

Objective Questions

Question 1.

If A and B are 2-rowed square matrices such that

$$(A+B)=\begin{pmatrix} 4 & -3 \\ 1 & 6 \end{pmatrix} \text{ and } (A-B)=\begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix} \text{ then } AB=?$$

A. $\begin{pmatrix} -7 & 5 \\ 1 & -5 \end{pmatrix}$

B. $\begin{pmatrix} 7 & -5 \\ 1 & 5 \end{pmatrix}$

C. $\begin{pmatrix} 7 & -1 \\ 5 & -5 \end{pmatrix}$

D. $\begin{pmatrix} 7 & -1 \\ -5 & 5 \end{pmatrix}$

Answer:

$$(A+B) = \begin{pmatrix} 4 & -3 \\ 1 & 6 \end{pmatrix} \text{-----1}$$

$$(A-B) = \begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix} \text{-----2}$$

$$1+2 \Rightarrow 2A = \begin{pmatrix} 4 & -3 \\ 1 & 6 \end{pmatrix} + \begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix}$$

$$\Rightarrow 2A = \begin{pmatrix} 2 & -4 \\ 6 & 8 \end{pmatrix}$$

Dividing the matrix by 2

$$\Rightarrow A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$$

$$1-2 \Rightarrow 2B = \begin{pmatrix} 4 & -3 \\ 1 & 6 \end{pmatrix} - \begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix}$$

$$\Rightarrow 2B = \begin{pmatrix} 6 & -2 \\ -4 & 4 \end{pmatrix}$$

Dividing the matrix by 2

$$\Rightarrow B = \begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}$$

$$A \times B = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \times 3 + (-2) \times (-2) & (1) \times (-1) + (-2) \times (2) \\ 3 \times 3 + 4 \times (-2) & 3 \times (-1) + 4 \times 2 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -5 \\ 1 & 5 \end{pmatrix}$$

Question 2.

$$\text{If } \begin{pmatrix} 3 & -2 \\ 5 & 6 \end{pmatrix} + 2A = \begin{pmatrix} 5 & 6 \\ -7 & 10 \end{pmatrix} \text{ then } A = ?$$

$$\text{A. } \begin{pmatrix} 1 & 3 \\ -5 & 4 \end{pmatrix}$$

$$\text{B. } \begin{pmatrix} -1 & 5 \\ -3 & 4 \end{pmatrix}$$

$$\text{C. } \begin{pmatrix} 1 & 4 \\ -6 & 2 \end{pmatrix}$$

D. none of these

Answer:

C

$$\begin{pmatrix} 3 & -2 \\ 5 & 6 \end{pmatrix} + 2A = \begin{pmatrix} 5 & 6 \\ -7 & 10 \end{pmatrix}$$

$$\Rightarrow 2A = \begin{pmatrix} 5 & 6 \\ -7 & 10 \end{pmatrix} - \begin{pmatrix} 3 & -2 \\ 5 & 6 \end{pmatrix}$$

$$\Rightarrow 2A = \begin{pmatrix} 2 & 8 \\ -12 & 4 \end{pmatrix}$$

Dividing the matrix by 2

$$\Rightarrow A = \begin{pmatrix} 1 & 4 \\ -6 & 2 \end{pmatrix}$$

Question 3.

If $A = \begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -3 \\ -6 & 2 \end{pmatrix}$ are such that $4A + 3X = 5B$ then $X = ?$

A. $\begin{pmatrix} 4 & -5 \\ -6 & 2 \end{pmatrix}$

B. $\begin{pmatrix} 4 & 5 \\ -6 & -2 \end{pmatrix}$

C. $\begin{pmatrix} -4 & 5 \\ 6 & -2 \end{pmatrix}$

D. none of these

Answer:

$$4A + 3X = 5B$$

$$\Rightarrow 4 \begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix} + 3X = 5 \begin{pmatrix} 4 & -3 \\ -6 & 2 \end{pmatrix}$$

$$\Rightarrow 3X = 5 \begin{pmatrix} 4 & -3 \\ -6 & 2 \end{pmatrix} - 4 \begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix}$$

$$\Rightarrow 3X = \begin{pmatrix} 20 & -15 \\ -30 & 10 \end{pmatrix} - \begin{pmatrix} 8 & 0 \\ -12 & 4 \end{pmatrix}$$

$$\Rightarrow 3X = \begin{pmatrix} 12 & -15 \\ -18 & 6 \end{pmatrix}$$

Dividing by 3

$$\Rightarrow X = \begin{pmatrix} 4 & -5 \\ -6 & 2 \end{pmatrix}$$

Question 4.

If $(A-2B)=\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$ and $(2A-3B)=\begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix}$ then $B=?$

A. $\begin{pmatrix} 6 & -4 \\ -3 & 3 \end{pmatrix}$

B. $\begin{pmatrix} -4 & 6 \\ -3 & -3 \end{pmatrix}$

C. $\begin{pmatrix} 4 & -6 \\ 3 & -3 \end{pmatrix}$

D. none of these

Answer:

B

$$(A-2B) = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$$

Multiplying equation by 2

$$2A-4B = \begin{pmatrix} 2 & -4 \\ 6 & 0 \end{pmatrix} \text{----- (i)}$$

$$2A-3B = \begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix} \text{----- (ii)}$$

(ii)-(i)

$$B = \begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix} - \begin{pmatrix} 2 & -4 \\ 6 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & 6 \\ 3 & -3 \end{pmatrix}$$

Question 5.

If $(2A - B) = \begin{pmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{pmatrix}$ and $(2B + A) = \begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix}$ then $A = ?$

A. $\begin{pmatrix} -3 & 2 & 1 \\ 2 & 1 & -1 \end{pmatrix}$

B. $\begin{pmatrix} 3 & 2 & -1 \\ 2 & -1 & 1 \end{pmatrix}$

C. $\begin{pmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{pmatrix}$

D. none of these

Answer:

$$(2A - B) = \begin{pmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{pmatrix}$$

Multiplying by 2

$$4A - 2B = \begin{pmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{pmatrix} \text{----- (i)}$$

$$2B + A = \begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix} \text{----- (ii)}$$

(i) + (ii)

$$5A = \begin{pmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix}$$

$$= \begin{pmatrix} 15 & 10 & 5 \\ -10 & 5 & -5 \end{pmatrix}$$

Dividing each element of the matrix by 5

$$A = \begin{pmatrix} 3 & 2 & 1 \\ -2 & 1 & -1 \end{pmatrix}$$

Question 6.

$$\text{If } 2 \begin{pmatrix} 3 & 4 \\ 5 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$$

A. $(x=-2, y=8)$

B. $(x=2, y=-8)$

C. $(x=3, y=-6)$

D. $(x=-3, y=6)$

Answer:

$$2 \begin{pmatrix} 3 & 4 \\ 5 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$$

To solve this problem we will use the comparison that is we will use that all the elements of L.H.S are equal to R.H.S .

$$= \begin{pmatrix} 6 & 8 \\ 10 & 2x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 8+y \\ 10 & 2x+1 \end{pmatrix}$$

Comparing with R.H.S

$$8+y = 0$$

$$y = -8$$

$$2x+1 = 5$$

$$2x = 4$$

$$x = 2$$

Question 7.

$$\text{If } \begin{pmatrix} x-y & 2x-y \\ 2x+z & 3z+w \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 5 & 13 \end{pmatrix} \text{ then}$$

A. $z=3, w=4$

B. $z=4, w=3$

C. $z=1, w=2$

D. $z=2, w=-1$

Answer:

A

By comparing L.H.S and R.H.S

$$x - y = -1 \text{ ----- i}$$

$$2x - y = 0 \text{ ----- ii}$$

$$2x + z = 5 \text{ ----- iii}$$

$$3z + w = 13 \text{ ----- iv}$$

Using i in equation ii

$$x = -1 + y$$

ii becomes, $-2 + 2y - y = 0$

$$y = 2$$

$$x = 1$$

Putting x in iii

$$2 + z = 5$$

$$z = 3$$

Putting z in iv

$$9 + w = 13$$

$$w = 4$$

Question 8.

If $\begin{pmatrix} x & y \\ 3y & x \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ then

A. $x=1, y=2$

B. $x=2, y=1$

C. $x=1, y=1$

D. none of these

Answer:

C

$$\begin{pmatrix} x & y \\ 3y & x \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} x \times 1 + y \times 2 \\ 3y \times 1 + x \times 2 \end{pmatrix}$$

$$= \begin{pmatrix} x + 2y \\ 3y + 2x \end{pmatrix}$$

Comparing with R.H.S

$$x + 2y = 3 \text{ ----- (i)}$$

$$2x + 3y = 5 \text{ ----- (ii)}$$

$$(i) \times 2 - (ii)$$

$$2x + 4y - 2x + 3y = 6 - 5$$

$$y = 1$$

Putting y in (i)

$$x + 2(1) = 3$$

$$x = 1$$

Question 9.

If the matrix $A = \begin{pmatrix} 3-2x & x+1 \\ 2 & 4 \end{pmatrix}$ is singular then $x = ?$

- A. 0
- B. 1
- C. -1
- D. -2

Answer:

When a given matrix is singular then the given matrix determinant is 0.

$$|A| = 0$$

$$\text{Given, } A = \begin{pmatrix} 3-2x & x+1 \\ 2 & 4 \end{pmatrix}$$

$$|A| = 0$$

$$4(3-2x) - 2(x+1) = 0$$

$$12 - 8x - 2x - 2 = 0$$

$$10 - 10x = 0$$

$$10x = 0$$

$$x = 1$$

Question 10.

If $A_\alpha = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}$ then $(A_\alpha)^2 = ?$

A. $\begin{pmatrix} \cos^2 \alpha & \sin^2 \alpha \\ -\sin^2 \alpha & \cos^2 \alpha \end{pmatrix}$

B. $\begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix}$

C. $\begin{pmatrix} 2\cos \alpha & 2\sin \alpha \\ -\sin \alpha & 2\cos \alpha \end{pmatrix}$

D. none of these

Answer:

Given, $A_\alpha = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}$

$$A_\alpha^2 = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}$$

$$= \begin{pmatrix} \cos \alpha \times \cos \alpha - \sin \alpha \times \sin \alpha & \cos \alpha \times \sin \alpha + \sin \alpha \times \cos \alpha \\ -\sin \alpha \times \cos \alpha - \cos \alpha \times \sin \alpha & -\sin \alpha \times \sin \alpha + \cos \alpha \times \cos \alpha \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2 \alpha - \sin^2 \alpha & \cos \alpha \sin \alpha + \sin \alpha \cos \alpha \\ -\sin \alpha \cos \alpha - \cos \alpha \sin \alpha & -\sin^2 \alpha + \cos^2 \alpha \end{pmatrix}$$

$$= \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix}$$

Question 11.

If $A = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}$ be such that $A + A' = I$, then $\alpha = ?$

A. π

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer:

L.H.S: $A + A' = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} + \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$

$$= \begin{pmatrix} \cos \alpha + \cos \alpha & \sin \alpha - \sin \alpha \\ -\sin \alpha + \sin \alpha & \cos \alpha + \cos \alpha \end{pmatrix}$$

$$= \begin{pmatrix} 2\cos a & 0 \\ 0 & 2\cos a \end{pmatrix}$$

This will be equal to $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

When $2\cos a = 1$

$$\cos a = \frac{1}{2}$$

$$a = \frac{\pi}{3}$$

Question 12.

If $A = \begin{pmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{pmatrix}$ is singular then $k = ?$

A. $\frac{16}{3}$

B. $\frac{34}{3}$

C. $\frac{33}{2}$

D. none of these

Answer:

When a given matrix is singular then the given matrix determinant is 0.

$$|A| = 0$$

Given,

$$A = \begin{pmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{pmatrix}$$

$$|A| = 0$$

$$1(-4k + 6) - k(-12 + 4) + 3(9 - 2k) = 0$$

$$-4k + 6 + 12k - 4k + 27 - 6k = 0$$

$$-2k + 33 = 0$$

$$k = \frac{33}{2}.$$

Question 13.

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $\text{adj } A = ?$

A. $\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$

B. $\begin{pmatrix} -d & b \\ c & -a \end{pmatrix}$

C. $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

D. $\begin{pmatrix} -d & -b \\ c & a \end{pmatrix}$

Answer:

To find $\text{adj } A$ we will first find the cofactor matrix

$$C_{11} = d \quad C_{12} = -c$$

$$C_{21} = -b \quad C_{22} = a$$

$$\text{Cofactor matrix } A = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$$

$$\text{Adj } A = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix},$$

$$= \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Question 14.

If $A = \begin{pmatrix} 2x & 0 \\ x & x \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$ then $x = ?$

A. 1

B. 2

C. $\frac{1}{2}$

D. -2

Answer:

We know that $A \times A^{-1} = I$

$$\begin{pmatrix} 2x & 0 \\ x & x \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2x \times 1 + 0 \times (-1) & 2x \times 0 + 0 \times 2 \\ x \times 1 + x \times (-1) & x \times 0 + x \times 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2x & 0 \\ 0 & 2x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

To satisfy the above condition $2x = 1$

$$x = \frac{1}{2}$$

Question 15.

If A and B are square matrices of the same order then $(A + B)(A - B) = ?$

A. $(A^2 - B^2)$

B. $A^2 + AB - BA - B^2$

C. $A^2 - AB + BA - B^2$

D. none of these

Answer:

Since A and B are square matrices of same order.

$$(A+B)(A-B) = A^2 - AB + BA - B^2$$

Question 16.

If A and B are square matrices of the same order then $(A + B)^2 = ?$

A. $A^2 + 2AB + B^2$

B. $A^2 + AB + BA + B^2$

C. $A^2 + 2BA + B^2$

D. none of these

Answer:

Since A and B are square matrices of same order.

$$(A + B)^2 = (A + B)(A + B)$$

$$= A^2 + AB + BA + B^2$$

Question 17.

If A and B are square matrices of the same order then $(A - B)^2 = ?$

A. $A^2 - 2AB + B^2$

B. $A^2 - AB - BA + B^2$

C. $A^2 - 2BA + B^2$

D. none of these

Answer:

Since A and B are square matrices of same order.

$$(A - B)^2 = (A - B)(A - B)$$

$$= A^2 - AB - BA + B^2$$

Question 18.

If A and B are symmetric matrices of the same order then $(AB - BA)$ is always

A. a symmetric matrix

B. a skew-symmetric matrix

C. a zero matrix

D. an identity matrix

Answer:

Given A and B are symmetric matrices

$$A' = A \text{ --- 1}$$

$$B' = B \text{ ---- 2}$$

$$\text{Now } (AB - BA)' = (AB)' - (BA)'$$

$$= B'A' - A'B'$$

$$[\because (AB)' = B'A']$$

$$= BA - AB \text{ [Using 1 and 2]}$$

$$\therefore (AB - BA)' = - (AB - BA)$$

AB-BA is a skew symmetric matrix.

Question 19.

Matrices A and B are inverse of each other only when

A. $AB=BA$

B. $AB=BA=0$

C. $AB=0, BA=I$

D. $AB=BA=I$

Answer:

$$A = B^{-1}$$

$$B=A^{-1}$$

We know that

$$AA^{-1} = I$$

$$(\text{Given } B=A^{-1})$$

$$AB = I \text{ ----- 1}$$

We know that

$$BB^{-1} = I$$

$$(\text{Given } A = B^{-1})$$

$$BA = I \text{ ----- 2}$$

From 1 and 2

$$AB = BA = I$$

Question 20.

For square matrices A and B of the same order, we have $\text{adj}(AB) = ?$

A. $(\text{adj } A)(\text{adj } B)$

B. $(\text{adj } B)(\text{adj } A)$

C. $|AB|$

D. none of these

Answer:

We know that $(AB)^{-1} = \text{adj}(AB) / |AB|$

$$\text{adj}(AB) = (AB)^{-1} \cdot |AB|$$

We also know that $(AB)^{-1} = B^{-1} \cdot A^{-1}$

$$|AB| = |A| |B|$$

Putting them in 1

$$\text{Adj}(AB) = B^{-1} \cdot A^{-1} \cdot |A| \cdot |B|$$

$$= (A^{-1} \cdot |A|) (B^{-1} |B|)$$

$$= \text{adj}(A) \text{adj}(B)$$

Since, $\text{adj}(A) = (A)^{-1} \cdot |A|$

$$\text{adj}(B) = (B)^{-1} \cdot |B|$$

Question 21.

If A is a 3-rowed square matrix and $|A|=4$ then $\text{adj}(\text{adj } A)=?$

- A. 4A
- B. 16A
- C. 64A
- D. none of these

Answer:

The property states that

$$\text{adj}(\text{adj } A) = |A|^{n-2} \cdot A$$

Here $n=3$

$$\text{adj}(\text{adj } A) = |4|^{3-2} \cdot A$$

$$= 4A$$

Question 22.

If A is a 3-rowed square matrix and $|A|=5$ then $|\text{adj } A|=?$

- A. 5
- B. 25
- C. 125
- D. none of these

Answer:

The property states that $|\text{adj } A| = |A|^{n-1}$

Here $n= 3$ and $|A|=5$

$$|\text{adj } A| = |5|^{3-1}$$

$$= |5|^2$$

$$= 25.$$

Question 23.

For any two matrices A and B,

A. $AB=BA$ is always true

B. $AB=BA$ is never true

C. sometimes $AB=BA$ and sometimes $AB \neq BA$

D. whenever AB exists, then BA exists

Answer:

If the two matrices A and B are of same order it is not necessary that in every situation $AB=BA$

$AB=BA=I$ only when $A=B^{-1}$

$B=A^{-1}$

Other time $AB \neq BA$

Question 24.

If $A \begin{pmatrix} 3 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$ then A=?

A. $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

B. $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

D. none of these

Answer:

The matrix on the R.H.S of the given matrix is of order 2×2 and the one given on left side is 2×2 . Therefore A has to be a 2×2 matrix.

Let $A = \begin{pmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 3a + b & 2a - b \\ 3c + d & 2c - d \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$$

$$3a + b = 4 \text{ ----- 1}$$

$$2a - b = 1 \text{ ----- 2}$$

$$3c + d = 2 \text{ ----- 3}$$

$$2c - d = 3 \text{ ----- 4}$$

Using 1 and 2

$$a = 1$$

$$b = 1$$

Using 3 and 4

$$c = 1$$

$$d = -1$$

$$\text{So A becomes } \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Question 25.

If A is an invertible square matrix then $|A^{-1}| = ?$

A. $|A|$

B. $\frac{1}{|A|}$

C. 1

D. 0

Answer:

B

We know that $AA^{-1} = \mathbf{I}$

Taking determinant both sides

$$|AA^{-1}| = |\mathbf{I}|$$

$$|A||A^{-1}| = |\mathbf{I}| \quad (|AB| = |A||B|)$$

$$|A||A^{-1}| = 1 \quad (|\mathbf{I}| = 1)$$

$$|A^{-1}| = \frac{1}{|A|}$$

Question 26.

If A and B are invertible matrices of the same order then $(AB)^{-1} = ?$

A. $(A^{-1} \times B^{-1})$

B. $(A \times B^{-1})$

C. $(A^{-1} \times B)$

D. $(B^{-1} \times A^{-1})$

Answer:

$$(AB)(AB)^{-1} = \mathbf{I}$$

$$A^{-1}(AB)(AB)^{-1} = \mathbf{I}A^{-1}$$

$$(A^{-1}A)B(AB)^{-1} = A^{-1}$$

$$\mathbf{I}B(AB)^{-1} = A^{-1}$$

$$B(AB)^{-1} = A^{-1}$$

$$B^{-1}B(AB)^{-1} = B^{-1}A^{-1}$$

$$\mathbf{I}(AB)^{-1} = B^{-1}A^{-1}$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

Question 27.

If A and B are two nonzero square matrices of the same order such that $AB=0$ then

A. $|A|=0$ or $|B|=0$

B. $|A|=0$ and $|B|=0$

C. $|A|\neq 0$ and $|B|\neq 0$

D. None of these

Answer:

As AB is a 0 matrix its determinant has to be 0.

So $|AB|=|A||B|=0$

So $|A|=|B|=0$

Question 28.

If A is a square matrix such that $|A|\neq 0$ and $A^2 - A + 2I = 0$ then $A^{-1}=?$

A. $(I-A)$

B. $(I+A)$

C. $\frac{1}{2}(I - A)$

D. $\frac{1}{2}(I + A)$

Answer:

$$A^2 - A + 2I = 0$$

Multiplying by A^{-1}

$$A^{-1}A^2 - A^{-1}A + 2I A^{-1} = 0$$

$$A - I + 2 A^{-1} = 0$$

$$A^{-1} = \frac{1}{2}(I - A)$$

Question 29.

If $A = \begin{pmatrix} 1 & \lambda & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{pmatrix}$ is not invertible then $\lambda=?$

A. 2

B. 1

C. -1

D. 0

Answer:

$$= \begin{pmatrix} 1 & \lambda & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{pmatrix}$$

$$|A|=0$$

$$1(2 \times 1 - 5 \times 1) - \lambda (1 \times 1 - 5 \times 2) + 2 (1 \times 1 - 2 \times 2) = 0$$

$$-3 + 9\lambda - 6 = 0$$

$$9\lambda = 9$$

$$\lambda = 1$$

Question 30.

$$\text{If } A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \text{ then } A^{-1} = ?$$

A. A

B. $-A$

C. $\text{Adj } A$

D. $-\text{adj } A$

Answer:

$$A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

$$|A| = \cos^2 \theta - (-\sin^2 \theta)$$

$$= \cos^2 \theta + (\sin^2 \theta)$$

$$= 1 \text{ ----- (I)}$$

We know that $A^{-1} = \frac{1}{|A|} \text{adj } A$

$= \text{adj } A$ [From I]

Question 31.

The matrix $A = \begin{pmatrix} ab & b^2 \\ -a^2 & -ab \end{pmatrix}$ is

- A. idempotent
- B. Orthogonal
- C. Nilpotent
- D. None of these

Answer:

Matrix A is said to be nilpotent since there exist a positive integer $k=1$ such that A^k is zero matrix.

Question 32.

The matrix $A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$ is

- A. Nonsingular
- B. Idempotent
- C. Nilpotent
- D. Orthogonal

Answer:

Here the diagonal value is $2+3-3= 1$

So the given matrix is idempotent.

Question 33.

If A is singular then $A(\text{adj}A)=?$

- A. A unit matrix
- B. A null matrix
- C. A symmetric matrix

D. None of these

Answer:

$$A(\text{adj}A) = A(|A| \times A^{-1})$$

Since determinant of singular matrix is always 0

$$A(\text{adj}A) = 0$$

So, it is a null matrix.

Question 34.

For any 2-rowed square matrix A, if $A(\text{adj}A) = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$ then the value of $|A|$ is

A. 0

B. 8

C. 64

D. 4

Answer:

$$(\text{adj}A) = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

$$= 8 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= |A|I$$

$$|A| = 8.$$

Question 35.

If $A = \begin{pmatrix} -2 & 3 \\ 1 & 1 \end{pmatrix}$ then $|A^{-1}| = ?$

A. -5

B. $-\frac{1}{5}$

C. $\frac{1}{25}$

D. 25

Answer:

$$A = \begin{pmatrix} -2 & 3 \\ 1 & 1 \end{pmatrix}$$

$$|A| = -2 - 3 = -5$$

We know that $|A^{-1}| = \frac{1}{|A|}$

$$= \frac{1}{-5}$$

Question 36.

If $A = \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$ and $A^2 + xI = yA$ then the values of x and y are

A. $x=6, y=6$

B. $x=8, y=8$

C. $x=5, y=8$

D. $x=6, y=8$

Answer:

$$A^2 + xI = yA$$

$$\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix} + x \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = y \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 16 & 8 \\ 56 & 32 \end{pmatrix} + x \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = y \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$$

$$8 \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix} + x \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = y \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$$

Comparing L.H.S and R.H.S

$$x=8 \quad y=8$$

Question 37.

If matrices A and B anticommute then

A. $AB=BA$

B. $AB=-BA$

C. $(AB)=(BA^{-1})$

D. None of these

Answer:

If A and B anticommute then $AB= -BA$

Question 38.

If $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ then $\text{adj } A = ?$

A. $\begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$

B. $\begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

C. $\begin{pmatrix} -1 & 2 \\ 3 & -5 \end{pmatrix}$

D. None of these

Answer:

To find $\text{adj } A$ we will first find the cofactor matrix

$$C_{11} = 3 \quad C_{12} = -1$$

$$C_{21} = -5 \quad C_{22} = 2$$

$$\text{Cofactor matrix } A = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$$

$$\text{Adj } A = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix},$$

$$= \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

Question 39.

If $A = \begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}$ and B is a square matrix of order 2 such that $AB=I$ then B=?

A. $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$

B. $\begin{pmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{pmatrix}$

C. $\begin{pmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}$

D. None of these

Answer:

$$B=I$$

$$B = A^{-1} \mid \text{-----} 1$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A \text{ ----- } 2$$

$$|A| = 3 \times 2 - (-4) \times (-1)$$

$$= 2$$

$$C_{11} = 2 \quad C_{12} = 1$$

$$C_{21} = 4 \quad C_{22} = 3$$

$$\text{Cofactor matrix } A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$$

$$\text{Adj } A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix},$$

$$= \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$$

Putting in 2

$$A^{-1} = \frac{1}{|2|} \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}$$

Putting in 1

$$B = A^{-1} I$$

$$= A^{-1}$$

$$= \begin{pmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}$$

Question 40.

If A and B are invertible square matrices of the same order then $(AB)^{-1} = ?$

A. AB^{-1}

B. $A^{-1}B$

C. $A^{-1}B^{-1}$

D. $B^{-1}A^{-1}$

Answer:

$$(AB)(AB)^{-1} = I$$

$$A^{-1}(AB)(AB)^{-1} = IA^{-1}$$

$$(A^{-1}A)B(AB)^{-1} = A^{-1}$$

$$IB(AB)^{-1} = A^{-1}$$

$$B(AB)^{-1} = A^{-1}$$

$$B^{-1}B(AB)^{-1} = B^{-1}A^{-1}$$

$$I(AB)^{-1} = B^{-1}A^{-1}$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

Question 41.

If $A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$, then $A^{-1} = ?$

A. $\begin{pmatrix} \frac{3}{7} & \frac{-1}{7} \\ \frac{1}{7} & \frac{2}{7} \end{pmatrix}$

B. $\begin{pmatrix} \frac{3}{7} & \frac{1}{7} \\ \frac{-1}{7} & \frac{2}{7} \end{pmatrix}$

C. $\begin{pmatrix} \frac{1}{3} & \frac{1}{7} \\ \frac{1}{7} & \frac{2}{7} \end{pmatrix}$

D. None of these

Answer:

$$A^{-1} = \frac{1}{|A|} \text{adj } A \text{ ----- } 1$$

$$|A| = 3 \times 2 - (1) \times (-1)$$

$$= 7$$

$$C_{11} = 3 \quad C_{12} = -1$$

$$C_{21} = 1 \quad C_{22} = 2$$

$$\text{Cofactor matrix } A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$$

$$\text{Adj } A = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix},$$

$$= \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

Putting in 1

$$A^{-1} = \frac{1}{|A|} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{7} & \frac{1}{7} \\ \frac{-1}{7} & \frac{2}{7} \end{pmatrix}$$

Question 42.

If $|A|=3$ and $A^{-1} = \begin{pmatrix} 3 & -1 \\ -5 & \frac{2}{3} \end{pmatrix}$ then $\text{adj } A = ?$

A. $\begin{pmatrix} 9 & 3 \\ -5 & -2 \end{pmatrix}$

B. $\begin{pmatrix} 9 & -3 \\ -5 & 2 \end{pmatrix}$

C. $\begin{pmatrix} -9 & 3 \\ 5 & -2 \end{pmatrix}$

D. $\begin{pmatrix} 9 & -3 \\ 5 & -2 \end{pmatrix}$

Answer:

$$^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{adj } A = |A| \times A^{-1}$$

$$= 3 \times \begin{pmatrix} 3 & -1 \\ -5 & \frac{2}{3} \end{pmatrix}$$

$$= \begin{pmatrix} 9 & -3 \\ -5 & 2 \end{pmatrix}$$

Question 43.

If A is an invertible matrix and $A^{-1} = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$ then A=?

A. $\begin{pmatrix} 6 & -4 \\ -5 & 3 \end{pmatrix}$

B. $\begin{pmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{1}{5} & \frac{1}{6} \end{pmatrix}$

C. $\begin{pmatrix} -3 & 2 \\ 5 & -3 \\ 2 & 2 \end{pmatrix}$

D. None of these

Answer:

y property of inverse

$$(A^{-1})^{-1} = A$$

$$(A^{-1})^{-1} = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}^{-1}$$

$$A = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}^{-1} \text{ ----- } 1$$

$$|A|^{-1} = 3 \times 6 - 4 \times 5$$

$$= -2$$

$$C_{11} = 6 \quad C_{12} = -5$$

$$C_{21} = -4 \quad C_{22} = 3$$

$$\text{Cofactor matrix } A = \begin{pmatrix} 6 & -5 \\ -4 & 3 \end{pmatrix}$$

$$\text{Adj } A = \begin{pmatrix} 6 & -4 \\ -5 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}^{-1} = \frac{1}{-2} \begin{pmatrix} 6 & -4 \\ -5 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 2 \\ \frac{5}{2} & \frac{-3}{2} \end{pmatrix}$$

Putting in 1

$$A = \begin{pmatrix} -3 & 2 \\ \frac{5}{2} & \frac{-3}{2} \end{pmatrix}$$

Question 44.

If $A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$ and $f(x) = 2x^2 - 4x + 5$ then $f(A) = ?$

A. $\begin{pmatrix} 19 & -32 \\ -16 & 51 \end{pmatrix}$

B. $\begin{pmatrix} 19 & -16 \\ -32 & 51 \end{pmatrix}$

C. $\begin{pmatrix} 19 & -11 \\ -27 & 51 \end{pmatrix}$

D. None of these

Answer:

$$f(A) = 2A^2 - 4A + 5I$$

$$A^2 = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix}$$

$$f(A) = 2A^2 - 4A + 5I$$

$$= 2 \begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix} - 4 \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} + 5 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 18 & -8 \\ -16 & 34 \end{pmatrix} - \begin{pmatrix} 4 & 8 \\ 16 & -12 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 19 & -16 \\ -32 & 51 \end{pmatrix}$$

Question 45.

If $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ then $A^2 - 4A = ?$

A. I

B. 5I

C. 3I

D. 0

Answer:

$$A^2 = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 16 \\ 8 & 17 \end{pmatrix}$$

$$A^2 - 4A = \begin{pmatrix} 9 & 16 \\ 8 & 17 \end{pmatrix} - 4 \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 16 \\ 8 & 17 \end{pmatrix} - \begin{pmatrix} 4 & 16 \\ 8 & 12 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= 5 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= 5I$$

Question 46.

If A is a 2-rowed square matrix and $|A|=6$ then $A \cdot \text{adj}A = ?$

A. $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$

B. $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

C. $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

D. None of these

Answer:

$$(\text{adj } A) = |A|I$$

$$= 6 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$

Question 47.

If A is an invertible square matrix and k is a non-negative real number then $(KA)^{-1} = ?$

A. $k \cdot A^{-1}$

B. $\frac{1}{k} \cdot A^{-1}$

C. $-k \cdot A^{-1}$

D. None of these

Answer:

By the property of inverse

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$(KA)^{-1} = A^{-1}K^{-1}$$

$$= \frac{1}{K} A^{-1}$$

Question 48.

If $A = \begin{pmatrix} 3 & 4 & 1 \\ 1 & 0 & -2 \\ -2 & -1 & 2 \end{pmatrix}$ then $A^{-1} = ?$

A. $\begin{pmatrix} 2 & 9 & -8 \\ -2 & 8 & 7 \\ -1 & 5 & -4 \end{pmatrix}$

B. $\begin{pmatrix} -2 & 9 & -8 \\ 2 & 8 & 7 \\ -1 & -5 & 4 \end{pmatrix}$

C. $\begin{pmatrix} -2 & -9 & -8 \\ 2 & 8 & 7 \\ -1 & -5 & -4 \end{pmatrix}$

D. None of these

Answer:

$$|A| = 3 \times (0 - 2) - 4 \times (2 - 4) + 1 \times (-1)$$

$$= -6 + 8 - 1$$

$$= 1$$

$$C_{11} = -2 \quad C_{12} = 2 \quad C_{13} = -1$$

$$C_{21} = -9 \quad C_{22} = 8 \quad C_{23} = -5$$

$$C_{31} = -8 \quad C_{32} = 7 \quad C_{33} = -4$$

$$\text{Cofactor } (A) = \begin{bmatrix} -2 & 2 & -1 \\ -9 & 8 & -5 \\ -8 & 7 & -4 \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} -2 & 2 & -1 \\ -9 & 8 & -5 \\ -8 & 7 & -4 \end{bmatrix}'$$

$$= \begin{bmatrix} -2 & -9 & -8 \\ 2 & 8 & 7 \\ -1 & -5 & -4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$= \frac{1}{1} \begin{bmatrix} -2 & -9 & -8 \\ 2 & 8 & 7 \\ -1 & -5 & -4 \end{bmatrix}$$

$$\begin{pmatrix} -2 & -9 & -8 \\ 2 & 8 & 7 \\ -1 & -5 & -4 \end{pmatrix}$$

Question 49.

If A is a square matrix then (A + A') is

- A. A null matrix
- B. An identity matrix
- C. A symmetric matrix
- D. A skew-symmetric matrix

Answer:

Let X = A + A'

$$X' = (A + A')'$$

$$= A' + (A')'$$

$$= A + A'$$

$$= X$$

Therefore (A + A') is symmetric matrix.

Question 50.

If A is a square matrix then (A - A') is

- A. A null matrix
- B. An identity matrix

C. A symmetric matrix

D. A skew-symmetric matrix

Answer:

Let $X = A - A'$

$$X' = (A - A')'$$

$$= A' - (A')'$$

$$= A' - A$$

$$= -(A - A')$$

$$= -X$$

Therefore $(A - A')$ is skew symmetric matrix.

Question 51.

If A is a 3-rowed square matrix and $|3A| = k|A|$ then $k = ?$

A. 3 B.9

C. 27 D.1

Answer:

Since the matrix is of order 3 so 3 will be taken common from each row or column.

So, $k = 27$

Tagging

Question 52.

Which one of the following is a scalar matrix?

A. $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

B. $\begin{pmatrix} 6 & 0 \\ 0 & 3 \end{pmatrix}$

$$c. \begin{pmatrix} -8 & 0 \\ 0 & -8 \end{pmatrix}$$

D. None of these

Answer:

$$= \begin{pmatrix} -8 & 0 \\ 0 & -8 \end{pmatrix}$$

$$= -8 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Since -8 could be taken common from each row or column. Hence C is a scalar matrix.

Question 53.

$$\text{If } A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \text{ and } B = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix} \text{ and}$$

$$(A + B)^2 = (A^2 + B^2) \text{ then}$$

$$A. a = 2, b = -3$$

$$B. a = -2, b = 3$$

$$C. a = 1, b = 4$$

D. none of these

Answer:

$$= \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} B = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$$

$$A+B = \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}$$

$$(A+B)^2 = \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix} \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}$$

$$= \begin{pmatrix} (1+a)^2 & 0 \\ (2+b)(1+a) - 4 - 2b & -4 \end{pmatrix}$$

$$= \begin{pmatrix} (1+a)^2 & 0 \\ 2+2a+b+ab-4-2b & 4 \end{pmatrix}$$

$$= \begin{pmatrix} (1+a)^2 & 0 \\ 2a+ab-b-2 & 4 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix} \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$$

$$= \begin{pmatrix} a^2 + b & a - 1 \\ ab - b & b + 1 \end{pmatrix}$$

$$(A + B)^2 = (A^2 + B^2)$$

$$\begin{pmatrix} (1+a)^2 & 0 \\ 2a+ab-b-2 & 4 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} a^2 + b & a - 1 \\ ab - b & b + 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 + a^2 + b & a - 1 \\ ab - b & b \end{pmatrix}$$

By comparison,

$$a - 1 = 0$$

$$a = 1$$

$$b = 4$$