Objective Questions

Question 1.
The direction ratios of two lines are 3, 2, -6 and 1, 2, 2, respectively. The acute angle between these lines is

A. $\cos^{-1}\left(\frac{5}{18}\right)$

B. $\cos^{-1}\left(\frac{3}{20}\right)$

C. $\cos^{-1}\left(\frac{5}{21}\right)$

D. $\cos^{-1}\left(\frac{8}{21}\right)$

Answer:
Direction ratio are given implies that we can write the parallel vector towards that line, lets consider the first parallel vector to be $|\vec{a}| = 3\hat{i} + 2\hat{j} - 6\hat{k}$ and second parallel vector be $|\vec{b}| = \hat{i} + 2\,\hat{j} + 2\hat{k}$.

For the angle, we can use the formula $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that, we need to find the magnitude of these vectors

$$|\vec{a}| = \sqrt{3^2 + 2^2 + (-6)^2}$$

$$= 7$$

$$|\vec{b}| = \sqrt{1 + 2^2 + 2^2}$$

$$= 3$$

$$\cos\alpha = \frac{(3\hat{i} + 2\hat{j} - 6\hat{k}).(\hat{i} + 2\,\hat{j} + 2\hat{k})}{7 \times 3}$$

$$\cos\alpha = \frac{3+4-12}{21}$$

$$\cos\alpha = \frac{-5}{21}$$

$$\alpha = \cos^{-1}\left(-\frac{5}{21}\right)$$

The negative sign does not affect anything in cosine as cosine is positive in the fourth quadrant.

$$\alpha = \cos^{-1}\left(\frac{5}{21}\right)$$

Question 2.
The direction ratios of two lines are a, b, c and (b – c), (c – a), (a – b) respectively. The angle between these lines is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. $\frac{3\pi}{4}$

Answer:
Direction ratio are given implies that we can write the parallel vector towards that line, lets consider the first parallel vector to be $|\vec{a}| = a\hat{i} + b\hat{j} + c\hat{k}$ and second parallel vector be $|\vec{b}| = (b-c)\hat{i} + (c-a)\,\hat{j} + (a-b)\hat{k}.$

For the angle, we can use the formula $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that, we need to find the magnitude of these vectors

$$|\vec{a}| = \sqrt{a^2 + b^2 + (c)^2}$$

$$= \sqrt{a^2 + b^2 + c^2}$$

$$|\vec{b}| = \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}$$

$$= \sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)}$$

$$\cos\alpha = \frac{(a\hat{i} + b\hat{j} + c\hat{k}).((b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k})}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$$

$$\cos\alpha = \frac{ab - ac + bc - ba + ca - cb}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$$

$$\cos\alpha = \frac{0}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$$

$$\alpha = \cos^{-1}(0)$$

$$\alpha = \frac{\pi}{2}$$

Question 3.

The angle between the lines $\frac{x-2}{2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{2} = \frac{z-5}{4}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\cos^{-1}\left(\frac{3}{8}\right)$

Answer:

Direction ratio are given implies that we can write the parallel vector towards those line, lets consider first parallel vector to be $|\vec{a}| = 2\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$ and second parallel vector be $|\vec{b}| = -\hat{\imath} + 2\,\hat{\jmath} + 4\hat{k}.$

For the angle we can use the formula $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that we need to find magnitude of these vectors

$$|\vec{a}| = \sqrt{3^2 + 2^2 + (7)^2}$$

$$= \sqrt{62}$$

$$|\vec{b}| = \sqrt{1 + 2^2 + 4^2}$$

$$= \sqrt{21}$$

$$\cos\alpha = \frac{(2\hat{\imath} + 7\hat{\jmath} - 3\hat{k}).(-\hat{\imath} + 2\,\hat{\jmath} + 4\hat{k})}{\sqrt{21} \times \sqrt{62}}$$

$$\cos\alpha = \frac{-2 + 14 - 12}{\sqrt{21} \times \sqrt{62}}$$

$$\cos\alpha = \frac{0}{\sqrt{21} \times \sqrt{62}}$$

$$\alpha = \cos^{-1} 0$$

Negative sign does not affect anything in cosine as cosine is positive in fourth quadrant

$$\alpha = \frac{\pi}{2}$$

Question 4.

If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular to each other then k = ?

A. $\frac{-5}{7}$

B. $\frac{5}{7}$

C. $\frac{10}{7}$

D. $\frac{-10}{7}$

Answer:
If the lines are perpendicular to each other then the angle between these lines will be

$\frac{\pi}{2}$, me the cosine will be 0

$\vec{a} = -3\hat{i} + 2k\hat{j} + 2\hat{k}$

$|\vec{a}| = \sqrt{3^2 + (2k)^2 + 2^2}$

$= \sqrt{13 + 4k^2}$

$\vec{b} = 3k\hat{i} + \hat{j} - 5\hat{k}$

$|\vec{b}| = \sqrt{(3k)^2 + 1 + 5^2}$

$= \sqrt{9k^2 + 26}$

$\cos\left(\frac{\pi}{2}\right) = \frac{(3k\hat{i} + \hat{j} - 5\hat{k}).(-3\hat{i} + 2k\hat{j} + 2\hat{k})}{\sqrt{13 + 4k^2} \times \sqrt{9k^2 + 26}}$

$0 = \frac{-9k + 2k - 10}{\sqrt{13 + 4k^2} \times \sqrt{9k^2 + 26}}$

$k = -\frac{10}{7}$

Question 5.

A line passes through the points A(2, -1, 4) and B(1, 2, -2). The equations of the line AB are

A. $\frac{x-2}{-1}=\frac{y+1}{2}=\frac{z-4}{-6}$

B. $\frac{x+2}{-1}=\frac{y+1}{2}=\frac{z-4}{6}$

C. $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-4}{6}$

D. none of these

Answer:
To write the equation of a line we need a parallel vector and a fixed point through which the line is passing

Parallel vector=$((2-1)\hat{\imath}+(-1-2)\hat{\jmath}+(4+2)\hat{k})$

$=\hat{\imath}-3\hat{\jmath}+6\hat{k}$

Or = $-(\hat{\imath}-3\hat{\jmath}+6\hat{k})$

Fixed point is $2\hat{\imath} - \hat{\jmath} + 4\hat{k}$

Equation

$$\frac{x-2}{1}=\frac{y-(-1)}{-3}=\frac{z-4}{6}$$

$$\frac{x-2}{1}=\frac{y+1}{-3}=\frac{z-4}{6}$$

Or

$$\frac{x-2}{-1}=\frac{y+1}{3}=\frac{z-4}{-6}$$

Question 6.

The angle between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$ is

A. $\cos^{-1}\left(\frac{3}{4}\right)$

B. $\cos^{-1}\left(\frac{5}{6}\right)$

C. $\cos^{-1}\left(\frac{2}{3}\right)$

D. $\frac{\pi}{3}$

Answer:
Direction cosine of the lines are given $2\hat{i}+2\hat{j}+\hat{k}$ and $4\hat{i}+\hat{j}+8\hat{k}$

$$\vec{a}=2\hat{i}+2\hat{j}+\hat{k}$$

$$|\vec{a}|=\sqrt{2^2+2^2+1}$$

$$|\vec{a}|=3$$

$$\vec{b}=4\hat{i}+\hat{j}+8\hat{k}$$

$$|\vec{b}|=\sqrt{4^2+1+8^2}$$

$$=9$$

$$\cos\alpha=\frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$$

$$\cos\alpha=\frac{(2\hat{i}+2\hat{j}+\hat{k}).(4\hat{i}+\hat{j}+8\hat{k})}{3\times 9}$$

$$\cos\alpha=\frac{8+8+2}{27}$$

$$\cos\alpha = \frac{2}{3}$$

Question 7.

The angle between the lines $\vec{r} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j} - 5\hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$ is

A. $\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$

B. $\cos^{-1}\left(\frac{6\sqrt{2}}{5}\right)$

C. $\cos^{-1}\left(\frac{5\sqrt{3}}{8}\right)$

D. $\cos^{-1}\left(\frac{5\sqrt{2}}{6}\right)$

Answer:

Let $\vec{a} = \hat{\imath} - \hat{\jmath} - 2\hat{k}$ and $\vec{b} = 3\hat{\imath} - 5\hat{\jmath} - 4\hat{k}$ and $|\vec{a}| = \sqrt{1 + 1 + 2^2} = \sqrt{6}$

$$|\vec{b}| = \sqrt{3^2 + 5^2 + 4^2} = 5\sqrt{2}$$

$$\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}| \times |\vec{b}|}$$

$$\cos\alpha = \frac{(3\hat{\imath} - 5\hat{\jmath} - 4\hat{k}).(\hat{\imath} - \hat{\jmath} - 2\hat{k})}{5\sqrt{2} \times \sqrt{6}}$$

$$\cos\alpha = \frac{3 + 5 + 8}{5\sqrt{12}}$$

$$\cos\alpha = \frac{8\sqrt{3}}{15}$$

Question 8.
A line is perpendicular to two lines having direction ratios 1, -2, -2 and 0, 2, 1. The direction cosines of the line are

A. $\frac{-2}{3}, \frac{1}{3}, \frac{2}{3}$

B. $\frac{2}{3}, \frac{1}{3}, \frac{-1}{3}$

C. $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$

D. none of these

Answer:
If a line is perpendicular to two given lines we can find out the parallel vector by cross product of the given two vectors.

$$\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$$

$$\vec{b} = 2\hat{j} + \hat{k}$$

$$\vec{a} \times \vec{b} = (\hat{i} - 2\hat{j} - 2\hat{k}) \times (2\hat{j} + \hat{k})$$

$$= 2\hat{i} - \hat{j} + 2\hat{k}$$

So the direction cosines are

$$\hat{n} = \frac{1}{\sqrt{2^2 + 1 + 2^2}}$$

$$\hat{n} = \frac{1}{3}$$

Direction cosine

$$\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$$

Question 9.

A line passes through the point A(5, -2, 4) and it is parallel to the vector $\left(2\hat{i}-\hat{j}+3\hat{k}\right)$. The vector equation of the line is

A. $\vec{r}=\left(2\hat{i}-\hat{j}+3\hat{k}\right)+\lambda\left(5\hat{i}-2\hat{j}+4\hat{k}\right)$

B. $\vec{r}=\left(5\hat{i}-2\hat{j}+4\hat{k}\right)+\lambda\left(2\hat{i}-\hat{j}+3\hat{k}\right)$

C. $\vec{r}\cdot\left(5\hat{i}-2\hat{j}+4\hat{k}\right)=\sqrt{14}$

D. none of these

Answer:
Fixed point is $5\hat{i}-2\hat{j}+4\hat{k}$ and parallel vector is $2\hat{i}-\hat{j}+3\hat{k}$

Equation $5\hat{i}-2\hat{j}+4\hat{k}+\alpha(2\hat{i}-\hat{j}+3\hat{k})$

Question 10.

The Cartesian equations of a line are $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-5}{-1}$. Its vector equation is

A. $\vec{r}=\left(-\hat{i}+2\hat{j}-5\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}-\hat{k}\right)$

B. $\vec{r}=\left(2\hat{i}+3\hat{j}-\hat{k}\right)+\lambda\left(\hat{i}-2\hat{j}+5\hat{k}\right)$

C. $\vec{r}=\left(\hat{i}-2\hat{j}+5\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}-4\hat{k}\right)$

D. none of these

Answer:
Dixed point (1,-2,5) and the parallel vector is $2\hat{i}+3\hat{j}-\hat{k}$

Equation ($\hat{i}$ -2$\hat{j}$ +5$\hat{k}$)+α $(2\hat{i}+3\hat{j}-\hat{k})$

Question 11.
A line passes through the point

A(-2, 4, -5) and is parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$. The vector equation of the line is

A. $\vec{r}=\left(-3\hat{i}+4\hat{j}-8\hat{k}\right)+\lambda\left(-2\hat{i}+4\hat{j}-5\hat{k}\right)$

B. $r = \left(-2\hat{i} + 4\hat{j} - 5\hat{k}\right) + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k})$

C. $\vec{r} = \left(3\hat{i} + 5\hat{j} + 6\hat{k}\right) + \lambda\left(-2\hat{i} + 4\hat{j} - 5\hat{k}\right)$

D. none of these

Answer:
Fixed point is $-2\hat{i} + 4\hat{j} - 5\hat{k}$ and the parallel vector is $3\hat{i} + 5\hat{j} + 6\hat{k}$

Equation is $r = \left(-2\hat{i} + 4\hat{j} - 5\hat{k}\right) + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k})$

Question 12.
The coordinates of the point where the line through the points A(5, 1, 6) and B(3, 4, 1) crosses the yz-plane is
A. (0, 17, -13)

B. $\left(0, \frac{-17}{2}, \frac{13}{2}\right)$

C. $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$

D. none of these

Answer:
We first need to find the equation of a line passing through the two given points

taking fixed point as $5\hat{i} + \hat{j} + 6\hat{k}$

and the parallel vector will be $(5-3)\hat{i} + (1-4)\hat{j} + (6-1)\hat{k} = 2\hat{i} - 3\hat{j} + 5\hat{k}$

equation of the line in cartesian form

$$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5}$$

Assume above equation to be equal to k, a constant

$$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = k$$

And y-z plane have x-coordinate as zero we may get

$$\frac{0-5}{2}=\frac{y-1}{-3}=\frac{z-6}{5}=k$$

$$k=-\frac{5}{2}$$

Now we can find y and z

$$\frac{y-1}{-3}=-\frac{5}{2}$$

$$y-1=\frac{15}{2}$$

$$y=\frac{17}{2}$$

$$\frac{z-6}{5}=-\frac{5}{2}$$

$$z-6=-\frac{25}{2}$$

$$z=\ -\frac{13}{2}$$

The coordinate where the line meets y-z plane is $\left(0,\frac{17}{2},-\frac{13}{2}\right)$

Question 13.
The vector equation of the x-axis is given by
A. $\vec{r}=\hat{i}$

B. $\vec{r}=\hat{j}+\hat{k}$

C. $\vec{r}=\lambda\hat{i}$

D. none of these

Answer:
Vector equation need a fixed point and a parallel vector

For x-axis fixed point can be anything ranging from negative to positive including origin

And parallel vector is î

Equation would be λ î

Question 14.

The Cartesian equations of a lines are $\frac{x-2}{2}=\frac{y+1}{3}=\frac{z-3}{-2}$. What is its vector equation?

A. $\vec{r}=\left(2\hat{i}+3\hat{j}-2\hat{k}\right)+\lambda\left(2\hat{i}-\hat{j}+3\hat{k}\right)$

B. $\vec{r}=\left(2\hat{i}-\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}-2\hat{k}\right)$

C. $\vec{r}=\left(2\hat{i}+3\hat{j}-2\hat{k}\right)$

D. none of these

Answer:
Fixed point is $2\hat{i}-\hat{j}+3\hat{k}$ and the vector is $2\hat{i}+3\hat{j}-2\hat{k}$

Equation $(2\hat{i}-\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}-2\hat{k})$

Question 15.

The angle between two lines having direction ratios 1, 1, 2 and $\left(\sqrt{3}-1\right), \left(-\sqrt{3}-1\right)$, 4 is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer:
Let $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = (\sqrt{3} - 1)\hat{\imath} + (-\sqrt{3} - 1)\hat{\jmath} + 4\hat{k}$

$$|\vec{a}| = \sqrt{6}\ |\vec{b}| = \sqrt{(4 - 2\sqrt{3}) + (4 + 2\sqrt{3}) + 16} = 2\sqrt{6}$$

$$\cos\alpha = \frac{(\hat{\imath} + \hat{\jmath} + 2\hat{k}).\left((\sqrt{3} - 1)\hat{\imath} + (-\sqrt{3} - 1)\hat{\jmath} + 4\hat{k}\right)}{\sqrt{6} \times 2\sqrt{6}}$$

$$\cos\alpha = \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{12}$$

$$\cos\alpha = \frac{1}{2}$$

$$\alpha = 60°$$

Question 16.
The straight line $\frac{x-2}{3} = \frac{y-3}{1} = \frac{z+1}{0}$ is
A. parallel to the x-axis

B. parallel to the y-axis

C. parallel to the z-axis

D. perpendicular to the z-axis

Answer:
It is perpendicular to z-axis because cos90° is 0 which implies that it makes 90° with z-axis

Question 17.
If a line makes angles α, β and γ with the x-axis, y-axis and z-axis respectively then ($\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$) = ?
A. 1

B. 3

C. 2

D. $\frac{3}{2}$

Answer:
$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1 - \cos^2\alpha + 1 - \cos\beta + 1 - \cos^2\gamma$

$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$ is the square of the direction ratios of all three axes which is always equal to 1

$= 3 - 1$

$= 2$

Question 18.
If (a_1, b_1, c_1) and (a_2, b_2, c_2) be the direction ratios of two parallel lines then
A. $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$

B. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

C. $a_1^2 + b_1^2 + c_1^2 = a_2^2 + b_2^2 + c_2^2$

D. $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Answer:
We know that if there is two parallel lines then their direction ratios must have a relation

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Question 19.
If the points A(-1, 3, 2), B(-4, 2, -2) and C(5, 5, λ) are collinear then the value of λ is
A. 5

B. 7

C. 8

D. 10

Answer:
Determinant of these point should be zero

$$\begin{vmatrix} -1 & 3 & 2 \\ -4 & 2 & -2 \\ 5 & 5 & \lambda \end{vmatrix} = 0$$

$$-1(2\lambda + 10) - 3(-4\lambda + 10) + 2(-20 - 10) = 0$$

$$10\lambda - 10 - 30 - 60 = 0$$

$$\lambda = 10$$