

Exercise 31

Question 1.

Find the mean (μ), variance (σ^2) and standard deviation (σ) for each of the following probability distributions:

(i)

	0	1	2	3
0	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

(ii)

	1	2	3	4
	0.4	0.3	0.2	0.1

(iii)

	-3	-1	0	2
	0.2	0.4	0.3	0.1

(iv)

	-2	-1	0	1	2
	0.1	0.2	0.4	0.2	0.1

Answer:

(i) Given :

	0	1	2	3
0	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

To find : mean (μ), variance (σ^2) and standard deviation (σ)

Formula used :

	x_1	x_2	x_3	x_4
()	(1)	(2)	(3)	(4)

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Standard deviation} = \sqrt{E(X^2) - E(X)^2}$$

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4)$$

$$\text{Mean} = E(X) = 0\left(\frac{1}{6}\right) + 1\left(\frac{1}{2}\right) + 2\left(\frac{3}{10}\right) + 3\left(\frac{1}{30}\right) = 0 + \frac{1}{2} + \frac{6}{10} + \frac{3}{30} = \frac{15+18+3}{30} = \frac{36}{30} = \frac{6}{5}$$

$$\text{Mean} = E(X) = \frac{6}{5} = 1.2$$

$$E(X)^2 = (1.2)^2 = 1.44$$

$$E(X^2) = \sum_{i=1}^{i=n} (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3) + (x_4)^2 \cdot P(x_4)$$

$$E(X^2) = (0)^2\left(\frac{1}{6}\right) + (1)^2\left(\frac{1}{2}\right) + (2)^2\left(\frac{3}{10}\right) + (3)^2\left(\frac{1}{30}\right) = 0 + \frac{1}{2} + \frac{12}{10} + \frac{9}{30} = \frac{15+36+9}{30} = \frac{60}{30}$$

$$E(X^2) = 2$$

$$\text{Variance} = E(X^2) - E(X)^2 = 2 - 1.44 = 0.56$$

$$\text{Variance} = E(X^2) - E(X)^2 = 0.56$$

$$\text{Standard deviation} = \sqrt{E(X^2) - E(X)^2} = \sqrt{0.56} = 0.74$$

$$\text{Mean} = 1.2$$

$$\text{Variance} = 0.56$$

$$\text{Standard deviation} = 0.74$$

(ii) Given :

	1	2	3	4
	0.4	0.3	0.2	0.1

To find : mean (μ), variance (σ^2) and standard deviation (σ)

Formula used :

	x_1	x_2	x_3	x_4
()	(1)	(2)	(3)	(4)

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Standard deviation} = \sqrt{E(X^2) - E(X)^2}$$

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4)$$

$$\text{Mean} = E(X) = 1(0.4) + 2(0.3) + 3(0.2) + 4(0.1) = 0.4 + 0.6 + 0.6 + 0.4 = 2$$

$$\text{Mean} = E(X) = 2$$

$$E(X)^2 = (2)^2 = 4$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3) + (x_4)^2 \cdot P(x_4)$$

$$E(X^2) = (1)^2(0.4) + (2)^2(0.3) + (3)^2(0.2) + (4)^2(0.1) = 0.4 + 1.2 + 1.8 + 1.6 = 5$$

$$E(X^2) = 5$$

$$\text{Variance} = E(X^2) - E(X)^2 = 5 - 4 = 1$$

$$\text{Variance} = E(X^2) - E(X)^2 = 1$$

$$\text{Standard deviation} = \sqrt{E(X^2) - E(X)^2} = \sqrt{1} = 1$$

Mean = 2

Variance = 1

Standard deviation = 1

(iii) Given :

	-3	-1	0	2
	0.2	0.4	0.3	0.1

To find : mean (μ), variance (σ^2) and standard deviation (σ)

Formula used :

	x_1	x_2	x_3	x_4
()	(1)	(2)	(3)	(4)

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Standard deviation} = \sqrt{E(X^2) - E(X)^2}$$

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4)$$

$$\text{Mean} = E(X) = -3(0.2) + (-1)(0.4) + 0(0.3) + 2(0.1) = -0.6 - 0.4 + 0 + 0.2 = -0.8$$

$$\text{Mean} = E(X) = -0.8$$

$$E(X)^2 = (-0.8)^2 = 0.64$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3) + (x_4)^2 \cdot P(x_4)$$

$$E(X^2) = (-3)^2(0.2) + (-1)^2(0.4) + (0)^2(0.3) + (2)^2(0.1) = 1.8 + 0.4 + 0 + 0.4 = 2.6$$

$$E(X^2) = 2.6$$

$$\text{Variance} = E(X^2) - E(X)^2 = 2.6 - 0.64 = 1.96$$

$$\text{Variance} = E(X^2) - E(X)^2 = 1.96$$

$$\text{Standard deviation} = \sqrt{E(X^2) - E(X)^2} = \sqrt{1.96} = 1.4$$

$$\text{Mean} = -0.8$$

$$\text{Variance} = 1.96$$

$$\text{Standard deviation} = 1.4$$

(iv) Given :

	-2	-1	0	1	2
	0.1	0.2	0.4	0.2	0.1

To find : mean (μ), variance (σ^2) and standard deviation (σ)

Formula used :

	x_1	x_2	x_3	x_4	x_5
	(1)	(2)	(3)	(4)	(5)

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Standard deviation} = \sqrt{E(X^2) - E(X)^2}$$

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4) + x_5 P(x_5)$$

$$\text{Mean} = E(X) = -2(0.1) + (-1)(0.2) + 0(0.4) + 1(0.2) + 2(0.1)$$

$$\text{Mean} = E(X) = -0.2 - 0.2 + 0 + 0.2 + 0.2 = 0$$

$$\text{Mean} = E(X) = 0$$

$$E(X)^2 = (0)^2 = 0$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 P(x_1) + (x_2)^2 P(x_2) + (x_3)^2 P(x_3) + (x_4)^2 P(x_4) + (x_5)^2 P(x_5)$$

$$E(X^2) = (-2)^2(0.1) + (-1)^2(0.2) + (0)^2(0.4) + (1)^2(0.2) + (2)^2(0.1)$$

$$E(X^2) = 0.4 + 0.2 + 0 + 0.2 + 0.4 = 1.2$$

$$E(X^2) = 1.2$$

$$\text{Variance} = E(X^2) - E(X)^2 = 1.2 - 0 = 1.2$$

$$\text{Variance} = E(X^2) - E(X)^2 = 1.2$$

$$\text{Standard deviation} = \sqrt{E(X^2) - E(X)^2} = \sqrt{1.2} = 1.095$$

Mean = 0

Variance = 1.2

Standard deviation = 1.095

Question 2.

Find the mean and variance of the number of heads when two coins are tossed simultaneously.

Answer:

Given : Two coins are tossed simultaneously

To find : mean (μ), variance (σ^2)

Formula used :

	x_1	x_2	x_3
()	(1)	(2)	(3)

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

When two coins are tossed simultaneously,

Total possible outcomes = TT , TH , HT , HH where H denotes head and T denotes tail.

$$P(0) = \frac{1}{4} \text{ (zero heads = 1 [TT])}$$

$$P(1) = \frac{2}{4} \text{ (one heads = 2 [HT , TH])}$$

$$P(2) = \frac{1}{4} \text{ (two heads = 1 [HH])}$$

The probability distribution table is as follows,

	0	1	2
()	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

$$\text{Mean} = E(X) = 0\left(\frac{1}{4}\right) + 1\left(\frac{2}{4}\right) + 2\left(\frac{1}{4}\right) = 0 + \frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$$

$$\text{Mean} = E(X) = 1$$

$$E(X)^2 = (1)^2 = 1$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3)$$

$$E(X^2) = (0)^2\left(\frac{1}{4}\right) + (1)^2\left(\frac{2}{4}\right) + (2)^2\left(\frac{1}{4}\right) = 0 + \frac{2}{4} + \frac{4}{4} = \frac{6}{4} = \frac{3}{2} = 1.5$$

$$E(X^2) = 1.5$$

$$\text{Variance} = E(X^2) - E(X)^2 = 1.5 - 1 = 0.5$$

$$\text{Variance} = E(X^2) - E(X)^2 = 0.5$$

$$\text{Mean} = 1$$

$$\text{Variance} = 0.5$$

Question 3.

Find the mean and variance of the number of tails when three coins are tossed simultaneously.

Answer:

Given : Three coins are tossed simultaneously

To find : mean (μ) and variance (σ^2)

Formula used :

	x_1	x_2	x_3	x_4
()	(1)	(2)	(3)	(4)

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

When three coins are tossed simultaneously,

Total possible outcomes = TTT , TTH , THT , HTT , THH , HTH , HHT , HHH where H denotes head and T denotes tail.

$$P(0) = \frac{1}{8} \text{ (zero tails = 1 [HHH])}$$

$$P(1) = \frac{3}{8} \text{ (one tail = 3 [HTH , THH , HHT])}$$

$$P(2) = \frac{3}{8} \text{ (two tail = 3 [HTT , THT , TTH])}$$

$$P(3) = \frac{1}{8} \text{ (three tails = 1 [TTT])}$$

The probability distribution table is as follows,

	0	1	2	3
()	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4)$$

$$\text{Mean} = E(X) = 0\left(\frac{1}{8}\right) + 1\left(\frac{3}{8}\right) + 2\left(\frac{3}{8}\right) + 3\left(\frac{1}{8}\right) = 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{3+6+3}{8} = \frac{12}{8} = \frac{3}{2}$$

$$\text{Mean} = E(X) = \frac{3}{2} = 1.5$$

$$E(X)^2 = (1.5)^2 = 2.25$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3) + (x_4)^2 \cdot P(x_4)$$

$$E(X^2) = (0)^2\left(\frac{1}{8}\right) + (1)^2\left(\frac{3}{8}\right) + (2)^2\left(\frac{3}{8}\right) + (3)^2\left(\frac{1}{8}\right) = 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8} = \frac{3+12+9}{8} = \frac{24}{8} = 3$$

$$E(X^2) = 3$$

$$\text{Variance} = E(X^2) - E(X)^2 = 3 - 2.25 = 0.75$$

$$\text{Variance} = E(X^2) - E(X)^2 = 0.75$$

$$\text{Mean} = 1.5$$

$$\text{Variance} = 0.75$$

Question 4.

A die is tossed twice. 'Getting an odd number on a toss' is considered a success. Find the probability distribution of a number of successes. Also, find the mean and variance of the number of successes.

Answer:

Given : A die is tossed twice and 'Getting an odd number on a toss' is considered a success.

To find : probability distribution of the number of successes and mean (μ) and variance (σ^2)

Formula used :

	x_1	x_2	x_3
()	(1)	(2)	(3)

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

When a die is tossed twice,

Total possible outcomes =

{(1,1) , (1,2) , (1,3) , (1,4) , (1,5) , (1,6)

(2,1) , (2,2) , (2,3) , (2,4) , (2,5) , (2,6)

(3,1) , (3,2) , (3,3) , (3,4) , (3,5) , (3,6)

(4,1) , (4,2) , (4,3) , (4,4) , (4,5) , (4,6)

(5,1) , (5,2) , (5,3) , (5,4) , (5,5) , (5,6)

(6,1) , (6,2) , (6,3) , (6,4) , (6,5) , (6,6)}

'Getting an odd number on a toss' is considered a success.

$$P(0) = \frac{9}{36} = \frac{1}{4} \text{ (zero odd numbers = 9)}$$

$$P(1) = \frac{18}{36} = \frac{1}{2} \text{ (one odd number = 18)}$$

$$P(2) = \frac{9}{36} = \frac{1}{4} \text{ (two odd numbers = 9)}$$

The probability distribution table is as follows,

	0	1	2
()	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$\text{Mean} = E(X) = 0\left(\frac{1}{4}\right) + 1\left(\frac{1}{2}\right) + 2\left(\frac{1}{4}\right) = 0 + \frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$$

$$\text{Mean} = E(X) = 1$$

$$E(X)^2 = (1)^2 = 1$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3)$$

$$E(X^2) = (0)^2\left(\frac{1}{4}\right) + (1)^2\left(\frac{1}{2}\right) + (2)^2\left(\frac{1}{4}\right) = 0 + \frac{2}{4} + \frac{4}{4} = \frac{6}{4} = \frac{3}{2} = 1.5$$

$$E(X^2) = 1.5$$

$$\text{Variance} = E(X^2) - E(X)^2 = 1.5 - 1 = 0.5$$

$$\text{Variance} = E(X^2) - E(X)^2 = 0.5$$

The probability distribution table is as follows,

	0	1	2
()	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Mean = 1

Variance = 0.5

Question 5.

A die is tossed twice. 'Getting a number greater than 4' is considered a success. Find the probability distribution of a number of successes. Also, find the mean and variance of the number of successes.

Answer:

Given : A die is tossed twice and 'Getting a number greater than 4 ' is considered a success.

To find : probability distribution of the number of successes and mean (μ) and variance (σ^2)

Formula used :

	x_1	x_2	x_3
()	(1)	(2)	(3)

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

When a die is tossed twice,

Total possible outcomes =

{(1,1) , (1,2) , (1,3) , (1,4) , (1,5) , (1,6)

(2,1) , (2,2) , (2,3) , (2,4) , (2,5) , (2,6)

(3,1) , (3,2) , (3,3) , (3,4) , (3,5) , (3,6)

(4,1) , (4,2) , (4,3) , (4,4) , (4,5) , (4,6)

(5,1) , (5,2) , (5,3) , (5,4) , (5,5) , (5,6)

(6,1) , (6,2) , (6,3) , (6,4) , (6,5) , (6,6)}

'Getting a number greater than 4' is considered a success.

$$P(0) = \frac{16}{36} = \frac{4}{9} \text{ (zero numbers greater than 4 = 16)}$$

$$P(1) = \frac{16}{36} = \frac{4}{9} \text{ (one number greater than 4 = 16)}$$

$$P(2) = \frac{4}{36} = \frac{1}{9} \text{ (two numbers greater than 4 = 4)}$$

The probability distribution table is as follows,

	0	1	2
()	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

$$\text{Mean} = E(X) = 0\left(\frac{4}{9}\right) + 1\left(\frac{4}{9}\right) + 2\left(\frac{1}{9}\right) = 0 + \frac{4}{9} + \frac{2}{9} = \frac{4+2}{9} = \frac{6}{9} = \frac{2}{3}$$

$$\text{Mean} = E(X) = \frac{2}{3}$$

$$E(X)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3)$$

$$E(X^2) = (0)^2 \left(\frac{4}{9}\right) + (1)^2 \left(\frac{4}{9}\right) + (2)^2 \left(\frac{1}{9}\right) = 0 + \frac{4}{9} + \frac{4}{9} = \frac{8}{9}$$

$$E(X^2) = \frac{8}{9}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{8}{9} - \frac{4}{9} = \frac{4}{9}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{4}{9}$$

The probability distribution table is as follows,

	0	1	2
()	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

$$\text{Mean} = \frac{2}{3}$$

$$\text{Variance} = \frac{4}{9}$$

Question 6.

A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of a number of successes, find the probability distribution of the number of successes. Also, find the mean and variance of a number of successes. [CBSE 2008]

Answer:

Given : A die is tossed twice and 'Getting a number greater than 4 ' is considered a success.

To find : probability distribution of the number of successes and mean (μ) and variance (σ^2)

Formula used :

	x_1	x_2	x_3	x_4	x_5
	(1)	(2)	(3)	(4)	(5)

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

When a die is tossed 4 times,

$$\text{Total possible outcomes} = 6^2 = 36$$

Getting a doublet is considered as a success

The possible doublets are (1,1) , (2,2) , (3,3) , (4,4) , (5,5) , (6,6)

Let p be the probability of success,

$$p = \frac{6}{36} = \frac{1}{6}$$

$$q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

$$q = \frac{5}{6}$$

since the die is thrown 4 times, $n = 4$

x can take the values of 1,2,3,4

$$P(x) = {}^n C_x p^x q^{n-x}$$

$$P(0) = {}^4C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$$

$$P(1) = {}^4C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^3 = \frac{500}{1296} = \frac{125}{324}$$

$$P(2) = {}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = \frac{150}{1296} = \frac{25}{216}$$

$$P(3) = {}^4C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^1 = \frac{20}{1296} = \frac{5}{324}$$

$$P(4) = {}^4C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^0 = \frac{1}{1296}$$

The probability distribution table is as follows,

	0	1	2	3	4
	$\frac{625}{1296}$	$\frac{125}{324}$	$\frac{25}{216}$	$\frac{5}{324}$	$\frac{1}{1296}$

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4) + x_5 P(x_5)$$

$$\text{Mean} = E(X) = 0\left(\frac{625}{1296}\right) + 1\left(\frac{125}{324}\right) + 2\left(\frac{25}{216}\right) + 3\left(\frac{5}{324}\right) + 4\left(\frac{1}{1296}\right)$$

$$\text{Mean} = E(X) = 0 + \frac{125}{324} + \frac{50}{216} + \frac{15}{324} + \frac{4}{1296} = \frac{500 + 300 + 60 + 4}{1296} = \frac{864}{1296} = \frac{2}{3}$$

$$\text{Mean} = E(X) = \frac{2}{3}$$

$$E(X)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3) + (x_4)^2 \cdot P(x_4) + (x_5)^2 \cdot P(x_5)$$

$$E(X^2) = (0)^2 \left(\frac{625}{1296} \right) + (1)^2 \left(\frac{125}{324} \right) + (2)^2 \left(\frac{25}{216} \right) + (3)^2 \left(\frac{5}{324} \right) + (4)^2 \left(\frac{1}{1296} \right)$$

$$E(X^2) = 0 + \frac{125}{324} + \frac{100}{216} + \frac{45}{324} + \frac{16}{1296} = \frac{500 + 600 + 180 + 16}{1296} = \frac{1296}{1296}$$

$$E(X^2) = 1$$

$$\text{Variance} = E(X^2) - E(X)^2 = 1 - \frac{4}{9} = \frac{5}{9}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{5}{9}$$

The probability distribution table is as follows,

	0	1	2	3	4
	$\frac{625}{1296}$	$\frac{125}{324}$	$\frac{25}{216}$	$\frac{5}{324}$	$\frac{1}{1296}$

$$\text{Mean} = \frac{2}{3}$$

$$\text{Variance} = \frac{5}{9}$$

Question 7.

A coin is tossed 4 times. Let X denote the number of heads. Find the probability distribution of X. also, find the mean and variance of X.

Answer:

Given : A coin is tossed 4 times

To find : probability distribution of X and mean (μ) and variance (σ^2)

Formula used :

	x_1	x_2	x_3	x_4	x_5
	(1)	(2)	(3)	(4)	(5)

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

A coin is tossed 4 times,

$$\text{Total possible outcomes} = 2^4 = 16$$

X denotes the number of heads

Let p be the probability of getting a head,

$$p = \frac{1}{2}$$

$$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

$$q = \frac{1}{2}$$

since the coin is tossed 4 times, $n = 4$

X can take the values of 1,2,3,4

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$P(0) = {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$P(1) = {}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3 = \frac{4}{16} = \frac{1}{4}$$

$$P(2) = {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 = \frac{6}{16} = \frac{3}{8}$$

$$P(3) = {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 = \frac{4}{16} = \frac{1}{4}$$

$$P(4) = {}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 = \frac{1}{16}$$

The probability distribution table is as follows,

	0	1	2	3	4
	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4) + x_5 P(x_5)$$

$$\text{Mean} = E(X) = 0\left(\frac{1}{16}\right) + 1\left(\frac{1}{4}\right) + 2\left(\frac{3}{8}\right) + 3\left(\frac{1}{4}\right) + 4\left(\frac{1}{16}\right)$$

$$\text{Mean} = E(X) = 0 + \frac{1}{4} + \frac{6}{8} + \frac{3}{4} + \frac{4}{16} = \frac{4 + 12 + 12 + 4}{16} = \frac{32}{16} = 2$$

$$\text{Mean} = E(X) = 2$$

$$E(X)^2 = (2)^2 = 4$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3) + (x_4)^2 \cdot P(x_4) + (x_5)^2 \cdot P(x_5)$$

$$E(X^2) = (0)^2 \left(\frac{1}{16}\right) + (1)^2 \left(\frac{1}{4}\right) + (2)^2 \left(\frac{3}{8}\right) + (3)^2 \left(\frac{1}{4}\right) + (4)^2 \left(\frac{1}{16}\right)$$

$$E(X^2) = 0 + \frac{1}{4} + \frac{12}{8} + \frac{9}{4} + \frac{16}{16} = \frac{0 + 4 + 24 + 36 + 16}{16} = \frac{80}{16} = 5$$

$$E(X^2) = 5$$

$$\text{Variance} = E(X^2) - E(X)^2 = 5 - 4 = 1$$

$$\text{Variance} = E(X^2) - E(X)^2 = 1$$

The probability distribution table is as follows,

	0	1	2	3	4
	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

$$\text{Mean} = 2$$

$$\text{Variance} = 1$$

Question 8.

Let X denote the number of times 'a total of 9' appears in two throws of a pair of dice. Find the probability distribution of X. Also, find the mean, variance and standard deviation of X.

Answer:

Given : Let X denote the number of times 'a total of 9' appears in two throws of a pair of dice

To find : probability distribution of X ,mean (μ) and variance (σ^2) and standard deviation

Formula used :

	x_1	x_2	x_3
()	(1)	(2)	(3)

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Standard deviation} = \sqrt{E(X^2) - E(X)^2}$$

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

When a die is tossed twice,

Total possible outcomes =

{(1,1) , (1,2) , (1,3) , (1,4) , (1,5) , (1,6)

(2,1) , (2,2) , (2,3) , (2,4) , (2,5) , (2,6)

(3,1) , (3,2) , (3,3) , (3,4) , (3,5) , (3,6)

(4,1) , (4,2) , (4,3) , (4,4) , (4,5) , (4,6)

(5,1) , (5,2) , (5,3) , (5,4) , (5,5) , (5,6)

(6,1) , (6,2) , (6,3) , (6,4) , (6,5) , (6,6)}

Let X denote the number of times 'a total of 9' appears in two throws of a pair of dice

$$p = \frac{4}{36} = \frac{1}{9}$$

$$q = 1 - \frac{1}{9} = \frac{8}{9}$$

Two dice are tossed twice, hence $n = 2$

$$P(0) = {}^2C_0 \left(\frac{1}{9}\right)^0 \left(\frac{8}{9}\right)^2 = \frac{64}{81}$$

$$P(1) = {}^2C_1 \left(\frac{1}{9}\right)^1 \left(\frac{8}{9}\right)^1 = \frac{16}{81}$$

$$P(2) = {}^2C_2 \left(\frac{1}{9}\right)^2 \left(\frac{8}{9}\right)^0 = \frac{1}{81}$$

The probability distribution table is as follows,

	0	1	2
()	$\frac{64}{81}$	$\frac{16}{81}$	$\frac{1}{81}$

$$\text{Mean} = E(X) = 0\left(\frac{64}{81}\right) + 1\left(\frac{16}{81}\right) + 2\left(\frac{1}{81}\right) = 0 + \frac{16}{81} + \frac{2}{81} = \frac{16+2}{81} = \frac{18}{81} = \frac{2}{9}$$

$$\text{Mean} = E(X) = \frac{2}{9}$$

$$E(X)^2 = \left(\frac{2}{9}\right)^2 = \frac{4}{81}$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3)$$

$$E(X^2) = (0)^2\left(\frac{64}{81}\right) + (1)^2\left(\frac{16}{81}\right) + (2)^2\left(\frac{1}{81}\right) = 0 + \frac{16}{81} + \frac{4}{81} = \frac{20}{81}$$

$$E(X^2) = \frac{20}{81}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{20}{81} - \frac{4}{81} = \frac{16}{81}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{16}{81}$$

$$\text{Standard deviation} = \sqrt{E(X^2) - E(X)^2} = \sqrt{\frac{16}{81}} = \frac{4}{9}$$

The probability distribution table is as follows,

	0	1	2
()	$\frac{64}{81}$	$\frac{16}{81}$	$\frac{1}{81}$

$$\text{Mean} = \frac{2}{9}$$

$$\text{Variance} = \frac{16}{81}$$

$$\text{Standard deviation} = \frac{4}{9}$$

Question 9.

There are 5 cards, numbers 1 to 5, one number on each card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two cards drawn. Find the mean and variance of X.

Answer:

Given : There are 5 cards, numbers 1 to 5, one number on each card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two cards drawn.

To find : mean (μ) and variance (σ^2) of X

Formula used :

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

There are 5 cards, numbers 1 to 5, one number on each card. Two cards are drawn at random without replacement.

X denote the sum of the numbers on two cards drawn

The minimum value of X will be 3 as the two cards drawn are 1 and 2

The maximum value of X will be 9 as the two cards drawn are 4 and 5

For X = 3 the two cards can be (1,2) and (2,1)

For X = 4 the two cards can be (1,3) and (3,1)

For X = 5 the two cards can be (1,4) , (4,1) , (2,3) and (3,2)

For X = 6 the two cards can be (1,5) , (5,1) , (2,4) and (4,2)

For X = 7 the two cards can be (3,4) , (4,3) , (2,5) and (5,2)

For X = 8 the two cards can be (5,3) and (3,5)

For X = 9 the two cards can be (4,5) and (5,4)

Total outcomes = 20

$$P(3) = \frac{2}{20} = \frac{1}{10}$$

$$P(4) = \frac{2}{20} = \frac{1}{10}$$

$$P(5) = \frac{4}{20} = \frac{1}{5}$$

$$P(6) = \frac{4}{20} = \frac{1}{5}$$

$$P(7) = \frac{4}{20} = \frac{1}{5}$$

$$P(8) = \frac{2}{20} = \frac{1}{10}$$

$$P(9) = \frac{2}{20} = \frac{1}{10}$$

The probability distribution table is as follows,

x_i	3	4	5	6	7	8	9
P_i	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4) + x_5 P(x_5) + x_6 P(x_6) + x_7 P(x_7)$$

$$\text{Mean} = E(X) = 3\left(\frac{1}{10}\right) + 4\left(\frac{1}{10}\right) + 5\left(\frac{1}{5}\right) + 6\left(\frac{1}{5}\right) + 7\left(\frac{1}{5}\right) + 8\left(\frac{1}{10}\right) + 9\left(\frac{1}{10}\right)$$

$$\text{Mean} = E(X) = \frac{3}{10} + \frac{4}{10} + \frac{5}{5} + \frac{6}{5} + \frac{7}{5} + \frac{8}{10} + \frac{9}{10} = \frac{3+4+10+12+14+8+9}{10} = \frac{60}{10} = 6$$

$$\text{Mean} = E(X) = 6$$

$$E(X)^2 = (6)^2 = 36$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3) + (x_4)^2 \cdot P(x_4) + (x_5)^2 \cdot P(x_5) + (x_6)^2 \cdot P(x_6) + (x_7)^2 \cdot P(x_7)$$

$$E(X^2) = (3)^2 \left(\frac{1}{10}\right) + (4)^2 \left(\frac{1}{10}\right) + (5)^2 \left(\frac{1}{5}\right) + (6)^2 \left(\frac{1}{5}\right) + (7)^2 \left(\frac{1}{5}\right) + (8)^2 \left(\frac{1}{10}\right) + (9)^2 \left(\frac{1}{10}\right)$$

$$E(X^2) = \frac{9}{10} + \frac{16}{10} + \frac{25}{5} + \frac{36}{5} + \frac{49}{5} + \frac{64}{10} + \frac{81}{10} = \frac{9+16+50+72+98+64+81}{10} = \frac{390}{10} = 39$$

$$E(X^2) = 39$$

$$\text{Variance} = E(X^2) - E(X)^2 = 39 - 36 = 3$$

$$\text{Variance} = E(X^2) - E(X)^2 = 3$$

$$\text{Mean} = 6$$

$$\text{Variance} = 3$$

Question 10.

Two cards are drawn from a well-shuffled pack of 52 cards. Find the probability distribution of a number of kings. Also, compute the variance for the number of kings. [CBSE 2007]

Answer:

Given : Two cards are drawn from a well-shuffled pack of 52 cards.

To find : probability distribution of the number of kings and variance (σ^2)

Formula used :

	x_1	x_2	x_3
(0)	(1)	(2)	(3)

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Two cards are drawn from a well-shuffled pack of 52 cards.

Let X denote the number of kings in the two cards

There are 4 king cards present in a pack of well-shuffled pack of 52 cards.

$$P(0) = \frac{{}^{48}C_2}{{}^{52}C_2} = \frac{48 \times 47}{52 \times 51} = \frac{188}{221}$$

$$P(1) = \frac{{}^4_1C \times {}^4_1C}{{}^{52}_2C} = \frac{48 \times 4 \times 2}{52 \times 51} = \frac{32}{221}$$

$$P(2) = \frac{{}^4_2C}{{}^{52}_2C} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$$

The probability distribution table is as follows,

	0	1	2
()	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$$\text{Mean} = E(X) = 0\left(\frac{188}{221}\right) + 1\left(\frac{32}{221}\right) + 2\left(\frac{1}{221}\right) = 0 + \frac{32}{221} + \frac{2}{221} = \frac{32+2}{221} = \frac{34}{221}$$

$$\text{Mean} = E(X) = \frac{34}{221}$$

$$E(X)^2 = \left(\frac{34}{221}\right)^2 = \frac{1156}{48841}$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3)$$

$$E(X^2) = (0)^2 \left(\frac{188}{221}\right) + (1)^2 \left(\frac{32}{221}\right) + (2)^2 \left(\frac{1}{221}\right) = 0 + \frac{32}{221} + \frac{4}{221} = \frac{36}{221}$$

$$E(X^2) = \frac{36}{221}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{36}{221} - \frac{1156}{48841} = \frac{7956 - 1156}{48841} = \frac{6800}{48841} = \frac{400}{2873}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{400}{2873}$$

The probability distribution table is as follows,

	0	1	2
()	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$$\text{Variance} = \frac{400}{2873}$$

Question 11.

A box contains 16 bulbs, out of which 4 bulbs are defective. Three bulbs are drawn at random from the box. Let X be the number of defective bulbs drawn. Find the mean and variance of X.

Answer:

Given : A box contains 16 bulbs, out of which 4 bulbs are defective. Three bulbs are drawn at random

To find : mean (μ) and variance (σ^2)

Formula used :

	x_1	x_2	x_3
()	(1)	(2)	(3)

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

A box contains 16 bulbs, out of which 4 bulbs are defective. Three bulbs are drawn at random

Let X denote the number of defective bulbs drawn

There are 4 defective bulbs present in 16 bulbs

$$P(0) = \frac{{}^{12}C_3}{{}^{16}C_3} = \frac{12 \times 11 \times 10}{16 \times 15 \times 14} = \frac{11}{28}$$

$$P(1) = \frac{{}^{12}C_2 \times {}^4C_1}{{}^{16}C_3} = \frac{12 \times 11 \times 4 \times 3 \times 2}{16 \times 15 \times 14 \times 2} = \frac{33}{70}$$

$$P(2) = \frac{{}^{12}C_1 \times {}^4C_2}{{}^{16}C_3} = \frac{12 \times 4 \times 3 \times 3 \times 2}{16 \times 15 \times 14 \times 2} = \frac{9}{70}$$

$$P(3) = \frac{{}^4C_3}{{}^{16}C_3} = \frac{4 \times 3 \times 2}{16 \times 15 \times 14} = \frac{1}{140}$$

The probability distribution table is as follows,

	0	1	2	3
()	$\frac{11}{28}$	$\frac{33}{70}$	$\frac{9}{70}$	$\frac{1}{140}$

$$\text{Mean} = E(X) = 0\left(\frac{11}{28}\right) + 1\left(\frac{33}{70}\right) + 2\left(\frac{9}{70}\right) + 3\left(\frac{1}{140}\right) = 0 + \frac{33}{70} + \frac{18}{70} + \frac{3}{140} = \frac{66 + 36 + 3}{140}$$

$$\text{Mean} = E(X) = \frac{105}{140} = \frac{3}{4}$$

$$E(X)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3)$$

$$E(X^2) = (0)^2 \left(\frac{11}{28}\right) + (1)^2 \left(\frac{33}{70}\right) + (2)^2 \left(\frac{9}{70}\right) + (3)^2 \left(\frac{1}{140}\right) = 0 + \frac{33}{70} + \frac{36}{70} + \frac{9}{140} = \frac{66 + 72 + 9}{140}$$

$$E(X^2) = \frac{147}{140}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{147}{140} - \frac{9}{16} = \frac{588 - 315}{560} = \frac{273}{560} = \frac{39}{80}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{39}{80}$$

$$\text{Mean} = E(X) = \frac{3}{4}$$

$$\text{Variance} = \frac{39}{80}$$

Question 12.

20% of the bulbs produced by a machine are defective. Find the probability distribution of the number of defective bulbs in a sample of 4 bulbs chosen at random. [CBSE 2004C]

Answer:

Given : 20% of the bulbs produced by a machine are defective.

To find probability distribution of a number of defective bulbs in a sample of 4 bulbs chosen at random.

Formula used :

The probability distribution table is given by ,

	x_1	x_2	x_3	x_4	x_5
	(1)	(2)	(3)	(4)	(5)

Where $P(x) = {}^nC_x p^x q^{n-x}$

Here p is the probability of getting a defective bulb.

$$q = 1 - p$$

Let the total number of bulbs produced by a machine be x

20% of the bulbs produced by a machine are defective.

$$\text{Number of defective bulbs produced by a machine} = \frac{20}{100} \times (x) = \frac{x}{5}$$

X denotes the number of defective bulbs in a sample of 4 bulbs chosen at random.

Let p be the probability of getting a defective bulb,

$$p = \frac{\frac{x}{5}}{x} = \frac{1}{5}$$

$$p = \frac{1}{5}$$

$$q = 1 - p = 1 - \frac{1}{5} = \frac{4}{5}$$

$$q = \frac{4}{5}$$

since 4 bulbs are chosen at random, $n = 4$

X can take the values of 0,1,2,3,4

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$P(0) = {}^4C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^4 = \frac{256}{625}$$

$$P(1) = {}^4C_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^3 = \frac{256}{625}$$

$$P(2) = {}^4C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^2 = \frac{96}{625}$$

$$P(3) = {}^4C_3 \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^1 = \frac{16}{625}$$

$$P(4) = {}^4C_4 \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^0 = \frac{1}{625}$$

The probability distribution table is as follows,

	0	1	2	3	4
	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Question 13.

Four bad eggs are mixed with 10 good ones. Three eggs are drawn one by one without replacement. Let X be the number of bad eggs drawn. Find the mean and variance of X.

Answer:

Given : Four bad eggs are mixed with 10 good ones. Three eggs are drawn one by one without replacement.

To find : mean (μ) and variance (σ^2)

Formula used :

	x_1	x_2	x_3
()	(1)	(2)	(3)

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Four bad eggs are mixed with 10 good ones. Three eggs are drawn one by one without replacement.

Let X denote the number of bad eggs drawn

There are 4 bad eggs present in 14 eggs

$$P(0) = \frac{{}^{10}_2C}{{}^{14}_3C} = \frac{10 \times 9 \times 8}{14 \times 13 \times 12} = \frac{30}{91}$$

$$P(1) = \frac{{}^{10}_2C \times {}^4_1C}{{}^{14}_3C} = \frac{10 \times 9 \times 4 \times 3 \times 2}{14 \times 13 \times 12 \times 2} = \frac{45}{91}$$

$$P(2) = \frac{{}^{10}_1C \times {}^4_2C}{{}^{14}_3C} = \frac{10 \times 4 \times 3 \times 3 \times 2}{14 \times 13 \times 12 \times 2} = \frac{15}{91}$$

$$P(3) = \frac{{}^4_3C}{{}^{14}_3C} = \frac{4 \times 3 \times 2}{14 \times 13 \times 12} = \frac{1}{91}$$

The probability distribution table is as follows,

	0	1	2	3
()	$\frac{30}{91}$	$\frac{45}{91}$	$\frac{15}{91}$	$\frac{1}{91}$

$$\text{Mean} = E(X) = 0\left(\frac{30}{91}\right) + 1\left(\frac{45}{91}\right) + 2\left(\frac{15}{91}\right) + 3\left(\frac{1}{91}\right) = 0 + \frac{45}{91} + \frac{30}{91} + \frac{3}{91} = \frac{45 + 30 + 3}{91}$$

$$\text{Mean} = E(X) = \frac{78}{91} = \frac{6}{7}$$

$$E(X)^2 = \left(\frac{6}{7}\right)^2 = \frac{36}{49}$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3)$$

$$E(X^2) = (0)^2 \left(\frac{30}{91}\right) + (1)^2 \left(\frac{45}{91}\right) + (2)^2 \left(\frac{15}{91}\right) + (3)^2 \left(\frac{1}{91}\right) = 0 + \frac{45}{91} + \frac{60}{91} + \frac{9}{91} = \frac{45 + 60 + 9}{91}$$

$$E(X^2) = \frac{114}{91}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{114}{91} - \frac{36}{49} = \frac{798 - 468}{637} = \frac{330}{637}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{330}{637}$$

$$\text{Mean} = E(X) = \frac{6}{7}$$

$$\text{Variance} = \frac{330}{637}$$

Question 14.

Four rotten oranges are accidentally mixed with 16 good ones. Three oranges are drawn at random from the mixed lot. Let X be the number of rotten oranges drawn. Find the mean and variance of X.

Answer:

Given : Four rotten oranges are mixed with 16 good ones. Three oranges are drawn one by one without replacement.

To find : mean (μ) and variance (σ^2)

Formula used :

	x_1	x_2	x_3
()	(1)	(2)	(3)

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Four rotten oranges are mixed with 16 good ones. Three oranges are drawn one by one without replacement.

Let X denote the number of rotten oranges drawn

There are 4 rotten oranges present in 20 oranges

$$P(0) = \frac{{}^{16}C_3}{{}^{20}C_3} = \frac{16 \times 15 \times 14}{20 \times 19 \times 18} = \frac{28}{57}$$

$$P(1) = \frac{{}^{16}C_2 \times {}^4C_1}{{}^{20}C_3} = \frac{16 \times 15 \times 4 \times 3 \times 2}{20 \times 19 \times 18 \times 2} = \frac{8}{19}$$

$$P(2) = \frac{{}^{16}C_1 \times {}^4C_2}{{}^{20}C_3} = \frac{16 \times 4 \times 3 \times 3 \times 2}{20 \times 19 \times 18 \times 2} = \frac{8}{95}$$

$$P(3) = \frac{{}^4C_3}{{}^{20}C_3} = \frac{4 \times 3 \times 2}{20 \times 19 \times 18} = \frac{1}{285}$$

The probability distribution table is as follows,

	0	1	2	3
()	$\frac{28}{57}$	$\frac{8}{19}$	$\frac{8}{95}$	$\frac{1}{285}$

$$\text{Mean} = E(X) = 0\left(\frac{28}{57}\right) + 1\left(\frac{8}{19}\right) + 2\left(\frac{8}{95}\right) + 3\left(\frac{1}{285}\right) = 0 + \frac{8}{19} + \frac{16}{95} + \frac{3}{285} = \frac{120 + 48 + 3}{285}$$

$$\text{Mean} = E(X) = \frac{171}{285} = \frac{3}{5}$$

$$E(X)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3)$$

$$E(X^2) = (0)^2 \left(\frac{28}{57}\right) + (1)^2 \left(\frac{8}{19}\right) + (2)^2 \left(\frac{8}{95}\right) + (3)^2 \left(\frac{1}{285}\right) = 0 + \frac{8}{19} + \frac{32}{95} + \frac{9}{285} = \frac{120 + 96 + 9}{285}$$

$$E(X^2) = \frac{225}{285} = \frac{15}{19}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{15}{19} - \frac{9}{25} = \frac{375 - 171}{475} = \frac{204}{475}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{204}{475}$$

$$\text{Mean} = E(X) = \frac{3}{5}$$

$$\text{Variance} = \frac{204}{475}$$

Question 15.

Three balls are drawn simultaneously from a bag containing 5 white and 4 red balls. Let X be the number of red balls drawn. Find the mean and variance of X.

Answer:

Given : Three balls are drawn simultaneously from a bag containing 5 white and 4 red balls.

To find : mean (μ) and variance (σ^2) of X

Formula used :

	x_1	x_2	x_3
()	(1)	(2)	(3)

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Three balls are drawn simultaneously from a bag containing 5 white and 4 red balls.

Let X be the number of red balls drawn.

$$P(0) = \frac{{}^5C_3}{{}^9C_3} = \frac{5 \times 4}{9 \times 8 \times 7} = \frac{5}{126}$$

$$P(1) = \frac{{}^5C_2 \times {}^4C_1}{{}^9C_3} = \frac{5 \times 4 \times 4 \times 3 \times 2}{9 \times 8 \times 7 \times 2} = \frac{10}{21}$$

$$P(2) = \frac{{}^5C_1 \times {}^4C_2}{{}^9C_3} = \frac{5 \times 4 \times 3 \times 3 \times 2}{9 \times 8 \times 7 \times 2} = \frac{5}{14}$$

$$P(3) = \frac{{}^4C_3}{{}^9C_3} = \frac{4 \times 3 \times 2}{9 \times 8 \times 7} = \frac{1}{21}$$

The probability distribution table is as follows,

	0	1	2	3
()	$\frac{5}{126}$	$\frac{10}{21}$	$\frac{5}{14}$	$\frac{1}{21}$

$$\text{Mean} = E(X) = 0\left(\frac{5}{126}\right) + 1\left(\frac{10}{21}\right) + 2\left(\frac{5}{14}\right) + 3\left(\frac{1}{21}\right) = 0 + \frac{10}{21} + \frac{10}{14} + \frac{3}{21} = \frac{20+30+6}{42}$$

$$\text{Mean} = E(X) = \frac{56}{42} = \frac{4}{3}$$

$$E(X)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3)$$

$$E(X^2) = (0)^2 \left(\frac{5}{126}\right) + (1)^2 \left(\frac{10}{21}\right) + (2)^2 \left(\frac{5}{14}\right) + (3)^2 \left(\frac{1}{21}\right) = 0 + \frac{10}{21} + \frac{20}{14} + \frac{9}{21} = \frac{20+60+18}{42}$$

$$E(X^2) = \frac{98}{42} = \frac{7}{3}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{7}{3} - \frac{16}{9} = \frac{21 - 16}{9} = \frac{5}{9}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{5}{9}$$

$$\text{Mean} = E(X) = \frac{4}{3}$$

$$\text{Variance} = \frac{5}{9}$$

Question 16.

Two cards are drawn without replacement from a well-shuffled deck of 52 cards. Let X be the number of face cards drawn. Find the mean and variance of X.

Answer:

Given : Two cards are drawn without replacement from a well-shuffled deck of 52 cards.

To find : mean (μ) and variance (σ^2) of X

Formula used :

	x_1	x_2	x_3
()	(1)	(2)	(3)

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Two cards are drawn without replacement from a well-shuffled deck of 52 cards.

Let X denote the number of face cards drawn

There are 12 face cards present in 52 cards

$$P(0) = \frac{{}^{40}_2C}{{}^{52}_2C} = \frac{40 \times 39}{52 \times 51} = \frac{10}{17}$$

$$P(1) = \frac{{}^{40}_1C \times {}^{12}_1C}{{}^{52}_2C} = \frac{40 \times 12 \times 2}{52 \times 51} = \frac{80}{221}$$

$$P(2) = \frac{{}^{12}_2C}{{}^{52}_2C} = \frac{12 \times 11}{52 \times 51} = \frac{11}{221}$$

The probability distribution table is as follows,

	0	1	2
()	$\frac{10}{17}$	$\frac{80}{221}$	$\frac{11}{221}$

$$\text{Mean} = E(X) = 0\left(\frac{10}{17}\right) + 1\left(\frac{80}{221}\right) + 2\left(\frac{11}{221}\right) = 0 + \frac{80}{221} + \frac{22}{221} = \frac{80+22}{221} = \frac{102}{221} = \frac{6}{13}$$

$$\text{Mean} = E(X) = \frac{6}{13}$$

$$E(X)^2 = \left(\frac{6}{13}\right)^2 = \frac{36}{169}$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3)$$

$$E(X^2) = (0)^2 \left(\frac{10}{17}\right) + (1)^2 \left(\frac{80}{221}\right) + (2)^2 \left(\frac{11}{221}\right) = 0 + \frac{80}{221} + \frac{44}{221} = \frac{80+44}{221}$$

$$E(X^2) = \frac{124}{221}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{124}{221} - \frac{36}{169} = \frac{1612 - 612}{2873} = \frac{1000}{2873}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{1000}{2873}$$

$$\text{Mean} = E(X) = \frac{6}{13}$$

$$\text{Variance} = \frac{1000}{2873}$$

Question 17.

Two cards are drawn one by one with replacement from a well-shuffled deck of 52 cards. Find the mean and variance of the number of aces.

Answer:

Given : Two cards are drawn with replacement from a well-shuffled deck of 52 cards.

To find : mean (μ) and variance (σ^2) of X

Formula used :

	x_1	x_2	x_3
()	(1)	(2)	(3)

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Mean} = E(X) = \sum_{i=1}^n x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Two cards are drawn with replacement from a well-shuffled deck of 52 cards.

Let X denote the number of ace cards drawn

There are 4 face cards present in 52 cards

X can take the value of 0,1,2.

$$P(0) = \frac{48}{52} \times \frac{48}{52} = \frac{144}{169}$$

$$P(1) = {}^2_1C \times \frac{4}{52} \times \frac{48}{52} = \frac{2 \times 4 \times 48}{52 \times 52} = \frac{24}{169}$$

$$P(2) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

The probability distribution table is as follows,

	0	1	2
()	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

$$\text{Mean} = E(X) = 0\left(\frac{144}{169}\right) + 1\left(\frac{24}{169}\right) + 2\left(\frac{1}{169}\right) = 0 + \frac{24}{169} + \frac{2}{169} = \frac{24+2}{169} = \frac{26}{169} = \frac{2}{13}$$

$$\text{Mean} = E(X) = \frac{2}{13}$$

$$E(X)^2 = \left(\frac{2}{13}\right)^2 = \frac{4}{169}$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3)$$

$$E(X^2) = (0)^2 \left(\frac{144}{169}\right) + (1)^2 \left(\frac{24}{169}\right) + (2)^2 \left(\frac{1}{169}\right) = 0 + \frac{24}{169} + \frac{4}{169} = \frac{28}{169}$$

$$E(X^2) = \frac{28}{169}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{28}{169} - \frac{4}{169} = \frac{24}{169}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{24}{169}$$

$$\text{Mean} = E(X) = \frac{2}{13}$$

$$\text{Variance} = \frac{24}{169}$$

Question 18.

Three cards are drawn successively with replacement from a well – shuffled deck of 52 cards. A random variable X denotes the number of hearts in the three cards drawn. Find the mean and variance of X.

Answer:

Given : Three cards are drawn successively with replacement from a well – shuffled deck of 52 cards.

To find : mean (μ) and variance (σ^2) of X

Formula used :

	x_1	x_2	x_3
()	(1)	(2)	(3)

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i)$$

$$\text{Variance} = E(X^2) - E(X)^2$$

$$\text{Mean} = E(X) = \sum_{i=1}^{i=n} x_i P(x_i) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$$

Three cards are drawn successively with replacement from a well – shuffled deck of 52 cards.

Let X be the number of hearts drawn.

Number of hearts in 52 cards is 13

$$P(0) = \frac{39}{52} \times \frac{39}{52} \times \frac{39}{52} = \frac{27}{64}$$

$$P(1) = {}^3C_1 \times \frac{13}{52} \times \frac{39}{52} \times \frac{39}{52} = \frac{27}{64}$$

$$P(2) = {}^3C_2 \times \frac{13}{52} \times \frac{13}{52} \times \frac{39}{52} = \frac{9}{64}$$

$$P(3) = \frac{13}{52} \times \frac{13}{52} \times \frac{13}{52} = \frac{1}{64}$$

The probability distribution table is as follows,

	0	1	2	3
()	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

$$\text{Mean} = E(X) = 0\left(\frac{27}{64}\right) + 1\left(\frac{27}{64}\right) + 2\left(\frac{9}{64}\right) + 3\left(\frac{1}{64}\right) = 0 + \frac{27}{64} + \frac{18}{64} + \frac{3}{64} = \frac{48}{64} = \frac{3}{4}$$

$$\text{Mean} = E(X) = \frac{3}{4}$$

$$E(X)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$E(X^2) = \sum_{i=1}^n (x_i)^2 \cdot P(x_i) = (x_1)^2 \cdot P(x_1) + (x_2)^2 \cdot P(x_2) + (x_3)^2 \cdot P(x_3)$$

$$E(X^2) = (0)^2 \left(\frac{27}{64}\right) + (1)^2 \left(\frac{27}{64}\right) + (2)^2 \left(\frac{9}{64}\right) + (3)^2 \left(\frac{1}{64}\right) = 0 + \frac{27}{64} + \frac{36}{64} + \frac{9}{64} = \frac{72}{64} = \frac{9}{8}$$

$$E(X^2) = \frac{9}{8}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{9}{8} - \frac{9}{16} = \frac{18-9}{16} = \frac{9}{16}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{9}{16}$$

$$\text{Mean} = E(X) = \frac{3}{4}$$

$$\text{Variance} = \frac{9}{16}$$

Question 19.

Five defective bulbs are accidentally mixed with 20 good ones. It is not possible to just look at a bulb and tell whether or not it is defective. Find the probability distribution from this lot.

Answer:

Given : Five defective bulbs are accidentally mixed with 20 good ones.

To find : probability distribution from this lot

Formula used :

	X_1	X_2	X_3	X_4	X_5
	(1)	(2)	(3)	(4)	(5)

Five defective bulbs are accidentally mixed with 20 good ones.

Total number of bulbs = 25

X denote the number of defective bulbs drawn

X can draw the value 0 , 1 , 2 , 3 , 4.

since the number of bulbs drawn is 4, $n = 4$

$$P(0) = P(\text{getting a no defective bulb}) = \frac{{}^{20}C_4}{{}^{25}C_4} = \frac{20 \times 19 \times 18 \times 17}{25 \times 24 \times 23 \times 22} = \frac{969}{2530}$$

$$P(1) = P(\text{getting 1 defective bulb and 3 good ones}) = \frac{{}^5_1C \times {}^{20}_3C}{{}^{25}_4C} = \frac{5 \times 20 \times 19 \times 18 \times 4}{25 \times 24 \times 23 \times 22}$$

$$P(1) = \frac{1140}{2530} = \frac{114}{253}$$

$$P(2) = P(\text{getting 2 defective bulbs and 2 good one}) = \frac{{}^5_2C \times {}^{20}_2C}{{}^{25}_4C}$$

$$P(2) = \frac{5 \times 4 \times 20 \times 19 \times 4 \times 3 \times 2}{25 \times 24 \times 23 \times 22 \times 2 \times 2} = \frac{380}{2530} = \frac{38}{253}$$

$$P(3) = P(\text{getting 3 defective bulbs and 1 good one}) = \frac{{}^5_3C \times {}^{20}_1C}{{}^{25}_4C} = \frac{5 \times 4 \times 20 \times 4 \times 3 \times 2}{25 \times 24 \times 23 \times 22 \times 2}$$

$$P(3) = \frac{40}{2530} = \frac{4}{253}$$

$$P(4) = P(\text{getting all defective bulbs}) = \frac{{}^5_4C}{{}^{25}_4C} = \frac{5 \times 4 \times 3 \times 2}{25 \times 24 \times 23 \times 22} = \frac{1}{2530}$$

$$P(4) = \frac{1}{2530}$$

The probability distribution table is as follows,

	0	1	2	3	4
	$\frac{969}{2530}$	$\frac{114}{253}$	$\frac{38}{253}$	$\frac{4}{253}$	$\frac{1}{2530}$