Exercise 5e

Question 1.
Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

Answer:

Let, A = $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$\text{Aug}\left[A \mid I\right] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right]$, where I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & 1 \end{array}\right] \xrightarrow{R_1 - 2R_2} \left[\begin{array}{cc|cc} 1 & 0 & 7 & -2 \\ 0 & 1 & -3 & 1 \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question 2.

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

Answer:

Let, A = $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$\text{Aug}\left[A|I\right] = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{array}\right]$, where I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -5 & -2 & 1 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{2}{5} & -\frac{1}{5} \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{5} & \frac{2}{5} \\ 0 & 1 & \frac{2}{5} & -\frac{1}{5} \end{array}\right]$$

Here, the matrix A is converted into the Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question 3.
Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$$

Answer:

Let, A = $\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A\,|\,I\right] = \left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ -3 & 1 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ -3 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ -1 & 6 & 1 & 1 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{cc|cc} 1 & 11 & 2 & 1 \\ -1 & 6 & 1 & 1 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{cc|cc} 1 & 11 & 2 & 1 \\ 0 & 17 & 3 & 2 \end{array}\right]$$

$$\xrightarrow{\frac{1}{17}R_2} \left[\begin{array}{cc|cc} 1 & 11 & 2 & 1 \\ 0 & 1 & \frac{3}{17} & \frac{2}{17} \end{array}\right] \xrightarrow{R_1-11R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{17} & -\frac{5}{17} \\ 0 & 1 & \frac{3}{17} & \frac{2}{17} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} \frac{1}{17} & -\frac{5}{17} \\ \frac{3}{17} & \frac{2}{17} \end{bmatrix} = \frac{1}{17}\begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question 4.
Using elementary row transformations, find the inverse of each of the following matrices:

$\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$

Answer:

Let, A = $\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\mathrm{Aug}\left[A \mid I\right] = \left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 0 & 11 & -2 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|cc} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 11 & -2 & 1 \end{array}\right] \xrightarrow{\frac{1}{11}R_2} \left[\begin{array}{cc|cc} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{2}{11} & \frac{1}{11} \end{array}\right]$$

$$\xrightarrow{R_1+\frac{3}{2}R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{5}{22} & \frac{3}{22} \\ 0 & 1 & -\frac{2}{11} & \frac{1}{11} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} \frac{5}{22} & \frac{3}{22} \\ -\frac{2}{11} & \frac{1}{11} \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question 5.

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$$

Answer:

Let, A = $\begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\mathrm{Aug}\left[A \mid I\right] = \left[\begin{array}{cc|cc} 4 & 0 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc} 4 & 0 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right] \xrightarrow{R_1 - 2R_2} \left[\begin{array}{cc|cc} 0 & -10 & 1 & -2 \\ 2 & 5 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|cc} 2 & 5 & 0 & 1 \\ 0 & -10 & 1 & -2 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|cc} 1 & \frac{5}{2} & 0 & \frac{1}{2} \\ 0 & -10 & 1 & -2 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{10}R_2} \left[\begin{array}{cc|cc} 1 & \frac{5}{2} & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{10} & \frac{1}{5} \end{array}\right] \xrightarrow{R_1 - \frac{5}{2}R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{4} & 0 \\ 0 & 1 & -\frac{1}{10} & \frac{1}{5} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ -\frac{1}{10} & \frac{1}{5} \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question 6.
Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$$

Answer:

Let, A = $\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\mathrm{Aug}\left[A \mid I\right] = \left[\begin{array}{cc|cc} 6 & 7 & 1 & 0 \\ 8 & 9 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{cc|cc} 6 & 7 & 1 & 0 \\ 8 & 9 & 0 & 1 \end{array}\right] \xrightarrow{R_2-R_1} \left[\begin{array}{cc|cc} 6 & 7 & 1 & 0 \\ 2 & 2 & -1 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|cc} 2 & 2 & -1 & 1 \\ 6 & 7 & 1 & 0 \end{array}\right] \xrightarrow{R_2-3R_1} \left[\begin{array}{cc|cc} 2 & 2 & -1 & 1 \\ 0 & 1 & 4 & -3 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|cc} 1 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 4 & -3 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{cc|cc} 1 & 0 & -\frac{9}{2} & \frac{7}{2} \\ 0 & 1 & 4 & -3 \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

A^{-1} = $\begin{bmatrix} -\frac{9}{2} & \frac{7}{2} \\ 4 & -3 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question 7.

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Answer:

Let, A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\mathrm{Aug}\left[A \mid I\right] = \left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - 3R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 + 4R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & 4 & -3 & 1 \end{array}\right] \xrightarrow{R_3 + R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array}\right] \xrightarrow{R_1 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array}\right] \xrightarrow{R_1 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question 8.
Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

Answer:

Let, $A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A \mid I\right] = \left[\begin{array}{ccc|ccc} 2 & -3 & 3 & 1 & 0 & 0 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 3 & -2 & 2 & 0 & 0 & 1 \end{array}\right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 2 & -3 & 3 & 1 & 0 & 0 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 3 & -2 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|ccc} 2 & -3 & 3 & 1 & 0 & 0 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 3 & -2 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|ccc} 2 & -3 & 3 & 1 & 0 & 0 \\ 0 & 5 & 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_1-R_3}\left[\begin{array}{ccc|ccc}1 & -4 & 4 & 2 & 0 & -1\\ 0 & 5 & 0 & -1 & 1 & 0\\ 1 & 1 & -1 & -1 & 0 & 1\end{array}\right]\xrightarrow{R_3-R_1}\left[\begin{array}{ccc|ccc}1 & -4 & 4 & 2 & 0 & -1\\ 0 & 5 & 0 & -1 & 1 & 0\\ 0 & 5 & -5 & -3 & 0 & 2\end{array}\right]\xrightarrow{R_3-R_2}\left[\begin{array}{ccc|ccc}1 & -4 & 4 & 2 & 0 & -1\\ 0 & 5 & 0 & -1 & 1 & 0\\ 0 & 0 & -5 & -2 & -1 & 2\end{array}\right]$$

$$\xrightarrow{R_1+R_2}\left[\begin{array}{ccc|ccc}1 & 1 & 4 & 1 & 1 & -1\\ 0 & 5 & 0 & -1 & 1 & 0\\ 0 & 0 & -5 & -2 & -1 & 2\end{array}\right]\xrightarrow{\frac{1}{5}R_2}\left[\begin{array}{ccc|ccc}1 & 1 & 4 & 1 & 1 & -1\\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0\\ 0 & 0 & -5 & -2 & -1 & 2\end{array}\right]\xrightarrow{-\frac{1}{5}R_3}\left[\begin{array}{ccc|ccc}1 & 1 & 4 & 1 & 1 & -1\\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0\\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5}\end{array}\right]$$

$$\xrightarrow{R_1-R_2}\left[\begin{array}{ccc|ccc}1 & 0 & 4 & \frac{6}{5} & \frac{4}{5} & -1\\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0\\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5}\end{array}\right]\xrightarrow{R_1-4R_3}\left[\begin{array}{ccc|ccc}1 & 0 & 0 & -\frac{2}{5} & 0 & \frac{3}{5}\\ 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} & 0\\ 0 & 0 & 1 & \frac{2}{5} & \frac{1}{5} & -\frac{2}{5}\end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1}=\begin{bmatrix}-\frac{2}{5} & 0 & \frac{3}{5}\\ -\frac{1}{5} & \frac{1}{5} & 0\\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5}\end{bmatrix}=-\frac{1}{5}\begin{bmatrix}2 & 0 & -3\\ 1 & -1 & 0\\ -2 & -1 & 2\end{bmatrix}\text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question 9.
Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix}3 & 0 & 2\\ 1 & 5 & 9\\ 6 & 4 & 7\end{bmatrix}$$

Answer:

Let, A = $\begin{bmatrix} 3 & 0 & 2 \\ 1 & 5 & 9 \\ 6 & 4 & 7 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A\,|\,I\right] = \left[\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 1 & 5 & 9 & 0 & 1 & 0 \\ 6 & 4 & 7 & 0 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 1 & 5 & 9 & 0 & 1 & 0 \\ 6 & 4 & 7 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 3 & 0 & 2 & 1 & 0 & 0 \\ 6 & 4 & 7 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 3 & 0 & 2 & 1 & 0 & 0 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 - 3R_1} \left[\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 0 & -15 & -25 & 1 & -3 & 0 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_2 + 4R_3} \left[\begin{array}{ccc|ccc} 1 & 5 & 9 & 0 & 1 & 0 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 4 & 3 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 - 4R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 0 & 55 & 26 & 12 & -15 \end{array}\right] \xrightarrow{\frac{1}{55}R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & -13 & -7 & -3 & 4 \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array}\right]$$

$$\xrightarrow{R_2 + 13R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 6 & 2 & 1 & -1 \\ 0 & 1 & 0 & -\frac{47}{55} & -\frac{9}{55} & \frac{25}{55} \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & \frac{157}{55} & \frac{64}{55} & -\frac{80}{55} \\ 0 & 1 & 0 & -\frac{47}{55} & -\frac{9}{55} & \frac{25}{55} \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array}\right]$$

$$\xrightarrow{R_1-6R_3}\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{55} & -\frac{8}{55} & \frac{10}{55} \\ 0 & 1 & 0 & -\frac{47}{55} & -\frac{9}{55} & \frac{25}{55} \\ 0 & 0 & 1 & \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} \frac{1}{55} & -\frac{8}{55} & \frac{10}{55} \\ -\frac{47}{55} & -\frac{9}{55} & \frac{25}{55} \\ \frac{26}{55} & \frac{12}{55} & -\frac{15}{55} \end{bmatrix} = -\frac{1}{55}\begin{bmatrix} -1 & 8 & -10 \\ 47 & 9 & -25 \\ -26 & -12 & 15 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question 10.
Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$$

Answer:

Let, $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\mathrm{Aug}\left[A\,|\,I\right]=\left[\begin{array}{ccc|ccc}1 & 2 & -3 & 1 & 0 & 0\\ 2 & 3 & 2 & 0 & 1 & 0\\ 3 & -3 & -4 & 0 & 0 & 1\end{array}\right]$$, where I = $$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc}1 & 2 & -3 & 1 & 0 & 0\\ 2 & 3 & 2 & 0 & 1 & 0\\ 3 & -3 & -4 & 0 & 0 & 1\end{array}\right]\xrightarrow{R_3-R_2}\left[\begin{array}{ccc|ccc}1 & 2 & -3 & 1 & 0 & 0\\ 2 & 3 & 2 & 0 & 1 & 0\\ 1 & -6 & -6 & 0 & -1 & 1\end{array}\right]\xrightarrow{R_2-R_1}\left[\begin{array}{ccc|ccc}1 & 2 & -3 & 1 & 0 & 0\\ 1 & 1 & 5 & -1 & 1 & 0\\ 1 & -6 & -6 & 0 & -1 & 1\end{array}\right]$$

$$\xrightarrow{R_3-R_2}\left[\begin{array}{ccc|ccc}1 & 2 & -3 & 1 & 0 & 0\\ 1 & 1 & 5 & -1 & 1 & 0\\ 0 & -7 & -11 & 1 & -2 & 1\end{array}\right]\xrightarrow{R_2-R_1}\left[\begin{array}{ccc|ccc}1 & 2 & -3 & 1 & 0 & 0\\ 0 & -1 & 8 & -2 & 1 & 0\\ 0 & -7 & -11 & 1 & -2 & 1\end{array}\right]\xrightarrow{-R_2}\left[\begin{array}{ccc|ccc}1 & 2 & -3 & 1 & 0 & 0\\ 0 & 1 & -8 & 2 & -1 & 0\\ 0 & -7 & -11 & 1 & -2 & 1\end{array}\right]$$

$$\xrightarrow{R_3+7R_2}\left[\begin{array}{ccc|ccc}1 & 2 & -3 & 1 & 0 & 0\\ 0 & 1 & -8 & 2 & -1 & 0\\ 0 & 0 & -67 & 15 & -9 & 1\end{array}\right]\xrightarrow{-\frac{1}{67}R_3}\left[\begin{array}{ccc|ccc}1 & 2 & -3 & 1 & 0 & 0\\ 0 & 1 & -8 & 2 & -1 & 0\\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67}\end{array}\right]$$

$$\xrightarrow{R_2+8R_3}\left[\begin{array}{ccc|ccc}1 & 2 & -3 & 1 & 0 & 0\\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67}\\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67}\end{array}\right]\xrightarrow{R_1-2R_2}\left[\begin{array}{ccc|ccc}1 & 0 & -3 & \frac{39}{67} & -\frac{10}{67} & \frac{16}{67}\\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67}\\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67}\end{array}\right]$$

$$\xrightarrow{R_1+3R_3}\left[\begin{array}{ccc|ccc}1 & 0 & 0 & -\frac{6}{67} & \frac{17}{67} & \frac{13}{67}\\ 0 & 1 & 0 & \frac{14}{67} & \frac{5}{67} & -\frac{8}{67}\\ 0 & 0 & 1 & -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67}\end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} -\frac{6}{67} & \frac{17}{67} & \frac{13}{67} \\ \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{bmatrix} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question 11.
Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$$

Answer:

Let, $A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A \middle| I\right] = \left[\begin{array}{ccc|ccc} 3 & -1 & -2 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 3 & -5 & 0 & 0 & 0 & 1 \end{array}\right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 3 & -1 & -2 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 3 & -5 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|ccc} 3 & -1 & -2 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -4 & 2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & -1 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -4 & 2 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2-2R_1}\left[\begin{array}{ccc|ccc}1 & -1 & -1 & 1 & -1 & 0\\ 0 & 2 & 1 & -2 & 3 & 0\\ 0 & -4 & 2 & -1 & 0 & 1\end{array}\right]\xrightarrow{R_3+2R_2}\left[\begin{array}{ccc|ccc}1 & -1 & -1 & 1 & -1 & 0\\ 0 & 2 & 1 & -2 & 3 & 0\\ 0 & 0 & 4 & -5 & 6 & 1\end{array}\right]\xrightarrow{\frac{1}{4}R_3}\left[\begin{array}{ccc|ccc}1 & -1 & -1 & 1 & -1 & 0\\ 0 & 2 & 1 & -2 & 3 & 0\\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4}\end{array}\right]$$

$$\xrightarrow{R_2-R_3}\left[\begin{array}{ccc|ccc}1 & -1 & -1 & 1 & -1 & 0\\ 0 & 2 & 0 & -\frac{3}{4} & \frac{6}{4} & -\frac{1}{4}\\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4}\end{array}\right]\xrightarrow{\frac{1}{2}R_2}\left[\begin{array}{ccc|ccc}1 & -1 & -1 & 1 & -1 & 0\\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8}\\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4}\end{array}\right]$$

$$\xrightarrow{R_1+R_2}\left[\begin{array}{ccc|ccc}1 & 0 & -1 & \frac{5}{8} & -\frac{2}{8} & -\frac{1}{8}\\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8}\\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4}\end{array}\right]\xrightarrow{R_1+R_3}\left[\begin{array}{ccc|ccc}1 & 0 & 0 & -\frac{5}{8} & \frac{10}{8} & \frac{1}{8}\\ 0 & 1 & 0 & -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8}\\ 0 & 0 & 1 & -\frac{5}{4} & \frac{6}{4} & \frac{1}{4}\end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1}=\begin{bmatrix}-\frac{5}{8} & \frac{10}{8} & \frac{1}{8}\\ -\frac{3}{8} & \frac{6}{8} & -\frac{1}{8}\\ -\frac{5}{4} & \frac{6}{4} & \frac{1}{4}\end{bmatrix}=-\frac{1}{8}\begin{bmatrix}5 & -10 & -1\\ 3 & -6 & 1\\ 10 & -12 & -2\end{bmatrix}\text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question 12.

Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

Answer:

Let, A = $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\mathrm{Aug}\left[A\,|\,I\right] = \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ -3 & 0 & -1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$, where I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ -3 & 0 & -1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3-2R_1} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ -3 & 0 & -1 & 0 & 1 & 0 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_2+3R_1} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 9 & -7 & 3 & 1 & 0 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & -5 & 4 & -2 & 0 & 1 \end{array}\right] \xrightarrow{-R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & 5 & -4 & 2 & 0 & -1 \end{array}\right] \xrightarrow{R_3-R_2} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & -3 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & -1 & -2 \end{array}\right]$$

$$\xrightarrow{R_2-4R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 & 5 & 9 \\ 0 & 1 & -1 & 1 & -1 & -2 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & -1 & -2 \\ 0 & 0 & 1 & -3 & 5 & 9 \end{array}\right]$$

$$\xrightarrow{R_2+R_3}\left[\begin{array}{ccc|ccc}1 & 3 & -2 & 1 & 0 & 0\\ 0 & 1 & 0 & -2 & 4 & 7\\ 0 & 0 & 1 & -3 & 5 & 9\end{array}\right]\xrightarrow{R_1+2R_3}\left[\begin{array}{ccc|ccc}1 & 3 & 0 & -5 & 10 & 18\\ 0 & 1 & 0 & -2 & 4 & 7\\ 0 & 0 & 1 & -3 & 5 & 9\end{array}\right]\xrightarrow{R_1-3R_2}\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & -2 & -3\\ 0 & 1 & 0 & -2 & 4 & 7\\ 0 & 0 & 1 & -3 & 5 & 9\end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question 13.
Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

Answer:
Let, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\mathrm{Aug}\left[A\,|\,I\right] = \left[\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 2 & 5 & 7 & 0 & 1 & 0\\ -2 & -4 & -5 & 0 & 0 & 1\end{array}\right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 7 & 0 & 1 & 0 \\ -2 & -4 & -5 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ -2 & -4 & -5 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3+2R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2-R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 9 & -2 & 2 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right] \xrightarrow{R_1-3R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -1 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} \text{ [Answer]}$$

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question 14.
Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Answer:

Let, $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\mathrm{Aug}\left[A \mid I\right] = \left[\begin{array}{ccc|ccc} 3 & 0 & -1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right], \text{ where } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} 3 & 0 & -1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 0 & 9 & 2 & -2 & 3 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2-2R_3} \left[\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3-4R_2} \left[\begin{array}{ccc|ccc} 1 & -3 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 0 & 1 & 8 & -12 & 9 \end{array}\right] \xrightarrow{R_1+R_3} \left[\begin{array}{ccc|ccc} 1 & -3 & 0 & 9 & -13 & 9 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 0 & 1 & 8 & -12 & 9 \end{array}\right]$$

$$\xrightarrow{R_1+3R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -4 & 3 \\ 0 & 1 & 0 & -2 & 3 & -2 \\ 0 & 0 & 1 & 8 & -12 & 9 \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$

Question 15.
Using elementary row transformations, find the inverse of each of the following matrices:

$$\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Answer:

Let, $A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Now we are going to write the Augmented Matrix followed by matrix A and the Identity matrix I, i.e.,

$$\text{Aug}\left[A\,|\,I\right] = \left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$, where I = $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now our job is to convert the matrix A into Identity Matrix. Therefore, the matrix we will get converting the matrix I will be our A^{-1}.

$$\left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{ccc|ccc} 0 & 3 & 5 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3-3R_2} \left[\begin{array}{ccc|ccc} 0 & 3 & 5 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{R_1\leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 3 & 5 & 1 & 1 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right] \xrightarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & -2 & -3 & 1 & -2 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{R_1-R_2} \left[\begin{array}{ccc|ccc} 0 & 2 & 3 & -1 & 2 & -1 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right] \xrightarrow{R_3+3R_1} \left[\begin{array}{ccc|ccc} 0 & 2 & 3 & -1 & 2 & -1 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & -3 & 3 & -2 \end{array}\right] \xrightarrow{R_1-2R_3} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 5 & -4 & 3 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & -3 & 3 & -2 \end{array}\right]$$

$$\xrightarrow{R_3-R_1} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 5 & -4 & 3 \\ 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -8 & 7 & -5 \end{array}\right] \xrightarrow{R_1\leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 5 & -4 & 3 \\ 0 & 1 & 0 & -8 & 7 & -5 \end{array}\right] \xrightarrow{R_2\leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -8 & 7 & -5 \\ 0 & 0 & 1 & 5 & -4 & 3 \end{array}\right]$$

Here, the matrix A is converted into Identity matrix. Therefore, we get the A^{-1} as,

$$A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$$ [Answer]

The value of A^{-1} is correct or not can be verified by the formula: $AA^{-1} = I$