Exercise 15a

Question 1.
Evaluate:

$\int \frac{dx}{x(x+2)}$

Answer:
Let $I = \int \frac{dx}{x(x+2)}$,

Putting $\frac{1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2} \ldots \ldots \ldots (1)$

Which implies A(x+2) + Bx = 1, putting x+2=0

Therefore x=-2,

And B = -0.5

Now put x=0, A= � ,

From equation (1), we get

$$\frac{1}{x(x+2)} = \frac{1}{2} \times \frac{1}{x} - \frac{1}{2} \times \frac{1}{x+2}$$

$$\int \frac{1}{x(x+2)} dx = \frac{1}{2} \int \frac{1}{x} dx - \frac{1}{2} \int \frac{1}{x+2} dx$$

$$= \frac{1}{2} \log|x| - \frac{1}{2} \log|x+2| + c$$

$$= \frac{1}{2} [\log|x| - \log|x+2|] + c$$

$$= \frac{1}{2} \log\left|\frac{x}{x+2}\right| + c$$

Question 2.
Evaluate:

$$\int \frac{(2x+1)}{(x+2)(x+3)} dx$$

Answer:
Let $I = \int \frac{(2x+1)}{(x+2)(x+3)} dx$,

Putting $\frac{2x+1}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3} \dots \dots \dots \dots (1)$

Which implies 2x=1 = A(x-3) + B(x+2)

Now put x-3=0, x=3

2×3+1=A(0)+B 3+2)

So $B = \frac{7}{5}$

Now put x+2=0 ,x=-2

-4+1=A(-2-3) + B(0)

So $A = \frac{3}{5}$

From equation (1), we get ,

$$\frac{2x+1}{(x+2)(x-3)} = \frac{3}{5} \times \frac{1}{x+2} + \frac{7}{5} \times \frac{1}{x-3}$$

$$\int \frac{2x+1}{(x+2)(x-3)} dx = \frac{3}{5} \int \frac{1}{x+2} dx + \frac{7}{5} \int \frac{1}{x-3} dx$$

$$= \frac{3}{5} \log|x+2| + \frac{7}{5} \log|x-3| + c$$

Question 3.
Evaluate:

$\int \frac{x}{(x+2)(3-2x)} dx$

Answer:
Let $I = \int \frac{x}{(x+2)(3-2x)} dx,$

Putting $\frac{x}{(x+2)(3-2x)} = \frac{A}{x+2} + \frac{B}{3-2x} \ldots \ldots \ldots (1)$

Which implies A(3-2x)+B(x+2)=x

Now put 3-2x=0

Therefore, $x = \frac{3}{2}$

$$A(0) + B\left(\frac{3}{2} + 2\right) = \frac{3}{2}$$

$$B\left(\frac{7}{2}\right) = \frac{3}{2}$$

$$B = \frac{3}{7}$$

Now put x+2=0

Therefore, x=-2

A(7)+B(0)=-2

$$A = \frac{-2}{7}$$

Now From equation (1) we get

$$\frac{x}{(x+2)(3-2x)} = \frac{-2}{7} \times \frac{1}{x+2} + \frac{3}{7} \times \frac{1}{3-2x}$$

$$\int \frac{x}{(x+2)(3-2x)} dx = \frac{-2}{7} \int \frac{1}{x+2} dx + \frac{3}{7} \int \frac{1}{3-2x} dx$$

$$= \frac{-2}{7} \log|x+2| + \frac{3}{7} \times \frac{1}{-2} \log|3-2x| + c$$

$$= \frac{-2}{7} \log|x+2| + \frac{3}{7} \times \frac{1}{-2} \log|3-2x| + c$$

$$= \frac{-2}{7} \log|x+2| - \frac{3}{14} \log|3-2x| + c$$

Question 4.
Evaluate:

$$\int \frac{dx}{x(x-2)(x-4)}$$

Answer:
Let $I = \int \frac{dx}{x(x-2)(x-4)}$,

Putting $\frac{1}{x(x-2)(x-4)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-4} \dots \dots (1)$

Which implies,

A(x-2)(x-4)+Bx(x-4)+Cx(x-2)=1

Now put x-2=0

Therefore, x=2

A(0)+B×2(2-4)+C(0)=1

B×2(-2)=1

$$B = -\frac{1}{4}$$

Now put x-4=0

Therefore, x=4

A(0)+B×(0)+C×4(4-2)=1

C×4(2)=1

$$C = \frac{1}{8}$$

Now put x=0

A(0-2)(0-4)+B(0)+C(0)=1

$$A = \frac{1}{8}$$

Now From equation (1) we get

$$\frac{1}{x(x-2)(x-4)} = \frac{1}{8} \times \frac{1}{x} - \frac{1}{4} \times \frac{1}{x-2} + \frac{1}{8} \times \frac{1}{x-4}$$

$$\int \frac{dx}{x(x-2)(x-4)} = \frac{1}{8}\int \frac{1}{x}dx - \frac{1}{4}\int \frac{1}{x-2}dx + \frac{1}{8}\int \frac{1}{x-4}dx$$

$$= \frac{1}{8}\log|x| - \frac{1}{4}\log|x-2| + \frac{1}{8}\log|x-4| + c$$

Question 5.
Evaluate:

$$\int \frac{(2x-1)}{(x-1)(x+2)(x-3)} dx$$

Answer:
Let $I = \int \frac{(2x-1)}{(x-1)(x+2)(x-3)} dx$

Putting $\frac{(2x-1)}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$ ... . (1)

Which implies,

A(x+2)(x-2)+B(x-1)(x-3)+C(x-1)(x+2)=2x-1

Now put x+2=0

Therefore, x=-2

A(0)+B(-2-1)(-2-3)+C(0)=2x-2-1

B(-3)(-5)=-5

$$B = -\frac{1}{3}$$

Now put x-3=0

Therefore, x=3

A(0)+B(0)+C(2)(5)=5

$$C = \frac{1}{2}$$

Now put x-1=0

Therefore, x=1

A(3)(-2)+B(0)+C(0)=1

$$A = -\frac{1}{6}$$

Now From equation (1) we get,

$$\frac{(2x-1)}{(x-1)(x+2)(x-3)} = \frac{-1}{6} \times \frac{1}{x-1} - \frac{1}{3} \times \frac{1}{x+2} + \frac{1}{2} \times \frac{1}{x-3}$$

$$\int \frac{(2x-1)}{(x-1)(x+2)(x-3)}dx = \frac{-1}{6}\int \frac{1}{x-1}dx - \frac{1}{3}\int \frac{1}{x+2}dx + \frac{1}{2}\int \frac{1}{x-3}dx$$

$$= \frac{-1}{6}\log|x-1| - \frac{1}{3}\log|x+2| + \frac{1}{2}\log|x-3| + c$$

Question 6.
Evaluate:

$$\int \frac{(2x-3)}{(x^2-1)(2x+3)}dx$$

Answer:
Let $I = \int \frac{(2x-3)}{(x^2-1)(2x+3)}dx$

Putting $\frac{(2x-3)}{(x-1)(x+1)(2x+3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{2x+3} \dots\dots(1)$

Which implies,

A(x+1)(2x+3)+B(x-1)(2x+3)+C(x-1)(x+1)=2x-3

Now put x+1=0

Therefore, x=-1

A(0)+B(-1-1)(-2+3)+C(0)=-2-3

$$B = -\frac{5}{2}$$

Now put x-1=0

Therefore, x=1

A(2)(2+3)+B(0)+C(0)=-1

$$A = -\frac{1}{10}$$

Now put 2x+3=0

Therefore, $x = -\frac{3}{2}$

$$A(0)+B(0)+C\left(\frac{-3}{2}-1\right)\left(\frac{-3}{2}+1\right)=2\left(\frac{-3}{2}\right)-3$$

$$C(\frac{-5}{2})(\frac{-1}{2})=-3-3$$

$$C=-\frac{24}{5}$$

.Now From equation (1) we get,

$$\frac{(2x-3)}{(x^2-1)(2x+3)}=\frac{-1}{10}\times\frac{1}{x-1}+\frac{5}{2}\times\frac{1}{x+1}-\frac{24}{5}\times\frac{1}{2x+3}$$

$$\int\frac{(2x-3)}{(x^2-1)(2x+3)}dx=\frac{-1}{10}\int\frac{1}{x-1}dx+\frac{5}{2}\int\frac{1}{x+1}dx-\frac{24}{5}\int\frac{1}{2x+3}dx$$

$$=\frac{-1}{10}\log|x-1|+\frac{5}{2}\log|x+1|-\frac{24}{5}\frac{\log|2x+3|}{2}+c$$

$$=\frac{-1}{10}\log|x-1|+\frac{5}{2}\log|x+1|-\frac{12}{5}\log|2x+3|+c$$

Question 7.
Evaluate:

$$\int\frac{(2x+5)}{(x^2-x-2)}dx$$

Answer:
Let $I=\int\frac{(2x+5)}{(x^2-x-2)}dx=\int\frac{(2x+5)}{(x-2)(x+1)}dx$

Putting $\frac{(2x+5)}{(x-2)(x+1)}=\frac{A}{x-2}+\frac{B}{x+1}\dots\dots(1)$

Which implies,

A(x+1)+B(x-2)=2x+5

Now put x+1=0

Therefore, x=-1

A(0)+B(-1-2)=3

B=-1

Now put x-2=0

Therefore, x=2

A(2+1)+B(0)=2×2+5=9

A=3

Now From equation (1) we get,

$$\frac{(2x+5)}{(x-2)(x+1)} = \frac{3}{x-2} + \frac{-1}{x+1}$$

$$\int \frac{(2x+5)}{(x-2)(x+1)} dx = \int \frac{3}{x-2} + \int \frac{-1}{x+1}$$

$$= 3\log|x-2| - \log|x+1| + c$$

Question 8.
Evaluate:

$$\int \frac{(x^2+5x+3)}{(x^2+3x+2)} dx$$

Answer:
Let $I = \int \frac{(x^2+5x+3)}{(x^2+3x+2)} dx = \int \frac{x^2+3x+2+2x+1}{(x^2+3x+2)} dx = \int \frac{x^2+3x+2}{(x^2+3x+2)} dx + \int \frac{2x+1}{(x^2+3x+2)} dx$

Which implies $I = \int dx + \int \frac{2x+1}{(x^2+3x+2)} dx$

Therefore, $I=x+I_1$

Where, $I_1=\int \frac{2x+1}{(x^2+3x+2)} dx$

Putting $\frac{(2x+1)}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \ldots \ldots (1)$

Which implies,

A(x+2)+B(x+1)=2x+1

Now put x+2=0

Therefore, x=-2

A(0)+B(-1)=-4+1

B=3

Now put x+1=0

Therefore, x=-1

A(-1+2)+B(0)=-2+1

A=-1

Now From equation (1) we get,

$$\frac{(2x+1)}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{3}{x+2}$$

$$\int \frac{(2x+1)}{(x+1)(x+2)} dx = -\int \frac{1}{x+1} dx + \int \frac{3}{x+2} dx$$

$$= -\log|x+1| + 3\log|x+2| + c$$

Question 9.
Evaluate:

$$\int \frac{(x^2+1)}{(x^2-1)} dx$$

Answer:
Let $I = \int \frac{x^2+1}{x^2-1} dx$

$$I = \int (1 + \frac{2}{x^2-1}) dx$$

$$I = \int dx + 2 \int \frac{1}{x^2-1} dx$$

$$I = x + 2 \times \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + c$$

$$I = x + \log \left| \frac{x-1}{x+1} \right| + c$$

Question 10.
Evaluate:

$$\int \frac{x^3}{(x^2-4)} dx$$

Answer:
Let $I = \int \frac{x^3}{x^2-4} dx$

$$I = \int x + \frac{4x}{x^2-4} dx$$

$$I = \int x\, dx + \int \frac{4x}{x^2-4} dx$$

$$= \frac{x^2}{2} + \int \frac{4x}{(x-2)(x+2)} dx$$

Let $I_1 = \int \frac{4x}{(x-2)(x+2)} dx$

So

$$I = \frac{x^2}{2} + I_1$$

Therefore $I_1 = \int \frac{4x}{x^2-4} dx$

Putting x^2-4=t

2xdx = dt

$$I_1 = 2\int \frac{dt}{t}$$

$$I_1 = 2log|x^2 - 4| + c$$

Putting the value of I_1 in I,

$$I = \frac{x^2}{2} + 2log|x^2 - 4| + c$$

Question 11.
Evaluate:

$$\int \frac{(3+4x-x^2)}{(x+2)(x-1)} dx$$

Answer:
Let $I = \int \frac{3+4x-x^2}{(x+2)(x-1)} dx$

$$= \int \left(-1 + \frac{5x+1}{(x+2)(x-1)}\right) dx$$

$$= \int -dx + \int \frac{5x+1}{(x+2)(x-1)} dx$$

$= -x + I_1$

$$I_1 = \int \frac{5x+1}{(x+2)(x-1)} dx$$

Put $\frac{5x+1}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$

A(x-1)+B(x+2)=5x+1

Now put x-1=0

Therefore, x=1

A(0)+B(1+2)=5+1=6

B=2

Now put x+2=0

Therefore, x=-2

A(-2-1)+B(0)=5×(-2)+1

A=3

Now From equation (1) we get,

$$\frac{5x+1}{(x+2)(x-1)} = \frac{3}{(x+2)} + \frac{2}{(x-1)}$$

$$\int \frac{5x+1}{(x+2)(x-1)} dx = 3\int \frac{1}{(x+2)} dx + 2\int \frac{1}{(x-1)} dx$$

$$3\log|x+2| + 2\log|x-1| + c$$

Therefore,

$$I = -x + 3log|x+2| + 2log|x-1| + c$$

Question 12.
Evaluate:

$$\int \frac{x^3}{(x-1)(x-2)} dx$$

Answer:

Let $I = \int \frac{x^3}{(x-1)(x-2)} dx$

$$= \int \left\{(x+3) + \frac{7x-6}{(x-1)(x-2)}\right\} dx$$

$$= \frac{x^2}{2} + 3x + \int \frac{7x-6}{(x-1)(x-2)} dx$$

$$= \frac{x^2}{2} + 3x + I_1 \dots\dots..(1)$$

Where,

$$I_1 = \int \frac{7x-6}{(x-1)(x-2)} dx$$

Putting $\frac{7x-6}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$..........(2)

A(x-2)+B(x-1)=7x-6

Now put x-2=0

Therefore, x=2

A(0)+B(2-1)=7×2-6

B=8

Now put x-1=0

Therefore, x=1

A(1-2)+B(0)=7-6=1

A=-1

Now From equation (2) we get,

$$\frac{7x-6}{(x-1)(x-2)} = \frac{-1}{x-1} + \frac{8}{x-2}$$

$$I_1 = \int \frac{7x-6}{(x-1)(x-2)} dx = -\int \frac{1}{x-1} dx + 8\int \frac{1}{x-2} dx$$

$$= -\log|x-1| + 8log|x-2| + c$$

Now From equation (1) we get,

$$I = \frac{x^2}{2} + 3x - \log|x-1| + 8log|x-2| + c$$

Question 13.
Evaluate:

$$\int \frac{(x^3-x-2)}{(1-x^2)} dx$$

Answer:
Let $I = \int \frac{(x^3-x-2)}{(1-x^2)} dx$

$$= \int \left(-x + \frac{-2}{1-x^2}\right) dx$$

$$= \int -xdx + (-2)\int \frac{1}{1-x^2} dx$$

$$= \frac{-x^2}{2} - log\left|\frac{1+x}{1-x}\right| + c$$

$$= \frac{-x^2}{2} + log\left|\frac{1-x}{1+x}\right| + c$$

Question 14.

Evaluate:

$$\frac{(2x+1)}{(4-3x-x^2)}dx$$

Answer:

Let $I = \int \frac{2x+1}{(4-3x-x^2)} dx$

$$= \int \frac{2x+1}{(1-x)(4+x)} dx$$

Putting $\frac{2x+1}{(1-x)(4+x)} = \frac{A}{1-x} + \frac{B}{4+x} \dots \dots \dots (1)$

A(4+x)+B(1-x)=2x+1

Now put 1-x=0

Therefore, x=1

A(5)+B(0)=3

$$A = \frac{3}{5}$$

Now put 4+x=0

Therefore, x=-4

A(0)+B(5)=-8+1=-7

$$B = \frac{-7}{5}$$

Now From equation (1) we get,

$$\frac{2x+1}{(1-x)(4+x)} = \frac{3}{5} \times \frac{1}{1-x} + \frac{-7}{5} \times \frac{1}{4+x}$$

$$\int \frac{2x+1}{(1-x)(4+x)} dx = \frac{3}{5} \int \frac{1}{1-x} dx + \frac{-7}{5} \int \frac{1}{4+x} dx$$

$$= \frac{-3}{5} log|1-x| - \frac{7}{5} log|4+x| + c$$

$$= -\frac{1}{5} [3log|1-x| + 7log|4+x|] + c$$

Question 15.
Evaluate:

$$\int \frac{2x}{(x^2+1)(x^2+3)} dx$$

Answer:
Put $x^2=t$

$2xdx=dt$

$$\int \frac{dt}{(1+t)(3+t)} = \frac{1}{2} \int \left(\frac{1}{1+t} - \frac{1}{3+t} \right) dt$$

$$\frac{1}{2} [log|1+t| - log|3+t|] + c = \frac{1}{2} log\left|\frac{1+t}{3+t}\right| + c$$

$$= \frac{1}{2} log\left|\frac{1+x^2}{3+x^2}\right| + c$$

Question 16.
Evaluate:

$$\int \frac{\cos x}{(\cos^2 x - \cos x - 2)} dx$$

Answer:

Let $I = \int \frac{cosx}{(1+sinx)(2+sinx)} dx$

Putting t=sin x

dt=cos x dx

$$I = \int \frac{dt}{(1+t)(2+t)},$$

Now putting, $\frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t} \ldots\ldots\ldots..(1)$

A(2+t)+B(1+t)=1

Now put t+1=0

Therefore, t=-1

A(2-1)+B(0)=1

A=1

Now put t+2=0

Therefore, t=-2

A(0)+B(-2+1)=1

B=-1

Now From equation (1) we get,

$$\frac{1}{(1+t)(2+t)} = \frac{1}{1+t} + \frac{-1}{2+t}$$

$$\int \frac{1}{(1+t)(2+t)} dt = \int \frac{1}{1+t} dt - \int \frac{1}{2+t} dt$$

$$= \log|1+t| - \log|t+2| + c$$

$$= log\left|\frac{t+1}{t+2}\right| + c$$

So,

$$I = \int \frac{cosx}{(1+sinx)(2+sinx)}dx = log\left|\frac{sinx+1}{sinx+2}\right| + c$$

Question 17.
Evaluate:

$$\int \frac{\sec^2 x}{(2+\tan x)(3+\tan x)}dx$$

Answer:
Let $I = \int \frac{sec^2 x}{(2+tanx)(3+tanx)} dx$

Putting t=tanx

dt=sec^{2}xdx

$$I = \int \frac{dt}{(2+t)(3+t)},$$

Now putting, $\frac{1}{(3+t)(2+t)} = \frac{A}{2+t} + \frac{B}{3+t} \ldots \ldots \ldots . . (1)$

A(3+t)+B(2+t)=1

Now put t+2=0

Therefore, t=-2

A(3-2)+B(0)=1

A=1

Now put t+3=0

Therefore, t=-3

A(0)+B(2-3)=1

B=-1

Now From equation (1) we get,

$$\frac{1}{(2+t)(3+t)}=\frac{1}{2+t}+\frac{-1}{3+t}$$

$$\int \frac{1}{(2+t)(3+t)}dt=\int \frac{1}{2+t}dt-\int \frac{1}{3+t}dt$$

$$=\log|2+t|-\log|t+3|+c$$

$$=log\left|\frac{t+2}{t+3}\right|+c$$

So,

$$I=\int \frac{sec^2 x}{(2+tanx)(3+tanx)}dx=log\left|\frac{tanx+2}{tanx+3}\right|+c$$

Question 18.
Evaluate:

$$\int \frac{\sin x \cos x}{(\cos^2 x-\cos x-2)}dx$$

Answer:
Let $I=\int \frac{sinx\ cosx}{cos^2 x-cosx-2}dx$

Putting t=cos x

dt=-sin x dx

$$I=\int \frac{(-dt)t}{t^2-t-2}=,-\int \frac{tdt}{(t+1)(t-2)},$$

Now putting, $\frac{-t}{(t+1)(t-2)} = \frac{A}{t+1} + \frac{B}{t-2} \dots \dots \dots . . (1)$

A(t-2)+B(t+1)=-t

Now put t-2=0

Therefore, t=2

A(0)+B(2+1)=-2

$$B = \frac{-2}{3}$$

Now put t+1=0

Therefore, t=-1

A(-1-2)+B(0)=1

$$A = \frac{-1}{3}$$

Now From equation (1) we get,

$$\frac{-t}{(t+1)(t-2)} = \frac{-1}{3} \times \frac{1}{t+1} - \frac{2}{3} \times \frac{1}{t-2}$$

$$\int \frac{-t}{(t+1)(t-2)} dt = \frac{-1}{3} \int \frac{1}{t+1} - \frac{2}{3} \int \frac{1}{t-2}$$

$$= \frac{-1}{3} \log|t+1| - \frac{2}{3} \log|t-2| + c$$

So,

$$I = \int \frac{sinx\ cosx}{cos^2 x - cosx - 2} dx = \frac{-1}{3} \log|cosx + 1| - \frac{2}{3} \log|cosx - 2| + c$$

Question 19.
Evaluate:

$$\int \frac{e^x}{\left(e^{2x}+5e^x+6\right)} dx$$

Answer:

Let $I = \int \frac{e^x}{e^{2x}+5e^x+6} dx$

Putting $t=e^x$

$dt=e^x\, dx$

$$I = \int \frac{dt}{(t^2+5t+6)},$$

Now putting, $\frac{1}{(t^2+5t+6)} = \frac{A}{2+t} + \frac{B}{3+t} \ldots \ldots \ldots . . (1)$

$A(3+t)+B(2+t)=1$

Now put $t+2=0$

Therefore, $t=-2$

$A(3-2)+B(0)=1$

$A=1$

Now put $t+3=0$

Therefore, $t=-3$

$A(0)+B(2-3)=1$

$B=-1$

Now From equation (1) we get,

$$\frac{1}{(2+t)(3+t)} = \frac{1}{2+t} + \frac{-1}{3+t}$$

$$\int \frac{1}{(2+t)(3+t)} dt = \int \frac{1}{2+t} dt - \int \frac{1}{3+t} dt$$

$$= \log|2+t| - \log|t+3| + c$$

$$= log\left|\frac{t+2}{t+3}\right| + c$$

$$= log\left|\frac{e^x+2}{e^x+3}\right| + c$$

Question 20.
Evaluate:

$$\int \frac{e^x}{(e^{3x}-3e^{2x}-e^x+3)} dx$$

Answer:

Let $I = \int \frac{e^x}{e^{3x}-3e^{2x}-e^x+3} dx$

Putting $t=e^x$

$dt=e^x\ dx$

$$I = \int \frac{dt}{(t^3-3t^2-t+3)} =, \int \frac{dt}{(t^2)(t-3)-(t-3)} = \int \frac{dt}{(t^2-1)(t-3)}$$

Now putting, $\frac{1}{(t-1)(t+1)(t-3)} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{t-3} \ldots \ldots \ldots . (1)$

A(t+1)(t-3)+B(t-1)(t-3)+C(t-1)(t+1)=1

Now put t+1=0

Therefore, t=-1

A(0)+B(-1-1)(-1-3)+C(0)=1

B(-2)(-4)=1

$$B = \frac{1}{8}$$

Now put t-1=0

Therefore, t=1

A(1+1)(1-3)+B(0)+C(0)=1

$$A = \frac{-1}{4}$$

Now put t-3=0

Therefore, t=3

A(0)+B(0)+C(3-1)(3+1)=1

$$C = \frac{1}{8}$$

Now From equation (1) we get,

$$\frac{1}{(t-1)(t+1)(t-3)} = \frac{-1}{4} \times \frac{1}{t-1} + \frac{1}{8} \times \frac{1}{t+1} + \frac{1}{8} \times \frac{1}{t-3}$$

$$\int \frac{1}{(t-1)(t+1)(t-3)} = \frac{-1}{4} \int \frac{1}{t-1} + \frac{1}{8} \int \frac{1}{t+1} + \frac{1}{8} \int \frac{1}{t-3}$$

$$= \frac{-1}{4} \log|t-1| + \frac{1}{8} \log|t+1| + \frac{1}{8} \log|t-3| + c$$

$$\int \frac{e^x}{e^{3x} - 3e^{2x} - e^x + 3} dx = \frac{-1}{4} \log|e^x - 1| + \frac{1}{8} \log|e^x + 1| + \frac{1}{8} \log|e^x - 3| + c$$

Question 21.
Evaluate:

$$\int \frac{2\log x}{x\left[2(\log x)^2 - \log x - 3\right]} dx$$

Answer:

Let $I = \int \frac{2logx}{x[2(logx)^2 - logx - 3]} dx$

Putting t=log x

dt=dx/x

$$I = \int \frac{2tdt}{(2t^2 - t - 3)},$$

Now putting, $\frac{2t}{(2t^2-t-3)} = \frac{A}{2t-3} + \frac{B}{t+1} \dots \dots \dots . . (1)$

A(t+1)+B(2t-3)=2t

Now put 2t-3=0

Therefore, $t = \frac{3}{2}$

$$A\left(\frac{3}{2} + 1\right) + B(0) = 2 \times \frac{3}{2} = 3$$

$$A = \frac{6}{5}$$

Now put t+1=0

Therefore, t=-1

A(0)+B(-2-3)=-2

$$B = \frac{2}{5}$$

Now From equation (1) we get,

$$\frac{2t}{(2t^2 - t - 3)} = \frac{6}{5} \times \frac{1}{2t - 3} + \frac{2}{5} \times \frac{1}{t + 1}$$

$$\int \frac{2t}{(2t^2 - t - 3)} dt = \frac{6}{5} \int \frac{1}{2t - 3} dt + \frac{2}{5} \int \frac{1}{t + 1} dt$$

$$= \frac{6}{5} \log \left| \frac{6}{5} \times \frac{log\ (2t - 3)}{2} \right| + \frac{2}{5} \log|\log x + 1| + c$$

$$\int \frac{2logx}{x[2(logx)^2 - logx - 3]} dx = \frac{3}{5} \log|2logx - 3| + \frac{2}{5} \log|logx + 1| + c$$

Question 22.
Evaluate:

$$\int \frac{\operatorname{cosec}^2 x}{(1 - \cot^2 x)} dx$$

Answer:
Let $I = \int \frac{cosec^2 x}{(1 - cot^2 x)} dx$

Putting t=cot x

dt=-cosec^{2}xdx

$$I = \int \frac{-dt}{(1 - t^2)} = -\int \frac{1}{(1 - t^2)} dt$$

$$= \frac{-1}{2} log \left| \frac{1 + cotx}{1 - cotx} \right| + c$$

Question 23.
Evaluate:

$$\int \frac{\sec^2 x}{(\tan^3 x + 4 \tan x)} dx$$

Answer:

Let $I = \int \frac{sec^2x}{(tan^3x+4tanx)}dx$

Putting t=tan x

dt=sec^2xdx

$$I = \int \frac{dt}{(t^3+4t)} = \int \frac{dt}{t(t^2+4)}$$

Now putting, $\frac{1}{t(t^2+4)} = \frac{A}{t} + \frac{Bt+C}{t^2+4} \ldots\ldots\ldots..(1)$

$A(t^2+4)+(Bt + C)t=1$

Putting t=0,

A(0+4)× B(0)=1

$$A = \frac{1}{4}$$

By equating the coefficients of t^2 and constant here,

A+B=0

$$\frac{1}{4} + B = 0$$

$$B = -\frac{1}{4}, C = 0$$

Now From equation (1) we get,

$$\int \frac{1}{t(t^2+4)}dt = \frac{1}{4}\int \frac{dt}{t} - \frac{1}{4}\int \frac{t}{t^2+4}dt$$

$$= \frac{1}{4} logt - \frac{1}{4} \times \frac{1}{2} \log(t^2 + 4) + c$$

$$= \frac{1}{4} \log tanx - \frac{1}{8} \log(tan^2 x + 4) + c$$

Question 24.
Evaluate:

$$\int \frac{\sin 2x}{(1 + \sin x)(2 + \sin x)} dx$$

Answer:

Let $I = \int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx$

Putting t=sin x

dt=cos x dx

$$I = \int \frac{2t}{(1 + t)(2 + t)} dt$$

Now putting, $\frac{2t}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t} \dots \dots \dots .. (1)$

A(2+t)+B(1+t)=2t

Now put t+2=0

Therefore, t=-2

A(0)+B(1-2)=-4

B=4

Now put t+1=0

Therefore, t=-1

A(2-)+B(0)=-2

A=-2

Now from equation (1), we get,

$$\frac{2t}{(1+t)(2+t)} = \frac{-2}{1+t} + \frac{4}{2+t}$$

$$\int \frac{2t}{(1+t)(2+t)} dt = -2\int \frac{1}{1+t} dt + 4\int \frac{1}{2+t} dt$$

$$= 4\log|2+t| - 2\log|1+t| + c$$

So,

$$\int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx = 4\log|2+t| - 2\log|1+t| + c$$

Question 25.
Evaluate:

$$\frac{e^x}{e^x(e^x-1)} dx$$

Answer:
Let $I = \int \frac{e^x}{e^x(e^x-1)} dx$

Putting t=e^x

dt=e^xdx

$$I = \int \frac{dt}{t(t-1)}$$

Now putting, $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1} \ldots\ldots(1)$

A(t-1)+Bt=1

Now put t-1=0

Therefore, t=1

A(0)+B(1) =1

B=1

Now put t=0

A(0-1)+B(0)=1

A=-1

Now From equation (1) we get,

$$\frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\int \frac{1}{t(t-1)} dt = -\int \frac{1}{t} dt + \int \frac{1}{t-1} dt$$

$$= -\log t + \log|t-1| + c$$

$$= log\left|\frac{t-1}{t}\right| + c$$

$$= log\left|\frac{e^x-1}{e^x}\right| + c$$

Question 26.
Evaluate:

$$\int \frac{dx}{x(x^4-1)}$$

Answer:

Let $I = \int \frac{dx}{x(x^4-1)} dx$

Putting $t=x^4$

$dt=4x^3dx$

$$I=\int\frac{x^3dx}{x^4(x^4-1)}=\frac{1}{4}\times\int\frac{dt}{t(t-1)}$$

Now putting, $\frac{1}{t(t-1)}=\frac{A}{t}+\frac{B}{t-1}\dots\dots..(1)$

$A(t-1)+Bt=1$

Now put $t-1=0$

Therefore, $t=1$

$A(0)+B(1)=1$

$B=1$

Now put $t=0$

$A(0-1)+B(0)=1$

$A=-1$

Now From equation (1) we get,

$$\frac{1}{t(t-1)}=\frac{-1}{t}+\frac{1}{t-1}$$

$$\frac{1}{4}\int\frac{1}{t(t-1)}dt=-\frac{1}{4}\int\frac{1}{t}dt+\frac{1}{4}\int\frac{1}{t-1}dt$$

$$=-\frac{1}{4}\log t+\frac{1}{4}\log|t-1|+c$$

$$=-\frac{1}{4}\log x^4+\frac{1}{4}\log|x^4-1|+c$$

$$= -\log|x| + \frac{1}{4}\log|x^4 - 1| + c$$

Question 27.
Evaluate:

$$\int \frac{(1-x^2)}{x(1-2x)}dx$$

Answer:

Let $I = \int \frac{(x^2-1)}{x(2x-1)}dx = \int \left(\frac{1}{2} + \frac{\left(\frac{1}{2}x-1\right)}{x(2x-1)}\right)dx = \int \frac{1}{2}dx + \int \frac{x}{x(2x-1)}dx - \int \frac{1}{x(2x-1)}dx$

$$I = \frac{1}{2}x + \frac{1}{2} \times \frac{\log|2x-1|}{2} - I_1 \ldots \ldots (1)$$

Where $I_1 = \int \frac{1}{x(2x-1)}dx$........(2)

Now putting, $\frac{1}{x(2x-1)} = \frac{A}{x} + \frac{B}{2x-1}$

A(2x-1)+Bx=1

Putting 2x-1=0

$$x = \frac{1}{2}$$

$$A(0) + B\left(\frac{1}{2}\right) = 1$$

B=2

Putting x=0,

A(0-1)+B(0)=1

A=-1

From equation (2), we get,

$$\frac{1}{x(2x-1)} = -\frac{1}{x} + \frac{2}{2x-1}$$

$$\int \frac{1}{x(2x-1)} dx = -\int \frac{1}{x} dx + 2\int \frac{1}{2x-1} dx$$

$$= -\log|x| + \frac{2\log|2x-1|}{2} + c$$

$$= \log|2x-1| - \log x + c$$

From equation (1),

$$I = \frac{1}{2}x + \frac{1}{4}\log|2x-1| - \log|2x-1| + logx + c$$

$$= \frac{1}{2}x - \frac{3}{4}\log|1-2x| + log|x| + c$$

Question 28.
Evaluate:

$$\int \frac{(x^2+x+1)}{(x+2)(x+1)^2} dx$$

Answer:

Let $I = \int \frac{x^2+x+1}{(x+2)(x+1)^2} dx$

Now putting, $\frac{x^2+x+1}{(x+2)(x+1)^2} = \frac{A}{(x+2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} \ldots \ldots . .(1)$

$A(x+1)^2+B(x+2)(x+1)+C(x+2)=x^2+x+1$

Now put x+1=0

Therefore, x=-1

A(0)+B(0)+C(-1+2) =1-1+1=1

C=1

Now put x+2=0

Therefore, x=-2

A(-2+1)^2+B(0)+C(0) =4-2+1=3

A=3

Equating the coefficient of x^2,A+B=1

3+B=1

B=-2

Form equation (1),we get,

$$\frac{x^2+x+1}{(x+2)(x+1)^2}=\frac{3}{(x+2)}-\frac{2}{(x+1)}+\frac{1}{(x+1)^2}$$

So,

$$\int\frac{x^2+x+1}{(x+2)(x+1)^2}dx=\int\frac{3}{(x+2)}dx-\int\frac{2}{(x+1)}dx+\int\frac{1}{(x+1)^2}dx$$

$$=3\log|x+2|-2\log|x+1|-\frac{1}{1+x}+c$$

Question 29.
Evaluate:

$$\int\frac{(2x+9)}{(x+2)(x-3)^2}dx$$

Answer:
Let $I=\int\frac{2x+9}{(x+2)(x-3)^2}dx$

Now putting, $\frac{2x+9}{(x+2)(x-3)^2} = \frac{A}{(x+2)} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2} \ldots \ldots . (1)$

$A(x-3)^2+B(x+2)(x-3)+C(x+2)=2x+9$

Now put x-3=0

Therefore, x=3

A(0)+B(0)+C(3+2) =6+9=15

C=3

Now put x+2=0

Therefore, x=-2

$A(-2-3)^2+B(0)+C(0) = -4+9=5$

$$A = \frac{1}{5}$$

Equating the coefficient of x^2,we get,

A+B=0

$$\frac{1}{5} + B = 0$$

$$B = -\frac{1}{5}$$

From equation (1), we get,

$$\frac{2x+9}{(x+2)(x-3)^2} = \frac{1}{5} \times \frac{1}{(x+2)} - \frac{1}{5} \times \frac{1}{(x-3)} + \frac{3}{(x-3)^2}$$

$$\int \frac{2x+9}{(x+2)(x-3)^2} dx = \frac{1}{5} \int \frac{1}{(x+2)} dx - \frac{1}{5} \int \frac{1}{(x-3)} dx + 3 \int \frac{1}{(x-3)^2} dx$$

$$= \frac{1}{5}\log|x+2| - \frac{1}{5}\log|x-3| - \frac{3}{x-3} + c$$

Question 30.
Evaluate:

$$\int \frac{(x^2+1)}{(x-1)^2(x+3)} dx$$

Answer:
Let $I = \int \frac{x^2+1}{(x+3)(x-1)^2} dx$

Now putting, $\frac{x^2+1}{(x+3)(x-1)^2} = \frac{A}{(x+3)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \ldots \ldots .. (1)$

$A(x-1)^2+B(x+3)(x-1)+C(x+3)=x^2+1$

Now put x-1=0

Therefore, x=1

A(0)+B(0)+C(4) =2

$$C = \frac{1}{2}$$

Now put x+3=0

Therefore, x=-3

$A(-3-1)^2+B(0)+C(0) =9+1=10$

$$A = \frac{5}{8}$$

By equating the coefficient of x^2, we get, A+B=1

$$\frac{5}{8} + B = 1$$

$B = 1 - \frac{5}{8} = \frac{3}{8}$

From equation (1), we get,

$$\frac{x^2+1}{(x+3)(x-2)^2} = \frac{5}{8} \times \frac{1}{(x+3)} + \frac{3}{8} \times \frac{1}{(x-2)} + \frac{1}{(x-2)^2}$$

$$\int \frac{x^2+1}{(x+3)(x-2)^2} dx = \frac{5}{8} \int \frac{1}{(x+3)} dx + \frac{3}{8} \int \frac{1}{(x-2)} dx + \int \frac{1}{(x-2)^2} dx$$

$$= \frac{5}{8} \log|x+3| + \frac{3}{8} \log|x-1| - \frac{1}{2(x-1)} + c$$

Question 31.
Evaluate:

$$\int \frac{(x^2+1)}{(x+3)(x-1)} dx$$

Answer:
Let $I = \int \frac{x^2+1}{(x-3)(x-1)^2} dx$

Now putting, $\frac{x^2+1}{(x-3)(x-1)^2} = \frac{A}{(x-3)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \ldots \ldots .. (1)$

$A(x-1)^2+B(x-3)(x-1)+C(x-3)=x^2+1$

Putting x-1=0,

X=1

A(0)+B(0)+C(1-3)=1+1

C=-1

Putting x-3=0,

X=3

A(3-1)^2+B(0)+C(0)=9+1

A(4)=10

$A = \frac{5}{2}$

Equating the coefficient of x^2

A+B=1

$\frac{5}{2} + B = 1$

$B = 1 - \frac{5}{2} = \frac{-3}{2}$

From (i) $\int \frac{x^2+1}{(x-3)(x-1)^2} dx = \frac{5}{2}\int \frac{1}{x-3} dx - \frac{3}{2}\int \frac{1}{x-1} dx - \int \frac{1}{(x-1)^2} dx$

$= \frac{5}{2} log|x-3| - \frac{3}{2}\log|x-1| + \frac{1}{x-1} + C$

Question 32.
Evaluate:

$\int \frac{(x^2+x+1)}{(x+2)(x^2+1)} dx$

Answer:
Let $I = \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$

Now putting, $\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$

A(x^2+1)+(Bx+C)(x+2) = x^2+x+1

Ax^2+A+Bx^2+Cx+2Bx+2C = x^2+x+1

$(A+B)x^2+(C+2B)x+(A+2C) = x^2+x+1$

Equating coefficients A+B=1.......(i)

A+2C=1

A=1-2C......(ii)

2B+C=1

2B=1-C

$$B = \frac{1-C}{2} \ldots\ldots..(iii)$$

$$(1-2C) + \frac{1-C}{2} = 1$$

2-4C+1-C=2

3-5C=2

-5C=-1

$$C = \frac{1}{5}$$

And $2B = 1 - \frac{1}{5} = \frac{4}{5}$

$$B = \frac{2}{5}$$

$$A = 1 - 2 \times \frac{1}{5}$$

$$= 1 - \frac{2}{5}$$

$= \frac{3}{5}$

$$I = \int \frac{x^2+x+1}{(x+2)(x^2+1)}dx = \int \frac{A}{(x+2)}dx + \int \frac{Bx+C}{(x^2+1)}dx$$

$$= \frac{3}{5} \times \int \frac{1}{(x+2)}dx + \frac{1}{5} \times \int \frac{2x+1}{(x^2+1)}dx$$

$$= \frac{3}{5}\log|x+2| + \frac{1}{5}I_1 + C_1$$

$$I_1 = \int \frac{2x+1}{(x^2+1)}dx = \int \frac{2x}{(x^2+1)}dx + \int \frac{1}{(x^2+1)}dx$$

$$= log|x^2+1| + tan^{-1}x + C_2$$

$$I = \int \frac{x^2+x+1}{(x+2)(x^2+1)}dx = \frac{3}{5}\log|x+2| + \frac{1}{5}log|x^2+1| + \frac{1}{5}tan^{-1}x + C$$

Question 33.
Evaluate:

$$\int \frac{2x}{(2x+1)^2}dx$$

Answer:
Let $I = \int \frac{2x}{(2x+1)^2}dx$

Now putting, $\frac{2x}{(2x+1)^2} = \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} \ldots\ldots\ldots\ldots (1)$

A(2x+1)+B = 2x

Putting 2x+1=0,

$$x = \frac{-1}{2}$$

A(0)+B=-1

B=-1

By equating the coefficient of x,

2A=2

A=1

From equation (1),we get,

$$\frac{2x}{(2x+1)^2}=\frac{1}{(2x+1)}-\frac{1}{(2x+1)^2}$$

$$\int\frac{2x}{(2x+1)^2}dx=\int\frac{1}{(2x+1)}dx-\int\frac{1}{(2x+1)^2}dx$$

$$=\frac{\log|2x+1|}{2}+\frac{1}{2(2x+1)}+c$$

$$=\frac{1}{2}\left[\log|2x+1|+\frac{1}{2x+1}\right]+c$$

Question 34.
Evaluate:

$$\int\frac{3x+1}{(x+2)(x-2)^2}dx$$

Answer:
Let $I=\int\frac{3x+1}{(x+2)(x-2)^2}dx$

Now putting, $\frac{3x+1}{(x+2)(x-2)^2}=\frac{A}{(x+2)}+\frac{B}{(x-2)}+\frac{C}{(x-2)^2}\ldots\ldots..(1)$

A(x-2)^2+B(x+2)(x-2)+C(x+2)=3x+1

Putting x-2=0,

X=2

A(0)+B(0)+C(2+1)=3×2+1

$$C = \frac{7}{4}$$

Putting x+2=0,

X=-2

A$(-4)^2$+B(0)+C(0)=-6+1=-5

$$A = \frac{-5}{16}$$

By equation the coefficient of x^2, we get, A+B=0

$$\frac{-5}{16} + B = 0$$

$$B = \frac{5}{16}$$

$$I = -\frac{5}{16} log|x+2| + \frac{5}{16} log|x-2| - \frac{7}{4(x-2)} + c$$

Question 35.
Evaluate:

$$\int \frac{(5x+8)}{x^2(3x+8)} dx$$

Answer:
Let $I = \int \frac{5x+8}{x^2(3x+8)} dx$

Now putting, $\frac{5x+8}{x^2(3x+8)} = \frac{A}{(3x+8)} + \frac{Bx+C}{x^2} \ldots \ldots (1)$

$Ax^2+(Bx + C)(3x+8) = 5x+8$

Putting $3x+8=0$,

$$x = -\frac{8}{3}$$

$$A\left(\frac{64}{9}\right) + B(0) = 5\left(-\frac{8}{3}\right) + 8$$

$$A\left(\frac{64}{9}\right) = \frac{-40 + 24}{3}$$

$$A\left(\frac{64}{9}\right) = \frac{-16}{3}$$

$$A = \frac{-3}{4}$$

By equating the coefficient of x^2 and constant term,

$A+3B=0$

$$\frac{-3}{4} + 3B = 0$$

$$3B = \frac{3}{4}$$

$$B = \frac{1}{4}$$

$8C=8$

$C=1$

From equation (1), we get,

$$\int \frac{5x + 8}{x^2(3x + 8)} dx = \frac{-3}{4} \times \int \frac{1}{(3x + 8)} dx + \frac{1}{4} \times \int \frac{x + 1}{x^2} dx$$

$$= \frac{-3}{4} \times \frac{\log(3x+8)}{3} + \frac{1}{4}\int \frac{x}{x^2}dx + \int \frac{1}{x^2}dx$$

$$= -\frac{1}{4}\log|3x+8| + \frac{1}{4}logx - \frac{1}{x} + c$$

Putting x+2=0,

X=-2

A$(-4)^2$+B(0)+C(0)=-6+1=-5

$$A = \frac{-5}{16}$$

Question 36.
Evaluate:

$$\int \frac{(5x^2 - 18x + 17)}{(x-1)^2 (2x-3)} dx$$

Answer:
Let $I = \int \frac{5x^2 18x+17}{(x-1)^2(2x-3)} dx$

Now putting, $\frac{5x^2 18x+17}{(x-1)^2(2x-3)} = \frac{A}{(2x-3)} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \ldots.(1)$

A$(x-1)^2$+B(2x-3)(x-1)+C(2x-3) = $5x^2$-18x+17

Putting x-1=0,

X=1

A(0)+B(0)+C(2-3)=5-18+17

C(-1)=4

Putting 2x-3=0,

$$x = \frac{3}{2}$$

$$A\left(\frac{3}{2} - 1\right)^2 + B(0) + C(0) = 5\left(\frac{3}{2}\right)^2 - 18\left(\frac{3}{2}\right) + 17$$

$$A\left(\frac{1}{4}\right) + 0 = 5 \times \frac{9}{4} - 27 + 17$$

$$A\left(\frac{1}{4}\right) = \frac{45}{4} - 10 = \frac{5}{4}$$

A=5

By equating the coefficient of x^2, we get ,

A+2B=5

5+2B=5

2B=0

B=0

From equation (1), we get,

$$\frac{5x^2 18x + 17}{(x-1)^2(2x-3)} = 5 \times \frac{1}{(2x-3)} + 0 - 4 \times \frac{1}{(x-1)^2}$$

$$\int \frac{5x^2 18x + 17}{(x-1)^2(2x-3)} dx = \frac{5}{2}\log(2x-3) + \frac{4}{x-1} + c$$

Question 37.
Evaluate:

$$\int \frac{8}{(x+2)(x^2+4)} dx$$

Answer:

Let $I = \int \frac{8}{(x+2)(x^2+4)} dx$

Now putting, $\frac{8}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{(x^2+4)} \dots\dots.(1)$

$A(x^2+4)+(Bx+C)(x+2)= 8$

Putting x+2=0,

X=-2

A(4+4)+0=8

A=1

By equating the coefficient of x^2 and constant term, A+B=0

1+B=0

B=-1

4A+2C=8

4×1+2C=8

2C=4

C=2

From equation (1),we get,

$$\frac{8}{(x+2)(x^2+4)} = \frac{1}{x+2} + \frac{-x+2}{(x^2+4)}$$

$$\int \frac{8}{(x+2)(x^2+4)} dx = \int \frac{1}{x+2} dx - \int \frac{x}{(x^2+4)} dx + 2\int \frac{1}{(x^2+4)} dx$$

$$= log|x+2| - \frac{1}{2}\log(x^2+4) + 2 \times \frac{1}{2} \times tan^{-1}\frac{x}{2} + c$$

$$= log|x+2| - \frac{1}{2}\log|x^2+4| + tan^{-1}\frac{x}{2} + c$$

Question 38.
Evaluate:

$$\int \frac{(3x+5)}{(x^3-x^2+x-1)}dx$$

Answer:
Let $I = \int \frac{3x+5}{(x^3-x^2+x-1)}dx$

Now putting, $\frac{3x+5}{(x^3-x^2+x-1)} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+1)} \ldots \ldots (1)$

$A(x^2+1)+(Bx+C)(x-1)=3x+5$

Putting x-1=0,

X=1

A(2)+B(0)=3+5=8

A=4

By equating the coefficient of x^2 and constant term, A+B=0

4+B=0

B=-4

A-C=5

4-C=5

C=-1

From equation (1),we get,

$$\frac{3x+5}{(x-1)(x^2+1)} = \frac{4}{x-1} + \frac{-4x-1}{(x^2+1)}$$

$$\int \frac{3x+5}{(x-1)(x^2+1)}dx = 4\int \frac{1}{x-1}dx - 4\int \frac{1}{(x^2+1)}dx - \int \frac{1}{(x^2+1)}dx$$

$$= 4\log(x-1) - \frac{4}{2}\log(x^2+1) - tan^{-1}x + c$$

$$= 4\log(x-1) - 2\log(x^2+1) - tan^{-1}x + c$$

Question 39.
Evaluate:

$$\int \frac{2x}{(x^2+1)(x^2+3)}dx$$

Answer:
Let $I = \int \frac{2x}{(x^2+1)(x^2+3)}dx$

Put $t=x^2$

dt=2xdx

Now putting, $\frac{1}{(t+1)(t+3)} = \frac{A}{t+1} + \frac{B}{t+3} \ldots\ldots(1)$

A(t+3) +B(t+1) = 1

Putting t+3=0,

X=-3

A(0) +B(-3+1)=1

$$B = -\frac{1}{2}$$

Putting t+1=0,

X=-1

A(-1+3)+B(0)=1

$$A = \frac{1}{2}$$

From equation(1),we get,

$$\frac{1}{(t+1)(t+3)} = \frac{1}{2} \times \frac{1}{t+1} - \frac{1}{2} \times \frac{1}{t+3}$$

$$\int \frac{1}{(t+1)(t+3)} dt = \frac{1}{2} \int \frac{1}{t+1} dt - \frac{1}{2} \int \frac{1}{t+3} dt$$

$$= \frac{1}{2} \log|t+1| - \frac{1}{2} \log|t+3| + c$$

$$\int \frac{2x}{(x^2+1)(x^2+3)} dx = \frac{1}{2} \log|x^2+1| - \frac{1}{2} \log|x^2+3| + c$$

Question 40.
Evaluate:

$$\int \frac{x^2}{(x^4-1)} dx$$

Answer:
Let $I = \int \frac{x^2}{(x^4-1)} dx$

Put t=x^2

dt=2xdx

Now putting, $\frac{x^2}{(x^4-1)} = \frac{t}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1} \ldots \ldots (1)$

A(t+1)+B(t-1) = t

Putting t+1=0,

t=-1

A(0)+B(-1-1)=-1

$$B = \frac{1}{2}$$

Putting t-1=0,

t=1

A(1+1)+B(0)=1

$$A = \frac{1}{2}$$

From equation(1),we get,

$$\frac{t}{(t-1)(t+1)} = \frac{1}{2} \times \frac{1}{t-1} + \frac{1}{2} \times \frac{1}{t+1}$$

$$\int \frac{x^2}{(x^4-1)} dt = \frac{1}{2} \int \frac{1}{x^2-1} dt + \frac{1}{2} \int \frac{1}{x^2+1} dt$$

$$= \frac{1}{2} \times \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + \frac{1}{2} tan^{-1} x + c$$

$$= \frac{1}{4} \log \left| \frac{x-1}{x+1} \right| + \frac{1}{2} tan^{-1} x + c$$

Question 41.

$\int \frac{dx}{(x^3-1)}$

Answer:

Let $I = \int \frac{dx}{x^3-1}$

Put $\frac{1}{x^3-1}=\frac{1}{(x-1)(x^2+x+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}\ldots\ldots\ldots..(1)$

$A(x^2+x+1)+(Bx+C)(x-1)=1$

Now putting x-1=0

X=1

A(1+1+1)+0=1

$$A=\frac{1}{3}$$

By equating the coefficient of x^2 and constant term, A+B=0

$$\frac{1}{3}+B=0$$

$$B=-\frac{1}{3}$$

A-C=1

$$\frac{1}{3}-C=1$$

$$C=\frac{1}{3}-1$$

$$C=\frac{-2}{3}$$

From the equation(1), we get,

$$\frac{1}{(x-1)(x^2+x+1)}=\frac{1}{3}\times\frac{1}{x-1}+\frac{-\frac{1}{3}x-\frac{2}{3}}{x^2+x+1}$$

$$I = \int \frac{1}{(x-1)(x^2+x+1)} dx$$

$$= \frac{1}{3}\int \frac{1}{x-1} dx - \frac{1}{3}\int \frac{x}{x^2+x+1} dx - \frac{2}{3}\int \frac{1}{x^2+x+1} dx$$

$$= \frac{1}{3} log|x-1| - \frac{1}{6}\int \frac{2x+1-1}{x^2+x+1} dx - \frac{2}{3}\int \frac{1}{x^2+x+1} dx$$

$$= \frac{1}{3} log|x-1| - \frac{1}{6}\int \frac{2x+1}{x^2+x+1} dx + \frac{1}{6}\int \frac{1}{x^2+x+1} dx - \frac{2}{3}\int \frac{1}{x^2+x+1} dx$$

Put $t=x^2+x+1$

$dt=(2x+1)dx$

$$I = \frac{1}{3} log|x-1| - \frac{1}{6}\int \frac{dt}{t} + \left(\frac{1}{6} - \frac{2}{3}\right)\int \frac{dx}{x^2+x+1}$$

$$= \frac{1}{3} log|x-1| - \frac{1}{6}\log t + \left(\frac{1-4}{6}\right)\int \frac{dx}{x^2+2\times\frac{1}{2}+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}$$

$$= \frac{1}{3} log|x-1| - \frac{1}{6} log|x^2+x+1| - \frac{1}{2}\times\frac{1}{\sqrt{3}/2} tan^{-1}\frac{x+1/2}{\sqrt{3}/2} + c$$

$$= \frac{1}{3} log|x-1| - \frac{1}{6} log|x^2+x+1| - \frac{1}{\sqrt{3}} tan^{-1}\frac{2x+1}{\sqrt{3}} + c$$

Question 42.

$\int \frac{dx}{(x^3+1)}$

Answer:

Let $I = \int \frac{dx}{x^3+1}$

Put $\frac{1}{x^3-1} = \frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \ldots\ldots\ldots(1)$

$A(x^2-x+1)+(Bx+C)(x+1)=1$

Now putting x+1=0

X=-1

A(1+1+1)+C(0)=1

$$A = \frac{1}{3}$$

By equating the coefficient of x^2 and constant term, A+B=0

$$\frac{1}{3} + B = 0$$

$$B = -\frac{1}{3}$$

A+C+=1

$$\frac{1}{3} + C = 1$$

$$C = 1 - \frac{1}{3}$$

$$C = \frac{2}{3}$$

From the equation(1), we get,

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{1}{3} \times \frac{1}{x+1} + \frac{-\frac{1}{3}x + \frac{2}{3}}{x^2-x+1}$$

$$I = \int \frac{1}{(x+1)(x^2-x+1)} dx$$
$$= \frac{1}{3}\int \frac{1}{x+1} dx - \frac{1}{3}\int \frac{x}{x^2-x+1} dx + \frac{2}{3}\int \frac{1}{x^2-x+1} dx$$

$$= \frac{1}{3} log|x+1| - \frac{1}{6}\int \frac{2x-1+1}{x^2-x+1} dx + \frac{2}{3}\int \frac{1}{x^2-x+1} dx$$

$$=\frac{1}{3}log|x+1|-\frac{1}{6}\int\frac{2x-1}{x^2-x+1}dx-\frac{1}{6}\int\frac{1}{x^2-x+1}dx+\frac{2}{3}\int\frac{1}{x^2-x+1}dx$$

$$=\frac{1}{3}log|x+1|-\frac{1}{6}log|x^2-x+1|-\frac{1}{2}\times\frac{1}{\sqrt{3}/2}tan^{-1}\frac{x-1/2}{\sqrt{3}/2}+c$$

$$=\frac{1}{3}log|x+1|-\frac{1}{6}log|x^2-x+1|+\frac{1}{\sqrt{3}}tan^{-1}\frac{2x-1}{\sqrt{3}}+c$$

Question 43.

$\int\frac{dx}{(x+1)^2(x^2+1)}$

Answer:

Let $I=\int\frac{dx}{(x^2+1)(x+1)^2}$

Put $\frac{1}{(x^2+1)(x+1)^2}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{Cx+D}{x^2+1}\dots\dots\dots..(1)$

$A(x+1)(x^2+1)+B(x^2+1)+(Cx+D)(x+1)^2=1$

Put x+1=0

X=-1

A(0)+B(1+1)+0=1

$$B=\frac{1}{2}$$

By equating the coefficient of x^2 and constant term, A+C=0

A+B+2C=0……(2)

$$A+2C=\frac{-1}{2}\dots\dots\dots.(3)$$

A+B+D=1

Solving (2) and (3),we get,

$$\frac{1}{(x^2+1)(x+1)^2}=\frac{1}{2}\times\frac{1}{x+1}+\frac{1}{2}\times\frac{1}{(x+1)^2}+\frac{-\frac{1}{2}x+0}{x^2+1}$$

$$\int\frac{1}{(x^2+1)(x+1)^2}dx=\frac{1}{2}\int\frac{1}{x+1}dx+\frac{1}{2}\int\frac{1}{(x+1)^2}dx-\frac{1}{2}\int\frac{x}{x^2+1}dx$$

$$=\frac{1}{2}log|x+1|-\frac{1}{2}\times\frac{1}{x+1}-\frac{1}{4}\log|x^2+1|+c$$

Question 44.

$$\int\frac{17}{(2x+1)(x^2+4)}dx$$

Answer:

Let $I=\int\frac{17}{(2x+1)(x^2+4)}dx$

Put $\frac{17}{(2x+1)(x^2+4)}=\frac{A}{2x+1}+\frac{Bx+C}{x^2+4}\ldots\ldots\ldots..(1)$

$A(x^2+4)+(Bx+C)(2x+1)=17$

Put 2x+1=0

$$x=-\frac{1}{2}$$

$$A\left(\frac{1}{4}+4\right)+0=17$$

$$A\left(\frac{17}{4}\right)=17$$

A=4

By equating the coefficient of x^2 and constant term,

A+2B=0

4+2B=0

B=-2

4A+C=17

4×4+C=17

C=1

From the equation(1), we get,

$$\frac{17}{(2x+1)(x^2+4)} = \frac{4}{2x+1} + \frac{-2x+1}{x^2+4}$$

$$\int \frac{17}{(2x+1)(x^2+4)} dx = 4\int \frac{1}{2x+1} dx - 2\int \frac{2x}{x^2+4} dx + \int \frac{1}{x^2+2^2} dx$$

$$= \frac{4\log|2x+1|}{2} - \log|x^2+4| + \frac{1}{2} tan^{-1}\frac{x}{2} + c$$

$$= 2\log|2x+1| - \log|x^2+4| + \frac{1}{2} tan^{-1}\frac{x}{2} + c$$

Question 45.

$\int \frac{dx}{(x^2+2)(x^2+4)}$

Answer:

Let $I = \int \frac{dx}{(x^2+2)(x^2+4)} dx$

Put $\frac{1}{(x^2+2)(x^2+4)} = \frac{1}{(t+2)(t+4)} = \frac{A}{t+2} + \frac{B}{t+4} \ldots\ldots\ldots..(1)$

A(t+4)+B(t+2) = 1

Put t+4=0

t=-4

A(0)+B(-4+2)=1

$$B = -\frac{1}{2}$$

Put t+2=0

t=-2

A(-2+4)+B(0)=1

$$A = \frac{1}{2}$$

From equation(1),we get,

$$\frac{1}{(t+2)(t+4)} = \frac{1}{2} \times \frac{1}{t+2} - \frac{1}{2} \times \frac{1}{t+4}$$

$$\int \frac{1}{(x^2+2)(x^2+4)} dx = \frac{1}{2} \int \frac{1}{x^2+2} dx - \frac{1}{2} \int \frac{1}{x^2+4} dx$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} tan^{-1} \frac{x}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{2} tan^{-1} \frac{x}{2} + c$$

$$= \frac{1}{4} tan^{-1} \frac{x}{\sqrt{2}} - \frac{1}{4} tan^{-1} \frac{x}{2} + c$$

Question 46.

$$\frac{x^2+1}{(x^2+4)(x^2+25)} dx$$

Answer:

Let $I = \int \frac{x^2+1}{(x^2+4)(x^2+25)} dx$

Putting $\frac{x^2+1}{(x^2+4)(x^2+25)} = \frac{t+1}{(t+4)(t+25)} = \frac{A}{t+4} + \frac{B}{t+25} \ldots \ldots \ldots (1)$

Where $t=x^2$

$(A+B)t+(25A+4B)=t+1$

$A+B=1$............(1)

$25A+4B=1$...........(2)

Solving equation (1)and(2), we get,

$$A=\frac{-1}{7}\ and\ B=\frac{8}{7}$$

Now,

$$\frac{t+1}{(t+4)(t+25)}=\frac{-1}{7}\times\frac{1}{t+4}+\frac{8}{7}\times\frac{1}{t+25}$$

$$\frac{x^2+1}{(x^2+4)(x^2+25)}=\frac{-1}{7}\times\frac{1}{x^2+4}+\frac{8}{7}\times\frac{1}{x^2+25}$$

$$\int\frac{x^2+1}{(x^2+4)(x^2+25)}dx=\frac{-1}{7}\int\frac{1}{x^2+2^2}dx+\frac{8}{7}\int\frac{1}{x^2+5^2}dx$$

$$=-\frac{1}{7}\times\frac{1}{2}tan^{-1}\left(\frac{x}{2}\right)+\frac{8}{7}\times\frac{1}{5}tan^{-1}\left(\frac{x}{5}\right)+c$$

$$=-\frac{1}{14}tan^{-1}\left(\frac{x}{2}\right)+\frac{8}{35}tan^{-1}\left(\frac{x}{5}\right)+c$$

Question 47.

$$\int\frac{dx}{\left(e^x-1\right)^2}$$

Answer:

putting $t=e^x-1$

$e^x=t+1$

$dt= e^x\ dx$

$$\frac{dt}{e^x} = dx$$

$$\frac{dt}{t+1} = dx$$

Put$\frac{1}{(1+t)t^2} = \frac{A}{t+1} + \frac{Bt+C}{t^2} \dots \dots (1)$

$A(t^2)+(Bt+C)(t+1)=1$

Put t+1=0

t=-1

A=1

Equating coefficients

A+B=0

1+B=0

B=-1

C=1

From equation (1),we get,

$$\frac{1}{(1+t)t^2} = \frac{1}{t+1} + \frac{-t+1}{t^2}$$

$$\int \frac{1}{(1+t)t^2} dt = \int \frac{1}{t+1} dt - \int \frac{t}{t^2} dt + \int \frac{1}{t^2} dt$$

$$= \log|t+1| - \int \frac{1}{t} dt + \int \frac{1}{t^2} dt$$

$$= \log|t+1| - log|t| - \frac{1}{t} + c$$

$$\int \frac{1}{(e^x - 1)^2} dx = log\,|e^x| - \log|e^x - 1| - \frac{1}{e^x - 1} + c$$

Question 48.

$\int \frac{dx}{x(x^5+1)}$

Answer:

Let $I = \int \frac{dx}{x(x^5+1)}$

Put t=x^5

dt=5x^4dx

$$\int \frac{dt}{\frac{(5x^4)}{x(t+1)}} = \frac{1}{5} \int \frac{dt}{x^5(t+1)} = \frac{1}{5} \int \frac{dt}{t(t+1)}$$

Putting $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1} \ldots \ldots ..(1)$

A(t+1)+Bt=1

Now put t+1=0

t=-1

A(0)+B(-1)=1

B=-1

Now put t=0

A(0+1)+B(0)=1

A=1

$$\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$$

$$\int \frac{1}{t(t+1)} dt = \int \frac{1}{t} dt - \int \frac{1}{t+1} dt$$

$$= logt - \log|t+1| + c$$

$$= log\left|\frac{t}{t+1}\right| + c$$

$$\int \frac{dx}{x(x^5+1)} = \frac{1}{5}\int \frac{dt}{t(t+1)} = \frac{1}{5} log\left|\frac{x^5}{x^5+1}\right| + c$$

$$= logx - \frac{1}{5} log|x^5+1| + c$$

Question 49.

$\int \frac{dx}{x(x^6+1)}$

Answer:

Let $I = \int \frac{dx}{x(x^6+1)}$

Put t=x^6

dt=6x^5dx

$$\int \frac{dt}{\frac{(6x^5)}{x(t+1)}} = \frac{1}{6}\int \frac{dt}{x^6(t+1)} = \frac{1}{6}\int \frac{dt}{t(t+1)}$$

Putting $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1} \ldots \ldots . . (1)$

A(t+1)+Bt=1

Now put t+1=0

t=-1

A(0)+B(-1)=1

B=-1

Now put t=0

A(0+1)+B(0)=1

A=1

$$\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$$

$$\int \frac{1}{t(t+1)} dt = \int \frac{1}{t} dt - \int \frac{1}{t+1} dt$$

$$= logt - \log|t+1| + c$$

$$= log\left|\frac{t}{t+1}\right| + c$$

$$\int \frac{dx}{x(x^6+1)} = \frac{1}{6} \int \frac{dt}{t(t+1)} = \frac{1}{6} log\left|\frac{x^6}{x^6+1}\right| + c$$

$$= logx - \frac{1}{6} log|x^6+1| + c$$

Question 50.

$$\int \frac{dx}{\sin x\,(3+2\cos x)}$$

Answer:

let $I = \int \frac{dx}{sinx\,(3+2cosx)}$

Put t=cosx

dt=-sinxdx

$$\frac{dt}{-sinx} = dx$$

$$I = \int \frac{dt}{\frac{-sinx}{sinx(3+2t)}}$$

$$= -\int \frac{dt}{sin^2x(3+2t)} = -\int \frac{dt}{(1-cos^2x)(3+2t)}$$

$$= -\int \frac{dt}{(1-t^2)(3+2t)}$$

$$\frac{1}{(1-t^2)(3+2t)} = \frac{1}{(1-t)(1+t)(3+2t)}$$

Putting $\frac{1}{(1-t)(1+t)(3+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{3+2t} \dots \dots .. (1)$

A(1+t)(3+2t)+B(1-t)(3+2t)+C(1+t)(1-t)=1

Now Putting 1+t=0

t=-1

A(0)+B(2)(3-2)+C(0)=1

$$B = \frac{1}{2}$$

Now Putting 1-t=0

t=1

A(2)(5)+B(0)+C(0)=1

$$A = \frac{1}{10}$$

Now Putting 3+2t=0

$$t = -\frac{3}{2}$$

$$A(0) + B(0) + C\left(1 - \frac{9}{4}\right) = 1$$

$$C = \frac{-4}{5}$$

$$\frac{1}{(1-t)(1+t)(3+2t)} = \frac{1}{10} \times \frac{1}{1-t} + \frac{1}{2} \times \frac{1}{1+t} - \frac{4}{5} \times \frac{1}{3+2t}$$

$$\int \frac{1}{(1-t)(1+t)(3+2t)} dt = \frac{1}{10} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{1+t} dt - \frac{4}{5} \int \frac{1}{3+2t} dt$$

$$= -\frac{1}{10} log|1-t| + \frac{1}{2} log|1+t| - \frac{4}{5} \times \frac{log|3+2t|}{2} + c$$

$$= -\frac{1}{10} log|1-cosx| + \frac{1}{2} log|1+cosx| - \frac{2}{5} log|3+2cosx| + c$$

Question 51.

$\int \frac{dx}{\cos x(5-4\sin x)}$

Answer:

let $I = \int \frac{dx}{cosx\,(5-4sinx)}$

Put t=sinx

dt=cosxdx

$$I = \int \frac{dt}{(1-sin^2x)(5-4t)} = \int \frac{dt}{(1-t^2)(5-4t)}$$

$$\frac{1}{(1-t^2)(5-4t)} = \frac{1}{(1-t)(1+t)(5-4t)}$$

Putting $\frac{1}{(1-t)(1+t)(5-4t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{5-4t} \dots \dots.(1)$

A(1+t)(5-4t)+B(1-t)(5-4t)+C(1+t)(1-t)=1

Now Putting 1+t=0

t=-1

A(0)+B(2)(9)+C(0)=1

$$B = \frac{1}{18}$$

Now Putting 1-t=0

t=1

A(2) +B(0)+C(0)=1

$$A = \frac{1}{2}$$

Now Putting 5-4t=0

$$t = \frac{5}{4}$$

$$A(0) + B(0) + C\left(1 - \frac{25}{16}\right) = 1$$

$$C = \frac{-16}{9}$$

From equation(1),we get,

$$\frac{1}{(1-t)(1+t)(5-4t)} = \frac{1}{2} \times \frac{1}{1-t} + \frac{1}{18} \times \frac{1}{1+t} - \frac{16}{9} \times \frac{1}{5-4t}$$

$$\int \frac{1}{(1-t)(1+t)(5-4t)} dt = \frac{1}{2} \int \frac{1}{1-t} dt + \frac{1}{18} \int \frac{1}{1+t} dt - \frac{16}{9} \int \frac{1}{5-4t} dt$$

$$= -\frac{1}{2} log|1-t| + \frac{1}{18} log|1+t| - \frac{16}{9} \times \frac{log|5-4t|}{-4} + c$$

$$= -\frac{1}{2} log|1-sinx| + \frac{1}{18} log|1+sinx| + \frac{4}{9} log|5-4sinx| + c$$

Question 52.

$\int \frac{dx}{\sin x \cos^2 x}$

Answer:

Let $I = \int \frac{1}{sinx \times cos^2 x} dx = \int \frac{sin^2 x + cos^2 x}{sinx \times cos^2 x} dx = \int \frac{sin^2 x}{sinx \times cos^2 x} dx + \int \frac{cos^2 x}{sinx \times cos^2 x} dx$

$$= \int \frac{sinx}{cos^2 x} dx + \int \frac{1}{sinx} dx$$

$$= \int (\tan x \sec x + cosec\ x) dx$$

$$= \sec x - \frac{1}{2} log cot^2 \frac{x}{2} = \sec x - \frac{1}{2} \log\left(\frac{1 + cosx}{1 - cosx}\right) + c$$

Question 53.

$\int \frac{\tan x}{(1 - \sin x)} dx$

Answer:

let $I = \int \frac{\tan x}{(1 - sinx)}$dx = $\int \frac{\sin x}{\cos x (1 - sinx)}$dx

Put t=sinx

dt=cosxdx

$$I = \int \frac{\sin x \times cosx}{cos^2 x\,(1 - sinx)} dx = \int \frac{tdt}{(1 - sin^2 x)(1 - t)} = \int \frac{tdt}{(1 - t^2)(1 - t)}$$

Putting $\frac{t}{(1-t)(1+t)(1-t)} = \frac{A}{1+t} + \frac{B}{1-t} + \frac{C}{(1-t)^2} \ldots \ldots (1)$

A(1+t)^2 +B(1-t)(1+t)+C(1+t)=t

Now Putting 1-t=0

t=1

A(0)+B(0)+C(1+1)=1

$$C = \frac{1}{2}$$

Now Putting 1+t=0

t=-1

A$(2)^2$ +B(0)+C(0)=-1

$$A = -\frac{1}{4}$$

By equating the coefficient of t^2,we get,A-B=0

$$\frac{-1}{4} - B = 0$$

$$B = -\frac{1}{4}$$

From equation(1),we get,

$$\frac{t}{(1-t)(1+t)(1-t)} = \frac{-1}{4} \times \frac{1}{1+t} - \frac{1}{4} \times \frac{1}{1-t} + \frac{1}{2} \times \frac{1}{(1-t)^2}$$

$$\int \frac{t}{(1-t)(1+t)(1-t)} dt = \frac{-1}{4} \int \frac{1}{1+t} dt - \frac{1}{4} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{(1-t)^2} dt$$

$$= \frac{-1}{4} \int \frac{1}{1+t} dt - \frac{1}{4} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{(1-t)^2} dt$$

$$= -\frac{1}{4} log|1+t| - \frac{1}{4} log|1-t| - \frac{1}{2} \times \frac{1}{1-t} + c$$

$$= -\frac{1}{4} log|1+sinx| - \frac{1}{4} log|1-sinx| - \frac{1}{2} \times \frac{1}{1-sinx} + c$$

Question 54.

$\int \frac{dx}{(\sin x + \sin 2x)}$

Answer:

let $I = \int \frac{dx}{(sinx+sin2x)} = \int \frac{dx}{(sinx+2sinxcosx)}$

Put t=cosx

dt=-sinxdx

$$\frac{-dt}{sinx} = dx$$

$$I = \int \frac{-dt}{sin^2 x\,(1+2t)} = \int \frac{dt}{(1-cos^2 x)(1+2t)} = \int \frac{dt}{(1-t^2)(1+2t)}$$

Putting $\frac{t}{(1-t)(1+t)(1+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{1+2t} \ldots \ldots .(1)$

A(1+t)(1+2t)+B(1-t)(1+2t)+C(1-t^2)=1

Putting 1+t=0

t=-1

A(0)+B(2)(1-2)+C(0)=1

$$B = -\frac{1}{2}$$

Putting 1-t=0

t=1

A(2)(3)+B(0)+C(0)=1

$$A = \frac{1}{6}$$

Putting 1+2t=0

$$t = -\frac{1}{2}$$

$$A(0) + B(0) + C\left(1 - \frac{1}{4}\right) = 1$$

$$C = \frac{4}{3}$$

From equation(1),we get,

$$\frac{1}{(1-t)(1+t)(1+2t)} = \frac{1}{6} \times \frac{1}{1-t} - \frac{1}{2} \times \frac{1}{1+t} + \frac{4}{3} \times \frac{1}{1+2t}$$

$$\int \frac{1}{(1-t)(1+t)(1+2t)} dt = \frac{1}{6}\int \frac{1}{1-t} dt - \frac{1}{2}\int \frac{1}{1+t} dt + \frac{4}{3}\int \frac{1}{1+2t} dt$$

$$= \frac{1}{6} log|1-t| - \frac{1}{2} log|1+t| + \frac{2}{3} log|1+2t| + c$$

$$= \frac{1}{6} log|1-cosx| - \frac{1}{2} log|1+cosx| + \frac{2}{3} log|1+2cosx| + c$$

Question 55.

$$\int \frac{x^2}{(x^4 - x^2 - 12)} dx$$

Answer:

Let $I = \int \frac{x^2}{(x^4 - x^2 - 12)} dx$

Putting $\frac{x^2}{(x^4-x^2-12)} = \frac{t}{t^2-t-12} = \frac{t}{(t-4)(t+3)} = \frac{A}{t-4} + \frac{B}{t+3} \ldots \ldots \ldots (1)$

Where t=x^2

A(t+3)+B(t-4)=t

Now put t+3=0

t=-3

A(0)+B(-7)=-3

$$B = \frac{3}{7}$$

Now put t-4=0

t=4

A(4+3)+B(0)=4

$$A = \frac{4}{7}$$

From equation(1)

$$\frac{t}{(t-4)(t+3)} = \frac{4}{7} \times \frac{1}{t-4} + \frac{3}{7} \times \frac{1}{t+3}$$

$$\frac{x^2}{(x^2-4)(x^2+3)} = \frac{4}{7} \times \frac{1}{x^2-2^2} + \frac{3}{7} \times \frac{1}{x^2+(\sqrt{3})^2}$$

$$\int \frac{x^2}{(x^2-4)(x^2+3)} dx = \frac{4}{7} \int \frac{1}{x^2-2^2} dx + \frac{3}{7} \int \frac{1}{x^2+\left(\sqrt{3}\right)^2} dx$$

$$= \frac{4}{7} \times \frac{1}{2} \times \frac{1}{2} log \left|\frac{x-2}{x+2}\right| + \frac{3}{7} \times \frac{1}{\sqrt{3}} tan^{-1} \frac{x}{\sqrt{3}} + c$$

$$= \frac{1}{7} log \left|\frac{x-2}{x+2}\right| + \frac{\sqrt{3}}{7} tan^{-1} \frac{x}{\sqrt{3}} + c$$

Question 56.

$$\int \frac{x^4}{(x^2+1)(x^2+9)(x^2+16)} dx$$

Answer:

Let $I = \int \frac{x^4}{(x^2+1)(x^2+9)(x^2+16)} dx$

Putting $\frac{(x^2)^2}{(x^2+1)(x^2+9)(x^2+16)} = \frac{t^2}{(t+1)(t+9)(t+16)} = \frac{A}{t+1} + \frac{B}{t+9} + \frac{C}{t+16} \ldots\ldots\ldots(1)$

Where $t=x^2$

$t^2=A(t+9)(t+16)+B(t+1)(t+16)+C(t+1)(t+9)$

Now put $t+1=0$

$t=-1$

$A(8)(15)+B(0)+C(0)=1$

$$A = \frac{1}{120}$$

Now put $t+9=0$

$t=-9$

$A(-9+9)(-9+16)+B(-9+1)(-9+16)+C(-9+1)(-9+9)=(-9)^2$

$A(0)+B(-56)+C(0)=81$

$$B = -\frac{81}{56}$$

Now put $t+16=0$

$t=-16$

$A(0)+B(0)+C(-15)(-7)=(-16)^2$

$A(0)+B(0)+C(105)=256$

$$C = \frac{256}{105}$$

From equation(1)

$$\frac{t^2}{(t+1)(t+9)(t+16)} = \frac{A}{t+1} + \frac{B}{t+9} + \frac{C}{t+16}$$

$$\int \frac{t^2}{(t+1)(t+9)(t+16)} dt = \int \left[\frac{\frac{1}{120}}{t+1} - \frac{\frac{81}{56}}{t+9} + \frac{\frac{256}{105}}{t+16} \right] dt$$

$$= \frac{1}{120}\int \frac{1}{t+1} dt - \frac{81}{56}\int \frac{1}{t+9} dt + \frac{256}{105}\int \frac{1}{t+16} dt$$

$$= \frac{1}{120}\int \frac{1}{x^2+1} dx - \frac{81}{56}\int \frac{1}{x^2+9} dx + \frac{256}{105}\int \frac{1}{x^2+16} dx$$

$$= \frac{1}{120}\int \frac{1}{x^2+1} dx - \frac{81}{56}\int \frac{1}{x^2+(3)^2} dx + \frac{256}{105}\int \frac{1}{x^2+(4)^2} dx$$

$$= \frac{1}{120} tan^{-1}x - \frac{81}{56} \times \frac{1}{3} tan^{-1}\left(\frac{x}{3}\right) + \frac{256}{105} \times \frac{1}{4} tan^{-1}\left(\frac{x}{4}\right) + c$$

$$= \frac{1}{120} tan^{-1}x - \frac{27}{56} tan^{-1}\left(\frac{x}{3}\right) + \frac{64}{105} tan^{-1}\left(\frac{x}{4}\right) + c$$

Question 57.

$\int \frac{\sin 2x}{(1-\cos 2x)(2-\cos 2x)} dx$

Answer:

let $I = \int \frac{sin2x}{(1-cos2x)(2-cos2x)} dx$

Put t=cos2x

dt=-2sin2xdx

$$I = \int \frac{-dt/2}{(1-t)(2-t)} = \frac{1}{2}\int \frac{dt}{(t-2)(1-t)}$$

Putting $\frac{1}{(t-2)(1-t)} = \frac{A}{t-2} + \frac{B}{1-t} \dots \dots . . (1)$

A(1-t)+B(t-2)=1

Putting 1-t=0

t=1

A(0)+B(1-2) =1

B=-1

Putting t-2=0

t=2

A(1-2)+B(0) =1

A=-1

From equation (1), we get,

$$\frac{1}{(t-2)(1-t)} = \frac{-1}{t-2} + \frac{-1}{1-t}$$

$$\int \frac{1}{(t-2)(1-t)} dt = \int \frac{1}{2-t} dt + \int \frac{1}{t-1} dt$$

$$= -log|2-t| + log|t-1| + c$$

$$= log|t-1| - log|2-t| + c$$

$$= log|cos2x - 1| - log|2 - cos2x| + c$$

Question 58.

$$\int \frac{2}{(1-x)(1+x^2)} dx$$

Answer:

Let $I = \int \frac{2}{(1-x)(1+x^2)} dx$

Put $\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{x^2+1} \dots \dots \dots . . (1)$

$A(1+x^2)+Bx(1-x)+C(1-x) =2$

Put x=1

2=2A+0+0

A=1

Put x=0

2=A+C

C=2-A

C=2-1=1

Putting x=2

We have 2=5A-2B-C

2=5×1-2B-1

2B=2

B=1

$$\frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x}{1+x^2} + \frac{1}{1+x^2}$$

$$\int \frac{2}{(1-x)(1+x^2)} dx = \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$-log|1-x|+\frac{1}{2}log|1+x^2|+tan^{-1}x+c$$

Question 59.

$$\int\frac{2x^2+1}{x^2\left(x^2+4\right)}dx$$

Answer:

Let $I=\int\frac{2x^2+1}{x^2(x^2+4)}dx$

Again let x^2=t

$$\frac{2t+1}{t(t+4)}=\frac{A}{t}+\frac{B}{(t+4)}\dots\dots..(1)$$

2t+1=A(t+4)+B(t)

Putting t=-4

2(-4)+1=A(-4+4)+B(-4)

-8+1=0-4B

-7=-4B

$$B=\frac{7}{4}$$

Putting t=0

2(0)+1=A(0+4)+B(0)

1=4A

$$A=\frac{1}{4}$$

$$\frac{2t+1}{t(t+4)}=\frac{\frac{1}{4}}{t}+\frac{\frac{7}{4}}{(t+4)}$$

$$\int \frac{2t+1}{t(t+4)}dt = \int \frac{2x^2+1}{x^2(x^2+4)}dx = \frac{1}{4}\int \frac{1}{x^2}dx + \frac{7}{4}\int \frac{1}{(x^2+2^2)}dx$$

$$= \frac{1}{4} \times \frac{(-1)}{x} + \frac{7}{4} \times \frac{1}{2} tan^{-1}\left(\frac{x}{2}\right) + c$$

$$I = \frac{-1}{4x} + \frac{7}{8} tan^{-1}\left(\frac{x}{2}\right) + c$$