DS & ALG.

28th March 2021

```
PRINT-OPTIMAL-PARENS(S, i, j).
  if i == j
      print "Ai"
  else print "("
      PRINT-OPTIMAL-PARENS(S, i, S[i,j] ).
      PRINT-OPTIMAL-PARENS( S, S[i,j]+1, j )
      print ")"
```

PRINT-OPTIMAL-PARENS(S, 1, n).

$p = \langle 10\ 50\ 20\ 100\ 15\ 30 \rangle$. $\langle p_0 - p_6 \rangle$ = 6 numbers.

$p_0\ p_1\ p_2\ p_3\ p_4\ p_5$

$A_1\ A_2\ A_3\ A_4\ A_5$

$A_i = p_{i-1} \times p_i$ where $i = 1, 2, 3, 4, 5$ $(n = 5)$.

$A_1 = 10 \times 50$ | $A_2 = 50 \times 20$ | $A_3 = 20 \times 100$ | $A_4 = 100 \times 15$ | $A_5 = 15 \times 30$

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p.length = 6, n = 5

Matrix-chain-Order (p)

m = 5×5, s = 5×5

for i = 1 to i = 5
  m[i,i] = 0

m	1	2	3	4	5
1	0	10000	30K	63K	
2		0	100K	45K	67.5K
3			0	30K	39K
4				0	45K
5					0

s	1	2	3	4	5
1		1	2	2	
2			2	2	4
3				3	4
4					4
5					

Iteration $l = 2$. $n = 5$

for $i = 1$ to $i = 4$

$i = 1$
$j = 2$

for $k = 1$ to $k = 1$.

for $k = 1$

$q = m[i, k] + m[k+1, j] + p_{i-1} \times p_k \times p_j$.

if $q < m[i, j]$.

$m[i, j] = q$.

$s[i, j] = k$.

$q = m[1, 1] + m[2, 2] + 10 \times 50 \times 20$

$i = 2$, $j = 3$

$k = 2$

$q = m[2][2] + m[3][3] + 50 \times 20 \times 100$

$i = 3$, $j = 4$.

$k = 3$

$q = m[3][3] + m[4][4] + 20 \times 100 \times 15$.

$i = 4$ | $j = 5$.

$k = 4$.

$q = m[4][4] + m[5][5] + 100 \times 15 \times 30$

$l = 3$

for $i = 1$ to $i = 3$

$i = 1$
$j = 3$

$i = 2$
$j = 4$

$i = 3$
$j = 5$.

$i=1, j=3$

$k=1$

$q = m[1][1] + m[2][3] + 50000$

$= 0 + 1L + 50$

$= \underline{1.5L}$

$k=2$

$q = m[1][2] + m[3][3] +$

$P_0 \times P_2 \times P_3$

$= 10000 + 0 + 20000$

$= 30000$

$i=2, j=4$

$k=2,$

$q = m[2][2] + m[3][4] +$

$P_1 \times P_2 \times P_4$

$= 30k + 15k$

$= 45000$

$k=3$

$q = m[2][3] + m[4][4]$

$+ P_1 \times P_3 \times P_7$

$= 1L + 75000$

$= 1.75L$

$i=3, \quad j=5$

$k=3$

$q = m[3,3] + m[4,5] +$

$P_2 \times P_3 \times P_5$

$= 0 + 45k + 60000$

$= 105000$

$k=4.$

$q = m[3,4] + m[5,5] +$

$P_2 \times P_4 \times P_5$

$= 30k + 0$

$= 30k + 9000$

$= 39000$

$i=4$

for $i=1$ to $i=2$

$i=1$ $\quad$ $i=2$

$j=4$ $\quad$ $j=5$

$i=1, j=4$

k=1

$q = m[1][1] + m[2][4] + P_0 \times P_1 \times P_4$

$= 0 + 45k + 7500$

$= 52500$

k=2

$q = m[1][2] + m[3][4] + P_0 \times P_2 \times P_4$

$q = 10000 + 30000 + 3000$

$= 43000$

k=3

$q = m[1][3] + m[4][4] + P_0 \times P_3 \times P_4$

$q = 30000 + 15000$

$= 45000$

$i = 2, j = 5$

k=2

$q = m[2][2] + m[3][5] + P_1 \times P_2 \times P_5$

$= 0 + 39k + 1500 \times 20$

$= 39k + 30000$

$= 69k$

k=3

$q = m[2][3] + m[4][5] + P_1 \times P_3 \times P_5$

$q = 1.4L +$

$q = 1.4L + 1500 \times 100$

$= 1.4L + 150000$

$= 290L$

k=4.

$q = m[2][4] + m[5][5] + P_1 \times P_4 \times P_5$

$= 45k$

$q = 45k + 1500 \times 15$

$= 45k + 22500$

$= 67500$

$l = 5$

for $i = 1$ to $i = 1$

$i = 1, j = 5$

k=1

$q = m[1,1] + m[2,5] + P_0 \times P_1 \times P_5$

k=2

$q = m[1,2] + m[3,5] + P_0 \times P_2 \times P_5$

k=3

$q = m[1,3] + m[4,5] + P_0 \times P_3 \times P_5$

k=4.

$q = m[1][4] + m[5][5] + P_0 \times P_4 \times P_5$

= 67500 + 300 × 50 = 67500 + 15000 = 82500

= 77500 + 300 × 20 = 77500 + 6000 = 83500

q = 75000 + 300 × 100 = 75000 + 30000 = 105000

q = 43000 + 300 × 15 = 43000 + 4500 = 47500

m

	1	2	3	4	5
1	0	10000	30K	43K	47.5
2		0	100K	45K	67.5K
3			0	30K	39K
4				0	45K
5					0

10×20 20×15

S	1	2	3	4	5
1		1	2	2	4
2			2	2	4
3				3	4
4					4
5					

10×15 × 15×30

P = ⟨ 10 50 20 100 15 30 ⟩

P_0 P_1 P_2 P_3 P_4 P_5

S	1	2	3	4	5
1		1	2	2	4
2			2	2	4
3				3	4
4					4
5					

P₀P(S, 1, 5) i=1, j=5 — S[i,j]=4

P₀P(S, 1, 4) i=1, j=4 — S[i,j]=2 ×

P₀P(S, 1, 2) i=1, j=2 — S[i,j]=1

P₀P(S, 1, 1) i=1 — ×

P₀P(2, 2) — ×

(((A1 A2)(A3 A4))A5)

(A1 · A2)(A3 · A4)(Aj

P₀P(3, 4)

P₀P(3, 3) 0

P₀P(4, 4) 0

P₀P(5, 5)

A5)

≥ 1000 + 30000 + 10×15×30

= 4500 + 1000 + 30000

= 35500

((((A1 · A2)(A3 A4))A5)

= 35500