Exercise 13a

Question 1.
Evaluate the following integrals:

$$\int (2x+9)^5 \, dx$$

Answer:
Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 2x + 9 = t ⇒ 2 dx = dt

$$\int t^5 (\frac{dt}{2}) = \frac{1}{2} \int t^5 dt = \frac{1}{2}\frac{t^6}{6} + c = \frac{t^6}{12} + c$$

$$= \frac{(2x+9)^6}{12} + c$$

Question 2.
Evaluate the following integrals:

Answer:
Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 7 – 3x = t ⇒ -3 dx = dt

$$\int t^4 (\frac{dt}{-3}) = \frac{1}{-3} \int t^4 dt = \frac{1}{-3}\frac{t^5}{5} + c = -\frac{t^5}{15} + c$$

$$= -\frac{(7-3x)^5}{15} + c$$

Question 3.
Evaluate the following integrals:

$$\int \sqrt{3x-5}\,dx$$

Answer:
Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 3x - 5 = t ⇒ 3 dx = dt

$$\int t^{0.5}\left(\frac{dt}{3}\right) = \frac{1}{3}\int t^{0.5} dt = \frac{1}{3} \times \frac{t^{1.5}}{1.5} + c = \frac{2}{1} \times \frac{t^{1.5}}{9} + c$$

$$= \frac{2(3x-5)^5}{9} + c$$

Question 4.
Evaluate the following integrals:

$$\int \frac{1}{\sqrt{4x+3}}\,dx$$

Answer:
Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 4x + 3 = t ⇒ 4 dx = dt

$$\int t^{-0.5}\left(\frac{dt}{4}\right) = \frac{1}{4}\int t^{-0.5} dt = \frac{1}{4} \times \frac{t^{0.5}}{0.5} + c = \frac{2}{4} \times \frac{t^{0.5}}{1} + c$$

$$= \frac{\sqrt{4x+3}}{2} + c$$

Question 5.
Evaluate the following integrals:

$$\int \frac{1}{\sqrt{3-4x}} dx$$

Answer:
Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 3 – 4x = t ⇒ -4 dx = dt

$$\int t^{-0.5}\left(\frac{dt}{-4}\right) = \frac{1}{-4}\int t^{-0.5}dt = \frac{1}{-4} \times \frac{t^{0.5}}{0.5} + c = \frac{2}{-4} \times \frac{t^{0.5}}{1} + c$$

$$= -\frac{\sqrt{3-4x}}{2} + c$$

Question 6.
Evaluate the following integrals:

$$\int \frac{1}{(2x-3)^{3/2}} dx$$

Answer:
Formula = $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$

Therefore ,

Put 2x – 3 = t ⇒ 2 dx = dt

$$\int t^{-\frac{3}{2}}\left(\frac{dt}{2}\right) = \frac{1}{2}\int t^{-\frac{3}{2}}dt = \frac{1}{2} \times \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + c = \frac{-2}{2} \times \frac{t^{-0.5}}{1} + c$$

$$= -\frac{1}{\sqrt{2x-3}} + c$$

Question 7.
Evaluate the following integrals:

$$\int e^{(2x-1)} dx$$

Answer:
Formula = $\int e^x dx = e^x + c$

Therefore ,

Put 2x – 1 = t ⇒ 2 dx = dt

$$\int e^t (\frac{dt}{2}) = \frac{1}{2} \int e^t dt = \frac{1}{2} \times e^t + c = \frac{e^{2x-1}}{2} + c$$

$$= \frac{e^{(2x-1)}}{2} + c$$

Question 8.
Evaluate the following integrals:

$$\int e^{(1-3x)} dx$$

Answer:
Formula = $\int e^x dx = e^x + c$

Therefore ,

Put 1 – 3x = t ⇒ -3 dx = dt

$$\int e^t (\frac{dt}{-3}) = \frac{1}{-3} \int e^t dt = \frac{1}{-3} \times e^t + c = \frac{e^{1-3x}}{-3} + c$$

$$= -\frac{e^{(1-3x)}}{3} + c$$

Question 9.
Evaluate the following integrals:

$$\int 3^{(2-3x)} dx$$

Answer:
Formula = $\int a^x dx = \frac{a^x}{\log a} + c$

Therefore ,

Put 2 – 3x = t ⇒ -3 dx = dt

$$\int 3^t (\frac{dt}{-3}) = \frac{1}{-3} \int 3^t dt = \frac{1}{-3} \times (\frac{3^t}{\log 3}) + c = \frac{3^t}{-3 \log 3} + c$$

$$= -\frac{3^{(2-3x)}}{3 \log 3} + c$$

Question 10.
Evaluate the following integrals:

$$\int \sin 3x \, dx$$

Answer:
Formula = $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put 3x = t ⇒ 3 dx = dt

$$\int \sin t \, (\frac{dt}{3}) = \frac{1}{3} \int \sin t \, dt = \frac{1}{3} \times (-\cos t) + c = \frac{-\cos 3x}{3} + c$$

$$= -\frac{\cos 3x}{3} + c$$

Question 11.
Evaluate the following integrals:

$$\int \cos(5 + 6x)\,dx$$

Answer:
Formula = $\int \cos x\, dx = \sin x + c$

Therefore ,

Put 5 + 6x = t ⇒ 6 dx = dt

$$\int \cos t\,(\frac{dt}{6}) = \frac{1}{6}\int \cos t\, dt = \frac{1}{6} \times (\sin t) + c = \frac{\sin 5 + 6x}{6} + c$$

$$= \frac{\sin(5 + 6x)}{6} + c$$

Question 12.
Evaluate the following integrals:

$$\int \sin x \sqrt{1 + \cos 2x}\, dx$$

Answer:
Formula $\int \cos x\, dx = \sin x + c$

$$1 + \cos 2x = 2cos^2\, x$$

Therefore ,

$$\int \sin x \sqrt{1 + \cos 2x}\, dx = \int \sin x \sqrt{2} \cos x + c$$

$$\int \sqrt{2} \sin x \cos x\;\, dx$$

Put sin x =t ⇒ cos x dx = dt

$$\int \sqrt{2}\sin x\cos x \; dx = \int \sqrt{2}t\, dt = \sqrt{2}\,\frac{t^2}{2} + c$$

$$= \frac{(\sin x)^2}{\sqrt{2}} + c$$

Question 13.
Evaluate the following integrals:

$$\int \mathrm{cosec}^2(2x+5)dx$$

Answer:
Formula $\int cosec^2 x\, dx = -\cot x + c$

Therefore ,

Put 2x + 5 =t ⇒ 2 dx = dt

$$\int cosec^2 t\,\frac{dt}{2} = -\frac{1}{2}\cot t + c = -\frac{1}{2}\cot(2x+5) + c$$

$$= -\frac{1}{2}\cot(2x+5) + c$$

Question 14.
Evaluate the following integrals:

$$\int \sin x \cos x \, dx$$

Answer:
Formula $\int \sin x\, dx = -\cos x + c$

Therefore ,

Put sin x =t ⇒ cos x dx = dt

$$\int t\,dt = \frac{t^2}{2} + c$$

$$= \frac{(\sin x)^2}{2} + c$$

Question 15.
Evaluate the following integrals:

$$\int \sin^3 x \cos x \, dx$$

Answer:
Formula $\int \sin x\,dx = -\cos x + c$

Therefore ,

Put sin x =t ⇒ cos x dx = dt

$$\int t^3\,dt = \frac{t^4}{4} + c$$

$$= \frac{(\sin x)^4}{4} + c$$

Question 16.
Evaluate the following integrals:

$$\int \left(\sqrt{\cos x}\right) \sin x \, dx$$

Answer:
Formula $\int \sin x\,dx = -\cos x + c$

Therefore ,

Put cos x =t ⇒ -sin x dx = dt

$$\int t^{0.5}(-1)dt = -\frac{t^{1.5}}{1.5} + c$$

$$= -\frac{2(\cos x)^{\frac{3}{2}}}{3} + c$$

Question 17.
Evaluate the following integrals:

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$$

Answer:
Formula $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c$ $\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

Therefore ,

Put $\sin^{-1} x$ =t ⇒ $\frac{1}{\sqrt{1-x^2}}$ dx = dt

$$\int t^1 dt = \frac{t^2}{2} + c$$

$$= \frac{(\sin^{-1} x)^2}{2} + c$$

Question 18.
Evaluate the following integrals:

$$\int \frac{\sin\left(2\tan^{-1} x\right)}{\left(1+x^2\right)} dx .$$

Answer:
Formula $\int \sin t \, dx = -\cos t + c$ $\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$

Therefore ,

Put $\tan^{-1} x = t \Rightarrow \frac{1}{1+x^2} dx = dt$

$$\int \sin 2t \, dt = \frac{-\cos 2t}{2} + c$$

$$= -\frac{\cos(2\tan^{-1} x)}{2} + c$$

Question 19.
Evaluate the following integrals:

$$\int \frac{\cos(\log x)}{x} dx$$

Answer:
Formula $\int \cos t \, dx = \sin t + c$ $\frac{d(\log x)}{dx} = \frac{1}{x}$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int \cos t \, dt = \sin t + c$$

$$= \sin(\log x) + c$$

Question 20.
Evaluate the following integrals:

$$\int \frac{\operatorname{cosec}^2(\log x)}{x} dx$$

Answer:
Formula $\int cosec^2 x \, dx = -\cot x + c$ $\frac{d(\log x)}{dx} = \frac{1}{x}$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int cosec^2 t \, \frac{dt}{1} = -\cot t + c = -\cot(\log x) + c$$

$$= -\cot(\log x) + c$$

Question 21.
Evaluate the following integrals:

$$\int \frac{1}{\mathrm{x} \log \mathrm{x}} \mathrm{dx}$$

Answer:
Formula $\frac{d(log x)}{dx} = \frac{1}{x} \int \frac{1}{x} dx = \log x$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int \frac{dt}{t} = \log t + c = \log(\log x) + c$$

$$= \log(\log x) + c$$

Question 22.
Evaluate the following integrals:

$$\int \frac{(\mathrm{x}+1)(\mathrm{x}+\log \mathrm{x})^2}{\mathrm{x}} \mathrm{dx}$$

Answer:
Formula $\frac{d(log x)}{dx} = \frac{1}{x} \int \frac{1}{x} dx = \log x$

$$\int \frac{(x+1)(x+\log x)^2}{x} dx = \int \frac{x+1}{x} \times \frac{(x+\log x)^2}{1} dx$$
$$= \int (1+\frac{1}{x}) \times \frac{(x+\log x)^2}{1} dx$$

Therefore ,

Put $x + \log x = t \Rightarrow (1+\frac{1}{x})dx = dt$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$= \frac{(x+\log x)^3}{3} + c$$

Question 23.
Evaluate the following integrals:

$$\int \frac{(\log x)^2}{x} dx$$

Answer:
Formula $\frac{d(\log x)}{dx} = \frac{1}{x}\ \int \frac{1}{x} dx = \log x$

Therefore ,

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\int t^2 dt = \frac{t^3}{3} + c = \frac{(\log x)^3}{3} + c$$

$$= \frac{(\log x)^3}{3} + c$$

Question 24.

Evaluate the following integrals:

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$$

Answer:

Formula $\int \cos t \, dx = \sin t + c$ $\frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$

Therefore ,

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$$\int \cos t \; 2dt = 2 \sin t + c$$

$$= 2 \sin(\sqrt{x}) + c$$

Question 25.

Evaluate the following integrals:

$$\int e^{\tan x} \sec^2 x \, dx$$

Answer:

Formula = $\int e^x dx = e^x + c$ $\frac{d(\tan x)}{dx} = sec^2 x$

Therefore ,

Put tan x = t ⇒ $sec^2 x$ dx = dt

$$\int e^t dt = e^t + c$$

$$= e^{\tan x} + c$$

Question 26.

Evaluate the following integrals:

$$\int e^{\cos^2 x} \sin 2x \, dx$$

Answer:
Formula = $\int e^x dx = e^x + c$ $\frac{d(cos^2 x)}{dx} = 2\cos x\,(-\sin x) = -\sin 2x$

Therefore ,

Put $cos^2 x$ = t ⇒ $-\sin 2x$ dx = dt

$$\int -e^t dt = -e^t + c$$

$$= -e^{cos^2 x} + c$$

Question 27.
Evaluate the following integrals:

$$\int \sin(ax+b)\cos(ax+b)\,dx$$

Answer:
Formula = $\int \sin x \, dx = -\cos x + c$

Therefore ,

Put ax+b = t ⇒ adx = dt

$$\int \sin t \cos t \frac{dt}{a} = \frac{1}{a}\int \sin t \cos t \, dt$$

Put sin t = z � cos t dt = dz

$$\frac{1}{a}\int z dz = \frac{1}{a} \times \frac{z^2}{2} + c$$

$$= \frac{(\sin ax + b)^2}{2a} + c$$

Question 28.
Evaluate the following integrals:

$$\int \cos^3 x \, dx$$

Answer:
Formula = $\int \cos x \, dx = \sin x + c$

$$\cos 3x = 3\cos x - 4cos^3 x$$

Therefore ,

$$\int (\frac{3\cos x}{4} - \frac{\cos 3x}{4}) dx = \frac{3\sin x}{4} - \frac{\sin 3x}{4 \times 3} + c$$

$$= \frac{3\sin x}{4} - \frac{\sin 3x}{12} + c$$

Question 29.
Evaluate the following integrals:

$$\int \frac{1}{x^2} e^{-1/x} dx$$

Answer:
Formula = $\int e^x dx = e^x + c$

Therefore ,

Put $-\frac{1}{x}$ = t ⇒ $\frac{1}{x^2} dx = dt$

$$\int e^t (dt) = \int e^t dt = e^t + c = e^{-\frac{1}{x}} + c$$

$= e^{-\frac{1}{x}} + c$

Question 30.
Evaluate the following integrals:

$$\int \frac{1}{x^2} \cos\left(\frac{1}{x}\right) dx$$

Answer:
Formula = $\int \cos x \, dx = \sin x + c$

Therefore ,

Put $-\frac{1}{x}$ = t ⇒ $\frac{1}{x^2} dx = dt$

$$\int \cos t \, (dt) = \int \cos t \, dt = \sin t + c = \sin(-\frac{1}{x}) + c$$

$$= -\sin\frac{1}{x} + c$$

Question 31.
Evaluate the following integrals:

$$\int \frac{dx}{\left(e^x + e^{-x}\right)}$$

Answer:
Formula = $\int e^x dx = e^x + c$

Therefore ,

$$\int \frac{e^x}{1 + e^{2x}} dx$$

Put e^x = t ⇒ $e^x dx = dt$

$$\int \frac{1}{1+t^2}(dt) = \int \frac{1}{1+t^2}dt = \tan^{-1}t + c$$

$$= \tan^{-1}(e^x) + c$$

Question 32.
Evaluate the following integrals:

$$\int \frac{e^{2x}}{\left(e^{2x}-2\right)} dx$$

Answer:
Formula = $\int e^x dx = e^x + c$

Therefore ,

Put $e^{2x} - 2$ = t ⇒ $2e^{2x}dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{2}\right) = \frac{1}{2}\int \frac{1}{t}dt = \frac{1}{2}\log t + c$$

$$= \frac{1}{2}\log(e^{2x} - 2) + c$$

Question 33.
Evaluate the following integrals:

$$\int \cot x \log(\sin x) dx$$

Answer:
Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sin x) = t ⇒ $\frac{\cos x}{\sin x}dx = dt$ � $\cot x \, dx = dt$

$$\int t\,dt = \frac{t^2}{2} + c$$

$$= \frac{(\log \sin x)^2}{2} + c$$

Question 34.
Evaluate the following integrals:

$$\int \frac{\cot x}{\log(\sin x)} dx$$

Answer:
Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sin x) = t ⇒ $\frac{\cos x}{\sin x} dx = dt$ � $\cot x\, dx = dt$

$$\int \frac{1}{t}\, dt = \log t + c$$

$$= \log(\log \sin x) + c$$

Question 35.
Evaluate the following integrals:

$$\int 2x \sin(x^2 + 1) dx$$

Answer:
Formula = $\int \sin x\, dx = -\cos x + c$

Therefore ,

Put $x^2 + 1 = t$ ⇒ $2x\, dx = dt$

$$\int \sin t \; dt = -\cos t + c$$

$$= -\cos(x^2 + 1) + c$$

Question 36.
Evaluate the following integrals:

$$\int \sec x \log(\sec x + \tan x) dx$$

Answer:
Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put log (sec x + tan x)= t

$$\frac{1}{\sec x + \tan x} \times (\sec x \tan x + sec^2 x) \; dx = dt$$

$$\frac{1}{\sec x + \tan x} \times \sec x \; (\sec x + \tan x) \; dx = dt$$

Sec x dx = dt

$$\int t \, dt = \frac{t^2}{2} + c$$

$$= \frac{(\log(\sec x + \tan x))^2}{2} + c$$

Question 37.
Evaluate the following integrals:

$$\int \frac{\tan \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}} dx$$

Answer:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

$\tan\sqrt{x}$= t

$$sec^2\sqrt{x} \times (\frac{1}{2\sqrt{x}})\, dx = dt$$

$$\int t\, dt = \frac{t^2}{2} + c$$

$$= \frac{(\tan\sqrt{x})^2}{2} + c$$

Question 38.
Evaluate the following integrals:

$$\int \frac{x \tan^{-1} x^2}{\left(1+x^4\right)} dx$$

Answer:
Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\tan^{-1} x^2$ = t ⇒ $\frac{1}{1+(x^2)^2} \times 2x \times dx = dt$ � $\frac{2x}{1+x^4} dx = dt$

$$\int t\left(\frac{dt}{2}\right) = \frac{1}{2}\int tdt = \frac{t^2}{4} + c$$

$$= \frac{(\tan^{-1} x^2)^2}{4} + c$$

Question 39.

Evaluate the following integrals:

$$\int \frac{x \sin^{-1} x^2}{\sqrt{1-x^4}} dx$$

Answer:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\sin^{-1} x^2$ = t ⇒ $\frac{1}{\sqrt{1-(x^2)^2}} \times 2x \times dx = dt$ � $\frac{2x}{\sqrt{1-x^4}} dx = dt$

$$\int t\left(\frac{dt}{2}\right) = \frac{1}{2}\int tdt = \frac{t^2}{4} + c$$

$$= \frac{(\sin^{-1} x^2)^2}{4} + c$$

Question 40.

Evaluate the following integrals:

$$\int \frac{1}{\left(\sqrt{1-x^2}\right)\sin^{-1} x} dx$$

Answer:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $\sin^{-1} x^1$ = t ⇒ $\frac{1}{\sqrt{1-(x^2)^1}} \times dx = dt$ � $\frac{1}{\sqrt{1-x^2}} dx = dt$

$$\int \frac{1}{t}\left(\frac{dt}{1}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log \sin^{-1} x + c$$

Question 41.
Evaluate the following integrals:

$$\int \frac{\sqrt{(2+\log x)}}{x} dx$$

Answer:
Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put 2 + log x = t ⇒ $\frac{1}{x} \times dx = dt$

$$\int \sqrt{t}\left(\frac{dt}{1}\right) = \int \sqrt{t} dt = \frac{2t^{1.5}}{3} + c$$

$$= \frac{2(2+\log x)^{\frac{3}{2}}}{3} + c$$

Question 42.
Evaluate the following integrals:

$$\int \frac{\sec^2 x}{(1+\tan x)} dx$$

Answer:
Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put 1 + tan x = t ⇒ $sec^2 x \times dx = dt$

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$= \log(1 + \tan x) + c$

Question 43.
Evaluate the following integrals:

$$\int \frac{\sin x}{(1 + \cos x)} dx$$

Answer:
Formula = $\int \cos x \, dx = \sin x + c$

Therefore ,

Put 1 + cos x = t ⇒ $-\sin x \times dx = dt$

$$\int \left(\frac{-dt}{t}\right) = -\int \frac{1}{t} dt = -\log t + c$$

$$= -\log(1 + \cos x) + c$$

Question 44.
Evaluate the following integrals:

$$\int \left(\frac{1 + \tan x}{1 - \tan x}\right) dx$$

Answer:
Formula = $\int \cos x \, dx = \sin x + c$

Therefore ,

$$\int \left(\frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}}\right) dx = \int \left(\frac{\cos x + \sin x}{\cos x - \sin x}\right) dx$$

Put cos x - sin x = t ⇒ (- cos x - sin x) dx = dt

$$\int \left(\frac{-dt}{t}\right) = -\int \frac{1}{t} dt = -\log t + c$$

$$= -\log(\cos x - \sin x) + c$$

Question 45.
Evaluate the following integrals:

i. $\int \frac{(1+\tan x)}{(x+\log \sec x)} dx$

ii. $\int \frac{(1-\sin 2x)}{\left(x+\cos^2 x\right)} dx$

Answer:
(i)

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put x + log (sec x) = t ⇒$1 + \frac{1}{\sec x} \times \sec x \tan x \, dx = dt$

$(1 + \tan x)dx = dt$

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(x + \log(\sec x)) + c$$

(ii)

Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put $x + cos^2 x$ = t ⇒$1 + 2\cos x \times (-\sin x)dx = dt$

$(1-\sin 2x)dx = dt$

$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$

$= \log(x + cos^2 x) + c$

Question 46.
Evaluate the following integrals:

$$\int \frac{\sin\ 2x}{\left(a^2 + b^2 \sin^2 x\right)} dx$$

Answer:
Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put $a^2 + b^2 sin^2 x = t$ � $b^2 \times 2 \sin x \times \cos x\, dx = dt$

$(b^2 \sin 2x)dx = dt$

$\int \frac{1}{t}\left(\frac{dt}{b^2}\right) = \frac{1}{b^2} \int \frac{1}{t} dt = \frac{1}{b^2} \log t + c$

$= \frac{1}{b^2} \log|a^2 + b^2 sin^2 x| + c$

Question 47.
Evaluate the following integrals:

$$\int \frac{\sin\ 2x}{\left(a^2 \cos^2 x + b^2 \sin^2 x\right)} dx$$

Answer:
Formula = $\int \frac{1}{x} dx = \log x + c$

Therefore ,

Put $a^2cos^2x + b^2sin^2x = t$

$$(a^2 \times 2\cos x \times (-\sin x) + b^2 \times 2\sin x \times \cos x)dx = dt$$

$$(b^2 - a^2)\sin 2x \;\; dx = dt$$

$$\int \frac{1}{t}\left(\frac{dt}{b^2 - a^2}\right) = \frac{1}{b^2 - a^2}\int \frac{1}{t}dt = \frac{1}{b^2 - a^2}\log t + c$$

$$= \frac{1}{b^2 - a^2}\log|a^2cos^2x + b^2sin^2x| + c$$

Question 48.
Evaluate the following integrals:

$$\int\left(\frac{2\cos x - 3\sin x}{3\cos x + 2\sin x}\right)dx$$

Answer:
Formula = $\int \cos x \, dx = \sin x + c$

Therefore ,

Put 3cos x + 2sin x = t ⇒ (2cos x - 3sin x) dx = dt

$$\int\left(\frac{dt}{t}\right) = \int \frac{1}{t}dt = \log t + c$$

$$= \log(3\cos x + 2\sin x) + c$$

Question 49.
Evaluate the following integrals:

$$\int \frac{4x}{\left(2x^2 + 3\right)}dx$$

Answer:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $2x^2 + 3 = t \Rightarrow (4x)\,dx = dt$

$$\int \left(\frac{dt}{t}\right) = \int \frac{1}{t} dt = \log t + c$$

$$= \log(2x^2 + 3) + c$$

Question 50.

Evaluate the following integrals:

$$\int \frac{(x+1)}{(x^2 + 2x - 3)} dx$$

Answer:

Formula = $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Therefore ,

Put $x^2+2x+3= t \Rightarrow (2x+2)\,dx = dt$ � $2(x+1)dx=dt$

$$\int \frac{1}{t}\left(\frac{dt}{2}\right) = \frac{1}{2}\int \frac{1}{t} dt = \frac{1}{2}\log t + c$$

$$= \frac{1}{2}\log(x^2 + 2x + 3) + c$$

Question 51.

Evaluate the following integrals:

$$\int \frac{(4x-5)}{(2x^2-5x+1)}dx$$

Answer:

To find: Value of $\int \frac{4x-5}{(2x^2-5x+1)}dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{4x-5}{(2x^2-5x+1)}dx$ ... (i)

Let $2x^2 - 5x + 1 = t$

$$\Rightarrow \frac{d(2x^2-5x+1)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow 4x - 5 = \frac{dt}{dx}$$

$\Rightarrow (4x - 5)dx = dt$

Putting this value in equation (i)

$$I = \int \frac{dt}{t} \;[\, 2x^2 - 5x + 1 = t \,]$$

$$I = \log|t| + c$$

$$I = \log|2x^2 - 5x + 1| + c$$

Ans) $\log|2x^2 - 5x + 1| + c$

Question 52.

Evaluate the following integrals:

$$\int \frac{\left(9x^2 - 4x + 5\right)}{\left(3x^3 - 2x^2 + 5x + 1\right)} dx$$

Answer:

To find: Value of $\int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{(9x^2 - 4x + 5)}{(3x^3 - 2x^2 + 5x + 1)} dx$ ... (i)

Let $3x^3 - 2x^2 + 5x + 1 = t$

$$\Rightarrow \frac{d(3x^3 - 2x^2 + 5x + 1)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow 9x^2 - 4x + 5 = \frac{dt}{dx}$$

$\Rightarrow (9x^2 - 4x + 5)dx = dt$

Putting this value in equation (i)

$$I = \int \frac{dt}{t} \; [\, 3x^3 - 2x^2 + 5x + 1 = t \,]$$

$$I = \log|t| + c$$

$$I = \log|3x^3 - 2x^2 + 5x + 1| + c$$

Ans) $\log|3x^3 - 2x^2 + 5x + 1| + c$

Question 53.

Evaluate the following integrals:

$$\int \frac{\sec x \ \operatorname{cosec} x}{\log(\tan x)} dx$$

Answer:

To find: Value of $\int \frac{\sec x \ \operatorname{cosec} x}{\log(\tan x)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\sec x \ \operatorname{cosec} x}{\log(\tan x)} dx$ ... (i)

Let $\log(\tan x)$ = t

$$\Rightarrow \frac{d(\log(\tan x))}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{d(\log(\tan x))}{d\tan x} \frac{d\tan x}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{\tan x} \sec^2 x = \frac{dt}{dx}$$

$$\Rightarrow \sec x \ \operatorname{cosec} x = \frac{dt}{dx}$$

⇒ $(\sec x \ \operatorname{cosec} x)$dx = dt

Putting this value in equation (i)

$$I = \int \frac{dt}{t} \ [\log(\tan x) = t]$$

$$I = \log|t| + c$$

$$I = \log|\log(\tan x)| + c$$

Ans) $\log|\log(\tan x)| + c$

Question 54.
Evaluate the following integrals:

$$\int \frac{(1+\cos x)}{(x+\sin x)^3} dx$$

Answer:
To find: Value of $\int \frac{(1+\cos x)}{(x+\sin x)^3} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{(1+\cos x)}{(x+\sin x)^3} dx$ ... (i)

Let $x + \sin x = t$

$$\Rightarrow \frac{d(x+\sin x)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{d(x)}{dx} + \frac{d(\sin x)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (1+\cos x) = \frac{dt}{dx}$$

$\Rightarrow (1+\cos x)dx = dt$

Putting this value in equation (i)

$$I = \int \frac{dt}{t^3} \; [\, x + \sin x = t \,]$$

$$\Rightarrow I = -\frac{1}{2t^2} + c$$

$$I = -\frac{1}{2(x + \sin x)^2} + c$$

Ans) $-\frac{1}{2(x + \sin x)^2} + c$

Question 55.
Evaluate the following integrals:

$$\int \frac{\sin x}{(1+\cos x)^2} dx$$

Answer:
To find: Value of $\int \frac{\sin x}{(1 + \cos x)^2} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1 + \cos x)^2} dx$ ... (i)

Let $1 + \cos x = t$

$$\Rightarrow \frac{d(1 + \cos x)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{d(1)}{dx} + \frac{d(\cos x)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (0 - \sin x) = \frac{dt}{dx}$$

$\Rightarrow$ (- $\sin x$)dx = dt

Putting this value in equation (i)

$$I = \int -\frac{dt}{t^2} \; [\, 1 + \cos x = t \,]$$

$$\Rightarrow I = \frac{1}{t} + c$$

$$I = \frac{1}{1 + \cos x} + c$$

Ans) $\frac{1}{1 + \cos x} + c$

Question 56.
Evaluate the following integrals:

$$\int \frac{(2x+3)}{\sqrt{x^2 + 3x - 2}} dx$$

Answer:
To find: Value of $\int \frac{(2x+3)}{\sqrt{x^2 + 3x - 2}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1 + \cos x)^2} dx$ ... (i)

Let $x^2 + 3x - 2 = t$

$$\Rightarrow (2x + 3) = \frac{dt}{dx}$$

$\Rightarrow$ $(2x + 3)$ dx = dt

Putting this value in equation (i)

$$I = \int \frac{dt}{\sqrt{t}} \; [\, x^2 + 3x - 2 = t \,]$$

$$\Rightarrow I = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$I = 2t^{\frac{1}{2}} + c$$

$$I = 2\sqrt{x^2 + 3x - 2} + c$$

Ans) $2\sqrt{x^2 + 3x - 2} + c$

Question 57.
Evaluate the following integrals:

$$\int \frac{(2x-1)}{\sqrt{x^2 - x - 1}} dx$$

Answer:
To find: Value of $\int \frac{(2x - 1)}{\sqrt{x^2 - x - 1}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin x}{(1 + \cos x)^2} dx$ ... (i)

Let $x^2 - x - 1 = t$

$$\Rightarrow \frac{d(x^2 - x - 1)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{d(x^2)}{dx} - \frac{d(x)}{dx} - \frac{d(1)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (2x - 1) = \frac{dt}{dx}$$

$\Rightarrow (2x - 1)\, dx = dt$

Putting this value in equation (i)

$$I = \int \frac{dt}{t^{\frac{1}{2}}} \; [\, x^2 - x - 1 = t \,]$$

$$\Rightarrow I = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\Rightarrow I = \frac{2\sqrt{t}}{1} + c$$

$$I = \frac{2\sqrt{x^2 - x - 1}}{1} + c$$

Ans) $2\sqrt{x^2 - x - 1} + c$

Question 58.
Evaluate the following integrals:

$$\int \frac{dx}{\left(\sqrt{x+a} + \sqrt{x+b}\right)}$$

Answer:
To find: Value of $\int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}}$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}}$ ... (i)

$$I = \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \times \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}}$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{\left(\sqrt{x+a}\right)^2 - \left(\sqrt{x+b}\right)^2} dx$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{(x+a) - (x+b)} dx$$

$$I = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{x+a-x-b} dx$$

$$I = \frac{1}{a-b}\left[\int \sqrt{x+a}\, dx - \int \sqrt{x+b}\, dx\right]$$

$$I = \frac{1}{a-b}\left[\int (x+a)^{\frac{1}{2}}\, dx - \int (x+b)^{\frac{1}{2}} dx\right]$$

$$I = \frac{1}{a-b}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$I = \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$$

Ans) $\frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$

Question 59.
Evaluate the following integrals:

$$\int \frac{dx}{\left(\sqrt{1-3x} - \sqrt{5-3x}\right)}$$

Answer:
To find: Value of $\int \frac{dx}{\sqrt{1-3x} - \sqrt{5-3x}}$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{dx}{\sqrt{1-3x} - \sqrt{5-3x}}$ ... (i)

$$I = \int \frac{dx}{\sqrt{1-3x} - \sqrt{5-3x}} \times \frac{\sqrt{1-3x} + \sqrt{5-3x}}{\sqrt{1-3x} + \sqrt{5-3x}}$$

$$I = \int \frac{\sqrt{1-3x} + \sqrt{5-3x}}{\left(\sqrt{1-3x}\right)^2 - \left(\sqrt{5-3x}\right)^2} dx$$

$$I = \int \frac{\sqrt{1-3x} + \sqrt{5-3x}}{(1-3x) - (5-3x)} dx$$

$$I = \int \frac{\sqrt{1-3x} + \sqrt{5-3x}}{1-3x-5+3x} dx$$

$$I = -\frac{1}{4}\left[\int \sqrt{1-3x}\, dx + \int \sqrt{5-3x}\, dx\right]$$

$$I = -\frac{1}{4}\left[\int (1-3x)^{\frac{1}{2}} dx + \int (5-3x)^{\frac{1}{2}} dx\right]$$

$$I = -\frac{1}{4}\left[\frac{(1-3x)^{\frac{3}{2}}}{\frac{3}{2}(-3)} + \frac{(5-3x)^{\frac{3}{2}}}{\frac{3}{2}(-3)}\right]$$

$$I = -\frac{2}{-9 \times 4}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$$

$$I = \frac{1}{18}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$$

Ans) $\frac{1}{18}\left[(1-3x)^{\frac{3}{2}} + (5-3x)^{\frac{3}{2}}\right] + c$

Question 60.

Evaluate the following integrals:

$$\int \frac{x^2}{\left(1+x^6\right)} dx$$

Answer:

To find: Value of $\int \frac{x^2}{(1+x^6)} dx$

Formula used: $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

We have, $I = \int \frac{x^2}{(1+x^6)} dx$ ... (i)

$$I = \int \frac{x^2}{1+(x^3)^2} dx$$

Let $x^3 = t$

$$\Rightarrow \frac{d(x^3)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (3x^2) = \frac{dt}{dx}$$

$$\Rightarrow (x^2)dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \frac{1}{3} \int \frac{dt}{1+t^2} \text{ [} 1 + \cos x = t \text{]}$$

$$\Rightarrow I = \frac{1}{3} \tan^{-1}(t) + c$$

$$I = \frac{1}{3} \tan^{-1}(x^3) + c$$

Ans) $\frac{1}{3}\tan^{-1}(x^3)+c$

Question 61.
Evaluate the following integrals:

$$\int \frac{x^3}{(1+x^8)}dx$$

Answer:

To find: Value of $\int \frac{x^3}{(1+x^8)}dx$

Formula used: $\int \frac{1}{1+x^2}dx = \tan^{-1}x$

We have, $I = \int \frac{x^3}{(1+x^8)}dx$ ... (i)

$$I = \int \frac{x^3}{1+(x^4)^2}dx$$

Let $x^4 = t$

$$\Rightarrow \frac{d(x^4)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (4x^3) = \frac{dt}{dx}$$

$$\Rightarrow (x^3)dx = \frac{dt}{4}$$

Putting this value in equation (i)

$$I = \frac{1}{4}\int \frac{dt}{1+t^2} \; [\, 1+\cos x = t \,]$$

$$\Rightarrow I = \frac{1}{4}\tan^{-1}(t) + c$$

$$I = \frac{1}{4}\tan^{-1}(x^4) + c$$

Ans) $\frac{1}{4}\tan^{-1}(x^4) + c$

Question 62.
Evaluate the following integrals:

$$\int \frac{x}{\left(1+x^4\right)} dx$$

Answer:
To find: Value of $\int \frac{x}{(1+x^4)} dx$

Formula used: $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

We have, $I = \int \frac{x}{(1+x^4)} dx$ ... (i)

$$I = \int \frac{x}{1 + (x^2)^2} dx$$

Let $x^2 = t$

$$\Rightarrow \frac{d(x^2)}{dx} = \frac{dt}{dx}$$

$$\Rightarrow (2x) = \frac{dt}{dx}$$

$$\Rightarrow (x)dx = \frac{dt}{2}$$

Putting this value in equation (i)

$$I = \frac{1}{2}\int \frac{dt}{1+t^2}\ [\,1 + \cos x = t\,]$$

$$\Rightarrow I = \frac{1}{2}\tan^{-1}(t) + c$$

$$I = \frac{1}{2}\tan^{-1}(x^2) + c$$

Ans) $\frac{1}{2}\tan^{-1}(x^2) + c$

Question 63.
Evaluate the following integrals:

$$\int \frac{x^5}{\sqrt{1+x^3}}\,dx$$

Answer:
To find: Value of $\int \frac{x^5}{\sqrt{1+x^3}}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int \frac{x^5}{\sqrt{1+x^3}}\,dx$ ... (i)

Let $1 + x^3 = t$

$\Rightarrow x^3 = t - 1$

$$\Rightarrow \frac{d(x^3)}{dx} = \frac{d(t-1)}{dx}$$

$$\Rightarrow \left(3x^2\right) = \frac{dt}{dx}$$

$$\Rightarrow x^2\,dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \int \frac{x^3 x^2}{\sqrt{1 + x^3}}\,dx$$

$$I = \int \frac{(t - 1)}{t^{\frac{1}{2}}}\frac{dt}{3}\ [\,1 + x^3 = t\,]$$

$$\Rightarrow I = \frac{1}{3}\int \frac{t}{t^{\frac{1}{2}}}dt - \frac{1}{3}\int \frac{1}{t^{\frac{1}{2}}}dt$$

$$\Rightarrow I = \frac{1}{3}\left[\int t^{\frac{1}{2}}dt - \int t^{-\frac{1}{2}}dt\right]$$

$$\Rightarrow I = \frac{1}{3}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right]$$

$$\Rightarrow I = \frac{2}{3}\left[\frac{(1 + x^3)^{\frac{3}{2}}}{3} - \frac{(1 + x^3)^{\frac{1}{2}}}{1}\right]$$

$$\Rightarrow I = \frac{2(1 + x^3)^{\frac{3}{2}}}{9} - \frac{2(1 + x^3)^{\frac{1}{2}}}{3} + c$$

Ans) $\frac{2(1 + x^3)^{\frac{3}{2}}}{9} - \frac{2(1 + x^3)^{\frac{1}{2}}}{3} + c$

Question 64.

Evaluate the following integrals:

$$\int \frac{x}{\sqrt{1+x}} dx$$

Answer:

To find: Value of $\int \frac{x}{\sqrt{1+x}} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{x}{\sqrt{1+x}} dx$ ... (i)

Let $1 + x = t$

$\Rightarrow x = t - 1$

$\Rightarrow dx = dt$

Putting this value in equation (i)

$$I = \int \frac{t-1}{\sqrt{t}} dx \; [\, 1 + x = t \,]$$

$$\Rightarrow I = \int \sqrt{t} dt - \int \frac{1}{\sqrt{t}} dt$$

$$\Rightarrow I = \left[\int t^{\frac{1}{2}} dt - \int t^{-\frac{1}{2}} dt \right]$$

$$\Rightarrow I = \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}} \right] + c$$

$$\Rightarrow I = 2 \left[\frac{(1+x)^{\frac{3}{2}}}{3} - \frac{(1+x)^{\frac{1}{2}}}{1} \right] + c$$

$$\Rightarrow I = \frac{2(1+x)^{\frac{3}{2}}}{3} - 2(1+x)^{\frac{1}{2}} + c$$

Ans) $\frac{2(1+x)^{\frac{3}{2}}}{3} - 2(1+x)^{\frac{1}{2}} + c$

Question 65.
Evaluate the following integrals:

$$\int \frac{1}{x\sqrt{x^4 - 1}} dx$$

Answer:
To find: Value of $\int \frac{1}{x\sqrt{x^4-1}} dx$

Formula used: $\int \frac{1}{x\sqrt{x^2 - 1}} dx = \sec^{-1} x + c$

We have, $I = \int \frac{1}{x\sqrt{x^4-1}} dx$ ... (i)

Multiplying numerator and denominator with x

$$I = \int \frac{x}{x^2\sqrt{(x^2)^2 - 1}} dx$$

Let $x^2 = t$

$$\Rightarrow 2x = \frac{dt}{dx}$$

$$\Rightarrow xdx = \frac{dt}{2}$$

Putting this value in equation (i)

$$I = \frac{1}{2}\int \frac{dt}{t\sqrt{t^2 - 1}} \quad [x^2 = t]$$

$\Rightarrow I = \frac{1}{2}\sec^{-1}t + c$

$\Rightarrow I = \frac{1}{2}\sec^{-1}(x^2) + c$

Ans) $\frac{1}{2}\sec^{-1}(x^2) + c$

Question 66.
Evaluate the following integrals:

$\int x\sqrt{-1}dx$

Answer:
To find: Value of $\int x\sqrt{x-1}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int x\sqrt{x-1}\,dx$ ... (i)

Let $x - 1 = t$

x = t + 1

$\Rightarrow dx = dt$

Putting this value in equation (i)

$I = \int (t+1)\sqrt{t}\,dt\ [\ x = t + 1\ ]$

$\Rightarrow I = \int t\sqrt{t}dx + \int \sqrt{t}\,dx$

$\Rightarrow I = \int t^{\frac{3}{2}}dx + \int t^{\frac{1}{2}}\,dx$

$$\Rightarrow I = \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{2}{5}(x - 1)^{\frac{5}{2}} + \frac{2}{3}(x - 1)^{\frac{3}{2}} + c$$

Ans) $\frac{2}{5}(x - 1)^{\frac{5}{2}} + \frac{2}{3}(x - 1)^{\frac{3}{2}} + c$

Question 67.
Evaluate the following integrals:

$$\int (1-x)\sqrt{1+x}\ dx$$

Answer:
To find: Value of $\int (1 - x)\sqrt{1 + x}\, dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int (1 - x)\sqrt{1 + x}\, dx$ ... (i)

Let $1 + x = t$

x = t - 1

$\Rightarrow dx = dt$

Putting this value in equation (i)

$$I = \int \{1 - (t - 1)\}\sqrt{t}\, dt\ [\ x = t - 1\ ]$$

$$\Rightarrow I = \int \{1 - t + 1\}\sqrt{t}\, dt$$

$$\Rightarrow I = \int \{2 - t\}\sqrt{t}\,dt$$

$$\Rightarrow I = \int 2\sqrt{t}\,dt - \int t\sqrt{t}\,dt$$

$$\Rightarrow I = 2\int t^{\frac{1}{2}}dx - \int t^{\frac{3}{2}}\,dx$$

$$\Rightarrow I = 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$\Rightarrow I = \frac{4}{3}(1 + x)^{\frac{3}{2}} - \frac{2}{5}(1 + x)^{\frac{5}{2}} + c$$

Ans) $\frac{4}{3}(1 + x)^{\frac{3}{2}} - \frac{2}{5}(1 + x)^{\frac{5}{2}} + c$

Question 68.
Evaluate the following integrals:

$$\int x\sqrt{x^2 - 1}\,dx$$

Answer:
To find: Value of $\int x\sqrt{x^2 - 1}\,dx$

Formula used: $\int x^n dx = \frac{1}{n + 1}x^{n + 1} + c$

We have, $I = \int x\sqrt{x^2 - 1}\,dx$ ... (i)

Let $x^2 - 1 = t$

$$\Rightarrow 2x = \frac{dt}{dx}$$

$$\Rightarrow xdx = \frac{dt}{2}$$

Putting this value in equation (i)

$$I = \int \frac{1}{2}\sqrt{t}\,dt\ [\ x = x^2 - 1\ ]$$

$$\Rightarrow I = \frac{1}{2}\int t^{\frac{1}{2}}\,dx$$

$$\Rightarrow I = \frac{1}{2}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{1}{3}\,t^{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{1}{3}\,(x^2 - 1)^{\frac{3}{2}} + c$$

Ans) $\frac{1}{3}\,(x^2 - 1)^{\frac{3}{2}} + c$

Question 69.
Evaluate the following integrals:

$$\int x\sqrt{3x-2}\,dx$$

Answer:
To find: Value of $\int x\sqrt{3x - 2}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int x\sqrt{3x - 2}\,dx$ ... (i)

Let $3x - 2 = t$

$$\Rightarrow 3x = t + 2$$

$$\Rightarrow x = \frac{t+2}{3}$$

$$\Rightarrow 3 = \frac{dt}{dx}$$

$$\Rightarrow dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \int \left(\frac{t+2}{3}\right)\sqrt{t}\,\frac{dt}{3}\ [\,t = 3x - 2\,]$$

$$\Rightarrow I = \frac{1}{9}\left[\int t^{\frac{3}{2}}\,dx + 2\int t^{\frac{1}{2}}\,dx\right]$$

$$\Rightarrow I = \frac{1}{9}\left[\frac{t^{\frac{5}{2}}}{\frac{5}{2}} + 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right] + c$$

$$\Rightarrow I = \frac{1}{9}\left[\frac{2}{5}(3x-2)^{\frac{5}{2}} + \frac{4}{3}(3x-2)^{\frac{3}{2}}\right] + c$$

$$\Rightarrow I = \frac{2}{45}(3x-2)^{\frac{5}{2}} + \frac{4}{27}(3x-2)^{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{2}{45}(3x-2)^{\frac{5}{2}} + \frac{4}{27}(3x-2)^{\frac{3}{2}} + c$$

Ans) $\frac{2}{45}(3x-2)^{\frac{5}{2}} + \frac{4}{27}(3x-2)^{\frac{3}{2}} + c$

Question 70.

Evaluate the following integrals:

$$\int \frac{dx}{x\cos^2(1+\log x)}$$

Answer:

To find: Value of $\int \frac{dx}{x\cos^2(1+\log x)}$

Formula used: $\int \sec^2 x\ dx = \tan x + c$

We have, $I = \int \frac{dx}{x\cos^2(1+\log x)}$ ... (i)

Let $1 + \log x = t$

$$\Rightarrow \frac{1}{x} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{x}dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{\cos^2(t)}\ [\,t = 1 + \log x\,]$$

$$\Rightarrow I = \int \sec^2 t\, dt$$

$$\Rightarrow I = \tan(t) + c$$

$$\Rightarrow I = \tan(1 + \log x) + c$$

Ans) $\tan(1 + \log x) + c$

Question 71.

Evaluate the following integrals:

$$\int x^2 \sin x^3 dx$$

Answer:
To find: Value of $\int x^2 \sin x^3 \, dx$

Formula used: $\int \sin x \, dx = -\cos x + c$

We have, $I = \int x^2 \sin x^3 \, dx$ ... (i)

Let $x^3 = t$

$$\Rightarrow 3x^2 = \frac{dt}{dx}$$

$$\Rightarrow x^2 dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \int \sin t \frac{dt}{3} \; [t = x^3]$$

$$\Rightarrow I = \frac{1}{3}\left[\int \sin t \, dt\right]$$

$$\Rightarrow I = \frac{1}{3}(-\cos t) + c$$

$$\Rightarrow I = \frac{1}{3}(-\cos x^3) + c$$

Ans) $\dfrac{-\cos x^3}{3} + c$

Question 72.

Evaluate the following integrals:

$$\int (2x+4)\sqrt{x^2+4x+3}\,dx$$

Answer:

To find: Value of $\int (2x+4)\sqrt{x^2+4x+3}\,dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int (2x+4)\sqrt{x^2+4x+3}\,dx$ ... (i)

Let $x^2 + 4x + 3 = t$

$$\Rightarrow (2x+4) = \frac{dt}{dx}$$

$$\Rightarrow (2x+4)dx = dt$$

Putting this value in equation (i)

$$I = \int \sqrt{t}\,dt\ [\ t = (2x+4)\ ]$$

$$\Rightarrow I = \int t^{\frac{1}{2}}\,dx$$

$$\Rightarrow I = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\Rightarrow I = \frac{2}{3}\left[(t)^{\frac{3}{2}}\right] + c$$

$$\Rightarrow I = \frac{2}{3}\left[(x^2+4x+3)^{\frac{3}{2}}\right] + c$$

Ans) $\frac{2}{3}\left[(x^2+4x+3)^{\frac{3}{2}}\right]+c$

Question 73.
Evaluate the following integrals:

$$\int \frac{\sin x}{(\sin x - \cos x)} dx$$

Answer:
To find: Value of $\int \frac{\text{sinx}}{(\text{sinx - cosx})}\, dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\text{sinx}}{(\text{sinx - cosx})}\, dx$ ... (i)

$$\Rightarrow I = \frac{1}{2}\int \frac{2\text{sinx}}{(\text{sinx - cosx})}\, dx$$

$$\Rightarrow I = \frac{1}{2}\int \frac{(\text{sinx} + \text{cosx}) + (\text{sinx - cosx})}{(\text{sinx - cosx})}\, dx$$

$$\Rightarrow I = \frac{1}{2}\int \frac{(\text{sinx} + \text{cosx})}{(\text{sinx - cosx})}\, dx + \frac{1}{2}\int \frac{(\text{sinx - cosx})}{(\text{sinx - cosx})}\, dx$$

Let $\text{sinx - cosx} = t$

$$\Rightarrow (\text{cosx} + \text{sinx}) = \frac{dt}{dx}$$

$$\Rightarrow (\text{cosx} + \text{sinx})dx = dt$$

Putting this value in equation (i)

$$I = \frac{1}{2}\int \frac{dt}{t} + \frac{1}{2}\int dx$$

$$\Rightarrow I = \frac{1}{2}\log|\sin x - \cos x| + \frac{1}{2}x + c$$

$$\Rightarrow I = \frac{x}{2} + \frac{1}{2}\log|\sin x - \cos x| + c$$

Ans) $\frac{x}{2} + \frac{1}{2}\log|\sin x - \cos x| + c$

Question 74.
Evaluate the following integrals:

$$\int \frac{dx}{(1 - \tan x)}$$

Answer:

To find: Value of $\int \frac{dx}{(1 - \tan x)}$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{dx}{(1 - \tan x)}$ ... (i)

$$\Rightarrow I = \int \frac{dx}{\left(1 - \frac{\sin x}{\cos x}\right)}$$

$$\Rightarrow I = \int \frac{dx}{\left(\frac{\cos x - \sin x}{\cos x}\right)}$$

$$\Rightarrow I = \frac{1}{2}\int \frac{2\cos x dx}{(\cos x - \sin x)}$$

$$I = \frac{1}{2}\int \frac{(\cos x + \sin x) + (\cos x - \sin x)dx}{(\cos x - \sin x)}$$

$$I = \frac{1}{2}\int \frac{(\cos x + \sin x)}{(\cos x - \sin x)} dx + \frac{1}{2}\int \frac{(\cos x - \sin x)}{(\cos x - \sin x)} dx$$

Let $(\cos x - \sin x) = t$

$$\Rightarrow (-\sin x - \cos x) = \frac{dt}{dx}$$

$$\Rightarrow (\sin x + \cos x) dx = -dt$$

Putting this value in equation (i)

$$I = -\frac{1}{2}\int \frac{dt}{(t)} dx + \frac{1}{2}\int dx$$

$$\Rightarrow I = -\frac{1}{2}\log|\cos x - \sin x| + \frac{1}{2}x + c$$

$$\Rightarrow I = \frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x| + c$$

Ans) $\frac{1}{2}x - \frac{1}{2}\log|\sin x - \cos x| + c$

Question 75.
Evaluate the following integrals:

$$\int \frac{dx}{(1 - \cot x)}$$

Answer:
To find: Value of $\int \frac{dx}{(1 - \cot x)}$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{dx}{(1 - \cot x)}$ ... (i)

$$\Rightarrow I = \int \frac{dx}{\left(1 - \frac{\cos x}{\sin x}\right)}$$

$$\Rightarrow I = \int \frac{dx}{\left(\frac{\sin x - \cos x}{\sin x}\right)}$$

$$\Rightarrow I = \frac{1}{2}\int \frac{2\sin x\,dx}{(\sin x - \cos x)}$$

$$I = \frac{1}{2}\int \frac{(\sin x + \cos x) + (\sin x - \cos x)dx}{(\sin x - \cos x)}$$

$$I = \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\sin x - \cos x)}dx + \frac{1}{2}\int \frac{(\sin x - \cos x)}{(\sin x - \cos x)}dx$$

Let $(\sin x - \cos x) = t$

$$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x + \sin x)dx = dt$$

Putting this value in equation (i)

$$I = \frac{1}{2}\int \frac{dt}{(t)}\,dx + \frac{1}{2}\int dx$$

$$\Rightarrow I = \frac{1}{2}\log|\sin x - \cos x| + \frac{1}{2}x + c$$

Ans) $\frac{1}{2}x + \frac{1}{2}\log|\sin x - \cos x| + c$

Question 76.

Evaluate the following integrals:

$$\int \frac{\cos 2x}{(\sin x + \cos x)} dx$$

Answer:

To find: Value of $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ ... (i)

$$\Rightarrow I = \int \frac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2} dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)^2} dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)} dx$$

Let $(\cos x + \sin x) = t$

$$\Rightarrow (-\sin x + \cos x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x - \sin x) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t}$$

$$\Rightarrow I = \log|t| + c$$

$$\Rightarrow I = \log|\cos x + \sin x| + c$$

Ans) $\log|\cos x + \sin x| + c$

Question 77.
Evaluate the following integrals:

$$\int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx$$

Answer:
To find: Value of $\int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{(\cos x - \sin x)}{(1 + \sin 2x)} dx$ ... (i)

$$\Rightarrow I = \int \frac{\cos x - \sin x}{\cos^2 x + \sin^2 x + 2\sin x \cos x} dx$$

$$\Rightarrow I = \int \frac{(\cos x - \sin x)}{(\cos x + \sin x)^2} dx$$

Let $(\sin x + \cos x) = t$

$$\Rightarrow (\cos x - \sin x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x - \sin x) dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t^2}$$

$$\Rightarrow I = -\frac{1}{t} + c$$

$$\Rightarrow I = -\frac{1}{\sin x + \cos x} + c$$

Ans) $\frac{-1}{\sin x + \cos x} + c$

Question 78.
Evaluate the following integrals:

$$\int \frac{(x+1)(x+\log x)^2}{x} dx$$

Answer:
To find: Value of $\int \frac{(x+1)(x+\log x)^2}{x} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{(x+1)(x+\log x)^2}{x} dx$ ... (i)

Let $(x + \log x) = t$

$$\Rightarrow \left(1 + \frac{1}{x}\right) = \frac{dt}{dx}$$

$$\Rightarrow \left(\frac{x+1}{x}\right) = \frac{dt}{dx}$$

Putting this value in equation (i)

$$I = \int t^2 dt$$

$$\Rightarrow I = \frac{t^3}{3} + c$$

$$\Rightarrow I = \frac{(x + \log x)^3}{3} + c$$

Ans) $\frac{(x + \log x)^3}{3} + c$

Question 79.
Evaluate the following integrals:

$$\int x \sin^3 x^2 \cos x^2 dx$$

Answer:
To find: Value of $\int x\sin^3 x^2 \cos x^2 dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int x\sin^3 x^2 \cos x^2 dx$ ... (i)

Let $(\sin x^2) = t$

$$\Rightarrow (\sin x^2 . 2x) = \frac{dt}{dx}$$

$$\Rightarrow (\sin x^2 . x)dx = \frac{dt}{2}$$

Putting this value in equation (i)

$$I = \int t^3 \frac{dt}{2}$$

$$I = \frac{1}{2} \int t^3 dt$$

$$\Rightarrow I = \frac{1}{2}\frac{t^4}{4} + c$$

$$\Rightarrow I = \frac{t^4}{8} + c$$

$$\Rightarrow I = \frac{\sin^4 x^2}{8} + c$$

Ans) $\frac{\sin^4 x^2}{8} + c$

Question 80.
Evaluate the following integrals:

$$\int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} dx$$

Answer:

To find: Value of $\int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} dx$

Formula used: $\int \frac{1}{\sqrt{1 - x^2}} dx = \sin^{-1} x + c$

We have, $I = \int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} dx$ ... (i)

Let $(\tan x) = t$

$$\Rightarrow (\sec^2 x) = \frac{dt}{dx}$$

$$\Rightarrow (\sec^2 x)dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{\sqrt{1 - t^2}}$$

$\Rightarrow I = \sin^{-1}(t) + c$

$\Rightarrow I = \sin^{-1}(\tan x) + c$

Ans) $\sin^{-1}(\tan x) + c$

Question 81.
Evaluate the following integrals:

$$\int e^{-x} \operatorname{cosec}^2\left(2e^{-x} + 5\right) dx$$

Answer:
To find: Value of $\int e^{-x} \operatorname{cosec}^2(2e^{-x} + 5)\, dx$

Formula used: $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

We have, $I = \int e^{-x} \operatorname{cosec}^2(2e^{-x} + 5)\, dx$ ... (i)

Let $(2e^{-x} + 5) = t$

$\Rightarrow (2e^{-x}(-1)) = \frac{dt}{dx}$

$\Rightarrow (e^{-x})dx = \frac{dt}{-2}$

Putting this value in equation (i)

$I = \int \operatorname{cosec}^2(t)\, \frac{dt}{-2}$

$I = \frac{1}{-2} \int \operatorname{cosec}^2(t)\, dt$

$\Rightarrow I = \frac{1}{-2}(-\cot t) + c$

$$\Rightarrow I = \frac{1}{2}\cot(2e^{-x} + 5) + c$$

Ans) $\frac{1}{2}\cot(2e^{-x} + 5) + c$

Question 82.

Evaluate the following integrals:

$$\int 2x\sec^3\left(x^2+3\right)\tan\left(x^2+3\right)dx$$

Answer:

To find: Value of $\int 2x\sec^3(x^2+3)\ \tan(x^2+3)dx$

Formula used: $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int 2x\sec^2(x^2+3)\sec(x^2+3)\ \tan(x^2+3)dx$ ... (i)

Let $\sec\left(x^2+3\right) = t$

$$\Rightarrow \sec(x^2+3) = \frac{dt}{dx}$$

$$\Rightarrow \sec(x^2+3)\tan(x^2+3).2x = \frac{dt}{dx}$$

$$\Rightarrow \sec(x^2+3)\tan(x^2+3).2x = \frac{dt}{dx}$$

Putting this value in equation (i)

$$I = \int t^2\,dt$$

$$\Rightarrow I = \frac{t^3}{3} + c$$

$$\Rightarrow I = \frac{\sec^3(x^2+3)}{3} + c$$

Ans) $\dfrac{\sec^3(x^2+3)}{3} + c$

Question 83.
Evaluate the following integrals:

$$\int \frac{\sin 2x}{(a + b\cos x)^2} dx$$

Answer:
To find: Value of $\int \frac{\sin 2x}{(a+b\cos x)^2} dx$

Formula used: (i) $\int \frac{1}{x} dx = \log|x| + c$

(ii) $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{\sin 2x}{(a+b\cos x)^2} dx$ ... (i)

$$I = \int \frac{2\sin x\cos x}{(a+b\cos x)^2} dx$$

Let $(a + b\cos x) = t$

$$\Rightarrow (\cos x) = \frac{t - a}{b}$$

$$\Rightarrow (\sin x)dx = \frac{dt}{-b}$$

Putting this value in equation (i)

$$I = \frac{2}{-b^2} \int \frac{t - a}{t^2} dt$$

$$I = \frac{2}{-b^2}\left[\int \frac{t}{t^2}\,dt - \int \frac{a}{t^2}\,dt\right]$$

$$I = \frac{2}{-b^2}\left[\int \frac{1}{t}\,dt - a\int \frac{1}{t^2}\,dt\right]$$

$$I = \frac{2}{-b^2}\left[\log|t| - a\left(-\frac{1}{t}\right) + c\right]$$

$$I = -\frac{2}{b^2}\left[\log|a + b\cos x| + \left(\frac{a}{a + b\cos x}\right)\right] + c$$

Ans) $-\frac{2}{b^2}\left[\log|a + b\cos x| + \left(\frac{a}{a + b\cos x}\right)\right] + c$

Question 84.
Evaluate the following integrals:

$$\int \frac{dx}{(3-5x)}$$

Answer:
To find: Value of $\int \frac{dx}{(3-5x)}$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{dx}{(3-5x)}$ ... (i)

Let $(3 - 5x) = t$

$$\Rightarrow (-5) = \frac{dt}{dx}$$

$$\Rightarrow dx = \frac{dt}{-5}$$

Putting this value in equation (i)

$$I = \int \frac{1}{t} \frac{dt}{-5}$$

$$I = \frac{1}{-5} \int \frac{dt}{t}$$

$$\Rightarrow I = \frac{1}{-5} \log |t| + c$$

$$\Rightarrow I = -\frac{1}{5} \log |3 - 5x| + c$$

Ans) $-\frac{1}{5} \log |3 - 5x| + c$

Question 85.
Evaluate the following integrals:

$$\int \sqrt{1+x}\, dx$$

Answer:
To find: Value of $\int \sqrt{1+x}\, dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \sqrt{1+x}\, dx$ ... (i)

Let $(1 + x) = t$

$$\Rightarrow dx = dt$$

Putting this value in equation (i)

$$I = \int \sqrt{t}\, dt$$

$$I = \int t^{\frac{1}{2}}\, dt$$

$$\Rightarrow I = \frac{2}{3}(1+x)^{\frac{3}{2}} + c$$

Ans) $\frac{2}{3}(1+x)^{\frac{3}{2}} + c$

Question 86.
Evaluate the following integrals:

$$\int x^2 e^{x^3} \cos\left(e^{x^3}\right) dx$$

Answer:
To find: Value of $\int x^2 e^{x^3} \cos(e^{x^3}) dx$

Formula used: $\int \cos x\, dx = \sin x + c$

We have, $I = \int x^2 e^{x^3} \cos(e^{x^3}) dx$ ... (i)

Let $e^{x^3} = t$

$$\Rightarrow e^{x^3} . 3x^2 = \frac{dt}{dx}$$

$$\Rightarrow e^{x^3} . x^2 . dx = \frac{dt}{3}$$

Putting this value in equation (i)

$$I = \int \cos(t)\, \frac{dt}{3}$$

$$I = \frac{\sin(t)}{3} + c$$

$$I = \frac{\sin\left(e^{x^3}\right)}{3} + c$$

Ans) $\frac{\sin\left(e^{x^3}\right)}{3} + c$

Question 87.
Evaluate the following integrals:

$$\int \frac{e^{m\ \tan^{-1} x}}{\left(1+x^2\right)} dx$$

Answer:

To find: Value of $\int \frac{e^{m\tan^{-1}x}\, dx}{(1+x^2)}$

Formula used: $\int e^t dx = e^t + c$

We have, $I = \int \frac{e^{m\tan^{-1}x}\, dx}{(1+x^2)}$ ... (i)

Let $\left(m\tan^{-1}x\right) = t$

$$\Rightarrow m\left(\frac{1}{1+x^2}\right) = \frac{dt}{dx}$$

$$\Rightarrow \left(\frac{1}{1+x^2}\right)dx = \frac{dt}{m}$$

Putting this value in equation (i)

$$I = \int e^t \frac{dt}{m}$$

$$\Rightarrow I = \frac{e^t}{m} + c$$

$$\Rightarrow I = \frac{e^{m\tan^{-1}x}}{m} + c$$

Ans) $\frac{e^{m\tan^{-1}x}}{m} + c$

Question 88.
Evaluate the following integrals:

$$\int \frac{(x+1)e^x}{\cos^2(xe^x)}dx$$

Answer:
To find: Value of $\int \frac{(x+1)e^x\,dx}{\cos^2(xe^x)}$

Formula used: $\int \sec^2 x\,dx = \tan x + c$

We have, $I = \int \frac{(x+1)e^x\,dx}{\cos^2(xe^x)}$ ... (i)

Let $(xe^x) = t$

$$\Rightarrow xe^x + e^x.1 = \frac{dt}{dx}$$

$$\Rightarrow e^x(x+1) = \frac{dt}{dx}$$

Putting this value in equation (i)

$$I = \int \frac{dt}{\cos^2(t)}$$

$$\Rightarrow I = \int \sec^2(t)\,dt$$

$\Rightarrow I = \tan(t) + c$

$\Rightarrow I = \tan(xe^x) + c$

Ans) $\tan(xe^x) + c$

Question 89.
Evaluate the following integrals:

$$\int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}} dx$$

Answer:

To find: Value of $\int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right) dx}{\sqrt{x}}$

Formula used: $\int \cos x \, dx = \sin x + c$

We have, $I = \int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right) dx}{\sqrt{x}}$ ... (i)

Let $\left(e^{\sqrt{x}}\right) = t$

$\Rightarrow e^{\sqrt{x}} \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$

Putting this value in equation (i)

$I = \int \cos(t) \, 2dt$

$I = 2\sin\left(e^{\sqrt{x}}\right) + c$

Ans) $2\sin\left(e^{\sqrt{x}}\right)+c$

Question 90.
Evaluate the following integrals:

$$\int\sqrt{e^x-1}\,dx$$

Answer:
To find: Value of $\int\sqrt{e^x-1}\ dx$

Formula used: $\int\frac{1}{x^2+1}dx = \tan^{-1}x+c$

We have, $I=\int\sqrt{e^x-1}\ dx$ ... (i)

Let $(e^x-1)=t^2$

$\Rightarrow e^x-1=t^2$

$\Rightarrow e^x=t^2+1$

$$\Rightarrow e^x=\frac{2tdt}{dx}$$

$$\Rightarrow dx=\frac{2tdt}{e^x}$$

$$\Rightarrow dx=\frac{2t}{t^2+1}dt$$

Putting this value in equation (i)

$$I=\int\sqrt{t^2}\,\frac{2t}{t^2+1}dt$$

$$\Rightarrow I=\int\frac{2t^2}{t^2+1}dt$$

$$\Rightarrow I = 2\int \frac{t^2 + 1 - 1}{t^2 + 1} dt$$

$$\Rightarrow I = 2\int \left(1 - \frac{1}{t^2 + 1}\right) dt$$

$$\Rightarrow I = 2\,[\,t - \tan^{-1} t] + c$$

$$\Rightarrow I = 2\,[\,\sqrt{e^x - 1} - \tan^{-1}\sqrt{e^x - 1}] + c$$

Ans) $2\,[\,\sqrt{e^x - 1} - \tan^{-1}\sqrt{e^x - 1}] + c$

Question 91.
Evaluate the following integrals:

Answer:
To find: Value of $\int \frac{dx}{(x - \sqrt{x})}$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{dx}{(x - \sqrt{x})}$ ... (i)

$$\Rightarrow I = \int \frac{dx}{\sqrt{x}(\sqrt{x} - 1)}$$

Let $(\sqrt{x} - 1) = t$

$$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$$

$$\Rightarrow \frac{1}{\sqrt{x}} dx = \frac{dt}{2}$$

Putting this value in equation (i)

$I = \int \frac{1}{t}\frac{dt}{2}$

$I = \frac{1}{2}\log|t| + c$

$I = \frac{1}{2}\log\left|\sqrt{x} - 1\right| + c$

Ans) $\frac{1}{2}\log\left|\sqrt{x} - 1\right| + c$

Question 92.
Evaluate the following integrals:

$$\int \frac{\sec^2\left(2\tan^{-1}x\right)}{\left(1+x^2\right)}dx$$

Answer:

To find: Value of $\int \frac{\sec^2(2\tan^{-1}x)}{(1+x^2)}dx$

Formula used: $\int \sec^2 x\,dx = \tan x + c$

We have, $I = \int \frac{\sec^2(2\tan^{-1}x)}{(1+x^2)}dx$ ... (i)

Let $2\tan^{-1}x = t$

$\Rightarrow \frac{2}{1+x^2} = \frac{dt}{dx}$

$\Rightarrow \frac{1}{1+x^2}dx = \frac{dt}{2}$

Putting this value in equation (i)

$$I = \int \sec^2 (t) \frac{dt}{2}$$

$$I = \frac{1}{2}\tan(t) + c$$

$$I = \frac{1}{2}\tan(2\tan^{-1} x) + c$$

Ans) $\frac{1}{2}\tan(2\tan^{-1} x) + c$

Question 93.
Evaluate the following integrals:

$$\left(\frac{1+\sin\ 2x}{x+\sin^2 x}\right)dx$$

Answer:
To find: Value of $\int \left(\frac{1+\sin 2x}{x+\sin^2 x}\right)dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \left(\frac{1+\sin 2x}{x+\sin^2 x}\right)dx$ ... (i)

Let $x + \sin^2 x = t$

$$\Rightarrow 1 + 2\sin x.\cos x = \frac{dt}{dx}$$

$$\Rightarrow (1 + \sin 2x)\ dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t}$$

$I = \log|t| + c$

$I = \log\left|x + \sin^2 x\right| + c$

Ans) $\log\left|x + \sin^2 x\right| + c$

Question 94.
Evaluate the following integrals:

$$\int\left(\frac{1 - \tan x}{x + \log \cos x}\right)dx$$

Answer:
To find: Value of $\int\left(\frac{1 - \tan x}{x + \log(\cos x)}\right)dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int\left(\frac{1 - \tan x}{x + \log(\cos x)}\right)dx$ ... (i)

Let $x + \log(\cos x) = t$

$$\Rightarrow 1 + \frac{1.(-\sin x)}{\cos x} = \frac{dt}{dx}$$

$$\Rightarrow 1 - \tan x = \frac{dt}{dx}$$

$$\Rightarrow (1 - \tan x)dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{dt}{t}$$

$I = \log|t| + c$

$I = \log|x + \log(\cos x)| + c$

Ans) $\log|x + \log(\cos x)| + c$

Question 95.
Evaluate the following integrals:

$$\int \frac{(1+\cot x)}{(x+\log \sin x)} dx$$

Answer:
To find: Value of $\int \left(\frac{1+\cot x}{x+\log(\sin x)}\right) dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \left(\frac{1+\cot x}{x+\log(\sin x)}\right) dx$ ... (i)

Let $x + \log(\sin x)$ = t

$$\Rightarrow 1 + \frac{1.(\cos x)}{\sin x} = \frac{dt}{dx}$$

$$\Rightarrow 1 + \cot x = \frac{dt}{dx}$$

$\Rightarrow (1 + \cot x)dx = dt$

Putting this value in equation (i)

$$I = \int \frac{dt}{t}$$

$I = \log|x + \log(\sin x)| + c$

$I = \log|x + \log(\sin x)| + c$

Ans) $\log|x + \log(\sin x)| + c$

Question 96.
Evaluate the following integrals:

$$\int \frac{\tan x \sec^2 x}{\left(1 - \tan^2 x\right)} dx$$

Answer:

To find: Value of $\int \frac{\tan x \sec^2 x}{\left(1 - \tan^2 x\right)} dx$

Formula used: $\int \frac{1}{x} dx = \log|x| + c$

We have, $I = \int \frac{\tan x \sec^2 x}{\left(1 - \tan^2 x\right)} dx$ ... (i)

Let $1 - \tan^2 x = t$

$\Rightarrow 0 - 2.\tan x.\sec^2 x = \frac{dt}{dx}$

$\Rightarrow (\tan x.\sec^2 x)dx = \frac{dt}{-2}$

$\Rightarrow (1 + \cot x)dx = dt$

Putting this value in equation (i)

$I = \int \frac{1}{t} \frac{dt}{(-2)}$

$I = \frac{1}{2} \log|t| + c$

$I = \frac{1}{2}\log|1 - \tan^2 x| + c$

Ans) $\frac{1}{2}\log|1 - \tan^2 x| + c$

Question 97.
Evaluate the following integrals:

$$\int \frac{\sin\left(2\tan^{-1}x\right)}{\left(1+x^2\right)}dx$$

Answer:
To find: Value of $\int \frac{\sin(2\tan^{-1}x)}{(1+x^2)}dx$

Formula used: $\int \sin x \, dx \ = \cos x + c$

We have, $I = \int \frac{\sin(2\tan^{-1}x)}{(1+x^2)}dx$ ... (i)

Let $2\tan^{-1}x = t$

$$\Rightarrow 2\frac{1}{1+x^2} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dx}{1+x^2} = \frac{dt}{2}$$

$$\Rightarrow (1 + \cot x)dx = dt$$

Putting this value in equation (i)

$$I = \int \sin(t)\frac{dt}{(2)}$$

$$I = -\frac{1}{2}\cos(t) + c$$

$$I = -\frac{1}{2}\cos(2\tan^{-1}x) + c$$

Ans) $-\frac{1}{2}\cos(2\tan^{-1}x) + c$

Question 98.
Evaluate the following integrals:

$$\int \frac{dx}{\left(x^{1/2} + x^{1/3}\right)}$$

Answer:

To find: Value of $\int \frac{dx}{\left(x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)}$

Formula used: (i) $\int \frac{1}{x} dx = \log|x| + c$

(ii) $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{dx}{\left(x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)}$ ... (i)

Let $x = t^6$

$$\Rightarrow x^{\frac{1}{6}} = t$$

$$\Rightarrow 6t^5\, dt = dx$$

Putting this value in equation (i)

$$I = \int \frac{6t^5\, dt}{\left(t^3 + t^2\right)}$$

$$I = \int \frac{6t^5\, dt}{t^2(t+1)}$$

$$I = 6\int \frac{t^3\, dt}{(t+1)}$$

$$I = 6\int \frac{t^3 + 1 - 1}{(t+1)} dt$$

$$I = 6\int \frac{(t+1)\,(t^2 - t + 1)}{(t+1)} dt - \int \frac{1}{(t+1)} dt$$

$$I = 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log|t+1|\right] + c$$

$$I = [2t^3 - 3t^2 + 6t - 6\log|t+1|] + c$$

$$I = \left[2\left(x^{\frac{1}{6}}\right)^3 - 3\left(x^{\frac{1}{6}}\right)^2 + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$$

$$I = \left[2\sqrt{x} - 3\left(x^{\frac{1}{3}}\right) + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$$

Ans) $\left[2\sqrt{x} - 3\left(x^{\frac{1}{3}}\right) + 6\left(x^{\frac{1}{6}}\right) - 6\log\left|\left(x^{\frac{1}{6}}\right) + 1\right|\right] + c$

Question 99.
Evaluate the following integrals:

$$\int \left(\sin^{-1} x\right)^2 dx$$

Answer:
To find: Value of $\int (\sin^{-1} x)^2 dx$

Formula used: $\int \sin x\, dx = \cos x + c$

We have, $I = \int (\sin^{-1} x)^2 dx$ ... (i)

Let $\sin^{-1} x = t$, x = sint ,

$$\Rightarrow \cos t = \sqrt{1 - x^2}$$

$$\Rightarrow \frac{1}{\sqrt{1 - x^2}} = \frac{dt}{dx}$$

$$\Rightarrow \sqrt{1 - x^2}\, dt = dx$$

$$\Rightarrow \sqrt{1 - (\sin t)^2}\, dt = dx$$

$$\Rightarrow \sqrt{1 - \sin^2 t}\, dt = dx$$

$$\Rightarrow \cos t\, dt = dx$$

Putting this value in equation (i)

$$I = \int t^2 \cos t\ dt$$

$$I = \int t^2 \cos t\ dt - \int \left[\frac{d(t^2)}{dt} \int \cos t\ dt\right] dt$$

$$I = t^2 \sin t - \int [2t. \sin t] dt$$

$$I = t^2 \sin t\ - 2\left\{\int t [\sin t] dt - \int \left[\frac{dt}{dt} \int \sin t\, dt\right] dt\right\}$$

$$I = t^2 \sin t\ - 2\left[-t\cos t + \int 1.\cos t\, dt\right]$$

$$I = t^2 \sin t\ + 2t\cos t - 2\sin t + c$$

$$I = (\sin^{-1}x)^2 x + 2(\sin^{-1}x)\sqrt{1 - x^2} - 2x + c$$

Ans) $(\sin^{-1}x)^2 x + 2(\sin^{-1}x)\sqrt{1 - x^2} - 2x + c$

Question 100.
Evaluate the following integrals:

$$\int \frac{2x \tan^{-1} x^2}{(1 + x^4)} dx$$

Answer:

To find: Value of $\int \frac{2x\tan^{-1}(x^2)}{(1 + x^4)} dx$

Formula used: $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$

We have, $I = \int \frac{2x\tan^{-1}(x^2)}{(1 + x^4)} dx$ ... (i)

Let $\tan^{-1}(x^2) = t$

$$\Rightarrow \frac{1}{1 + (x^2)^2} . 2x = \frac{dt}{dx}$$

$$\Rightarrow \frac{2x}{1 + x^4} dx = dt$$

Putting this value in equation (i)

$$I = \int t. dt$$

$$I = \frac{t^2}{2} + c$$

$$I = \frac{\{\tan^{-1}(x^2)\}^2}{2} + c$$

Ans) $\frac{\{\tan^{-1}(x^2)\}^2}{2} + c$

Question 101.
Evaluate the following integrals:

$$\int \frac{(x^2+1)}{(x^4+1)} dx$$

Answer:
To find: Value of $\int \frac{(x^2+1)}{(x^4+1)} dx$

Formula used: $\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

We have, $I = \int \frac{(x^2+1)}{(x^4+1)} dx$ ... (i)

Dividing Numerator and Denominator by x^2,

$$I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2} + 2 - 2\right)} dx$$

$$I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2 - 2.x.\frac{1}{x} + \left(\frac{1}{x}\right)^2 + 2\right)} dx$$

$$I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(\left(x-\frac{1}{x}\right)^2 + (\sqrt{2})^2\right)} dx$$

Let $x - \frac{1}{x} = t$

$$\Rightarrow \left(1 + \frac{1}{x^2}\right)dx = dt$$

Putting this value in equation (i)

$$I = \int \frac{1}{(t)^2 + (\sqrt{2})^2} dt$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{t}{\sqrt{2}}\right) + c$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x - \frac{1}{x}}{\sqrt{2}}\right) + c$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{2}x}\right) + c$$

Ans) $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{2}x}\right) + c$

Question 102.
Evaluate the following integrals:

$$\int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}} dx$$

Answer:
To find: Value of $\int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}} dx$

Formula used: $\int \frac{1}{\sqrt{1 - x^2}} dx = \sin^{-1} x + c$

We have, $I = \int \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}} dx$ ... (i)

Let (sinx – cosx) = t

$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$

$\Rightarrow (\cos x + \sin x)\, dx = dt$

$\Rightarrow t^2 = \sin^2 x - 2\sin x . \cos x + \cos^2 x$

$\Rightarrow t^2 = 1 - 2\sin x.\cos x$

$\Rightarrow 2\sin x.\cos x = 1 - t^2$

$\Rightarrow \sin 2x = 1 - t^2$

Putting this value in equation (i)

$\Rightarrow I = \int \frac{dt}{\sqrt{1 - t^2}}$

$I = \sin^{-1} t$

$I = \sin^{-1}(\sin x - \cos x)$

Let $\sin^{-1}(\sin x - \cos x) = \theta$

$\Rightarrow I = \sin^{-1}(\sin x - \cos x) = \theta$ ... (ii)

$\Rightarrow \sin\theta = \sin x - \cos x$

Now if $\sin\theta = \sin x - \cos x$

Then $\cos\theta = \sqrt{1 - (\sin x - \cos x)^2}$

$\Rightarrow \cos\theta = \sqrt{1 - (\sin^2 x - 2\sin x.\cos x + \cos^2 x)}$

$\Rightarrow \cos\theta = \sqrt{1 - (1 - 2\sin x.\cos x)}$

$\Rightarrow \cos\theta = \sqrt{2\sin x.\cos x)}$

Now $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Now $\tan\theta = \frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}\right)$$

Comparing the value θ from eqn. (ii)

$$I = \theta = \tan^{-1}\left(\frac{\sin x - \cos x}{\sqrt{2\sin x.\cos x)}}\right)$$

Dividing Numerator and denominator from cosx

$$I = \theta = \tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x)}}\right)$$

Ans.) $\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x)}}\right)$