Exercise 5c

Question 1.

Compute AB and BA, which ever exists when

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}$$

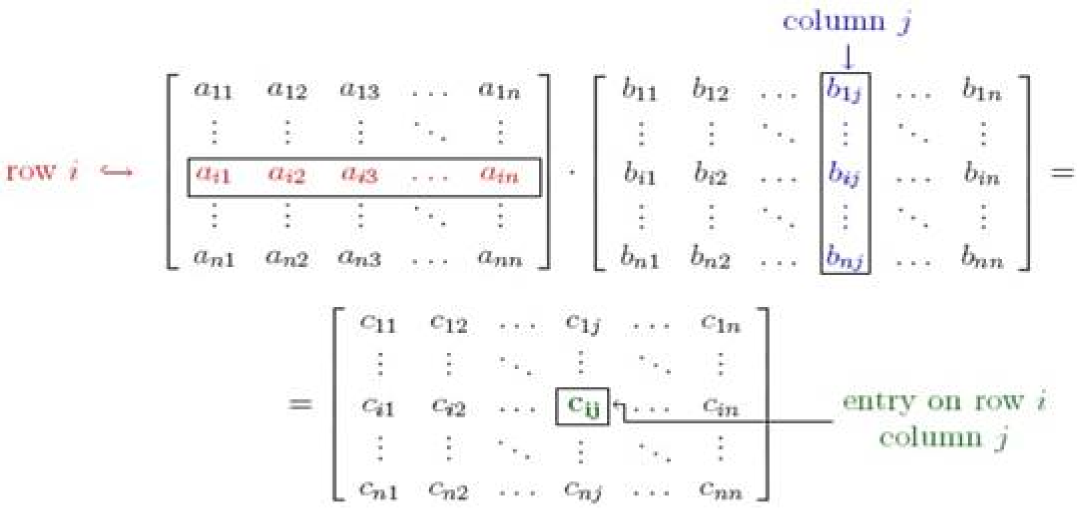
Answer:

Given : $A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}$

Matrix A is of order 3 $\times$ 2, and Matrix B is of order 2 $\times$ 2

To find : matrix AB and BA

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix AB, a = 3,b = c = 2,d = 2 ,thus matrix AB is of order 3 $\times$ 2

Matrix AB = $\begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix} \times \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2(-2)+(-1)(0) & 2(3)+\ (-1)(4) \\ 3(-2)+0(0) & 3(3)+0(4) \\ -1(-2)+4(0) & -1(3)+4(4) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} -4+0 & 6-4 \\ -6+0 & 9+0 \\ 2+0 & -3+16 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ -6 & 9 \\ 2 & 13 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} -4 & 2 \\ -6 & 9 \\ 2 & 13 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} -4 & 2 \\ -6 & 9 \\ 2 & 13 \end{bmatrix}$

For matrix BA, a = 3,b = c = 2,d = 2 ,thus matrix BA exists, if and only if d=a

But 3 $\neq$ 2

Thus matrix BA does not exist

Question 2.
Compute AB and BA, which ever exists when

$A = \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix}$

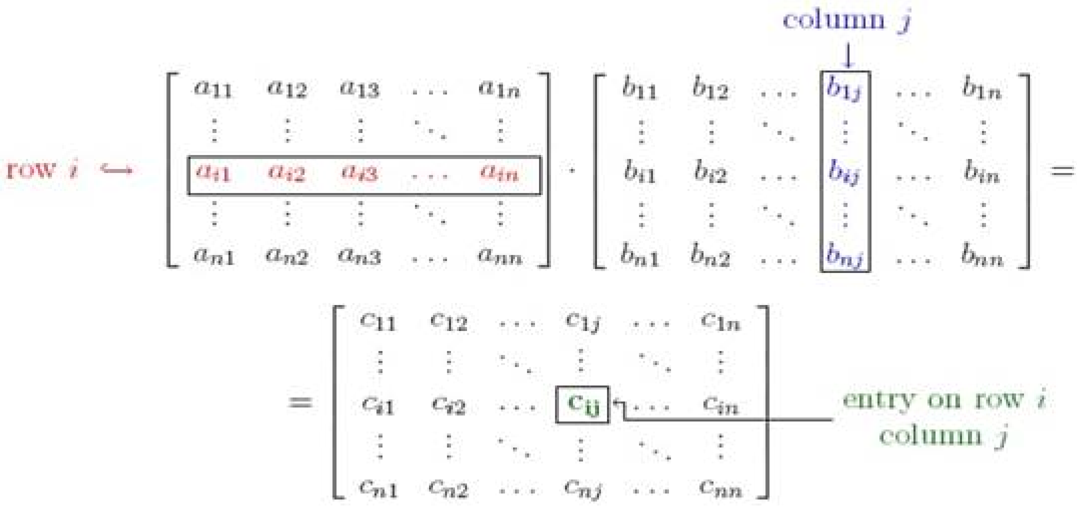
Answer:

Given : $A = \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix}$

Matrix A is of order 3 × 2, and Matrice B is of order 3 × 3

To find : matrix AB and BA

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrice of order c × d ,then matrice AB exists and is of order a × d ,if and only if b = c

If A is a matrix of order a × b and B is a matrice of order c × d ,then matrice BA exists and is of order c × b ,if and only if d = a

For matrix AB, a = 3,b = 2,c = 3,d = 3 ,thus matrix AB does not exist, as d ≠ a

But 2 ≠ 3

Thus matrix AB does not exist

For matrix BA, a = 3,b = 2,c = 3,d = 3 ,thus matrix BA is of order 3 × 2

as d = a = 3

Matrix BA =
$$\begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix} =$$
$$\begin{bmatrix} 3(-1)-2(-2)+1(-3) & 3(1)-2(2)+1(3) \\ 0(-1)+1(-2)+2(-3) & 0(1)+1(2)+2(3) \\ -3(-1)+4(-2)-5(-3) & -3(1)+4(2)-5(3) \end{bmatrix}$$

Matrix BA = $\begin{bmatrix} -3+4-3 & 3-4+3 \\ 0-2-6 & 0+2+6 \\ 3-8+15 & -3+8-15 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -8 & 8 \\ 10 & -10 \end{bmatrix}$

Matrix $\mathrm{BA} = \begin{bmatrix} -2 & 2 \\ -8 & 8 \\ 10 & -10 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} -2 & 2 \\ -8 & 8 \\ 10 & -10 \end{bmatrix}$

Question 3.
Compute AB and BA, which ever exists when

$\mathrm{A} = \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix}$ and $\mathrm{B} = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix}$

Answer:

Given : $\mathrm{A} = \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix}$ and $\mathrm{B} = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix}$

Matrix A is of order 2×3 and Matrix B is of order 3×2

To find : matrices AB and BA

Formula used :

$$
\begin{array}{cc}
 & \\
\text{row } i \hookrightarrow &
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\boxed{a_{i1}} & a_{i2} & a_{i3} & \dots & a_{in} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn}
\end{bmatrix}
\end{array}
\cdot
\begin{array}{c}
\text{column } j \\ \downarrow \\
\begin{bmatrix}
b_{11} & b_{12} & \dots & b_{1j} & \dots & b_{1n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
b_{i1} & b_{i2} & \dots & b_{ij} & \dots & b_{in} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
b_{n1} & b_{n2} & \dots & b_{nj} & \dots & b_{nn}
\end{bmatrix}
\end{array}
=
$$

$$
=
\begin{bmatrix}
c_{11} & c_{12} & \dots & c_{1j} & \dots & c_{1n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
c_{i1} & c_{i2} & \dots & \boxed{c_{ij}} & \dots & c_{in} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
c_{n1} & c_{n2} & \dots & c_{nj} & \dots & c_{nn}
\end{bmatrix}
\quad \begin{array}{c}\text{entry on row } i \\ \text{column } j\end{array}
$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots\dots\dots\dots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$,then matrix AB exists and is of order $a \times d$,if and only if $b = c$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$,then matrix BA exists and is of order $c \times b$,if and only if $d = a$

For matrix AB, $a = 2, b = 3, c = 3, d = 2$,matrix AB exists and is of order 2×2,as

$b = c = 3$

Matrix AB = $\begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0(1)+1(-1)-5(0) & 0(3)+1(0)-5(5) \\ 2(1)+4(-1)+0(0) & 2(3)+4(0)+0(5) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 0-1-0 & 0+0-25 \\ 2-4+0 & 6+0+0 \end{bmatrix} = \begin{bmatrix} -1 & -25 \\ -2 & 6 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} -1 & -25 \\ -2 & 6 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} -1 & -25 \\ -2 & 6 \end{bmatrix}$

For matrix BA, $a = 2, b = 3, c = 3, d = 2$,matrix BA exists and is of order 3×3,as

$d = a = 2$

Matrix BA = $\begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & -5 \\ 2 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1(0)+3(2) & 1(1)+3(4) & 1(-5)+3(0) \\ -1(0)+0(2) & -1(1)+0(4) & -1(-5)+0(0) \\ 0(0)+5(2) & 0(1)+5(4) & 0(-5)+5(0) \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 0+6 & 1+12 & -5+0 \\ 0+0 & -1+0 & 5+0 \\ 0+10 & 0+20 & 0+0 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 6 & 13 & -5 \\ 0 & -1 & 5 \\ 10 & 20 & 0 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 6 & 13 & -5 \\ 0 & -1 & 5 \\ 10 & 20 & 0 \end{bmatrix}$

Question 4.
Compute AB and BA, which ever exists when

A = [1 2 3 4] and $B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

Answer:

Given : A = [1 2 3 4] and $B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

Matrix A is of order 1 $\times$ 4 and Matrix B is of order 4 $\times$ 1

To find : matrices AB and BA

Formula used :

$$\text{row } i \hookrightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nj} & \cdots & b_{nn} \end{bmatrix} \end{array} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \cdots & \boxed{c_{ij}} & \cdots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} \begin{array}{l} \text{entry on row } i \\ \text{column } j \end{array}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix AB, a = 1,b = 4,c = 4,d = 1 ,matrix AB exists and is of order 1 $\times$ 1,as

b = c = 4

Matrix AB = $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = [1(1) + 2(2) + 3(3) + 4(4)]$

Matrix AB = $[1 + 4 + 9 + 16]$ = $[30]$

Matrix AB = $[30]$

Matrix AB = $[30]$

For matrix BA, a = 1,b = 4,c = 4,d = 1 ,matrix BA exists and is of order 4 $\times$ 4,as

d = a = 1

Matrix BA = $\begin{bmatrix}1\\2\\3\\4\end{bmatrix} \times \begin{bmatrix}1 & 2 & 3 & 4\end{bmatrix} = \begin{bmatrix}1(1) & 1(2) & 1(3) & 1(4)\\2(1) & 2(2) & 2(3) & 2(4)\\3(1) & 3(2) & 3(3) & 3(4)\\4(1) & 4(2) & 4(3) & 4(4)\end{bmatrix}$

Matrix BA = $\begin{bmatrix}1 & 2 & 3 & 4\\2 & 4 & 6 & 8\\3 & 6 & 9 & 12\\4 & 8 & 12 & 16\end{bmatrix}$

Matrix BA = $\begin{bmatrix}1 & 2 & 3 & 4\\2 & 4 & 6 & 8\\3 & 6 & 9 & 12\\4 & 8 & 12 & 16\end{bmatrix}$

Question 5.
Compute AB and BA, which ever exists when

$A = \begin{bmatrix}2 & 1\\3 & 2\\-1 & 1\end{bmatrix}$ and $B = \begin{bmatrix}1 & 0 & 1\\-1 & 2 & 1\end{bmatrix}$

Answer:

Given : $A = \begin{bmatrix}2 & 1\\3 & 2\\-1 & 1\end{bmatrix}$ and $B = \begin{bmatrix}1 & 0 & 1\\-1 & 2 & 1\end{bmatrix}$

Matrix A is of order 3 $\times$ 2 and Matrix B is of order 2 $\times$ 3

To find : matrices AB and BA

Formula used :

$$\text{row } i \hookrightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \cdot \begin{array}{cccccc} & & & \text{column } j & & \\ & & & \downarrow & & \\ \end{array} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nj} & \cdots & b_{nn} \end{bmatrix} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \cdots & \boxed{c_{ij}} & \cdots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} \quad \begin{array}{c} \text{entry on row } i \\ \text{column } j \end{array}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix AB, a = 3,b = 2,c = 2,d = 3 ,matrix AB exists and is of order 3 $\times$ 3,as

b = c = 2

Matrix AB = $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & 0+2 & -1+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$

For matrix BA, a = 3,b = 2,c = 2,d = 3 ,matrix BA exists and is of order 2 $\times$ 2,as d = a = 3

Matrix BA = $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1(2)+0(3)+1(-1) & 1(1)+0(2)+1(1) \\ -1(2)+2(3)+1(-1) & -1(1)+2(2)+1(1) \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 2+0-1 & 1+0+1 \\ -2+6-1 & -1+4+1 \end{bmatrix}$ = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Question 6.
Show that AB ≠ BA in each of the following cases :

$A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

Answer:
Given : $A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

Matrix A is of order 2 $\times$ 2 and Matrix B is of order 2 $\times$ 2

To show : matrix AB $\neq$ BA

Formula used :

$$\text{row } i \hookrightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nj} & \cdots & b_{nn} \end{bmatrix} \end{array} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \cdots & \boxed{c_{ij}} & \cdots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} \begin{array}{c} \text{entry on row } i \\ \text{column } j \end{array}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix AB, a = 2,b = c = 2,d = 2 ,thus matrix AB is of order 2 $\times$ 2

Matrix AB = $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5(2)-1(3) & 5(1)-1(4) \\ 6(2)+7(3) & 6(1)+7(4) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$

For matrix BA, a = 2,b = c = 2,d = 2 ,thus matrix BA is of order 2 $\times$ 2

Matrix BA= $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 2(5)+1(6) & 2(-1)+1(7) \\ 3(5)+4(6) & 3(-1)+4(7) \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$ and Matrix AB = $\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$

Matrix AB $\neq$ BA

Question 7.
Show that AB ≠ BA in each of the following cases :

$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

Answer:

Given : $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

Matrix A is of order 3 $\times$ 3, and Matrix B is of order 3 $\times$ 3

To show : matrix AB $\neq$ BA

The formula used :
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$
$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix AB, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3 $\times$ 3

Matrix AB =

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{bmatrix}$$

Matrix AB = $\begin{bmatrix} -1+0+6 & 1-2+9 & 0+2+12 \\ 0+0+0 & 0-1+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

For matrix BA, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3 $\times$ 3

Matrix BA=

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} -1(1)+1(0)+0(1) & -1(2)+1(1)+0(1) & -1(3)+1(0)+0(0) \\ 0(1)-1(0)+1(1) & 0(2)-1(1)+1(1) & 0(3)-1(0)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{bmatrix}$$

Matrix BA = $\begin{bmatrix} -1+0+0 & -2+1+0 & -3+0+0 \\ 0-1+1 & 0-1+1 & 0+0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -3 \\ 0 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} -1 & -1 & -3 \\ 0 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} -1 & -1 & -3 \\ 0 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$ and Matrix AB = $\begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

Matrix AB $\neq$ BA

Question 8.

Show that AB = BA in each of the following cases:

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \text{ and } B = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

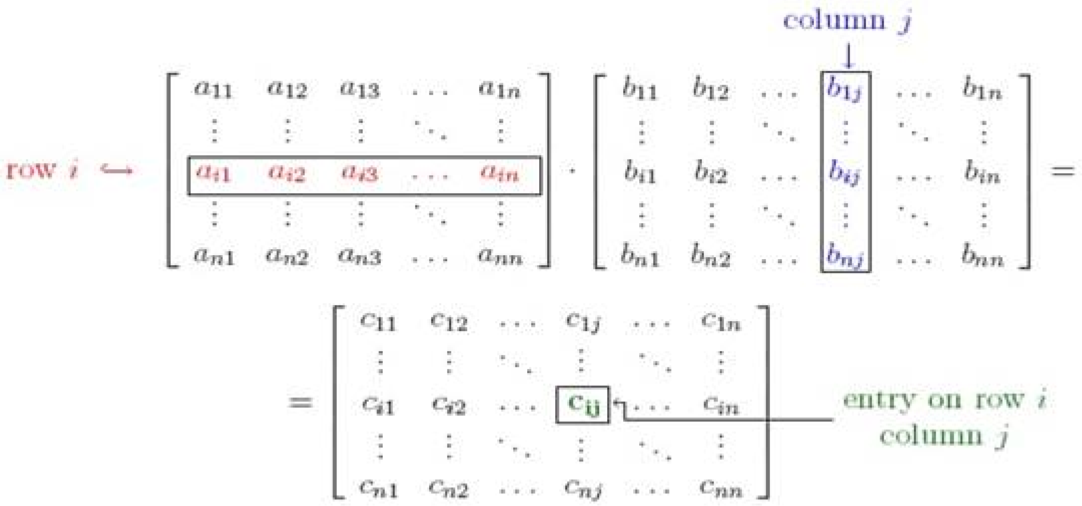
Answer:

Given : $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$

Matrix A is of order 2 $\times$ 2 and Matrix B is of order 2 $\times$ 2

To show : matrix AB = BA

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix AB, a = 2,b = c = 2,d = 2 ,thus matrix AB is of order 2 $\times$ 2

Matrix AB =

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} \cos\emptyset & -\sin\emptyset \\ \sin\emptyset & \cos\emptyset \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\emptyset - \sin\theta\sin\emptyset & -\cos\theta\sin\emptyset - \sin\theta\sin\emptyset \\ \sin\theta\cos\emptyset + \cos\theta\sin\emptyset & -\sin\theta\sin\emptyset + \cos\theta\cos\emptyset \end{bmatrix}$$

Matrix AB = $\begin{bmatrix} \cos\theta\cos\emptyset - \sin\theta\sin\emptyset & -\cos\theta\sin\emptyset - \sin\theta\sin\emptyset \\ \sin\theta\cos\emptyset + \cos\theta\sin\emptyset & -\sin\theta\sin\emptyset + \cos\theta\cos\emptyset \end{bmatrix}$

Matrix AB = $\begin{bmatrix} \cos\theta\cos\emptyset - \sin\theta\sin\emptyset & -\cos\theta\sin\emptyset - \sin\theta\sin\emptyset \\ \sin\theta\cos\emptyset + \cos\theta\sin\emptyset & -\sin\theta\sin\emptyset + \cos\theta\cos\emptyset \end{bmatrix}$

For matrix BA, a = 2,b = c = 2,d = 2 ,thus matrix BA is of order 2 $\times$ 2

Matrix BA=

$$\begin{bmatrix} \cos\emptyset & -\sin\emptyset \\ \sin\emptyset & \cos\emptyset \end{bmatrix} \times \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\emptyset\cos\theta - \sin\emptyset\sin\theta & -\cos\emptyset\sin\theta - \sin\emptyset\cos\theta \\ \sin\emptyset\cos\theta + \cos\emptyset\sin\theta & -\sin\emptyset\sin\theta + \cos\emptyset\cos\theta \end{bmatrix}$$

Matrix BA = $\begin{bmatrix} \cos\theta\cos\emptyset - \sin\theta\sin\emptyset & -\cos\theta\sin\emptyset - \sin\theta\sin\emptyset \\ \sin\theta\cos\emptyset + \cos\theta\sin\emptyset & -\sin\theta\sin\emptyset + \cos\theta\cos\emptyset \end{bmatrix}$

Matrix BA = Matrix AB = $\begin{bmatrix} \cos\theta\cos\emptyset - \sin\theta\sin\emptyset & -\cos\theta\sin\emptyset - \sin\theta\sin\emptyset \\ \sin\theta\cos\emptyset + \cos\theta\sin\emptyset & -\sin\theta\sin\emptyset + \cos\theta\cos\emptyset \end{bmatrix}$

Thus Matrix AB = BA

Question 9.
Show that AB = BA in each of the following cases:

$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$

Answer:

Given : $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$

Matrix A is of order 3 $\times$ 3 and Matrix B is of order 3 $\times$ 3

To show : matrix AB $\neq$ BA

Formula used :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix AB, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3 $\times$ 3

Matrix AB = $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \times \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$ =

$$\begin{bmatrix} 1(10)+2(-11)+1(-9) & 1(-4)+2(5)+1(-5) & 1(-1)+2(0)+1(1) \\ 3(10)+4(-11)+2(-9) & 3(-4)+4(5)+2(-5) & 3(-1)+4(0)+2(1) \\ 1(10)+3(-11)+2(-9) & 1(-4)+3(5)+2(-5) & 1(-1)+3(0)+2(1) \end{bmatrix}$$

Matrix AB = $\begin{bmatrix} 10-22-9 & -4+10-5 & -1+0+1 \\ 30-44-18 & -12+20-10 & -3+0+2 \\ 10-33-18 & -4+15-10 & -1+0+2 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ -32 & -2 & -1 \\ -41 & 1 & 1 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} -3 & 1 & 0 \\ -32 & -2 & -1 \\ -41 & 1 & 1 \end{bmatrix}$

For matrix BA, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3 $\times$ 3

Matrix BA=

$$\begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} 10(1) - 4(3) - 1(1) & 10(2) - 4(4) - 1(3) & 10(1) - 4(2) - 1(2) \\ -11(1) + 5(3) + 0(1) & -11(2) + 5(4) + 0(3) & -11(1) + 5(2) + 0(2) \\ 9(1) - 5(3) + 1(1) & 9(2) - 5(4) + 1(3) & 9(1) - 5(2) + 1(2) \end{bmatrix}$$

Matrix BA = $\begin{bmatrix} 10 - 12 - 1 & 20 - 16 - 3 & 10 - 8 - 2 \\ -11 + 15 + 0 & -22 + 20 + 0 & -11 + 10 + 0 \\ 9 - 15 + 1 & 18 - 20 + 3 & 9 - 10 + 2 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ -4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix}$

Matrix AB $\neq$ BA

Question 10.
Show that AB = BA in each of the following cases:

$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$

Answer:

Given : $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$

Matrix A is of order 3 $\times$ 3 and Matrix B is of order 3 $\times$ 3

To show : matrix AB = BA

Formula used :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix AB, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3 $\times$ 3

Matrix AB = $\begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} =$

$$\begin{bmatrix} 1(-2)+3(-1)-1(-6) & 1(3)+3(2)-1(9) & 1(-1)+3(-1)-1(-4) \\ 2(-2)+2(-1)-1(-6) & 2(3)+2(2)-1(9) & 2(-1)+2(-1)-1(-4) \\ 3(-2)+0(-1)-1(-6) & 3(3)+0(2)-1(9) & 3(-1)+0(-1)-1(-4) \end{bmatrix}$$

Matrix AB = $\begin{bmatrix} -2-3+6 & 3+6-9 & -1-3+4 \\ -4-2+6 & 6+4-9 & -2-2+4 \\ -6+0+6 & 9+0-9 & -3+0+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

For matrix BA, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3 $\times$ 3

Matrix BA = $\begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} -2(1)+3(2)-1(3) & -2(3)+3(2)-1(0) & -2(-1)+3(-1)-1(-1) \\ -1(1)+2(2)-1(3) & -1(3)+2(2)-1(0) & -1(-1)+2(-1)-1(-1) \\ -6(1)+9(2)-4(3) & -6(3)+9(2)-4(0) & -6(-1)+9(-1)-4(-1) \end{bmatrix}$

Matrix BA = $\begin{bmatrix} -2+6-3 & -6+6+0 & 2-3+1 \\ -1+2-3 & -3+4+0 & 1-2+1 \\ -6+18-12 & -18+18+0 & 6-9+4 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Matrix AB = Matrix BA = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Question 11.

If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, shown that AB = A and BA = B.

Answer:

Given : $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$,

Matrix A is of order 3 $\times$ 3 and Matrix B is of order 3 $\times$ 3

To show : matrix AB = A, BA = B

Formula used :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix AB, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3 $\times$ 3

Matrix AB = $\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \times \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} =$

$$\begin{bmatrix} 2(2) - 3(-1) - 5(1) & 2(-2) - 3(3) - 5(-2) & 2(-4) - 3(4) - 5(-3) \\ -1(2) + 4(-1) + 5(1) & -1(-2) + 4(3) + 5(-2) & -1(-4) + 4(4) + 5(-3) \\ 1(2) - 3(-1) - 4(1) & 1(-2) - 3(3) - 4(-2) & 1(-4) - 3(4) - 4(-3) \end{bmatrix}$$

Matrix AB = $\begin{bmatrix} 4+3-5 & -4-9+10 & -8-12+15 \\ -2-4+5 & +2+12-10 & 4+16-15 \\ 2+3-4 & -2-9+8 & -4-12+12 \end{bmatrix} = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ = Matrix A

Matrix AB = Matrix A

For matrix BA, a = 3,b = c = 3,d = 3 ,thus matrix AB is of order 3 $\times$ 3

Matrix BA = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 2(2)-2(-1)-4(1) & 2(-3)-2(4)-4(-3) & 2(-5)-2(5)-4(-4) \\ -1(2)+3(-1)+4(1) & -1(-3)+3(4)+4(-3) & -1(-5)+3(5)+4(-4) \\ 1(2)-2(-1)-3(1) & 1(-3)-2(4)-3(-3) & 1(-5)-2(5)-3(-4) \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 4+2-4 & -6-8+12 & -10-10+16 \\ -2-3+4 & +3+12-12 & +5+15-16 \\ 2+2-3 & -3-8+9 & -5-10+12 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

Matrix BA = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ = Matrix B

Matrix BA = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ = Matrix B

MATRIX AB = A and MATRIX BA = B

Question 12.

If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, show that AB is a zero matrix.

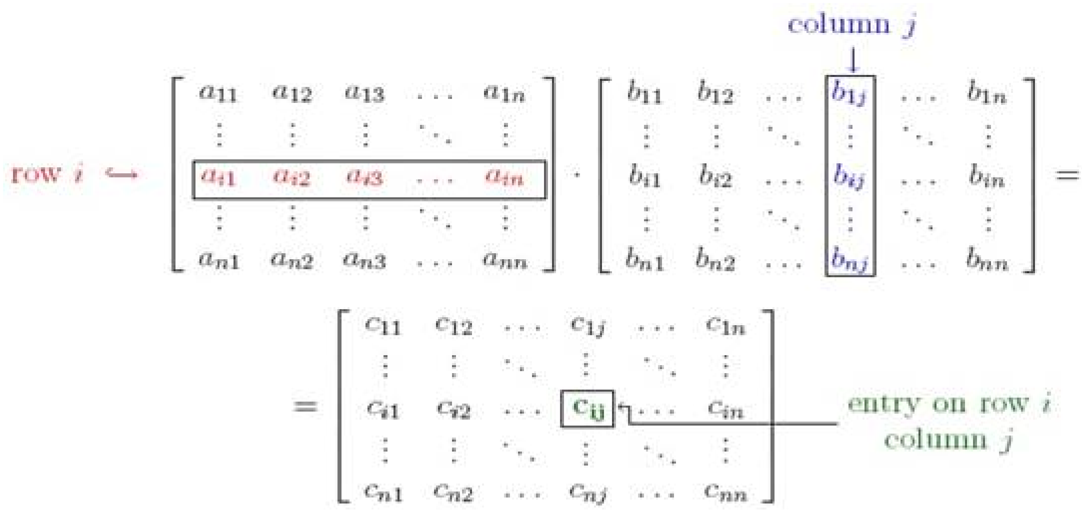
Answer:

Given : $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

Matrix A is of order 3 $\times$ 3 , matrix B is of order 3 $\times$ 3 and matrix C is of order 3 $\times$ 3

To show : AB is a zero matrix

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 \times a^2 + c \times ab - b \times ac & 0 \times ab + c \times b^2 - b \times bc & 0 \times ac + c \times bc - b \times c^2 \\ -c \times a^2 + 0 \times ab + a \times ac & -c \times ab + 0 \times b^2 + a \times bc & -c \times ac + 0 \times bc + a \times c^2 \\ b \times a^2 - a \times ab + \times ac & b \times ab - a \times b^2 + \times bc & b \times ac - a \times bc + \times c^2 \end{bmatrix}$$

$$= \begin{bmatrix} abc - abc & b^2c - b^2c & bc^2 - bc^2 \\ -a^2c + a^2c & -abc + abc & -ac^2 + ac^2 \\ a^2b - a^2b & ab^2 - ab^2 & abc - abc \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

= 0 matrix

Hence, Proved

Question 13.
For the following matrices, verify that A(BC) = (AB)C :

$A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and C = [1 -2]

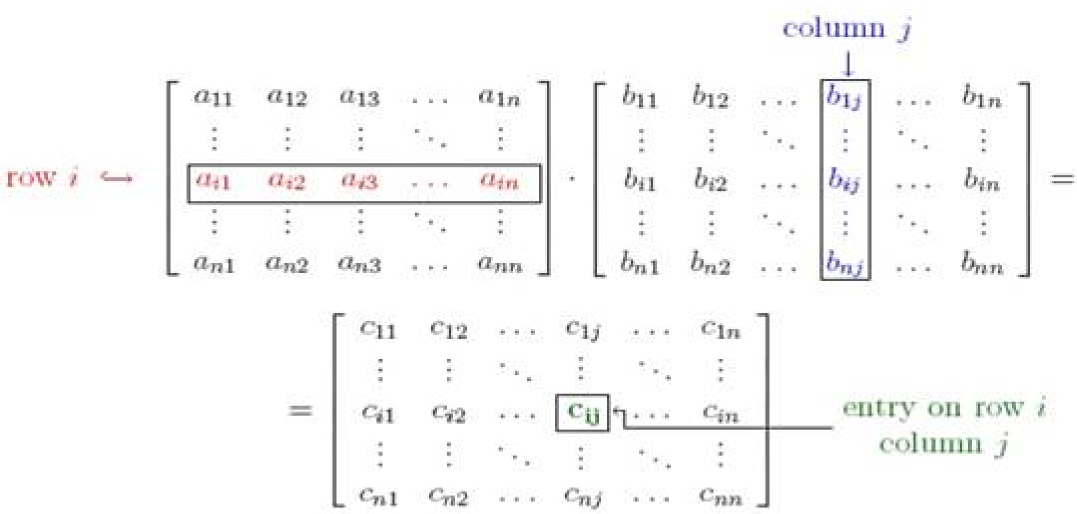
Answer:

Given : $A = \begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and C = [1 -2]

Matrix A is of order 2 $\times$ 3 , matrix B is of order 3 $\times$ 1 and matrix C is of order 1 $\times$ 2

To show : matrix A(BC) = (AB)C

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

For matrix BC, a = 3,b = c = 1,d = 2 ,thus matrix BC is of order 3 $\times$ 2

Matrix BC = $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \times \begin{bmatrix} 1 & -2 \end{bmatrix} = \begin{bmatrix} 1(1) & 1(-2) \\ 1(1) & 1(-2) \\ 2(1) & 2(-2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix}$

Matrix BC = $\begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix}$

For matrix A(BC),a = 2 ,b = c = 3 ,d = 2 ,thus matrix A(BC) is of order 2 x 2

Matrix A(BC) = $\begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & -2 \\ 1 & -2 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 2(1)+3(1)-1(2) & 2(-2)+3(-2)-1(-4) \\ 3(1)+0(1)+2(2) & 3(-2)+0(-2)+2(-4) \end{bmatrix}$

Matrix A(BC) = $\begin{bmatrix} 2+3-2 & -4-6+4 \\ 3+0+4 & -6+0-8 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$

Matrix A(BC) = $\begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$

Matrix A(BC) = $\begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$

For matrix AB, a = 2,b = c = 3,d = 1 ,thus matrix BC is of order 2 $\times$ 1

Matrix AB = $\begin{bmatrix} 2 & 3 & -1 \\ 3 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2(1)+3(1)-1(2) \\ 3(1)+0(1)+2(2) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 2+3-2 \\ 3+0+4 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

For matrix (AB)C, a = 2,b = c = 1,d = 2 ,thus matrix (AB)C is of order 2 $\times$ 2

Matrix (AB)C = $\begin{bmatrix} 3 \\ 7 \end{bmatrix} \times \begin{bmatrix} 1 & -2 \end{bmatrix}$ = $\begin{bmatrix} 3(1) & 3(-2) \\ 7(1) & 7(-2) \end{bmatrix}$ = $\begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$

Matrix (AB)C = $\begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$

Matrix A(BC) = (AB)C = $\begin{bmatrix} 3 & -6 \\ 7 & -14 \end{bmatrix}$

Question 14.
Verify that A(B + C) = (AB + AC), when

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}.$$

Answer:

Given : $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$.

Matrix A is of order 2 $\times$ 2 , matrix B is of order 2 $\times$ 2 and matrix C is of order 2 $\times$ 2

To verify : A(B + C) = (AB + AC)

Formula used :

$$\begin{array}{r} \\ \\ \text{row } i \hookrightarrow \\ \\ \\ \end{array} \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nj} & \cdots & b_{nn} \end{bmatrix} \end{array} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \cdots & \boxed{c_{ij}} & \cdots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} \quad \begin{array}{c} \text{entry on row } i \\ \text{column } j \end{array}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

B + C = $\begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2+1 & 0-1 \\ 1+0 & -3+1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$

B + C = $\begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$

Matrix A(B + C) is of order 2 x 2

A(B + C) = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1(3)+2(1) & 1(-1)+2(-2) \\ 3(3)+4(1) & 3(-1)+4(-2) \end{bmatrix}$

A(B + C) = $\begin{bmatrix} 3+2 & -1-4 \\ 9+4 & -3-8 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$

A(B + C) = $\begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$

For matrix AB, a = b = c = d = 2 ,matrix AB is of order 2 x 2

Matrix AB = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 1(2)+2(1) & 1(0)+2(-3) \\ 3(2)+4(1) & 3(0)+4(-3) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 2+2 & 0-6 \\ 6+4 & 0-12 \end{bmatrix}$ = $\begin{bmatrix} 4 & -6 \\ 10 & -12 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 4 & -6 \\ 10 & -12 \end{bmatrix}$

For matrix AC, a = b = c = d = 2 ,matrix AC is of order 2 x 2

Matrix AC = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ = $\begin{bmatrix} 1(1)+2(0) & 1(-1)+2(1) \\ 3(1)+4(0) & 3(-1)+4(1) \end{bmatrix}$

Matrix AC = $\begin{bmatrix} 1+0 & -1+2 \\ 3+0 & -3+4 \end{bmatrix}$ = $\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

Matrix AC = $\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

Matrix AB + AC = $\begin{bmatrix} 4 & -6 \\ 10 & -12 \end{bmatrix}$ + $\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$ = $\begin{bmatrix} 4+1 & -6+1 \\ 10+3 & -12+1 \end{bmatrix}$ = $\begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$

Matrix AB + AC = A(B + C) = $\begin{bmatrix} 5 & -5 \\ 13 & -11 \end{bmatrix}$

A(B + C) = (AB + AC)

Question 15.
Verify that A(B + C) = (AB + AC), when

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}.$$

Answer:

Given : $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}.$

Matrix A is of order 3 $\times$ 2 , matrix B is of order 2 $\times$ 2 and matrix C is of order 2 $\times$ 2

To verify : A(B + C) = (AB + AC)

Formula used :

$$\begin{array}{r} \\ \\ \text{row } i \hookrightarrow \\ \\ \\ \end{array} \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \boxed{a_{i1} \quad a_{i2} \quad a_{i3} \quad \cdots \quad a_{in}} & & & & \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nj} & \cdots & b_{nn} \end{bmatrix} \end{array} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \cdots & \boxed{c_{ij}} & \cdots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} \begin{array}{l} \\ \\ \text{entry on row } i \\ \text{column } j \\ \\ \end{array}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

$$B + C = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5-1 & -3+2 \\ 2+3 & 1+4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 5 & 5 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 4 & -1 \\ 5 & 5 \end{bmatrix}$$

For Matrix A(B + C), a = 3,b = c = d = 2,thus matrix A(B + C) is of order 3 x 2

$$A(B + C) = \begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & -1 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 2(4)+3(5) & 2(-1)+3(5) \\ -1(4)+4(5) & -1(-1)+4(5) \\ 0(4)+1(5) & 0(-1)+1(5) \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} 8+15 & -2+15 \\ -4+20 & 1+20 \\ 0+5 & 0+5 \end{bmatrix} = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$$

For matrix AB, a = 3, b = c = d = 2 ,matrix AB is of order 3 x 2

Matrix AB = $\begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(5)+3(2) & 2(-3)+3(1) \\ -1(5)+4(2) & -1(-3)+4(1) \\ 0(5)+1(2) & 0(-3)+1(1) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 10+6 & -6+3 \\ -5+8 & 3+4 \\ 0+2 & 0+1 \end{bmatrix} = \begin{bmatrix} 16 & -3 \\ 3 & 7 \\ 2 & 1 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 16 & -3 \\ 3 & 7 \\ 2 & 1 \end{bmatrix}$

For matrix AC, a = 3, b = c = d = 2 ,matrix AC is of order 3 x 2

Matrix AC = $\begin{bmatrix} 2 & 3 \\ -1 & 4 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2(-1)+3(3) & 2(2)+3(4) \\ -1(-1)+4(3) & -1(2)+4(4) \\ 0(-1)+1(3) & 0(2)+1(4) \end{bmatrix}$

Matrix AC = $\begin{bmatrix} -2+9 & 4+12 \\ 1+12 & -2+16 \\ 0+3 & 0+4 \end{bmatrix} = \begin{bmatrix} 7 & 16 \\ 13 & 14 \\ 3 & 4 \end{bmatrix}$

Matrix AC = $\begin{bmatrix} 7 & 16 \\ 13 & 14 \\ 3 & 4 \end{bmatrix}$

Matrix AB + AC = $\begin{bmatrix} 16 & -3 \\ 3 & 7 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 7 & 16 \\ 13 & 14 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 16+7 & 16-3 \\ 3+13 & 7+21 \\ 2+3 & 1+4 \end{bmatrix} = \begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$

Matrix AB + AC = A(B + C) = $\begin{bmatrix} 23 & 13 \\ 16 & 21 \\ 5 & 5 \end{bmatrix}$

A(B + C) = (AB + AC)

Question 16.

If $A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$; verify that A(B – C) = (AB – AC).

Answer:

Given : $A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$;

Matrix A is of order 3 $\times$ 3; matrix B is of order 3 $\times$ 3 and matrix C is of order 3 $\times$ 3

To verify : A(B – C) = (AB – AC).

The formula used :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix BA exists and is of order c $\times$ b ,if and only if d = a

B - C = $\begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0-1 & 5-5 & -4-2 \\ -2+1 & 1-1 & 3-0 \\ 1-0 & 0+1 & 2-1 \end{bmatrix}$

B - C = $\begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

For Matrix A(B - C), a = 3,b = c = d = 3,thus matrix A(B + C) is of order 3 x 3

A(B - C) = $\begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

$$A(B - C) = \begin{bmatrix} 1(-1) + 0(-1) - 2(1) & 1(0) + 0(0) - 2(1) & 1(-6) + 0(3) - 2(1) \\ 3(-1) - 1(-1) + 0(1) & 3(0) - 1(0) + 0(1) & 3(-6) - 1(3) + 0(1) \\ -2(-1) + 1(-1) + 1(1) & -2(0) + 1(0) + 1(1) & -2(-6) + 1(3) + 1(1) \end{bmatrix}$$

$$A(B - C) = \begin{bmatrix} -1 + 0 - 2 & 0 + 0 - 2 & -6 + 0 - 2 \\ -3 + 1 + 0 & 0 + 0 + 0 & -18 - 3 + 0 \\ 2 - 1 + 1 & 0 + 0 + 1 & 12 + 3 + 1 \end{bmatrix} = \begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$$

$$A(B - C) = \begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$$

For matrix AB, a = 3, b = c = d = 3 ,matrix AB is of order 3 x 3

$$\text{Matrix AB} = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 1(0) + 0(-2) - 2(1) & 1(5) + 0(1) - 2(0) & 1(-4) + 0(3) - 2(2) \\ 3(0) - 1(-2) + 0(1) & 3(5) - 1(1) + 0(0) & 3(-4) - 1(3) + 0(2) \\ -2(0) + 1(-2) + 1(1) & -2(5) + 1(1) + 1(0) & -2(-4) + 1(3) + 1(2) \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} 0 + 0 - 2 & 5 + 0 + 0 & -4 + 0 - 4 \\ 0 + 2 + 0 & 15 - 1 + 0 & -12 - 3 + 0 \\ 0 - 2 + 1 & -10 + 1 + 0 & 8 + 3 + 2 \end{bmatrix} = \begin{bmatrix} -2 & 5 & -8 \\ 2 & 14 & -15 \\ -1 & -9 & 13 \end{bmatrix}$$

$$\text{Matrix AB} = \begin{bmatrix} -2 & 5 & -8 \\ 2 & 14 & -15 \\ -1 & -9 & 13 \end{bmatrix}$$

For matrix AC, a = 3, b = c = d = 3 ,matrix AC is of order 3 x 3

$$\text{Matrix AC} = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\text{Matrix AC} = \begin{bmatrix} 1(1) + 0(-1) - 2(0) & 1(5) + 0(1) - 2(-1) & 1(2) + 0(0) - 2(1) \\ 3(1) - 1(-1) + 0(0) & 3(5) - 1(1) + 0(-1) & 3(2) - 1(0) + 0(1) \\ -2(1) + 1(-1) + 1(0) & -2(5) + 1(1) + 1(-1) & -2(2) + 1(0) + 1(1) \end{bmatrix}$$

$$\text{Matrix AC} = \begin{bmatrix} 1 + 0 + 0 & 5 + 0 + 2 & 2 + 0 - 2 \\ 3 + 1 + 0 & 15 + 1 + 0 & 6 + 0 + 0 \\ -2 - 1 + 0 & -10 + 1 - 1 & -4 + 0 + 1 \end{bmatrix} = \begin{bmatrix} 1 & 7 & 0 \\ 4 & 16 & 6 \\ -3 & -10 & -3 \end{bmatrix}$$

Matrix AC = $\begin{bmatrix} 1 & 7 & 0 \\ 4 & 16 & 6 \\ -3 & -10 & -3 \end{bmatrix}$

Matrix AB - AC = $\begin{bmatrix} -2 & 5 & -8 \\ 2 & 14 & -15 \\ -1 & -9 & 13 \end{bmatrix} - \begin{bmatrix} 1 & 7 & 0 \\ 4 & 16 & 6 \\ -3 & -10 & -3 \end{bmatrix} = \begin{bmatrix} -2-1 & 5-7 & -8-0 \\ 2-4 & 14-16 & -15-6 \\ -1+3 & -9+10 & 13+3 \end{bmatrix}$

Matrix AB - AC = $\begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$

A(B – C) = (AB – AC) = $\begin{bmatrix} -3 & -2 & -8 \\ -2 & 0 & -21 \\ 2 & 1 & 16 \end{bmatrix}$

Question 17.

If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, show that $A^2 = O$.

Answer:

Given : $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$,

Matrix A is of order 2×2

To show : $A^2 = O$

Formula used :

$$\begin{array}{rcccl} & & & \text{column } j & \\ \text{row } i \hookrightarrow & \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} & \cdot & \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nj} & \cdots & b_{nn} \end{bmatrix} & = \end{array}$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \cdots & \boxed{c_{ij}} & \cdots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} \quad \begin{array}{c} \text{entry on row } i \\ \text{column } j \end{array}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$$A^2 = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \times \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} = \begin{bmatrix} ab(ab) + b^2(-a^2) & ab(b^2) + b^2(-ab) \\ -a^2(ab) - ab(-a^2) & -a^2(b^2) - ab(-ab) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A^2 = O$

Question 18.

If $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, show that $A^2 = A$.

Answer:

Given : $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$,

Matrix A is of order 3 $\times$ 3

To show : $A^2 = A$

Formula used :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$$A^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2(2)-2(-1)-4(1) & 2(-2)-2(3)-4(-2) & 2(-4)-2(4)-4(-3) \\ -1(2)+3(-1)+4(1) & -1(-2)+3(3)+4(-2) & -1(-4)+3(4)+4(-3) \\ 1(2)-2(-1)-3(1) & 1(-2)-2(3)-3(-2) & 1(-4)-2(4)-3(-3) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4+2-4 & -4-6+8 & -8-8+12 \\ -2-3+4 & 2+9-8 & 4+12-12 \\ 2+2-3 & -2-6+6 & -4-8+9 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$A^2 = A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

Question 19.

If $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$, show that $A^2 = I$.

Answer:

Given : $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$,

Matrix A is of order 3 $\times$ 3

To show : $A^2 = I$

Formula used :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$
$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} & a_{11}b_{13} + a_{12}b_{23} + a_{13}b_{33} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} & a_{21}b_{13} + a_{22}b_{23} + a_{23}b_{33} \\ a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31} & a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32} & a_{31}b_{13} + a_{32}b_{23} + a_{33}b_{33} \end{bmatrix}$$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$$A^2 = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} \times \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4(4) - 1(3) - 4(3) & 4(-1) - 1(0) - 4(-1) & 4(-4) - 1(-4) - 4(-3) \\ 3(4) + 0(3) - 4(3) & 3(-1) + 0(0) - 4(-1) & 3(-4) + 0(-4) - 4(-3) \\ 3(4) - 1(3) - 3(3) & 3(-1) - 1(0) - 3(-1) & 3(-4) - 1(-4) - 3(-3) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 16 - 3 - 12 & -4 + 0 + 4 & -16 + 4 + 12 \\ 12 + 0 - 12 & -3 + 0 + 4 & -12 + 0 + 12 \\ 12 - 3 - 9 & -3 + 0 + 3 & -12 + 4 + 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 20.

If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find ($3A^2 - 2B + I$).

Answer:

Given : $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$,

Matrix A is of order 2 $\times$ 2, Matrix B is of order 2 $\times$ 2

To find : $3A^2 - 2B + I$

Formula used :

$$\begin{array}{rc} & \\ \text{row } i \hookrightarrow & \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \end{array} \cdot \begin{array}{c} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nj} & \cdots & b_{nn} \end{bmatrix} \end{array} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \cdots & \boxed{c_{ij}} & \cdots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix}$$

entry on row i
column j

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$A^2 = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 2(2)-1(3) & 2(-1)-1(2) \\ 3(2)+2(3) & 3(-1)+2(2) \end{bmatrix} = \begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix}$

$3A^2 = 3 \times \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix}$

$3A^2 = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix}$

$2B = 2 \times \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$

$2B = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3-0+1 & -12-8+0 \\ 36+2+0 & 3-14+1 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$

Question 21.

If $A = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$ then find $(-A^2 + 6A)$.

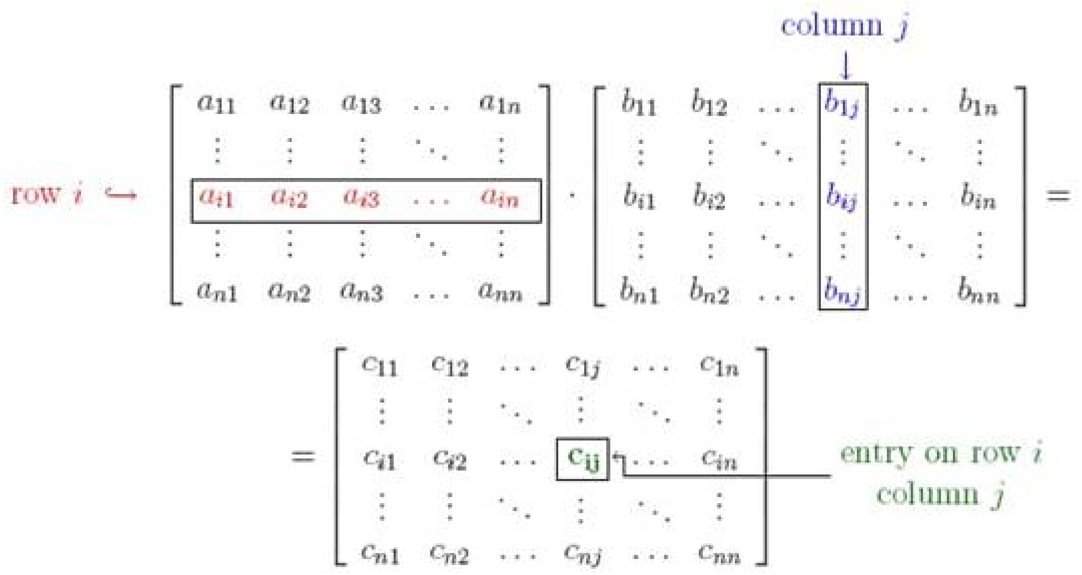
Answer:

Given : $A = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$

Matrix A is of order 2 × 2.

To find : $-A^2 + 6A$

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a × b and B is a matrix of order c × d ,then matrix AB exists and is of order a × d ,if and only if b = c

$A^2 = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 2(2)-2(-3) & 2(-2)-2(4) \\ -3(2)+4(-3) & -3(-2)+4(4) \end{bmatrix} = \begin{bmatrix} 4+6 & -4-8 \\ -6-12 & 6+16 \end{bmatrix}$

$A^2 = \begin{bmatrix} 10 & -12 \\ -18 & 22 \end{bmatrix}$

$-A^2 = -\begin{bmatrix} 10 & -12 \\ -18 & 22 \end{bmatrix} = \begin{bmatrix} -10 & 12 \\ 18 & -22 \end{bmatrix}$

$6A = 6 \times \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix}$

$6A = \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix}$

$-A^2 + 6A = \begin{bmatrix} -10 & 12 \\ 18 & -22 \end{bmatrix} + \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix} = \begin{bmatrix} -10+12 & 12-12 \\ 18-18 & -22+24 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$-A^2 + 6A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

Question 22.

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that (A2 – 5A + 7I) = O.

Answer:

Given : $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$,

Matrix A is of order 2×2.

To show : $A^2 - 5A + 7I = 0$

Formula used :

$$\text{row } i \hookrightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \begin{array}{cccccc} \phantom{b_{11}} & \phantom{b_{12}} & & \text{column } j \downarrow & & \phantom{b_{1n}} \end{array} \\ \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nj} & \cdots & b_{nn} \end{bmatrix} \end{array} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \cdots & \boxed{c_{ij}} & \cdots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} \quad \text{entry on row } i \text{ column } j$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = 5 \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$7I = 7 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 - 5A + 7I = 0$$

Question 23.

Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^3 - 4A^2 + A = O$.

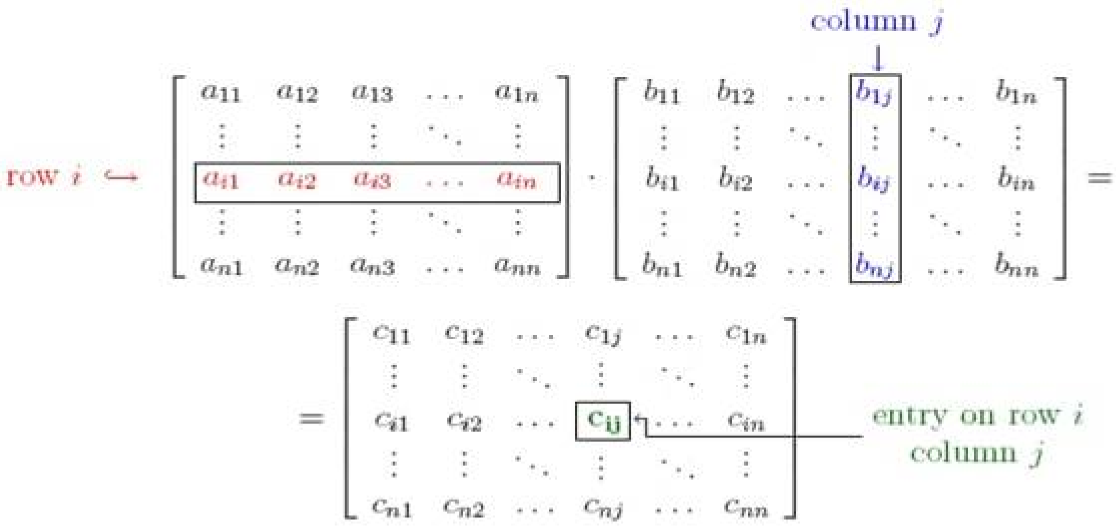
Answer:

Given : $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

Matrix A is of order 2×2.

To show : $A^3 - 4A^2 + A = 0$

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^2 and A^3 are matrices of order 2 x 2.

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2(2)+3(1) & 2(3)+3(2) \\ 1(2)+2(1) & 1(3)+2(2) \end{bmatrix} = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7(2)+12(1) & 7(3)+12(2) \\ 4(2)+7(1) & 4(3)+7(2) \end{bmatrix} = \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix}$$

$A^3 = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$

$4A^2 = 4 \times \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix}$

$4A^2 = \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix}$

$A^3 - 4A^2 + A = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 26 - 28 + 2 & 45 - 48 + 3 \\ 15 - 16 + 1 & 26 - 28 + 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^3 - 4A^2 + A = 0$

Question 24.

If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find k so that $A^2 = kA - 2I$.

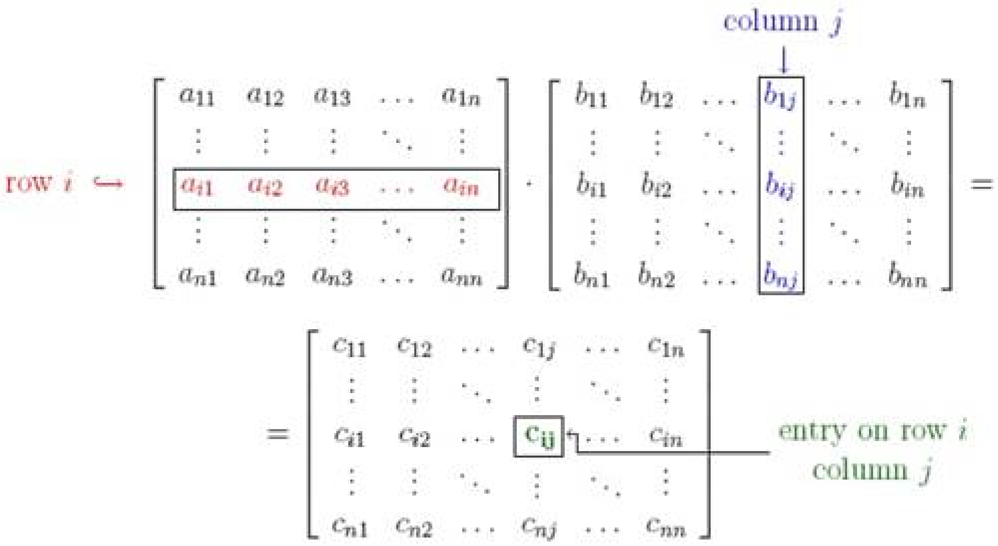
Answer:

Given : $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, $A^2 = kA - 2I$.

Matrix A is of order 2×2.

To find : k

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^2 is a matrix of order 2 x 2.

$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3(3)-2(4) & 3(-2)-2(-2) \\ 4(3)-2(4) & 4(-2)-2(-2) \end{bmatrix} = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

$kA = k \times \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}$

$kA - 2I = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - 2 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$

It is the given that $A^2 = kA - 2I$

$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$

Equating like terms,

$3k - 2 = 1$

$3k = 1 + 2 = 3$

$3k = 3$

$k = \frac{3}{3} = 1$

$k = 1$

Question 25.
For the following matrices, verify that A(BC) = (AB)C :

$A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$

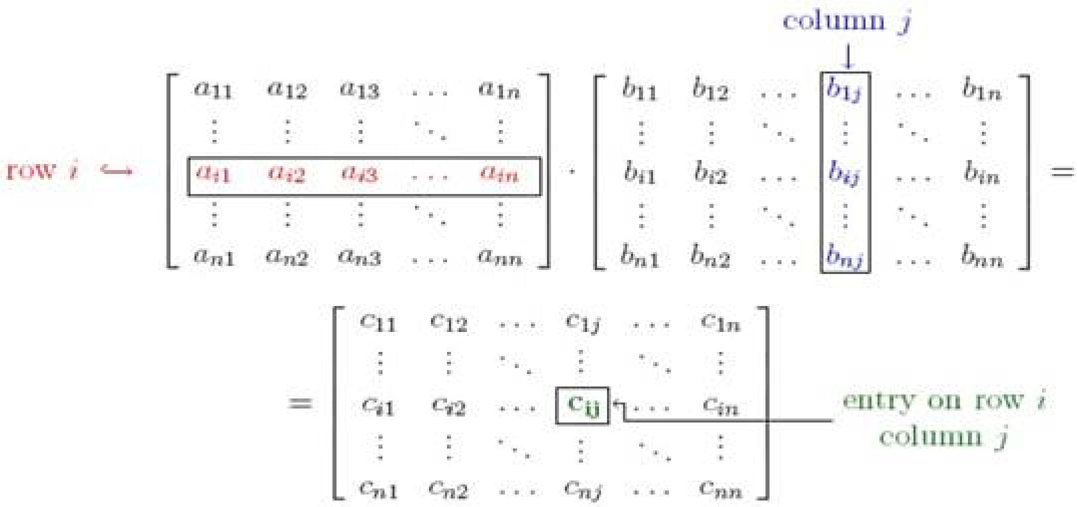
Answer:

Given : $A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$

Matrix A is of order 2×3 , matrix B is of order 3×3 and matrix C is of order 3×1

To show : matrix A(BC) = (AB)C

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$,then matrix AB exists and is of order $a \times d$,if and only if b = c

If A is a matrix of order $a \times b$ and B is a matrix of order $c \times d$,then matrix BA exists and is of order $c \times b$,if and only if d = a

For matrix BC, a = 3,b = c = 3,d = 1 ,thus matrix BC is of order 3×1

Matrix BC = $\begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 2(1)+3(4)+0(5) \\ 1(1)+0(4)+4(5) \\ 1(1)-1(4)+2(5) \end{bmatrix} = \begin{bmatrix} 2+12+0 \\ 1+0+20 \\ 1-4+10 \end{bmatrix}$

Matrix BC = $\begin{bmatrix} 14 \\ 21 \\ 7 \end{bmatrix}$

For matrix A(BC),a = 2 ,b = c = 3 ,d = 1 ,thus matrix A(BC) is of order 2 x 1

Matrix A(BC) = $\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 14 \\ 21 \\ 7 \end{bmatrix} = \begin{bmatrix} 1(14)+2(21)+5(7) \\ 0(14)+1(21)+3(7) \end{bmatrix} = \begin{bmatrix} 14+42+35 \\ 0+21+21 \end{bmatrix}$

Matrix A(BC) = $\begin{bmatrix} 91 \\ 42 \end{bmatrix}$

Matrix A(BC) = $\begin{bmatrix} 91 \\ 42 \end{bmatrix}$

For matrix AB, a = 2,b = c = 3,d = 3 ,thus matrix BC is of order 2 $\times$ 3

Matrix AB =$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 1(2)+2(1)+5(1) & 1(3)+2(0)+5(-1) & 1(0)+2(4)+5(2) \\ 0(2)+1(1)+3(1) & 0(3)+1(0)+3(-1) & 0(0)+1(4)+3(2) \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 2+2+5 & 3+0-5 & 0+8+10 \\ 0+1+3 & 0+0-3 & 0+4+6 \end{bmatrix} = \begin{bmatrix} 9 & -2 & 18 \\ 4 & -3 & 10 \end{bmatrix}$

Matrix AB = $\begin{bmatrix} 9 & -2 & 18 \\ 4 & -3 & 10 \end{bmatrix}$

For matrix (AB)C, a = 2,b = c = 3,d = 1 ,thus matrix (AB)C is of order 2 $\times$ 1

Matrix (AB)C = $\begin{bmatrix} 9 & -2 & 18 \\ 4 & -3 & 10 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 9(1)-2(4)+18(5) \\ 4(1)-3(4)+10(5) \end{bmatrix}$

Matrix (AB)C = $\begin{bmatrix} 9-8+90 \\ 4-12+50 \end{bmatrix} = \begin{bmatrix} 91 \\ 42 \end{bmatrix}$

Matrix (AB)C = $\begin{bmatrix} 91 \\ 42 \end{bmatrix}$

Matrix A(BC) = (AB)C = $\begin{bmatrix} 91 \\ 42 \end{bmatrix}$

Question 26.

If $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$, find f(A), where $f(x) = x^2 - 2x + 3$.

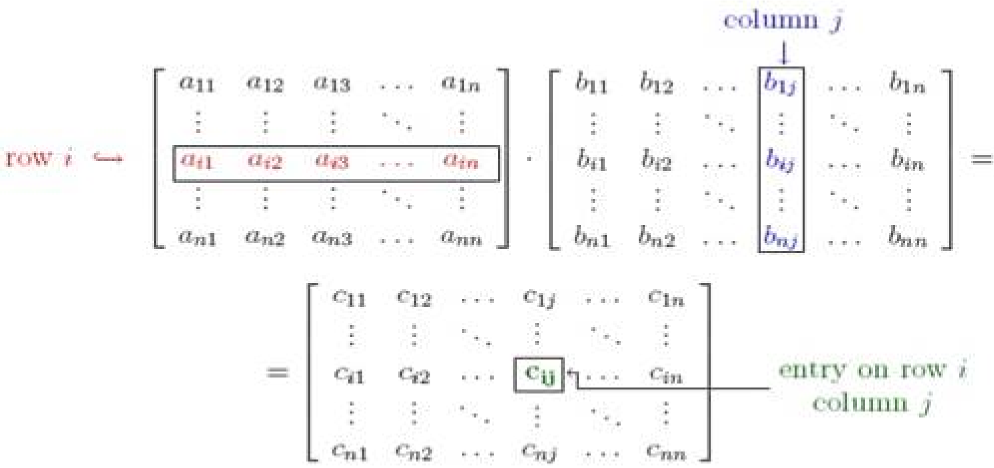
Answer:

Given : $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$, and $f(x) = x^2 - 2x + 3$.

Matrix A is of order 2×2.

To find : f(A)

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^2 is a matrix of order 2 x 2.

$f(x) = x^2 - 2x + 3$

$f(A) = A^2 - 2A + 3I$

$$A^2 = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -1(-1) + 2(3) & -1(2) + 2(1) \\ 3(-1) + 1(3) & 3(2) + 1(1) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 + 6 & -2 + 2 \\ -3 + 3 & 6 + 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$A^2 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$2A = 2 \times \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix}$

$2A = \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix}$

$3I = 3 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$3I = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$f(A) = A^2 - 2A + 3I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 7+2+3 & -4+0 \\ 0-6+0 & 7-2+3 \end{bmatrix}$

$f(A) = A^2 - 2A + 3I = \begin{bmatrix} 12 & -4 \\ -6 & 8 \end{bmatrix}$

$f(A) = A^2 - 2A + 3I = \begin{bmatrix} 12 & -4 \\ -6 & 8 \end{bmatrix}$

Question 27.

If $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and $f(x) = 2x^3 + 4x + 5$, find f(A).

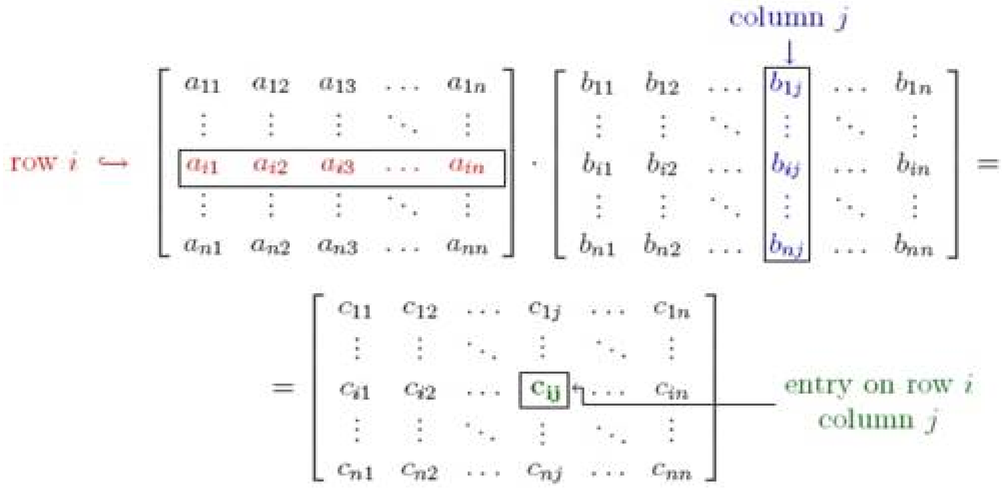
Answer:

Given : $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and $f(x) = 2x^3 + 4x + 5$

Matrix A is of order 2×2.

To find : f(A)

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^3 is a matrix of order 2 x 2.

$f(x) = 2x^3 + 4x + 5$

$f(A) = 2A^3 + 4A + 5I$

$A^2 = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 1(1)+2(4) & 1(2)+2(-3) \\ 4(1)-3(4) & 4(2)-3(-3) \end{bmatrix} = \begin{bmatrix} 1+8 & 2-6 \\ 4-12 & 8+9 \end{bmatrix}$

$A^2 = \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}$

$A^3 = \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 9(1)-4(4) & 9(2)-4(-3) \\ -8(1)+17(4) & -8(2)+17(-3) \end{bmatrix}$

$A^3 = \begin{bmatrix} 9-16 & 18+12 \\ -8+68 & -16-51 \end{bmatrix} = \begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix}$

$A^3 = \begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix}$

$2A^3 = 2 \times \begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix} = \begin{bmatrix} -14 & 60 \\ 120 & -134 \end{bmatrix}$

$2A^3 = \begin{bmatrix} -14 & 60 \\ 120 & -134 \end{bmatrix}$

$4A = 4 \times \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix}$

4A = $\begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix}$

5I = 5 × $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

5I = $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$2A^3$ + 4A + 5I = $\begin{bmatrix} -14 & 60 \\ 120 & -134 \end{bmatrix}$ + $\begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix}$ + $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ = $\begin{bmatrix} -14+4+5 & 60+8+0 \\ 120+16+0 & -134-12+5 \end{bmatrix}$

f(A) = $2A^3$ + 4A + 5I = $\begin{bmatrix} -5 & 68 \\ 136 & -141 \end{bmatrix}$

f(A) = $2A^3$ + 4A + 5I = $\begin{bmatrix} -5 & 68 \\ 136 & -141 \end{bmatrix}$

Question 28.
Find the values of x and y, when

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Answer:
Given : $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

To find : x and y

Formula used :

$$\text{row } i \hookrightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nj} & \cdots & b_{nn} \end{bmatrix} \end{array} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \cdots & \boxed{c_{ij}} & \cdots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} \quad \begin{array}{c} \text{entry on row } i \\ \text{column } j \end{array}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

The resulting matrix obtained on multiplying $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \end{bmatrix}$ is of order 2 × 1

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x - 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2x - 3y \\ x + y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Equating similar terms,

2x – 3y = 1 equation 1

x + y = 3 equation 2

equation 1 + 3(equation 2) and solving the above equations,

$$\begin{array}{l} 2x - 3y = 1 \\ + \\ \underline{3x + 3y} = 9 \\ 5x = 10 \end{array}$$

$$x = \frac{10}{5} = 2$$

x = 2 , substituting x = 2 in equation 2,

2 + y = 3

y = 3 – 2 = 1

x = 2 and y = 1

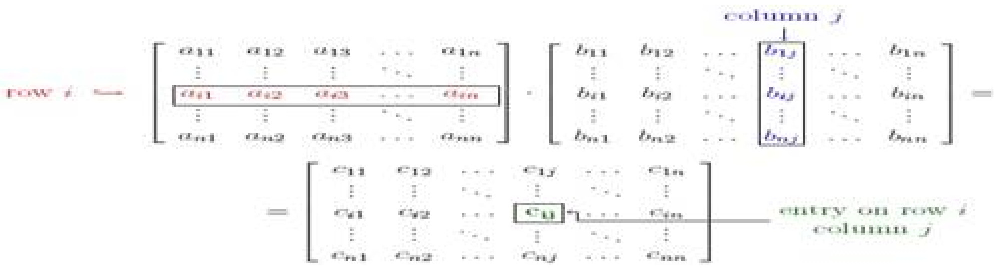
Question 29.
Solve for x and y, when

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}.$$

Answer:
Given : $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}.$

To find : x and y

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

The resulting matrix obtained on multiplying $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \end{bmatrix}$ is of order 2 × 1

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x - 4y \\ x + 2y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 3x - 4y \\ x + 2y \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$$

Equating similar terms,

3x – 4y = 3 equation 1

x + 2y = 11 equation 2

equation 1 + 2(equation 2) and solving the above equations,

$$\begin{array}{l} 3x - 4y = 3 \\ + \\ \underline{2x + 4y = 22} \\ 5x = 3 + 22 = 25 \end{array}$$

5x = 25

$x = \frac{25}{5} = 5$

x = 5 , substituting x = 2 in equation 2,

5 + 2y = 11

2y =11 – 5 = 6

2y = 6

$y = \frac{6}{2} = 3$

x = 5 and y = 3

Question 30.

If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + xI = yA$.

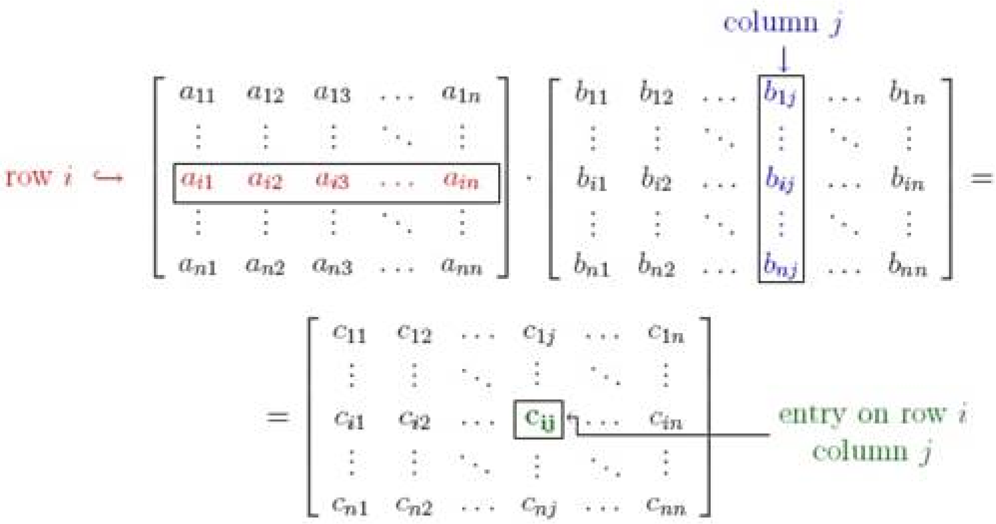
Answer:

Given : $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, $A^2 + xI = yA$.

A is a matrix of order 2 x 2

To find : x and y

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

A^2 is a matrix of order 2 x 2

$$A^2 = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 3(3)+1(7) & 3(1)+1(5) \\ 7(3)+5(7) & 7(1)+5(5) \end{bmatrix} = \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}$$

$$xI = x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$$

xI = $\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$

A^2 + xI = $\begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 16+x & 8+0 \\ 56+0 & 32+x \end{bmatrix} = \begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix}$

A^2 + xI = $\begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix}$

yA = y × $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$

yA = $\begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$

It is given that A^2 + xI = yA,

$\begin{bmatrix} 16+x & 8 \\ 56 & 32+x \end{bmatrix} = \begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$

Equating similar terms in the given matrices,

16 + x = 3y and 8 = y,

hence y = 8

Substituting y = 8 in equation 16 + x = 3y

16 + x = 3 × 8 = 24

16 + x = 24

x = 24 – 16 = 8

x = 8

x = 8, y = 8

Question 31.

If $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the value of a and b such that $A^2 + aA + bI = O$.

Answer:

Given : $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, $A^2 + aA + bI = O$

A is a matrix of order 2 x 2

To find : a and b

Formula used :

$$\begin{array}{cc} & \\ \text{row } i \hookrightarrow & \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \boxed{a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \end{array} \cdot \begin{array}{c} \text{column } j \\ \downarrow \\ \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nj} & \cdots & b_{nn} \end{bmatrix} \end{array} =$$

$$= \begin{array}{cc} \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \cdots & \boxed{c_{ij}} & \cdots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} & \begin{array}{c} \text{entry on row } i \\ \text{column } j \end{array} \end{array}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

A^2 is a matrix of order 2 x 2

$A^2 = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3(3) + 2(1) & 3(2) + 2(1) \\ 1(3) + 1(1) & 1(2) + 1(1) \end{bmatrix} = \begin{bmatrix} 9 + 2 & 6 + 2 \\ 3 + 1 & 2 + 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

$aA = a \times \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3a & 2a \\ 1a & 1a \end{bmatrix}$

bI = b × $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = $\begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}$

bI = $\begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}$

A^2 + aA + bI = $\begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$ + $\begin{bmatrix} 3a & 2a \\ 1a & 1a \end{bmatrix}$ + $\begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}$ = $\begin{bmatrix} 11 + 3a + b & 8 + 2a + 0 \\ 4 + a + 0 & 3 + a + b \end{bmatrix}$

A^2 + aA + bI = $\begin{bmatrix} 11 + 3a + b & 8 + 2a \\ 4 + a & 3 + a + b \end{bmatrix}$

It is given that A^2 + aA + bI = 0

$\begin{bmatrix} 11 + 3a + b & 8 + 2a \\ 4 + a & 3 + a + b \end{bmatrix}$ = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Equating similar terms in the matrices,we get

4 + a = 0 and 3 + a + b = 0

a = 0 – 4 = -4

a = -4

substituting a = -4 in 3 + a + b = 0

3 – 4 + b = 0

-1 + b = 0

b = 0 + 1 = 1

b = 1

a = -4 and b = 1

Question 32.

Find the matrix A such that $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}. \mathrm{A} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$.

Answer:

Given : $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}. A = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}.$

To find : matrix A

Formula used :

$$\text{row } i \hookrightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \begin{array}{cccccc} & & & \text{column } j & & \\ & & & \downarrow & & \end{array} \\ \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nj} & \cdots & b_{nn} \end{bmatrix} \end{array} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \cdots & \boxed{c_{ij}} & \cdots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} \quad \begin{array}{c} \text{entry on row } i \\ \text{column } j \end{array}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots\dots\dots\dots\dots\dots + a_{in}b_{nj}$

IF XA = B, then $A = X^{-1}B$

$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} . A = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} \times \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

To find $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1}$

Determinant of given matrix = $\begin{vmatrix} 5 & -7 \\ -2 & 3 \end{vmatrix}$ = 5(3) – (-7)(-2) = 15 – 14 = 1

Adjoint of matrix $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} = \frac{1}{1} \times \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} \times \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \times \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 3(-16)+7(7) & 3(-6)+7(2) \\ 2(-16)+5(7) & 2(-6)+5(2) \end{bmatrix} = \begin{bmatrix} -48+49 & -18+14 \\ -32+35 & -12+10 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix}$$

Question 33.

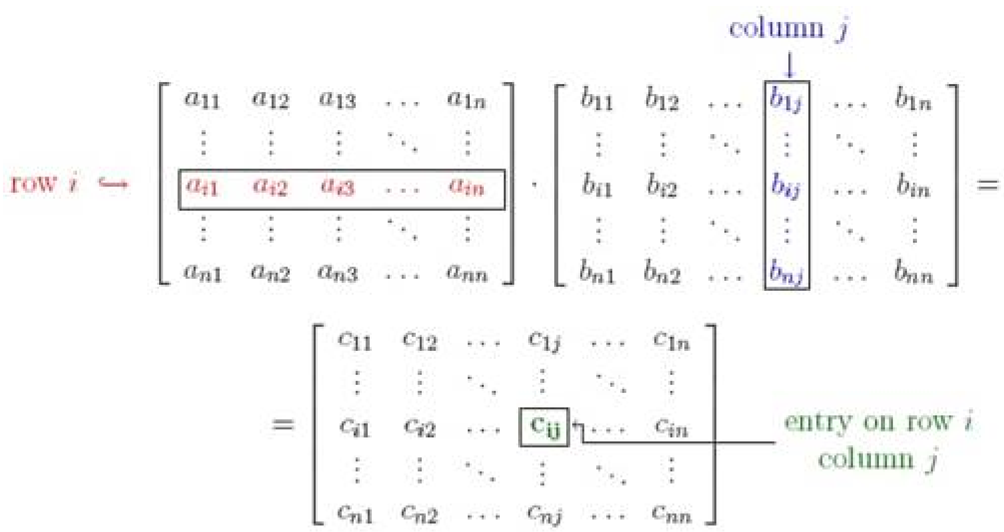
Find the matrix A such that A. $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$.

Answer:

Given : A. $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$.

To find : matrix A

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

IF AX = B, then $A = BX^{-1}$

A. $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$

$A = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1}$

To find $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1}$

Determinant of given matrix = $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$ = 5(2) – (4)(3) = 10 – 12 = -2

Adjoint of matrix $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \frac{1}{-2} \times \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} = \frac{1}{-2} . \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \frac{1}{-2} . \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \frac{1}{-2} . \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

$A = \frac{1}{-2} \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix} \times \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} = \frac{1}{-2} . \begin{bmatrix} 0(5) - 4(-4) & 0(-3) - 4(2) \\ 10(5) + 3(-4) & 10(-3) + 3(2) \end{bmatrix}$

$A = \frac{1}{-2} . \begin{bmatrix} 0 + 16 & 0 - 8 \\ 50 - 12 & -30 + 6 \end{bmatrix} = \frac{1}{-2} . \begin{bmatrix} 16 & -8 \\ 38 & -24 \end{bmatrix} = \begin{bmatrix} -8 & 4 \\ -19 & 12 \end{bmatrix}$

$A = \begin{bmatrix} -8 & 4 \\ -19 & 12 \end{bmatrix}$

$A = \begin{bmatrix} -8 & 4 \\ -19 & 12 \end{bmatrix}$

Question 34.

If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = (A^2 + B^2)$ then find the values of a and b.

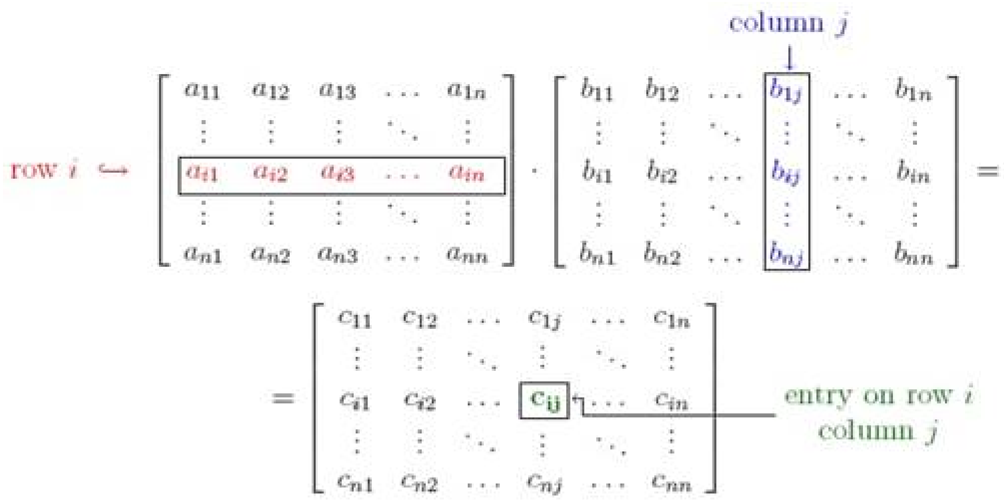
Answer:

Given : $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix}$

$(A + B)^2 = (A^2 + B^2)$

To find : a and b

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$A + B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} 1+a & -1-1 \\ 2+b & -1-1 \end{bmatrix} = \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix}$

$A + B = \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix}$

$(A + B)^2 = \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix} \times \begin{bmatrix} 1+a & -2 \\ 2+b & -2 \end{bmatrix} = \begin{bmatrix} (1+a)(1+a) - 2(2+b) & (1+a)(-2) - 2(-2) \\ (2+b)(1+a) - 2(2+b) & (2+b)(-2) - 2(-2) \end{bmatrix}$

$(A + B)^2 = \begin{bmatrix} 1 + a^2 + 2a - 4 - 2b & -2 - 2a + 4 \\ 2 + 2a + b + ab - 4 - 2b & -4 - 2b + 4 \end{bmatrix} = \begin{bmatrix} a^2 + 2a - 2b - 3 & 2 - 2a \\ 2a - b + ab - 2 & -2b \end{bmatrix}$

$(A + B)^2 = \begin{bmatrix} a^2 + 2a - 2b - 3 & 2 - 2a \\ 2a - b + ab - 2 & -2b \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1(1) - 1(2) & 1(-1) - 1(-1) \\ 2(1) - 1(2) & 2(-1) - 1(-1) \end{bmatrix} = \begin{bmatrix} 1 - 2 & -1 + 1 \\ 2 - 2 & -2 + 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$

$B^2 = \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix} \times \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a(a) - 1(b) & a(-1) - 1(-1) \\ b(a) - 1(b) & b(-1) - 1(-1) \end{bmatrix} = \begin{bmatrix} a^2 - b & -a + 1 \\ ab - b & -b + 1 \end{bmatrix}$

$B^2 = \begin{bmatrix} a^2 - b & -a + 1 \\ ab - b & -b + 1 \end{bmatrix}$

$(A^2 + B^2) = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a^2 - b & -a + 1 \\ ab - b & -b + 1 \end{bmatrix} = \begin{bmatrix} -1 + a^2 - b & -a + 1 \\ ab - b & -b + 1 \end{bmatrix}$

$(A^2 + B^2) = \begin{bmatrix} -1 + a^2 - b & -a + 1 \\ ab - b & -b + 1 \end{bmatrix}$

It is given that $(A + B)^2 = (A^2 + B^2)$

$\begin{bmatrix} a^2 + 2a - 2b - 3 & 2 - 2a \\ 2a - b + ab - 2 & -2b \end{bmatrix} = \begin{bmatrix} -1 + a^2 - b & -a + 1 \\ ab - b & -b + 1 \end{bmatrix}$

Equating similar terms in the given matrices we get,

2 – 2a = -a + 1 and -2b = -b + 1

2 – 1 = -a + 2a and -2b + b = 1

1 = a and -b = 1

a = 1 and b = -1

Question 35.

If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$, show that F(x) . F(y) = F(x + y).

Answer:

Given : $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$,

To show : F(x) . F(y) = F(x + y).

Formula used :

$$\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\times\begin{bmatrix}b_{11} & b_{12} & b_{13}\\ b_{21} & b_{22} & b_{23}\\ b_{31} & b_{32} & b_{33}\end{bmatrix}$$

$$=\begin{bmatrix}a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33}\\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33}\\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33}\end{bmatrix}$$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

F(x) = $\begin{bmatrix}\text{cosx} & -\text{sinx} & 0\\ \text{sinx} & \text{cosx} & 0\\ 0 & 0 & 1\end{bmatrix}$

F(y) = $\begin{bmatrix}\text{cosy} & -\text{siny} & 0\\ \text{siny} & \text{cosy} & 0\\ 0 & 0 & 1\end{bmatrix}$

F(x + y) = $\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$

F(x) . F(y) = $\begin{bmatrix}\text{cosx} & -\text{sinx} & 0\\ \text{sinx} & \text{cosx} & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}\text{cosy} & -\text{siny} & 0\\ \text{siny} & \text{cosy} & 0\\ 0 & 0 & 1\end{bmatrix}$

=
$$\begin{bmatrix}\text{cosx}(\text{cosy})-\text{sinx}(\text{siny})+0(0) & \text{cosx}(-\text{siny})-\text{sinx}(\text{cosy})+0(0) & \text{cosx}(0)-\text{sinx}(0)+0(1)\\ \text{sinx}(\text{cosy})+\text{cosx}(\text{siny})+0(0) & \text{sinx}(-\text{siny})+\text{cosx}(\text{cosy})+0(0) & \text{sinx}(0)+\text{cosx}(0)+0(1)\\ 0(\text{cosy})+0(\text{siny})+1(0) & 0(-\text{siny})+0(\text{cosy})+1(0) & 0(0)+0(0)+1(1)\end{bmatrix}$$

F(x) . F(y) = $\begin{bmatrix} \cos x \cos y - \sin x \sin y & -\cos x \sin y - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

We know that,

cosx(cosy) – sinx (siny) = cos(x+y) and -cosx(siny) - sinx(cosy) = -sin(x+y)

F(x) . F(y) = $\begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

F(x + y) = F(x) . F(y) = $\begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

F(x + y) = F(x) . F(y)

Question 36.

If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, show that $A^2 = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

Answer:

Given : $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$,

To show : $A^2 = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

Formula used :

$$\begin{array}{c} \\ \text{row } i \hookrightarrow \end{array} \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \begin{array}{ccccccc} & & & \text{column } j & & \\ & & & \downarrow & & \end{array} \\ \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nj} & \cdots & b_{nn} \end{bmatrix} \end{array} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \cdots & \boxed{c_{ij}} & \cdots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} \quad \begin{array}{c} \text{entry on row } i \\ \text{column } j \end{array}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \times \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\alpha(\cos\alpha) + \sin\alpha(-\sin\alpha) & \cos\alpha(\sin\alpha) + \sin\alpha(\cos\alpha) \\ -\sin\alpha(\cos\alpha) + \cos\alpha(-\sin\alpha) & -\sin\alpha(\cos\alpha) + \cos\alpha(\cos\alpha) \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & -2\sin\alpha\cos\alpha \\ -2\sin\alpha\cos\alpha & -\sin^2\alpha + \cos^2\alpha \end{bmatrix}$

We know that cos2α = $\cos^2\alpha - \sin^2\alpha$ and sin2α = $2\sin\alpha\cos\alpha$

$A^2 = \begin{bmatrix} \cos2\alpha & -\sin2\alpha \\ -\sin2\alpha & \cos2\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos2\alpha & -\sin2\alpha \\ -\sin2\alpha & \cos2\alpha \end{bmatrix}$

Question 37.

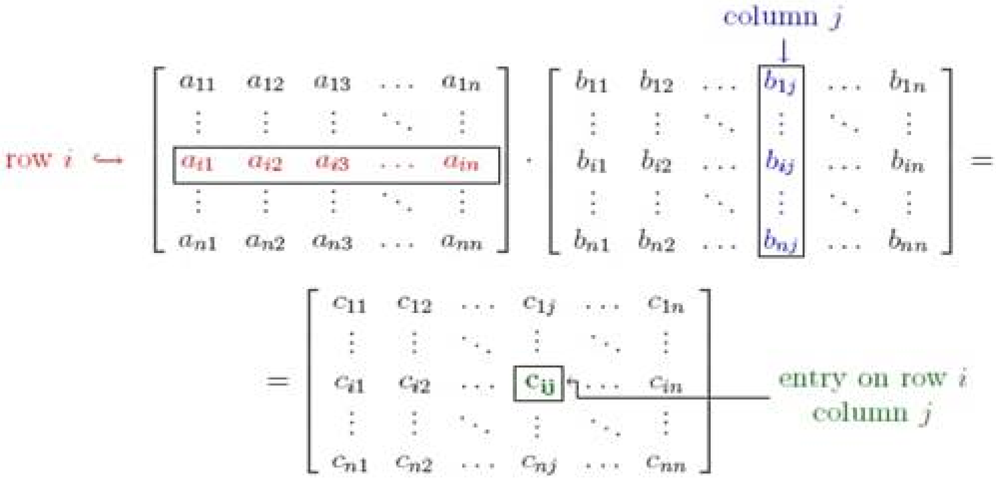
If $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = O,$ find x.

Answer:

Given : $\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix}\begin{bmatrix}1\\-2\\3\end{bmatrix} = O,$

To find : x

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$$\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix}\begin{bmatrix}1\\-2\\3\end{bmatrix} = 0$$

$$\begin{bmatrix}1 & x & 1\end{bmatrix} \times \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix} = \begin{bmatrix}1(1)+x(4)+1(3) & 1(2)+x(5)+1(2) & 1(3)+x(6)+1(5)\end{bmatrix}$$

$$\begin{bmatrix}1 & x & 1\end{bmatrix} \times \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix} = \begin{bmatrix}1+4x+3 & 2+5x+2 & 6x+8\end{bmatrix}$$

$$\begin{bmatrix}1 & x & 1\end{bmatrix} \times \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\3 & 2 & 5\end{bmatrix} = \begin{bmatrix}4x+4 & 5x+4 & 6x+8\end{bmatrix}$$

$$[1 \quad x \quad 1]\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 3 & 2 & 5\end{bmatrix}\begin{bmatrix}1\\ -2\\ 3\end{bmatrix} = [4x+4 \quad 5x+4 \quad 6x+8] \times \begin{bmatrix}1\\ -2\\ 3\end{bmatrix}$$

$$[4x+4 \quad 5x+4 \quad 6x+8] \times \begin{bmatrix}1\\ -2\\ 3\end{bmatrix} = [(4x+4)(1)+(5x+4)(-2)+(6x+8)(3)]$$

$$[4x+4 \quad 5x+4 \quad 6x+8] \times \begin{bmatrix}1\\ -2\\ 3\end{bmatrix} = [4x+4-10x-8+18x+24] = [12x+20]$$

$$[1 \quad x \quad 1]\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 3 & 2 & 5\end{bmatrix}\begin{bmatrix}1\\ -2\\ 3\end{bmatrix} = [12x+20] = 0$$

12x + 20 = 0

12x = -20

$x = \frac{-20}{12} = \frac{-5}{3}$

$x = \frac{-5}{3}$

Question 38.

If $\begin{bmatrix}x & 4 & 1\end{bmatrix}\begin{bmatrix}2 & 1 & 2\\ 1 & 0 & 2\\ 0 & 2 & -4\end{bmatrix}\begin{bmatrix}x\\ 4\\ -1\end{bmatrix} = O$, find x.

Answer:

Given : $\begin{bmatrix}x & 4 & 1\end{bmatrix}\begin{bmatrix}2 & 1 & 2\\ 1 & 0 & 2\\ 0 & 2 & -4\end{bmatrix}\begin{bmatrix}x\\ 4\\ -1\end{bmatrix} = O,$

To find : x

Formula used :

$$\text{row } i \hookrightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \cdot \begin{array}{c} \begin{array}{cccccc} & & & \text{column } j & & \\ & & & \downarrow & & \end{array} \\ \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & b_{i2} & \cdots & b_{ij} & \cdots & b_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nj} & \cdots & b_{nn} \end{bmatrix} \end{array} =$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{i1} & c_{i2} & \cdots & \boxed{c_{ij}} & \cdots & c_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} \quad \begin{array}{c} \text{entry on row } i \\ \text{column } j \end{array}$$

Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$$[x \quad 4 \quad 1]\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$[x \quad 4 \quad 1]\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} = [x(2) + 4(1) + 1(0) \quad x(1) + 4(0) + 1(2) \quad x(2) + 4(2) + 1(-4)]$$

$$[x \quad 4 \quad 1]\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} = [2x + 4 \quad x + 2 \quad 2x + 4]$$

$$[2x + 4 \quad x + 2 \quad 2x + 4]\begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = [(2x + 4)(x) + 4(x + 2) + (2x + 4)(-1)]$$

$$[x \quad 4 \quad 1]\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = [2x^2 + 4x + 4x + 8 - 2x - 4] = [2x^2 + 6x + 4] = 0$$

$2x^2 + 6x + 4 = 0$

$x^2 + 3x + 2 = 0$

$(x + 1)(x + 2) = 0$

$x + 1 = 0$ or $x + 2 = 0$

x = -1 or x = -2

x = -1 or x = -2

Question 39.
Find the values of a and b for which

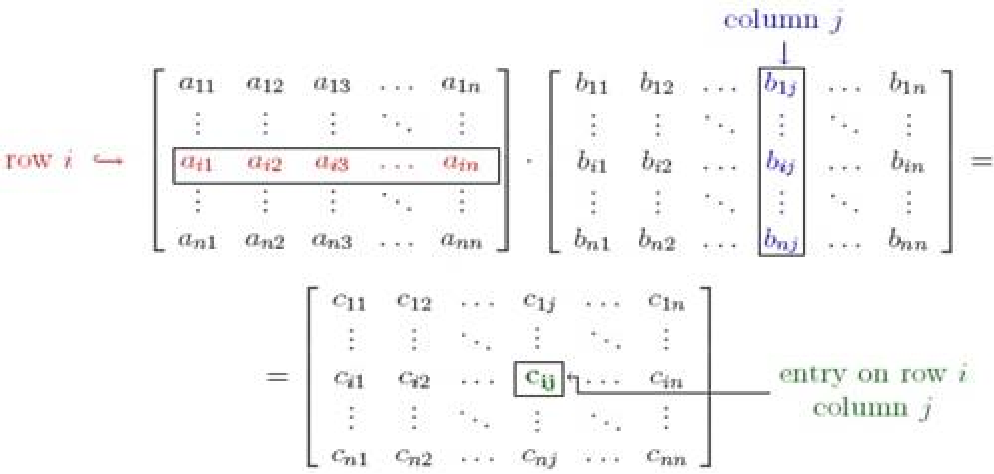
$$\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}.$$

Answer:

Given : $\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}.$

To find : a and b

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

If A is a matrix of order a $\times$ b and B is a matrix of order c $\times$ d ,then matrix AB exists and is of order a $\times$ d ,if and only if b = c

$$\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} a(2) + b(-1) \\ -a(2) + 2b(-1) \end{bmatrix} = \begin{bmatrix} 2a - b \\ -2a - 2b \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$\begin{bmatrix} 2a - b \\ -2a - 2b \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

Equating similar terms,

2a – b = 5

-2a – 2b = 4

Adding the above two equations,we get

-3b = 9

b = $\frac{9}{-3}$ = -3

b = -3

substituting b = -3 in 2a – b = 5,we get

2a + 3 = 5

2a = 5 – 3 = 2

a = 1

a = 1 and b = -3

Question 40.

If $A = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}$, find f(A), where f(x) = x2 – 5x + 7.

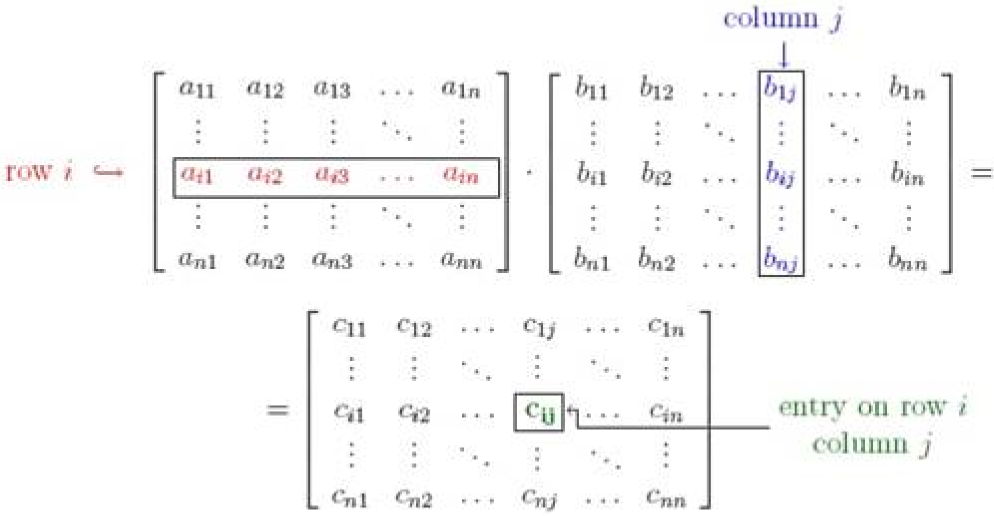
Answer:

Given : $A = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}$, and f(x) = x^2 – 5x + 7.

Matrix A is of order 2 $\times$ 2.

To find : f(A)

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

A^2 is a matrix of order 2 x 2.

$f(x) = x^2 - 5x + 7$

$f(A) = A^2 - 5A + 7I$

$A^2 = \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} \times \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 3(3)+4(-4) & 3(4)+4(-3) \\ -4(3)-3(-4) & -4(4)-3(-3) \end{bmatrix} = \begin{bmatrix} 9-16 & 12-12 \\ -12+12 & -16+9 \end{bmatrix}$

$A^2 = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$

$5A = 5 \times \begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix}$

$5A = \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix}$

$7I = 7 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

f(A) = $A^2 - 5A + 7I = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} - \begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} -7-15+7 & 0-20+0 \\ 0+20+0 & -7+15+7 \end{bmatrix}$

f(A) = $A^2 - 5A + 7I = \begin{bmatrix} -15 & -20 \\ 20 & 15 \end{bmatrix}$

f(A) = $A^2 - 5A + 7I = \begin{bmatrix} -15 & -20 \\ 20 & 15 \end{bmatrix}$

Question 41.

If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for all n $\in$ N.

Answer:

Given : $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$,

Matrix A is of order 2 $\times$ 2.

To prove : $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

Proof :

A = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Let us assume that the result holds for A^{n-1}

$A^{n-1} = \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix}$

We need to prove that the result holds for A^n by mathematical induction .

$A^n = A^{n-1} \times A = \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1(1)+(n-1)(0) & 1(1)+(n-1)(1) \\ 0(1)+1(0) & 0(1)+1(1) \end{bmatrix}$

$A^n = \begin{bmatrix} 1+0 & 1+n-1 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

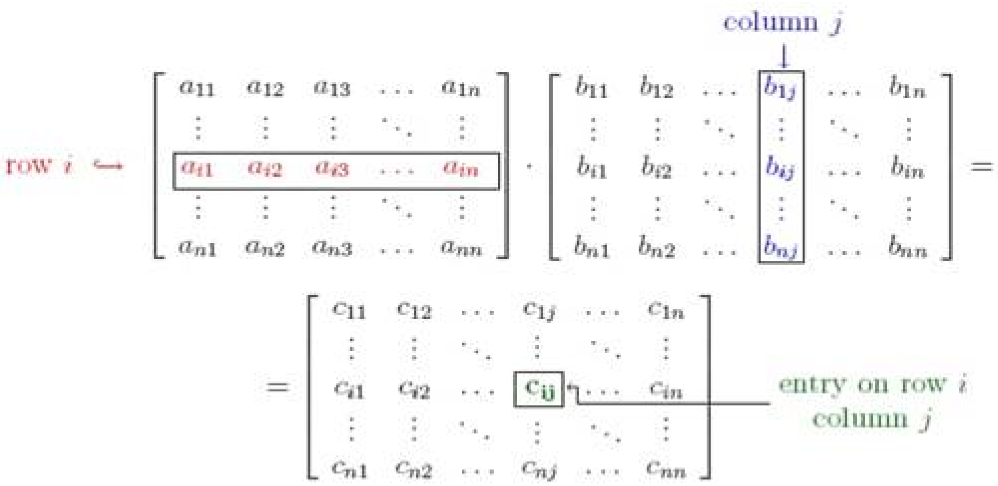
Question 42.
Given an example of two matrices A and B such that

A ≠ O, B ≠ O, AB = O and BA ≠ O.

Answer:
Given : A ≠ 0,B ≠ 0 ,AB = 0, BA ≠ 0

To Find : matrix A and B

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

Let A = $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

A ≠ 0,B ≠ 0

AB = $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1(0)+0(1) & 1(0)+0(0) \\ 0(0)+0(1) & 0(0)+0(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

AB = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ = 0

BA = $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0(1)+0(0) & 0(0)+0(0) \\ 1(1)+0(0) & 1(0)+0(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$$BA = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

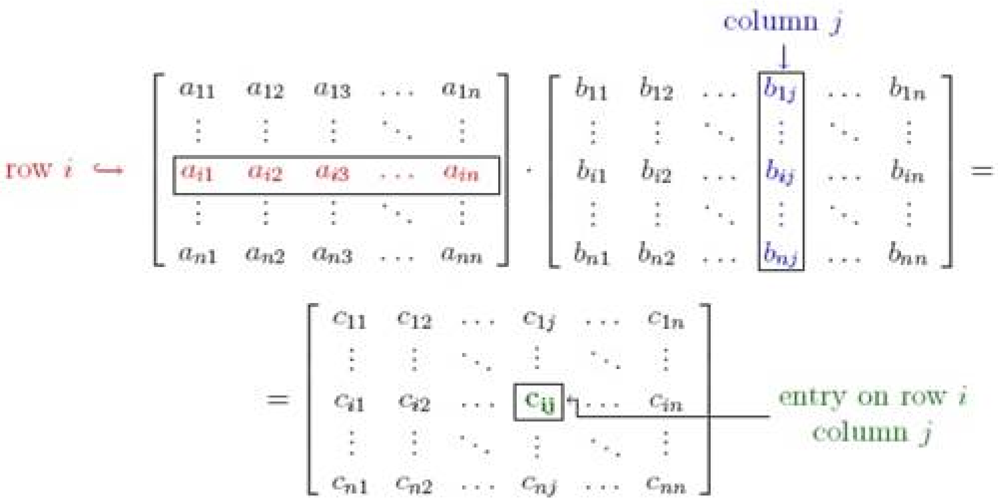
Question 43.
Give an example of three matrices A, B, C such that

AB = AC but B ≠ C.

Answer:
Given : AB = AC and B ≠ C.

To Find : matrix A and B

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

B ≠ C

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1(0) + 0(1) & 1(0) + 0(0) \\ 0(0) + 0(1) & 0(0) + 0(0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1(0)+0(0) & 1(0)+0(1) \\ 0(0)+0(0) & 0(0)+0(1) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AC = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

AB = AC = 0

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Question 44.

If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $(3A^2 - 2B + I)$.

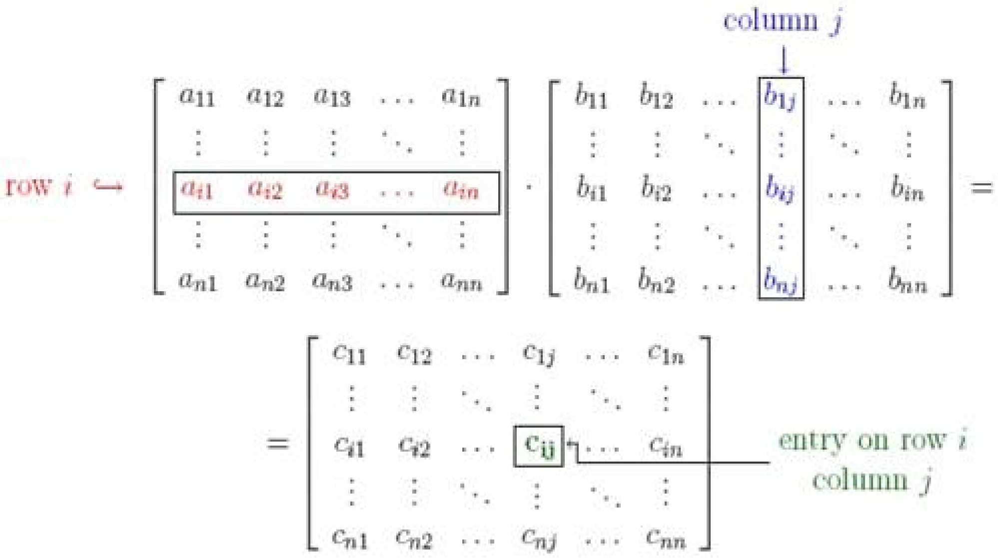
Answer:

Given : $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$,

Matrices A and B are of order 2×2.

To find : $(3A^2 - 2B + I)$.

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots\dots\dots\dots\dots\dots + a_{in}b_{nj}$

A^2 is a matrix of order 2 x 2.

A = $\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 1(1)+0(-1) & 1(0)+0(7) \\ -1(1)+7(-1) & -1(0)+7(7) \end{bmatrix}$

$A^2 = \begin{bmatrix} 1+0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$

$3A^2 = 3 \times \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ -24 & 147 \end{bmatrix}$

$3A^2 = \begin{bmatrix} 3 & 0 \\ -24 & 147 \end{bmatrix}$

$2B = 2 \times \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$

$2B = \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 3 & 0 \\ -24 & 147 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3-0+1 & 0-8+0 \\ -24+2+0 & 147-14+1 \end{bmatrix}$

$3A^2 - 2B + I = \begin{bmatrix} 4 & -8 \\ -22 & 134 \end{bmatrix}$

Question 45.

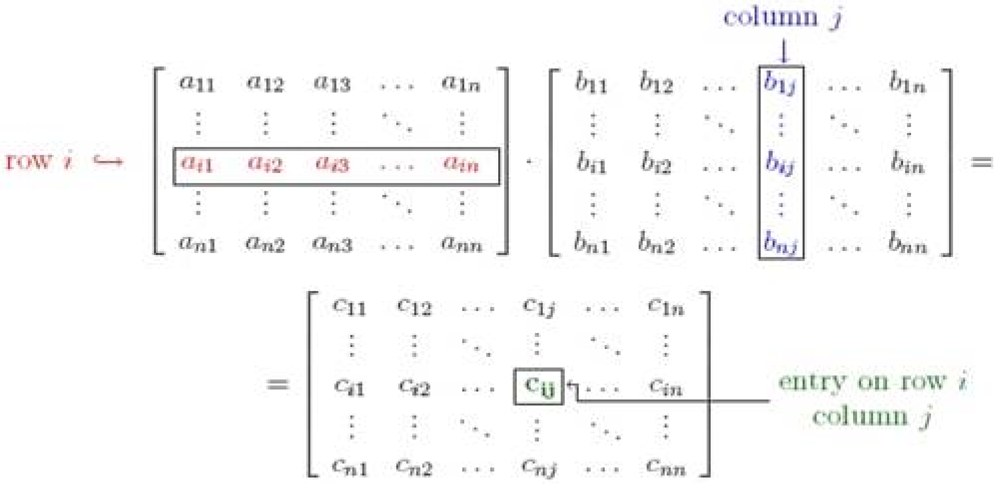
If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, find the value of x.

Answer:

Given : $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$,

To find : x

Formula used :



Where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots\ldots\ldots\ldots\ldots\ldots + a_{in}b_{nj}$

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 2(1) + 3(-2) & 2(-3) + 3(4) \\ 5(1) + 7(-2) & 5(-3) + 7(4) \end{bmatrix} = \begin{bmatrix} 2 - 6 & -6 + 12 \\ 5 - 14 & -15 + 28 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

Equating similar terms in the two matrices, we get

x = 13

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