Exercise 10b

Question 1.
Differentiate each of the following w.r.t. x:

(i) e^{4x}

(ii) e^{-5x}

(iii) $(e)^{x^3}$

Answer:
(i) Let $y = e^{4x}$ $z = 4x$

Formula : $\frac{d(e^x)}{dx} = e^x$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= (e^{4x}) \times 4$$

$$= 4e^{4x}$$

(ii) Let $y = e^{-5x}$ $z = -5x$

Formula : $\frac{d(e^x)}{dx} = e^x$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= (e^{-5x}) \times (-5)$$

$$= -5e^{-5x}$$

(iii) Let y = $(e)^{x^3}$ z = x^3

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left((e)^{x^3}\right) \times 3x^2$$

$$= 3x^2(e)^{x^3}$$

Question 2.
Differentiate each of the following w.r.t. x:

(i) $e^{2/x}$

(ii) $e^{\sqrt{x}}$

(iii) $e^{-2\sqrt{x}}$

Answer:
(i) Let y = $e^{2/x}$ z = 2/x

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(e^{2/x}\right) \times \left(\frac{-2}{x^2}\right)$$

$$= \frac{-2}{x^2} \times e^{\frac{2}{x}}$$

(ii) Let y $=e^{\sqrt{x}}$ z = $\sqrt{x}$

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(e^{\sqrt{x}}\right) \times \left(\frac{1}{2} \times x^{-0.5}\right) = \left(e^{\sqrt{x}}\right) \times \left(\frac{1}{2 \times \sqrt{x}}\right)$$

$$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

(iii) Let y $=e^{-2\sqrt{x}}$ z = $-2\sqrt{x}$

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(e^{-2\sqrt{x}}\right) \times \left(-2 \times \frac{1}{2} \times x^{-0.5}\right) = \left(e^{-2\sqrt{x}}\right) \times \left(\frac{-1}{\sqrt{x}}\right)$$

$$= \frac{-e^{-2\sqrt{x}}}{\sqrt{x}}$$

Question 3.
Differentiate each of the following w.r.t. x:

(i) $e^{\cot x}$

(ii) $e^{-\sin 2x}$

(iii) $e^{\sqrt{\sin x}}$

Answer:
(i) Let y =$e^{\cot x}$ z = $\cot x$

Formula : $\frac{d(e^x)}{dx} = e^x$, $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= (e^{\cot x}) \times (-\operatorname{cosec}^2 x)$$

$$= -\operatorname{cosec}^2 x \; e^{\cot x}$$

(ii) Let y =$e^{-\sin 2x}$ z = $-\sin 2x$

Formula : $\frac{d(e^x)}{dx} = e^x$, $\frac{d(\sin x)}{dx} = \cos x$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(e^{-\sin 2x}\right) \times (-\cos 2x \times 2)$$

$$= (-2 \cos 2x)\, e^{-\sin 2x}$$

(iii) Let y =$e^{\sqrt{\sin x}}$ z = $\sqrt{\sin x}$

Formula : $\frac{d(e^x)}{dx} = e^x$, $\frac{d(\sin x)}{dx} = \cos x$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(e^{\sqrt{\sin x}}\right) \times \left(\frac{1}{2} \times (\sin x)^{-0.5} \times \cos x\right) = \left(e^{\sqrt{\sin x}}\right) \times \left(\frac{1 \times \cos x}{2\sqrt{\sin x}}\right)$$

$$= \frac{\cos x}{2\sqrt{\sin x}} e^{\sqrt{\sin x}}$$

Question 4.
Differentiate each of the following w.r.t. x:

(i) tan (log x)

(ii) log (sec x)

(iii) log (sin (x/2))

Answer:
(i) Let y =tan(log x) z = log x

Formula : $\frac{d(\tan x)}{dx} = \sec^2 x, \frac{d(\log x)}{dx} = 1/x$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= (\sec^2 \log x) \times \left(\frac{1}{x}\right)$$

$$= \frac{\sec^2 (\log x)}{x}$$

(ii) Let y = log (sec x) z = sec x

Formula : $\frac{d(\sec x)}{dx} = \sec x \times \tan x, \frac{d(\log x)}{dx} = 1/x$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(\frac{1}{\sec x}\right)(\sec x \times \tan x)$$

$$= \tan x$$

(iii) Let y = log (sin (x/2)) z = sin (x/2)

Formula : $\frac{d(\sin x)}{dx} = \cos x, \frac{d(\log x)}{dx} = 1/x$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(\frac{1}{\sin (x/2)}\right)\left(\cos (x/2) \times \frac{1}{2}\right)$$

$$= \frac{1}{2} \times \cot (x/2)$$

Question 5.
Differentiate each of the following w.r.t. x:

(i)$\log_3 x$

(ii) 2^{-x}

(iii) 3^{x+2}

Answer:
(i) Let y = $\log_3 x$

Formula : $\log_a b = \frac{\log b}{\log a}, \frac{d(\log x)}{dx} = 1/x$

Therefore y =$\frac{\log x}{\log 3}$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dx}$$

$$= \left(\frac{1}{\log 3}\right)\left(\frac{1}{x}\right)$$

$$= \frac{1}{x(\log 3)}$$

(ii) Let y = 2^{-x} z = -x

Formula : $\frac{d(a^x)}{dx} = a^x (\log a)$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= (2^{-x}) \times (\log 2)(-1)$$

$$= -2^{-x}(\log 2)$$

(iii) Let y = 3^{x+2} z = x

Therefore Y = $3^2 \times 3^x$

Formula : $\frac{d(a^x)}{dx} = a^x (\log a)$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= 9(3^x) \times (\log 3)$$

Question 6.
Differentiate each of the following w.r.t. x:

(i) $\log\left(x+\frac{1}{x}\right)$

(ii) log (sin (3x))

(iii) $\log\left(x+\sqrt{1+x^2}\right)$

Answer:
(i) Let y = $\log(x+\frac{1}{x})$ z = $x+\frac{1}{x}$

Formula : $\frac{d(\log x)}{dx}=\frac{1}{x}, \frac{d(x^n)}{dx}=n\times x^{n-1}$

According to chain rule of differentiation

$$dy/_{dx}=\frac{dy}{dz}\times\frac{dz}{dx}$$

$$=\left(\frac{1}{x+\frac{1}{x}}\right)\times(1-\frac{1}{x^2})$$

$$=(\frac{x}{x^2+1})\times(\frac{x^2-1}{x^2})$$

$$=\left(\frac{x^2-1}{x(x^2+1)}\right)$$

(ii) Let y = log (sin (3x)) z = sin (3x)

Formula : $\frac{d(\sin x)}{dx}=\cos x, \frac{d(\log x)}{dx}=1/x$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(\frac{1}{\sin(3x)}\right)(\cos(3x) \times 3)$$

$$= 3 \times \cot(3x)$$

(iii) Let $y = \log(x + \sqrt{1+x^2})$ $z = x + \sqrt{1+x^2}$

Formula : $\frac{d(\log x)}{dx} = \frac{1}{x}$, $\frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(\frac{1}{x + \sqrt{1+x^2}}\right) \times \left(1 + \frac{1}{2}(1+x^2)^{-0.5}\, 2x\right)$$

$$= \left(\frac{1}{x + \sqrt{1+x^2}}\right) \times \left(1 + \frac{x}{1}(1+x^2)^{-0.5}\right)$$

$$= \left(\frac{1}{x + \sqrt{1+x^2}}\right) \times \left(1 + \frac{x}{\sqrt{1+x^2}}\right)$$

$$= \left(\frac{1}{x + \sqrt{1+x^2}}\right) \times \left(\frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}}\right)$$

$$= \left(\frac{1}{\sqrt{1+x^2}}\right)$$

Question 7.
Differentiate each of the following w.r.t. x:

$$e^{\sqrt{x}} \log x$$

Answer:
Let y = $e^{\sqrt{x}}\log x$, z = $e^{\sqrt{x}}$ and w = log (x)

Formula : $\frac{d(e^x)}{dx} = e^x$, $\frac{d(\log x)}{dx} = \frac{1}{x}$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\log (x) \times (e^{\sqrt{x}}) \times \frac{1}{2\sqrt{x}}] + [e^{\sqrt{x}} \times \frac{1}{x}]$$

$$= e^{\sqrt{x}} \times [\frac{\log (x)}{2\sqrt{x}} + \frac{1}{x}]$$

$$= e^{\sqrt{x}} \times [\frac{\sqrt{x}\log (x)}{2x} + \frac{2}{2x}]$$

$$= e^{\sqrt{x}} \times [\frac{2 + \sqrt{x}\log (x)}{2x}]$$

Question 8.
Differentiate each of the following w.r.t. x:

$$\log \sin \sqrt{x^2 + 1}$$

Answer:
Let y = $\log \sin \sqrt{1 + x^2}$, z = $\sin \sqrt{1 + x^2}$

Formula : $\frac{d(\sin x)}{dx} = \cos x$, $\frac{d(\log x)}{dx} = \frac{1}{x}$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= [\frac{1}{\sin\sqrt{1+x^2}}] \times [\cos\sqrt{1+x^2}] \times [\frac{1}{2} \times \frac{1}{\sqrt{1+x^2}} \times 2x]$$

$$= [\cot\sqrt{1+x^2}] \times [\frac{1}{1} \times \frac{1}{\sqrt{1+x^2}} \times x]$$

$$= \frac{x}{\sqrt{x^2+1}} \cot\sqrt{x^2+1}$$

Question 9.
Differentiate each of the following w.r.t. x:

$e^{2x} \sin 3x$

Answer:
Let $y = e^{2x} \sin 3x$, $z = e^{2x}$ and $w = \sin 3x$

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\sin x)}{dx} = \cos x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\sin 3x \times (2 \times e^{2x})] + [e^{2x} \times 3\cos 3x]$$

$$= e^{2x} \times [2\sin 3x + 3\cos 3x]$$

Question 10.
Differentiate each of the following w.r.t. x:

$e^{3x} \cos 2x$

Answer:
Let $y = e^{3x} \cos 2x$, $z = e^{3x}$ and $w = \cos 2x$

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cos x)}{dx} = -\sin x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\cos 2x \times (3 \times e^{3x})] + [e^{3x} \times (-2 \sin 2x)]$$

$$= e^{3x} \times [3 \cos 2x - 2 \sin 2x]$$

Question 11.
Differentiate each of the following w.r.t. x:

e^{-5x} cot 4x

Answer:
Let $y = e^{-5x}$ cot 4x , $z = e^{-5x}$ and w = cot 4x

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\cot 4x \times (-5e^{-5x})] + [e^{-5x} \times (-4 \operatorname{cosec}^2 4x)]$$

$$= -e^{-5x} \times [5 \cot 4x + 4 \operatorname{cosec}^2 4x]$$

Question 12.
Differentiate each of the following w.r.t. x:

e^x log (sin 2x)

Answer:
Let $y = e^x$ log (sin 2x), $z = e^x$ and w = log (sin 2x)

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(\log x)}{dx} = \frac{1}{x}$ and $\frac{d(\sin x)}{dx} = \cos x$

According to product rule of differentiation

$$dy/dx = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\log(\sin 2x) \times (e^x)] + [e^x \times \frac{1}{\sin 2x} \times 2\cos 2x]$$

$$= e^x \times [\log(\sin 2x) + \frac{2\cos 2x}{\sin 2x}]$$

$$= e^x \times [\log(\sin 2x) + 2\cot 2x]$$

Question 13.
Differentiate each of the following w.r.t. x:

$$\log(\operatorname{cosec} x - \cot x)$$

Answer:
Let y = $\log(\operatorname{cosec} x - \cot x)$, z =$(\operatorname{cosec} x - \cot x)$

Formula :

$$\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cot x \,, \frac{d(\log x)}{dx} = \frac{1}{x} \text{ and } \frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= [\frac{1}{(\operatorname{cosec} x - \cot x)}] \times [-\operatorname{cosec} x \cot x - (-\operatorname{cosec}^2 x)]$$

$$= [\frac{1}{(\operatorname{cosec} x - \cot x)}] \times [-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x]$$

$$= [\frac{1}{(\operatorname{cosec} x - \cot x)}] \times [\operatorname{cosec} x(\operatorname{cosec} x - \cot x)]$$

= cosec x

Question 14.
Differentiate each of the following w.r.t. x:

$$\log\left(\sec\frac{x}{2}+\tan\frac{x}{2}\right)$$

Answer:
Let y = $\log(\sec\frac{x}{2}+\tan\frac{x}{2})$, z =$(\sec\frac{x}{2}+\tan\frac{x}{2})$

Formula :

$$\frac{d(\sec x)}{dx}=\sec x\tan x\,,\frac{d(\log x)}{dx}=\frac{1}{x}\text{ and }\frac{d(\tan x)}{dx}=\sec^2 x$$

According to chain rule of differentiation

$$dy/_{dx}=\frac{dy}{dz}\times\frac{dz}{dx}$$

$$=[\frac{1}{(\sec\frac{x}{2}+\tan\frac{x}{2})}]\times[(\sec\frac{x}{2}\tan\frac{x}{2}\times\frac{1}{2})+(\sec^2\frac{x}{2}\times\frac{1}{2})]$$

$$=[\frac{1}{(\sec\frac{x}{2}+\tan\frac{x}{2})}]\times[\frac{1}{2}\sec\frac{x}{2}(\sec\frac{x}{2}+\tan\frac{x}{2})]$$

$$=\frac{1}{2}\sec\frac{x}{2}$$

Question 15.
Differentiate each of the following w.r.t. x:

$$\sqrt{\frac{1+e^x}{1-e^x}}$$

Answer:

Let y = $\sqrt{\frac{1+e^x}{1-e^x}}$, u =$1+e^x$, v =$1-e^x$, z=$\frac{1+e^x}{1-e^x}$

Formula : $\frac{d(e^x)}{dx} = e^x$

According to quotient rule of differentiation

If z = $\frac{u}{v}$

$$dz/_{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1-e^x) \times (e^x) - (1+e^x) \times (-e^x)}{(1-e^x)^2}$$

$$= \frac{e^x - e^{2x} + e^x + e^{2x}}{(1-e^x)^2}$$

$$= \frac{2e^x}{(1-e^x)^2}$$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[\frac{1}{2} \times \left(\frac{1+e^x}{1-e^x}\right)^{\frac{1}{2}-1}\right] \times \left[\frac{2e^x}{(1-e^x)^2}\right]$$

$$= \left[\frac{e^x}{1} \times \left(\frac{1+e^x}{1}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(1-e^x)^{2-\frac{1}{2}}}\right]$$

$$= \left[\frac{e^x}{(1+e^x)^{\frac{1}{2}} \times (1-e^x)^{2-\frac{1}{2}}}\right]$$

$$= \left[\frac{e^x}{(1+e^x)^{\frac{1}{2}} \times (1-e^x)^{\frac{1}{2}} \times (1-e^x)^1}\right]$$

$$= \left[\frac{e^x}{((1+e^x)(1-e^x))^{\frac{1}{2}} \times (1-e^x)^1}\right]$$

$$= \frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}$$

Question 16.
Differentiate each of the following w.r.t. x:

$$\frac{e^x + e^{-x}}{e^x - e^{-x}}$$

Answer:
Let $y = \frac{e^x+e^{-x}}{e^x-e^{-x}}$, $u = e^x + e^{-x}$, $v = e^x - e^{-x}$

Formula : $\frac{d(e^x)}{dx} = e^x$

According to quotient rule of differentiation

If $y = \frac{u}{v}$

$$dy/dx = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(e^x - e^{-x}) \times (e^x - e^{-x}) - (e^x + e^{-x}) \times (e^x + e^{-x})}{(e^x - e^{-x})^2}$$

$$= \frac{(e^x - e^{-x})^2 - (e^x + e^{-x})^2}{(e^x - e^{-x})^2}$$

$$= \frac{(e^x - e^{-x} + e^x + e^{-x})(e^x - e^{-x} - e^x - e^{-x})}{(e^x - e^{-x})^2}$$

$(a^2 - b^2 = (a - b)(a + b))$

$$= \frac{(2\,e^x)\,(-2e^{-x})}{(e^x - e^{-x})^2}$$

$$= \frac{-4}{(e^x - e^{-x})^2}$$

Question 17.
Differentiate each of the following w.r.t. x:

$$xe^{\sqrt{\sin x}}$$

Answer:
Let $y = xe^{\sqrt{\sin x}}$, $z = x$ and $w = e^{\sqrt{\sin x}}$

Formula : $\frac{d(e^x)}{dx} = e^x$, $\frac{d(\sin x)}{dx} = \cos x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\,e^{\sqrt{\sin x}} \times (1)] + [x \times e^{\sqrt{\sin x}} \times \frac{1}{2} \times \frac{1}{\sqrt{\sin x}} \times \cos x\,]$$

$$= e^{\sqrt{\sin x}} \times [1 + \frac{x \cos x}{2\sqrt{\sin x}}]$$

Question 18.
Differentiate each of the following w.r.t. x:

$$e^{\sin x} \sin(e^x)$$

Answer:
Let $y = e^{\sin x} \sin e^x$, $z = e^{\sin x}$ and $w = \sin e^x$

Formula : $\frac{d(e^x)}{dx} = e^x$, $\frac{d(\sin x)}{dx} = \cos x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\sin e^x \times (e^{\sin x} \times \cos x)] + [e^{\sin x} \times \cos e^x \times e^x]$$

$$= e^{\sin x}[(\sin e^x \times \cos x) + (\cos e^x \times e^x)]$$

$$= e^{\sin x}(e^x \cos e^x + \cos x \sin e^x)$$

Question 19.
Differentiate each of the following w.r.t. x:

$$e^{\sqrt{1-x^2}} \tan x$$

Answer:
Let y = $e^{\sqrt{1-x^2}} \tan x$, z = $e^{\sqrt{1-x^2}}$ and w = $\tan x$

Formula : $\frac{d(e^x)}{dx} = e^x$, $\frac{d(\tan x)}{dx} = \sec^2 x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\tan x \times \left(e^{\sqrt{1-x^2}} \times \frac{1}{2} \times \frac{1}{\sqrt{1-x^2}} \times (-2x)\right)] + [e^{\sqrt{1-x^2}} \times \sec^2 x]$$

$$= e^{\sqrt{1-x^2}} \times \left[\sec^2 x - \frac{x \tan x}{\sqrt{1-x^2}}\right]$$

Question 20.

Differentiate each of the following w.r.t. x:

$$\frac{e^x}{1+\cos x}$$

Answer:

Let $y = \frac{e^x}{1+\cos x}$, $u = e^x$, $v = 1 + \cos x$

Formula: $\frac{d(e^x)}{dx} = e^x, \frac{d(\cos x)}{dx} = -\sin x$

According to quotient rule of differentiation

If $y = \frac{u}{v}$

$$dy/dx = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1+\cos x) \times (e^x) - (e^x) \times (-\sin x)}{(1+\cos x)^2}$$

$$= \frac{e^x(1+\cos x + \sin x)}{(1+\cos x)^2}$$

Question 21.

Differentiate each of the following w.r.t. x:

$$x^3 e^x \cos x$$

Answer:

Let $y = x^3 e^x \cos x$, $z = x^3$ and $w = e^x \cos x$

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cos x)}{dx} = -\sin x$

$$\frac{dw}{dx} = [\cos x \times (e^x)] + [e^x \times (-\sin x)] = e^x[\cos x - \sin x]$$

According to product rule of differentiation

$$dy/dx = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [e^x \cos x \times (3x^2)] + [x^3 \times (e^x[\cos x - \sin x])]$$

$$= e^x x^2 \times [3\cos x + x\cos x - x\sin x]$$

$$= e^x x^2 (x\cos x - x\sin x + 3\cos x)$$

Question 22.
Differentiate each of the following w.r.t. x:

$e^{x \cos x}$

Answer:
Let $y = e^{x\cos x}$, $z = x\cos x$

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\cos x)}{dx} = -\sin x$

$\frac{dz}{dx} = [\cos x \times (1)] + [x \times (-\sin x)] = [\cos x - x\sin x]$ (Using product rule)

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= [e^{x\cos x}] \times [\cos x - x\sin x]$$

$$= e^{x\cos x}(\cos x - x\sin x)$$