Exercise 25a

Question 1.
Prove that

i. $[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$

ii. $[\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = [\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = [\hat{\jmath} \quad \hat{\imath} \quad \hat{k}] = -1$

Answer:
i. $[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$

Let, $\hat{\imath}, \hat{\jmath}, \hat{k}$ be unit vectors in the direction of positive X-axis, Y-axis, Z-axis respectively.

Hence,

$Magnitude\ of\ \hat{\imath}\ is\ 1 \Rightarrow\ |\hat{\imath}| = 1$

$Magnitude\ of\ \hat{\jmath}\ is\ 1 \Rightarrow\ |\hat{\jmath}| = 1$

$Magnitude\ of\ \hat{k}\ is\ 1 \Rightarrow |\hat{k}| = 1$

To Prove :

$[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$

Formulae :

a) Dot Products :

i) $\hat{\imath}\,.\hat{\imath} = \ \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

ii) $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$

b) Cross Products :

i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$

iii) $\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}, \hat{\imath} \times \hat{k} = -\hat{\jmath}$

c) Scalar Triple Product :

$[\bar{a} \quad \bar{b} \quad \bar{c}] = \bar{a} \,.\, (\bar{b} \times \bar{c})$

Now,

(i) $[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = \hat{\imath} \,.\, (\hat{\jmath} \times \hat{k})$

$= \hat{\imath} \,.\, \hat{\imath}$ .... $(\because \hat{\jmath} \times \hat{k} = \hat{\imath})$

= 1 .... $(\because \hat{\imath} \,.\, \hat{\imath} = 1)$

$\therefore \ [\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = 1$ .... eq(1)

(ii) $[\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = \hat{\jmath} \,.\, (\hat{k} \times \hat{\imath})$

$= \hat{\jmath} \,.\, \hat{\jmath}$ .... $(\because \hat{k} \times \hat{\imath} = \hat{\jmath})$

= 1 .... $(\because \hat{\jmath} \,.\, \hat{\jmath} = 1)$

$\therefore \ [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = 1$ .... eq(2)

(iii) $[\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = \hat{k} \,.\, (\hat{\imath} \times \hat{\jmath})$

$= \hat{k} \,.\, \hat{k}$ .... $(\because \hat{\imath} \times \hat{\jmath} = \hat{k})$

= 1 .... $(\because \hat{k} \,.\, \hat{k} = 1)$

$\therefore \ [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$ .... eq(3)

From eq(1), eq(2) and eq(3),

$[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$

Hence Proved.

Notes :

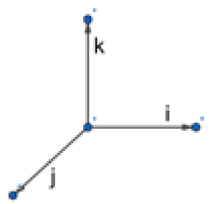
1. A cyclic change of vectors in a scalar triple product does not change its value i.e.

$[\bar{a} \quad \bar{b} \quad \bar{c}] = [\bar{b} \quad \bar{c} \quad \bar{a}] = [\bar{c} \quad \bar{a} \quad \bar{b}]$

2. Scalar triple product of unit vectors taken in a clockwise direction is 1, and that of unit vectors taken in anticlockwise direction is -1

$[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = 1$

$[\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = -1$



ii. $[\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = [\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = [\hat{\jmath} \quad \hat{\imath} \quad \hat{k}] = -1$

Let, $\hat{\imath}, \hat{\jmath}, \hat{k}$ be unit vectors in the direction of positive X-axis, Y-axis, Z-axis respectively.

Hence,

$Magnitude\ of\ \hat{\imath}\ is\ 1 \Rightarrow\ |\hat{\imath}| = 1$

$Magnitude\ of\ \hat{\jmath}\ is\ 1 \Rightarrow\ |\hat{\jmath}| = 1$

$Magnitude\ of\ \hat{k}\ is\ 1 \Rightarrow\ |\hat{k}| = 1$

To Prove :

$[\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = [\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = [\hat{\jmath} \quad \hat{\imath} \quad \hat{k}] = -1$

Formulae :

a) Dot Products :

i) $\hat{\imath}\,.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

ii) $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$

b) Cross Products :

i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$

iii) $\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}, \hat{\imath} \times \hat{k} = -\hat{\jmath}$

c) Scalar Triple Product :

$[\bar{a} \quad \bar{b} \quad \bar{c}] = \bar{a}\,.\left(\bar{b} \times \bar{c}\right)$

Answer :

(i) $[\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = \hat{\imath}\,.\left(\hat{k} \times \hat{\jmath}\right)$

$= \hat{\imath}\,.(-\hat{\imath})$ ………… $\left(\because \hat{k} \times \hat{\jmath} = -\hat{\imath}\right)$

$= -\hat{\imath}\,.\hat{\imath}$

= -1 ………… $(\because \hat{\imath}\,.\hat{\imath} = 1)$

$\therefore\ [\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = -1$ ………… eq(1)

(ii) $[\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = \hat{k}\,.(\hat{\jmath} \times \hat{\imath})$

$= \hat{k}\,.\left(-\hat{k}\right)$ ………… $\left(\because \hat{\jmath} \times \hat{\imath} = -\hat{k}\right)$

$= -\hat{k}\,.\hat{k}$

= -1 .... $(\because \hat{k} . \hat{k} = 1)$

$\therefore$ $[\hat{k} \quad \hat{j} \quad \hat{\imath}] = -1$ .... eq(2)

(iii) $[\hat{j} \quad \hat{\imath} \quad \hat{k}] = \hat{j} . (\hat{\imath} \times \hat{k})$

$= \hat{j} . (-\hat{j})$ .... $(\because \hat{\imath} \times \hat{k} = -\hat{j})$

$= -\hat{j} . \hat{j}$

= -1 .... $(\because \hat{j} . \hat{j} = 1)$

$\therefore$ $[\hat{j} \quad \hat{\imath} \quad \hat{k}] = -1$ .... eq(3)

From eq(1), eq(2) and eq(3),

$[\hat{\imath} \quad \hat{k} \quad \hat{j}] = [\hat{k} \quad \hat{j} \quad \hat{\imath}] = [\hat{j} \quad \hat{\imath} \quad \hat{k}] = -1$

Hence Proved.

Notes :

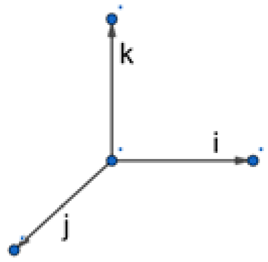
1. A cyclic change of vectors in a scalar triple product does not change its value i.e.

$[\bar{a} \quad \bar{b} \quad \bar{c}] = [\bar{b} \quad \bar{c} \quad \bar{a}] = [\bar{c} \quad \bar{a} \quad \bar{b}]$

2. Scalar triple product of unit vectors taken in a clockwise direction is 1, and that of unit vectors taken in anticlockwise direction is -1

$[\hat{\imath} \quad \hat{j} \quad \hat{k}] = 1$

$[\hat{k} \quad \hat{j} \quad \hat{\imath}] = -1$



Question 2.

Find $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$, when

i. $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$.

ii. $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

iii. $\vec{a} = 2\hat{i} - 3\hat{j}, \vec{b} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{k}$

Answer:

i. $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

Given Vectors :

1) $\bar{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

2) $\bar{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

3) $\bar{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

$\bar{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\bar{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

$= 2(2 \times 2 - 1 \times 1) - 1((-1) \times 2 - 3 \times 1) + 3((-1) \times 1 - 3 \times 2)$

$= 2(3) - 1(-5) + 3(-7)$

= 6 + 5 – 21

= - 10

$$\boxed{\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -10}$$

ii. $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

Given Vectors :

1) $\bar{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

2) $\bar{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

3) $\bar{c} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\bar{c} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$= 2(2 \times 2 - (-1) \times (-1)) - (-3)(1 \times 2 - 3 \times (-1)) + 4(1 \times (-1) - 3 \times 2)$

$= 2(3) + 3(5) + 4(-7)$

= 6 + 15 – 28

= - 7

$$\boxed{\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -7}$$

iii. $\vec{a} = 2\hat{i} - 3\hat{j}$, $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{k}$

Given Vectors :

1) $\bar{a} = 2\hat{\imath} - 3\hat{\jmath}$

2) $\bar{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$

3) $\bar{c} = 3\hat{\imath} - \hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} - 3\hat{\jmath} + 0\hat{k}$

$\bar{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = 3\hat{\imath} + 0\hat{\jmath} - \hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$$

$= 2(1 \times (-1) - (-1) \times 0) - (-3)(1 \times (-1) - 3 \times (-1)) + 0(1 \times 0 - 3 \times 1)$

$= 2(-1) + 3(2) + 0$

= - 2 + 6

= 4

$$\boxed{\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 4}$$

Question 3.
Find the volume of the parallelepiped whose conterminous edges are represented by the vectors

i. $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$

ii. $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}, \vec{b} = -5\hat{i} + 7\hat{j} - 3\hat{k}, \vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$

iii. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{j} + \hat{k}$

iv. $\bar{a} = 6\hat{\imath}, \bar{b} = 2\hat{\jmath}, \bar{c} = 5\hat{k}$

Answer:
i. $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$$\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$$

$$\bar{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$$

$$\bar{c} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

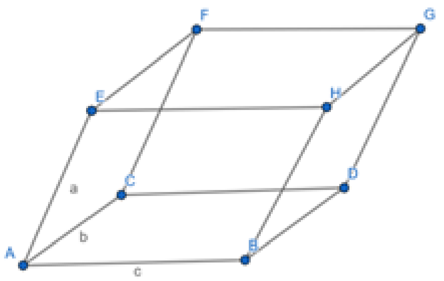
Answer :

Volume of parallelopiped with coterminous edges

$\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{c} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= 1((-1) \times (-1) - 2 \times 1) - 1(1 \times (-1) - 1 \times 1) + 1(1 \times 2 - 1 \times (-1))$$

= 1(-1) -1(-2) + 1(3)

= -1+2+3

= 4

Therefore,

$$\boxed{\text{Volume of parallelepiped } = 4 \text{ cubic unit}}$$

ii. $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}, \vec{b} = -5\hat{i} + 7\hat{j} - 3\hat{k}, \vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$

$\bar{b} = -5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$

$\bar{c} = 7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

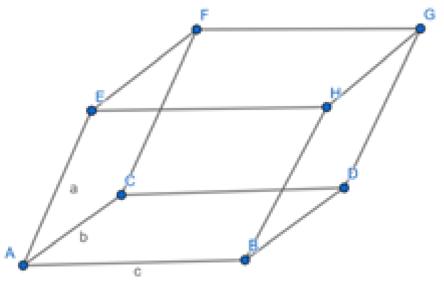
Answer :

Volume of parallelopiped with coterminous edges

$\bar{a} = -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$

$\bar{b} = -5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$

$\bar{c} = 7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & 7 & 5 \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix}$$

$$= -3\big(7 \times (-3) - (-5) \times (-3)\big) - 7\big((-5) \times (-3) - 7 \times (-3)\big) + 5((-5) \times (-5) - 7 \times 7)$$

= -3(-36) -7(36) + 5(-24)

= 108 – 252 – 120

= -264

As volume is never negative

Therefore,

$$\boxed{\text{Volume of parallelepiped} = 264 \text{ cubic unit}}$$

iii. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{j} + \hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = \hat{\jmath} + \hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

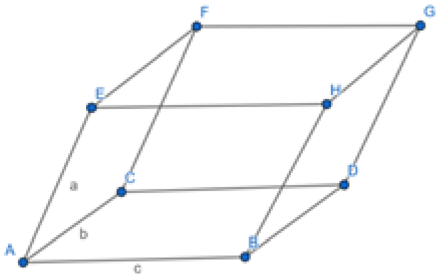
Answer :

Volume of parallelopiped with coterminous edges

$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = 0\hat{\imath} + \hat{\jmath} + \hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 1(1 \times 1 - 1 \times (-1)) - (-2)(2 \times 1 - 0 \times (-1)) + 3(2 \times 1 - 0 \times 1)$$

= 1(2) +2(2) + 3(2)

= 2 + 4 + 6

= 12

Therefore,

$$\boxed{\text{Volume of parallelepiped} = 12 \text{ cubic unit}}$$

iv. $\bar{a} = 6\hat{\imath}, \bar{b} = 2\hat{\jmath}, \bar{c} = 5\hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = 6\hat{\imath}$

$\bar{b} = 2\hat{\jmath}$

$\bar{c} = 5\hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

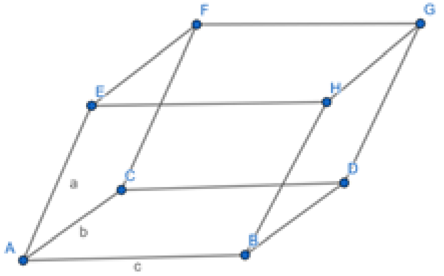
Answer :

Volume of parallelopiped with coterminous edges

$$\bar{a} = 6\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$$

$$\bar{b} = 0\hat{\imath} + 2\hat{\jmath} + 0\hat{k}$$

$$\bar{c} = 0\hat{\imath} + 0\hat{\jmath} + 5\hat{k}$$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{vmatrix}$$

$$= 6(2 \times 5 - 0 \times 0) - 0(0 \times 5 - 0 \times 0) + 0(0 \times 0 - 0 \times 2)$$

= 6(10) + 0 + 0

= 60

Therefore,

$$\boxed{\text{Volume of parallelepiped} = 60 \text{ cubic unit}}$$

Question 4.

Show that the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, when

i. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

ii. $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = 7\hat{j} + 3\hat{k}$

iii. $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} - 4\hat{j} + 7\hat{k}$

Answer:

i. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

Given Vectors :

$$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$$

$$\bar{b} = -2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$$

$$\bar{c} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

i.e. $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Formulae :

1) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$$

$$\bar{b} = -2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$$

$$\bar{c} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$$

$$= 1(3 \times 5 - (-3) \times (-4)) - (-2)((-2) \times 5 - 1 \times (-4)) + 3((-2) \times (-3) - 3 \times 1)$$

= 1(3) + 2(-6) + 3(3)

= 3 – 12 +9

= 0

$$\boxed{\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0}$$

Hence, the vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Note : For coplanar vectors $\bar{a}, \bar{b}, \bar{c}$,

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

ii. $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = 7\hat{j} + 3\hat{k}$

Given Vectors :

$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 7\hat{\jmath} + 3\hat{k}$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

i.e. $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 7\hat{\jmath} + 3\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & 7 & 3 \end{vmatrix}$$

$= 1((-1) \times 3 - 7 \times (-1)) - 3(2 \times 3 - 0 \times (-1)) + 1(2 \times 7 - 0 \times (-1))$

= 1(4) – 3(6) + 1(14)

= 4 – 18 + 14

= 0

$$\boxed{\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0}$$

Hence, the vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Note : For coplanar vectors $\bar{a}, \bar{b}, \bar{c}$,

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

iii. $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} - 4\hat{j} + 7\hat{k}$

Given Vectors :

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

i.e. $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -1 & 2 \\ 1 & 2 & -3 \\ 3 & -4 & 7 \end{vmatrix}$$

$= 2(2 \times 7 - (-3) \times (-4)) - (-1)(1 \times 7 - 3 \times (-3)) + 2(1 \times (-4) - 3 \times 2)$

= 2(2) + 1(16) + 2(-10)

= 4 + 16 -20

= 0

$$\boxed{\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0}$$

Hence, the vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Note : For coplanar vectors $\bar{a}, \bar{b}, \bar{c}$,

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Question 5.

Find the value of λ for which the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, when

i. $\vec{a} = \left(2\hat{i} - \hat{j} + \hat{k}\right), \vec{b} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and $\vec{c} = \left(3\hat{i} + \lambda\hat{j} + 5\hat{k}\right)$

ii. $\vec{a} = \lambda\hat{i} - 10\hat{j} - 5\hat{k}, \vec{b} = -7\hat{i} - 5\hat{j}$ and $\vec{c} = \hat{i} - 4\hat{j} - 3\hat{k}$

iii. $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$

Answer:

i. .$\vec{a} = \left(2\hat{i} - \hat{j} + \hat{k}\right), \vec{b} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$. and $\vec{c} = \left(3\hat{i} + \lambda\hat{j} + 5\hat{k}\right)$

Given : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Where,

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{c} = 3\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$

To Find : value of λ

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{c} = 3\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & \lambda & 5 \end{vmatrix}$$

$= 2(2 \times 5 - 3 \times \lambda) - (-1)(1 \times 5 - 3 \times 3) + 1(1 \times \lambda - 3 \times 2)$

$= 2(10 - 3\lambda) - 4 + 1(\lambda - 6)$

$= 20 - 6\lambda - 4 + \lambda - 6$

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 10 - 5\lambda$ ....eq(2)

From eq(1) and eq(2),

$$10 - 5\lambda = 0$$

$$\therefore 5\lambda = 10$$

$$\boxed{\therefore \lambda = 2}$$

ii. $\vec{a} = \lambda\hat{i} - 10\hat{j} - 5\hat{k}, \vec{b} = -7\hat{i} - 5\hat{j}$ and $\vec{c} = \hat{i} - 4\hat{j} - 3\hat{k}$

Given : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Where,

$$\bar{a} = \lambda\hat{\imath} - 10\hat{\jmath} - 5\hat{k}$$

$$\bar{b} = -7\hat{\imath} - 5\hat{\jmath}$$

$$\bar{c} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$$

To Find : value of λ

Formulae :

1) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = \lambda\hat{\imath} - 10\hat{\jmath} - 5\hat{k}$

$\bar{b} = -7\hat{\imath} - 5\hat{\jmath} + 0\hat{k}$

$\bar{c} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} \lambda & -10 & -5 \\ -7 & -5 & 0 \\ 1 & -4 & -3 \end{vmatrix}$$

$$= \lambda((-5) \times (-3) - 0 \times (-4)) - (-10)\big((-7) \times (-3) - 0 \times 1\big) + (-5)((-7) \times (-4) - 1 \times (-5))$$

$= \lambda(15) + 10(21) - 5(33)$

$= 15\lambda + 45$

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 15\lambda + 45$ ....eq(2)

From eq(1) and eq(2),

$15\lambda + 45 = 0$

$\therefore 15\lambda = 45$

$\boxed{\therefore \lambda = -3}$

iii. $\cdot\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}\cdot$ and $\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$

Given : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Where,

$\bar{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = \lambda\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$

To Find : value of λ

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = \lambda\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix}$$

$= 1(1 \times \lambda - (-1) \times (-1)) - (-1)(2 \times \lambda - (-1) \times \lambda) + 1(2 \times (-1) - \lambda \times 1)$

$= 1(\lambda - 1) + 1(3\lambda) + 1(-\lambda - 2)$

$= \lambda - 1 + 3\lambda - 2 - \lambda$

$= 3\lambda - 3$

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 3\lambda - 3$ ....eq(2)

From eq(1) and eq(2),

$3\lambda - 3 = 0$

$\therefore 3\lambda = 3$

$\boxed{\therefore \lambda = 1}$

Question 6.

If $\vec{a} = \left(2\hat{i} - \hat{j} + \hat{k}\right), \vec{b} = \left(\hat{i} - 3\hat{j} - 5\hat{k}\right)$ and $\vec{c} = \left(3\hat{i} - 4\hat{j} - \hat{k}\right)$, find $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ and interpret the result.

Answer:
Given Vectors :

$$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$$

$$\bar{b} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$$

$$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} - \hat{k}$$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$

$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} - \hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -1 & 1 \\ 1 & -3 & -5 \\ 3 & -4 & -1 \end{vmatrix}$$

$$= 2((-3) \times (-1) - (-4) \times (-5)) - (-1)((-1) \times 1 - 3 \times (-5)) + 1((-4) \times 1 - 3 \times (-3))$$

$$= 2(-17) + 1(14) + 1(5)$$

= - 34 + 14 + 5

= - 15

$$\boxed{\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -15}$$

Question 7.

The volume of the parallelepiped whose edges are $\left(-12\hat{i} + \lambda\hat{k}\right), \left(3\hat{j} - \hat{k}\right)$ and $\left(2\hat{i} + \hat{j} - 15\hat{k}\right)$ is 546 cubic units. Find the value of λ.

Answer:
Given :

1) Coterminous edges of parallelepiped are

$\bar{a} = -12\hat{\imath} + \lambda\hat{k}$

$\bar{b} = 3\hat{\jmath} - \hat{k}$

$\bar{c} = 2\hat{\imath} + \hat{\jmath} - 15\hat{k}$

2) Volume of parallelepiped,

V = 546 cubic unit

To Find : value of λ

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

Given volume of parallelepiped,

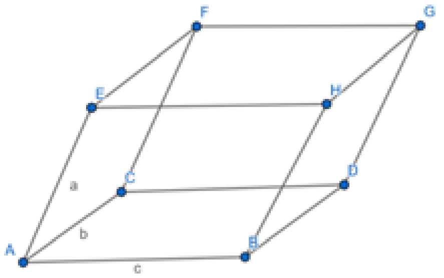
V = 546 cubic unit ....eq(1)

Volume of parallelopiped with coterminous edges

$\bar{a} = -12\hat{\imath} + \lambda\hat{k}$

$\bar{b} = 3\hat{\jmath} - \hat{k}$

$\bar{c} = 2\hat{\imath} + \hat{\jmath} - 15\hat{k}$



$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$= \begin{vmatrix} -12 & 0 & \lambda \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix}$

$= -12(3 \times (-15) - 1 \times (-1)) - 0 + \lambda(0 \times 1 - 3 \times 2)$

$= 528 - 0 - 6\,\lambda$

$= 528 - 6\,\lambda$

$\therefore V = (528 - 6\,\lambda)$ cubic unit ....eq(2)

From eq(1) and eq(2)

$528 - 6\,\lambda = 546$

$\therefore -6\,\lambda = 18$

$$\boxed{\therefore\ \lambda = -3}$$

Question 8.

Show that the vectors $\vec{a} = \left(\hat{i} + 3\hat{j} + \hat{k}\right), \vec{b} = \left(2\hat{i} - \hat{j} - \hat{k}\right)$ and $\vec{c} = \left(7\hat{j} + 3\hat{k}\right)$ are parallel to the same plane.

{HINT: Show that $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$}

Answer:
Given Vectors :

$$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$$

$$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$$

$$\bar{c} = 7\hat{\jmath} + 3\hat{k}$$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are parallel to same plane.

Formulae :

1) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

Vectors will be parallel to the same plane if they are coplanar.

For vectors $\bar{a}, \bar{b}, \bar{c}$ to be coplanar, $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Now, for given vectors,

$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 7\hat{\jmath} + 3\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & 7 & 3 \end{vmatrix}$$

$= 1(3 \times (-1) - 7 \times (-1)) - 3(2 \times 3 - 0 \times (-1)) + 1(2 \times 7 - 0 \times (-1))$

= 1(4) – 3(6) + 1(14)

= 4 – 18 + 14

= 0

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Hence, given vectors are parallel to the same plane.

Question 9.

If the vectors $\left(a\hat{i} + a\hat{j} + c\hat{k}\right), \left(\hat{i} + \hat{k}\right)$ and $\left(c\hat{i} + c\hat{j} + b\hat{k}\right)$ be coplanar, show that $c^2 = ab$.

Answer:
Given : vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar. Where,

$$\bar{a} = a\hat{\imath} + a\hat{\jmath} + c\hat{k}$$

$$\bar{b} = \hat{\imath} + \hat{k}$$

$$\bar{c} = c\hat{\imath} + c\hat{\jmath} + b\hat{k}$$

To Prove : $c^2 = ab$

Formulae :

1) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore\ [\bar{a}\quad \bar{b}\quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = a\hat{\imath} + a\hat{\jmath} + c\hat{k}$

$\bar{b} = \hat{\imath} + \hat{k}$

$\bar{c} = c\hat{\imath} + c\hat{\jmath} + b\hat{k}$

$$[\bar{a}\quad \bar{b}\quad \bar{c}] = \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix}$$

$= a(0 \times b - c \times 1) - a(1 \times b - 1 \times c) + c(1 \times c - 0 \times c)$

= a.(- c) – a.(b - c) + c(c)

= – ac – ab + ac + c^2

= - ab + c^2

$\therefore\ [\bar{a}\quad \bar{b}\quad \bar{c}] = -ab + c^2$ ....eq(2)

From eq(1) and eq(2),

- ab + c^2 = 0

Therefore,

$\boxed{c^2 = ab}$

Hence proved.

Note : Three vectors $\bar{a}, \bar{b}$ & $\bar{c}$ are coplanar if and only if

$[\bar{a}\quad \bar{b}\quad \bar{c}] = 0$

Question 10.

Show that the four points with position vectors $\left(4\hat{i}+8\hat{j}+12\hat{k}\right)$, $\left(2\hat{i}+4\hat{j}+6\hat{k}\right)$, $\left(3\hat{i}+5\hat{j}+4\hat{k}\right)$ and $\left(5\hat{i}+8\hat{j}+5\hat{k}\right)$ are coplanar.

Answer:
Given :

Let A, B, C & D be four points with position vectors $\bar{a}, \bar{b}, \bar{c}$ & $\bar{d}$.

Therefore,

$$\bar{a} = 4\hat{\imath} + 8\hat{\jmath} + 12\hat{k}$$

$$\bar{b} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$$

$$\bar{c} = 3\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$$

$$\bar{d} = 5\hat{\imath} + 8\hat{\jmath} + 5\hat{k}$$

To Prove : Points A, B, C & D are coplanar.

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then vector $\overline{AB}$ is given by,

$$\overline{AB} = \bar{b} - \bar{a}$$

$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

2) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$\bar{a} = 4\hat{\imath} + 8\hat{\jmath} + 12\hat{k}$

$\bar{b} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$

$\bar{c} = 3\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$

$\bar{d} = 5\hat{\imath} + 8\hat{\jmath} + 5\hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (4-2)\hat{\imath} + (8-4)\hat{\jmath} + (12-6)\hat{k}$

$\therefore\ \overline{BA} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (4-3)\hat{\imath} + (8-5)\hat{\jmath} + (12-4)\hat{k}$

$\therefore\ \overline{CA} = \hat{\imath} + 3\hat{\jmath} + 8\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (4-5)\hat{\imath} + (8-8)\hat{\jmath} + (12-5)\hat{k}$

$\therefore\ \overline{DA} = -\hat{\imath} + 0\hat{\jmath} + 7\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$

$\overline{CA} = \hat{\imath} + 3\hat{\jmath} + 8\hat{k}$

$\overline{DA} = -\hat{\imath} + 0\hat{\jmath} + 7\hat{k}$

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} 2 & 4 & 6 \\ 1 & 3 & 8 \\ -1 & 0 & 7 \end{vmatrix}$$

$= 2(3 \times 7 - 0 \times 8) - 4(1 \times 7 - (-1) \times 8) + 6(1 \times 0 - (-1) \times 3)$

= 2(21) – 4(15) + 6(3)

= 42 – 60 + 18

= 0

$\therefore [\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Hence, vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if $[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Question 11.

Show that the four points with position vectors $\left(6\hat{i}-7\hat{j}\right), \left(16\hat{i}-19\hat{j}-4\hat{k}\right), \left(3\hat{j}-6\hat{k}\right)$ and $\left(2\hat{i}-5\hat{j}+10\hat{k}\right)$ are coplanar.

Answer:

Given :

Let A, B, C & D be four points with position vectors $\bar{a}, \bar{b}, \bar{c}$ & $\bar{d}$.

Therefore,

$\bar{a} = 6\hat{\imath} - 7\hat{\jmath}$

$\bar{b} = 16\hat{\imath} - 19\hat{\jmath} - 4\hat{k}$

$\bar{c} = 3\hat{\jmath} - 6\hat{k}$

$\bar{d} = 2\hat{\imath} - 5\hat{\jmath} + 10\hat{k}$

To Prove : Points A, B, C & D are coplanar.

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then vector $\overline{AB}$ is given by,

$$\overline{AB} = \bar{b} - \bar{a}$$

$$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$$

2) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$$\bar{a} = 6\hat{\imath} - 7\hat{\jmath}$$

$$\bar{b} = 16\hat{\imath} - 19\hat{\jmath} - 4\hat{k}$$

$\bar{c} = 3\hat{j} - 6\hat{k}$

$\bar{d} = 2\hat{\imath} - 5\hat{j} + 10\hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (6 - 16)\hat{\imath} + (-7 + 19)\hat{j} + (0 + 4)\hat{k}$

$\therefore\ \overline{BA} = -10\hat{\imath} + 12\hat{j} + 4\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (6 - 0)\hat{\imath} + (-7 - 3)\hat{j} + (0 + 6)\hat{k}$

$\therefore\ \overline{CA} = 6\hat{\imath} - 10\hat{j} + 6\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (6 - 2)\hat{\imath} + (-7 + 5)\hat{j} + (0 - 10)\hat{k}$

$\therefore\ \overline{DA} = 4\hat{\imath} - 2\hat{j} - 10\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -10\hat{\imath} + 12\hat{j} + 4\hat{k}$

$\overline{CA} = 6\hat{\imath} - 10\hat{j} + 6\hat{k}$

$\overline{DA} = 4\hat{\imath} - 2\hat{j} - 10\hat{k}$

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} -10 & 12 & 4 \\ 6 & -10 & 6 \\ 4 & -2 & -10 \end{vmatrix}$$

$$= -10\big((-10) \times (-10) - (-2) \times 6\big) - 12(6 \times (-10) - 4 \times 6) + 4(6 \times (-2) - (-10) \times 4)$$

= -10(112) – 12(-84) + 4(28)

= -1120 + 1008 + 112

= 0

∴ $[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Hence, vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if $[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Question 12.

Find the value of λ for which the four points with position vectors $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$, $\left(3\hat{i}-\hat{j}+2\hat{k}\right)$, $\left(-2\hat{i}+\lambda\hat{j}+\hat{k}\right)$ and $\left(6\hat{i}-4\hat{j}+2\hat{k}\right)$ are coplanar.

Ans. λ = 3

Answer:

Given :

Let, A, B, C & D be four points with given position vectors

$\bar{a} = 1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\bar{c} = -2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$

$\bar{d} = 6\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$

To Find : value of λ

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then vector $\overline{AB}$ is given by,

$\overline{AB} = \bar{b} - \bar{a}$

$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

2) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$\bar{a} = 1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\bar{c} = -2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$

$\bar{d} = 6\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (1-3)\hat{\imath} + (2+1)\hat{\jmath} + (3-2)\hat{k}$

$\therefore\ \overline{BA} = -2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (1+2)\hat{\imath} + (2-\lambda)\hat{\jmath} + (3-1)\hat{k}$

$\therefore\ \overline{CA} = 3\hat{\imath} + (2-\lambda)\hat{\jmath} + 2\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (1-6)\hat{\imath} + (2+4)\hat{\jmath} + (3-2)\hat{k}$

$\therefore\ \overline{DA} = -5\hat{\imath} + 6\hat{\jmath} + \hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -2\hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\overline{CA} = 3\hat{\imath} + (2-\lambda)\hat{\jmath} + 2\hat{k}$

$\overline{DA} = -5\hat{\imath} + 6\hat{\jmath} + \hat{k}$

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} -2 & 3 & 1 \\ 3 & (2-\lambda) & 2 \\ -5 & 6 & 1 \end{vmatrix}$$

$$= -2\big((2-\lambda)\times 1 - 2\times 6\big) - 3(3\times 1 - 2\times(-5)) + 1(6\times 3 - (2-\lambda)\times(-5))$$

= -2(- λ – 10) – 3(13) + 1(28 – 5λ)

= 2λ + 20 – 39 + 28 – 5λ

= 9 – 3λ

$\therefore [\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 9 - 3\lambda$ ....eq(4)

Four points A, B, C & D are coplanar if and only if

$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$ ....eq(5)

From eq(4) and eq(5)

9 – 3λ = 0

3λ = 9

$\boxed{\lambda = 3}$

Question 13.

Find the value of λ for which the four points with position vectors $\left(-\hat{j}+\hat{k}\right)$, $\left(2\hat{i}-\hat{j}-\hat{k}\right)$, $\left(\hat{i}+\lambda\hat{j}+\hat{k}\right)$ and $\left(3\hat{j}+3\hat{k}\right)$ are coplanar.

Answer:

Given :

Let, A, B, C & D be four points with given position vectors

$\bar{a} = -\hat{j} + \hat{k}$

$\bar{b} = 2\hat{i} - \hat{j} - \hat{k}$

$\bar{c} = \hat{i} + \lambda\hat{j} + \hat{k}$

$\bar{d} = 3\hat{j} + 3\hat{k}$

To Find : value of λ

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then vector $\overline{AB}$ is given by,

$\overline{AB} = \bar{b} - \bar{a}$

$(b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$

2) Scalar Triple Product:

If

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$$\bar{a} = -\hat{j} + \hat{k}$$

$$\bar{b} = 2\hat{\imath} - \hat{j} - \hat{k}$$

$$\bar{c} = \hat{\imath} + \lambda\hat{j} + \hat{k}$$

$$\bar{d} = 3\hat{j} + 3\hat{k}$$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$$\overline{BA} = \bar{a} - \bar{b}$$

$$= (0 - 2)\hat{\imath} + (-1 + 1)\hat{j} + (1 + 1)\hat{k}$$

$\therefore\ \overline{BA} = -2\hat{\imath} + 0\hat{j} + 2\hat{k}$ ....eq(1)

$$\overline{CA} = \bar{a} - \bar{c}$$

$$= (0 - 1)\hat{\imath} + (-1 - \lambda)\hat{j} + (1 - 1)\hat{k}$$

$\therefore\ \overline{CA} = -\hat{\imath} + (-1 - \lambda)\hat{j} + 0\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (0-0)\hat{\imath} + (-1-3)\hat{\jmath} + (1-3)\hat{k}$

$\therefore\ \overline{DA} = 0\hat{\imath} - 4\hat{\jmath} - 2\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -2\hat{\imath} + 0\hat{\jmath} + 2\hat{k}$

$\overline{CA} = -\hat{\imath} + (-1-\lambda)\hat{\jmath} + 0\hat{k}$

$\overline{DA} = 0\hat{\imath} - 4\hat{\jmath} - 2\hat{k}$

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} -2 & 0 & 2 \\ -1 & (-1-\lambda) & 0 \\ 0 & -4 & -2 \end{vmatrix}$$

$$= -2\big((-1-\lambda)\times(-2) - (-4)\times 0\big) - 0((-1)\times(-2) - 0\times 0) + 2((-1)\times(-4) - (-1-\lambda)\times 0)$$

= -2(2 + 2λ) – 0 + 2(4)

= - 4 - 4λ + 8

= 4 – 4λ

$\therefore [\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 4 - 4\lambda$ ....eq(4)

Four points A, B, C & D are coplanar if and only if

$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$ ....eq(5)

From eq(4) and eq(5)

4 – 4λ = 0

4λ = 4

$\boxed{\lambda = 1}$

Question 14.
Using vector method, show that the points A(4, 5, 1), B(0, -1, -1), C(3, 9, 4) and D(-4, 4, 4) are coplanar.

Answer:
Given Points :

A ≡ (4, 5, 1)

B ≡ (0, -1, -1)

C ≡ (3, 9, 4)

D ≡ (-4, 4, 4)

To Prove : Points A, B, C & D are coplanar.

Formulae :

4) Position Vectors :

If A is a point with co-ordinates (a_1, a_2, a_3)

then its position vector is given by,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

5) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then vector $\overline{AB}$ is given by,

$$\overline{AB} = \bar{b} - \bar{a}$$

$$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$$

6) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

7) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given points,

A ≡ (4, 5, 1)

B ≡ (0, -1, -1)

C ≡ (3, 9, 4)

D ≡ (-4, 4, 4)

Position vectors of above points are,

$\bar{a} = 4\hat{\imath} + 5\hat{\jmath} + \hat{k}$

$\bar{b} = 0\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}$

$\bar{d} = -4\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (4-0)\hat{\imath} + (5+1)\hat{\jmath} + (1+1)\hat{k}$

$\therefore\ \overline{BA} = 4\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (4-3)\hat{\imath} + (5-9)\hat{\jmath} + (1-4)\hat{k}$

$\therefore\ \overline{CA} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (4+4)\hat{\imath} + (5-4)\hat{\jmath} + (1-4)\hat{k}$

$\therefore\ \overline{DA} = 8\hat{\imath} + 1\hat{\jmath} - 3\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = 4\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$

$\overline{CA} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

$\overline{DA} = 8\hat{\imath} + 1\hat{\jmath} - 3\hat{k}$

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} 4 & 6 & 2 \\ 1 & -4 & -3 \\ 8 & 1 & -3 \end{vmatrix}$$

$$= 4((-4) \times (-3) - 1 \times (-3)) - 6(1 \times (-3) - (-3) \times 8) + 2(1 \times 1 - (-4) \times 8)$$

= 4(15) – 6(21) + 2(33)

= 60 – 126 + 66

= 0

$\therefore [\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Hence, vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if $[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Question 15.
Find the value of λ for which the points A(3, 2, 1), B(4, λ, 5), C(4, 2, -2) and D(6, 5, -1) are coplanar.

Ans. λ = 5

Answer:
Given :

Points A, B, C & D are coplanar where,

A ≡ (3, 2, 1)

B ≡ (4, λ, 5)

C ≡ (4, 2, -2)

D ≡ (6, 5, -1)

To Find : value of λ

Formulae :

1) Position Vectors :

If A is a point with co-ordinates (a_1, a_2, a_3)

then its position vector is given by,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

2) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then vector $\overline{AB}$ is given by,

$$\overline{AB} = \bar{b} - \bar{a}$$

$$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$$

3) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

4) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given points,

A ≡ (3, 2, 1)

B ≡ (4, λ, 5)

C ≡ (4, 2, -2)

D ≡ (6, 5, -1)

Position vectors of above points are,

$$\bar{a} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$$

$$\bar{b} = 4\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$$

$$\bar{c} = 4\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$$

$$\bar{d} = 6\hat{\imath} + 5\hat{\jmath} - \hat{k}$$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$$\overline{BA} = \bar{a} - \bar{b}$$

$$= (3 - 4)\hat{\imath} + (2 - \lambda)\hat{\jmath} + (1 - 5)\hat{k}$$

$\therefore\ \overline{BA} = -\hat{\imath} + (2-\lambda)\hat{\jmath} - 4\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (3-4)\hat{\imath} + (2-2)\hat{\jmath} + (1+2)\hat{k}$

$\therefore\ \overline{CA} = -\hat{\imath} + 0\hat{\jmath} + 3\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (3-6)\hat{\imath} + (2-5)\hat{\jmath} + (1+1)\hat{k}$

$\therefore\ \overline{DA} = -3\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -\hat{\imath} + (2-\lambda)\hat{\jmath} - 4\hat{k}$

$\overline{CA} = -\hat{\imath} + 0\hat{\jmath} + 3\hat{k}$

$\overline{DA} = -3\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

$$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = \begin{vmatrix} -1 & (2-\lambda) & -4 \\ -1 & 0 & 3 \\ -3 & -3 & 2 \end{vmatrix}$$

$$= -1\big(0\times 2 - 3\times(-3)\big) - (2-\lambda)(2\times(-1) - (-3)\times 3) - 4((-1)\times(-3) - (-3)\times 0)$$

= - 1(9) – (2 - λ).(7) – 4(3)

= - 9 – 14 + 7λ – 12

= 7λ – 35

$\therefore [\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 7\lambda - 35$ .... eq(4)

But points A, B, C & D are coplanar if and only if

$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$ ………… eq(5)

From eq(4) and eq(5)

$7\lambda - 35 = 0$

$\therefore 7\lambda = 35$