^2 x} dx &= \int \left(\frac{\sin^3 x}{\sin^2 x \cos^2 x} - \frac{\cos^3 x}{\sin^2 x \cos^2 x} \right) dx \\
&= \int (\sin x \sec^2 x - \cos x \csc^2 x) dx \\
&= \int (\tan x \sec x - \cot x \csc x) dx \\
&= \sec x + \csc x + C
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q24

$$I = \int \frac{5 \cos^3 x + 6 \sin^3 x}{2 \sin^2 x \cos^2 x} dx$$

Now,

$$\begin{aligned}
I &= \int \frac{5 \cos^3 x + 6 \sin^3 x}{2 \sin^2 x \cos^2 x} dx \\
&= \int \frac{5 \cos^3 x}{2 \sin^2 x \cos^2 x} dx + \int \frac{6 \sin^3 x}{2 \sin^2 x \cos^2 x} dx \\
&= \frac{5}{2} \int \frac{\cos x}{\sin^2 x} dx + 3 \int \frac{\sin x}{\cos^2 x} dx \\
&= \frac{5}{2} \int \cot x \cosec x dx + 3 \int \sec x \tan x dx \\
&= + \frac{-5}{2} \cosec x + 3 \sec x + C
\end{aligned}$$

$$\therefore I = \frac{-5}{2} \cosec x + 3 \sec x + C$$

Indefinite Integrals Ex 19.2 Q25

$$\begin{aligned}
& \int (\tan x + \cot x)^2 dx \\
&= \int (\tan^2 x + \cot^2 x + 2 \tan x \cot x) dx \\
&= \int \left(\sec^2 x - 1 + \csc^2 x - 1 + \frac{2 \times 1}{\cot x} \cot x \right) dx \\
&= \int (\sec^2 x + \csc^2 x) dx \\
&= \int \sec^2 x dx + \int \csc^2 x dx \\
&= \tan x - \cot x + C \\
\therefore \quad & \int (\tan x + \cot x)^2 dx = \tan x - \cot x + C
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q26

$$\begin{aligned}
& \int \frac{1 - \cos 2x}{1 + \cos 2x} dx \\
&= \int \frac{2 \sin^2 x}{2 \cos^2 x} dx \\
&= \int \tan^2 x dx \\
&= \int (\sec^2 x - 1) dx \\
&= \int \sec^2 x dx - \int dx \\
&= \tan x - x + C \\
\therefore \quad & \int \frac{1 - \cos 2x}{1 + \cos 2x} dx = \tan x - x + C
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q27

$$\begin{aligned}
& \int \frac{\cos x}{1 - \cos x} dx \\
&= \int \frac{\cos x (1 + \cos x)}{(1 - \cos x)(1 + \cos x)} dx \\
&= \int \frac{\cos x + \cos^2 x}{1 - \cos^2 x} dx \\
&= \int \frac{\cos x + \cos^2 x}{\sin^2 x} dx \\
&= \int \frac{\cos x}{\sin^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x} dx \\
&= \int \cot x \times \operatorname{cosec} x dx + \int (\operatorname{cosec}^2 x - 1) dx \\
&= -\operatorname{cosec} x - \cot x - x + c \\
\\
&\therefore \int \frac{\cos x}{1 - \cos x} dx = -\operatorname{cosec} x - \cot x - x + c
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q28

$$\begin{aligned}
& \int \frac{\cos^2 x - \sin^2 x}{\sqrt{1 + \cos 4x}} dx \\
&= \int \frac{\cos^2 x - \sin^2 x}{\sqrt{2 \cos^2 2x}} dx \\
&= \frac{1}{\sqrt{2}} \int \frac{\cos^2 x - \sin^2 x}{\cos 2x} dx \\
&= \frac{1}{\sqrt{2}} \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x - \sin^2 x} dx \quad [\because \cos 2x = \cos^2 x - \sin^2 x] \\
&= \frac{1}{\sqrt{2}} \int 1 dx \\
&= \frac{x}{\sqrt{2}} + c \\
\\
&\therefore \int \frac{\cos^2 x - \sin^2 x}{\sqrt{1 + \cos 4x}} dx = \frac{x}{\sqrt{2}} + c
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q29

$$\begin{aligned}
& \int \frac{1}{1 - \cos x} dx \\
&= \int \frac{1}{1 - \cos x} \times \frac{1 + \cos x}{1 + \cos x} \times dx \\
&= \int \frac{1 + \cos x}{1 - \cos^2 x} dx \\
&= \int \frac{1 + \cos x}{\sin^2 x} dx \\
&= \int \frac{1}{\sin^2 x} dx + \int \frac{\cos x}{\sin^2 x} dx \\
&= \int \operatorname{cosec}^2 x dx + \int \cot x \times \operatorname{cosec} x dx \\
&= -\operatorname{cot} x - \operatorname{cosec} x + c \\
\\
&\therefore \int \frac{1}{1 - \cos x} dx = -\operatorname{cot} x - \operatorname{cosec} x + c.
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q30

$$\begin{aligned}
& \int \frac{1}{1 - \sin x} dx \\
&= \int \frac{1}{1 - \sin x} \times \frac{1 + \sin x}{1 + \sin x} dx \\
&= \int \frac{1 + \sin x}{1 - \sin^2 x} dx \\
&= \int \frac{1 + \sin x}{\cos^2 x} dx \\
&= \int \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} \right) dx \\
&= \int \frac{1}{\cos^2 x} dx + \int \frac{\sin x}{\cos^2 x} dx \\
&= \int \sec^2 x dx + \int \tan x \sec x dx \\
&= \tan x + \sec x + c \\
\therefore \quad & \int \frac{1}{1 - \sin x} dx = \tan x + \sec x + c.
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q31

$$\begin{aligned}
& \int \frac{\tan x}{\sec x + \tan x} dx \\
&= \int \frac{\tan x}{\sec x + \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x} dx \\
&= \int \frac{\tan x (\sec x - \tan x)}{\sec^2 x - \tan^2 x} dx \\
&= \int (\tan x \sec x - \tan^2 x) dx \\
&= \int \sec x \tan x dx - \int (\sec^2 x - 1) dx \\
&= \int \sec x \tan x dx - \int \sec^2 x dx + \int dx \\
&= \sec x - \tan x + x + c \\
\therefore \quad & \int \frac{\tan x}{\sec x + \tan x} dx = \sec x - \tan x + x + c.
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q32

$$\begin{aligned}
& \int \frac{\csc x}{\csc x - \cot x} dx \\
&= \int \frac{\csc x}{\csc x - \cot x} \times \frac{\csc x + \cot x}{\csc x + \cot x} dx \\
&= \int \frac{\csc x (\csc x + \cot x)}{\csc^2 x - \cot^2 x} dx \\
&= \int (\csc^2 x + \csc x \cot x) dx \\
&= \int \csc^2 x dx + \int \csc x dx \\
&= -\cot x - \csc x + c \\
\therefore \quad & \int \frac{\csc x}{\csc x - \cot x} dx = -\cot x - \csc x + c.
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q33

$$\begin{aligned}
& \int \frac{1}{1 + \cos 2x} dx \\
&= \int \frac{1}{2 \cos^2 x} dx \\
&= \frac{1}{2} \int \sec^2 x dx \\
&= \frac{1}{2} \times \tan x + c \\
&= \frac{\tan x}{2} + c \\
\therefore \quad & \int \frac{1}{1 + \cos 2x} dx = \frac{1}{2} \tan x + c.
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q34

$$\begin{aligned}
& \int \frac{1}{1 - \cos 2x} dx \\
&= \int \frac{1}{2 \sin^2 x} \times dx \\
&= \frac{1}{2} \int \csc^2 x \times dx \\
&= \frac{-1}{2} \times \cot x + c \\
&= \frac{-1 \cot x}{2} + c \\
\therefore \quad & \int \frac{1}{1 - \cos 2x} = \frac{-1}{2} \cot x + c.
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q35

$$\begin{aligned}
& \int \tan^{-1} \left[\frac{\sin 2x}{1 + \cos 2x} \right] dx \\
&= \int \tan^{-1} \left[\frac{2 \sin x \cos x}{2 \cos^2 x} \right] dx \\
&= \int \tan^{-1} \left[\frac{\sin x}{\cos x} \right] dx \\
&= \int \tan^{-1} (\tan x) dx \\
&= \int x dx \\
&= \frac{x^2}{2} + c \\
\therefore \quad & \int \tan^{-1} \left[\frac{\sin 2x}{1 + \cos 2x} \right] dx = \frac{x^2}{2} + c.
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q36

$$\begin{aligned}
& \int \cos^{-1} (\sin x) dx \\
&= \int \cos^{-1} \left[\cos \left(\frac{\pi}{2} - x \right) \right] dx \\
&= \int \left(\frac{\pi}{2} - x \right) dx \\
&= \frac{\pi}{2} \int dx - \int x dx \\
&= \frac{\pi}{2} \times x - \frac{x^2}{2} + c \\
\therefore \quad & \int \cos^{-1} (\sin x) dx = \frac{\pi}{2} \times x - \frac{x^2}{2} + c.
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q37

$$\begin{aligned}
& \int \cos^{-1} (\sin x) dx \\
&= \int \cot^{-1} \left[\frac{\sin 2x}{1 - \cos 2x} \right] dx \\
&= \int \cot^{-1} \left(\frac{\cos x}{\sin x} \right) dx \\
&= \int \cot^{-1} (\cot x) dx \\
&= \int x dx \\
&= \frac{x^2}{2} + c \\
\therefore \quad & \int \cot^{-1} \left[\frac{\sin 2x}{1 - \cos 2x} \right] dx = \frac{x^2}{2} + c.
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q38

$$\begin{aligned}
& \int \sin^{-1} \left(\frac{2 \tan x}{1 + \tan^2 x} \right) dx \\
&= \int \sin^{-1} (\sin 2x) dx \quad \left[\because \sin 2x = \frac{2 \tan x}{1 + \tan^2 x} \right] \\
&= \int 2x dx \\
&= 2 \int x dx \\
&= \frac{2x^2}{2} + c \\
&= x^2 + c \\
\therefore \quad & \int \sin^{-1} \left(\frac{2 \tan x}{1 + \tan^2 x} \right) dx = x^2 + c.
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q39

$$\begin{aligned}
& \int \frac{(x^3 + 8)(x - 1)}{x^2 - 2x + 4} dx \\
&= \int \frac{(x+2)(x^2 - 2x + 4)(x-1)}{x^2 - 2x + 4} dx \\
&= \int (x+2)(x-1) dx \\
&= \int (x^2 - x + 2x - 2) dx \\
&= \int (x^2 + x - 2) dx \\
&= \frac{x^3}{3} + \frac{x^2}{2} - 2x + c \\
\therefore \quad & \int \frac{(x^3 + 8)(x - 1)}{x^2 - 2x + 4} dx = \frac{x^3}{3} + \frac{x^2}{2} - 2x + c.
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q40

$$\begin{aligned}
& \int (a \tan x + b \cot x)^2 dx \\
&= \int (a^2 \tan^2 x + b^2 \cot^2 x + 2ab \tan x \cot x) dx \\
&= \int [a^2 (\sec^2 x - 1) + b^2 (\cosec^2 x - 1) + 2ab] dx \\
&= \int [a^2 \sec^2 x - a^2 + b^2 \cosec^2 x - b^2 + 2ab] dx \\
&= a^2 \tan x - a^2 x - b^2 \cot x - b^2 x + 2abx + c \\
&= a^2 \tan x - b^2 \cot x - (a^2 + b^2 - 2ab)x + c \\
\therefore \quad & \int (a \tan x + b \cot x)^2 dx = a^2 \tan x - b^2 \cot x - (a^2 + b^2 - 2ab)x + c.
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q41

$$\begin{aligned}
& \int \frac{x^3 - 3x^2 + 5x - 7 + x^2 a^x}{2x^2} dx \\
&= \frac{1}{2} \int \frac{x^3}{x^2} dx - \frac{3}{2} \int \frac{x^2}{x^2} dx + \frac{5}{2} \int x \frac{x}{x^2} dx - \frac{7}{2} \int x^{-2} dx + \frac{1}{2} \int \frac{x^2 a^x}{x^2} dx \\
&= \frac{1}{2} \times \frac{x^2}{2} - \frac{3}{2} x + \frac{5}{2} \log x - \frac{7}{2} x^{-1} + \frac{1}{2} \frac{a^x}{\log a} + c \\
&= \frac{1}{2} \left[\frac{x^2}{2} - 3x + 5 \log x + \frac{7}{x} + \frac{a^x}{\log a} \right] + c \\
\therefore \quad & \int \frac{x^3 - 3x^2 + 5x - 7 + x^2 a^x}{2x^2} dx = \frac{1}{2} \left[\frac{x^2}{2} - 3x + 5 \log x + \frac{7}{x} + \frac{a^x}{\log a} \right] + c
\end{aligned}$$

Indefinite Integrals Ex 19.2 Q42

$$\begin{aligned}
 \frac{\cos x}{1+\cos x} &= \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} & \left[\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \text{ and } \cos x = 2 \cos^2 \frac{x}{2} - 1 \right] \\
 &= \frac{1}{2} \left[1 - \tan^2 \frac{x}{2} \right] \\
 \therefore \int \frac{\cos x}{1+\cos x} dx &= \frac{1}{2} \int \left(1 - \tan^2 \frac{x}{2} \right) dx \\
 &= \frac{1}{2} \int \left(1 - \sec^2 \frac{x}{2} + 1 \right) dx \\
 &= \frac{1}{2} \int \left(2 - \sec^2 \frac{x}{2} \right) dx \\
 &= \frac{1}{2} \left[2x - \frac{\tan \frac{x}{2}}{\frac{1}{2}} \right] + C \\
 &= x - \tan \frac{x}{2} + C
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q43

$$\begin{aligned}
 \frac{1-\cos x}{1+\cos x} &= \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} & \left[2\sin^2 \frac{x}{2} = 1 - \cos x \text{ and } 2\cos^2 \frac{x}{2} = 1 + \cos x \right] \\
 &= \tan^2 \frac{x}{2} \\
 &= \left(\sec^2 \frac{x}{2} - 1 \right) \\
 \therefore \int \frac{1-\cos x}{1+\cos x} dx &= \int \left(\sec^2 \frac{x}{2} - 1 \right) dx \\
 &= \left[\frac{\tan \frac{x}{2}}{\frac{1}{2}} - x \right] + C \\
 &= 2\tan \frac{x}{2} - x + C
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q44

$$\begin{aligned}
 &\int \left\{ 3\sin x - 4\cos x + \frac{5}{\cos^2 x} - \frac{6}{\sin^2 x} + \tan^2 x - \cot^2 x \right\} dx \\
 &= 3\int \sin x dx - 4\int \cos x dx + 5\int \sec^2 x dx - 6\int \csc^2 x dx + \int \tan^2 x dx - \int \cot^2 x dx \\
 &= 3\int \sin x dx - 4\int \cos x dx + 5\int \sec^2 x dx - 6\int \csc^2 x dx + \int (\sec^2 x - 1) dx - \int (\csc^2 x - 1) dx \\
 &= 3\int \sin x dx - 4\int \cos x dx + 6\int \sec^2 x dx - 7\int \csc^2 x dx \\
 &= -3\cos x - 4\sin x + 6\tan x + 7\cot x + C
 \end{aligned}$$

Indefinite Integrals Ex 19.2 Q45

It is given that $f'(x) = x - \frac{1}{x^2}$

$$\begin{aligned}\therefore \int f'(x) dx &= \int \left(x - \frac{1}{x^2} \right) dx \\ \Rightarrow f(x) &= \int x dx - \int \frac{1}{x^2} dx \\ &= \frac{x^2}{2} + x^{-1} + C \\ &= \frac{x^2}{2} + \frac{1}{x} + C \\ \Rightarrow f(x) &= \frac{x^2}{2} + \frac{1}{x} + C \quad \text{---(i)}\end{aligned}$$

Now,

$$\begin{aligned}f(1) &= \frac{1}{2} && [\text{given}] \\ \Rightarrow \frac{1^2}{2} + \frac{1}{1} + C &= \frac{1}{2} \\ \Rightarrow C &= -1\end{aligned}$$

Putting $C = -1$ in (i), we get

$$f(x) = \frac{x^2}{2} + \frac{1}{x} - 1.$$

Indefinite Integrals Ex 19.2 Q46

It is given that $f'(x) = x + b$

$$\therefore \int f'(x) dx = \int (x + b) dx$$

$$\Rightarrow f(x) = \frac{x^2}{2} + bx + c \quad \text{---(i)}$$

Since,

$$f(1) = 5$$

$$\therefore \frac{1^2}{2} + b \times 1 + c = 5$$

$$\Rightarrow \frac{1}{2} + b + c = 5$$

$$\Rightarrow b + c = \frac{9}{2} \quad \text{---(ii)}$$

and, $f(2) = 13$

$$\Rightarrow \frac{(2)^2}{2} + b \times 2 + c = 13$$

$$\Rightarrow 2 + 2b + c = 13$$

$$\Rightarrow 2b + c = 11 \quad \text{---(iii)}$$

Subtracting equation (ii) from equation (iii), we get

$$b = 11 - \frac{9}{2}$$

$$\Rightarrow b = \frac{13}{2}$$

Putting $b = \frac{13}{2}$ in equation (ii), we get

$$\frac{13}{2} + c = \frac{9}{2}$$

$$\Rightarrow c = \frac{9}{2} - \frac{13}{2}$$

$$\Rightarrow c = \frac{9 - 13}{2} = \frac{-4}{2} = -2$$

Putting $b = \frac{13}{2}$ and $c = -2$ in equation (i), we get

$$f(x) = \frac{x^2}{x} + \frac{13}{2}x - 2$$

$$\therefore f(x) = \frac{x^2}{2} + \frac{13}{2}x - 2$$

Indefinite Integrals Ex 19.2 Q47

We have,

$$f'(x) = 8x^3 - 2x$$

$$\Rightarrow f(x) = \int f'(x) dx = \int (8x^3 - 2x) dx$$

$$\Rightarrow f(x) = \int (8x^3 - 2x) dx$$

$$= \int 8x^3 dx - \int 2x dx$$

$$= \frac{8x^4}{4} - \frac{2x^2}{2} + c$$

$$= 2x^4 - x^2 + c$$

$$\Rightarrow f(x) = 2x^4 - x^2 + c \quad \text{---(i)}$$

Since, $f(2) = 8$

$$\therefore f(2) = 2(2)^4 - (2)^2 + c = 8$$

$$\Rightarrow 32 - 4 + c = 8$$

$$\Rightarrow 28 + c = 8$$

$$\Rightarrow c = -20$$

Putting $c = -20$ in equation (i), we get

$$f(x) = 2x^4 - x^2 - 20$$

Hence, $f(x) = 2x^4 - x^2 - 20$.

Indefinite Integrals Ex 19.2 Q48

We have,

$$\begin{aligned} f(x) &= \int f'(x) dx \\ \Rightarrow f(x) &= \int (a \sin x + b \cos x) dx \\ &= -a \cos x + b \sin x + c \\ \therefore f(x) &= -a \cos x + b \sin x + c \end{aligned} \quad \text{--- (i)}$$

Since,

$$\begin{aligned} f'(0) &= 4 \\ \therefore f'(0) &= a \sin 0 + b \cos 0 = 4 \\ \Rightarrow a \times 0 + b \times 1 &= 4 \\ \Rightarrow b &= 4 \end{aligned}$$

Now,

$$\begin{aligned} f(0) &= 3 \\ \therefore f(0) &= -a \cos 0 + b \sin 0 + c = 3 \\ \Rightarrow -a + 0 + c &= 3 \\ \Rightarrow c - a &= 3 \end{aligned} \quad \text{--- (ii)}$$

$$\begin{aligned} \text{and, } f\left(\frac{\pi}{2}\right) &= 5 \\ \therefore f\left(\frac{\pi}{2}\right) &= -a \cos\left(\frac{\pi}{2}\right) + b \sin\left(\frac{\pi}{2}\right) + c = 5 \\ \Rightarrow -a \times 0 + b \times 1 + c &= 5 \\ \Rightarrow b + c &= 5 \\ \Rightarrow 4 + c &= 5 \quad [\because b = 4] \\ \Rightarrow c &= 5 - 4 \\ \Rightarrow c &= 1 \end{aligned}$$

Putting $c = 1$ in equation (ii), we get

$$\begin{aligned} 1 - a &= 3 \\ \Rightarrow -a &= 3 - 1 \\ \Rightarrow -a &= 2 \\ \Rightarrow a &= -2 \end{aligned}$$

Putting $a = -2$, $b = 4$ and $c = 1$ in equation (i), we get

$$\begin{aligned} f(x) &= -(-2) \cos x + 4 \sin x + 1 \\ \Rightarrow f(x) &= 2 \cos x + 4 \sin x + 1 \end{aligned}$$

Hence, $f(x) = 2 \cos x + 4 \sin x + 1$

Indefinite Integrals Ex 19.2 Q49

We have,

$$\begin{aligned} f(x) &= \sqrt{x} + \frac{1}{\sqrt{x}} \\ \Rightarrow \int f(x) dx &= \int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx \\ &= \int x^{\frac{1}{2}} dx + \int x^{-\frac{1}{2}} dx \\ &= \frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c \end{aligned}$$

Hence, the primitive or anti-derivative of $f(x) = \frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$.

Ex 19.3

Indefinite Integrals Ex 19.3 Q1

Let $I = \int [2x - 3]^5 + \sqrt{3x + 2} dx$. Then,

$$\begin{aligned} I &= \int (2x - 3)^5 dx + \int (3x + 2)^{\frac{1}{2}} dx \\ &= \frac{(2x - 3)^6}{2 \times 6} + \frac{(3x + 2)^{\frac{3}{2}}}{3 \times \frac{3}{2}} + C \\ &= \frac{(2x - 3)^6}{12} + \frac{2}{9} (3x + 2)^{\frac{3}{2}} + C \end{aligned}$$

$$\therefore I = \frac{(2x - 3)^6}{12} + \frac{2}{9} (3x + 2)^{\frac{3}{2}} + C$$

Indefinite Integrals Ex 19.3 Q2

Let $I = \int \left[\frac{1}{(7x - 5)^3} + \frac{1}{\sqrt{5x - 4}} \right] dx$. Then,

$$\begin{aligned} I &= \int (7x - 5)^{-3} dx + \int (5x - 4)^{-\frac{1}{2}} dx \\ &= \frac{1}{7 \times (-2)} + \frac{(5x - 4)^{\frac{1}{2}}}{5 \times \frac{1}{2}} + C \\ &= -\frac{(7x - 5)^{6-2}}{14} + \frac{2}{5} \sqrt{5x - 4} + C \end{aligned}$$

$$\therefore I = -\frac{1}{14} (7x - 5)^{-2} + \frac{2}{5} \sqrt{5x - 4} + C.$$

Indefinite Integrals Ex 19.3 Q3

Let $I = \int \frac{1}{2 - 3x} + \frac{1}{\sqrt{3x - 2}} dx$. Then,

$$\begin{aligned} I &= \int \frac{1}{2 - 3x} dx + \int \frac{1}{\sqrt{3x - 2}} dx \\ &= \frac{\log|2 - 3x|}{-3} + \frac{2}{3} (3x - 2)^{\frac{1}{2}} C \\ &= \frac{-1}{3} \log|2x - 3| + \frac{2}{3} \sqrt{3x - 2} + C \end{aligned}$$

Indefinite Integrals Ex 19.3 Q4

Let $I = \int \frac{x+3}{(x+1)^4} dx$. Then,

$$\begin{aligned} I &= \int \frac{x+1+2}{(x+1)^4} dx \\ &= \int \frac{x+1}{(x+1)^4} dx + 2 \int \frac{1}{(x+1)^4} dx \\ &= \int \frac{1}{(x+1)^3} dx + 2 \int \frac{1}{(x+1)^4} dx \\ &= \int (x+1)^{-3} dx + 2 \int (x+1)^{-4} dx \\ &= \frac{(x+1)^{-2}}{-2} + 2 \frac{(x+1)^{-3}}{-3} + C \\ &= -\frac{1}{2} \times \frac{1}{(x+1)^2} - \frac{2}{3} \times \frac{1}{(x+1)^3} + C \end{aligned}$$

$$\therefore I = \frac{-1}{2(x+1)^2} - \frac{2}{3(x+1)^3} + C$$

Indefinite Integrals Ex 19.3 Q5

Let $I = \int \frac{1}{\sqrt{x+1} + \sqrt{x}} dx$. Then,

$$\begin{aligned} I &= \int \frac{1}{\sqrt{x+1} + \sqrt{x}} \times \frac{\sqrt{x+1} - \sqrt{x}}{\sqrt{x+1} - \sqrt{x}} \times dx \\ &= \int \frac{\sqrt{x+1} - \sqrt{x}}{(\sqrt{x+1})^2 - (\sqrt{x})^2} \times dx \\ &= \int \frac{\sqrt{x+1} - \sqrt{x}}{x+1-x} \times dx \\ &= \int (\sqrt{x+1} - \sqrt{x}) \times dx \\ &= \int (x+1)^{\frac{1}{2}} dx - \int x^{\frac{1}{2}} dx \\ &= \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c \end{aligned}$$

$$\therefore I = \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.3 Q6

Let $I = \int \frac{1}{\sqrt{2x+3} + \sqrt{2x-3}} dx$. Then,

$$\begin{aligned} I &= \int \frac{1}{\sqrt{2x+3} + \sqrt{2x-3}} \times \frac{\sqrt{2x+3} - \sqrt{2x-3}}{\sqrt{2x+3} - \sqrt{2x-3}} \times dx \\ &= \int \frac{\sqrt{2x+3} - \sqrt{2x-3}}{(\sqrt{2x+3})^2 - (\sqrt{2x-3})^2} \times dx \\ &= \int \frac{\sqrt{2x+3} - \sqrt{2x-3}}{2x+3-2x+3} \times dx \\ &= \frac{1}{6} \int (2x+3)^{\frac{1}{2}} dx - \frac{1}{6} \int (2x-3)^{\frac{1}{2}} dx \\ &= \frac{1}{6} \times \frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2} \times 2} - \frac{1}{6} \times \frac{(2x-3)^{\frac{3}{2}}}{\frac{3}{2} \times 2} + c \\ &= \frac{1}{18} \times (2x+3)^{\frac{3}{2}} - \frac{1}{18} (2x-3)^{\frac{3}{2}} + c \end{aligned}$$

$$\therefore I = \frac{1}{18}(2x+3)^{\frac{3}{2}} - \frac{1}{18}(2x-3)^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.3 Q7

Let $I = \int \frac{2x}{(2x+1)^2} dx$. Then,

$$\begin{aligned} I &= \int \frac{2x+1-1}{(2x+1)^2} \times dx \\ &= \int \frac{2x+1}{(2x+1)^2} \times dx - \int \frac{1}{(2x+1)^2} \times dx \\ &= \int \frac{1}{2x+1} \times dx - \int (2x+1)^{-2} \times dx \\ &= \frac{1}{2} \log|2x+1| - \frac{(2x+1)^{-1}}{-1 \times 2} + c \\ &= \frac{1}{2} \log|2x+1| + \frac{1}{2} \times \frac{1}{2x+1} + c \end{aligned}$$

$$\therefore I = \frac{1}{2} \log|2x+1| + \frac{1}{2(2x+1)} + c.$$

Indefinite Integrals Ex 19.3 Q8

Let $I = \int \frac{1}{\sqrt{x+a} + \sqrt{x+b}} dx$. Then,

$$\begin{aligned} I &= \int \frac{1}{\sqrt{x+a} + \sqrt{x+b}} \times \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}} \times dx \\ &= \int \frac{\sqrt{x+a} - \sqrt{x+b}}{x+a-x-b} \times dx \\ &= \int \frac{\sqrt{x+a} - \sqrt{x+b}}{a-b} \times dx \\ &= \frac{1}{a-b} \left[\frac{2}{3}(x+a)^{\frac{3}{2}} - \frac{2}{3}(x+b)^{\frac{3}{2}} \right] + c \\ &= \frac{2}{3(a-b)} \left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}} \right] + c \\ \therefore I &= \frac{2}{3(a-b)} \left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}} \right] + c. \end{aligned}$$

Indefinite Integrals Ex 19.3 Q9

Let $I = \int \sin \sqrt{1+c \cos 2x} dx$

$$\begin{aligned} I &= \int \sin x \times \sqrt{2 \cos^2 x} \times dx \\ &= \int \sin x \times \sqrt{2} \times \cos x \times dx \\ &= \sqrt{2} \int \sin x \times \cos x \times dx \\ &= \frac{\sqrt{2}}{2} \int 2 \sin x \times \cos x \times dx \\ &= \frac{\sqrt{2}}{2} \int \sin 2x \times dx \\ &= \frac{\sqrt{2}}{2} \times \frac{-\cos 2x}{2} + c \\ &= \frac{-1}{2\sqrt{2}} \times \cos 2x + c \end{aligned}$$

$$\therefore I = \frac{-1}{2\sqrt{2}} \times \cos 2x + c$$

Indefinite Integrals Ex 19.3 Q10

Let $I = \int \frac{1+\cos x}{1-\cos x} dx$. Then,

$$\begin{aligned} I &= \int \frac{\frac{2 \cos^2 \frac{x}{2}}{2}}{\frac{2 \sin^2 \frac{x}{2}}{2}} \times dx \\ &= \int \frac{\cos^2 \frac{x}{2}}{\sin^2 \frac{x}{2}} \times dx \\ &= \int \cot^2 \frac{x}{2} \times dx \\ &= \int \left(\operatorname{cosec}^2 \frac{x}{2} - 1 \right) dx \\ &= \frac{-\cot \frac{x}{2}}{\frac{1}{2}} - x + c \\ &= -2 \cot \frac{x}{2} - x + c \end{aligned}$$

Indefinite Integrals Ex 19.3 Q11

Let $I = \int \frac{1 - \cos x}{1 + \cos x} dx$. Then,

$$\begin{aligned}
 I &= \int \frac{\frac{2 \sin^2 \frac{x}{2}}{2}}{\frac{2 \cos^2 \frac{x}{2}}{2}} \times dx \\
 &= \int \frac{\sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2}} \times dx \\
 &= \int \tan^2 \frac{x}{2} dx \\
 &= \int \left(\sec^2 \frac{x}{2} - 1 \right) dx \\
 &= \frac{\tan \frac{x}{2}}{\frac{1}{2}} - x + c \\
 &= 2 \tan \frac{x}{2} - x + c
 \end{aligned}$$

Indefinite Integrals Ex 19.3 Q12

Let $I = \int \frac{1}{1 - \sin \frac{x}{2}} dx$. Then,

$$\begin{aligned}
 I &= \int \frac{1}{1 - \sin \frac{x}{2}} \times \frac{1 + \sin \frac{x}{2}}{1 + \sin \frac{x}{2}} dx \\
 &= \int \frac{1 + \sin \frac{x}{2}}{1 - \sin^2 \frac{x}{2}} \times dx \\
 &= \int \frac{1 + \sin \frac{x}{2}}{\cos^2 \frac{x}{2}} \times dx \\
 &= \int \frac{1}{\cos^2 \frac{x}{2}} dx + \int \frac{\sin \frac{x}{2}}{\cos^2 \frac{x}{2}} dx \\
 &= \int \sec^2 \frac{x}{2} dx + \int \sec \frac{x}{2} \tan \frac{x}{2} dx \\
 &= \frac{\tan \frac{x}{2}}{\frac{1}{2}} + \frac{\sec \frac{x}{2}}{\frac{1}{2}} + c \\
 &= 2 \tan \frac{x}{2} + 2 \sec \frac{x}{2} + c
 \end{aligned}$$

$$\therefore I = 2 \left(\tan \frac{x}{2} + \sec \frac{x}{2} \right) + c$$

Indefinite Integrals Ex 19.3 Q13

Let $I = \int \frac{1}{1 + \cos 3x} dx$. Then,

$$\begin{aligned}
I &= \int \frac{1}{1 + \cos 3x} \times \frac{1 - \cos 3x}{1 - \cos 3x} dx \\
&= \int \frac{1 - \cos 3x}{1 - \cos^2 3x} dx \\
&= \int \frac{1 - \cos 3x}{\sin^2 3x} dx \\
&= \int \left(\frac{1}{\sin^2 3x} - \frac{\cos 3x}{\sin^2 3x} \right) dx \\
&= \int (\csc^2 3x - \csc 3x \cot 3x) dx \\
&= \frac{-\cot 3x}{3} + \frac{\csc 3x}{3} + C \\
&= \frac{-1}{3} \times \frac{\cos 3x}{\sin 3x} + \frac{1}{3} \times \frac{1}{\sin 3x} + C \\
&= \frac{1 - \cos 3x}{3 \sin 3x} + C
\end{aligned}$$

$$\therefore I = \frac{1 - \cos 3x}{3 \sin 3x} + C.$$

Indefinite Integrals Ex 19.3 Q14

Consider $I = \int (e^x + 1)^3 e^x dx$

let $(e^x + 1) = t \rightarrow e^x dx = dt$

$$\begin{aligned}
I &= \int (e^x + 1)^3 e^x dx \\
&= \int (t)^3 dt \\
&= \frac{t^3}{3} + C \\
&= \frac{(e^x + 1)^3}{3} + C
\end{aligned}$$

Indefinite Integrals Ex 19.3 Q15

Let $I = \int \left(e^x + \frac{1}{e^x} \right)^2 dx$. Then,

$$\begin{aligned}
I &= \int \left(e^x + \frac{1}{e^x} \right)^2 dx \\
&= \int \left(e^{2x} + \frac{1}{e^{2x}} + 2 \right) dx \\
&= \frac{e^{2x}}{2} - \frac{1}{2} e^{-2x} + 2x + C
\end{aligned}$$

$$\therefore I = \frac{1}{2} \times e^{2x} + 2x - \frac{1}{2} \times e^{-2x} + C$$

Indefinite Integrals Ex 19.3 Q16

Let $I = \int \frac{1 + \cos 4x}{\cot x - \tan x} dx$. Then,

$$\begin{aligned}
 I &= \int \frac{2 \cos^2 2x}{\cos x - \frac{\sin x}{\cos x}} dx \\
 &= \int \frac{2 \cos^2 2x}{\cos^2 x - \sin^2 x} \frac{\sin x \cos x}{\sin x \cos x} dx \\
 &= \int \frac{2 \cos^2 2x \times \sin x \cos x}{\cos^2 x - \sin^2 x} dx \\
 &= \int \frac{\cos^2 2x \times \sin 2x}{\cos^2 2x} dx \\
 &= \int \cos 2x \times \sin 2x \times dx \\
 &= \frac{1}{2} \int 2 \sin 2x \cos 2x dx \\
 &= \frac{1}{2} \int [\sin(2x + 2x) + \sin(2x - 2x)] dx \\
 &= \frac{1}{2} \int (\sin 4x + \sin 0) dx \\
 &= \frac{1}{2} \int (\sin 4x + 0) dx \\
 &= \frac{1}{2} \int \sin 4x dx \\
 &= -\frac{1}{2} \times \frac{\cos 4x}{4} + C \\
 &= -\frac{1}{8} \times \cos 4x + C
 \end{aligned}$$

Indefinite Integrals Ex 19.3 Q17

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} dx. \text{ Then,} \\
 I &= \int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} \times \frac{\sqrt{x+3} + \sqrt{x+2}}{\sqrt{x+3} + \sqrt{x+2}} dx \\
 &= \int \frac{\sqrt{x+3} + \sqrt{x+2}}{x+3 - x-2} dx \\
 &= \int \left[(x+3)^{\frac{1}{2}} + (x+2)^{\frac{1}{2}} \right] dx \\
 &= \frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{(x+2)^{\frac{3}{2}}}{\frac{3}{2}} + C \\
 &= \frac{2}{3} \times (x+3)^{\frac{3}{2}} + \frac{2}{3} (x+2)^{\frac{3}{2}} + C \\
 &= \frac{2}{3} \left\{ (x+3)^{\frac{3}{2}} + (x+2)^{\frac{3}{2}} \right\} + C
 \end{aligned}$$

$$\therefore I = \frac{2}{3} \left\{ (x+3)^{\frac{3}{2}} + (x+2)^{\frac{3}{2}} \right\} + C$$

Indefinite Integrals Ex 19.3 Q18

$$\tan^2(2x-3) = \sec^2(2x-3) - 1$$

$$\text{Let } 2x-3 = t$$

$$\Rightarrow 2dx = dt$$

$$\begin{aligned}
 \Rightarrow \int \tan^2(2x-3) dx &= \int [(\sec^2 t) - 1] dt \\
 &= \frac{1}{2} \int (\sec^2 t) dt - \int 1 dt \\
 &= \frac{1}{2} \int \sec^2 t dt - \int 1 dt \\
 &= \frac{1}{2} \tan t - x + C \\
 &= \frac{1}{2} \tan(2x-3) - x + C
 \end{aligned}$$

Indefinite Integrals Ex 19.3 Q19

$$\begin{aligned}
\text{Consider } I &= \int \frac{1}{\cos^2 x (1 - \tan x)^2} dx \\
&= \int \frac{1}{\cos^2 x \left(1 - \frac{\sin x}{\cos x}\right)^2} dx \\
&= \int \frac{1}{(\cos x - \sin x)^2} dx \\
&= \int \frac{1}{1 - \sin 2x} dx \\
&= \int \frac{1}{1 + \cos\left(\frac{\pi}{2} + 2x\right)} dx \\
&= \int \frac{1}{2 \cos^2\left(\frac{\pi}{4} + x\right)} dx \\
&= \frac{1}{2} \int \sec^2\left(\frac{\pi}{4} + x\right) dx
\end{aligned}$$

Ex 19.4

Indefinite Integrals Ex 19.4 Q1

$$\text{Let } I = \int \frac{x^2 + 5x + 2}{x+2} dx$$

Using long division method, we have

$$\begin{aligned}\frac{x^2 + 5x + 2}{x+2} &= x + 3 - \frac{4}{x+2} \\ \therefore I &= \int \frac{x^2 + 5x + 2}{x+2} = \int \left(x + 3 - \frac{4}{x+2} \right) dx \\ \Rightarrow I &= \int x dx + 3 \int dx - 4 \int \frac{1}{x+2} dx \\ &= \frac{x^2}{2} + 3x - 4 \log|x+2| + c \\ \therefore I &= \frac{x^2}{2} + 3x - 4 \log|x+2| + c\end{aligned}$$

Indefinite Integrals Ex 19.4 Q2

$$\text{Let } I = \int \frac{x^3}{x-2} dx$$

Using long division method, we have

$$\begin{aligned}\frac{x^3}{x-2} &= x^2 + 2x + 4 + \frac{8}{x-2} \\ \therefore I &= \int \left(x^2 + 2x + 4 + \frac{8}{x-2} \right) dx \\ &= \int x^2 dx + 2 \int x dx + 4 \int dx + 8 \int \frac{1}{x-2} dx \\ &= \frac{x^3}{3} + \frac{2x^2}{2} + 4x + 8 \log|x-2| + c \\ &= \frac{x^3}{3} + x^2 + 4x + 8 \log|x-2| + c \\ \therefore I &= \frac{x^3}{3} + x^2 + 4x + 8 \log|x-2| + c\end{aligned}$$

Indefinite Integrals Ex 19.4 Q3

$$\text{Let } I = \int \frac{x^2 + x + 5}{3x+2} dx$$

Using long division method, we have

$$\begin{aligned}\frac{x^2 + x + 5}{3x+2} &= \frac{x}{3} + \frac{1}{9} + \frac{43}{9} \times \frac{1}{3x+2} \\ \therefore I &= \int \left(\frac{x}{3} + \frac{1}{9} + \frac{43}{9} \times \frac{1}{3x+2} \right) dx \\ &= \frac{x^2}{6} + \frac{1}{9} \times x + \frac{43}{9 \times 3} \log|3x+2| + c \\ &= \frac{x^2}{6} + \frac{1}{9} \times x + \frac{43}{27} \log|3x+2| + c \\ \therefore I &= \frac{x^2}{6} + \frac{1}{9} \times x + \frac{43}{27} \log|3x+2| + c\end{aligned}$$

Indefinite Integrals Ex 19.4 Q4

Let $I = \int \frac{2x+3}{(x-1)^2} dx$. Then,

$$\begin{aligned}
I &= \int \frac{2x+2-2+3}{(x-1)^2} dx \\
&= \int \frac{2x-2+5}{(x-1)^2} dx \\
&= 2 \int \frac{(x-1)}{(x-1)^2} dx + 5 \int \frac{1}{(x-1)^2} dx \\
&= 2 \int \frac{1}{x-1} dx + 5 \int (x-1)^{-2} dx \\
&= 2 \log|x-1| + 5 \times \frac{(x-1)^{-1}}{-1} + c \\
&= 2 \log|x-1| - \frac{5}{x-1} + c
\end{aligned}$$

$\therefore I = 2 \log|x-1| - \frac{5}{x-1} + c.$

Indefinite Integrals Ex 19.4 Q5

Let $I = \int \frac{x^2+3x-1}{(x+1)^2} dx$. Then,

$$\begin{aligned}
I &= \int \frac{x^2+x+2x-1}{(x+1)^2} dx \\
&= \int \frac{x(x+1)+2x-1}{(x+1)^2} dx \\
&= \int \frac{x(x+1)}{(x+1)^2} dx + \int \frac{2x-1}{(x+1)^2} dx \\
&= \int \frac{x}{x+1} dx + \int \frac{\sqrt{2x+2-2-1}}{(x+1)^2} dx \\
&= \int \frac{x+1-1}{x+1} dx + \int \frac{2(x+1)-3}{(x+1)^2} dx \\
&= \int \frac{x+1}{x+1} dx - \int \frac{1}{x+1} dx + \int \frac{2(x+1)}{(x+1)^2} dx - 3 \int \frac{1}{(x+1)^2} dx \\
&= \int dx - \int \frac{1}{x+1} dx + 2 \int \frac{1}{x+1} dx - 3 \int (x+1)^{-2} dx \\
&= x - \log|x+1| + 2 \log|x+1| + \frac{3}{x+1} + c \\
&= x + \log|x+1| + \frac{3}{x+1} + c
\end{aligned}$$

$\therefore I = x + \log|x+1| + \frac{3}{x+1} + c$

Indefinite Integrals Ex 19.4 Q6

Let $I = \int \frac{2x-1}{(x-1)^2} dx$. Then,

$$\begin{aligned}
I &= \int \frac{2x-1+2-2}{(x-1)^2} dx \\
&= \int \frac{2x-2+1}{(x-1)^2} dx \\
&= \int \frac{2(x-1)}{(x-1)^2} dx + 1 \int \frac{1}{(x-1)^2} dx \\
&= 2 \int \frac{1}{x-1} dx + \int (x-1)^{-2} dx \\
&= 2 \log|x-1| - (x-1)^{-1} + c \\
&= 2 \log|x-1| - \frac{1}{x-1} + c
\end{aligned}$$

$\therefore I = \frac{-1}{x-1} + 2 \log|x-1| + c.$

Ex 19.5

Indefinite Integrals Ex 19.5 Q1

$$\text{Let } I = \int \frac{x+1}{\sqrt{2x+3}} dx$$

$$\text{Let } x+1 = \lambda(2x+3) + \mu$$

On equating the coefficients of like powers of x on both sides, we get

$$\begin{aligned} I &= 2\lambda, \quad 3\lambda + \mu = 1 \\ \Rightarrow \quad \lambda &= \frac{1}{2} \text{ and } 3 \times \frac{1}{2} + \mu = 1 \\ \Rightarrow \quad \lambda &= \frac{1}{2} \text{ and } \mu = \frac{-1}{2} \end{aligned}$$

Replacing $x+1$ by $\lambda(2x+3) + \mu$ in the given integral, we get

$$\begin{aligned} I &= \int \frac{\lambda(2x+3) + \mu}{\sqrt{2x+3}} dx \\ &= \int \frac{\lambda(2x+3)}{\sqrt{2x+3}} dx + \mu \int \frac{1}{\sqrt{2x+3}} dx \\ &= \lambda \int (2x+3)^{\frac{1}{2}} dx + \mu \int (2x+3)^{-\frac{1}{2}} dx \\ &= \lambda \frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2} \times 2} + \mu \frac{(2x+3)^{\frac{1}{2}}}{2 \times \frac{1}{2}} + c \\ &= \frac{1}{2} \times \frac{(2x+3)^{\frac{3}{2}}}{3} + \left(\frac{-1}{2}\right) \times (2x+3)^{\frac{1}{2}} + c \quad \left[\because \lambda = \frac{1}{2}, \mu = \frac{-1}{2} \right] \\ &= \frac{(2x+3)^{\frac{3}{2}}}{6} - \frac{(2x+3)^{\frac{1}{2}}}{2} + c \end{aligned}$$

$$\therefore I = \frac{1}{6} \times (2x+3)^{\frac{3}{2}} - \frac{1}{2} (2x+3)^{\frac{1}{2}} + c.$$

Indefinite Integrals Ex 19.5 Q2

$$\text{Let } I = \int x \sqrt{x+2} dx. \text{ Then,}$$

$$\begin{aligned} I &= \int \{(x+2)-2\}x + 2dx \quad [\because x = (x+2)-2] \\ \Rightarrow \quad I &= \int \left\{ (x+2)^{\frac{3}{2}} - 2(x+2)^{\frac{1}{2}} \right\} dx \\ \Rightarrow \quad I &= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + c \end{aligned}$$

Indefinite Integrals Ex 19.5 Q3

$$\text{Let } I = \int \frac{x-1}{\sqrt{x+4}} dx. \text{ Then,}$$

$$\begin{aligned} I &= \int \frac{x+4-4-1}{\sqrt{x+4}} \\ &= \int \frac{x+4-5}{\sqrt{x+4}} dx \\ &= \int \frac{x+4}{\sqrt{x+4}} dx - 5 \int \frac{1}{\sqrt{x+4}} dx \\ &= \int (x+4)^{\frac{1}{2}} dx - 5 \int (x+4)^{-\frac{1}{2}} dx \\ &= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 5 \frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + c \\ &= \frac{2}{3} \times (x+4)^{\frac{3}{2}} - 10 (x+4)^{\frac{1}{2}} + c \\ \therefore I &= \frac{2}{3} \times (x+4)^{\frac{3}{2}} - 10 (x+4)^{\frac{1}{2}} + c \end{aligned}$$

Indefinite Integrals Ex 19.5 Q4

$$\text{Let } I = \int (x+2) \sqrt{3x+5} dx$$

Let $x+2 = \lambda(3x+5) + \mu$ on equating the coefficients of like powers of x on both sides, we get

$$\begin{aligned} 3\lambda &= 1 \quad \text{and} \quad 5\lambda + \mu = 2 \\ \Rightarrow \lambda &= \frac{1}{3} \quad \text{and} \quad 5 \times \frac{1}{3} + \mu = 2 \\ \Rightarrow \lambda &= \frac{1}{3} \quad \text{and} \quad \mu = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \therefore I &= \int \{\lambda(3x+5) + \mu\} \sqrt{3x+5} dx \\ &= \lambda \int (3x+5) \sqrt{3x+5} dx + \mu \int \sqrt{3x+5} dx \\ &= \lambda \int (3x+5)^{\frac{3}{2}} dx + \mu \int (3x+5)^{\frac{1}{2}} dx \\ &= \lambda \times \frac{(3x+5)^{\frac{5}{2}}}{\frac{5}{2} \times 3} + \mu \times \frac{(3x+5)^{\frac{3}{2}}}{3 \times \frac{3}{2}} + c \\ &= \frac{1}{3} \times \frac{2}{15} \times (3x+5)^{\frac{5}{2}} + \frac{1}{3} \times \frac{2}{9} (3x+5)^{\frac{3}{2}} + c \\ &= \frac{2}{45} \times (3x+5)^{\frac{5}{2}} + \frac{2}{27} \times (3x+5)^{\frac{3}{2}} + c \\ &= \frac{2}{9} \times (3x+5)^{\frac{3}{2}} \left[\frac{1}{5} \times (3x+5)^1 + \frac{1}{3} \right] + c \\ &= \frac{2}{9} \times (3x+5)^{\frac{3}{2}} \left[\frac{3(3x+5)+5}{15} \right] + c \\ &= \frac{2}{9} \times (3x+5)^{\frac{3}{2}} \frac{(9x+15+5)}{15} + c \\ &= \frac{2}{135} \times (3x+5)^{\frac{3}{2}} (9x+20) + c \end{aligned}$$

$$\therefore I = \frac{2}{135} \times (9x+20)(3x+5)^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.5 Q5

$$\text{Let } I = \int \frac{2x+1}{\sqrt{3x+2}} dx$$

Let $2x+1 = \lambda(3x+2) + \mu$ on equating the coefficients of like powers of x on both sides, we get

$$\begin{aligned} 3\lambda &= 2 \quad \text{and} \quad 2\lambda + \mu = 1 \\ \Rightarrow \lambda &= \frac{2}{3} \quad \text{and} \quad 2 \times \frac{2}{3} + \mu = 1 \\ \Rightarrow \lambda &= \frac{2}{3} \quad \text{and} \quad \mu = \frac{-1}{3} \end{aligned}$$

$$\begin{aligned} \therefore I &= \int \frac{\lambda(3x+2) + \mu}{\sqrt{3x+2}} dx \\ &= \lambda \int \frac{3x+2}{\sqrt{3x+2}} dx + \mu \int \frac{1}{\sqrt{3x+2}} dx \\ &= \lambda \int (3x+2)^{\frac{1}{2}} dx + \mu \int (3x+2)^{-\frac{1}{2}} dx \\ &= \lambda \times \frac{(3x+2)^{\frac{3}{2}}}{\frac{3}{2} \times 3} + \mu \times \frac{(3x+2)^{\frac{1}{2}}}{\frac{1}{2} \times 3} + c \\ &= \frac{2}{3} \times \frac{2}{9} \times (3x+2)^{\frac{3}{2}} - \frac{1}{3} \times \frac{2}{3} \times (3x+2)^{\frac{1}{2}} + c \\ &= \frac{4}{27} \times (3x+2)^{\frac{3}{2}} - \frac{2}{9} \times (3x+2)^{\frac{1}{2}} + c \\ &= \frac{2}{9} \times \sqrt{3x+2} \left[\frac{2}{3} \times (3x+2) - 1 \right] + c \\ &= \frac{2}{9} \times \sqrt{3x+2} \left[\frac{6x+4-3}{3} \right] + c \\ &= \frac{2}{27} \times \sqrt{3x+2} (6x+1) + c \end{aligned}$$

$$\therefore I = \frac{2}{27} \times (6x+1) \sqrt{3x+2} + c.$$

Indefinite Integrals Ex 19.5 Q6

$$\text{Let } I = \int \frac{3x+5}{\sqrt{7x+9}} dx$$

Let $3x+5 = \lambda(7x+9) + \mu$ on equating the coefficients of like powers of x on both sides, we get

$$\begin{aligned} 7\lambda &= 3 \quad \text{and} \quad 9\lambda + \mu = 5 \\ \Rightarrow \lambda &= \frac{3}{7} \quad \text{and} \quad 9 \times \frac{3}{7} + \mu = 5 \\ \Rightarrow \lambda &= \frac{3}{7} \quad \text{and} \quad \mu = \frac{8}{7} \end{aligned}$$

$$\begin{aligned} \therefore I &= \int \frac{\lambda(7x+9) + \mu}{\sqrt{7x+9}} dx \\ &= \lambda \int \frac{7x+9}{\sqrt{7x+9}} dx + \mu \int \frac{1}{\sqrt{7x+9}} dx \\ &= \lambda \int (7x+9)^{\frac{1}{2}} dx + \mu \int (7x+9)^{-\frac{1}{2}} dx \\ &= \lambda \times \frac{(7x+9)^{\frac{3}{2}}}{\frac{3}{2} \times 7} + \mu \times \frac{(7x+9)^{\frac{1}{2}}}{\frac{1}{2} \times 7} + c \\ &= \frac{3}{7} \times \frac{2}{21} \times (7x+9)^{\frac{3}{2}} + \frac{8}{7} \times \frac{2}{7} \times (7x+9)^{\frac{1}{2}} + c \\ &= \frac{2}{49} \times (7x+9)^{\frac{3}{2}} + \frac{16}{49} \times (7x+9)^{\frac{1}{2}} + c \\ &= \frac{2}{49} \times (7x+9)^{\frac{1}{2}} [7x+9+8] + c \\ &= \frac{2}{49} \times (7x+9)^{\frac{1}{2}} [7x+17] + c \\ &= \frac{2}{49} \times (7x+17) \sqrt{7x+9} + c \end{aligned}$$

Indefinite Integrals Ex 19.5 Q7

Let $I = \int \frac{x}{\sqrt{x+4}} dx$. Then,

$$\begin{aligned} I &= \int \frac{x+4-4}{\sqrt{x+4}} dx \\ &= \int \frac{x+4}{\sqrt{x+4}} dx - 4 \int \frac{1}{\sqrt{x+4}} dx \\ &= \int (x+4)^{\frac{1}{2}} dx - 4 \int (x+4)^{-\frac{1}{2}} dx \\ &= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 4 \frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= \frac{2(x+4)^{\frac{3}{2}}}{3} - 8(x+4)^{\frac{1}{2}} + C \\ &= 2(x+4)^{\frac{1}{2}} \left[\frac{1}{3}(x+4) - 4 \right] + C \\ &= 2(x+4)^{\frac{1}{2}} \left[\frac{(x+4)-12}{3} \right] + C \\ &= \frac{2}{3}(x+4)^{\frac{1}{2}} [x-8] + C \end{aligned}$$

$$\therefore I = \frac{2}{3} \times (x-8) \sqrt{x+4} + C.$$

Indefinite Integrals Ex 19.5 Q8

Let $I = \int \frac{2-3x}{\sqrt{1+3x}} \times dx$. Then,

$$\begin{aligned} I &= \int \frac{2-3x-1+1}{\sqrt{1+3x}} \times dx \\ &= \int \frac{-3x-1+3}{\sqrt{1+3x}} \times dx \\ &= \int -\frac{(3x+1)}{\sqrt{1+3x}} \times dx + 3 \int \frac{1}{\sqrt{1+3x}} dx \\ &= -1 \int \frac{1+3x}{\sqrt{1+3x}} \times dx + 3 \int \frac{1}{\sqrt{1+3x}} dx \\ &= -1 \int (1+3x)^{\frac{1}{2}} dx + 3 \int (1+3x)^{-\frac{1}{2}} dx \\ &= -1 \times \frac{(1+3x)^{\frac{3}{2}}}{\frac{3}{2} \times 3} + 3 \times \frac{(1+3x)^{\frac{1}{2}}}{\frac{1}{2} \times 3} + C \\ &= -\frac{2}{9} \times (1+3x)^{\frac{3}{2}} + 2(1+3x)^{\frac{1}{2}} + C \\ &= 2(1+3x)^{\frac{1}{2}} \left[-\frac{1}{9}(1+3x)^1 + 1 \right] + C \\ &= 2(1+3x)^{\frac{1}{2}} \left[\frac{-1-3x+9}{9} \right] + C \\ &= 2(1+3x)^{\frac{1}{2}} \left[\frac{8-3x}{9} \right] + C \\ &= \frac{2}{9} \sqrt{1+3x} (8-3x) + C \end{aligned}$$

$$\therefore I = \frac{2}{9} (8-3x) \sqrt{1+3x} + C$$

Indefinite Integrals Ex 19.5 Q9

Let $I = \int 5x + 3\sqrt{2x - 1} dx$

Let $5x + 3 = \lambda(2x - 1) + \mu$ comparing both sides, we get

$$\begin{aligned}2\lambda &= 5 \quad \text{and} \quad -\lambda + \mu = 3 \\ \Rightarrow \lambda &= \frac{5}{2} \quad \text{and} \quad -\frac{5}{2} + \mu = 3 \\ \Rightarrow \lambda &= \frac{5}{2} \quad \text{and} \quad \mu = \frac{11}{2}\end{aligned}$$

$$\begin{aligned}\therefore I &= \int \{\lambda(2x - 1) + \mu\} \sqrt{2x - 1} dx \\ &= \lambda \int (2x - 1) \sqrt{2x - 1} dx + \mu \int \sqrt{2x - 1} dx \\ &= \lambda \int (2x - 1)^{\frac{3}{2}} dx + \mu \int (2x - 1)^{\frac{1}{2}} dx \\ &= \lambda \frac{(2x - 1)^{\frac{5}{2}}}{\frac{5}{2} \times 2} + \mu \frac{(2x - 1)^{\frac{3}{2}}}{\frac{3}{2} \times 2} \\ &= \lambda \frac{(2x - 1)^{\frac{5}{2}}}{5} + \mu \frac{(2x - 1)^{\frac{3}{2}}}{3} + C \\ &= \frac{5}{2} \times \frac{(2x - 1)^{\frac{5}{2}}}{5} + \frac{11}{2} \times \frac{(2x - 1)^{\frac{3}{2}}}{3} + C \\ &= \frac{(2x - 1)^{\frac{5}{2}}}{2} + \frac{11}{6} \times (2x - 1)^{\frac{3}{2}} + C \\ &= \frac{1}{2} (2x - 1)^{\frac{3}{2}} \left[(2x - 1) + \frac{11}{3} \right] + C \\ &= \frac{1}{2} \times (2x - 1)^{\frac{3}{2}} \left[\frac{6x + 8}{3} \right] + C \\ &= \frac{1}{2} \times (2x - 1)^{\frac{3}{2}} \times 2 \frac{(3x + 4)}{3} + C \\ &= (2x - 1)^{\frac{3}{2}} \times \frac{(3x + 4)}{3} + C \\ &= \frac{1}{3} \times (3x + 4) (2x - 1)^{\frac{3}{2}} + C\end{aligned}$$

$$\therefore I = \frac{1}{3} \times (3x + 4) (2x - 1)^{\frac{3}{2}} + C.$$

Ex 19.6

Indefinite Integrals Ex 19.6 Q1

$$\begin{aligned}\sin^2(2x+5) &= \frac{1-\cos(2(2x+5))}{2} = \frac{1-\cos(4x+10)}{2} \\ \Rightarrow \int \sin^2(2x+5) dx &= \int \frac{1-\cos(4x+10)}{2} dx \\ &= \frac{1}{2} \int 1 dx - \frac{1}{2} \int \cos(4x+10) dx \\ &= \frac{1}{2}x - \frac{1}{2} \left(\frac{\sin(4x+10)}{4} \right) + C \\ &= \frac{1}{2}x - \frac{1}{8} \sin(4x+10) + C\end{aligned}$$

Indefinite Integrals Ex 19.6 Q2

We need to evaluate $\int \sin^3(2x+1) dx$
by using the formula $\rightarrow \sin 3\theta = -4 \sin^3 \theta + 3 \sin \theta$

$$\begin{aligned}\therefore \sin^3(2x+1) &= \frac{3 \sin(2x+1) - \sin 3(2x+1)}{4} \\ &= \int \frac{3 \sin(2x+1) - \sin 3(2x+1)}{4} dx \\ &= \int \frac{3 \sin(2x+1) - \sin 3(2x+1)}{4} dx \\ &= -\frac{3}{8} \cos(2x+1) + \frac{1}{24} \cos 3(2x+1) + C\end{aligned}$$

Indefinite Integrals Ex 19.6 Q3

Evaluate the integral as follows

$$\begin{aligned}1 \int \cos^4 2x dx &= \int (\cos^2 2x)^2 dx \\ &= \int \left(\frac{1}{2}(\cos 4x + 1) \right)^2 dx \\ &= \int \left(\frac{1}{4}(\cos^2 4x) + \frac{1}{4} + \frac{\cos 4x}{2} \right) dx \\ &= \int \left(\frac{1}{4} \left(\frac{1}{2}(\cos 8x + 1) \right) + \frac{1}{4} + \frac{\cos 4x}{2} \right) dx \\ &= \int \frac{1}{8} \left(\cos 8x + \frac{3}{8} + \frac{\cos 4x}{2} \right) dx \\ &= \frac{1}{64} \sin 8x + \frac{3}{8}x + \frac{1}{8} \sin 4x + C\end{aligned}$$

Indefinite Integrals Ex 19.6 Q4

Let $I = \int \sin^2 bx dx$. Then,

$$\begin{aligned}I &= \int \frac{1 - \cos 2bx}{2} x dx \\ &= \frac{1}{2} \int x dx - \frac{1}{2} \int \cos 2bx x dx \\ &= \frac{1}{2} \cdot \frac{x^2}{2} - \frac{1}{2} \frac{\sin(2bx)}{2b} + C \\ \therefore I &= \frac{x^2}{4} - \frac{\sin 2bx}{4b} + C\end{aligned}$$

Indefinite Integrals Ex 19.6 Q5

Let $I = \int \sin^2 \frac{x}{2} dx$. Then,

$$\begin{aligned}
 I &= \frac{1}{2} \int 2 \sin^2 \frac{x}{2} dx \\
 &= \frac{1}{2} \int (1 - \cos x) dx \quad [\because \cos 2x = 1 - 2 \sin^2 x] \\
 &= \frac{1}{2} \int dx - \frac{1}{2} \int \cos x dx \\
 &= \frac{1}{2} x - \frac{1}{2} \times \sin x + c \\
 &= \frac{1}{2} (x - \sin x) + c
 \end{aligned}$$

$$\therefore I = \frac{1}{2} (x - \sin x) + c.$$

Indefinite Integrals Ex 19.6 Q6

We have,

$$\begin{aligned}
 \int \cos^2 \frac{x}{2} dx &= \frac{1}{2} \int 2 \cos^2 \frac{x}{2} dx \\
 &= \frac{1}{2} \int (1 + \cos x) dx \\
 &= \frac{1}{2} \int dx + \frac{1}{2} \int \cos x dx \\
 &= \frac{1}{2} x + \frac{1}{2} \sin x + c \\
 &= \frac{1}{2} (x + \sin x) + c
 \end{aligned}$$

$$\therefore \int \cos^2 \frac{x}{2} dx = \frac{1}{2} (x + \sin x) + c.$$

Indefinite Integrals Ex 19.6 Q7

Let $I = \int \cos^2 nx dx$. Then,

$$\begin{aligned}
 I &= \frac{1}{2} \int 2 \cos^2 nx dx \\
 &= \frac{1}{2} \int [1 + \cos 2nx] dx \\
 &= \frac{1}{2} \int \left[x + \frac{\sin 2nx}{2n} \right] + c \\
 &= \frac{x}{2} + \frac{1}{4n} \times \sin 2nx + c
 \end{aligned}$$

$$\therefore I = \frac{x}{2} + \frac{1}{4n} \times \sin 2nx + c.$$

Indefinite Integrals Ex 19.6 Q8

Let $I = \int \sin \sqrt{1 - \cos 2x} dx$. Then,

$$\begin{aligned} I &= \int \sin x \times \sqrt{2 \sin^2 x} \times dx \\ &= \int \sin x \times \sqrt{2} \times \sin x dx \\ &= \sqrt{2} \int \sin^2 x \times dx \\ &= \frac{\sqrt{2}}{2} \int 2 \sin^2 x \times dx \\ &= \frac{\sqrt{2}}{2} \left[x - \frac{\sin 2x}{2} \right] + c \\ &= \frac{\sqrt{2}x}{2} - \frac{\sqrt{2}}{4} \times \sin 2x + c \\ &= \frac{1}{\sqrt{2}} \times x - \frac{\sin 2x}{2\sqrt{2}} + c \\ \therefore I &= \frac{1}{\sqrt{2}} \times x - \frac{\sin 2x}{2\sqrt{2}} + c. \end{aligned}$$

Ex 19.7

Indefinite Integrals Ex 19.7 Q1

Let $I = \int \sin 4x \cos 7x dx$. Then,

$$\begin{aligned} I &= \frac{1}{2} \int 2 \sin 4x \times \cos 7x dx \\ &= \frac{1}{2} \int (\sin 11x + \sin(-3x)) dx \\ &= \frac{1}{2} \int \sin 11x dx - \frac{1}{2} \int \sin 3x dx \\ &= \frac{-1}{2 \times 11} \times \cos 11x + \frac{1}{2 \times 3} \cos 3x + c \\ &= -\frac{1}{22} \times \cos 11x + \frac{1}{6} \times \cos 3x + c \\ \therefore I &= -\frac{1}{22} \times \cos 11x + \frac{1}{6} \times \cos 3x + c. \end{aligned}$$

Indefinite Integrals Ex 19.7 Q2

Let $I = \int \cos 3x \cos 4x dx$. Then,

$$\begin{aligned} I &= \frac{1}{2} \int (2 \cos 3x \cos 4x) dx \\ &= \frac{1}{2} \int (\cos 7x + \cos(-x)) dx \\ &= \frac{1}{2} \int \cos 7x dx + \frac{1}{2} \int \cos x dx \quad [\because \cos(-x) = \cos x] \\ &= \frac{\sin 7x}{2 \times 7} + \frac{\sin x}{2} + c \\ &= \frac{1}{14} \times \sin 7x + \frac{1}{2} \sin x + c \\ \therefore I &= \frac{1}{14} \times \sin 7x + \frac{1}{2} \sin x + c. \end{aligned}$$

Indefinite Integrals Ex 19.7 Q3

Let $I = \int \cos mx \cos nx dx$ $m \neq n$. Then,

$$\begin{aligned} I &= \frac{1}{2} \int 2 \cos mx \cos nx dx \\ &= \frac{1}{2} \int [\cos(m+n)x + \cos(m-n)x] dx \\ &= \frac{1}{2} \times \frac{\sin(m+n)x}{m+n} + \frac{1}{2} \times \frac{\sin(m-n)x}{m-n} + c \\ \therefore I &= \frac{1}{2} \left[\frac{\sin(m+n)x}{m+n} + \frac{\sin(m-n)x}{m-n} \right] + c. \end{aligned}$$

Indefinite Integrals Ex 19.7 Q4

We have,

$$\begin{aligned} &\int \sin mx \cos nx dx, m \neq n \\ &= \frac{1}{2} \int 2 \sin mx \cos nx dx \\ &= \frac{1}{2} \int [\sin(m+n)x + \sin(m-n)x] dx \\ &= \frac{1}{2} \times \left[\frac{-\cos(m+n)x}{m+n} - \frac{\cos(m-n)x}{m-n} \right] + c \\ \therefore \int \sin mx \cos nx &= \frac{1}{2} \left[\frac{-\cos(m+n)x}{m+n} - \frac{\cos(m-n)x}{m-n} \right] + c. \end{aligned}$$

Ex 19.8

Indefinite Integrals Ex 19.8 Q1

We have,

$$\begin{aligned} \int \frac{1}{\sqrt{1-\cos 2x}} dx &= \int \frac{1}{\sqrt{2 \sin^2 x}} dx \\ &= \int \frac{1}{\sqrt{2} \sin x} dx \\ &= \frac{1}{\sqrt{2}} \int \csc x dx \\ &= \frac{1}{\sqrt{2}} \log \left| \tan \frac{x}{2} \right| + c \end{aligned}$$

$$\therefore \int \frac{1}{\sqrt{1-\cos 2x}} dx = \frac{1}{\sqrt{2}} \log \left| \tan \frac{x}{2} \right| + c$$

Indefinite Integrals Ex 19.8 Q2

We have,

$$\begin{aligned} \int \frac{1}{\sqrt{1+\cos x}} dx &= \int \frac{1}{\sqrt{2 \cos^2 \frac{x}{2}}} dx \\ &= \int \frac{1}{\sqrt{2} \cos \frac{x}{2}} dx \\ &= \frac{1}{\sqrt{2}} \int \sec \frac{x}{2} dx \\ &= \frac{1}{\sqrt{2}} \int \csc \left(\frac{\pi}{2} + \frac{x}{2} \right) dx \\ &= \frac{2}{\sqrt{2}} \log \left| \tan \left(\frac{\pi}{4} + \frac{x}{4} \right) \right| + c \end{aligned}$$

$$\therefore \int \frac{1}{\sqrt{1+\cos x}} dx = \sqrt{2} \log \left| \tan \left(\frac{\pi}{4} + \frac{x}{4} \right) \right| + c$$

Indefinite Integrals Ex 19.8 Q3

Let $I = \int \frac{1+\cos 2x}{\sqrt{1-\cos 2x}} dx$ then,

$$\begin{aligned} I &= \int \frac{\sqrt{2 \cos^2 \frac{x}{2}}}{\sqrt{2 \sin^2 \frac{x}{2}}} dx \\ &= \int \sqrt{\cot^2 x} dx \\ &= \int \cot x dx \\ &= \log |\sin x| + c \quad [\because \int \cot x dx = \log |\sin x| + c] \end{aligned}$$

$$I = \log |\sin x| + c$$

Indefinite Integrals Ex 19.8 Q4

Let $I = \int \frac{1-\cos x}{\sqrt{1+\cos x}} dx$ then,

$$\begin{aligned} I &= \int \frac{\sqrt{2 \sin^2 \frac{x}{2}}}{\sqrt{2 \cos^2 \frac{x}{2}}} dx \\ &= \int \sqrt{\tan^2 \frac{x}{2}} dx \\ &= \int \tan \frac{x}{2} dx \\ &= -2 \log \left| \cos \frac{x}{2} \right| + c \quad [\because \int \tan x dx = \log |\cos x| + c] \end{aligned}$$

$$\therefore I = -2 \log \left| \cos \frac{x}{2} \right| + c$$

Indefinite Integrals Ex 19.8 Q5

$$\begin{aligned}
 \text{Let } I &= \int \frac{\sec x}{\sec 2x} dx, \quad \text{then,} \\
 I &= \int \frac{\frac{1}{\cos x}}{\frac{1}{\cos 2x}} dx \\
 &= \int \frac{\cos 2x}{\cos x} dx \\
 &= \int \frac{2\cos^2 x - 1}{\cos x} dx \\
 &= \int 2\cos x dx - \int \frac{1}{\cos x} dx \\
 &= 2\int \cos x dx - \int \sec x dx \\
 &= 2\sin x - \log|\sec x + \tan x| + C
 \end{aligned}$$

$$\therefore I = 2\sin x - \log|\sec x + \tan x| + C$$

Indefinite Integrals Ex 19.8 Q6

$$\begin{aligned}
 \frac{\cos 2x}{(\cos x + \sin x)^2} &= \frac{\cos 2x}{\cos^2 x + \sin^2 x + 2\sin x \cos x} = \frac{\cos 2x}{1 + \sin 2x} \\
 \therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx &= \int \frac{\cos 2x}{(1 + \sin 2x)} dx \\
 \text{Let } 1 + \sin 2x &= t \\
 \Rightarrow 2\cos 2x dx &= dt \\
 \therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx &= \frac{1}{2} \int \frac{dt}{t} \\
 &= \frac{1}{2} \log|t| + C \\
 &= \frac{1}{2} \log|1 + \sin 2x| + C \\
 &= \frac{1}{2} \log|(\sin x + \cos x)^2| + C \\
 &= \log|\sin x + \cos x| + C
 \end{aligned}$$

Indefinite Integrals Ex 19.8 Q7

$$\begin{aligned}
 \text{Let } I &= \int \frac{\sin(x-a)}{\sin(x-b)} dx \text{ then} \\
 I &= \int \frac{\sin(x-a+b-b)}{\sin(x-b)} dx \\
 &= \int \frac{\sin(x-b+b-a)}{\sin(x-b)} dx \\
 &= \int \frac{\sin(x-b)\cos(b-a) + \cos(x-b)\sin(b-a)}{\sin(x-b)} dx \\
 &= \int (\cos(b-a) + \cot(x-b)\sin(b-a)) dx \\
 &= \cos(b-a) \int dx + \sin(b-a) \int \cot(x-b) dx \\
 &= x \cos(b-a) + \sin(b-a) \log|\sin(x-b)| + C
 \end{aligned}$$

$$\therefore I = x \cos(b-a) + \sin(b-a) \log|\sin(x-b)| + C$$

Indefinite Integrals Ex 19.8 Q8

Let $I = \int \frac{\sin(x - \alpha)}{\sin(x + \alpha)} dx$ then,

$$\begin{aligned}
I &= \int \frac{\sin(x - \alpha + \alpha - \alpha)}{\sin(x + \alpha)} dx \\
&= \int \frac{\sin(x + \alpha - 2\alpha)}{\sin(x + \alpha)} dx \\
&= \int \frac{\sin(x + \alpha) \cos 2\alpha - \cos(x + \alpha) \sin 2\alpha}{\sin(x + \alpha)} dx \\
&= \int \left[\frac{\sin(x + \alpha) \cos 2\alpha}{\sin(x + \alpha)} - \frac{\cos(x + \alpha) \sin 2\alpha}{\sin(x + \alpha)} \right] dx \\
&= \int (\cos 2\alpha - \cot(x + \alpha) \sin 2\alpha) dx \\
&= \cos 2\alpha \int dx - \sin 2\alpha \int \cot(x + \alpha) dx \\
&= x \cos 2\alpha - \sin 2\alpha \log |\sin(x + \alpha)| + c
\end{aligned}$$

$$\therefore I = x \cos 2\alpha - \sin 2\alpha \log |\sin(x + \alpha)| + c$$

Indefinite Integrals Ex 19.8 Q9

$$\begin{aligned}
\text{Let } I &= \int \frac{1 + \tan x}{1 - \tan x} dx \\
I &= \int \frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}} dx \\
&= \int \frac{\cos x + \sin x}{\cos x - \sin x} dx
\end{aligned}$$

$$\Rightarrow I = \int \frac{\cos x + \sin x}{\cos x - \sin x} dx \quad \text{--- --- (i)}$$

Let $\cos x - \sin x = t$ then,
 $d(\cos x - \sin x) = dt$

$$\begin{aligned}
\Rightarrow & (-\sin x - \cos x) dx = dt \\
\Rightarrow & -(\sin x + \cos x) dx = dt \\
\Rightarrow & dx = -\frac{dt}{\sin x + \cos x}
\end{aligned}$$

Putting $\cos x - \sin x = t$ and $dx = \frac{-dt}{\sin x + \cos x}$ in equation (i), we get

$$\begin{aligned}
I &= \int \frac{\cos x + \sin x}{t} \times \frac{-dt}{\sin x + \cos x} \\
&= -\int \frac{dt}{t} \\
&= -\log |t| + c \\
&= -\log |\cos x - \sin x| + c \\
\therefore I &= -\log |\cos x - \sin x| + c
\end{aligned}$$

Indefinite Integrals Ex 19.8 Q10

Let $I = \int \frac{\cos x}{\cos(x-a)} dx$ then,

$$\begin{aligned}
 I &= \int \frac{\cos(x+a-a)}{\cos(x-a)} dx \\
 &= \int \frac{\cos(x-a+a)}{\cos(x-a)} dx \\
 &= \int \frac{\cos(x-a)\cos a - \sin(x-a)\sin a}{\cos(x-a)} dx \\
 &= \int \frac{\cos(x-a)\cos a}{\cos(x-a)} dx - \int \frac{\sin(x-a)\sin a}{\cos(x-a)} dx \\
 &= \cos a dx - \sin a \int \tan(x-a) dx \\
 &= x \cos a - \sin a \log|\sec(x-a)| + c
 \end{aligned}$$

Indefinite Integrals Ex 19.8 Q11

Let $I = \int \sqrt{\frac{1-\sin 2x}{1+\sin 2x}} dx$ then,

$$\begin{aligned}
 I &= \int \sqrt{\frac{1-\cos(\frac{\pi}{2}-2x)}{1+\cos(\frac{\pi}{2}-2x)}} dx \\
 &= \int \sqrt{\frac{2\sin^2(\frac{\pi}{4}-x)}{2\cos^2(\frac{\pi}{4}-x)}} dx \\
 &= \int \sqrt{\tan^2(\frac{\pi}{4}-x)} dx \\
 &= \int \tan(\frac{\pi}{4}-x) dx \\
 &= \log|\cos(\frac{\pi}{4}-x)| + c
 \end{aligned}$$

Indefinite Integrals Ex 19.8 Q12

Let $I = \int \frac{e^{3x}}{e^{3x}+1} dx$ ---- (i)

Let $e^{3x}+1=t$, then,
 $d(e^{3x}+1)=dt$

$$\begin{aligned}
 \Rightarrow 3e^{3x}dx &= dt \\
 \Rightarrow dx &= \frac{dt}{3e^{3x}}
 \end{aligned}$$

Putting $e^{3x}+1=t$ and $dx = \frac{dt}{3e^{3x}}$ in equation (i), we get

$$\begin{aligned}
 I &= \int \frac{e^{3x}}{t} \times \frac{dt}{3e^{3x}} \\
 &= \frac{1}{3} \int \frac{dt}{t} \\
 &= \frac{1}{3} \log|t| + c
 \end{aligned}$$

$$= \frac{1}{3} \log|3e^{3x}+1| + c$$

$$\therefore = \frac{1}{3} \log|3e^{3x}+1| + c$$

Indefinite Integrals Ex 19.8 Q13

$$\text{Let } I = \int \frac{\sec x \tan x}{3 \sec x + 5} dx \quad \dots \dots \dots (i)$$

Let $3 \sec x + 5 = t$, then,

$$\begin{aligned}\Rightarrow & d(3 \sec x + 5) = dt \\ \Rightarrow & 3 \sec x \tan x dx = dt \\ \Rightarrow & dx = \frac{dt}{3 \sec x \tan x}\end{aligned}$$

Putting $3 \sec x \tan x dx = t$ and $dx = \frac{dt}{3 \sec x \tan x}$ in equation (i), we get

$$\begin{aligned}I &= \int \frac{\sec x \tan x}{t} \times \frac{dt}{3 \sec x \tan x} \\ &= \frac{1}{3} \int \frac{1}{t} dt \\ &= \frac{1}{3} \log|t| + c \\ &= \frac{1}{3} \log|3 \sec x + 5| + c\end{aligned}$$

Indefinite Integrals Ex 19.8 Q14

$$\text{Let } I = \int \frac{1 - \cot x}{1 + \cot x} dx \text{ then,}$$

$$\begin{aligned}I &= \int \frac{\frac{1 - \cos x}{\sin x}}{\frac{1 + \cos x}{\sin x}} dx \\ &= \int \frac{\sin x - \cos x}{\sin x + \cos x} dx \\ \Rightarrow & I = \int \frac{\sin x - \cos x}{\sin x + \cos x} dx \quad \dots \dots \dots (i) \\ \text{Let } & \sin x + \cos x = t, \text{ then,} \\ & d(\sin x + \cos x) = dt \\ \Rightarrow & (\cos x - \sin x) dx = dt \\ \Rightarrow & -(\sin x - \cos x) dx = dt \\ \Rightarrow & dx = -\frac{dt}{\sin x - \cos x} \\ \text{Putting } & \sin x + \cos x = t \text{ and } dx = -\frac{dt}{\sin x - \cos x} \text{ in equation (i), we get,} \\ I &= \int \frac{\sin x - \cos x}{t} \times \frac{-dt}{\sin x - \cos x} \\ &= \int \frac{-dt}{t} \\ &= -\log|t| + c \\ &= -\log|\sin x + \cos x| + c\end{aligned}$$

Indefinite Integrals Ex 19.8 Q15

$$\text{Let } I = \int \frac{\sec x \cosec x}{\log(\tan x)} dx \quad \text{then,}$$

$$\begin{aligned} \text{Let } \log(\tan x) &= t \quad \text{then,} \\ d[\log(\tan x)] &= dt \end{aligned}$$

$$\begin{aligned} \Rightarrow \sec x \cosec x dx &= dt & \left[\because \frac{d}{dx}(\log \tan x) = \sec x \cosec x \right] \\ \Rightarrow dx &= \frac{dt}{\sec x \cosec x} \end{aligned}$$

Putting $\log(\tan x) = t$ and $dx = \frac{dt}{\sec x \cosec x}$ in equation (i), we get,

$$\begin{aligned} I &= \int \frac{\sec x \cosec x}{t} \times \frac{dt}{\sec x \cosec x} \\ &= \int \frac{dt}{t} \\ &= \log|t| + c \\ &= \log|\log \tan x| + c \end{aligned}$$

Indefinite Integrals Ex 19.8 Q16

$$\text{Let } I = \int \frac{1}{x(3+\log x)} dx \quad \text{--- (i)}$$

$$\begin{aligned} \text{Let } 3 + \log x &= t \quad \text{then,} \\ d(3 + \log x) &= dt \\ \Rightarrow \frac{1}{x} dx &= dt \\ \Rightarrow dx &= x dt \end{aligned}$$

Putting $3 + \log x = t$ and $dx = x dt$ in equation (i), we get,

$$\begin{aligned} I &= \int \frac{1}{x \times t} \times x dt \\ &= \int \frac{dt}{t} \\ &= \log|t| + c \\ &= \log|(3 + \log x)| + c \\ \therefore I &= \log|(3 + \log x)| + c \end{aligned}$$

Indefinite Integrals Ex 19.8 Q17

$$\text{Let } I = \int \frac{e^x + 1}{e^x + x} dx \quad \dots \quad (i)$$

$$\begin{aligned} & \text{Let } e^x + x = t \quad \text{then,} \\ & d(e^x + x) = dt \\ \Rightarrow & (e^x + x)dx = dt \\ \Rightarrow & dx = \frac{dt}{e^x + 1} \end{aligned}$$

Putting $e^x + x = t$ and $dx = \frac{dt}{e^x + 1}$ in equation (i), we get,

$$\begin{aligned} I &= \int \frac{e^x + 1}{t} \times \frac{dt}{e^x + 1} \\ &= \int \frac{dt}{t} \\ &= \log|t| + c \\ &= \log|e^x + x| + c \\ \therefore I &= \log|e^x + x| + c \end{aligned}$$

Indefinite Integrals Ex 19.8 Q18

$$\text{Let } I = \int \frac{1}{x \log x} dx \quad \dots \quad (i)$$

$$\begin{aligned} & \text{Let } \log x = t \quad \text{then,} \\ & d(\log x) = dt \\ \Rightarrow & \frac{1}{x} dx = dt \\ \Rightarrow & dx = x dt \end{aligned}$$

Putting $\log x = t$ and $dx = x dt$ in equation (i), we get,

$$\begin{aligned} I &= \int \frac{1}{x \times t} \times x dt \\ &= \int \frac{dt}{t} \\ &= \log|t| + c \\ &= \log|(\log x)| + c \\ \therefore I &= \log|(\log x)| + c \end{aligned}$$

Indefinite Integrals Ex 19.8 Q19

$$\text{Let } I = \int \frac{\sin 2x}{a \cos^2 x + b \sin^2 x} dx \quad \dots \dots \dots (i)$$

Let $a \cos^2 x + b \sin^2 x = t$ then,

$$d(a \cos^2 x + b \sin^2 x) = dt$$

$$[a(2 \cos x (-\sin x)) + b(2 \sin x \cos x)] dx = dt$$

$$\Rightarrow [-a(2 \sin x \cos x) + b(2 \sin x \cos x)] dx = dt$$

$$\Rightarrow [-a \sin 2x + b \sin 2x] dx = dt$$

$$\Rightarrow \sin 2x (b - a) dx = dt$$

$$\Rightarrow dx = \frac{dt}{(b - a) \sin 2x}$$

Putting $a \cos^2 x + b \sin^2 x = t$ and $dx = \frac{dt}{(b - a) \sin 2x}$ in equation (i), we get,

$$I = \int \frac{\sin 2x}{t} \times \frac{dt}{(b - a) \sin 2x}$$

$$= \frac{1}{b - a} \int \frac{dt}{t}$$

$$= \frac{1}{b - a} \log|t| + c$$

$$= \frac{1}{b - a} \log|a \cos^2 x + b \sin^2 x| + c$$

Indefinite Integrals Ex 19.8 Q20

$$\text{Let } I = \int \frac{\cos x}{2 + 3 \sin x} dx \quad \dots \dots \dots (i)$$

Let $2 + 3 \sin x = t$ then,

$$d(2 + 3 \sin x) = dt$$

$$3 \cos x dx = dt$$

$$\Rightarrow dx = \frac{dt}{3 \cos x}$$

Putting $2 + 3 \sin x = t$ and $dx = \frac{dt}{3 \cos x}$ in equation (i), we get,

$$I = \int \frac{\cos x}{t} \times \frac{dt}{3 \cos x}$$

$$= \frac{1}{3} \int \frac{dt}{t}$$

$$= \frac{1}{3} \log|t| + c$$

$$= \frac{1}{3} \log|2 + 3 \sin x| + c$$

Indefinite Integrals Ex 19.8 Q21

$$\text{Let } I = \int \frac{1 - \sin x}{x + \cos x} dx \quad \text{--- (i)}$$

$$\begin{aligned}\text{Let } x + \cos x &= t \quad \text{then,} \\ d(x + \cos x) &= dt\end{aligned}$$

$$\Rightarrow (1 - \sin x)dx = dt$$

$$\Rightarrow dx = \frac{dt}{1 - \sin x}$$

Putting $x + \cos x = t$ and $dx = \frac{dt}{1 - \sin x}$ in equation (i), we get,

$$\begin{aligned}I &= \int \frac{1 - \sin x}{t} \times \frac{dt}{1 - \sin x} \\&= \int \frac{dt}{t} \\&= \log|t| + c \\&= \log|x + \cos x| + c\end{aligned}$$

$$\therefore I = \log|x + \cos x| + c$$

Indefinite Integrals Ex 19.8 Q22

$$\text{Let } I = \int \frac{a}{b + ce^x} dx \quad \text{then,}$$

$$I = \int \frac{a}{e^x \left[\frac{b}{e^x} + c \right]} dx$$

$$\Rightarrow I = \int \frac{a}{e^x [be^{-x} + c]} dx \quad \text{--- (i)}$$

$$\text{Let } be^{-x} + c = t \quad \text{then,}$$

$$d(be^{-x} + c) = dt$$

$$\Rightarrow -be^{-x}dx = dt$$

$$\Rightarrow dx = \frac{-dt}{be^{-x}}$$

$$= -\frac{e^x dt}{b}$$

Putting $be^{-x} + c = t$ and $dx = \frac{-e^x dt}{b}$ in equation (i), we get,

$$\begin{aligned}I &= \int \frac{a}{e^x \times t} \times \frac{-e^x dt}{b} \\&= -\frac{a}{b} \int \frac{dt}{t} \\&= -\frac{a}{b} \log|t| + c\end{aligned}$$

$$= -\frac{a}{b} \log|be^{-x} + c| + c$$

Indefinite Integrals Ex 19.8 Q23

Let $I = \int \frac{1}{e^x + 1} dx$ then,

$$I = \int \frac{1}{e^x \left[1 + \frac{1}{e^x} \right]} dx$$

$$\Rightarrow I = \int \frac{1}{e^x \left[1 + e^{-x} \right]} dx \quad \text{--- (i)}$$

Let $1 + e^{-x} = t$ then,

$$d(1 + e^{-x}) = dt$$

$$\Rightarrow -e^{-x} dx = dt$$

$$\Rightarrow dx = \frac{-dt}{e^{-x}}$$

$$dx = -dt \times e^x$$

Putting $1 + e^{-x} = t$ and $dx = -e^x dt$ in equation (i), we get,

$$I = \int \frac{1}{e^x \times t} \times -e^x dt$$

$$= - \int \frac{dt}{t}$$

$$= -\log|t| + c$$

$$= -\log|1 + e^{-x}| + c$$

Indefinite Integrals Ex 19.8 Q24

$$\text{Let } I = \int \frac{\cot x}{\log \sin x} dx \quad \text{--- (i)}$$

Let $\log \sin x = t$ then,

$$d(\log \sin x) = dt$$

$$\Rightarrow \frac{\cos x}{\sin x} dx = dt$$

$$\Rightarrow \cot x dx = dt$$

$$\Rightarrow dx = \frac{dt}{\cot x}$$

Putting $\log \sin x = t$ and $dx = \frac{dt}{\cot x}$ in equation (i), we get,

$$I = \int \frac{\cot x}{t} \times \frac{dt}{\cot x}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log|\log \sin x| + c$$

Indefinite Integrals Ex 19.8 Q25

$$\text{Let } I = \int \frac{e^{2x}}{e^{2x} - 2} dx \quad \dots \quad (i)$$

$$\begin{aligned}\text{Let } e^{2x} - 2 &= t \quad \text{then,} \\ dt &= d(e^{2x} - 2) = dt\end{aligned}$$

$$\begin{aligned}\Rightarrow 2e^{2x}dx &= dt \\ \Rightarrow dx &= \frac{dt}{2e^{2x}}\end{aligned}$$

Putting $e^{2x} - 2 = t$ and $dx = \frac{dt}{2e^{2x}}$ in equation (i), we get,

$$\begin{aligned}I &= \int \frac{2e^{2x}}{t} \times \frac{dt}{2e^{2x}} \\ &= \frac{1}{2} \int \frac{dt}{t} \\ &= \frac{1}{2} \log|t| + C \\ &= \frac{1}{2} \log|e^{2x} - 2| + C\end{aligned}$$

$$\therefore = \frac{1}{2} \log|e^{2x} - 2| + C$$

Indefinite Integrals Ex 19.8 Q26

$$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} = \frac{2\cos x - 3\sin x}{2(3\cos x + 2\sin x)}$$

$$\text{Let } 3\cos x + 2\sin x = t$$

$$(-3\sin x + 2\cos x)dx = dt$$

$$\begin{aligned}\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} dx &= \int \frac{dt}{2t} \\ &= \frac{1}{2} \int \frac{1}{t} dt \\ &= \frac{1}{2} \log|t| + C \\ &= \frac{1}{2} \log|2\sin x + 3\cos x| + C\end{aligned}$$

Indefinite Integrals Ex 19.8 Q27

$$\text{Let } I = \int \frac{\cos 2x + x + 1}{x^2 + \sin 2x + 2x} dx \dots\dots\dots (i)$$

$$\begin{aligned} &\text{Let } x^2 + \sin 2x + 2x = t \quad \text{then,} \\ &d(x^2 + \sin 2x + 2x) = dt \end{aligned}$$

$$\begin{aligned} &\Rightarrow (2x + 2 \cos 2x + 2) dx = dt \\ &\Rightarrow 2(\cos 2x + x + 1) dx = dt \\ &\Rightarrow dx = \frac{dt}{2(\cos 2x + x + 1)} \end{aligned}$$

Putting $x^2 + \sin 2x + 2x = t$ and $dx = \frac{dt}{2(\cos 2x + x + 1)}$ in equation (i), we get,

$$\begin{aligned} I &= \int \frac{\cos 2x + x + 1}{t} \times \frac{dt}{2(\cos 2x + x + 1)} \\ &= \frac{1}{2} \int \frac{dt}{t} \\ &= \frac{1}{2} \log|t| + c \\ &= \frac{1}{2} \log|x^2 + \sin 2x + 2x| + c \end{aligned}$$

$$\therefore I = \frac{1}{2} \log|x^2 + \sin 2x + 2x| + c$$

Indefinite Integrals Ex 19.8 Q29

$$\text{Let } I = \int \frac{-\sin x + 2 \cos x}{2 \sin x + \cos x} dx \dots\dots\dots (i)$$

$$\begin{aligned} &\text{Let } 2 \sin x + \cos x = t \quad \text{then,} \\ &d(2 \sin x + \cos x) = dt \end{aligned}$$

$$\begin{aligned} &\Rightarrow (2 \cos x - \sin x) dx = dt \\ &\Rightarrow dx = \frac{dt}{- \sin x + 2 \cos x} \end{aligned}$$

Putting $2 \sin x + \cos x = t$ and $dx = \frac{dt}{- \sin x + 2 \cos x}$ in equation (i), we get,

$$\begin{aligned} I &= \int \frac{-\sin x + 2 \cos x}{t} \times \frac{dt}{- \sin x + 2 \cos x} \\ &= \int \frac{dt}{t} \\ &= \log|t| + c \\ &= \log|2 \sin x + \cos x| + c \end{aligned}$$

$$\therefore I = \log|2 \sin x + \cos x| + c$$

Indefinite Integrals Ex 19.8 Q30

$$\int \frac{\cos 4x - \cos 2x}{\sin 4x - \sin 2x} dx$$

$$= -\int \frac{2 \sin 3x \sin x}{2 \cos 3x \sin x} dx$$

$$= -\int \frac{\sin 3x}{\cos 3x} dx$$

Putting $\cos 3x = t$, and $-3 \sin 3x dx = dt$

$$= \frac{1}{3} \int \frac{dt}{t}$$

$$= \frac{1}{3} \log|t| + C$$

$$= \frac{1}{3} \log|\cos 3x| + C$$

Indefinite Integrals Ex 19.8 Q31

$$\text{Let } I = \int \frac{\sec x}{\log(\sec x + \tan x)} dx \quad \dots \dots \dots (i)$$

Let $\log(\sec x + \tan x) = t$ then,

$$d[\log(\sec x + \tan x)] = dt$$

$$\Rightarrow \sec x dx = dt \quad \left[\because \frac{d}{dx} [\log(\sec x + \tan x)] = \sec x \right]$$

$$\Rightarrow dx = \frac{dt}{\sec x}$$

Putting $\log(\sec x + \tan x) = t$ and $dx = \frac{dt}{\sec x}$ in equation (i), we get,

$$I = \int \frac{\sec x}{t} \times \frac{dt}{\sec x}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + C$$

$$= \log|\log(\sec x + \tan x)| + C$$

$$\therefore I = \log|\log(\sec x + \tan x)| + C$$

Indefinite Integrals Ex 19.8 Q32

$$\text{Let } I = \int \frac{\cosec x}{\log \tan \frac{x}{2}} dx \quad \dots \dots \dots (i)$$

Let $\log \tan \frac{x}{2} = t$ then,

$$d \left[\log \tan \frac{x}{2} \right] = dt$$

$$\Rightarrow \cosec x dx = dt$$

$$\Rightarrow dx = \frac{dt}{\cosec x}$$

Putting $\log \tan \frac{x}{2} = t$ and $dx = \frac{dt}{\cosec x}$ in equation (i), we get,

$$\begin{aligned} I &= \int \frac{\cosec x}{t} \times \frac{dt}{\cosec x} \\ &= \int \frac{dt}{t} \\ &= \log|t| + c \\ &= \log \left| \log \tan \frac{x}{2} \right| + c \end{aligned}$$

$$\therefore I = \log \left| \log \tan \frac{x}{2} \right| + c$$

Indefinite Integrals Ex 19.8 Q33

$$\text{Let } I = \int \frac{1}{x \log x \log(\log x)} dx \quad \dots \dots \dots (i)$$

Let $\log(\log x) = t$ then,

$$d[\log(\log x)] = dt$$

$$\begin{aligned} \Rightarrow \frac{1}{x} \times \frac{1}{\log x} dx &= dt \\ \Rightarrow dx &= x \log x dt \end{aligned}$$

Putting $\log(\log x) = t$ and $dx = x \log x dt$ in equation (i), we get,

$$\begin{aligned} I &= \int \frac{1}{x \log x t} \times x \log x dt \\ &= \int \frac{1}{t} dt \\ &= \log|t| + c \\ &= \log|\log(\log x)| + c \end{aligned}$$

$$\therefore I = \log|\log(\log x)| + c$$

Indefinite Integrals Ex 19.8 Q34

$$\text{Let } I = \int \frac{\csc^2 x}{1 + \cot x} dx \quad \dots \quad (i)$$

$$\begin{aligned} \text{Let } 1 + \cot x &= t \quad \text{then,} \\ d[1 + \cot x] &= dt \end{aligned}$$

$$\begin{aligned} \Rightarrow -\csc^2 x dx &= dt \\ \Rightarrow dx &= -\frac{dt}{\csc^2 x} \end{aligned}$$

Putting $1 + \cot x = t$ and $dx = -\frac{dt}{\csc^2 x}$ in equation (i), we get,

$$\begin{aligned} I &= \int \frac{\csc^2 x}{t} \times -\frac{dt}{\csc^2 x} \\ &= -\int \frac{1}{t} dt \\ &= -\log|t| + c \\ &= -\log|1 + \cot x| + c \end{aligned}$$

$$\therefore I = -\log|1 + \cot x| + c$$

Indefinite Integrals Ex 19.8 Q35

$$\text{Let } I = \int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx \quad \dots \quad (i)$$

$$\begin{aligned} \text{Let } 10^x + x^{10} &= t \quad \text{then,} \\ d(10^x + x^{10}) &= dt \end{aligned}$$

$$\begin{aligned} \Rightarrow (10^x \log_e 10 + 10x^9) dx &= dt \\ \Rightarrow dx &= \frac{dt}{10x^9 + 10^x \log_e 10} \end{aligned}$$

Putting $10^x + x^{10} = t$ and $dx = \frac{dt}{10x^9 + 10^x \log_e 10}$ in equation (i), we get,

$$\begin{aligned} I &= \int \frac{10x^9 + 10^x \log_e 10}{t} \times \frac{dt}{10x^9 + 10^x \log_e 10} \\ &= \int \frac{dt}{t} \\ &= \log|t| + c \\ &= \log|10^x + x^{10}| + c \end{aligned}$$

$$\therefore I = \log|10^x + x^{10}| + c$$

Indefinite Integrals Ex 19.8 Q36

Let $I = \int \frac{1 - \sin 2x}{x + \cos^2 x} dx \dots \dots \dots (i)$

Let $x + \cos^2 x = t \quad \text{then,}$
 $d(x + \cos^2 x) = dt$

$$\Rightarrow (1 - 2 \cos x \sin x) dx = dt$$

$$\Rightarrow dx = \frac{dt}{1 - 2 \cos x \sin x}$$

Putting $x + \cos^2 x = t$ and $dx = \frac{dt}{1 - 2 \cos x \sin x}$ in equation (i), we get

$$I = \int \frac{1 - \sin 2x}{t} \times \frac{dt}{1 - 2 \cos x \sin x}$$

$$= \int \frac{1 - \sin 2x}{t} \times \frac{dt}{1 - \sin 2x}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log|x + \cos^2 x| + c$$

$$\therefore I = \log|x + \cos^2 x| + c$$

Indefinite Integrals Ex 19.8 Q37

Let $I = \int \frac{1 + \tan x}{x + \log \sec x} dx \dots \dots \dots (i)$

Let $x + \log \sec x = t \quad \text{then,}$
 $d(x + \log \sec x) = dt$

$$\Rightarrow (1 + \tan x) dx = dt \quad \left[\because \frac{d}{dx}(\log \sec x) = \tan x \right]$$

$$\Rightarrow dx = \frac{dt}{1 + \tan x}$$

Putting $x + \log \sec x = t$ and $dx = \frac{dt}{1 + \tan x}$ in equation (i), we get,

$$I = \int \frac{1 + \tan x}{t} \times \frac{dt}{1 + \tan x}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$\Rightarrow I = \log|x + \log \sec x| + c$$

Indefinite Integrals Ex 19.8 Q38

Let $I = \int \frac{\sin 2x}{a^2 + b^2 \sin^2 x} dx \dots \text{(i)}$

Let $a^2 + b^2 \sin^2 x = t \quad \text{then,}$
 $d(a^2 + b^2 \sin^2 x) = dt$

$$\Rightarrow b^2(2 \sin x \cos x) dx = dt$$

$$\begin{aligned}\Rightarrow dx &= \frac{dt}{b^2(2 \sin x \cos x)} \\ &= \frac{dt}{b^2 \sin 2x}\end{aligned}$$

Putting $a^2 + b^2 \sin^2 x = t$ and $dx = \frac{dt}{b^2 \sin 2x}$ in equation (i), we get,

$$\begin{aligned}I &= \int \frac{\sin 2x}{t} \times \frac{dt}{b^2 \sin 2x} \\ &= \frac{1}{b^2} \int \frac{dt}{t} \\ &= \frac{1}{b^2} \log|t| + c \\ &= \frac{1}{b^2} \log|a^2 + b^2 \sin^2 x| + c\end{aligned}$$

$$\Rightarrow I = \frac{1}{b^2} \log|a^2 + b^2 \sin^2 x| + c$$

Indefinite Integrals Ex 19.8 Q39

Let $I = \int \frac{x+1}{x(x+\log x)} dx \dots \text{(i)}$

Let $(x + \log x) = t \quad \text{then,}$
 $d(x + \log x) = dt$

$$\Rightarrow \left(1 + \frac{1}{x}\right) dx = dt$$

$$\Rightarrow \left(\frac{x+1}{x}\right) dx = dt$$

$$\Rightarrow dx = \frac{x}{x+1} dt$$

Putting $(x + \log x) = t$ and $dx = \frac{x}{x+1} dt$ in equation (i), we get,

$$\begin{aligned}I &= \int \frac{x+1}{x \times t} \times \frac{x}{x+1} dt \\ &= \int \frac{dt}{t} \\ &= \log|t| + c \\ &= \log|x + \log x| + c\end{aligned}$$

$$\Rightarrow I = \log|x + \log x| + c$$

Indefinite Integrals Ex 19.8 Q40

$$\text{Let } I = \int \frac{1}{\sqrt{1-x^2} (2+3\sin^{-1}x)} dx \quad \text{--- (i)}$$

$$\begin{aligned}\text{Let } 2+3\sin^{-1}x &= t \quad \text{then,} \\ d(2+3\sin^{-1}x) &= dt\end{aligned}$$

$$\begin{aligned}\Rightarrow \quad 3 \times \frac{1}{\sqrt{1-x^2}} dx &= dt \\ \Rightarrow \quad dx &= \frac{\sqrt{1-x^2}}{3} dt\end{aligned}$$

Putting $2+3\sin^{-1}x = t$ and $dx = \frac{\sqrt{1-x^2}}{3} dt$ in equation (i), we get,

$$\begin{aligned}I &= \int \frac{\sqrt{1-x^2}}{3} \times \frac{1}{\sqrt{1-x^2} t} dt \\ &= \frac{1}{3} \int \frac{dt}{t} \\ &= \frac{1}{3} \log|t| + c \\ &= \frac{1}{3} \log|2+3\sin^{-1}x| + c\end{aligned}$$

$$\Rightarrow \quad I = \frac{1}{3} \log|2+3\sin^{-1}x| + c$$

Indefinite Integrals Ex 19.8 Q41

$$\text{Let } I = \int \frac{\sec^2 x}{\tan x + 2} dx \quad \text{--- (i)}$$

$$\begin{aligned}\text{Let } \tan x + 2 &= t \quad \text{then,} \\ d(\tan x + 2) &= dt\end{aligned}$$

$$\begin{aligned}\Rightarrow \quad \sec^2 x dx &= dt \\ \Rightarrow \quad dx &= \frac{1}{\sec^2 x} dt\end{aligned}$$

Putting $\tan x + 2 = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get,

$$\begin{aligned}I &= \int \frac{\sec^2 x}{t} \times \frac{1}{\sec^2 x} dt \\ &= \int \frac{dt}{t} \\ &= \log|t| + c \\ &= \log|\tan x + 2| + c\end{aligned}$$

$$\Rightarrow \quad I = \log|\tan x + 2| + c$$

Indefinite Integrals Ex 19.8 Q42

Let $I = \int \frac{2 \cos 2x + \sec^2 x}{\sin 2x + \tan x - 5} dx \dots\dots\dots (i)$

Let $\sin 2x + \tan x - 5 = t$ then,

$$d(\sin 2x + \tan x - 5) = dt$$

$$\Rightarrow (2 \cos 2x + \sec^2 x) dx = dt$$

$$\Rightarrow dx = \frac{1}{2 \cos 2x + \sec^2 x} dt$$

Putting $\sin 2x + \tan x - 5 = t$ and $dx = \frac{dt}{2 \cos 2x + \sec^2 x}$ in equation (i), we get,

$$\begin{aligned} I &= \int \frac{2 \cos 2x + \sec^2 x}{t} \times \frac{1}{2 \cos 2x + \sec^2 x} dt \\ &= \int \frac{dt}{t} \\ &= \log|t| + c \\ &= \log|\sin 2x + \tan x - 5| + c \end{aligned}$$

$$\therefore I = \log|\sin 2x + \tan x - 5| + c$$

Indefinite Integrals Ex 19.8 Q43

Let $I = \int \frac{\sin 2x}{\sin 5x \sin 3x} dx$ then,

$$\begin{aligned} I &= \int \frac{\sin(5x - 3x)}{\sin 5x \sin 3x} dx \\ &= \int \frac{\sin 5x \cos 3x - \cos 5x \sin 3x}{\sin 5x \sin 3x} dx \\ &= \int \frac{\sin 5x \cos 3x}{\sin 5x \sin 3x} dx - \int \frac{\cos 5x \sin 3x}{\sin 5x \sin 3x} dx \\ &= \int \frac{\cos 3x}{\sin 3x} dx - \int \frac{\cos 5x}{\sin 5x} dx \\ &= \int \cot 3x dx - \int \cot 5x dx \\ &= \frac{1}{3} \log|\sin 3x| - \frac{1}{5} \log|\sin 5x| + c \end{aligned}$$

$$\therefore I = \frac{1}{3} \log|\sin 3x| - \frac{1}{5} \log|\sin 5x| + c$$

Indefinite Integrals Ex 19.8 Q44

Let $I = \int \frac{1 + \cot x}{x + \log \sin x} dx \dots\dots\dots (i)$

Let $x + \log \sin x = t$ then,

$$d(x + \log \sin x) = dt$$

$$\begin{aligned} \Rightarrow (1 + \cot x) dx &= dt & \left[\because \frac{d}{dx} (\log \sin x) = \cot x \right] \\ \Rightarrow dx &= \frac{dt}{1 + \cot x} \end{aligned}$$

Putting $x + \log \sin x = t$ and $dx = \frac{dt}{1 + \cot x}$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{1 + \cot x}{t} \times \frac{dt}{1 + \cot x} \\ &= \int \frac{dt}{t} \\ &= \log|t| + c \\ &= \log|x + \log \sin x| + c \end{aligned}$$

$$\therefore I = \log|x + \log \sin x| + c$$

Indefinite Integrals Ex 19.8 Q45

$$\text{Let } I = \int \frac{1}{\sqrt{x}(\sqrt{x}+1)} dx \quad \dots \quad (i)$$

$$\begin{aligned} \text{Let } \sqrt{x} + 1 &= t \quad \text{then,} \\ d(\sqrt{x} + 1) &= dt \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad \frac{1}{2\sqrt{x}} dx &= dt \\ \Rightarrow \quad dx &= 2\sqrt{x} dt \end{aligned}$$

Putting $\sqrt{x} + 1 = t$ and $dx = 2\sqrt{x} dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{1}{\sqrt{x}t} \times 2\sqrt{x} dt \\ &= 2 \int \frac{dt}{t} \\ &= 2 \log|t| + C \\ &= 2 \log|\sqrt{x} + 1| + C \\ \therefore \quad I &= 2 \log|\sqrt{x} + 1| + C \end{aligned}$$

Indefinite Integrals Ex 19.8 Q46

$$\text{Let } I = \int \tan 2x \tan 3x \tan 5x dx \quad \dots \quad (i)$$

Now,

$$\begin{aligned} \tan(5x) &= \tan(2x + 3x) \\ &= \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x} \\ \Rightarrow \quad \tan 5x &= \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x} \\ \Rightarrow \quad \tan 5x - \tan 2x \tan 3x \tan 5x &= \tan 2x + \tan 3x \\ \Rightarrow \quad \tan 5x - \tan 2x - \tan 3x &= \tan 2x \tan 3x \tan 5x \quad \dots \quad (ii) \end{aligned}$$

Using equation (i) and equation (ii), we get

$$\begin{aligned} I &= \int [\tan 5x - \tan 2x - \tan 3x] dx \\ &= \frac{1}{5} \log|\sec 5x| - \frac{1}{2} \log|\sec 2x| - \frac{1}{3} \log|\sec 3x| + C \\ \therefore \quad I &= \frac{1}{5} \log|\sec 5x| - \frac{1}{2} \log|\sec 2x| - \frac{1}{3} \log|\sec 3x| + C \end{aligned}$$

Indefinite Integrals Ex 19.8 Q47

Since,

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\therefore \quad \tan(x + \theta - x) = \frac{\tan(x + \theta) - \tan x}{1 + \tan(x + \theta) \tan x}$$

$$\begin{aligned} \Rightarrow \quad 1 + \tan(x + \theta) \tan x &= \frac{\tan(x + \theta) - \tan x}{\tan \theta} \\ \Rightarrow \quad \int 1 + \tan(x + \theta) \tan x dx &= \\ &= \frac{1}{\tan \theta} \left[\int \tan(x + \theta) dx - \int \tan x dx \right] \\ &= \frac{1}{\tan \theta} \left[-\log|\cos(x + \theta)| + \log|\cos x| \right] + C \\ &= \frac{1}{\tan \theta} \left[\log|\cos x| - \log|\cos(x + \theta)| \right] + C \\ &= \frac{1}{\tan \theta} \log \left| \frac{\cos x}{\cos(x + \theta)} \right| + C \end{aligned}$$

Indefinite Integrals Ex 19.8 Q48

$$\begin{aligned}
\text{Consider } I &= \int \left(\frac{\sin 2x}{\sin\left(x - \frac{\pi}{6}\right) \sin\left(x + \frac{\pi}{6}\right)} \right) dx \\
&= \int \left(\frac{\sin 2x}{\left(\frac{3}{4} \sin^2 x - \frac{1}{4} \cos^2 x\right)} \right) dx \\
&= \int \left(\frac{\sin 2x}{\left(\frac{3}{4}(1 - \cos^2 x) - \frac{1}{4} \cos^2 x\right)} \right) dx \\
&= \int \left(\frac{\sin 2x}{\left(\frac{3}{4} - \cos^2 x\right)} \right) dx \\
\text{let } \cos^2 x &= t \rightarrow \sin 2x dx = -dt \\
I &= \int \left(\frac{-dt}{\left(\frac{3}{4} - t\right)} \right) \\
I &= \log \left| \sin^2 x - \frac{1}{4} \right| + C
\end{aligned}$$

Indefinite Integrals Ex 19.8 Q49

$$\begin{aligned}
&\int \frac{e^{x-1} + x^{e-1}}{e^x + x^e} dx \\
&= \frac{1}{e} \int \frac{e^x + ex^{e-1}}{e^x + x^e} dx \\
\text{Let } e^x + x^e &= u \\
\Rightarrow &\quad \left(e^x + ex^{e-1} \right) dx = du \\
&= \frac{1}{e} \int \frac{1}{4} du = \frac{1}{e} \log |u| + C \\
&= \frac{1}{e} \log |e^x + x^e| + C
\end{aligned}$$

Indefinite Integrals Ex 19.8 Q50

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{\sin x \cos^2 x} dx, \quad \text{then,} \\
I &= \int \frac{\sin^2 x + \cos^2 x}{\sin x \cos^2 x} dx \\
&= \int \frac{\sin^2 x}{\sin x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin x \cos^2 x} dx \\
&= \int \sec x \tan x dx + \int \cosec x dx \\
&= \sec x + \log \left| \tan \frac{x}{2} \right| + C
\end{aligned}$$

$$\therefore I = \sec x + \log \left| \tan \frac{x}{2} \right| + C$$

Indefinite Integrals Ex 19.8 Q51

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{\cos 3x - \cos x} dx, \quad \text{then,} \\
 I &= \int \frac{\sin^2 x + \cos^2 x}{-2 \sin 2x \sin x} dx \\
 &= \int \frac{\sin^2 x + \cos^2 x}{-4 \sin^2 x \cos x} dx \\
 &= -\frac{1}{4} \int \left[\frac{\sin^2 x}{\sin^2 x \cos x} + \frac{\cos^2 x}{\sin^2 x \cos x} \right] dx \\
 &= -\frac{1}{4} \int [\sec x + \operatorname{cosec} x \cot x] dx \\
 &= -\frac{1}{4} [\log |\sec x + \tan x| - \operatorname{cosec} x] + c
 \end{aligned}$$

$$\therefore I = \frac{1}{4} [\operatorname{cosec} x - \log |\sec x + \tan x|] + c$$

EX 19.9

Indefinite Integrals Ex 19.9 Q1

$$\text{Let } I = \int \frac{\log x}{x} dx$$

$$\begin{aligned}\text{Let } \log x &= t && \text{then,} \\ d(\log x) &= dt\end{aligned}$$

$$\begin{aligned}\Rightarrow \frac{1}{x} dx &= dt \\ \Rightarrow dx &= x dt\end{aligned}$$

Putting $\log x = t$ and $dx = x dt$, we get

$$\begin{aligned}I &= \int \frac{t}{x} \times x dt \\ &= \int t dt \\ &= \frac{t^2}{2} + c \\ &= \frac{(\log x)^2}{2} + c\end{aligned}$$

$$\therefore I = \frac{(\log x)^2}{2} + c$$

Indefinite Integrals Ex 19.9 Q2

$$\text{Let } I = \int \frac{\log\left(1 + \frac{1}{x}\right)}{x(1+x)} dx \quad \text{--- (i)}$$

$$\begin{aligned} &\text{Let } \log\left(1 + \frac{1}{x}\right) = t \quad \text{then,} \\ &d\left[\log\left(1 + \frac{1}{x}\right)\right] = dt \end{aligned}$$

$$\begin{aligned} &\Rightarrow \frac{1}{1 + \frac{1}{x}} \times \frac{-1}{x^2} dx = dt \\ &\Rightarrow \frac{1}{x+1} \times \frac{-1}{x^2} dx = dt \\ &\Rightarrow \frac{-x}{x^2(x+1)} dx = -dt \\ &\Rightarrow \frac{dx}{x(x+1)} = -dt \end{aligned}$$

Putting $\log\left(1 + \frac{1}{x}\right) = t$ and $\frac{dx}{x(x+1)} = -dt$ in equation (i), we get

$$\begin{aligned} I &= \int t \times -dt \\ &= -\frac{t^2}{2} + C \\ &= -\frac{1}{2} \left[\log\left(1 + \frac{1}{x}\right) \right]^2 + C \end{aligned}$$

$$\therefore I = -\frac{1}{2} \left[\log\left(1 + \frac{1}{x}\right) \right]^2 + C$$

Let $I = \int \frac{\log\left(1 + \frac{1}{x}\right)}{x(1+x)} dx \quad \text{--- (i)}$

$$\begin{aligned} &\text{Let } \log\left(1 + \frac{1}{x}\right) = t \quad \text{then,} \\ &d\left[\log\left(1 + \frac{1}{x}\right)\right] = dt \\ &\Rightarrow \frac{1}{1 + \frac{1}{x}} \times \frac{-1}{x^2} dx = dt \\ &\Rightarrow \frac{1}{x+1} \times \frac{-1}{x^2} dx = dt \\ &\Rightarrow \frac{-x}{x^2(x+1)} dx = dt \\ &\Rightarrow \frac{dx}{x(x+1)} = -dt \end{aligned}$$

Putting $\log\left(1 + \frac{1}{x}\right) = t$ and $\frac{dx}{x(x+1)} = -dt$ in equation (i), we get

$$\begin{aligned} I &= \int t \times -dt \\ &= -\frac{t^2}{2} + C \\ &= -\frac{1}{2} \left[\log\left(1 + \frac{1}{x}\right) \right]^2 + C \end{aligned}$$

$$\therefore I = -\frac{1}{2} \left[\log\left(1 + \frac{1}{x}\right) \right]^2 + C$$

Indefinite Integrals Ex 19.9 Q3

$$\text{Let } I = \int \frac{(1+\sqrt{x})^2}{\sqrt{x}} dx$$

$$\begin{aligned} \text{Let } & (1+\sqrt{x}) = t & \text{then,} \\ & dt = dt \end{aligned}$$

$$\begin{aligned} \Rightarrow & \frac{1}{2\sqrt{x}} dx = dt \\ \Rightarrow & dx = dt \times 2\sqrt{x} \end{aligned}$$

Putting $(1+\sqrt{x}) = t$ and $dx = dt \times 2\sqrt{x}$, we get

$$\begin{aligned} I &= \int \frac{t^2}{\sqrt{x}} \times dt \times 2\sqrt{x} \\ &= 2 \int t^2 dt \\ &= 2 \times \frac{t^3}{3} + C \\ &= \frac{2}{3} [1+\sqrt{x}]^3 + C \end{aligned}$$

$$\therefore I = \frac{2}{3} (1+\sqrt{x})^3 + C$$

Indefinite Integrals Ex 19.9 Q4

$$\text{Let } I = \int \sqrt{1+e^x} e^x dx \quad (i)$$

$$\begin{aligned} \text{Let } & 1+e^x = t & \text{then,} \\ & dt = e^x dx \end{aligned}$$

$$\begin{aligned} \Rightarrow & e^x dx = dt \\ \Rightarrow & dx = \frac{dt}{e^x} \end{aligned}$$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i), we get

$$\begin{aligned} I &= \int \sqrt{t} e^x \frac{dt}{e^x} \\ &= \int t^{\frac{1}{2}} dt \\ &= \frac{2}{3} \times \frac{t^{\frac{3}{2}}}{3} + C \end{aligned}$$

$$= \frac{2}{3} (1+e^x)^{\frac{3}{2}} + C$$

Indefinite Integrals Ex 19.9 Q5

Let $I = \int \sqrt[3]{\cos^2 x} \sin x dx \dots \dots \dots (i)$

Let $\cos x = t \quad \text{then,}$
 $d(\cos x) = dt$

$$\Rightarrow -\sin x dx = dt$$

$$\Rightarrow dx = -\frac{dt}{\sin x}$$

Putting $\cos x = t$ and $dx = -\frac{dt}{\sin x}$ in equation (i), we get

$$\begin{aligned} I &= \int \sqrt[3]{t^2} \sin x \times \frac{-dt}{\sin x} \\ &= -\int t^{\frac{2}{3}} \sin x \frac{dt}{\sin x} \\ &= -\int t^{\frac{2}{3}} dt \\ &= -\frac{3}{5} \times \frac{t^{\frac{5}{3}}}{5} + c \\ &= -\frac{3}{5} (\cos x)^{\frac{5}{3}} + c \end{aligned}$$

$$\therefore I = -\frac{3}{5} (\cos x)^{\frac{5}{3}} + c$$

Indefinite Integrals Ex 19.9 Q6

Let $I = \int \frac{e^x}{(1+e^x)^2} dx \dots \dots \dots (i)$

Let $1+e^x = t \quad \text{then,}$
 $d(1+e^x) = dt$

$$\Rightarrow e^x dx = dt$$

$$\Rightarrow dx = \frac{dt}{e^x}$$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{e^x}{t^2} \times \frac{dt}{e^x} \\ &= \int \frac{dt}{t^2} \\ &= \int t^{-2} dt \\ &= -t^{-1} + c \\ &= -\frac{1}{t} + c \\ &= -\frac{1}{1+e^x} + c \end{aligned}$$

$$\therefore I = -\frac{1}{1+e^x} + c$$

Indefinite Integrals Ex 19.9 Q7

Let $I = \int \cot^3 x \cosec^2 x dx \dots \dots \dots (i)$

Let $\cot x = t \quad \text{then,}$
 $d(\cot x) = dt$

$$\Rightarrow -\cosec^2 x dx = dt$$

$$\Rightarrow dx = -\frac{dt}{\cosec^2 x}$$

Putting $\cot x = t$ and $dx = -\frac{dt}{\cosec^2 x}$ in equation (i), we get

$$I = \int t^3 \cosec^2 x \times \frac{-dt}{\cosec^2 x}$$

$$= -\int t^3 dt$$

$$= -\frac{t^4}{4} + c$$

$$= -\frac{\cot^4 x}{4} + c$$

$$\therefore I = -\frac{\cot^4 x}{4} + c$$

Indefinite Integrals Ex 19.9 Q8

Let $I = \int \frac{\{e^{\sin^{-1} x}\}^2}{\sqrt{1-x^2}} dx \dots \dots \dots (i)$

Let $\sin^{-1} x = t \quad \text{then,}$
 $d(\sin^{-1} x) = dt$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\Rightarrow dx = \sqrt{1-x^2} dt$$

Putting $\sin^{-1} x = t$ and $dx = \sqrt{1-x^2} dt$ in equation (i), we get

$$I = \int \frac{\{e^t\}^2}{\sqrt{1-x^2}} \times \sqrt{1-x^2} dt$$

$$= \int e^{2t} dt$$

$$= \frac{e^{2t}}{2} + c$$

$$= \frac{e^{2\sin^{-1} x}}{2} + c$$

$$\therefore I = \frac{\{e^{\sin^{-1} x}\}^2}{2} + c$$

Indefinite Integrals Ex 19.9 Q9

$$\text{Let } I = \int \frac{1 + \sin x}{\sqrt{x - \cos x}} dx \quad \dots \dots \dots (i)$$

$$\begin{aligned} \text{Let } x - \cos x &= t && \text{then,} \\ d(x - \cos x) &= dt \end{aligned}$$

$$\begin{aligned} \Rightarrow [1 - (-\sin x)]dx &= dt \\ \Rightarrow (1 + \sin x)dx &= dt \end{aligned}$$

Putting $x - \cos x = t$ and $(1 + \sin x)dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{dt}{\sqrt{t}} \\ &= \int t^{-\frac{1}{2}} dt \\ &= 2t^{\frac{1}{2}} + c \\ &= 2(x - \cos x)^{\frac{1}{2}} + c \end{aligned}$$

$$\therefore I = 2\sqrt{x - \cos x} + c$$

Indefinite Integrals Ex 19.9 Q10

$$\text{Let } I = \int \frac{1}{\sqrt{1-x^2} (\sin^{-1} x)^2} dx \quad \dots \dots \dots (i)$$

$$\begin{aligned} \text{Let } \sin^{-1} x &= t && \text{then,} \\ d(\sin^{-1} x) &= dt \end{aligned}$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$$

Putting $\sin^{-1} x = t$ and $\frac{1}{\sqrt{1-x^2}} dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{dt}{t^2} \\ &= \int t^{-2} dt \\ &= -t^{-1} + c \\ &= \frac{-1}{t} + c \\ &= \frac{-1}{\sin^{-1} x} + c \end{aligned}$$

$$\therefore I = \frac{-1}{\sin^{-1} x} + c$$

Indefinite Integrals Ex 19.9 Q11

$$\text{Let } I = \int \frac{\cot x}{\sqrt{\sin x}} dx \quad \dots \dots \dots \text{(i)}$$

Let $\sin x = t$ then,
 $d(\sin x) = dt$

$$\Rightarrow \cos x dx = dt$$

$$\begin{aligned} \text{Now, } I &= \int \frac{\cot x}{\sqrt{\sin x}} dx \\ &= \int \frac{\cos x}{\sin x \sqrt{\sin x}} dx \\ &= \int \frac{\cos x}{(\sin x)^{\frac{3}{2}}} dx \end{aligned}$$

$$\Rightarrow \int \frac{\cos x}{(\sin x)^{\frac{3}{2}}} dx \quad \dots \dots \dots \text{(ii)}$$

Putting $\sin x = t$ and $\cos x dx = dt$ in equation (ii), we get

$$\begin{aligned} I &= \int \frac{dt}{t^{\frac{3}{2}}} \\ &= \int t^{-\frac{3}{2}} dt \\ &= -2t^{-\frac{1}{2}} + C \\ &= \frac{-2}{\sqrt{t}} + C \\ &= \frac{-2}{\sqrt{\sin x}} + C \end{aligned}$$

$$\therefore I = \frac{-2}{\sqrt{\sin x}} + C$$

Indefinite Integrals Ex 19.9 Q12

$$\text{Let } I = \int \frac{\tan x}{\sqrt{\cos x}} dx$$

$$\therefore I = \int \frac{\sin x}{\cos x \sqrt{\cos x}} dx \\ = \int \frac{\sin x}{(\cos x)^{\frac{3}{2}}} dx$$

$$\Rightarrow I = \int \frac{\sin x}{(\cos x)^{\frac{3}{2}}} dx \quad \text{--- --- (i)}$$

$$\text{Let } \cos x = t \quad \text{then,} \\ d(\cos x) = dt$$

$$\Rightarrow -\sin x dx = dt \\ \Rightarrow \sin x dx = -dt$$

Putting $\cos x = t$ and $\sin x dx = -dt$ in equation (i), we get

$$I = \int \frac{-dt}{t^{\frac{3}{2}}} \\ = -\int t^{-\frac{3}{2}} dt \\ = -\left[-2t^{-\frac{1}{2}} \right] + C \\ = \frac{2}{t^{\frac{1}{2}}} + C \\ = \frac{2}{\sqrt{\cos x}} + C$$

$$\therefore I = \frac{2}{\sqrt{\cos x}} + C$$

Indefinite Integrals Ex 19.9 Q13

$$\text{Let } I = \int \frac{\cos^3 x}{\sqrt{\sin x}} dx$$

$$\therefore I = \int \frac{\cos^2 x \cos x}{\sqrt{\sin x}} dx \\ = \int \frac{(1 - \sin^2 x) \cos x}{\sqrt{\sin x}} dx$$

$$\therefore I = \int \frac{(1 - \sin^2 x)}{\sqrt{\sin x}} \cos x dx \quad \dots \dots \dots (i)$$

$$\text{Let } \sin x = t \quad \text{then,} \\ d(\sin x) = dt$$

$$\Rightarrow \cos x dx = dt$$

Putting $\sin x = t$ and $\cos x dx = dt$ in equation (i), we get

$$I = \int \frac{1 - t^2}{\sqrt{t}} dt \\ = \int \left(t^{\frac{-1}{2}} - t^2 \times t^{\frac{-1}{2}} \right) dt \\ = \int \left(t^{\frac{-1}{2}} - t^{\frac{3}{2}} \right) dt \\ = 2t^{\frac{1}{2}} - \frac{2}{5}t^{\frac{5}{2}} + C \\ \Rightarrow I = 2(\sin x)^{\frac{1}{2}} - \frac{2}{5}(\sin x)^{\frac{5}{2}} + C$$

$$\therefore I = 2\sqrt{\sin x} - \frac{2}{5}(\sin x)^{\frac{5}{2}} + C$$

Indefinite Integrals Ex 19.9 Q14

$$\text{Let } I = \int \frac{\sin^3 x}{\sqrt{\cos x}} dx$$

$$\therefore I = \int \frac{\sin^2 x \sin x}{\sqrt{\cos x}} dx$$

$$\Rightarrow I = \int \frac{(1 - \cos^2 x)}{\sqrt{\cos x}} \sin x dx \quad \text{--- (i)}$$

$$\text{Let } \cos x = t \quad \text{then,}$$

$$d(\cos x) = dt$$

$$\Rightarrow -\sin x dx = dt$$

$$\Rightarrow \sin x dx = -dt$$

Putting $\cos x = t$ and $\sin x dx = -dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{(1 - t^2)}{\sqrt{t}} \times -dt \\ &= \int \frac{t^2 - 1}{\sqrt{t}} dt \\ &= \int \left(\frac{t^2}{t^{\frac{1}{2}}} - \frac{1}{t^{\frac{1}{2}}} \right) dx \\ &= \int \left(t^{\frac{3}{2}} - t^{-\frac{1}{2}} \right) dt \\ &= \int \left(t^{\frac{3}{2}} - t^{-\frac{1}{2}} \right) dt \\ &= \frac{2}{5} t^{\frac{5}{2}} - 2t^{\frac{1}{2}} + c \\ \therefore I &= \frac{2}{5} \cos^{\frac{5}{2}} x - 2 \cos^{\frac{1}{2}} x + c \end{aligned}$$

$$\therefore I = \frac{2}{5} \cos^{\frac{5}{2}} x - 2\sqrt{\cos x} + c$$

Indefinite Integrals Ex 19.9 Q15

$$\text{Let } I = \int \frac{1}{\sqrt{\tan^{-1} x}} \left(1 + x^2 \right)^{-\frac{1}{2}} dx \quad \text{--- (i)}$$

$$\text{Let } \tan^{-1} x = t \quad \text{then,}$$

$$d(\tan^{-1} x) = dt$$

$$\Rightarrow \frac{1}{1+x^2} dx = dt$$

Putting $\tan^{-1} x = t$ and $\frac{1}{1+x^2} dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{1}{\sqrt{t}} dt \\ &= \int t^{-\frac{1}{2}} dt \\ &= 2t^{\frac{1}{2}} + c \\ &= 2\sqrt{\tan^{-1} x} + c \end{aligned}$$

$$\therefore I = 2\sqrt{\tan^{-1} x} + c$$

Indefinite Integrals Ex 19.9 Q16

$$\begin{aligned}
 \text{Let } I &= \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx \\
 &= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx \\
 &= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx \\
 &= \int \frac{\sec^2 x dx}{\sqrt{\tan x}}
 \end{aligned}$$

Let $\tan x = t \Rightarrow \sec^2 x dx = dt$

$$\begin{aligned}
 \therefore I &= \int \frac{dt}{\sqrt{t}} \\
 &= 2\sqrt{t} + C \\
 &= 2\sqrt{\tan x} + C
 \end{aligned}$$

Indefinite Integrals Ex 19.9 Q17

$$\text{Let } I = \int \frac{1}{x} (\log x)^2 dx \quad \text{--- (i)}$$

$$\begin{aligned}
 \text{Let } \log x &= t \quad \text{then,} \\
 d(\log x) &= dt
 \end{aligned}$$

$$\Rightarrow \frac{1}{x} dx = dt$$

Putting $\log x = t$ and $\frac{1}{x} dx = dt$ in equation (i), we get

$$\begin{aligned}
 I &= \int t^2 dt \\
 &= \frac{t^3}{3} + C \\
 &= \frac{(\log x)^3}{3} + C
 \end{aligned}$$

$$\therefore I = \frac{1}{3} (\log x)^3 + C$$

Indefinite Integrals Ex 19.9 Q18

$$\text{Let } I = \int \sin^5 x \cos x dx \quad \text{--- (i)}$$

$$\begin{aligned}
 \text{Let } \sin x &= t \quad \text{then,} \\
 d(\sin x) &= dt
 \end{aligned}$$

$$\Rightarrow \cos x dx = dt$$

Putting $\sin x = t$ and $\cos x dx = dt$ in equation (i), we get

$$\begin{aligned}
 I &= \int t^5 dt \\
 &= \frac{t^6}{6} + C \\
 &= \frac{\sin^6 x}{6} + C
 \end{aligned}$$

$$\therefore I = \frac{1}{6} \sin^6 x + C$$

Indefinite Integrals Ex 19.9 Q19

$$\text{Let } I = \int \tan^{\frac{3}{2}} x \sec^2 x dx \dots \text{(i)}$$

$$\begin{aligned}\text{Let } \tan x &= t && \text{then,} \\ d(\tan x) &= dt\end{aligned}$$

$$\Rightarrow \sec^2 x dx = dt$$

Putting $\tan x = t$ and $\sec^2 x dx = dt$ in equation (i), we get

$$\begin{aligned}I &= \int t^{\frac{3}{2}} dt \\ &= \frac{2}{5} t^{\frac{5}{2}} + C \\ &= \frac{2}{5} (\tan x)^{\frac{5}{2}} + C\end{aligned}$$

$$\therefore I = \frac{2}{5} \tan^{\frac{5}{2}} x + C$$

Indefinite Integrals Ex 19.9 Q20

$$\text{Let } I = \int \frac{x^3}{(x^2+1)^3} x \, dx \quad \text{--- (i)}$$

$$\begin{aligned}\text{Let } 1+x^2 &= t && \text{then,} \\ d(1+x^2) &= dt\end{aligned}$$

$$\begin{aligned}\Rightarrow 2x \, dx &= dt \\ \Rightarrow x \, dx &= \frac{dt}{2}\end{aligned}$$

Putting $1+x^2 = t$ and $x \, dx = \frac{dt}{2}$ in equation (i), we get

$$\begin{aligned}I &= \int \frac{x^2}{t^3} \times \frac{dt}{2} \\ &= \frac{1}{2} \int \frac{(t-1)}{t^3} dt \quad [\because 1+x^2 = t] \\ &= \frac{1}{2} \int \left[\left(\frac{1}{t^3} - \frac{1}{t^2} \right) dt \right] \\ &= \frac{1}{2} \int (t^{-2} - t^{-3}) dt \\ &= \frac{1}{2} \left[-1t^{-1} - \frac{1}{-2} t^{-2} \right] + C \\ &= \frac{1}{2} \left[-\frac{1}{t} + \frac{1}{2t^2} \right] + C \\ &= -\frac{1}{2t} + \frac{1}{4t^2} + C \\ &= -\frac{1}{2(1+x^2)} + \frac{1}{4(1+x^2)^2} + C \\ &= \frac{-2(1+x^2)+1}{4(1+x^2)^2} + C \\ &= \frac{-2-2x^2+1}{4(1+x^2)^2} + C \\ &= \frac{-2x^2-1}{4(1+x^2)^2} + C \\ &= -\frac{(1+2x^2)}{4(x^2+1)^2} + C \\ \therefore I &= -\frac{(1+2x^2)}{4(x^2+1)^2} + C\end{aligned}$$

Indefinite Integrals Ex 19.9 Q21

Let $x^2 + x + 1 = t$
 $(2x + 1)dx = dt$

$$\begin{aligned} & \int (4x+2)\sqrt{x^2+x+1} dx \\ &= \int 2\sqrt{t} dt \\ &= 2 \int \sqrt{t} dt \\ &= 2 \left(\frac{\frac{3}{2}}{\frac{3}{2}} \right) + C \\ &= \frac{4}{3}(x^2+x+1)^{\frac{3}{2}} + C \end{aligned}$$

Indefinite Integrals Ex 19.9 Q22

Let $I = \int \frac{4x+3}{\sqrt{2x^3+3x+1}} dx \dots \text{(i)}$

Let $2x^3 + 3x + 1 = t$ then,
 $d(2x^3 + 3x + 1) = dt$
 $\Rightarrow (4x+3)dx = dt$

Putting $2x^3 + 3x + 1 = t$ and $(4x+3)dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{dt}{\sqrt{t}} \\ &= \int t^{\frac{-1}{2}} dt \\ &= 2t^{\frac{1}{2}} + C \\ &= 2\sqrt{t} + C \end{aligned}$$

$\therefore I = 2\sqrt{2x^3+3x+1} + C$

Indefinite Integrals Ex 19.9 Q23

Let $I = \int \frac{1}{1+\sqrt{x}} dx \dots \text{(i)}$

Let $x = t^2$ then,
 $dx = d(t^2)$

$\Rightarrow dx = 2t dt$

Putting $x = t^2$ and $dx = 2t dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{2t}{1+\sqrt{t^2}} dt \\ &= \int \frac{2t}{1+t} dt \\ &= 2 \int \frac{t}{1+t} dt \\ &= 2 \int \frac{1+t-1}{1+t} dt \\ &= 2 \left[\int \frac{1+t}{1+t} dt - \int \frac{1}{1+t} dt \right] \\ &= 2 \left[dt - 2 \int \frac{1}{1+t} dt \right] \\ &= 2t - 2 \log(1+t) + c \\ &= 2\sqrt{x} - 2 \log(1+\sqrt{x}) + c \end{aligned}$$

$\therefore I = 2\sqrt{x} - 2 \log(1+\sqrt{x}) + c$

Indefinite Integrals Ex 19.9 Q24

Let $I = \int e^{\cos^2 x} \sin 2x dx \dots \text{(i)}$

Let $\cos^2 x = t$ then,
 $d(\cos^2 x) = dt$

$$\begin{aligned} \Rightarrow -2 \cos x \sin x dx &= dt \\ \Rightarrow -\sin 2x dx &= dt \\ \Rightarrow \sin 2x dx &= -dt \end{aligned}$$

Putting $\cos^2 x = t$ and $\sin 2x dx = -dt$ in equation (i), we get

$$\begin{aligned} I &= \int e^t (-dt) \\ &= -e^t + c \\ &= -e^{\cos^2 x} + c \end{aligned}$$

$\therefore I = -e^{\cos^2 x} + c$

Indefinite Integrals Ex 19.9 Q25

$$\text{Let } I = \int \frac{1 + \cos x}{(x + \sin x)^3} dx \quad \dots \quad (i)$$

$$\text{Let } x + \sin x = t \quad \text{then,}$$

$$d(x + \sin x) = dt$$

$$\Rightarrow (1 + \cos x) dx = dt$$

Putting $x + \sin x = t$ and $(1 + \cos x) dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{dt}{t^3} \\ &= \int t^{-3} dt \\ &= \frac{t^{-2}}{-2} + C \\ &= -\frac{1}{2t^2} + C \\ &= \frac{-1}{2(x + \sin x)^2} + C \end{aligned}$$

$$\therefore I = \frac{-1}{2(x + \sin x)^2} + C$$

Indefinite Integrals Ex 19.9 Q26

$$\begin{aligned} \frac{\cos x - \sin x}{1 + \sin 2x} &= \frac{\cos x - \sin x}{(\sin^2 x + \cos^2 x) + 2 \sin x \cos x} \\ &\quad [\sin^2 x + \cos^2 x = 1; \sin 2x = 2 \sin x \cos x] \\ &= \frac{\cos x - \sin x}{(\sin x + \cos x)^2} \end{aligned}$$

$$\text{Let } \sin x + \cos x = t$$

$$\therefore (\cos x - \sin x) dx = dt$$

$$\begin{aligned} \Rightarrow \int \frac{\cos x - \sin x}{1 + \sin 2x} dx &= \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx \\ &= \int \frac{dt}{t^2} \\ &= \int t^{-2} dt \\ &= -t^{-1} + C \\ &= -\frac{1}{t} + C \\ &= -\frac{1}{\sin x + \cos x} + C \end{aligned}$$

Indefinite Integrals Ex 19.9 Q27

Let $I = \int \frac{\sin 2x}{(a+b \cos 2x)^2} dx \dots\dots\dots (i)$

Let $a+b \cos 2x = t$ then,
 $d(a+b \cos 2x) = dt$

$$\Rightarrow b(-2 \sin 2x) dx = dt$$

$$\Rightarrow \sin 2x dx = -\frac{dt}{2b}$$

Putting $a+b \cos 2x = t$ and $\sin 2x dx = -\frac{dt}{2b}$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{1}{t^2} \times \frac{-dt}{2b} \\ &= \frac{-1}{2b} \int t^{-2} dt \\ &= -\frac{1}{2b} (-1t^{-1}) + c \\ &= \frac{1}{2bt} + c \\ &= \frac{1}{2b(a+b \cos 2x)} + c \end{aligned}$$

$\therefore I = \frac{1}{2b(a+b \cos 2x)} + c$

Indefinite Integrals Ex 19.9 Q28

Let $I = \int \frac{\log x^2}{x} dx \dots\dots\dots (i)$

Let $\log x = t$ then,
 $d(\log x) = dt$

$$\begin{aligned} \Rightarrow \frac{1}{x} dx &= dt \\ \Rightarrow \frac{dx}{x} &= dt \\ \text{Now, } I &= \int \frac{\log x^2}{x} dx \\ &= \int \frac{2 \log x}{x} dx \\ &= 2 \int \frac{\log x}{x} dx \dots\dots\dots (ii) \end{aligned}$$

Putting $\log x = t$ and $\frac{dx}{x} = dt$ in equation (ii), we get

$$\begin{aligned} I &= 2 \int t dt \\ &= \frac{2t^2}{2} + c \\ &= t^2 + c \end{aligned}$$

$\therefore I = (\log x)^2 + c$

Indefinite Integrals Ex 19.9 Q29

$$\text{Let } I = \int \frac{\sin x}{(1 + \cos x)^2} dx \quad \text{(i)}$$

$$\begin{aligned}\text{Let } 1 + \cos x &= t \quad \text{then,} \\ d(1 + \cos x) &= dt\end{aligned}$$

$$\begin{aligned}\Rightarrow -\sin x dx &= dt \\ \Rightarrow \sin x dx &= -dt\end{aligned}$$

Putting $1 + \cos x = t$ and $\sin x dx = -dt$ in equation (ii), we get

$$\begin{aligned}I &= \int \frac{-dt}{t^2} \\ &= -\int t^{-2} dt \\ &= -(-1t^{-1}) + C \\ &= \frac{1}{t} + C \\ &= \frac{1}{1 + \cos x} + C \\ \therefore I &= \frac{1}{1 + \cos x} + C\end{aligned}$$

Indefinite Integrals Ex 19.9 Q30

$$\text{Let } \log \sin x = t$$

$$\begin{aligned}\Rightarrow \frac{1}{\sin x} \cdot \cos x dx &= dt \\ \therefore \cot x dx &= dt\end{aligned}$$

$$\begin{aligned}\Rightarrow \int \cot x \log \sin x dx &= \int t dt \\ &= \frac{t^2}{2} + C \\ &= \frac{1}{2} (\log \sin x)^2 + C\end{aligned}$$

Indefinite Integrals Ex 19.9 Q31

$$\text{Let } I = \int \sec x \cdot \log(\sec x + \tan x) dx \quad \text{(i)}$$

$$\begin{aligned}\text{Let } \log(\sec x + \tan x) &= t \quad \text{then,} \\ d[\log(\sec x + \tan x)] &= dt\end{aligned}$$

$$\Rightarrow \sec x dx = dt \quad \left[\because \frac{d}{dx} [\log(\sec x + \tan x)] = \sec x \right]$$

Putting $\log(\sec x + \tan x) = t$ and $\sec x dx = dt$ in equation (i), we get

$$\begin{aligned}I &= \int t dt \\ &= \frac{t^2}{2} + C \\ &= \frac{1}{2} [\log(\sec x + \tan x)]^2 + C\end{aligned}$$

$$\therefore I = \frac{1}{2} [\log(\sec x + \tan x)]^2 + C$$

Indefinite Integrals Ex 19.9 Q32

Let $I = \int \csc x \log(\csc x - \cot x) dx \dots \text{(i)}$

Let $\log(\csc x - \cot x) = t \quad \text{then,}$
 $dx [\log(\csc x - \cot x)] = dt$

$$\Rightarrow \csc x dx = dt \quad \left[\because \frac{d}{dx} (\log(\csc x - \cot x)) = \csc x \right]$$

Putting $\log(\csc x - \cot x) = t$ and $\csc x dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int t dt \\ &= \frac{t^2}{2} + c \end{aligned}$$

$$\therefore I = \frac{1}{2} [\log(\csc x - \cot x)]^2 + c$$

Indefinite Integrals Ex 19.9 Q33

Let $I = \int x^3 \cos x^4 dx \dots \text{(i)}$

Let $x^4 = t$ then,
 $dx (x^4) = dt$

$$\begin{aligned} \Rightarrow 4x^3 dx &= dt \\ \Rightarrow x^3 &= \frac{dt}{4} \end{aligned}$$

Putting $x^4 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$$\begin{aligned} I &= \int \cos t \frac{dt}{4} \\ &= \frac{1}{4} \sin t + c \end{aligned}$$

$$\therefore I = \frac{1}{4} \sin x^4 + c$$

Indefinite Integrals Ex 19.9 Q34

Let $I = \int x^3 \sin x^4 dx \dots \text{(i)}$

Let $x^4 = t$ then,
 $d(x^4) = dt$

$$\begin{aligned} \Rightarrow 4x^3 dx &= dt \\ \Rightarrow x^3 &= \frac{dt}{4} \end{aligned}$$

Putting $x^4 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$$\begin{aligned} I &= \int \sin t \frac{dt}{4} \\ &= \frac{1}{4} \int \sin t dt \\ &= -\frac{1}{4} \cos t + c \\ &= -\frac{1}{4} \cos x^4 + c \end{aligned}$$

$$\therefore I = -\frac{1}{4} \cos x^4 + c$$

Indefinite Integrals Ex 19.9 Q35

$$\text{Let } I = \int \frac{x \sin^{-1} x^2}{\sqrt{1-x^4}} dx \quad \text{--- (i)}$$

$$\begin{aligned} \text{Let } \sin^{-1} x^2 &= t \quad \text{then,} \\ d(\sin^{-1} x^2) &= dt \end{aligned}$$

$$\Rightarrow 2x \times \frac{1}{\sqrt{1-x^4}} dx = dt$$

$$\Rightarrow \frac{x}{\sqrt{1-x^4}} dx = \frac{dt}{2}$$

Putting $\sin^{-1} x^2 = t$ and $\frac{x}{\sqrt{1-x^4}} dx = \frac{dt}{2}$ in equation (i), we get

$$\begin{aligned} I &= \int t \frac{dt}{2} \\ &= \frac{1}{2} \times \frac{t^2}{2} + c \\ &= \frac{1}{4} (\sin^{-1} x^2)^2 + c \end{aligned}$$

$$\therefore I = \frac{1}{4} (\sin^{-1} x^2)^2 + c$$

Indefinite Integrals Ex 19.9 Q36

$$\text{Let } I = \int x^3 \sin(x^4 + 1) dx \quad \text{--- (i)}$$

$$\begin{aligned} \text{Let } x^4 + 1 &= t \quad \text{then,} \\ d(x^4 + 1) &= dt \end{aligned}$$

$$\Rightarrow x^4 dx = dt$$

$$\Rightarrow x^3 dx = \frac{dt}{4}$$

Putting $x^4 + 1 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$$\begin{aligned} I &= \int \sin t \frac{dt}{4} \\ &= -\frac{1}{4} \cos t + c \\ &= -\frac{1}{4} \cos(x^4 + 1) + c \end{aligned}$$

$$\therefore I = -\frac{1}{4} \cos(x^4 + 1) + c$$

Indefinite Integrals Ex 19.9 Q37

$$\text{Let } I = \int \frac{(x+1)e^x}{\cos^2(xe^x)} dx \quad \text{--- (i)}$$

$$\text{Let } xe^x = t \quad \text{then,} \\ d(xe^x) = dt$$

$$\Rightarrow (e^x + xe^x)dx = dt \\ \Rightarrow (x+1)e^x dx = dt$$

Putting $xe^x = t$ and $(x+1)e^x dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{dt}{\cos^2 t} \\ &= \int \sec^2 t dt \\ &= \tan t + c \\ &= \tan(xe^x) + c \end{aligned}$$

$$\therefore I = \tan(xe^x) + c$$

Indefinite Integrals Ex 19.9 Q38

$$\text{Let } I = \int x^2 e^{x^3} \cos(e^{x^3}) dx \quad \text{--- (i)}$$

$$\text{Let } e^{x^3} = t \quad \text{then,} \\ d(e^{x^3}) = dt$$

$$\Rightarrow 3x^2 e^{x^3} dx = dt \\ \Rightarrow x^2 e^{x^3} dx = \frac{dt}{3}$$

Putting $e^{x^3} = t$ and $x^2 e^{x^3} dx = \frac{dt}{3}$ in equation (i), we get

$$\begin{aligned} I &= \int \cos t \frac{dt}{3} \\ &= \frac{\sin t}{3} + c \\ &= \frac{\sin(e^{x^3})}{3} + c \end{aligned}$$

$$\therefore I = \frac{1}{3} \sin(e^{x^3}) + c$$

Indefinite Integrals Ex 19.9 Q39

$$\text{Let } I = \int 2x \sec^3(x^2 + 3) \tan(x^2 + 3) dx \dots \text{(i)}$$

$$\begin{aligned} \text{Let } & \sec(x^2 + 3) = t & \text{then,} \\ & d[\sec(x^2 + 3)] = dt \end{aligned}$$

$$\Rightarrow 2x \sec(x^2 + 3) \tan(x^2 + 3) dx = dt$$

Putting $\sec(x^2 + 3) = t$ and $2x \sec(x^2 + 3) \tan(x^2 + 3) dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int t^2 dt \\ &= \frac{t^3}{3} + C \\ &= \frac{1}{3} [\sec(x^2 + 3)]^3 + C \end{aligned}$$

$$\therefore I = \frac{1}{3} [\sec(x^2 + 3)]^3 + C$$

Indefinite Integrals Ex 19.9 Q40

$$\frac{(x+1)(x+\log x)^2}{x} = \left(\frac{x+1}{x}\right)(x+\log x)^2 = \left(1 + \frac{1}{x}\right)(x+\log x)^2$$

$$\text{Let } (x+\log x) = t$$

$$\begin{aligned} \Rightarrow & \left(1 + \frac{1}{x}\right) dx = dt \\ \Rightarrow & \int \left(1 + \frac{1}{x}\right) (x+\log x)^2 dx = \int t^2 dt \\ &= \frac{t^3}{3} + C \\ &= \frac{1}{3} (x+\log x)^3 + C \end{aligned}$$

Indefinite Integrals Ex 19.9 Q41

Let $I = \int \tan x \sec^2 x \sqrt{1 - \tan^2 x} dx \dots \text{(i)}$

Let $1 - \tan^2 x = t$ then,

$$d(1 - \tan^2 x) = dt$$

$$\Rightarrow -2 \tan x \sec^2 x dx = dt$$

$$\Rightarrow \tan x \sec^2 x dx = \frac{-dt}{2}$$

Putting $1 - \tan^2 x = t$ and $\tan x \sec^2 x dx = -\frac{dt}{2}$ in equation (i), we get

$$I = \int \sqrt{t} \times \frac{-dt}{2}$$

$$= -\frac{1}{2} \int t^{\frac{1}{2}} dt$$

$$= -\frac{1}{2} \times \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= -\frac{1}{3} t^{\frac{3}{2}} + C$$

$$\therefore I = -\frac{1}{3} [1 - \tan^2 x]^{\frac{3}{2}} + C$$

Indefinite Integrals Ex 19.9 Q42

Let $I = \int \log x \frac{\sin(1 + (\log x)^2)}{x} dx \dots \text{(i)}$

Let $1 + (\log x)^2 = t$ then,

$$d(1 + (\log x)^2) = dt$$

$$\Rightarrow 2 \log x \frac{1}{x} dx = dt$$

$$\Rightarrow \frac{\log x}{x} dx = \frac{dt}{2}$$

Putting $1 + (\log x)^2 = t$ and $\frac{\log x}{x} dx = \frac{dt}{2}$ in equation (i), we get

$$I = \int \sin t \times \frac{dt}{2}$$

$$= \frac{1}{2} \int \sin t dt$$

$$\therefore I = -\frac{1}{2} \cos t + C$$

$$= -\frac{1}{2} \cos[1 + (\log x)^2] + C$$

$$\therefore I = -\frac{1}{2} \cos[1 + (\log x)^2] + C$$

Indefinite Integrals Ex 19.9 Q43

$$\text{Let } I = \int \frac{1}{x^2} \cos^2 \left(\frac{1}{x} \right) dx \quad \dots \dots \dots (i)$$

$$\text{Let } \frac{1}{x} = t \text{ then,}$$

$$d\left(\frac{1}{x}\right) = dt$$

$$\Rightarrow \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \frac{1}{x^2} dx = -dt$$

Putting $\frac{1}{x} = t$ and $\frac{1}{x^2} dx = -dt$ in equation (i), we get

$$\begin{aligned} I &= \int \cos^2 t (-dt) \\ &= - \int \cos^2 t dt \\ &= - \int \frac{\cos^2 2t + 1}{2} dt \\ &= - \frac{1}{2} \int \cos 2t dt - \frac{1}{2} \int dt \\ &= - \frac{1}{2} \times \frac{\sin 2t}{2} - \frac{1}{2} t + c \\ \therefore I &= - \frac{1}{4} \sin 2t - \frac{1}{2} t + c \\ &= - \frac{1}{4} \sin 2 \times \frac{1}{x} - \frac{1}{2} \times \frac{1}{x} + c \end{aligned}$$

$$\therefore I = - \frac{1}{4} \sin \left(\frac{2}{x} \right) - \frac{1}{2} \left(\frac{1}{x} \right) + c$$

Indefinite Integrals Ex 19.9 Q44

$$\text{Let } I = \int \sec^4 x \tan x dx \quad \dots \dots \dots (i)$$

$$\begin{aligned} \text{Let } \tan x &= t && \text{then,} \\ d(\tan x) &= dt \end{aligned}$$

$$\Rightarrow \sec^2 x dx = dt$$

$$\Rightarrow dx = \frac{dt}{\sec^2 x}$$

Putting $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get

$$\begin{aligned} I &= \int \sec^4 x \tan x \frac{dt}{\sec^2 x} \\ &= \int \sec^2 x t dt \\ &= \int (1 + \tan^2 x) t dt \\ &= \int (1 + t^2) t dt \\ &= \int (t + t^3) dt \\ &= \frac{t^2}{2} + \frac{t^4}{4} + c \\ &= \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + c \end{aligned}$$

$$\therefore I = \frac{1}{2} \tan^2 x + \frac{1}{4} \tan^4 x + c$$

Indefinite Integrals Ex 19.9 Q45

$$\text{Let } I = \int \frac{e^{\sqrt{x}} \cos(e^{\sqrt{x}})}{\sqrt{x}} dx \quad \dots \quad (i)$$

$$\begin{aligned} \text{Let } e^{\sqrt{x}} &= t && \text{then,} \\ d(e^{\sqrt{x}}) &= dt \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad e^{\sqrt{x}} \frac{1}{2\sqrt{x}} dx &= dt \\ \Rightarrow \quad \frac{e^{\sqrt{x}}}{\sqrt{x}} dx &= 2dt \end{aligned}$$

Putting $e^{\sqrt{x}} = t$ and $\frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$ in equation (i), we get

$$\begin{aligned} I &= \int \cos t \times 2dt \\ &= 2 \int \cos t dt \\ &= 2 \sin t + c \\ &= 2 \sin(e^{\sqrt{x}}) + c \end{aligned}$$

$$\therefore I = 2 \sin(e^{\sqrt{x}}) + c$$

Indefinite Integrals Ex 19.9 Q46

$$\text{Let } I = \int \frac{\cos^5 x}{\sin x} dx \quad \dots \quad (i)$$

$$\begin{aligned} \text{Let } \sin x &= t && \text{then,} \\ d(\sin x) &= dt \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad \cos x dx &= dt \\ \Rightarrow \quad dx &= \frac{dt}{\cos x} \end{aligned}$$

Putting $\sin x = t$ and $dx = \frac{dt}{\cos x}$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{\cos^5 x}{t} \times \frac{dt}{\cos x} \\ &= \int \frac{\cos^4 x}{t} dt \\ &= \int \frac{(1 - \sin^2 x)^2}{t} dt \\ &= \int \frac{(1 - t^2)^2}{t} dt \\ &= \int \frac{1 + t^4 - 2t^2}{t} dt \\ &= \int \frac{1}{t} dt + \int \frac{t^4}{t} dt - 2 \int \frac{t^2}{t} dt \\ &= \log|t| + \frac{t^4}{4} - \frac{2t^2}{2} + c \\ &= \log|\sin x| + \frac{\sin^4 x}{4} - \sin^2 x + c \end{aligned}$$

$$\therefore I = \frac{1}{4} \sin^4 x - \sin^2 x + \log|\sin x| + c$$

Indefinite Integrals Ex 19.9 Q47

$$\begin{aligned} \text{Let } \sqrt{x} &= t \\ \Rightarrow \frac{1}{2\sqrt{x}} dx &= dt \\ \Rightarrow \frac{1}{\sqrt{x}} dx &= 2dt \end{aligned}$$

$$\begin{aligned} \text{Therefore,} \\ \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx \\ = 2 \int \sin t dt \\ = -2 \cos t + C \\ = -2 \cos \sqrt{x} + C \end{aligned}$$

Indefinite Integrals Ex 19.9 Q48

$$\text{Let } I = \int \frac{(x+1)e^x}{\sin^2(xe^x)} dx \quad \dots \dots \dots (i)$$

$$\begin{aligned} \text{Let } xe^x &= t && \text{then,} \\ d(xe^x) &= dt \end{aligned}$$

$$\begin{aligned} \Rightarrow (xe^x + e^x) dx &= dt \\ \Rightarrow (x+1)e^x dx &= dt \end{aligned}$$

Putting $xe^x = t$ and $(x+1)e^x dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{dt}{\sin^2 t} \\ &= \int \csc^2 t dt \\ &= -\cot t + C \\ &= -\cot(xe^x) + C \end{aligned}$$

$$\therefore I = -\cot(xe^x) + C$$

Indefinite Integrals Ex 19.9 Q49

$$\text{Let } I = \int 5^{x+\tan^{-1}x} \left(\frac{x^2+2}{x^2+1} \right) dx \quad \text{--- (i)}$$

$$\text{Let } x + \tan^{-1}x = t \quad \text{then,}$$

$$d(x + \tan^{-1}x) = dt$$

$$\Rightarrow \left(1 + \frac{1}{1+x^2} \right) dx = dt$$

$$\Rightarrow \left(\frac{1+x^2+1}{1+x^2} \right) dx = dt$$

$$\Rightarrow \frac{(x^2+2)}{(x^2+1)} dx = dt$$

Putting $x + \tan^{-1}x = t$ and $\left(\frac{x^2+2}{x^2+1} \right) dx = dt$ in equation (i),

we get

$$\begin{aligned} I &= \int 5^t dt \\ &= \frac{5^t}{\log 5} + C \\ &= \frac{5^{x+\tan^{-1}x}}{\log 5} + C \end{aligned}$$

Indefinite Integrals Ex 19.9 Q50

$$\text{Let } I = \int \frac{e^{m \sin^{-1}x}}{\sqrt{1-x^2}} dx \quad \text{--- (i)}$$

$$\text{Let } m \sin^{-1}x = t \quad \text{then,}$$

$$d(m \sin^{-1}x) = dt$$

$$\Rightarrow m \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = \frac{dt}{m}$$

Putting $m \sin^{-1}x = t$ and $\frac{dx}{\sqrt{1-x^2}} = \frac{dt}{m}$ in equation (i),

we get

$$\begin{aligned} I &= \int e^t \frac{dt}{m} \\ &= \frac{1}{m} e^t + C \\ &= \frac{1}{m} e^{m \sin^{-1}x} + C \end{aligned}$$

$$\therefore I = \frac{1}{m} e^{m \sin^{-1}x} + C$$

Indefinite Integrals Ex 19.9 Q51

Let $\sqrt{x} = t$

$$\begin{aligned}\Rightarrow \frac{1}{2\sqrt{x}} dx &= dt \\ \Rightarrow \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx &= 2 \int \cos t dt \\ &= 2 \sin t + C \\ &= 2 \sin \sqrt{x} + C\end{aligned}$$

Indefinite Integrals Ex 19.9 Q52

Let $I = \int \frac{\sin(\tan^{-1} x)}{1+x^2} dx \dots\dots\dots (i)$

Let $\tan^{-1} x = t$ then,

$$d(\tan^{-1} x) = dt$$

$$\Rightarrow \frac{1}{1+x^2} dx = dt$$

Putting $\tan^{-1} x = t$ and $\frac{dx}{1+x^2} = dt$ in equation (i),
we get

$$\begin{aligned}I &= \int \sin t dt \\ &= -\cos t + C \\ &= -\cos(\tan^{-1} x) + C\end{aligned}$$

Indefinite Integrals Ex 19.9 Q53

Let $I = \int \frac{\sin(\log x)}{x} dx \dots\dots\dots (i)$

Let $\log x = t$ then,
 $d(\log x) = dt$

$$\Rightarrow \frac{1}{x} dx = dt$$

Putting $\log x = t$ and $\frac{1}{x} dx = dt$ in equation (i),
we get

$$\begin{aligned}I &= \int \sin t dt \\ &= -\cos t + C \\ &= -\cos(\log x) + C\end{aligned}$$

$$\therefore I = -\cos(\log x) + C$$

Indefinite Integrals Ex 19.9 Q54

Let $\tan^{-1}x = t$

Differentiating the above function with respect to, w, we have,

$$\begin{aligned}\frac{1}{1+x^2} dx &= dt \\ \Rightarrow \int \frac{e^{mtan^{-1}x}}{1+x^2} &= \int e^{mt} \times dt \\ \Rightarrow \int \frac{e^{mtan^{-1}x}}{1+x^2} &= \frac{e^{mt}}{m}\end{aligned}$$

Resubstituting the value of t in the above solution, we have,

$$\Rightarrow \int \frac{e^{mtan^{-1}x}}{1+x^2} = \frac{e^{mtan^{-1}x}}{m} + C$$

Indefinite Integrals Ex 19.9 Q55

$$\text{Let } I = \int \frac{x}{\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}} dx$$

$$\begin{aligned}\therefore I &= \int \frac{x}{\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}} \times \frac{\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2}}{\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2}} dx \\ &= \int \frac{x(\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2})}{x^2 + a^2 - x^2 + a^2} dx \\ &= \int \frac{x}{2a^2} (\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2}) dx \\ \therefore I &= \frac{1}{2a^2} \int x (\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2}) dx \quad \text{--- --- --- (i)}$$

$$\text{Let } x^2 = t \quad \text{then,}$$

$$d(x^2) = dt$$

$$\Rightarrow 2x dx = dt$$

$$\Rightarrow x dx = \frac{dt}{2}$$

Putting $x^2 = t$ and $x dx = \frac{dt}{2}$ in equation (i),

we get

$$\begin{aligned}I &= \frac{1}{2a^2} \left[\int (\sqrt{t + a^2} - \sqrt{t - a^2}) \frac{dt}{2} \right] \\ &= \frac{1}{4a^2} \left[\frac{2}{3}(t + a^2)^{\frac{3}{2}} - \frac{2}{3}(t - a^2)^{\frac{3}{2}} \right] + C \\ \therefore I &= \frac{1}{4a^2} \left[\frac{2}{3}(x^2 + a^2)^{\frac{3}{2}} - \frac{2}{3}(x^2 - a^2)^{\frac{3}{2}} \right] + C \\ &= \frac{1}{6a^2} \left[(x^2 + a^2)^{\frac{3}{2}} - (x^2 - a^2)^{\frac{3}{2}} \right] + C\end{aligned}$$

Indefinite Integrals Ex 19.9 Q56

$$\text{Let } I = \int x \frac{\tan^{-1} x^2}{1+x^4} dx \dots \text{(i)}$$

$$\text{Let } \tan^{-1} x^2 = t \quad \text{then,}$$

$$d(\tan^{-1} x^2) = dt$$

$$\Rightarrow \frac{1 \times 2x}{1 + (x^2)^2} dx = dt$$

$$\Rightarrow \frac{1 \times x}{1 + x^4} dx = \frac{dt}{2}$$

Putting $\tan^{-1} x^2 = t$ and $\frac{x}{1+x^4} dx = \frac{dt}{2}$ in equation (i), we get

$$\begin{aligned} I &= \int t \frac{dx}{2} \\ &= \frac{1}{2} \int t dt \\ &= \frac{1}{2} \times \frac{t^2}{2} + c \\ \therefore I &= \frac{t^2}{4} + c \\ &= \frac{(\tan^{-1} x^2)^2}{4} + c \end{aligned}$$

$$\therefore I = \frac{1}{4} (\tan^{-1} x^2)^2 + c$$

Indefinite Integrals Ex 19.9 Q57

$$\text{Let } I = \int \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}} dx \dots \text{(i)}$$

$$\text{Let } \sin^{-1} x = t \quad \text{then,}$$

$$d(\sin^{-1} x) = dt$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$$

Putting $\sin^{-1} x = t$ and $\frac{1}{\sqrt{1-x^2}} dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int t^3 dt \\ &= \frac{t^4}{4} + c \end{aligned}$$

$$\therefore I = \frac{1}{4} (\sin^{-1} x)^4 + c$$

Indefinite Integrals Ex 19.9 Q58

$$\text{Let } I = \int \frac{\sin(2 + 3\log x)}{x} dx \quad \dots \quad (1)$$

$$\text{Let } 2 + 3 \log x = t \quad \text{then,}$$

$$d(2 + 3 \log x) = dt$$

$$\Rightarrow \frac{3}{x} dx = dt$$

Putting $2+3\log x = t$ and $\frac{dx}{x} = \frac{dt}{3}$ in equation (i), we get

$$\begin{aligned}I &= \int \sin t \frac{dt}{3} \\&= \frac{1}{3}(-\cos t) + C \\&= -\frac{1}{3}\cos(2+3\log x) + C\end{aligned}$$

$$\therefore I = -\frac{1}{3} \cos(2 + 3 \log x) + c$$

Indefinite Integrals Ex 19.9 Q59

$$\text{Let } I = \int x e^{x^2} dx \dots \dots \dots (i)$$

$$\text{Let } x^2 = t \quad \text{then,}$$

$$d(x^2) = dt$$

$$\Rightarrow x \, dx = dt$$

Putting $x^2 = t$ and $x dx = \frac{dt}{2}$ in equation (i), we get

$$\begin{aligned}I &= \int e^t \frac{dt}{2} \\&= \frac{1}{2}e^t + c \\&= \frac{1}{2}e^{x^2} + c\end{aligned}$$

$$\therefore I = \frac{1}{2}e^{x^2} + c$$

Indefinite Integrals Ex 19.9 Q60

Let $I = \int \frac{e^{2x}}{1+e^x} dx \dots (i)$

Let $1+e^x = t \quad \text{then,}$
 $d(1+e^x) = dt$

$$\Rightarrow e^x dx = dt$$

$$\Rightarrow dx = \frac{dt}{e^x}$$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i),
we get

$$\begin{aligned} I &= \int \frac{e^{2x}}{t} \times \frac{dt}{e^x} \\ &= \int \frac{e^x}{t} dt \\ &= \int \frac{t-1}{t} dt \\ &= \int \left(\frac{t}{t} - \frac{1}{t} \right) dt \\ &= t - \log|t| + c \\ &= (1+e^x) - \log|1+e^x| + c \end{aligned}$$

$\therefore I = 1+e^x - \log|1+e^x| + c$

Indefinite Integrals Ex 19.9 Q61

Let $I = \int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx \dots (i)$

Let $\sqrt{x} = t \quad \text{then,}$
 $d(\sqrt{x}) = dt$

$$\begin{aligned} \Rightarrow \frac{1}{2\sqrt{x}} dx &= dt \\ \Rightarrow dx &= 2\sqrt{x} dt \\ \Rightarrow dx &= 2tdt \quad [\because \sqrt{x} = t] \end{aligned}$$

Putting $\sqrt{x} = t$ and $dx = 2tdt$ in equation (i),
we get

$$\begin{aligned} I &= \int \frac{\sec^2 t}{t} \times 2tdt \\ &= 2 \int \sec^2 t dt \\ &= 2 \tan t + c \\ &= 2 \tan \sqrt{x} + c \end{aligned}$$

$\therefore I = 2 \tan \sqrt{x} + c$

Indefinite Integrals Ex 19.9 Q62

$$\begin{aligned}
\tan^3 2x \sec 2x &= \tan^2 2x \tan 2x \sec 2x \\
&= (\sec^2 2x - 1) \tan 2x \sec 2x \\
&= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x \\
\therefore \int \tan^3 2x \sec 2x \, dx &= \int \sec^2 2x \tan 2x \sec 2x \, dx - \int \tan 2x \sec 2x \, dx \\
&= \int \sec^2 2x \tan 2x \sec 2x \, dx - \frac{\sec 2x}{2} + C
\end{aligned}$$

Let $\sec 2x = t$

$$\therefore 2 \sec 2x \tan 2x \, dx = dt$$

$$\begin{aligned}
\therefore \int \tan^3 2x \sec 2x \, dx &= \frac{1}{2} \int t^2 dt - \frac{\sec 2x}{2} + C \\
&= \frac{t^3}{6} - \frac{\sec 2x}{2} + C \\
&= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C
\end{aligned}$$

Indefinite Integrals Ex 19.9 Q63

$$\text{Let } I = \int \frac{x + \sqrt{x+1}}{x+2} dx \quad \dots \text{(i)}$$

$$\text{Let } x+1 = t^2 \quad \text{then,}$$

$$dx = 2t dt$$

$$\Rightarrow dx = 2t dt$$

Putting $x+1 = t^2$ and $dx = 2t dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{x + \sqrt{x+1}}{x+2} 2t dt \\ &= 2 \int \frac{\left(t^2 - 1\right) + t}{\left(t^2 - 1\right) + 2} t dt \quad [\because x+1 = t^2] \\ &= 2 \int \frac{t^2 + t - 1}{t^2 + 1} t dt \\ &= 2 \int \frac{t^3 + t^2 - t}{t^2 + 1} dt \\ &= 2 \left[\int \frac{t^3}{t^2 + 1} dt + \int \frac{t^2}{t^2 + 1} dt - \int \frac{t}{t^2 + 1} dt \right] \\ \therefore I &= 2 \left[\int \frac{t^3}{t^2 + 1} dt + \int \frac{t^2}{t^2 + 1} dt - \int \frac{t}{t^2 + 1} dt \right] \quad \dots \text{(ii)} \end{aligned}$$

$$\begin{aligned} \text{Let } I_1 &= \int \frac{t^3}{t^2 + 1} dt \\ I_2 &= \int \frac{t^2}{t^2 + 1} dt \\ \text{and } I_3 &= \int \frac{t}{t^2 + 1} dt \end{aligned}$$

$$\begin{aligned} \text{Now, } I_1 &= \int \frac{t^3}{t^2 + 1} dt \\ &= \int \left(t - \frac{t}{t^2 + 1} \right) dt \\ &= \frac{t^2}{2} - \frac{1}{2} \log(t^2 + 1) \\ \therefore I &= \frac{t^2}{2} - \frac{1}{2} \log(t^2 + 1) + c_1 \quad \dots \text{(iii)} \end{aligned}$$

$$\begin{aligned} \text{Since, } I_2 &= \int \frac{t^2}{t^2 + 1} dt \\ &= \int \frac{t^2 + 1 - 1}{t^2 + 1} dt \\ &= \int \frac{t^2 + 1}{t^2 + 1} dt - \int \frac{1}{t^2 + 1} dt \\ &= \int dt - \int \frac{1}{t^2 + 1} dt \\ \Rightarrow I_2 &= t - \tan^{-1}(t^2) + c_2 \quad \dots \text{(iv)} \end{aligned}$$

$$\text{and, } I_3 = \int \frac{t}{t^2 + 1} dt \\ = \frac{1}{2} \log(1+t^2) + C_3 \quad \dots \dots \dots (v)$$

Using equations (ii), (iii), (iv) and (v), we get

$$\begin{aligned} I &= 2 \left[\frac{t^2}{2} - \frac{1}{2} \log(t^2 + 1) + C_1 + t - \tan^{-1}(t^2) + C_2 - \frac{1}{2} \log(1+t^2) + C_3 \right] \\ &= 2 \left[\frac{t^2}{2} + t - \tan^{-1}(t^2) - \log(1+t^2) + C_1 + C_2 + C_3 \right] \\ &= 2 \left[\frac{t^2}{2} + t - \tan^{-1}(t^2) - \log(1+t^2) + C_4 \right] \quad [\text{Putting } C_1 + C_2 + C_3 = C_4] \\ &= t^2 + 2t - 2 \tan^{-1}(t^2) - 2 \log(1+t^2) + 2C_4 \\ &= (x+1) + 2\sqrt{x+1} - 2 \tan^{-1}(\sqrt{x+1}) - 2 \log(1+x+1) + 2C_4 \\ &= (x+1) + 2\sqrt{x+1} - 2 \tan^{-1}(\sqrt{x+1}) - 2 \log(x+2) + C \quad [\text{Putting } 2C_4 = C] \end{aligned}$$

$$\therefore I = (x+1) + 2\sqrt{x+1} - 2 \tan^{-1}(\sqrt{x+1}) - 2 \log(x+2) + C$$

Indefinite Integrals Ex 19.9 Q64

$$\text{Let } I = \int 5^{5^x} 5^{5^x} 5^x dx \quad \dots \dots \dots (i)$$

$$\begin{aligned} \text{Let } 5^{5^x} &= t \text{ then,} \\ d\left(5^{5^x}\right) &= dt \end{aligned}$$

$$\begin{aligned} \Rightarrow 5^{5^x} 5^{5^x} 5^x &(\log 5)^3 dx = dt \\ \Rightarrow 5^{5^x} 5^{5^x} 5^x dx &= \frac{dt}{(\log 5)^3} \end{aligned}$$

Putting $5^{5^x} = t$ and $5^{5^x} 5^{5^x} 5^x dx = \frac{dt}{(\log 5)^3}$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{dt}{(\log 5)^3} \\ &= \frac{1}{(\log 5)^3} \int dt \\ &= \frac{t}{(\log 5)^3} + C \end{aligned}$$

$$\therefore I = \frac{5^{5^x}}{(\log 5)^3} + C$$

Indefinite Integrals Ex 19.9 Q65

$$\text{Let } I = \int \frac{1}{x\sqrt{x^4 - 1}} dx \quad \dots \quad (i)$$

$$\begin{aligned} \text{Let } x^2 &= t \text{ then,} \\ d(x^2) &= dt \end{aligned}$$

$$\begin{aligned} \Rightarrow 2x \, dx &= dt \\ \Rightarrow dx &= \frac{dt}{2x} \end{aligned}$$

Putting $x^2 = t$ and $dx = \frac{dt}{2x}$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{1}{x\sqrt{t^2 - 1}} \times \frac{dt}{2x} \\ &= \frac{1}{2} \int \frac{1}{x^2\sqrt{t^2 - 1}} dt \\ &= \frac{1}{2} \int \frac{1}{t\sqrt{t^2 - 1}} dt \\ &= \frac{1}{2} \sec^{-1} t + c \\ &= \frac{1}{2} \sec^{-1} x^2 + c \end{aligned}$$

$$\therefore I = \frac{1}{2} \sec^{-1}(x^2) + c$$

Indefinite Integrals Ex 19.9 Q66

$$\text{Let } I = \int \sqrt{e^x - 1} dx \quad \dots \quad (i)$$

$$\begin{aligned} \text{Let } e^x - 1 &= t^2 \quad \text{then,} \\ d(e^x - 1) &= dt(t^2) \end{aligned}$$

$$\begin{aligned} \Rightarrow e^x \, dx &= 2t \, dt \\ \Rightarrow dx &= \frac{2t}{e^x} dt \\ \Rightarrow dx &= \frac{2t}{t^2 + 1} dt \quad [\because e^x - 1 = t^2] \end{aligned}$$

Putting $e^x - 1 = t^2$ and $dx = \frac{2t \, dt}{t^2 + 1}$ in equation (i), we get

$$\begin{aligned} I &= \int \sqrt{t^2} \times \frac{2t \, dt}{t^2 + 1} \\ &= 2 \int \frac{t \times t}{t^2 + 1} dt \\ &= 2 \int \frac{t^2}{t^2 + 1} dt \\ &= 2 \int \frac{t^2 + 1 - 1}{t^2 + 1} dt \\ &= 2 \int \left[\frac{t^2 + 1}{t^2 + 1} - \frac{1}{t^2 + 1} \right] dt \\ &= 2 \int dt - 2 \int \frac{1}{t^2 + 1} dt \\ &= 2t - 2 \tan^{-1}(t) + c \\ &= 2\sqrt{(e^x - 1)} - 2 \tan^{-1}(\sqrt{e^x - 1}) + c \end{aligned}$$

$$\therefore I = 2\sqrt{e^x - 1} - 2 \tan^{-1} \sqrt{e^x - 1} + c$$

Indefinite Integrals Ex 19.9 Q67

$$I = \int \frac{1}{(x+1)(x^2+2x+2)} dx$$

$$= \int \frac{1}{(x+1)((x+1)^2+1)} dx$$

Let $x+1 = \tan u$

$$\Rightarrow dx = \sec^2 u du$$

$$\therefore I = \int \frac{\sec^2 u}{\tan u (\tan^2 u + 1)} du$$

$$= \int \frac{\cos u}{\sin u} du$$

$$= \log |\sin u| + C$$

$$= \log \left| \frac{\tan u}{\sec^2 u} \right| + C$$

$$= \log \left| \frac{x+1}{\sqrt{x^2+2x+2}} \right| + C$$

Indefinite Integrals Ex 19.9 Q68

$$\text{Let } I = \int \frac{x^5}{\sqrt{1+x^3}} dx \dots (i)$$

$$\text{Let } 1+x^3 = t^2 \quad \text{then,}$$

$$d(1+x^3) = d(t^2)$$

$$\Rightarrow 3x^2 dx = dt \cdot 2t$$

$$\Rightarrow dx = \frac{dt}{3x^2} \cdot \frac{1}{2t}$$

Putting $1+x^3 = t^2$ and $dx = \frac{dt}{3x^2} \cdot \frac{1}{2t}$ in equation (i),
we get

$$\begin{aligned} I &= \int \frac{x^5}{\sqrt{t^2}} \times \frac{2t}{3x^2} dt \\ &= \int \frac{x^5}{t} \times \frac{2t}{3x^2} dt \\ &= \frac{2}{3} \int x^3 dt \\ &= \frac{2}{3} \int (t^2 - 1) dt \\ &= \frac{2}{3} \times \frac{t^3}{3} - \frac{2}{3} t + C \end{aligned}$$

$$\therefore I = \frac{2}{9} (1+x^3)^{\frac{3}{2}} - \frac{2}{3} \sqrt{1+x^3} + C$$

Indefinite Integrals Ex 19.9 Q69

Let $I = \int 4x^3 \sqrt{5-x^2} dx \dots \dots (i)$

Let $5-x^2 = t^2$ then,
 $d(5-x^2) = t^2 dt$

$$\Rightarrow -2x dx = 2t dt$$

$$\Rightarrow dx = \frac{-t}{x} dt$$

Putting $5-x^2 = t^2$ and $dx = \frac{-t}{x} dt$ in equation (i),
we get

$$\begin{aligned} I &= \int 4x^3 \sqrt{t^2} \times \frac{-t}{x} dt \\ &= -4 \int x^2 t \times t dt \\ &= -4 \int (5-t^2) t^2 dt \quad [\because 5-x^2 = t^2] \\ &= -4 \int (5t^2 - t^4) dt \\ &= -20 \times \frac{t^3}{3} + 4 \frac{t^5}{5} + C \\ &= \frac{-20}{3} \times t^3 + \frac{4}{5} \times t^5 + C \\ &= \frac{-20}{3} \times (5-x^2)^{\frac{3}{2}} + \frac{4}{5} \times (5-x^2)^{\frac{5}{2}} + C \end{aligned}$$

$$\therefore I = \frac{-20}{3} \times (5-x^2)^{\frac{3}{2}} + \frac{4}{5} \times (5-x^2)^{\frac{5}{2}} + C$$

Indefinite Integrals Ex 19.9 Q70

Let $I = \int \frac{1}{\sqrt{x+x}} dx \dots \dots (i)$

Let $\sqrt{x} = t$ then,
 $d(\sqrt{x}) = dt$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow dx = 2\sqrt{x} dt$$

Putting $\sqrt{x} = t$ and $2\sqrt{x} dt = dx$ in equation (i),
we get

$$\begin{aligned} I &= \int \frac{1}{t+t^2} 2t \times dt \quad [\because \sqrt{x} = t \Rightarrow x = t^2] \\ &= \int \frac{2t}{t(1+t)} dt \\ &= 2 \int \frac{t}{(1+t)} dt \\ &= 2 \log|1+t| + C \\ &= 2 \log|1+\sqrt{x}| + C \end{aligned}$$

$$\therefore I = 2 \log|1+\sqrt{x}| + C$$

Indefinite Integrals Ex 19.9 Q71

$$\frac{1}{x^2(x^4+1)^{\frac{3}{4}}}$$

Multiplying and dividing by x^{-3} , we obtain

$$\begin{aligned} \frac{x^{-3}}{x^2 \cdot x^{-3}(x^4+1)^{\frac{3}{4}}} &= \frac{x^{-3}(x^4+1)^{-\frac{3}{4}}}{x^2 \cdot x^{-3}} \\ &= \frac{(x^4+1)^{-\frac{3}{4}}}{x^5 \cdot (x^4)^{-\frac{3}{4}}} \\ &= \frac{1}{x^5} \left(\frac{x^4+1}{x^4} \right)^{-\frac{3}{4}} \\ &= \frac{1}{x^5} \left(1 + \frac{1}{x^4} \right)^{-\frac{3}{4}} \end{aligned}$$

$$\text{Let } \frac{1}{x^4} = t \Rightarrow -\frac{4}{x^5} dx = dt \Rightarrow \frac{1}{x^5} dx = -\frac{dt}{4}$$

$$\begin{aligned} \therefore \int \frac{1}{x^2(x^4+1)^{\frac{3}{4}}} dx &= \int \frac{1}{x^5} \left(1 + \frac{1}{x^4} \right)^{-\frac{3}{4}} dx \\ &= -\frac{1}{4} \int (1+t)^{-\frac{3}{4}} dt \\ &= -\frac{1}{4} \left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}} \right] + C \\ &= -\frac{1}{4} \left(\frac{1 + \frac{1}{x^4}}{\frac{1}{4}} \right)^{\frac{1}{4}} + C \\ &= -\left(1 + \frac{1}{x^4} \right)^{\frac{1}{4}} + C \end{aligned}$$

Indefinite Integrals Ex 19.9 Q72

$$\text{Let } I = \int \frac{\sin^5 x}{\cos^4 x} dx \quad \text{--- (i)}$$

Let $\cos x = t$ then,
 $d(\cos x) = dt$

$$\Rightarrow -\sin x dx = dt$$

$$\Rightarrow dx = -\frac{dt}{\sin x}$$

Putting $\cos x = t$ and $dx = -\frac{dt}{\sin x}$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{\sin^5 x}{t^4} \times -\frac{dt}{\sin x} \\ &= -\int \frac{\sin^4 x}{t^4} dt \\ &= -\int \frac{(1 - \cos^2 x)^2}{t^4} dt \\ &= -\int \frac{(1 - t^2)^2}{t^4} dt \\ &= -\int \frac{1 + t^4 - 2t^2}{t^4} dt \\ &= -\int \left(\frac{1}{t^4} + \frac{t^4}{t^4} - \frac{2t^2}{t^4} \right) dt \\ &= -\int (t^{-4} + 1 - 2t^{-2}) dt \\ &= -\left[\frac{t^{-3}}{-3} + t - 2 \frac{t^{-1}}{-1} \right] + C \\ &= -\left[-\frac{1}{3} \times \frac{1}{t^3} + t + \frac{2}{t} \right] + C \\ &= \frac{1}{3} \times \frac{1}{t^3} - t - \frac{2}{t} + C \\ &= \frac{1}{3} \times \frac{1}{\cos^3 x} - \cos x - \frac{2}{\cos x} + C \end{aligned}$$

$$\therefore I = -\cos x - \frac{2}{\cos x} + \frac{1}{3 \cos^3 x} + C$$

Ex 19.10

Indefinite Integrals Ex 19.10 Q1

$$\text{Let } I = \int x^2 \sqrt{x+2} dx$$

Substituting $x+2 = t$ and $dx = dt$ we get,

$$\begin{aligned} I &= \int (t-2)^2 \sqrt{t} dt \\ &= \int (t^2 + 4 - 4t) \sqrt{t} dt \\ &= \int \left(t^{\frac{5}{2}} - 4t^{\frac{3}{2}} + 4t^{\frac{1}{2}} \right) dt \\ &= \frac{2}{7} t^{\frac{7}{2}} - \frac{8}{5} t^{\frac{5}{2}} + \frac{8}{3} t^{\frac{3}{2}} + C \\ &= \frac{2}{7} (x+2)^{\frac{7}{2}} - \frac{8}{5} (x+2)^{\frac{5}{2}} + \frac{8}{3} (x+2)^{\frac{3}{2}} + C \\ \therefore I &= \frac{2}{7} (x+2)^{\frac{7}{2}} - \frac{8}{5} (x+2)^{\frac{5}{2}} + \frac{8}{3} (x+2)^{\frac{3}{2}} + C \end{aligned}$$

Indefinite Integrals Ex 19.10 Q2

$$\text{Let } I = \int \frac{x^2}{\sqrt{x-1}} dx$$

Substituting $x-1 = t$ and $dx = dt$ we get,

$$\begin{aligned} I &= \int \frac{(t+1)^2}{\sqrt{t}} dt \\ &= \int \frac{t^2 + 1 + 2t}{\sqrt{t}} dt \\ &= \int \left(t^{\frac{3}{2}} + t^{\frac{-1}{2}} + 2t^{\frac{-1}{2}} \right) dt \\ &= \frac{2}{5} t^{\frac{5}{2}} + 2t^{\frac{1}{2}} + \frac{4}{3} t^{\frac{3}{2}} + C \\ &= \frac{6t^{\frac{5}{2}} + 30t^{\frac{3}{2}} + 20t^{\frac{1}{2}}}{15} + C \\ &= \frac{2}{15} t^{\frac{1}{2}} (3t^2 + 15 + 10t) + C \\ &= \frac{2}{15} \sqrt{x-1} (3(x-1)^2 + 15 + 10(x-1)) + x \\ &= \frac{2}{15} \sqrt{x-1} (3(x^2 + 1 - 2x) + 15 + 10x - 10) + C \\ &= \frac{2}{15} \sqrt{x-1} (3x^2 + 3 - 6x + 15 + 10x - 10) + C \\ &= \frac{2}{15} \sqrt{x-1} (3x^2 + 4x + 8) + C \\ \therefore I &= \frac{2}{15} (3x^2 + 4x + 8) \sqrt{x-1} + C \end{aligned}$$

Indefinite Integrals Ex 19.10 Q3

$$\text{Let } I = \int \frac{x^2}{\sqrt{3x+4}} dx$$

Substituting $3x+4 = t$ and $dx = \frac{dt}{3}$ we get,

$$\begin{aligned} I &= \int \frac{\left(\frac{t-4}{3}\right)^2}{\sqrt{t}} \times \frac{dt}{3} & \left[\because x = \frac{t-4}{3} \right] \\ &= \int \frac{(t-4)^2}{9\sqrt{t}t^3} dt \\ &= \frac{1}{27} \int \frac{t^2 + 16 - 8t}{\sqrt{t}} dt \\ &= \frac{1}{27} \int \left(t^{\frac{3}{2}} - 8t^{\frac{1}{2}} + 16t^{\frac{1}{2}} - 16t^{-\frac{1}{2}} \right) dt \\ &= \frac{1}{27} \left[\frac{2}{5}t^{\frac{5}{2}} - \frac{16}{3}t^{\frac{3}{2}} + 16t^{\frac{1}{2}} \right] + c \\ &= \frac{2}{135}(3x+4)^{\frac{5}{2}} - \frac{16}{81}(3x+4)^{\frac{3}{2}} + \frac{32}{27}(3x+4)^{\frac{1}{2}} + c \\ \therefore I &= \frac{2}{135}(3x+4)^{\frac{5}{2}} - \frac{16}{81}(3x+4)^{\frac{3}{2}} + \frac{32}{27}(3x+4)^{\frac{1}{2}} + c \end{aligned}$$

Indefinite Integrals Ex 19.10 Q4

$$\text{Let } I = \int \frac{2x-1}{(x-1)^2} dx$$

Substituting $x-1 = t$ and $dx = dt$, we get

$$\begin{aligned} I &= \int \frac{2(t+1)}{t^2} dt \\ &= \int \frac{2t+2-1}{t^2} dt \\ &= \int \frac{2t+1}{t^2} dt \\ &= \int \left(\frac{2t}{t^2} + \frac{1}{t^2} \right) dt \\ &= 2 \int \frac{1}{t} dt + \int t^{-2} dt \\ &= 2 \log|t| - t^{-1} + c \\ &= 2 \log|x-1| - \frac{1}{x-1} + c \end{aligned}$$

$$\int \frac{2x-1}{(x-1)^2} dx = 2 \log|x-1| - \frac{1}{x-1} + c$$

Indefinite Integrals Ex 19.10 Q5

$$\text{Let } I = \int (2x^2 + 3) \sqrt{x+2} dx$$

Substituting $x+2 = t$ and $dx = dt$, we get

$$\begin{aligned} I &= \int [2(t-2)^2 + 3] \sqrt{t} dt \\ &= \int [2(t^2 + 4 - 4t) + 3] \sqrt{t} dt \\ &= \int [2t^2 + 8 - 8t + 3] \sqrt{t} dt \\ &= \int \left(2t^{\frac{5}{2}} + 11t^{\frac{1}{2}} - 8t^{\frac{3}{2}} \right) dt \\ &= \frac{4}{7}t^{\frac{7}{2}} + \frac{22}{3}t^{\frac{3}{2}} - \frac{16}{5}t^{\frac{5}{2}} + c \\ &= \frac{4}{7}(x+2)^{\frac{7}{2}} - \frac{16}{5}(x+1)^{\frac{5}{2}} + \frac{22}{3}(x+2)^{\frac{3}{2}} + c \end{aligned}$$

$$\therefore I = \frac{4}{7}(x+2)^{\frac{7}{2}} - \frac{16}{5}(x+1)^{\frac{5}{2}} + \frac{22}{3}(x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.10 Q6

$$\text{Let } I = \int \frac{x^2 + 3x + 1}{(x+1)^2} dx$$

Substituting $x+1 = t$ and $dx = dt$, we get

$$\begin{aligned} I &= \int \frac{(t-1)^2 + 3(t-1) + 1}{t^2} dt \\ &= \int \frac{t^2 + 1 - 2t + 3t - 3 + 1}{t^2} dt \\ &= \int \frac{t^2 + t - 1}{t^2} dt \\ &= \int \left(\frac{t^2}{t^2} + \frac{t}{t^2} - \frac{1}{t^2} \right) dt \\ &= \int \left(1 + \frac{1}{t} - t^{-2} \right) dt \\ &= t + \log|t| + t^{-1} + C \\ &= t + \log|t| + \frac{1}{t} + C \\ &= (x+1) + \log|x+1| + \frac{1}{x+1} + C \end{aligned}$$

Indefinite Integrals Ex 19.10 Q7

$$\text{Let } I = \int \frac{x^2}{\sqrt{1-x}} dx$$

Substituting $1-x = t$ and $dx = -dt$, we get

$$\begin{aligned} I &= \int \frac{(1-t)^2}{\sqrt{t}} \times -dt \\ &= - \int \frac{1+t^2-2t}{\sqrt{t}} \times dt \\ &= - \int \left(\frac{-1}{t^{\frac{1}{2}}} + t^{\frac{3}{2}} - 2t^{\frac{1}{2}} \right) dt \\ &= - \left[2t^{\frac{1}{2}} + \frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}} \right] + C \\ &= - \left[\frac{30t^{\frac{1}{2}} + 6t^{\frac{5}{2}} - 20t^{\frac{3}{2}}}{15} \right] + C \\ &= - \frac{2t^{\frac{1}{2}}}{15} \left[15 + 3t^2 - 10t \right] + C \\ &= - \frac{2}{15} \sqrt{1-x} \left[15 + 3(1-x)^2 - 10(1-x) \right] + C \\ &= - \frac{2}{15} \sqrt{1-x} \left(15 + 3(1+x^2 - 2x) - 10 + 10x \right) + C \\ &= - \frac{2}{15} \sqrt{1-x} \left(5 + 3 + 3x^2 - 6x + 10x \right) + C \\ &= - \frac{2}{15} \sqrt{1-x} \left(3x^2 + 4x + 8 \right) + C \\ &= - \frac{2}{15} (3x^2 + 4x + 8) \sqrt{1-x} + C \end{aligned}$$

$$\therefore I = - \frac{2}{15} (3x^2 + 4x + 8) \sqrt{1-x} + C$$

Indefinite Integrals Ex 19.10 Q8

$$\text{Let } I = \int x(1-x)^{23} dx$$

Substituting $1-x = t$ and $dx = -dt$, we get

$$\begin{aligned}I &= -\int (1-t)t^{23} dt \\&= -\int (t^{23} - t^{24}) dt \\&= -\left(\frac{t^{24}}{24} - \frac{t^{25}}{25}\right) + C \\&= \frac{t^{25}}{25} - \frac{t^{24}}{24} + C \\&= \frac{(1-x)^{25}}{25} - \frac{(1-x)^{24}}{24} + C \\\\therefore I &= \frac{(1-x)^{25}}{25} - \frac{(1-x)^{24}}{24} + C. \\&= \frac{1}{600}(1-x)^{24}[24(1-x) - 25] + C \\&= \frac{1}{600}(1-x)^{24}[24 - 24x - 25] + C \\&= \frac{1}{600}(1-x)^{24}[-1 - 24x] + C \\&= \frac{1}{600}(1-x)^{24} \times -[1 + 24x] + C \\&= -\frac{1}{600}(1-x)^{24}(1+24x) + C\end{aligned}$$

Ex 19.11

Indefinite Integrals Ex 19.11 Q1

$$\text{Let } I = \int \tan^3 x \sec^2 x dx \quad \text{---(i)}$$

Let $\tan x = t$. Then

$$d(\tan x) = dt$$

$$\Rightarrow \sec^2 x dx = dt$$

$$\Rightarrow dx = \frac{dt}{\sec^2 x}$$

Putting $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get

$$I = \int t^3 \sec^2 x \times \frac{dt}{\sec^2 x}$$

$$= \int t^3 dt$$

$$= \frac{t^{3+1}}{3+1} + C$$

$$= \frac{t^4}{4} + C$$

$$= \frac{(\tan x)^4}{4} + C$$

$$\therefore I = \frac{(\tan x)^4}{4} + C$$

$$= \frac{1}{4} \times \tan^4 x + C.$$

Indefinite Integrals Ex 19.11 Q2

$$\text{Let } I = \int \tan x \sec^4 x dx. \text{ Then}$$

$$I = \int \tan x \sec^2 x \sec^2 x dx$$

$$= \int \tan x (1 + \tan^2 x) \sec^2 x dx$$

$$\Rightarrow I = \int (\tan x + \tan^3 x) \sec^2 x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int (t + t^3) dt$$

$$= \frac{t^2}{2} + \frac{t^4}{4} + C$$

$$= \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + C$$

$$\therefore I = \frac{1}{2} \times \tan^2 x + \frac{1}{4} \times \tan^4 x + C.$$

Indefinite Integrals Ex 19.11 Q3

$$\text{Let } I = \int \tan^5 x \sec^4 x dx. \text{ Then}$$

$$I = \int \tan^4 x \sec^2 x \sec^2 x dx$$

$$= \int \tan^5 x (1 + \tan^2 x) \sec^2 x dx$$

$$= \int (\tan^5 x + \tan^7 x) \sec^2 x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int (t^5 + t^7) dt$$

$$= \frac{t^6}{6} + \frac{t^8}{8} + C$$

$$= \frac{(\tan x)^6}{6} + \frac{(\tan x)^8}{8} + C$$

$$\therefore I = \frac{1}{6} \times \tan^6 x + \frac{1}{8} \times \tan^8 x + C.$$

Indefinite Integrals Ex 19.11 Q4

Let $I = \int \sec^6 x \tan x dx$. Then

$$I = \int \sec^5 x (\sec x \tan x) dx$$

Substituting $\sec x = t$ and $\sec x \tan x = dt$, we get

$$I = \int t^5 dt$$

$$= \frac{t^6}{6} + C$$

$$= \frac{(\sec x)^6}{6} + C$$

$$\therefore I = \frac{1}{6} \sec^6 x + C$$

Indefinite Integrals Ex 19.11 Q5

Let $I = \int \tan^5 x dx$. Then

$$I = \int \tan^2 x \tan^3 x dx$$

$$= \int (\sec^2 x - 1) \tan^3 x dx$$

$$= \int \sec^2 x \tan^3 x dx - \int \tan^3 x dx$$

$$= \int \sec^2 x \tan^3 x dx - \int (\sec^2 x - 1) \tan x dx$$

$$= \int \sec^2 x \tan^3 x dx - \int \sec^2 x \tan x dx + \int \tan x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$ in first two integral, we get

$$I = \int t^3 dt - \int t dt + \int \tan x dx$$

$$= \frac{t^4}{4} - \frac{t^2}{2} + \log|\sec x| + C$$

$$= \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log|\sec x| + C$$

$$\therefore I = \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log|\sec x| + C$$

Indefinite Integrals Ex 19.11 Q6

Let $I = \int \sqrt{\tan x} \sec^4 x dx$. Then

$$I = \int \sqrt{\tan x} \sec^2 x \sec^2 x dx$$

$$= \int \sqrt{\tan x} (1 + \tan^2 x) \sec^2 x dx$$

$$= \int \tan x^{\frac{1}{2}} (1 + \tan^2 x) \sec^2 x dx$$

$$\Rightarrow I = \int \left(\tan x^{\frac{1}{2}} + \tan x^{\frac{5}{2}} \right) \sec^2 x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \left(t^{\frac{1}{2}} + t^{\frac{5}{2}} \right) dt$$

$$= \frac{2}{3} t^{\frac{3}{2}} + \frac{2}{7} t^{\frac{7}{2}} + C$$

$$= \frac{2}{3} (\tan x)^{\frac{3}{2}} + \frac{2}{7} (\tan x)^{\frac{7}{2}} + C$$

$$\therefore I = \frac{2}{3} \tan^{\frac{3}{2}} x + \frac{2}{7} \tan^{\frac{7}{2}} x + C$$

Indefinite Integrals Ex 19.11 Q7

Let $I = \int \sec^4 2x dx$. Then

$$\begin{aligned} I &= \int \sec^2 2x \sec^2 2x dx \\ &= \int (1 + \tan^2 2x) \sec^2 2x dx \\ &= \int (\sec^2 2x + \sec^2 2x \tan^2 2x) dx \\ \Rightarrow I &= \int \sec^2 2x dx + \int \sec^2 2x \tan^2 2x dx \\ \Rightarrow I &= \int \sec^2 2x \tan^2 2x dx + \int \sec^2 2x dx \end{aligned}$$

Substituting $\tan 2x = t$ and $\sec^2 2x dx = \frac{dt}{2}$ in first integral, we get

$$\begin{aligned} I &= \int t^2 \frac{dt}{2} + \int \sec^2 2x dx \\ &= \frac{1}{2} \times \frac{t^3}{3} + \frac{1}{2} \tan 2x + c \\ \Rightarrow I &= \frac{1}{6} \tan^3 2x + \frac{1}{2} \tan 2x + c \\ \therefore I &= \frac{1}{2} \tan 2x + \frac{1}{6} \tan^3 2x + c \end{aligned}$$

Indefinite Integrals Ex 19.11 Q8

Let $I = \int \csc^4 3x dx$. Then

$$\begin{aligned} I &= \int \csc^2 3x \csc^2 3x dx \\ &= \int (1 + \cot^2 3x) \csc^2 3x dx \\ &= \int (\csc^2 3x + \cot^2 3x \csc^2 3x) dx \\ \Rightarrow I &= \int \csc^2 3x dx + \int \cot^2 3x \csc^2 3x dx \end{aligned}$$

Substituting $\cot 3x = t$ and $\csc^2 3x dx = -dt$ in 2nd integral, we get

$$\begin{aligned} I &= \int \csc^2 3x dx - \int t^2 \frac{dt}{3} \\ &= \frac{-1}{3} \cot 3x - \frac{t^3}{9} + c \\ &= \frac{-1}{3} \cot 3x - \frac{\cot^3 3x}{9} + c \\ \therefore I &= \frac{-1}{3} \cot 3x - \frac{1}{9} \cot^3 3x + c \end{aligned}$$

Indefinite Integrals Ex 19.11 Q9

Let $I = \int \cot^n x \csc^2 x dx$, $n \neq -1$ --- (i)

Let $\cot x = t$. Then

$$\begin{aligned} d(\cot x) &= dt \\ \Rightarrow -\csc^2 x dx &= dt \\ \Rightarrow \csc^2 x dx &= -dt \end{aligned}$$

Putting $\cot x = t$ and $\csc^2 x dx = -dt$ in equation (i), we get

$$\begin{aligned} I &= \int t^n \times (-dt) \\ &= -\frac{t^{n+1}}{n+1} + c \\ \Rightarrow I &= -\frac{(\cot x)^{n+1}}{n+1} + c \end{aligned}$$

Indefinite Integrals Ex 19.11 Q10

Let $I = \int \cot^5 x \csc^4 x dx$. Then,

$$\begin{aligned} I &= \int \cot^5 x \csc^2 x \csc^2 x dx \\ &= \int \cot^5 x (1 + \cot^2 x) \csc^2 x dx \\ \Rightarrow I &= \int (\cot^5 x + \cot^7 x) \csc^2 x dx \end{aligned}$$

Substituting $\cot x = t$ and $-\csc^2 x dx = dt$, we get

$$\begin{aligned} I &= \int (t^5 + t^7) (-dt) \\ &= -\frac{t^6}{6} - \frac{t^8}{8} + C \\ &= -\frac{\cot^6 x}{6} - \frac{\cot^8 x}{8} + C \\ \therefore I &= -\frac{\cot^6 x}{6} - \frac{\cot^8 x}{8} + C \end{aligned}$$

Indefinite Integrals Ex 19.11 Q11

Let $I = \int \cot^5 x dx$. Then,

$$\begin{aligned} I &= \int \cot^3 x \times \cot^2 x dx \\ &= \int \cot^3 x \times (\csc^2 x - 1) dx \\ &= \int \cot^3 x \csc^2 x dx - \int \cot^3 x dx \\ &= \int \cot^3 x \csc^2 x dx - \int (\csc^2 x - 1) \cot x dx \\ &= \int \cot^3 x \csc^2 x dx - \int \csc^2 x \cot x dx + \int \cot x dx \\ \Rightarrow I &= \int \cot^3 x \csc^2 x dx - \int \csc^2 x \cot x dx + \int \cot x dx \end{aligned}$$

Substituting $\cot x = t$ and $-\csc^2 x dx = dt$ in first two integral, we get

$$\begin{aligned} I &= \int t^3 (-dt) - \int t \times (-dt) + \int \cot x dx \\ &= -\frac{t^4}{4} + \frac{t^2}{2} + \log |\sin x| + C \\ &= -\frac{1}{4} \cot^4 x + \frac{1}{2} \cot^2 x + \log |\sin x| + C \\ \therefore I &= -\frac{1}{4} \cot^4 x + \frac{1}{2} \cot^2 x + \log |\sin x| + C \end{aligned}$$

Indefinite Integrals Ex 19.11 Q12

Let $I = \int \cot^6 x dx$. Then,

$$\begin{aligned} I &= \int \cot^2 x \times \cot^4 x dx \\ &= \int (\csc^2 x - 1) \times \cot^4 x dx \\ &= \int (\csc^2 x \cot^4 x - \cot^4 x) dx \\ &= \int \csc^2 x \cot^4 x dx - \int \cot^4 x dx \\ &= \int \csc^2 x \cot^4 x dx - \int \cot^2 x (\csc^2 - 1) dx \\ &= \int \csc^2 x \cot^4 x dx - \int \cot^2 x \csc^2 x dx + \int \cot^2 x dx \\ \Rightarrow I &= \int \csc^2 x \cot^4 x dx - \int \cot^2 x \csc^2 x dx + \int (\csc^2 x - 1) dx \end{aligned}$$

Substituting $\cot x = t$ and $-\csc^2 x dx = dt$ in first two integral, we get

$$\begin{aligned} I &= \int t^4 (-dt) - \int t^2 (-dt) + \int \csc^2 x dx - \int dx \\ &= -\frac{t^5}{5} + \frac{t^3}{3} - \cot x - x + C \\ &= -\frac{\cot^5 x}{5} + \frac{\cot^3 x}{3} - \cot x - x + C \\ \therefore I &= -\frac{1}{5} \times \cot^5 x + \frac{1}{3} \times \cot^3 x - \cot x - x + C \end{aligned}$$

Ex 19.12

Indefinite Integrals Ex 19.12 Q1

$$\text{Let } I = \int \sin^4 x \cos^3 x dx$$

Here, power of $\cos x$ is odd, so we substitute

$$\begin{aligned} & \sin x = t \\ \Rightarrow & \cos x dx = dt \\ \Rightarrow & dx = \frac{dt}{\cos x} \\ \therefore & I = \int t^4 \cos^3 x \frac{dt}{\cos x} \\ &= \int t^4 \cos^2 x dt \\ &= \int t^4 (1 - \sin^2 x) dt \\ &= \int t^4 (1 - t^2) dt \\ &= \int (t^4 - t^6) dt \\ &= \frac{t^5}{5} - \frac{t^7}{7} + C \end{aligned}$$

$$\therefore I = \frac{1}{5} \times \sin^5 x - \frac{1}{7} \times \sin^7 x + C$$

Indefinite Integrals Ex 19.12 Q2

$$\text{Let } I = \int \sin^5 x dx. \text{ Then}$$

$$\begin{aligned} I &= \int \sin^3 x \sin^2 x dx \\ &= \int \sin^3 x (1 - \cos^2 x) dx \\ &= \int (\sin^3 x - \sin^3 x \cos^2 x) dx \\ &= \int [\sin x (1 - \cos^2 x) - \sin^3 x \cos^2 x] dx \\ &= \int (\sin x - \sin x \cos^2 x - \sin^3 x \cos^2 x) dx \\ \Rightarrow I &= \int \sin x dx - \int \sin x \cos^2 x dx - \int \sin^3 x \cos^2 x dx \end{aligned}$$

Putting $\cos x = t$ and $-\sin x dx = dt$ in 2nd and 3rd integrals, we get

$$\begin{aligned} I &= \int \sin x dx - \int t^2 (-dt) + \int \sin^2 x t^2 dt \\ &= \int \sin x dx + \int t^2 dt + \int (1 - \cos^2 x) t^2 dt \\ &= \int \sin x dx + \int t^2 dt + \int (1 - t^2) t^2 dt \\ &= -\cos x + \frac{t^3}{3} + \frac{t^3}{3} - \frac{t^5}{5} + C \\ &= -\cos x + \frac{2}{3} t^3 - \frac{1}{5} t^5 + C \\ &= -\cos x + \frac{2}{3} (\cos^3 x) - \frac{1}{5} (\cos^5 x) + C \end{aligned}$$

$$\therefore I = - \left[\cos x - \frac{2}{3} \cos^3 x + \frac{1}{5} \cos^5 x \right] + C$$

Indefinite Integrals Ex 19.12 Q3

Let $I = \int \cos^5 x dx$. Then

$$\begin{aligned} I &= \int \cos^2 x \cos^3 x dx \\ &= \int (1 - \sin^2 x) \cos^3 x dx \\ &= \int \cos^3 x dx - \int \sin^2 x \cos^3 x dx \\ &= \int \cos^2 x \cos x dx - \int \sin^2 x (1 - \sin^2 x) \cos x dx \\ &= \int (\cos x - \sin^2 x \cos x) dx - \int (\sin^2 x \cos x - \sin^4 x \cos x) dx \\ \Rightarrow I &= \int \cos x dx - 2 \int \sin^2 x \cos x dx + \int \sin^4 x \cos x dx \end{aligned}$$

Putting $\sin x = t$ and $\cos x dx = dt$ in 2nd and 3rd integrals, we get

$$\begin{aligned} I &= \int \cos x dx - 2 \int t^2 dt + \int t^4 dt \\ &= \sin x - \frac{2}{3}t^3 + \frac{t^5}{5} + C \\ &= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C \\ \therefore I &= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C \end{aligned}$$

Indefinite Integrals Ex 19.12 Q4

Let $I = \int \sin^5 x \cos x dx$ --- (i)

Let $\sin x = t$. Then,

$$\begin{aligned} d(\sin x) &= dt \\ \Rightarrow \cos x dx &= dt \end{aligned}$$

Putting $\sin x = t$ and $\cos x dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int t^5 dt \\ &= \frac{t^6}{6} + C \\ &= \frac{\sin^6 x}{6} + C \\ \therefore I &= \frac{1}{6}\sin^6 x + C \end{aligned}$$

Indefinite Integrals Ex 19.12 Q5

Let $I = \int \sin^3 x \cos^6 x dx$

Here, power of $\sin x$ is odd, so we substitute

$$\begin{aligned} \cos x &= t \\ \Rightarrow -\sin x dx &= dt \\ \therefore I &= \int \sin^2 x t^6 (-dt) \\ &= - \int (1 - \cos^2 x) t^6 dt \\ &= - \int (t^6 - t^8) dt \\ &= - \frac{t^7}{7} + \frac{t^9}{9} + C \\ &= - \frac{\cos^7 x}{7} + \frac{\cos^9 x}{9} + C \end{aligned}$$

$$\therefore I = - \frac{1}{7} \cos^7 x + \frac{1}{9} \cos^9 x + C$$

Indefinite Integrals Ex 19.12 Q6

Let $I = \int \cos^7 x dx$. Then

$$\begin{aligned}
 I &= \int \cos^6 x \cos x dx \\
 &= \int (\cos^2 x)^3 \cos x dx \\
 &= \int (1 - \sin^2 x)^3 \cos x dx \\
 &= \int [1 - \sin^6 x - 3\sin^2 x + 3\sin^4 x] \cos x dx \\
 &= \int [\cos x - \sin^6 x \cos x - 3\sin^2 x \cos x + 3\sin^4 x \cos x] dx + c \\
 \Rightarrow I &= \int \cos x dx - \int \sin^6 x \cos x dx - 3 \int \sin^2 x \cos x dx + 3 \int \sin^4 x \cos x dx
 \end{aligned}$$

Putting $\sin x = t$ and $\cos x dx = dt$ in 2nd and 3rd and 4th integral, we get

$$\begin{aligned}
 I &= \int \cos x dx - \int t^6 dt - 3 \int t^2 dt + 3 \int t^4 dt \\
 &= \sin x - \frac{t^7}{7} - \frac{3}{3} t^3 + \frac{3}{5} t^5 + c \\
 &= \sin x - \frac{1}{7} \sin^7 x - \sin^3 x + \frac{3}{5} \sin^5 x + c \\
 \therefore I &= \sin x - \sin^3 x + \frac{3}{5} \sin^5 x - \frac{1}{7} \sin^7 x + c
 \end{aligned}$$

Indefinite Integrals Ex 19.12 Q7

Let $I = \int x \cos^3 x^2 \sin x^2 dx$

Let $\cos x^2 = t$. Then

$$\begin{aligned}
 d(\cos x^2) &= dt \\
 \Rightarrow -2x \sin x^2 x &= dt \\
 \Rightarrow x \sin x^2 dx &= -\frac{dt}{2} \\
 \therefore I &= \int t^3 \times \frac{-dt}{2} \\
 &= -\frac{t^4}{8} + c \\
 &= -\frac{1}{8} \cos^4 x^2 + c \\
 \therefore I &= -\frac{1}{8} \cos^4 x^2 + c
 \end{aligned}$$

Indefinite Integrals Ex 19.12 Q8

Let $I = \int \sin^7 x dx$. Then

$$\begin{aligned}
 I &= \int \sin^6 x \sin x dx \\
 &= \int (\sin^2 x)^3 \sin x dx \\
 &= \int (1 - \cos^2 x)^3 \sin x dx \\
 &= \int (1 - \cos^6 x + 3\cos^4 x - 3\cos^2 x) \sin x dx \\
 \Rightarrow I &= \int \sin x dx - \int \cos^6 x \sin x dx + 3 \int \cos^4 x \sin x dx - 3 \int \cos^2 x \sin x dx \\
 \text{Putting } \cos x &= t \text{ and } -\sin x dx = dt \text{ in 2ne, 3rd and 4th integral, we get} \\
 I &= \int \sin x dx - \int t^6 (-dt) + 3 \int t^4 (-dt) - 3 \int t^2 (-dt) \\
 &= -\cos x + \frac{t^7}{7} - \frac{3}{5} t^5 + \frac{3}{3} t^3 + c \\
 &= -\cos x + \frac{\cos^7 x}{7} - \frac{3}{5} \cos^5 x + \cos^3 x + c \\
 \therefore I &= -\cos x + \cos^3 x - \frac{3}{5} \cos^5 x + \frac{1}{7} \cos^7 x + c
 \end{aligned}$$

Indefinite Integrals Ex 19.12 Q9

Let $I = \int \sin^3 x \cos^5 x dx$. Then

Let $\cos x = t$. Then

$$\begin{aligned} d(\cos x) &= dt \\ \Rightarrow -\sin x dx &= dt \\ \Rightarrow dx &= \frac{-dt}{\sin x} \\ \therefore I &= \int \sin^3 x t^5 \frac{-dt}{\sin x} \\ &= -\int \sin^2 x t^5 dt \\ &= -\int (1 - \cos^2 x) t^5 dt \\ &= -\int (1 - t^2) t^5 dt \\ &= -\int (t^5 - t^7) dt \\ &= -\frac{t^6}{6} + \frac{t^8}{8} + C \\ &= -\frac{1}{6} \cos^6 x + \frac{1}{8} \cos^8 x + C \\ \therefore I &= \frac{-1}{6} \cos^6 x + \frac{1}{8} \cos^8 x + C \end{aligned}$$

Indefinite Integrals Ex 19.12 Q10

$$\text{Let } I = \int \frac{1}{\sin^4 x \cos^2 x} dx \quad \text{--- (i)}$$

Then, $I = \int \sin^{-4} x \cos^{-2} x dx$

Since $-4 - 2 = -6$, which is even integer. So, we divide both numerator and denominator by $\cos^6 x$.

$$\begin{aligned} \therefore I &= \int \frac{\frac{1}{\cos^6 x}}{\frac{\sin^4 x \cos^2 x}{\cos^6 x}} dx \\ &= \int \frac{\sec^6 x}{\frac{\sin^4 x}{\cos^4 x}} dx \\ &= \int \frac{\sec^6 x}{\tan^4 x} dx \\ &= \int \frac{\sec^4 x \times \sec^2 x}{\tan^4 x} dx \\ &= \int \frac{(\sec^2 x)^2 \times \sec^2 x}{\tan^4 x} dx \\ &= \int \frac{(1 + \tan^2 x)^2 \times \sec^2 x}{\tan^4 x} dx \\ \Rightarrow I &= \int \frac{(1 + \tan^4 x + 2 \tan^2 x) \times \sec^2 x}{\tan^4 x} dx \quad \text{--- (ii)} \end{aligned}$$

Let $\tan x = t$. Then,

$$\begin{aligned} d(\tan x) &= dt \\ \Rightarrow \sec^2 x dx &= dt \\ \Rightarrow dx &= \frac{dt}{\sec^2 x} \end{aligned}$$

Putting $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{(1 + t^4 + 2t^2)}{t^4} \times \sec^2 x \times \frac{dt}{\sec^2 x} \\ &= \int (t^{-4} + 1 + 2t^{-2}) dt \\ &= -\frac{t^{-3}}{3} + t - 2t^{-1} + C \\ &= -\frac{1}{3t^3} + t - \frac{2}{t} + C \\ &= -\frac{1}{3 \tan^3 x} + \tan x - \frac{2}{\tan x} + C \\ &= -\frac{1}{3} \times \cot^3 x + \tan x - 2 \times \cot x + C \\ \therefore I &= \frac{-1}{3} \times \cot^3 x - 2 \cot x + \tan x + C \end{aligned}$$

Indefinite Integrals Ex 19.12 Q11

$$\text{Let } I = \int \frac{1}{\sin^3 x \cos^5 x} dx$$

---(i)

$$\text{Then, } I = \int \sin^{-3} x \cos^{-5} x dx$$

Since $-3 - 5 = -8$, which is even integer. So, we divide both numerator and denominator by $\cos^8 x$.

$$\begin{aligned} \therefore I &= \int \frac{\frac{1}{\cos^8 x}}{\frac{\sin^3 x \cos^5 x}{\cos^8 x}} dx \\ &= \int \frac{\sec^8 x}{\tan^3 x} dx \\ &= \int \frac{\sec^8 x}{\tan^3 x} \sec^2 x dx \\ &= \int \frac{(\sec^2 x)^3}{\tan^3 x} \sec^2 x dx \\ &= \int \frac{(1 + \tan^2 x)^3}{\tan^3 x} \times \sec^2 x dx \\ \Rightarrow I &= \int \frac{(1 + \tan^6 x + 3 \tan^4 x + 3 \tan^2 x) \times \sec^2 x}{\tan^3 x} dx \end{aligned}$$

---(ii)

Let $t = \tan x$. Then,

$$\begin{aligned} d(\tan x) &= dt \\ \Rightarrow \sec^2 x dx &= dt \\ \therefore I &= \int \frac{(1 + t^6 + 3t^4 + 3t^2)}{t^3} dt \\ &= \int (t^{-3} + t^3 + 3t + 3t^{-1}) dt \\ &= -\frac{t^{-2}}{2} + \frac{t^4}{4} + \frac{3}{2} t^2 + 3 \log t + c \\ &= -\frac{1}{2t^2} + \frac{t^4}{4} + \frac{3}{2} t^2 + 3 \log t + c \\ &= -\frac{1}{2} \times \frac{1}{\tan^2 x} + \frac{\tan^4 x}{4} + \frac{3}{2} \times \tan^2 x + 3 \log |\tan x| + c \\ \therefore I &= \frac{-1}{2 \tan^2 x} + 3 \log |\tan x| + \frac{3}{2} \tan^2 x + \frac{1}{4} \times \tan^4 x + c \end{aligned}$$

Indefinite Integrals Ex 19.12 Q12

$$\text{Let } I = \int \frac{1}{\sin^3 x \cos x} dx$$

---(i)

$$\text{Then, } I = \int \sin^{-3} x \cos^{-1} x dx$$

Since $-3 - 1 = -4$, which is even integer. So, we divide both numerator and denominator by $\cos^4 x$.

$$\begin{aligned} \therefore I &= \int \frac{\frac{1}{\cos^4 x}}{\frac{\sin^3 x \cos x}{\cos^4 x}} dx \\ &= \int \frac{\sec^4 x}{\tan^3 x} dx \\ &= \int \frac{\sec^4 x}{\tan^3 x} \sec^2 x dx \\ &= \int \frac{(\sec^2 x)^3}{\tan^3 x} \sec^2 x dx \\ &= \int \frac{(1 + \tan^2 x)^3}{\tan^3 x} \times \sec^2 x dx \end{aligned}$$

Putting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$\begin{aligned} I &= \int \frac{1+t^2}{t^3} dt \\ &= \int \left(t^{-3} + \frac{1}{t} \right) dt \\ &= -\frac{t^{-2}}{2} + \log |t| + c \\ &= -\frac{1}{2t^2} + \log |t| + c \\ &= -\frac{1}{2 \tan^2 x} + \log |\tan x| + c \end{aligned}$$

Indefinite Integrals Ex 19.12 Q13

$$\begin{aligned}
 \frac{1}{\sin x \cos^3 x} &= \frac{\sin^2 x + \cos^2 x}{\sin x \cos^3 x} \\
 &= \frac{\sin x}{\cos^3 x} + \frac{1}{\sin x \cos x} \\
 &= \tan x \sec^2 x + \frac{1 \cos^2 x}{\sin x \cos x} \\
 &= \tan x \sec^2 x + \frac{\sec^2 x}{\tan x}
 \end{aligned}$$

$$\therefore \int \frac{1}{\sin x \cos^3 x} dx = \int \tan x \sec^2 x dx + \int \frac{\sec^2 x}{\tan x} dx$$

Let $\tan x = t \Rightarrow \sec^2 x dx = dt$

$$\begin{aligned}
 \Rightarrow \int \frac{1}{\sin x \cos^3 x} dx &= \int t dt + \int \frac{1}{t} dt \\
 &= \frac{t^2}{2} + \log|t| + C \\
 &= \frac{1}{2} \tan^2 x + \log|\tan x| + C
 \end{aligned}$$

Ex 19.13

Indefinite Integrals Ex 19.13 Q1

$$\begin{aligned} \text{Let } I &= \int \frac{x}{\sqrt{x^4 + a^4}} dx \\ &= \int \frac{x}{\sqrt{(x^2)^2 + (a^2)^2}} dx \end{aligned}$$

Let $x^2 = t$

$$\Rightarrow 2x dx = dt$$

$$\Rightarrow x dx = \frac{dt}{2}$$

$$I = \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + (a^2)^2}}$$

$$= \frac{1}{2} \log \left| t + \sqrt{t^2 + (a^2)^2} \right| + C$$

[Since $\int \frac{1}{\sqrt{x^2 + a^2}} dx = \log |x + \sqrt{x^2 + a^2}| + C$]

$$I = \frac{1}{2} \log \left| x^2 + \sqrt{(x^2)^2 + (a^2)^2} \right| + C$$

$$I = \frac{1}{2} \log \left| x^2 + \sqrt{x^4 + a^4} \right| + C$$

Indefinite Integrals Ex 19.13 Q2

Let $\tan x = t$

$$\Rightarrow \sec^2 x dx = dt$$

$$\begin{aligned} \Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx &= \int \frac{dt}{\sqrt{t^2 + 2^2}} \\ &= \log \left| t + \sqrt{t^2 + 4} \right| + C \\ &= \log \left| \tan x + \sqrt{\tan^2 x + 4} \right| + C \end{aligned}$$

Indefinite Integrals Ex 19.13 Q3

$$\text{Let } I = \int \frac{e^x}{\sqrt{16 - e^{2x}}} dx$$

Let $e^x = t$

$$\Rightarrow e^x dx = dt$$

$$I = \int \frac{dt}{\sqrt{(4)^2 - t^2}}$$

$$= \sin^{-1} \left(\frac{t}{4} \right) + C$$

[Since $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + C$]

$$I = \sin^{-1} \left(\frac{e^x}{4} \right) + C$$

Indefinite Integrals Ex 19.13 Q4

$$\text{Let } I = \int \frac{\cos x}{\sqrt{4 + \sin^2 x}} dx$$

Let $\sin x = t$

$$\Rightarrow \cos x dx = dt$$

$$I = \int \frac{dt}{\sqrt{(2)^2 + t^2}}$$

$$= \log \left| t + \sqrt{(2)^2 + t^2} \right| + C$$

[Since $\int \frac{1}{\sqrt{a^2 + x^2}} dx = \log |x + \sqrt{a^2 + x^2}| + C$]

$$I = \log \left| \sin x + \sqrt{4 + \sin^2 x} \right| + C$$

Indefinite Integrals Ex 19.13 Q5

Let $I = \int \frac{\sin x}{\sqrt{4 \cos^2 x - 1}} dx$

Let $2 \cos x = t$
 $\Rightarrow -2 \sin x dx = dt$
 $\Rightarrow \sin x dx = -\frac{dt}{2}$
 $I = -\frac{1}{2} \int \frac{dt}{\sqrt{t^2 - 1}}$
 $= -\frac{1}{2} \log|t + \sqrt{t^2 - 1}| + C$ [Since $\int \frac{1}{\sqrt{t^2 - a^2}} dt = \log|t + \sqrt{t^2 - a^2}| + C$]

$$I = -\frac{1}{2} \log|2 \cos x + \sqrt{4 \cos^2 x - 1}| + C$$

Indefinite Integrals Ex 19.13 Q6

Let $I = \int \frac{x}{\sqrt{4 - x^4}} dx$

Let $x^2 = t$
 $\Rightarrow 2x dx = dt$
 $\Rightarrow x dx = \frac{dt}{2}$
 $I = \frac{1}{2} \int \frac{dt}{\sqrt{(2)^2 - t^2}}$
 $= \frac{1}{2} \sin^{-1}\left(\frac{t}{2}\right) + C$ [Since $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$]

$$I = \frac{1}{2} \sin^{-1}\left(\frac{x^2}{2}\right) + C$$

Indefinite Integrals Ex 19.13 Q7

Let $I = \int \frac{1}{x \sqrt{4 - 9(\log x)^2}} dx$

Let $3 \log x = t$
 $\Rightarrow \frac{3}{x} dx = dt$
 $\Rightarrow \frac{1}{x} dx = \frac{dt}{3}$
 $I = \frac{1}{3} \int \frac{dt}{\sqrt{(2)^2 - t^2}}$
 $= \frac{1}{3} \sin^{-1}\left(\frac{t}{2}\right) + C$ [Since $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$]

$$I = \frac{1}{3} \sin^{-1}\left(\frac{3 \log x}{2}\right) + C$$

Indefinite Integrals Ex 19.13 Q8

Let $I = \int \frac{\sin 8x}{\sqrt{9 + (\sin 4x)^4}} dx$

Let $\sin^2 4x = t$
 $\Rightarrow 2 \sin 4x \cos 4x (4) dx = dt$
 $\Rightarrow 4 \sin 8x dx = dt$
 $\Rightarrow \sin 8x dx = \frac{dt}{4}$
 $I = \frac{1}{4} \int \frac{dt}{\sqrt{(3)^2 + t^2}}$
 $= \frac{1}{4} \log|t + \sqrt{(3)^2 + t^2}| + C$ [Since $\int \frac{1}{\sqrt{a^2 + x^2}} dx = \log|x + \sqrt{a^2 + x^2}| + C$]

$$I = \frac{1}{4} \log|\sin^2 4x + \sqrt{9 + \sin^4 4x}| + C$$

Indefinite Integrals Ex 19.13 Q9

$$\begin{aligned}
& \text{Let } I = \int \frac{\cos 2x}{\sqrt{\sin^2 2x + 8}} dx \\
& \text{Let } \sin 2x = t \\
& \Rightarrow 2 \cos 2x dx = dt \\
& \Rightarrow \cos 2x dx = \frac{dt}{2} \\
& I = \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + (2\sqrt{2})^2}} \\
& = \frac{1}{2} \log \left| t + \sqrt{t^2 + (2\sqrt{2})^2} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log \left| x + \sqrt{x^2 + a^2} \right| + c \right]
\end{aligned}$$

$$I = \frac{1}{2} \log \left| \sin 2x + \sqrt{\sin^2 2x + 8} \right| + c$$

Indefinite Integrals Ex 19.13 Q10

$$\begin{aligned}
& \text{Let } I = \int \frac{\sin 2x}{\sqrt{\sin^4 x + 4 \sin^2 x - 2}} dx \\
& \text{Let } \sin^2 x = t \\
& \Rightarrow 2 \sin x \cos x dx = dt \\
& \Rightarrow \sin 2x dx = dt \\
& I = \int \frac{dt}{\sqrt{t^2 + 4t - 2}} \\
& = \int \frac{dt}{\sqrt{t^2 + 2t(2) + (2)^2 - (2)^2 - 2}} \\
& = \int \frac{dt}{\sqrt{(t+2)^2 - 6}} \\
& \text{Let } t+2 = u \\
& dt = du \\
& = \int \frac{du}{\sqrt{u^2 - (\sqrt{6})^2}} \\
& = \log \left| u + \sqrt{u^2 - 6} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| + c \right] \\
& = \log \left| t+2 + \sqrt{(t+2)^2 - 6} \right| + c
\end{aligned}$$

$$I = \log \left| \sin^2 x + 2 + \sqrt{\sin^4 x + 4 \sin^2 x - 2} \right| + c$$

Indefinite Integrals Ex 19.13 Q11

$$\begin{aligned}
& \int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx = \\
& \text{let } t = \cos^2 x \rightarrow -dt = 2 \cos x \sin x dx \\
& \int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx = \int \frac{-1}{\sqrt{t^2 - (1-t) + 2}} dt \\
& = \int \frac{-1}{\sqrt{t^2 + t + 1}} dt = \int \frac{-1}{\sqrt{t^2 + t + \frac{1}{4} + \frac{3}{4}}} dt \\
& = \int \frac{-1}{\sqrt{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}}} dt = -\log \left| \left(t + \frac{1}{2}\right) + \sqrt{t^2 + t + 1} \right| \\
& = -\log \left| \cos^2 x + \frac{1}{2} + \sqrt{\cos^4 x + \cos^2 x + 1} \right| + C
\end{aligned}$$

Indefinite Integrals Ex 19.13 Q12

$$\begin{aligned}
 \text{Let } I &= \int \frac{\cos x}{\sqrt{4 - \sin^2 x}} dx \\
 \text{Let } \sin x &= t \\
 \Rightarrow \cos x dx &= dt \\
 &= \int \frac{dt}{\sqrt{(2)^2 - t^2}} \\
 &= \sin^{-1} \left(\frac{t}{2} \right) + C \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + C \right]
 \end{aligned}$$

$$I = \sin^{-1} \left(\frac{\sin x}{2} \right) + C$$

Indefinite Integrals Ex 19.13 Q13

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{x^{\frac{2}{3}} \sqrt{x^{\frac{2}{3}} - 4}} dx \\
 \text{Let } x^{\frac{1}{3}} &= t \\
 \Rightarrow \frac{1}{3} x^{-\frac{2}{3}} dx &= dt \\
 \Rightarrow \frac{1}{3} x^{-\frac{2}{3}} dx &= dt \\
 \Rightarrow \frac{dx}{x^{\frac{2}{3}}} &= 3dt \\
 I &= 3 \int \frac{dt}{\sqrt{t^2 - (2)^2}} \\
 &= 3 \log \left| t + \sqrt{t^2 - 4} \right| + C \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| + C \right]
 \end{aligned}$$

$$I = 3 \log \left| x^{\frac{1}{3}} + \sqrt{x^{\frac{2}{3}} - 4} \right| + C$$

Indefinite Integrals Ex 19.13 Q14

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{\sqrt{(1-x^2)} \left[9 + (\sin^{-1} x)^2 \right]} dx \\
 \text{Let } \sin^{-1} x &= t \\
 \Rightarrow \frac{1}{\sqrt{1-x^2}} dx &= dt \\
 I &= \int \frac{dt}{\sqrt{(3)^2 + t^2}} \\
 &= \log \left| t + \sqrt{9+t^2} \right| + C \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2+x^2}} dx = \log \left| x + \sqrt{a^2+x^2} \right| + C \right]
 \end{aligned}$$

$$I = \log \left| \sin^{-1} x + \sqrt{9 + (\sin^{-1} x)^2} \right| + C$$

Indefinite Integrals Ex 19.13 Q15

$$\begin{aligned} \text{Let } I &= \int \frac{\cos x}{\sqrt{\sin^2 x - 2 \sin x - 3}} dx \\ \text{Let } \sin x &= t \\ \Rightarrow \cos x dx &= dt \\ &= \int \frac{dt}{\sqrt{t^2 - 2t - 3}} \\ &= \int \frac{dt}{\sqrt{t^2 - 2t + 1^2 - 1^2 - 3}} \\ &= \int \frac{dt}{\sqrt{(t-1)^2 - 4}} \end{aligned}$$

Let $t-1 = u$

$$\begin{aligned} \Rightarrow dt &= du \\ I &= \int \frac{du}{\sqrt{u^2 - 4}} \\ &= \log|u + \sqrt{u^2 - 4}| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log|x + \sqrt{x^2 - a^2}| + c \right] \\ &= \log|t-1 + \sqrt{(t-1)^2 + 4}| + c \end{aligned}$$

$$I = \log|\sin x - 1 + \sqrt{\sin^2 x - 2 \sin x - 3}| + c$$

Indefinite Integrals Ex 19.13 Q16

$$\begin{aligned} \text{Let } I &= \int \sqrt{\cosec x - 1} dx \\ &= \int \sqrt{\frac{1 - \sin x}{\sin x}} dx \\ &= \int \sqrt{\frac{(1 - \sin x) + (1 + \sin x)}{\sin x(1 + \sin x)}} dx \\ &= \int \sqrt{\frac{\cos^2 x}{\sin^2 x + \sin x}} dx \\ &= \int \frac{\cos x}{\sqrt{\sin^2 x + \sin x}} dx \end{aligned}$$

Let $\sin x = t$

$$\begin{aligned} \Rightarrow \cos x dx &= dt \\ &= \int \frac{dt}{\sqrt{t^2 + t}} \\ &= \int \frac{dt}{\sqrt{t^2 + 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} \\ &= \int \frac{dt}{\sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} \end{aligned}$$

Let, $t + \frac{1}{2} = u$

$$\begin{aligned} \Rightarrow dt &= du \\ &= \int \frac{du}{\sqrt{u^2 - \left(\frac{1}{2}\right)^2}} \\ &= \log|u + \sqrt{u^2 - \left(\frac{1}{2}\right)^2}| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log|x + \sqrt{x^2 - a^2}| + c \right] \\ &= \log\left(t + \frac{1}{2} + \sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\right) + c \end{aligned}$$

$$I = \log\left|\sin x + \frac{1}{2} + \sqrt{\sin^2 x + \sin x}\right| + c$$

Indefinite Integrals Ex 19.13 Q17

To evaluate the following integral follow the steps:

$$\int \frac{\sin x - \cos x}{\sqrt{\sin 2x}} dx = \int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}} dx$$

Let $\sin x + \cos x = t$ therefore $(\cos x - \sin x)dx = dt$

Now

$$\begin{aligned}\int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}} dx &= - \int \frac{dt}{\sqrt{t^2 - 1}} \\ &= -\ln|t + \sqrt{t^2 - 1}| + c \\ &= -\ln|\sin x + \cos x + \sqrt{\sin 2x}| + c\end{aligned}$$

Ex 19.14

Indefinite Integrals Ex 19.14 Q1

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{a^2 - b^2 x^2} dx \\
 &= \frac{1}{b^2} \int \frac{1}{\frac{a^2}{b^2} - x^2} dx \\
 &= \frac{1}{b^2} \int \frac{1}{\left(\frac{a}{b}\right)^2 - x^2} dx \\
 I &= \frac{1}{b^2} \times \frac{1}{2 \times \left(\frac{a}{b}\right)} \log \left| \frac{\frac{a}{b} + x}{\frac{a}{b} - x} \right| + c \quad \left[\text{Since } \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{x+a}{x-a} \right| + c \right] \\
 I &= \frac{1}{2ab} \log \left| \frac{a+bx}{a-bx} \right| + c
 \end{aligned}$$

Indefinite Integrals Ex 19.14 Q2

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{a^2 x^2 - b^2} dx \\
 &= \frac{1}{a^2} \int \frac{1}{x^2 - \frac{b^2}{a^2}} dx \\
 &= \frac{1}{a^2} \int \frac{1}{x^2 - \left(\frac{b}{a}\right)^2} dx \\
 I &= \frac{1}{a^2} \times \frac{1}{2 \times \left(\frac{b}{a}\right)} \times \log \left| \frac{x - \frac{b}{a}}{x + \frac{b}{a}} \right| + c \quad \left[\text{Since } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right] \\
 I &= \frac{1}{2ab} \log \left| \frac{ax - b}{ax + b} \right| + c
 \end{aligned}$$

Indefinite Integrals Ex 19.14 Q3

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{a^2 x^2 + b^2} dx \\
 &= \frac{1}{a^2} \int \frac{1}{x^2 + \frac{b^2}{a^2}} dx \\
 &= \frac{1}{a^2} \int \frac{1}{x^2 + \left(\frac{b}{a}\right)^2} dx \\
 I &= \frac{1}{a^2} \times \frac{1}{\left(\frac{b}{a}\right)} \tan^{-1} \left(\frac{x}{\frac{b}{a}} \right) + c \quad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right] \\
 I &= \frac{1}{ab} \tan^{-1} \left(\frac{ax}{b} \right) + c
 \end{aligned}$$

Indefinite Integrals Ex 19.14 Q4

Let $I = \int \frac{x^2 - 1}{x^2 - 4} dx$

Let $I = \int \frac{x^2 - 1}{x^2 + 4} dx$

$$= \int \frac{(x^2 + 4) - 4 - 1}{x^2 + 4} dx$$

$$= \int \frac{x^2 + 4}{x^2 + 4} dx - \int \frac{5}{x^2 + 4} dx$$

$$= \int dx - 5 \int \frac{1}{x^2 + (2)^2} dx$$

$$I = x - 5 \times \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C \quad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) \right]$$

$$I = x - \frac{5}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$$

Indefinite Integrals Ex 19.14 Q5

Let $2x = t$

$\Rightarrow 2dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{1+4x^2}} dx = \frac{1}{2} \int \frac{dt}{\sqrt{1+t^2}}$$

$$= \frac{1}{2} \left[\log|t + \sqrt{t^2 + 1}| \right] + C \quad \left[\int \frac{1}{\sqrt{x^2 + a^2}} dx = \log|x + \sqrt{x^2 + a^2}| \right]$$

$$= \frac{1}{2} \log|2x + \sqrt{4x^2 + 1}| + C$$

Indefinite Integrals Ex 19.14 Q6

Let $I = \int \frac{1}{\sqrt{a^2 + b^2x^2}} dx$

Let $bx = t$

$\Rightarrow \quad bdx = dt$

$dx = \frac{dt}{b}$

$I = \frac{1}{b} \int \frac{1}{\sqrt{a^2 + t^2}} dt$

$$I = \frac{1}{b} \log|t + \sqrt{a^2 + t^2}| + C \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log|x + \sqrt{a^2 + x^2}| + C \right]$$

$$I = \frac{1}{b} \log|bx + \sqrt{a^2 + b^2x^2}| + C \quad [\text{since } t = bx]$$

Indefinite Integrals Ex 19.14 Q7

Let $I = \int \frac{1}{\sqrt{a^2 - b^2x^2}} dx$

Let $bx = t$

$\Rightarrow \quad bdx = dt$

$dx = \frac{dt}{b}$

so, $I = \frac{1}{b} \int \frac{1}{\sqrt{a^2 - t^2}} dt$

$$I = \frac{1}{b} \sin^{-1}\left(\frac{t}{a}\right) + C \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C \right]$$

$$I = \frac{1}{b} \sin^{-1}\left(\frac{bx}{a}\right) + C \quad [\text{since } bx = t]$$

Indefinite Integrals Ex 19.14 Q8

$$\text{Let } I = \int \frac{1}{\sqrt{(2-x)^2 + 1}} dx$$

$$\text{Let } 2-x = t$$

$$\Rightarrow -dx = dt$$

$$dx = -dt$$

$$\text{so, } I = - \int \frac{1}{\sqrt{t^2 + 1}} dt$$

$$I = -\log|t + \sqrt{t^2 + 1}| + c$$

$$\left[\text{since } \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log|x + \sqrt{x^2 + a^2}| + c \right]$$

$$I = -\log|(2-x) + \sqrt{(2-x)^2 + 1}| + c \quad [\text{since } t = (2-x)]$$

Indefinite Integrals Ex 19.14 Q9

$$\text{Let } I = \int \frac{1}{\sqrt{(2-x)^2 - 1}} dx$$

$$\text{Let } 2-x = t$$

$$\Rightarrow -dx = dt$$

$$dx = -dt$$

$$\text{so, } I = - \int \frac{1}{\sqrt{t^2 - 1}} dt$$

$$I = -\log|t + \sqrt{t^2 - 1}| + c$$

$$\left[\text{since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log|x + \sqrt{x^2 - a^2}| + c \right]$$

$$I = -\log|(2-x) + \sqrt{(2-x)^2 - 1}| + c \quad [\text{since } t = (2-x)]$$

Indefinite Integrals Ex 19.14 Q10

$$\text{Let } I = \int \frac{x^4 + 1}{x^2 + 1} dx$$

$$I = \int \frac{(x^2 + 1)^2 - 2x^2}{x^2 + 1} dx \quad [a^2 + b^2 = (a+b)^2 - 2ab]$$

$$I = \int \frac{(x^2 + 1)^2}{x^2 + 1} dx - \int \frac{2x^2}{x^2 + 1} dx$$

$$I = \int (x^2 + 1) dx - \int \frac{2x^2 + 2 - 2}{x^2 + 1} dx$$

$$I = \int (x^2 + 1) dx - \int \frac{2(x^2 + 1)}{x^2 + 1} dx + 2 \int \frac{1}{x^2 + 1} dx$$

$$I = \int (x^2 + 1) dx - \int 2 dx + 2 \int \frac{1}{x^2 + 1} dx$$

$$I = \frac{x^3}{3} + x - 2x + 2 \times \tan^{-1}(x) + c \quad \left[\text{since } \int \frac{1}{x^2 + 1} dx = \tan^{-1}(x) + c \right]$$

$$I = \frac{x^3}{3} - x + 2 \tan^{-1}(x) + c$$

Ex 19.15

Indefinite Integrals Ex 19.15 Q1

$$\text{Let } I = \int \frac{1}{4x^2 + 12x + 5} dx$$

$$\begin{aligned} &= \frac{1}{4} \int \frac{1}{x^2 + 3x + \frac{5}{4}} dx \\ &= \frac{1}{4} \int \frac{1}{x^2 + 2 \times x \times \left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{5}{4}} dx \\ I &= \frac{1}{4} \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - 1} dx \end{aligned}$$

$$\begin{aligned} \text{Let } \left(x + \frac{3}{2}\right) &= t \quad \dots \text{(i)} \\ \Rightarrow \quad dx &= dt \\ \text{so,} \quad & \end{aligned}$$

$$\begin{aligned} I &= \frac{1}{4} \int \frac{1}{t^2 - 1} dt \\ I &= \frac{1}{4} \times \frac{1}{2 \times \{1\}} \log |t - 1| + c \quad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right] \\ I &= \frac{1}{8} \log \left| \frac{x + \frac{3}{2} - 1}{x + \frac{3}{2} + 1} \right| + c \quad [\text{using (i)}] \\ I &= \frac{1}{8} \log \left| \frac{2x + 1}{2x + 5} \right| + c \end{aligned}$$

Indefinite Integrals Ex 19.15 Q2

$$\text{Let } I = \int \frac{1}{x^2 - 10x + 34} dx$$

$$\begin{aligned} &= \int \frac{1}{x^2 - 2x \times 5 + 5^2 - 5^2 + 34} dx \\ &= \int \frac{1}{(x - 5)^2 + 9} dx \end{aligned}$$

$$\begin{aligned} \text{Let } (x-5) &= t \quad \dots \text{(i)} \\ \Rightarrow \quad dx &= dt \\ \text{so,} \quad & \end{aligned}$$

$$\begin{aligned} I &= \int \frac{1}{t^2 + 3^2} dt \\ I &= \frac{1}{3} \tan^{-1} \left(\frac{t}{3} \right) + c \quad \left[\text{Since, } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right] \\ I &= \frac{1}{3} \tan^{-1} \left(\frac{x-5}{3} \right) + c \quad [\text{using (i)}] \end{aligned}$$

Indefinite Integrals Ex 19.15 Q3

$$\begin{aligned}
\int \frac{1}{1+x-x^2} dx &= \int \frac{1}{-(x^2-x-1)} dx \\
&\text{adding and subtracting } \frac{1}{4} \text{ in the denominator to make it a perfect square} \\
&= \int \frac{1}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}\right)} dx \\
&= \int \frac{1}{-\left[\left(x-\frac{1}{2}\right)^2-1-\frac{1}{4}\right]} dx \\
&= \int \frac{1}{-\left[\left(x-\frac{1}{2}\right)^2-1-\frac{1}{4}\right]} dx = \int \frac{1}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{5}{4}\right]} dx \\
&= \int \frac{1}{\left(\frac{\sqrt{5}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2} dx \\
&= \frac{1}{2\left(\frac{\sqrt{5}}{2}\right)} \log \left| \frac{\frac{\sqrt{5}}{2} + \left(x-\frac{1}{2}\right)}{\frac{\sqrt{5}}{2} - \left(x-\frac{1}{2}\right)} \right| \\
&= \frac{1}{\sqrt{5}} \log \left| \frac{\sqrt{5}+2x-1}{\sqrt{5}-2x+1} \right| \\
&= \frac{1}{\sqrt{5}} \log \left| \frac{\sqrt{5}-1+2x}{\sqrt{5}+1-2x} \right|
\end{aligned}$$

Indefinite Integrals Ex 19.15 Q4

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{2x^2 - x - 1} dx \\
&= \frac{1}{2} \int \frac{1}{x^2 - \frac{x}{2} - \frac{1}{2}} dx \\
&= \frac{1}{2} \int \frac{1}{x^2 - 2x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - \frac{1}{2}} dx \\
&= \frac{1}{2} \int \frac{1}{\left(x - \frac{1}{4}\right)^2 - \frac{9}{16}} dx
\end{aligned}$$

$$\begin{aligned}
\text{Let } x - \frac{1}{4} &= t \\
\Rightarrow \quad dx &= dt
\end{aligned}$$

$$\begin{aligned}
I &= \frac{1}{2} \int \frac{1}{t^2 - \left(\frac{3}{4}\right)^2} dt \\
I &= \frac{1}{2} \times \frac{1}{2 \times \left(\frac{3}{4}\right)} \log \left| \frac{t - \frac{3}{4}}{t + \frac{3}{4}} \right| + C \quad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C \right] \\
I &= \frac{1}{3} \log \left| \frac{x - \frac{1}{4} - \frac{3}{4}}{x - \frac{1}{4} + \frac{3}{4}} \right| + C
\end{aligned}$$

$$I = \frac{1}{3} \log \left| \frac{x-1}{2x+1} \right| + C$$

Indefinite Integrals Ex 19.15 Q5

We have $x^2 + 6x + 13 = x^2 + 6x + 3^2 - 3^2 + 13 = (x + 3)^2 + 4$

Sol, $\int \frac{dx}{x^2 + 6x + 13} = \int \frac{1}{(x + 3)^2 + 2^2} dx$

Let $x + 3 = t$. Then $dx = dt$

Therefore, $\int \frac{dx}{x^2 + 6x + 13} = \int \frac{dt}{t^2 + 2^2} = \frac{1}{2} \tan^{-1} \frac{t}{2} + C$ [by 7.4 (3)]
 $= \frac{1}{2} \tan^{-1} \frac{x+3}{2} + C$

Ex 19.16

Indefinite Integrals Ex 19.16 Q1

$$\text{Let } I = \int \frac{\sec^2 x}{1 - \tan^2 x} dx$$

Let $\tan x = t$

$$\Rightarrow \sec^2 x dx = dt$$

$$I = \int \frac{dt}{(1)^2 - t^2}$$

$$= \frac{1}{2(1)} \log \left| \frac{1+t}{1-t} \right| + c \quad \left[\text{Since, } \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c \right]$$

$$I = \frac{1}{2} \log \left| \frac{1 + \tan x}{1 - \tan x} \right| + c$$

Indefinite Integrals Ex 19.16 Q2

$$\text{Let } I = \int \frac{e^x}{1 + e^{2x}} dx$$

Let $\tan e^x = t$

$$\Rightarrow e^x dx = dt$$

$$\text{so, } I = \int \frac{dt}{1 + t^2}$$

$$= \tan^{-1}(t) + c \quad \left[\text{Since, } \int \frac{1}{1+x^2} dx = \tan^{-1} x + c \right]$$

$$I = \tan^{-1}(e^x) + c$$

Indefinite Integrals Ex 19.16 Q3

$$\text{Let } I = \int \frac{\cos x}{\sin^2 x + 4 \sin x + 5} dx$$

Let $\sin x = t$

$$\Rightarrow \cos x dx = dt$$

$$\text{so, } I = \int \frac{dx}{t^2 + 4t + 5}$$

$$= \int \frac{dt}{t^2 + 2t(2) + (2)^2 - (2)^2 + 5}$$

$$= \int \frac{dt}{(t+2)^2 + 1}$$

Again, Let $(t+2) = u$

$$dt = du$$

$$I = \int \frac{du}{u^2 + 1}$$

$$= \tan^{-1}(u) + c \quad \left[\text{Since, } \int \frac{1}{x^2 + 1} dx = \tan^{-1} x + c \right]$$

$$I = \tan^{-1}(t+2) + c$$

$$I = \tan^{-1}(\sin x + 2) + c$$

Indefinite Integrals Ex 19.16 Q4

$$\text{Let } I = \int \frac{e^x}{e^{2x} + 5e^x + 6} dx$$

Let $e^x = t$

$$\Rightarrow e^x dx = dt$$

so, $I = \int \frac{dt}{t^2 + 5t + 6}$

$$= \int \frac{dt}{t^2 + 2t\left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 6}$$

$$= \int \frac{dt}{\left(t + \frac{5}{2}\right)^2 - \frac{1}{4}}$$

Put $\left(t + \frac{5}{2}\right) = u$

$$\Rightarrow dt = du$$

$$I = \int \frac{du}{u^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{2\left(\frac{1}{2}\right)} \log \left| \frac{u - \frac{1}{2}}{u + \frac{1}{2}} \right| + c \quad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \left| \frac{x-a}{x+a} \right| + c \right]$$

$$I = \log \left| \frac{2u - 1}{2u + 1} \right| + c$$

$$I = \log \left| \frac{2\left(t + \frac{5}{2}\right) - 1}{2\left(t + \frac{5}{2}\right) + 1} \right| + c$$

$$I = \log \left| \frac{e^x + 2}{e^x + 3} \right| + c$$

Indefinite Integrals Ex 19.16 Q5

$$\text{Let } I = \int \frac{e^{3x}}{4e^{6x} - 9} dx$$

Let $e^{3x} = t$

$$\Rightarrow 3e^{3x} dx = dt$$

$$\Rightarrow e^{3x} dx = \frac{dt}{3}$$

$$I = \frac{1}{3} \int \frac{dt}{4t^2 - 9}$$

$$= \frac{1}{12} \int \frac{dt}{t^2 - \frac{9}{4}}$$

$$= \frac{1}{12} \int \frac{dt}{t^2 - \left(\frac{3}{2}\right)^2}$$

$$= \frac{1}{12} \times \frac{1}{2\left(\frac{3}{2}\right)} \log \left| \frac{t - \frac{3}{2}}{t + \frac{3}{2}} \right| + c \quad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \left| \frac{x-a}{x+a} \right| + c \right]$$

$$I = \frac{1}{36} \log \left| \frac{2t - 3}{2t + 3} \right| + c$$

$$I = \frac{1}{36} \log \left| \frac{2e^{3x} - 3}{2e^{3x} + 3} \right| + c$$

Indefinite Integrals Ex 19.16 Q6

$$\begin{aligned} \text{Let } I &= \int \frac{dx}{e^x + e^{-x}} \\ &= \int \frac{dx}{e^x + \frac{1}{e^x}} \\ &= \int \frac{e^x dx}{(e^x)^2 + 1} \end{aligned}$$

$$\begin{aligned} \text{Let } e^x &= t \\ \Rightarrow e^x dx &= dt \\ I &= \int \frac{dt}{t^2 + 1} \\ I &= \tan^{-1} t + c \quad \left[\text{Since } \int \frac{1}{1+x^2} dx = \tan^{-1} x + c \right] \\ I &= \tan^{-1}(e^x) + c \end{aligned}$$

Indefinite Integrals Ex 19.16 Q7

$$\begin{aligned} \text{Let } I &= \int \frac{x}{x^4 + 2x^2 + 3} dx \\ \text{Let } x^2 &= t \\ \Rightarrow 2x dx &= dt \\ \Rightarrow x dx &= \frac{dt}{2} \\ I &= \frac{1}{2} \int \frac{dt}{t^2 + 2t + 3} \\ &= \frac{1}{2} \int \frac{dt}{t^2 + 2t + 1 - 1 + 3} \\ &= \frac{1}{2} \int \frac{dt}{(t+1)^2 + 2} \\ \text{put } t+1 &= u \\ \Rightarrow dt &= du \\ I &= \frac{1}{2} \int \frac{du}{u^2 + (\sqrt{2})^2} \\ &= \frac{1}{2} \times \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + c \quad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c \right] \\ I &= \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{t+1}{\sqrt{2}}\right) + c \\ I &= \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x^2 + 1}{\sqrt{2}}\right) + c \end{aligned}$$

Indefinite Integrals Ex 19.16 Q8

$$\begin{aligned} \text{Let } I &= \int \frac{3x^5}{1+x^{12}} dx \\ &= \int \frac{3x^5}{1+(x^6)^2} dx \\ \text{Let } x^6 &= t \\ \Rightarrow 6x^5 dx &= dt \\ \Rightarrow x^5 dx &= \frac{dt}{6} \\ I &= \frac{3}{6} \int \frac{dt}{1+t^2} \\ &= \frac{1}{2} \tan^{-1}(t) + c \quad \left[\text{Since } \int \frac{1}{x^2 + 1} dx = \tan^{-1} x + c \right] \\ I &= \frac{1}{2} \tan^{-1}(x^6) + c \end{aligned}$$

Indefinite Integrals Ex 19.16 Q9

$$\begin{aligned}
 \text{Let } I &= \int \frac{x^2}{x^6 - a^6} dx \\
 &= \int \frac{x^2}{(x^3)^2 - (a^3)^2} dx \\
 \text{Let } x^3 &= t \\
 \Rightarrow 3x^2 dx &= dt \\
 \Rightarrow x^2 dx &= \frac{dt}{3} \\
 \text{so, } I &= \frac{1}{3} \int \frac{dt}{t^2 - (a^3)^2} \\
 &= \frac{1}{3} \times \frac{1}{2a^3} \log \left| \frac{t - a^3}{t + a^3} \right| + c \quad \left[\text{Since } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c \right]
 \end{aligned}$$

$$I = \frac{1}{6a^3} \log \left| \frac{x^3 - a^3}{x^3 + a^3} \right| + c$$

Indefinite Integrals Ex 19.16 Q10

$$\begin{aligned}
 \text{Let } I &= \int \frac{x^2}{x^6 + (a^3)^2} dx \\
 &= \int \frac{x^2}{(x^3)^2 + (a^3)^2} dx \\
 \text{Let } x^3 &= t \\
 \Rightarrow 3x^2 dx &= dt \\
 \Rightarrow x^2 dx &= \frac{dt}{3} \\
 \text{so, } I &= \frac{1}{3} \int \frac{dt}{t^2 + (a^3)^2} \\
 &= \frac{1}{3} \times \frac{1}{a^3} \tan^{-1} \left(\frac{t}{a^3} \right) + c \quad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right]
 \end{aligned}$$

$$I = \frac{1}{3a^3} \tan^{-1} \left(\frac{x^3}{a^3} \right) + c$$

Indefinite Integrals Ex 19.16 Q11

$$\begin{aligned} \text{Let } I &= \int \frac{1}{x(x^6+1)} dx \\ &= \int \frac{x^5}{x^6(x^6+1)} dx \end{aligned}$$

Let $x^6 = t$

$$\Rightarrow 6x^5 dx = dt$$

$$\Rightarrow x^5 dx = \frac{dt}{6}$$

$$I = \frac{1}{6} \int \frac{dt}{t(t+1)}$$

$$= \frac{1}{6} \int \frac{dt}{t^2+t}$$

$$= \frac{1}{6} \int \frac{dt}{t^2 + 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{6} \int \frac{dt}{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$\text{Let } t + \frac{1}{2} = u$$

$$\Rightarrow dt = du$$

$$I = \frac{1}{6} \int \frac{du}{u^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{6} \times \frac{1}{2\left(\frac{1}{2}\right)} \log \left| \frac{u - \frac{1}{2}}{u + \frac{1}{2}} \right| + C \quad \left[\text{Since } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C \right]$$

$$I = \frac{1}{6} \log \left| \frac{t + \frac{1}{2} - \frac{1}{2}}{t + \frac{1}{2} + \frac{1}{2}} \right| + C$$

$$I = \frac{1}{6} \log \left| \frac{x^6}{x^6 + 1} \right| + C$$

Indefinite Integrals Ex 19.16 Q12

$$\text{Let } I = \int \frac{x}{x^4 - x^2 + 1} dx$$

$$\text{Let } x^2 = t$$

$$\Rightarrow 2x dx = dt$$

$$\Rightarrow x dx = \frac{dt}{2}$$

$$\text{so, } I = \frac{1}{2} \int \frac{dt}{t^2 - t + 1}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 - 2t \times \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$$

$$= \frac{1}{2} \int \frac{dt}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)}$$

$$\text{Let } t - \frac{1}{2} = u$$

$$\Rightarrow dt = du$$

$$I = \frac{1}{2} \int \frac{du}{u^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{2} \times \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1} \left(\frac{u}{\frac{\sqrt{3}}{2}} \right) + C \quad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C \right]$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{t - \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) + C$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x^2 - 1}{\sqrt{3}} \right) + C$$

Indefinite Integrals Ex 19.16 Q13

$$\text{Let } I = \int \frac{x}{3x^4 - 18x^2 + 11} dx$$

$$= \frac{1}{3} \int \frac{x}{x^4 - 6x^2 + \frac{11}{3}} dx$$

Let $x^2 = t$

$$\Rightarrow 2x dx = dt$$

$$\Rightarrow x dx = \frac{dt}{2}$$

$$I = \frac{1}{3} \times \frac{1}{2} \int \frac{dt}{t^2 - 6t + \frac{11}{3}}$$

$$= \frac{1}{6} \int \frac{dt}{t^2 - 2t(3) + (3)^2 - (3)^2 + \frac{11}{3}}$$

$$= \frac{1}{6} \int \frac{dt}{(t-3)^2 - \left(\frac{16}{3}\right)}$$

Let $t-3 = u$

$$\Rightarrow dt = du$$

$$I = \frac{1}{6} \int \frac{du}{u^2 - \left(\frac{4}{\sqrt{3}}\right)^2}$$

$$= \frac{1}{6} \times \frac{1}{2 \left(\frac{4}{\sqrt{3}}\right)} \log \left| \frac{u - \frac{4}{\sqrt{3}}}{u + \frac{4}{\sqrt{3}}} \right| + c \quad \left[\text{Since } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right]$$

$$I = \frac{\sqrt{3}}{48} \log \left| \frac{t-3 - \frac{4}{\sqrt{3}}}{t-3 + \frac{4}{\sqrt{3}}} \right| + c$$

$$I = \frac{\sqrt{3}}{48} \log \left| \frac{x^2 - 3 - \frac{4}{\sqrt{3}}}{x^2 - 3 + \frac{4}{\sqrt{3}}} \right| + c$$

Indefinite Integrals Ex 19.16 Q14

To evaluate the following integral follow the steps:

Let $e^x = t$ therefore $e^x dx = dt$

Now

$$\begin{aligned} \int \frac{e^x}{(1+e^x)(2+e^x)} dx &= \int \frac{dt}{(1+t)(2+t)} \\ &= \int \frac{dt}{(1+t)} - \int \frac{dt}{(2+t)} \\ &= \ln|1+t| - \ln|2+t| + c \\ &= \ln \left| \frac{1+t}{2+t} \right| + c \\ &= \ln \left| \frac{1+e^x}{2+e^x} \right| + c \end{aligned}$$

Ex 19.17

Indefinite Integrals Ex 19.17 Q1

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{\sqrt{2x - x^2}} dx \\
 &= \int \frac{1}{\sqrt{-[x^2 - 2x]}} dx \\
 &= \int \frac{1}{\sqrt{-[x^2 - 2x(1) + 1^2 - 1^2]}} dx \\
 &= \int \frac{1}{\sqrt{-[(x-1)^2 - 1]}} dx \\
 &= \int \frac{1}{\sqrt{1 - (x-1)^2}} dx
 \end{aligned}$$

Let $(x-1) = t$
 $\Rightarrow dx = dt$
so, $I = \int \frac{1}{\sqrt{1-t^2}} dt$

$$= \sin^{-1} t + C \quad \left[\text{Since } \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C \right]$$

$$I = \sin^{-1}(x-1) + C$$

Indefinite Integrals Ex 19.17 Q2

$8+3x-x^2$ can be written as $8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$.

Therefore,

$$\begin{aligned}
 &8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right) \\
 &= \frac{41}{4}-\left(x-\frac{3}{2}\right)^2 \\
 \Rightarrow &\int \frac{1}{\sqrt{8+3x-x^2}} dx = \int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}} dx
 \end{aligned}$$

Let $x-\frac{3}{2}=t$

$\therefore dx = dt$

$$\begin{aligned}
 \Rightarrow &\int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}} dx = \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-t^2}} dt \\
 &= \sin^{-1}\left(\frac{t}{\sqrt{41}}\right) + C \\
 &= \sin^{-1}\left(\frac{x-\frac{3}{2}}{\sqrt{41}}\right) + C \\
 &= \sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right) + C
 \end{aligned}$$

Indefinite Integrals Ex 19.17 Q3

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{\sqrt{5 - 4x - 2x^2}} dx \\
&= \int \frac{1}{\sqrt{-2[x^2 + 2x - \frac{5}{2}]}} dx \\
&= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-[x^2 + 2x + (1)^2 - (1)^2 - \frac{5}{2}]}} dx \\
&= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-(x+1)^2 - \frac{7}{2}}} dx \\
&= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\frac{7}{2} - (x+1)^2}} dx
\end{aligned}$$

Let $(x+1) = t$

$$\Rightarrow dx = dt$$

$$\begin{aligned}
\text{so, } I &= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\frac{7}{2}\right)^2 - t^2}} dt \\
&= \frac{1}{\sqrt{2}} \sin^{-1} \left(\frac{t}{\frac{\sqrt{7}}{2}} \right) + C \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + C \right]
\end{aligned}$$

$$I = \frac{1}{\sqrt{2}} \sin^{-1} \left(\frac{\sqrt{2}}{\sqrt{7}} \times (x+1) \right) + C$$

Indefinite Integrals Ex 19.17 Q4

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{\sqrt{3x^2 + 5x + 7}} dx \\
&= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 + \frac{5}{3}x + \frac{7}{3}}} dx \\
&= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 + 2x \left(\frac{5}{6}\right) + \left(\frac{5}{6}\right)^2 - \left(\frac{5}{6}\right)^2 + \frac{7}{3}}} dx \\
&= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{\left(x + \frac{5}{6}\right)^2 - \frac{59}{36}}} dx
\end{aligned}$$

Let $\left(x + \frac{5}{6}\right) = t$

$$\Rightarrow dx = dt$$

$$\begin{aligned}
I &= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{t^2 - \left(\frac{\sqrt{59}}{6}\right)^2}} dt \\
&= \frac{1}{\sqrt{3}} \log \left| t + \sqrt{t^2 - \left(\frac{\sqrt{59}}{6}\right)^2} \right| + C \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log|x + \sqrt{x^2 - a^2}| + C \right] \\
I &= \frac{1}{\sqrt{3}} \log \left| x + \frac{5}{6} + \sqrt{\left(x + \frac{5}{6}\right)^2 - \left(\frac{\sqrt{59}}{6}\right)^2} \right| + C
\end{aligned}$$

$$I = \frac{1}{\sqrt{3}} \log \left| x + \frac{5}{6} + \sqrt{x^2 + \frac{5x}{3} + \frac{7}{3}} \right| + C$$

Indefinite Integrals Ex 19.17 Q5

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{\sqrt{(x-\alpha)(\beta-x)}} dx, (\text{as } \beta > \alpha) \\
&= \int \frac{1}{\sqrt{-x^2 + x(\alpha+\beta) - \alpha\beta}} dx \\
&= \int \frac{1}{\sqrt{-\left[x^2 - 2x\left(\frac{\alpha+\beta}{2}\right) + \left(\frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha+\beta}{2}\right)^2 + \alpha\beta\right]}} dx \\
&= \int \frac{1}{\sqrt{\left[\left(x - \frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha+\beta}{2}\right)^2\right]}} dx, \quad [\because \beta > \alpha]
\end{aligned}$$

$$\text{Let } \left(x - \frac{\alpha+\beta}{2}\right) = t$$

$$\Rightarrow dx = dt$$

$$\begin{aligned}
I &= \int \frac{1}{\sqrt{\left(\frac{\beta-\alpha}{2}\right)^2 - t^2}} dt \\
&= \sin^{-1} \left(\frac{t}{\frac{\beta-\alpha}{2}} \right) + C \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + C \right] \\
I &= \sin^{-1} \left(\frac{2\left(x - \frac{\alpha+\beta}{2}\right)}{\beta-\alpha} \right) + C \\
I &= \sin^{-1} \left(\frac{2x - \alpha - \beta}{\beta-\alpha} \right) + C
\end{aligned}$$

Indefinite Integrals Ex 19.17 Q6

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{\sqrt{7 - 3x - 2x^2}} dx \\
&= \int \frac{1}{\sqrt{-2[x^2 + \frac{3}{2}x - \frac{7}{2}]}} dx \\
&= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-[x^2 + 2x(\frac{3}{4}) + (\frac{3}{4})^2 - (\frac{3}{4})^2 - \frac{7}{2}]}} dx \\
&= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-[(x - \frac{3}{4})^2 - \frac{65}{16}]}} dx \\
&= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2 - (x + \frac{3}{4})^2}} dx
\end{aligned}$$

Let $\left(x + \frac{3}{4}\right) = t$

$$\Rightarrow dx = dt$$

$$\begin{aligned}
I &= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2 - t^2}} dt \\
&= \frac{1}{\sqrt{2}} \sin^{-1} \left(\frac{t}{\frac{\sqrt{41}}{4}} \right) + C \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + C \right]
\end{aligned}$$

$$I = \frac{1}{\sqrt{2}} \sin^{-1} \left(\frac{4(x + \frac{3}{4})}{\sqrt{65}} \right) + C$$

$$I = \frac{1}{\sqrt{2}} \sin^{-1} \left(\frac{4x + 3}{\sqrt{65}} \right) + C$$

Indefinite Integrals Ex 19.17 Q7

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{\sqrt{16 - 6x - x^2}} dx \\
&= \int \frac{1}{\sqrt{-[x^2 + 6x - 16]}} dx \\
&= \int \frac{1}{\sqrt{-[x^2 + 2x(3) + (3)^2 - (3)^2 - 16]}} dx \\
&= \int \frac{1}{\sqrt{-[(x + 3)^2 - 25]}} dx \\
&= \int \frac{1}{\sqrt{25 - (x + 3)^2}} dx
\end{aligned}$$

Let $(x + 3) = t$

$$\Rightarrow dx = dt$$

$$\begin{aligned}
I &= \int \frac{1}{\sqrt{5^2 - t^2}} dt \\
&= \sin^{-1} \left(\frac{t}{5} \right) + C \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + C \right]
\end{aligned}$$

$$I = \sin^{-1} \left(\frac{x + 3}{5} \right) + C$$

Indefinite Integrals Ex 19.17 Q8

$7 - 6x - x^2$ can be written as $7 - (x^2 + 6x + 9 - 9)$.

Therefore,

$$\begin{aligned} & 7 - (x^2 + 6x + 9 - 9) \\ & = 16 - (x^2 + 6x + 9) \\ & = 16 - (x+3)^2 \\ & = (4)^2 - (x+3)^2 \\ \therefore \int \frac{1}{\sqrt{7-6x-x^2}} dx &= \int \frac{1}{\sqrt{(4)^2-(x+3)^2}} dx \end{aligned}$$

Let $x+3=t$

$$\Rightarrow dx = dt$$

$$\begin{aligned} \Rightarrow \int \frac{1}{\sqrt{(4)^2-(x+3)^2}} dx &= \int \frac{1}{\sqrt{(4)^2-(t)^2}} dt \\ &= \sin^{-1}\left(\frac{t}{4}\right) + C \\ &= \sin^{-1}\left(\frac{x+3}{4}\right) + C \end{aligned}$$

Indefinite Integrals Ex 19.17 Q9

$$\begin{aligned} \text{We have } \int \frac{dx}{\sqrt{5x^2 - 2x}} &= \int \frac{dx}{\sqrt{5\left(x^2 - \frac{2x}{5}\right)}} \\ &= \frac{1}{\sqrt{5}} \int \frac{dx}{\sqrt{\left(x - \frac{1}{5}\right)^2 - \left(\frac{1}{5}\right)^2}} \quad (\text{completing the square}) \end{aligned}$$

Put $x - \frac{1}{5} = t$. Then $dx = dt$.

$$\begin{aligned} \text{Therefore, } \int \frac{dx}{\sqrt{5x^2 - 2x}} &= \frac{1}{\sqrt{5}} \int \frac{dt}{\sqrt{t^2 - \left(\frac{1}{5}\right)^2}} \\ &= \frac{1}{\sqrt{5}} \log \left| t + \sqrt{t^2 - \left(\frac{1}{5}\right)^2} \right| + C \quad [\text{by 7.4 (4)}] \\ &= \frac{1}{\sqrt{5}} \log \left| x - \frac{1}{5} + \sqrt{x^2 - \frac{2x}{5}} \right| + C \end{aligned}$$

Ex 19.18

Indefinite Integrals Ex 19.18 Q1

$$\begin{aligned}
 \text{Let } I &= \int \frac{x}{\sqrt{x^4 + a^4}} dx \\
 &= \int \frac{x}{\sqrt{(x^2)^2 + (a^2)^2}} dx \\
 \text{Let } x^2 &= t \\
 \Rightarrow 2x dx &= dt \\
 \Rightarrow x dx &= \frac{dt}{2} \\
 I &= \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + (a^2)^2}} \\
 &= \frac{1}{2} \log \left| t + \sqrt{t^2 + (a^2)^2} \right| + C \quad \left[\text{Since } \int \frac{1}{\sqrt{t^2 + a^2}} dt = \log \left| t + \sqrt{t^2 + a^2} \right| + C \right] \\
 I &= \frac{1}{2} \log \left| x^2 + \sqrt{(x^2)^2 + (a^2)^2} \right| + C
 \end{aligned}$$

$$I = \frac{1}{2} \log \left| x^2 + \sqrt{x^4 + a^4} \right| + C$$

Indefinite Integrals Ex 19.18 Q2

$$\begin{aligned}
 \text{Let } \tan x &= t \\
 \Rightarrow \sec^2 x dx &= dt \\
 \Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx &= \int \frac{dt}{\sqrt{t^2 + 2^2}} \\
 &= \log \left| t + \sqrt{t^2 + 4} \right| + C \\
 &= \log \left| \tan x + \sqrt{\tan^2 x + 4} \right| + C
 \end{aligned}$$

Indefinite Integrals Ex 19.18 Q3

$$\begin{aligned}
 \text{Let } I &= \int \frac{e^x}{\sqrt{16 - e^{2x}}} dx \\
 \text{Let } e^x &= t \\
 \Rightarrow e^x dx &= dt \\
 I &= \int \frac{dt}{\sqrt{(4)^2 - t^2}} \\
 &= \sin^{-1} \left(\frac{t}{4} \right) + C \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + C \right] \\
 I &= \sin^{-1} \left(\frac{e^x}{4} \right) + C
 \end{aligned}$$

Indefinite Integrals Ex 19.18 Q4

$$\begin{aligned}
 \text{Let } I &= \int \frac{\cos x}{\sqrt{4 + \sin^2 x}} dx \\
 \text{Let } \sin x &= t \\
 \Rightarrow \cos x dx &= dt \\
 I &= \int \frac{dt}{\sqrt{(2)^2 + t^2}} \\
 &= \log \left| t + \sqrt{(2)^2 + t^2} \right| + C \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log \left| x + \sqrt{a^2 + x^2} \right| + C \right] \\
 I &= \log \left| \sin x + \sqrt{4 + \sin^2 x} \right| + C
 \end{aligned}$$

Indefinite Integrals Ex 19.18 Q5

$$\begin{aligned} \text{Let } I &= \int \frac{\sin x}{\sqrt{4 \cos^2 x - 1}} dx \\ \text{Let } 2 \cos x &= t \\ \Rightarrow -2 \sin x dx &= dt \\ \Rightarrow \sin x dx &= -\frac{dt}{2} \\ I &= -\frac{1}{2} \int \frac{dt}{\sqrt{t^2 - 1}} \\ &= -\frac{1}{2} \log |t + \sqrt{t^2 - 1}| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{t^2 - a^2}} dt = \log |t + \sqrt{t^2 - a^2}| + c \right] \end{aligned}$$

$$I = -\frac{1}{2} \log |2 \cos x + \sqrt{4 \cos^2 x - 1}| + c$$

Indefinite Integrals Ex 19.18 Q6

$$\begin{aligned} \text{Let } I &= \int \frac{x}{\sqrt{4 - x^4}} dx \\ \text{Let } x^2 &= t \\ \Rightarrow 2x dx &= dt \\ \Rightarrow x dx &= \frac{dt}{2} \\ I &= \frac{1}{2} \int \frac{dt}{\sqrt{(2)^2 - t^2}} \\ &= \frac{1}{2} \sin^{-1} \left(\frac{t}{2} \right) + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + c \right] \end{aligned}$$

$$I = \frac{1}{2} \sin^{-1} \left(\frac{x^2}{2} \right) + c$$

Indefinite Integrals Ex 19.18 Q7

$$\begin{aligned} \text{Let } I &= \int \frac{1}{x \sqrt{4 - 9 (\log x)^2}} dx \\ \text{Let } 3 \log x &= t \\ \Rightarrow \frac{3}{x} dx &= dt \\ \Rightarrow \frac{1}{x} dx &= \frac{dt}{3} \\ I &= \frac{1}{3} \int \frac{dt}{\sqrt{(2)^2 - t^2}} \\ &= \frac{1}{3} \sin^{-1} \left(\frac{t}{2} \right) + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + c \right] \\ I &= \frac{1}{3} \sin^{-1} \left(\frac{3 \log x}{2} \right) + c \end{aligned}$$

Indefinite Integrals Ex 19.18 Q8

$$\begin{aligned}
 \text{Let } I &= \int \frac{\sin 8x}{\sqrt{9 + (\sin 4x)^4}} dx \\
 \text{Let } \sin^2 4x &= t \\
 \Rightarrow 2 \sin 4x \cos 4x (4) dx &= dt \\
 \Rightarrow 4 \sin 8x dx &= dt \\
 \Rightarrow \sin 8x dx &= \frac{dt}{4} \\
 I &= \frac{1}{4} \int \frac{dt}{\sqrt{(3)^2 + t^2}} \\
 &= \frac{1}{4} \log \left| t + \sqrt{(3)^2 + t^2} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log \left| x + \sqrt{a^2 + x^2} \right| + c \right]
 \end{aligned}$$

$$I = \frac{1}{4} \log \left| \sin^2 4x + \sqrt{9 + \sin^4 4x} \right| + c$$

Indefinite Integrals Ex 19.18 Q9

$$\begin{aligned}
 \text{Let } I &= \int \frac{\cos 2x}{\sqrt{\sin^2 2x + 8}} dx \\
 \text{Let } \sin 2x &= t \\
 \Rightarrow 2 \cos 2x dx &= dt \\
 \Rightarrow \cos 2x dx &= \frac{dt}{2} \\
 I &= \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + (2\sqrt{2})^2}} \\
 &= \frac{1}{2} \log \left| t + \sqrt{t^2 + (2\sqrt{2})^2} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log \left| x + \sqrt{x^2 + a^2} \right| + c \right]
 \end{aligned}$$

$$I = \frac{1}{2} \log \left| \sin 2x + \sqrt{\sin^2 2x + 8} \right| + c$$

Indefinite Integrals Ex 19.18 Q10

$$\begin{aligned}
 \text{Let } I &= \int \frac{\sin 2x}{\sqrt{\sin^4 x + 4 \sin^2 x - 2}} dx \\
 \text{Let } \sin^2 x &= t \\
 \Rightarrow 2 \sin x \cos x dx &= dt \\
 \Rightarrow \sin 2x dx &= dt \\
 \Rightarrow I &= \int \frac{dt}{\sqrt{t^2 + 4t - 2}} \\
 &= \int \frac{dt}{\sqrt{t^2 + 2t(2) + (2)^2 - (2)^2 - 2}} \\
 &= \int \frac{dt}{\sqrt{(t+2)^2 - 6}} \\
 \text{Let } t+2 &= u \\
 dt &= du \\
 &= \int \frac{du}{\sqrt{u^2 - (\sqrt{6})^2}} \\
 &= \log \left| u + \sqrt{u^2 - 6} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| + c \right] \\
 &= \log \left| t+2 + \sqrt{(t+2)^2 - 6} \right| + c \\
 I &= \log \left| \sin^2 x + 2 + \sqrt{\sin^4 x + 4 \sin^2 x - 2} \right| + c
 \end{aligned}$$

Indefinite Integrals Ex 19.18 Q11

$$\begin{aligned}
& \int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx = \\
& \text{let } t = \cos^2 x \rightarrow -dt = 2 \cos x \sin x dx \\
& \int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx = \int \frac{-1}{\sqrt{t^2 - (1-t) + 2}} dt \\
& = \int \frac{-1}{\sqrt{t^2 + t + \frac{1}{4} + \frac{3}{4}}} dt = \int \frac{-1}{\sqrt{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}}} dt \\
& = \int \frac{-1}{\sqrt{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}}} dt = -\log \left| \left(t + \frac{1}{2}\right) + \sqrt{t^2 + t + 1} \right| \\
& = -\log \left| \left(\cos^2 x + \frac{1}{2}\right) + \sqrt{\cos^4 x + \cos^2 x + 1} \right| + C
\end{aligned}$$

Indefinite Integrals Ex 19.18 Q12

$$\begin{aligned}
\text{Let } I &= \int \frac{\cos x}{\sqrt{4 - \sin^2 x}} dx \\
\text{Let } \sin x &= t \\
\Rightarrow \cos x dx &= dt \\
&= \int \frac{dt}{\sqrt{(2)^2 - t^2}} \\
&= \sin^{-1} \left(\frac{t}{2} \right) + C \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + C \right]
\end{aligned}$$

$$I = \sin^{-1} \left(\frac{\sin x}{2} \right) + C$$

Indefinite Integrals Ex 19.18 Q13

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{x^{\frac{2}{3}} \sqrt{x^{\frac{2}{3}} - 4}} dx \\
\text{Let } x^{\frac{1}{3}} &= t \\
\Rightarrow \frac{1}{3} x^{-\frac{2}{3}} dx &= dt \\
\Rightarrow \frac{1}{3} x^{-\frac{2}{3}} dx &= dt \\
\Rightarrow \frac{dx}{x^{\frac{2}{3}}} &= 3dt \\
I &= 3 \int \frac{dt}{\sqrt{t^2 - (2)^2}} \\
&= 3 \log \left| t + \sqrt{t^2 - 4} \right| + C \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| + C \right]
\end{aligned}$$

$$I = 3 \log \left| x^{\frac{1}{3}} + \sqrt{x^{\frac{2}{3}} - 4} \right| + C$$

Indefinite Integrals Ex 19.18 Q14

Let $I = \int \frac{1}{\sqrt{(1-x^2)[9+(\sin^{-1}x)^2]}} dx$

Let $\sin^{-1}x = t$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$I = \int \frac{dt}{\sqrt{[3]^2 + t^2}}$$

$$= \log|t + \sqrt{9+t^2}| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2+x^2}} dx = \log|x + \sqrt{a^2+x^2}| + c \right]$$

$$I = \log|\sin^{-1}x + \sqrt{9+(\sin^{-1}x)^2}| + c$$

Indefinite Integrals Ex 19.18 Q15

Let $I = \int \frac{\cos x}{\sqrt{\sin^2 x - 2 \sin x - 3}} dx$

Let $\sin x = t$

$$\Rightarrow \cos x dx = dt$$

$$= \int \frac{dt}{\sqrt{t^2 - 2t - 3}}$$

$$= \int \frac{dt}{\sqrt{t^2 - 2t + (1)^2 - (1)^2 - 3}}$$

$$= \int \frac{dt}{\sqrt{(t-1)^2 - (2)^2}}$$

Let $t-1 = u$

$$\Rightarrow dt = du$$

$$I = \int \frac{du}{\sqrt{u^2 - (2)^2}}$$

$$= \log|u + \sqrt{u^2 - 4}| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log|x + \sqrt{x^2 - a^2}| + c \right]$$

$$= \log|t-1 + \sqrt{(t-1)^2 + 4}| + c$$

$$I = \log|\sin x - 1 + \sqrt{\sin^2 x - 2 \sin x - 3}| + c$$

Indefinite Integrals Ex 19.18 Q16

$$\begin{aligned}
\text{Let } I &= \int \sqrt{\csc x - 1} dx \\
&= \int \sqrt{\frac{1 - \sin x}{\sin x}} dx \\
&= \int \sqrt{\frac{(1 - \sin x) + (1 + \sin x)}{\sin x (1 + \sin x)}} dx \\
&= \int \sqrt{\frac{\cos^2 x}{\sin^2 x + \sin x}} dx \\
&= \int \frac{\cos x}{\sqrt{\sin^2 x + \sin x}} dx
\end{aligned}$$

Let $\sin x = t$

$$\begin{aligned}
\Rightarrow \cos x dx &= dt \\
&= \int \frac{dt}{\sqrt{t^2 + t}} \\
&= \int \frac{dt}{\sqrt{t^2 + 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} \\
&= \int \frac{dt}{\sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}
\end{aligned}$$

Let, $t + \frac{1}{2} = u$

$$\begin{aligned}
\Rightarrow dt &= du \\
&= \int \frac{du}{\sqrt{u^2 - \left(\frac{1}{2}\right)^2}} \\
&= \log \left| u + \sqrt{u^2 - \left(\frac{1}{2}\right)^2} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| + c \right] \\
&= \log \left| \left(t + \frac{1}{2}\right) + \sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} \right| + c
\end{aligned}$$

$$I = \log \left| \sin x + \frac{1}{2} + \sqrt{\sin^2 x + \sin x} \right| + c$$

Indefinite Integrals Ex 19.18 Q17

To evaluate the following integral follow the steps:

$$\int \frac{\sin x - \cos x}{\sqrt{\sin 2x}} dx = \int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}} dx$$

Let $\sin x + \cos x = t$ therefore $(\cos x - \sin x) dx = dt$

Now

$$\begin{aligned}
\int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}} dx &= - \int \frac{dt}{\sqrt{t^2 - 1}} \\
&= - \ln \left| t + \sqrt{t^2 - 1} \right| + c \\
&= - \ln \left| \sin x + \cos x + \sqrt{\sin 2x} \right| + c
\end{aligned}$$

Ex 19.19

Indefinite Integrals Ex 19.19 Q1

$$\text{Let } I = \int \frac{x}{x^2 + 3x + 2} dx$$

$$\begin{aligned}\text{Let } x &= \lambda \frac{d}{dx}(x^2 + 3x + 2) + \mu \\ &= \lambda(2x + 3) + \mu \\ x &= (2\lambda)x + (3\lambda + \mu)\end{aligned}$$

Comparing the coefficients of like powers of x,

$$2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$3\lambda + \mu = 0 \Rightarrow 3\left(\frac{1}{2}\right) + \mu = 0$$

$$\mu = -\frac{3}{2}$$

$$\text{so, } I = \int \frac{\frac{1}{2}(2x + 3) - \frac{3}{2}}{x^2 + 3x + 2} dx$$

$$I = \frac{1}{2} \int \frac{2x + 3}{x^2 + 3x + 2} dx - \frac{3}{2} \int \frac{1}{x^2 + 3x + 2} dx$$

$$= \frac{1}{2} \int \frac{2x + 3}{x^2 + 3x + 2} dx - \frac{3}{2} \int \frac{1}{x^2 + 2x \left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2} dx$$

$$I = \frac{1}{2} \int \frac{2x + 3}{x^2 + 3x + 2} dx - \frac{3}{2} \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$$

$$= \frac{1}{2} \log|x^2 + 3x + 2| - \frac{3}{2} \times \frac{1}{2\left(\frac{1}{2}\right)} \log \left| \frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}} \right| + c \quad \left[\text{since, } \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right]$$

$$I = \frac{1}{2} \log|x^2 + 3x + 2| - \frac{3}{2} \log \left| \frac{x+1}{x+2} \right| + c$$

Indefinite Integrals Ex 19.19 Q2

$$\text{Let } I = \int \frac{x+1}{x^2+x+3} dx$$

$$\text{Let } x+1 = \lambda \frac{d}{dx}(x^2+x+3) + \mu$$

$$x+1 = \lambda(2x+1) + \mu$$

$$x+1 = (2\lambda)x + (\lambda+\mu)$$

Comparing the coefficients of like powers of x ,

$$2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$\lambda + \mu = 1 \Rightarrow \left(\frac{1}{2}\right) + \mu = 0$$

$$\mu = -\frac{1}{2}$$

$$\text{so, } I = \int \frac{\frac{1}{2}(2x+1) + \frac{1}{2}}{x^2+x+3} dx$$

$$I = \frac{1}{2} \int \frac{2x+1}{x^2+x+3} dx + \frac{1}{2} \int \frac{1}{x^2+2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+3} dx$$

$$I = \frac{1}{2} \int \frac{2x+1}{x^2+x+3} dx + \frac{1}{2} \int \frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{11}{4}\right)} dx$$

$$I = \frac{1}{2} \int \frac{2x+1}{x^2+x+3} dx + \frac{1}{2} \int \frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{11}}{2}\right)^2} dx$$

$$= \frac{1}{2} \log|x^2+x+3| + \frac{1}{2} \times \frac{1}{\sqrt{11}} \tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{11}}{2}}\right) + C \quad \left[\text{since, } \int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C \right]$$

$$I = \frac{1}{2} \log|x^2+x+3| + \frac{1}{\sqrt{11}} \tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right) + C$$

Indefinite Integrals Ex 19.19 Q3

$$\text{Let } I = \int \frac{x-3}{x^2+2x-4} dx$$

$$\text{Let } x-3 = \lambda \frac{d}{dx}(x^2+2x-4) + \mu$$

$$= \lambda(2x+2) + \mu$$

$$x-3 = (2\lambda)x + (2\lambda+\mu)$$

Comparing the coefficients of like powers of x ,

$$2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$2\lambda + \mu = -3 \Rightarrow 2\left(\frac{1}{2}\right) + \mu = -3$$

$$\mu = -4$$

$$\text{so, } I = \int \frac{\frac{1}{2}(2x+2)-4}{x^2+2x-4} dx$$

$$I = \frac{1}{2} \int \frac{2x+2}{x^2+2x-4} dx - 4 \int \frac{1}{x^2+2x+(1)^2-(1)^2-4} dx$$

$$I = \frac{1}{2} \int \frac{2x+2}{x^2+2x-4} dx - 4 \int \frac{1}{(x+1)^2-\left(\sqrt{5}\right)^2} dx$$

$$I = \frac{1}{2} \log|x^2+2x-4| - 4 \times \frac{1}{2\sqrt{5}} \log \left| \frac{x+1-\sqrt{5}}{x+1+\sqrt{5}} \right| + C \quad \left[\text{since, } \int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C \right]$$

$$I = \frac{1}{2} \log|x^2+2x-4| - \frac{2}{\sqrt{5}} \log \left| \frac{x+1-\sqrt{5}}{x+1+\sqrt{5}} \right| + C$$

Indefinite Integrals Ex 19.19 Q4

$$\text{Let } I = \int \frac{2x - 3}{x^2 + 6x + 13} dx$$

$$\begin{aligned}\text{Let } 2x - 3 &= \lambda \frac{d}{dx}(x^2 + 6x + 13) + \mu \\ &= \lambda(2x + 6) + \mu \\ 2x - 3 &= (2\lambda)x + (6\lambda + \mu)\end{aligned}$$

Comparing the coefficients of like powers of x,

$$\begin{aligned}2\lambda &= 2 \Rightarrow \lambda = 1 \\ 6\lambda + \mu &= -3 \Rightarrow 6(1) + \mu = -3 \\ \mu &= -9\end{aligned}$$

$$\text{so, } I = \int \frac{1(2x + 6) - 9}{x^2 + 6x + 13} dx$$

$$I = \int \frac{2x + 6}{x^2 + 6x + 13} dx - 9 \int \frac{1}{x^2 + 2x(3) + (3)^2 - (3)^2 + 13} dx$$

$$I = \int \frac{2x + 6}{x^2 + 6x + 13} dx - 9 \int \frac{1}{(x+3)^2 + (2)^2} dx$$

$$= \log|x^2 + 6x + 13| - 9 \times \frac{1}{2} \tan^{-1}\left(\frac{x+3}{2}\right) + C \quad \left[\text{since, } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C \right]$$

$$I = \log|x^2 + 6x + 13| - \frac{9}{2} \tan^{-1}\left(\frac{x+3}{2}\right) + C$$

Indefinite Integrals Ex 19.19 Q5

$$\begin{aligned}\text{Let } I &= \int \frac{x^2}{x^2 + 7x + 10} dx \\ &= \int \left\{ 1 - \frac{7x + 10}{x^2 + 7x + 10} \right\} dx \\ I &= x - \int \frac{7x + 10}{x^2 + 7x + 10} dx + C_1 \quad \dots (1)\end{aligned}$$

$$\text{Let } I_1 = \int \frac{7x + 10}{x^2 + 7x + 10} dx$$

$$\begin{aligned}\text{Let } 7x + 10 &= \lambda \frac{d}{dx}(x^2 + 7x + 10) + \mu \\ &= \lambda(2x + 7) + \mu \\ 7x + 10 &= (2\lambda)x + 7\lambda + \mu\end{aligned}$$

Comparing the coefficients of like powers of x,

$$\begin{aligned}7 &= 2\lambda \Rightarrow \lambda = \frac{7}{2} \\ 7\lambda + \mu &= 10 \Rightarrow 7\left(\frac{7}{2}\right) + \mu = 10 \\ \mu &= -\frac{29}{2}\end{aligned}$$

$$\text{so, } I = \int \frac{\frac{1}{6}(6x - 4) - \frac{1}{3}}{3x^2 - 4x + 3} dx$$

$$I = \frac{1}{6} \int \frac{6x - 4}{3x^2 - 4x + 3} dx - \frac{1}{9} \int \frac{1}{x^2 - \frac{4}{3}x + 1} dx$$

$$I = \frac{1}{6} \int \frac{6x - 4}{3x^2 - 4x + 3} dx - \frac{1}{9} \int \frac{1}{x^2 - 2x\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^2 + (2)^2} dx$$

$$I = \frac{1}{6} \int \frac{6x - 4}{3x^2 - 4x + 3} dx - \frac{1}{9} \int \frac{1}{\left(x - \frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2} dx$$

$$= \frac{1}{6} \log|3x^2 - 4x + 3| - \frac{1}{9} \times \frac{1}{\left(\frac{\sqrt{5}}{3}\right)} \tan^{-1}\left(\frac{x - \frac{2}{3}}{\frac{\sqrt{5}}{3}}\right) + C \quad \left[\text{since, } \int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C \right]$$

$$I = \frac{1}{6} \log|3x^2 - 4x + 3| - \frac{\sqrt{5}}{15} \tan^{-1}\left(\frac{3x - 2}{\sqrt{5}}\right) + C$$

Indefinite Integrals Ex 19.19 Q6

We need to evaluate the integral $\int \frac{2x}{2+x-x^2} dx$

write the numerator in the following form

$$2x = \lambda \left\{ \frac{d}{dx} (2+x-x^2) \right\} + \mu$$

$$\text{i.e. } 2x = \lambda(-2x+1) + \mu$$

Equating the coefficients will give the values of λ, μ

$$\lambda = -1, \mu = 1$$

$$\begin{aligned} \int \frac{2x}{2+x-x^2} dx &= \int \frac{\lambda(-2x+1) + \mu}{2+x-x^2} dx \\ &= \int \frac{-1(-2x+1) + 1}{2+x-x^2} dx \\ &= \int \frac{-1(-2x+1)}{2+x-x^2} dx + \int \frac{1}{2+x-x^2} dx \\ &= -\log|2+x-x^2| + \int \frac{1}{2+x-x^2} dx \\ &= -\log|2+x-x^2| - \int \frac{1}{(x^2-x-2)} dx \end{aligned}$$

$$\begin{aligned} &= -\log|2+x-x^2| - \int \frac{1}{\left(x^2-x+\frac{1}{4}-2-\frac{1}{4}\right)} dx \\ &= -\log|2+x-x^2| - \int \frac{1}{\left(x^2-x+\frac{1}{4}-\frac{9}{4}\right)} dx \\ &= -\log|2+x-x^2| - \int \frac{1}{\left(\left(x-\frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right)} dx \\ &= -\log|2+x-x^2| - \frac{1}{3} \log \left| \frac{\left(x-\frac{1}{2}\right) - \left(\frac{3}{2}\right)}{\left(x-\frac{1}{2}\right) + \left(\frac{3}{2}\right)} \right| + C \\ &= -\log|2+x-x^2| - \frac{1}{3} \log \left| \frac{(x-2)}{(x+1)} \right| + C \end{aligned}$$

Indefinite Integrals Ex 19.19 Q7

$$\text{Let } I = \int \frac{1-3x}{3x^2+4x+2} dx$$

$$\begin{aligned}\text{Let } 1-3x &= \lambda \frac{d}{dx}(3x^2+4x+2) + \mu \\ &= \lambda(6x+4) + \mu \\ 1-3x &= (6\lambda)x + (4\lambda + \mu)\end{aligned}$$

Comparing the coefficients of like powers of x ,

$$6\lambda = -3 \quad \Rightarrow \quad \lambda = -\frac{1}{2}$$

$$4\lambda + \mu = 1 \quad \Rightarrow \quad 4\left(-\frac{1}{2}\right) + \mu = 1$$

$$\mu = 3$$

$$\text{so, } I = \int \frac{-\frac{1}{2}(6x+4)+3}{3x^2+4x+2} dx$$

$$I = -\frac{1}{2} \int \frac{6x+4}{3x^2+4x+2} dx + 3 \int \frac{1}{3x^2+4x+2} dx$$

$$I = -\frac{1}{2} \int \frac{6x+4}{3x^2+4x+2} dx + \frac{3}{3} \int \frac{1}{x^2 + \frac{4}{3}x + \frac{2}{3}} dx$$

$$I = -\frac{1}{2} \int \frac{6x+4}{3x^2+4x+2} dx + \int \frac{1}{x^2 + 2x\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^2 + \frac{2}{3}} dx$$

$$= -\frac{1}{2} \int \frac{6x+4}{3x^2+4x+2} dx + \int \frac{1}{\left(x + \frac{2}{3}\right)^2 + \frac{2}{9}} dx$$

$$I = -\frac{1}{2} \int \frac{6x+4}{3x^2+4x+2} dx + \int \frac{1}{\left(x + \frac{2}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2} dx$$

$$= -\frac{1}{2} \log|3x^2+4x+2| + \frac{3}{\sqrt{2}} \tan^{-1} \left(\frac{x+\frac{2}{3}}{\frac{\sqrt{2}}{3}} \right) + C \quad \left[\text{since, } \int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C \right]$$

$$I = -\frac{1}{2} \log|3x^2+4x+2| + \frac{3}{\sqrt{2}} \tan^{-1} \left(\frac{3x+2}{\sqrt{2}} \right) + C$$

Indefinite Integrals Ex 19.19 Q8

$$\text{Let } I = \int \frac{2x+5}{x^2-x-2} dx$$

$$\begin{aligned}\text{Let } 2x+5 &= \lambda \frac{d}{dx}(x^2-x-2) + \mu \\ &= \lambda(2x-1) + \mu \\ 2x+5 &= (2\lambda)x - \lambda + \mu\end{aligned}$$

Comparing the coefficients of like powers of x ,

$$\begin{aligned}2\lambda &= 2 \quad \Rightarrow \quad \lambda = 1 \\ -\lambda + \mu &= 5 \quad \Rightarrow \quad -1 + \mu = 5 \\ \mu &= 6\end{aligned}$$

$$\text{so, } I = \int \frac{(2x-1)+6}{x^2-x-2} dx$$

$$I = \int \frac{(2x-1)}{x^2-x-2} dx + 6 \int \frac{1}{x^2-2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-2} dx$$

$$I = \int \frac{2x-1}{x^2-x-2} dx + 6 \int \frac{1}{\left(x-\frac{1}{2}\right)^2-\frac{9}{4}} dx$$

$$I = \int \frac{2x-1}{x^2-x-2} dx + 6 \int \frac{1}{\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2} dx$$

$$I = \log|x^2-x-2| + \frac{6}{2\left(\frac{3}{2}\right)} \log \left| \frac{x-\frac{1}{2}-\frac{3}{2}}{x-\frac{1}{2}+\frac{3}{2}} \right| + c \quad \left[\text{since, } \int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right]$$

$$I = \log|x^2-x-2| + 2 \log \left| \frac{x-2}{x+1} \right| + c$$

Indefinite Integrals Ex 19.19 Q9

$$\text{Let } I = \int \frac{ax^3 + bx}{x^4 + c^2} dx$$

$$\text{Let } ax^3 + bx = \lambda \frac{d}{dx} (x^4 + c^2) + \mu$$

$$ax^3 + bx = \lambda(4x^3) + \mu$$

Comparing the coefficients of like powers of x

$$4\lambda = a \Rightarrow \lambda = \frac{a}{4}$$

$$\mu = 0 \Rightarrow \mu = 0$$

$$\text{so, } I = \int \frac{\frac{a}{4}(4x^3) + bx}{x^4 + c^2} dx$$

$$I = \frac{a}{4} \int \frac{4x^3}{x^4 + c^2} dx + b \int \frac{x}{(x^2)^2 + c^2} dx$$

$$I = \frac{a}{4} \int \frac{4x^3}{x^4 + c^2} dx + \frac{b}{2} \int \frac{2x}{(x^2)^2 + c^2} dx$$

$$= \frac{a}{4} \log|x^4 + c^2| + \frac{b}{2} I_1 \quad \text{--- (i)}$$

Now,

$$I_1 = \int \frac{2x}{(x^2)^2 + c^2} dx$$

$$\text{Put } x^2 = t$$

$$\Rightarrow 2x dx = dt$$

$$I_1 = \int \frac{1}{(t)^2 + c^2} dt$$

$$= \frac{1}{c} \tan^{-1}\left(\frac{t}{c}\right) + c_1$$

$$I_1 = \frac{1}{c} \tan^{-1}\left(\frac{x^2}{c}\right) + c_1 \quad \text{--- (ii)}$$

Using equation (ii) in equation (i),

$$I = \frac{a}{4} \log|x^4 + c^2| + \frac{b}{2c} \tan^{-1}\left(\frac{x^2}{c}\right) + k$$

K = Integration constant

Indefinite Integrals Ex 19.19 Q10

$$\text{Let } I = \int \frac{x+2}{2x^2+6x+5} dx$$

$$\begin{aligned}\text{Let } x+2 &= \lambda \frac{d}{dx}(2x^2+6x+5) + \mu \\ &= \lambda(4x+6) + \mu \\ x+2 &= (4\lambda)x + (6\lambda + \mu)\end{aligned}$$

Comparing the coefficients of like powers of x,

$$\begin{aligned}4\lambda &= 1 &\Rightarrow \lambda &= \frac{1}{4} \\ 6\lambda + \mu &= 2 &\Rightarrow 6\left(\frac{1}{4}\right) + \mu &= 2 \\ \mu &= \frac{1}{2}\end{aligned}$$

$$\text{so, } I = \int \frac{\frac{1}{4}(4x+6) + \frac{1}{2}}{2x^2+6x+5} dx$$

$$\begin{aligned}I &= \frac{1}{4} \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{2} \int \frac{1}{2x^2+6x+5} dx \\ I &= \frac{1}{4} \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4} \int \frac{1}{x^2+3x+\frac{5}{2}} dx\end{aligned}$$

$$\begin{aligned}I &= \frac{1}{4} \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4} \int \frac{1}{x^2+2x\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{5}{2}} dx \\ &= \frac{1}{4} \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4} \int \frac{1}{\left(x+\frac{3}{2}\right)^2 + \frac{1}{4}} dx\end{aligned}$$

$$\begin{aligned}I &= \frac{1}{4} \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4} \int \frac{1}{\left(x+\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} dx \\ I &= \frac{1}{4} \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4} \times \frac{1}{\frac{1}{2}} \tan^{-1}\left(\frac{x+\frac{3}{2}}{\frac{1}{2}}\right) + c dx \quad \left[\text{since, } \int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c\right]\end{aligned}$$

$$I = \frac{1}{4} \log|2x^2+6x+5| + \frac{1}{2} \tan^{-1}(2x+3) + c$$

Indefinite Integrals Ex 19.19 Q11

$$\begin{aligned}
 \text{Let } I &= \int \frac{(3\sin x - 2)\cos x}{5 - \cos^2 x - 4\sin x} dx \\
 \therefore I &= \int \frac{(3\sin x - 2)\cos x}{5 - (1 - \sin^2 x) - 4\sin x} dx \\
 \Rightarrow I &= \int \frac{(3\sin x - 2)\cos x}{5 - 1 + \sin^2 x - 4\sin x} dx \\
 \text{Substitute } \sin x &= t \\
 \Rightarrow \cos x \, dx &= dt \\
 \text{Thus,} \\
 I &= \int \frac{(3t - 2)}{4 + t^2 - 4t} dt \\
 \Rightarrow I &= \int \frac{(3t - 2)}{t^2 - 4t + 4} dt \\
 \Rightarrow I &= \int \frac{(3t - 2)}{(t - 2)^2} dt
 \end{aligned}$$

Now let us separate the integrand into the simplest form using partial fractions.

$$\begin{aligned}
 \frac{(3t - 2)}{(t - 2)^2} &= \frac{A}{(t - 2)} + \frac{B}{(t - 2)^2} \\
 &= \frac{A(t - 2) + B}{(t - 2)^2} \\
 &= \frac{At - 2A + B}{(t - 2)^2}
 \end{aligned}$$

$$\Rightarrow 3t - 2 = At - 2A + B$$

Comparing the coefficients, we have,

$$A = 3$$

and

$$-2A + B = -2$$

Substituting the value of A=3 in the above equation, we have,

$$\Rightarrow -2 \times 3 + B = -2$$

$$\Rightarrow -6 + B = -2$$

$$\Rightarrow B = 6 - 2$$

$$\Rightarrow B = 4$$

Thus, $I = \int \frac{(3t - 2)}{(t - 2)^2} dt$ becomes,

$$\begin{aligned}
 I &= \int \frac{3}{(t - 2)} dt + \int \frac{4}{(t - 2)^2} dt \\
 &= 3 \log|t - 2| - 4 \left(\frac{1}{t - 2} \right) + C \\
 &= 3 \log|2 - t| + 4 \left(\frac{1}{2 - t} \right) + C
 \end{aligned}$$

Now substituting $t = \sin x$, we have,

$$I = 3 \log|2 - \sin x| + 4 \left(\frac{1}{2 - \sin x} \right) + C$$

Indefinite Integrals Ex 19.19 Q12

$$\text{Let } I = \int \frac{5x - 2}{1 + 2x + 3x^2} dx$$

Rewriting the numerator we have,

$$5x - 2 = A \frac{d}{dx}(1 + 2x + 3x^2) + B$$

$$\Rightarrow 5x - 2 = A(2 + 6x) + B$$

$$\Rightarrow 5x - 2 = 6xA + 2A + B$$

Comparing the coefficients, we have,

$$6A = 5 \text{ and } 2A + B = -2$$

$$\Rightarrow A = \frac{5}{6}$$

Substituting the value of A in $2A + B = -2$, we have,

$$2 \times \frac{5}{6} + B = -2$$

$$\Rightarrow \frac{10}{6} + B = -2$$

$$\Rightarrow B = -2 - \frac{10}{6}$$

$$\Rightarrow B = \frac{-12 - 10}{6}$$

$$\Rightarrow B = \frac{-22}{6}$$

$$\Rightarrow B = \frac{-11}{3}$$

$$5x - 2 = \frac{5}{6}(2 + 6x) - \frac{11}{3}$$

Thus, $I = \int \frac{5x - 2}{1 + 2x + 3x^2} dx$ becomes,

$$\begin{aligned} I &= \int \frac{\left[\frac{5}{6}(2 + 6x) - \frac{11}{3} \right]}{3x^2 + 2x + 1} dx \\ &= \frac{5}{6} \int \frac{(2 + 6x)}{3x^2 + 2x + 1} dx - \frac{11}{3} \int \frac{dx}{3x^2 + 2x + 1} \\ &= \frac{5}{6} \log(3x^2 + 2x + 1) - \frac{11}{3 \times 3} \int \frac{dx}{x^2 + \frac{2}{3}x + \frac{1}{3}} + C \\ &= \frac{5}{6} \log(3x^2 + 2x + 1) - \frac{11}{9} \int \frac{dx}{x^2 + \frac{2}{3}x + \left(\frac{4}{3}\right)^2 + \frac{1}{3} - \left(\frac{4}{3}\right)^2} + C \\ &= \frac{5}{6} \log(3x^2 + 2x + 1) - \frac{11}{9} \int \frac{dx}{\left(x + \frac{1}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2} + C \\ &= \frac{5}{6} \log(3x^2 + 2x + 1) - \frac{11}{9} \times \frac{1}{\frac{\sqrt{2}}{3}} \tan^{-1} \left[\frac{\left(x + \frac{1}{3}\right)}{\frac{\sqrt{2}}{3}} \right] + C \\ &= \frac{5}{6} \log(3x^2 + 2x + 1) - \frac{11}{9} \times \frac{3}{\sqrt{2}} \tan^{-1} \left[\frac{\left(3x + 1\right)}{\frac{\sqrt{2}}{3}} \right] + C \\ &= \frac{5}{6} \log(3x^2 + 2x + 1) - \frac{11}{3\sqrt{2}} \tan^{-1} \left[\frac{3x + 1}{\sqrt{2}} \right] + C \end{aligned}$$

Ex 19.20

Indefinite Integrals Ex 19.20 Q1

$$\begin{aligned} \text{Let } I &= \int \frac{x^2 + x + 1}{x^2 - x} dx \\ &= \int \left[1 + \frac{2x + 1}{x^2 - x} \right] dx \\ &= x + \int \frac{2x + 1}{x^2 - x} dx + c_1 \quad \dots \text{(i)} \end{aligned}$$

$$I_1 = \int \frac{2x + 1}{x^2 - x} dx$$

$$\begin{aligned} \text{Let } 2x + 1 &= \lambda \frac{d}{dx} (x^2 - x) + \mu \\ &= \lambda (2x - 1) + \mu \\ 2x + 1 &= (2\lambda)x - \lambda + \mu \end{aligned}$$

Comparing the coefficients of like powers of x,

$$\begin{aligned} 2 &= 2\lambda \Rightarrow \lambda = 1 \\ -\lambda + \mu &= 1 \Rightarrow \mu = 2 \end{aligned}$$

$$\begin{aligned} \text{so, } I_1 &= \int \frac{(2x - 1) + 2}{x^2 - x} dx \\ I &= \int \frac{2x - 1}{x^2 - x} dx + 2 \int \frac{1}{x^2 - x} dx \end{aligned}$$

$$\begin{aligned} I &= \int \frac{2x - 1}{x^2 - x} dx + 2 \int \frac{1}{x^2 - 2x \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx \\ &= \int \frac{2x - 1}{x^2 - x} dx + 2 \int \frac{1}{\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx \end{aligned}$$

$$I = \log|x^2 - x| + 2 \times \frac{1}{2\left(\frac{1}{2}\right)} \log \left| \frac{x - \frac{1}{2} - \frac{1}{2}}{x - \frac{1}{2} + \frac{1}{2}} \right| + c_1 \quad \left[\text{since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right]$$

$$I_1 = \log|x^2 - x| + 2 \log \left| \frac{x-1}{x} \right| + c_2 \quad \dots \text{(ii)}$$

Using equation (i) and (ii)

$$I = x + \log|x^2 - x| + 2 \log\left|\frac{x-1}{x}\right| + c$$

$$\text{Let } 2x+1 = \lambda \frac{d}{dx}(x^2 - x) + \mu$$

$$= \lambda(2x-1) + \mu$$

$$2x+1 = (2\lambda)x - \lambda + \mu$$

Comparing the coefficients of like powers of x,

$$2 = 2\lambda \Rightarrow \lambda = 1$$

$$-\lambda + \mu = 1 \Rightarrow \mu = 2$$

$$\text{so, } I_1 = \int \frac{(2x-1)+2}{x^2-x} dx$$

$$I = \int \frac{2x-1}{x^2-x} dx + 2 \int \frac{1}{x^2-x} dx$$

$$I = \int \frac{2x-1}{x^2-x} dx + 2 \int \frac{1}{x^2 - 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$$

$$= \int \frac{2x-1}{x^2-x} dx + 2 \int \frac{1}{\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$$

$$I = \log|x^2 - x| + 2 \times \frac{1}{2\left(\frac{1}{2}\right)} \log \left| \frac{x - \frac{1}{2} - \frac{1}{2}}{x - \frac{1}{2} + \frac{1}{2}} \right| + c_1 \quad \left[\text{since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right]$$

$$I_1 = \log|x^2 - x| + 2 \log\left|\frac{x-1}{x}\right| + c_2 \quad \text{--- (ii)}$$

Using equation (i) and (ii)

$$I = x + \log|x^2 - x| + 2 \log\left|\frac{x-1}{x}\right| + c$$

Indefinite Integrals Ex 19.20 Q2

$$\text{Let } I = \int \frac{x^2+x-1}{x^2+x-6} dx$$

$$= \int \left[1 + \frac{5}{x^2+x-6} \right] dx$$

$$I = x + \int \frac{5}{x^2+x-6} dx + c_1 \quad \text{--- (i)}$$

$$\text{Let } I_1 = 5 \int \frac{1}{x^2+x-6} dx$$

$$= 5 \int \frac{1}{x^2 + 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 6} dx$$

$$= 5 \int \frac{1}{\left(x + \frac{1}{2}\right)^2 - \left(\frac{5}{2}\right)^2} dx$$

$$= 5 \frac{1}{2\left(\frac{5}{2}\right)} \log \left| \frac{x + \frac{1}{2} - \frac{5}{2}}{x + \frac{1}{2} + \frac{5}{2}} \right| + c_2 \quad \left[\text{since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right]$$

$$I_1 = \log \left| \frac{x-2}{x+3} \right| + c_2 \quad \text{--- (ii)}$$

Using equation (i) and (ii)

$$I = x + \log \left| \frac{x-2}{x+3} \right| + c$$

Indefinite Integrals Ex 19.20 Q3

$$\begin{aligned}
\text{Let } I &= \int \frac{1-x^2}{x(1-2x)} dx \\
&= \int \frac{1-x^2}{x-2x^2} dx \\
&= \int \frac{x^2-1}{2x^2-x} dx \\
&= \int \left[\frac{1}{2} + \frac{\frac{x}{2}-1}{2x^2-x} \right] dx
\end{aligned}$$

$$I = \frac{1}{2}x + \int \frac{\frac{x}{2}-1}{2x^2-x} dx + c_1 \quad \dots \dots (i)$$

$$\text{Let } I_1 = \int \frac{\frac{x}{2}-1}{2x^2-x} dx$$

$$\text{Let } \frac{x}{2}-1 = \lambda \frac{d}{dx}(2x^2-x) + \mu$$

$$= \lambda(4x-1) + \mu$$

$$\frac{x}{2}-1 = (4\lambda)x - \lambda + \mu$$

Comparing the coefficients of like powers of x ,

$$\begin{aligned}
\frac{1}{2} = 4\lambda &\Rightarrow \lambda = \frac{1}{8} \\
-\lambda + \mu = -1 &\Rightarrow -\left(\frac{1}{8}\right) + \mu = -1 \\
\mu = -\frac{7}{8}
\end{aligned}$$

$$\text{so, } I_1 = \int \frac{\frac{1}{8}(4x-1)-\frac{7}{8}}{2x^2-x} dx$$

$$I = \frac{1}{8} \int \frac{4x-1}{2x^2-x} dx - \frac{7}{8} \int \frac{1}{2\left(x^2-\frac{x}{2}\right)} dx$$

$$I = \frac{1}{8} \int \frac{4x-1}{2x^2-x} dx - \frac{7}{16} \int \frac{1}{x^2-2x\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2} dx$$

$$= \frac{1}{8} \int \frac{4x-1}{2x^2-x} dx - \frac{7}{16} \int \frac{1}{\left(x-\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2} dx$$

$$I_1 = \frac{1}{8} \log|2x^2-x| - \frac{7}{16} \times \frac{1}{2\left(\frac{1}{4}\right)} \log \left| \frac{x-\frac{1}{4}-\frac{1}{4}}{x-\frac{1}{4}+\frac{1}{4}} \right| + c_2 \quad \left[\text{since, } \int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log|x-a|+c \right]$$

$$I = \frac{1}{8} \log|x| + \frac{1}{8} \log|2x-1| - \frac{7}{8} \log|1-2x| + \frac{7}{8} \log 2 + \frac{7}{8} \log|x| + c$$

$$I_1 = \log|x| - \frac{3}{4} \log|1-2x| + c_3 \quad \dots \dots (ii) \quad \left[\text{say, } c_3 = c_2 + \frac{7}{8} \log 2 \right]$$

Using equation (i) and (ii)

$$I = \frac{1}{2}x + \log|x| - \frac{3}{4} \log|1-2x| + c$$

Indefinite Integrals Ex 19.20 Q4

Here the integrand $\frac{x^2+1}{x^2-5x+6}$ is not proper rational function, so we divide x^2+1 by x^2-5x+6 and find the

$$\frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{x^2-5x+6} = 1 + \frac{5x-5}{(x-2)(x-3)}$$

$$\text{Let } \frac{5x-5}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

So that

Equating the coefficients of x and constant terms on both sides, we get $A+B=5$ and $3A+2B=5$. Solving these we get $A=-5$ and $B=10$

$$\text{Thus } \frac{x^2+1}{x^2-5x+6} = 1 - \frac{5}{x-2} + \frac{10}{x-3}$$

$$\text{Therefore, } \int \frac{x^2+1}{(x+1)^2(x+3)} dx = \int dx - 5 \int \frac{1}{x-2} dx + 10 \int \frac{x}{x-3} dx \\ = x - 5 \log|x-2| + 10 \log|x-3| + C.$$

Indefinite Integrals Ex 19.20 Q5

$$\begin{aligned} \text{Let } I &= \int \frac{x^2}{x^2 + 7x + 10} dx \\ &= \int \left[1 - \frac{7x + 10}{x^2 + 7x + 10} \right] dx \\ &= x - \int \frac{7x + 10}{x^2 + 7x + 10} dx + c_1 \quad \dots \text{(i)} \end{aligned}$$

$$\text{Let } I_1 = \int \frac{7x + 10}{x^2 + 7x + 10} dx$$

$$\begin{aligned} \text{Let } 7x + 10 &= \lambda \frac{d}{dx}(x^2 + 7x + 10) + \mu \\ &= \lambda(2x + 7) + \mu \\ 7x + 10 &= (2\lambda)x + 7\lambda + \mu \end{aligned}$$

Comparing the coefficients of like powers of x ,

$$\begin{aligned} 7 &= 2\lambda \Rightarrow \lambda = \frac{7}{2} \\ 7\lambda + \mu &= 10 \Rightarrow 7\left(\frac{7}{2}\right) + \mu = 10 \\ \mu &= -\frac{29}{2} \end{aligned}$$

$$\begin{aligned} \text{so, } I_1 &= \int \frac{\frac{7}{2}(2x + 7) - \frac{29}{2}}{x^2 + 7x + 10} dx \\ &= \frac{7}{2} \int \frac{(2x + 7)}{x^2 + 7x + 10} dx - \frac{29}{2} \int \frac{1}{x^2 + 2x\left(\frac{7}{2}\right) + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 10} dx \\ I_1 &= \frac{7}{2} \int \frac{2x + 7}{x^2 + 7x + 10} dx - \frac{29}{2} \int \frac{1}{\left(x + \frac{7}{2}\right)^2 - \left(\frac{3}{2}\right)^2} dx \end{aligned}$$

$$\begin{aligned} I_1 &= \frac{2}{7} \log|x^2 + 7x + 10| - \frac{29}{2} \times \frac{1}{2\left(\frac{3}{2}\right)} \log \left| \frac{x + \frac{7}{2} - \frac{3}{2}}{x + \frac{7}{2} + \frac{3}{2}} \right| + c_2 \quad \left[\text{since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right] \\ &= \frac{7}{2} \log|x^2 + 7x + 10| - \frac{29}{6} \log \left| \frac{x+2}{x+5} \right| + c_2 \quad \dots \text{(ii)} \end{aligned}$$

Using equation (i) and (ii)

$$I = x - \frac{7}{2} \log|x^2 + 7x + 10| + \frac{29}{6} \log \left| \frac{x+2}{x+5} \right| + c$$

Indefinite Integrals Ex 19.20 Q6

$$\begin{aligned} \text{Let } I &= \int \frac{x^2 + x + 1}{x^2 - x + 1} dx \\ &= \int \left[1 + \frac{2x}{x^2 - x + 1} \right] dx \\ I &= x + \int \frac{2x}{x^2 - x + 1} dx + c_1 \quad \dots \dots \text{(i)} \end{aligned}$$

$$\begin{aligned} \text{Let } I &= \int \frac{2x}{x^2 - x + 1} dx \\ \text{Let } 2x &= \lambda \frac{d}{dx}(x^2 - x + 1) + \mu \\ &= \lambda(2x - 1) + \mu \\ 2x &= (2\lambda)x - \lambda + \mu \end{aligned}$$

Comparing the coefficients of like powers of x,

$$\begin{aligned} 2 &= 2\lambda \Rightarrow \lambda = 1 \\ -\lambda + \mu &= 0 \Rightarrow -1 + \mu = 0 \\ \mu &= 1 \end{aligned}$$

$$\begin{aligned} \text{so, } I_1 &= \int \frac{(2x - 1) + 1}{x^2 - x + 1} dx \\ &= \int \frac{(2x - 1)}{x^2 - x + 1} dx + \int \frac{1}{x^2 - 2x \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1} dx \\ I_1 &= \int \frac{2x - 1}{x^2 - x + 1} dx + \int \frac{1}{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx \end{aligned}$$

$$\begin{aligned} &= \log|x^2 - x + 1| + \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) + c_2 \quad \left[\text{since, } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right] \\ &= \log|x^2 - x + 1| + \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2x - 1}{\sqrt{3}} \right) + c_2 \quad \dots \dots \text{(ii)} \end{aligned}$$

Using equation (i) and (ii)

$$I = x + \log|x^2 - x + 1| + \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2x - 1}{\sqrt{3}} \right) + c$$

Indefinite Integrals Ex 19.20 Q7

$$\begin{aligned}
 \text{Let } I &= \int \frac{(x-1)^2}{x^2+2x+2} dx \\
 &= \int \frac{x^2-2x+1}{x^2+2x+2} dx \\
 &= \int \left[1 - \frac{4x+1}{x^2+2x+2} \right] dx \\
 I &= x - \int \frac{4x+1}{x^2+2x+2} dx + c_1 \quad \dots \dots \text{(i)}
 \end{aligned}$$

$$\text{Let } I = \int \frac{4x+1}{x^2+2x+2} dx$$

$$\begin{aligned}
 \text{Let } 4x+1 &= \lambda \frac{d}{dx}(x^2+2x+2) + \mu \\
 &= \lambda(2x+2) + \mu \\
 &= (2\lambda)x + (2\lambda + \mu)
 \end{aligned}$$

Comparing the coefficients of like powers of x,

$$\begin{aligned}
 4 &= 2\lambda \Rightarrow \lambda = 2 \\
 2\lambda + \mu &= 1 \Rightarrow 2(2) + \mu = 1 \\
 \mu &= -3
 \end{aligned}$$

$$\begin{aligned}
 \text{so, } I_1 &= \int \frac{2(2x+2)-3}{x^2+2x+2} dx \\
 &= 2 \int \frac{(2x+2)}{x^2+2x+2} dx - 3 \int \frac{1}{x^2-2x+(1)^2-(1)^2+2} dx \\
 I_1 &= 2 \int \frac{2x+2}{x^2+2x+2} dx - 3 \int \frac{1}{(x+1)^2+(1)^2} dx
 \end{aligned}$$

$$I_1 = 2 \log|x^2+2x+2| - 3 \tan^{-1}(x+1) + c_2 \quad \dots \dots \text{(ii)} \quad \left[\text{since, } \int \frac{1}{x^2+1} dx = \tan^{-1} x + c \right]$$

Using equation (i) and (ii)

$$I = x - 2 \log|x^2+2x+2| + 3 \tan^{-1}(x+1) + c$$

Indefinite Integrals Ex 19.20 Q8

$$\begin{aligned} \text{Let } I &= \int \frac{x^3 + x^2 + 2x + 1}{x^2 - x + 1} dx \\ &= \int \left[x + 2 + \frac{3x - 1}{x^2 - x + 1} \right] dx \\ &= \frac{x^2}{2} + 2x + \int \frac{3x - 1}{x^2 - x + 1} dx + c_1 \quad \dots \text{(i)} \end{aligned}$$

$$\begin{aligned} \text{Let } I_1 &= \int \frac{3x - 1}{x^2 - x + 1} dx \\ \text{Let } 3x - 1 &= \lambda \frac{d}{dx}(x^2 - x + 1) + \mu \\ &= \lambda(2x - 1) + \mu \\ 3x - 1 &= (2\lambda)x - \lambda + \mu \end{aligned}$$

Comparing the coefficients of like powers of x,

$$\begin{aligned} 3 &= 2\lambda \Rightarrow \lambda = \frac{3}{2} \\ -\lambda + \mu &= -1 \Rightarrow -\left(\frac{3}{2}\right) + \mu = -1 \\ \mu &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{so, } I_1 &= \int \frac{\frac{3}{2}(2x - 1) + \frac{1}{2}}{x^2 - x + 1} dx \\ &= \frac{3}{2} \int \frac{(2x - 1)}{x^2 - x + 1} dx + \frac{1}{2} \int \frac{1}{x^2 - 2x \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1} dx \\ I_1 &= \frac{3}{2} \int \frac{2x - 1}{x^2 - x + 1} dx + \frac{1}{2} \int \frac{1}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx \end{aligned}$$

$$\begin{aligned} I_1 &= \frac{3}{2} \log|x^2 - x + 1| + \frac{1}{2} \times \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x + \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) + c_2 \quad \left[\text{since, } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right] \\ I_1 &= \frac{3}{2} \log|x^2 - x + 1| + \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x + 1}{\sqrt{3}} \right) + c_2 \quad \dots \text{(ii)} \end{aligned}$$

Using equation (i) and (ii)

$$I = \frac{x^2}{2} + 2x + \frac{3}{2} \log|x^2 - x + 1| + \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x + 1}{\sqrt{3}} \right) + c$$

Indefinite Integrals Ex 19.20 Q9

$$\begin{aligned} \text{Let } I &= \int \frac{x^2(x^4 + 4)}{(x^2 + 4)} dx \\ &= \int \frac{x^6 + 4x^2}{(x^2 + 4)} dx \\ &= \int \left[x^4 - 4x^2 + 20 - \frac{80}{x^2 + 4} \right] dx \\ I &= \frac{x^5}{5} - \frac{4x^3}{3} + 20x - 80 \int \frac{1}{x^2 + 4} dx + c_1 \quad \dots \text{(i)} \end{aligned}$$

$$\begin{aligned} \text{Let } I_1 &= \int \frac{1}{x^2 + 4} dx \\ &= \int \frac{1}{x^2 + (2)^2} dx \\ I_1 &= \frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) + c_2 \quad \dots \text{(ii)} \quad \left[\text{since, } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right] \end{aligned}$$

Using equation (i) and (ii)

$$I = \frac{x^5}{5} - \frac{4x^3}{3} + 20x - \frac{80}{2} \tan^{-1} \left(\frac{x}{2} \right) + c$$

$$I = \frac{x^5}{5} - \frac{4x^3}{3} + 20x - 40 \tan^{-1} \left(\frac{x}{2} \right) + c$$

Indefinite Integrals Ex 19.20 Q10

$$\begin{aligned} \text{Let } I &= \int \frac{x^2}{x^2 + 6x + 12} dx \\ &= \int \left[1 - \frac{6x + 12}{x^2 + 6x + 12} \right] dx \\ &= x - \int \frac{6x + 12}{x^2 + 6x + 12} dx + c_1 \quad \text{--- (i)} \end{aligned}$$

$$\text{Let } I_1 = \int \frac{6x + 12}{x^2 + 6x + 12} dx$$

$$\begin{aligned} \text{Let } 6x + 12 &= \lambda \frac{d}{dx}(x^2 + 6x + 12) + \mu \\ &= \lambda(2x + 6) + \mu \\ 6x + 12 &= (2\lambda)x + 6\lambda + \mu \end{aligned}$$

Comparing the coefficients of like powers of x,

$$\begin{aligned} 6 &= 2\lambda \Rightarrow \lambda = 3 \\ 6\lambda + \mu &= 12 \Rightarrow 6(3) + \mu = 12 \\ &\mu = -6 \end{aligned}$$

$$\begin{aligned} \text{so, } I_1 &= \int \frac{3(2x + 6) - 6}{x^2 + 6x + 12} dx \\ &= 3 \int \frac{(2x + 6)}{x^2 + 6x + 12} dx - 6 \int \frac{1}{x^2 + 2x(3) + (3)^2 - (3)^2 + 12} dx \\ I_1 &= 3 \int \frac{2x + 6}{x^2 + 6x + 12} dx + 6 \int \frac{1}{(x+3)^2 + (\sqrt{3})^2} dx \\ I_1 &= 3 \log|x^2 + 6x + 12| + \frac{6}{\sqrt{3}} \tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c_2 \quad \left[\text{since, } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c \right] \\ I_1 &= 3 \log|x^2 + 6x + 12| + 2\sqrt{3} \tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c_2 \quad \text{--- (ii)} \end{aligned}$$

Using equation (i) and (ii)

$$I = x - 3 \log|x^2 + 6x + 12| + 2\sqrt{3} \tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c$$

Ex 19.21

Indefinite Integrals Ex 19.21 Q1

$$\text{Let } I = \int \frac{x}{\sqrt{x^2 + 6x + 10}} dx$$

$$\begin{aligned} \text{Let } x &= \lambda \frac{d}{dx} (x^2 + 6x + 10) + \mu \\ &= \lambda (2x + 6) + \mu \\ x &= (2\lambda)x + 6\lambda + \mu \end{aligned}$$

Comparing the coefficients of like powers of x,

$$\begin{aligned} 2\lambda &= 1 &\Rightarrow \lambda &= \frac{1}{2} \\ 6\lambda + \mu &= 0 &\Rightarrow 6\left(\frac{1}{2}\right) + \mu &= 0 \\ \mu &= -3 \end{aligned}$$

$$\text{so, } I_1 = \int \frac{\frac{1}{2}(2x + 6) - 3}{\sqrt{x^2 + 6x + 10}} dx$$

$$\begin{aligned} &= \frac{1}{2} \int \frac{(2x + 6)}{\sqrt{x^2 + 6x + 10}} dx - 3 \int \frac{1}{\sqrt{x^2 + 2x(3) + (3)^2 - (3)^2 + 10}} dx \\ I_1 &= \frac{1}{2} \int \frac{2x + 6}{\sqrt{x^2 + 6x + 10}} dx - 3 \int \frac{1}{\sqrt{(x+3)^2 + 1^2}} dx \end{aligned}$$

$$I_1 = \frac{1}{2} \left[2\sqrt{x^2 + 6x + 10} \right] - 3 \log \left| x + 3 + \sqrt{(x+3)^2 + 1} \right| + C \quad \left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C, \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log \left| x + \sqrt{x^2 + a^2} \right| + C \right]$$

$$I = \sqrt{x^2 + 6x + 10} - 3 \log \left| x + 3 + \sqrt{x^2 + 6x + 10} \right| + C$$

Indefinite Integrals Ex 19.21 Q2

$$\text{Let } I = \int \frac{2x + 1}{\sqrt{x^2 + 2x - 1}} dx$$

$$\begin{aligned} \text{Let } 2x + 1 &= \lambda \frac{d}{dx} (x^2 + 2x - 1) + \mu \\ &= \lambda (2x + 2) + \mu \\ 2x + 1 &= (2\lambda)x + 2\lambda + \mu \end{aligned}$$

Comparing the coefficients of like powers of x,

$$\begin{aligned} 2\lambda &= 2 &\Rightarrow \lambda &= 1 \\ 2\lambda + \mu &= 1 &\Rightarrow 2(1) + \mu &= 1 \\ \mu &= -1 \end{aligned}$$

$$\text{so, } I = \int \frac{(2x + 2) - 1}{\sqrt{x^2 + 2x - 1}} dx$$

$$\begin{aligned} &= \int \frac{(2x + 2)}{\sqrt{x^2 + 2x - 1}} dx - \int \frac{1}{\sqrt{x^2 + 2x + (1)^2 - (1)^2 - 1}} dx \\ I &= \int \frac{2x + 2}{\sqrt{x^2 + 2x - 1}} dx - \int \frac{1}{\sqrt{(x+1)^2 - (\sqrt{2})^2}} dx \end{aligned}$$

$$I = \left(2\sqrt{x^2 + 2x - 1} \right) - \log \left| x + 1 + \sqrt{(x+1)^2 - (\sqrt{2})^2} \right| + C \quad \left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C, \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| + C \right]$$

$$I = 2\sqrt{x^2 + 2x - 1} - \log \left| x + 1 + \sqrt{x^2 + 2x - 1} \right| + C$$

Indefinite Integrals Ex 19.21 Q3

$$\text{Let } I = \int \frac{x+1}{\sqrt{4+5x-x^2}} dx$$

$$\begin{aligned}\text{Let } x+1 &= \lambda \frac{d}{dx} (4+5x-x^2) + \mu \\ &= \lambda(5-2x) + \mu \\ x &= (-2\lambda)x + 5\lambda + \mu\end{aligned}$$

Comparing the coefficients of like powers of x ,

$$\begin{aligned}-2\lambda &= 1 & \Rightarrow \lambda &= -\frac{1}{2} \\ 5\lambda + \mu &= 1 & \Rightarrow 5\left(-\frac{1}{2}\right) + \mu &= 1 \\ \mu &= \frac{7}{2}\end{aligned}$$

$$\text{so, } I = \int \frac{-\frac{1}{2}(5-2x) + \frac{7}{2}}{\sqrt{4+5x-x^2}} dx$$

$$= -\frac{1}{2} \int \frac{(5-2x)}{\sqrt{4+5x-x^2}} dx + \frac{7}{2} \int \frac{1}{\sqrt{-[x^2 - 5x - 4]}} dx$$

$$I = -\frac{1}{2} \int \frac{5-2x}{\sqrt{4+5x-x^2}} dx + \frac{7}{2} \int \frac{1}{\sqrt{-[x^2 - 2x\left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 4]}} dx$$

$$I = -\frac{1}{2} \int \frac{5-2x}{\sqrt{4+5x-x^2}} dx + \frac{7}{2} \int \frac{1}{\sqrt{-\left[\left(x - \frac{5}{2}\right)^2 - \left(\frac{\sqrt{41}}{2}\right)^2\right]}} dx$$

$$I = -\frac{1}{2} \int \frac{5-2x}{\sqrt{4+5x-x^2}} dx + \frac{7}{2} \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2 - \left(x - \frac{5}{2}\right)^2}} dx$$

$$I = -\frac{1}{2} \left(2\sqrt{4+5x-x^2} \right) + \frac{7}{2} \sin^{-1} \left(\frac{x - \frac{5}{2}}{\frac{\sqrt{41}}{2}} \right) + c \quad \left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c, \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + c \right]$$

$$I = -\sqrt{4+5x-x^2} + \frac{7}{2} \sin^{-1} \left(\frac{2x-5}{\sqrt{41}} \right) + c$$

Indefinite Integrals Ex 19.21 Q4

$$\text{Let } I = \int \frac{6x-5}{\sqrt{3x^2-5x+1}} dx$$

$$\text{Let } 3x^2-5x+1 = t$$

$$(6x-5)dx = dt$$

$$I = \int \frac{dt}{\sqrt{t}}$$

$$= 2\sqrt{t} + c$$

$$I = 2\sqrt{3x^2-5x+1} + c$$

Indefinite Integrals Ex 19.21 Q5

$$\text{Let } I = \int \frac{3x+1}{\sqrt{5-2x-x^2}} dx$$

$$\begin{aligned}\text{Let } 3x+1 &= \lambda \frac{d}{dx}(5-2x-x^2) + \mu \\ &= \lambda(-2-2x) + \mu \\ 3x+1 &= (-2\lambda)x - 2\lambda + \mu\end{aligned}$$

Comparing the coefficients of like powers of x ,

$$\begin{aligned}-2\lambda &= 3 & \Rightarrow \lambda &= -\frac{3}{2} \\ -2\lambda + \mu &= 1 & \Rightarrow -2\left(-\frac{3}{2}\right) + \mu &= 1 \\ && \mu &= -2\end{aligned}$$

$$\begin{aligned}\text{so, } I &= \int \frac{-\frac{3}{2}(-2-2x)-2}{\sqrt{5-2x-x^2}} dx \\ &= -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{[-x^2-2x-(1)^2-5]}} dx \\ I &= -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{[-x^2-2x+(1)^2-(1)^2-5]}} dx \\ I &= -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{[(x+1)^2-(\sqrt{6})^2]}} dx \\ I &= -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{[(\sqrt{6})^2-(x+1)^2]}} dx \\ I &= -\frac{3}{2} \times 2\sqrt{5-2x-x^2} - 2 \sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + C & \left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C, \int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C \right]\end{aligned}$$

$$I = -3\sqrt{5-2x-x^2} - 2 \sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + C$$

Indefinite Integrals Ex 19.21 Q6

$$\text{Let } I = \int \frac{x}{\sqrt{8+x-x^2}} dx$$

$$\begin{aligned}\text{Let } x &= \lambda \frac{d}{dx}(8+x-x^2) + \mu \\ &= \lambda(1-2x) + \mu \\ x &= (-2\lambda)x + \lambda + \mu\end{aligned}$$

Comparing the coefficients of like powers of x ,

$$\begin{aligned}-2\lambda &= 1 & \Rightarrow \lambda &= -\frac{1}{2} \\ \lambda + \mu &= 0 & \Rightarrow \left(-\frac{1}{2}\right) + \mu &= 0 \\ \mu &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\text{so, } I &= \int \frac{-\frac{1}{2}(1-2x)+\frac{1}{2}}{\sqrt{8+x-x^2}} dx \\ &= -\frac{1}{2} \int \frac{(1-2x)}{\sqrt{8+x-x^2}} dx + \frac{1}{2} \int \frac{1}{\sqrt{[-x^2+x-8]}} dx \\ I &= -\frac{1}{2} \int \frac{(1-2x)}{\sqrt{8+x-x^2}} dx + \frac{1}{2} \int \frac{1}{\sqrt{[x^2-2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-8]}} dx \\ I &= -\frac{1}{2} \int \frac{(1-2x)}{\sqrt{8+x-x^2}} dx + \frac{1}{2} \int \frac{1}{\sqrt{[\left(x-\frac{1}{2}\right)^2-\left(\frac{33}{4}\right)^2]}} dx \\ I &= -\frac{1}{2} \int \frac{(1-2x)}{\sqrt{8+x-x^2}} dx + \frac{1}{2} \int \frac{1}{\sqrt{\left(\frac{\sqrt{33}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}} dx \\ I &= -\frac{1}{2} \times 2\sqrt{8+x-x^2} + \frac{1}{2} \sin^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{33}}{2}}\right) + C & \left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C, \int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C \right]\end{aligned}$$

$$I = -\sqrt{8+x-x^2} + \frac{1}{2} \sin^{-1}\left(\frac{2x-1}{\sqrt{33}}\right) + C$$

Indefinite Integrals Ex 19.21 Q7

Let $I = \int \frac{x+2}{\sqrt{x^2+2x-1}} dx$
 Let $x+2 = \lambda \frac{d}{dx}(x^2+2x-1) + \mu$
 $x+2 = \lambda(2x+2) + \mu$
 $x+2 = (2\lambda)x + 2\lambda + \mu$

Comparing the coefficients of like powers of x ,

$$\begin{aligned} 2\lambda &= 1 & \Rightarrow & \lambda = \frac{1}{2} \\ 2\lambda + \mu &= 2 & \Rightarrow & 2\left(\frac{1}{2}\right) + \mu = 2 \\ && \Rightarrow & \mu = 1 \end{aligned}$$

so, $I_1 = \int \frac{\frac{1}{2}(2x+2)+1}{\sqrt{x^2+2x-1}} dx$
 $= \frac{1}{2} \int \frac{(2x+2)}{\sqrt{x^2+2x-1}} dx + \int \frac{1}{\sqrt{x^2+2x+(1)^2-(1)^2-1}} dx$
 $I = \frac{1}{2} \int \frac{2x+2}{\sqrt{x^2+2x-1}} dx + \int \frac{1}{\sqrt{(x+1)^2 - (\sqrt{2})^2}} dx$
 $I = \frac{1}{2} 2\sqrt{x^2+2x-1} + \log|x+1 + \sqrt{(x+1)^2 - (\sqrt{2})^2}| + c$ $\left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x}, \int \frac{1}{\sqrt{x^2-a^2}} dx = \log|x + \sqrt{x^2-a^2}| + c \right]$

$$I = \sqrt{x^2+2x-1} + \log|x+1 + \sqrt{x^2+2x-1}| + c$$

Indefinite Integrals Ex 19.21 Q8

$$\begin{aligned} \text{Let } x+2 &= A \frac{d}{dx}(x^2-1) + B & \dots(1) \\ \Rightarrow x+2 &= A(2x) + B \end{aligned}$$

Equating the coefficients of x and constant term on both sides, we obtain

$$2A = 1 \Rightarrow A = \frac{1}{2}$$

$$B = 2$$

From (1), we obtain

$$(x+2) = \frac{1}{2}(2x) + 2$$

$$\begin{aligned} \text{Then, } \int \frac{x+2}{\sqrt{x^2-1}} dx &= \int \frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}} dx \\ &= \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} dx + \int \frac{2}{\sqrt{x^2-1}} dx & \dots(2) \end{aligned}$$

$$\ln \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} dx, \text{ let } x^2-1=t \Rightarrow 2xdx=dt$$

$$\begin{aligned} \ln \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} dx, \text{ let } x^2-1=t &\Rightarrow 2xdx=dt \\ \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} dx &= \frac{1}{2} \int \frac{dt}{\sqrt{t}} \\ &= \frac{1}{2} [2\sqrt{t}] \\ &= \sqrt{t} \\ &= \sqrt{x^2-1} \end{aligned}$$

$$\text{Then, } \int \frac{2}{\sqrt{x^2-1}} dx = 2 \int \frac{1}{\sqrt{x^2-1}} dx = 2 \log|x + \sqrt{x^2-1}|$$

From equation (2), we obtain

$$\int \frac{x+2}{\sqrt{x^2-1}} dx = \sqrt{x^2-1} + 2 \log|x + \sqrt{x^2-1}| + C$$

Indefinite Integrals Ex 19.21 Q9

$$\begin{aligned}
& \int \frac{x-1}{\sqrt{x^2+1}} dx = \\
& \int \frac{x-1}{\sqrt{x^2+1}} dx = \int \frac{x}{\sqrt{x^2+1}} - \frac{1}{\sqrt{x^2+1}} dx = \frac{1}{2} \int \frac{2x}{\sqrt{x^2+1}} dx - \int \frac{1}{\sqrt{x^2+1}} dx \\
& = \frac{1}{2} \int \frac{d}{dx} (x^2 + x + 1) dx = \frac{1}{2} (2\sqrt{x^2+1}) - \int \frac{1}{\sqrt{x^2+1}} dx = \sqrt{x^2+1} - \ln|x + \sqrt{x^2+1}| + C \\
& = \sqrt{x^2+1} - \ln|x + \sqrt{x^2+1}| + C
\end{aligned}$$

Indefinite Integrals Ex 19.21 Q10

$$\text{Let } I = \int \frac{x}{\sqrt{x^2+x+1}} dx$$

$$\begin{aligned}
\text{Let } x &= \lambda \frac{d}{dx} (x^2 + x + 1) + \mu \\
&= \lambda (2x+1) + \mu \\
x &= (2\lambda)x + \lambda + \mu
\end{aligned}$$

Comparing the coefficients of like powers of x,

$$\begin{aligned}
2\lambda &= 1 \quad \Rightarrow \quad \lambda = \frac{1}{2} \\
\lambda + \mu &= 0 \quad \Rightarrow \quad \left(\frac{1}{2}\right) + \mu = 0 \\
&\Rightarrow \quad \mu = -\frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
\text{so, } I &= \int \frac{\frac{1}{2}(2x+1) + \frac{1}{2}}{\sqrt{x^2+x+1}} dx \\
&= \frac{1}{2} \int \frac{(2x+1)}{\sqrt{x^2+x+1}} dx - \frac{1}{2} \int \frac{1}{\sqrt{x^2+2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}} dx \\
I &= \frac{1}{2} \int \frac{2x+1}{\sqrt{x^2+x+1}} dx - \frac{1}{2} \int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2}} dx \\
I &= \frac{1}{2} \times 2\sqrt{x^2+x+1} - \frac{1}{2} \log|x + \frac{1}{2} + \sqrt{\left(x+\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2}| + C \quad \left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C, \int \frac{1}{\sqrt{x^2-a^2}} dx = \log|x + \sqrt{x^2-a^2}| + C \right]
\end{aligned}$$

$$I = \sqrt{x^2+x+1} - \frac{1}{2} \log|x + \frac{1}{2} + \sqrt{x^2+x+1}| + C$$

Indefinite Integrals Ex 19.21 Q11

$$\text{Let } I = \int \frac{x+1}{\sqrt{x^2+1}} dx$$

$$\begin{aligned}
\text{Let } x+1 &= \lambda \frac{d}{dx} (x^2 + 1) + \mu \\
x+1 &= \lambda (2x) + \mu
\end{aligned}$$

Comparing the coefficients of like powers of x,

$$\begin{aligned}
2\lambda &= 1 \quad \Rightarrow \quad \lambda = \frac{1}{2} \\
&\Rightarrow \quad \mu = 1
\end{aligned}$$

$$\begin{aligned}
\text{so, } I &= \int \frac{\frac{1}{2}(2x)+1}{\sqrt{x^2+1}} dx \\
&= \frac{1}{2} \int \frac{(2x)}{\sqrt{x^2+1}} dx + \int \frac{1}{\sqrt{x^2+1}} dx \\
I &= \frac{1}{2} \times 2\sqrt{x^2+1} + \log|x + \sqrt{x^2+1}| + C \quad \left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C, \int \frac{1}{\sqrt{x^2+a^2}} dx = \log|x + \sqrt{x^2-a^2}| + C \right]
\end{aligned}$$

$$I = \sqrt{x^2+1} + \log|x + \sqrt{x^2+1}| + C$$

Indefinite Integrals Ex 19.21 Q12

$$\text{Let } I = \int \frac{2x+5}{\sqrt{x^2+2x+5}} dx$$

$$\begin{aligned}\text{Let } 2x+5 &= \lambda \frac{d}{dx}(x^2+2x+5) + \mu \\ &= \lambda(2x+2) + \mu \\ 2x+5 &= \{2\lambda\}x + 2\lambda + \mu\end{aligned}$$

Comparing the coefficients of like powers of x ,

$$\begin{aligned}2\lambda &= 2 \quad \Rightarrow \quad \lambda = 1 \\ 2\lambda + \mu &= 5 \quad \Rightarrow \quad 2(1) + \mu = 5 \\ &\Rightarrow \quad \mu = 3\end{aligned}$$

$$\text{so, } I = \int \frac{(2x+2)+3}{\sqrt{x^2+2x+5}} dx$$

$$= \int \frac{(2x+3)}{\sqrt{x^2+2x+5}} dx + 3 \int \frac{1}{\sqrt{x^2+2x+(1)^2-(1)^2+5}} dx$$

$$I = \int \frac{2x+3}{\sqrt{x^2+2x+5}} dx + 3 \int \frac{1}{\sqrt{(x+1)^2+(2)^2}} dx$$

$$I = 2\sqrt{x^2+2x+5} + 3 \log|x+1+\sqrt{(x+1)^2+(2)^2}| + C \quad \left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C, \int \frac{1}{\sqrt{x^2+a^2}} dx = \log|x+\sqrt{x^2+a^2}| + C \right]$$

$$I = 2\sqrt{x^2+2x+5} + 3 \log|x+1+\sqrt{x^2+2x+5}| + C$$

Indefinite Integrals Ex 19.21 Q13

$$\text{Let } I = \int \frac{3x+1}{\sqrt{5-2x-x^2}} dx$$

$$\begin{aligned}\text{Let } 3x+1 &= \lambda \frac{d}{dx}(5-2x-x^2) + \mu \\ &= \lambda(-2-2x) + \mu \\ 3x+1 &= (-2\lambda)x - 2\lambda + \mu\end{aligned}$$

Comparing the coefficients of like powers of x ,

$$\begin{aligned}-2\lambda &= 3 \quad \Rightarrow \quad \lambda = -\frac{3}{2} \\ -2\lambda + \mu &= 1 \quad \Rightarrow \quad -2\left(-\frac{3}{2}\right) + \mu = 1 \\ &\Rightarrow \quad \mu = -2\end{aligned}$$

$$\text{so, } I = \int \frac{-\frac{3}{2}(-2-2x)-2}{\sqrt{5-2x-x^2}} dx$$

$$= -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{[-x^2+2x-5]}} dx$$

$$I = -\frac{3}{2} \int \frac{-2-2x}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{[-x^2+2x+(1)^2-(1)^2+5]}} dx$$

$$I = -\frac{3}{2} \int \frac{-2-2x}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{[(x+1)^2-(\sqrt{6})^2]}} dx$$

$$I = -\frac{3}{2} \int \frac{-2-2x}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{(\sqrt{6})^2-(x+1)^2}} dx$$

$$I = -\frac{3}{2} \times 2\sqrt{5-2x-x^2} - 2 \sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + C \quad \left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C, \int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C \right]$$

$$I = -3\sqrt{5-2x-x^2} - 2 \sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + C$$

Indefinite Integrals Ex 19.21 Q14

$$\begin{aligned} \text{Let } I &= \int \frac{1-x}{\sqrt{1+x}} dx \\ &= \int \frac{1-x}{\sqrt{1+x}} \times \frac{1-x}{1-x} dx \\ &= \int \frac{1-x}{\sqrt{1-x^2}} dx \end{aligned}$$

$$\begin{aligned} \text{Let } 1-x &= \lambda \frac{d}{dx} (1-x^2) + \mu \\ &= \lambda (-2x) + \mu \\ 1-x &= (-2\lambda)x + \mu \end{aligned}$$

Comparing the coefficients of like powers of x ,

$$\begin{aligned} -2\lambda &= -1 &\Rightarrow \lambda &= \frac{1}{2} \\ &&\Rightarrow \mu &= 1 \end{aligned}$$

$$\begin{aligned} \text{so, } I &= \int \frac{\frac{1}{2}(-2x) + 1}{\sqrt{1-x^2}} dx \\ &= \frac{1}{2} \int \frac{(-2x)}{\sqrt{1-x^2}} dx + \int \frac{1}{\sqrt{1-x^2}} dx \\ I &= \frac{1}{2} \times 2\sqrt{1-x^2} + \sin^{-1} x + C & \left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C, \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + C \right] \end{aligned}$$

$$I = \sqrt{1-x^2} + \sin^{-1} x + C$$

Indefinite Integrals Ex 19.21 Q15

$$\text{Let } I = \int \frac{2x+1}{\sqrt{x^2+4x+3}} dx$$

$$\begin{aligned} \text{Let } 2x+1 &= \lambda \frac{d}{dx} (x^2+4x+3) + \mu \\ &= \lambda (2x+4) + \mu \\ 2x+1 &= (2\lambda)x + 4\lambda + \mu \end{aligned}$$

Comparing the coefficients of like powers of x ,

$$\begin{aligned} 2\lambda &= 2 &\Rightarrow \lambda &= 1 \\ 4\lambda + \mu &= 1 &\Rightarrow 4(1) + \mu &= 1 \\ &&\Rightarrow \mu &= -3 \end{aligned}$$

$$\begin{aligned} \text{so, } I &= \int \frac{(2x+4)-3}{\sqrt{x^2+4x+3}} dx \\ &= \int \frac{(2x+4)}{\sqrt{x^2+4x+3}} dx - 3 \int \frac{1}{\sqrt{x^2+2x(2)+(2)^2-(2)^2+3}} dx \\ &= \int \frac{(2x+4)}{\sqrt{x^2+4x+3}} dx - 3 \int \frac{1}{\sqrt{(x+2)^2-1}} dx \\ I &= 2\sqrt{x^2+4x+3} - 3 \log|x+2+\sqrt{(x+2)^2-1}| + C & \left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C, \int \frac{1}{\sqrt{x^2-a^2}} dx = \log|x+\sqrt{x^2-a^2}| + C \right] \end{aligned}$$

$$I = 2\sqrt{x^2+4x+3} - 3 \log|x+2+\sqrt{x^2+4x+3}| + C$$

Indefinite Integrals Ex 19.21 Q16

$$\text{Let } I = \int \frac{2x+3}{\sqrt{x^2+4x+5}} dx$$

$$\begin{aligned} \text{Let } 2x+3 &= \lambda \frac{d}{dx} (x^2+4x+5) + \mu \\ &= \lambda (2x+4) + \mu \\ 2x+3 &= (2\lambda)x + 4\lambda + \mu \end{aligned}$$

Comparing the coefficients of like powers of x ,

$$\begin{aligned} 2\lambda &= 2 &\Rightarrow \lambda &= 1 \\ 4\lambda + \mu &= 3 &\Rightarrow 4(1) + \mu &= 3 \\ &&\Rightarrow \mu &= -1 \end{aligned}$$

$$\begin{aligned} \text{so, } I &= \int \frac{(2x+4)-1}{\sqrt{x^2+4x+5}} dx \\ &= \int \frac{(2x+4)}{\sqrt{x^2+4x+5}} dx - \int \frac{1}{\sqrt{x^2+2x(2)+(2)^2-(2)^2+5}} dx \\ &= \int \frac{(2x+4)}{\sqrt{x^2+4x+5}} dx - \int \frac{1}{\sqrt{(x+2)^2+(1)^2}} dx \\ I &= 2\sqrt{x^2+4x+5} - \log|x+2+\sqrt{(x+2)^2+1}| + C & \left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C, \int \frac{1}{\sqrt{x^2+a^2}} dx = \log|x+\sqrt{x^2+a^2}| + C \right] \end{aligned}$$

$$I = 2\sqrt{x^2+4x+5} - \log|x+2+\sqrt{x^2+4x+5}| + C$$

Indefinite Integrals Ex 19.21 Q17

$$\begin{aligned}
& \int \frac{5x+3}{\sqrt{x^2+4x+10}} dx \\
\rightarrow & \text{let } 5x+3 = \lambda(2x+4) + \mu \\
\lambda = & \frac{5}{2}, \mu = -7 \\
\int \frac{\lambda(2x+4)+\mu}{\sqrt{x^2+4x+10}} dx &= \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}} dx \\
&= \int \frac{\frac{5}{2}(2x+4)}{\sqrt{x^2+4x+10}} dx - \int \frac{7}{\sqrt{x^2+4x+10}} dx \\
&= \int \frac{\frac{5}{2}dt}{\sqrt{t}} - \int \frac{7}{\sqrt{(x+2)^2+6}} dx \\
&= 5\sqrt{x^2+4x+10} - 7\log|x+2| + \sqrt{x^2+4x+10} + C
\end{aligned}$$

Indefinite Integrals Ex 19.21 Q18

$$\begin{aligned}
\text{Let } I &= \int \frac{x+2}{\sqrt{x^2+2x+3}} \\
x+2 &= A \frac{d}{dx}[x^2+2x+3] + B
\end{aligned}$$

$$\Rightarrow x+2 = 2Ax+2A+B$$

Comparing the coefficients, we have,

$$2A=1 \text{ and } 2A+B=2$$

$$\Rightarrow A = \frac{1}{2}$$

Substituting the value of A in $2A+B=2$, we have,

$$2 \times \frac{1}{2} + B = 2$$

$$\Rightarrow 1 + B = 2$$

$$\Rightarrow B = 2 - 1$$

$$\Rightarrow B = 1$$

Thus we have,

$$x+2 = \frac{1}{2}[2x+2] + 1$$

Hence,

$$\begin{aligned}
I &= \int \frac{x+2}{\sqrt{x^2+2x+3}} dx \\
&= \int \frac{\left[\frac{1}{2}[2x+2]+1\right]}{\sqrt{x^2+2x+3}} dx \\
&= \int \frac{\left[\frac{1}{2}[2x+2]\right]}{\sqrt{x^2+2x+3}} dx + \int \frac{dx}{\sqrt{x^2+2x+3}} \\
&= \frac{1}{2} \int \frac{[2x+2]}{\sqrt{x^2+2x+3}} dx + \int \frac{dx}{\sqrt{x^2+2x+3}}
\end{aligned}$$

Substituting $t=x^2+2x+3$ and $dt=2x+2$

in the first integrand, we have,

$$\begin{aligned}
I &= \frac{1}{2} \int \frac{dt}{\sqrt{t}} + \int \frac{dx}{\sqrt{x^2+2x+3}} \\
&= \frac{1}{2} \times 2\sqrt{t} + \int \frac{dx}{\sqrt{x^2+2x+1+2}} + C \\
&= \sqrt{t} + \int \frac{dx}{\sqrt{(x+1)^2+(\sqrt{2})^2}} + C \\
I &= \sqrt{x^2+2x+3} + \log \left[|x+1| + \sqrt{(x+1)^2 + (\sqrt{2})^2} \right] + C \\
\Rightarrow I &= \sqrt{x^2+2x+3} + \log \left[|x+1| + \sqrt{x^2+2x+3} \right] + C
\end{aligned}$$

Ex 19.22

Indefinite Integrals Ex 19.22 Q1

Let $I = \int \frac{1}{4 \cos^2 x + 9 \sin^2 x} dx$

Diving numerator and denominator by $\cos^2 x$

$$= \int \frac{\frac{1}{\cos^2 x}}{4 + 9 \tan^2 x} dx$$

$$I = \int \frac{\sec^2 x}{4 + 9 \tan^2 x} dx$$

Let $\tan x = t$

$$\sec^2 x dx = dt$$

$$I = \int \frac{dt}{4 + 9(t)^2}$$

$$= \int \frac{dt}{4 + (3t)^2}$$

Let $3t = u$

$$3dt = du$$

$$I = \frac{1}{3} \int \frac{du}{(2)^2 + (u)^2}$$

$$= \frac{1}{3} \times \frac{1}{2} \times \tan^{-1} \left(\frac{u}{2} \right) + c$$

$$I = \frac{1}{6} \tan^{-1} \left(\frac{3t}{2} \right) + c$$

$$I = \frac{1}{6} \tan^{-1} \left(\frac{3 \tan x}{2} \right) + c$$

Indefinite Integrals Ex 19.22 Q2

Let $I = \int \frac{1}{4 \sin^2 x + 5 \cos^2 x} dx$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{4 \tan^2 x + 5} dx$$

$$= \int \frac{\sec^2 x}{4 \tan^2 x + 5} dx$$

Let $\tan x = t$

$$\sec^2 x dx = dt$$

$$I = \int \frac{dt}{4 + 9(t)^2}$$

$$= \int \frac{dt}{4t^2 + 5}$$

Let $2t = u$

$$2dt = du$$

$$I = \frac{1}{2} \int \frac{du}{(4)^2 + (\sqrt{5})^2}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{5}} \times \tan^{-1} \left(\frac{u}{\sqrt{5}} \right) + c$$

$$= \frac{1}{2\sqrt{5}} \tan^{-1} \left(\frac{2t}{\sqrt{5}} \right) + c$$

$$I = \frac{1}{2\sqrt{5}} \tan^{-1} \left(\frac{2 \tan x}{\sqrt{5}} \right) + c$$

Indefinite Integrals Ex 19.22 Q3

$$\text{Let } I = \int \frac{2}{2 + \sin 2x} dx$$

$$= \int \frac{2}{2 + 2 \sin x \cos x} dx$$

Divide numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x} + \frac{\sin x \cos x}{\cos^2 x}} dx$$

$$= \int \frac{\sec^2 x}{\sec^2 x + \tan x} dx$$

$$I = \int \frac{\sec^2 x}{1 + \tan^2 x + \tan x} dx$$

Let $\tan x = t$

$$\sec^2 x dx = dt$$

$$I = \int \frac{dt}{t^2 + t + 1}$$

$$= \int \frac{dt}{t^2 + 2t \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$$

$$I = \int \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1} \left(\frac{t + \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) + C$$

$$= \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2t + 1}{\sqrt{3}} \right) + C$$

$$I = \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2 \tan x + 1}{\sqrt{3}} \right) + C$$

Indefinite Integrals Ex 19.22 Q4

$$\text{Let } I = \int \frac{\cos x}{\cos 3x} dx$$

$$= \int \frac{\cos x}{4 \cos^3 x - 3 \cos x} dx$$

Diving numerator and denominator by $\cos^3 x$

$$I = \int \frac{\frac{\cos x}{\cos^3 x}}{\frac{4 \cos^3 x}{\cos^3 x} + \frac{3 \cos x}{\cos^3 x}} dx$$

$$= \int \frac{\sec^2 x}{4 - 3 \sec^2 x} dx$$

$$= \int \frac{\sec^2 x}{4 - 3(1 + \tan^2 x)} dx$$

$$= \int \frac{\sec^2 x}{4 - 3 - 3 \tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{1 - 3 \tan^2 x} dx$$

$$\text{Let } \tan x = t$$

$$\sec^2 x dx = dt$$

$$I = \int \frac{dt}{1 - 3t^2}$$

$$= \int \frac{dt}{1 - (\sqrt{3}t)^2}$$

$$\text{Let } \sqrt{3}t = u$$

$$\sqrt{3}dt = du$$

$$= \int \frac{du}{(1)^2 - (4)^2}$$

$$= \frac{1}{2\sqrt{3}} \log \left| \frac{u+1}{1-u} \right| + c$$

$$= \frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3}t+1}{1-\sqrt{3}t} \right| + c$$

$$I = \frac{1}{2\sqrt{3}} \log \left| \frac{1+\sqrt{3} \tan x}{1-\sqrt{3} \tan x} \right| + c$$

Indefinite Integrals Ex 19.22 Q5

$$\text{Let } I = \int \frac{1}{1 + 3 \sin^2 x} dx$$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x} + \frac{3 \sin^2 x}{\cos^2 x}} dx$$

$$= \int \frac{\sec^2 x}{\sec^2 x + 3 \tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{1 + \tan^2 x + 3 \tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{1 + 4 \tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{1 + (2 \tan x)^2} dx$$

$$\text{Let } 2 \tan x = t$$

$$2 \sec^2 x dx = dt$$

$$I = \frac{1}{2} \int \frac{dt}{1+t^2}$$

$$= \frac{1}{2} \tan^{-1} t + c$$

$$I = \frac{1}{2} \tan^{-1} (2 \tan x) + c$$

Indefinite Integrals Ex 19.22 Q6

Let $I = \int \frac{1}{3 + 2 \cos^2 x} dx$

Diving numerator and denominator by $\cos^2 x$

$$\begin{aligned} I &= \int \frac{\frac{1}{\cos^2 x}}{\frac{3}{\cos^2 x} + \frac{2 \cos^2 x}{\cos^2 x}} dx \\ &= \int \frac{\sec^2 x}{3 \sec^2 x + 2} dx \\ &= \int \frac{\sec^2 x}{3(1 + \tan^2 x) + 2} dx \\ &= \int \frac{\sec^2 x}{3 + 3 \tan^2 x + 2} dx \\ &= \int \frac{\sec^2 x}{5 + 3 \tan^2 x} dx \\ \text{Let } &\sqrt{3} \tan x = t \\ \sqrt{3} \sec^2 x dx &= dt \\ I &= \frac{1}{\sqrt{3}} \int \frac{dt}{(\sqrt{5})^2 + t^2} \\ &= \frac{1}{\sqrt{3} \times \sqrt{5}} \tan^{-1} \left(\frac{t}{\sqrt{5}} \right) + c \end{aligned}$$

$$I = \frac{1}{\sqrt{15}} \tan^{-1} \left(\frac{\sqrt{3} \tan x}{\sqrt{5}} \right) + c$$

Indefinite Integrals Ex 19.22 Q7

$$\begin{aligned} \text{Let } I &= \int \frac{1}{(\sin x - 2 \cos x)(2 \sin x + \cos x)} dx \\ &= \int \frac{1}{2 \sin^2 x + \sin x \cos x - 4 \sin x \cos x - 2 \cos^2 x} dx \\ &= \int \frac{1}{2 \sin^2 x - 3 \sin x \cos x - 2 \cos^2 x} dx \end{aligned}$$

Diving numerator and denominator by $\cos^2 x$

$$\begin{aligned} I &= \int \frac{\sec^2 x}{2 \tan^2 x - 3 \tan x - 2} dx \\ \text{Let } &\tan x = t \\ \sec^2 x dx &= dt \\ I &= \int \frac{dt}{2t^2 - 3t - 2} \\ &= \frac{1}{2} \int \frac{dt}{t^2 - \frac{3}{2}t - 1} \\ &= \frac{1}{2} \int \frac{dt}{t^2 - 2t \left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 - 1} \\ I &= \frac{1}{2} \int \frac{dt}{\left(t - \frac{3}{4}\right)^2 - \left(\frac{5}{4}\right)^2} \\ &= \frac{1}{2} \times \frac{1}{2 \left(\frac{5}{4}\right)} \log \left| \frac{t - \frac{3}{4} - \frac{5}{4}}{t - \frac{3}{4} + \frac{5}{4}} \right| + c \\ &= \frac{1}{5} \log \left| \frac{t - 2}{2t + 1} \right| + c \end{aligned}$$

$$I = \frac{1}{5} \log \left| \frac{\tan x - 2}{2 \tan x + 1} \right| + c$$

Indefinite Integrals Ex 19.22 Q8

Let $I = \int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

Diving numerator and denominator by $\cos^4 x$

$$I = \int \frac{2 \tan x \sec^2 x}{\tan^4 x + 1} dx$$

Let $\tan^2 x = t$
 $2 \tan x \sec^2 x dx = dt$

$$I = \int \frac{dt}{t^2 + 1}$$

$$= \tan^{-1} t + c$$

$$I = \tan^{-1}(\tan^2 x) + c$$

Indefinite Integrals Ex 19.22 Q9

Let $I = \int \frac{1}{\cos x (\sin x + 2 \cos x)} dx$
 $= \int \frac{1}{\sin x \cos x + 2 \cos^2 x} dx$

Diving numerator and denominator by $\cos^2 x$,

$$I = \int \frac{\sec^2 x}{\tan x + 2} dx$$

Let $2 + \tan x = t$
 $\sec^2 x dx = dt$
 $I = \int \frac{dt}{t}$
 $= \log|t| + c$

$$I = \log|2 + \tan x| + c$$

Indefinite Integrals Ex 19.22 Q10

Let $I = \int \frac{1}{\sin^2 x + \sin 2x} dx$
 $= \int \frac{1}{\sin^2 x + 2 \sin x \cos x} dx$

Diving numerator and denominator by $\cos^2 x$,

$$I = \int \frac{\sec^2 x}{\tan^2 x + 2 \tan x} dx$$

Let $\tan x = t$
 $\sec^2 x dx = dt$
 $= \int \frac{dt}{t^2 + 2t + (1)^2 - (1)^2}$
 $= \int \frac{dt}{(t+1)^2 - (1)^2}$
 $= \frac{1}{2} \log \left| \frac{t+1-1}{t+1+1} \right| + c$
 $= \frac{1}{2} \log \left| \frac{t}{t+2} \right| + c$

$$I = \frac{1}{2} \log \left| \frac{\tan x}{\tan x + 2} \right| + c$$

Indefinite Integrals Ex 19.22 Q11

$$\begin{aligned} \text{Let } I &= \int \frac{1}{\cos 2x + 3 \sin^2 x} dx \\ &= \int \frac{1}{2 \cos^2 x - 1 + 3 \sin^2 x} dx \end{aligned}$$

Diving numerator and denominator by $\cos^2 x$,

$$\begin{aligned} I &= \int \frac{\sec^2 x}{2 - \sec^2 x + 3 \tan^2 x} dx \\ &= \int \frac{\sec^2 x}{2 - (1 + \tan^2 x)^2 + 3 \tan^2 x} dx \\ &= \int \frac{\sec^2 x}{2 - 1 - \tan^2 x + 3 \tan^2 x} dx \\ &= \int \frac{dt}{1 + 2 \tan^2 x} \end{aligned}$$

$$\begin{aligned} \text{Let } \sqrt{2} \tan x &= t \\ \sqrt{2} \sec^2 x dx &= dt \\ I &= \frac{1}{\sqrt{2}} \int \frac{1}{1+t^2} dt \\ &= \frac{1}{\sqrt{2}} \tan^{-1} t + C \\ I &= \frac{1}{\sqrt{2}} \tan^{-1} (\sqrt{2} \tan x) + C \end{aligned}$$

Ex 19.23

Indefinite Integrals Ex 19.23 Q1

Let $I = \int \frac{1}{5 + 4 \cos x} dx$

Put $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$\begin{aligned} I &= \int \frac{1}{5 + 4 \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} dx \\ &= \int \frac{1 + \tan^2 \frac{x}{2}}{5 \left(1 + \tan^2 \frac{x}{2} \right) + 4 \left(1 - \tan^2 \frac{x}{2} \right)} dx \\ &= \int \frac{\sec^2 \frac{x}{2}}{5 + 5 \tan^2 \frac{x}{2} + 4 - 4 \tan^2 \frac{x}{2}} dx \\ &= \int \frac{\sec^2 \frac{x}{2}}{9 + \tan^2 \frac{x}{2}} dx \\ \text{Let } \tan \frac{x}{2} &= t \\ \frac{1}{2} \sec^2 \frac{x}{2} dx &= dt \\ &= \int \frac{2dt}{(3)^2 + t^2} \\ &= 2 \times \frac{1}{3} \tan^{-1}(t) + C \end{aligned}$$

$$I = \frac{2}{3} \tan^{-1} \left(\frac{\tan \frac{x}{2}}{3} \right) + C$$

Indefinite Integrals Ex 19.23 Q2

$$\text{Let } I = \int \frac{1}{5 - 4 \sin x} dx$$

$$\text{Put } \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$= \int \frac{1}{5 - 4 \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{5 \left(1 + \tan^2 \frac{x}{2} \right) - 4 \left(2 \tan \frac{x}{2} \right)} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{5 + 5 \tan^2 \frac{x}{2} - 8 \tan \frac{x}{2}} dx$$

$$\text{Let } \tan \frac{x}{2} = t$$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$= \int \frac{2dt}{5t^2 - 8t + 5}$$

$$= \frac{2}{5} \int \frac{dt}{t^2 - \frac{8}{5}t + 1}$$

$$= \frac{2}{5} \int \frac{dt}{t^2 - 2t \left(\frac{4}{5} \right) + \left(\frac{4}{5} \right)^2 - \left(\frac{4}{5} \right)^2 + 1}$$

$$I = \frac{2}{5} \int \frac{dt}{\left(t - \frac{4}{5} \right)^2 + \left(\frac{3}{5} \right)^2}$$

$$= \frac{2}{5} \times \frac{1}{\frac{3}{5}} \tan^{-1} \left(\frac{t - \frac{4}{5}}{\frac{3}{5}} \right) + C$$

$$= \frac{2}{3} \tan^{-1} \left(\frac{5t - 4}{3} \right) + C$$

$$I = \frac{2}{3} \tan^{-1} \left(\frac{5 \tan \frac{x}{2} - 4}{3} \right) + C$$

Indefinite Integrals Ex 19.23 Q3

$$\text{Let } I = \int \frac{1}{1 - 2 \sin x} dx$$

$$\text{Put } \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$I = \int \frac{1}{1 - 2 \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2} - 4 \tan \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{\tan^2 \frac{x}{2} - 4 \tan \frac{x}{2} + 1} dx$$

$$\text{Let } \tan \frac{x}{2} = t$$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{t^2 - 4t + 1}$$

$$= \int \frac{2dt}{t^2 - 2t(2) + (2)^2 - (2)^2 + 1}$$

$$I = 2 \int \frac{dt}{(t-2)^2 + 3}$$

$$= 2 \int \frac{dt}{(t-2)^2 + (\sqrt{3})^2}$$

$$= 2 \times \frac{1}{2\sqrt{3}} \log \left| \frac{t-2-\sqrt{3}}{t-2+\sqrt{3}} \right| + C$$

$$I = \frac{1}{\sqrt{3}} \log \left| \frac{\tan \frac{x}{2} - 2 - \sqrt{3}}{\tan \frac{x}{2} - 2 + \sqrt{3}} \right| + C$$

Indefinite Integrals Ex 19.23 Q4

Let $I = \int \frac{1}{4\cos x - 1} dx$

Put $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{4 \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) - 1} dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{4 \left(1 - \tan^2 \frac{x}{2} \right) - \left(1 + \tan^2 \frac{x}{2} \right)} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{4 - 4 \tan^2 \frac{x}{2} - 1 - \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{3 - 5 \tan^2 \frac{x}{2}} dx$$

Let $\sqrt{5} \tan \frac{x}{2} = t$

$$\frac{\sqrt{5}}{2} \sec^2 \frac{x}{2} dt = dt$$

$$I = \int \frac{dt}{(\sqrt{3})^2 - t^2}$$

$$I = \frac{1}{\sqrt{15}} \log \left| \frac{\sqrt{3} + \sqrt{5} \tan \frac{x}{2}}{\sqrt{3} - \sqrt{5} \tan \frac{x}{2}} \right| + c$$

Indefinite Integrals Ex 19.23 Q5

Let $I = \int \frac{1}{1 - \sin x + \cos x} dx$

Put $\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{1 - \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} + \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2} - 2 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{2 - 2 \tan \frac{x}{2}} dx$$

Let $\tan \frac{x}{2} = t$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$= \frac{1}{2} \int \frac{dt}{1-t}$$

$$= -\log|1-t| + c$$

$$I = -\log \left| 1 - \tan \frac{x}{2} \right| + c$$

Indefinite Integrals Ex 19.23 Q6

$$\text{Let } I = \int \frac{1}{3+2\sin x + \cos x} dx$$

$$\text{Put } \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \quad \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\begin{aligned} I &= \int \frac{1}{3 + 2 \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) + \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} dx \\ &= \int \frac{\left(1 + \tan^2 \frac{x}{2} \right)}{3 \left(1 + \tan^2 \frac{x}{2} \right) + 4 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} dx \\ &= \int \frac{\sec^2 \frac{x}{2}}{3 + 3 \tan^2 \frac{x}{2} + 4 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} dx \\ &= \int \frac{\sec^2 \frac{x}{2}}{2 \tan^2 \frac{x}{2} + 4 \tan \frac{x}{2} + 4} dx \end{aligned}$$

$$\text{Let } \tan \frac{x}{2} = t$$

$$\begin{aligned} \frac{1}{2} \sec^2 \frac{x}{2} dx &= dt \\ I &= \frac{1}{2} \int \frac{dt}{t^2 + 2t + 2} \\ &= \int \frac{dt}{t^2 + 2t + 1 - 1 + 2} \\ I &= \int \frac{dt}{(t+1)^2 + 1^2} \\ &= \tan^{-1}(t+1) + C \end{aligned}$$

$$I = \tan^{-1} \left(\tan \frac{x}{2} + 1 \right) + C$$

Indefinite Integrals Ex 19.23 Q7

Let $I = \int \frac{1}{13 + 3 \cos x + 4 \sin x} dx$

Put $\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \quad \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{13 + 3 \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) + 4 \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} dx$$

$$= \int \frac{\left(1 + \tan^2 \frac{x}{2} \right)}{13 \left(1 + \tan^2 \frac{x}{2} \right) + 3 - 3 \tan^2 \frac{x}{2} + 8 \tan \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{16 + 13 \tan^2 \frac{x}{2} - 3 \tan^2 \frac{x}{2} + 8 \tan \frac{x}{2}} dx$$

Let $\tan \frac{x}{2} = t$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{16 + 10t^2 + 8t}$$

$$= \frac{2}{10} \int \frac{dt}{t^2 + \frac{4}{5}t + \frac{8}{5}}$$

$$I = \frac{1}{5} \int \frac{dt}{t^2 + 2t \left(\frac{2}{5} \right)^2 + \left(\frac{2}{5} \right)^2 - \left(\frac{2}{5} \right)^2 + \frac{8}{5}}$$

$$= \frac{1}{5} \int \frac{dt}{\left(t + \frac{2}{5} \right)^2 + \left(\frac{6}{5} \right)^2}$$

$$= \frac{1}{5} \times \frac{1}{\left(\frac{6}{5} \right)} \tan^{-1} \left(\frac{t + \frac{2}{5}}{\frac{6}{5}} \right) + C$$

$$= \frac{1}{6} \tan^{-1} \left(\frac{5t + 2}{6} \right) + C$$

$$I = \frac{1}{6} \tan^{-1} \left(\frac{5 \tan \frac{x}{2} + 2}{6} \right) + C$$

Indefinite Integrals Ex 19.23 Q8

$$\text{Let } I = \int \frac{1}{\cos x - \sin x} dx$$

$$\text{Put } \cos x = \frac{1 - \tan^2 \frac{x}{2}}{2}, \sin x = \frac{2 \tan \frac{x}{2}}{2}$$

$$\begin{aligned} I &= \int \left(\frac{\frac{1}{1 - \tan^2 \frac{x}{2}}}{\frac{1 + \tan^2 \frac{x}{2}}{2}} - \frac{\frac{2 \tan \frac{x}{2}}{2}}{\frac{1 + \tan^2 \frac{x}{2}}{2}} \right) dx \\ &= \int \frac{\left(1 + \tan^2 \frac{x}{2} \right)}{1 - \tan^2 \frac{x}{2} - 2 \tan \frac{x}{2}} dx \\ &= \int \frac{\sec^2 \frac{x}{2}}{1 - \tan^2 \frac{x}{2} - 2 \tan \frac{x}{2}} dx \end{aligned}$$

$$\text{Let } \tan \frac{x}{2} = t$$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$\begin{aligned} I &= \int \frac{2dt}{1 - t^2 - 2t} \\ &= - \int \frac{2dt}{t^2 + 2t - 1} \\ I &= - \int \frac{2dt}{t^2 + 2t + 1 - 1 - 1} \\ I &= - \int \frac{2dt}{(t+1)^2 - (\sqrt{2})^2} \\ &= \int \frac{2dt}{(\sqrt{2})^2 - (t+1)^2} \\ &= \frac{2}{2\sqrt{2}} \log \left| \frac{\sqrt{2} + t + 1}{\sqrt{2} - t - 1} \right| + C \end{aligned}$$

$$I = \frac{1}{\sqrt{2}} \log \left| \frac{\sqrt{2} + \tan \frac{x}{2} + 1}{\sqrt{2} - \tan \frac{x}{2} - 1} \right| + C$$

Indefinite Integrals Ex 19.23 Q9

$$\text{Let } I = \int \frac{1}{\sin x + \cos x} dx$$

$$\text{Put } \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$I = \int \frac{1}{\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} + \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} + 1 - \frac{\tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} dx$$

$$\text{Let } \tan \frac{x}{2} = t$$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{2t+1-t^2}$$

$$= -2 \int \frac{dt}{t^2 - 2t - 1}$$

$$I = -2 \int \frac{dt}{t^2 - 2t + 1 - 1 - 1}$$

$$I = -2 \int \frac{dt}{(t-1)^2 - (\sqrt{2})^2}$$

$$= 2 \int \frac{2dt}{(\sqrt{2})^2 - (t-1)^2}$$

$$= \frac{2}{2\sqrt{2}} \log \left| \frac{\sqrt{2} + t - 1}{\sqrt{2} - t + 1} \right| + C$$

$$I = \frac{1}{\sqrt{2}} \log \left| \frac{\sqrt{2} + \tan \frac{x}{2} - 1}{\sqrt{2} - \tan \frac{x}{2} + 1} \right| + C$$

Indefinite Integrals Ex 19.23 Q10

Let $I = \int \frac{1}{5 - 4 \cos x} dx$

Put $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{5 - 4 \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} dx$$

$$= \int \frac{\left(1 + \tan^2 \frac{x}{2} \right)}{5 + 5 \tan^2 \frac{x}{2} - 4 + 4 \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{9 \tan^2 \frac{x}{2} + 1} dx$$

Let $3 \tan \frac{x}{2} = t$

$$\frac{3}{2} \sec^2 \frac{x}{2} dx = dt$$

$$I = \frac{2}{3} \int \frac{dt}{t^2 + 1}$$

$$= \frac{2}{3} \tan^{-1}(t) + C$$

$$I = \frac{2}{3} \tan^{-1} \left(3 \tan \frac{x}{2} \right) + C$$

Indefinite Integrals Ex 19.23 Q11

Let $I = \int \frac{1}{2 + \sin x + \cos x} dx$

Put $\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{2 + \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) + \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} dx$$

$$= \int \frac{\left(1 + \tan^2 \frac{x}{2} \right)}{2 + 2 \tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{\tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} + 3} dx$$

Let $\tan \frac{x}{2} = t$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{t^2 + 2t + 3}$$

$$= 2 \int \frac{dt}{t^2 + 2t + 1 - 1 + 3}$$

$$I = 2 \int \frac{dt}{(t+1)^2 + (\sqrt{2})^2}$$

$$= \frac{2}{\sqrt{2}} \tan^{-1} \left(\frac{t+1}{\sqrt{2}} \right) + C$$

$$I = \sqrt{2} \tan^{-1} \left(\frac{\tan \frac{x}{2} + 1}{\sqrt{2}} \right) + C$$

Indefinite Integrals Ex 19.23 Q12

Let $I = \int \frac{1}{\sin x + \sqrt{3} \cos x} dx$

Put $\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{\left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) + \sqrt{3} \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} dx$$

$$= \int \frac{\left(1 + \tan^2 \frac{x}{2} \right)}{2 \tan \frac{x}{2} + \sqrt{3} - \sqrt{3} \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{2 \tan \frac{x}{2} + \sqrt{3} - \sqrt{3} \tan^2 \frac{x}{2}} dx$$

Let $\tan \frac{x}{2} = t$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{2t + \sqrt{3} - \sqrt{3}t^2}$$

$$= -\frac{2}{\sqrt{3}} \int \frac{dt}{t^2 - \frac{2}{\sqrt{3}}t + \left(\frac{1}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2 - 1}$$

$$I = -\frac{2}{\sqrt{3}} \int \frac{dt}{\left(t - \frac{1}{\sqrt{3}}\right)^2 - \left(\frac{2}{\sqrt{3}}\right)^2}$$

$$= \frac{2}{\sqrt{3}} \int \frac{dt}{\left(\frac{2}{\sqrt{3}}\right)^2 - \left(t - \frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{2}{\sqrt{3}} \times \frac{1}{2\left(\frac{2}{\sqrt{3}}\right)} \log \left| \frac{\frac{2}{\sqrt{3}} + t + \frac{1}{\sqrt{3}}}{\frac{2}{\sqrt{3}} - t + \frac{1}{\sqrt{3}}} \right| + c$$

$$I = \frac{1}{2} \log \left| \frac{\sqrt{3}t + 1}{3 - \sqrt{3}t} \right| + c$$

$$I = \frac{1}{2} \log \left| \frac{1 + \sqrt{3} \tan \frac{x}{2}}{3 - \sqrt{3} \tan \frac{x}{2}} \right| + c$$

Indefinite Integrals Ex 19.23 Q13

Let $I = \int \frac{1}{\sqrt{3} \sin x + \cos x} dx$

Let $\sqrt{3} = r \cos \theta, \text{ and } 1 = r \sin \theta$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6}$$

$$r = \sqrt{3+1} = 2$$

$$I = \int \frac{1}{r \cos \theta \sin x + r \sin \theta \cos x} dx$$

$$= \frac{1}{r} \int \frac{1}{\sin(x + \theta)} dx$$

$$= \frac{1}{2} \int \csc(x + \theta) dx$$

$$= \frac{1}{2} \log \left| \tan \left(\frac{x}{2} + \frac{\theta}{2} \right) \right| + c$$

$$I = \frac{1}{2} \log \left| \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) \right| + c$$

Indefinite Integrals Ex 19.23 Q14

Let $I = \int \frac{1}{\sin x - \sqrt{3} \cos x} dx$

Let $1 = r \cos \theta$, and $\sqrt{3} = r \sin \theta$
 $r = \sqrt{3+1} = 2$
 $\tan \theta = \sqrt{3}$
 $\theta = \frac{\pi}{3}$

$$I = \int \frac{1}{r \cos \theta \sin x - r \sin \theta \cos x} dx$$

$$= \frac{1}{2} \int \frac{1}{\sin(x - \theta)} dx$$

$$= \frac{1}{2} \int \csc(x - \theta) dx$$

$$= \frac{1}{2} \log \left| \tan \left(\frac{x}{2} - \frac{\theta}{2} \right) \right| + c$$

$$I = \frac{1}{2} \log \left| \tan \left(\frac{x}{2} - \frac{\pi}{6} \right) \right| + c$$

Indefinite Integrals Ex 19.23 Q15

Let $I = \int \frac{1}{5 + 7 \cos x + \sin x} dx$

Put $\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$, $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

Now,

$$I = \int \frac{1}{5 + \frac{7(1 - \tan^2 \frac{x}{2})}{1 + \tan^2 \frac{x}{2}} + \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} dx$$

$$= \int \frac{\left(1 + \tan^2 \frac{x}{2} \right)}{5 \left(1 + \tan^2 \frac{x}{2} \right) + 7 - 7 \tan^2 \frac{x}{2} + 2 \tan \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{-2 \tan^2 \frac{x}{2} + 12 + 2 \tan \frac{x}{2}} dx$$

Let $\tan \frac{x}{2} = t$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{-2t^2 + 12 + 2t}$$

$$= - \int \frac{dt}{t^2 - t - 6}$$

$$= - \int \frac{dt}{t^2 - 2t \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right)^2 - \left(\frac{1}{2} \right)^2 - 6}$$

$$= - \int \frac{dt}{\left(t - \frac{1}{2} \right)^2 - \left(\frac{5}{2} \right)^2}$$

$$= - \frac{1}{2 \left(\frac{5}{2} \right)} \log \left| \frac{t - \frac{1}{2} - \frac{5}{2}}{t - \frac{1}{2} + \frac{5}{2}} \right| + c$$

$$= - \frac{1}{5} \log \left| \frac{t - 3}{t + 2} \right| + c$$

$$I = \frac{1}{5} \log \left| \frac{\tan \frac{x}{2} + 2}{\tan \frac{x}{2} - 3} \right| + c$$

Ex 19.24

Indefinite Integrals Ex 19.24 Q1

$$\begin{aligned} \text{Let } I &= \int \frac{1}{1 - \cot x} dx \\ &= \int \frac{1}{1 - \frac{\cos x}{\sin x}} dx \\ &= \int \frac{\sin x}{\sin x - \cos x} dx \end{aligned}$$

$$\begin{aligned} \text{Let } \sin x &= \lambda \frac{d}{dx} (\sin x - \cos x) + \mu (\sin x - \cos x) + v \\ \sin x &= \lambda \frac{d}{dx} (\cos x + \sin x) + \mu (\sin x - \cos x) + v \\ \sin x &= \cos(\lambda - \mu) + \sin x(\lambda + \mu) + v \end{aligned}$$

Comparing the coefficients of $\sin x$ & $\cos x$ on the both the sides,

$$\lambda + \mu = 1 \quad \dots (1)$$

$$\lambda - \mu = 1 \quad \dots (2)$$

$$v = 0 \quad \dots (3)$$

Equation (1), (2), (3) gives

$$\begin{aligned} \lambda &= \frac{1}{2}, \mu = \frac{1}{2}, v = 0 \\ I &= \int \frac{\frac{1}{2}(\cos x + \sin x) + \frac{1}{2}(\sin x - \cos x)}{(\sin x - \cos x)} dx \\ &= \frac{1}{2} \int \frac{(\cos x + \sin x)}{(\sin x - \cos x)} dx + \frac{1}{2} \int dx \\ I &= \frac{1}{2} \log |\sin x - \cos x| + \frac{1}{2}x + C \end{aligned}$$

Indefinite Integrals Ex 19.24 Q2

$$\begin{aligned} \text{Let } I &= \int \frac{1}{1 - \tan x} dx \\ &= \int \frac{1}{1 - \frac{\sin x}{\cos x}} dx \\ &= \int \frac{\cos x}{\cos x - \sin x} dx \end{aligned}$$

$$\begin{aligned} \text{Let } \cos x &= \lambda \frac{d}{dx} (\cos x - \sin x) + \mu (\cos x - \sin x) + v \\ &= \lambda \frac{d}{dx} (-\sin x - \cos x) + \mu (\cos x - \sin x) + v \\ \cos x &= \sin x(-\lambda - \mu) + \cos x(-\lambda + \mu) + v \end{aligned}$$

Comparing the coefficients of $\cos x$ & $\sin x$ on the both the sides,

$$-\lambda - \mu = 0 \quad \dots (1)$$

$$-\lambda + \mu = 1 \quad \dots (2)$$

$$v = 0 \quad \dots (3)$$

Equation (1), (2), (3) gives

$$\begin{aligned} \lambda &= -\frac{1}{2}, \mu = \frac{1}{2}, v = 0 \\ I &= \int \frac{-\frac{1}{2}(-\sin x - \cos x) + \frac{1}{2}(\cos x - \sin x)}{(\cos x - \sin x)} dx \\ &= \frac{1}{2} \int \frac{(\sin x + \cos x)}{(\cos x - \sin x)} dx + \frac{1}{2} \int dx \\ I &= -\frac{1}{2} \log |\cos x - \sin x| + \frac{1}{2}x + C \end{aligned}$$

$$I = \frac{1}{2}x - \frac{1}{2} \log |\cos x - \sin x| + C$$

Indefinite Integrals Ex 19.24 Q3

Let $I = \int \frac{3+2\cos x + 4\sin x}{2\sin x + \cos x + 3} dx$

Let $3+2\cos x + 4\sin x = \lambda \frac{d}{dx}(2\sin x + \cos x + 3) + \mu(2\sin x + \cos x + 3) + \nu$
 $3+2\cos x + 4\sin x = \lambda(2\cos x - \sin x) + \mu(2\sin x + \cos x + 3) + \nu$
 $3+2\cos x + 4\sin x = (-\lambda + 2\mu)\sin x + (2\lambda + \mu)\cos x + 3\mu + \nu$

Comparing the coefficients of $\sin x$ & $\cos x$ on the both the sides,

$$-\lambda + 2\mu = 4 \quad \dots \dots \dots (1)$$

$$2\lambda + \mu = 2 \quad \dots \dots \dots (2)$$

$$2\mu + \nu = 3 \quad \dots \dots \dots (3)$$

Solving equation (1), (2) and (3), we get

$$\lambda = 0, \mu = 2, \nu = -3$$

$$\begin{aligned} I &= \int \frac{2(2\sin x + \cos x + 3) - 3}{(2\sin x + \cos x + 3)} dx \\ &= 2 \int dx - 3 \int \frac{1}{2\sin x + \cos x + 3} dx \\ I &= 2x - 3I_1 + C_1 \end{aligned} \quad \dots \dots \dots (4)$$

Let $I_1 = \int \frac{1}{2\sin x + \cos x + 3} dx$

Put $\sin x = \frac{2\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$\begin{aligned} I_1 &= \int \frac{1}{2\left(\frac{2\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) + \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) + 3} dx \\ &= \int \frac{\left(1 + \tan^2 \frac{x}{2}\right)}{4\tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2} + 3\left(1 + \tan^2 \frac{x}{2}\right)} dx \\ &= \int \frac{\sec^2 \frac{x}{2}}{2\tan^2 \frac{x}{2} + 4\tan \frac{x}{2} + 4} dx \end{aligned}$$

Let $\tan \frac{x}{2} = t$

$$\frac{1}{2}\sec^2 \frac{x}{2} dt = dt$$

$$I_1 = \int \frac{2dt}{2t^2 + 4t + 4}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 + 2t + 2}$$

$$= \int \frac{dt}{t^2 + 2t + 1 - 1 + 2}$$

$$= \int \frac{dt}{(t+1)^2 + 1}$$

$$= \tan^{-1}(t+1) + C_2$$

$$= \tan^{-1}\left(\tan \frac{x}{2} + 1\right) + C_2$$

Now, using equation (1),

$$I = 2x - 3 \tan^{-1}\left(\tan \frac{x}{2} + 1\right) + C$$

Indefinite Integrals Ex 19.24 Q4

$$\begin{aligned} \text{Let } I &= \int \frac{1}{p+q \tan x} dx \\ &= \int \frac{1}{p+q \left(\frac{\sin x}{\cos x} \right)} dx \\ &= \int \frac{\cos x}{p \cos x + q \sin x} dx \end{aligned}$$

$$\begin{aligned} \text{Let } \cos x &= \lambda \frac{d}{dx} (p \cos x + q \sin x) + \mu (p \cos x + q \sin x) + v \\ \cos x &= \lambda (-p \sin x + q \cos x) + \mu (p \cos x + q \sin x) + v \\ \cos x &= (-p\lambda + q\mu) \sin x + (q\lambda + p\mu) \cos x + v \end{aligned}$$

Comparing the coefficients of $\sin x, \cos x$ on the both the sides,

$$-p\lambda + q\mu = 0 \quad \dots \dots \dots (1)$$

$$q\lambda + p\mu = 1 \quad \dots \dots \dots (2)$$

$$v = 0 \quad \dots \dots \dots (3)$$

Solving equation (1), (2) and (3),

$$\lambda = \frac{q}{(p^2 + q^2)}$$

$$\mu = \frac{p}{(p^2 + q^2)}$$

$$v = 0$$

Now,

$$\begin{aligned} I &= \int \frac{q}{(p^2 + q^2)} \frac{(-p \sin x + q \cos x)}{(p \cos x + q \sin x)} dx + \int \frac{p}{(p^2 + q^2)} \frac{(p \cos x + q \sin x)}{(p \cos x + q \sin x)} dx \\ I &= \frac{q}{(p^2 + q^2)} (\log |p \cos x + q \sin x|) + \frac{p}{(p^2 + q^2)} x + C \end{aligned}$$

Indefinite Integrals Ex 19.24 Q5

$$\text{Let } I = \int \frac{5 \cos x + 6}{2 \cos x + \sin x + 3} dx$$

$$\begin{aligned} \text{Let } (5 \cos x + 6) &= \lambda \frac{d}{dx} (2 \cos x + \sin x + 3) + \mu (2 \cos x + \sin x + 3) + v \\ (5 \cos x + 6) &= \lambda (-2 \sin x + \cos x) + \mu (2 \cos x + \sin x + 3) + v \\ (5 \cos x + 6) &= (-2\lambda + \mu) \sin x + (\lambda + 2\mu) \cos x + (3\mu + v) \end{aligned}$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$-2\lambda + \mu = 0 \quad \dots \dots \dots (1)$$

$$\lambda + 2\mu = 5 \quad \dots \dots \dots (2)$$

$$3\mu + v = 6 \quad \dots \dots \dots (3)$$

Solving equation (1), (2) and (3),

$$\lambda = 1$$

$$\mu = 2$$

$$v = 0$$

Now,

$$I = \int \frac{(-2 \sin x + \cos x)}{(2 \cos x + \sin x + 3)} dx + 2 \int dx$$

$$I = \log |2 \cos x + \sin x + 3| + 2x + C$$

Indefinite Integrals Ex 19.24 Q6

Let $I = \int \frac{2 \sin x + 3 \cos x}{3 \sin x + 4 \cos x} dx$

Let $(2 \sin x + 3 \cos x) = \lambda \frac{d}{dx}(3 \sin x + 4 \cos x) + \mu(3 \sin x + 4 \cos x) + v$

$$(2 \sin x + 3 \cos x) = \lambda(3 \cos x - 4 \sin x) + \mu(3 \sin x + 4 \cos x) + v$$

$$(2 \sin x + 3 \cos x) = (3\lambda + 4\mu) \cos x + (-4\lambda + 3\mu) \sin x + v$$

Camparing the coefficients of $\sin x, \cos x$ on the both the sides,

$$3\lambda + 4\mu = 3 \quad \dots \dots \dots (1)$$

$$-4\lambda + 3\mu = 2 \quad \dots \dots \dots (2)$$

$$v = 0 \quad \dots \dots \dots (3)$$

Solving the equation (1), (2) and (3),

$$\lambda = \frac{1}{25}$$

$$\mu = \frac{18}{25}$$

$$v = 0$$

$$I = \frac{1}{25} \int \frac{(3 \cos x - 4 \sin x)}{(3 \sin x + 4 \cos x)} dx + \frac{18}{25} \int dx$$

$$I = \frac{1}{25} \log|3 \sin x + 4 \cos x| + \frac{18}{25} x + c$$

Indefinite Integrals Ex 19.24 Q7

Let $I = \int \frac{1}{3 + 4 \cot x} dx$

$$= \int \frac{\sin x}{3 \sin x + 4 \cos x} dx$$

Let $\sin x = \lambda \frac{d}{dx}(3 \sin x + 4 \cos x) + \mu(3 \sin x + 4 \cos x) + v$

$$\sin x = \lambda(3 \cos x - 4 \sin x) + \mu(3 \sin x + 4 \cos x) + v$$

$$\sin x = (3\lambda + 4\mu) \cos x + (-4\lambda + 3\mu) \sin x + v$$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$3\lambda + 4\mu = 0 \quad \dots \dots \dots (1)$$

$$-4\lambda + 3\mu = 1 \quad \dots \dots \dots (2)$$

$$v = 0 \quad \dots \dots \dots (3)$$

Solving the equation (1), (2) and (3), we get

$$\lambda = -\frac{4}{25}$$

$$\mu = \frac{3}{25}$$

$$v = 0$$

$$I = -\frac{4}{25} \int \frac{(3 \cos x - 4 \sin x)}{(3 \sin x + 4 \cos x)} dx + \frac{3}{25} \int dx$$

$$I = -\frac{4}{25} \log|3 \sin x + 4 \cos x| + \frac{3}{25} x + c$$

Indefinite Integrals Ex 19.24 Q8

$$\begin{aligned}
 \text{Let } I &= \int \frac{2 \tan x + 3}{3 \tan x + 4} dx \\
 &= \int \frac{2 \sin x + 3 \cos x}{3 \sin x + 4 \cos x} dx \\
 \text{Let } 2 \sin x + 3 \cos x &= \lambda \frac{d}{dx}(3 \sin x + 4 \cos x) + \mu(3 \sin x + 4 \cos x) + v \\
 2 \sin x + 3 \cos x &= \lambda(3 \cos x - 4 \sin x) + \mu(3 \sin x + 4 \cos x) + v \\
 2 \sin x + 3 \cos x &= (3\lambda + 4\mu) \cos x + (-4\lambda + 3\mu) \sin x + v
 \end{aligned}$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$3\lambda + 4\mu = 3 \quad \dots \dots \dots (1)$$

$$-4\lambda + 3\mu = 2 \quad \dots \dots \dots (2)$$

$$v = 0$$

Solving the equation (1), (2) and (3),

$$\mu = \frac{18}{25}$$

$$\lambda = \frac{1}{25}$$

$$v = 0$$

$$I = \frac{1}{25} \int \frac{(3 \cos x - 4 \sin x)}{(3 \sin x + 4 \cos x)} dx + \frac{18}{25} \int dx$$

$$I = \frac{18}{25} x + \frac{1}{25} \log|3 \sin x + 4 \cos x| + c$$

Indefinite Integrals Ex 19.24 Q9

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{4 + 3 \tan x} dx \\
 &= \int \frac{\cos x}{4 \cos x + 3 \sin x} dx \\
 \text{Let } \cos x &= \lambda \frac{d}{dx}(4 \cos x + 3 \sin x) + \mu(4 \cos x + 3 \sin x) + v \\
 \cos x &= \lambda(-4 \sin x + 3 \cos x) + \mu(4 \cos x + 3 \sin x) + v \\
 \cos x &= (-4\lambda + 3\mu) \sin x + (3\lambda + 4\mu) \cos x + v
 \end{aligned}$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$-4\lambda + 3\mu = 0 \quad \dots \dots \dots (1)$$

$$3\lambda + 4\mu = 1 \quad \dots \dots \dots (2)$$

$$v = 0 \quad \dots \dots \dots (3)$$

Solving the equation (1), (2) and (3),

$$\lambda = \frac{3}{25}$$

$$\mu = \frac{4}{25}$$

$$v = 0$$

$$I = \int \frac{3}{25} \frac{(-4 \sin x + 3 \cos x)}{(4 \cos x + 3 \sin x)} dx + \frac{4}{25} \int dx$$

$$I = \frac{3}{25} \log|4 \cos x + 3 \sin x| + \frac{4}{25} x + c$$

Indefinite Integrals Ex 19.24 Q10

$$\begin{aligned} \text{Let } I &= \int \frac{8 \cot x + 1}{3 \cot x + 2} dx \\ &I = \int \frac{8 \cos x + \sin x}{3 \cos x + 2 \sin x} dx \\ \text{Let } 8 \cos x + \sin x &= \lambda \frac{d}{dx}(3 \cos x + 2 \sin x) + \mu(3 \cos x + 2 \sin x) + v \\ 8 \cos x + \sin x &= \lambda(-3 \sin x + 2 \cos x) + \mu(3 \cos x + 2 \sin x) + v \\ 8 \cos x + \sin x &= (-3\lambda + 2\mu) \sin x + (2\lambda + 3\mu) \cos x + v \end{aligned}$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$\begin{aligned} 2\lambda + 3\mu &= 8 \quad \dots \dots \dots (1) \\ -3\lambda + 2\mu &= 1 \quad \dots \dots \dots (2) \\ v &= 0 \quad \dots \dots \dots (3) \end{aligned}$$

Solving equation (1), (2) and (3),

$$\begin{aligned} \lambda &= 1 \\ \mu &= 2 \\ v &= 0 \\ I &= \int \frac{(-3 \sin x + 2 \cos x)}{(3 \cos x + 2 \sin x)} dx + 2 \int dx \end{aligned}$$

$$I = \log|3 \cos x + 2 \sin x| + 2x + C$$

Indefinite Integrals Ex 19.24 Q11

$$\begin{aligned} \text{Let } I &= \int \frac{4 \sin x + 5 \cos x}{5 \sin x + 4 \cos x} dx \\ \text{Let } 4 \sin x + 5 \cos x &= \lambda \frac{d}{dx}(5 \sin x + 4 \cos x) + \mu(5 \sin x + 4 \cos x) + v \\ 4 \sin x + 5 \cos x &= \lambda(5 \cos x - 4 \sin x) + \mu(5 \sin x + 4 \cos x) + v \\ 4 \sin x + 5 \cos x &= (5\lambda + 4\mu) \cos x + (-4\lambda + 5\mu) \sin x + v \end{aligned}$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$\begin{aligned} -4\lambda + 5\mu &= 4 \quad \dots \dots \dots (1) \\ 5\lambda + 4\mu &= 5 \quad \dots \dots \dots (2) \\ v &= 0 \quad \dots \dots \dots (3) \end{aligned}$$

Solving equation (1), (2) and (3),

$$\begin{aligned} \lambda &= \frac{9}{41} \\ \mu &= \frac{40}{41} \\ v &= 0 \end{aligned}$$

Now,

$$I = \frac{40}{41} \int dx + \frac{9}{41} \int \frac{(5 \cos x - 4 \sin x)}{(5 \sin x + 4 \cos x)} dx$$

$$I = \frac{40}{41} x + \frac{9}{41} \log|5 \sin x + 4 \cos x| + C$$

Ex 19.25

Indefinite Integrals Ex 19.25 Q1

Let $I = \int x \cos x dx$

Using integration by parts,

$$\begin{aligned} I &= x \int \cos x dx - \int (1 \times \int \cos x dx) dx + c \\ &= x \sin x - \int \sin x dx + c \end{aligned}$$

$$I = x \sin x + \cos x + c$$

Indefinite Integrals Ex 19.25 Q2

Let $I = \int \log(x+1) dx$
 $= \int 1 \times \log(x+1) dx$

Using integration by parts,

$$\begin{aligned} I &= \log(x+1) \int 1 dx - \int \left(\frac{1}{x+1} \times \int 1 dx \right) dx + c \\ &= x \log(x+1) - \int \left(\frac{x}{x+1} \right) dx + c \\ &= x \log(x+1) - \int \left(1 - \frac{1}{x+1} \right) dx + c \end{aligned}$$

$$I = x \log(x+1) - x + \log(x+1) + c$$

Indefinite Integrals Ex 19.25 Q3

Let $I = \int x^3 \log x dx$

Using integration by parts,

$$\begin{aligned} I &= \log x \int x^3 dx - \int \left(\frac{1}{x} \times \int x^3 dx \right) dx + c \\ &= \frac{x^4}{4} \log x - \int \frac{x^4}{4x} dx + c \\ &= \frac{x^4}{4} \log x - \frac{1}{4} \int x^3 dx + c \\ &= \frac{x^4}{4} \log x - \frac{1}{4} \int \frac{x^4}{4} dx + c \end{aligned}$$

$$I = \frac{x^4}{4} \log x - \frac{1}{16} x^4 + c$$

Indefinite Integrals Ex 19.25 Q4

Take first function as x and second function as e^x . The integral of the second function is e^x .
Therefore, $\int x e^x dx = x e^x - \int 1 \cdot e^x dx = x e^x - e^x + C$.

Indefinite Integrals Ex 19.25 Q5

Let $I = \int x e^{2x} dx$

Using integration by parts,

$$\begin{aligned} I &= x \int e^{2x} dx - \int (1 \times \int e^{2x} dx) dx + c \\ &= \frac{x e^{2x}}{2} - \int \left(\frac{e^{2x}}{2} \right) dx + c \\ &= \frac{x e^{2x}}{2} - \frac{e^{2x}}{4} + c \end{aligned}$$

$$I = \left(\frac{x}{2} - \frac{1}{4} \right) e^{2x} + c$$

Indefinite Integrals Ex 19.25 Q6

Let $I = \int x^2 e^{-x} dx$

Using integration by parts,

$$\begin{aligned} I &= x^2 \int e^{-x} dx - \int (2x) (e^{-x}) dx \\ &= -x^2 e^{-x} - \int (2x) (-e^{-x}) dx \\ &= -x^2 e^{-x} + 2 \int x e^{-x} dx \\ &= -x^2 e^{-x} + 2 \left[x \int e^{-x} dx - \int (1 \times e^{-x}) dx \right] \\ &= -x^2 e^{-x} + 2 \left[x (-e^{-x}) - \int (-e^{-x}) dx \right] \\ &= -x^2 e^{-x} - 2x e^{-x} + 2 \int e^{-x} dx \\ I &= -x^2 e^{-x} - 2x e^{-x} - 2 e^{-x} + c \\ I &= -e^{-x} (x^2 + 2x + 2) + c \end{aligned}$$

Indefinite Integrals Ex 19.25 Q7

Let $I = \int x^2 \cos x dx$

Using integration by parts,

$$\begin{aligned} I &= x^2 \int \cos x dx - \int (2x) (\cos x) dx \\ &= x^2 \sin x - 2 \int (x) (\sin x) dx \\ &= x^2 \sin x - 2 \left[x \int \sin x dx - \int (1 \times \sin x) dx \right] \\ &= x^2 \sin x - 2 \left[x (-\cos x) - \int (-\cos x) dx \right] \\ &= x^2 \sin x + 2x \cos x - 2 \int (\cos x) dx \end{aligned}$$

$$I = x^2 \sin x + 2x \cos x - 2 \sin x + c$$

Indefinite Integrals Ex 19.25 Q8

Let $I = \int x^2 \cos 2x dx$

Using integration by parts,

$$\begin{aligned} I &= x^2 \int \cos 2x dx - \int (2x) (\cos 2x) dx \\ &= x^2 \frac{\sin 2x}{2} - 2 \int x \left(\frac{\sin 2x}{2} \right) dx \\ &= \frac{1}{2} x^2 \sin 2x - \int x \sin 2x dx \\ &= \frac{1}{2} x^2 \sin 2x - \left[x \int \sin 2x dx - \int (1 \sin 2x) dx \right] \\ &= \frac{1}{2} x^2 \sin 2x - \left[x \left(-\frac{\cos 2x}{2} \right) - \int \left(-\frac{\cos 2x}{2} \right) dx \right] \\ &= \frac{1}{2} x^2 \sin 2x + \frac{x}{2} \cos 2x - \frac{1}{2} \int (\cos 2x) dx \end{aligned}$$

$$I = \frac{1}{2} x^2 \sin 2x + \frac{x}{2} \cos 2x - \frac{1}{4} \sin 2x + c$$

Indefinite Integrals Ex 19.25 Q9

Let $I = \int x \sin 2x dx$

Using integration by parts,

$$\begin{aligned} I &= x \int \sin 2x dx - \int (1) (\sin 2x) dx \\ &= x \left(-\frac{\cos 2x}{2} \right) - \int \left(-\frac{\cos 2x}{2} \right) dx \\ &= -\frac{x}{2} \cos 2x + \frac{1}{2} \int \cos 2x dx \\ &= -\frac{x}{2} \cos 2x + \frac{1}{2} \frac{\sin 2x}{2} + c \\ I &= -\frac{x}{2} \cos 2x + \frac{1}{4} \sin 2x + c \end{aligned}$$

Indefinite Integrals Ex 19.25 Q10

$$\text{Let } I = \int \frac{\log(\log x)}{x} dx$$

$$= \int \left(\frac{1}{x} \right) (\log(\log x)) dx$$

Using integration by parts,

$$I = \log \log x \int \frac{1}{x} dx - \int \left(\frac{1}{x \log x} \int \frac{1}{x} dx \right) dx$$

$$= \log x \times \log(\log x) - \int \left(\frac{1}{x \log x} \log x \right) dx$$

$$= \log x \times \log(\log x) - \int \frac{1}{x} dx$$

$$= \log x \times \log(\log x) - \log x + c$$

$$I = \log x (\log \log x - 1) + c$$

Indefinite Integrals Ex 19.25 Q11

$$\text{Let } I = \int x^2 \cos x dx$$

Using integration by parts,

$$I = x^2 \int \cos x dx - \int (2x \int \cos x dx) dx$$

$$= x^2 \sin x - 2 \int x \sin x dx$$

$$= x^2 \sin x - 2 [x \int \sin x dx - \int (1 \int \sin x dx) dx]$$

$$= x^2 \sin x - 2 [x (-\cos x) - \int (-\cos x) dx]$$

$$= x^2 \sin x + 2x \cos x - 2 \int (\cos x) dx$$

$$I = x^2 \sin x + 2x \cos x - 2 \sin x + c$$

Indefinite Integrals Ex 19.25 Q12

$$\text{Let } I = \int x \csc^2 x dx$$

Using integration by parts,

$$I = x \int \csc^2 x dx - \int (1 \int \csc^2 x dx) dx$$

$$= -x \cot x + \int \cot x dx$$

$$= -x \cot x + \log |\sin x| + c$$

Indefinite Integrals Ex 19.25 Q13

$$\text{Let } I = \int x \cos^2 x dx$$

Using integration by parts,

$$I = x \int \cos^2 x dx - \int (1 \int \cos^2 x dx) dx$$

$$= x \int \left(\frac{\cos 2x + 1}{2} \right) dx - \int \left(\int \left(\frac{1 + \cos 2x}{2} \right) dx \right) dx$$

$$= \frac{x}{2} \left[\frac{\sin 2x}{2} + x \right] - \frac{1}{2} \int \left(x + \frac{\sin 2x}{2} \right) dx$$

$$= \frac{x}{4} \sin 2x + \frac{x^2}{2} - \frac{1}{2} \times \frac{x^2}{2} - \frac{1}{4} \left(-\frac{\cos 2x}{2} \right) + c$$

$$I = \frac{x}{4} \sin 2x + \frac{x^2}{4} + \frac{1}{8} \cos 2x + c$$

Indefinite Integrals Ex 19.25 Q14

Let $I = \int x^n \log x \, dx$

Using integration by parts,

$$\begin{aligned} I &= \log x \int x^2 \, dx - \int \left(\frac{1}{x} \int x^2 \, dx \right) dx \\ &= \frac{x^{n+1}}{n+1} \log x - \int \left(\frac{1}{x} \times \frac{x^{n+1}}{n+1} \right) dx \\ &= \frac{x^{n+1}}{n+1} \log x - \int \left(\frac{x^n}{n+1} \right) dx \end{aligned}$$

$$I = \frac{x^{n+1}}{n+1} \log x - \frac{1}{(n+1)^2} x^{n+1} + C$$

Indefinite Integrals Ex 19.25 Q15

$$\int \frac{\log x}{x^n} \, dx = \int (\log x) \left(\frac{1}{x^n} \right) dx$$

by integration by parts

$$\begin{aligned} \int (\log x) \left(\frac{1}{x^n} \right) dx &= \log x \int \left(\frac{1}{x^n} \right) dx - \int \left(\frac{d(\log x)}{dx} \right) \left(\int \left(\frac{1}{x^n} \right) dx \right) dx \\ &= \log x \left(\frac{x^{1-n}}{1-n} \right) - \int \frac{1}{x} \left(\frac{x^{1-n}}{1-n} \right) dx = \log x \left(\frac{x^{1-n}}{1-n} \right) - \int \left(\frac{x^{-n}}{1-n} \right) dx \\ &= \log x \left(\frac{x^{1-n}}{1-n} \right) - \left(\frac{1}{1-n} \right) \left(\frac{x^{1-n}}{1-n} \right) = \log x \left(\frac{x^{1-n}}{1-n} \right) - \left(\frac{x^{1-n}}{(1-n)^2} \right) + C \end{aligned}$$

Indefinite Integrals Ex 19.25 Q16

$$\begin{aligned} \text{Let } I &= \int x^2 \sin^2 x \, dx \\ &= \int x^2 \left(\frac{1 - \cos 2x}{2} \right) dx \\ &= \int \frac{x^2}{2} dx - \int \left(\frac{x^2 \cos 2x}{2} \right) dx \\ &= \frac{x^3}{6} - \frac{1}{2} \left[\int x^2 \cos 2x \, dx \right] \\ &= \frac{x^3}{6} - \frac{1}{2} \left[x^2 \int \cos 2x \, dx - \int (2x \int \cos 2x \, dx) \, dx \right] \\ &= \frac{x^3}{6} - \frac{1}{2} \left(x^2 \frac{\sin 2x}{2} \right) + \frac{1}{2} \times 2 \int \left(x \frac{\sin 2x}{2} \right) dx \\ &= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x + \frac{1}{2} \left[x \int \sin 2x \, dx - \int (1 \int \sin 2x \, dx) \, dx \right] \\ &= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x + \frac{1}{2} \left[x \left(-\frac{\cos 2x}{2} \right) - \int \left(-\frac{\cos 2x}{2} \right) dx \right] \\ &= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{4} \frac{\sin 2x}{2} + C \\ I &= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + C \end{aligned}$$

Indefinite Integrals Ex 19.25 Q17

$$\text{Let } I = \int 2x^3 e^{x^2} x \, dx$$

$$\text{Let } x^2 = t$$

$$2x \, dx = dt$$

$$I = \int t x e^t dt$$

Using integration by parts,

$$\begin{aligned} &= t \int e^t dt - \int (1 \times \int e^t dt) dt \\ &= t e^t - \int e^t dt \\ &= t e^t - e^t + C \\ &= e^t (t - 1) + C \end{aligned}$$

$$I = e^{x^2} (x^2 - 1) + C$$

Indefinite Integrals Ex 19.25 Q18

Let $I = \int x^3 \cos x^2 dx$

Let $x^2 = t$

$2x dx = dt$

$I = \frac{1}{2} \int t \cos t dt$

Using integration by parts,

$$= \frac{1}{2} [t \int \cos t dt - \int (1 \times \int \cos t dt) dt]$$

$$= \frac{1}{2} [t \sin t - \int \sin t dt]$$

$$= \frac{1}{2} [t \sin t + \cos t] + c$$

$$I = \frac{1}{2} [x^2 \sin x^2 + \cos x^2] + c$$

Indefinite Integrals Ex 19.25 Q19

Let $I = \int x \sin x \cos x dx$

$$= \int \frac{x}{2} (2 \sin x \cos x) dx$$

$$= \frac{1}{2} \int x \sin 2x dx$$

Using integration by parts,

$$= \frac{1}{2} [x \int \sin 2x dx - \int (1 \times \int \sin 2x dx) dx]$$

$$= \frac{1}{2} \left[x \left(\frac{-\cos 2x}{2} \right) - \int \left(\frac{-\cos 2x}{2} \right) dx \right]$$

$$= -\frac{1}{4} x \cos 2x + \frac{1}{4} \int \cos 2x dx$$

$$I = -\frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + c$$

Indefinite Integrals Ex 19.25 Q20

Let $I = \int \sin x (\log \cos x) dx$

Let $\cos x = t$

$-\sin x dx = dt$

$I = -\int \log t dt$

$= -\int 1 \times \log t dt$

Using integration by parts,

$$= -\left[\log t dt - \int \left(\frac{1}{t} \times \int dt \right) dt \right]$$

$$= -\left[t \log t - \int \frac{1}{t} \times t dt \right]$$

$$= -[t \log t - \int dt]$$

$$= -[t \log t - t + c_1]$$

$$= t(1 - \log t) + c$$

$$I = \cos x (1 - \log \cos x) + c$$

Indefinite Integrals Ex 19.25 Q21

Let $I = \int (\log x)^2 x dx$

Using integration by parts,

$$\begin{aligned} &= (\log x)^2 \int x dx - \int \left(2(\log x) \left(\frac{1}{x} \right) \int x dx \right) dx \\ &= \frac{x^2}{2} (\log x)^2 - 2 \int (\log x) \left(\frac{1}{x} \right) \left[\frac{x^2}{2} \right] dx \\ &= \frac{x^2}{2} (\log x)^2 - \left[\log x \int x dx - \int \left(\frac{1}{x} \int x dx \right) dx \right] \\ &= \frac{x^2}{2} (\log x)^2 - \left[\frac{x^2}{2} \log x - \int \left(\frac{1}{x} \times \frac{x^2}{2} \right) dx \right] \\ &= \frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{1}{2} \int x dx \\ &= \frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{1}{4} x^2 + c \end{aligned}$$

$$I = \frac{x^2}{2} \left[(\log x)^2 - \log x + \frac{1}{2} \right] + c$$

Indefinite Integrals Ex 19.25 Q22

Let $I = \int e^{\sqrt{x}} dx$

Let $\sqrt{x} = t$

$x = t^2$

$dx = 2t dt$

$I = 2 \int e^t t dt$

$I = 2 \left[t \int e^t dt - \int (1 \int e^t dt) dt \right]$

$I = 2 \left[t e^t - \int e^t dt \right]$

$= 2 \left[t e^t - e^t \right] + c$

$= 2e^t (t - 1) + c$

$$I = 2e^{\sqrt{x}} (\sqrt{x} - 1) + c$$

Indefinite Integrals Ex 19.25 Q23

Let $I = \int \frac{\log(x+2)}{(x+2)^2} dx$

Let $\frac{1}{x+2} = t$

$-\frac{1}{(x+2)^2} dx = dt$

$I = -\int \log \left(\frac{1}{t} \right) dt$

$= -\int \log t^{-1} dt$

$= -\int 1 \times \log t dt$

Using integration by parts,

$$\begin{aligned} I &= \log t \int dt - \int \left(\frac{1}{t} \int dt \right) dt \\ &= t \log t - \int \left(\frac{1}{t} \times t \right) dt \\ &= t \log t - \int dt \\ &= t \log t - t + c \\ &= \frac{1}{x+2} \left(\log(x+2)^{-1} - 1 \right) + c \end{aligned}$$

$$I = \frac{-1}{x+2} - \frac{\log(x+2)}{x+2} + c$$

Indefinite Integrals Ex 19.25 Q24

$$\begin{aligned} \text{Let } I &= \int \frac{x + \sin x}{1 + \cos x} dx \\ &= \int \frac{x}{2 \cos^2 \frac{x}{2}} dx + \int \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} dx \\ &= \frac{1}{2} \int x \sec^2 \frac{x}{2} dx + \int \tan \frac{x}{2} dx \end{aligned}$$

Using integration by parts,

$$\begin{aligned} &= \frac{1}{2} \left[x \int \sec^2 \frac{x}{2} dx - \int \left(1 \int \sec^2 \frac{x}{2} dx \right) dx \right] + \int \tan \frac{x}{2} dx \\ &= \frac{1}{2} \left[2x \tan \frac{x}{2} - 2 \int \tan \frac{x}{2} dx \right] + \int \tan \frac{x}{2} dx + c \\ &= x \tan \frac{x}{2} - \int \tan \frac{x}{2} dx + \int \tan \frac{x}{2} dx + c \end{aligned}$$

$$I = x \tan \frac{x}{2} + c$$

Indefinite Integrals Ex 19.25 Q25

$$\begin{aligned} \text{Let } I &= \int \log_{10} x dx \\ &= \int \frac{\log x}{\log 10} dx \\ &= \frac{1}{\log 10} \int 1 \times \log x dx \end{aligned}$$

Using integration by parts,

$$\begin{aligned} &= \frac{1}{\log 10} \left[\log x \int dx - \int \left(\frac{1}{x} \int dx \right) dx \right] \\ &= \frac{1}{\log 10} \left[x \log x - \int \left(\frac{x}{x} \right) dx \right] \\ &= \frac{1}{\log 10} [x \log x - x] \end{aligned}$$

$$I = \frac{x}{\log 10} (\log x - 1)$$

Indefinite Integrals Ex 19.25 Q26

$$\begin{aligned} \text{Let } I &= \int \cos \sqrt{x} dx \\ \sqrt{x} &= t \\ x &= t^2 \\ dx &= 2t dt \\ &= \int 2t \cos t dt \\ I &= 2 \int t \cos t dt \\ I &= 2 \left[t \int \cos t dt - \int (1 \int \cos t dt) dt \right] \\ &= 2 \left[t \sin t - \int \sin t dt \right] \\ &= 2 \left[t \sin t + \cos t \right] + c \end{aligned}$$

$$I = 2 \left[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x} \right] + c$$

Indefinite Integrals Ex 19.25 Q27

$$\text{Let } I = \int \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx$$

Let us substitute, $t = \cos^{-1} x$

$$\Rightarrow dt = \frac{-1}{\sqrt{1-x^2}} dx$$

Also, $\cos t = x$

Thus,

$$I = - \int t \cos t dt$$

Now let us solve this by the 'by parts' method.

Let $u = t$; $du = dt$

$$\int \cos t dt = \int dv$$

$$\Rightarrow \sin t = v$$

$$\text{Thus, } I = - \left[tsint - \int \sin t dt \right]$$

$$\Rightarrow I = - \left[tsint + \cos t \right] + C$$

Substituting the value $t = \cos^{-1} x$, we have,

$$I = - \left[\cos^{-1} x \sin x + x \right] + C$$

$$\Rightarrow I = - \left[\cos^{-1} x \sqrt{1-x^2} + x \right] + C$$

Indefinite Integrals Ex 19.25 Q29

$$\text{Let } I = \int \cosec^3 x dx$$

$$= \int \cosec x - \cosec^2 x dx$$

Using integration by parts,

$$= \cosec x \int \cosec^2 x dx + \left(\cosec x \cot x \int \cosec^2 x dx \right) dx$$

$$= \cosec x \times (-\cot x) + \int \cosec x \cot x (-\cot x) dx$$

$$= -\cosec x \cot x - \int \cosec x \cot^2 x dx$$

$$= -\cosec x \cot x - \int \cosec x (\cosec^2 x - 1) dx$$

$$= -\cosec x \cot x - \int \cosec^3 x dx + \int \cosec x dx$$

$$I = -\cosec x \cot x - I + \log \left| \tan \frac{x}{2} \right| + C_1$$

$$2I = -\cosec x \cot x + \log \left| \tan \frac{x}{2} \right| + C_1$$

$$I = -\frac{1}{2} \cosec x \cot x + \frac{1}{2} \log \left| \tan \frac{x}{2} \right| + C$$

Indefinite Integrals Ex 19.25 Q30

$$\text{Let } I = \int \sec^{-1} \sqrt{x} dx$$

$$\text{Let } \sqrt{x} = t$$

$$x = t^2$$

$$dx = 2t dt$$

$$I = \int 2t \sec^{-1} t dt$$

$$= 2 \left[\sec^{-1} t | t dt - \int \left(\frac{1}{t \sqrt{t^2 - 1}} \right) dt \right]$$

$$= 2 \left[\frac{t^2}{2} \sec^{-1} t - \int \left(\frac{t}{2t \sqrt{t^2 - 1}} \right) dt \right]$$

$$= t^2 \sec^{-1} t - \int \frac{t}{\sqrt{t^2 - 1}} dt$$

$$= t^2 \sec^{-1} t - \frac{1}{2} \int \frac{2t}{\sqrt{t^2 - 1}} dt$$

$$= t^2 \sec^{-1} t - \frac{1}{2} \times 2 \sqrt{t^2 - 1} + C$$

$$I = x \sec^{-1} \sqrt{x} - \sqrt{x-1} + C$$

Indefinite Integrals Ex 19.25 Q31

$$\begin{aligned}
& \int \sin^{-1} \sqrt{x} dx = \\
& \text{let } x = t^2 \rightarrow dx = 2tdt \\
& \int \sin^{-1} \sqrt{x} dx = \int \sin^{-1} \sqrt{t^2} 2tdt = \int \sin^{-1} t 2tdt \\
& = \sin^{-1} t \int 2tdt - \left(\int \frac{d \sin^{-1} t}{dt} \left(\int 2tdt \right) dt \right) \\
& = \sin^{-1} t (t^2) - \int \frac{1}{\sqrt{1-t^2}} (t^2) dt \\
& \text{Let's solve } \int \frac{1}{\sqrt{1-t^2}} (t^2) dt \\
& \int \frac{1}{\sqrt{1-t^2}} (t^2) dt = \int \frac{t^2-1+1}{\sqrt{1-t^2}} dt = \int \frac{t^2-1}{\sqrt{1-t^2}} dt + \int \frac{1}{\sqrt{1-t^2}} dt \\
& \text{we know that, value of } \int \frac{1}{\sqrt{1-t^2}} dt = \sin^{-1} t \\
& \text{Remaining integral to evaluate is } \int \frac{t^2-1}{\sqrt{1-t^2}} dt = \int -\sqrt{1-t^2} dt \\
& \text{sub } t = \sin u, dt = \cos u du \\
& \int -\sqrt{1-t^2} dt = \int -\cos^2 u du = - \int \left[\frac{1+\cos 2u}{2} \right] du \\
& = -\frac{u}{2} - \frac{\sin 2u}{4} \\
& \text{Substitute back } u = \sin^{-1} t \text{ and } t = \sqrt{x} \\
& = -\frac{\sin^{-1} \sqrt{x}}{2} - \frac{\sin(2\sin^{-1} \sqrt{x})}{4} \\
& \int \sin^{-1} \sqrt{x} dx = x \sin^{-1} \sqrt{x} - \frac{\sin^4 \sqrt{x}}{2} - \frac{\sin(2\sin^{-1} \sqrt{x})}{4} \\
& \sin(2\sin^{-1} \sqrt{x}) = 2\sqrt{x}\sqrt{1-x} \\
& \int \sin^{-1} \sqrt{x} dx = x \sin^{-1} \sqrt{x} - \frac{\sin^4 \sqrt{x}}{2} - \frac{\sqrt{x}(1-x)}{2}
\end{aligned}$$

Indefinite Integrals Ex 19.25 Q32

$$\begin{aligned}
\text{Let } I &= \int x \tan^2 x dx \\
&= \int x (\sec^2 x - 1) dx \\
&= \int x \sec^2 x dx - \int x dx \\
&= \left[x \int \sec^2 x dx - \left(\int (1) \sec^2 x dx \right) dx \right] - \frac{x^2}{2} \\
&= x \tan x - \int \tan x dx - \frac{x^2}{2} \\
I &= x \tan x - \log \sec x - \frac{x^2}{2} + C
\end{aligned}$$

Indefinite Integrals Ex 19.25 Q33

$$\begin{aligned}
\text{Let } I &= \int x \left(\frac{\sec 2x - 1}{\sec 2x + 1} \right) dx \\
&= \int x \left(\frac{1 - \cos 2x}{1 + \cos 2x} \right) dx \\
&= \int x \left(\frac{\sec^2 x}{\cos^2 x} \right) dx \\
&= \int x \tan^2 x dx \\
&= \int x (\sec^2 x - 1) dx \\
&= \int x \sec^2 x dx - \int dx \\
&= \left[x \int \sec^2 x dx - \left(\int (1) \sec^2 x dx \right) dx \right] - \frac{x^2}{2} \\
&= x \tan x - \int \tan x dx - \frac{x^2}{2} \\
I &= x \tan x - \log \sec x - \frac{x^2}{2} + C
\end{aligned}$$

Indefinite Integrals Ex 19.25 Q34

Let $I = \int (x+1)e^x \log(xe^x) dx$

Let $xe^x = t$

$$(1 \cdot e^x + xe^x)dx = dt$$

$$(x+1)e^x dx = dt$$

$$I = \int \log t dt$$

$$= \int 1 \cdot \log t dt$$

$$= \log t \int dt - \int \left(\frac{1}{t} \right) dt$$

$$= t \log t - \int \left(\frac{1}{t} \right) dt$$

$$= t \log t - \int dt$$

$$= t \log t - t + c$$

$$= t(\log t - 1) + c$$

$$I = xe^x (\log xe^x - 1) + c$$

Indefinite Integrals Ex 19.25 Q35

Let $I = \int \sin^{-1}(3x - 4x^3) dx$

Let $x = \sin \theta$

$$dx = \cos \theta d\theta$$

$$= \int \sin^{-1}(3 \sin \theta - 4 \sin^3 \theta) \cos \theta d\theta$$

$$= \int \sin^{-1}(\sin 3\theta) \cos \theta d\theta$$

$$= \int 3\theta \cos \theta d\theta$$

$$= 3[\theta \int \cos \theta d\theta - \int (1 \int \cos \theta d\theta) d\theta]$$

$$= 3[\theta \sin \theta - \int \sin \theta d\theta]$$

$$= 3[\theta \sin \theta + \cos \theta] + c$$

$$I = 3[x \sin^{-1} x + \sqrt{1-x^2}] + c$$

Indefinite Integrals Ex 19.25 Q36

Let $x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$

$$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2 \tan \theta}{1+\tan^2 \theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$$\Rightarrow \int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx = \int 2\theta \cdot \sec^2 \theta d\theta = 2 \int \theta \cdot \sec^2 \theta d\theta$$

Integrating by parts, we obtain

$$2 \left[\theta \cdot \int \sec^2 \theta d\theta - \int \left(\frac{d}{d\theta} \theta \right) \int \sec^2 \theta d\theta d\theta \right]$$

$$= 2 \left[\theta \cdot \tan \theta - \int \tan \theta d\theta \right]$$

$$= 2 \left[\theta \tan \theta + \log |\cos \theta| \right] + C$$

$$= 2 \left[x \tan^{-1} x + \log \left| \frac{1}{\sqrt{1+x^2}} \right| \right] + C$$

$$= 2x \tan^{-1} x + 2 \log(1+x^2)^{\frac{1}{2}} + C$$

$$= 2x \tan^{-1} x + 2 \left[-\frac{1}{2} \log(1+x^2) \right] + C$$

$$= 2x \tan^{-1} x - \log(1+x^2) + C$$

Indefinite Integrals Ex 19.25 Q37

$$\begin{aligned}
\text{Let } I &= \int \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) dx \\
\text{Let } x &= \tan \theta \\
dx &= \sec^2 \theta d\theta \\
I &= \int \tan^{-1} \left(\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right) \sec^2 \theta d\theta \\
&= \int \tan^{-1} (\tan 3\theta) \sec^2 \theta d\theta \\
&= \int 3\theta \sec^2 \theta d\theta \\
&= 3 \left[\theta \int \sec^2 \theta d\theta - \int (1 \int \sec^2 \theta d\theta) d\theta \right] \\
&= 3 \left[\theta \tan \theta - \int \tan \theta d\theta \right] \\
&= 3 \left[\theta \tan \theta + \log \sec \theta \right] + c \\
&= 3 \left[x \tan^{-1} x - \log \sqrt{1+x^2} \right] + c
\end{aligned}$$

$$I = 3x \tan^{-1} x - \frac{3}{2} \log |1+x^2| + c$$

Indefinite Integrals Ex 19.25 Q38

$$\begin{aligned}
\text{Let } I &= \int x^2 \sin^{-1} x dx \\
I &= \sin^{-1} x \int x^2 dx - \int \left(\frac{1}{\sqrt{1-x^2}} \int x^2 dx \right) dx \\
&= \frac{x^3}{3} \sin^{-1} x - \int \frac{x^3}{3\sqrt{1-x^2}} dx \\
I &= \frac{x^3}{3} \sin^{-1} x - \frac{1}{3} I_1 + c_1 \quad \dots \dots \dots (1) \\
I_1 &= \int \frac{x^3}{\sqrt{1-x^2}} dx \\
\text{Let } 1-x^2 &= t^2 \\
-2x dx &= 2t dt \\
-x dx &= t dt \\
I_1 &= - \int \frac{(1-t^2)t dt}{t} \\
&= \int (t^2 - 1) dt \\
&= \frac{t^3}{3} - t + c_2 \\
&= \frac{(1-x^2)^{\frac{3}{2}}}{3} - (1-x^2)^{\frac{1}{2}} + c_2
\end{aligned}$$

Now,

$$I = \frac{x^3}{3} \sin^{-1} x - \frac{1}{9} (1-x^2)^{\frac{3}{2}} + \frac{1}{3} (1-x^2)^{\frac{1}{2}} + c$$

Indefinite Integrals Ex 19.25 Q39

$$\begin{aligned}
\text{Let } I &= \int \frac{\sin^{-1} x}{x^2} dx \\
&= \int \left(\frac{1}{x^2} \right) (\sin^{-1} x) dx \\
I &= \left[\sin^{-1} x \int \frac{1}{x^2} dx - \int \left(\frac{1}{\sqrt{1-x^2}} \int \frac{1}{x^2} dx \right) dx \right] \\
&= \sin^{-1} x \left(-\frac{1}{x} \right) - \int \frac{1}{\sqrt{1-x^2}} \left(-\frac{1}{x} \right) dx \\
I &= -\frac{1}{x} \sin^{-1} x + \int \frac{1}{x \sqrt{1-x^2}} dx \\
I &= -\frac{1}{x} \sin^{-1} x + I_1 \quad \dots \quad (1)
\end{aligned}$$

Where,

$$\begin{aligned}
I_1 &= \int \frac{1}{x \sqrt{1-x^2}} dx \\
\text{Let } 1-x^2 &= t^2 \\
-2x dx &= 2t dt \\
I_1 &= \int \frac{x}{x^2 \sqrt{1-x^2}} dx \\
&= - \int \frac{t dt}{(1-t^2) \sqrt{t}} \\
&= - \int \frac{dt}{(1-t^2)} \\
&= \int \frac{1}{t^2-1} dt \\
&= \frac{1}{2} \log \left| \frac{t-1}{t+1} \right| \\
&= \frac{1}{2} \log \left| \frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1} \right| + C_1
\end{aligned}$$

Now,

$$\begin{aligned}
I &= -\frac{\sin^{-1} x}{x} + \frac{1}{2} \log \left| \frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1} \right| \left| \frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}-1} \right| + C \\
&= -\frac{\sin^{-1} x}{x} + \frac{1}{2} \log \left| \frac{(\sqrt{1-x^2}-1)^2}{1-x^2-1} \right| + C \\
&= -\frac{\sin^{-1} x}{x} + \frac{1}{2} \log \left| \frac{(\sqrt{1-x^2}-1)^2}{-x^2} \right| + C \\
&= -\frac{\sin^{-1} x}{x} + \log \left| \frac{\sqrt{1-x^2}-1}{-x} \right| + C \\
I &= -\frac{\sin^{-1} x}{x} + \log \left| \frac{1-\sqrt{1-x^2}}{x} \right| + C
\end{aligned}$$

Indefinite Integrals Ex 19.25 Q40

$$\begin{aligned}
\text{Let } I &= \int \frac{x^2 \tan^{-1} x}{1+x^2} dx \\
\text{Let } \tan^{-1} x &= t \\
x &= \tan t \\
\frac{1}{1+x^2} dx &= dt \\
I &= \int t \tan^2 t dt \\
&= \int t (\sec^2 t - 1) dt \\
&= \int (t \sec^2 t - t) dt \\
&= \int t \sec^2 t dt - \int t dt \\
&= \left[t \int \sec^2 t dt - \int (1 \int \sec^2 t dt) dt \right] - \frac{t^2}{2} \\
&= \left[t \tan t - \int \tan t dt \right] - \frac{t^2}{2} \\
&= t \tan t - \log |\sec t| - \frac{t^2}{2} + C \\
&= x \tan^{-1} x - \log \sqrt{1+x^2} - \frac{\tan^2 x}{2} + C
\end{aligned}$$

$$I = x \tan^{-1} x - \frac{1}{2} \log |1+x^2| - \frac{\tan^2 x}{2} + C$$

Indefinite Integrals Ex 19.25 Q41

$$\begin{aligned}
\text{Let } I &= \int \cos^{-1}(4x^3 - 3x) dx \\
\text{Let } x &= \cos \theta \\
dx &= -\sin \theta d\theta \\
I &= - \int \cos^{-1}(4\cos^3 \theta - 3\cos \theta) \sin \theta d\theta \\
&= - \int \cos^{-1}(\cos 3\theta) \sin \theta d\theta \\
&= - \int 3\theta \sin \theta d\theta \\
&= -3[\theta \sin \theta - \int (1 \sin \theta) d\theta] \\
&= -3[-\theta \cos \theta + \int \cos \theta d\theta] \\
&= 3\theta \cos \theta - 3 \sin \theta + C
\end{aligned}$$

$$I = 3x \cos^{-1} x - 3\sqrt{1-x^2} + C$$

Indefinite Integrals Ex 19.25 Q42

$$\begin{aligned}
\text{Let } I &= \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) dx \\
\text{Let } x &= \tan t \\
dx &= \sec^2 t dt \\
I &= \int \cos^{-1}\left(\frac{1-\tan^2 t}{1+\tan^2 t}\right) \sec^2 t dt \\
&= \int \cos^{-1}(\cos 2t) \sec^2 t dt \\
&= \int 2t \sec^2 t dt \\
&= 2 \left[t \int \sec^2 t dt - \int (1 \int \sec^2 t dt) dt \right] \\
&= 2 \left[t \tan^2 t - \int \tan t dt \right] \\
&= 2 \left[t \tan^2 t - \log |\sec t| \right] + C \\
&= 2 \left[x \tan^{-1} x - \log \sqrt{1+x^2} \right] + C
\end{aligned}$$

$$I = 2x \tan^{-1} x - \log |1+x^2| + C$$

Indefinite Integrals Ex 19.25 Q43

$$\begin{aligned}
\text{Let } I &= \int \tan^{-1} \left(\frac{2x}{1-x^2} \right) dx \\
\text{Let } x &= \tan \theta \\
dx &= \sec^2 \theta d\theta \\
I &= \int \tan^{-1} \left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right) \sec^2 \theta d\theta \\
&= \int \tan^{-1} (\tan 2\theta) \sec^2 \theta d\theta \\
&= \int 2\theta \sec^2 \theta d\theta \\
&= 2 \left[\theta \int \sec^2 \theta d\theta - \int (1 \int \sec^2 \theta d\theta) d\theta \right] \\
&= 2[\theta \tan \theta - \int \tan \theta d\theta] \\
&= 2[\theta \tan \theta - \log \sec \theta] + c \\
&= 2 \left[x \tan^{-1} x - \log \sqrt{1+x^2} \right] + c
\end{aligned}$$

$$I = 2x \tan^{-1} x - \log |1+x^2| + c$$

Indefinite Integrals Ex 19.25 Q44

$$\begin{aligned}
\text{Let } I &= \int (x+1) \log x dx \\
&= \log x \int (x+1) dx - \int \left(\frac{1}{x} \int (x+1) dx \right) dx \\
&= \left(\frac{x^2}{2} + x \right) \log x - \int \frac{1}{x} \left(\frac{x^2}{2} + x \right) dx \\
&= \left(\frac{x^2}{2} + x \right) \log x - \frac{1}{2} \int x dx - \int dx \\
&= \left(x + \frac{x^2}{2} \right) \log x - \frac{1}{2} \times \frac{x^2}{2} - x + c
\end{aligned}$$

$$I = \left(x + \frac{x^2}{2} \right) \log x - \left(\frac{x^2}{4} + x \right) + c$$

Indefinite Integrals Ex 19.25 Q45

$$\begin{aligned}
\text{Let } I &= \int x^2 \tan^{-1} x dx \\
&= \tan^{-1} x \int x^2 dx - \int \left(\frac{1}{1+x^2} \int x^2 dx \right) \\
&= \tan^{-1} x \left(\frac{x^3}{3} \right) - \frac{1}{3} \int \frac{x^3}{1+x^2} dx \\
&= \frac{1}{3} x^3 \tan^{-1} x - \frac{1}{3} \int \left(x - \frac{x}{1+x^2} \right) dx \\
&= \frac{1}{3} x^3 \tan^{-1} x - \frac{1}{3} \times \frac{x^2}{2} + \frac{1}{3} \int \frac{x}{1+x^2} dx
\end{aligned}$$

$$I = \frac{1}{3} x^3 \tan^{-1} x - \frac{1}{6} x^2 + \frac{1}{6} \log |1+x^2| + c$$

Indefinite Integrals Ex 19.25 Q46

$$\begin{aligned}
\text{Let } I &= \int (e^{\log x} + \sin x) \cos x \, dx \\
&= \int (x + \sin x) \cos x \, dx \\
&= \int x \cos x \, dx + \int \sin x \cos x \, dx \\
&= [x \int \cos x \, dx - \int (1 \int \cos x \, dx) dx] + \frac{1}{2} \int \sin 2x \, dx \\
&= [x \sin x - \int \sin x \, dx] + \frac{1}{2} \left(-\frac{\cos 2x}{2} \right) + c
\end{aligned}$$

$$I = x \sin x + \cos x - \frac{1}{4} \cos 2x + c$$

$$I = x \sin x + \cos x - \frac{1}{4} [1 - 2 \sin^2 x] + c$$

$$I = x \sin x + \cos x - \frac{1}{4} + \frac{1}{2} \sin^2 x + c$$

$$I = x \sin x + \cos x + \frac{1}{2} \sin^2 x + c - \frac{1}{4}$$

$$I = x \sin x + \cos x + \frac{1}{2} \sin^2 x + k, \text{ where } k = c - \frac{1}{4}$$

Indefinite Integrals Ex 19.25 Q47

$$\text{Let } I = \int \frac{x \tan^{-1} x}{\sqrt[3]{1+x^2}} \, dx$$

$$\text{Let } \tan^{-1} x = t$$

$$\frac{1}{1+x^2} dx = dt$$

$$I = \int \frac{t \tan t}{\sqrt{1+\tan^2 t}} dt$$

$$= \int \frac{t \tan t}{\sec t} dt$$

$$= \int t \frac{\sin t}{\cos t} \csc t dt$$

$$= \int t \sin t dt$$

$$= [t \int \sin t dt - \int (1 \int \sin t dt) dt]$$

$$= [-t \cos t + \int \cos t dt]$$

$$= [-t \cos t + \sin t] + c$$

$$I = -\frac{\tan^{-1} x}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$$

Indefinite Integrals Ex 19.25 Q48

$$\text{Let } I = \int \tan^{-1} (\sqrt{x}) \, dx$$

$$\text{Let } x = t^2$$

$$dx = 2t \, dt$$

$$I = \int 2t \tan^{-1} t \, dt$$

$$= 2 \left[\tan^{-1} t \int t \, dt - \int \left(\frac{1}{1+t^2} \int t \, dt \right) dt \right]$$

$$= 2 \left[\frac{t^2}{2} \tan^{-1} t - \int \frac{t^2}{2(1+t^2)} dt \right]$$

$$= t^2 \tan^{-1} t - \int \frac{t^2+1-1}{1+t^2} dt$$

$$= t^2 \tan^{-1} t - \int \left(1 - \frac{1}{1+t^2} \right) dt$$

$$= t^2 \tan^{-1} t - t + \tan^{-1} t + c$$

$$= (t^2 + 1) \tan^{-1} t - t + c$$

$$I = (x+1) \tan^{-1} \sqrt{x} - \sqrt{x} + c$$

Indefinite Integrals Ex 19.25 Q49

$$\begin{aligned}
\int x^3 \tan^{-1} x \, dx &= \\
\int x^3 \tan^{-1} x \, dx &= \tan^{-1} x \int x^3 \, dx - \left(\int \frac{d \tan^{-1} x}{dx} \left(\int x^3 \, dx \right) dx \right) \\
&= \tan^{-1} x \frac{x^4}{4} - \left(\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx \right) \\
&= \tan^{-1} x \frac{x^4}{4} - \left(\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx \right) \\
\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx &= \frac{1}{4} \left[\int \frac{1}{1+x^2} dx + (x^2 - 1) dx \right] \\
\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx &= \frac{1}{4} \left[\tan^{-1} x + \frac{x^3}{3} - x \right] \\
\int x^3 \tan^{-1} x \, dx &= \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \left[\tan^{-1} x + \frac{x^3}{3} - x \right] + C
\end{aligned}$$

Indefinite Integrals Ex 19.25 Q50

$$\begin{aligned}
\text{Let } I &= \int x \sin x \cos 2x \, dx \\
&= \frac{1}{2} \int x (2 \sin x \cos 2x) \, dx \\
&= \frac{1}{2} \int x (\sin(x+2x) - \sin(2x-x)) \, dx \\
&= \frac{1}{2} \int x (\sin 3x - \sin x) \, dx \\
&= \frac{1}{2} \left[x \int (\sin 3x - \sin x) \, dx - \int (1)(\sin 3x - \sin x) \, dx \right] \\
&= \frac{1}{2} \left[x \left(\frac{-\cos 3x}{3} + \cos x \right) - \int \left(-\frac{\cos 3x}{3} + \cos x \right) \, dx \right] \\
I &= \frac{1}{2} \left[-x \frac{\cos 3x}{3} + x \cos x + \frac{1}{9} \sin 3x - \sin x \right] + C
\end{aligned}$$

Indefinite Integrals Ex 19.25 Q51

$$\begin{aligned}
\text{Let } I &= \int (\tan^{-1} x^2) x \, dx \\
\text{Let } x^2 &= t \\
2x \, dx &= dt \\
I &= \frac{1}{2} \int \tan^{-1} t \, dt \\
&= \frac{1}{2} \int \tan^{-1} t \, dt \\
&= \frac{1}{2} \left[\tan^{-1} t \int dt - \left(\int \frac{1}{1+t^2} \int dt \right) dt \right] \\
&= \frac{1}{2} \left[t \tan^{-1} t - \int \frac{t}{1+t^2} dt \right] \\
&= \frac{1}{2} t \tan^{-1} t - \frac{1}{4} \int \frac{2t}{1+t^2} dt \\
&= \frac{1}{2} t \tan^{-1} t - \frac{1}{4} \log|1+t^2| + C \\
I &= \frac{1}{2} x^2 \tan^{-1} x^2 - \frac{1}{4} \log|1+x^4| + C
\end{aligned}$$

Indefinite Integrals Ex 19.25 Q52

Let first function be $\sin^{-1} x$ and second function be $\frac{x}{\sqrt{1-x^2}}$.

First we find the integral of the second function, i.e., $\int \frac{xdx}{\sqrt{1-x^2}}$.

Put $t = 1 - x^2$. Then $dt = -2x dx$

$$\text{Therefore, } \int \frac{xdx}{\sqrt{1-x^2}} = -\frac{1}{2} \int \frac{dt}{\sqrt{t}} = -\sqrt{t} = -\sqrt{1-x^2}$$

$$\text{Hence, } \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = (\sin^{-1} x) \left(-\sqrt{1-x^2} \right) - \int \frac{1}{\sqrt{1-x^2}} \left(-\sqrt{1-x^2} \right) dx \\ = -\sqrt{1-x^2} \sin^{-1} x + x + C = x - \sqrt{1-x^2} \sin^{-1} x + C$$

Indefinite Integrals Ex 19.25 Q53

$$\text{Let } I = \int \sin^3 \sqrt{x} dx$$

$$\sqrt{x} = t$$

$$x = t^2$$

$$dx = 2t dt$$

$$I = 2 \int t \sin^3 t dt$$

$$= 2 \int t \left(\frac{3 \sin t - \sin 3t}{4} \right) dt$$

$$= \frac{1}{2} \int t (3 \sin t - \sin 3t) dt$$

Using integration by parts,

$$I = \frac{1}{2} \left[t \left(-3 \cos t + \frac{1}{3} \cos 3t \right) - \int \left(-3 \cos t + \frac{\cos 3t}{3} \right) dt \right] \\ = \frac{1}{2} \left[\frac{-9t \cos t + t \cos 3t}{3} - \left\{ -3 \sin t + \frac{\sin 3t}{9} \right\} \right] + C \\ = \frac{1}{2} \left[\frac{-9t \cos t + t \cos 3t}{3} + \frac{27 \sin t - 3 \sin 3t}{9} \right] + C \\ = \frac{1}{18} [-27t \cos t + 3t \cos 3t + 27 \sin t - 3 \sin 3t] + C$$

$$I = \frac{1}{18} [3\sqrt{x} \cos 3\sqrt{x} + 27 \sin \sqrt{x} - 27\sqrt{x} \cos \sqrt{x} - 3 \sin 3\sqrt{x}] + C$$

Indefinite Integrals Ex 19.25 Q54

$$\text{Let } I = \int x \sin^3 x dx$$

$$= \int x \left(\frac{3 \sin x - \sin 3x}{4} \right) dx$$

$$= \frac{1}{4} \int x (3 \sin x - \sin 3x) dx$$

Using integration by parts,

$$I = \frac{1}{4} \left[x \int (3 \sin x - \sin 3x) dx - \int \left(x \int (3 \sin x - \sin 3x) dx \right) dx \right] \\ = \frac{1}{4} \left[x \left(-3 \cos x + \frac{\cos 3x}{3} \right) - \int \left(-3 \cos x + \frac{\cos 3x}{3} \right) dx \right] \\ = \frac{1}{4} \left[-3x \cos x + \frac{x \cos 3x}{3} + 3 \sin x - \frac{\sin 3x}{9} \right] + C$$

$$I = \frac{1}{36} [3x \cos 3x - 27x \cos x + 27 \sin x - \sin 3x] + C$$

Indefinite Integrals Ex 19.25 Q55

$$\begin{aligned}
\text{Let } I &= \int \cos^3 \sqrt{x} dx \\
\text{Let } x &= t^2 \\
dx &= 2tdt \\
&= 2 \int t \cos^3 t dt \\
&= 2 \int t \left(\frac{3 \cos t + \cos 3t}{4} \right) dt \\
&= \frac{1}{2} \int t (3 \cos t + \cos 3t) dt
\end{aligned}$$

Using integration by parts,

$$\begin{aligned}
I &= \frac{1}{2} \left[t \left(3 \sin t + \frac{1}{3} \sin 3t \right) + \int \left(1 \times 3 \sin t + \frac{\sin 3t}{3} \right) dt \right] \\
&= \frac{1}{2} \left[t \left(\frac{9 \sin t + \sin 3t}{3} \right) + 3 \cos t + \frac{\cos 3t}{9} \right] + c \\
&= \frac{1}{18} [27t \sin t + 3t \sin 3t + 9 \cos t + \cos 3t] + c \\
I &= \frac{1}{18} [27\sqrt{x} \sin \sqrt{x} + 3\sqrt{x} \sin 3\sqrt{x} + 9 \cos \sqrt{x} + \cos 3\sqrt{x}] + c
\end{aligned}$$

Indefinite Integrals Ex 19.25 Q56

$$\begin{aligned}
\text{Let } I &= \int x \cos^3 x dx \\
&= \int x \left(\frac{3 \cos x + \cos 3x}{4} \right) dx \\
&= \frac{1}{4} \int x (3 \cos x + \cos 3x) dx
\end{aligned}$$

Using integration by parts,

$$\begin{aligned}
I &= \frac{1}{4} \left[x \int (3 \cos x + \cos 3x) dx - \int (1 \int (3 \cos x + \cos 3x) dx) dx \right] \\
&= \frac{1}{4} \left[x \left(3 \sin x + \frac{\sin 3x}{3} \right) - \int \left(3 \sin x + \frac{\sin 3x}{3} \right) dx \right] \\
&= \frac{1}{4} \left[3x \sin x + \frac{x \sin 3x}{3} + 3 \cos x + \frac{\cos 3x}{9} \right] + c \\
I &= \frac{3x \sin x}{4} + \frac{x \sin 3x}{12} + \frac{3 \cos x}{4} + \frac{\cos 3x}{36} + c
\end{aligned}$$

Indefinite Integrals Ex 19.25 Q57

$$\begin{aligned}
\text{Let } I &= \int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx \text{ and } x = \cos \theta \Rightarrow dx = -\sin \theta d\theta \\
\therefore I &= \int \tan^{-1} \left(\tan \frac{\theta}{2} \right) (-\sin \theta) d\theta \\
&= -\frac{1}{2} \int \theta \sin \theta d\theta
\end{aligned}$$

$$Let \theta = u \text{ and } \sin \theta d\theta = v \text{ so that } \sin \theta = \int v d\theta$$

$$Then, \int uv dx = u \int (v dx) - \left(\int \frac{du}{dx} \int v dx \right) dx$$

$$Hence, I = -\frac{1}{2} (-\theta \cos \theta - \int -\cos \theta d\theta)$$

$$= -\frac{1}{2} (-\theta \cos \theta + \sin \theta) + c$$

$$= -\frac{1}{2} (-\theta \cos \theta + \sqrt{1 - \cos^2 \theta}) + c$$

$$= -\frac{1}{2} (-x \cos^{-1} x + \sqrt{1 - x^2}) + c$$

Indefinite Integrals Ex 19.25 Q58

Let $I = \int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$

Let $x = a \tan^2 \theta$
 $dx = 2a \tan \theta \sec^2 \theta d\theta$

$$I = \int \left(\sin^{-1} \sqrt{\frac{a \tan^2 \theta}{a + a \tan^2 \theta}} \right) (2a \tan \theta \sec^2 \theta) d\theta$$

$$= \int \left(\sin^{-1} \sqrt{\frac{\tan^2 \theta}{\sec^2 \theta}} \right) (2a \tan \theta \sec^2 \theta) d\theta$$

$$= \int \sin^{-1} (\sin \theta) (2a \tan \theta \sec^2 \theta) d\theta$$

$$= \int 2a \tan \theta \sec^2 \theta d\theta$$

$$= 2a \theta (\tan \theta \sec^2 \theta) d\theta$$

$$= 2a \left[\theta \int \tan \theta \sec^2 \theta d\theta - \int (\int \tan \theta \sec^2 \theta d\theta) d\theta \right]$$

$$= 2a \left[\theta \frac{\tan^2 \theta}{2} - \int \frac{\tan^2 \theta}{2} d\theta \right]$$

$$= a\theta \tan^2 \theta - \frac{2a}{2} \int (\sec^2 \theta - 1) d\theta$$

$$= a\theta \tan^2 \theta - a \tan \theta + a\theta + C$$

$$= a \left(\tan^{-1} \sqrt{\frac{x}{a}} \right) \frac{x}{a} - a \sqrt{\frac{x}{a}} + a \tan^{-1} \sqrt{\frac{x}{a}} + C$$

$$I = x \tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{ax} + a \tan^{-1} \sqrt{\frac{x}{a}} + C$$

Indefinite Integrals Ex 19.25 Q59

Let $I = \int \frac{x^3 \sin^{-1} x^2}{\sqrt{1-x^4}} dx$

Let $\sin^{-1} x^2 = t$
 $\frac{1}{\sqrt{1-x^4}} (2x) dx = dt$

$$I = \int \frac{x^2 \sin^{-1} x^2}{\sqrt{1-x^2}} x dx$$

$$= \int (\sin t) t \frac{dt}{2}$$

$$= \frac{1}{2} \int t \sin t dt$$

$$= \frac{1}{2} [t \int \sin t dt - \int (1 \sin t) dt]$$

$$= \frac{1}{2} [t(-\cos t) - \int (-\cos t) dt]$$

$$= \frac{1}{2} [-t \cos t + \sin t] + C$$

$$I = \frac{1}{2} \left[x^2 - \sqrt{1-x^4} \sin^{-1} x^2 \right] + C$$

Indefinite Integrals Ex 19.25 Q60

$$\text{Let } I = \int \frac{x^2 \sin^{-1} x}{\sqrt{1-x^2}} dx$$

$$\text{Let } \sin^{-1} x = t$$
$$\frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\begin{aligned} I &= \int \frac{\sin^2 t \times t}{(1-\sin^2 t)} dt \\ &= \int \frac{t \sin^2 t}{\cos^2 t} dt \\ &= \int t \tan^2 t dt \\ &= \int t (\sec^2 t - 1) dt \\ &= \int t \sec^2 t dt - \frac{t^2}{2} \\ &= t \int \sec^2 t dt - \left[\int 1 dt \right] - \frac{t^2}{2} \\ &= t \tan t - \int \tan t dt - \frac{t^2}{2} \\ &= t \tan t - \log |\sec t| - \frac{t^2}{2} + C \end{aligned}$$

$$I = \frac{x}{\sqrt{1-x^2}} \sin^{-1} x + \log |1-x^2| - \frac{1}{2} (\sin^{-1} x)^2 + C$$

Ex 19.26

Indefinite Integrals Ex 19.26 Q1

$$\begin{aligned} \text{Let } I &= \int e^x (\cos x - \sin x) dx \\ &= \int e^x \cos x dx - \int e^x \sin x dx \end{aligned}$$

Integrating by parts

$$\begin{aligned} &= e^x \cos x - \int e^x \left(\frac{d}{dx} \cos x \right) dx - \int e^x \sin x dx \\ &= e^x \cos x + \int e^x \sin x dx - \int e^x \sin x dx \\ &= e^x \cos x + c \\ \therefore \quad \int e^x (\cos x - \sin x) dx &= e^x \cos x + c \end{aligned}$$

Indefinite Integrals Ex 19.26 Q2

$$\begin{aligned} I &= \int e^x (x^{-2} - 2x^{-3}) dx \\ &= \int e^x x^{-2} dx - 2 \int e^x x^{-3} dx \end{aligned}$$

Integrating by parts

$$\begin{aligned} &= e^x x^{-2} - \int e^x \left(\frac{d}{dx} (x^{-2}) \right) dx - 2 \int e^x x^{-3} dx \\ &= e^x x^{-2} + 2 \int e^x x^{-3} dx - 2 \int e^x x^{-3} dx \\ &= \frac{e^x}{x^2} + c \\ \int e^x \left(\frac{1}{x^2} - \frac{2}{x^3} \right) dx &= \frac{e^x}{x^2} + c \end{aligned}$$

Indefinite Integrals Ex 19.26 Q3

$$\begin{aligned}
& e^x \left(\frac{1+\sin x}{1+\cos x} \right) \\
&= e^x \left(\frac{\frac{\sin^2 x}{2} + \frac{\cos^2 x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right) \\
&= \frac{e^x \left(\frac{\sin x}{2} + \frac{\cos x}{2} \right)^2}{2 \cos^2 \frac{x}{2}} \\
&= \frac{1}{2} e^x \cdot \left(\frac{\frac{\sin x}{2} + \frac{\cos x}{2}}{\cos \frac{x}{2}} \right)^2 \\
&= \frac{1}{2} e^x \left[\tan \frac{x}{2} + 1 \right]^2 \\
&= \frac{1}{2} e^x \left(1 + \tan \frac{x}{2} \right)^2 \\
&= \frac{1}{2} e^x \left[1 + \tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right] \\
&= \frac{1}{2} e^x \left[\sec^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right] \\
& \frac{e^x (1+\sin x) dx}{(1+\cos x)} = e^x \left[\frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right] \quad \dots(1)
\end{aligned}$$

$$\text{Let } \tan \frac{x}{2} = f(x) \Rightarrow f'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

From equation (1), we obtain

$$\int \frac{e^x (1+\sin x)}{(1+\cos x)} dx = e^x \tan \frac{x}{2} + C$$

Indefinite Integrals Ex 19.26 Q4

$$I = \int e^x \{ \cot x - \operatorname{cosec}^2 x \} dx$$

$$= \int e^x \cot x dx - \int e^x \operatorname{cosec}^2 x dx$$

Integrating by parts

$$\begin{aligned}
&= e^x \cot x - \int e^x \left(\frac{d}{dx} \cot x \right) dx - \int e^x \operatorname{cosec}^2 x dx \\
&= e^x \cot x + \int e^x \operatorname{cosec}^2 x dx - \int e^x \operatorname{cosec}^2 x dx \\
&= e^x \cot x + C
\end{aligned}$$

$$\int e^x \{ \cot x - \operatorname{cosec}^2 x \} dx = e^x \cot x + C$$

Indefinite Integrals Ex 19.26 Q5

$$\text{Let } I = \int e^x \frac{1}{2x} dx - \int e^x \frac{1}{2x^2} dx$$

Integrating by parts

$$\begin{aligned} &= \frac{e^x}{2x} - \int e^x \left(\frac{d}{dx} \left(\frac{1}{2x} \right) \right) dx - \int \frac{e^x}{2x^2} dx \\ &= \frac{e^x}{2x} + \int \frac{e^x}{2x^2} dx - \int \frac{e^x}{2x^2} dx \\ &= \frac{e^x}{2x} + c \end{aligned}$$

Indefinite Integrals Ex 19.26 Q6

$$\text{Let } I = \int e^x \sec x (1 + \tan x) dx$$

$$= \int e^x \sec x dx + \int e^x \sec x \tan x dx$$

Integrating by parts

$$\begin{aligned} &= e^x \sec x - \int e^x \left(\frac{d}{dx} \sec x \right) dx + \int e^x \sec x \tan x dx \\ &= e^x \sec x - \int e^x \sec x \tan x dx + \int e^x \sec x \tan x dx \\ &= e^x \sec x + c \end{aligned}$$

$$\therefore \int e^x \sec x (1 + \tan x) dx = e^x \sec x + c$$

Indefinite Integrals Ex 19.26 Q7

$$\text{Let } I = \int e^x (\tan x - \log \cos x) dx$$

$$= \int e^x \tan x dx - \int e^x \log \cos x dx$$

Integrating by parts

$$\begin{aligned} &= \int e^x \tan x dx - \left\{ e^x \log \cos x - \int e^x \left(\frac{d}{dx} \log \cos x \right) dx \right\} \\ &= \int e^x \tan x dx - \left\{ e^x \log \cos x + \int e^x \tan x dx \right\} \\ &= \int e^x \tan x dx - e^x \log \cos x - \int e^x \tan x dx + c \\ &= -e^x \log \cos x + c \end{aligned}$$

$$= e^x \log \sec x + c \quad [\because \log \sec x = -\log \cos x]$$

Indefinite Integrals Ex 19.26 Q8

$$\text{Let } I = \int e^x [\sec x + \log(\sec x + \tan x)] dx$$

$$= \int e^x \sec x dx + \int e^x \log(\sec x + \tan x) dx$$

Integrating by parts

$$\begin{aligned} &= \int e^x \sec x dx + e^x \log(\sec x + \tan x) - \int e^x \left\{ \frac{d}{dx} \log(\sec x + \tan x) \right\} dx \\ &= \int e^x \sec x dx + e^x \log(\sec x + \tan x) - \int e^x \sec x dx \\ &= e^x \log(\sec x + \tan x) + c \end{aligned}$$

Indefinite Integrals Ex 19.26 Q9

$$\text{Let } I = \int e^x (\cot x + \log \sin x) dx$$

$$= \int e^x \cot x dx + \int e^x \log \sin x dx$$

Integrating by parts

$$\begin{aligned} &= \int e^x \log \sin x dx + \int e^x \cot x dx \\ &= (\log \sin x) e^x - \int e^x \left(\frac{d}{dx} \log \sin x \right) dx + \int e^x \cot x dx \\ &= e^x \log \sin x - \int e^x \cot x dx + \int e^x \cot x dx \\ &= e^x \log \sin x + c \end{aligned}$$

Indefinite Integrals Ex 19.26 Q10

$$\text{Let } I = \int e^x \frac{x+1-2}{(x+1)^3} dx$$

$$\begin{aligned} &= \int e^x \left\{ \frac{1}{(x+1)^2} + \frac{-2}{(x+1)^3} \right\} dx \\ &= \int e^x \frac{1}{(x+1)^2} dx + \int e^x \frac{(-2)}{(x+1)^3} dx \end{aligned}$$

Integrating by parts

$$\begin{aligned} &= e^x \frac{1}{(x+1)^2} - \int e^x \left(\frac{d}{dx} (x+1)^{-2} \right) dx + \int e^x \frac{(-2)}{(x+1)^3} dx \\ &= e^x \frac{1}{(x+1)^2} - \int e^x \frac{(-2)}{(x+1)^3} dx + \int e^x \frac{(-2)}{(x+1)^3} dx \\ &= e^x \frac{1}{(x+1)^2} + c \end{aligned}$$

Indefinite Integrals Ex 19.26 Q11

$$\text{Let } I = \int e^x \left(\frac{\sin 4x - 4}{2 \sin^2 2x} \right) dx$$

$$\begin{aligned} &= \int e^x \left\{ \frac{2 \sin 2x \cos 2x}{2 \sin^2 2x} - \frac{4}{2 \sin^2 2x} \right\} dx \\ &= \int e^x (\cot 2x - 2 \operatorname{cosec}^2 2x) dx \\ &= \int e^x \cot 2x dx - 2 \int e^x \operatorname{cosec}^2 2x dx \end{aligned}$$

Integrating by parts

$$\begin{aligned} &= e^x \cot 2x - \int e^x \frac{d}{dx} (\cot 2x) dx - 2 \int e^x \operatorname{cosec}^2 2x dx \\ &= e^x \cot 2x + 2 \int e^x \operatorname{cosec}^2 2x dx - 2 \int e^x \operatorname{cosec}^2 2x dx \\ &= e^x \cot 2x + c \end{aligned}$$

Indefinite Integrals Ex 19.26 Q12

$$\text{Let } I = \int \frac{2-x}{(1-x)^2} e^x dx$$

$$\begin{aligned}&= \int e^x \left\{ \frac{(1-x)+1}{(1-x)^2} \right\} dx \\&= \int e^x \left\{ \frac{1}{1-x} + \frac{1}{(1-x)^2} \right\} dx\end{aligned}$$

$$\text{Here, } f(x) = \frac{1}{1-x} \text{ and } f'(x) = \frac{1}{(1-x)^2}$$

And we know that,

$$\begin{aligned}\int e^{ax} (af(x) + f'(x)) dx &= e^{ax} f(x) + c \\ \therefore \int e^x \left\{ \frac{1}{1-x} + \frac{1}{(1-x)^2} \right\} dx &= e^x \cdot \frac{1}{1-x} + c\end{aligned}$$

Hence,

$$I = \frac{e^x}{1-x} + c$$

Indefinite Integrals Ex 19.26 Q13

$$\text{Let } I = \int e^x \frac{1+x}{(2+x)^2} dx$$

$$\begin{aligned}&= \int e^x \left(\frac{x+2-1}{(2+x)^2} \right) dx \\&= \int e^x \left\{ \frac{1}{x+2} - \frac{1}{(x+2)^2} \right\} dx \\&= \int e^x \frac{1}{x+2} dx - \int e^x \frac{1}{(x+2)^2} dx\end{aligned}$$

Integrating by parts

$$\begin{aligned}&= e^x \frac{1}{x+2} - \int e^x \left(\frac{d}{dx} \left(\frac{1}{x+2} \right) \right) dx - \int e^x \frac{1}{(x+2)^2} dx \\&= e^x \frac{1}{x+2} + \int e^x \frac{1}{(x+2)^2} dx - \int e^x \frac{1}{(x+2)^2} dx \\&= \frac{e^x}{x+2} + c\end{aligned}$$

Indefinite Integrals Ex 19.26 Q14

$$\text{Let } I = \int \frac{\sqrt{1-\sin x}}{1+\cos x} e^{-\frac{x}{2}} dx$$

$$\begin{aligned}\text{Put } & \frac{x}{2} = t \\ \Rightarrow & x = 2t \\ dx &= 2dt\end{aligned}$$

$$\begin{aligned}\therefore & \int \frac{\sqrt{1-\sin x}}{1+\cos x} e^{-\frac{x}{2}} dx \\ &= 2 \int \frac{\sqrt{1-\sin 2t}}{1+\cos 2t} e^{-t} dt \quad [\because \sin^2 t + \cos^2 t = 1] \\ &= 2 \int \frac{\sqrt{\sin^2 t + \cos^2 t - 2 \sin t \cos t}}{1+\cos 2t} e^{-t} dt \\ &= 2 \int \frac{\sqrt{(\cos t - \sin t)^2}}{2 \cos^2 t} e^{-t} dt \\ &= 2 \int \frac{(\cos t - \sin t)}{2 \cos^2 t} e^{-t} dt \\ &= \int (\sec t - \tan t \sec t) e^{-t} dt \\ &= \int \sec t e^{-t} dt - \int \tan t \sec t e^{-t} dt\end{aligned}$$

Integrating by parts

$$\begin{aligned}&= e^{-t} \sec t + \int e^{-t} \frac{d}{dt} (\sec t) dt - \int \tan t \sec t e^{-t} dt \\ &= -e^{-t} \sec t + \int e^{-t} \sec t \tan t dt - \int \sec t \tan t e^{-t} dt \\ &= -e^{-t} \sec t + c\end{aligned}$$

Putting the value of t

$$= -e^{-\frac{x}{2}} \sec \frac{x}{2} + c$$

Indefinite Integrals Ex 19.26 Q15

We have,

$$I = \int e^x \left(\log x + \frac{1}{x} \right) dx$$

We know that

$$\int e^{ax} (af(x) + f'(x)) dx = e^{ax} f(x) + c$$

$$\text{Here } f(x) = \log x \text{ and } f'(x) = \frac{1}{x}$$

$$\therefore \int e^x \left(\log x + \frac{1}{x} \right) dx = e^x \log x + c$$

Indefinite Integrals Ex 19.26 Q16

We have,

$$\begin{aligned}
 I &= \int e^x \left(\log x + \frac{1}{x^2} \right) dx \\
 &= \int e^x \left(\log x + \frac{1}{x} - \frac{1}{x} + \frac{1}{x^2} \right) dx \\
 &= \int e^x \left(\log x - \frac{1}{x} \right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2} \right) dx
 \end{aligned}$$

Integrating by parts

$$\begin{aligned}
 &= e^x \left(\log x - \frac{1}{x} \right) - \int e^x \frac{d}{dx} \left(\log x - \frac{1}{x} \right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2} \right) dx \\
 &= e^x \left(\log x - \frac{1}{x} \right) - \int e^x \left(\frac{1}{x} + \frac{1}{x^2} \right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2} \right) dx \\
 &= e^x \left(\log x - \frac{1}{x} \right) + C
 \end{aligned}$$

Indefinite Integrals Ex 19.26 Q17

We have,

$$\begin{aligned}
 I &= \int \frac{e^x}{x} \left\{ x (\log x)^2 + 2 \log x \right\} dx \\
 &= \int e^x \left\{ (\log x)^2 + \frac{2}{x} \log x \right\} dx \\
 &= \int e^x (\log x)^2 dx + 2 \int \frac{e^x}{x} \log x dx
 \end{aligned}$$

Integrating by parts

$$\begin{aligned}
 &= e^x (\log x)^2 - \int e^x \frac{d}{dx} (\log x)^2 dx + 2 \int e^x \frac{1}{x} \log x dx \\
 &= e^x (\log x)^2 - \int e^x \frac{2 \log x}{x} dx + 2 \int e^x \frac{\log x}{x} dx \\
 &= e^x (\log x)^2 + C
 \end{aligned}$$

Indefinite Integrals Ex 19.26 Q18

$$\begin{aligned}
 \text{Let } I &= \int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}} \right) dx \\
 I &= \int e^x \sin^{-1} x dx + \int e^x \frac{1}{\sqrt{1-x^2}} dx
 \end{aligned}$$

Integrating by parts

$$\begin{aligned}
 &= e^x \sin^{-1} x - \int e^x \left(\frac{d}{dx} (\sin^{-1} x) \right) dx + \int e^x \frac{1}{\sqrt{1-x^2}} dx \\
 &= e^x \sin^{-1} x - \int e^x \frac{1}{\sqrt{1-x^2}} dx + \int e^x \frac{1}{\sqrt{1-x^2}} dx \\
 &= e^x \sin^{-1} x + C
 \end{aligned}$$

Indefinite Integrals Ex 19.26 Q19

$$\begin{aligned} \text{Let } I &= \int e^{2x} (-\sin x + 2 \cos x) dx \\ &= -\int e^{2x} \sin x dx + 2 \int e^{2x} \cos x dx \end{aligned}$$

Applying by parts in the 2nd integrand

$$\begin{aligned} \therefore I &= -\int e^{2x} \sin x dx + 2 \left\{ \frac{1}{2} e^{2x} \cos x + \int \frac{1}{2} e^{2x} \sin x dx \right\} \\ &= -\int e^{2x} \sin x dx + e^{2x} \cos x + \int e^{2x} \sin x dx + c \\ &= e^{2x} \cos x + c \end{aligned}$$

Thus,

$$I = e^{2x} \cos x + c$$

Indefinite Integrals Ex 19.26 Q20

$$\text{Let } I = \int e^x \left(\tan^{-1} x + \frac{1}{1+x^2} \right) dx$$

$$\text{Here, } f(x) = \tan^{-1} x \text{ and } f'(x) = \frac{1}{1+x^2}$$

And we know that,

$$\int e^{ax} (af(x) + f'(x)) dx = e^{ax} f(x) + c$$

$$\therefore \int e^x \left(\tan^{-1} x + \frac{1}{1+x^2} \right) dx = e^x \tan^{-1} x + c$$

Thus,

$$I = e^x \tan^{-1} x + c$$

Indefinite Integrals Ex 19.26 Q21

$$\begin{aligned} \text{Let } I &= \int e^x \left(\frac{\sin x \cos x - 1}{\sin^2 x} \right) dx \\ &= \int e^x (\cot x - \operatorname{cosec}^2 x) dx \\ &= \int e^x (\cot x + (-\operatorname{cosec}^2 x)) dx \end{aligned}$$

$$\therefore \int e^{ax} (af(x) + f'(x)) dx = e^{ax} f(x) + c$$

$$\text{Here } f(x) = \cot x$$

$$\Rightarrow f'(x) = -\operatorname{cosec}^2 x$$

$$\therefore \int e^x (\cot x - \operatorname{cosec}^2 x) dx = e^x \cot x + c$$

Thus,

$$I = e^x \cot x + c$$

Indefinite Integrals Ex 19.26 Q22

$$\text{Let } I = \int \{\tan(\log x) + \sec^2(\log x)\} dx$$

Let $\log x = z$

$$\Rightarrow x = e^z$$

$$\Rightarrow dx = e^z dz$$

$$\therefore I = \int \{\tan z + \sec^2 z\} e^z dz$$

$$\text{Here, } f(z) = \tan z \text{ and } f'(z) = \sec^2 z$$

And we know that

$$\int e^{az} (af(x) + f'(x)) dx = e^{ax} f(x) + C$$

$$\therefore \int e^z \{\tan z + \sec^2 z\} dz = e^z \tan z + C$$

$$\therefore I = x \tan(\log x) + C$$

Indefinite Integrals Ex 19.26 Q23

$$\text{Let } I = \int \frac{e^x (x - 4)}{(x - 2)^3} dx$$

$$= \int e^x \left\{ \frac{(x - 2) - 2}{(x - 2)^3} \right\} dx$$

$$= \int e^x \left\{ \frac{1}{(x - 2)^2} - \frac{2}{(x - 2)^3} \right\} dx$$

$$\text{Here, } f(x) = \frac{1}{(x - 2)^2} \text{ and } f'(x) = \frac{-2}{(x - 2)^3}$$

And we know that,

$$\int e^{ax} (af(x) + f'(x)) dx = e^{ax} f(x) + C$$

$$\therefore \int e^x \left\{ \frac{1}{(x - 2)^2} - \frac{2}{(x - 2)^3} \right\} dx = \frac{e^x}{(x - 2)^2} + C$$

$$\therefore I = \frac{e^x}{(x - 2)^2} + C$$

Indefinite Integrals Ex 19.26 Q24

$$\text{Let } I = \int e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$$

We have, $\cos 2x = 1 - 2\sin^2 x$

$$\begin{aligned} I &= \int e^{2x} \left(\frac{1 - \sin 2x}{1 - (1 - 2\sin^2 x)} \right) dx \\ &= \int e^{2x} \left(\frac{1 - \sin 2x}{2\sin^2 x} \right) dx \\ &= \int e^{2x} \left(\frac{\csc^2 x}{2} - \frac{2\sin x \cos x}{2\sin^2 x} \right) dx \\ &= \int e^{2x} \left(\frac{\csc^2 x}{2} - \frac{\cos x}{\sin x} \right) dx \\ &= \int e^{2x} \left(\frac{\csc^2 x}{2} - \cot x \right) dx \\ &= \frac{1}{2} \int e^{2x} \csc^2 x dx - \int e^{2x} \cot x dx \end{aligned}$$

That is

$$I = I_1 + I_2, \text{ where, } I_1 = \frac{1}{2} \int e^{2x} \csc^2 x dx \text{ and } I_2 = - \int e^{2x} \cot x dx$$

$$\text{Consider } I_1 = \frac{1}{2} \int e^{2x} \csc^2 x dx$$

Take e^{2x} as the first function and $\csc^2 x$ as the second function.

So, $u = e^{2x}$; $du = 2e^{2x} dx$

and

$$\int \csc^2 x dx = \int dv$$

$$\Rightarrow v = -\cot x$$

$$I_1 = \frac{1}{2} [e^{2x}(-\cot x) - \int (-\cot x) 2e^{2x} dx]$$

$$\Rightarrow I_1 = \frac{1}{2} [e^{2x}(-\cot x) + 2 \int \cot x e^{2x} dx]$$

$$\Rightarrow I_1 = \frac{1}{2} [e^{2x}(-\cot x)] + \int \cot x e^{2x} dx$$

Thus,

$$I = \frac{1}{2} [e^{2x}(-\cot x)] + \int \cot x e^{2x} dx - \int e^{2x} \cot x dx$$

$$= I = \frac{1}{2} [e^{2x}(-\cot x)] + C$$

Ex 19.27

Indefinite Integrals Ex 19.27 Q1

$$\text{Let } I = \int e^{ax} \cos bx dx$$

Integrating by parts,

$$\begin{aligned} I &= e^{ax} \frac{\sin bx}{b} - a \int e^{ax} \frac{\sin bx}{b} dx \\ &= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \int e^{ax} \sin bx dx \\ &= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \left[-e^{ax} \frac{\cos bx}{b} + \int ae^{ax} \frac{\cos bx}{b} dx \right] \\ &= \frac{1}{b} e^{ax} \sin bx + \frac{a^2}{b^2} e^{ax} \cos bx - \frac{a^2}{b^2} \int e^{ax} \cos bx dx \\ \Rightarrow I &= \frac{e^{ax}}{b^2} [b \sin bx + a \cos bx] - \frac{a^2}{b^2} I + c \\ \Rightarrow I &= \left\{ \frac{a^2 + b^2}{b^2} \right\} \frac{e^{ax}}{b^2} [b \cos bx + a \sin bx] + c \end{aligned}$$

Thus,

$$I = \frac{e^{ax}}{a^2 + b^2} [a \cos bx + b \sin bx] + c$$

Indefinite Integrals Ex 19.27 Q2

$$\text{Let } I = \int e^{ax} \sin(bx + c) dx$$

$$\begin{aligned} \Rightarrow I &= -e^{ax} \frac{\cos(bx + c)}{b} + \int ae^{ax} \frac{\cos(bx + c)}{b} dx \\ &= -\frac{1}{b} e^{ax} \cos(bx + c) + \frac{a}{b} \int e^{ax} \cos(bx + c) dx \\ &= -\frac{1}{b} e^{ax} \cos(bx + c) + \frac{a}{b} \left[\int e^{ax} \frac{\sin(bx + c)}{b} - \int ae^{ax} \frac{\sin(bx + c)}{b} dx \right] + c_1 \\ &= \frac{e^{ax}}{b^2} \{a \sin(bx + c) - b \cos(bx + c)\} - \frac{a^2}{b^2} \int e^{ax} \sin(bx + c) dx + c_1 \\ \Rightarrow I &= \frac{e^{ax}}{b^2} \{a \sin(bx + c) - b \cos(bx + c)\} - \frac{a^2}{b^2} I + c_1 \\ \Rightarrow I &= \left\{ \frac{a^2 + b^2}{b^2} \right\} \frac{e^{ax}}{b^2} \{a \sin(bx + c) - b \cos(bx + c)\} + c_1 \\ \Rightarrow I &= \frac{e^{ax}}{a^2 + b^2} \{a \sin(bx + c) - b \cos(bx + c)\} + c_1 \end{aligned}$$

Indefinite Integrals Ex 19.27 Q3

Let $\log x = t$

$$\begin{aligned}\Rightarrow \frac{1}{x} dx &= dt \\ \Rightarrow dx &= xdt \\ \Rightarrow dx &= e^t dt\end{aligned}$$

$$\therefore I = \int \cos(\log x) dx = \int e^t \cos t dt$$

We know that

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \cos bx + b \sin bx\} + C$$

Here $a = 1, b = 1$

$$\text{So, } I = \frac{e^t}{2} \{\cos t + \sin t\} + C$$

Hence,

$$I = \int \cos(\log x) dx = \frac{e^{\log x}}{2} \{\cos(\log x) + \sin(\log x)\} + C$$

$$\Rightarrow I = \frac{x}{2} \{\cos(\log x) + \sin(\log x)\} + C$$

Indefinite Integrals Ex 19.27 Q4

$$\text{Let } I = \int e^{2x} \cos(3x + 4) dx$$

Integrating by parts

$$\begin{aligned}I &= e^{2x} \frac{\sin(3x + 4)}{3} - \int 2e^{2x} \frac{\sin(3x + 4)}{3} dx \\ &= \frac{1}{3} e^{2x} \sin(3x + 4) - \frac{2}{3} \int e^{2x} \sin(3x + 4) dx \\ &= \frac{1}{3} e^{2x} \sin(3x + 4) - \frac{2}{3} \left[-e^{2x} \frac{\cos(3x + 4)}{3} + \int 2e^{2x} \frac{\cos(3x + 4)}{3} dx \right] + C \\ I &= \frac{e^{2x}}{9} \{2 \cos(3x + 4) + 3 \sin(3x + 4)\} + C\end{aligned}$$

Hence,

$$I = \frac{e^{2x}}{13} \{2 \cos(3x + 4) + 3 \sin(3x + 4)\} + C$$

Indefinite Integrals Ex 19.27 Q5

Let $I = \int e^{2x} \sin x \cos x dx$

$$\begin{aligned} &= \frac{1}{2} \int e^{2x} 2 \sin x \cos x dx \\ &= \frac{1}{2} \int e^{2x} \sin 2x dx \end{aligned}$$

We know that

$$\begin{aligned} \int e^{ax} \sin bx dx &= \frac{e^{ax}}{a^2 + b^2} \{a \sin bx - b \cos bx\} + c \\ \Rightarrow \int e^{2x} \sin 2x dx &= \frac{e^{2x}}{8} \{2 \sin 2x - 2 \cos 2x\} + c \end{aligned}$$

$$\therefore I = \frac{1}{2} \cdot \frac{e^{2x}}{8} \{2 \sin 2x - 2 \cos 2x\} + c$$

$$\therefore I = \frac{e^{2x}}{8} \{\sin 2x - \cos 2x\} + c$$

Indefinite Integrals Ex 19.27 Q6

$$\text{Let } I = \int e^{2x} \sin x dx \quad \dots(1)$$

Integrating by parts, we obtain

$$\begin{aligned} I &= \sin x \int e^{2x} dx - \int \left(\frac{d}{dx} \sin x \right) \int e^{2x} dx dx \\ &\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} dx \\ &\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \int e^{2x} \cos x dx \end{aligned}$$

Again integrating by parts, we obtain

$$\begin{aligned} I &= \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\cos x \int e^{2x} dx - \int \left(\frac{d}{dx} \cos x \right) \int e^{2x} dx dx \right] \\ &\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[\cos x \cdot \frac{e^{2x}}{2} - \int (-\sin x) \frac{e^{2x}}{2} dx \right] \\ &\Rightarrow I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\frac{e^{2x} \cos x}{2} + \frac{1}{2} \int e^{2x} \sin x dx \right] \\ &\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} I \quad [\text{From (1)}] \end{aligned}$$

$$\begin{aligned} &\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} I \\ &\Rightarrow I + \frac{1}{4} I = \frac{e^{2x} \cdot \sin x}{2} - \frac{e^{2x} \cos x}{4} \\ &\Rightarrow \frac{5}{4} I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} \\ &\Rightarrow I = \frac{4}{5} \left[\frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} \right] + C \\ &\Rightarrow I = \frac{e^{2x}}{5} [2 \sin x - \cos x] + C \end{aligned}$$

Indefinite Integrals Ex 19.27 Q8

Let $I = \int e^x \sin^2 x dx$

$$\begin{aligned} &= \frac{1}{2} \int e^x 2 \sin^2 x dx \\ &= \frac{1}{2} \int e^x (1 - \cos 2x) dx \\ &= \frac{1}{2} \int e^x dx - \frac{1}{2} \int e^x \cos 2x dx \end{aligned}$$

$$\therefore \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \cos bx - b \sin bx\} + c$$

$$\therefore I = \frac{1}{2} \left[e^x - \frac{e^x}{5} \{\cos 2x + 2 \sin 2x\} \right] + c$$

$$\therefore I = \frac{e^x}{2} - \frac{e^x}{10} \{\cos 2x + 2 \sin 2x\} + c$$

Indefinite Integrals Ex 19.27 Q9

Let $I = \int \frac{1}{x^3} \sin(\log x) dx$

Let $\log x = t$

$$\begin{aligned} \Rightarrow \frac{1}{x} dx &= dt \\ \Rightarrow dx &= e^t dt \end{aligned}$$

$$\therefore I = \int e^{-2t} \sin t dt$$

We know that,

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \sin bx - b \cos bx\} + c$$

$$\therefore \int e^{-2t} \sin t dt = \frac{e^{-2t}}{5} \{-2 \sin t - \cos t\} + c$$

$$\therefore I = \frac{x^{-2}}{5} \{-2 \sin(\log x) - \cos(\log x)\} + c$$

Hence,

$$\int \frac{1}{x^3} \sin(\log x) dx = \frac{-1}{5x^2} \{2 \sin(\log x) + \cos(\log x)\} + c$$

Indefinite Integrals Ex 19.27 Q10

Let $I = \int e^{2x} \cos^2 x dx$

$$\begin{aligned} &= \frac{1}{2} \int e^{2x} 2 \cos^2 x dx \\ &= \frac{1}{2} \int e^{2x} (1 + \cos 2x) dx \\ &= \frac{1}{2} \int e^{2x} dx + \frac{1}{2} \int e^{2x} \cos 2x dx \end{aligned}$$

$$\therefore \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \cos bx - b \sin bx\} + c$$

$$\therefore I = \frac{1}{4} e^{2x} + \frac{1}{2} \frac{e^{2x}}{8} \{2 \cos 2x + 2 \sin 2x\} + c$$

Hence,

$$I = \frac{e^{2x}}{4} + \frac{e^{2x}}{16} \{2 \cos 2x + 2 \sin 2x\} + c$$

or

$$I = \frac{e^{2x}}{4} + \frac{e^{2x}}{8} \{\cos 2x + \sin 2x\} + c$$

Indefinite Integrals Ex 19.27 Q11

Let $I = \int e^{-2x} \sin x dx$

$$\therefore \int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \sin bx - b \cos bx\} + c$$

$$\therefore I = \frac{e^{-2x}}{5} \{-2 \sin x - \cos x\} + c$$

Indefinite Integrals Ex 19.27 Q12

Let $I = \int x^2 e^{x^3} \cos x^3 dx$

$$\text{Let } x^3 = t$$

$$\Rightarrow 3x^2 dx = dt$$

$$\therefore I = \frac{1}{3} \int e^t \cos t dt$$

$$\therefore \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \cos bx + b \sin bx\} + c$$

$$\therefore I = \frac{1}{3} \left\{ \frac{e^t}{2} (\cos t + \sin t) \right\} + c$$

$$\therefore I = \frac{1}{3} \left\{ \frac{e^{x^3}}{2} (\cos x^3 + \sin x^3) \right\} + c$$

Ex 19.28

Indefinite Integrals Ex 19.28 Q1

$$\int \sqrt{3+2x-x^2} dx = \int \sqrt{4-(x-1)^2} dx$$

Let $x-1=t$, so that $dx=dt$

$$\text{Thus, } \int \sqrt{3+2x-x^2} dx = \int \sqrt{4-t^2} dt$$

$$= \frac{1}{2} t \sqrt{4-t^2} + \frac{1}{2} \sin^{-1}\left(\frac{t}{2}\right) + C$$

$$= \frac{1}{2}(x-1)\sqrt{3+2x-x^2} + 2\sin^{-1}\left(\frac{x-1}{2}\right) + C$$

Indefinite Integrals Ex 19.28 Q2

$$\text{Let } I = \int \sqrt{x^2+x+1} dx$$

$$= \int \sqrt{x^2+x+\frac{1}{4}+\frac{3}{4}} dx$$

$$= \int \sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx$$

$$= \frac{\left(x+\frac{1}{2}\right)}{2} \sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \frac{\left(\frac{\sqrt{3}}{2}\right)^2}{2} \cdot \log \left| \left(x+\frac{1}{2}\right) + \sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \right| + C$$

$$= \left(\frac{2x+1}{4}\right) \sqrt{x^2+x+1} + \frac{3}{8} \log \left| \frac{2x+1}{2} + \sqrt{x^2+x+1} \right| + C$$

$$I = \left(\frac{2x+1}{4}\right) \sqrt{x^2+x+1} + \frac{3}{8} \log \left| 2x+1 + \sqrt{x^2+x+1} \right| + C$$

Indefinite Integrals Ex 19.28 Q3

$$\text{Let } I = \int \sqrt{x-x^2} dx$$

$$= \int \sqrt{\frac{1}{4} - \frac{1}{4} + x - x^2} dx \quad \left[\text{Add and subtract } \frac{1}{4} \right]$$

$$= \int \sqrt{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2} - x\right)^2} dx$$

$$= -\left(\frac{1-2x}{4}\right) \sqrt{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2} - x\right)^2} - \frac{\left(\frac{1}{2}\right)^2}{2} \sin^{-1} \left(\frac{1-2x}{\frac{1}{2}} \right) + C$$

$$I = \left(\frac{2x-1}{4}\right) \sqrt{x-x^2} + \frac{1}{8} \sin^{-1}(2x-1) + C$$

Indefinite Integrals Ex 19.28 Q4

$$\text{Let } I = \int \sqrt{1+x-2x^2} dx$$

$$\begin{aligned} &= \sqrt{2} \int \sqrt{\frac{1}{2} + \frac{x}{2} - x^2} dx \\ &= \sqrt{2} \int \sqrt{\frac{9}{16} - \left(\frac{1}{16} - \frac{x}{2} + x^2\right)} dx \\ &= \sqrt{2} \int \sqrt{\left(\frac{3}{4}\right)^2 - \left(x - \frac{1}{4}\right)^2} dx \\ &= \sqrt{2} \left\{ \frac{\left(x - \frac{1}{4}\right)}{2} \sqrt{\frac{1}{2} + \frac{x}{2} - x^2} + \frac{9}{32} \sin^{-1} \left(\frac{x - \frac{1}{4}}{\frac{3}{4}} \right) \right\} + C \\ I &= \frac{1}{8} (4x - 1) \sqrt{1+x-2x^2} + \frac{9\sqrt{2}}{32} \sin^{-1} \left(\frac{4x-1}{3} \right) + C \end{aligned}$$

Indefinite Integrals Ex 19.28 Q5

$$\text{Let } I = \int \cos x \sqrt{4 - \sin^2 x} dx$$

$$\begin{aligned} \text{Let } \sin x &= t \\ \Rightarrow \quad \cos x dx &= dt \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad I &= \int \sqrt{4 - t^2} dt \\ &= \int \sqrt{2^2 - t^2} dt \\ &= \frac{t}{2} \sqrt{2^2 - t^2} + \frac{4}{2} \sin^{-1} \frac{t}{2} + C \\ \therefore \quad I &= \frac{1}{2} \sin x \sqrt{4 - \sin^2 x} + 2 \sin^{-1} \left(\frac{\sin x}{2} \right) + C \end{aligned}$$

Indefinite Integrals Ex 19.28 Q6

$$\text{Let } I = \int e^x \sqrt{e^{2x} + 1} dx$$

$$\begin{aligned} \text{Let } e^x &= t \\ \Rightarrow \quad e^x dx &= dt \end{aligned}$$

$$\begin{aligned} \therefore \quad I &= \int \sqrt{t^2 + 1^2} dt \\ &= \frac{t}{2} \sqrt{t^2 + 1} + \frac{1}{2} \log \left| t + \sqrt{t^2 + 1} \right| + C \\ \therefore \quad I &= \frac{e^x}{2} \sqrt{e^{2x} + 1} + \frac{1}{2} \log \left| e^x + \sqrt{e^{2x} + 1} \right| + C \end{aligned}$$

Indefinite Integrals Ex 19.28 Q7

$$\text{Let } I = \int \sqrt{3^2 - x^2}$$

We know that,

$$\begin{aligned} \int \sqrt{a^2 - x^2} &= \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C \\ \therefore \quad I &= \frac{x}{2} \sqrt{9 - x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} + C \end{aligned}$$

Indefinite Integrals Ex 19.28 Q8

$$\text{Let } I = \int \sqrt{16x^2 + 25} dx$$

$$\begin{aligned} &= \int \sqrt{(4x)^2 + 5^2} dx \\ &= 4 \int \sqrt{x^2 + \left(\frac{5}{4}\right)^2} dx \\ &= 4 \left\{ \frac{x}{2} \sqrt{x^2 + \left(\frac{5}{4}\right)^2} + \frac{\left(\frac{5}{4}\right)^2}{2} \log \left| x + \sqrt{x^2 + \left(\frac{5}{4}\right)^2} \right| + C \right\} \\ \therefore I &= 2x \sqrt{x^2 + \frac{25}{16}} + \frac{25}{8} \log \left| x + \sqrt{x^2 + \frac{25}{16}} \right| + C \end{aligned}$$

Indefinite Integrals Ex 19.28 Q9

$$\text{Let } I = \int \sqrt{4x^2 - 5} dx$$

$$\begin{aligned} &= 2 \int \sqrt{x^2 - \left(\frac{\sqrt{5}}{2}\right)^2} dx \\ &= 2 \left\{ \frac{x}{2} \sqrt{x^2 - \frac{5}{4}} - \frac{5}{8} \log \left| x + \sqrt{x^2 - \frac{5}{4}} \right| + C \right\} \\ \therefore I &= x \sqrt{x^2 - \frac{5}{4}} - \frac{5}{4} \log \left| x + \sqrt{x^2 - \frac{5}{4}} \right| + C \end{aligned}$$

Indefinite Integrals Ex 19.28 Q10

$$\text{Let } I = \int \sqrt{2x^2 + 3x + 4} dx$$

$$\begin{aligned} &= \sqrt{2} \int \sqrt{x^2 + \frac{3}{2}x + 2} dx \\ &= \sqrt{2} \int \sqrt{x^2 + \frac{3}{2}x + \frac{9}{16} + \frac{23}{16}} dx \\ &= \sqrt{2} \int \sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2} dx \\ &= \sqrt{2} \left\{ \frac{\left(x + \frac{3}{4}\right)}{2} \sqrt{x^2 + \frac{3}{2}x + 2} + \frac{23}{32} \cdot \log \left| \left(x + \frac{3}{4}\right) + \sqrt{x^2 + \frac{3}{2}x + 2} \right| + C \right\} \\ \therefore I &= \frac{4x+3}{8} \sqrt{2x^2 + 3x + 4} + \frac{23\sqrt{2}}{32} \cdot \log \left| \left(x + \frac{3}{4}\right) + \sqrt{x^2 + \frac{3}{2}x + 2} \right| + C \end{aligned}$$

Indefinite Integrals Ex 19.28 Q11

$$\text{Let } I = \int \sqrt{3 - 2x - 2x^2} dx$$

$$\begin{aligned}
&= \sqrt{2} \int \sqrt{\frac{3}{2} - x - x^2} dx \\
&= \sqrt{2} \int \sqrt{\frac{7}{4} - \left(\frac{1}{4} + x + x^2\right)} dx \quad [\text{Adding and subtracting } \frac{1}{4}] \\
&= \sqrt{2} \int \sqrt{\left(\frac{\sqrt{7}}{2}\right)^2 - \left(x + \frac{1}{2}\right)^2} dx \\
&= \sqrt{2} \left\{ \frac{x + \frac{1}{2}}{2} \sqrt{\frac{3}{2} - x - x^2} + \frac{7}{8} \sin^{-1} \left(\frac{x + \frac{1}{2}}{\frac{\sqrt{7}}{2}} \right) + C \right\} \\
\therefore I &= \frac{2x + 1}{4} \sqrt{3 - 2x - 2x^2} + \frac{7\sqrt{2}}{8} \sin^{-1} \left(\frac{2x + 1}{\sqrt{7}} \right) + C
\end{aligned}$$

Indefinite Integrals Ex 19.28 Q12

$$\begin{aligned}
\text{Let } x^2 &= t \\
\Rightarrow 2x dx &= dt \\
\therefore I &= \frac{1}{2} \int \sqrt{t^2 + 1^2} dt \\
&= \frac{1}{2} \left\{ \frac{t}{2} \sqrt{t^2 + 1} + \frac{1}{2} \log \left| t + \sqrt{t^2 + 1} \right| \right\} + C \\
\therefore I &= \frac{1}{2} \left\{ \frac{x^2}{2} \sqrt{x^4 + 1} + \frac{1}{2} \log \left| x^2 + \sqrt{x^4 + 1} \right| \right\} + C
\end{aligned}$$

Indefinite Integrals Ex 19.28 Q13

$$\begin{aligned}
\text{Let } I &= \int x^2 \sqrt{a^6 - x^6} dx \\
\text{Let } x^3 &= t \\
\Rightarrow 3x^2 dx &= dt \\
\therefore I &= \frac{1}{3} \int \sqrt{a^6 - t^2} dt \\
&= \frac{1}{3} \left\{ \frac{t}{2} \sqrt{a^6 - t^2} + \frac{a^6}{2} \sin^{-1} \left(\frac{t}{a^3} \right) \right\} + C \\
\therefore I &= \frac{x^3}{6} \sqrt{a^6 - x^6} + \frac{a^6}{6} \sin^{-1} \left(\frac{x^3}{a^3} \right) + C
\end{aligned}$$

Indefinite Integrals Ex 19.28 Q14

$$\begin{aligned}
\text{Let } I &= \int \frac{\sqrt{16 + (\log x)^2}}{x} dx \\
\text{Let } \log x &= t \\
\Rightarrow \frac{1}{x} dx &= dt \\
\therefore I &= \int \sqrt{16 + t^2} dt \\
&= \int \sqrt{4^2 + t^2} dt \\
&= \frac{t}{2} \sqrt{16 + t^2} + \frac{16}{2} \log \left| t + \sqrt{16 + t^2} \right| + C \\
\therefore I &= \frac{\log x}{2} \sqrt{16 + (\log x)^2} + 8 \log \left| \log x + \sqrt{16 + (\log x)^2} \right| + C
\end{aligned}$$

Indefinite Integrals Ex 19.28 Q15

$$\text{Let } I = \int \sqrt{2ax - x^2} dx$$

$$\begin{aligned}
&= \int \sqrt{a^2 - (a^2 - 2ax + x^2)} dx && [\text{Adding and subtracting } a^2] \\
&= \int \sqrt{a^2 - (a - x)^2} dx \\
&= \int \sqrt{a^2 - (x - a)^2} dx \\
&= \frac{(x - a)}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x - a}{a} \right) + C \\
\therefore I &= \frac{1}{2} (x - a) \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x - a}{a} \right) + C
\end{aligned}$$

Indefinite Integrals Ex 19.28 Q16

$$\text{Let } I = \int \sqrt{3 - x^2} dx$$

$$\begin{aligned}
&= \int \sqrt{(\sqrt{3})^2 - x^2} dx \\
I &= \frac{x}{2} \sqrt{3 - x^2} + \frac{3}{2} \sin^{-1} \left(\frac{x}{\sqrt{3}} \right) + C
\end{aligned}$$

Ex 19.29

Indefinite Integrals Ex 19.29 Q1

$$\text{Let } I = \int (x+1) \sqrt{x^2 - x + 1} dx \quad \dots \dots (1)$$

$$\begin{aligned} \text{Let } x+1 &= \lambda \frac{d}{dx}(x^2 - x + 1) + \mu \\ &= \lambda(2x - 1) + \mu \end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned} 2\lambda &= 1 \Rightarrow \lambda = \frac{1}{2} \\ -\lambda + \mu &= 1 \\ \Rightarrow \mu &= 1 + \lambda = 1 + \frac{1}{2} = \frac{3}{2} \therefore \mu = \frac{3}{2} \end{aligned}$$

So,

$$\begin{aligned} I &= \int \left(\frac{1}{2}(2x-1) + \frac{3}{2} \right) \sqrt{x^2 - x + 1} dx \\ &= \frac{1}{2} \int (2x-1) \sqrt{x^2 - x + 1} dx + \frac{3}{2} \int \sqrt{x^2 - x + 1} dx \end{aligned}$$

$$\begin{aligned} \text{Let } x^2 - x + 1 &= t \\ \Rightarrow (2x-1) dx &= dt \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} \int \sqrt{t} dt + \frac{3}{2} \int \sqrt{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx \\ &= \frac{1}{2} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3}{2} \int \frac{\left(x - \frac{1}{2}\right)}{2} \sqrt{x^2 - x + 1} + \frac{3}{8} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1} \right| \\ \Rightarrow I &= \frac{1}{3} t^{\frac{3}{2}} + \frac{3}{8} (2x-1) \sqrt{x^2 - x + 1} + \frac{9}{16} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1} \right| + C \end{aligned}$$

Hence,

$$I = \frac{1}{3} (x^2 - x + 1)^{\frac{3}{2}} + \frac{3}{8} (2x-1) \sqrt{x^2 - x + 1} + \frac{9}{16} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1} \right| + C$$

Indefinite Integrals Ex 19.29 Q2

$$\text{Let } I = \int (x+1) \sqrt{2x^2 + 3} dx$$

$$\begin{aligned}\text{Let } x+1 &= \lambda \frac{d}{dx} (2x^2 + 3) + \mu \\ &= \lambda (4x) + \mu\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}4\lambda &= 1 \Rightarrow \lambda = \frac{1}{4} \\ \mu &= 1\end{aligned}$$

$$\therefore I = \int \frac{1}{4} (4x) \sqrt{2x^2 + 3} dx + \int 1 \cdot \sqrt{2x^2 + 3} dx$$

$$\begin{aligned}\text{Let } 2x^2 + 3 &= t \\ \Rightarrow 4x dx &= dt\end{aligned}$$

$$\begin{aligned}I &= \frac{1}{4} \int \sqrt{t} dt + \sqrt{2} \int \sqrt{x^2 + \frac{3}{2}} dx \\ &= \frac{1}{4} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \sqrt{2} \left\{ \frac{x}{2} \sqrt{x^2 + \frac{3}{2}} + \frac{3}{4} \log \left| x + \sqrt{x^2 + \frac{3}{2}} \right| \right\} + C\end{aligned}$$

Hence,

$$I = \frac{1}{6} (2x^2 + 3)^{\frac{3}{2}} + \frac{x}{2} \sqrt{2x^2 + 3} + \frac{3}{2\sqrt{2}} \log \left| x + \sqrt{x^2 + \frac{3}{2}} \right| + C$$

Indefinite Integrals Ex 19.29 Q3

$$\text{Let } I = \int (2x - 5) \sqrt{2 + 3x - x^2} dx$$

$$\begin{aligned}\text{Let } 2x - 5 &= \lambda \frac{d}{dx} (2 + 3x - x^2) + \mu \\ &= \lambda(3 - 2x) + \mu\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}-2\lambda &= 2 \Rightarrow \lambda = -1 \\ 3\lambda + \mu &= -5 \Rightarrow \mu = -5 - 3\lambda = -2\end{aligned}$$

$$\therefore \mu = -2$$

So,

$$\begin{aligned}I &= \left[(-1(3 - 2x) - 2) \sqrt{2 + 3x - x^2} \right] dx \\ &= - \int (3 - 2x) \sqrt{2 + 3x - x^2} dx - 2 \int \sqrt{2 + 3x - x^2} dx\end{aligned}$$

$$\text{Let } 2 + 3x - x^2 = t$$

$$\Rightarrow (3 - 2x) dx = dt$$

$$\begin{aligned}I &= - \int \sqrt{t} dt - 2 \int \sqrt{\frac{17}{4} - \left(\frac{9}{4} - 3x - x^2\right)} dx \\ &= - \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2 \int \sqrt{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2} dx \\ \Rightarrow I &= - \frac{2}{3} (2 + 3x - x^2)^{\frac{3}{2}} - 2 \int \frac{\left(x - \frac{3}{2}\right)}{2} \sqrt{2 + 3x - x^2} + \frac{17}{8} \sin^{-1} \left(\frac{x - \frac{3}{2}}{\frac{\sqrt{17}}{2}} \right) + C\end{aligned}$$

Hence,

$$I = - \frac{2}{3} (2 + 3x - x^2)^{\frac{3}{2}} - \frac{(2x - 3)}{2} \sqrt{2 + 3x - x^2} - \frac{17}{8} \sin^{-1} \left(\frac{2x - 3}{\sqrt{17}} \right) + C$$

Indefinite Integrals Ex 19.29 Q4

$$\text{Let } I = \int (x+2) \sqrt{x^2+x+1} dx$$

$$\begin{aligned}\text{Let } x+2 &= \lambda \frac{d}{dx}(x^2+x+1) + \mu \\ &= \lambda(2x+1) + \mu\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}2\lambda &= 1 \quad \Rightarrow \quad \lambda = \frac{1}{2} \\ \lambda + \mu &= 2 \quad \Rightarrow \quad \mu = 2 - \lambda = \frac{3}{2} \\ \therefore \quad \mu &= \frac{3}{2}\end{aligned}$$

$$\begin{aligned}\therefore \quad I &= \int \left(\frac{1}{2}(2x+1) + \frac{3}{2} \right) \sqrt{x^2+x+1} dx \\ &= \frac{1}{2} \int (2x+1) \sqrt{x^2+x+1} + \frac{3}{2} \int \sqrt{x^2+x+1} dx\end{aligned}$$

$$\begin{aligned}\text{Let } x^2+x+1 &= t \\ (2x+1)dx &= dt\end{aligned}$$

$$\begin{aligned}\therefore \quad I &= \frac{1}{2} \int \sqrt{t} dt + \frac{3}{2} \int \sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx \\ \Rightarrow \quad I &= \frac{1}{2} \cdot \frac{3}{2} t^{\frac{3}{2}} + \frac{3}{2} \int \frac{\left(x+\frac{1}{2}\right)}{2} \sqrt{x^2+x+1} + \frac{3}{8} \log \left| \left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1} \right| + C\end{aligned}$$

Hence,

$$I = \frac{1}{3} (x^2+x+1)^{\frac{3}{2}} + \frac{3}{8} (2x+1) \sqrt{x^2+x+1} + \frac{9}{16} \log \left| \left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1} \right| + C$$

Indefinite Integrals Ex 19.29 Q5

$$\text{Let } I = \int (4x+1) \sqrt{x^2 - x - 2} dx$$

$$\begin{aligned}\text{Let } 4x+1 &= \lambda \frac{d}{dx}(x^2 - x - 2) + \mu \\ &= \lambda(2x-1) + \mu\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}2\lambda &= 4 \Rightarrow \lambda = 2 \\ -\lambda + \mu &= 1 \Rightarrow \mu = 3\end{aligned}$$

So,

$$\begin{aligned}I &= \int (2(2x-1) + 3) \sqrt{x^2 - x - 2} dx \\ &= 2 \int (2x-1) \sqrt{x^2 - x - 2} dx + 3 \int \sqrt{x^2 - x - 2} dx\end{aligned}$$

$$\text{Let } x^2 - x - 2 = t$$

$$(2x-1)dx = dt$$

$$\begin{aligned}\therefore I &= 2 \int \sqrt{t} dt + 3 \int \sqrt{\left(x - \frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2} dx \\ \Rightarrow I &= 2 \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + 3 \left[\frac{\left(x - \frac{1}{2}\right)}{2} \sqrt{x^2 - x - 2} - \frac{9}{8} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 2} \right| \right] + C\end{aligned}$$

Hence,

$$I = \frac{4}{3} (x^2 - x - 2)^{\frac{3}{2}} + \frac{3}{4} (2x-1) \sqrt{x^2 - x - 2} - \frac{27}{8} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 2} \right| + C$$

Indefinite Integrals Ex 19.29 Q6

$$\text{Let } I = \int (x-2) \sqrt{2x^2 - 6x + 5} dx$$

$$\begin{aligned}\text{Let } x-2 &= \lambda \frac{d}{dx}(2x^2 - 6x + 5) + \mu \\ &= \lambda(4x - 6) + \mu\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}4\lambda &= 1 \Rightarrow \lambda = \frac{1}{4} \\ -6\lambda + \mu &= -2 \Rightarrow \mu = -2 + 6\lambda = -\frac{2}{4} = -\frac{1}{2} \\ \therefore \mu &= -\frac{1}{2}\end{aligned}$$

So,

$$\begin{aligned}I &= \int \left(\frac{1}{4}(4x-6) + \left(-\frac{1}{2} \right) \right) \sqrt{2x^2 - 6x + 5} dx \\ &= \frac{1}{4} \int (4x-6) \sqrt{2x^2 - 6x + 5} dx - \frac{1}{2} \int \sqrt{2x^2 - 6x + 5} dx\end{aligned}$$

$$\text{Let } 2x^2 - 6x + 5 = t$$

$$(4x-6)dx = dt$$

$$\begin{aligned}\therefore I &= \frac{1}{4} \int \sqrt{t} dt - \frac{\sqrt{2}}{2} \int \sqrt{x^2 - 3x + \frac{5}{2}} dx \\ \Rightarrow I &= \frac{1}{4} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{\sqrt{2}} \int \sqrt{\left(x - \frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} dx \\ &= \frac{1}{6} (2x^2 - 6x + 5)^{\frac{3}{2}} - \frac{1}{\sqrt{2}} \left\{ \frac{\left(x - \frac{3}{2}\right)}{2} \sqrt{x^2 - 3x + \frac{5}{2}} + \frac{1}{8} \log \left| \left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + \frac{5}{2}} \right| \right\} + C\end{aligned}$$

Hence,

$$I = \frac{1}{6} (2x^2 - 6x + 5)^{\frac{3}{2}} - \frac{1}{8} (2x-3) \sqrt{2x^2 - 6x + 5} - \frac{1}{8\sqrt{2}} \log \left| \left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + \frac{5}{2}} \right| + C$$

Indefinite Integrals Ex 19.29 Q7

$$\text{Let } I = \int (x+1) \sqrt{x^2+x+1} dx$$

$$\begin{aligned}\text{Let } x+1 &= \lambda \frac{d}{dx}(x^2+x+1) + \mu \\ &= \lambda(2x+1) + \mu\end{aligned}$$

Equating similar terms, we get,

$$2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$\lambda + \mu = 1 \Rightarrow \mu = \frac{1}{2}$$

So,

$$\begin{aligned}I &= \int \left(\frac{1}{2}(2x+1) + \frac{1}{2} \right) \sqrt{x^2+x+1} dx \\ &= \frac{1}{2} \int (2x+1) \sqrt{x^2+x+1} dx + \frac{1}{2} \int \sqrt{x^2+x+1} dx\end{aligned}$$

$$\text{Let } x^2+x+1 = t$$

$$\Rightarrow (2x+1) dx = dt$$

$$\begin{aligned}&= \frac{1}{2} \int \sqrt{t} dt + \frac{1}{2} \int \sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx \\ &= \frac{1}{2} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{2} \int \frac{\left(x+\frac{1}{2}\right)}{2} \sqrt{x^2+x+1} + \frac{3}{8} \log \left| \left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1} \right| + C\end{aligned}$$

$$\Rightarrow I = \frac{1}{3} (x^2+x+1)^{\frac{3}{2}} + \frac{1}{8} (2x+1) \sqrt{x^2+x+1} + \frac{3}{16} \log \left| \left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1} \right| + C$$

Hence,

$$I = \frac{1}{3} (x^2+x+1)^{\frac{3}{2}} + \frac{1}{8} (2x+1) \sqrt{x^2+x+1} + \frac{3}{16} \log \left| \left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1} \right| + C$$

Indefinite Integrals Ex 19.29 Q8

$$\text{Let } I = \int (2x+3) \sqrt{x^2 + 4x + 3} dx$$

$$\begin{aligned}\text{Let } (2x+3) &= \lambda \frac{d}{dx}(x^2 + 4x + 3) + \mu \\ &= \lambda(2x+4) + \mu\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}\lambda &= 1 \text{ and } 4\lambda + \mu = 3 \\ \Rightarrow \mu &= -1\end{aligned}$$

So,

$$\begin{aligned}I &= \int ((2x+4) + (-1)) \sqrt{x^2 + 4x + 3} dx \\ &= \int (2x+4) \sqrt{x^2 + 4x + 3} dx - \int \sqrt{x^2 + 4x + 3} dx\end{aligned}$$

$$\begin{aligned}\text{Let } x^2 + 4x + 3 &= t \\ \Rightarrow (2x+4) dx &= dt\end{aligned}$$

$$\begin{aligned}\therefore I &= \int \sqrt{t} dt - \int \sqrt{(x+2)^2 - 1} dx \\ &= \frac{3}{2} t^{\frac{3}{2}} - \frac{(x+2)}{2} \sqrt{x^2 + 4x + 3} + \frac{1}{2} \log |(x+2) + \sqrt{x^2 + 4x + 3}| + c\end{aligned}$$

Hence,

$$I = \frac{2}{3} (x^2 + 4x + 3)^{\frac{3}{2}} - \left(\frac{x+2}{2} \right) \sqrt{x^2 + 4x + 3} + \frac{1}{2} \log |(x+2) + \sqrt{x^2 + 4x + 3}| + c$$

Indefinite Integrals Ex 19.29 Q9

$$\text{Let } I = \int (2x-5) \sqrt{x^2 - 4x + 3} dx$$

$$\begin{aligned}\text{Let } (2x-5) &= \lambda \frac{d}{dx}(x^2 - 4x + 3) + \mu \\ &= \lambda(2x-4) + \mu\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}\lambda &= 1 \text{ and } -4\lambda + \mu = -5 \\ \Rightarrow \mu &= -1\end{aligned}$$

So,

$$\begin{aligned}I &= \int ((2x-4) - 1) \sqrt{x^2 - 4x + 3} dx \\ &= \int (2x-4) \sqrt{x^2 - 4x + 3} dx - \int \sqrt{x^2 - 4x + 3} dx\end{aligned}$$

$$\begin{aligned}\text{Let } x^2 - 4x + 3 &= t \\ \Rightarrow 2x - 4 dx &= dt\end{aligned}$$

$$\begin{aligned}I &= \int \sqrt{t} dt - \int \sqrt{(x-2)^2 - 1} dx \\ &= \frac{3}{2} t^{\frac{3}{2}} - \frac{(x-2)}{2} \sqrt{x^2 - 4x + 3} + \frac{1}{2} \log |(x-2) + \sqrt{x^2 - 4x + 3}| + c\end{aligned}$$

Thus,

$$I = \frac{2}{3} (x^2 - 4x + 3)^{\frac{3}{2}} - \frac{1}{2} (x-2) \sqrt{x^2 - 4x + 3} + \frac{1}{2} \log |(x-2) + \sqrt{x^2 - 4x + 3}| + c$$

Indefinite Integrals Ex 19.29 Q10

Let $I = \int x\sqrt{x^2 + x} dx$

$$\begin{aligned}\text{Let } x &= \lambda \frac{d}{dx} (x^2 + x) + \mu \\ &= \lambda (2x + 1) + \mu\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}2\lambda &= 1 \quad \Rightarrow \quad \lambda = \frac{1}{2} \\ \lambda + \mu &= 0 \quad \Rightarrow \quad \mu = -\frac{1}{2}\end{aligned}$$

So,

$$\begin{aligned}I &= \int \left(\frac{1}{2}(2x+1) - \frac{1}{2} \right) \sqrt{x^2+x} dx \\ &= \frac{1}{2} \int (2x+1) \sqrt{x^2+x} - \frac{1}{2} \int \sqrt{x^2+x} dx\end{aligned}$$

$$\begin{aligned}\text{Let } x^2 + x &= t \\ \Rightarrow (2x+1) dx &= dt\end{aligned}$$

So,

$$\begin{aligned}I &= \frac{1}{2} \int \sqrt{t} dt - \frac{1}{2} \int \sqrt{\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx \\ I &= \frac{1}{2} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2} \left\{ \frac{\left(x + \frac{1}{2}\right)}{2} \sqrt{x^2+x} - \frac{1}{8} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2+x} \right| \right\} + C\end{aligned}$$

Hence,

$$I = \frac{1}{3} (x^2 + x)^{\frac{3}{2}} - \frac{1}{8} \left(x + \frac{1}{2} \right) \sqrt{x^2+x} + \frac{1}{16} \log \left| \left(x + \frac{1}{2} \right) + \sqrt{x^2+x} \right| + C$$

Indefinite Integrals Ex 19.29 Q11

Consider the integral $I = \int (x - 3)\sqrt{x^2 + 3x - 18} dx$

$$\text{Let us express } x - 3 = \lambda \frac{d}{dx} [x^2 + 3x - 18] + \mu$$

$$\Rightarrow x - 3 = \lambda[2x + 3] + \mu$$

$$\Rightarrow x - 3 = 2\lambda x + 3\lambda + \mu$$

Comparing the coefficients, we have,

$$2\lambda = 1 \text{ and } 3\lambda + \mu = -3$$

$$\Rightarrow \lambda = \frac{1}{2} \text{ and } 3 \times \frac{1}{2} + \mu = -3$$

$$\Rightarrow \lambda = \frac{1}{2} \text{ and } \mu = -3 - \frac{3}{2}$$

$$\Rightarrow \lambda = \frac{1}{2} \text{ and } \mu = -\frac{9}{2}$$

Then

$$x - 3 = \lambda[2x + 3] + \mu$$

Now the integral $I = \int (x - 3)\sqrt{x^2 + 3x - 18} dx$

$$= \int \left[\frac{1}{2}[2x + 3] - \frac{9}{2} \right] \sqrt{x^2 + 3x - 18} dx$$

$$I = \frac{1}{2} \int [2x + 3] \sqrt{x^2 + 3x - 18} dx - \frac{9}{2} \int \sqrt{x^2 + 3x - 18} dx$$

$$\Rightarrow I = I_1 + I_2$$

$$\text{where, } I_1 = \frac{1}{2} \int [2x + 3] \sqrt{x^2 + 3x - 18} dx \text{ and}$$

$$I_2 = -\frac{9}{2} \int \sqrt{x^2 + 3x - 18} dx$$

Let us consider the integral, I_1 :

$$I_1 = \frac{1}{2} \int [2x + 3] \sqrt{x^2 + 3x - 18} dx$$

$$\text{Substituting, } x^2 + 3x - 18 = t$$

$$\Rightarrow (2x + 3)dx = dt$$

Thus,

$$I_1 = \frac{1}{2} \int \sqrt{t} dt$$

$$= \frac{1}{2} \times \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$\begin{aligned}
&= \frac{1}{2} \times \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C \\
&= \frac{1}{2} \times \frac{2}{3} \times t^{\frac{3}{2}} + C \\
&= \frac{1}{3} \times t^{\frac{3}{2}} + C \\
&= \frac{1}{3} \times (x^2 + 3x - 18)^{\frac{3}{2}} + C
\end{aligned}$$

Now consider the integral

$$\begin{aligned}
I_2 &= -\frac{9}{2} \int \sqrt{x^2 + 3x - 18} \, dx \\
&= -\frac{9}{2} \int \sqrt{x^2 + 2 \times \frac{3}{2}x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 18} \, dx \\
&= -\frac{9}{2} \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - 18} \, dx \\
&= -\frac{9}{2} \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9+18}{4}\right)} \, dx \\
&= -\frac{9}{2} \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9+72}{4}\right)} \, dx \\
&= -\frac{9}{2} \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{81}{4}\right)} \, dx \\
&= -\frac{9}{2} \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} \, dx \\
&= -\frac{9}{2} \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} \, dx
\end{aligned}$$

We know that $\int \sqrt{x^2 - a^2} \, dx = \frac{1}{2}x\sqrt{x^2 - a^2} - \frac{1}{2}a^2 \log |x + \sqrt{x^2 - a^2}| + C$

$$\begin{aligned}
\therefore I_2 &= -\frac{9}{2} \left[\frac{1}{2} \left(x + \frac{3}{2} \right) \sqrt{\left(x + \frac{3}{2} \right)^2 - \left(\frac{9}{2} \right)^2} - \frac{1}{2} \left(\frac{9}{2} \right)^2 \log \left| \left(x + \frac{3}{2} \right) + \sqrt{\left(x + \frac{3}{2} \right)^2 - \left(\frac{9}{2} \right)^2} \right| \right] + C \\
&= -\frac{9}{4} \left\{ \left(\frac{2x+3}{2} \right) \sqrt{x^2 + 3x - 18} - \left(\frac{729}{4} \right) \log \left| \left(x + \frac{3}{2} \right) + \sqrt{x^2 + 3x - 18} \right| \right\} + C \\
&= -\frac{9}{8} (2x+3) \sqrt{x^2 + 3x - 18} + \frac{729}{16} \log \left| \left(x + \frac{3}{2} \right) + \sqrt{x^2 + 3x - 18} \right| + C
\end{aligned}$$

Thus, $I = \frac{1}{3} (x^2 + 3x - 18)^{\frac{3}{2}} - \frac{9}{8} (2x+3) \sqrt{x^2 + 3x - 18} + \frac{729}{16} \log \left| \left(x + \frac{3}{2} \right) + \sqrt{x^2 + 3x - 18} \right| + C$

Indefinite Integrals Ex 19.29 Q12

$$\int (x+3)\sqrt{3-4x-x^2}dx$$

$$\text{Let } x+3 = A \frac{d}{dx}(3-4x-x^2) + B$$

$$x+3 = A(-4-2x) + B$$

$$x+3 = -2Ax + B - 4A$$

$$-2A = 1, B - 4A = 3$$

$$A = -\frac{1}{2},$$

$$B = 4 \times \left(-\frac{1}{2}\right) + 3 = 1$$

$$\int (x+3)\sqrt{3-4x-x^2}dx$$

$$x+3 = -\frac{1}{2}(-4-2x) + 1$$

$$\int \left[-\frac{1}{2}(-4-2x) + 1 \right] \sqrt{3-4x-x^2} dx$$

$$= -\frac{1}{2} \int (-4-2x) \sqrt{3-4x-x^2} dx + \int \sqrt{3-4x-x^2} dx$$

$$= I_1 + I_2, \dots, (i)$$

$$I_1 = -\frac{1}{2} \int (-4-2x) \sqrt{3-4x-x^2} dx$$

$$\text{Let } z = 3-4x-x^2$$

$$dz = -4-2x$$

$$I_1 = -\frac{1}{2} \int \sqrt{z} dz$$

$$= -\frac{1}{2} \left[\frac{z^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]$$

$$= -\frac{1}{2} \left[\frac{z^{\frac{3}{2}}}{\frac{3}{2}} \right]$$

$$= - \left[\frac{(3-4x-x^2)^{\frac{3}{2}}}{3} \right]$$

$$I_2 = \int \sqrt{3-4x-x^2} dx$$

$$= \int \sqrt{3-(x^2+4x+4)+4} dx$$

$$= \int \sqrt{7-(x^2+4x+4)} dx$$

$$= \int \sqrt{(\sqrt{7})^2 - (x+2)^2} dx$$

$$= \frac{(x+2)\sqrt{(\sqrt{7})^2 - (x+2)^2}}{2} + \frac{1}{2}(\sqrt{7})^2 \tan^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + C$$

$$= \frac{(x+2)\sqrt{3-4x-x^2}}{2} + \frac{7}{2} \tan^{-1}\left(\frac{x+2}{\sqrt{7}}\right)$$

From (i),

$$= I_1 + I_2$$

$$= -\frac{1}{3}(3-4x-x^2)^{\frac{3}{2}} + \frac{1}{2}(x+2)\sqrt{3-4x-x^2} + \frac{7}{2} \tan^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + C$$

Ex 19.30

Indefinite Integrals Ex 19.30 Q1

$$\text{Let } \int \frac{2x+1}{(x+1)(x-2)} dx = \frac{A}{(x+1)} + \frac{B}{(x-2)}$$

$$\Rightarrow 2x+1 = A(x-2) + B(x+1)$$

Put $x = 2$

$$\Rightarrow 5 = 3B \Rightarrow B = \frac{5}{3}$$

Put $x = -1$

$$\Rightarrow -1 = -3A \Rightarrow A = \frac{1}{3}$$

So,

$$\begin{aligned} \int \frac{2x+1}{(x+1)(x-2)} dx &= \frac{1}{3} \int \frac{dx}{x+1} + \frac{5}{3} \int \frac{dx}{x-2} \\ &= \frac{1}{3} \log|x+1| + \frac{5}{3} \log|x-2| + C \end{aligned}$$

Thus,

$$I = \frac{1}{3} \log|x+1| + \frac{5}{3} \log|x-2| + C$$

Indefinite Integrals Ex 19.30 Q2

$$\text{Let } \int \frac{1}{x(x-2)(x-4)} dx = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-4}$$

$$\Rightarrow 1 = A(x-2)(x-4) + B(x)(x-4) + Cx(x-2)$$

Put $x = 0$

$$\Rightarrow 1 = 8A \Rightarrow A = \frac{1}{8}$$

Put $x = 2$

$$\Rightarrow 1 = -4B \Rightarrow B = -\frac{1}{4}$$

Put $x = 4$

$$\Rightarrow 1 = 8C \Rightarrow C = \frac{1}{8}$$

So,

$$\begin{aligned} \int \frac{1}{x(x-2)(x-4)} dx &= \frac{1}{8} \int \frac{dx}{x} + \left(-\frac{1}{4}\right) \int \frac{dx}{x-2} + \frac{1}{8} \int \frac{dx}{x-4} \\ &= \frac{1}{8} \log|x| - \frac{1}{4} \log|x-2| + \frac{1}{8} \log|x-4| + c \\ &= \frac{1}{8} \log \left| \frac{x(x-4)}{(x-2)^2} \right| + c \end{aligned}$$

Thus,

$$I = \frac{1}{8} \log \left| \frac{x(x-4)}{(x-2)^2} \right| + c$$

Indefinite Integrals Ex 19.30 Q3

$$\text{Let } I = \int \frac{x^2+x-1}{x^2+x-6} dx$$

$$= \int 1 + \frac{5}{x^2+x-6} dx$$

$$\Rightarrow I = \int dx + \int \frac{5dx}{(x+3)(x-2)} \quad \dots \quad (1)$$

$$\text{Let } \frac{5}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$$

$$\Rightarrow 5 = A(x-2) + B(x+3)$$

Put $x = 2$

$$\Rightarrow 5 = 5B \Rightarrow B = 1$$

Put $x = -3$

$$\Rightarrow 5 = -5A \Rightarrow A = -1$$

$$\therefore I = \int dx + \int \frac{-dx}{x+3} + \int \frac{dx}{x-2} \\ = x - \log|x+3| + \log|x-2| + c$$

Hence,

$$I = x - \log|x+3| + \log|x-2| + c$$

Indefinite Integrals Ex 19.30 Q4

$$\text{Let } I = \int \frac{3+4x-x^2}{(x+2)(x-1)} dx$$

$$= \int -1 + \frac{5x+1}{(x+2)(x-1)} dx$$

$$\Rightarrow I = -\int dx + \int \frac{5x+1}{(x+2)(x-1)} dx \quad \dots \dots (1)$$

$$\text{Let } \frac{5x+1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$\Rightarrow 5x+1 = A(x-1) + B(x+2)$$

Put $x = 1$

$$\Rightarrow 6 = 3B \Rightarrow B = 2$$

Put $x = -2$

$$\Rightarrow -9 = -3A \Rightarrow A = 3$$

So,

$$I = -\int dx + 3 \int \frac{dx}{x+2} + 2 \int \frac{dx}{x-1}$$

$$I = -x + 3 \log|x+2| + 2 \log|x-1| + c$$

Indefinite Integrals Ex 19.30 Q5

$$\text{Let } I = \int \frac{x^2+1}{x^2-1} dx$$

$$\begin{aligned} &= \int 1 + \frac{2}{x^2-1} dx \\ &= \int dx + \int \frac{2dx}{(x+1)(x-1)} \\ &= \int dx + \int \frac{-1}{x+1} + \frac{1}{x-1} dx \end{aligned}$$

$$= x - \log|x+1| + \log|x-1| + c$$

$$I = x + \log \left| \frac{x-1}{x+1} \right| + c$$

Indefinite Integrals Ex 19.30 Q6

$$\text{Let } I = \int \frac{x^2}{(x-1)(x-2)(x-3)} dx = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$\Rightarrow x^2 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

Put $x = 1$

$$\Rightarrow 1 = 2A \Rightarrow A = \frac{1}{2}$$

Put $x = 2$

$$\Rightarrow 4 = -B \Rightarrow B = -4$$

Put $x = 3$

$$\Rightarrow 9 = 2C \Rightarrow C = \frac{9}{2}$$

Thus,

$$\begin{aligned} I &= \int \frac{x^2}{(x-1)(x-2)(x-3)} dx = \frac{1}{2} \int \frac{dx}{x-1} - 4 \int \frac{dx}{x-2} + \frac{9}{2} \int \frac{dx}{x-3} \\ &= \frac{1}{2} \log|x-1| - 4 \log|x-2| + \frac{9}{2} \log|x-3| + c \end{aligned}$$

Hence,

$$I = \frac{1}{2} \log|x-1| - 4 \log|x-2| + \frac{9}{2} \log|x-3| + c$$

Indefinite Integrals Ex 19.30 Q7

$$\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$$

$$\text{Let } \frac{5x}{(x+1)(x+2)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x-2)}$$

$$5x = A(x+2)(x-2) + B(x+1)(x-2) + C(x+1)(x+2) \quad \dots(1)$$

Substituting $x = -1, -2, \text{ and } 2$ respectively in equation (1), we obtain

$$A = \frac{5}{3}, B = -\frac{5}{2}, \text{ and } C = \frac{5}{6}$$

$$\begin{aligned} \therefore \frac{5x}{(x+1)(x+2)(x-2)} &= \frac{5}{3(x+1)} - \frac{5}{2(x+2)} + \frac{5}{6(x-2)} \\ \Rightarrow \int \frac{5x}{(x+1)(x^2-4)} dx &= \frac{5}{3} \int \frac{1}{(x+1)} dx - \frac{5}{2} \int \frac{1}{(x+2)} dx + \frac{5}{6} \int \frac{1}{(x-2)} dx \\ &= \frac{5}{3} \log|x+1| - \frac{5}{2} \log|x+2| + \frac{5}{6} \log|x-2| + C \end{aligned}$$

Indefinite Integrals Ex 19.30 Q8

$$\text{Let } I = \int \frac{x^2 + 1}{x(x^2 - 1)} dx = \int \frac{x^2 + 1}{x(x+1)(x-1)} dx$$

$$\text{Let } \frac{x^2 + 1}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$$

$$\Rightarrow x^2 + 1 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1)$$

Put $x = 0$

$$\Rightarrow 1 = -A \Rightarrow A = -1$$

Put $x = -1$

$$\Rightarrow 2 = 2B \Rightarrow B = 1$$

Put $x = 1$

$$\Rightarrow 2 = 2C \Rightarrow C = 1$$

Thus,

$$I = -\int \frac{dx}{x} + \int \frac{dx}{x+1} + \int \frac{dx}{x-1}$$

$$= -\log|x| + \log|x+1| + \log|x-1| + c$$

$$I = \log \left| \frac{x^2 - 1}{x} \right| + c$$

Indefinite Integrals Ex 19.30 Q9

$$\text{Let } I = \int \frac{2x-3}{(x^2-1)(2x+3)} dx = \int \frac{2x-3}{(x+1)(x-1)(2x+3)} dx$$

$$\text{Let } \frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(2x+3)}$$

$$\Rightarrow 2x-3 = A(x-1)(2x+3) + B(x+1)(2x+3) + C(x^2-1)$$

Put $x = -1$

$$\Rightarrow -5 = -2A \Rightarrow A = \frac{5}{2}$$

Put $x = 1$

$$\Rightarrow -1 = 10B \Rightarrow B = -\frac{1}{10}$$

Put $x = -\frac{3}{2}$

$$\Rightarrow -6 = \frac{5}{4}C \Rightarrow C = -\frac{24}{5}$$

Thus,

$$I = \frac{5}{2} \int \frac{dx}{x+1} - \frac{1}{10} \int \frac{dx}{x-1} - \frac{24}{5} \int \frac{dx}{2x+3}$$

$$= \frac{5}{2} \log|x+1| - \frac{1}{10} \log|x-1| - \frac{24}{5} \cdot \frac{1}{2} \log|2x+3| + c$$

Hence,

$$I = \frac{5}{2} \log|x+1| - \frac{1}{10} \log|x-1| - \frac{12}{5} \log|2x+3| + c$$

Indefinite Integrals Ex 19.30 Q10

$$\begin{aligned} \text{Let } I &= \int \frac{x^3}{(x-1)(x-2)(x-3)} dx \\ &= \int 1 + \frac{6x^2 - 9x + 6}{(x-1)(x-2)(x-3)} dx \end{aligned}$$

$$\text{Let } \frac{6x^2 - 11x + 6}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$\Rightarrow 6x^2 - 11x + 6 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

Put $x = 1$

$$\Rightarrow 1 = 2A \Rightarrow A = \frac{1}{2}$$

Put $x = 2$

$$\Rightarrow 8 = -B \Rightarrow B = -8$$

Put $x = 3$

$$\Rightarrow 27 = 2C \Rightarrow C = \frac{27}{2}$$

Thus,

$$\begin{aligned} I &= \int dx + \frac{1}{2} \int \frac{dx}{x-1} - 8 \int \frac{dx}{x-2} + \frac{27}{2} \int \frac{dx}{x-3} \\ &= x + \frac{1}{2} \log|x-1| - 8 \log|x-2| + \frac{27}{2} \log|x-3| + c \end{aligned}$$

Hence,

$$I = x + \frac{1}{2} \log|x-1| - 8 \log|x-2| + \frac{27}{2} \log|x-3| + c$$

Indefinite Integrals Ex 19.30 Q11

$$\text{Let } \int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx = \frac{A}{1+\sin x} + \frac{B}{2+\sin x}$$

$$\Rightarrow \sin 2x = A(2+\sin x) + B(1+\sin x)$$

$$\Rightarrow 2\sin x \cos x = (2A+B) + (A+B)\sin x$$

Equating similar terms, we get,

$$2A + B = 0 \Rightarrow B = -2A \text{ and}$$

$$A + B = 2\cos x \Rightarrow -A = 2\cos x$$

$$\Rightarrow A = -2\cos x$$

$$\text{and } B = +4\cos x$$

Thus,

$$\begin{aligned} I &= \int -\frac{2\cos x}{1+\sin x} dx + \int \frac{4\cos x}{2+\sin x} dx \\ &= -2 \log|1+\sin x| + 4 \log|2+\sin x| + c \end{aligned}$$

$$I = \log \left| \frac{(2+\sin x)^4}{(1+\sin x)^2} \right| + c$$

Indefinite Integrals Ex 19.30 Q12

$$\text{Let } \int \frac{2x}{(x^2+1)(x^2+3)} dx = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+3}$$

$$\Rightarrow 2x = (Ax+B)(x^2+3) + (Cx+D)(x^2+1) \\ = (A+C)x^3 + (B+D)x^2 + (3A+C)x + (3B+D)$$

Equating similar terms, we get,

$$A+C=0, B+D=0, 3A+C=2 \text{ and } 3B+D=0$$

$$\Rightarrow A=-C, B=D=0, 2A=2 \Rightarrow A=1 \& C=-1$$

Thus,

$$I = \int \frac{x dx}{x^2+1} - \int \frac{x dx}{x^2+3} \\ = \frac{1}{2} \log|x^2+1| - \frac{1}{2} \log|x^2+3| + c$$

$$I = \frac{1}{2} \log \left| \frac{x^2+1}{x^2+3} \right| + c$$

Indefinite Integrals Ex 19.30 Q13

$$\text{Let } \int \frac{1}{x \log x (2+\log x)} dx = \frac{A}{x \log x} + \frac{B}{x(2+\log x)}$$

$$\Rightarrow 1 = A(2+\log x) + B \log x$$

$$\text{Put } x=1$$

$$\Rightarrow 1 = 2A \Rightarrow A = \frac{1}{2}$$

$$\text{Put } x=10^{-2}$$

$$\Rightarrow 1 = -2B \Rightarrow B = -\frac{1}{2}$$

Thus,

$$I = \frac{1}{2} \int \frac{dx}{x \log x} + \left(-\frac{1}{2} \right) \int \frac{dx}{x(2+\log x)} \\ = \frac{1}{2} \log|\log x| - \frac{1}{2} \log|2+\log x| + c$$

$$I = \frac{1}{2} \log \left| \frac{\log x}{2+\log x} \right| + c$$

Indefinite Integrals Ex 19.30 Q15

$$\text{Let } \frac{ax^2 + bx + c}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$$

$$\Rightarrow ax^2 + bx + c = A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b)$$

Put $x = a$

$$\Rightarrow a^3 + ba + c = (a-b)(a-c)A \Rightarrow A = \frac{a^3 + ba + c}{(a-b)(a-c)}$$

Put $x = b$

$$\Rightarrow ab^2 + b^2 + c = (b-a)(b-c)B \Rightarrow B = \frac{ab^2 + b^2 + c}{(b-a)(b-c)}$$

Put $x = c$

$$\Rightarrow ac^2 + bc + c = (c-a)(c-b)C \Rightarrow C = \frac{ac^2 + bc + c}{(c-a)(c-b)}$$

Thus,

$$I = \frac{a^3 + ba + c}{(a-b)(a-c)} \int \frac{dx}{x-a} + \frac{ab^2 + b^2 + c}{(b-a)(b-c)} \int \frac{dx}{x-b} + \frac{ac^2 + bc + c}{(c-a)(c-b)} \int \frac{dx}{x-c}$$

Hence,

$$I = \frac{a^3 + ba + c}{(a-b)(a-c)} \log|x-a| + \frac{ab^2 + b^2 + c}{(b-a)(b-c)} \log|x-b| + \frac{ac^2 + bc + c}{(c-a)(c-b)} \log|x-c| + C$$

Indefinite Integrals Ex 19.30 Q16

Consider the integral

$$I = \int \frac{x}{(x^2 + 1)(x - 1)} dx$$

Now let us separate the fraction $\frac{x}{(x^2 + 1)(x - 1)}$

through partial fractions.

$$\begin{aligned} \frac{x}{(x^2 + 1)(x - 1)} &= \frac{A}{x - 1} + \frac{Bx + C}{x^2 + 1} \\ \Rightarrow \frac{x}{(x^2 + 1)(x - 1)} &= \frac{A(x^2 + 1) + (Bx + C)(x - 1)}{(x^2 + 1)(x - 1)} \\ \Rightarrow x &= A(x^2 + 1) + (Bx + C)(x - 1) \end{aligned}$$

$$\Rightarrow x = Ax^2 + A + Bx^2 - Bx + Cx - C$$

Comparing the coefficients, we have,

$$A + B = 0, -B + C = 1 \text{ and } A - C = 0$$

Solving the equations, we get,

$$\begin{aligned} \Rightarrow A &= \frac{1}{2}, B = -\frac{1}{2} \text{ and } C = \frac{1}{2} \\ \Rightarrow \frac{x}{(x^2 + 1)(x - 1)} &= \frac{A}{x - 1} + \frac{Bx + C}{x^2 + 1} \\ \Rightarrow \frac{x}{(x^2 + 1)(x - 1)} &= \frac{1}{2} \times \frac{1}{x - 1} - \frac{1}{2} \times \frac{x - 1}{x^2 + 1} \\ \Rightarrow \frac{x}{(x^2 + 1)(x - 1)} &= \frac{1}{2(x - 1)} - \frac{x}{2(x^2 + 1)} + \frac{1}{2(x^2 + 1)} \end{aligned}$$

Thus, we have,

$$\begin{aligned} I &= \int \frac{x}{(x^2 + 1)(x - 1)} dx \\ &= \int \left[\frac{1}{2(x - 1)} - \frac{x}{2(x^2 + 1)} + \frac{1}{2(x^2 + 1)} \right] dx \\ &= \int \frac{dx}{2(x - 1)} - \int \frac{x dx}{2(x^2 + 1)} + \int \frac{dx}{2(x^2 + 1)} \\ &= \frac{1}{2} \int \frac{dx}{(x - 1)} - \frac{1}{2} \int \frac{x dx}{(x^2 + 1)} + \frac{1}{2} \int \frac{dx}{(x^2 + 1)} \\ &= \frac{1}{2} \int \frac{dx}{(x - 1)} - \frac{1}{2} \times \frac{1}{2} \int \frac{2x dx}{(x^2 + 1)} + \frac{1}{2} \int \frac{dx}{(x^2 + 1)} \\ &= \frac{1}{2} \log|x - 1| - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \tan^{-1} x + C \end{aligned}$$

Indefinite Integrals Ex 19.30 Q17

$$\text{Let } I = \int \frac{1}{(x-1)(x+1)(x+2)} dx = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2}$$

$$\Rightarrow 1 = A(x+1)(x+2) + B(x-1)(x+2) + C(x^2 - 1)$$

Put $x = 1$

$$\Rightarrow 1 = 6A \Rightarrow A = \frac{1}{6}$$

Put $x = -1$

$$\Rightarrow 1 = -2B \Rightarrow B = -\frac{1}{2}$$

Put $x = -2$

$$\Rightarrow 1 = 3C \Rightarrow C = \frac{1}{3}$$

So,

$$I = \frac{1}{6} \int \frac{dx}{x-1} - \frac{1}{2} \int \frac{dx}{x+1} + \frac{1}{3} \int \frac{dx}{x+2}$$

$$I = \frac{1}{6} \log|x-1| - \frac{1}{2} \log|x+1| + \frac{1}{3} \log|x+2| + C$$

Indefinite Integrals Ex 19.30 Q18

Consider the integral

$$I = \int \frac{x^2}{(x^2 + 4)(x^2 + 9)} dx$$

Now let us separate the fraction $\frac{x^2}{(x^2 + 4)(x^2 + 9)}$

through partial fractions.

Substitute $x^2 = t$

$$\begin{aligned} \frac{x^2}{(x^2 + 4)(x^2 + 9)} &= \frac{t}{(t + 4)(t + 9)} \\ \Rightarrow \frac{t}{(t + 4)(t + 9)} &= \frac{A}{t + 4} + \frac{B}{t + 9} \\ \Rightarrow \frac{t}{(t + 4)(t + 9)} &= \frac{A(t + 9) + B(t + 4)}{(t + 4)(t + 9)} \\ \Rightarrow t &= A(t + 9) + B(t + 4) \\ \Rightarrow t &= At + 9A + Bt + 4B \end{aligned}$$

Comparing the coefficients, we have,

$$A + B = 1 \text{ and } 9A + 4B = 0$$

$$\Rightarrow A = -\frac{4}{5} \text{ and } B = \frac{9}{5}$$

$$\begin{aligned} \Rightarrow \frac{x^2}{(x^2 + 4)(x^2 + 9)} &= -\frac{4}{5(t + 4)} + \frac{9}{5(t + 9)} \\ \Rightarrow \frac{x^2}{(x^2 + 4)(x^2 + 9)} &= -\frac{4}{5(x^2 + 4)} + \frac{9}{5(x^2 + 9)} \end{aligned}$$

Thus, we have,

$$\begin{aligned} I &= \int \frac{x^2}{(x^2 + 4)(x^2 + 9)} dx \\ &= \int \left[-\frac{4}{5(x^2 + 4)} + \frac{9}{5(x^2 + 9)} \right] dx \\ &= -\int \frac{4dx}{5(x^2 + 4)} + \int \frac{9dx}{5(x^2 + 9)} \\ &= -\frac{4}{5} \int \frac{dx}{x^2 + 4} + \frac{9}{5} \int \frac{dx}{x^2 + 9} \\ &= -\frac{4}{5} \times \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + \frac{9}{5} \times \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C \\ &= -\frac{2}{5} \tan^{-1}\left(\frac{x}{2}\right) + \frac{3}{5} \tan^{-1}\left(\frac{x}{3}\right) + C \end{aligned}$$

Indefinite Integrals Ex 19.30 Q19

$$\text{Let } \int \frac{5x^2 - 1}{x(x-1)(x+1)} dx = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$\Rightarrow 5x^2 - 1 = A(x^2 - 1) + B(x+1)x + C(x-1)x$$

Put $x = 0$

$$\Rightarrow -1 = -A \Rightarrow A = 1$$

Put $x = +1$

$$\Rightarrow 4 = 2B \Rightarrow B = 2$$

Put $x = -1$

$$\Rightarrow 4 = 2C \Rightarrow C = 2$$

So,

$$\begin{aligned} I &= \int \frac{dx}{x} + \int \frac{2dx}{x-1} + \int \frac{2dx}{x+1} \\ &= \log|x| + 2\log|x-1| + 2\log|x+1| + C \end{aligned}$$

$$I = \log|x(x^2-1)^2|$$

Indefinite Integrals Ex 19.30 Q20

$$\begin{aligned} \text{Let } I &= \int \frac{x^2 + 6x - 8}{x^3 - 4x} dx \\ \Rightarrow I &= \int \frac{x^2 + 6x - 8}{x(x+2)(x-2)} dx \end{aligned}$$

Now,

$$\begin{aligned} \text{Let } \frac{x^2 + 6x - 8}{x(x+2)(x-2)} &= \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2} \\ \Rightarrow x^2 + 6x - 8 &= A(x^2 - 4) + B(x-2)x + C(x+2)x \\ \text{Put } x = 0 & \end{aligned}$$

$$\Rightarrow -8 = -4A \Rightarrow A = 2$$

$$\text{Put } x = -2$$

$$\Rightarrow -16 = 8B \Rightarrow B = -2$$

$$\text{Put } x = 2$$

$$\Rightarrow 8 = 8C \Rightarrow C = 1$$

Thus,

$$\begin{aligned} I &= \int \frac{2dx}{x} - \int \frac{2dx}{x+2} + \int \frac{dx}{x-2} \\ &= 2\log|x| - 2\log|x+2| + \log|x-2| + c \end{aligned}$$

$$\therefore I = \log \left| \frac{x^2(x-2)}{(x+2)^2} \right| + c$$

Indefinite Integrals Ex 19.30 Q21

$$\begin{aligned} \text{Let } \int \frac{x^2 + 1}{(2x+1)(x^2-1)} dx &= \frac{A}{2x+1} + \frac{Bx+C}{x^2-1} \\ \Rightarrow x^2 + 1 &= A(x^2 - 1) + (Bx + C)(2x + 1) \\ &= (A + 2B)x^2 + (B + 2C)x + (-A + C) \end{aligned}$$

Equating similar terms, we get,

$$A + 2B = 1, B + 2C = 0 \text{ and } -A + C = 1$$

Solving we get,

$$A = -\frac{5}{3}, \quad B = \frac{4}{3}, \quad C = -\frac{2}{3}$$

Thus,

$$\begin{aligned} I &= -\frac{5}{3} \int \frac{dx}{2x+1} + \frac{4}{3} \int \frac{x-2}{x^2-1} dx \\ &= -\frac{5}{3} \int \frac{dx}{2x+1} + \frac{2}{3} \int \frac{2xdx}{x^2-1} - \frac{2}{3} \int \frac{dx}{x^2-1} \\ &= -\frac{5}{3} \int \frac{dx}{2x+1} + \frac{2}{3} \int \frac{2x-1}{(x+1)(x-1)} dx \\ &= -\frac{5}{3} \int \frac{dx}{2x+1} + \frac{2}{3} \left[\int \left(\frac{\frac{3}{2}}{(x+1)} + \frac{\frac{1}{2}}{(x-1)} \right) dx \right] \end{aligned}$$

$$I = -\frac{5}{6} \log|2x+1| + \log|x+1| + \frac{1}{3} \log|x-1| + c$$

Indefinite Integrals Ex 19.30 Q22

$$\begin{aligned} \text{Let } I &= \int \frac{dx}{x[6(\log x)^2 + 7\log x + 2]} \\ &= \int \frac{1}{x(2\log x + 1)(3\log x + 2)} dx \end{aligned}$$

Now,

$$\text{Let } \frac{1}{x(2\log x + 1)(3\log x + 2)} = \frac{A}{x(2\log x + 1)} + \frac{B}{x(3\log x + 2)}$$

$$\Rightarrow 1 = A(3\log x + 2) + B(2\log x + 1)$$

$$\text{Put } x = 10^{-\frac{1}{2}}$$

$$\Rightarrow 1 = \frac{1}{2}A \Rightarrow A = 2$$

$$\text{Put } x = 10^{-\frac{2}{3}}$$

$$\Rightarrow 1 = -\frac{1}{3}B \Rightarrow B = -3$$

$$\begin{aligned} \therefore I &= \int \frac{2dx}{x(2\log x + 1)} - \int \frac{3dx}{x(3\log x + 2)} \\ &= \log|2\log x + 1| - \log|3\log x + 2| + C \end{aligned}$$

$$\therefore I = \log \left| \frac{2\log x + 1}{3\log x + 2} \right| + C$$

Indefinite Integrals Ex 19.30 Q23

$$\frac{1}{x(x^n + 1)}$$

Multiplying numerator and denominator by x^{n-1} , we obtain

$$\frac{1}{x(x^n + 1)} = \frac{x^{n-1}}{x^{n-1}x(x^n + 1)} = \frac{x^{n-1}}{x^n(x^n + 1)}$$

$$\text{Let } x^n = t \Rightarrow x^{n-1}dx = dt$$

$$\therefore \int \frac{1}{x(x^n + 1)} dx = \int \frac{x^{n-1}}{x^n(x^n + 1)} dx = \frac{1}{n} \int \frac{1}{t(t+1)} dt$$

$$\text{Let } \frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)}$$

$$1 = A(1+t) + Bt \quad \dots(1)$$

Substituting $t = 0, -1$ in equation (1), we obtain

$$A = 1 \text{ and } B = -1$$

$$\therefore \frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{(1+t)}$$

$$\begin{aligned} \Rightarrow \int \frac{1}{x(x^n + 1)} dx &= \frac{1}{n} \int \left\{ \frac{1}{t} - \frac{1}{(t+1)} \right\} dt \\ &= \frac{1}{n} \left[\log|t| - \log|t+1| \right] + C \\ &= -\frac{1}{n} \left[\log|x^n| - \log|x^n + 1| \right] + C \\ &= \frac{1}{n} \log \left| \frac{x^n}{x^n + 1} \right| + C \end{aligned}$$

Indefinite Integrals Ex 19.30 Q24

$$\begin{aligned} \text{Let } I &= \int \frac{x}{(x^2 - a^2)(x^2 - b^2)} dx = \frac{Ax + B}{(x^2 - a^2)} + \frac{Cx + D}{(x^2 - b^2)} \\ \Rightarrow x &= (Ax + B)(x^2 - b^2) + (Cx + D)(x^2 - a^2) \\ &= (A + C)x^3 + (B + D)x^2 + (-Ab^2 - Ca^2)x + (-Bb^2 - Da^2) \\ \Rightarrow A + C &= 0, B + D = 0, -Ab^2 - Ca^2 = 1, -Bb^2 - Da^2 = 0 \end{aligned}$$

$$\text{We get } B = 0, D = 0, C = \frac{1}{b^2 - a^2}, A = -\frac{1}{b^2 - a^2}$$

Thus,

$$I = -\frac{1}{b^2 - a^2} \int \frac{x dx}{x^2 - a^2} + \frac{1}{b^2 - a^2} \int \frac{x dx}{x^2 - b^2}$$

$$I = -\frac{1}{2(b^2 - a^2)} \log|x^2 - a^2| + \frac{1}{2(b^2 - a^2)} \log|x^2 - b^2| + C$$

Indefinite Integrals Ex 19.30 Q25

Consider the integral

$$I = \int \frac{x^2 + 1}{(x^2 + 4)(x^2 + 25)} dx$$

Let $y = x^2$

Thus,

$$\begin{aligned} \frac{x^2 + 1}{(x^2 + 4)(x^2 + 25)} &= \frac{y + 1}{(y + 4)(y + 25)} \\ \Rightarrow \frac{y + 1}{(y + 4)(y + 25)} &= \frac{A}{y + 4} + \frac{B}{y + 25} \\ \Rightarrow \frac{y + 1}{(y + 4)(y + 25)} &= \frac{A(y + 25) + B(y + 4)}{(y + 4)(y + 25)} \end{aligned}$$

$$\Rightarrow y + 1 = Ay + 25A + By + 4B$$

Comparing the coefficients, we have,

$$A + B = 1 \text{ and } 25A + 4B = 1$$

Solving the above equations, we have,

$$A = \frac{-1}{7} \text{ and } B = \frac{8}{7}$$

$$\text{Thus, } \int \frac{x^2 + 1}{(x^2 + 4)(x^2 + 25)} dx$$

$$\begin{aligned} &= \int \frac{\frac{-1}{7}}{x^2 + 4} dx + \int \frac{\frac{8}{7}}{x^2 + 25} dx \\ &= \frac{-1}{7} \int \frac{1}{x^2 + 4} dx + \frac{8}{7} \int \frac{1}{x^2 + 25} dx \\ &= \frac{-1}{7} \times \frac{1}{2} \tan^{-1} \frac{x}{2} + \frac{8}{7} \times \frac{1}{5} \tan^{-1} \frac{x}{5} + C \\ &= \frac{-1}{14} \tan^{-1} \frac{x}{2} + \frac{8}{35} \tan^{-1} \frac{x}{5} + C \end{aligned}$$

Indefinite Integrals Ex 19.30 Q26

$$\begin{aligned} \text{Let } I &= \int \frac{x^3 + x + 1}{x^2 - 1} dx \\ &= \int \left(x + \frac{2x + 1}{x^2 - 1} \right) dx \end{aligned}$$

Now,

$$\text{Let } \frac{2x + 1}{x^2 - 1} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$\Rightarrow 2x + 1 = A(x - 1) + B(x + 1)$$

Put $x = 1$

$$\Rightarrow 3 = 2B \Rightarrow B = \frac{3}{2}$$

Put $x = -1$

$$\Rightarrow -1 = -2A \Rightarrow A = \frac{1}{2}$$

$$\therefore I = \int x dx + \frac{1}{2} \int \frac{dx}{x+1} + \frac{3}{2} \int \frac{dx}{x-1}$$

$$I = \frac{x^2}{2} + \frac{1}{2} \log|x+1| + \frac{3}{2} \log|x-1| + C$$

Indefinite Integrals Ex 19.30 Q27

$$\text{Let } \frac{3x - 2}{(x+1)^2(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+3)}$$

$$\begin{aligned} \Rightarrow 3x - 2 &= A(x+1)(x+3) + B(x+3) + C(x+1)^2 \\ &= (A+C)x^2 + (4A+B+2C)x + (3A+3B+C) \end{aligned}$$

Equating similar terms, we get,

$$A + C = 0 \Rightarrow A = -C$$

$$4A + B + 2C = 3 \Rightarrow B = -2C = 3$$

$$3A + 3B + C = -2 \Rightarrow 3B - 2C = -2$$

Solving, we get, $B = -\frac{5}{2}$, $C = -\frac{11}{4}$ & $A = \frac{11}{4}$

Thus,

$$I = \frac{11}{4} \int \frac{dx}{x+1} - \frac{5}{2} \int \frac{dx}{(x+1)^2} - \frac{11}{4} \int \frac{dx}{x+3}$$

$$I = \frac{11}{4} \log|x+1| + \frac{5}{2(x+1)} - \frac{11}{4} \log|x+3| + C$$

Indefinite Integrals Ex 19.30 Q28

$$\text{Let } \frac{2x+1}{(x+2)(x-3)^2} = \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

$$\Rightarrow 2x+1 = A(x-3)^2 + B(x+2)(x-3) + C(x+2)$$

$$= (A+B)x^2 + (-6A-B+C)x + (9A-6B+2C)$$

Equating similar terms, we get,

$$\begin{aligned} A+B &= 0 & \Rightarrow A &= -B \\ -6A-B+C &= 2 & \Rightarrow 5B+C &= 2 \\ 9A-6B+2C &= 1 \Rightarrow -15B+2C &= 1 \end{aligned}$$

$$\text{Solving, we get, } B = \frac{3}{25}, C = \frac{7}{5}, A = -\frac{3}{25}$$

Thus,

$$I = -\frac{3}{25} \int \frac{dx}{x+2} + \frac{3}{25} \int \frac{dx}{x-3} + \frac{7}{5} \int \frac{dx}{(x-3)^2}$$

$$I = -\frac{3}{25} \log|x+2| + \frac{3}{25} \log|x-3| - \frac{7}{5(x-3)} + C$$

Indefinite Integrals Ex 19.30 Q29

$$\text{Let } \frac{x^2+1}{(x-2)^2(x+3)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+3}$$

$$\Rightarrow x^2+1 = A(x-2)(x+3) + B(x+3) + C(x-2)^2$$

$$= (A+C)x^2 + (A+B-4C)x + (-6A+3B+4C)$$

Equating similar terms, we get,

$$A+C=1, A+B-4C=0, -6A+3B+4C=1$$

$$\text{Solving, we get, } A = \frac{3}{5}, B = 1, C = \frac{2}{5}$$

Thus,

$$I = \frac{3}{5} \int \frac{dx}{x-2} + \int \frac{dx}{(x-2)^2} + \frac{2}{5} \int \frac{dx}{x+3}$$

$$I = \frac{3}{5} \log|x-2| - \frac{1}{(x-2)} + \frac{2}{5} \log|x+3| + C$$

Indefinite Integrals Ex 19.30 Q30

$$\text{Let } \frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

Substituting $x = 1$, we obtain

$$B = \frac{1}{3}$$

Equating the coefficients of x^2 and constant term, we obtain

$$A + C = 0$$

$$-2A + 2B + C = 0$$

On solving, we obtain

$$A = \frac{2}{9} \text{ and } C = -\frac{2}{9}$$

$$\begin{aligned} \therefore \frac{x}{(x-1)^2(x+2)} &= \frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} - \frac{2}{9(x+2)} \\ \Rightarrow \int \frac{x}{(x-1)^2(x+2)} dx &= \frac{2}{9} \int \frac{1}{(x-1)} dx + \frac{1}{3} \int \frac{1}{(x-1)^2} dx - \frac{2}{9} \int \frac{1}{(x+2)} dx \\ &= \frac{2}{9} \log|x-1| + \frac{1}{3} \left(\frac{-1}{x-1} \right) - \frac{2}{9} \log|x+2| + C \\ &= \frac{2}{9} \log \left| \frac{x-1}{x+2} \right| - \frac{1}{3(x-1)} + C \end{aligned}$$

Indefinite Integrals Ex 19.30 Q31

$$\text{Let } \frac{x^2}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$\begin{aligned} \Rightarrow x^2 &= A(x+1)^2 + B(x-1)(x+1) + C(x-1) \\ &= (A+B)x^2 + (2A+C)x + (A-B-C) \end{aligned}$$

Equating similar terms,

$$A + B = 1, 2A + C = 0, A - B - C = 0$$

$$\text{Solving, we get, } A = \frac{1}{4}, B = \frac{3}{4}, C = -\frac{1}{2}$$

Thus,

$$\begin{aligned} I &= \frac{1}{4} \int \frac{dx}{x-1} + \frac{3}{4} \int \frac{dx}{x+1} - \frac{1}{2} \int \frac{dx}{(x+1)^2} \\ &= \frac{1}{4} \log|x-1| + \frac{3}{4} \log|x+1| + \frac{1}{2(x+1)} + C \end{aligned}$$

$$I = \frac{1}{4} \log|x-1| + \frac{3}{4} \log|x+1| + \frac{1}{2(x+1)} + C$$

Indefinite Integrals Ex 19.30 Q32

$$\text{Let } \frac{x^2+x-1}{(x+1)^2(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+2)}$$

$$\Rightarrow x^2+x-1 = A(x+1)(x+2) + B(x+2) + C(x+1)^2$$

$$= (A+C)x^2 + (3A+B+2C)x + (2A+2B+C)$$

Equating similar terms

$$A+C = 1, 3A+B+2C = 1, 2A+2B+C = -1$$

Solving, we get, $A = 0, B = -1, C = 1$

Thus,

$$I = 0 \int \frac{dx}{x+1} + (-1) \int \frac{dx}{(x+1)^2} + 1 \int \frac{dx}{(x+2)}$$

$$= + \frac{1}{x+1} + \log|x+2| + c$$

$$I = \frac{1}{x+1} + \log|x+2| + c$$

Indefinite Integrals Ex 19.30 Q33

$$\text{Let } \frac{2x^2+7x-3}{x^2(2x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(2x+1)}$$

$$\Rightarrow 2x^2+7x-3 = Ax(2x+1) + B(2x+1) + Cx^2$$

Equating similar terms, we get,

$$2A+C = 2, A+2B = 7, B = -3$$

Solving, we get, $A = 13, C = -24$

Thus,

$$I = \int \frac{13dx}{x} - \int \frac{3dx}{x^2} - 24 \int \frac{dx}{2x+1}$$

$$I = 13\log|x| + \frac{3}{x} - 12\log|2x+1| + c$$

Indefinite Integrals Ex 19.30 Q34

$$\text{Let } I = \int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x} dx$$

$$= \int \frac{5x^2 + 20x + 6}{x(x+1)^2} dx$$

Now,

$$\text{Let } \frac{5x^2 + 20x + 6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$\Rightarrow 5x^2 + 20x + 6 = A(x+1)^2 + Bx(x+1) + Cx$$

Equating similar terms, we get,

$$A + B = 5, 2A + B + C = 20, A = 6$$

Solving, we get, $B = -1, C = 9$

Thus,

$$I = \int \frac{6dx}{x} - 1 \int \frac{dx}{x+1} + 9 \int \frac{dx}{(x+1)^2}$$

$$\therefore I = 6 \log|x| - \log|x+1| - \frac{9}{x+1} + C$$

Indefinite Integrals Ex 19.30 Q35

$$\text{Let } \frac{18}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$$

$$\Rightarrow 18 = A(x^2+4) + (Bx+C)(x+2)$$

$$18 = (A+B)x^2 + (2B+C)x + (4A+2C)$$

Equating similar terms, we get,

$$A + B = 0, 2B + C = 0, 4A + 2C = 18$$

$$\text{Solving, we get, } A = \frac{9}{4}, B = -\frac{9}{4}, C = \frac{9}{2}$$

Thus,

$$I = \frac{9}{4} \int \frac{dx}{x+2} + \left(-\frac{9}{4}\right) \int \frac{x}{x^2+4} dx + \frac{9}{2} \int \frac{dx}{x^2+4}$$

$$I = \frac{9}{4} \log|x+2| - \frac{9}{8} \log|x^2+4| + \frac{9}{4} \tan^{-1}\left(\frac{x}{2}\right) + C \quad \left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\frac{x}{a} \right]$$

Indefinite Integrals Ex 19.30 Q36

$$\text{Let } \frac{5}{(x^2+1)(x+2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+2}$$

$$\Rightarrow 5 = (Ax+B)(x+2) + C(x^2+1)$$

Equating similar terms, we get,

$$A+C=0, 2A+B=0, 2B+C=5$$

Solving, we get, $A=-1, B=2, C=1$

Thus,

$$\begin{aligned} I &= \int \frac{-x+2}{x^2+1} dx + \int \frac{dx}{x+2} \\ &= \int \frac{-xdx}{x^2+1} + 2 \int \frac{dx}{x^2+1} + \int \frac{dx}{x+2} \end{aligned}$$

$$I = -\frac{1}{2} \log|x^2+1| + 2 \tan^{-1} x + \log|x+2| + C$$

Indefinite Integrals Ex 19.30 Q37

$$\text{Let } \frac{x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$\Rightarrow x = A(x^2+1) + (Bx+C)(x+1)$$

Equating similar terms, we get,

$$A+B=0, B+C=1, A+C=0$$

$$\text{Solving, we get, } A=-\frac{1}{2}, B=\frac{1}{2}, C=\frac{1}{2}$$

Thus,

$$I = -\frac{1}{2} \int \frac{dx}{x+1} + \frac{1}{2} \int \frac{xdx}{x^2+1} + \frac{1}{2} \int \frac{dx}{x^2+1}$$

$$I = -\frac{1}{2} \log|x+1| + \frac{1}{4} \log|x^2+1| + \frac{1}{2} \tan^{-1} x + C$$

Indefinite Integrals Ex 19.30 Q38

$$\text{Let } I = \int \frac{dx}{1+x+x^2+x^3}$$

$$\Rightarrow I = \int \frac{dx}{(x^2+1)(x+1)}$$

Now,

$$\text{Let } \frac{1}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$$

$$\Rightarrow 1 = (Ax+B)(x+1) + C(x^2+1)$$

Equating similar terms, we get,

$$A+C=0, A+B=0, B+C=1$$

$$\text{Solving, we get, } A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$$

Thus,

$$I = -\frac{1}{2} \int \frac{x dx}{x^2+1} + \frac{1}{2} \int \frac{dx}{x^2+1} + \frac{1}{2} \int \frac{dx}{x+1}$$

$$I = -\frac{1}{4} \log|x^2+1| + \frac{1}{2} \tan^{-1} x + \frac{1}{2} \log|x+1| + c$$

Indefinite Integrals Ex 19.30 Q39

$$\text{Let } \frac{1}{(x+1)^2(x^2+1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+1}$$

$$\begin{aligned} \Rightarrow 1 &= A(x+1)(x^2+1) + B(x^2+1) + (Cx+D)(x+1)^2 \\ &= (A+C)x^3 + (A+B+2C+D)x^2 + (A+C+2D)x + (A+B+D) \end{aligned}$$

Equating similar terms, we get,

$$A+C=0, A+B+2C+D=0, A+C+2D=0, A+B+D=1$$

$$\text{Solving, we get, } A = \frac{1}{2}, B = \frac{1}{2}, C = -\frac{1}{2}, D = 0$$

Thus,

$$I = \frac{1}{2} \int \frac{dx}{x+1} + \frac{1}{2} \int \frac{dx}{(x+1)^2} - \frac{1}{2} \int \frac{x dx}{x^2+1}$$

$$I = \frac{1}{2} \log|x+1| - \frac{1}{2(x+1)} - \frac{1}{4} \log|x^2+1| + c$$

Indefinite Integrals Ex 19.30 Q40

$$\text{Let } I = \int \frac{2x}{x^3 - 1} dx = \int \frac{2x}{(x-1)(x^2+x+1)} dx$$

Now,

$$\text{Let } \frac{2x}{(x-1)(x^2+x+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{x^2+x+1}$$

$$\Rightarrow 2x = A(x^2+x+1) + (Bx+C)(x-1)$$

$$= (A+B)x^2 + (A-B+C)x + (A-C)$$

Equating similar terms,

$$A+B=0, A-B+C=2, A-C=0,$$

$$\text{Solving, we get, } A = \frac{2}{3}, B = -\frac{2}{3}, C = \frac{2}{3}$$

Thus,

$$\begin{aligned} I &= \frac{2}{3} \int \frac{dx}{x-1} - \frac{2}{3} \int \frac{(x-1)dx}{x^2+x+1} \\ &= \frac{2}{3} \int \frac{dx}{x-1} - \frac{2}{3} \cdot \frac{1}{2} \int \frac{(2x-2)dx}{x^2+x+1} \\ \Rightarrow I &= \frac{2}{3} \int \frac{dx}{x-1} - \frac{1}{3} \int \frac{2x+1}{x^2+x+1} dx + \int \frac{dx}{x^2+x+1} \\ &= \frac{2}{3} \int \frac{dx}{x-1} - \frac{1}{3} \int \frac{2x+1}{x^2+x+1} dx + \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\ &= \frac{2}{3} \log|x-1| - \frac{1}{3} \log|x^2+x+1| + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + C \end{aligned}$$

Hence,

$$I = \frac{2}{3} \log|x-1| - \frac{1}{3} \log|x^2+x+1| + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$$

Indefinite Integrals Ex 19.30 Q41

$$\text{Let } \frac{1}{(x^2+1)(x^2+4)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{x^2+4}$$

$$\Rightarrow 1 = (Ax+B)(x^2+4) + (Cx+D)(x^2+1)$$

$$= (A+C)x^3 + (B+D)x^2 + (4A+C)x + 4B+D$$

Equating similar terms, we get,

$$A+C=0, B+D=0, 4A+C=0, 4B+D=1$$

$$\text{Solving, we get, } A=0, B=\frac{1}{3}, C=0, D=-\frac{1}{3}$$

Thus,

$$I = \int \frac{\frac{1}{3}dx}{x^2+1} - \int \frac{\frac{1}{3}dx}{x^2+4}$$

$$= \frac{1}{3} \tan^{-1} x - \frac{1}{6} \tan^{-1} \left(\frac{x}{2} \right) + C \quad \left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} \right]$$

$$\therefore I = \frac{1}{3} \tan^{-1} x - \frac{1}{6} \tan^{-1} \left(\frac{x}{2} \right) + C$$

Indefinite Integrals Ex 19.30 Q42

$$\text{Let } \frac{x^2}{(x^2+1)(3x^2+4)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{3x^2+4}$$

$$\Rightarrow x^2 = (Ax+B)(3x^2+4) + (Cx+D)(x^2+1)$$

$$= (3A+C)x^3 + (3B+D)x^2 + (4A+C)x + 4B+D$$

Equating similar terms, we get,

$$3A+C=0, 3B+D=1, 4A+C=0, 4B+D=0$$

$$\text{Solving, we get, } A=0, B=-1, C=0, D=4$$

Thus,

$$I = \int \frac{-dx}{x^2+1} + \int \frac{4dx}{3x^2+4}$$

$$= -\tan^{-1} x + \frac{4}{3} \int \frac{dx}{x^2 + \left(\frac{2}{\sqrt{3}}\right)^2}$$

$$= -\tan^{-1} x + \frac{4}{3} \cdot \frac{\sqrt{3}}{2} \tan^{-1} \left(\frac{\sqrt{3}x}{2} \right) + C$$

$$I = \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{\sqrt{3}x}{2} \right) - \tan^{-1} x + C$$

Indefinite Integrals Ex 19.30 Q43

To evaluate the integral follow the steps:

$$\int \frac{3x+5}{x^3-x^2-x+1} dx$$

$$\text{Let } \frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$$

$$3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

For $x=1$ $B=4$

$$\text{For } x=-1 C=\frac{1}{2}$$

$$\text{For } x=0 A=-\frac{1}{2}$$

Therefore

$$\begin{aligned} \int \frac{3x+5}{(x-1)^2(x+1)} dx &= -\frac{1}{2} \int \frac{dx}{x-1} + 4 \int \frac{dx}{(x-1)^2} + \frac{1}{2} \int \frac{dx}{x+1} \\ &= -\frac{1}{2} \ln|x-1| - \frac{4}{(x-1)} + \frac{1}{2} \ln|x+1| + c \\ &= \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| - \frac{4}{(x-1)} + c \end{aligned}$$

Indefinite Integrals Ex 19.30 Q44

$$\text{Let } I = \int \frac{x^3-1}{x^3+x} dx$$

$$\begin{aligned} &= \int 1 - \frac{x+1}{x^3+x} dx \\ &= \int dx - \int \frac{x+1}{x^3+x} dx \end{aligned}$$

$$\text{Let } \frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$\Rightarrow x+1 = A(x^2+1) + (Bx+C)x$$

$$= (A+B)x^2 + (B+C)x + A$$

Equating similar terms, we get,

$$A+B=0, C=1, A=1$$

Solving, we get, $A=1, B=-1, C=1$

Thus,

$$I = -\int \frac{dx}{x} - \int \frac{-x+1}{x^2+1} dx + \int dx$$

$$= -\log|x| + \int \frac{x dx}{x^2+1} - \int \frac{dx}{x^2+1} + \int dx$$

$$\therefore I = x - \log|x| + \frac{1}{2} \log|x^2+1| - \tan^{-1}x + c$$

$$\therefore I = x - \log|x| + \frac{1}{2} \log|x^2+1| - \tan^{-1}x + c$$

Indefinite Integrals Ex 19.30 Q45

To evaluate the integral follow the steps:

$$\int \frac{x^2+x+1}{(x+1)^2(x+2)} dx$$

$$\text{Let } \frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$$

$$x^2+x+1 = A(x+1)(x+2) + B(x+2) + C(x+1)^2$$

$$\text{For } x=-1 \quad B=1$$

$$\text{For } x=-2 \quad C=3$$

$$\text{For } x=0 \quad A=-2,$$

Therefore

$$\begin{aligned} \int \frac{x^2+x+1}{(x+1)^2(x+2)} dx &= -2 \int \frac{dx}{x+1} + \int \frac{dx}{(x+1)^2} + 3 \int \frac{dx}{x+2} \\ &= -2 \ln|x+1| - \frac{1}{x+1} + 3 \ln|x+2| + c \end{aligned}$$

Indefinite Integrals Ex 19.30 Q46

$$\text{Let } \frac{1}{x(x^4+1)} = \frac{A}{x} + \frac{Bx^3+Cx^2+Dx+E}{x^4+1}$$

$$\Rightarrow \begin{aligned} 1 &= A(x^4+1) + (Bx^3+Cx^2+Dx+E)x \\ &= (A+B)x^4 + Cx^3 + Dx^2 + Ex + A \end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned} A+B &= 0, \quad C=0, \quad D=0, \quad E=0, \quad A=1 \\ \therefore B &= -1 \end{aligned}$$

Thus,

$$\begin{aligned} I &= \int \frac{dx}{x} + \int -\frac{x^3 dx}{x^4+1} \\ &= \log|x| - \frac{1}{4} \log|x^4+1| + c \end{aligned}$$

$$I = \frac{1}{4} \log \left| \frac{x^4}{x^4+1} \right| + c$$

Indefinite Integrals Ex 19.30 Q47

Consider the integral

$$I = \int \frac{1}{x(x^3 + 8)} dx$$

Rewriting the above integrand, we have,

$$\begin{aligned} I &= \int \frac{x^2}{x^3(x^3 + 8)} dx \\ &= \frac{1}{3} \int \frac{3x^2}{x^3(x^3 + 8)} dx \end{aligned}$$

Now substituting $x^3 = t$, we have,

$$3x^2 dx = dt$$

$$\Rightarrow I = \frac{1}{3} \int \frac{dt}{t(t+8)}$$

Let us separate the integrand by partial fractions.

Thus,

$$\begin{aligned} \frac{1}{t(t+8)} &= \frac{A}{t} + \frac{B}{t+8} \\ \Rightarrow \frac{1}{t(t+8)} &= \frac{A(t+8) + Bt}{t(t+8)} \\ \Rightarrow 1 &= A(t+8) + Bt \\ \Rightarrow 1 &= At + 8A + Bt \end{aligned}$$

Comparing the coefficients, we have,

$$A+B=0 \text{ and } 8A=1$$

$$\Rightarrow A = \frac{1}{8} \text{ and } B = -\frac{1}{8}$$

Therefore,

$$\begin{aligned} I &= \frac{1}{3} \int \frac{dt}{t(t+8)} \\ &= \frac{1}{3} \int \left[\frac{\frac{1}{8}}{t} - \frac{\frac{1}{8}}{t+8} \right] dt \\ &= \frac{1}{3} \times \frac{1}{8} \int \frac{dt}{t} dt - \frac{1}{3} \times \frac{1}{8} \int \frac{dt}{t+8} \\ &= \frac{1}{24} \log t - \frac{1}{24} \times \log(t+8) + C \\ &= \frac{1}{24} \log x^3 - \frac{1}{24} \times \log(x^3 + 8) + C \\ &= \frac{3}{24} \log x - \frac{1}{24} \times \log(x^3 + 8) + C \\ &= \frac{1}{8} \log x - \frac{1}{24} \times \log(x^3 + 8) + C \end{aligned}$$

Indefinite Integrals Ex 19.30 Q48

$$\text{Let } \frac{3}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$$

$$\Rightarrow 3 = A(1+x^2) + (Bx+C)(1-x) \\ = (A-B)x^2 + (B-C)x + (A+C)$$

Equating similar terms, we get,

$$A - B = 0, B - C = 0, A + C = 3$$

Solving we get,

$$A = C = \frac{3}{2} \text{ and } B = \frac{3}{2}$$

Thus,

$$I = \frac{3}{2} \int \frac{dx}{1-x} + \frac{3}{2} \int \frac{x dx}{1+x^2} + \frac{3}{2} \int \frac{dx}{1+x^2} \\ = -\frac{3}{2} \log|1-x| + \frac{3}{2} \log|1+x^2| + \frac{3}{2} \tan^{-1} x + C \\ I = \frac{3}{4} \left[\log \left| \frac{1+x^2}{(1-x)^2} \right| + 2 \tan^{-1} x + C \right]$$

Indefinite Integrals Ex 19.30 Q49

Let

$$\begin{aligned} \sin x &= t \\ \Rightarrow \cos x &= dt \\ \therefore \int \frac{\cos x}{(1-\sin x)^3 (2+\sin x)} &= \int \frac{1}{(1-t)^3 (2+t)} dt \end{aligned}$$

$$\text{Let } f(t) = \frac{1}{(1-t)^3 (2+t)}$$

Then, suppose

$$\begin{aligned} \frac{1}{(1-t)^3 (2+t)} &= \frac{A}{1-t} + \frac{B}{(1-t)^2} + \frac{C}{(1-t)^3} + \frac{D}{2+t} \\ \Rightarrow 1 &= A(1-t)^2(2+t) + B(1-t)(2+t) + C(2+t) + D(1-t)^3 \end{aligned}$$

Put $t = 1$

$$1 = 3C$$

$$\Rightarrow C = \frac{1}{3}$$

Put $t = -2$

$$1 = 27D$$

$$\Rightarrow D = \frac{1}{27}$$

Similarly, we can find that $A = \frac{-1}{27}$ and $B = \frac{+1}{9}$

$$\begin{aligned} \therefore \int \frac{1}{(1-t)^3 (2+t)} dt &= \frac{-1}{27} \int \frac{1}{1-t} dt + \frac{1}{9} \int \frac{dt}{(1-t)^2} + \frac{1}{3} \int \frac{dt}{(1-t)^3} + \frac{1}{27} \int \frac{dt}{2+t} \\ &= \frac{-1}{27} \log|1-t| + \frac{1}{9(1-t)} + \frac{1}{6(1-t)^2} + \frac{1}{27} \log|2+t| + C \end{aligned}$$

Putting $t = \sin x$, we get

$$\begin{aligned} \int \frac{\cos x}{(1-\sin x)^3 (2+\sin x)} dx \\ = \frac{-1}{27} \log|1-\sin x| + \frac{1}{9(1-\sin x)} + \frac{1}{6(1-\sin x)^2} + \frac{1}{27} \log|2+\sin x| + C \end{aligned}$$

Indefinite Integrals Ex 19.30 Q50

Consider the integral

$$I = \int \frac{2x^2 + 1}{x^2(x^2 + 4)} dx$$

Now let us separate the fraction $\frac{2x^2 + 1}{x^2(x^2 + 4)}$ through partial fractions.

Substitute $x^2 = t$, then

$$\begin{aligned} \frac{2x^2 + 1}{x^2(x^2 + 4)} &= \frac{2t + 1}{t(t + 4)} \\ \Rightarrow \frac{2t + 1}{t(t + 4)} &= \frac{A}{t} + \frac{B}{t + 4} \\ \Rightarrow \frac{2t + 1}{t(t + 4)} &= \frac{A(t + 4) + Bt}{t(t + 4)} \\ \Rightarrow 2t + 1 &= A(t + 4) + Bt \\ \Rightarrow 2t + 1 &= At + 4A + Bt \end{aligned}$$

Comparing the coefficients, we have,

$$A + B = 2 \text{ and } 4A = 1$$

$$\begin{aligned} \Rightarrow A &= \frac{1}{4} \text{ and } B = \frac{7}{4} \\ \Rightarrow \frac{2x^2 + 1}{x^2(x^2 + 4)} &= \frac{1}{4x^2} + \frac{7}{4(x^2 + 4)} \end{aligned}$$

Thus, we have,

$$\begin{aligned} I &= \int \frac{2x^2 + 1}{x^2(x^2 + 4)} dx \\ &= \frac{1}{4} \int \frac{dx}{x^2} + \frac{7}{4} \int \frac{dx}{(x^2 + 4)} \\ &= -\frac{1}{4x} + \frac{7}{4} \times \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C \\ &= -\frac{1}{4x} + \frac{7}{8} \tan^{-1}\left(\frac{x}{2}\right) + C \end{aligned}$$

Indefinite Integrals Ex 19.30 Q51

To evaluate the integral follow the steps:

$$\int \frac{\cos x}{(1 - \sin x)(2 - \sin x)} dx$$

Let $1 - \sin x = t$ and

$$-\cos x dx = dt$$

Therefore

$$\begin{aligned} -\int \frac{dt}{t(1+t)} &= -\int \left(\frac{1}{t} - \frac{1}{t+1} \right) dt \\ &= \ln|t+1| - \ln|t| + C \\ &= \ln \left| \frac{t+1}{t} \right| + C \\ &= \ln \left| \frac{2 - \sin x}{1 - \sin x} \right| + C \end{aligned}$$

Indefinite Integrals Ex 19.30 Q52

$$\text{Let } \frac{2x+1}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)}$$

$$\Rightarrow 2x+1 = A(x-3) + B(x-2)$$

$$= (A+B)x + (-3A-2B)$$

Equating similar terms, we get,

$$A+B = 2, \text{ and } -3A-2B = 1$$

Thus,

$$I = -5 \int \frac{dx}{x-2} + 7 \int \frac{dx}{x-3}$$

$$= -5 \log|x-2| + 7 \log|x-3| + C$$

$$I = \log \left| \frac{(x-3)^7}{(x-2)^5} \right| + C$$

Indefinite Integrals Ex 19.30 Q53

$$\text{Let } x^2 = y$$

$$\text{Then } \frac{1}{(y+1)(y+2)} = \frac{A}{y+1} + \frac{B}{y+2}$$

$$\Rightarrow 1 = A(y+2) + B(y+1)$$

$$= (A+B)y + (2A+B)$$

Equating similar terms, we get,

$$A+B = 0, \text{ and } 2A+B = 1$$

Solving, we get,

Thus,

$$I = \int \frac{dx}{x^2+1} - \int \frac{dx}{x^2+2}$$

$$I = \tan^{-1} x - \frac{1}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + C$$

Indefinite Integrals Ex 19.30 Q54

To evaluate the integral follow the steps:

$$\int \frac{1}{x(x^4-1)} dx$$

$$\text{Let } \frac{1}{x(x^4-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} + \frac{D}{x^2+1}$$

$$1 = A(x+1)(x-1)(x^2+1) + Bx(x-1)(x^2+1) + Cx(x+1)(x^2+1) + Dx(x+1)(x-1)$$

For $x=0$ $A = -1$,

$$\text{For } x=1 \quad C = \frac{1}{4}$$

$$\text{For } x=-1 \quad B = \frac{1}{4}$$

$$\text{For } x=2 \quad D = \frac{1}{4}$$

Therefore

$$\begin{aligned} \int \frac{1}{x(x^4-1)} dx &= -\int \frac{1}{x} dx + \frac{1}{4} \int \frac{dx}{x+1} + \frac{1}{4} \int \frac{dx}{x-1} + \frac{1}{4} \int \frac{dx}{x^2+1} \\ &= -\ln|x| + \frac{1}{4} \ln|(x+1)| + \frac{1}{4} \ln|(x-1)| + \frac{1}{4} \ln|(x^2+1)| + c \\ &= \frac{1}{4} \ln \left| \frac{x^4-1}{x^4} \right| + c \end{aligned}$$

Indefinite Integrals Ex 19.30 Q55

To evaluate the integral follow the steps:

$$\int \frac{1}{(x^4-1)} dx$$

$$\text{Let } \frac{1}{(x^4-1)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{x^2+1}$$

$$1 = A(x-1)(x^2+1) + B(x+1)(x^2+1) + C(x+1)(x-1)$$

$$\text{For } x=1 \quad B = \frac{1}{4}$$

$$\text{For } x=-1 \quad A = -\frac{1}{4}$$

$$\text{For } x=0 \quad C = -\frac{1}{2}$$

Therefore

$$\begin{aligned} \int \frac{1}{(x^4-1)} dx &= -\frac{1}{4} \int \frac{dx}{x+1} + \frac{1}{4} \int \frac{dx}{x-1} - \frac{1}{2} \int \frac{dx}{x^2+1} \\ &= -\frac{1}{4} \ln|(x+1)| + \frac{1}{4} \ln|(x-1)| - \frac{1}{2} \tan^{-1} x + c \\ &= \frac{1}{4} \ln \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \tan^{-1} x + c \end{aligned}$$

Indefinite Integrals Ex 19.30 Q57

$$\begin{aligned}
 \text{Let } I &= \int \frac{dx}{\cos x (5 - 4 \sin x)} \\
 &= \int \frac{\cos x dx}{\cos^2 x (5 - 4 \sin x)} \\
 &= \int \frac{\cos x dx}{(1 - \sin^2 x)(5 - 4 \sin x)}
 \end{aligned}$$

$$\text{Let } \sin x = t \Rightarrow \cos x dx = dt$$

$$\therefore I = \int \frac{dt}{(1 - t^2)(5 - 4t)}$$

Now,

$$\text{Let } \frac{1}{(1 - t^2)(5 - 4t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{5-4t}$$

$$\Rightarrow 1 = A(1+t)(5-4t) + B(1-t)(5-4t) + C(1-t^2)$$

$$\text{Put } t = 1$$

$$\Rightarrow 1 = 2A \Rightarrow A = \frac{1}{2}$$

$$\text{Put } t = -1$$

$$\Rightarrow 1 = 18B \Rightarrow B = \frac{1}{18}$$

$$\text{Put } t = \frac{5}{4}$$

$$\Rightarrow 1 = -\frac{9C}{16} \Rightarrow C = -\frac{16}{9}$$

Thus,

$$\begin{aligned}
 I &= \frac{1}{2} \int \frac{dt}{1-t} + \frac{1}{18} \int \frac{dt}{1+t} - \frac{16}{9} \int \frac{dt}{5-4t} \\
 &= -\frac{1}{2} \log|1-t| + \frac{1}{18} \log|1+t| + \frac{4}{9} \log|5-4t| + c
 \end{aligned}$$

Hence,

$$I = -\frac{1}{2} \log|1-\sin x| + \frac{1}{18} \log|1+\sin x| + \frac{4}{9} \log|5-4\sin x| + c$$

Indefinite Integrals Ex 19.30 Q58

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{\sin x (3+2 \cos x)} dx \\
 &= \int \frac{\sin x dx}{\sin^2 x (3+2 \cos x)} \\
 &= \int \frac{\sin x dx}{(1-\cos^2 x)(3+2 \cos x)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Let } \cos x &= t \\
 \Rightarrow -\sin x dx &= dt
 \end{aligned}$$

$$\therefore I = \int \frac{dt}{(t^2-1)(3+2t)}$$

Now,

$$\text{Let } \frac{1}{(t^2-1)(3+2t)} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{3+2t}$$

$$\Rightarrow 1 = A(t+1)(3+2t) + B(t-1)(3+2t) + C(t^2-1)$$

Put $t = 1$

$$\Rightarrow 1 = 10A \Rightarrow A = \frac{1}{10}$$

Put $t = -1$

$$\Rightarrow 1 = -2B \Rightarrow B = -\frac{1}{2}$$

Put $t = -\frac{3}{2}$

$$\Rightarrow 1 = \frac{5}{4}C \Rightarrow C = \frac{4}{5}$$

Thus,

$$\begin{aligned}
 I &= \frac{1}{10} \int \frac{dt}{t-1} - \frac{1}{2} \int \frac{dt}{t+1} + \frac{5}{4} \int \frac{dt}{3+2t} \\
 &= \frac{1}{10} \log|t-1| - \frac{1}{2} \log|t+1| + \frac{2}{5} \log|3+2t| + C
 \end{aligned}$$

Hence,

$$I = \frac{1}{10} \log|\cos x - 1| - \frac{1}{2} \log|\cos x + 1| + \frac{2}{5} \log|3+2 \cos x| + C$$

Indefinite Integrals Ex 19.30 Q59

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{\sin x + \sin 2x} dx \\
 &= \int \frac{dx}{\sin x + 2 \sin x \cos x} \\
 &= \int \frac{\sin x dx}{(1 - \cos^2 x) + 2(1 - \cos^2 x) \cos x}
 \end{aligned}$$

$$\text{Let } \cos x = t \Rightarrow -\sin x dx = dt$$

$$\begin{aligned}
 \therefore I &= \int \frac{dt}{(t^2 - 1) + 2(t^2 - 1)t} \\
 &= \int \frac{dt}{(t^2 - 1)(1 + 2t)}
 \end{aligned}$$

$$\text{Let } \int \frac{1}{(t^2 - 1)(1 + 2t)} = \frac{A}{t - 1} + \frac{B}{t + 1} + \frac{C}{1 + 2t}$$

$$\Rightarrow 1 = A(t+1)(1+2t) + B(t-1)(1+2t) + C(t^2-1)$$

Put $t = 1$

$$\Rightarrow 1 = 6A \Rightarrow A = \frac{1}{6}$$

Put $t = -1$

$$\Rightarrow 1 = 2B \Rightarrow B = \frac{1}{2}$$

Put $t = -\frac{1}{2}$

$$\Rightarrow 1 = -\frac{3}{4}C \Rightarrow C = -\frac{4}{3}$$

Thus,

$$\begin{aligned}
 I &= \frac{1}{6} \int \frac{dt}{t-1} + \frac{1}{2} \int \frac{dt}{t+1} - \frac{4}{3} \int \frac{dt}{1+2t} \\
 &= \frac{1}{6} \log|t-1| + \frac{1}{2} \log|t+1| - \frac{2}{3} \log|1+2t| + c
 \end{aligned}$$

Hence,

$$I = \frac{1}{6} \log|\cos x - 1| + \frac{1}{2} \log|\cos x + 1| - \frac{2}{3} \log|1 + 2 \cos x| + c$$

Indefinite Integrals Ex 19.30 Q60

$$\begin{aligned}
\text{Let } I &= \int \frac{x+1}{x(1+xe^x)} dx \\
&= \int \frac{(x+1)(1+xe^x - xe^x)}{x(1+xe^x)} dx \\
&= \int \frac{(x+1)(1+xe^x)}{x(1+xe^x)} dx - \int \frac{(x+1)xe^x}{x(1+xe^x)} dx \\
&= \int \frac{(x+1)}{x} dx - \int \frac{e^x(x+1)}{1+xe^x} dx \\
&= \int \frac{(x+1)e^x}{xe^x} dx - \int \frac{e^x(x+1)}{1+xe^x} dx \\
&= \log|x e^x| - \log|1+x e^x| + c
\end{aligned}$$

$$\therefore I = \log \left| \frac{x e^x}{1+x e^x} \right| + c$$

Indefinite Integrals Ex 19.30 Q61

$$f(x) = \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$$

Now,

$$\begin{aligned}
&\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} \\
&= \frac{x^4 + 3x^2 + 2}{x^4 + 7x^2 + 12} \\
&= \frac{(x^4 + 7x^2 + 12) - 4x^2 - 10}{x^4 + 7x^2 + 12} \\
&= 1 - \frac{4x^2 + 10}{x^4 + 7x^2 + 12}
\end{aligned}$$

Now,

$$\frac{4x^2 + 10}{x^4 + 7x^2 + 12} = \frac{4x^2 + 10}{(x^2+3)(x^2+4)}$$

$$\text{Let } \frac{4x^2 + 10}{(x^2+3)(x^2+4)} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+4}$$

$$\Rightarrow 4x^2 + 10 = (Ax+B)(x^2+4) + (Cx+D)(x^2+3)$$

Let $x = 0$, we get

$$10 = 4B + 3D \quad \text{--- (i)}$$

If $x = 1$, we get

$$14 = 5(A+B) + 4(C+D) = 5A + 5B + 4C + 4D \quad \text{--- (ii)}$$

If $x = -1$, we get

$$14 = 5(-A+B) + 4(-C+D) = -5A + 5B - 4C + 4D \quad \text{--- (iii)}$$

Applying (ii) and (iii), we get

$$28 = 10B + 8D$$

$$\Rightarrow 14 = 5B + 4D \quad \text{--- (iv)}$$

From (i), we get

$$10 = 4B + 3D \quad \text{--- (i)}$$

Multiplying equation (iv) by 3 and (i) by 4 and subtracting, we get

$$42 - 40 = 15B - 16B$$

$$\Rightarrow 2 = -B$$

$$\text{or } B = -2 \quad \text{--- (v)}$$

Putting value of B in (i), we get

$$10 = 4(-2) + 3D$$

$$\frac{10+8}{3} = D$$

$$\Rightarrow D = 6 \quad \text{--- (vi)}$$

Comparing coefficients of x^3 in

$$4x^2 + 10 = (Ax + B)(x^2 + 4) + (Cx + D)(x^2 + 3), \text{ we get,}$$

$$0 = A + C \quad \text{---(vii)}$$

Comparing coefficients of x , we get

$$0 = 4A + 3C \quad \text{---(viii)}$$

$$\Rightarrow A = C = 0$$

$$\therefore f(x) = 1 - \frac{(-2)}{x^2 + 3} - \frac{6}{x^2 + 4}$$

$$= 1 + \frac{2}{x^2 + 3} - \frac{6}{x^2 + 4}$$

$$\therefore \int f(x) dx = \int 1 + \frac{2}{x^2 + 3} - \frac{6}{x^2 + 4} dx$$

$$= x + \frac{2}{\sqrt{3}} \tan^{-1} x \frac{x}{\sqrt{3}} - 3 \tan^{-1} \frac{x}{2} + c$$

Indefinite Integrals Ex 19.30 Q62

Let $x^2 = y$

$$\therefore \frac{4x^4 + 3}{(x^2 + 2)(x^2 + 3)(x^2 + 4)} = \frac{4y^2 + 3}{(y + 2)(y + 3)(y + 4)}$$

Now,

$$\text{Let } \frac{4y^2 + 3}{(y + 2)(y + 3)(y + 4)} = \frac{A}{y + 2} + \frac{B}{y + 3} + \frac{C}{y + 4}$$

$$\Rightarrow 4y^2 + 3 = A(y + 3)(y + 4) + B(y + 2)(y + 4) + C(y + 2)(y + 3)$$

$$= (A + B + C)y^2 + (7A + 6B + 5C)y + 12A + 8B + 6C$$

Equating similar terms,

$$A + B + C = 4, \quad 7A + 6B + 5C = 0, \quad 12A + 8B + 6C = 3$$

Solving, we get

$$A = \frac{19}{2}, \quad B = -39, \quad C = \frac{67}{2}$$

Thus,

$$I = \frac{19}{2} \int \frac{dx}{x^2 + 2} + (-39) \int \frac{dx}{x^2 + 3} + \frac{67}{2} \int \frac{dx}{x^2 + 4}$$

$$\Rightarrow I = \frac{19}{2\sqrt{2}} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) - \frac{39}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) + \frac{67}{4} \tan^{-1} \left(\frac{x}{2} \right) + c$$

Hence,

$$I = \frac{19}{2\sqrt{2}} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) - \frac{39}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) + \frac{67}{4} \tan^{-1} \left(\frac{x}{2} \right) + c$$

Indefinite Integrals Ex 19.30 Q63

$$\begin{aligned}
\frac{x^4}{(x-1)(x^2+1)} &= \frac{x^4}{x^3 - x^2 + x - 1} \\
&= \frac{x(x^3 - x^2 + x - 1) + 1(x^3 - x^2 + x - 1) + 1}{x^3 - x^2 + x - 1} \\
&= x + 1 + \frac{1}{(x-1)(x^2+1)}
\end{aligned}$$

Now, suppose

$$\begin{aligned}
\frac{1}{(x-1)(x^2+1)} &= \frac{A}{x-1} + \frac{Bx+C}{x^2+1} \\
\Rightarrow 1 &= A(x^2+1) + (Bx+C)(x-1)
\end{aligned}$$

Put $x = 1$

$$1 = 2A$$

$$\Rightarrow A = \frac{1}{2}$$

Put $x = 0$

$$1 = A - C$$

$$\Rightarrow C = A - 1 = -\frac{1}{2}$$

Put $x = -1$

$$1 = 2A + 2B - 2C = 2(A - C) + 2B$$

$$\Rightarrow 1 = 2 + 2B$$

$$\Rightarrow 2B = -1$$

$$\Rightarrow B = -\frac{1}{2}$$

$$\begin{aligned}
\therefore \int \frac{x^4}{(x-1)(x^2+1)} dx &= \int x dx + \int 1 dx + \frac{1}{2} \int \frac{1}{x-1} dx - \frac{1}{2} \int \frac{x+1}{x^2+1} dx \\
&= \frac{x^2}{2} + x + \frac{1}{2} \log|x-1| - \frac{1}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + C
\end{aligned}$$

Ex 19.31

Indefinite Integrals Ex 19.31 Q1

$$\begin{aligned} I &= \int \frac{x^2 + 1}{x^4 + x^2 + 1} dx \\ &= \int \frac{1 + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx \end{aligned}$$

Dividing numerator and denominator by x^2

$$= \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + 3} dx$$

Let $x - \frac{1}{x} = t \Rightarrow \left(1 + \frac{1}{x^2}\right)dx = dt$

$$\therefore I = \int \frac{dt}{t^2 + 3}$$
$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + C$$

$$\therefore I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{3}x}\right) + C$$

Indefinite Integrals Ex 19.31 Q2

$$\int \sqrt{\cot \theta} d\theta$$

Let $\cot \theta = x^2$

$$\Rightarrow -\cos \theta d\theta = 2x dx$$

$$\Rightarrow d\theta = \frac{-2x}{\cos \theta} dx$$

$$= \frac{-2x}{1 + \cot^2 \theta} dx$$

$$= \frac{-2x}{1 + x^4} dx$$

$$\therefore I = -\int \frac{2x^2}{1 + x^4} dx$$

$$= -\int \frac{2}{x^2 + x^2} dx$$

Dividing numerator and denominator by x^2

$$= -\int \frac{1 + \frac{1}{x^2} + 1 - \frac{1}{x^2}}{x^2 + \frac{1}{x^2}} dx$$

$$= -\int \frac{\left(1 + \frac{1}{x^2}\right) dx}{\left(x - \frac{1}{x}\right)^2 + 2} - \int \frac{\left(1 - \frac{1}{x^2}\right) dx}{\left(x + \frac{1}{x}\right)^2 - 2}$$

Let $x - \frac{1}{x} = t \Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$

and $x + \frac{1}{x} = z \Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dz$

$$\Rightarrow I = -\int \frac{dt}{t^2 + 2} - \int \frac{dz}{z^2 - 2}$$

$$= -\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{t}{\sqrt{2}} \right) - \frac{1}{2\sqrt{2}} \log \left| \frac{z - \sqrt{2}}{z + \sqrt{2}} \right| + C$$

$$= -\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right) - \frac{1}{2\sqrt{2}} \log \left| \frac{x^2 + 1 - \sqrt{2}x}{x^2 + 1 + \sqrt{2}x} \right| + C$$

$$I = -\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\cot \theta - 1}{\sqrt{2} \cot \theta} \right) - \frac{1}{2\sqrt{2}} \log \left| \frac{\cot \theta + 1 - \sqrt{2} \cot \theta}{\cot \theta + 1 + \sqrt{2} \cot \theta} \right| + C$$

Indefinite Integrals Ex 19.31 Q3

$$\text{Let } I = \int \frac{x^2 + 9}{x^4 + 81} dx$$

Dividing numerator and denominator by x^2

$$I = \int \frac{1 + \frac{9}{x^2}}{x^2 + \frac{81}{x^2}} dx$$

$$= \int \frac{1 + \frac{9}{x^2}}{\left(x - \frac{9}{x}\right)^2 + 18} dx$$

Let $\left(x - \frac{9}{x}\right) = t \Rightarrow \left(1 + \frac{9}{x^2}\right) dx = dt$

$$\therefore I = \int \frac{dt}{t^2 + 18}$$

$$\Rightarrow I = \frac{1}{3\sqrt{2}} \tan^{-1} \left(\frac{t}{3\sqrt{2}} \right) + C$$

Thus,

$$I = \frac{1}{3\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 9}{3\sqrt{2}x} \right) + C$$

Indefinite Integrals Ex 19.31 Q4

$$\text{Let } I = \int \frac{1}{x^4 + x^2 + 1} dx$$

Dividing numerator and denominator by x^2

$$\begin{aligned} I &= \int \frac{\frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx \\ &= \frac{1}{2} \int \frac{1 + \frac{1}{x^2} - 1 + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx \\ &= \frac{1}{2} \left\{ \int \frac{\left(1 + \frac{1}{x^2}\right) dx}{\left(x - \frac{1}{x}\right)^2 + 3} - \int \frac{\left(1 - \frac{1}{x^2}\right) dx}{\left(x + \frac{1}{x}\right)^2 - 1} \right\} \end{aligned}$$

$$\text{Let } \left(x - \frac{1}{x}\right) = t$$

$$\Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$$

$$\text{and } \left(x + \frac{1}{x}\right) = z$$

$$\Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dz$$

$$\therefore I = \frac{1}{2} \int \frac{dt}{t^2 + 3} - \frac{1}{2} \int \frac{dz}{z^2 - 1}$$

$$\Rightarrow I = \frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) - \frac{1}{4} \log|z - 1| + c$$

$$\Rightarrow I = \frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{3}x}\right) - \frac{1}{4} \log\left|\frac{x^2 + 1 - x}{x^2 + 1 + x}\right| + c$$

Indefinite Integrals Ex 19.31 Q5

$$\text{Let } I = \int \frac{x^2 - 3x + 1}{x^4 + x^2 + 1} dx$$

Dividing numerator and denominator by x^2

$$\begin{aligned} \therefore I &= \int \frac{\frac{1}{x^2} - \frac{3}{x} + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx \\ &= \int \frac{\left(1 + \frac{1}{x^2}\right) dx}{\left(x - \frac{1}{x}\right)^2 + 3} - \int \frac{3x}{x^4 + x^2 + 1} dx \end{aligned}$$

$$\text{Let } \left(x - \frac{1}{x}\right) = t$$

$$\Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt \quad [\text{For Ist part}]$$

$$\text{Let } x^2 = z$$

$$\Rightarrow 2x dx = dz \quad [\text{For IIInd part}]$$

$$\therefore I = \int \frac{dt}{t^2 + 3} - \frac{3}{2} \int \frac{dz}{z^2 + z + 1}$$

$$\Rightarrow = \int \frac{dt}{t^2 + 3} - \frac{3}{2} \int \frac{dz}{\left(z + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$\Rightarrow = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) - \frac{3}{2} \times \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{z + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

$$\Rightarrow I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{3}x}\right) - \sqrt{3} \tan^{-1}\left(\frac{2z + 1}{\sqrt{3}}\right) + c$$

Hence,

$$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{3}x}\right) - \sqrt{3} \tan^{-1}\left(\frac{2x^2 + 1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.31 Q6

$$\text{Let } I = \int \frac{x^2 + 1}{x^4 - x^2 + 1} dx$$

Dividing numerator and denominator by x^2

$$\begin{aligned}\therefore I &= \int \frac{\left(1 + \frac{1}{x^2}\right)}{x^2 - 1 + \frac{1}{x^2}} dx \\ &= \int \frac{\left(1 + \frac{1}{x^2}\right) dx}{\left(x - \frac{1}{x}\right)^2 + 1}\end{aligned}$$

$$\text{Let } \left(x - \frac{1}{x}\right) = t$$

$$\Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$$

$$\begin{aligned}\Rightarrow I &= \int \frac{dt}{t^2 + 1} \\ &= \tan^{-1} t + C\end{aligned}$$

$$\therefore I = \tan^{-1} \left(\frac{x^2 - 1}{x} \right) + C$$

Indefinite Integrals Ex 19.31 Q7

$$\text{Let } I = \int \frac{x^2 - 1}{x^4 + 1} dx$$

Dividing numerator and denominator by x^2

$$\begin{aligned}\therefore I &= \int \frac{\left(1 - \frac{1}{x^2}\right)}{x^2 + \frac{1}{x^2}} dx \\ &= \int \frac{\left(1 - \frac{1}{x^2}\right) dx}{\left(x + \frac{1}{x}\right)^2 - 2}\end{aligned}$$

$$\text{Let } \left(x + \frac{1}{x}\right) = t$$

$$\Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dt$$

$$\begin{aligned}\therefore I &= \int \frac{dt}{t^2 - 2} \\ &= \frac{1}{2\sqrt{2}} \log \left| \frac{t - \sqrt{2}}{t + \sqrt{2}} \right| + C\end{aligned}$$

So,

$$I = \frac{1}{2\sqrt{2}} \log \left| \frac{x^2 + 1 - \sqrt{2}x}{x^2 + 1 + \sqrt{2}x} \right| + C$$

Indefinite Integrals Ex 19.31 Q8

$$\text{Let } I = \int \frac{x^2 + 1}{x^4 + 7x^2 + 1} dx$$

Dividing numerator and denominator by x^2

$$\begin{aligned} \therefore I &= \int \frac{\left(1 + \frac{1}{x^2}\right)}{x^2 + 7 + \frac{1}{x^2}} dx \\ &= \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + 9} dx \end{aligned}$$

$$\text{Let } \left(x - \frac{1}{x}\right) = t$$

$$\Rightarrow \left(1 + \frac{1}{x^2}\right)dx = dt$$

$$\therefore I = \int \frac{dt}{t^2 + 9}$$

$$= \frac{1}{3} \tan^{-1} \left| \frac{t}{3} \right| + C$$

Hence,

$$I = \frac{1}{3} \tan^{-1} \left(\frac{x^2 - 1}{3x} \right) + C$$

Indefinite Integrals Ex 19.31 Q9

$$\begin{aligned} \text{Let } I &= \int \frac{(x - 1)^2}{x^4 + x^2 + 1} dx \\ &= \int \frac{x^2 - 2x + 1}{x^4 + x^2 + 1} dx \end{aligned}$$

Dividing numerator and denominator by x^2

$$\begin{aligned} \therefore I &= \int \frac{1 - \frac{2}{x} + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx \\ &= \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + 3} dx - \int \frac{2x}{x^4 + x^2 + 1} dx \end{aligned}$$

$$\text{Let } \left(x - \frac{1}{x}\right) = t$$

$$\Rightarrow \left(1 + \frac{1}{x^2}\right)dx = dt \quad [\text{For 1st part}]$$

$$\text{Let } x^2 = z$$

$$\Rightarrow 2x dx = dz \quad [\text{For 2nd part}]$$

$$\begin{aligned} \therefore I &= \int \frac{dt}{t^2 + 3} - \int \frac{dz}{z^2 + z + 1} \\ &= \int \frac{dt}{t^2 + 3} - \int \frac{dz}{\left(z + \frac{1}{2}\right)^2 + \frac{3}{4}} \end{aligned}$$

$$\Rightarrow I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{t}{\sqrt{3}} \right) - \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{z + \frac{1}{2}}{\sqrt{3}} \right) + C$$

$$\Rightarrow I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x} \right) - \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2x^2 + 1}{\sqrt{3}} \right) + C$$

Hence,

$$I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x} \right) - \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2x^2 + 1}{\sqrt{3}} \right) + C$$

Indefinite Integrals Ex 19.31 Q10

$$\text{Let } I = \int \frac{1}{x^4 + 3x^2 + 1} dx$$

Dividing numerator and denominator by x^2

$$\begin{aligned}\therefore I &= \int \frac{\frac{1}{x^2}}{x^2 + 3 + \frac{1}{x^2}} dx \\ &= \frac{1}{2} \int \frac{\left(1 + \frac{1}{x^2}\right) - \left(1 - \frac{1}{x^2}\right)}{x^2 + 3 + \frac{1}{x^2}} dx \\ &= \frac{1}{2} \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + 5} dx - \frac{1}{2} \int \frac{\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 + 1} dx\end{aligned}$$

$$\text{Let } \left(x - \frac{1}{x}\right) = t$$

$$\Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$$

$$\text{and } x + \frac{1}{x} = z$$

$$\Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dz$$

$$\therefore I = \frac{1}{2} \int \frac{dt}{t^2 + 5} - \frac{1}{2} \int \frac{dz}{z^2 + 1}$$

$$= \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{t}{\sqrt{5}}\right) - \frac{1}{2} \tan^{-1}(z) + c$$

Hence,

$$I = \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{5}x}\right) - \frac{1}{2} \tan^{-1}\left(\frac{x^2 + 1}{x}\right) + c$$

Indefinite Integrals Ex 19.31 Q11

Consider the integral

$$I = \int \frac{1}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x} dx$$

Divide both the numerator and the denominator by $\cos^4 x$, we have,

$$\begin{aligned} I &= \int \frac{\frac{\cos^4 x}{\cos^4 x}}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x} dx \\ &= \int \frac{\sec^4 x}{\tan^4 x + \tan^2 x + 1} dx \\ &= \int \frac{\sec^2 x \times \sec^2 x}{\tan^4 x + \tan^2 x + 1} dx \\ &= \int \frac{(\tan^2 x + 1) \times \sec^2 x}{\tan^4 x + \tan^2 x + 1} dx \end{aligned}$$

Substituting $\tan x = t$; $\sec^2 x dx = dt$

Thus,

$$\begin{aligned} I &= \int \frac{(1+t^2)dt}{t^4 + t^2 + 1} \\ &= \int \frac{\left(1 + \frac{1}{t^2}\right)dt}{\left(t^2 + \frac{1}{t^2} + 1\right)} \\ &= \int \frac{\left(1 + \frac{1}{t^2}\right)dt}{\left(t^2 + \frac{1}{t^2} - 2 + 2 + 1\right)} \\ &= \int \frac{\left(1 + \frac{1}{t^2}\right)dt}{\left(t - \frac{1}{t}\right)^2 + 3} \\ &= \int \frac{\left(1 + \frac{1}{t^2}\right)dt}{\left(t - \frac{1}{t}\right)^2 + 3} \end{aligned}$$

Substituting $z = t - \frac{1}{t}$; $dz = \left(1 + \frac{1}{t^2}\right)dt$

$$\begin{aligned} I &= \int \frac{dz}{z^2 + 3} \\ \Rightarrow I &= \int \frac{dz}{z^2 + (\sqrt{3})^2} \\ I &= \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{z}{\sqrt{3}} \right) + C \\ &= \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{t - \frac{1}{t}}{\sqrt{3}} \right) + C \\ &= \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{\tan x - \frac{1}{\tan x}}{\sqrt{3}} \right) + C \\ I &= \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{\tan x - \cot x}{\sqrt{3}} \right) + C \end{aligned}$$

Ex 19.32

Indefinite Integrals Ex 19.32 Q1

$$\text{Let } I = \int \frac{1}{(x-1)\sqrt{x+2}} dx$$

$$\text{Let } x+2 = t^2$$

$$\begin{aligned}\therefore I &= \int \frac{2tdt}{(t^2-3)t} \\ &= 2 \int \frac{dt}{t^2-3} \\ &= \frac{2}{2\sqrt{3}} \log \left| \frac{t-\sqrt{3}}{t+\sqrt{3}} \right| + c\end{aligned}$$

Thus,

$$I = \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{x-2} - \sqrt{3}}{\sqrt{x+2} + \sqrt{3}} \right| + c$$

Indefinite Integrals Ex 19.32 Q2

$$\text{Let } I = \int \frac{1}{(x-1)\sqrt{2x+3}} dx$$

$$\text{Let } I = \int \frac{1}{(x-1)\sqrt{2x+3}} dx$$

$$\text{Let } 2x+3 = t^2$$

$$\Rightarrow 2dx = 2tdt$$

$$\begin{aligned}\therefore I &= \int \frac{t dt}{\left(\frac{t^2-3}{2}-1\right)t} \\ &= 2 \int \frac{dt}{t^2-5} \\ &= \frac{2}{2\sqrt{5}} \log \left| \frac{t-\sqrt{5}}{t+\sqrt{5}} \right| + c\end{aligned}$$

Thus,

$$I = \frac{1}{\sqrt{5}} \log \left| \frac{\sqrt{2x+3} - \sqrt{5}}{\sqrt{2x+3} + \sqrt{5}} \right| + c$$

Indefinite Integrals Ex 19.32 Q3

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{(x-1)\sqrt{x+2}} dx \\
 I &= \int \frac{(x-1)+2}{(x-1)\sqrt{x+2}} dx \\
 I &= \int \frac{dx}{\sqrt{x+2}} + 2 \int \frac{dx}{(x-1)\sqrt{x+2}}
 \end{aligned} \tag{A}$$

Now,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_1$$

and,

$$\int \frac{dx}{(x-1)\sqrt{x+2}}$$

$$\text{Let } x+2 = t^2$$

$$\Rightarrow dx = 2tdt$$

$$\begin{aligned}
 \therefore \int \frac{dx}{(x-1)\sqrt{x+2}} &= 2 \int \frac{tdt}{(t^2-3)t} = 2 \int \frac{dt}{t^2-3} \\
 &= \frac{2 \times 1}{2\sqrt{3}} \log \left| \frac{t-\sqrt{3}}{t+\sqrt{3}} \right| + c_2 \\
 &= \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}} \right| + c_2
 \end{aligned}$$

Thus, from (A),

$$I = 2\sqrt{x+2} + c_1 + \frac{2}{\sqrt{3}} \log \left| \frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}} \right| + c_2$$

Hence,

$$I = 2\sqrt{x+2} + \frac{2}{\sqrt{3}} \log \left| \frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}} \right| + c$$

Indefinite Integrals Ex 19.32 Q4

$$\begin{aligned}
 \text{Let } I &= \int \frac{x^2}{(x-1)\sqrt{x+2}} dx \\
 &= \int \frac{(x^2-1+1)}{(x-1)\sqrt{x+2}} dx \\
 &= \int \frac{(x+1)(x-1)}{(x-1)\sqrt{x+2}} dx + \int \frac{dx}{(x-1)\sqrt{x+2}} \\
 &= \int \frac{(x+1)}{\sqrt{x+2}} dx + \int \frac{dx}{(x-1)\sqrt{x+2}} \\
 &= \int \frac{(x+2)-1}{\sqrt{x+2}} dx + \int \frac{dx}{(x-1)\sqrt{x+2}} \\
 I &= \int \sqrt{x+2} dx - \int \frac{dx}{\sqrt{x+2}} + \int \frac{dx}{(x-1)\sqrt{x+2}}
 \end{aligned} \tag{A}$$

Now,

$$\int \sqrt{x+2} dx = \frac{2}{3}(x+2)^{\frac{3}{2}} + c_1$$

and,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_2$$

$$\int \frac{dx}{(x-1)\sqrt{x+2}}$$

$$\text{Let } x+2 = t^2$$

$$\Rightarrow dx = 2tdt$$

$$\begin{aligned}
 \therefore 2 \int \frac{tdt}{(t^2-3)t} &= 2 \int \frac{dt}{t^2-3} \\
 &= \frac{2}{2\sqrt{3}} \log \left| \frac{t-\sqrt{3}}{t+\sqrt{3}} \right| + c_3
 \end{aligned}$$

$$\therefore \int \frac{dx}{(x-1)\sqrt{x+2}} = \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}} \right| + c_3$$

Thus, from (A)

$$I = \frac{2}{3}(x+2)^{\frac{3}{2}} - 2\sqrt{x+2} + \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}} \right| + c$$

[when $c = c_1 + c_2 + c_3$]

Indefinite Integrals Ex 19.32 Q5

$$\begin{aligned}
 \text{Let } I &= \int \frac{x}{(x-3)\sqrt{x+1}} dx \\
 &= \int \frac{(x-3)+3}{(x-3)\sqrt{x+1}} dx \\
 I &= \int \frac{dx}{\sqrt{x+1}} + 3 \int \frac{dx}{(x-3)\sqrt{x+1}}
 \end{aligned} \tag{A}$$

Now,

$$\int \frac{dx}{\sqrt{x+1}} = 2\sqrt{x+1} + c_1$$

and,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_2$$

$$\int \frac{dx}{(x-3)\sqrt{x+1}}$$

Let $x+1 = t^2$

$$\Rightarrow dx = 2tdt$$

$$\begin{aligned}
 \therefore \int \frac{dx}{(x-3)\sqrt{x+1}} &= 2 \int \frac{tdt}{(t^2-4)t} \\
 &= 2 \left| \frac{dt}{t^2-4} \right| \\
 &= \frac{2}{2 \times 2} \log \left| \frac{t-2}{t+2} \right| + c_2
 \end{aligned}$$

$$\therefore \int \frac{dx}{(x-3)\sqrt{x+1}} = \frac{1}{2} \log \left| \frac{\sqrt{x+1}-2}{\sqrt{x+1}+2} \right| + c_2$$

Thus, from (A)

$$I = 2\sqrt{x+1} + \frac{3}{2} \log \left| \frac{\sqrt{x+1}-2}{\sqrt{x+1}+2} \right| + c \quad [\text{when } c = c_1 + c_2]$$

Indefinite Integrals Ex 19.32 Q6

$$\text{Let } I = \int \frac{x}{(x^2 + 1)\sqrt{x}} dx$$

$$\text{Let } x = t^2$$

$$\Rightarrow dx = 2tdt$$

$$\therefore 2 \int \frac{tdt}{(t^2 + 1)t}$$

$$= 2 \left| \frac{dt}{t^4 + 1} \right|$$

Dividing numerator and denominator by t^2

$$\begin{aligned} I &= 2 \int \frac{\frac{t}{t^2}}{\left(t^2 + \frac{1}{t^2}\right)} dt \\ &= \int \frac{\left(1 + \frac{1}{t^2}\right) - \left(1 - \frac{1}{t^2}\right)}{\left(t^2 + \frac{1}{t^2}\right)} dt \\ &= \int \frac{\left(1 + \frac{1}{t^2}\right)}{\left(t - \frac{1}{t}\right)^2 + 2} dt - \int \frac{\left(1 - \frac{1}{t^2}\right)}{\left(1 + \frac{1}{t}\right)^2 - 2} dt \end{aligned}$$

$$\text{Let } t - \frac{1}{t} = z$$

$$\Rightarrow \left(1 + \frac{1}{t^2}\right) dt = dz \quad [\text{For Ist part}]$$

and,

$$t + \frac{1}{t} = y$$

$$\Rightarrow \left(1 - \frac{1}{t^2}\right) dt = dy \quad [\text{For IInd part}]$$

$$\begin{aligned} I &= \int \frac{dz}{z^2 + 2} - \int \frac{dy}{y^2 - 2} \\ &= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{z}{\sqrt{2}} \right) - \frac{1}{2\sqrt{2}} \log \left| \frac{y - \sqrt{2}}{y + \sqrt{2}} \right| + C \\ &= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{t^2 - 1}{\sqrt{2}t} \right) - \frac{1}{2\sqrt{2}} \log \left| \frac{x+1 - \sqrt{2}x}{x+1 + \sqrt{2}x} \right| + C \end{aligned}$$

Indefinite Integrals Ex 19.32 Q7

$$\text{Let } I = \int \frac{x}{(x^2 + 2x + 2)\sqrt{x+1}} dx$$

$$\text{Let } x+1 = t^2$$

$$\begin{aligned} \Rightarrow dx &= 2tdt \\ &= 2 \int \frac{(t^2 - 1)tdt}{(t^4 + 1)t} \\ &= 2 \int \frac{(t^2 - 1)dt}{(t^4 + 1)} \\ &= 2 \int \frac{\left(1 - \frac{1}{t^2}\right)dt}{t^2 + \frac{1}{t^2}} \\ &= 2 \int \frac{\left(1 - \frac{1}{t^2}\right)dt}{\left(t + \frac{1}{t}\right)^2 - 2} \end{aligned}$$

$$\text{Let } t + \frac{1}{t} = y$$

$$\begin{aligned} \Rightarrow \left(1 - \frac{1}{t^2}\right)dt &= dy \\ \therefore I &= 2 \int \frac{dy}{y^2 - 2} \\ &= \frac{2}{2\sqrt{2}} \log \left| \frac{y - \sqrt{2}}{y + \sqrt{2}} \right| + c \end{aligned}$$

Thus,

$$I = \frac{1}{\sqrt{2}} \log \left| \frac{t^2 + 1 - \sqrt{2}t}{t^2 + 1 + \sqrt{2}t} \right| + c$$

Hence,

$$I = \frac{1}{\sqrt{2}} \log \left| \frac{x + 2 - \sqrt{2(x+1)}}{x + 2 + \sqrt{2(x+1)}} \right| + c$$

Indefinite Integrals Ex 19.32 Q8

$$\text{Let } I = \int \frac{1}{(x-1)\sqrt{x^2+1}} dx$$

$$\text{Let } x-1 = \frac{1}{t}$$

$$\begin{aligned} \Rightarrow dx &= -\frac{1}{t^2} dt \\ \therefore I &= - \int \frac{\frac{1}{t^2} dt}{\frac{1}{t} \sqrt{\left(\frac{1}{t} + 1\right)^2 + 1}} \\ &= - \int \frac{dt}{\sqrt{2t^2 + 2t + 1}} \\ &= - \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{t^2 + t + \frac{1}{2}}} \\ &= - \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{\left(t + \frac{1}{2}\right)^2 + \frac{1}{4}}} \end{aligned}$$

$$\therefore I = - \frac{1}{\sqrt{2}} \log \left| \left(t + \frac{1}{2} \right) + \sqrt{\left(t + \frac{1}{2} \right)^2 + \frac{1}{4}} \right| + c \quad \left[\text{When } t = \frac{1}{x-1} \right]$$

Indefinite Integrals Ex 19.32 Q9

Let $I = \int \frac{1}{(x+1)\sqrt{x^2+x+1}} dx$

Let $x+1 = \frac{1}{t}$

$\Rightarrow dx = -\frac{1}{t^2} dt$

$\therefore I = -\int \frac{\frac{1}{t^2} dt}{\frac{1}{t}\sqrt{\left(\frac{1}{t^2} + \frac{1}{t} - 1\right)}}$

$= -\int \frac{dt}{\sqrt{1+t-t^2}}$

$= -\int \frac{dt}{\sqrt{\frac{5}{4} - \left(\frac{1}{4} - t + t^2\right)}}$

$= -\int \frac{dt}{\sqrt{\frac{5}{4} - \left(t - \frac{1}{2}\right)^2}}$

$= -\sin^{-1}\left(\frac{t - \frac{1}{2}}{\frac{\sqrt{5}}{2}}\right) + C$

$\therefore I = -\sin^{-1}\left(\frac{2t-1}{\sqrt{5}}\right) + C \quad \left[\text{When } t = \frac{1}{x+1}\right]$

Indefinite Integrals Ex 19.32 Q10

Let $I = \int \frac{1}{(x^2-1)\sqrt{x^2+1}} dx$

Let $x = \frac{1}{t}$

$\Rightarrow dx = -\frac{1}{t^2} dt$

$\therefore I = -\int \frac{\frac{1}{t^2} dt}{\left(\frac{1}{t^2} - 1\right)\sqrt{\left(\frac{1}{t^2} + 1\right)}}$

$= -\int \frac{tdt}{\left(1-t^2\right)\sqrt{1+t^2}}$

Let $1+t^2 = u^2$

$\Rightarrow 2tdt = 2udu$

$I = \int \frac{u du}{(u^2-2)u}$

$= \int \frac{du}{u^2-2}$

$\therefore I = \frac{1}{2\sqrt{2}} \log \left| \frac{u-\sqrt{2}}{u+\sqrt{2}} \right| + C$

$= \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{1+t^2}-\sqrt{2}}{\sqrt{1+t^2}+\sqrt{2}} \right| + C$

Hence,

$$I = -\frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2}x + \sqrt{x^2+1}}{\sqrt{2}x - \sqrt{x^2+1}} \right| + C$$

Indefinite Integrals Ex 19.32 Q11

$$I = \int \frac{x}{(x^2 + 4)\sqrt{x^2 + 1}} dx$$

Let $x^2 + 1 = u^2$

$$\Rightarrow 2xdx = 2u du$$

$$\begin{aligned} \therefore I &= \int \frac{u}{(u^2 + 3)u} du \\ &= \int \frac{1}{u^2 + 3} du \\ &= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{u}{\sqrt{3}}\right) + C \\ &= \frac{1}{\sqrt{3}} \tan^{-1}\left(\sqrt{\frac{x^2 + 1}{3}}\right) + C \end{aligned}$$

Indefinite Integrals Ex 19.32 Q12

$$\text{Let } I = \int \frac{1}{(1+x^2)\sqrt{1-x^2}} dx$$

$$\text{Let } x = \frac{1}{t}$$

$$\Rightarrow dx = -\frac{1}{t^2} dt$$

$$\begin{aligned} \therefore I &= \int \frac{-\frac{1}{t^2} dt}{\left(\frac{1}{t^2} + 1\right)\sqrt{\left(1 - \frac{1}{t^2}\right)}} \\ &= -\int \frac{tdt}{(t^2 + 1)\sqrt{t^2 - 1}} \end{aligned}$$

$$\text{Let } t^2 - 1 = u^2$$

$$\Rightarrow 2tdt = 2udu$$

$$\begin{aligned} I &= -\int \frac{udu}{(u^2 + 2)u} \\ &= -\int \frac{du}{u^2 + 2} \\ &= -\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C \\ &= -\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\sqrt{t^2 - 1}}{\sqrt{2}}\right) + C \end{aligned}$$

Thus,

$$I = -\frac{1}{\sqrt{2}} \tan^{-1}\left(\sqrt{\frac{1-x^2}{2x^2}}\right) + C$$

Indefinite Integrals Ex 19.32 Q13

$$\text{Let } I = \int \frac{1}{(2x^2 + 3)\sqrt{x^2 - 4}} dx$$

$$\text{Let } x = \frac{1}{t}$$

$$\Rightarrow dx = -\frac{1}{t^2} dt$$

$$\therefore I = \int \frac{-\frac{1}{t^2} dt}{\left(\frac{2}{t^2} + 3\right) \sqrt{\left(\frac{1}{t^2} - 4\right)}} \\ = -\int \frac{tdt}{(2 + 3t^2)\sqrt{1 - 4t^2}}$$

$$\text{Let } 1 - 4t^2 = u^2$$

$$\Rightarrow -8tdt = 2udu$$

$$\therefore I = \frac{1}{4} \int \frac{udu}{\frac{(11 - 3u^2)}{4} u} \\ = \frac{1}{3} \int \frac{du}{\frac{11}{3} - u^2}$$

$$= \frac{1}{2\sqrt{33}} \log \left| \frac{u - \sqrt{\frac{11}{3}}}{u + \sqrt{\frac{11}{3}}} \right| + C \\ = \frac{1}{2\sqrt{33}} \log \left| \frac{\sqrt{1 - 4t^2} - \sqrt{\frac{11}{3}}}{\sqrt{1 - 4t^2} + \sqrt{\frac{11}{3}}} \right|$$

Hence,

$$I = \frac{1}{2\sqrt{33}} \log \left| \frac{\sqrt{11}x + \sqrt{3x^2 - 12}}{\sqrt{11}x - \sqrt{3x^2 - 12}} \right| + C$$

Indefinite Integrals Ex 19.32 Q14

$$I = \int \frac{x}{(x^2 + 4)\sqrt{x^2 + 9}} dx$$

$$\text{Let } x^2 + 9 = u^2$$

$$\Rightarrow 2xdx = 2u du$$

$$\therefore I = \int \frac{u}{(u^2 - 5)u} du \\ = \int \frac{du}{u^2 - 5} \\ = \frac{1}{2\sqrt{5}} \log \left(\frac{u - \sqrt{5}}{u + \sqrt{5}} \right) + C \\ = \frac{1}{2\sqrt{5}} \log \left(\frac{\sqrt{x^2 + 9} - \sqrt{5}}{\sqrt{x^2 + 9} + \sqrt{5}} \right) + C$$