Exercise 27f

Question 1.
If a line has direction ratios 2, -1, -2 then what are its direction cosines?

Answer:
Given : A line has direction ratios 2, -1, -2

To find : Direction cosines of the line

Formula used : If (l,m,n) are the direction ratios of a given line then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$

Here l = 2 , m = -1 , n = -2

Direction cosines of the line with direction ratios 2, -1, -2 is

$$\frac{2}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-1}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-2}{\sqrt{2^2+(-1)^2+(-2)^2}}$$

$$= \frac{2}{\sqrt{4+1+4}}, \frac{-1}{\sqrt{4+1+4}}, \frac{-2}{\sqrt{4+1+4}} = \frac{2}{\sqrt{9}}, \frac{-1}{\sqrt{9}}, \frac{-2}{\sqrt{9}}$$

$$= \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$$

Direction cosines of the line with direction ratios 2, -1, -2 is $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$

Question 2.
Find the direction cosines of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

Answer:
Given : A line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

To find : Direction cosines of the line

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}$ then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$

The line is $\frac{x-4}{-2}=\frac{y-0}{6}=\frac{z-1}{-3}$

Here l = -2 , m = 6 , n = -3

Direction cosines of the line $\frac{x-4}{-2}=\frac{y-0}{6}=\frac{z-1}{-3}$ is

$$\frac{-2}{\sqrt{(-2)^2+(6)^2+(-3)^2}}, \frac{6}{\sqrt{(-2)^2+(6)^2+(-3)^2}}, \frac{-3}{\sqrt{(-2)^2+(6)^2+(-3)^2}}$$

$$=\frac{-2}{\sqrt{4+36+9}}, \frac{6}{\sqrt{4+36+9}}, \frac{-3}{\sqrt{4+36+9}}=\frac{-2}{\sqrt{49}}, \frac{6}{\sqrt{49}}, \frac{-3}{\sqrt{49}}$$

$$=\frac{-2}{7}, \frac{6}{7}, \frac{-3}{7}$$

Direction cosines of the line $\frac{x-4}{-2}=\frac{y-0}{6}=\frac{z-1}{-3}$ is $\frac{-2}{7}, \frac{6}{7}, \frac{-3}{7}$

Question 3.

If the equations of a line are $\frac{3-x}{-3}=\frac{y+2}{-2}=\frac{z+2}{6}$, find the direction cosines of a line parallel to the given line.

Answer:

Given : A line $\frac{3-x}{-3}=\frac{y+2}{-2}=\frac{z+2}{6}$,

To find : Direction cosines of the line parallel to $\frac{3-x}{-3}=\frac{y+2}{-2}=\frac{z+2}{6}$,

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}$ then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$

The line is $\frac{x-3}{3}=\frac{y+2}{-2}=\frac{z+2}{6}$

Parallel lines have same direction ratios and direction cosines

Here l = 3 , m = -2 , n = 6

Direction cosines of the line $\frac{x-3}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$ is

$$\frac{3}{\sqrt{(3)^2+(-2)^2+(6)^2}}, \frac{-2}{\sqrt{(3)^2+(-2)^2+(6)^2}}, \frac{6}{\sqrt{(3)^2+(-2)^2+(6)^2}}$$

$$= \frac{3}{\sqrt{9+4+36}}, \frac{-2}{\sqrt{9+4+36}}, \frac{6}{\sqrt{9+4+36}} = \frac{3}{\sqrt{49}}, \frac{-2}{\sqrt{49}}, \frac{6}{\sqrt{49}}$$

$$= \frac{3}{7}, \frac{-2}{7}, \frac{6}{7}$$

Direction cosines of the line parallel to the line $\frac{x-3}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$ is

$$\frac{3}{7}, \frac{-2}{7}, \frac{6}{7}$$

Question 4.

Write the equations of a line parallel to the line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$ and passing through the point (1, -2, 3).

Answer:

Given : A line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$

To find : equations of a line parallel to the line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$ and passing through the point (1, -2, 3).

Formula used : If a line is given by $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ then equation of parallel

line passing through the point (p,q,r) is given by $\frac{x-p}{l} = \frac{y-q}{m} = \frac{z-r}{n}$

Here l = -3 , m = 2 , n = 6 and p = 1 , q = -2 , r = 3

The line parallel to the line $\frac{x-2}{-3}=\frac{y+3}{2}=\frac{z+5}{6}$ and passing through the point (1,-2,3)

is given by

$\frac{x-1}{-3}=\frac{y-(-2)}{2}=\frac{z-3}{6}$

$\frac{x-1}{-3}=\frac{y+2}{2}=\frac{z-3}{6}$

The line parallel to the line $\frac{x-2}{-3}=\frac{y+3}{2}=\frac{z+5}{6}$ and passing through the point

(1,-2,3) is given by $\frac{x-1}{-3}=\frac{y+2}{2}=\frac{z-3}{6}$

Question 5.
Find the Cartesian equations of the line which passes through the point (-2, 4, -5) and which is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

Answer:
Given : A line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

To find : equations of a line parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

and passing through the point (-2, 4, -5).

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}$ then equation of parallel

line passing through the point (p,q,r) is given by $\frac{x-p}{l}=\frac{y-q}{m}=\frac{z-r}{n}$

The given line is $\frac{x+3}{3}=\frac{y-4}{-5}=\frac{z+8}{6}$

Here l = 3 , m = -5 , n = 6 and p = -2 , q = 4 , r = -5

The line parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by

$\frac{x-(-2)}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$

$\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$

The line parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$

Question 6.

Write the vector equation of a line whose Cartesian equations are $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$.

Answer:

Given : A line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$.

To find : vector equation of a line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$.

Formula used : If a line is given by $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n} = \lambda$ then vector equation of the line is given by $\vec{r} = a\vec{\imath} + b\vec{\jmath} + c\vec{k} + \lambda\,(l\vec{\imath} + m\vec{\jmath} + n\vec{k})$

Here a = 5 , b = -4 , c = 6 and l = 3 , m = 7 , n = -2

Substituting the above values,we get

$\vec{r} = 5\vec{\imath} - 4\vec{\jmath} + 6\vec{k} + \lambda\,(3\vec{\imath} + 7\vec{\jmath} - 2\vec{k})$

The vector equation of a line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$. is given by

$\vec{r}$ = 5$\vec{\imath}$ - 4$\vec{\jmath}$ + 6$\vec{k}$ + λ (3$\vec{\imath}$ + 7$\vec{\jmath}$ - 2$\vec{k}$)

Question 7.

The Cartesian equations of a line are $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$. Write the vector equation of the line.

Answer:

Given : A line $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$.

To find : vector equation of a line $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$.

Formula used : If a line is given by $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ = λ then vector equation of the line is given by $\vec{r}$ = a$\vec{\imath}$ + b$\vec{\jmath}$ + c$\vec{k}$ + λ (l$\vec{\imath}$ + m$\vec{\jmath}$ + n$\vec{k}$)

The given line is $\frac{x-3}{-5} = \frac{y+4}{7} = \frac{z-3}{2}$

Here a = 3 , b = -4 , c = 3 and l = -5 , m = 7 , n = 2

Substituting the above values,we get

$\vec{r}$ = 3$\vec{\imath}$ - 4$\vec{\jmath}$ + 3$\vec{k}$ + λ (-5$\vec{\imath}$ + 7$\vec{\jmath}$ + 2$\vec{k}$)

The vector equation of a line is given by $\frac{x-3}{-5} = \frac{y+4}{7} = \frac{z-3}{2}$

$\vec{r}$ = 3$\vec{\imath}$ - 4$\vec{\jmath}$ + 3$\vec{k}$ + λ (-5$\vec{\imath}$ + 7$\vec{\jmath}$ + 2$\vec{k}$)

Question 8.

Write the vector equation of a line passing through the point (1, -1, 2) and parallel to the line whose equations are $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$.

Answer:

Given : A line $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z+1}{-2}$.

To find : vector equation of a line passing through the point (1, -1, 2) and parallel

to the line whose equations are $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z+1}{-2}$.

Formula used : If a line is parallel to $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}$ and passing through the point (p,q,r) then vector equation of the line is given by $\vec{r}$ = p$\vec{i}$ + q$\vec{j}$ + r$\vec{k}$ + λ (l$\vec{i}$ + m$\vec{j}$ + n$\vec{k}$)

The given line is $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z+1}{-2}$

Here p = 1 , q = -1 , c = 2 and l = 1 , m = 2 , n = 2

Substituting the above values,we get

$\vec{r}$ = 1$\vec{i}$ - 1$\vec{j}$ + 2$\vec{k}$ + λ (1$\vec{i}$ + 2$\vec{j}$ + 2$\vec{k}$)

The vector equation of a line passing through the point (1, -1, 2) and

parallel to the line whose equations are $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z+1}{-2}$. is given by

$\vec{r}$ = $\vec{i}$ - $\vec{j}$ + 2$\vec{k}$ + λ ($\vec{i}$ + 2$\vec{j}$ + 2$\vec{k}$)

Question 9.
If P(1, 5, 4) and Q(4, 1, -2) be two given points, find the direction ratios of PQ.

Answer:
Given : P(1, 5, 4) and Q(4, 1, -2) be two given points

To find : direction ratios of PQ

Formula used : if P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) be two given points then direction ratios of PQ is given by $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$

$x_1 = 1$, $y_1 = 5$, $z_1 = 4$ and $x_2 = 4$, $y_2 = 1$, $z_2 = -2$

Direction ratios of PQ is given by $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$

Direction ratios of PQ is given by $4 - 1, 1 - 5, -2 - 4$

Direction ratios of PQ is given by $3, -4, -6$

Direction ratios of PQ is given by $3, -4, -6$

Question 10.

The equations of a line are $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$. Find the direction cosines of a line parallel to this line.

Answer:

Given : A line $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$.

To find : Direction cosines of the line parallel to $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$.

Formula used : If a line is given by $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$

The line is $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$

Parallel lines have same direction ratios and direction cosines

Here $l = -2$, $m = 2$, $n = 1$

Direction cosines of the line $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$ is

$$\frac{-2}{\sqrt{(-2)^2+(2)^2+(1)^2}}, \frac{2}{\sqrt{(-2)^2+(2)^2+(1)^2}}, \frac{1}{\sqrt{(-2)^2+(2)^2+(1)^2}}$$

$= \frac{-2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{1}{\sqrt{4+4+1}} = \frac{-2}{\sqrt{9}}, \frac{2}{\sqrt{9}}, \frac{1}{\sqrt{9}}$

$= \frac{-2}{3}, \frac{2}{3}, \frac{1}{3}$

Direction cosines of the line parallel to the line $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$ is

$\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}$

Question 11.

The Cartesian equations of a line are $\frac{x-1}{2} = \frac{y+2}{3} = \frac{5-z}{1}$. Find its vector equation.

Answer:

Given : A line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{5-z}{1}$.

To find : vector equation of a line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{5-z}{1}$.

Formula used : If a line is given by $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n} = \lambda$ then vector equation of the line is given by $\vec{r} = a\vec{i} + b\vec{j} + c\vec{k} + \lambda\,(l\vec{i} + m\vec{j} + n\vec{k})$

The given line is $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-5}{-1}$

Here a = 1 , b = -2 , c = 5 and l = 2 , m = 3 , n = -1

Substituting the above values,we get

$\vec{r} = 1\vec{i} - 2\vec{j} + 5\vec{k} + \lambda\,(2\vec{i} + 3\vec{j} - 1\vec{k})$

The vector equation of a line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{5-z}{1}$. is given by

$\vec{r} = 1\vec{i} - 2\vec{j} + 5\vec{k} + \lambda\,(2\vec{i} + 3\vec{j} - 1\vec{k})$

Question 12.
Find the vector equation of a line passing through the point (1, 2, 3) and parallel to the vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.

Answer:
Given : A vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.

To find : vector equation of a line passing through the point (1, 2, 3) and parallel

to the vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.

Formula used : If a line is parallel to the vector $(l\vec{i} + m\vec{j} + n\vec{k})$

and passing through the point (p,q,r) then vector equation of the line is given by

$\vec{r} = p\vec{i} + q\vec{j} + r\vec{k} + \lambda\ (l\vec{i} + m\vec{j} + n\vec{k})$

Here p = 1 , q = 2 , c = 3 and l = 3 , m = 2 , n = -2

Substituting the above values,we get

$\vec{r} = 1\vec{i} + 2\vec{j} + 3\vec{k} + \lambda\ (3\vec{i} + 2\vec{j} - 2\vec{k})$

The vector equation of a line passing through the point (1, 2, 3) and

parallel to the vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.is $\vec{r} = \vec{i} + 2\vec{j} + 3\vec{k} + \lambda\ (3\vec{i} + 2\vec{j} - 2\vec{k})$

Question 13.
The vector equation of a line is $\vec{r}=\left(2\hat{i}+\hat{j}-4\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$. Find its Cartesian equation.

Answer:
Given : The vector equation of a line is $\vec{r}=\left(2\hat{i}+\hat{j}-4\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$.

To find : Cartesian equation of the line

Formula used : If the vector equation of the line is given by

$\vec{r} = p\vec{\imath} + q\vec{\jmath} + r\vec{k} + \lambda\,(l\vec{\imath} + m\vec{\jmath} + n\vec{k})$ then its Cartesian equation is given by

$\frac{x-p}{l} = \frac{y-q}{m} = \frac{z-r}{n}$

The vector equation of a line is $\vec{r} = \left(2\hat{i} + \hat{j} - 4\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} - \hat{k}\right)$.

Here p = 2, q = 1, r = -4 and l = 1,m = -1,n = -1

Cartesian equation is given by

$\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z-(-4)}{-1}$

$\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z+4}{-1}$

Cartesian equation of the line is given by $\frac{x-2}{1} = \frac{y-1}{-1} = \frac{z+4}{-1}$

Question 14.
Find the Cartesian equation of a line which passes through the point (-2, 4, -5) and which is parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

Answer:
Given : A line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

To find : cartesian equations of a line parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

and passing through the point (-2, 4, -5).

Formula used : If a line is given by $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ then equation of parallel line passing through the point (p,q,r) is given by $\frac{x-p}{l}=\frac{y-q}{m}=\frac{z-r}{n}$

The given line is $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$

Here l = 3 , m = 5 , n = 6 and p = -2 , q = 4 , r = -5

The line parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by

$$\frac{x-(-2)}{3}=\frac{y-4}{5}=\frac{z-(-5)}{6}$$

$$\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$$

The line parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by $\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$

Question 15.
Find the Cartesian equation of a line which passes through the point having position vector $\left(2\hat{i}-\hat{j}+4\hat{k}\right)$ and is in the direction of the vector $\left(\hat{i}+2\hat{j}-\hat{k}\right)$.

Answer:
Given : A line which passes through the point having position vector $\left(2\hat{i}-\hat{j}+4\hat{k}\right)$

and is in the direction of the vector $\left(\hat{i}+2\hat{j}-\hat{k}\right)$.

To find : cartesian equations of a line

Formula used : If a line which passes through the point having position vector

$p\vec{i} + q\vec{j} + r\vec{k}$ and is in the direction of the vector $l\vec{i} + m\vec{j} + n\vec{k}$ then its Cartesian

equation is given by

$$\frac{x-p}{l} = \frac{y-q}{m} = \frac{z-r}{n}$$

A line which passes through the point having position vector $\left(2\hat{i} - \hat{j} + 4\hat{k}\right)$

and is in the direction of the vector $\left(\hat{i} + 2\hat{j} - \hat{k}\right)$.

Here l = 1 , m = 2 , n = -1 and p = 2 , q = -1 , r = 4

$$\frac{x-2}{1} = \frac{y-(-1)}{2} = \frac{z-4}{-1}$$

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$$

The Cartesian equation of a line which passes through the point having

position vector $\left(2\hat{i} - \hat{j} + 4\hat{k}\right)$ and is in the direction of the vector $\left(\hat{i} + 2\hat{j} - \hat{k}\right)$. is

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$$

Question 16.

Find the angle between the lines $\vec{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and

$\vec{r} = \left(7\hat{i} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$.

Answer:

Given : the lines $\vec{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and

$\vec{r} = \left(7\hat{i} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$.

To find : angle between the lines

Formula used : If the lines are $a\vec{i} + b\vec{j} + c\vec{k} + \lambda(p\vec{i} + q\vec{j} + r\vec{k})$ and $d\vec{i} + e\vec{j} + f\vec{k} +$

$\lambda(l\vec{i} + m\vec{j} + n\vec{k})$ then the angle between the lines 'θ' is given by

$$\theta = \cos^{-1}\frac{pl + qm + rn}{\sqrt{p^2+q^2+r^2}\sqrt{l^2+m^2+n^2}}$$

the lines $\vec{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and $\vec{r} = \left(7\hat{i} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$.

Here p = 3 , q = 2 , r = 6 and l = 1 , m = 2 , n = 2

$$\theta = \cos^{-1}\frac{3(1)+2(2)+6(2)}{\sqrt{3^2+2^2+6^2}\sqrt{1^2+2^2+2^2}} = \cos^{-1}\frac{3+4+12}{\sqrt{9+4+36}\sqrt{1+4+4}}$$

$$\theta = \cos^{-1}\frac{3+4+12}{\sqrt{49}\sqrt{9}} = \cos^{-1}\frac{19}{7\times 3} = \cos^{-1}\frac{19}{21}$$

$$\theta = \cos^{-1}\frac{19}{21}$$

The angle between the lines $\vec{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and $\vec{r} = \left(7\hat{i} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$. is $\cos^{-1}\frac{19}{21}$

Question 17.

Find the angle between the lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

Answer:

Given : the lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

To find : angle between the lines

Formula used : If the lines are $\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$ and $\frac{x-c}{l} = \frac{y-d}{m} = \frac{z-e}{n}$

then the angle between the lines 'θ' is given by

$$\theta = \cos^{-1}\frac{pl + qm + rn}{\sqrt{p^2 + q^2 + r^2}\sqrt{l^2 + m^2 + n^2}}$$

The lines are $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

Here p = 3 , q = 5 , r = 4 and l = 1 , m = 1 , n = 2

$$\theta = \cos^{-1}\frac{3(1)+5(1)+4(2)}{\sqrt{3^2+5^2+4^2}\sqrt{1^2+1^2+2^2}} = \cos^{-1}\frac{3+5+8}{\sqrt{9+25+16}\sqrt{1+1+4}}$$

$$\theta = \cos^{-1}\frac{3+5+8}{\sqrt{50}\sqrt{6}} = \cos^{-1}\frac{16}{10\sqrt{3}} = \cos^{-1}\frac{8}{5\sqrt{3}}$$

$$\theta = \cos^{-1}\frac{8\sqrt{3}}{15}$$

The angle between the lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

is $\cos^{-1}\frac{8\sqrt{3}}{15}$

Question 18.

Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are at right angles.

Answer:

Given : the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$.

To prove : the lines are at right angles.

Formula used : If the lines are $\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$ and $\frac{x-c}{l} = \frac{y-d}{m} = \frac{z-e}{n}$

then the angle between the lines ‘θ’ is given by

$$\theta = \cos^{-1}\frac{pl + qm + rn}{\sqrt{p^2 + q^2 + r^2}\sqrt{l^2 + m^2 + n^2}}$$

The lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$.

Here p = 7 , q = -5 , r = 1 and l = 1 , m = 2 , n = 3

$\theta = \cos^{-1}\frac{7(1)+(-5)(2)+1(3)}{\sqrt{7^2+(-5)^2+1}\sqrt{1^2+2^2+3^2}} = \cos^{-1}\frac{7-10+3}{\sqrt{49+25+1}\sqrt{1+4+9}}$

$\theta = \cos^{-1}\frac{0}{\sqrt{75}\sqrt{14}} = \cos^{-1}0$ = 90°

θ = 90°

The Lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are at right angles.

Question 19.
The direction ratios of a line are 2, 6, -9. What are its direction cosines?

Answer:
Given : A line has direction ratios 2, 6 , -9

To find : Direction cosines of the line

Formula used : If (l,m,n) are the direction ratios of a given line then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$

Here l = 2 , m = 6 , n = -9

Direction cosines of the line with direction ratios 2, 6 , -9 is

$\frac{2}{\sqrt{2^2+6^2+(-9)^2}}, \frac{6}{\sqrt{2^2+6^2+(-9)^2}}, \frac{-9}{\sqrt{2^2+6^2+(-9)^2}}$

$= \frac{2}{\sqrt{4+36+81}}, \frac{6}{\sqrt{4+36+81}}, \frac{-9}{\sqrt{4+36+81}} = \frac{2}{\sqrt{121}}, \frac{6}{\sqrt{121}}, \frac{-9}{\sqrt{121}}$

$= \frac{2}{11}, \frac{6}{11}, \frac{-9}{11}$

Direction cosines of the line with direction ratios 2, 6, -9 is $\frac{2}{11}, \frac{6}{11}, \frac{-9}{11}$

Question 20.
A line makes angles 90^o, 135^o and 45^o with the positive directions of x-axis, y-axis and z-axis respectively. what are the direction cosines of the line?

Answer:
Given : A line makes angles 90^o, 135^o and 45^o with the positive directions of x-axis, y-axis and z-axis respectively.

To find : Direction cosines of the line

Formula used : If a line makes angles α^o, β^o and γ^o with the positive directions of x-axis, y-axis and z-axis respectively. then direction cosines are given by $\cos\alpha$, $\cos(180° - \beta)$, $\cos(180° - \gamma)$

$\alpha = 90^o$, $\beta = 135^o$ and $\gamma = 45^o$

Direction cosines of the line is

$\cos 90°$, $\cos(180° - 135°)$, $\cos(180° - 45°)$

$\cos 90°$, $\cos 45°$, $\cos(135°)$

0, $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Direction cosines of the line is 0, $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Question 21.
What are the direction cosines of the y-axis?

Answer:
To find : Direction cosines of the y- axis

Formula used : If a line makes angles α^o, β^o and γ^o with the positive directions of x-axis, y-axis and z-axis respectively. then direction cosines are given by $\cos\alpha$, $\cos\ \beta$, $\cos\gamma$

y-axis makes 90^o with the x and z axes α = 90°, β = 0 ° and γ = 90 °

Direction cosines of the line is

$\cos 90°$, $\cos 0°$, $\cos 90°$

0, 1, 0

Direction cosines of the line is 0, 1 , 0

Question 22.

What are the direction cosines of the vector $\left(2\hat{i} + \hat{j} - 2\hat{k}\right)$?

Answer:

Given : A vector $\left(2\hat{i} + \hat{j} - 2\hat{k}\right)$?

To find : Direction cosines of the vector

Formula used : If a vector is $l\vec{i} + m\vec{j} + n\vec{k}$ then direction cosines are given by $\frac{l}{\sqrt{l^2 + m^2 + n^2}}$, $\frac{m}{\sqrt{l^2 + m^2 + n^2}}$, $\frac{n}{\sqrt{l^2 + m^2 + n^2}}$

Here l = 2 , m = 1 , n = -2

Direction cosines of the line with direction ratios 2, 1, -2 is

$$\frac{2}{\sqrt{2^2 + (1)^2 + (-2)^2}}, \frac{1}{\sqrt{2^2 + (1)^2 + (-2)^2}}, \frac{-2}{\sqrt{2^2 + (1)^2 + (-2)^2}}$$

$$= \frac{2}{\sqrt{4+1+4}}, \frac{1}{\sqrt{4+1+4}}, \frac{-2}{\sqrt{4+1+4}} = \frac{2}{\sqrt{9}}, \frac{1}{\sqrt{9}}, \frac{-2}{\sqrt{9}}$$

$$= \frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$$

Direction cosines of the vector is $\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$

Question 23.

What is the angle between the vector $\vec{r} = \left(4\hat{i} + 8\hat{j} + \hat{k}\right)$ and the x-axis?

Answer:

Given : the vector $\vec{r} = \left(4\hat{i} + 8\hat{j} + \hat{k}\right)$

To find : angle between the vector and the x-axis

Formula used : If the vector $l\vec{i} + m\vec{j} + n\vec{k}$ and x-axis then the angle between the

lines 'θ' is given by

$$\theta = \cos^{-1}\frac{l}{\sqrt{l^2 + m^2 + n^2}}$$

Here l = 4 , m = 8 , n = 1

$$\theta = \cos^{-1}\frac{4}{\sqrt{4^2 + 8^2 + 1^2}} = \cos^{-1}\frac{4}{\sqrt{16 + 64 + 1}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{81}} = \cos^{-1}\frac{4}{9}$$

$$\theta = \cos^{-1}\frac{4}{9}$$

The angle between the vector and the x-axis is $\cos^{-1}\frac{4}{9}$