Exercise 5d

Question 1.

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 7 & -4 \end{bmatrix}$, verify that (A′)′ = A.

Answer:

Transpose of a matrix is obtained by interchanging the rows and the columns of matrix A. It is denoted by A′.

e.g. $A_{12} = A_{21}$

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 7 & -4 \end{bmatrix}$$

Hence transpose of matrix A is,

$$A' = \begin{bmatrix} 2 & 0 \\ -3 & 7 \\ 5 & -4 \end{bmatrix}$$

$$(A')' = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 7 & -4 \end{bmatrix}$$

(A')' = A

Hence, Proved.

Question 2.

If $A = \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 4 & -6 \end{bmatrix}$, verify that (2A)′ = 2A′.

Answer:

Given $A = \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 4 & -6 \end{bmatrix}$

To Prove: (2A)′ = 2A′

Proof: Let us consider, B = 2A

Now, $B = 2\begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 4 & -6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & 10 \\ -4 & 0 \\ 8 & -12 \end{bmatrix}$

LHS $\Rightarrow$ $B' = \begin{bmatrix} 6 & -4 & 8 \\ 10 & 0 & -12 \end{bmatrix}$

Again to find RHS, we will find the transpose of matrix A

$A' = \begin{bmatrix} 3 & -2 & 4 \\ 5 & 0 & -6 \end{bmatrix}$

RHS = 2A′

$\Rightarrow 2\begin{bmatrix} 3 & -2 & 4 \\ 5 & 0 & -6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 & -4 & 8 \\ 10 & 0 & -12 \end{bmatrix}$

LHS = RHS

Hence proved.

Question 3.

If $A = \begin{bmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{bmatrix}$, verify that (A + B)′ = (A′ + B′).

Answer:

Given $A = \begin{bmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{bmatrix}$

To Prove: (A + B)′ = A′ + B′

Proof: Let us consider C = A + B

$$C = \begin{bmatrix} 3 & 2 & -1 \\ -5 & 0 & -6 \end{bmatrix} + \begin{bmatrix} -4 & -5 & -2 \\ 3 & 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -3 & -3 \\ -2 & 1 & 2 \end{bmatrix}$$

Now LHS = C′

$$\Rightarrow \begin{bmatrix} -1 & -2 \\ -3 & 1 \\ -3 & 2 \end{bmatrix}$$

To find RHS, we will find transpose of matrix A and B

$$A' = \begin{bmatrix} 3 & -5 \\ 2 & 0 \\ -1 & -6 \end{bmatrix} \text{ And } B' = \begin{bmatrix} -4 & 3 \\ -5 & 1 \\ -2 & 8 \end{bmatrix}$$

RHS = A′ + B′

$$\Rightarrow \begin{bmatrix} 3 & -5 \\ 2 & 0 \\ -1 & -6 \end{bmatrix} + \begin{bmatrix} -4 & 3 \\ -5 & 1 \\ -2 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -2 \\ -3 & 1 \\ -3 & 2 \end{bmatrix}$$

LHS = RHS

Hence proved.

Question 4.

If $P = \begin{bmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{bmatrix}$ and $P = \begin{bmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{bmatrix}$, verify that (P + Q)′ = (P′ + Q′).

Answer:

Given $P = \begin{bmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{bmatrix}$ and $Q = \begin{bmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{bmatrix}$

To Prove: (P + Q)′ = P′ + Q′

Proof: Let us consider R = P + Q,

$$R = \begin{bmatrix} 3 & 4 \\ 2 & -1 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} 7 & -5 \\ -4 & 0 \\ 2 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & -1 \\ -2 & -1 \\ 2 & 11 \end{bmatrix}$$

LHS = R $\Rightarrow$(P + Q)′

$$LHS = \begin{bmatrix} 10 & -2 & 2 \\ -1 & -1 & 11 \end{bmatrix}$$

To find RHS, we will first find the transpose of matrix P and Q

$P' = \begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 5 \end{bmatrix}$ And $Q' = \begin{bmatrix} 7 & -4 & 2 \\ -5 & 0 & 6 \end{bmatrix}$

RHS = P′ + Q′

$$\Rightarrow \begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 5 \end{bmatrix} + \begin{bmatrix} 7 & -4 & 2 \\ -5 & 0 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & -2 & 2 \\ -1 & -1 & 11 \end{bmatrix}$$

LHS = RHS

Hence proved.

Question 5.

If $A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$, show that (A + A′) is symmetric.

Answer:

Given $A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$

To Prove: A + A′ is symmetric.(Note:A matrix P is symmetric if P′ = P)

Proof: We will find A′,

$$A' = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$$

Now let us take P = A + A′

$$P = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$$

Now $P' = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$

$$\Rightarrow P' = P$$

Hence A + A′ is a symmetric matrix.

Question 6.

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, show that (A + A′) is skew-symmetric.

Answer:

Given $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

To prove: A-A′ is a skew-symmetric matrix.(Note: A matrix P is skew-symmetric if P′ = -P)

Proof: First we will find the transpose of matrix A

$$A' = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

Let us take P = A-A′

$$P = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$$P = ' \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$$

$\Rightarrow$P′ = P

Hence A-A′ is a skew symmetric matrix.

Question 7.

Show that the matrix $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ is skew-symmetric.

HINT: Show that A′ = -A.

Answer:

Given $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

To Prove: A is a skew symmetric matrix.

Proof: As for a matrix to be skew symmetric A′ = -A

We will find A′.

$$A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$= - \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

$\Rightarrow$A′ = -A

So A is A skew symmetric matrix.

Question 8.

Express the matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

Answer:

Given $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$, As for a symmetric matrix A′ = A hence

A + A′ = 2A

A $= \frac{1}{2}(A + A') \Rightarrow P$ (Symmetric Matrix)

Similarly for a skew symmetric matrix since A′ = -A hence

A-A′ = 2A

A $= \frac{1}{2}(A - A') \Rightarrow Q$ (Skew Symmetric Matrix)

So a matrix can be represented as a sum of a symmetric matrix P and skew symmetric matrix Q.

First, we will find the transpose of matrix A,

$$A' = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

Now using the above formulas,

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 4 & 2 \\ 2 & 8 \end{bmatrix}$$

$$P = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$$

$$Q = \frac{1}{2}\left(\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

Hence A = P + Q

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ [Matrix A as the sum of P and Q]

$$\Rightarrow \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$$

Question 9.

Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

Answer:

Given $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$,to express as sthe um of symmetric matrix P and skew symmetric matrix Q.

A = P + Q

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$,we will find transpose of matrix A

$$A' = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

Now using the above formulas

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 6 & -3 \\ -3 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$

Hence A = P + Q

$$\Rightarrow \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$ [Matrix A as the sum of P and Q]

$$\Rightarrow \begin{bmatrix} 3 & \frac{-8}{2} \\ \frac{2}{2} & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

Question 10.

Express the matrix $A = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Answer:

Given $A = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$, to express as sum of symmetric matrix P and skew symmetric matrix Q.

A = P + Q

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$,

First, we find A′

$$A' = \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix}$$

Now using the above mentioned formulas

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} -2 & 7 & 8 \\ 7 & 6 & 4 \\ 8 & 4 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & \frac{7}{2} & 4 \\ \frac{7}{2} & 3 & 2 \\ 4 & 2 & 9 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 7 \\ 5 & 3 & 0 \\ 1 & 4 & 9 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & 3 & -6 \\ -3 & 0 & 4 \\ 6 & -4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & \frac{3}{2} & -3 \\ \frac{-3}{2} & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix}$$

Now A = P + Q

$$\Rightarrow \begin{bmatrix} -1 & \frac{7}{2} & 4 \\ \frac{7}{2} & 3 & 2 \\ 4 & 2 & 9 \end{bmatrix} + \begin{bmatrix} 0 & \frac{3}{2} & -3 \\ \frac{-3}{2} & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix}$$ [Matrix A as sum of P and Q]

$$\Rightarrow \begin{bmatrix} -1 & \frac{10}{2} & 1 \\ \frac{4}{2} & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{bmatrix}$$

Question 11.

Express the matrix A as the sum of a symmetric and a skew-symmetric matrix, where

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}.$$

Answer:

Given $A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}$, to express as sum of symmetric matrix P and skew symmetric matrix Q

A = P + Q

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$,

First we will find A',

$$A' = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix}$$

Now using above mentioned formulas,

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 6 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 1 & 2 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 3 & 2 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & -3 & -1 \\ 3 & 0 & 4 \\ 1 & -4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & \frac{-3}{2} & \frac{-1}{2} \\ \frac{3}{2} & 0 & 2 \\ \frac{1}{2} & -2 & 0 \end{bmatrix}$$

Now A = P + Q

$$\Rightarrow \begin{bmatrix} 3 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-3}{2} & \frac{-1}{2} \\ \frac{3}{2} & 0 & 2 \\ \frac{1}{2} & -2 & 0 \end{bmatrix}$$ [Matrix A as sum of P and Q]

$$\Rightarrow \begin{bmatrix} 3 & \frac{-2}{2} & 0 \\ \frac{4}{2} & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}$$

Question 12.

Express the matrix $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$ as sum of two matrices such that one is symmetric and the other is skew-symmetric.

Answer:

Given $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$, to express as sum of symmetric matrix P and skew symmetric matrix Q.

A = P + Q

Where $P = \frac{1}{2}(A + A')$ and $Q = \frac{1}{2}(A - A')$,

First we will find A′

$$A' = \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}$$

Now using above mentioned formulas

$$P = \frac{1}{2}(A + A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$\Rightarrow \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix}\right)$$

$$\Rightarrow \frac{1}{2}\begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & \frac{-3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 0 \end{bmatrix}$$

Now A = P + Q

$$\Rightarrow \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 1 & \frac{9}{2} \\ \frac{5}{2} & \frac{9}{2} & 7 \end{bmatrix} + \begin{bmatrix} 0 & -1 & \frac{5}{2} \\ 1 & 0 & \frac{-3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$$

Question 13.
For each of the following pairs of matrices A and B, verify that (AB)′ = (B′ A′) :

$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$

Answer:
Let us take C = AB

$$C = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+6 & 4+15 \\ 2+8 & 8+20 \end{bmatrix}$$

$$C = \begin{bmatrix} 7 & 19 \\ 10 & 28 \end{bmatrix}$$

$$\text{LHS} \Rightarrow C' = \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$$

To find RHS we will find transpose of matrix A and B,

$$A' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ And } B' = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$$

RHS = B′A′

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+6 & 2+8 \\ 3+16 & 8+20 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$$

LHS = RHS

Hence proved.

Question 14.
For each of the following pairs of matrices A and B, verify that (AB)′ = (B′ A′) :

$$A = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$$

Answer:
Let us take C = AB

$$C = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3+(-2) & -9+1 \\ 2+(-4) & -6+2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -8 \\ -2 & -4 \end{bmatrix}$$

$$\text{LHS} \Rightarrow C' = \begin{bmatrix} 1 & -2 \\ -8 & -4 \end{bmatrix}$$

To find RHS we will find transpose of matrix A and B,

$B' = \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}$ And $A' = \begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix}$

RHS = B′A′

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -1 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3+(-2) & 2+(-4) \\ -9+1 & -6+2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ -8 & -4 \end{bmatrix}$$

LHS = RHS

Hence proved.

Question 15.
For each of the following pairs of matrices A and B, verify that (AB)′ = (B′ A′) :

$A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and B = [-2 -1 -4]

Answer:

Let us take C = AB

$$C = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 4 \\ -4 & -2 & -8 \\ -6 & -3 & -12 \end{bmatrix}$$

LHS = C′

$$\Rightarrow \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$$

To find RHS we will find transpose of matrix A and B,

$A' = \begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$ And $B' = \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix}$

RHS = B′A′

$$\Rightarrow \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$$

LHS = RHS

Hence proved.

Question 16.
For each of the following pairs of matrices A and B, verify that (AB)′ = (B′ A′) :

$$A = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{bmatrix}$$

Answer:
Let us take C = AB

$$C = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3+4+3 & 4+2+0 \\ 12+(-10)+(-6) & -16+(-5) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 6 \\ -4 & -21 \end{bmatrix}$$

LHS = C′

$$\Rightarrow \begin{bmatrix} 4 & -4 \\ 6 & -21 \end{bmatrix}$$

To find RHS we will find transpose of matrix A and B,

$$A' = \begin{bmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{bmatrix} \text{ And } B' = \begin{bmatrix} 3 & 2 & -1 \\ -4 & 1 & 0 \end{bmatrix}$$

RHS = B′A′

$$\Rightarrow \begin{bmatrix} 3 & 2 & -1 \\ -4 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -3+4+3 & 12+(-10)+(-6) \\ 4+2 & -16+(-5) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & -4 \\ 6 & -21 \end{bmatrix}$$

LHS = RHS

Hence proved.

Question 17.

If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, show that A′A = I.

Answer:

Given $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, We will find A′

$$A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

LHS = A′A

$$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha.\sin\alpha + (-\sin\alpha.\cos\alpha) \\ \sin\alpha.\cos\alpha + (-\cos\alpha.\sin\alpha) & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ [Using $\cos^2\alpha + \sin^2\alpha = 1$ and commutative law a.b = b.a i.e. $\sin\alpha.\cos\alpha = \cos\alpha.\sin\alpha$)]

RHS = I $\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

LHS = RHS

Hence proved.

Question 18.
If matrix A = [1 2 3], write AA′.

Answer:
Given $A =$ [1 2 3]

We will find A′ to calculate AA′,

$$A' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Now

$$AA' = [123]\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow$[1 + 4 + 9]

$\Rightarrow$[14]