

Radix sort and Shell Sort

High level view

8	5	0
2	9	5
1	1	9
9	8	7
1	7	8

→

8	5	0
2	9	5
9	8	7
1	7	8
1	1	9

→

1	1	9
8	5	0
1	7	8
9	8	7
2	9	5

→

1	1	9
1	7	8
2	9	5
8	5	0
9	8	7

[1] Determine the number with most digits.

[2] Separate out the digits.

$$\lceil \lg_2 N \rceil + 1$$

$$5 \quad 3 \quad 1 \quad 2^0 = 1$$

$$\lg_2 1 = 0$$

$$N \geq 1$$

$$\lg_2 N$$

$$\lceil \lg_2 N \rceil + 1$$

$$\lceil \lg_2 8 \rceil = 2$$

$$N = 1$$

$$\lg_2 1 = 0$$

$$0 + 1 = 1$$

$$\lg_2 2 = 1$$

$$1 + 1 = 2$$

$$\lfloor \lg_2 N \rfloor + 1$$

10

$$\lfloor \lg_2 N \rfloor + 1$$

$$\lfloor \lg_2 1 \rfloor = 0 + 1 = 1$$

$$\lfloor \lg_2 2 \rfloor = 1 + 1 = 2$$

$$\lfloor \lg_2 3 \rfloor = 1 + 1 = 2$$

$$\lfloor \lg_2 4 \rfloor = 2 + 1 = 3$$

[2] n

int q;

int r;

while ((q = n / 10) >= 1)

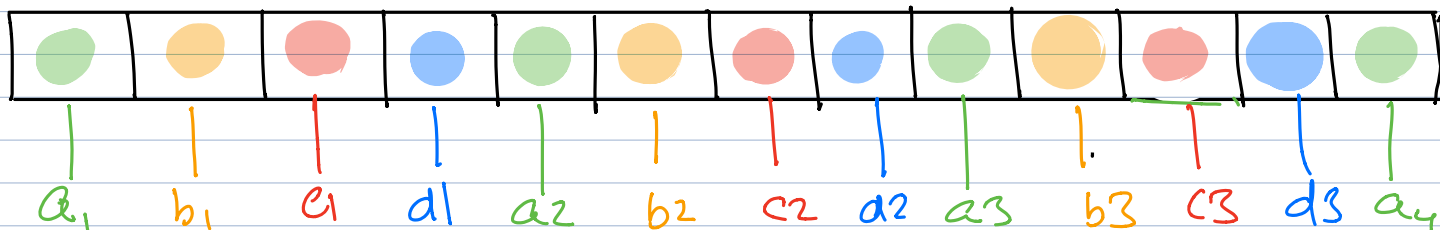
{
 r = n % 10;
 if (q == 0)
 break;
}

↳
kth place.

for (i = 0; i < k; i++)
{
 Ⓟ = n % 10;
 n = n / 10;
}

kth place.
—————→

Shell sort:



$\langle a_1, a_2, a_3, a_4 \rangle$ sorted \wedge $\langle b_1, b_2, b_3 \rangle$ sorted \wedge

\wedge $\langle c_1, c_2, c_3 \rangle$ sorted \wedge $\langle d_1, d_2, d_3 \rangle$ sorted $\Rightarrow A$ is sorted.

→ Partition: Correctness. \rightarrow

```
1) int partition(int *a, int p, int r)
{
    int i, j, pivot, tmp;
    pivot = a[r];
    i = p-1;
    for (j=p; j<r; ++j)
    {
        if (a[j] <= pivot)
        {
            i = i+1;
            a[i] <-> a[j]
        }
    }
    a[i+1] <-> a[r];
    return (i+1);
}
```

Post Cond: $\exists q (p \leq q \leq r)$

$a[p \dots q-1] \leq a[q]$

$a[q+1 \dots r] > a[q]$.

Invariant Cond:

$a[p \dots i] \leq \text{pivot}$

$a[i+1 \dots j-1] > \text{pivot}$

$\text{Inv}(i, j) : \forall k (p \leq k \leq i) (a[k] \leq \text{pivot})$

$\forall k (i+1 \leq k < j) (a[k] > \text{pivot})$.

Initialization: $i = p-1, j = p$.

$a[p \dots i] \equiv a[p \dots p-1] \equiv \text{empty array range}$

$a[i+1 \dots j-1] \equiv a[p \dots p-1] \equiv \text{empty array range}.$

Maintenance Phase:

Assume for $j=j_0$, (i has become \hat{i}).

$$\left. \begin{array}{l} a[p \dots \hat{i}_0] \leq \text{pivot} \\ a[\hat{i}_0+1 \dots j_0-1] > \text{pivot} \end{array} \right\} \equiv \text{True.}$$

when iteration runs with value of i as \hat{i}_0 and value of j as j_0 . (we need to establish that the invariant holds for the updated value of j & i).

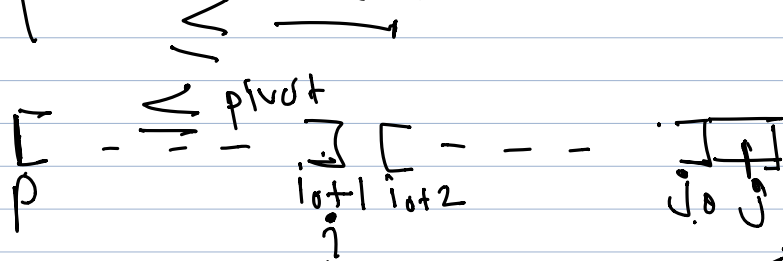
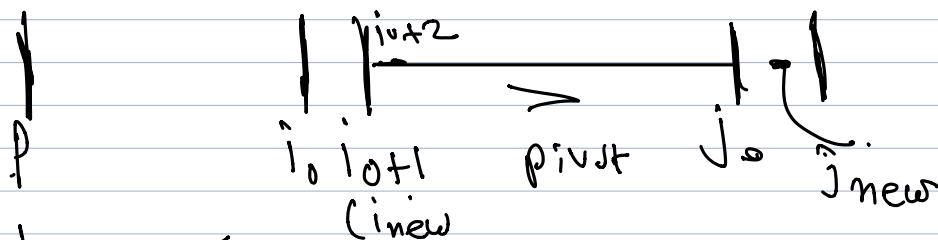
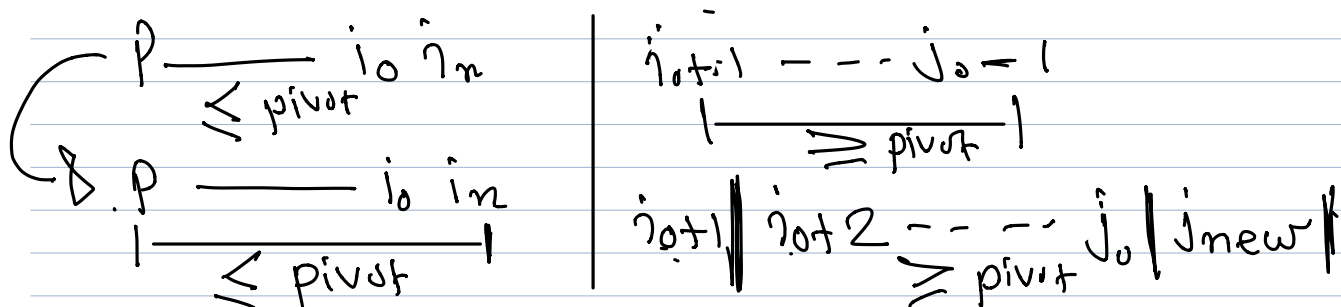
Case (i) : $a[j_0] \leq \text{pivot}$.

$$i_{\text{new}} \leftarrow \hat{i}_0 + 1$$

$$a[i_{\text{new}}] \leftarrow a[j_0]$$

$$a[j_0] \leftarrow a[i_{\text{new}}]$$

$$j_{\text{new}} \leftarrow j_0 + 1$$



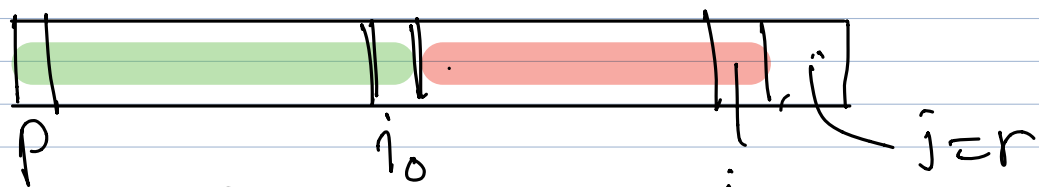
$$a[p \dots i_0+1] \leq \text{pivot}$$

$$a[i_0+2 \dots j_0] > \text{pivot}$$

$$a[p \dots i] \leq \text{pivot}$$

$$a[i+1 \dots j-1] > \text{pivot}$$

$$\hat{j} = r$$



after $Q[i_0+1] \leftarrow \text{arg}_{r,j-1}$

