Exercise 10a

Question 1.
Differentiate each of the following w.r.t. x:

sin 4x

Answer:
Formulae:

- $\frac{d}{dx}(\sin x) = \cos x$

- $\frac{d}{dx}(kx) = k$

Let,

y = sin 4x

and u = 4x

therefore, y = sin u

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\sin u) \cdot \frac{d}{dx}(4x)$$

$$= \cos u \cdot 4 \text{ .... } \left(\because \frac{d}{dx}(\sin x) = \cos x \;\&\; \frac{d}{dx}(kx) = k\right)$$

= cos 4x . 4

= 4 cos 4x

Question 2.

Differentiate each of the following w.r.t. x:

cos 5x

Answer:
<u>Formulae</u>:

- $\frac{d}{dx}(\cos x) = -\sin x$

- $\frac{d}{dx}(kx) = k$

Let,

y = cos 5x

and u = 5x

therefore, y= cos u

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\cos u) . \frac{d}{dx}(5x)$$

$$= -\sin u . 5 \text{ .... } \left(\because \frac{d}{dx}(\cos x) = -\sin x \ \& \ \frac{d}{dx}(kx) = k\right)$$

= - sin 5x . 5

= - 5 sin 5x

Question 3.
Differentiate each of the following w.r.t. x:

tan 3x

Answer:
<u>Formulae</u>:

- $\frac{d}{dx}(\tan x) = \sec^2 x$

- $\frac{d}{dx}(kx) = k$

Let,

y = tan 3x

and u = 3x

therefore, y= tan u

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\tan u) . \frac{d}{dx}(3x)$

$= \sec^2 u \ .3$ .... $\left(\because \frac{d}{dx}(\tan x) = \sec^2 x \ \& \ \frac{d}{dx}(kx) = k\right)$

$= \sec^2 3x \ . \ 3$

$= 3 \sec^2 3x$

Question 4.
Differentiate each of the following w.r.t. x:

$\cos x^3$

Answer:
<u>Formulae</u>:

• $\frac{d}{dx}(\cos x) = -\sin x$

• $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

y = cos x^3

and u = x^3

therefore, y= cos u

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ ………… By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\cos u).\frac{d}{dx}(x^3)$$

$= -\sin u\ .3x^2$ ………… $\left(\because \frac{d}{dx}(\cos x) = -\sin x \;\&\; \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

= - sin x^3 . $3x^2$

= - $3x^2$ sin x^3

Question 5.
Differentiate each of the following w.r.t. x:

$\cot^2 x$

Answer:
<u>Formulae</u>:

• $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

• $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

y = $\cot^2 x$

and u = cot x

therefore, y= u^2

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(u^2) \cdot \frac{d}{dx}(\cot x)$

$= 2\,u\,.(-\operatorname{cosec}^2 x)$ .... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x\right)$

= 2 cot x .(- cosec^2 x)

= - 2cot x . cosec^2 x

Question 6.
Differentiate each of the following w.r.t. x:

$\tan^3 x$

Answer:
<u>Formulae</u>:

- $\frac{d}{dx}(\tan x) = \sec^2 x$

- $\frac{d}{dx}(x^n) = n.\,x^{n-1}$

Let,

y = $\tan^3$ x

and u = tan x

therefore, y= u^3

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ ………… By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(u^3).\frac{d}{dx}(\tan x)$

$= 3\,u^2\ .\sec^2 x$ ………… $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1}\ \&\ \frac{d}{dx}(\tan x) = \sec^2 x\right)$

= 3 $\tan^2$ x .($\sec^2$ x)

= 3 $\tan^2$ x . $\sec^2$ x

Question 7.
Differentiate each of the following w.r.t. x:

$\cot\sqrt{x}$

Answer:
<u>Formulae</u>:

$\cdot \frac{d}{dx}(\cot x) = -\ \mathrm{cosec}^2 x$

$\cdot \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Let,

$y\ = \cot\sqrt{x}$

and $u\ = \sqrt{x}$

therefore, y= cot u

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ ………… By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\cot u) . \frac{d}{dx}(\sqrt{x})$$

$$= -\operatorname{cosec}^2 u \ . \frac{1}{2\sqrt{x}} \text{…………} \left(\because \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x \ \& \ \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}\right)$$

$$= -\operatorname{cosec}^2 \sqrt{x} . \frac{1}{2\sqrt{x}}$$

$$= \frac{-1}{2\sqrt{x}} \operatorname{cosec}^2 \sqrt{x}$$

Question 8.
Differentiate each of the following w.r.t. x:

$$\sqrt{\tan x}$$

Answer:
Formulae:

- $\frac{d}{dx}(\tan x) = \sec^2 x$

- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Let,

$$y = \sqrt{\tan x}$$

and u = tan x

therefore, $y = \sqrt{u}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}\left(\sqrt{u}\right) . \frac{d}{dx}\left(\tan x\right)$

$= \frac{1}{2\sqrt{u}} . \sec^2 x$ .... $\left(\because \frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}} \;\&\; \frac{d}{dx}\left(\tan x\right) = \sec^2 x\right)$

$= \frac{1}{2\sqrt{\tan x}} . \sec^2 x$

$= \frac{\sec^2 x}{2\sqrt{\tan x}}$

Question 9.
Differentiate each of the following w.r.t. x:

$(5 + 7x)^6$

Answer:
<u>Formulae</u>:

- $\frac{d}{dx}\left(x^n\right) = n.x^{n-1}$

- $\frac{d}{dx}\left(kx\right) = k$

- $\frac{d}{dx}\left(k\right) = 0$

- $\frac{d}{dx}\left(u + v\right) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = (5+7x)^6$

and u = (5+7x)

therefore, y = u^6

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(u^6) . \frac{d}{dx}(5 + 7x)$$

$= 6.(u)^5 . \left(\frac{d}{dx}(5) + \frac{d}{dx}(7x)\right)$ .... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \; \& \; \frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

= 6. $(5+7x)^5$. (0+7) .... $\left(\because \frac{d}{dx}(k) = 0 \; \& \; \frac{d}{dx}(kx) = k\right)$

= 42. $(5+7x)^5$

Question 10.
Differentiate each of the following w.r.t. x:

$(3 - 4x)^5$

Answer:
<u>Formulae</u>:

- $\frac{d}{dx}(x^n) = n.x^{n-1}$
- $\frac{d}{dx}(kx) = k$
- $\frac{d}{dx}(k) = 0$
- $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = (3\text{-}4x)^5$

and $u = (3\text{-}4x)$

therefore, $y = u^5$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(u^5).\frac{d}{dx}(3 - 4x)$$

$= 5.(u)^4 . \left(\frac{d}{dx}(3) + \frac{d}{dx}(-4x)\right)$ .... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= 5.(3\text{-}4x)^4.(0\text{-}4)$ .... $\left(\because \frac{d}{dx}(k) = 0 \;\&\; \frac{d}{dx}(kx) = k\right)$

$= -20(3\text{-}4x)^4$

Question 11.
Differentiate each of the following w.r.t. x:

$(2x^2 - 3x + 4)^5$

Answer:
<u>Formulae</u>:

- $\frac{d}{dx}(x^n) = n.x^{n-1}$

- $\frac{d}{dx}(kx) = k$

- $\frac{d}{dx}(k) = 0$

- $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = (2x^2 - 3x + 4)^5$

and $u = (2x^2 - 3x + 4)$

therefore, $y = u^5$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(u^5) . \frac{d}{dx}(2x^2 - 3x + 4)$$

$$= 5.(u)^4 . \left(\frac{d}{dx}(2x^2) + \frac{d}{dx}(-3x) + \frac{d}{dx}(4)\right) \ldots\ldots\ldots\ldots$$
$$\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$$

$= 5. (2x^2 - 3x + 4)^4. (4x\text{-}3\text{+}0)$ .... $\left(\because \frac{d}{dx}(kx) = k \;\&\; \frac{d}{dx}(k) = 0\right)$

$= 5. (2x^2 - 3x + 4)^4 (4x\text{-}3)$

Question 12.
Differentiate each of the following w.r.t. x:

$(ax^2 + bx + c)^6$

Answer:
<u>Formulae</u>:

- $\frac{d}{dx}(x^n) = n.x^{n-1}$

- $\frac{d}{dx}(kx) = k$

- $\frac{d}{dx}(k) = 0$

$\cdot \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

y = (ax^2 + bx + c)^6

and u = (ax^2 + bx + c)

therefore, y = u^6

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(u^6) . \frac{d}{dx}(ax^2 + bx + c)$$

$$= 6.(u)^5 . \left(\frac{d}{dx}(ax^2) + \frac{d}{dx}(bx) + \frac{d}{dx}(c)\right)$$

$$= 6.(ax^2 + bx + c)^5 . \frac{d}{dx}(ax^2 + bx + c) \text{ .... } \left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$$

= 6. (ax^2 + bx + c)^5. (2ax+b+0) .... $\left(\because \frac{d}{dx}(kx) = k \;\&\; \frac{d}{dx}(k) = 0\right)$

Question 13.
Differentiate each of the following w.r.t. x:

$$\frac{1}{(x^2 - 3x + 5)^3}$$

Answer:
<u>Formulae</u>:

$\cdot \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

- $\frac{d}{dx}(x^n) = n.x^{n-1}$

- $\frac{d}{dx}(kx) = k$

- $\frac{d}{dx}(k) = 0$

- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$$y = \frac{1}{(x^2 - 3x + 5)^3}$$

Let, u = $(x^2\text{-}3x\text{+}5)^3$

Therefore, $y = \frac{1}{u}$

For u = $(x^2\text{-}3x\text{+}5)^3$

Let, v = $(x^2\text{-}3x\text{+}5)$

Therefore, u = $(v)^3$

Therefore, $y = \frac{1}{v^3}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dv} . \frac{dv}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}\left(\frac{1}{u}\right).\frac{d}{dv}(v)^3.\frac{d}{dx}(x^2 - 3x + 5)$$

$$= \frac{-1}{u^2}.3v^2.\left(\frac{d}{dx}(x^2) + \frac{d}{dx}(-3x) + \frac{d}{dx}(5)\right)$$

$$\text{............} \left(\because \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}, \frac{d}{dx}(x^n) = n.x^{n-1} \ \& \ \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$$

$$= \frac{-1}{(x^2-3x+5)^6}.3(x^2-3x+5)^2.(2x-3+0) \text{ .... } \left(\because \frac{d}{dx}(kx) = k \ \& \ \frac{d}{dx}(k) = 0\right)$$

$$= \frac{-3}{(x^2-3x+5)^4}.(2x-3)$$

$$= \frac{-3(2x-3)}{(x^2-3x+5)^4}$$

Question 14.
Differentiate each of the following w.r.t. x:

$$\sqrt{\frac{a^2-x^2}{a^2+x^2}}$$

Answer:
Formulae:

- $\frac{d}{dx}(x^n) = n.x^{n-1}$

- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

- $\frac{d}{dx}(k) = 0$

- $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{(v)^2}$

Let,

$$y = \sqrt{\frac{a^2-x^2}{a^2+x^2}}$$

and $u = \frac{a^2 - x^2}{a^2 + x^2}$

$\therefore y = \sqrt{u}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ ………… By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}\left(\sqrt{u}\right) \cdot \frac{d}{dx}\left(\frac{a^2 - x^2}{a^2 + x^2}\right)$$

$$= \frac{1}{2\sqrt{u}}\left(\frac{(a^2+x^2)\frac{d}{dx}(a^2-x^2) - (a^2-x^2)\frac{d}{dx}(a^2+x^2)}{(a^2+x^2)^2}\right) \text{.............}$$

$$\left(\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{d}{dx}(u) - u \cdot \frac{d}{dx}(v)}{(v)^2} \ \& \ \frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{\frac{a^2-x^2}{a^2+x^2}}}\left(\frac{(a^2+x^2).(-2x) - (a^2-x^2).(2x)}{(a^2+x^2)^2}\right) \text{.............} \left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \ \& \ \frac{d}{dx}(k) = 0\right)$$

$$= \frac{\sqrt{a^2+x^2}}{2\sqrt{a^2-x^2}} \cdot (2x)\left(\frac{-a^2 - x^2 - a^2 + x^2}{(a^2+x^2)^2}\right)$$

$$= \frac{(a^2+x^2)^{1/2}}{2(a^2-x^2)^{1/2}} \cdot (2x) \cdot \frac{-2a^2}{(a^2+x^2)^2}$$

$$= \frac{-2a^2x}{(a^2-x^2)^{1/2}\,(a^2+x^2)^{2-\frac{1}{2}}}$$

$$= \frac{-2a^2x}{(a^2-x^2)^{1/2} \cdot (a^2+x^2)^{3/2}}$$

Question 15.
Differentiate each of the following w.r.t. x:

$$\sqrt{\frac{1+\sin x}{1-\sin x}}$$

Answer:
Formulae:

• $1 - \sin^2 x = \cos^2 x$

• $\frac{d}{dx}(\sec x) = \sec x . \tan x$

• $\frac{d}{dx}(\tan x) = \sec^2 x$

Let,

$$y = \sqrt{\frac{1 + \sin x}{1 - \sin x}}$$

Multiplying numerator and denominator by (1+sin x),

$$\therefore y = \sqrt{\frac{1 + \sin x}{1 - \sin x} . \frac{1 + \sin x}{1 + \sin x}}$$

$$= \sqrt{\frac{(1 + \sin x)^2}{1 - \sin^2 x}}$$

$$= \sqrt{\frac{(1+\sin x)^2}{\cos^2 x}} \text{ .... } (1 - \sin^2 x = \cos^2 x)$$

$$= \frac{1 + \sin x}{\cos x}$$

$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

y = sec x + tan x

Differentiating above equation w.r.t. x,

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sec x + \tan x)$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sec x) + \frac{d}{dx}(\tan x)$$

$$= \sec x.\tan x + \sec^2 x \;............ \left(\because \frac{d}{dx}(\sec x) = \sec x.\tan x \;\&\; \frac{d}{dx}(\tan x) = \sec^2 x\right)$$

$$= \sec x\;(\tan x + \sec x)$$

Question 16.
Differentiate each of the following w.r.t. x:

$\cos^2 x^3$

Answer:
<u>Formulae</u>:

- $\frac{d}{dx}(\cos x) = -\sin x$

- $\frac{d}{dx}(x^n) = n.x^{n-1}$

- 2 sin x. cos x = sin 2x

Let,

$y = \cos^2 x^3$

and $u = x^3$

therefore, $y = \cos^2 u$

let, $v = \cos u$

therefore, $y = v^2$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv} . \frac{dv}{du} . \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^2) . \frac{d}{du}(\cos u) . \frac{d}{dx}(x^3)$$

$$= 2\,v\ .(-\sin u).3x^2 \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(x^n) = n.x^{n-1}\ \&\ \frac{d}{dx}(\cos x) = -\sin x\right)$$

$$= -\,2\cos u\ .\sin u.3x^2$$

$$= -\ \sin 2u.3x^2 \ldots\ldots\ldots\ldots (\because 2\sin x.\cos x = \sin 2x)$$

$$= -\ \sin 2x^3.3x^2$$

Question 17.
Differentiate each of the following w.r.t. x:

$\sec^3 (x^2+1)$

Answer:
<u>Formulae</u>:

- $\frac{d}{dx}(\sec x) = \sec x.\tan x$

- $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

$y = \sec^3 (x^2+1)$

and $u = x^2+1$

therefore, $y = \sec^3 u$

let, $v = \sec u$

therefore, $y = v^3$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv} . \frac{dv}{du} . \frac{du}{dx}$ ………… By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^3) . \frac{d}{du}(\sec u) . \frac{d}{dx}(x^2 + 1)$$

$$= 3v^2 . (\sec u . \tan u) . 2x \text{ …………} \left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \ \& \ \frac{d}{dx}(\sec x) = \sec x . \tan x\right)$$

$$= 3 \sec^2 u . (\sec u . \tan u) . 2x$$

$$= 6x . \sec^3 u . \tan u$$

$$= 6x . \sec^3 (x^2 + 1) . \tan(x^2 + 1)$$

Question 18.
Differentiate each of the following w.r.t. x:

$$\sqrt{\cos 3x}$$

Answer:
<u>Formulae</u>:

- $\frac{d}{dx}(\cos x) = -\sin x$

- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

- $\frac{d}{dx}(kx) = k$

Let,

$$y = \sqrt{\cos 3x}$$

and u = 3x

therefore, $y = \sqrt{\cos u}$

let, v = cos u

therefore, $y = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}\left(\sqrt{v}\right) \cdot \frac{d}{du}(\cos u) \cdot \frac{d}{dx}(3x)$$

$$= \frac{1}{2\sqrt{v}} \cdot (-\sin u) \cdot 3 \text{ .... } \left(\because \frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}, \frac{d}{dx}(\cos x) = -\sin x \;\&\; \frac{d}{dx}(kx) = k\right)$$

$$= \frac{-3}{2} \cdot \frac{\sin u}{\sqrt{\cos u}}$$

$$= \frac{-3}{2} \cdot \frac{\sin 3x}{\sqrt{\cos 3x}}$$

Question 19.
Differentiate each of the following w.r.t. x:

$\sqrt[3]{\sin 2x}$

Answer:
<u>Formulae</u>:

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(x^n) = n.x^{n-1}$
- $\frac{d}{dx}(kx) = k$

Let,

$$y = \sqrt[3]{\sin 2x}$$

and u = 2x

therefore, $y = \sqrt[3]{\sin u}$

let, v = sin u

therefore, $y = \sqrt[3]{v} = v^{3/2}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}\left(v^{1/3}\right).\frac{d}{du}(\sin u).\frac{d}{dx}(2x)$$

$$= \frac{1}{3}v^{-2/3}.(\cos u).2 \text{ .... } \left(\because \frac{d}{dx}(x^n) = n.x^{n-1}, \frac{d}{dx}(\sin x) = \cos x \ \& \ \frac{d}{dx}(kx) = k\right)$$

$$= \frac{2}{3}\frac{\cos u}{v^{2/3}}.$$

$$= \frac{2}{3}\frac{\cos u}{(\sin u)^{2/3}}$$

$$= \frac{2}{3}\frac{\cos 2x}{(\sin 2x)^{2/3}}$$

Question 20.
Differentiate each of the following w.r.t. x:

$$\sqrt{1+\cot x}$$

Answer:

Formulae:

- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

- $\frac{d}{dx}(k) = 0$

- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = \sqrt{1+\cot x}$

and u = 1+ cot x

therefore, $y = \sqrt{u}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\sqrt{u}) \cdot \frac{d}{dx}(1+\cot x)$$

$$= \frac{1}{2\sqrt{u}} \cdot \left(\frac{d}{dx}(1) + \frac{d}{dx}(\cot x)\right) \text{............} \left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \;\&\; \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$$

$$= \frac{1}{2\sqrt{1+\cot x}} \cdot (0 - \operatorname{cosec}^2 x) \text{ .}$$

$$= \frac{-1}{2} \frac{\operatorname{cosec}^2 x}{\sqrt{1+\cot x}}$$

Question 21.

Differentiate each of the following w.r.t. x:

$$\operatorname{cosec}^3 \frac{1}{x^2}$$

Answer:
Formulae:

- $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x . \cot x$

- $\frac{d}{dx}(x^n) = n . x^{n-1}$

Let,

$$y = \operatorname{cosec}^3 \frac{1}{x^2}$$

and $u = \frac{1}{x^2}$

therefore, y= cosec^3 u

let, v = cosec u

therefore, y= v^3

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv} . \frac{dv}{du} . \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^3) . \frac{d}{du}(\operatorname{cosec} u) . \frac{d}{dx}\left(\frac{1}{x^2}\right)$$

$$= 3v^2 . (-\operatorname{cosec} u . \cot u) . \frac{d}{dx}(x^{-2})$$

.... $\left(\because \frac{d}{dx}(x^n) = n . x^{n-1} \ \& \ \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x . \cot x\right)$

$= 3\,\text{cosec}^2 u\,.\,(-\text{cosec}\,u.\cot u)\,.\,(-2x^{-3})$

$= 3\,\text{cosec}^3 u.\cot u\ \ (2\frac{1}{x^3})$

$= \frac{6}{x^3}\,.\,\text{cosec}^3\left(\frac{1}{x^2}\right).\cot\left(\frac{1}{x^2}\right)$

Question 22.
Differentiate each of the following w.r.t. x:

$\sqrt{\sin x^3}$

Answer:
<u>Formulae</u>:

- $\frac{d}{dx}(\sin x) = \cos x$

- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

- $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

$y\ = \sqrt{\sin x^3}$

and u = x^3

therefore, $y\ = \sqrt{\sin u}$

let, v = sin u

therefore, $y\ = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(\sqrt{v}).\frac{d}{du}(\sin u).\frac{d}{dx}(x^3)$$

$$= \frac{1}{2\sqrt{v}}.(\cos u).3x^2$$

.... $\left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}, \frac{d}{dx}(x^n) = n.x^{n-1} \,\&\, \frac{d}{dx}(\sin x) = \cos x\right)$

$$= \frac{1}{2\sqrt{\sin u}}.(\cos u).3x^2$$

$$= \frac{3}{2}x^2.\frac{\cos x^3}{\sqrt{\sin x^3}}$$

Question 23.
Differentiate each of the following w.r.t. x:

$\sqrt{x \sin x}$

Answer:
Formulae:

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}(kx) = k$
- $\frac{d}{dx}(u.v) = u.\frac{d}{dx}(v) + v.\frac{d}{dx}(u)$

Let,

$y = \sqrt{x.\sin x}$

and u = x. sin x

therefore, $y = \sqrt{u}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}\left(\sqrt{u}\right).\frac{d}{dx}(x.\sin x)$$

$$= \frac{1}{2\sqrt{u}}.\left(x.\frac{d}{dx}(\sin x) + \sin x.\frac{d}{dx}(x)\right)$$

.... $\left(\because \frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}} \ \& \ \frac{d}{dx}(u.v) = u.\frac{d}{dx}(v) + v.\frac{d}{dx}(u)\right)$

$= \frac{1}{2\sqrt{x.\sin x}}.(x.(\cos x) + \sin x.(1))$ .... $\left(\because \frac{d}{dx}(kx) = k \ \& \ \frac{d}{dx}(\sin x) = \cos x\right)$

$$= \frac{(x.\cos x + \sin x)}{2\sqrt{x.\sin x}}$$

Question 24.
Differentiate each of the following w.r.t. x:

$$\sqrt{\cot\sqrt{x}}$$

Answer:
<u>Formulae</u>:

- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

- $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$

Let,

$$y = \sqrt{\cot\sqrt{x}}$$

And $u = \sqrt{x}$

therefore, $y = \sqrt{\cot u}$

let, v = cot u

therefore, $y = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}\left(\sqrt{v}\right) \cdot \frac{d}{du}(\cot u) \cdot \frac{d}{dx}\left(\sqrt{x}\right)$$

$$= \frac{1}{2\sqrt{v}} \cdot \left(-\operatorname{cosec}^2 u\right) \cdot \frac{1}{2\sqrt{x}}$$

.... $\left(\because \frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}} \;\&\; \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2\right)$

$$= \frac{1}{2\sqrt{\cot u}} \cdot \left(-\operatorname{cosec}^2 u\right) \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{\cot\sqrt{x}}} \cdot \left(-\operatorname{cosec}^2\sqrt{x}\right) \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{-\operatorname{cosec}^2\sqrt{x}}{4\sqrt{x}\sqrt{\cot\sqrt{x}}}$$

Question 25.

Differentiate each of the following w.r.t. x:

$\cot^3 x^2$

Answer:
Formulae:

- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

- $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

y = $\cot^3 x^2$

and u = x^2

therefore, y= $\cot^3 u$

let, v = cot u

therefore, y= v^3

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ ………… By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^3).\frac{d}{du}(\cot u).\frac{d}{dx}(x^2)$$

$$= 3v^2 .(-\operatorname{cosec}^2 u).2x \text{ ………… } \left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x\right)$$

$$= 3\cot^2 u .(-\operatorname{cosec}^2 u).2x$$

$$= -6x.\cot^2 u .\operatorname{cosec}^2 u$$

$$= -6x.\cot^2(x^2) .\operatorname{cosec}^2(x^2)$$

Question 26.
Differentiate each of the following w.r.t. x:

$$\cos\left(\sin\sqrt{ax+b}\right)$$

Answer:
Formulae:

- $\frac{d}{dx}(\cos x) = -\sin x$

- $\frac{d}{dx}(\sin x) = \cos x$

- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$$y = \cos\left(\sin\sqrt{ax+b}\right)$$

and u = ax + b

therefore, $y = \cos\left(\sin\sqrt{u}\right)$

let, $v = \sqrt{u}$

therefore, $y = \cos(\sin v)$

let, $w = \sin v$

therefore, $y = \cos w$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dw} \cdot \frac{dw}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dw}(\cos w) \cdot \frac{d}{dv}(\sin v) \cdot \frac{d}{du}(\sqrt{u}) \cdot \frac{d}{dx}(ax + b)$$

$$= (-\sin w) \cdot (\cos v) \cdot \left(\frac{1}{2\sqrt{u}}\right) \cdot \left(\frac{d}{dx}(ax) + \frac{d}{dx}(b)\right)$$

.... $\left(\because \frac{d}{dx}(\cos x) = -\sin x, \frac{d}{dx}(\sin x) = \cos x, \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \,\&\, \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$$= (-\sin(\sin v)) \cdot (\cos \sqrt{u}) \cdot \left(\frac{1}{2\sqrt{ax + b}}\right) \cdot (a + 0)$$

$$= (-\sin(\sin \sqrt{u})) \cdot (\cos \sqrt{ax + b}) \cdot \left(\frac{1}{2\sqrt{ax + b}}\right) \cdot (a)$$

$$= \left(\frac{-a \cdot \cos \sqrt{ax + b}}{2\sqrt{ax + b}}\right) \cdot \left(\sin(\sin\sqrt{ax + b})\right)$$

Question 27.
Differentiate each of the following w.r.t. x:

$$\sqrt{\operatorname{cosec}(x^3 + 1)}$$

Answer:
<u>Formulae</u>:

- $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$

- $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$

- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$\cdot \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = \sqrt{\text{cosec}\,(x^3+1)}$

and u = x^3 + 1

therefore, $y = \sqrt{\text{cosec}\,u}$

let, $v = \text{cosec}\,u$

therefore, $y = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$ ………… By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(\sqrt{v}) . \frac{d}{du}(\text{cosec}\,u) . \frac{d}{dx}(x^3+1)$$

$$= \frac{1}{2\sqrt{v}} . (-\text{cosec}\,u . \cot u) . \left(\frac{d}{dx}(x^3) + \frac{d}{dx}(1)\right)$$

………… $\left(\because \frac{d}{dx}(\text{cosec}\,x) = -\text{cosec}\,x . \cot x , \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \;\&\; \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$$= \frac{1}{2\sqrt{\text{cosec}\,u}} . (-\text{cosec}(x^3+1) . \cot(x^3+1)) . (3x^2+0)$$

………… $\left(\because \frac{d}{dx}(x^n) = n . x^{n-1}\right)$

$$= \frac{1}{2\sqrt{\text{cosec}(x^3+1)}} . (-\text{cosec}(x^3+1) . \cot(x^3+1)) . (3x^2)$$

$$= \frac{-3x^2}{2} . \sqrt{\operatorname{cosec}(x^3 + 1)} . \cot(x^3 + 1)$$

Question 28.
Differentiate each of the following w.r.t. x:

sin 5x cos 3x

Answer:
<u>**Formulae**</u>**:**

• $(2\sin a . \cos b) = \sin(a + b) + \sin(a - b)$

• $\frac{d}{dx}(\sin x) = \cos x$

• $\frac{d}{dx}(kx) = k$

• $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

y = sin 5x. cos 3x

$$y = \frac{1}{2}(2 \sin 5x . \cos 3x)$$

$$y = \frac{1}{2}(\sin(5x + 3x) + \sin(5x - 3x)) \text{ .... } (\because (2\sin a . \cos b) = \sin(a + b) + \sin(a - b))$$

$$y = \frac{1}{2}(\sin(8x) + \sin(2x))$$

Differentiating above equation w.r.t. x,

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}(\sin(8x) + \sin(2x))\right)$$

$\therefore \frac{dy}{dx} = \frac{1}{2}\left(\frac{d}{dx}\sin 8x + \frac{d}{dx}\sin 2x\right)$ .... $\left(\because \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= \frac{1}{2}(8\cos 8x + 2\cos 2x)$ .... $\left(\because \frac{d}{dx}(\sin x) = \cos x \ \& \ \frac{d}{dx}(kx) = k\right)$

$= 4\cos 8x + \cos 2x$

Question 29.
Differentiate each of the following w.r.t. x:

sin $2x$ sin x

Answer:
Formulae:

• $(2\sin a . \sin b) = \cos(a-b) - \cos(a+b)$

• $\frac{d}{dx}(\cos x) = -\sin x$

• $\frac{d}{dx}(kx) = k$

• $\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$

Let,

y = sin 2x. sin x

$y = \frac{1}{2}(2\sin 2x. \sin x)$

$y = \frac{1}{2}(\cos(2x-x) - \cos(2x+x))$ .... $(\because (2\sin a . \sin b) = \cos(a-b) - \cos(a+b))$

$y = \frac{1}{2}(\cos x - \cos 3x)$

Differentiating above equation w.r.t. x,

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}(\cos x - \cos 3x)\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\left(\frac{d}{dx}\cos x - \frac{d}{dx}\cos 3x\right) \text{............} \left(\because \frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}\right)$$

$$= \frac{1}{2}(-\sin x + 3\sin 3x) \text{............} \left(\because \frac{d}{dx}(\cos x) = -\sin x \ \& \ \frac{d}{dx}(kx) = k\right)$$

$$= \frac{3}{2}\sin 3x - \frac{1}{2}\sin x$$

Question 30.
Differentiate each of the following w.r.t. x:

cos $4x$ cos $2x$

Answer:
<u>Formulae</u>:

- $(2\cos a \,.\cos b\,) = \cos(a + b) + \cos(a - b)$

- $\frac{d}{dx}(\cos x) = -\sin x$

- $\frac{d}{dx}(kx) = k$

- $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

y = cos 4x. cos 2x

$$y = \frac{1}{2}(2\cos 4x.\cos 2x)$$

$y = \frac{1}{2}(\cos(4x + 2x) + \cos(4x - 2x))$ ....
$(\because (2\cos a \,.\cos b\,) = \cos(a + b) + \cos(a - b))$

$$y = \frac{1}{2}(\cos 6x + \cos 2x)$$

Differentiating above equation w.r.t. x,

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}(\cos 6x + \cos 2x)\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\left(\frac{d}{dx}\cos 6x + \frac{d}{dx}\cos 2x\right) \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$$

$$= \frac{1}{2}(-6\sin 6x - 2\sin 2x) \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(\cos x) = -\sin x \ \& \ \frac{d}{dx}(kx) = k\right)$$

= -3 sin 6x – sin 2x

= - (3 sin 6x + sin 2x)

Question 31.

Find $\frac{dy}{dx}$, when:

$$y = \sin\left(\frac{1+x^2}{1-x^2}\right)$$

Answer:

Formulae:

- $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$
- $\frac{1+\tan^2 x}{1-\tan^2 x} = \cos 2x$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$

• $1 + \tan^2 x = \sec^2 x$

Given,

$$y = \sin\left(\frac{1 + x^2}{1 - x^2}\right)$$

Put x = tan a

Therefore, $\frac{dx}{da} = \sec^2 a$ .... eq (1)

$$y = \sin\left(\frac{1 + \tan^2 a}{1 - \tan^2 a}\right)$$

y = sin (cos 2a) .... $\left(\because \frac{1 + \tan^2 x}{1 - \tan^2 x} = \cos 2x\right)$

Differentiating above equation w.r.t. a ,

$$\frac{dy}{da} = \frac{d}{da}(\sin(\cos 2a))$$

$= (\cos(\cos 2a))\frac{d}{da}(\cos 2a)$ .... $\left(\because \frac{d}{dx}(\sin x) = \cos x\right)$

$= (\cos(\cos 2a)).(-\sin 2a).\frac{d}{da}(2a)$ .... $\left(\because \frac{d}{dx}(\cos x) = -\sin x\right)$

$= (-2\sin 2a).(\cos(\cos 2a))$

$= -2\left(\frac{2\tan a}{1 + \tan^2 a}\right).\left(\cos\left(\frac{1 + \tan^2 a}{1 - \tan^2 a}\right)\right)$ .... $\left(\because \frac{1 + \tan^2 x}{1 - \tan^2 x} = \cos 2x \;\&\; \frac{2\tan x}{1 + \tan^2 x} = \sin 2x\right)$

But, x = tan a

$$\frac{dy}{da} = -2\left(\frac{2x}{1 + x^2}\right).\left(\cos\left(\frac{1 + x^2}{1 - x^2}\right)\right)$$

$\frac{dy}{da} = \left(\frac{-4x}{1+x^2}\right).\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right)$ .... eq (2)

Now,

$\frac{dy}{dx} = \frac{dy}{da}.\frac{da}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \left(\frac{-4x}{1+x^2}\right).\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right).\frac{1}{\sec^2 a}$ .... from eq (1) & eq (2)

$= \left(\frac{-4x}{1+x^2}\right).\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right).\frac{1}{1+\tan^2 a}$ .... $(\because 1 + \tan^2 x = \sec^2 x)$

$= \left(\frac{-4x}{1+x^2}\right).\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right).\frac{1}{1+x^2}$ .... $(\because x = \tan a)$

$$\therefore \frac{dy}{dx} = \frac{-4x}{(1+x^2)^2}.\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right)$$

Question 32.

Find $\frac{dy}{dx}$, when:

$$y = \frac{(\sin x + x^2)}{\cot 2x}$$

Answer:

<u>Formulae</u>:

- $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{(v)^2}$

- $\frac{d}{dx}(\sin x) = \cos x$

- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

- $\frac{d}{dx}(x^n) = n.x^{n-1}$

Given,

$$y = \frac{\sin x + x^2}{\cot 2x}$$

Differentiating above equation w.r.t. x ,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin x + x^2}{\cot 2x}\right)$$

$$= \frac{\cot 2x.\frac{d}{dx}(\sin x + x^2) - (\sin x + x^2).\frac{d}{dx}(\cot 2x)}{(\cot 2x)^2} \text{............} \left(\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{(v)^2}\right)$$

$$= \frac{\cot 2x.(\cos 2x + 2x) - (\sin x + x^2).(-2\text{cosec}^2 2x)}{(\cot 2x)^2}$$

$$\text{............} \left(\because \frac{d}{dx}(\sin x) = \cos x, \frac{d}{dx}(x^n) = n.x^{n-1} \ \& \ \frac{d}{dx}(\cot x) = -\text{cosec}^2\right)$$

$$= \frac{(\cos 2x + 2x)}{\cot 2x} - \frac{(\sin x + x^2).(-2\,\text{cosec}^2 2x)}{(\cot 2x)^2}$$

$$= \tan 2x.(\cos 2x + 2x) + \frac{(\sin x + x^2).\left(\frac{2}{\sin^2 x}\right)}{\frac{\cos^2 x}{\sin^2 x}}$$

$$= \tan 2x.(\cos 2x + 2x) + \frac{2(\sin x + x^2)}{\cos^2 x}$$

$$= \tan 2x.(\cos 2x + 2x) + 2\sec^2 2x.(\sin x + x^2)$$

$$\therefore \frac{dy}{dx} = \tan 2x.(\cos 2x + 2x) + 2\sec^2 2x.(\sin x + x^2)$$

Question 33.

If $y = \frac{(\cos x - \sin x)}{(\cos x + \sin x)}$, prove that $\frac{dy}{dx} + y^2 + 1 = 0$.

Answer:
Formulae:

- $\frac{\sin x}{\cos x} = \tan x$

- $\frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4} - x\right)$

- $\frac{d}{dx}(\tan x) = \sec^2 x$

- $\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$

- $\frac{d}{dx}(kx) = k$

- $\frac{d}{dx}(k) = 0$

- $\tan^2 x + 1 = \sec^2 x$

Given,

$$y = \frac{(\cos x - \sin x)}{(\cos x + \sin x)}$$

Dividing numerator and denominator by cosx,

$$y = \frac{\left(1 - \frac{\sin x}{\cos x}\right)}{\left(1 + \frac{\sin x}{\cos x}\right)}$$

$$y = \frac{1-\tan x}{1+\tan x} \;\ldots\ldots\ldots\ldots \left(\because \frac{\sin x}{\cos x} = \tan x\right)$$

$$y = \tan\left(\frac{\pi}{4} - x\right) \;\ldots\ldots\ldots\ldots \left(\because \frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4} - x\right)\right)$$

Differentiating above equation w.r.t. x ,

$$\frac{dy}{dx} = \frac{d}{dx}\tan\left(\frac{\pi}{4} - x\right)$$

$$= \sec^2\left(\frac{\pi}{4} - x\right).\frac{d}{dx}\left(\frac{\pi}{4} - x\right) \text{............} \left(\because \frac{d}{dx}(\tan x) = \sec^2 x\right)$$

$$= \sec^2\left(\frac{\pi}{4} - x\right).\left(\frac{d}{dx}\left(\frac{\pi}{4}\right) - \frac{d}{dx}(x)\right) \text{............} \left(\because \frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}\right)$$

$$= \sec^2\left(x + \frac{\pi}{4}\right).(0 - 1) \text{............} \left(\because \frac{d}{dx}(kx) = k \;\&\; \frac{d}{dx}(k) = 0\right)$$

$$= -\sec^2\left(x + \frac{\pi}{4}\right)$$

$$\therefore \frac{dy}{dx} = -\sec^2\left(x + \frac{\pi}{4}\right)$$

Now,

$$\frac{dy}{dx} + y^2 + 1 = -\sec^2\left(x + \frac{\pi}{4}\right) + \left(\tan^2\left(x + \frac{\pi}{4}\right) + 1\right)$$

$$= -\sec^2\left(x + \frac{\pi}{4}\right) + \left(\sec^2\left(x + \frac{\pi}{4}\right)\right) \text{............} (\because \tan^2 x + 1 = \sec^2 x)$$

= 0

$$\therefore \frac{dy}{dx} + y^2 + 1 = 0$$

Hence Proved.

Question 34.

If $y = \frac{(\cos x + \sin x)}{(\cos x - \sin x)}$, prove that $\frac{dy}{dx} = \sec^2\left(x + \frac{\pi}{4}\right)$.

Answer:

<u>Formulae</u>:

- $\frac{\sin x}{\cos x} = \tan x$

- $\frac{1+\tan x}{1-\tan x} = \tan\left(x+\frac{\pi}{4}\right)$

- $\frac{d}{dx}(\tan x) = \sec^2 x$

- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

- $\frac{d}{dx}(kx) = k$

- $\frac{d}{dx}(k) = 0$

Given,

$$y = \frac{(\cos x + \sin x)}{(\cos x - \sin x)}$$

Dividing numerator and denominator by cosx,

$$y = \frac{\left(1+\frac{\sin x}{\cos x}\right)}{\left(1-\frac{\sin x}{\cos x}\right)}$$

$$y = \frac{1+\tan x}{1-\tan x} \text{............} \left(\because \frac{\sin x}{\cos x} = \tan x\right)$$

$$y = \tan\left(x+\frac{\pi}{4}\right) \text{............} \left(\because \frac{1+\tan x}{1-\tan x} = \tan\left(x+\frac{\pi}{4}\right)\right)$$

Differentiating above equation w.r.t. x ,

$$\frac{dy}{dx} = \frac{d}{dx}\tan\left(x+\frac{\pi}{4}\right)$$

$$= \sec^2\left(x+\frac{\pi}{4}\right).\frac{d}{dx}\left(x+\frac{\pi}{4}\right) \text{............} \left(\because \frac{d}{dx}(\tan x) = \sec^2 x\right)$$

$$= \sec^2\left(x + \frac{\pi}{4}\right).\left(\frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{\pi}{4}\right)\right) \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}\,(u + v) = \frac{du}{dx} + \frac{dv}{dx}\right)$$

$$= \sec^2\left(x + \frac{\pi}{4}\right).(1 + 0) \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}\,(kx) = k \;\&\; \frac{d}{dx}\,(k) = 0\right)$$

$$= \sec^2\left(x + \frac{\pi}{4}\right)$$

$$\therefore \frac{dy}{dx} = \sec^2\left(x + \frac{\pi}{4}\right)$$

Hence Proved.