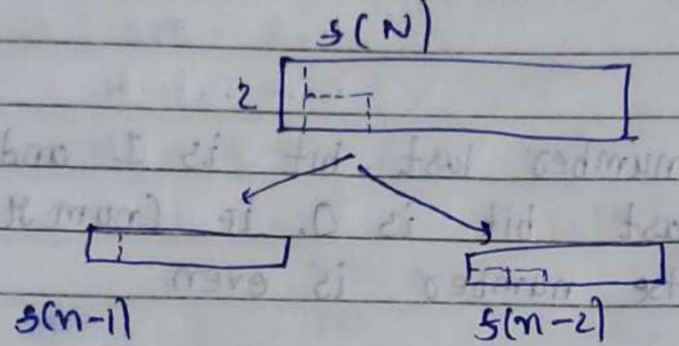
Fibonucci Series

$F(n) = F(n-1) + F(n-2)$

→ Sample problem

① Find number of ways to build a wall of size $2 \times N$, with bricks of size 1×2 and 2×1.



→ How to compute

→ Recursion
→ DP
→ Dp with space optimisation (3 variable)
→ Matrix Exponentiation → $O(k^3 \log n) = O(\log N)$
→ Binet's Formula → $\dfrac{(\phi)^n - (-\phi)^{-n}}{\sqrt{5}}$
→ Code Fores hack
↓ blog no → 14516

Zechandorf's theorem

→ Any number can be written in a unique way as sum of one or more Fibo number such that so some doesn't include any two consecutive number.

$$100 = 89 + 8 + 3$$
$$77 = 55 + 21 + 1$$
$$17 = 13 + 3 + 1$$

→ **Binomial coefficients :-**

$\binom{n}{k}$ → ways to select k items out of n possible.

$$^nC_k = \frac{n!}{(n-k)! \cdot k!}$$

→ $$^nC_k = {}^nC_{n-k}$$

→ $$C(n, 0) = C(n, n) = 1$$

$$\boxed{^nC_k = {}^{n-1}C_{k-1} + {}^{n-1}C_k}$$

k →

n ↓

1

1 1

1 2 1

⇒ Binomial co-efficients use to find ways in grids.

* Catalan series ÷

0 1 2 32

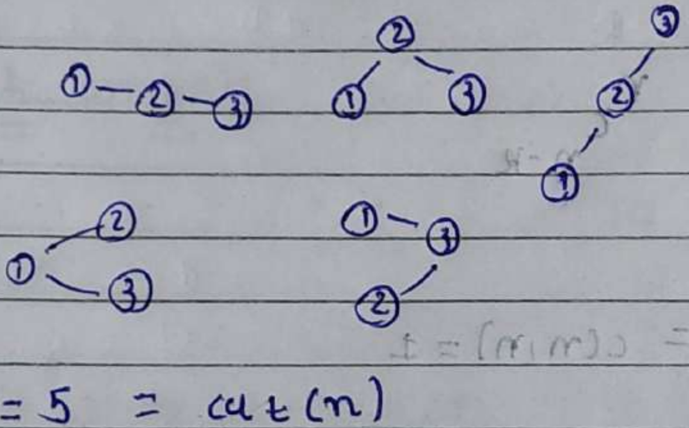
1, 1, 2, 5, 14, 42, 132, 429, 1430 ....

$$cat(n) = \frac{{}^{2n}C_n}{(n+1)} = \frac{(2n)!}{(n+1)!\,(n)!}$$

$cat(0) = 0$

Q → Count the no of BST's with n nodes.

$N = 3$



ans = 5 = cat(n)

→ number of expressions containing n pairs of parentheses which are correctly matched.

For $n = 3$, ans = 5

((())), ()(()), ()()(), (())(), (()())

$$\boxed{f(n) = \sum_{i=1}^{n} f(i-1) + f(N-i)}$$ → using dp

* Binary Exponentiation :

→ 2^{10} ⇒ , $10 = (8 + 2)$
↑ In binary = 1010

→ Ex → $5^{117} = ?$

117 in binary = 1110101

$117 = 2^0 + 2^2 + 2^4 + 2^5 + 2^6$

$5^{117} = 5^{(2^0 + 2^2 + 2^4 + 2^5 + 2^6)}$

$= (5^1 \cdot 5^4 \cdot 5^{16} \cdot 5^{32} \cdot 5^{64})$

→ We can squre again and again,

$5^1 \to 5^2 \to 5^4 \to 5^8 \to 5^{16} \to 5^{32} \to 5^{64}$.

→ And we have to do multiplication when bit is 1 (number is prime) and devide number by 2.

Time complexity → $O(\log(b))$.

* Therome :

→ you have a, b, c Find $(a \times b) \% c$.
$a, b, c \leq 10^{15}$

cout << ((a × b) % c); Not work
$\uparrow 10^{15} \rightarrow 10^{15} \rightarrow 10^{30}$

→ $((a \% c) \cdot (b \% c)) \% c$

$a = b = 10^{15} - 1$
$c = 10^{15}$
} This not work.

Binary Exponati

a^b

$\underbrace{a \times a \times a \times \ldots a}_{b \text{ times}}$

Binary multiplication

$a \times b$

$\underbrace{a + a + \ldots + a + a}_{b \text{ times}}$

→ IF $a \times b$
2×10.

$= (2 \times 8) + (2 \times 2)$
$= 20$