Exercise 2c

Question 1.
Prove that the function f: R → R : $f(x)= 2x$ is one-one and onto.

Answer:
To prove: function is one-one and onto

Given: f: R → R : $f(x)= 2x$

We have,

$f(x) = 2x$

For, $f(x_1) = f(x_2)$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$

When, $f(x_1) = f(x_2)$ then $x_1 = x_2$

$\therefore$ f(x) is one-one

$f(x) = 2x$

Let $f(x) = y$ such that $y \in R$

$\Rightarrow y = 2x$

$$\Rightarrow x = \frac{y}{2}$$

Since $y \in R$,

$$\Rightarrow \frac{y}{2} \in R$$

⇒ x will also be a real number, which means that every value of y is associated with some x

∴ f(x) is onto

Hence Proved

Question 2.
Prove that the function f: N → N : f (x)=3x is one-one and into.

Answer:
To prove: function is one-one and into

Given: f: N → N : f(x)= 3x

We have,

f(x) = 3x

For, $f(x_1) = f(x_2)$

⇒ $3x_1 = 3x_2$

⇒ $x_1 = x_2$

When, $f(x_1) = f(x_2)$ then $x_1 = x_2$

∴ f(x) is one-one

f(x) = 3x

Let f(x) = y such that $y \in N$

⇒ y = 3x

⇒ $x = \frac{y}{3}$

If y = 1,

$\Rightarrow x = \frac{1}{3}$

But as per question $x \in N$, hence x can not be $\frac{1}{3}$

Hence f(x) is into

Hence Proved

Question 3.
Show that the function $f : R \to R : f(x) = x^2$ is neither one-one nor onto.

Answer:
To prove: function is neither one-one nor onto

Given: $f : R \to R : f(x) = x^2$

Solution: We have,

$f(x) = x^2$

For, $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$ or, $x_1 = -x_2$

Since x_1 doesn't has unique image

$\therefore$ f(x) is not one-one

$f(x) = x^2$

Let f(x) = y such that $y \in R$

$\Rightarrow y = x^2$

$\Rightarrow x = \sqrt{y}$

If y = -1, as $y \in R$

Then x will be undefined as we cannot place the negative value under the square root

Hence f(x) is not onto

Hence Proved

Question 4.
Show that the function $f : N \to N : f(x) = x^2$ is one-one and into.

Answer:
To prove: function is one-one and into

Given: $f : N \to N : f(x) = x^2$

Solution: We have,

$f(x) = x^2$

For, $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$

Here we can't consider $x_1 = -x_2$ as $x \in N$, we can't have negative values

$\therefore$ f(x) is one-one

$f(x) = x^2$

Let f(x) = y such that $y \in N$

$\Rightarrow y = x^2$

$\Rightarrow x = \sqrt{y}$

If y = 2, as $y \in N$

Then we will get the irrational value of x, but $x \in N$

Hence f(x) is not into

Hence Proved

Question 5.
Show that the function f : R → R : f (x) = x^4 is neither one-one nor onto.

Answer:
To prove: function is neither one-one nor onto

Given: f : R → R : f (x) = x^4

We have,

$f(x) = x^4$

For, $f(x_1) = f(x_2)$

$\Rightarrow x_1^4 = x_2^4$

$\Rightarrow (x_1^4 - x_2^4) = 0$

$\Rightarrow (x_1^2 - x_2^2)(x_1^2 + x_2^2) = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2)(x_1^2 + x_2^2) = 0$

$\Rightarrow x_1 = x_2$ or, $x_1 = -x_2$ or, $x_1^2 = -x_2^2$

We are getting more than one value of x_1 (no unique image)

$\therefore$ f(x) is not one-one

$f(x) = x^4$

Let f(x) = y such that $y \in R$

$\Rightarrow y = x^4$

$\Rightarrow x = \sqrt[4]{y}$

If y = -2, as $y \in R$

Then x will be undefined as we can't place the negative value under the square root

Hence f(x) is not onto

Hence Proved

Question 6.
Show that the function f : Z → Z : f (x) = x^3 is one-one and into.

Answer:
To prove: function is one-one and into

Given: f : Z → Z : f (x) = x^3

Solution: We have,

$f(x) = x^3$

For, $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

When, $f(x_1) = f(x_2)$ then $x_1 = x_2$

$\therefore$ f(x) is one-one

$f(x) = x^3$

Let f(x) = y such that $y \in Z$

$\Rightarrow y = x^3$

$\Rightarrow x = \sqrt[3]{y}$

If y = 2, as $y \in Z$

Then we will get an irrational value of x, but $x \in Z$

Hence f(x) is into

Hence Proved

Question 7.
Let R_0 be the set of all nonzero real numbers. Then, show that the function $f : R_0 \rightarrow R_0 : f(x) = \frac{1}{x}$ is one-one and onto.

Answer:
To prove: function is one-one and onto

Given: $f : R_0 \rightarrow R_0 : f(x) = \frac{1}{x}$

We have,

$f(x) = \frac{1}{x}$

For, $f(x_1) = f(x_2)$

$\Rightarrow \frac{1}{x_1} = \frac{1}{x_2}$

$\Rightarrow x_1 = x_2$

When, $f(x_1) = f(x_2)$ then $x_1 = x_2$

∴ f(x) is one-one

$$f(x) = \frac{1}{x}$$

Let f(x) = y such that $y \in R_0$

$$\Rightarrow y = \frac{1}{x}$$

$$\Rightarrow x = \frac{1}{y}$$

Since $y \in R_0$,

$$\Rightarrow \frac{1}{y} \in R_0$$

⇒ x will also $\in R_0$, which means that every value of y is associated with some x

∴ f(x) is onto

Hence Proved

Question 8.
Show that the function f : R → R : f(x) = 1 + x^2 is many-one into.

Answer:
To prove: function is many-one into

Given: f : R → R : f(x) = 1 + x^2

We have,

$f(x) = 1 + x^2$

For, $f(x_1) = f(x_2)$

$\Rightarrow 1 + x_1^2 = 1 + x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$

$\Rightarrow x_1 = x_2$ or, $x_1 = -x_2$

Clearly x_1 has more than one image

$\therefore$ f(x) is many-one

$f(x) = 1 + x^2$

Let f(x) = y such that $y \in R$

$\Rightarrow y = 1 + x^2$

$\Rightarrow x^2 = y - 1$

$\Rightarrow x = \sqrt{y-1}$

If y = 3, as $y \in R$

Then x will be undefined as we can't place the negative value under the square root

Hence f(x) is into

Hence Proved

Question 9.

Let $f : R \to R : f(x) = \dfrac{2x - 7}{4}$ be an invertible function. Find f^{-1}.

Answer:
To find: f^{-1}

Given: $f: R \to R : f(x) = \frac{2x-7}{4}$

We have,

$$f(x) = \frac{2x-7}{4}$$

Let f(x) = y such that $y \in R$

$$\Rightarrow y = \frac{2x-7}{4}$$

$\Rightarrow 4y = 2x - 7$

$\Rightarrow 4y + 7 = 2x$

$$\Rightarrow x = \frac{4y+7}{2}$$

$$\Rightarrow f^{-1} = \frac{4y+7}{2}$$

Ans) $f^{-1}(y) = \frac{4y+7}{2}$ for all $y \in R$

Question 10.
Let f : R → R : f(x) = 10x + 3. Find f^{-1}.

Answer:
To find: f^{-1}

Given: f : R → R : f(x) = 10x + 3

We have,

f(x) = 10x + 3

Let f(x) = y such that $y \in R$

$\Rightarrow y = 10x + 3$

$\Rightarrow y - 3 = 10x$

$\Rightarrow x = \dfrac{y - 3}{10}$

$\Rightarrow f^{-1} = \dfrac{y - 3}{10}$

Ans) $f^{-1}(y) = \dfrac{y - 3}{10}$ for all $y \in R$

Question 11.

$$f : R \rightarrow R : f(x) = \begin{cases} 1, \text{ if } x \text{ is rational} \\ -1, \text{ if } x \text{ is rational} \end{cases}$$

Show that f is many-one and into.

Answer:
To prove: function is many-one and into

Given: $f : R \rightarrow R : f(x) = \begin{cases} 1, \text{ if } x \text{ is rational} \\ -1, \text{ if } x \text{ is irrational} \end{cases}$

We have,

f(x) = 1 when x is rational

It means that all rational numbers will have same image i.e. 1

$\Rightarrow$ f(2) = 1 = f (3) , As 2 and 3 are rational numbers

Therefore f(x) is many-one

The range of function is [{-1},{1}] but codomain is set of real numbers.

Therefore f(x) is into

Question 12.
Let f $(x) = x + 7$ and $g(x) = x - 7$, $x \in R$. Find (f o g) (7).

Answer:
To find: (f o g) (7)

Formula used: f o g = f(g(x))

Given: (i) f $(x) = x + 7$

(ii) g $(x) = x - 7$

We have,

f o g = f(g(x)) = f(x – 7) = [(x – 7) + 7]

$\Rightarrow$ x

(f o g) (x) = x

(f o g) (7) = 7

Ans). (f o g) (7) = 7

Question 13.
Let $f : R \rightarrow R$ and $g : R \rightarrow R$ defined by $f(x) = x^2$ and $g(x) = (x + 1)$. Show that g o f ≠ f o g.

Answer:
To prove: g o f ≠ f o g

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) $f : R \rightarrow R : f(x) = x^2$

(ii) $g : R \rightarrow R : g(x) = (x + 1)$

We have,

$f \circ g = f(g(x)) = f(x + 7)$

$f \circ g = (x + 7)^2 = x^2 + 14x + 49$

$g \circ f = g(f(x)) = g(x^2)$

$g \circ f = (x^2 + 1) = x^2 + 1$

Clearly $g \circ f \neq f \circ g$

Hence Proved

Question 14.
Let $f : R \to R : f(x) = (3 - x^3)^{1/3}$. Find f o f.

Answer:
To find: f o f

Formula used: (i) $f \circ f = f(f(x))$

Given: (i) $f : R \to R : f(x) = (3 - x^3)^{1/3}$

We have,

$f \circ f = f(f(x)) = f((3 - x^3)^{1/3})$

$f \circ f = [3 - \{(3 - x^3)^{1/3}\}^3]^{1/3}$

$= [3 - (3 - x^3)]^{1/3}$

$= [3 - 3 + x^3]^{1/3}$

$= [x^3]^{1/3}$

$= x$

Ans) $f \circ f\ (x) = x$

Question 15.
Let f : R → R : f(x) = 3x + 2, find f{f(x)}.

Answer:
To find: f{f(x)}

Formula used: (i) f o f = f(f(x))

Given: (i) f : R → R : f(x) = 3x + 2

We have,

f{f(x)} = f(f(x)) = f(3x + 2)

f o f =3(3x + 2) + 2

= 9x + 6 + 2

= 9x + 8

Ans) f{f(x)} = 9x + 8

Question 16.
Let f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)). Write down g o f.

Answer:
To find: g o f

Formula used: g o f = g(f(x))

Given: (i) f = {(1, 2), (3, 5), (4, 1)}

(ii) g = {(1, 3), (2, 3), (5, 1)}

We have,

gof(1) = g(f(1)) = g(2) = 3

gof(3) = g(f(3)) = g(5) = 1

gof(4) = g(f(4)) = g(1) = 3

Ans) g o f = {(1, 3), (3, 1), (4, 3)}

Question 17.
Let A = {1, 2, 3, 4} and f = {(1, 4), (2, 1) (3, 3), (4, 2)}. Write down (f o f).

Answer:
To find: f o f

Formula used: f o f = f(f(x))

Given: (i) f = {(1, 4), (2, 1) (3, 3), (4, 2)}

We have,

fof(1) = f(f(1)) = f(4) = 2

fof(2) = f(f(2)) = f(1) = 4

fof(3) = f(f(3)) = f(3) = 3

fof(4) = f(f(4)) = f(2) = 1

Ans) f o f = {(1, 2), (2, 4), (3, 3), (4, 1)}

Question 18.
Let f(x) = $8x^3$ and g(x) = $x^{1/3}$. Find g o f and f o g.

Answer:
To find: g o f and f o g

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) $f(x) = 8x^3$

(ii) $g(x) = x^{1/3}$

We have,

$g \circ f = g(f(x)) = g(8x^3)$

$g \circ f = (8x^3)^{\frac{1}{3}} = 2x$

$f \circ g = f(g(x)) = f(x^{1/3})$

$f \circ g = 8\left(x^{\frac{1}{3}}\right)^3 = 8x$

Ans) $g \circ f = 2x$ and $f \circ g = 8x$

Question 19.
Let $f : R \to R : f(x) = 10x + 7$. Find the function $g : R \to R : g \circ f = f \circ g = I_g$.

Answer:
To find: the function $g : R \to R : g \circ f = f \circ g = I_g$

Formula used: (i) $g \circ f = g(f(x))$

(ii) $f \circ g = f(g(x))$

Given: $f : R \to R : f(x) = 10x + 7$

We have,

$f(x) = 10x + 7$

Let $f(x) = y$

$\Rightarrow y = 10x + 7$

$\Rightarrow y - 7 = 10x$

$\Rightarrow x = \dfrac{y - 7}{10}$

Let $g(y) = \dfrac{y-7}{10}$ where g: R → R

g o f = g(f(x)) = g(10x + 7)

$= \dfrac{(10x + 7) - 7}{10}$

= x

$= I_g$

f o g = f(g(x)) = $f\left(\dfrac{x-7}{10}\right)$

$= 10\left(\dfrac{x-7}{10}\right) + 7$

= x – 7 + 7

= x

Clearly g o f = f o g = I_g

Ans). $g(x) = \dfrac{x - 7}{10}$

Question 20.
Let A = {1, 2, 3), B = {4, 5, 6, 7) and let f = {(1, 4), (2,5), (3, 6)} be a function from A to B. State whether f is one-one.

Answer:
To state: Whether f is one-one

Given: f = {(1, 4), (2,5), (3, 6)}

Here the function is defined from A → B

For a function to be one-one if the images of distinct elements of A under f are distinct

i.e. 1,2 and 3 must have a distinct image.

From f = {(1, 4), (2, 5), (3, 6)} we can see that 1, 2 and 3 have distinct image.

Therefore f is one-one

Ans) f is one-one