Convolutional Networks

ADVANCED ARTIFICIAL INTELLIGENCE

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Convolutional networks

- Convolutional networks also known as convolutional neural networks (CNN), are a specialized kind of neural network for processing data with grid-like topology.
  - Time-series data, which can be thought of as a 1D grid taking samples at regular time intervals
  - Image data, which can be thought of as a 2D grid of pixels.
- The name "convolutional neural network" indicates that the network employs a mathematical operation called **convolution.**

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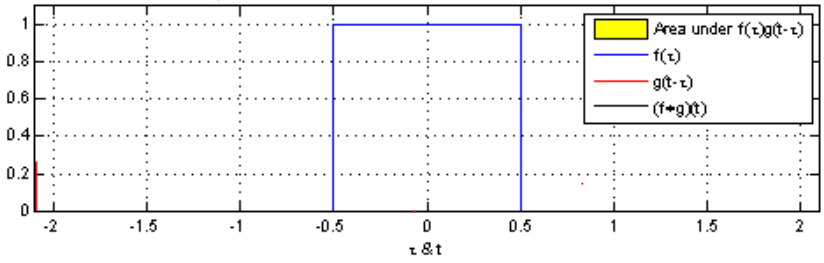
The Convolution Operation

- The convolution of f and g is written $f * g$
  - $(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$
  - f: input
  - g: kernel
  - $(f * g)$: feature map



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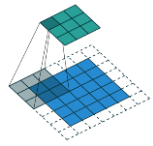
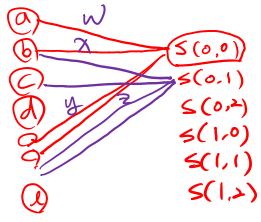
The Convolution Operation

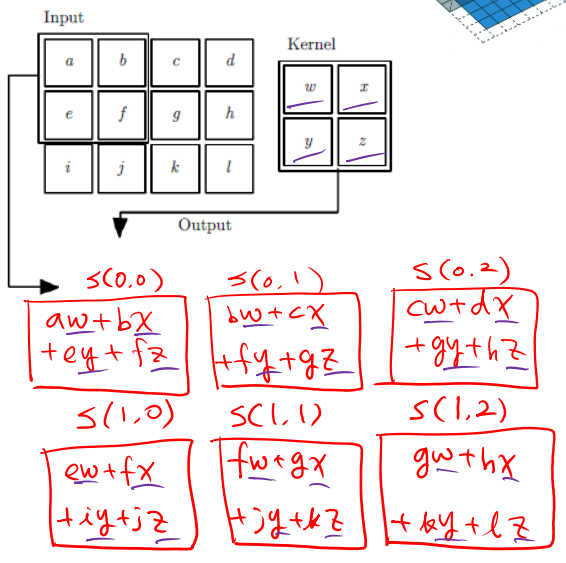


- Discretized two-dimensional
  - $S(i,j) = (I * K)(i,j)$

$= \sum_m \sum_n I(i+m, j+n) K(m,n)$

- I: input
- K: kernel





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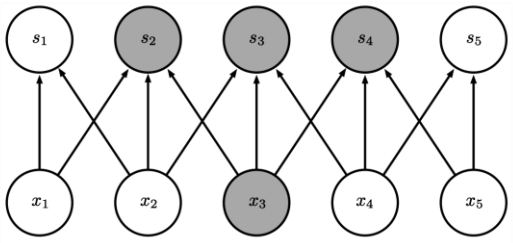
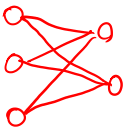
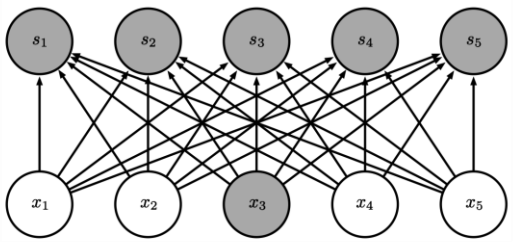
Sparse interactions

- This is accomplished by making the kernel smaller than the input



Sparse connections due to small convolution kernel





Dense connections

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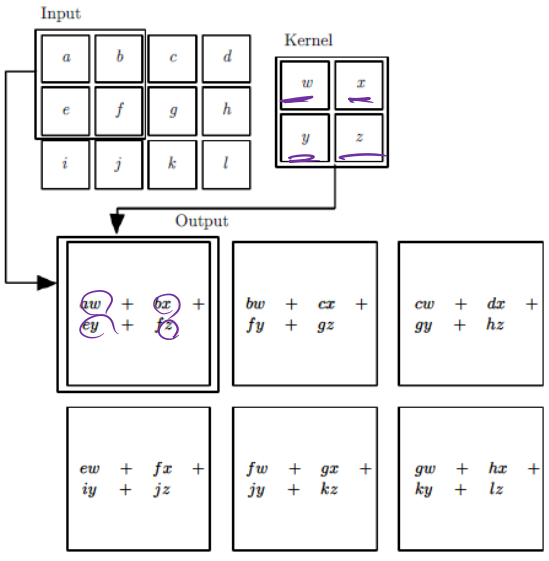
Parameter Sharing

- Using the same parameter for more than one function in a model.
- Each member of the kernel is used at every position of the input



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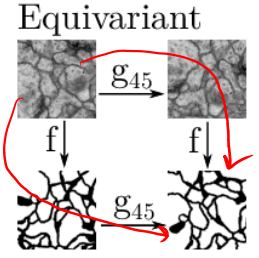
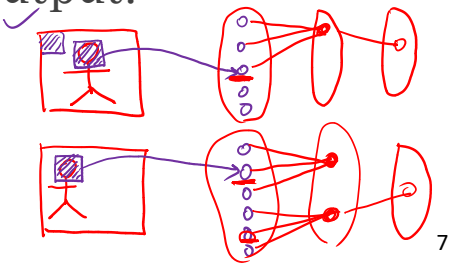
Equivariance

- A function f is equivariant to a function g
  - if $f(g(x)) = g(f(x))$
- With images, convolution creates a 2-D map of where certain features appear in the input.
- If we move the object in the input, its representation will move the same amount in the output.





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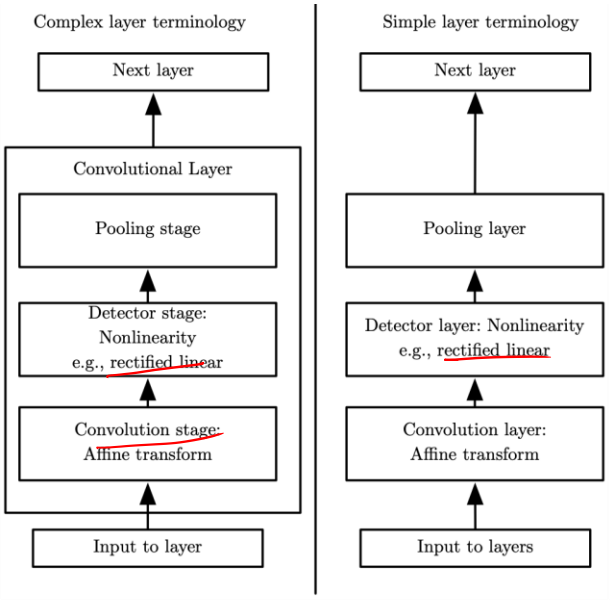
Convolutional Network Components



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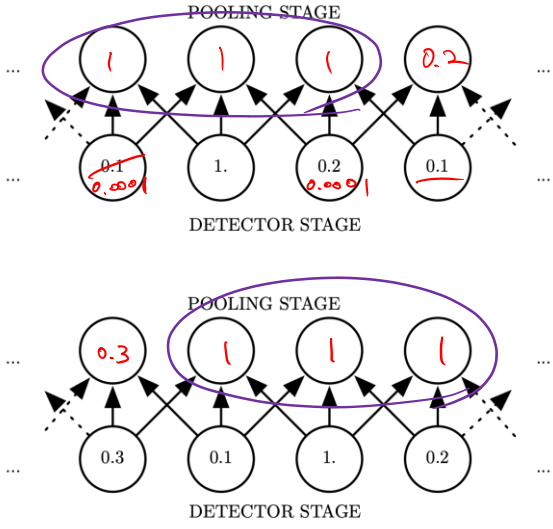
Pooling

- Pooling helps to make the representation become approximately **invariant** to small translations of the input
- Invariance to local translation can be a very useful property if we care more about whether some feature is present than exactly where it is



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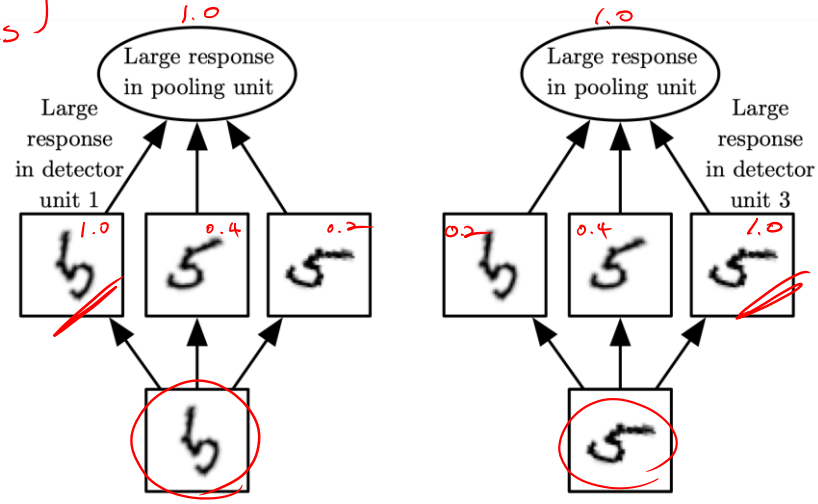
Invariance to Learned Transformations

- Example of learned invariances
  - All filters (kernels) are intended to detect a hand-written 5.



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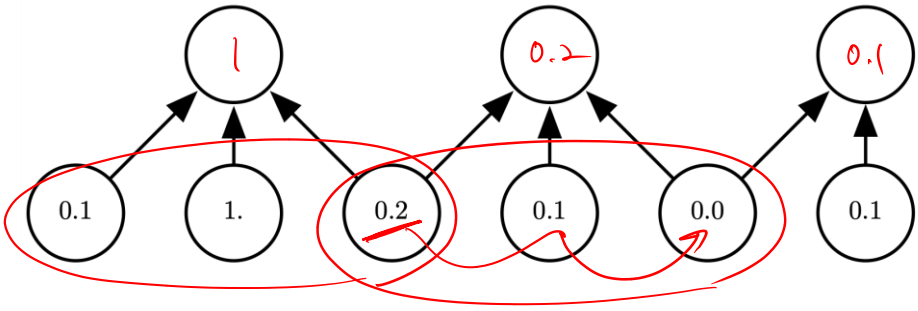
Pooling with Downsampling

- Max-pooling with a pool width of three and a stride between pools of two
  - This reduces the representation size by a factor of two, which reduces the computational and statistical burden on the next layer



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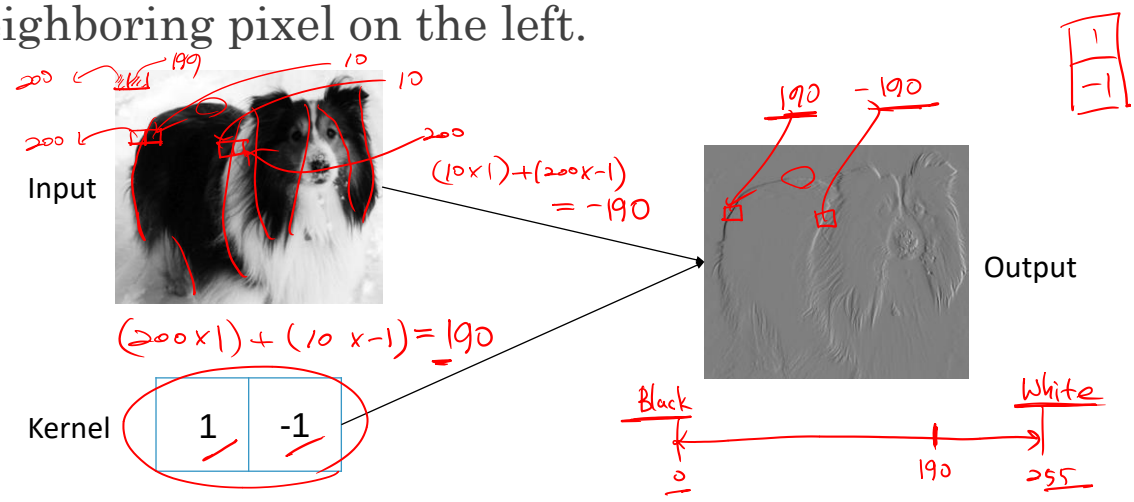
Edge Detection by Convolution

- The image on the right was formed by taking each pixel in the original image and subtracting the value of its neighboring pixel on the left.



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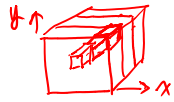
Variants of the Basic Convolution Function

- Convolution with a single kernel can only extract one kind of feature
- The input is usually not just a grid of real values
  - A color image has a ___R, G, B___ intensity at each pixel
  - When working with images, we usually think of the input and output of the convolution as being 3-D ___tensors___, with one index into the different ___channels___ and two indices into the spatial ___coordinates___ of each channel.



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Convolution with Stride

- We may want to skip over some positions of the kernel in order to reduce the computational cost
  - We can think of this as downsampling the output of the full convolution function

Input

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

Kernel

x	y
w	z

$(4\times4) \rightarrow (3\times3) \rightarrow \cdots (1\times1)$

Output

$ax+by+ew+fz$	$cx+dy+gw+hz$
$ix+jy+mw+nz$	$kx+ly+ow+pz$

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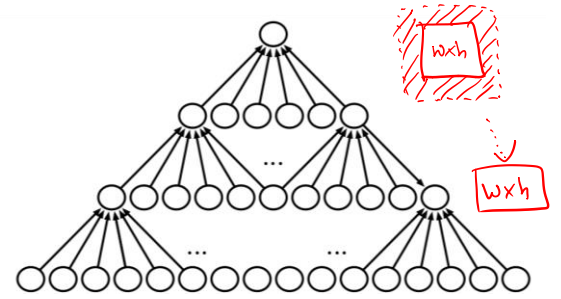
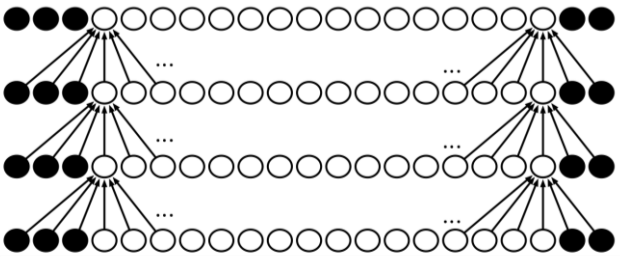
Zero Padding Controls Size

- Without zero padding, the width of the representation shrinks by one pixel less than the kernel width
- Zero padding the input allows us to control the kernel width and the size of the output independently



Without zero padding



With zero padding

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Valid and Same padding in keras

- "valid" means no padding
  - The size of the output shrinks at each layer
  - As layers are added, the spatial dimension of the network will eventually drop to 1 x 1
- "same" results in padding the input such that the output has the same length as the original input.
  - The network can contain as many convolutional layers as the available hardware can support

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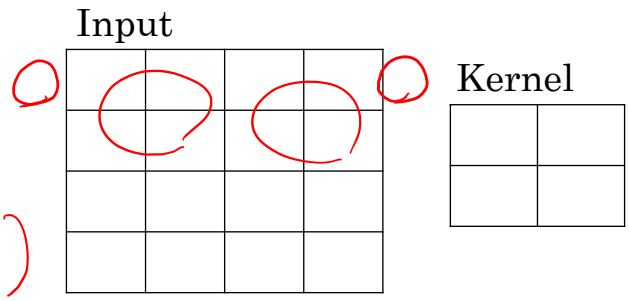
Dimensions

- Let padding P and stride S
- The dimension of an input
  - $H \times W$
- The dimension of a kernel (filter)
  - $FH \times FW$
- The dimension of an output
  - $OH = \left\lfloor \frac{H + 2P - FH}{S} \right\rfloor + 1$
  - $OW = \left\lfloor \frac{W + 2P - FW}{S} \right\rfloor + 1$



W=4 S=2 OW=2

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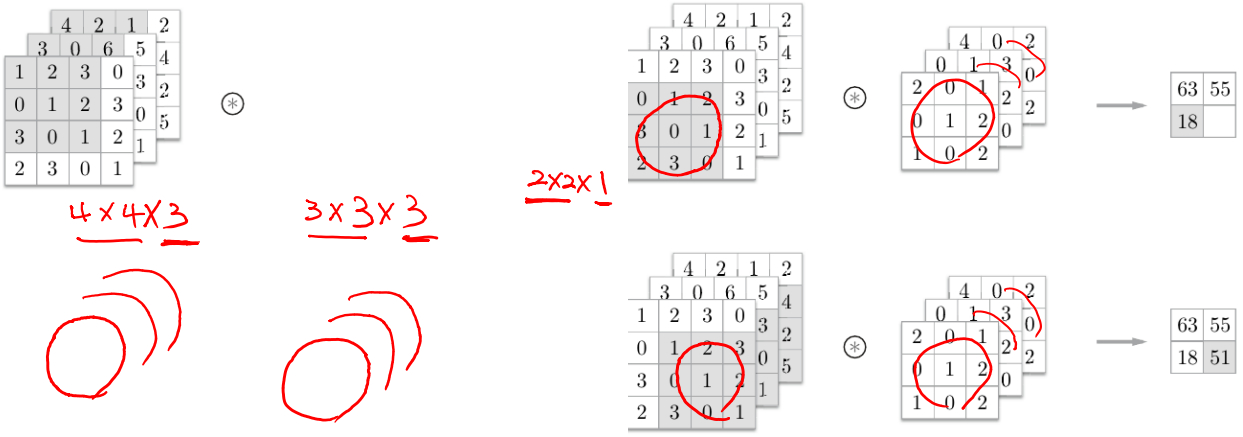
Convolutions on multi-channel

- 3D tensor input and kernel
  - Assume $p = 0$ and $s = 1$



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Convolutions on multi-channel

- 3D tensor input and kernel
  - Assume $p = 0$ and $s = 1$

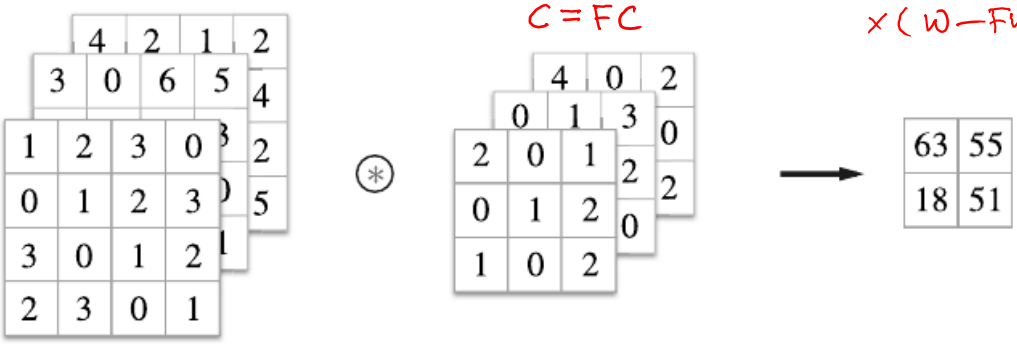
$OH = \left\lfloor \frac{H + 2p - FH}{s} \right\rfloor + 1$

Input
$H \times W \times C$

Kernel
$FH \times FW \times FC$

$C = FC$

Output
$(H - FH + 1) \times (W - FW + 1) \times 1$



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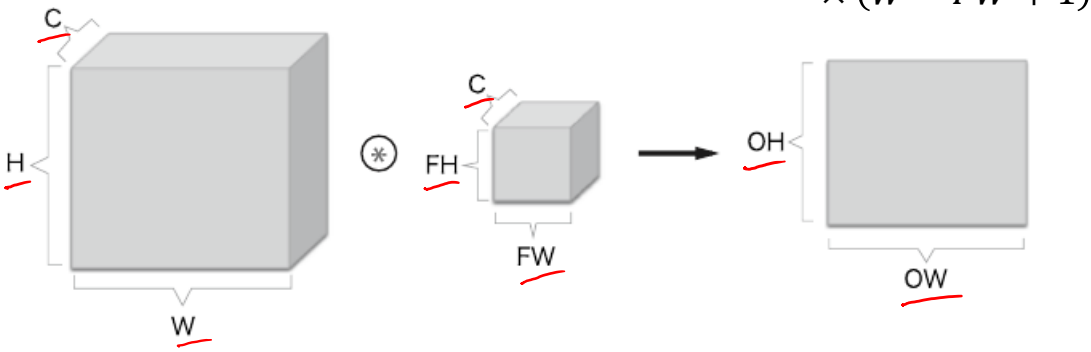
Convolutions on multi-channel

- 3D tensor input and kernel
  - Assume $p = 0$ and $s = 1$

Input
$H \times W \times C$

Kernel
$FH \times FW \times FC$

Output
$(H - FH + 1) \times (W - FW + 1)$



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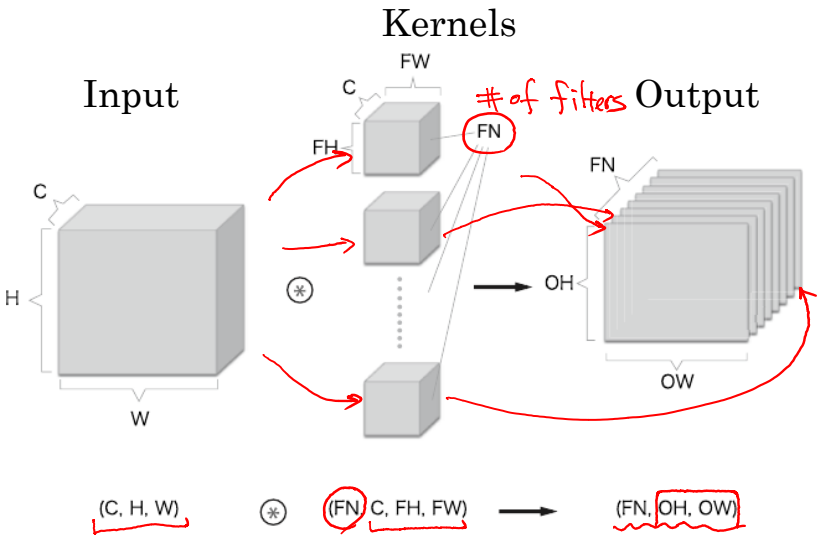
Convolutions on multi-channel / multi-kernels

- 3D tensor input and kernels (filters)



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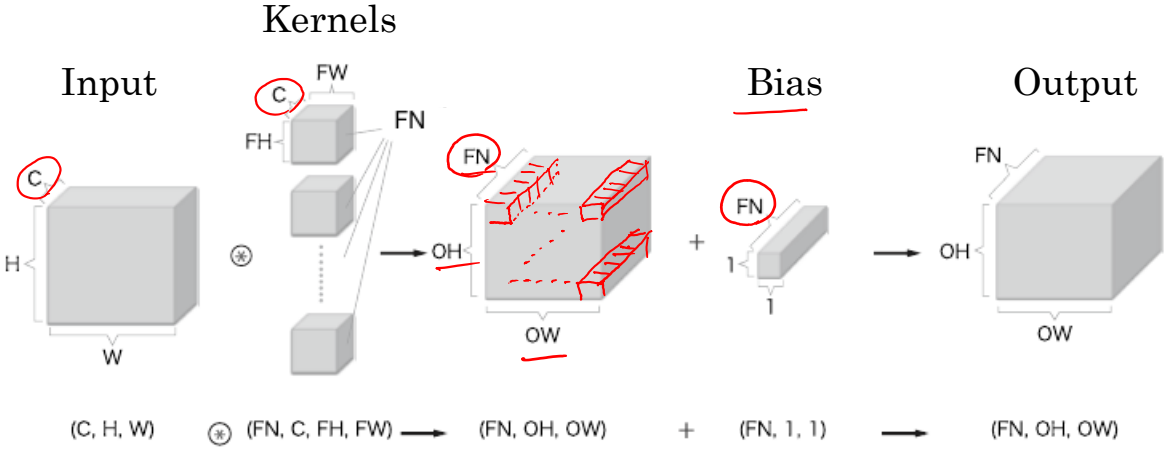
Convolutions on multi-channel / multi-kernels

- 3D tensor input and kernel (filter)



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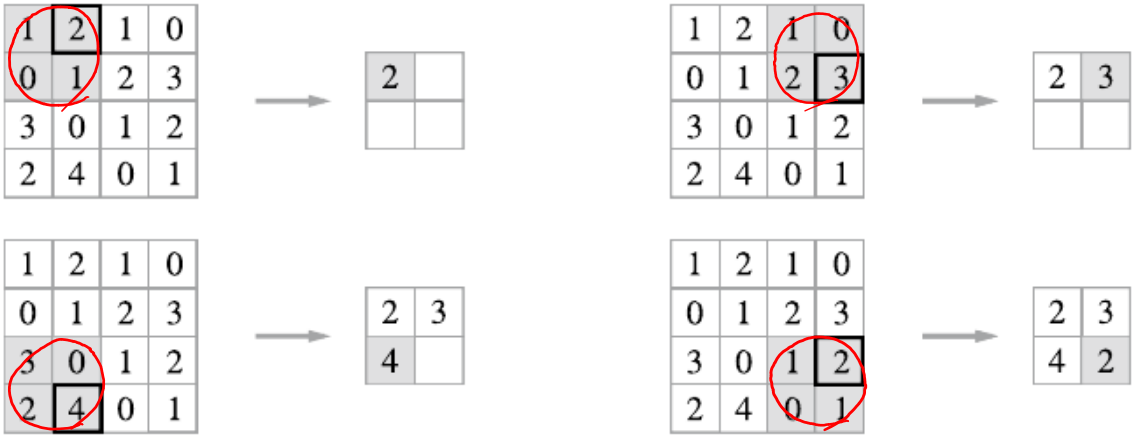
Max Pooling

- No parameters to be trained



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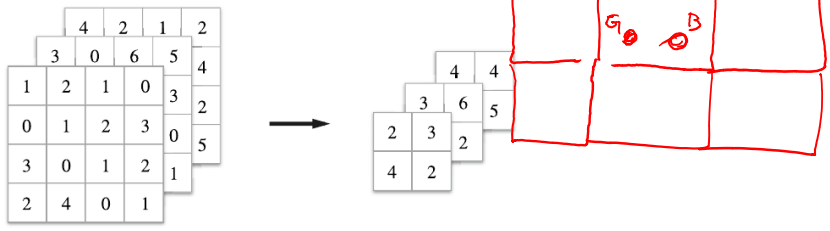
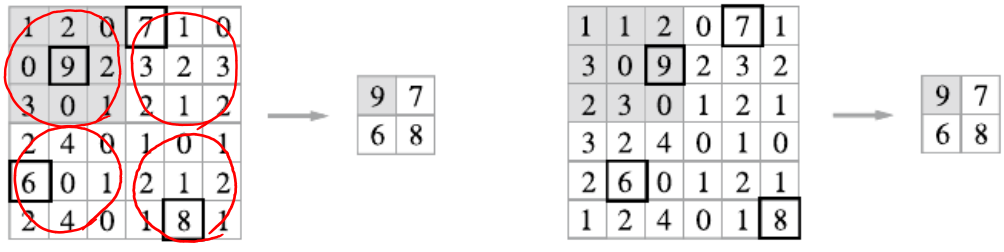
Max Pooling

- Same number of channels



- Invariant to small translations of the input



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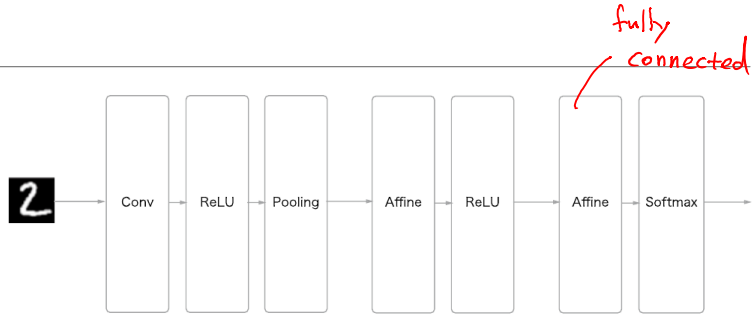
CNN example

- Design
  - Input: 28x28x1
  - Conv: 5x5x1x30
    - Padding: 0
    - Stride: 1
  - Pooling: 2x2x1x30
    - Stride: 2



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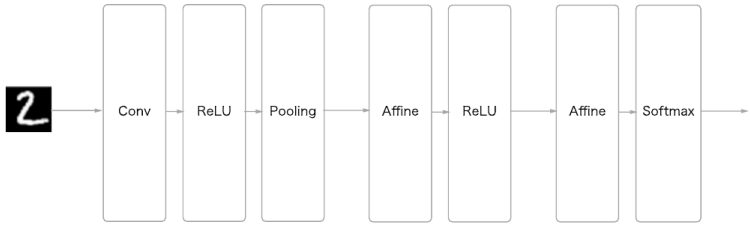
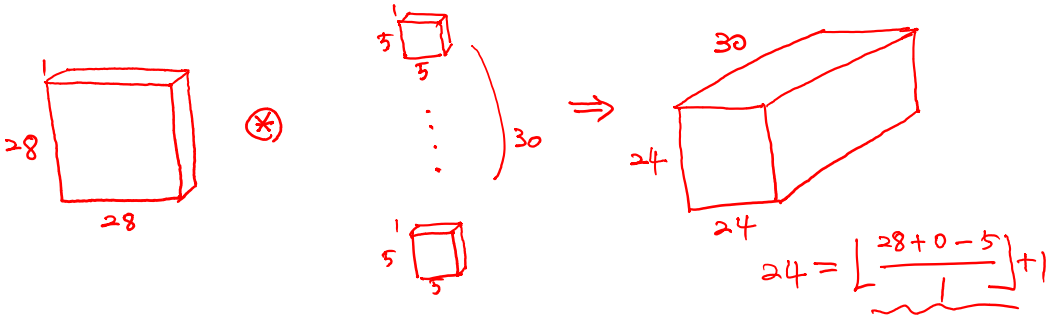
CNN example

$$OH = \left\lfloor \frac{H + 2P - FH}{S} \right\rfloor + 1$$

- Design
  - Input: 28 × 28 × 1
  - Conv: 5 × 5 × 1 × 30
    - Padding: 0
    - Stride: 1





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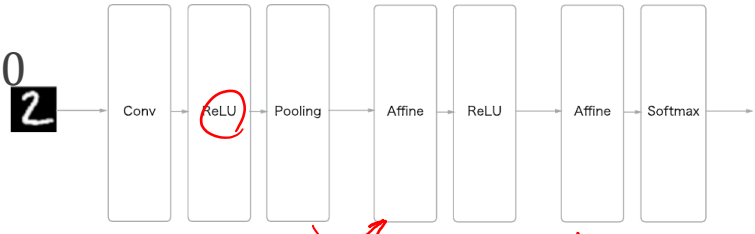
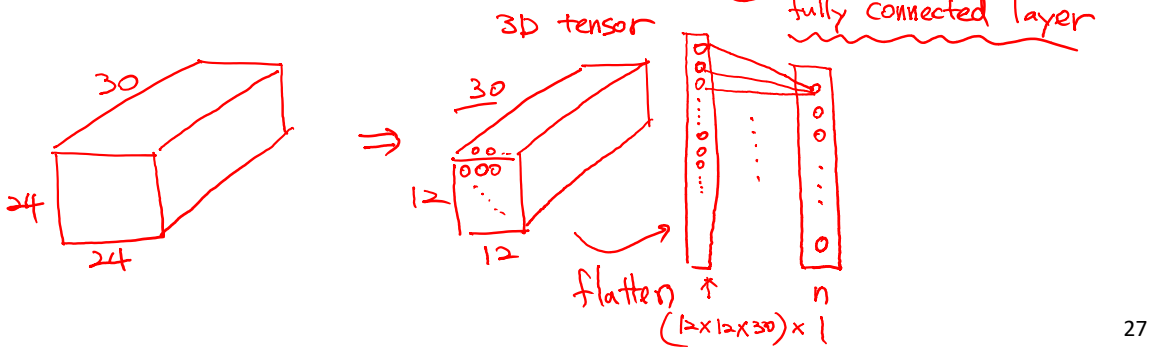
CNN example

- Design
  - Pooling: $2 \times 2 \times 1 \times 30$
    - Stride: 2





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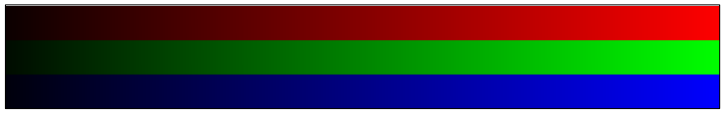
Bitmap

- Image pixels are generally stored with a variable number of bits per pixel which identify its color, the color depth.
  - Grayscale 8 bits
    - 0: Black, $2^8 - 1 = 255$: White

  - RGB 24 bits



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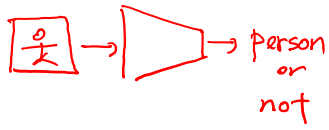
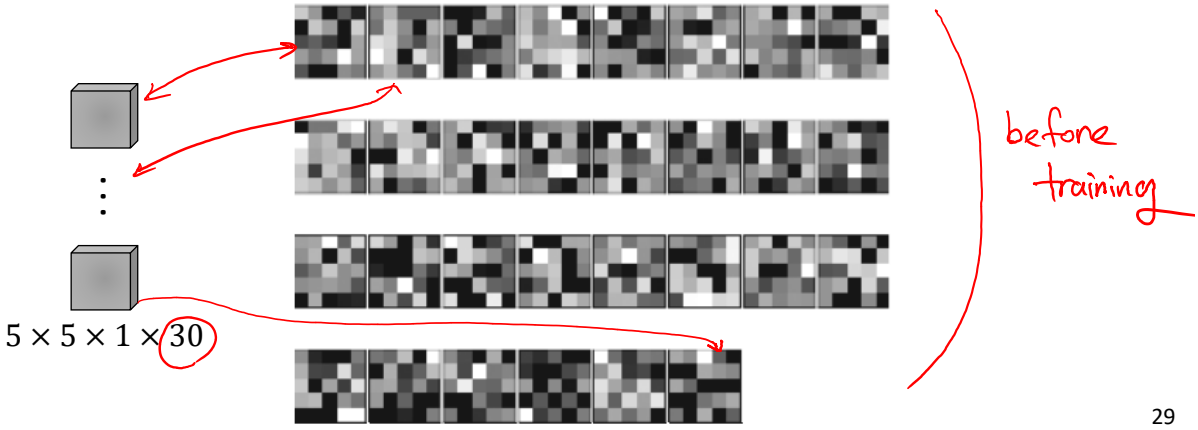
Visualize CNN

- Conv: $5 \times 5 \times 1 \times 30$
- Parameters: random initial numbers
  - Normalize between 0 ~ 255





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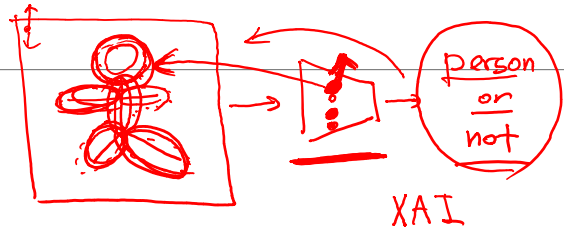
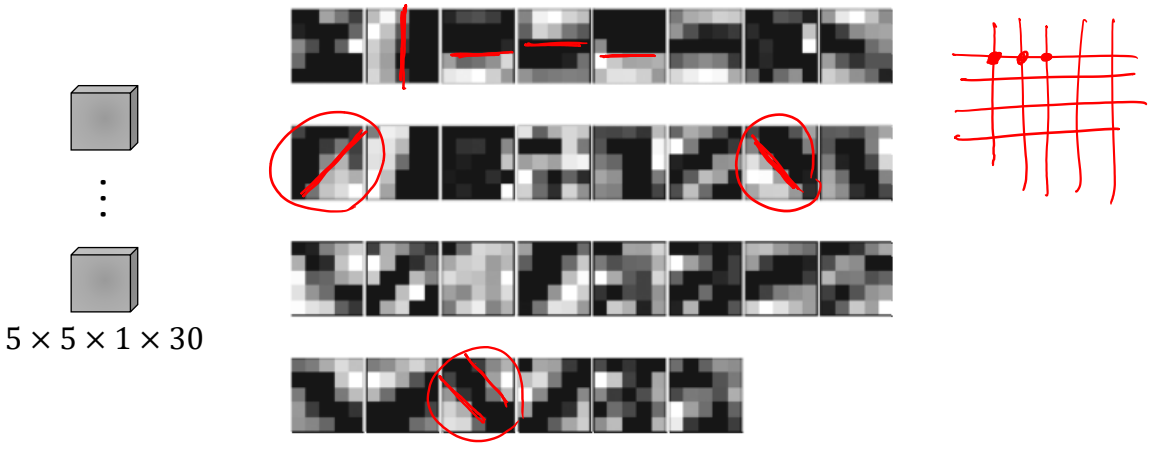
Visualize CNN

- Conv: $5 \times 5 \times 1 \times 30$
- Parameters: after training
  - Normalize between 0 ~ 255





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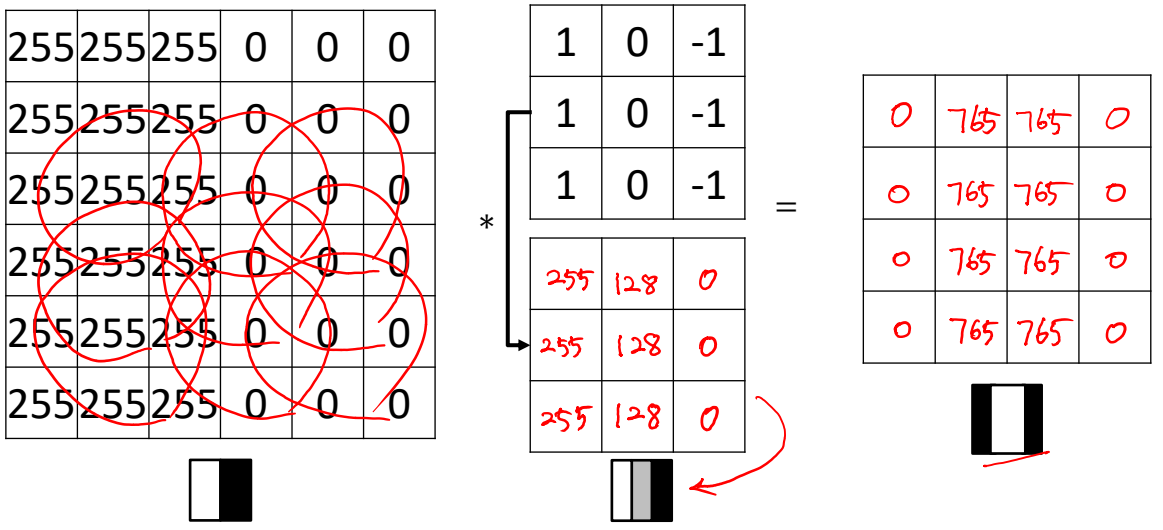
Vertical edge detection

- How CNN detect vertical edge?



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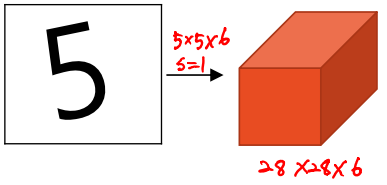
LeNet (1998)

- Input: $32 \times 32 \times 3$
- Conv1: $5 \times 5 \times 3 \times 6 \rightarrow 5 \times 5 \times 6$

$$OH = \left\lfloor \frac{H + 2P - FH}{S} \right\rfloor + 1$$

  - Stride: 1 , Padding: 0
  - Output: $28 \times 28 \times 6$



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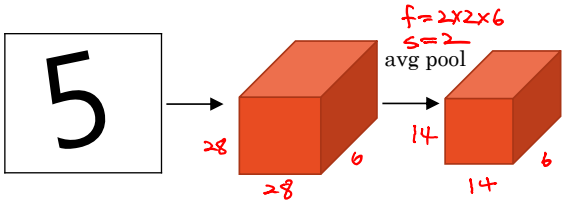
LeNet (1998)

- Pool1: 2 x 2 x 6
  - Stride: 2
  - Output: 14 x 14 x 6



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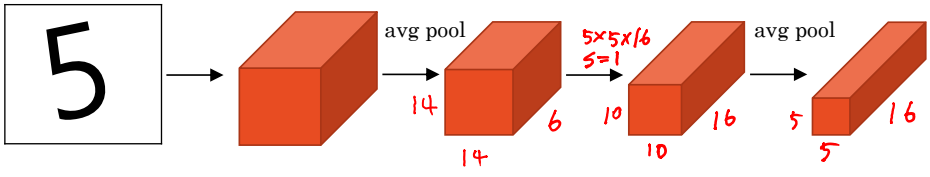
LeNet (1998)

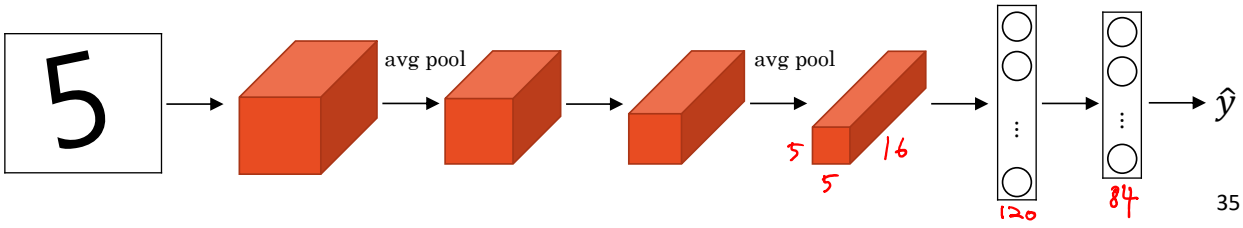
- Conv2: 5 x 5 x 16
  - Stride: 1 , Padding: 0
  - Output: 10 x 10 x 16
- Pool2: 5 x 5 x 16



34

LeNet (1998)

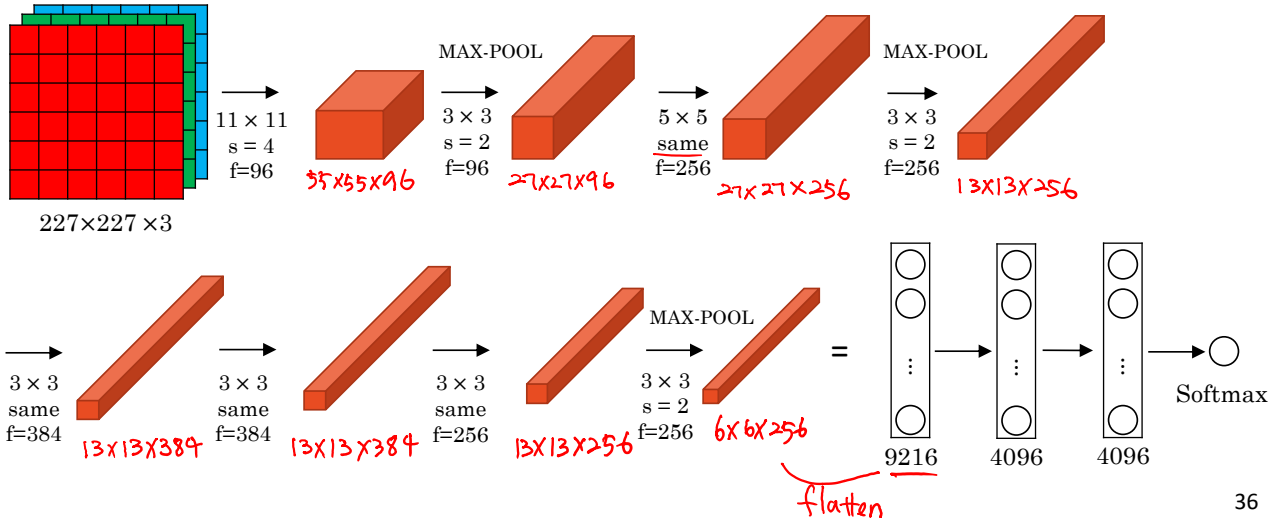
- Pool2: 5 × 5 × 16
- H3: 120
- H4: 84
- Softmax



35

AlexNet (2012)

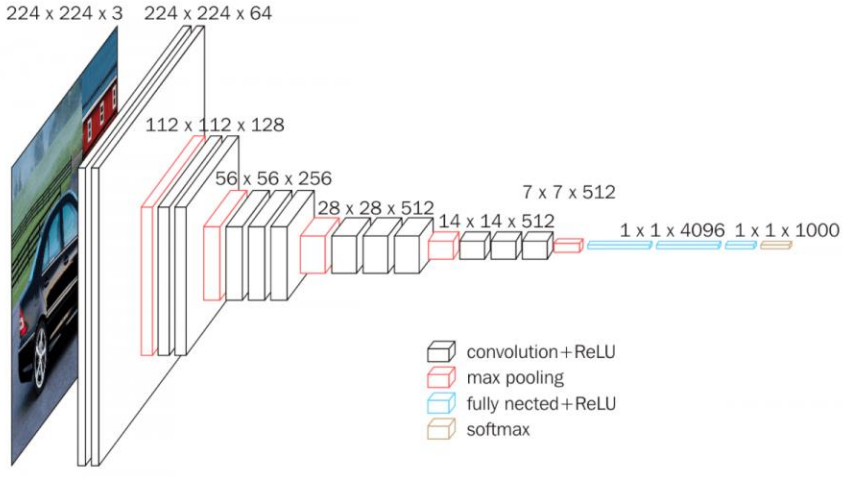
- Upgrade version of LeNet



36

VGG-16 (2015)

```
model.add(Conv2D(filters=64,kernel_size=(3,3),padding="same", activation="relu"))
model.add(Conv2D(filters=64,kernel_size=(3,3),padding="same", activation="relu"))
model.add(MaxPool2D(pool_size=(2,2),strides=(2,2)))
```

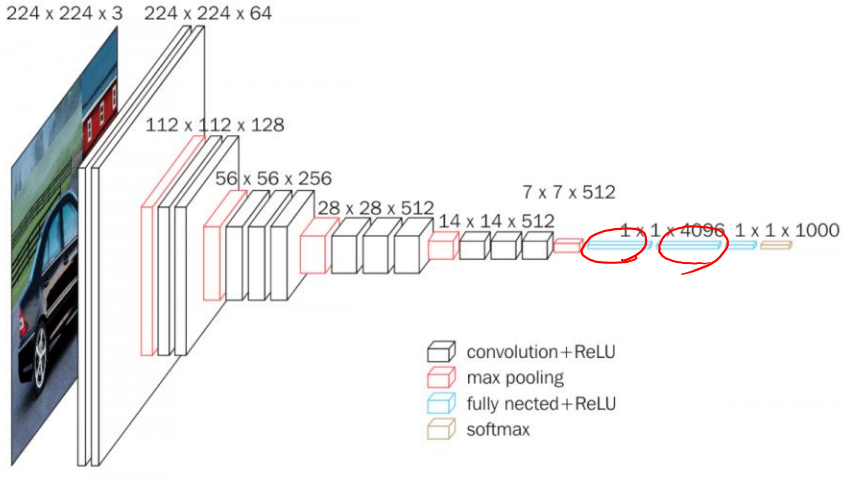


37

37

VGG-16 (2015)

```
model.add(Flatten())
model.add(Dense(units=4096,activation="relu"))
model.add(Dense(units=4096,activation="relu"))
```



38

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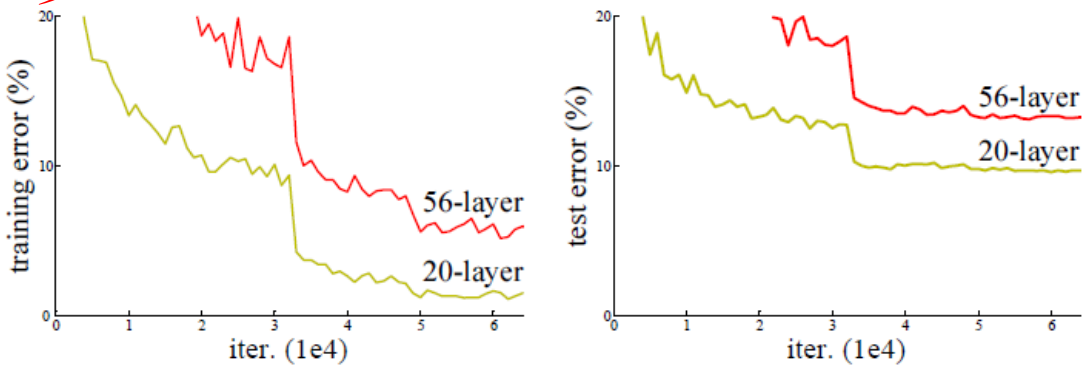
Deep Residual Learning

- Deeper neural networks are better in general?
  - Adding more layers to a suitably deep model leads to **(higher/lower)** training and test error



43

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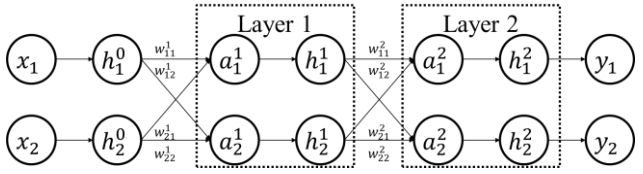
Deep Neural Networks

- Can a DNN learn an identity function? $\vec{y} = \vec{x}$
  - Yes, but it's not easy.
  - $y_1 = x_1$



$= h_1^2 = g(a_1^2)$

$\hookrightarrow w_{11}^2 h_1^1 + w_{21}^2 h_2^1$

$g(a_1^1)$ $g(a_2^1)$ $w_{21}^1 h_1^0 + w_{22}^1 h_2^0$

$\llcorner w_{11}^1 h_1^0 + w_{21}^1 h_2^0$

x_1 x_2 x_1 x_2

$= g\left(w_{11}^2 g(w_{11}^1 x_1 + w_{21}^1 x_2) + w_{21}^2 g(w_{21}^1 x_1 + w_{22}^1 x_2)\right) = x_1$

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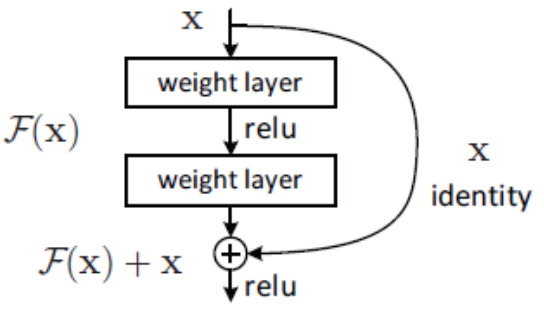
Deep Residual Network

- Shortcut Connections
  - $a^k =$ $w^k h^{k-1} + b^{k-1}$
  - Plain block
    - $h^k =$ $g(a^k)$
  - Residual block
    - $h^k =$ $g(a^k + h^{k-1})$

$= g((w^k h^{k-1} + b^{k-1}) + h^{k-1})$

$w \to 0, b \to 0$

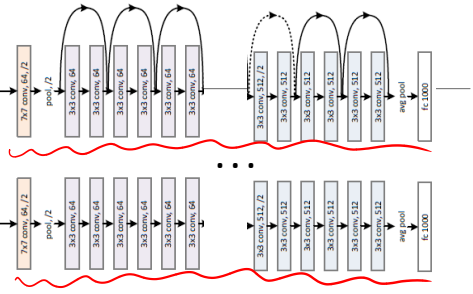


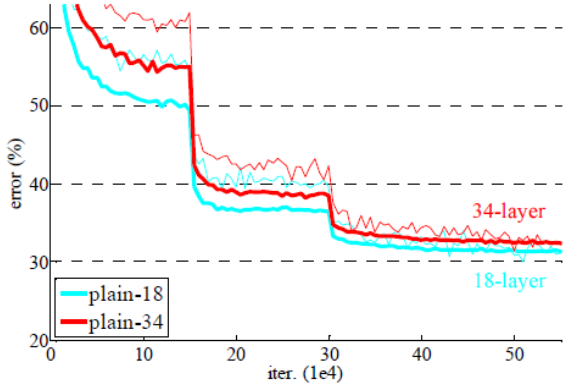
45

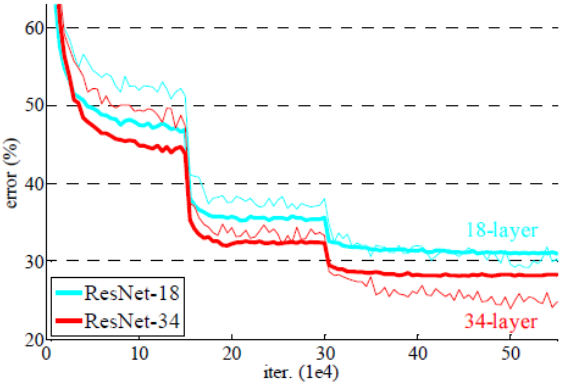
45

Deep Residual Network

- Experiments
  - Plain net of 18 and 34
  - Residual net of 18 and 34







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ILSVRC

- The ImageNet Large Scale Visual Recognition Challenge (ILSVRC) evaluates algorithms for object detection and image classification at large scale.

Siberian husky

Eskimo dog

Year	Error
2012	16.4%
2013	11.7%
2014	6.67%

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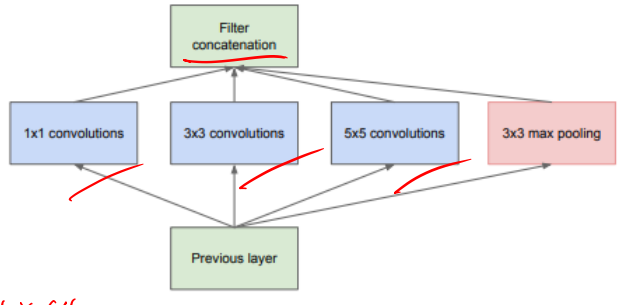
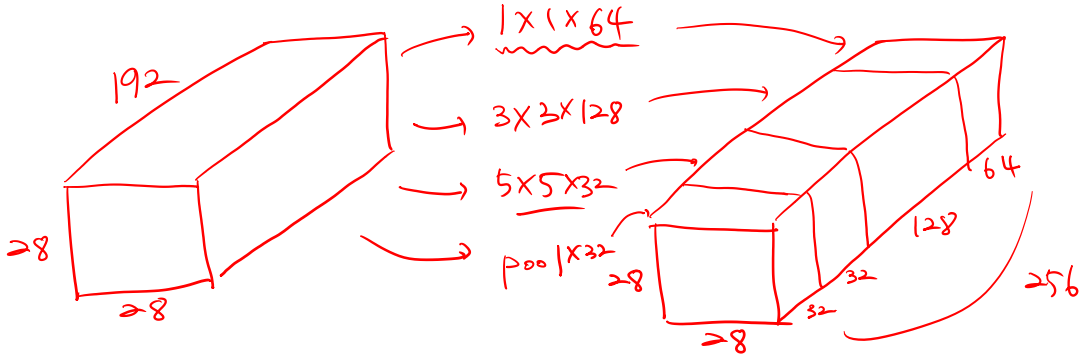
GoogLeNet

- Inception architecture





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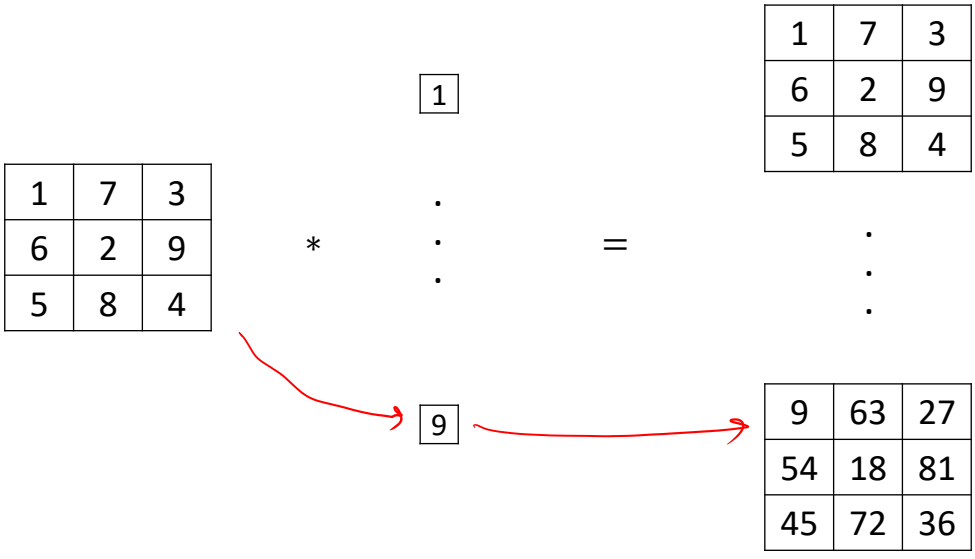
1 × 1 Convolution

- Meaning of using 1 × 1 convolution



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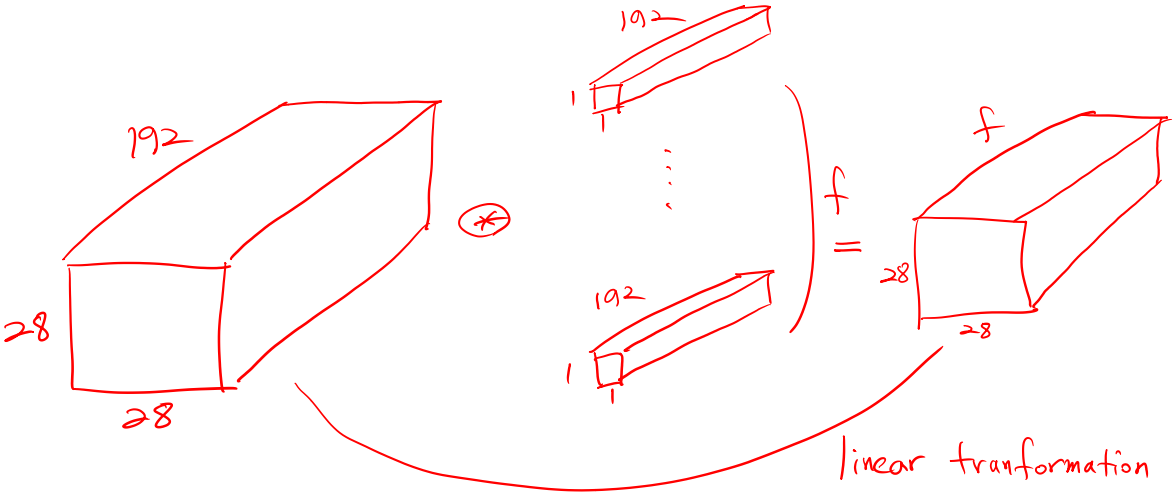
1 × 1 Convolution

- Depth reduction

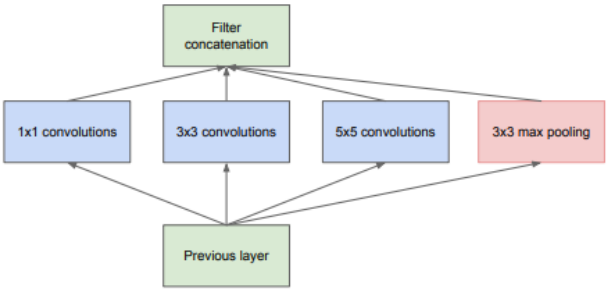


50

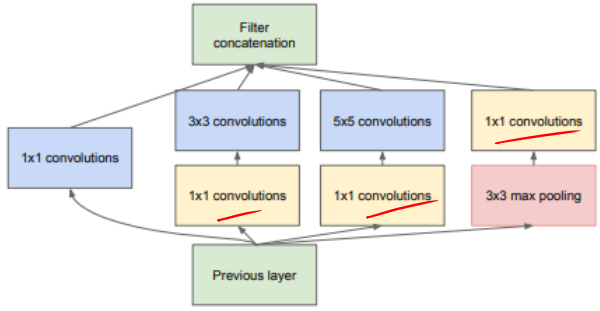
50

The problem of inception network

- Computational cost



Naïve version



dimension reduction

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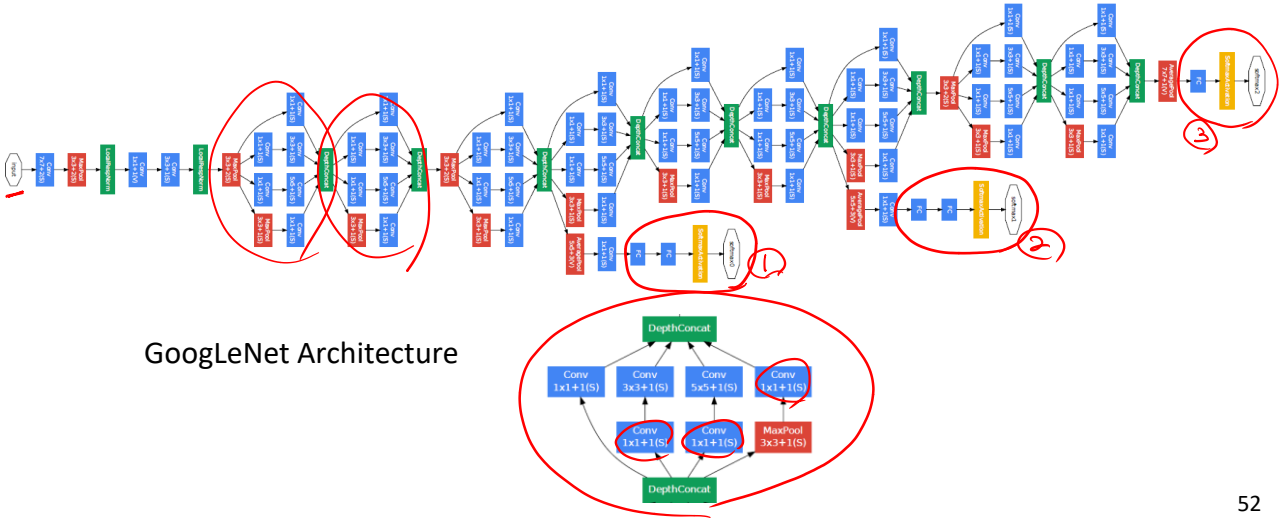
ILSVRC 2014

- GoogLeNet



GoogLeNet Architecture

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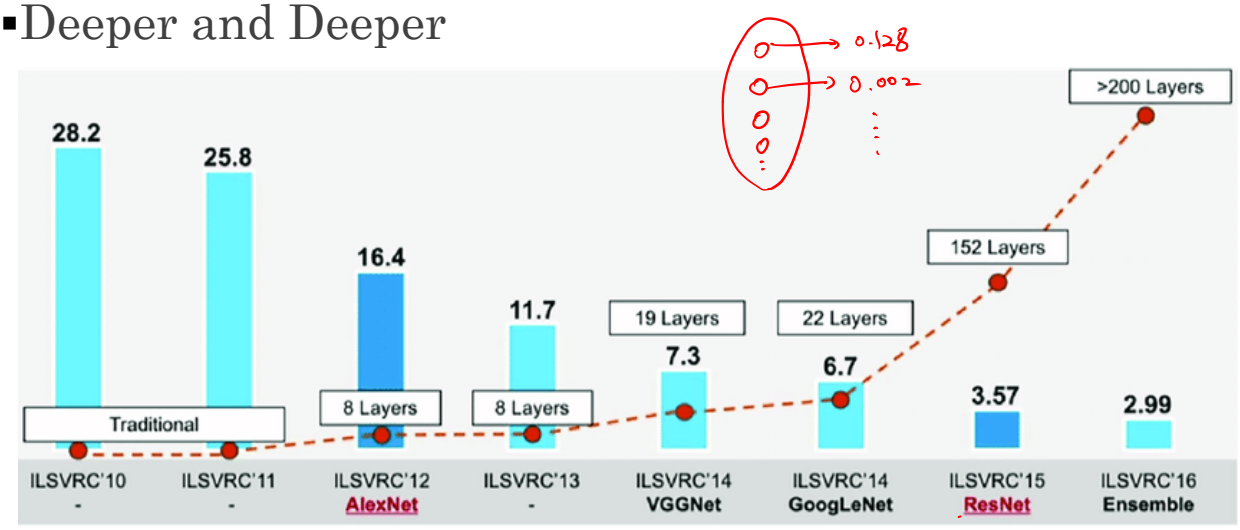
ILSVRC

- Deeper and Deeper



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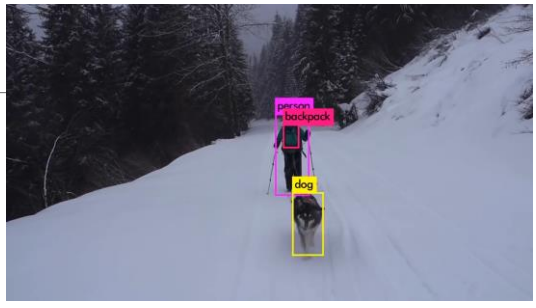
Object Detection

- Detecting instances of semantic objects of a certain class
  - Classification

    Car
  - Detection

    Car

    Location



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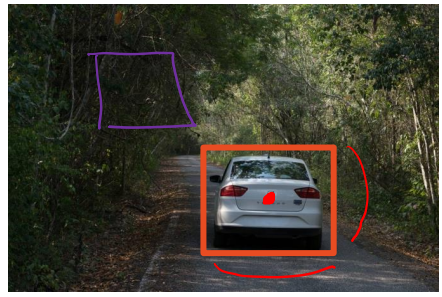
Object Detection

- Bounding box
  - $\vec{y} = [b_x, b_y, b_h, b_w, c_1, c_2, \cdots, c_n]^T$
  - c_i: 1 if its class is i
- Loss function
  - $L(\vec{y}, \hat{\vec{y}}) = \|\vec{y} - \hat{\vec{y}}\|_2^2$



$[\_, \_, \_, \_, 0.02, 0.123, 0.134, \cdots]$

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Object Detection

- Just when you think you have the correct answers, things get changed
- Bounding box
  - $\vec{y} = [p_c, b_x, b_y, b_h, b_w, c_1, c_2, \cdots, c_n]^T$
  - p_c: Image has the object
  - c_i: 1 if its class is i

- Loss function
  - $L(\vec{y}, \hat{\vec{y}}) = \begin{cases} \|\vec{y} - \hat{\vec{y}}\|_2^2 & \text{if } p_c = 1 \\ (y_1 - \hat{y}_1)^2 & \text{O.W} \end{cases}$ (p_c, $\hat{p}_c$) $\Rightarrow \hat{p}_c^2 \to 0$

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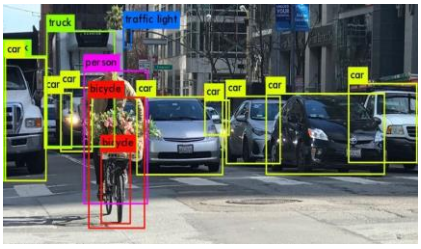
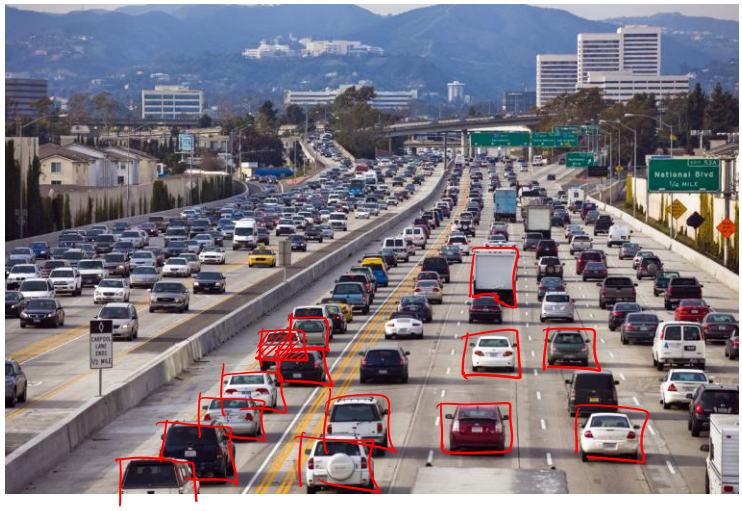
Multiple Objects

- In practice, labeling job is painful.





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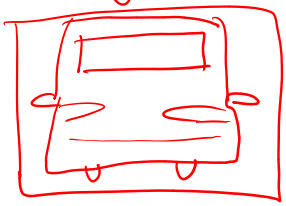
Can we utilize CIFAR-10 dataset?

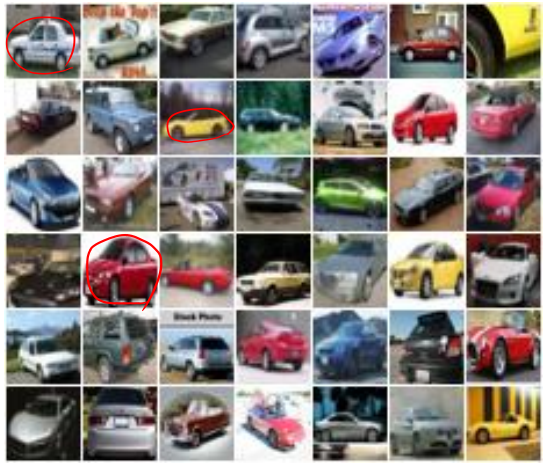
- We need a magic function s.t. $f(\vec{x}) = \hat{\vec{y}}$
  - $\vec{x}$: Image
  - $\hat{\vec{y}}$: Class label + bounding box

Test Image



Car!

Training Images



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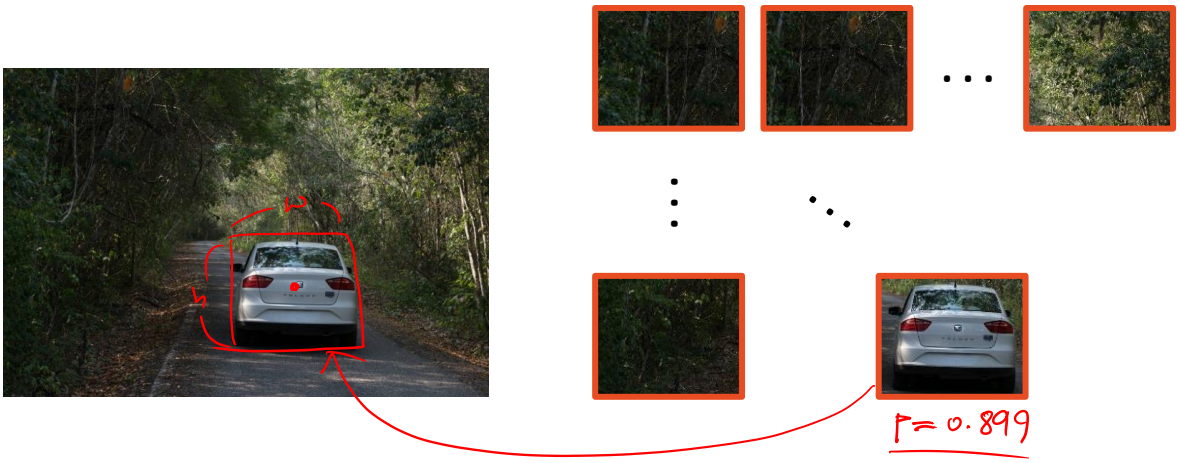
Sliding window

- Slide a box image and classify each image inside a box



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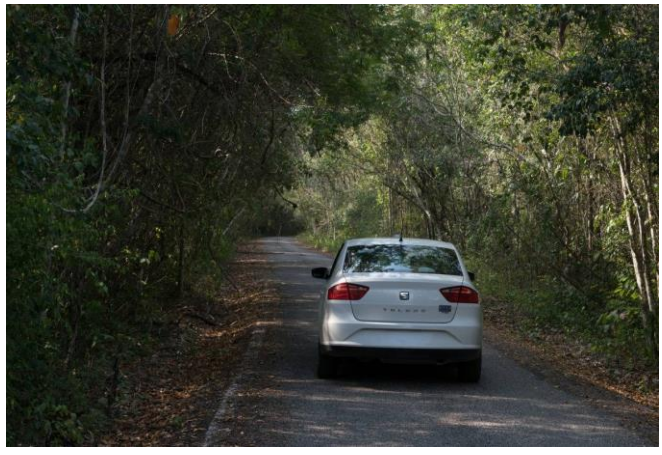
Sliding window

- Questions?
  - Shape of the boxes
  - Size of the boxes
  - Step interval



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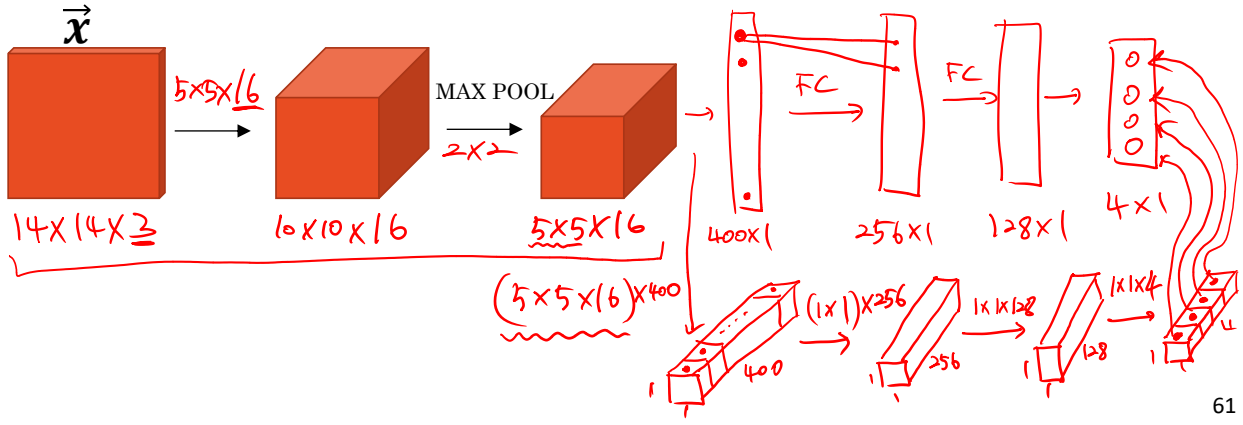
Fully connected layers

- Typical CNN architectures include
  - Convolutional layers (Pooling layers)
  - Fully connected layers (Softmax layer)



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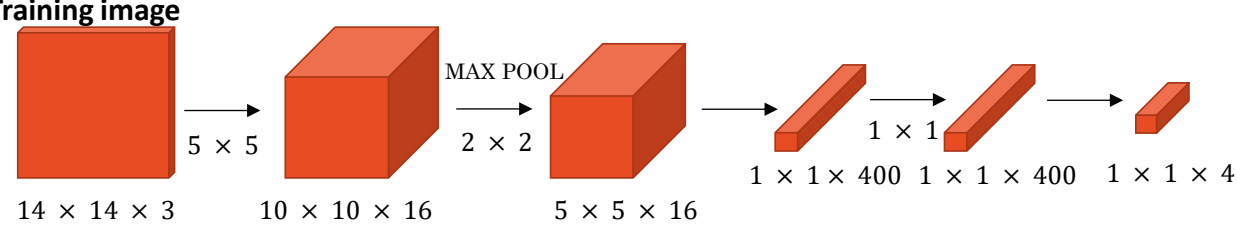
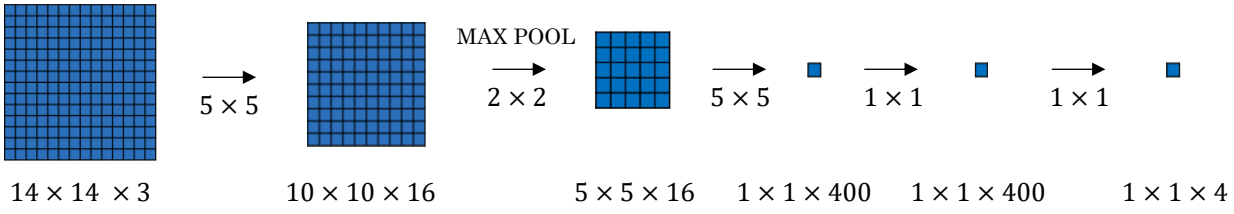
ConvNets for detection

- Fast/cheap implementation of sliding window

Training image



Simplified diagram



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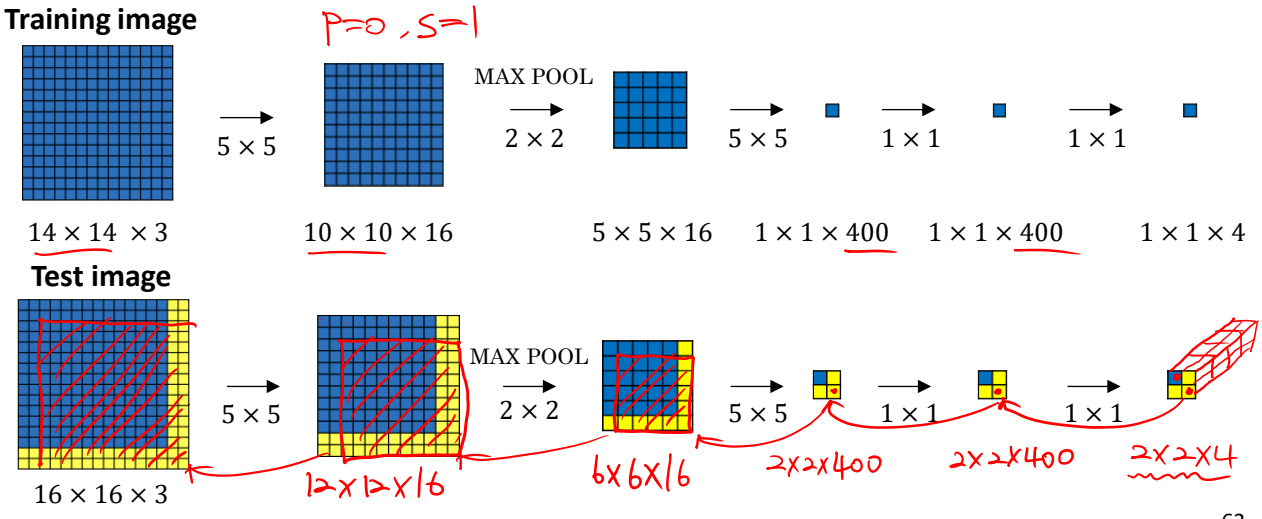
ConvNets for detection

$OH = \left\lfloor \frac{W+2P-FW}{S} \right\rfloor + 1$

- Fast/cheap implementation of sliding window



Sermanet, et al. (2013). Overfeat: Integrated recognition, localization and detection using convolutional networks

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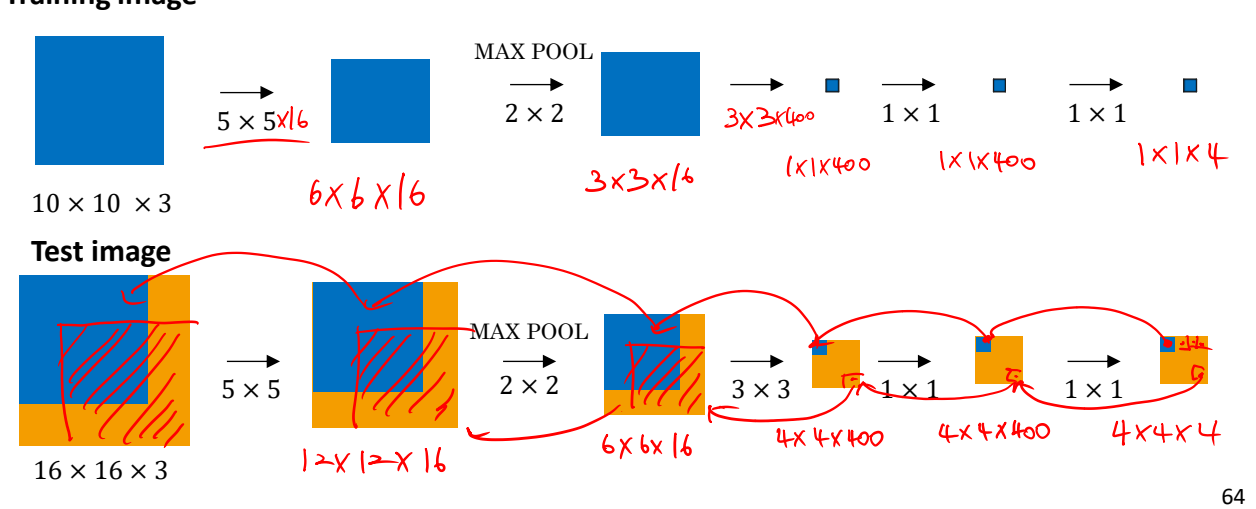
ConvNets for detection

- How we can change the window size and step interval?



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Problem of sliding window detection

- What if the bounding boxes are not matched perfectly?
  - YOLO

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Problem of sliding window detection

- What if the bounding boxes are not matched perfectly?
  - YOLO (You Only Live Once)

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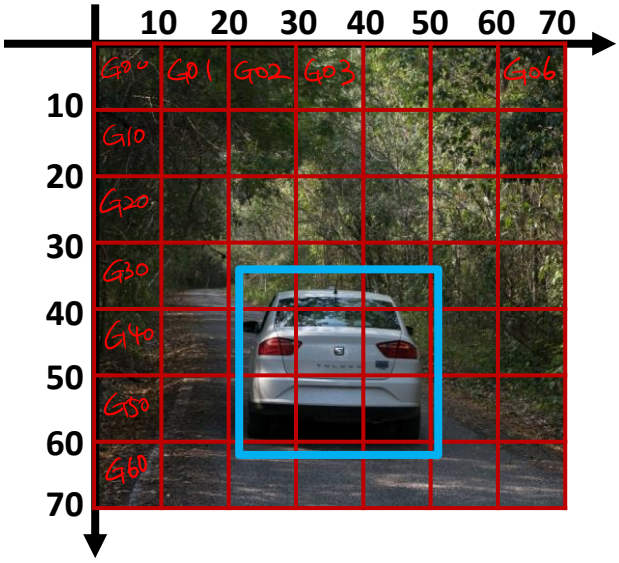
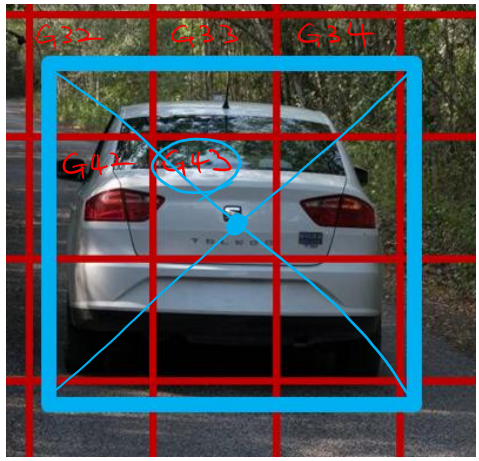
YOLO

- To us, YOLO stands for "You Only Look Once"





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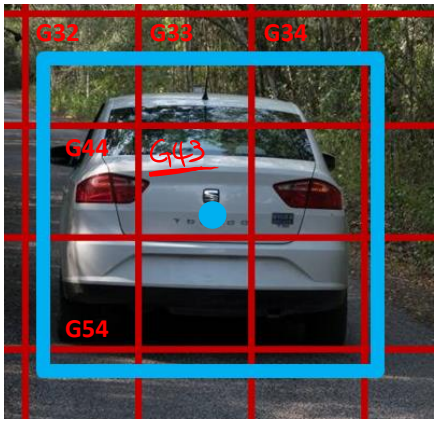
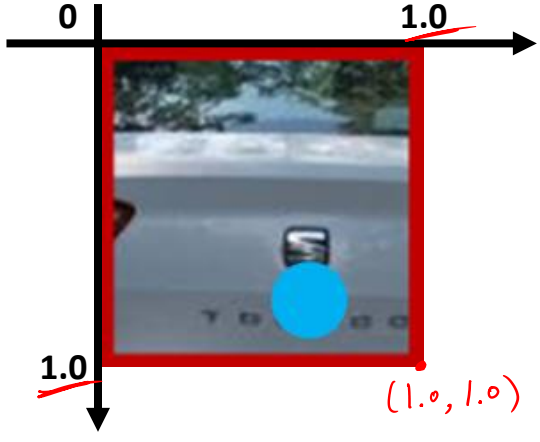
YOLO

- Labeling for the training set
  - The object is assigned to only one grid cell which includes the center of the bounding box





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YOLO

- Labeling for the training set
  - The object is assigned to only one grid cell

$\vec{y} = [P_c, b_x, b_y, b_h, b_w, c_1, c_2 \cdots]^T$ (car)

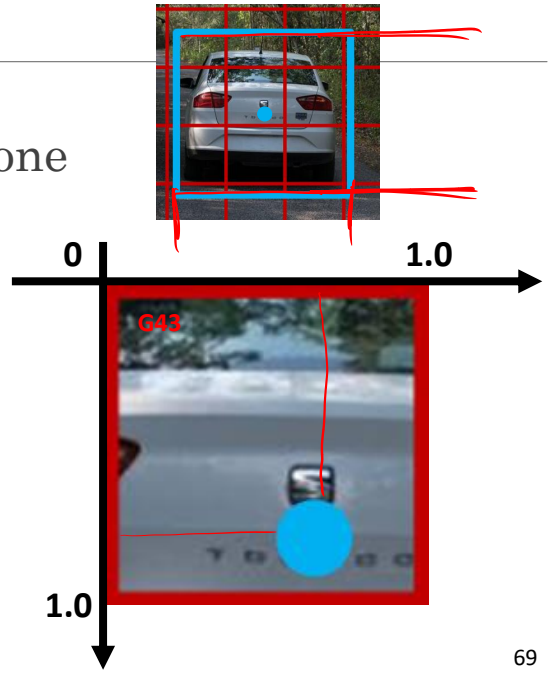
$\vec{y}_{43} = [1, 0.9, 0.8, 2.5, 2.9, 0, 1, \cdots]^T$

$\vec{y}_{42} = [0, ?, ?, ? \cdots]^T$

$\vec{y}_{53} = [0, ?, ?, ? \cdots]^T$

$|\vec{y}| \times 7 \times 7 = |\vec{Y}|$

$(5+4)$



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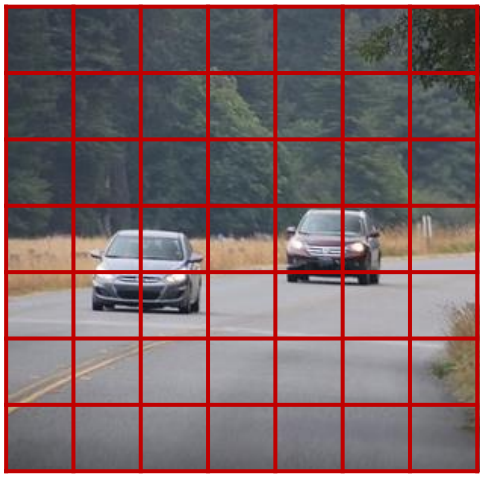
YOLO

- At test phase
  - Multiple grids may include the same object.



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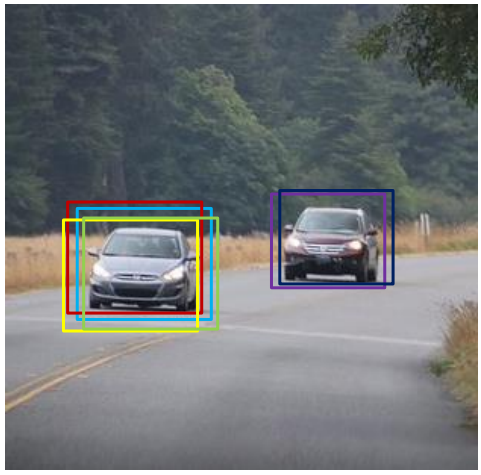
YOLO

- Evaluation of bounding boxes



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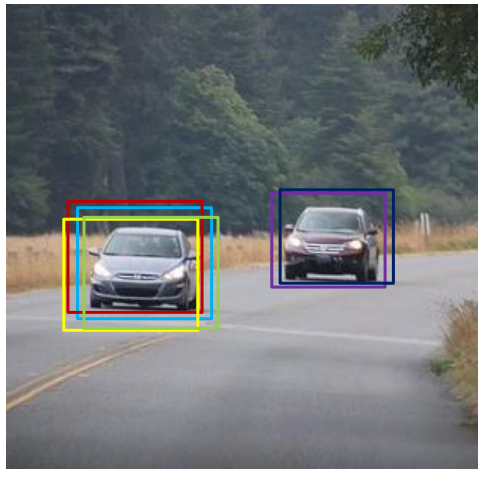
YOLO

- Non-maximal suppression
  - Drop boxes s.t. $p_c < threshold$
  - Group boxes that are overlapped
    - Based on intersection of union (IoU) of A and B
    - $IoU = \frac{A \cap B}{A \cup B}$
    - For instance, $IoU > 0.5$
  - For each group, keep a box whose p_c is largest



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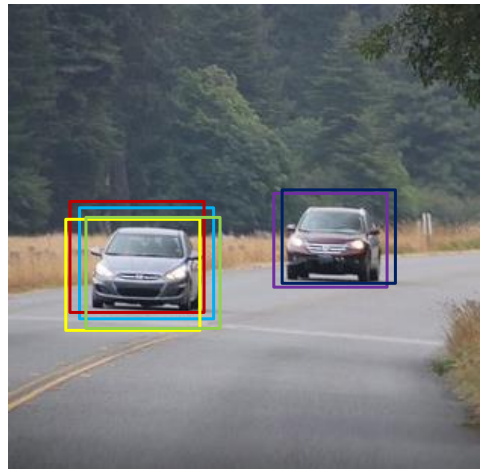
YOLO

- Non-maximal suppression
  - $\vec{y}_{red} = [0.96, \dots]^T$
  - $\vec{y}_{blue} = [0.97, \dots]^T$
  - $\vec{y}_{green} = [0.94, \dots]^T$
  - $\vec{y}_{yellow} = [0.93, \dots]^T$
  - $\vec{y}_{navy} = [0.95, \dots]^T$
  - $\vec{y}_{purple} = [0.98, \dots]^T$



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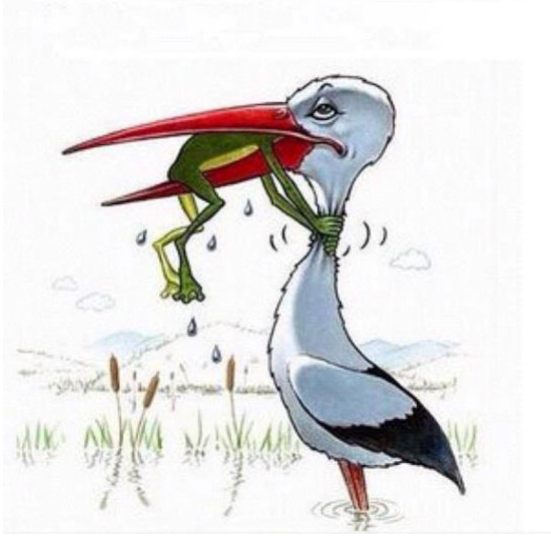
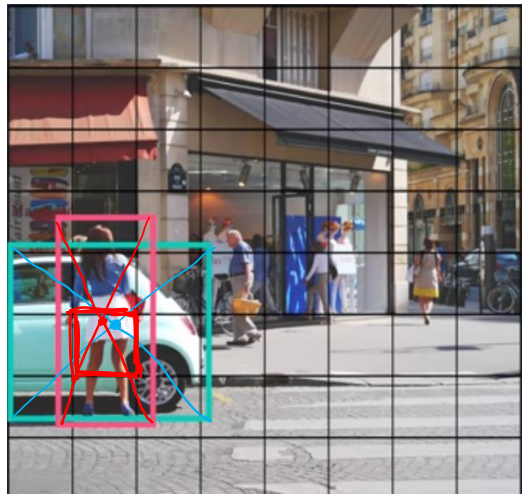
Never enough

- It ain't over till it's over





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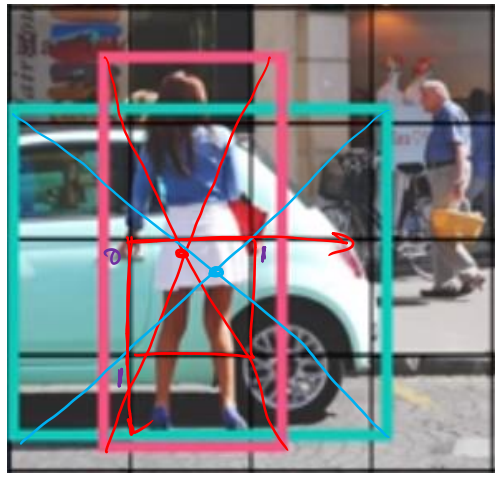
Anchor Boxes

- A grid can have multiple objects.

$$\vec{y} = \begin{bmatrix} p_c, b_x, b_y, b_h, b_w \\ p_c, b_x, b_y, b_h, b_w \\ c_1, c_2, \cdots \end{bmatrix}^T$$

ex) $\vec{y} = \begin{bmatrix} 1, 0.4, 0.1, 3.1, 1.4 \\ 1, 0.8, 0.3, 2.8, 3.1 \\ 0, 0, \cdots 1, \cdots, 1, 0 \end{bmatrix}$

↑ human ↑ car



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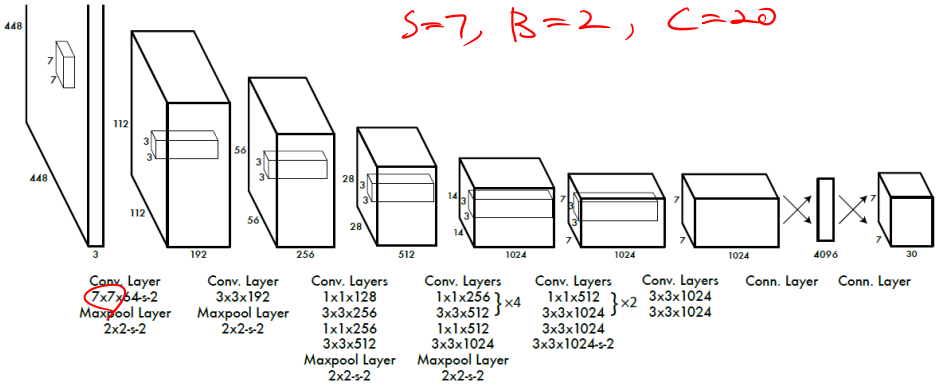
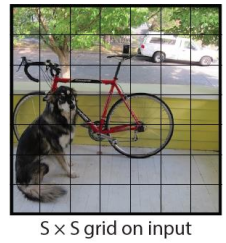
YOLO

- YOLO divides the image into an $S \times S$ grid and for each grid cell predicts B bounding boxes, confidence for those boxes, and C class probabilities. These predictions are encoded as an $S \times S \times (B \times 5 + C)$ tensor.

$S=7, B=2, C=20$





S × S grid on input

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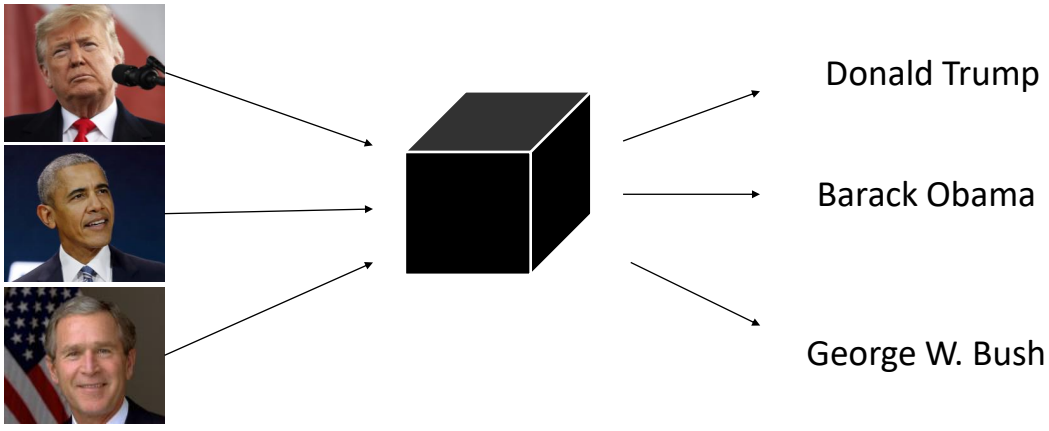
Face Recognition

- Identifying or verifying a person from visual data.
  - Input: image / video
  - Output: id / name

Closed set recognition



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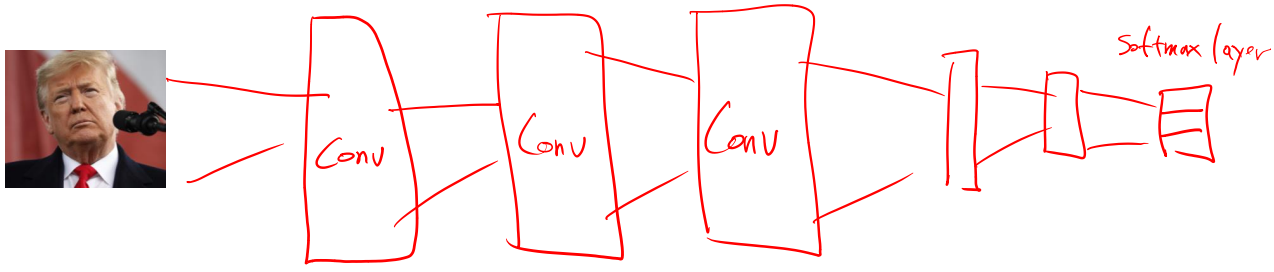
Face Recognition

- Assume we have enough labeled data
- Network architecture (1st trial)
  - Convolutional layers (Dense layers)
  - Softmax layer



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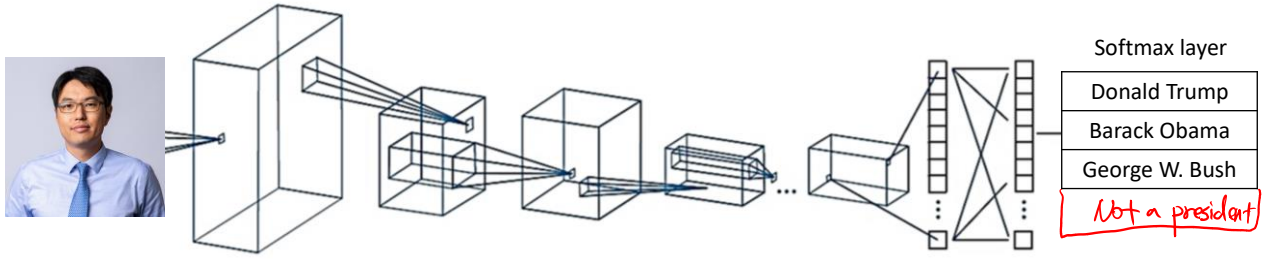
Face Recognition

- Assume we have enough labeled data
- Network architecture (2nd trial)
  - Convolutional layers (Dense layers)
  - Softmax layer



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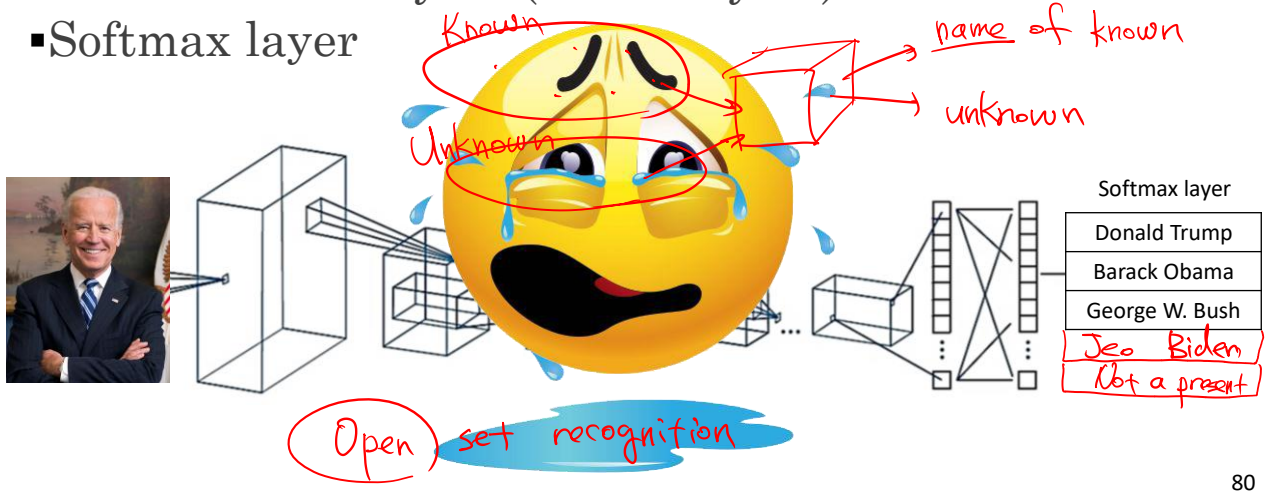
Face Recognition

- Network architecture (3rd trial)
  - Convolutional layers (Dense layers)
  - Softmax layer



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Metric based recognition

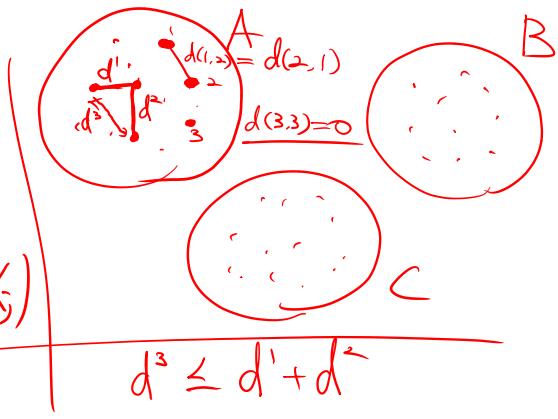
- Let $\vec{X}_i$ is an image of i.
- Assume we have a distance function $d(\vec{X}_i, \vec{X}_j)$ such that

$d(X_i, X_j) \geq 0$

$d(X_i, X_j) = 0 \Leftrightarrow X_i = X_j$

$d(X_i, X_j) = d(X_j, X_i)$

$d(X_i, X_j) \leq d(X_i, X_k) + d(X_k, X_j)$



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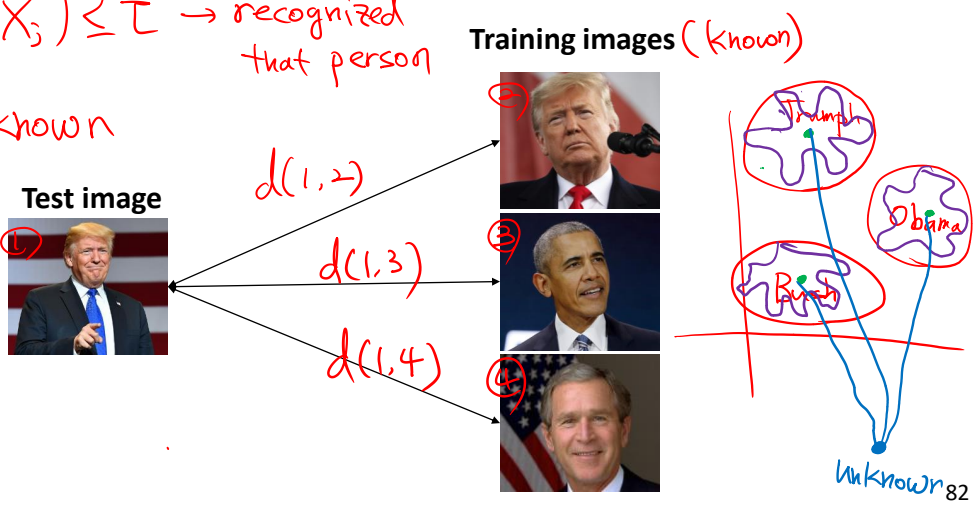
Metric based recognition

- Assume we have a metric $d(\vec{X}_i, \vec{X}_j)$

If $d(X_i, X_j) \leq \tau$ → recognized that person

else ⇒ unknown



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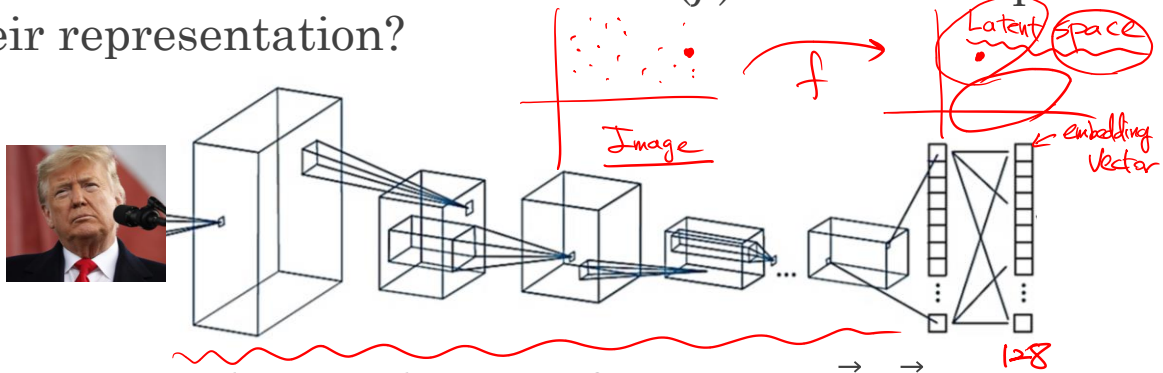
Representation Learning

- Can we utilize a neural network (f) to encode inputs to their representation?



- How can we have a distance function $d(\vec{X}_i, \vec{X}_j)$?

$d(X_i, X_j) = \| f(X_i) - f(X_j) \|_2^2 \quad \| X_i - X_j \|_2^2$

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Embedding Vector

- We need to train a network to encode inputs such that
  - $d(\vec{X}_i, \vec{X}_j) = \|f(\vec{X}_i) - f(\vec{X}_j)\|_2^2$ is a distance between $\vec{X}_i$ and $\vec{X}_j$

$d(X_1, X_2) < d(X_1, X_3)$
$d(X_1, X_2) < d(X_1, X_4)$
$d(X_3, X_4) < d(X_3, X_1)$
$d(X_3, X_4) < d(X_3, X_2)$
$d(X_1, X_2) \; ? \; d(X_3, X_4)$

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Triplet Loss

- We want to ensure that an image $\vec{X}^a$ (anchor) of a specific person is closer to all other images $\vec{X}^p$ (positive) of the same person than it is to any image $\vec{X}^n$ (negative) of any other person.

$d(X^a, X^p) < d(X^a, X^n)$ for all pairs

$d(X^a, X^p) + \alpha < d(X^a, X^n)$ α is a margin.

$\|f(X^a) - f(X^p)\|_2^2 + \alpha - \|f(X^a) - f(X^n)\|_2^2 < 0$

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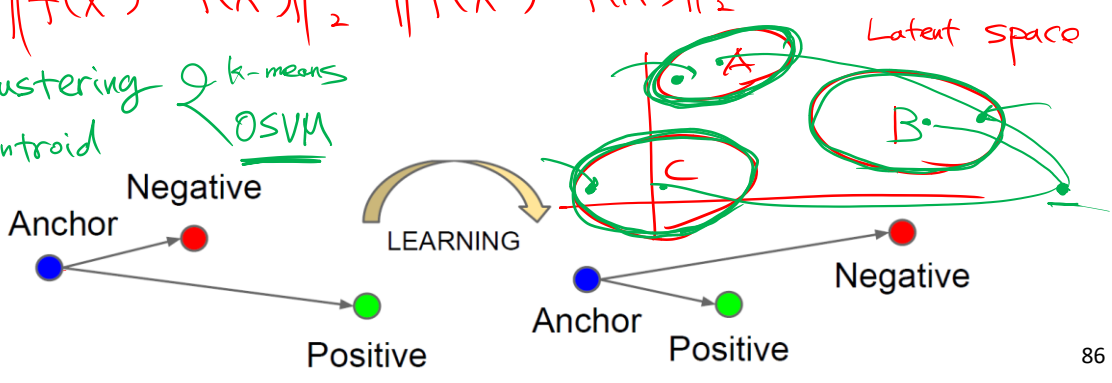
Triplet Loss

- Triplet Loss minimizes the distance between an anchor and a positive and maximizes the distance between the anchor and a negative

$L = \|f(X^a) - f(X^p)\|_2^2 - \|f(X^a) - f(X^n)\|_2^2 + \alpha$

1. Clustering & k-means
2. Centroid

OSVM



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