Exercise 32

Question 1.
A coin is tossed 6 times. Find the probability of getting at least 3 heads.

Answer:
As the coin is tossed 6 times the total number of outcomes will be 2^6

And we know that the favourable outcomes of getting at least 3 heads will be ${}^6c_3 + {}^6c_4 + {}^6c_5 + {}^6c_6$

Thus, the probability of getting at least 3 heads will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$$\Rightarrow = \frac{\binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6}}{2^6}$$

$$\Rightarrow = \frac{21}{32}$$

Question 2.
A coin is tossed 5 times. What is the probability that a head appears an even number of times?

Answer:
As the coin is tossed 5 times the total number of outcomes will be $2^5 = 32$.

And we know that the favourable outcomes of a head appearing even number of times will be,

That either the head appears 0, 2 or 4 times so,

The respective probabilities will be:- ${}^5C_0 + {}^5C_2 + {}^5C_4 = 16$

Thus, the probability

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$\Rightarrow = \frac{16}{32} = \frac{1}{2}$

Hence, the probability is $1/2$.

Question 3.
7 coins are tossed simultaneously. What is the probability that a tail appears an odd number of times?

Answer:
As 7 coins are tossed simultaneously the total number of outcomes are 2^7=128.

The favourable number of outcomes that a tail appears an odd number of times will be, ${}^7C_1 + {}^7C_3 + {}^7C_5 + {}^7C_7 = 64$.

Thus, the probability

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$$= \frac{64}{128}$$

$$= \frac{1}{2}$$

Hence, the probability is $1/2$.

Question 4.
A coin is tossed 6 times. Find the probability of getting

(i) exactly 4 heads

(ii) at least 1 heads

(iii) at most 4 heads

Answer:
(i) As the coin is tossed 6 times the total number of outcomes will be 2^6= 64

And we know that the favourable outcomes of getting exactly 4 heads will be ${}^6C_4$ = 15

Thus, the probability of getting exactly 4 heads will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

⇒ 15/64

(ii) As the coin is tossed 6 times the total number of outcomes will be 2^6= 64

And we know that the favourable outcomes of getting at least 1 heads will be ${}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6 = 63$

Thus, the probability of getting at least 1 head will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

⇒ 63/64

(iii) As the coin is tossed 6 times the total number of outcomes will be 2^6= 64

And we know that the favourable outcomes of getting at most 4 heads will be ${}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 = 57$

Thus, the probability of getting at most 4 heads will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

⇒ 57/64

Question 5.
10 coins are tossed simultaneously. Find the probability of getting

(i) exactly 3 heads

(ii) not more than 4 heads

(iii) at least 4 heads

Answer:
(i) As 10 coins are tossed simultaneously the total number of outcomes are 2^{10}=1024.

the favourable outcomes of getting exactly 3 heads will be

$^{10}C_3 = 120$

Thus, the probability

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$$= \frac{120}{1024}$$

$$= \frac{15}{128}$$

Hence, the probability is $\frac{15}{128}$.

(ii) As 10 coins are tossed simultaneously the total number of outcomes are 2^{10}=1024.

the favourable outcomes of getting not more than 4 heads will be

$^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 = 386$

Thus, the probability

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$$= \frac{386}{1024}$$

$$\Rightarrow \frac{193}{512}$$

Hence, the probability is $\frac{193}{512}$.

(iii) As 10 coins are tossed simultaneously the total number of outcomes are 2^{10}=1024.

the favourable outcomes of getting at least 4 heads will be

${}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} = 848$

Thus, the probability

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$$= \frac{848}{1024}$$

$$\Rightarrow \frac{53}{64}$$

Hence, the probability is $\frac{53}{64}$.

Question 6.
A die is thrown 6 times. If 'getting an even number' is a success, find the probability of getting

(i) exactly 5 successes

(ii) at least 5 successes

(iii) at most 5 successes

Answer:
(i) Using Bernoulli's Trial P(Success=x) = ${}^{n}C_x . p^x . q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the die is thrown 6 times the total number of outcomes will be 6^6.

And we know that the favourable outcomes of getting exactly 5 successes will be, either getting 2, 4 or 6 i.e., 1/6 probability of each, total, $\frac{3}{6}$ probability, p = $\frac{1}{2}$,q = $\frac{1}{2}$

The probability of success is $\frac{3}{6}$ and of failure is also $\frac{3}{6}$.

Thus, the probability of getting exactly 5 successes will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$\Rightarrow {}^6C_5 \frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}$

$\Rightarrow {}^6C_5.\frac{1}{64}$

$\Rightarrow \frac{3}{32}$

(ii) Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the die is thrown 6 times the total number of outcomes will be 6^6.

And we know that the favourable outcomes of getting at least 5 successes will be, either getting 2, 4 or 6 i.e, 1/6 probability of each, total, $\frac{3}{6}$ probability, p = $\frac{3}{6}$, q = $\frac{3}{6}$

The probability of success is $\frac{3}{6}$ and of failure is also $\frac{3}{6}$.

Thus, the probability of getting at least 5 successes will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$\Rightarrow ({}^6C_5 + {}^6C_6)\ \frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}$

$\Rightarrow ({}^6C_5 + {}^6C_6).\frac{1}{64}$

$\Rightarrow \frac{7}{64}$

(iii) Using Bernoulli's Trial P(Success=x) = ${}^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

As the die is thrown 6 times the total number of outcomes will be 6^6.

And we know that the favourable outcomes of getting at most 5 successes will be, either getting 2, 4 or 6 i.e, 1/6 probability of each, total, $\frac{3}{6}$ probability of success .

The probability of success is $\frac{3}{6}$ and of failure is also $\frac{3}{6}$.

Thus, the probability of getting at most 5 successes will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$\Rightarrow ({}^{6}C_{0} + {}^{6}C_{1} + {}^{6}C_{2} + {}^{6}C_{3} + {}^{6}C_{4} + {}^{6}C_{5}).\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}.\frac{3}{6}$

$\Rightarrow ({}^{6}C_{0} + {}^{6}C_{1} + {}^{6}C_{2} + {}^{6}C_{3} + {}^{6}C_{4} + {}^{6}C_{5}).\frac{1}{64}$

$\Rightarrow \frac{63}{64}$

Question 7.
A die is thrown 4 times. 'Getting a 1 or a 6' is considered a success, Find the probability of getting

(i) exactly 3 successes

(ii) at least 2 successes

(iii) at most 2 successes

Answer:
Using Bernoulli's Trial P(Success=x) = ${}^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

We know that the favourable outcomes of getting exactly 3 successes will be, either getting 1 or a 6 i.e, total, $\frac{2}{6}$probability

The probability of success is $\frac{2}{6}$ and of failure is $\frac{4}{6}$.

Thus, the probability of getting exactly 3 successes will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$\Rightarrow ({}^4C_3)\, \frac{2}{6}.\frac{2}{6}.\frac{2}{6}.\frac{4}{6}$

$\Rightarrow ({}^4C_3).\frac{2}{81}$

$\Rightarrow \frac{8}{81}$

(ii) Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

We know that the favourable outcomes of getting at least 2 successes will be, either getting 1 or a 6 i.e, total, $\frac{2}{6}$probability

The probability of success is $\frac{2}{6}$ and of failure is $\frac{4}{6}$.

Thus, the probability of getting at least 2 successes will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$\Rightarrow ({}^4C_2)\,)\, \frac{2}{6}.\frac{2}{6}.\frac{4}{6}.\frac{4}{6} + ({}^4C_3)\,)\, \frac{2}{6}.\frac{2}{6}.\frac{2}{6}.\frac{4}{6} + ({}^4C_4)\, \frac{2}{6}.\frac{2}{6}.\frac{2}{6}.\frac{2}{6}$

$\Rightarrow \frac{33}{81}$

$\Rightarrow \frac{11}{27}$

(iii) Using Bernoulli's Trial P(Success=x) = ${}^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

We know that the favourable outcomes of getting at most 2 successes will be, either getting 1 or a 6 i.e, total, $\frac{2}{6}$probability

The probability of success is $\frac{2}{6}$ and of failure is $\frac{4}{6}$.

Thus, the probability of getting at most 2 successes will be

$$= \frac{The\ favourable\ outcomes}{The\ total\ number\ of\ outcomes}$$

$\Rightarrow ({}^{4}C_{0})\ \frac{4}{6}.\frac{4}{6}.\frac{4}{6}.\frac{4}{6} + ({}^{4}C_{1})\ \frac{2}{6}.\frac{4}{6}.\frac{4}{6}.\frac{4}{6} + ({}^{4}C_{2})\ \frac{2}{6}.\frac{2}{6}.\frac{4}{6}.\frac{4}{6}$

$\Rightarrow \frac{72}{81}$

$\Rightarrow \frac{8}{9}$

Question 8.
Find the probability of a 4 turning up at least once in two tosses of a fair die.

Answer:
The total outcomes = 36,

The favourable outcomes are (1,4), (2,4), (3,4), (4,4), (5,4), (6,4), (4,1), (4,2), (4,3) ,(4,5), (4,6)

Thus, the probability = favourable outcomes/total outcomes

$\Rightarrow \frac{11}{36}$

Question 9.

A pair of dice is thrown 4 times. If ‘getting a doublet’ is considered a success, find the probability of getting 2 successes.

Answer:
As the pair of die is thrown 4 times,

The total number of outcomes = 36

Using Bernoulli’s Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

q = $\frac{5}{6}$

probability of 2 successes = $^4C_2.(\frac{1}{6})^2(\frac{5}{6})^2$

$\Rightarrow \frac{25}{216}$

Question 10.
A pair of dice is thrown 7 times. If ‘getting a total of 7’ is considered a success, find the probability of getting

(i) no success

(ii) exactly 6 successes

(iii) at least 6 successes

(iv) at most 6 successes

Answer:
(i) Using Bernoulli’s Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =7

the favourable outcomes ,

(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

q = $\frac{5}{6}$

probability of no success = ${}^{7}C_{0}.(\frac{1}{6})^{0}(\frac{5}{6})^{7}$

⇒ $(\frac{5}{6})^{7}$

(ii) Using Bernoulli's Trial P(Success=x) = ${}^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =7

the favourable outcomes ,

(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

q = $\frac{5}{6}$

probability of exactly 6 successes = ${}^{7}C_{6}.(\frac{1}{6})^{6}(\frac{5}{6})^{1}$

⇒ $35.(\frac{1}{6})^{7}$

(iii) Using Bernoulli's Trial P(Success=x) = ${}^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =7

the favourable outcomes ,

(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

$q = \frac{5}{6}$

probability of at least 6 successes =

${}^7C_6.(\frac{1}{6})^6(\frac{5}{6})^1 + {}^7C_7.(\frac{1}{6})^7(\frac{5}{6})^0$

$\Rightarrow 36.(\frac{1}{6})^7$

$\Rightarrow (\frac{1}{6})^5$

(iv) Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =7

the favourable outcomes ,

(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)

The probability of success = p = $\frac{6}{36} = \frac{1}{6}$

$q = \frac{5}{6}$

probability of at least 6 successes =

${}^7C_0.(\frac{1}{6})^0(\frac{5}{6})^7 + {}^7C_1.(\frac{1}{6})^1(\frac{5}{6})^6 + {}^7C_2.(\frac{1}{6})^2(\frac{5}{6})^5 + {}^7C_3.(\frac{1}{6})^3(\frac{5}{6})^4 + {}^7C_4.(\frac{1}{6})^4(\frac{5}{6})^3 + {}^7C_5.(\frac{1}{6})^5(\frac{5}{6})^2 + {}^7C_6.(\frac{1}{6})^6(\frac{5}{6})^1$

$\Rightarrow (1- (\frac{1}{6})^7)$

Question 11.
There are 6% defective items in a large bulk of times. Find the probability that a sample of 8 items will include not more than one detective item.

Answer:
Using Bernoulli's Trial P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =8

The probability of success, i.e. the bulb is defective = p = $\frac{6}{100} = \frac{6}{100}$

q = $1 - \frac{6}{100} = \frac{94}{100}$

probability of that there is not more than one defective piece=

P(0 defective items) + P(1 defective item) =

$^8C_0.(\frac{6}{100})^0(\frac{94}{100})^8 + {}^8C_1.(\frac{6}{100})^1(\frac{94}{100})^7$

$\Rightarrow ((\frac{47}{50})^7 \times (\frac{71}{50}))$

Question 12.
In a box containing 60 bulbs, 6 are defective. What is the probability that out of a sample of 5 bulbs

(i) none is defective

(ii) exactly 2 are defective

Answer:
(i) Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

The probability of success, i.e. the bulb is defective = p = $\frac{6}{60} = \frac{1}{10}$

q = $1 - \frac{1}{10} = \frac{9}{10}$

probability of that no bulb is defective piece=

P(0 defective items) =

$^5C_0.(\frac{1}{10})^0(\frac{9}{10})^5$

$\Rightarrow ((\frac{9}{10})^5)$

(ii) Using Bernoulli's Trial P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

The probability of success, i.e. the bulb is defective = p = $\frac{6}{60} = \frac{1}{10}$

q = $1 - \frac{1}{10} = \frac{9}{10}$

probability of that there are exactly 2 defective pieces=

P(2 defective items) =

$^5C_2.(\frac{1}{10})^2(\frac{9}{10})^3$

$\Rightarrow ((\frac{729}{10000}))$

Question 13.
The probability that a bulb produced by a factory will fuse after 6 months of use is 0.05. find the probability that out of 5 such bulbs

(i) none will fuse after 6 months of use

(ii) at least one will fuse after 6 months of use

(iii) not more than one will fuse after 6 months of use

Answer:
(i) The probability that the bulb will fuse = 0.05 = p

The probability that the bulb will not fuse = 1-0.05 = 0.95 = q

Using Bernoulli's we have,

P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

Probability that none will fuse =

$^5C_0.(0.05)^0(0.95)^5$

$\Rightarrow (0.95)^5$

(ii) The probability that the bulb will fuse = 0.05 = p

The probability that the bulb will not fuse = 1-0.05 = 0.95 = q

Using Bernoulli's we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

Probability that at least one will fuse = P(1) + P(2) + P(3) + P(4) + P(5)

$^5C_1.(0.05)^1(0.95)^4 + {}^5C_2.(0.05)^2(0.95)^3 + {}^5C_3.(0.05)^3(0.95)^2 + {}^5C_4.(0.05)^4(0.95)^1 + {}^5C_5.(0.05)^5(0.95)^0$

$\Rightarrow (1-(0.95)^5)$

(iii) The probability that the bulb will fuse = 0.05 = p

The probability that the bulb will not fuse = 1-0.05 = 0.95 = q

Using Bernoulli's we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =5

Probability that not more than one will fuse = P(0) + P(1)

$^5C_0.(0.05)^0(0.95)^5 + {}^5C_1.(0.05)^1(0.95)^4$

$\Rightarrow (1.20).(0.95)^5$

Question 14.
In the items produced by a factory, there are 10% defective items. A sample of 6 items is randomly chosen. Find the probability that this sample contains.

(i) exactly 2 defective items

(ii) not more than 2 defective items

(iii) at least 3 defective items

Answer:
(i) The probability that the item is defective = $\frac{1}{10}$ = p

The probability that the bulb will not fuse = $1 - \frac{1}{10} = \frac{9}{10}$ = q

Using Bernoulli's we have,

P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =6

The probability that exactly 2 defective items are,

$\Rightarrow {}^6C_2.(\frac{1}{10})^2(\frac{9}{10})^4$

$$\Rightarrow \frac{3}{20} \times \left(\frac{9}{10}\right)^4$$

(ii) The probability that the item is defective = $\frac{1}{10}$ = p

The probability that the bulb will not fuse = $1 - \frac{1}{10} = \frac{9}{10}$ = q

Using Bernoulli's we have,

P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =6

The probability that not more than 2 defective items are,

$\Rightarrow {}^6C_0.(\frac{1}{10})^0(\frac{9}{10})^6 + {}^6C_1.(\frac{1}{10})^1(\frac{9}{10})^5 + {}^6C_2.(\frac{1}{10})^2(\frac{9}{10})^4$

$\Rightarrow \left(\frac{81+54+15}{10^6}\right).(9^4) = \frac{150\times 9^4}{10^6}$

(iii) The probability that the item is defective = $\frac{1}{10}$ = p

The probability that the bulb will not fuse = 1— $\frac{1}{10}$= $\frac{9}{10}$ = q

Using Bernoulli's we have,

P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =6

The probability of at least 3 defective items are,

P(3) + P(4) + P(5) + P(6)

$\Rightarrow {}^6C_3.(\frac{1}{10})^3(\frac{9}{10})^3 + {}^6C_4.(\frac{1}{10})^4(\frac{9}{10})^2 + {}^6C_5.(\frac{1}{10})^5(\frac{9}{10})^1 + {}^6C_6.(\frac{1}{10})^6(\frac{9}{10})^0$

$\Rightarrow \frac{15850}{10^6}$

Question 15.
Assume that on an average one telephone number out of 15, called between 3 p.m. on weekdays, will be busy. What is the probability that if six randomly selected telephone numbers are called, at least 3 of them will be busy?

Answer:
The probability that the called number is busy is $\frac{1}{15}$

Using Bernoulli's Trial we have,

P(Success=x) = ${}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =6

The probability that at least three of them will be busy is:-

P(0) + P(1) + P(2) + P(3)

$\Rightarrow {}^6C_0(\frac{1}{15})^0(\frac{14}{15})^6 + {}^6C_1(\frac{1}{15})^1(\frac{14}{15})^5 + {}^6C_2(\frac{1}{15})^2(\frac{14}{15})^4 + {}^6C_3(\frac{1}{15})^3(\frac{14}{15})^3$

$\Rightarrow 1 - (\frac{14}{15})^4.(\frac{59}{45})$

Question 16.
Three cars participate in a race. The probability that any one of them has an accident is 0.1. Find the probability that all the cars reach the finishing line without any accident.

Answer:
The probability that any one of them has an accident is 0.1.

The probability any car reaches safely is 0.9.

The probability that all the cars reach the finishing line without any accident is = (0.9)(0.9)(0.9) = 0.729

Question 17.
Past records show that 80% of the operations performed by a certain doctor were successful. If the doctor performs 4 operations in a day, what is the probability that at least 3 operations will be successful?

Answer:
The probability that the operations performed are successful is = 0.8

The probability that at least three operations are successful is = P(3) + P(4)

$\Rightarrow {}^4C_3(0.8)^3(0.2)^1 + {}^4C_4(0.8)^4(0.2)^0$

$\Rightarrow \frac{512}{625}$

Question 18.

The probability of a man hitting a target is (1/4). If he fires 7 times, what is the probability of his hitting the target at least twice?

Answer:
Using Bernoulli's Trial we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =7

p = � q = �

The probability of hitting the target at least twice is = P(2) + P(3) + P(4) + P(5) + P(6) + P(7)

⇒ 1-(P(0) + P(1))

$\Rightarrow 1 - ({}^7C_0(\frac{1}{4})^0(\frac{3}{4})^7 + {}^7C_1(\frac{1}{4})^1(\frac{3}{4})^6)$

$\Rightarrow 1 - (\frac{10}{4})(\ (\frac{3}{4})^6)$

$\Rightarrow \frac{4547}{8192}$

Question 19.
In a hurdles race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is (5/6). What is the probability that he will knock down fewer than 2 hurdles?

Answer:
The probability that the hurdle will be cleared is 5/6

Using Bernoulli's Trial we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =10

p = 5/6 q = 1/6

Probability that he will knock down fewer than 2 hurdles is =

P(0) + P(1)

$\Rightarrow {}^{10}C_0(\frac{1}{6})^0(\frac{5}{6})^{10} + {}^{10}C_1(\frac{1}{6})^1(\frac{5}{6})^9$

$\Rightarrow \frac{5^{10}}{2\times 6^9}$

Question 20.
A man can hit a bird, once in 3 shots. On this assumption he fires 3 shots. What is the chance that at least one bird is hit?

Answer:
The probability that the bird will be shot, is 1/3

Using Bernoulli's Trial we have,

$P(Success=x) = {}^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =3

p = 1/3 q =2/3

Probability that he will hit at least one bird is =

P(1) + P(2) + P(3)

$\Rightarrow {}^3C_1(\frac{1}{3})^1(\frac{2}{3})^2 + {}^3C_2(\frac{1}{3})^2(\frac{2}{3})^1 + {}^3C_3(\frac{1}{3})^3(\frac{2}{3})^0$

$\Rightarrow \frac{19}{27}$

Question 21.
If the probability that a man aged 60 will live to be 70 is 0.65, what is the probability that out of 10 men, now 60, at least 8 will live to be 70?

Answer:
The probability that a man aged 60 will live to be 70 is 0.65

Using Bernoulli's Trial we have,

$P(\text{Success}=x) = {}^nC_x.p^x.q^{(n-x)}$

$x=0, 1, 2, \ldots\ldots\ldots n$ and $q = (1-p)$, $n = 8$

$p = 0.65$ $q = 0.35$

Probability that out of 10 men, now 60, at least 8 will live to be 70 is: P(8) + P(9) + P(10)

${}^{10}C_8(0.65)^8(0.35)^2 + {}^{10}C_9(0.65)^9(0.35)^1 + {}^{10}C_{10}(0.65)^{10}(0.35)^0$

$\Rightarrow 0.2615$

Question 22.
A bag contains 5 white, 7 red 8 black balls. If four balls are drawn one by one with replacement, what is the probability that

(i) None is white

(ii) All are white

(iii) At least one is white

Answer:
(i) Balls are drawn at random,

So, the probability that none is white is,

In a trial the probability of selecting a non-white ball is $\frac{15}{20}$

So, in 4 trials it will be,

$\Rightarrow \left(\frac{15}{20}\right)\left(\frac{15}{20}\right)\left(\frac{15}{20}\right)\left(\frac{15}{20}\right) = \frac{81}{256}$

(ii) Balls are drawn at random,

So, the probability that all are white is,

In a trial the probability of selecting a white ball is $\frac{5}{20}$

So, in 4 trials it will be,

$\Rightarrow (\frac{5}{20})(\frac{5}{20})(\frac{5}{20})(\frac{5}{20}) = \frac{1}{256}$

(iii) Balls are drawn at random,

So, the probability that at least one is white is,

In a trial the probability of selecting a white ball is $\frac{5}{20}$

So, in 4 trials the probability that at least one is white is,

Selecting a white and then choosing from the rest,

$\Rightarrow 1-\frac{81}{256}$ that no ball is white

is $\frac{175}{256}$.

Question 23.
A policeman fires 6 bullets at a burglar. The probability that the burglar will be hit by a bullet is 0.6. what is the probability that burglar is still unhurt?

Answer:
The probability that the burglar will be hit by a bullet is 0.6.

Using Bernoulli's Trial we have,

$P(Success=x) = {}^{n}C_{x}.p^{x}.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =6

p = 0.6 q =0.4

The probability that the burglar is unhurt is,

${}^{6}C_{0}(0.6)^{0}(0.4)^{6}$

$\Rightarrow 0.004096$

Question 24.
A die is tossed thrice. A success is 1 or 6 on a toss. Find the mean and variance of successes.

Answer:
Using Bernoulli's Trial we have,

P(Success=x) = $^nC_x.p^x.q^{(n-x)}$

x=0, 1, 2, ...n and q = (1-p), n =3

p = 2/6 = 1/3, q = 4/6 = 2/3

P(x = 0) = P (no success) = P (all failures) = $(\frac{2}{3})(\frac{2}{3})(\frac{2}{3}) = \frac{8}{27}$

P(x = 1) = P (1 success and 2 failures) = $^3C_1(\frac{1}{3})^1(\frac{2}{3})^2 = \frac{12}{27}$

P(x = 2) = P (2 success and 1 failure) = $^3C_2(\frac{1}{3})^2(\frac{2}{3})^1 = \frac{6}{27}$

P(x = 3) = P (all 3 success) = $^3C_3(\frac{1}{3})^3(\frac{2}{3})^0 = \frac{1}{27}$

∴ The probability distribution of the random variable x is -

x : 0 1 2 3

P(x) : $\frac{8}{27}$ $\frac{12}{27}$ $\frac{6}{27}$ $\frac{1}{27}$

x_1 p_1 p_1x_1 $p_1x_1^2$

0 $\frac{8}{27}$ 0 0

1 $\frac{12}{27}$ $\frac{12}{27}$ $\frac{12}{27}$

2 $\frac{6}{27}$ $\frac{12}{27}$ $\frac{24}{27}$

$3 \frac{1}{27} \frac{3}{27} \frac{9}{27}$

$1 \frac{45}{27}$

Mean $\mu = \Sigma p_1 x_1 = 1$

Variance = $\sigma^2 = \Sigma p_1 x_1^2 - \mu$

⇒ 5/3 - 1/1

⇒ 2/3

Question 25.
A die is thrown 100 times. Getting an even number is considered a success. Find the mean and variance of success.

Answer:
Probability of getting an even number is = 3/6 = 1/2

Probability of getting an odd number is = 3/6 = 1/2

Variance = npq

⇒ $100 \times \frac{1}{2} \times \frac{1}{2}$

⇒ 25

Question 26.
Determine the binomial distribution whose mean is 9 and variance is 6?

Answer:
Mean = np = 9

Variance = npq = 6

⇒ $q = \frac{6}{9} = \frac{2}{3}$

$\Rightarrow p = 1 - \frac{6}{9} = \frac{1}{3}$

$\Rightarrow n = 27$

Binomial distribution

$${}^{27}C_r \cdot \left(\frac{1}{3}\right)^r \cdot \left(\frac{2}{3}\right)^{(27-r)}$$ where r = 0, 1, 2, 3, ..., 27

Question 27.
Find the binomial distribution whose mean is 5 and variance is 2.5.

Answer:
Mean = np = 5

Variance = npq = 2.5

$\Rightarrow q = \frac{2.5}{5} = \frac{1}{2}$

$\Rightarrow p = 1 - \frac{1}{2} = \frac{1}{2}$

$\Rightarrow n = 10$

Probability distribution is:-

$${}^{10}C_r \cdot \left(\frac{1}{2}\right)^r \cdot \left(\frac{1}{2}\right)^{(10-r)}, 0 \le r \le 10$$

Question 28.
The mean and variance of a binomial distribution are 4 and (4/3) respectively. Find P(X≥ 1).

Answer:
Mean = np = 4

Variance = npq = 4/3

$\Rightarrow q = \frac{1}{3}$

$\Rightarrow p = 1 - \frac{1}{3} = \frac{2}{3}$

$\Rightarrow n = 6$

The probability $(X \geq 1)$ is

$^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5 + {}^6C_1(\frac{2}{3})^1(\frac{1}{3})^5$

$= \frac{728}{729}$

Question 29.
For a binomial distribution, the mean is 6 and the standard deviation is $\sqrt{2}$. Find the probability of getting 5 successes.

Answer:
Mean = np = 6

Variance = npq = 2

$\Rightarrow q = \frac{1}{3}$

$\Rightarrow p = 1 - \frac{1}{3} = \frac{2}{3}$

$\Rightarrow n = 9$

The probability of getting 5 successes,

$^9C_5(\frac{2}{3})^5(\frac{1}{3})^4$

Question 30.
In a binomial distribution, the sum and the product of the mean and the variance are (25/3) and (50/3) respectively. Find the distribution.

Answer:
Mean + Variance = np + npq = np(1 + q) = 25/3

Variance = n^2p^2q = n2 = 50/3 ...(i)

$n^2p^2(1 + q)^2$ = 625/9 ...(ii)

Dividing (i) by (ii), we get,

$$\frac{q}{(q\ +\ 1)^2} = \frac{\frac{50}{3}}{\frac{625}{9}} = \frac{6}{25}$$

⇒ 6q2-13q + 6 = 0

⇒ q = 2/3 or 3/2

⇒ But as q can not be greater than 1 thus, q = 2/3.

⇒ p = 1/3

⇒ n = 15

Binomial distribution,

$${}^{15}C_r \cdot \left(\frac{1}{3}\right)^r \cdot \left(\frac{2}{3}\right)^{(15-r)}$$

Question 31.
Obtain the binomial distribution whose mean is 10 and standard deviation is $2\sqrt{2}$.

Answer:
Mean is 10,

Standard deviation is $2\sqrt{2}$

So, variance is σ^2 i.e. 8

Thus,

Mean = np = 10

Variance = npq = 8

$\Rightarrow q = \frac{4}{5}$

$\Rightarrow p = 1 - \frac{4}{5} = \frac{1}{5}$

$\Rightarrow n = 50$

Thus, the binomial distribution is

$$. \ {}^{50}C_r . \left(\frac{1}{5}\right)^r . \left(\frac{4}{5}\right)^{(50-r)}, 0 \le r \le 50$$

Question 32.
Bring out the fallacy, if any, in the following statement:

‘The mean of a binomial distribution is 6 and its variance is 9’

Answer:
Variance can not be greater than mean as then, q wll be greater than 1, which is not possible.

As, np = 6 and npq = 9

q = 3/2 ...(not possible)