



Topics covered:

- Course Flow
- Flow charts
- Pseudocode

Q. How to solve a programming problem?

→ Given the problem,

- ① Understand the problem. → Add 2 numbers
- ② Check the given values. → 2 variables. Data Types?
- ③ Figure out an approach → $a+b = \text{my answer}$.
 - This comes from practice and past coding experience.
- ④ Code! → `int ans = a+b; cout << ans << endl;`

WARNING

Agar code samajh na aaye to



Aage sab kuch cover hoga.

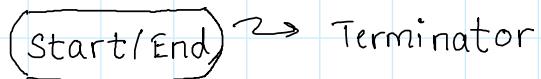
Given some problem **P**, say you 'think' of some solution, ki aise aise karenge, etc. Now write down this crude solution on paper, not necessarily in correct syntax (code ki bhasha). Now your idea is on paper. Convert this rough work, also called 'pseudocode' into a program in a programming language of your choice, say C++.

Pseudocode: A very simple and high-level (upar-upar ka) form of computer language that is used in program design.

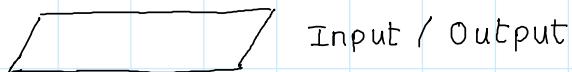
A Flowchart is a diagrammatic representation of an approach. This draws out all the steps of your approach in order.

Components:

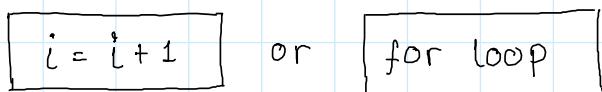
① Terminator: Specifies the start and end of a program.



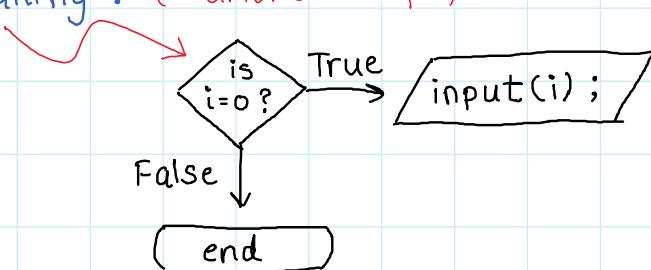
② Parallelogram: For taking input or showing output.



③ Process: Operations and processes ke liye.



④ Decision Making: (Diamond Shape)



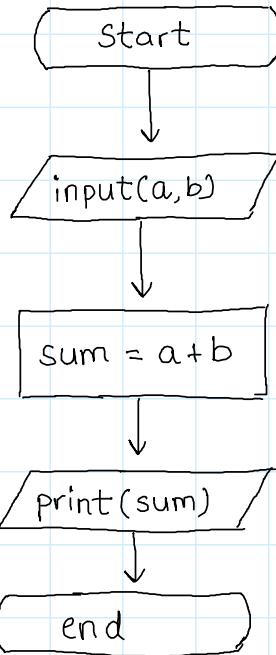
⑤ Circle: Connectors (To be covered when we discuss functions / methods)

⑥ Arrows: Code ka pravaah dikhane ke liye.

(Upar flowchart dekho ↑)

EXAMPLE :

Flow chart for adding 2 numbers.



Pseudocode for adding 2 numbers:



- input 2 numbers a and b.
- Let sum = a+b
- Print out sum



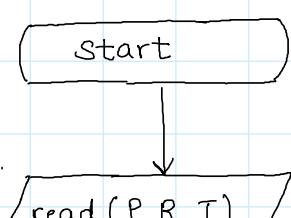
- Read a and read b.
- Sum variable is a+b
- sum chaapo

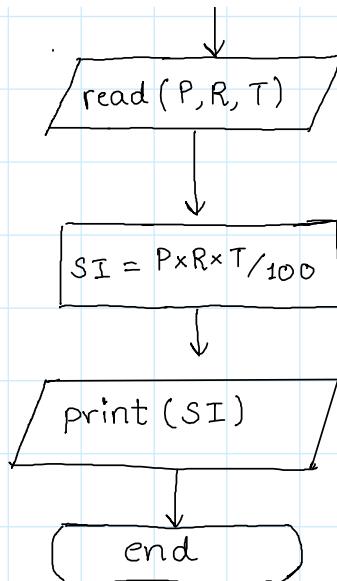
Both pseudocodes are OK. No pseudocode is wrong as long as the logic is same / similar.

EXAMPLE: Calculate simple interest.

$$SI = \frac{P \times R \times T}{100}$$

(P - Principal Amount)
(R - Rate of Interest)
(T - Time)

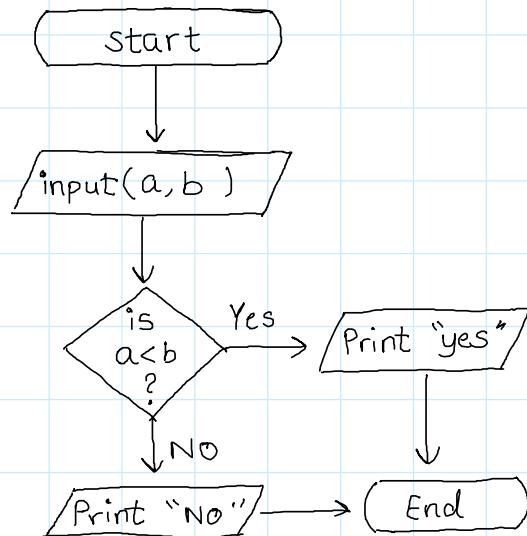




Pseudocode

- read P, R and T
- Make $SI = P \times R \times T / 100$
- Print SI

EXAMPLE: Determine if $a < b$



PSEUDOCODE:

- Read a and b.
- if $a < b$
 then print Yes
else
 print No

NEW CONCEPT: % (modulo) operator

Gives the remainder after division a/b .

$\therefore a \% b = \text{Remainder of } a/b$.

Eg: $5 \% 2 = 1$

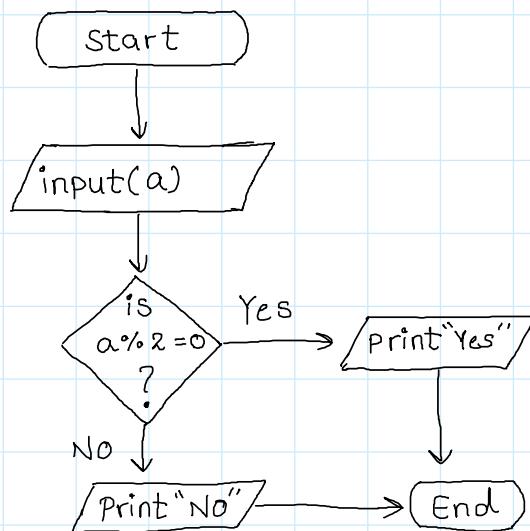
$$6 \% 4 = 2$$

$$8 \% 4 = 0$$

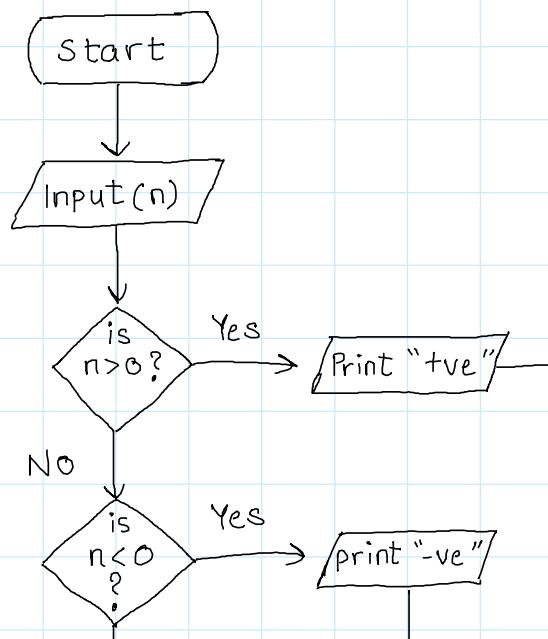
$$* 4 \% 9 = 4 \quad \text{when } a > b, a \% b = a.$$

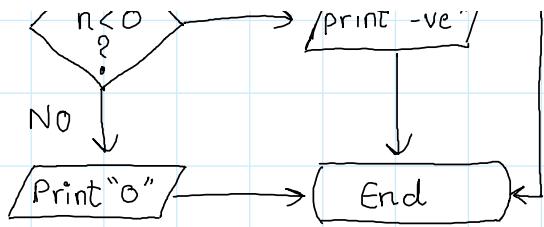
\therefore If $n \% 2 = 0$, n is even
else n is odd.

EXAMPLE: Check if n is even or odd.



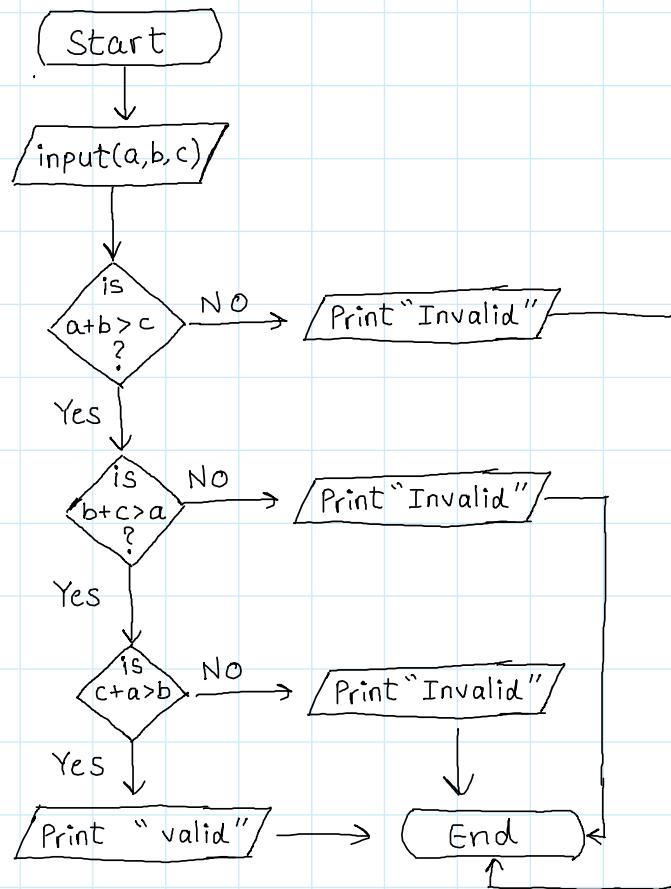
EXAMPLE: Is n positive, negative or zero.





Homework : Check if a given triangle is valid.

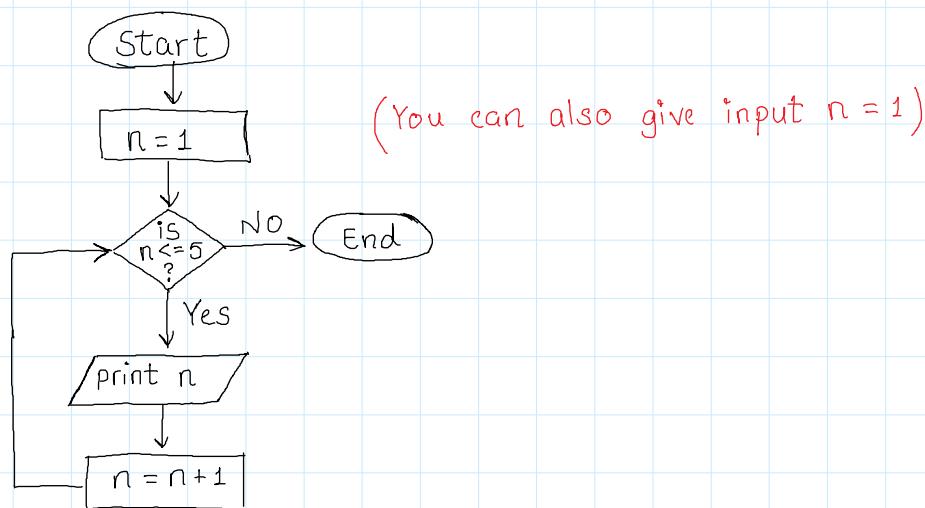
Hint - $a+b>c$, $b+c>a$ & $c+a>b$



Let Variable $n = 1$.

Now make n go from 1 to 5.

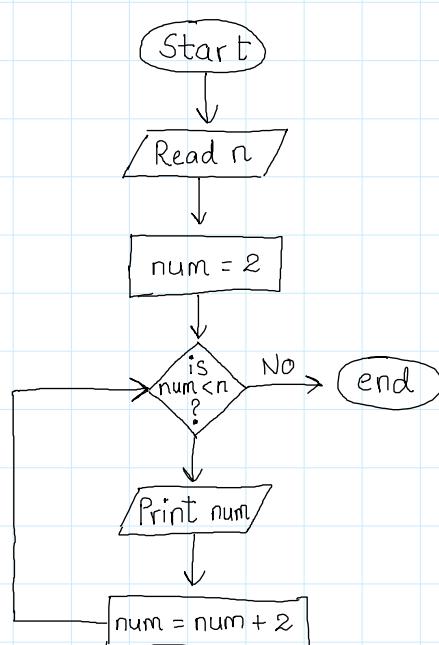
Soln.



This is where we use loops to continuously perform some action while updating some value.

EXAMPLE : Print even numbers b/w 1 and n . (exclusive)

Hint - Even numbers start from 2 and occur alternatively.



Pseudocode

```

→ Input(n)
→ Let num = 2
→ while num < n,
  →   print num
  →   num = num + 2
→ End
  
```

EXAMPLE : Print all odd numbers from 1 to n (inclusive)
(Homework hai)

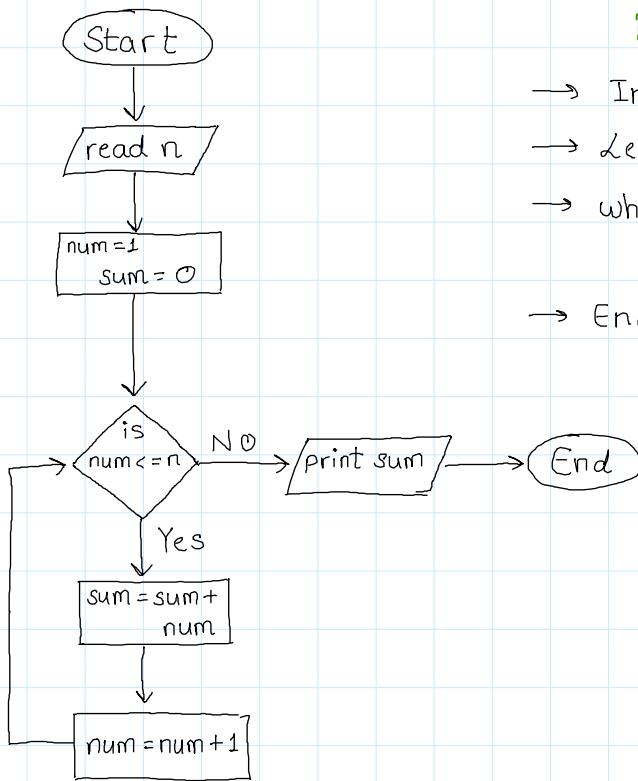
Pseudocode

```

→ Input (n)
→ Let a = 1
→ while a < 1, print(a)
      a=a+2.
→ End

```

EXAMPLE : Find sum from 1 to n.



Pseudocode

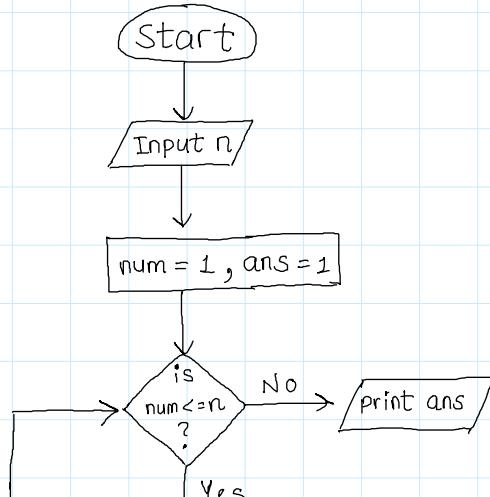
```

→ Input (n)
→ Let num = 1 and sum = 0
→ while num <= n,
      sum = sum + num
      num = num + 1
→ End

```

Homework : Find n!

Hint - $n! = n \times (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1$

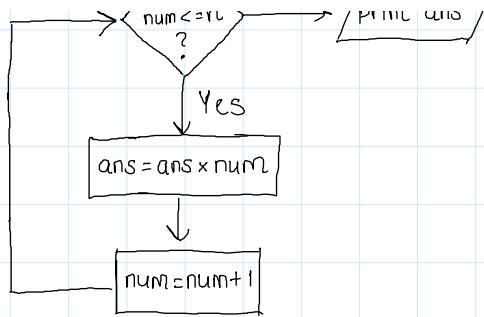


Pseudocode

```

→ Input (n)
→ Let num = 1 and ans = 1
→ while num <= n,
      ans = ans × num
      num = num + 1
→ End

```



Example: Check if n is prime.

Input: 5

Output: Yes

Prime n :

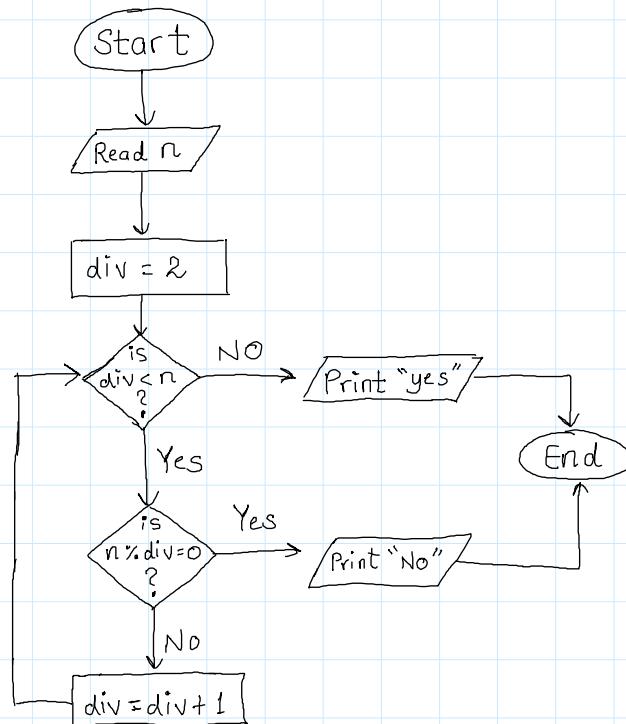
$n \% \text{(any number from 2 to } n-1) \neq 0$

Input: 9

Output: No

Written as

$\neq 0$ in C++



Pseudocode

```

→ Input (n)
→ Let div = 2
→ while div < n,
   if n % div = 0
      print "No"
      exit
   else
      div = div + 1
   → Print "Yes"

```

What is a Programming Language?

- Jaise khaana khaane ke liye mummy/papa ko bolna padhta hai, vaise hi we must instruct our computer to perform some task for us.
- More formally, a programming language is a way to communicate with a computer. It is a formal language which consists of sets of strings that produce various kinds of machine output.

Eg: C, C++, Java, Python, R, GO, etc.

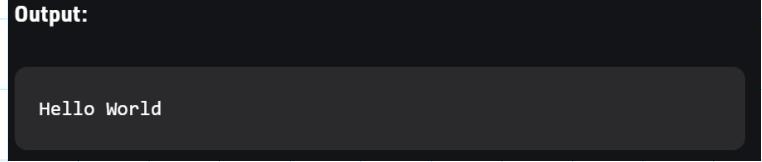
```
#include<iostream>

using namespace std;

// main function -
// where the execution of program begins
int main()
{
    // prints hello world
    cout<<"Hello World";

    return 0;
}
```

Output:



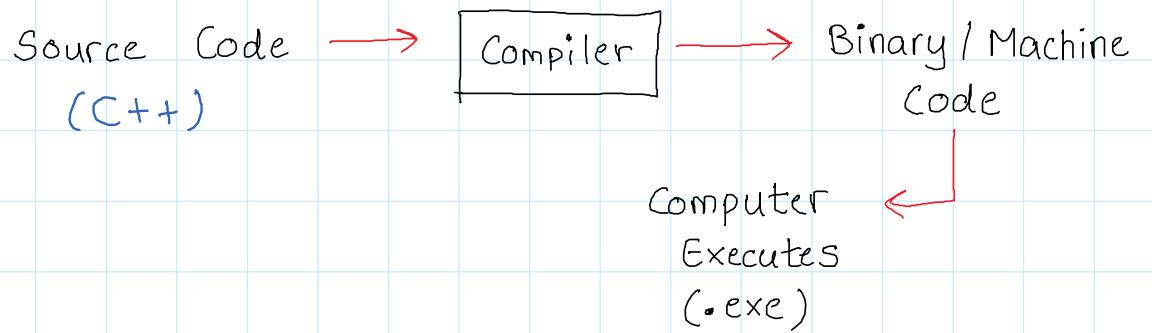
```
Hello World
```

⇒ This code in C++ instructs our computer to print "Hello World" on our screen.

Source: <https://www.geeksforgeeks.org/writing-first-c-program-hello-world-example/>

- Every language must be written following some rules called syntax of that language.
- A computer essentially only understands binary codes of 0s and 1s. A compiler processes the statements of a programming language into Machine Code (Binary).

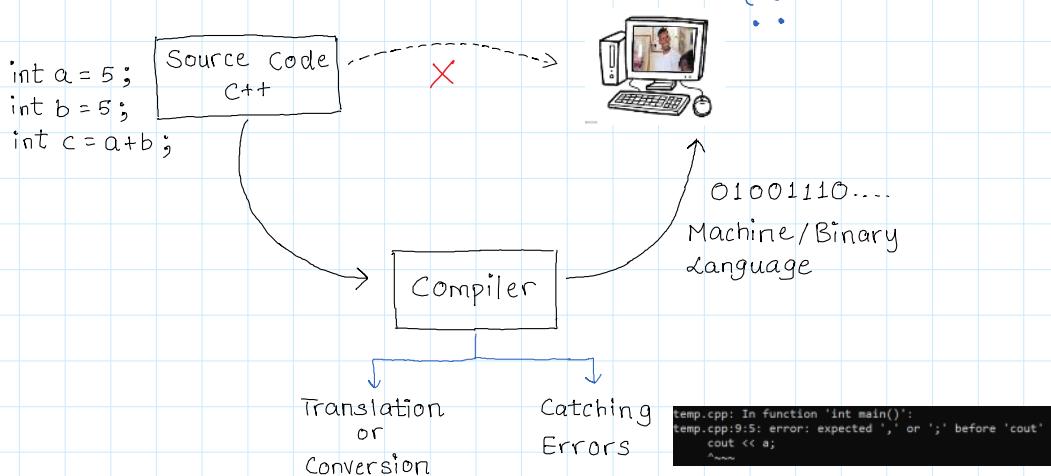




Read more about Compilers here: <https://www.geeksforgeeks.org/introduction-to-compilers/>

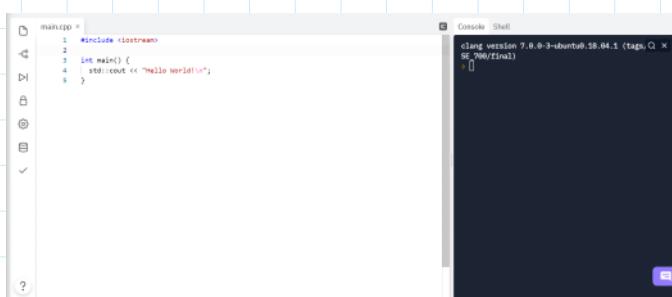


As discussed earlier :



IDEs (Integrated Development Environment) :

An environment that helps you write, run and even debug (in some cases) code in a programming language.
Eg: VS CODE, Code Studio, Eclipse, NetBeans, etc.



First Program in C++ : (Namaste Duniya)

- ① Our program will find `int main(){` and start executing
from there.
- ② `int main(){}` → These brackets show the 'scope' of the
main function i.e. the code which belongs to / is defined
within `int main()` function.

[More about scope of a function](#)

- ③ In C++, we use 'cout' to print something.
Eg: `cout << "Namaste Duniya";`
- ④ This cout is already defined in a file (header file)

which must be included before using cout.

⇒ `#include <file_name>` is a preprocessor directive

which runs before the program is compiled and includes the file to be used later in the source code.

A file called iostream has cout defined in it so:

`#include <iostream>`

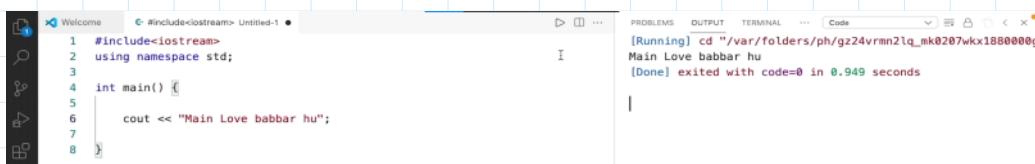
Hint: i/o means input/output.

⑤ Namespaces : [Stack Overflow Question](#)

`using namespace std;`

⑥ Using cout :

We use '`<<`' after 'cout' to display something to Standard Output (your screen) within std namespace.

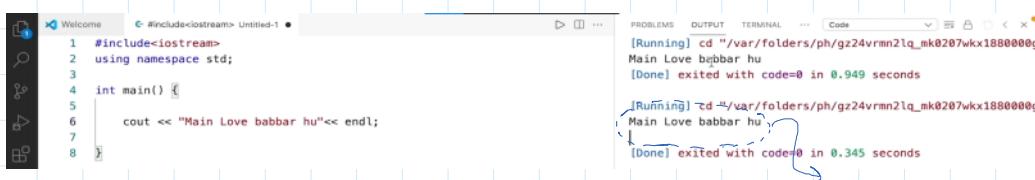


```
#include<iostream>
using namespace std;
int main() {
    cout << "Main Love babbar hu";
}
```

⑦ endl : Used to enter new line. Just like ENTER.

endl is like "`\n`" which is a new line escape sequence character used in various languages including C++.

`cout << "Namaste Duniya" << endl;`



```
#include<iostream>
using namespace std;
int main() {
    cout << "Main Love babbar hu" << endl;
}
```

⑧ `endl;` ; is used to terminate statements.

DATA TYPES : Different types of data to be stored in memory. Eg- integer, float, character, double, etc.

Eg- int: Stores integers like -5, 0, 8, etc.

char: Single character like 'a', '+', '\$', '7', etc.

float: Floating point values like -2.014, 1.0000, 6.7800

Different data types use different amounts of memory. Amount of memory used also depends on the architecture of your CPU.

Data Type	Meaning	Size (in Bytes)
int	Integer	2 or 4
float	Floating-point	4
double	Double Floating-point	8
char	Character	1
wchar_t	Wide Character	2
bool	Boolean	1
void	Empty	0

[Source: Programiz](#)

Character : A 1-byte (= 8 bits) data type that takes 1 character.

```
char ch = 'a';
```

Boolean : True / False . Take 1 bit and 1 : True
0 : False

```
bool isGood = 1;
bool isBad = false;
```

Float / Double : Float takes 4/8 bytes
Double takes 8 bytes.

```
float num1 = 1.2;
double num2 = 2.4;
```

[Know more about C++ Data Types](#)

Variable Naming / Nomenclature :

- ① Can contain alphabets, numbers and underscores.
- ② Cannot start with a number.
- ③ Cannot be keywords like int, cout, double, bool, etc.
- ④ Case sensitive
- ⑤ Cannot contain special symbols like %, \$, !, #, etc.

WARNING: Don't use same variable name multiple times.

```
6 int a = 123;
7
8 cout << a << endl;
9
10 char a = 'v';
11
12 cout << a << endl;
```

```
[1:1] cd "/var/folders/ph/gz24vrnn2lq_mk0207wkh188000gn/T/"
[1:1] CodeRunnerFile.cpp:10:10: error: redefinition of 'a' with a
[1:1]     ^~~~~
[1:1] CodeRunnerFile.cpp:6:9: note: previous definition is here
[1:1]     ^~~~~
[1:1]     int a = 123;
```

Using different data types in code :

```

1 #include<iostream>
2 using namespace std;
3
4 int main() {
5
6     int a = 123;
7
8     cout << a << endl;
9
10    char b = 'v';
11
12    cout << b << endl;
13
14    bool bl = true;
15    cout << bl << endl;
16
17    float f = 1.2;
18    cout << f << endl;
19
20    double d = 1.23;
21    cout << d << endl;
22
23 }

```

[Running] cd "/var/folders/ph/gz24vrmm2lq_mk0207wkhx1880000g
123
v
1
1.2
1.23
[Done] exited with code=0 in 0.595 seconds
[Running] cd "/var/folders/ph/gz24vrmm2lq_mk0207wkhx1880000g
123
v
1
1.2
1.23
[Done] exited with code=0 in 0.551 seconds
[Running] cd "/var/folders/ph/gz24vrmm2lq_mk0207wkhx1880000g
123
v
1
1.2
1.23
[Done] exited with code=0 in 0.537 seconds

Check the size of different Data Types for your system using `sizeof(variable_name)`;

```

1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int a = 4;
6     double b = 1.90;
7     char c = 'a';
8     int sizeInteger = sizeof(a), sizeDouble = sizeof(b), sizeChar = sizeof(c);
9     cout << "Size of an integer is " << sizeInteger << " bytes" << endl;
10    cout << "Size of a double is " << sizeDouble << " bytes" << endl;
11    cout << "Size of a char is " << sizeChar << " bytes" << endl;
12 }

```

Console Shell
> clang++-7 -fno-rtti -std=c++17 -o main mainQ.x
> ./main
Size of an integer is 4 bytes
Size of a double is 8 bytes
Size of a char is 1 bytes
>

HOW IS DATA STORED IN MEMORY ?

Eg: `int a = 8;` // int takes 4 bytes = 32 bits

In binary, $8 = 1000$ (4 bits needed)

\therefore `int a` 00000000 00000000 00000000 00001000 } 32 bits

Eg: `int b = 5;`

b
5

4 bytes.
address = 100 (assume)
100,101,102,103
4 bytes are consumed

Eg: `char c = 'a';`

Characters are mapped to the standard ASCII values

<code>'a'</code> \rightarrow 97	<code>'A'</code> \rightarrow 65
<code>'b'</code> \rightarrow 98	<code>'B'</code> \rightarrow 66
<code>'c'</code> \rightarrow 99	<code>'C'</code> \rightarrow 67
<code>' '</code> \rightarrow 32	<code>' '</code> \rightarrow 32
<code>'Z'</code> \rightarrow 90	<code>'Z'</code> \rightarrow 90

ASCII Table
(Homework)

`char c = 'a';`

ASCII → 97 → Binary → 1100001 → Store

0 1 1 0 0 0 1
1 byte



Conversion of one data type to another (if it is valid) is called Type Casting.

Example:

```
int a = 'a';
```

variable a will store the ASCII value of 'a' = 97.

Example:

```
char ch = 98;
```

98 will automatically get typecasted to its corresponding character.

This automatic typecasting is called implicit typecasting.

```
int a = 'a';
cout << a << endl;
char ch = 98;
cout << ch << endl;
```

```
[Running] cd "/var/folders/ph/gz24vrmn2lq_mk0207wkh1880000g
97
b
```

What if we type cast from int to char but the value is too large to be stored in char?

Soln: A warning is thrown and the last byte from the original value is given to the character.

```
char ch1 = 123456;
cout << ch1 << endl;
```

```
[Running] cd "/var/folders/ph/gz24vrmn2lq_mk0207wkh1880000g
tempCodeRunnerFile.cpp:33:16: warning: implicit conversion
      char ch1 = 123456;
                  ^~~~ ^~~~
1 warning generated.
```

How are negative numbers stored?

Soln: The first bit tells us whether the number is positive or negative.

First Bit → 0 means Positive
→ 1 means Negative

Steps to store -5 in binary format:

- ① Ignore the -ve sign. (5)
- ② Write the binary representation of 5.
- ③ Take its 2's complement and store it.

Example: a = -5.

- ① $-5 \rightarrow 5$ (Ignore the -ve sign)
- ② $5 \rightarrow 0101 \rightarrow \underbrace{00\ldots}_{29 \text{ Zeros}} 0101$ (Binary)

③ 2's complement = 1's complement + 1.

$$\begin{array}{r}
 5 \rightarrow 00\ldots 0101 \\
 1's \text{ compl.} \rightarrow 11\ldots 1010 \quad (\text{Flip the bits}) \\
 + \underline{\hspace{2cm}} 1 \\
 2's \text{ compl.} \rightarrow \textcircled{11\ldots 1011}
 \end{array}$$

$$+ \underline{\quad \quad \quad 1}$$

2's compl. \rightarrow 11.....1011
29 Ones

Displaying Negative Number :

- ① Take 2's complement of the stored number

Stored : 11.....1011
 ↓
 This shows -ve

$$\begin{array}{r} 11\dots 1011 \\ 1's \text{ comp. } 00\dots 0100 \\ 2's \text{ comp. } 00\dots 0101 = 5 \end{array}$$

-5 print ho gaya!



If we stored numbers as it is without using 2's complement, then

1	0	0	---	0	0
---	---	---	-----	---	---

and

0	0	0	---	0	0
---	---	---	-----	---	---

will be equal &
thus waste space.

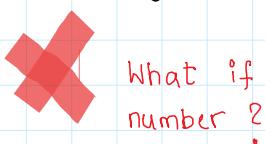


Store only Positive Integers

The default signed representation allows us to store both positive & negative values.

To store only positive integers, we use unsigned..

Eg: unsigned int a = 10 ;



Example: unsigned int a = -112 ;
`cout << a << endl ;`

Output:

4294967184 ??

Explanation :

We tried to store -112.

$-112 = 2's \text{ Complement of } 112.$

$112 = \underbrace{00\dots 0}_{25 \text{ Zeros}} 1110000.$

1's compl. = 11.....10001111
 $+ \underline{\quad \quad \quad 1}$

2's Compl. = 11.....10010000

Unsigned int uses all 32 bits to store the value and the MSB (=1) will make the value.

An unsigned int does not use the 2's complement again to display the number.

Thus, 11.....10010000. gets printed as it is in decimal.
25 ones

Binary to Decimal converter

From	To
Binary	Decimal
Enter binary number	
1111111111111111111111110010000 2	
= Convert	Reset
Decimal number	
4294967184 10	
Decimal from signed 2's complement	
-112 10	
Source	

Therefore ,

```
unsigned int a = -112; → [Running] cd "/var/folders/ph/gz24vrmn2lq_mk0207wkh1880000g  
cout<< a << endl;
```



Basic Arithmetic Operators:

$+, -, *, /, \%$

Caution

- ① $\text{int} / \text{int} = \text{int}$ (Floor value of answer)

Examples: $5/2 = 1$.

$$3/5 = 0$$

$$9/2 = 4$$

- ② $\text{int} / \text{float} \quad \} \text{ float}$
 $\text{float} / \text{int} \quad \}$

- $\text{double} / \text{int} \quad \} \text{ double}$
 $\text{int} / \text{double}$

`cout << 5.0/2 << endl;`

Output:

2.5

```

42 float a = 2.0/5;
43 cout << a << endl;
44
45 cout << 2.0/5 << endl;
46
47

```

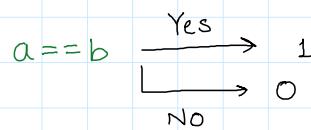
[Running] cd "/var/folders/ph/gz24vrmn2lq_mk0207wkh1880000g
0.4
0.4

[Done] exited with code=0 in 0.591 seconds

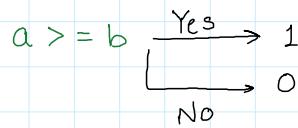
Relational Operators :

$=, >, <, <=, >, <, !=$

- ① Is $a = b$?



- ② Is a greater than or equal to b ?



Untitled-1

```

49 int a = 2;
50 int b = 3;
51
52 bool first = (a==b);
53 cout << first << endl;
54
55 bool second = (a>b);
56 cout << second << endl;
57
58 bool third = (a);
59 cout << third << endl;
60
61 bool fourth = (a>=b);
62 cout << fourth << endl;
63
64 bool fifth = (a<b);
65 cout << fifth << endl;
66
67 bool sixth = (a!=b);
68 cout << sixth << endl;
69

```

[Running] cd "/var/folders/ph/gz24vrmn2lq_mk0207wkh1880000g
0

[Done] exited with code=0 in 0.851 seconds

[Running] cd "/var/folders/ph/gz24vrmn2lq_mk0207wkh1880000g
0
0
1
0
1
1

[Done] exited with code=0 in 1.124 seconds

Logical Operators :

$\&&, ||, !$
(AND) (OR) (NOT)

- ① Logical AND :

All conditions must be true for the output to be true.

Example : $\text{int } a = 5, b = 10, c = 15;$

```
cout << ((a>0) && (b!=0) && (c<=15));
```

Output :

1

② Logical OR :

At least 1 condition must be true for the output to be true.

Example : int a=5, b=10, c=15;

```
cout << ((a>5) || (b<10) || (c >=15));
```

Output :

1

③ Logical NOT :

Inverts the logic. True \rightleftharpoons False

Non-zero \rightleftharpoons Zero

Example : int a=10, b=0;

```
cout << (!a) << endl;
```

```
cout << (!b) << endl;
```

Output :

0

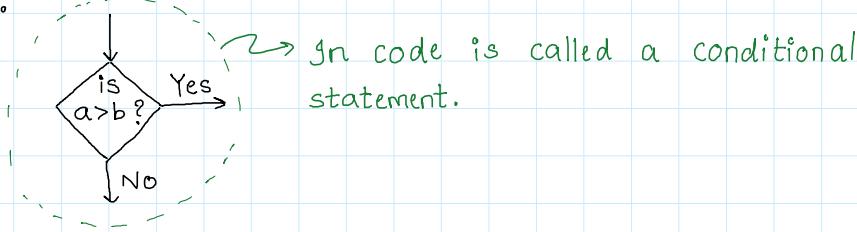
1

Point out what was not covered in Typecasting and try to code yourself.

Conditionals



Recall :



Example :

I/P : a, b

O/P : $\begin{cases} a & \text{if } a > b \\ b & \text{otherwise} \end{cases}$

Conditionals like these are solved using if statements.

`if (~~~) {`

~~~~~

~~~~~

~~~~~

}

If the condition within ( ) is true, then execute the entire code block within { }.

Thus,

```
if (a > b){  
    cout << a << endl;  
}  
if (a <= b){  
    cout << b << endl;  
}
```

In the above example, instead of checking again for  $b > a$  in the second if block, we know that if the first if condition does not get fulfilled, then the second block must be executed no matter what. This can be achieved using an if - else block.

*If this is fulfilled, else block won't execute*

```
if (a > b){  
    cout << a << endl;  
}  
  
If if block doesn't execute, then this else block will execute.  
else{  
    cout << b << endl;  
}
```

NEW CONCEPT :

`(cin >> n)` → Waits for user to give input and assigns it to n at its address.

Example -

```
int a;
cin >> a;
```

- ① variable a will get initialized with a random, 'garbage' value.
- ② Program waits for user to give input (an integer) to a and stores a with that value.

```
inputAndOutput.cpp
1 #include <iostream>
2 using namespace std;
3
4 int main(void)
5 {
6     int a;
7     cout << "Value of a just after initialization = " << a << endl;
8     cin >> a;
9     cout << "Value of a just after cin statement = " << a << endl;
10    return 0;
11 }

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

Try the new cross-platform PowerShell https://aka.ms/pscore6

PS C:\Users\ [REDACTED] ; if ($?) { g++ InputAndOutput.cpp -o InputAndOutput } ; if ($?) {
.\InputAndOutput }
Value of a just after initialization = 4281131
```

Fig ①

```
inputAndOutput.cpp
1 #include <iostream>
2 using namespace std;
3
4 int main(void)
5 {
6     int a;
7     cout << "Value of a just after initialization = " << a << endl;
8     cin >> a;
9     cout << "Value of a just after cin statement = " << a << endl;
10    return 0;
11 }

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

Try the new cross-platform PowerShell https://aka.ms/pscore6

PS C:\Users\ [REDACTED] ; if ($?) { g++ InputAndOutput.cpp -o InputAndOutput } ; if ($?) {
.\InputAndOutput }
Value of a just after initialization = 4281131
5
Value of a just after cin statement = -1
```

Fig ②

Back to if - else :

Here, the if block doesn't execute and the program exits without printing anything

①

```
InputAndOutput.cpp
1 #include <iostream>
2 using namespace std;
3
4 int main(void)
5 {
6     int a;
7     cin >> a;
8
9     if(a > 0) {
10         cout << "a is positive" << endl;
11     }
12     return 0;
13 }
```

```
PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

{ .\InputAndOutput }
5
a is positive
```

②

```
InputAndOutput.cpp
1 #include <iostream>
2 using namespace std;
3
4 int main(void)
5 {
6     int a;
7     cin >> a;
8
9     if(a > 0) {
10         cout << "a is positive" << endl;
11     }
12     return 0;
13 }
```

```
PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

-1
PS
```

③

```
InputAndOutput.cpp
1 #include <iostream>
2 using namespace std;
3
4 int main(void)
5 {
6     int a;
7     cin >> a;
8 }
```

④

```
InputAndOutput.cpp
1 #include <iostream>
2 using namespace std;
3
4 int main(void)
5 {
6     int a;
7     cin >> a;
8 }
```

```

3
4 int main(void)
5 {
6     int a;
7     cin >> a;
8
9     if(a > 0) {
10         cout << "a is positive" << endl;
11     }
12     else {
13         cout << "a is not positive" << endl;
14     }
15     return 0;
16 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
.\InputAndOutput
5
a is positive
```

```

3
4 int main(void)
5 {
6     int a;
7     cin >> a;
8
9     if(a > 0) {
10         cout << "a is positive" << endl;
11     }
12     else {
13         cout << "a is not positive" << endl;
14     }
15     return 0;
16 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
.\InputAndOutput
-2
a is not positive
```

Program to compare two numbers :

```

C++ InputAndOutput.cpp X
1 #include <iostream>
2 using namespace std;
3
4 int main()
5 {
6     int a, b;
7     cout << "Enter the value of a: ";
8     cin >> a;
9     cout << "Enter the value of b: ";
10    cin >> b;
11
12    if(a > b) {
13        cout << "a is greater than b" << endl;
14    }
15    if (b > a) {
16        cout << "b is greater than a" << endl;
17    }
18    return 0;
19 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
.\InputAndOutput
Enter the value of a: 6
Enter the value of b: 4
a is greater than b
```

```

C++ InputAndOutput.cpp X
1 #include <iostream>
2 using namespace std;
3
4 int main()
5 {
6     int a, b;
7     cout << "Enter the value of a: ";
8     cin >> a;
9     cout << "Enter the value of b: ";
10    cin >> b;
11
12    if(a > b) {
13        cout << "a is greater than b" << endl;
14    }
15    if (b > a) {
16        cout << "b is greater than a" << endl;
17    }
18    return 0;
19 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
.\InputAndOutput
Enter the value of a: 3
Enter the value of b: 6
b is greater than a
```

Note : `cin` ignores `ENTER(\n)`, `TAB(\t)` and `SPACE( )` while taking input. These are called whitespace characters. Use `cin.get()` to read these whitespace characters.

Using nested if - else:

```

1 #include <iostream>
2 using namespace std;
3
4 int main()
5 {
6     int a;
7     cin >> a;
8
9     if(a > 0) {
10         cout << "a is positive" << endl;
11     }
12     else {
13         cout << "a is not positive" << endl;
14     }
15     return 0;
16 }
```

In this `else` block, we can see that either `a` will be negative or `0`. So we can further break down this `else` block.

```

1 #include <iostream>
2 using namespace std;
3
4 int main()
5 {
6     int a;
7     cin >> a;
8     if(a > 0) {
9         cout << "a is positive" << endl;
10    }
11    else {
12        if (a == 0) {
13            cout << "a is 0" << endl;
14        }
15        else {
16            cout << "a is negative" << endl;
17        }
18    }
19    return 0;
20 }

```

PROBLEMS    OUTPUT    TERMINAL    DEBUG CONSOLE

```
.\InputAndOutput 
```

5  
a is positive

```

1 #include <iostream>
2 using namespace std;
3
4 int main()
5 {
6     int a;
7     cin >> a;
8     if(a > 0) {
9         cout << "a is positive" << endl;
10    }
11    else {
12        if (a == 0) {
13            cout << "a is 0" << endl;
14        }
15        else {
16            cout << "a is negative" << endl;
17        }
18    }
19    return 0;
20 }

```

PROBLEMS    OUTPUT    TERMINAL    DEBUG CONSOLE

```
.\InputAndOutput 
```

0  
a is 0

```

1 #include <iostream>
2 using namespace std;
3
4 int main()
5 {
6     int a;
7     cin >> a;
8     if(a > 0) {
9         cout << "a is positive" << endl;
10    }
11    else {
12        if (a == 0) {
13            cout << "a is 0" << endl;
14        }
15        else {
16            cout << "a is negative" << endl;
17        }
18    }
19    return 0;
20 }

```

PROBLEMS    OUTPUT    TERMINAL    DEBUG CONSOLE

```
.\InputAndOutput 
```

-9  
a is negative



Code ganda hote jaa raha hai. Ek-do baar aur nested if - else kar diya to coding chorni padhegi.

Solution : else if

```

if (~~~) {
}
else if (~~~) {
}
else {
}

```

```

int a ;
cout<<" enter the value of a "<<endl;
cin>>a;

if(a>0) {
    cout<<" A is positive" << endl;
}
else if(a<0) {
    cout<<" A is negative" << endl;
}
else {
    cout<<" A is 0" << endl;
}

```

Note : else if and else are optional.  
else can be used as a default case.

Homework : Output ??

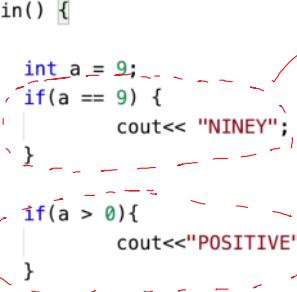
①

```

1 #include <iostream>
2 using namespace std;
3
4 int main() {
5
6     int a = 9;
7     if(a == 9) {
8         cout<< "NINEY";
9     }
10
11    if(a > 0){
12        cout<<"POSITIVE";
13    }
14    else
15    {
16        cout<<"NEGATIVE";
17    }
18 }

```

Answer : NINEYPOSITIVE.



```

1 #include <iostream>
2 using namespace std;
3
4 int main() {
5
6     int a = 2;
7     int b = a+1;
8
9     if((a=3)==b) {
10         cout<<a;
11     }
12     else
13     {
14         cout<<a+1;
15     }
16 }
```

Answer : 3

$a$  is assigned 3.  
Now,  $a == b$  is true.

```

1 #include <iostream>
2 using namespace std;
3
4 int main() {
5
6     int a = 24;
7
8     if(a > 20){
9         cout<< "Love ";
10    }
11    else if(a == 24) {
12        cout<<"Lovely";
13    }
14    else
15    {
16        cout<<"Babbar";
17    }
18    cout<<a;
19 }
```

Answer : Love24

#### ④ Code Homework :

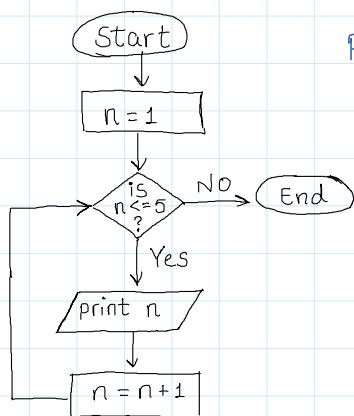
```

1 #include <iostream>
2 using namespace std;
3
4 int main()
5 {
6     char a;
7     cin >> a;
8     // 'A' is 65
9     // 'a' is 97
10    // '0' is 48
11    if(a >= 'A' && a <= 'Z') {
12        cout << "This is upper case" << endl;
13    }
14    else if(a >= 'a' && a <= 'z') {
15        cout << "This is lower case" << endl;
16    }
17    else if(a >= '0' && a <= '9') {
18        cout << "This is a digit" << endl;
19    }
20    return 0;
21 }
```

We can also use ASCII values

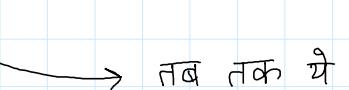


Recollect from flowcharts :



Print from 1 to 5.

while loop :

`while (condition) {`   
`}` 

जब तक ये तुर्स हैं  
 तब तक ये करते रहो

while the condition is true , keep on executing the block

Example :

Print 1 to N .

```

1 #include <iostream>
2 using namespace std;
3
4 int main()
5 {
6     int n;
7     cin >> n;
8     int i = 1;
9     while(i <= n) {
10         cout << i << " ";
11         i = i + 1;
12     }
13     return 0;
14 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE  
 .\InputAndOutput  
 8  
 1 2 3 4 5 6 7 8

Example : Find sum from 1 to n .

```

1 #include <iostream>
2 using namespace std;
3
4 int main()
5 {
6     int n;
7     cin >> n;
8     int sum = 0;
9     int i = 1;
10    while(i <= n) {
11        sum = sum + i;
12        i = i + 1;
13    }
14    cout << "Sum from " << 1 << " to " << n << " = " << sum << endl;
15    return 0;
16 }

```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
{ .\InputAndOutput }
8
Sum from 1 to 8 = 36
```

Example : Find sum of all even numbers from 1 to n.

```

1 #include <iostream>
2 using namespace std;
3
4 int main()
5 {
6     int n;
7     cin >> n;
8     int sum = 0;
9     int i = 2;
10    while(i <= n) {
11        if(i % 2 == 0)
12            sum = sum + i;
13        i = i + 1;
14    }
15    cout << "Sum from " << 1 << " to " << n << " = " << sum << endl;
16    return 0;
17 }

```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
8
Sum from 1 to 8 = 20
```

Homework : Fahrenheit to Celsius.

$$C = \frac{5}{9} (F - 32)$$

```

1 #include <iostream>
2 using namespace std;
3
4 int main()
5 {
6     float fahrenheit;
7     cin >> fahrenheit;
8     float celsius = (5.0/9) * (fahrenheit - 32);
9     cout << fahrenheit << " F = " << celsius << " C" << endl;
10
11    return 0;
12 }

```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
{ .\InputAndOutput }
100
100 F = 37.7778 C
```

## Example: Prime or not ?

```
1 #include <iostream>
2 using namespace std;
3
4 int main()
5 {
6     int n;
7     cin >> n;
8     int i = 2;
9     while(i < n) {
10         if(n % i == 0) {
11             cout << "Not Prime" << endl;
12             return 0;
13         }
14         i = i + 1;
15     }
16     cout << "Prime" << endl;
17     return 0;
18 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE  
{ .\InputAndOutput }  
14  
Not Prime

```
1 #include <iostream>
2 using namespace std;
3
4 int main()
5 {
6     int n;
7     cin >> n;
8     int i = 2;
9     while(i < n) {
10         if(n % i == 0) {
11             cout << "Not Prime" << endl;
12             return 0;
13         }
14         i = i + 1;
15     }
16     cout << "Prime" << endl;
17     return 0;
18 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE  
{ .\InputAndOutput }  
11  
Prime

## Patterns

07 January 2022 20:46



Example :

★ ★ ★ ★  
★ ★ ★ ★  
★ ★ ★ ★  
★ ★ ★ ★

}  $n=4$  (4 rows, 4 columns)

For every row, print 'row' num.  
of columns or stars.

★ ★ ★  
★ ★ ★  
★ ★ ★

} here  $n=3$

```
1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int i = 0;
8     while(i < n) { // i = 0, 1, 2, 3, ..., n-1 which is basically n times
9         int j = 0;
10        while(j < n) {
11            cout << "* ";
12            j = j + 1;
13        }
14        cout << endl; // after printing one row, we need to enter the next row so endl
15        i = i + 1;
16    }
17    return 0;
18 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

.\InputAndOutput

3  
\* \* \*  
\* \* \*  
\* \* \*



Ye code samajh nahi aa raha 😭

Don't worry, initially thoda difficult lag sakta hai BUT  
the next video has 18 problems solved end to end so  
keep going 😊

Example : For  $n=3$ ,

1 1 1  
2 2 2  
3 3 3

```
1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     while(i <= n) {
9         int j = 1;
10        while(j <= n) {
11            cout << i << " ";
12            j = j + 1;
13        }
14        cout << endl;
15        i = i + 1;
16    }
17    return 0;
18 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

4  
1 1 1 1  
2 2 2 2  
3 3 3 3  
4 4 4 4



Q1. Print 1 2 3 4

1 2 3 4      for  $n = 4$   
 1 2 3 4  
 1 2 3 4

Answer :

```

1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1; // Try printing in reverse (solved in the video)
8     while(i <= n) {
9         int j = 1;
10        while(j <= n) {
11            cout << j << " ";
12            j = j + 1;
13        }
14        cout << endl;
15        i = i + 1;
16    }
17    return 0;
18 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```

4
1 2 3 4
1 2 3 4
1 2 3 4
1 2 3 4
```

```

1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     // Try printing in reverse (solved in the video)
9     while(i <= n) {
10        int j = 1;
11        while(j <= n) {
12            cout << j << " ";
13            j = j + 1;
14        }
15        cout << endl;
16        i = i + 1;
17    }
18    return 0;
19 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```

rk\Coding\Patterns\" ; if ($?) { g++ Pattern.cpp -o Pattern
3
1 2 3
1 2 3
1 2 3
```

Homework : For  $n = 4$ ,

4 3 2 1  
 4 3 2 1  
 4 3 2 1  
 4 3 2 1

Q2. Print :

1 2 3      for  $n = 3$   
 4 5 6  
 7 8 9

Logic - maintain a variable, incrementing it by 1 after every cout.

Answer :

```

1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     int toPrint = 1;
9     while(i <= n) {
10        int j = 1;
11        while(j <= n) {
12            cout << toPrint << " ";
13            toPrint = toPrint + 1;
14            j = j + 1;
15        }
16        cout << endl;
17        i = i + 1;
18    }
19    return 0;
20 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```

\rk\Coding\Patterns\" ; if ($?) { g++ P
3
1 2 3
4 5 6
7 8 9
```

```

1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     int toPrint = 1;
9     while(i <= n) {
10        int j = 1;
11        while(j <= n) {
12            cout << toPrint << " ";
13            toPrint = toPrint + 1;
14            j = j + 1;
15        }
16        cout << endl;
17        i = i + 1;
18    }
19    return 0;
20 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```

rk\Coding\Patterns\" ; if ($?) { g++ Pa
4
1 2 3 4
5 6 7 8
9 10 11 12
13 14 15 16
```

Try printing :

9 8 7      for  $n = 3$ .  
 6 5 4  
 3 2 1

3 2 1

16 15 14 13  
12 11 10 9  
8 7 6 5  
4 3 2 1

{ Hint: Starting point has  
a relation with n }

Q3. Print : 

for  $n = 4$ .

{ Hint: We are printing '\*' , row number of times where  $row=1,2,\dots$  }

Answer :

```
1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int row = 1;
8     while(row <= n) {
9         int column = 1;
10        while(column <= row) {
11            cout << "* ";
12            column = column + 1;
13        }
14        cout << endl;
15        row = row + 1;
16    }
17    return 0;
18 }
```

```
1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int row = 1;
8     while(row <= n) {
9         int column = 1;
10        while(column <= row) {
11            cout << "* ";
12            column = column + 1;
13        }
14        cout << endl;
15        row = row + 1;
16    }
17    return 0;
18 }
```

Q4. Print : 1  
2 2  
3 3 3  
4 4 4 4

- for  $n = 4$ .

(Easy)

Answer :

```
1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int row = 1;
8     while(row <= n) {
9         int column = 1;
10        while(column <= row) {
11            cout << row << " ";
12            column = column + 1;
13        }
14        cout << endl;
15        row = row + 1;
16    }
17    return 0;
18 }
```

```
1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int row = 1;
8     while(row <= n) {
9         int column = 1;
10        while(column <= row) {
11            cout << row << " ";
12            column = column + 1;
13        }
14        cout << endl;
15        row = row + 1;
16    }
17    return 0;
18 }
```

Homework - Print : 1  
2 3  
4 5 6  
7 8 9 10

for  $n = 4$ .

Answer :

```
1 #include <iostream>
```

7 8 9 10

Answer :

```
1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int row = 1, toPrint = 1;
8     while(row <= n) {
9         int column = 1;
10        while(column <= row) {
11            cout << toPrint << " ";
12            toPrint = toPrint + 1;
13            column = column + 1;
14        }
15        cout << endl;
16        row = row + 1;
17    }
18    return 0;
}
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
4
1
2 3
4 5 6
7 8 9 10
```

Q5 Print 1 for  $n = 4$ .

2 3  
3 4 5  
4 5 6 7

Logic - We are starting from  $i$  for every row  $i$ .  
Keep incrementing  $i$  times for each row.

Answer :

```
1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int row = 1, toPrint = 1;
8     while(row <= n) {
9         int column = 1;
10        toPrint = row; // start printing from row
11        while(column <= row) {
12            cout << toPrint << " ";
13            toPrint = toPrint + 1;
14            column = column + 1;
15        }
16        cout << endl;
17        row = row + 1;
18    }
19    return 0;
}
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
4
1
2 3
3 4 5
4 5 6 7
```

Homework : Solve the above question without using the extra variable `toPrint`.

Logic - Column starts from 1 for every row. Change it to start from  $row$  and go till  $row + row = row * 2$  (exclusive)

Answer :

```
1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int row = 1;
8     while(row <= n) {
9         int column = row;
10        while(column < row * 2) {
11            cout << column << " ";
12            column = column + 1;
13        }
14        cout << endl;
15        row = row + 1;
16    }
17    return 0;
18 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
rk\Coding\Patterns\" ; if ($?) { g++ Pattern.cpp
4
1
2 3
3 4 5
4 5 6 7
```

Q6. Print

1  
2 1  
3 2 1  
4 3 2 1

{ Hint: starting from row number  
and decrementing.

Logic : column = 1 2 3 4

row = 1      1  
row = 2      2 1  
row = 3      3 2 1  
row = 4      4 3 2 1

Subtract column number from row  
number and add 1.  
 $(row - column + 1)$

Answer :

```
1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int row = 1;
8     while(row <= n) {
9         int column = 1;
10        while(column <= row) {
11            cout << (row - column + 1) << " ";
12            column = column + 1;
13        }
14        cout << endl;
15        row = row + 1;
16    }
17    return 0;
18 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
rk\Coding\Patterns\" ; if ($?) { g++ Pattern.cpp -o Pat
5
1
2 1
3 2 1
4 3 2 1
5 4 3 2 1
```



Q7 Print : A A A      for n = 3.  
 B B B  
 C C C

Answer :

```

4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     while(i <= n) {
9         int j = 1;
10        while(j <= n) {
11            // you can also print 'A' + i - 1 because with an int
12            // 'A' will get typecasted to 65
13            cout << (char)(65 + i - 1) << " ";
14            j = j + 1;
15        }
16        i = i + 1;
17        cout << endl;
18    }
19    return 0;
20 }

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE
rk\Coding\Patterns\" ; if ($?) { g++ Pattern.cpp -o Pattern } ; if ($?) {
4
A A A A
B B B B
C C C C
D D D D

```

Q8 Print : A B C      for n = 3  
 A B C  
 A B C      (Homework Code)

Answer :

```

3
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     while(i <= n) {
9         int j = 1;
10        char ch = 'A';
11        while(j <= n) {
12            cout << ch << " ";
13            ch = ch + 1;
14            j = j + 1;
15        }
16        i = i + 1;
17        cout << endl;
18    }
19    return 0;
20 }

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE
rk\Coding\Patterns\" ; if ($?) { g++ Pattern.cpp -o Pattern } ; if ($?) {
3
A B C
A B C
A B C

```

Q9 Print : A B C      for n = 3  
 D E F  
 G H I

Logic is same as with numbers

Answer :

```
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     char ch = 'A';
9     while(i <= n) {
10         int j = 1;
11         while(j <= n) {
12             cout << ch << " ";
13             ch = ch + 1;
14             j = j + 1;
15         }
16         i = i + 1;
17     }
18 }
19 return 0;
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
rk\Coding\Patterns\" ; if ($?) { g++ Pattern.cpp -o Pattern
3
A B C
D E F
G H I
```

Q10 Print : A B C  
B C D  
C D E

Logic is same as with numbers

{ 'A' + row - 1  
for row = 1, 2, ...  
increment this

Answer :

```
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     while(i <= n) {
9         char ch = 'A' + i - 1;
10        int j = 1;
11        while(j <= n) {
12            cout << ch << " ";
13            ch = ch + 1;
14            j = j + 1;
15        }
16        i = i + 1;
17        cout << endl;
18    }
19 }
20 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
rk\Coding\Patterns\" ; if ($?) { g++ Pattern.cpp -o Pattern
4
A B C D
B C D E
C D E F
D E F G
```

```
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     while(i <= n) {
9         int j = 1;
10        while(j <= n) {
11            char ch = 'A' + i + j - 2;
12            cout << ch << " ";
13            j = j + 1;
14        }
15        i = i + 1;
16        cout << endl;
17    }
18 }
19 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
rk\Coding\Patterns\" ; if ($?) { g++ Pattern.cpp -o Pattern
4
A B C D
B C D E
C D E F
D E F G
```

Q11. Print : A for n = 3  
B B  
C C C

Answer :

```
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     while(i <= n) {
9         int j = 1;
10        char ch = 'A' + i - 1;
11        while(j <= i) {
12            cout << ch << " ";
13            j = j + 1;
14        }
15        i = i + 1;
16        cout << endl;
17    }
18    return 0;
19 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
rk\Coding\Patterns\" ; if ($?) { g++ Pattern.cpp -o
4
A
B B
C C C
D D D D
```

q12 Print : A (Try yourself)  
B C  
D E F  
G H I J

Logic same as with numbers homework question.

Answer :

```
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     char ch = 'A';
9     while(i <= n) {
10        int j = 1;
11        while(j <= i) {
12            cout << ch << " ";
13            ch = ch + 1;
14            j = j + 1;
15        }
16        i = i + 1;
17        cout << endl;
18    }
19    return 0;
20 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
4
A
B C
D E F
G H I J
```

q13 Print : A for n = 4  
B C  
C D E  
D E F G

Logic is same as numbers.

Answer:

```
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     while(i <= n) {
9         int j = 1;
10        char ch = 'A' + i - 1;
11        while(j <= i) {
12            cout << ch << " ";
13            ch = ch + 1;
14            j = j + 1;
15        }
16        i = i + 1;
17        cout << endl;
18    }
19    return 0;
20 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
rk\Coding\Patterns\" ; if ($?) { g++ Pattern.cpp -o
4
```

The Babbar Answer

```
1 #include<iostream>
2 using namespace std;
3
4 int main() {
5
6     int n;
7     cin>>n;
8
9     int row = 1;
10
11    while(row <= n) {
12
13        int col = 1;
14
15        while(col <= row) {
16            char ch = ('A' + row + col - 2);
17            cout<<ch;
18            col = col + 1;
19        }
20
21        cout<<endl;
22        row = row + 1;
23    }
}
```

```

    }
20 }

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE
rk\Coding\Patterns\" ; if ($?) { g++ Pattern.cpp
4
A
B C
C D E
D E F G

```

(Homework)

Q14 Print : D

C D  
B C D  
A B C D

Logic - We are starting from

$ch = ('A' + \text{rows} - 1)$  in row 1.  
 $ch = ('A' + \text{rows} - 2)$  in row 2.  
 $ch = ('A' + \text{rows} - 3)$  in row 3.

⋮ ⋮ ⋮ ⋮ ⋮ ⋮ ⋮ ⋮ ⋮  
| | | | | | | | | |  
 $ch = ('A' + \text{rows} - i)$  for row  $i$ .

Answer :

```

4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     while(i <= n) {
9         char ch = 'A' + n - i;
10        int j = 1;
11        while(j <= i) {
12            cout << ch << " ";
13            ch = ch + 1;
14            j = j + 1;
15        }
16        i = i + 1;
17        cout << endl;
18    }
19    return 0;
20 }

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE
rk\Coding\Patterns\" ; if ($?) { g++ Pattern.cpp
4
D
C D
B C D
A B C D

```

Q15 Print :

☆  
☆☆  
☆☆☆  
☆☆☆☆

for  $n = 4$

(Notice the number of spaces and its relation with the row number,  $i$ )

Logic - for row  $i$ , we are first printing  $n-i$  spaces and then printing  $i$  stars.

Answer :

```
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     while(i <= n) {
9         int space = 1;
10        while(space <= n - i) {
11            cout << " ";
12            space = space + 1;
13        }
14        int j = 1;
15        while(j <= i) {
16            cout << "*";
17            j = j + 1;
18        }
19        i = i + 1;
20        cout << endl;
21    }
22    return 0;
23 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

\Work\Coding\Patterns\" ; if (\$?) { g++

```
4
*
**
***
****
```

Q16. Print :

★ ★ ★ ★  
★ ★ ★  
★ ★  
★

(Homework)

Answer :

```
1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     while(i <= n) {
9         int j = 1;
10        while(j <= n - i + 1) {
11            cout << "* ";
12            j = j + 1;
13        }
14        i = i + 1;
15        cout << endl;
16    }
17    return 0;
18 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

\Work\Coding\Patterns\" ; if (\$?) { g++ Pattern.cpp -o Pattern

```
4
* * *
* *
* *
*
```

Q17. Print :

★ ★ ★ ★  
★ ★ ★  
★ ★  
★

for  $n = 4$

(Homework)

Logic : Printing  $i-1$  spaces &  $n-i+1$  stars for  $i^{th}$  row.

Answer :

```
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     while(i <= n) {
9         int j = 1, space = 1;
10        while(space < i) {
11            cout << " ";
12            space = space + 1; // space++ is also used
13        }
14        while(j <= n - i + 1) {
15            cout << "*";
16            j = j + 1;
17        }
18        i = i + 1;
19        cout << endl;
20    }
21    return 0;
22 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

\Work\Coding\Patterns\" ; if (\$?) { g++ Pattern.cpp -o Pattern

```
4
*****
 ***
 **
 *
```

Q18 Print : 1 1 1 1      for n = 4      (Homework)

```

2 2 2
3 3
4

```

Answer :

```

4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     while(i <= n) {
9         int j = 1, space = 1;
10        while(space < i) {
11            cout << " ";
12            space = space + 1; // space++ is also used
13        }
14        while(j <= n - i + 1) {
15            cout << i;
16            j = j + 1;
17        }
18        i = i + 1;
19        cout << endl;
20    }
21    return 0;
22 }

```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
\Work\Coding\Patterns\" ; if ($?) { g++ Pattern.cpp -o Pat
4
1111
222
33
4
```

Q19 Print : 1      for n = 4      (Homework)

```

2 2
3 3 3
4 4 4 4

```

Answer :

```

4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     while(i <= n) {
9         int j = 1, space = 1;
10        while(space <= n - i) {
11            cout << " ";
12            space = space + 1; // space++ is also used
13        }
14        while(j <= i) {
15            cout << i;
16            j = j + 1;
17        }
18        i = i + 1;
19        cout << endl;
20    }
21    return 0;
22 }

```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

```
\Work\Coding\Patterns\" ; if ($?) { g++ Pattern.cpp -o Pat
4
1
22
333
4444
```

Q20. Print : 1      for n = 4

```

2 3
4 5 6
7 8 9 10

```

Logic - Print space ( $n-i$ ) times and print num. Then increment num.

Answer -

```
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1, num = 1;
8     while(i <= n) {
9         int j = 1, space = 1;
10        while(space <= n - i) {
11            cout << " ";
12            space = space + 1;
13        }
14        while(j <= i) {
15            cout << num << " ";
16            num = num + 1;
17            j = j + 1;
18        }
19        i = i + 1;
20        cout << endl;
21    }
22    return 0;
23 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE

4

1  
2 3  
4 5 6  
7 8 9 10

Q21 Print :

1 for  $n = 4$   
1 2 1  
1 2 3 2 1  
1 2 3 4 3 2 1

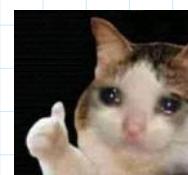
Logic - Use 2 while loops for each row  $i$ .  
One for printing  $(n-i)$  " " (double spaces)  
One for printing from 1 to  $i$ .  
Other for printing from  $(i-1)$  to 1.

Answer :

```
1 #include <iostream>
2 using namespace std;
3
4 int main() {
5     int n;
6     cin >> n;
7     int i = 1;
8     while(i <= n) {
9         int j = 1, space = 1;
10        while(space <= n - i) {
11            cout << " ";
12            space = space + 1;
13        }
14        while(j <= i) {
15            cout << j << " ";
16            j = j + 1;
17        }
18        j = i - 1;
19        while(j >= 1) {
20            cout << j << " ";
21            j = j - 1;
22        }
23        i = i + 1;
24        cout << endl;
25    }
26    return 0;
27 }
```

TAGDA HOMEWORK :

Print : 1 2 3 4 5 5 4 3 2 1 for  $n = 5$   
1 2 3 4 ⋆ ⋆ 4 3 2 1  
1 2 3 ⋆ ⋆ ⋆ ⋆ 3 2 1  
1 2 ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ 2 1  
1 ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ ⋆ 1



Answer :

```
4  int main() {
5      int n;
6      cin >> n;
7      int i = 1;
8      while(i <= n) {
9          // part 1: numbers from i to n - i + 1
10         int j = 1;
11         while(j <= n - i + 1) {
12             cout << j << " ";
13             j = j + 1;
14         }
15         // part 2: Stars (i-1)*2 times
16         j = 1;
17         while(j <= (i-1)*2) {
18             cout << "* ";
19             j = j + 1;
20         }
21         // part 3: numbers from n - i + 1 to 1
22         j = n - i + 1;
23         while(j >= 1) {
24             cout << j << " ";
25             j = j - 1;
26         }
27         i = i + 1;
28     }
29 }
30
31 }
```



### ① Bitwise AND : (&)

Eg:  $2 \& 3 = 010 \& 011 = 010$

Both bits must be 1 to get 1.

| x | y | AND |
|---|---|-----|
| 0 | 0 | 0   |
| 0 | 1 | 0   |
| 1 | 0 | 0   |
| 1 | 1 | 1   |

Eg:  $5 \& 7 : 101$

$$\begin{array}{r} & \underline{111} \\ \& 101 \\ & \end{array} = \textcircled{5}$$

### ② Bitwise OR : (|)

Eg:  $2 | 5 : 010$

$$\begin{array}{r} & \underline{101} \\ 101 \\ & \end{array}$$

If at least one of the bits is 1 then the output will be 1.

| x | y | OR |
|---|---|----|
| 0 | 0 | 0  |
| 0 | 1 | 1  |
| 1 | 0 | 1  |
| 1 | 1 | 1  |

Eg:  $3 | 6 : 011$

$$\begin{array}{r} & \underline{110} \\ 110 \\ & \end{array}$$

### ③ Bitwise NOT : (~)

Eg: (consider 4 byte representation)

$$a = 2 = \overbrace{000\dots00}^{30\text{ bits}} 10$$

$$\sim a = \sim 2 = \textcircled{1}11\dots1101$$

Signed Bit

Two's complement :  $000\dots0010$

$$\begin{array}{r} + \end{array}$$

| x | NOT |
|---|-----|
| 0 | 1   |
| 1 | 0   |



$$000\ldots0011 = \textcircled{-3}$$

#### ④ Bitwise XOR : (^)

| x | y | XOR | If different bits → 1<br>If same bits → 0 |
|---|---|-----|-------------------------------------------|
| 0 | 0 | 0   |                                           |
| 0 | 1 | 1   |                                           |
| 1 | 0 | 1   |                                           |
| 1 | 1 | 0   |                                           |

Eg:  $5^7 = \begin{array}{r} 101 \\ ^{\wedge} \quad 111 \\ \hline 010 \end{array} = \textcircled{2}$

```

1 #include<iostream>
2 using namespace std;
3
4
5 int main() {
6     int a = 4;
7     int b = 6;
8
9     cout<<" a&b " << (a&b) << endl;
10    cout<<" a|b " << (a|b) << endl;
11    cout<<" ~a " << ~a << endl;
12    cout<<" a^b " << (a^b) << endl;
13
14 }
15

```

lovebabbar@192 ~ % cd /Users/lovebabbar/Dev/g++ BitwiseOperators
BitwiseOperators && "/Users/lovebabbar/"BitwiseOperators
a&b 6
a|b -5
~a -5
a^b 2
lovebabbar@192 ~ % []

#### ⑤ Left Shift : (<<)

Eg:  $5 << 1 \rightarrow$  Left shift 5, 1 time  
 $= 101 << 1 \rightarrow 00\ldots0101 << 1$   
 $= 00\ldots1010 = \textcircled{10} = 5 \times 2^1$

Eg:  $3 << 2 \rightarrow 00\ldots011 << 2$   
 $= 00\ldots1100 = \textcircled{12} = 3 \times 2^2$

In most cases we multiply with power of 2.  
But in some cases, this isn't true.

Eg:  $0100\ldots00 << 1 = 1000\ldots00$   
Positive → Negative ??

So << is ok for smaller numbers.

#### ⑥ Right Shift : (>>)

Eg:  $15 >> 1 = 000\ldots1111 >> 1$   
 $= 000\ldots0111 = \textcircled{7}$

Eg:  $5 >> 2 = 000\dots0101 >> 2$   
 $000\dots0001 = 1$

Padding in  $<<$  and  $>>$  for POSITIVE numbers is done with 0.

For NEGATIVE numbers, padding is compiler dependent.

```
cout<< (17>>1)<<endl;
cout<< (17>>2) <<endl;
cout<< (19<<1) <<endl;
cout<< (21<<2) <<endl;
```

```
8
4
38
84
lovebabbar@192 ~ %
```

## ⑦ Increment/Decrement :

→ We can write  $i = i + 1$  as  $i++$  or  $++i$ .

$i++$  is called Post-Increment  
 $++i$  is called Pre-Increment.

→ We can write  $i = i - 1$  as  $i--$  or  $--i$ .

$i--$  is called Post-decrement  
 $--i$  is called Pre-decrement.

→  $a = a + b$  is same as  $a += b$ .

→  $a = a - b$  is same as  $a -= b$ .

Post-Increment: The value gets used first and then increments.

Eg:  $\text{int } i = 3, a = 2;$

```
int sum = a + (i++);
sum = 2 + 3;
sum = 5;
Now i is 4.
```

Pre-Increment: The value gets incremented first and then gets used.

Eg:  $\text{int } i = 3, a = 2;$

```

int sum = a + (++i);    i has become 4
sum = 2 + 4 ;
sum = 6 ;

```

**Post-Decrement**: The value gets used first and then decrements.

Eg: int i = 3, a = 2 ;

```

int sum = a + (i--) ;
sum = 2 + 3 ;
sum = 5 ;
Now i is 2 .

```

**Pre-Decrement**: The value gets decremented first and then gets used.

Eg: int i = 3, a = 2 ;

```

int sum = a + (--i);    i has become 2
sum = 2 + 2 ;
sum = 4 ;

```

```

20 int i = 7;
21 cout << (++i) << endl;
22 // 8
23 cout << (i++) << endl;
24 // 8 , i = 9
25 cout << (i--) << endl;
26 // 9 , i = 8
27 cout << (--i) << endl;
28 // 7, i = 7
29

```

```

8
8
9
7
lovebabbar@192 ~ %

```

Q1

```

1 #include <iostream>
2 using namespace std;
3 int main()
4 {
5     int a, b = 1;
6     a = 10;
7     if (++a)
8         cout << b;
9     else
10        cout << ++b;
11 }

```

Answer : 1

Q2

```
1 #include <iostream>
2 using namespace std;
3 int main()
4 {
5     int a = 1;
6     int b = 2;
7
8     if(a-- > 0 && ++b > 2 ){
9         cout << "Stage1 - Inside If ";
10    } else{
11        cout << "Stage2 - Inside else ";
12    }
13    cout << a << " " << b << endl;
14 }
```

Answer: Stage1 - Inside If O 3

Q3

```
1 #include <iostream>
2 using namespace std;
3 int main()
4 {
5     int a = 1;
6     int b = 2;
7
8     if(a-- > 0 || ++b > 2 ){
9         cout << "Stage1 - Inside If ";
10    } else{
11        cout << "Stage2 - Inside else ";
12    }
13    cout << a << " " << b << endl;
14 }
```

Answer : Stage1 - Inside If O 2

Hint: Only one of the conditions must be true for || so it won't check  $++b > 2$ .

Q3

```
1 #include <iostream>
2 using namespace std;
3 int main()
4 {
5     int number = 3;
6     cout << (25 * (++number) );
7 }
```

Answer : 100

Q4

```
1 #include <iostream>
2 using namespace std;
3 int main()
4 {
5     int a = 1;
6     int b = a++;
7     int c = ++a;
8     cout << b ;
9     cout << c;
10 }
```

Answer : 13



## Example Code :

```

1 #include<iostream>
2 using namespace std;
3
4 int main() {
5
6     int n ;
7     cout<<" enter the value of n" <<endl;
8     cin >> n;
9
10    cout<<"printing count from 1 to n" << endl;
11
12    for(int i = 1; i<=n; i++) {
13        cout<< i << endl;
14    }
15
16
17
18 }

```

```

lovebabbar@192 ~ % cd "/Users/lovebabbar/" && g++ forLoop.cpp -o forLoop &
& "/Users/lovebabbar/"forLoop
enter the value of n
5
printing count from 1 to n
1
2
3
4
5
lovebabbar@192 ~ %

```

Structure : `for (int i=0 ; i < n ; i++)`

initialization , cond. to check , Updation  
before every  
loop/iteration.

{  
~~~~~  
~~~~~  
}

Not Necessary to specify.

Example : `for(;;) = ?`

```

1 #include<iostream>
2 using namespace std;
3
4 int main() {
5
6     int n ;
7     cout<<" enter the value of n" <<endl;
8     cin >> n;
9
10    cout<<"printing count from 1 to n" << endl;
11    int i = 1;
12    for(;; i++) {
13        cout<< i << endl;
14        i++;
15    }
16
17
18 }

```

```

lovebabbar@192 ~ % cd "/Users/lovebabbar/" && g++ forLoop.cpp -o forLoop &
& "/Users/lovebabbar/"forLoop
enter the value of n
→ printing count from 1 to n
1
2
3
4
5
lovebabbar@192 ~ %

```

You can declare all the 3 parts outside the parenthesis () .

WRONG :

```

11 int i = 1;
12 for(;;) {
13     if(i<=n) {
14         cout<< i << endl;
15     }
16
17     i++;
18 }

```



The for loop doesn't know when to stop.

Solution : `break;`

Gets you out of the current loop.

```

int n ;
cout<<" enter the value of n" << endl;
cin >> n;

cout<<"printing count from 1 to n" << endl;
int i = 1;
for( ; ; ) {
    if(i<=n) {
        cout<< i << endl;
    }
    else{
        break;
    }

    i++;
}

```

enter the value of n  
5  
printing count from 1 to n  
1  
2  
3  
4  
5  
lovebabbar@192 ~ %

Example: Using multiple inits, conditions and updations.

```

for(int a = 0 , b =1, c = 2; a>=0 && b>=1 && c>=2; a--,b--, c--){
    cout<<a << " << b << " <<c << endl;
}

```

Question: Print 1 to n.

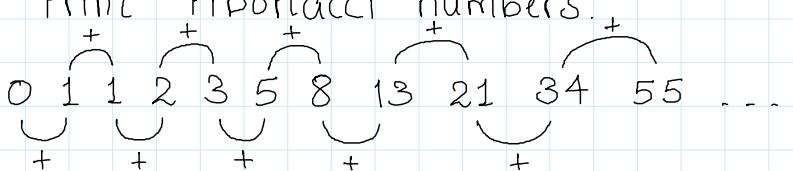
```

1 #include<iostream>
2 using namespace std;
3
4 int main() {
5
6     int n ;
7     cout<<" enter the value of n" << endl;
8     cin >> n;
9
10    int sum = 0;
11
12    for(int i=1; i<=n; i++ ) {
13        //sum = sum + i;
14        sum += i;
15    }
16
17    cout<< sum << endl;
18
19 }

```

lovebabbar@192 ~ % cd "/Users/lovebabbar/" && g++ forSum.cpp -o forSum && ./forSum  
"/Users/lovebabbar/"forSum  
enter the value of n  
5  
15  
lovebabbar@192 ~ %

Question: Print Fibonacci numbers.



Solution :

```

1 #include<iostream>
2 using namespace std;
3
4 int main() {
5
6     int n = 10;
7
8     int a = 0;
9     int b = 1;
10    cout<<a <<" " <<b<<" ";
11    for(int i = 1; i<=n; i++ ) {
12
13        int nextNumber = a+b;
14        cout<<nextNumber<<" ";
15
16        a = b;
17        b = nextNumber;
18
19
20
21
22
23
24 }

```

lovebabbar@192 ~ % cd "/Users/lovebabbar/" && g++ forFib.cpp -o forFib && ./forFib  
0 1 1 2 3 5 8 13 21 34 55 89 I  
lovebabbar@192 ~ %

Question: Print if prime. (Logic already covered)

Ex : I/P - 7

O/P - Yes

Solution :

```
1 #include<iostream>
2 using namespace std;
3
4 int main() {
5
6     int n ;
7     cout<<" enter the value of n" <<endl;
8     cin >> n;
9
10    bool isPrime = 1 ;
11
12    for(int i = 2; i<n; i++) {
13
14        //rem = 0, Not a Prime
15        if(n%i == 0) {
16            //cout<<" Not a Prime Number" << endl;
17            isPrime = 0;
18            break;
19        }
20
21
22        if(isPrime == 0) {
23            cout<<" Not a Prime Number" << endl;
24        }
25        else
26        {
27            cout<<"is a Prime Number " << endl;
28        }
29
30    }
31 }
```

```
lovebabbar@192 ~ % cd "/Users/lovebabbar/" && g++ forPrime.cpp -o forPrime
&& "/Users/lovebabbar/"forPrime
enter the value of n
101
is a Prime Number
```

break ka brother continue

continue : Used to skip an iteration of the current loop.

It skips the remaining block of code for that iteration.

Code example :

```
1 #include<iostream>
2 using namespace std;
3
4 int main() {
5
6     for(int i=0; i<5; i++) {
7
8         cout<< " HI " << endl;
9         cout<< " Hey " << endl;
10        continue;
11
12        cout<< "Reply toh karte " << endl;
13
14    }
15
16
17 }
```

```
lovebabbar@192 ~ % cd "/Users/lovebabbar/" && g++ continue.cpp -o continue
&& "/Users/lovebabbar/"continue
HI
Hey
HI
Hey
HI
Hey
HI
Hey
HI
Hey
lovebabbar@192 ~ %
```

Output Questions :

Q1.

```
#include<iostream>
using namespace std;
int main() {
    for(int i = 0; i<=5; i++) {
        cout<< i << " ";
        i++;
    }
}
```

Output : 0 2 4

Q1.

```
#include<iostream>
using namespace std;
int main() {
    for(int i = 0; i<=5; i++) {
        cout<< i << " ";
        i++;
    }
}
```

Output : 0 2 4

Q2.

```
#include<iostream>
using namespace std;
int main() {
    for(int i = 0; i<=5; i--) {
        cout<< i << " ";
        i++;
    }
}
```

Output : 0 0 0 0 . . .

(infinite times)

Q3.

```
#include<iostream>
using namespace std;
int main() {
    for(int i = 0; i<=15; i += 2 ) {
        cout<< i << " ";
        if( i&1 ){
            continue;
        }
        i++;
    }
}
```

Output : 0 3 5 7 9 11 13 15

Q4

```
#include<iostream>
using namespace std;
int main() {
    for(int i=0;i<5;i++) {
        for(int j=i;j<=5;j++) {
            cout<< i << " " << j << endl;
        }
    }
}
```

Output :

0 0  
0 1  
0 2  
0 3  
0 4  
0 5  
1 1  
1 2  
1 3  
1 4  
1 5  
2 2  
2 3  
2 4  
2 5  
3 3  
3 4  
3 5  
4 4  
4 5

Q5

```
#include<iostream>
using namespace std;
int main() {
    for(int i=0;i<5;i++) {
        for(int j=i;j<=5;j++) {
            if(i+j == 10) {
                break;
            }
            cout<<i << " " << j << endl;
        }
    }
}
```

Output :

|   |   |
|---|---|
| 0 | 0 |
| 0 | 1 |
| 0 | 2 |
| 0 | 3 |
| 0 | 4 |
| 0 | 5 |
| 1 | 1 |
| 1 | 2 |
| 1 | 3 |
| 1 | 4 |
| 1 | 5 |
| 2 | 2 |
| 2 | 3 |
| 2 | 4 |
| 2 | 5 |
| 3 | 3 |
| 3 | 4 |
| 3 | 5 |
| 4 | 4 |
| 4 | 5 |



**Scope:** The life-time of a variable. Where does the variable exist and after what line of code will it get destroyed.  
In short, its accessibility.

Example :

```
1 #include<iostream>
2 using namespace std;
3
4
5 int main() {
6
7     int a = 3;
8     cout << a << endl;
9
10    if(true) {
11        cout << a << endl;
12    }
13
14
15 }
```

a is accessible throughout the main() function, after it is declared.

Example :

```
1 #include<iostream>
2 using namespace std;
3
4
5 int main() {
6
7     int a = 3;
8     cout << a << endl;
9
10    if(true) {
11        int a = 5;
12        cout << a << endl;
13    }
14
15
16 }
```

This is 'local' to int main().

This is 'local' to if block, and is only accessible within the if block.

This will print out 3  
5

Example :

```
1 #include<iostream>
2 using namespace std;
3
4
5 int main() {
6
7     int a = 3;
8     cout << a << endl;
9
10    if(true) {
11        int b = 5;
12        cout << b << endl;
13    }
14
15
16    cout << b << endl;
17
18 }
```



Example :

```
int i = 8;
cout << b << endl;

for(; i<8; i++) {
    cout << " HI " << endl;
}
```



Kuch Nahi (Empty)

```
for(; i<8; i++) {  
    cout<<" HI " << endl;  
}
```

The `i` outside the for loop gets used.



## Precedence Table:

(No need to mug up)

| Precedence order | Operator                                   | Associativity |
|------------------|--------------------------------------------|---------------|
| 1                | () [] →                                    | Left to right |
| 2                | ++ -- (unary) ! ~ * & sizeof               | Right to left |
| 3                | * / %                                      | Left to right |
| 4                | + -                                        | Left to right |
| 5                | << >>                                      | Left to right |
| 6                | < <= > >=                                  | Left to right |
| 7                | == !=                                      | Left to right |
| 8                | & (bitwise AND)                            | Left to right |
| 9                | ^ (bitwise XOR)                            | Left to right |
| 10               | (bitwise OR)                               | Left to right |
| 11               | && (logical AND)                           | Left to right |
| 12               | (logical OR)                               | Left to right |
| 13               | ? : (conditional)                          | Right to left |
| 14               | = += -= *= /= %=<br>(assignment operators) | Right to left |
| 15               | , (comma Operator)                         | Left to right |

Just like BODMAS prioritizes Brackets, we can also prioritize calculations using Brackets.



Q1. Subtract the product and sum of the digits of an integer.

Soln :

```
class Solution {
public:
    int subtractProductAndSum(int n) {
        int product = 1, sum = 0;
        while(n) {
            product = product * (n % 10); // Can also use *=
            sum += (n % 10);
            n /= 10; // same as n = n / 10;
        }
        return product - sum;
    }
};
```



Success Details >

Runtime: 0 ms, faster than 100.00% of C++ online submissions for Subtract the Product and Sum of Digits of an Integer.

Memory Usage: 5.9 MB, less than 23.16% of C++ online submissions for Subtract the Product and Sum of Digits of an Integer.

Q2. Count number of 1 bits.

Soln :

```
class Solution {
public:
    int hammingWeight(uint32_t n) {
        int ans = 0;
        while(n) {
            if(n & 1) {
                ans++;
            }
            n = n >> 1;
        }
        return ans;
    }
};
```

Q3. Reverse integer.

Soln :

```
class Solution {
public:
    int reverse(int x) {
        int rev = 0;
        while (x != 0) {
            int pop = x % 10;
            x /= 10;
            if (rev > INT_MAX/10 || (rev == INT_MAX / 10 && pop > 7)) return 0;
            if (rev < INT_MIN/10 || (rev == INT_MIN / 10 && pop < -8)) return 0;
            rev = rev * 10 + pop;
        }
        return rev;
    }
};
```

## Q4. Complement of Base 10

Soln :

```
class Solution {
public:
    int bitwiseComplement(int n) {
        if(n == 0) return 1;

        int ans = 0, fac = 1;

        while(n != 0){
            int bit = n % 2 == 0;
            ans += fac * bit;
            fac *= 2;
            n /= 2;
        }
        return ans;
    }
};
```

## Q5 Number Compliment.

Soln :

```
class Solution {
public:
    int findComplement(int num) {
        int msb = (int)(log2(num));
        if(num == 0) {
            return 1;
        }

        int sum = 0;
        for(int i=msb; i>=0;i--) {
            if(num &(1<<i)){
                continue;
            }
            else {
                sum+=pow(2,i);
            }
        }
        return sum;
    }
};
```

## Q6. Binary to Decimal.

Soln :

```
1 #include <iostream>
2 using namespace std;
3
4 int main()
5 {
6     int num = 1010001;
7     int dec_value = 0;
8     int base = 1;
9     int temp = num;
10
11    while (temp) {
12        int last_digit = temp % 10;
13        temp = temp / 10;
14
15        dec_value += last_digit * base;
16
17        base = base * 2;
18    }
19    cout << dec_value << endl;
20    return 0;
21 }
```



## Decimal to Binary :

$$\textcircled{1} \quad 10 \rightarrow \frac{10}{2} = 5, \frac{5}{2} = 2, \frac{2}{2} = 1, \frac{1}{2} = 0$$

↓      ↓      ↓      ↓  
(Remainder) 0    1    0    1  
Finish      ←      Start

$$10 \rightarrow \begin{smallmatrix} 3 & 2 & 2 & 1 & 0 \\ 2 & 2 & 2 & 1 & 2 \end{smallmatrix}$$

Steps : i) Divide number by 2  
ii) Store the remainder.  
iii) Repeat for the dividend.

| Division       | Remainder |
|----------------|-----------|
| $\frac{10}{2}$ | 0         |
| $\frac{5}{2}$  | 1         |
| $\frac{2}{2}$  | 0         |
| $\frac{1}{2}$  | 1         |

Reverse = 1010

  
Khatam Bye Bye Tata Goodbye

g.  $n = 7$  to Binary

| Division      | Remainder |
|---------------|-----------|
| $\frac{7}{2}$ | 1         |
| $\frac{3}{2}$ | 1         |
| $\frac{1}{2}$ | 1         |

Reverse = 111

  
Khatam Bye Bye Tata Goodbye

\textcircled{2}  $n = 5 \rightarrow \text{Binary ?}$

Logic :  $n \& 1 = 0$  if  $n$  is even.

$n \& 1 = 1$  if  $n$  is odd

Thus, a bit is 1 if bit  $\& 1 = 1$   
a bit is 0 if bit  $\& 1 = 0$

Thus, (any\_bit) & 1 = (any\_bit)

∴ Last bit of 5 =  $5 \& 1 = 1$

Check  $5 = 101 \Rightarrow 101$

$$\begin{array}{r} & \cancel{\&} \\ & | \\ \underline{\quad} & (1) \end{array}$$

Right shift 5 by 1  $\Rightarrow n = 2 \quad (010)$

Second last bit of 5 =  $2 \& 1 = 0$

Right shift 2 by 1  $\Rightarrow n = 1 \quad (001)$

First bit of 5 =  $1 \& 1 = 1$

Right shift 1 by 1  $\Rightarrow n = 0 \quad (000)$

Khatam !

```
1 #include<iostream>
2 #include<math.h>
3 using namespace std;
4
5
6 int main() {
7
8     int n;
9     cin >> n;
10
11
12     int ans = 0;
13     int i = 0;
14     while(n != 0) {
15
16         int bit = n & 1;
17
18         ans = (bit * pow(10, i)) + ans;
19
20         n = n >> 1;
21         i++;           I
22
23     }
24
25     cout << "Answer is " << ans << endl;
26
27
28 }
```

Example :  $n = 6$

①  $ans = 0, i = 0$

②  $bit = 6 \& 1 = 0 \quad (\text{even})$

③  $ans = 10^0 \times 0 + ans = 0, n = 3, i = 1$

④  $bit = 3 \& 1 = 1 \quad (\text{odd})$

⑤  $ans = (10^1 \times 1) + ans = 10, n = 1, i = 2$

⑥  $bit = 1 \& 1 = 1 \quad (\text{odd})$

$$\textcircled{7} \quad \text{ans} = (10^2 \times 1) + \text{ans} = 100 + 10 = 110, n=0$$

$$\text{ans} = 110$$

Negative Decimal to Binary : (Homework)

-6 → Binary ??

We have discussed the logic about the storage & display of negative numbers earlier.

$$\begin{array}{r} -6 \rightarrow 6 \rightarrow 000\dots00110 \rightarrow 111\dots11001 \\ \qquad\qquad\qquad + 1 \\ \hline 111\dots11010 \end{array}$$

Think what is  $111\dots11010$  equal to if it is unsigned?

$$\begin{array}{l} \underbrace{111\dots11010}_{29 \text{ bits}} = 4294967290 \\ = 2^{32} - 6 \end{array}$$

$$\therefore -6 \xrightarrow{\text{Binary}} \text{Binary of } (2^{32} - 6) \rightarrow \underbrace{111\dots11010}_{29}$$

We can't express this in any data type.

Assuming we have 2 byte (= 16 bit) integers:

```

1 #include <iostream>
2 #include <math.h>
3 using namespace std;
4
5 int main(void)
6 {
7     long long int n;
8     cin >> n;
9     unsigned long long int i = 0, ans = 0;
10    if(n < 0) {
11        n = pow(2, 16) + n;
12    }
13    cout << n << endl;
14    while(n) {
15        int lastBit = n & 1;
16        ans = (pow(10, i) * lastBit) + ans;
17        n = n >> 1;
18        i++;
19        cout << ans << endl;
20    }
21    cout << ans << endl;
22    return 0;
23 }
```

(Might need)  
online IDE

Binary to Decimal :

$$\begin{aligned}
 101011 &= 2^0 \times 1 + 2^1 \times 1 + 2^2 \times 0 + \\
 2^5 &+ 2^4 & 2^3 \times 1 + 2^4 \times 0 + 2^5 \times 1 \\
 &= 1 + 2 + 0 + 8 + 0 + 32 \\
 &= \boxed{43}
 \end{aligned}$$

Logic:  $n \& 1 = \begin{cases} 0, & \text{don't do anything} \\ 1, & \text{multiply with } 2^i \end{cases}$   
 $i++, n >> 1.$

We will give `int n = 10010;` but it is actually a decimal number so last bit =  $n \% 10;$

```

1 #include<iostream>
2 #include<math.h>
3 using namespace std;
4
5
6 int main() {
7
8     int n;
9     cin >> n;
10
11    int i = 0, ans = 0;
12
13    while( n != 0) {
14
15        int digit = n % 10;
16
17        if( digit == 1) {
18            ans = ans + pow(2, i);
19        }
20
21        n = n/10;
22        i++;
23
24    }
25    cout<< ans << endl;
26
27
28 }

```



## Q1- Reverse Integer

Example test cases : I/P :- 123  
O/P :- 321

I/P :- -123  
O/P :- -321

I/P :- -2147483646  
O/P :- 0

(Because the reversed integer exceeds the range of int.)

Hint : Check for overflow in every iteration before or after updating ans.

### Babbar Code

```

1 class Solution {
2 public:
3     int reverse(int x) {
4         int ans = 0;
5         while(x) {
6             int digit = x % 10;
7             if((ans > INT_MAX/10) || (ans < INT_MIN/10))
8                 return 0;
9             ans = ans * 10 + digit;
10            x /= 10;
11        }
12        return ans;
13    }
14 };

```

### Using long long int

```

1 class Solution {
2 public:
3     int reverse(int x) {
4         long long int ans = 0;
5         while(x) {
6             int lastDigit = x % 10;
7             ans = ans*10 + lastDigit;
8             if(ans > INT_MAX || ans < INT_MIN)
9                 return 0;
10            x /= 10;
11        }
12        ans = x < 0 ? -ans : ans;
13        return ans;
14    }
15 };

```

## Q2 Complement of Base-10 integer.

Approach -1 :  $n = 5 = 101$

$$\begin{array}{r} 101 \\ \times \frac{1}{100} \\ \hline 100 \end{array}, \quad \begin{array}{r} 100 \\ \times \frac{1}{110} \\ \hline 110 \end{array}, \quad \begin{array}{r} 110 \\ \times \frac{1}{010} \\ \hline 010 \end{array} \Rightarrow \textcircled{2}$$

Let  $x = 1$ , till  $x \leq n$ , do  $n^x$  and then  $x \ll 1$ .

```

class Solution {
public:
    int bitwiseComplement(int n) {
        if(n == 0)
            return 1;
        int x = 1;
        while(x <= n) {
            n = n ^ x;
            x = x << 1;
        }
        return n;
    }
};

```

Approach - 2 : Babbar Method. (Video dekho)

```

class Solution {
public:
    int bitwiseComplement(int n) {
        int m = n;
        int mask = 0;
        if(n == 0)
            return 1;

        while(m != 0) {
            mask = (mask << 1) | 1;
            m = m >> 1;
        }

        int ans = (~n) & mask;
        return ans;
    }
};

```

Q3 Power of Two.

Hint : Any negative integer is not a power of 2.

①

```

class Solution {
public:
    bool isPowerOfTwo(int n) {
        for(int i = 0; i <= 30; i++) {
            int ans = pow(2,i);

            if(ans == n)
            {
                return true;
            }
        }
        return false;
    }
};

```

→ Brute Force

Improved  
Brute Force

(2)

```
class Solution {
public:
    bool isPowerOfTwo(int n) {
        int temp = 1;
        while(n != temp && temp < INT_MAX/2) {
            temp*=2;
        }
        return n == temp;
    }
};
```

OR

```
class Solution {
public:
    bool isPowerOfTwo(int n) {
        int ans = 1;
        for(int i = 0; i <= 30; i++) {
            //cout<<" ans "<<ans <<endl;
            if(ans == n)
            {
                return true;
            }
            if(ans < INT_MAX/2)
                ans = ans * 2;
        }
        return false;
    }
};
```

(3) Homework (Most efficient)

```
class Solution {
public:
    bool isPowerOfTwo(int n) {
        if(n <= 0)
            return false;
        return (n & (n - 1)) == 0;
    }
};
```

Hint:  $n \& (n-1)$  flips the rightmost SET bit.

Eg:  $n = 18 \Rightarrow 000\ldots010010$

$18 \& (17) \Rightarrow 000\ldots010010$   
 $\& \underline{000\ldots010001}$   
 $000\ldots010000$



Switch statement is a cleaner way of representing various cases for the same variable as compared to an if - else statement.

If you are repeatedly using if - else if for representing different cases of a variable like :

```
if( ) {  
}  
else if( ) {  
}  
}  
else if( ) {  
}  
}  
:  
else {  
}
```

Then you probably should switch to switch - case statements

## SYNTAX :

```
switch (variable-name) {  
    case value_1 :  
        ~~~~~~  
        ~~~~~~  
    break; ← optional
```

case value\_2 :

~~~~~  
~~~~~

break; ← optional.

| | | |  
| | | |  
| | | |  
| | | |

default :

~~~~~  
~~~~~

}



If you didn't understand the syntax, then don't worry and have patience. Everything will be made clear with the code implementation.

A simple usecase of switch - case

```
char ch = '1';

cout << endl;
switch( ch ) {

    case 1: cout << "First" << endl;
               break;

    case '1': cout << "character one" << endl;
               break;

    default: cout << " It is default case" << endl;
}
```

↓ Gives

```
lovebabbar@192 ~ % cd "/Users/lovebabbar/" && g++ switch.
ch && "/Users/lovebabbar/"switch

character one
```

Here, character ch is being checked for cases 1 and '1'.

Clearly, since ch = '1', the second case's condition is fulfilled and we get "character one" as output.

If `char ch = 'a'`, then we will go to the default case if you have made one. Thus, default is optional.

### Break :

Using `break` is optional for each case. If `break` is not used, all the cases starting from the first case whose condition was fulfilled, will get executed.

If `break` is used, only that case will get executed whose condition is fulfilled.

→ Without `break`:

```
char ch = 'a';
int num = 1;

cout << endl;
switch( num ) {

    case 1: cout << "First" << endl;
    cout << " First again " << endl;
    //break;

    case '1': cout << "character one" << endl;
    break;

    default: cout << " It is default case" << endl;
}
```

We will stop  
here if case '1' or  
case 1 is executed.

↓ Gives

```
lovebabbar@192 ~ % cd "/Users/lovebabbar/" && g++ switch.
ch && "/Users/lovebabbar/"switch

First
First again
character one
```

→ With `break`:

```
char ch = '1';
// int num = 1;

cout << endl;
switch( ch ) {

    case 1: cout << "First" << endl;
    cout << " First again " << endl;
    break;

    case '1': cout << "character one" << endl;
    break;

    default: cout << " It is default case" << endl;
}
```

| ↴ |

```
}
```

↓ Gives

```
lovebabbar@192 ~ % cd "/Users/lovebabbar/" && g++ switch.  
ch && "/Users/lovebabbar/"switch  
character one
```

## NESTED SWITCH - CASE EXAMPLE :

```
char ch = '1';  
int num = 1;  
switch(ch) {  
    case 1:  
        cout << "First Case" << endl;  
        break;  
    case '1':  
        cout << "Second Case" << endl;  
        switch(num) {  
            case 0:  
                cout << "num = 0" << endl;  
                break;  
            case 1:  
                cout << "num = 1" << endl;  
                break;  
            case 2:  
                cout << "num = 2" << endl;  
                break;  
            default:  
                cout << "don't know what num is" << endl;  
                break;  
        }  
        break;  
    default:  
        cout << "don't know what ch is" << endl;  
}
```

Notice this  
break. for  
case '1'

↓ Gives

```
Output.txt  
1 Second Case  
2 num = 1  
3
```

## Continue in Switch - Case :

Not possible/ valid to use continue statement in switch - case because we are not going through any iterations, thus there is nothing to skip.

## Mini - Calculator :

```

int a, b;

cout << " Enter the value of a " << endl;
cin >> a;
cout << " Enter the value of b " << endl;
cin >> b;
char op;
cout << " Enter the Operation you want to perform" << endl;
cin >> op;

switch( op ) {

    case '+': cout << (a+b) << endl;
                  break;

    case '-': cout << (a-b) << endl;
                  break;

    case '*': cout << (a*b) << endl;
                  break;

    case '/': cout << (a/b) << endl;
                  break;

    case '%': cout << (a%b) << endl;
                  break;

    default: cout << "Please enter a valid Operation " << endl;
}

}

```

## Homework Solution :

```

#include <iostream>
using namespace std;

int main(void)
{
    int money;
    cin >> money;

    switch (1) {
        case 1:
            cout << "No of 100 Rupee Notes = " << money/100 << endl;
            money = money % 100;

        case 2:
            cout << "No of 50 Rupee Notes = " << money/50 << endl;
            money = money % 50;

        case 3:
            cout << "No of 20 Rupee Notes = " << money/20 << endl;
            money = money % 20;

        case 4:
            cout << "No of 1 Rupee Notes= " << money << endl;
            money = money % 1; // unnecessary

    }
    return 0;
}

```

Output.txt

```

1 1330
1 No of 100 Rupee Notes = 13
2 No of 50 Rupee Notes = 0
3 No of 20 Rupee Notes = 1
4 No of 1 Rupee Notes= 10
5

```

## Functions :

Functions are code blocks / programs that store a code 'template' / 'blueprint' to be used again and again without making the code bulky. Any change that has to be made

is done within the function to reflect it everywhere. Functions make code readable, less bulky, less buggy.

Eg: Find power -  $2^5, 3^7, 4^6$

- ① Write the code of power 3 times and each time, take input for a and b. ( $a^b$ )
- ② Write a 'power' function and call it 3 times for different values of a and b as arguments.

```
int power() {
    int a, int b;
    cin >> a >> b;

    int ans = 1;

    for(int i = 1; i<=b; i++) {
        ans = ans * a;
    }

    return ans;
}
```

Calling power(a,b) in main() function.

```
int ans = power();
cout << " answer is " << ans << endl;
```

Output :

```
lovebabbar@192 ~ % cd "/Users/lovebabbar/" && g++ function1 && "/Users/lovebabbar/"function1
12 2
answer is 144
```

Check Even :

```
bool isEven(int a) {
    //odd
    if(a&1) {
        return 0;
    }
    else { //Even
        return 1;
    }
}
```

Find  ${}^n C_r$  :

-

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

We will need another function that finds out factorial of a number for us.

```
int factorial(int n) {
    int fact = 1;
    for(int i = 1; i<=n; i++) {
        fact = fact * i;
    }
    return fact;
}

int nCr(int n, int r) {
    int num = factorial(n);
    int denom = factorial(r) * factorial(n-r);
    return num/denom;
}
```

When  $nCr(5, 2)$  is called in main() function,

```
int num = factorial(5); // n=5 for this call
int denom = factorial(2) * factorial(3); // r=2
return num / denom;
```

for factorial(5), n=5 is passed to the function and it returns 120. Similar is the case for n = 2, 3.

## Counting Program :

Note: If you don't want to return anything, use void as the return type.

```

void printCounting(int n) {
    for(int i=1; i<=n; i++) {
        cout << i << " ";
    }
    cout << endl;
}

int main() {
    int n;
    cin >> n;

    printCounting(n);

    return 0;
}

```

```

lovebabbar@192 ~ % cd "/Users/lovebabbar/" && g++ counting && "/Users/lovebabbar/"counting
10
1 2 3 4 5 6 7 8 9 10

```

## Check Prime :

Return type ←  
void/int/float/char/bool

```

// 1 -> Prime no.
// 0 -> Not a Prime no.
bool isPrime(int n) {
    for( int i = 2; i<n; i++) {
        //divide hogya h , not a prime no.
        if(n%i == 0) {
            return 0;
        }
    }
    return 1;
}

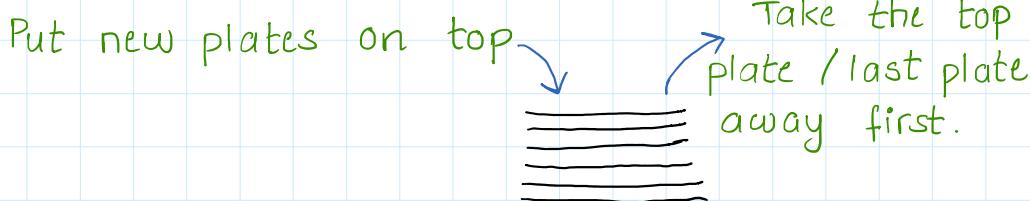
```

function parameters :  
variables local to  
this function that  
will receive some  
value from the fun.  
where it's called.

return statements

## Function Call Stack :

Stack is a Last-In-First-Out 'thing'. Example : Stack of plates.

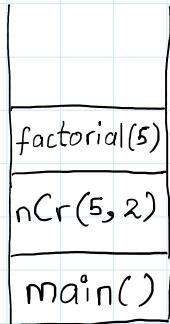


Last In

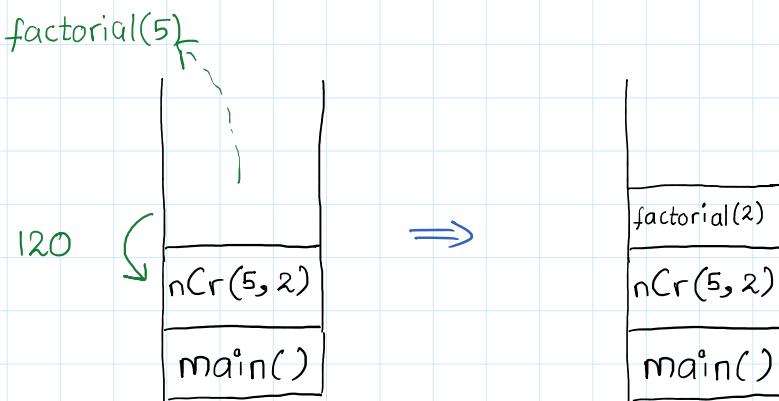
First Out

Functions that are called, get stored in a stack like structure.

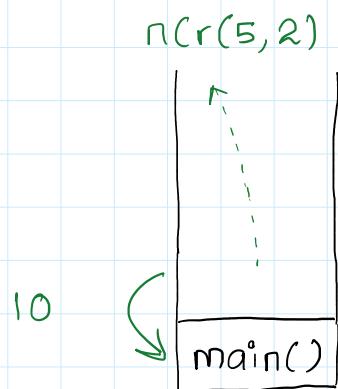
Eg: `main()`  $\xrightarrow{\text{Call}}$  `nCr(5, 2)`  $\xrightarrow{\text{Call}}$  `factorial(5)`



`factorial(5)` returns 120 and then `factorial(2)` is called.



and so on ... till `nCr(5, 2)` returns 10.



## Homework Questions :

Q1. Print  $3 * n + 7$  A.P.

```
#include <iostream>
using namespace std;

int ap(int n) {
    return n*3 + 7;
}

int main(void)
{
    int n;
    cin >> n;
    cout << ap(n) << endl;
    return 0;
}
```

1 16  
1 55

Q2. Print total no. of SET bits in A & B.

```
#include <iostream>
using namespace std;

int setBits(int num) {
    int ans = 0;
    while(num) {
        num = num & (num-1);
        ans++;
    }
    return ans;
}

int main(void)
{
    int a, b;
    cin >> a >> b;
    cout << setBits(a) + setBits(b) << endl;
    return 0;
}
```

1 11  
2 2  
1 4  
2

Q3. Fibonacci Number

```
#include <iostream>
using namespace std;

int fibonacciNumber(int n) {
    if(n == 1 || n == 2)
        return n-1;
    int ans = 1, a = 0, b = 1;
    for(int i=3;i<=n;i++) {
        ans = a + b;
        a = b;
        b = ans;
    }
    return ans;
}

int main(void)
{
    int n;
    cin >> n;
    cout << fibonacciNumber(n) << endl;
    return 0;
}
```

1 8  
1 13  
2

Call By Value :

When we pass a variable (not its pointer / address) to a function, we pass its value instead of passing the entire

variable (original variable). This is call by value where we pass the copy of the variable instead of the original variable present at some memory address.

If we call fibonacciNumber(6), then we are just giving the value 6 in place of n everywhere in the funct.

```
int fibonacciNumber(int n) {
    if(n == 1 || n == 2)
        return n-1;
    int ans = 1, a = 0, b = 1;
    for(int i=3;i<=n;i++) {
        ans = a + b;
        a = b;
        b = ans;
    }
    return ans;
```

If in main() function,

```
int n = 6;
cout << fibonacciNumber(n);
```

And change the value of n in fibonacciNumber(), then we will not change the value of the original n in the main() function.

```
#include <iostream>
using namespace std;

void fun(int n) {
    cout << "value of copy in fun() before changing = " << n << endl;
    n += 10;
    cout << "value of copy in fun() after changing = " << n << endl;
}

int main(void)
{
    int n;
    cin >> n;
    cout << "value of real n before function call = " << n << endl;
    fun(n);
    cout << "value of real n after function call = " << n << endl;

    return 0;
}
```

1 10

Output.txt

```
1 value of real n before function call = 10
2 value of copy in fun() before changing = 10
3 value of copy in fun() after changing = 20
4 value of real n after function call = 10
5
```



Arrays help you store variables of the same data type at contiguous memory locations.

Store 10 integers. (Suppose int takes 4 bytes)

|          |                                               |
|----------|-----------------------------------------------|
| array:   | 20   11   4   -2   5   17   11   19   91   83 |
| address: | 200 204 208 212 216 220 224 228 232 236       |

Start address ↗

The contiguous memory blocks help us when we try accessing the elements of the array.

Given the start address (address of the array in memory), I can access all the elements by simply incrementing the address by 4 (size of data type).

Suppose our array is named arr. We represent an array as arr[ ].

To access the elements of an array, we use indices (plural of index). We follow a 0-based indexing in most programming languages

|                                               |     |     |     |     |     |     |     |     |     |
|-----------------------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0                                             | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
| 20   11   4   -2   5   17   11   19   91   83 |     |     |     |     |     |     |     |     |     |
| 200                                           | 204 | 208 | 212 | 216 | 220 | 224 | 228 | 232 | 236 |

$n = 10$

$\therefore \text{arr}[0] = 20, \text{arr}[6] = 11$

Thus, arrays are a 'data structure' that help storing multiple variables of the same data type in contiguous memory locations

## DECLARATION OF ARRAYS :

1. Declaring an array to store 100 integers :

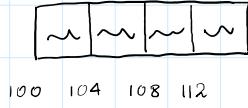
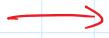
`int arr[100];`

2. Declaring an array to store 100 booleans:

```
bool isGood[100];
```

Now, in examples ① and ②, arr and isGood are the addresses of the respective arrays and point to the memory location where the first element is stored.

Eg: int arr[4];



~: garbage value

where arr[0] is 100 - 103 (4 bytes)

arr[1] is 104 - 107 (4 bytes)

arr[2] is 108 - 111 (4 bytes)

arr[3] is 112 - 115 (4 bytes)

If you print arr, you will get 100 in output.

```
cout << arr;
```

STDOUT: 100

Which is the address of the first element arr[0].

```
cout << &arr[0];
```

STDOUT: 100

```
cout << arr[1];
```

// Suppose arr[]: 

|   |   |    |   |
|---|---|----|---|
| 4 | 8 | 11 | 1 |
|---|---|----|---|

STDOUT: 8

0 1 2 3

## INITIALIZATION:

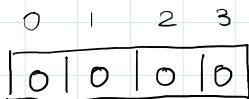
① int num[5] = {5, 18, 40, 12}

0 1 2 3

|   |    |    |    |
|---|----|----|----|
| 5 | 18 | 40 | 12 |
|---|----|----|----|

② Initializing an array with 0.

```
int arr[4] = {0};
```



③ Initializing an array with any number. (Homework)

```
int arr[10];
std::fill(arr, arr+10, 27); // std:: is optional if you have
                           ↑   ↑   ↑
                           start address end address value
                           declared the namespace as
                           discussed in previous lectures.
```

OR

```
int arr[10];
std::fill_n(arr, 10, 27);
                           ↑   ↑   ← Value
                           start address No.of elements
                           from the start address
                           to fill
```

## ACCESSING ELEMENTS OUT OF RANGE :

```
//declare
int number[15];

//accessing an array
cout << "Value at 14 index " << number[14] << endl;
cout << "Value at 20 index " << number[20] << endl;
```



⚠️ Accessing index out of range will throw an error.

Here, you can only access elements from number[0] to number[14].

```
lovebabbar@192 ~ % cd "/Users/lovebabbar/" && g++ arraysIntro.cpp -o arraysIntro && "/Users/lovebabbar/"arraysIntro
arraysIntro.cpp:17:37: warning: array index 20 is past the end of the array (which contains 15 elements) [-Warray-bounds]
    cout << "Value at 20 index " << number[20] << endl;
                           ^ ~~~~~
arraysIntro.cpp:12:5: note: array 'number' declared here
    int number[15];
               ^
1 warning generated.
Value at 14 index 1
Value at 20 index 1
```

## PRINTING AN ARRAY :

```
//print the array
for(int i = 0; i < n; i++) {
    cout << third[i] << " ";
}
```

where  $n$  is the size of the array. See here that our loop is running from 0 to  $n-1$  (not  $n$ ). Name of our array is `third`.

Printing array using a function :

```
void printArray(int arr[], int size) {
    cout << " printing the array " << endl;
    //print the array
    for(int i = 0; i < size; i++) {
        cout << arr[i] << " ";
    }
    cout << " printing DONE " << endl;
}
```

We have to pass the size because `int arr[]` in the function declaration is simply the address of the array and gives us no information about the size.

Also, if the array has space for 1000 elements but I am storing just 6, then I can pass size as 6 to the function.

## MAXIMUM / MINIMUM IN AN ARRAY :

Any value we enter in our array would be less than or equal to max.

```
int getMax(int num[], int n) {
    int max = INT_MIN;
    for(int i = 0; i < n; i++) {
        if(num[i] > max){
            max = num[i];
        }
    }
    //returning max value
    return max;
}
```

```
int getMin(int num[], int n) {
    int min = INT_MAX;
    for(int i = 0; i < n; i++) {
        if(num[i] < min){
            min = num[i];
        }
    }
    //returning min value
    return min;
}
```

```

int main() {
    int size;
    cin >> size;

    int num[100];

    //taking input in array
    for(int i = 0; i<size; i++) {
        cin >> num[i];
    }

    cout << " Maximum value is " << getMax(num, size) << endl;
    cout << " Minimum value is " << getMin(num, size) << endl;

    return 0;
}

```

**CAUTION:** Never declare a variable-sized array. Always specify a constant size while declaring your array even if it turns out to be very large as compared to your required array size.

## ARRAYS ARE ALWAYS PASSED BY REFERENCE :

When we pass an array to a function like this :

func (int arr[]) → address of arr, say 100.

We are not passing a copy of the array unlike variables, since array name 'arr' is the address of the array, thus we pass the address of the array to the function allowing it to make changes to the original array.

This is called passing by reference of array.

**HOMEWORK:** Print sum of all elements of an array.

```

#include <iostream>
using namespace std;

int getSumOfArray(int arr[], int n) {
    int sum = 0;
    for(int i=0; i<n; i++) {
        sum += arr[i];
    }
    return sum;
}

int main(void)
{
    int n;
    cin >> n;
    int arr[1000];
    for(int i=0; i<n; i++) {
        cin >> arr[i];
    }
    int sum = getSumOfArray(arr, n);
    cout << "Sum of all elements of the array = " << sum << endl;
    return 0;
}

```

1 5  
2 -4 9 0 11 2

Output.txt  
1 Sum of all elements of the array = 18  
2

## LINEAR SEARCH :

```

bool search(int arr[], int size, int key) {

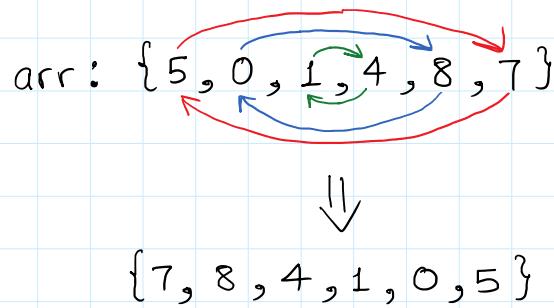
    for( int i = 0; i<size; i++ ) {

        if( arr[i] == key) {
            return 1;
        }
    }
    return 0;
}

```

Traversing the array and comparing each element with the search key.

## REVERSE AN ARRAY :



```

void reverse(int arr[], int n) {

    int start = 0;
    int end = n-1;

    while(start<=end) {
        swap(arr[start], arr[end]);
        start++;
        end--;
    }
}

```



## Q1. Swap Alternate.

**Approach :** Given an array, say  $[1, 2, 3, 4, 5]$ , we will run a for loop from  $0$  to  $n-2$  and swap  $\text{arr}[i]$  with  $\text{arr}[i+1]$ . If we reach the last element, we will stop, so check if  $(i+1)^{\text{th}}$  element < size of array.

**Dry Run :**  $\text{arr}[] : \{1, 2, 3, 4, 5\}$

- ①  $\{ \underset{i}{\cancel{1}}, 2, 3, 4, 5 \} \Rightarrow \{ 2, 1, 3, 4, 5 \}$
- ②  $\{ 2, \underset{i}{\cancel{1}}, 3, 4, 5 \} \Rightarrow \{ 2, 1, 4, 3, 5 \}$
- ③  $\{ 2, 1, 4, 3, 5 \} \quad \underline{\text{Stop}}$

**Code :**

```
void swapAlternate(int arr[], int size) {
    for(int i = 0; i<size; i+=2) {
        if(i+1 < size) {
            swap(arr[i], arr[i+1]);
        }
    }
}
```

## SWAP FUNCTION :

If  $a = 5$ ,  $b = 10$ , I can't simply do  $a = b$ ,  $b = a$  to swap them. Suppose I did this:

```
int a = 5, b = 10;
a = b; // a becomes 10
b = a; // b becomes 10
```

$$a = 10 \quad b = 10$$

Use a third variable to temporarily store the value of  $a$  and then change  $a$  to  $b$ .

```

int a = 5, b = 10;
int temp = a; // temp = 5.
a = b; // a becomes 10
b = temp; // b becomes 5.

```

a = 10    b = 5

```

void swap(int *a, int *b) {
    // pointers to numbers are given
    int temp = *a; // Store the value at address a
    *a = *b; // Change the value at address a to the value at address b
    *b = temp; // Change the value at address b to the original value of a
}

```

## Q2. Find Unique Element.

**Approach:** Since all numbers except any one occur twice, thus if there are m numbers that occur twice and one that occurs once, there will have  $2m+1$  numbers.

The XOR of same numbers is 0. Thus  $5^5 = 1$ .

We can find XOR of all numbers, which will be the only element occurring once.

**Dry Run:** arr[] : {1, 2, 8, 2, 1, 7, 7}

$$1^2^8^2^1^7^7 = 0^8 = \boxed{8}$$

**Code:**

```

int findUnique(int *arr, int size)
{
    int ans = 0;

    for(int i = 0; i < size; i++) {
        ans = ans ^ arr[i];
    }
    return ans;
}

```

## Q3 Unique Number of Occurrences. (Homework)

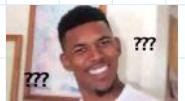
↑ We can use an unordered map that uses the concept of



We can use an unordered map that uses the concept of hashing. Learn about it and then use it by yourself.

```
bool uniqueOccurrences(vector<int>& arr) {  
    vector<int> occurrence(2001, 0);  
    // ith index is occurrence of (i-1000)  
  
    for(int i=0; i<arr.size(); i++) {  
        occurrence[arr[i]+1000]++;  
    }  
    // check if all non-zero elements are unique  
    sort(occurrence.begin(), occurrence.end());  
    // if any adjacent non-zero neighbours are equal, then we have repeating occurrences  
    for(int i=0;i<2000;i++) {  
        if(occurrence[i] != 0 && occurrence[i] == occurrence[i+1])  
            return false;  
    }  
    return true;  
}
```

```
bool uniqueOccurrences(vector<int>& arr) {  
    unordered_map<int, int> m;  
    for(int i=0;i<arr.size();i++) {  
        m[arr[i]]++;  
    }  
    // insert the occurrences into a set that will store them in increasing order  
    set<int> occ;  
    for(auto x: m) {  
        occ.insert(x.second);  
    }  
    // if the set has the same number of elements as the map then there were all unique occurrences  
    return occ.size() == m.size();  
}
```



## Q4 Find Duplicates

Approach 1 : XOR all the elements of the array with  $1, 2, 3, 4, \dots, n-1$ .

Approach 2 : Subtract the sum of first  $n-1$  natural nos. from the sum of all elements of the array.

Dry Run :  $\text{arr}[]: \{4, 2, 1, 3, 1\}$

$n-1$  Natural Numbers :  $1, 2, 3, 4$

$$\text{XOR} : (4^2^1^3^1) \oplus (1^2^3^4) = \boxed{1}$$

$$\text{sum}(\text{arr}) = 4 + 2 + 1 + 3 + 1 = 11.$$

$$\text{sum of } (n-1) \text{ natural nos.} = 1 + 2 + 3 + 4 = 10$$

$$\frac{(n-1)*n}{2}$$

- ①

- ②

$$\text{eq.} ① - \text{eq.} ② \stackrel{e}{=}$$

$$\begin{array}{r}
 4 + 2 + 1 + 3 + 1 \\
 - 1 + 2 + 3 + 4 \\
 \hline
 1
 \end{array}$$

Code:

```

int findDuplicate(vector<int> &arr)
{
    int ans = 0;

    for(int i = 0; i<arr.size(); i++) {
        ans = ans^arr[i];
    }

    for(int i = 1; i<arr.size(); i++) {
        ans = ans^i;
    }
    return ans;
}

```

Homework: Find all duplicates in an array.

- Approaches :
- ① For every element, check if there exists a duplicate element in  $\text{arr}[i+1 \dots n-1]$ , if yes then store it in answer array.
  - ② Sort the array and check for adjacent elements if they are equal.
  - ③ Track frequency using an unordered map (HashMap) and traverse it to get all the repeating elements.
  - ④ Treat the element at  $\text{arr}[i]$  as the next index to jump on and mark this new element as negative to show that you have already been to  $\text{arr}[i]$ .  
If you visit an element and found it negative, then you have visited it twice.



MOST EFFICIENT APPROACH'S DRY RUN:

$i$   
 $\text{arr}[] : \{4, 3, 2, 5, 2\}$

$\text{arr}[i] = 4$  for  $i=0$ , visit  $\text{arr}[4-1] = \text{arr}[3] = 5$

and mark it -ve

⋮  
 $\{4, 3, 2, -5, 3\}$

$\text{arr}[i] = 3$  for  $i=1$ , visit  $\text{arr}[3-1] = \text{arr}[2] = 2$   
and mark it -ve.

$\{4, 3, -2, -5, 3\}$

$\text{arr}[i] = 2$  for  $i=2$ , visit  $\text{arr}[2-1] = \text{arr}[1] = 3$   
and mark it -ve.

$\{4, -3, -2, -5, 3\}$

$\text{arr}[i] = 5$  for  $i=3$ , visit  $\text{arr}[5-1] = \text{arr}[4] = 3$   
and mark it -ve.

$\{4, 3, -2, -5, -3\}$

$\text{arr}[i] = 3$  for  $i=4$ , visit  $\text{arr}[3-1] = \text{arr}[2] = -2$   
Since it's already negative, we have  
visited  $\text{arr}[2]$  twice and thus 3 has  
been used twice to check for  $\text{arr}[3-1]$

∴ 3 is occurring twice.

Code :

```
vector<int> findDuplicates(vector<int>& nums) {
    vector<int> ans;
    for(int i=0; i<nums.size(); i++) {
        // visit nums[abs(nums[i]) - 1]
        // abs is used because the current element might be negative
        if(nums[abs(nums[i]) - 1] < 0) {
            ans.push_back(abs(nums[i]));
        }
        else {
            nums[abs(nums[i]) - 1] = -nums[abs(nums[i]) - 1];
        }
    }
    return ans;
}
```

## Q5 Find Intersection.

Approach : Maintain two trackers  $i$  and  $j$  for the first and second array respectively.  
At any point of time if :

- ①  $\text{arr}[i] < \text{arr}[j] \rightarrow$  increment  $i$
- ②  $\text{arr}[i] > \text{arr}[j] \rightarrow$  increment  $j$
- ③  $\text{arr}[i] == \text{arr}[j] \rightarrow$  include in answer and increment both  $i$  and  $j$ .

Code :

```
vector<int> findArrayIntersection(vector<int> &arr1, int n, vector<int> &arr2, int m)
{
    vector<int> ans;
    int i = 0, j = 0;
    while(i < arr1.size() && j < arr2.size()) {
        if(arr1[i] < arr2[j]) {
            i++;
        }
        else if(arr1[i] > arr2[j]) {
            j++;
        }
        else {
            ans.push_back(arr1[i]);
            i++;
            j++;
        }
    }
    return ans;
}
```

## Q6. Pair Sum.

Approach : For the  $i^{th}$  element, check if its sum with all  $i+1, i+2, \dots, n-1$  elements  $= sum$ . If no then continue, if yes then push the  $i^{th}$  &  $j^{th}$  elements in an answer vector.

Dry Run :  $\text{arr}[] : \{1, 2, 3, 4\}$   
 $sum = 5$ .

- ①  $\overset{i}{\text{arr}}[] : \{1, 2, 3, 4\}$

Check  $\text{arr}[j]$  for all  $j = 1$  to  $3$ .

$\text{arr}[i] = 1, 1 + \text{arr}[j] == 5$ .

This is satisfied for  $j = 3, 1 + 4 = 5$ .

$\Rightarrow \text{ans.push\_back}(\{1, 4\})$

- ②  $\overset{i}{\text{arr}}[] : \{1, 2, 3, 4\}$

Check  $\text{arr}[j]$  for all  $j = 2$  to  $3$ .

$\text{arr}[i] = 2, 2 + \text{arr}[j] == 5$ .

This is satisfied for  $j = 2, 2 + 3 = 5$ .

$\rightarrow \dots . . . ^{c} - ^{n} \downarrow$

$\Rightarrow \text{ans.push\_back}(\{2, 3\})$

(3)  $\text{arr}[] : \{1, 2, 3, 4\}$

Check  $\text{arr}[j]$  for all  $j = 3$  to 3.

$\text{arr}[i] = 3, 3 + \text{arr}[j] == 5$ .

This is not satisfied for any pair.

Code :

```
vector<vector<int>> pairSum(vector<int> &arr, int s){  
    vector<vector<int>> ans;  
  
    for(int i=0; i<arr.size(); i++)  
    {  
        for(int j = i+1; j<arr.size(); j++) {  
            if(arr[i] + arr[j] == s)  
            {  
                vector<int> temp;  
                temp.push_back(min(arr[i], arr[j]));  
                temp.push_back(max(arr[i], arr[j]));  
                ans.push_back(temp);  
            }  
        }  
    }  
    sort(ans.begin(), ans.end());  
    return ans;  
}
```

Homework : Find triplet sum.

Same Approach as Find Pair Sum.

Code :

```
#include <set>  
vector<vector<int>> findTriplets(vector<int> arr, int n, int K) {  
    vector<vector<int>> ans;  
    set<vector<int>> s;  
    for(int i=0; i<n; i++) {  
        for(int j = i + 1; j < n; j++) {  
            for(int k = j + 1; k < n; k++) {  
                if(arr[i]+arr[j]+arr[k] == K) {  
                    vector<int> temp;  
                    temp.push_back(arr[i]);  
                    temp.push_back(arr[j]);  
                    temp.push_back(arr[k]);  
                    sort(temp.begin(), temp.end());  
                    s.insert(temp);  
                }  
            }  
        }  
    }  
    for(auto x: s) {  
        ans.push_back(x);  
    }  
    return ans;  
}
```

## Q7. Sort 0, 1

Approach : ① Traverse array & update count of 0 and 1.  
Again, traverse the array and fill 0s for the first numZero indices and 1 for the remaining indices.

② Maintain 2 pointing variables  $i$  and  $j$ ,  $i=0$  &  $j=n-1$ . In any condition, if :

- i)  $\text{arr}[i] == 0 \rightarrow i++$
- ii)  $\text{arr}[j] == 1 \rightarrow j--$
- iii)  $\text{arr}[i] == 1 \& \& \text{arr}[j] == 0 \rightarrow \text{swap them}$   
 $i++, j--$

Dry Run :  $\text{arr}[] = \{1, 0, 0, 1, 1, 1, 0, 1\}$

$\begin{matrix} i \\ \downarrow \\ \{1, 0, 0, 1, 1, 1, 0, 1\} \end{matrix} \rightarrow \text{decrement } j;$

$\begin{matrix} i \\ \downarrow \\ \{1, 0, 0, 1, 1, 1, 0, 1\} \end{matrix} \rightarrow \text{swap}(\text{arr}[i++], \text{arr}[j--]);$

$\begin{matrix} i \\ \downarrow \\ \{1, 0, 0, 1, 1, 1, 0, 1\} \end{matrix} \rightarrow \text{increment } i;$

$\begin{matrix} i \\ \downarrow \\ \{1, 0, 0, 1, 1, 1, 0, 1\} \end{matrix} \rightarrow \text{increment } i;$

$\begin{matrix} i \\ \downarrow \\ \{1, 0, 0, 1, 1, 1, 0, 1\} \end{matrix} \rightarrow \text{decrement } j;$

$\begin{matrix} i \\ \downarrow \\ \{1, 0, 0, 1, 1, 1, 0, 1\} \end{matrix} \rightarrow \text{decrement } j;$

$\begin{matrix} i \\ \downarrow \\ \{1, 0, 0, 1, 1, 1, 0, 1\} \end{matrix} \rightarrow$



Code :

```
void printArr(int arr[], int n) {
    for(int i=0; i<n; i++) {
        cout << arr[i] << " ";
    }
    cout << endl;
}

void sortOneTwo(int arr[], int n) {
    int i = 0, j = n-1;
    while(i < j) {
        if(arr[i] == 0) {
            i++;
        } else if(arr[j] == 1) {
            j--;
        } else if(arr[i] == 1 && arr[j] == 0) {
            swap(arr[i], arr[j]);
            i++;
            j--;
        }
    }
}
```

```
1 1 0 0 0 1 0 3
1 0 0 0 0 1 1 1
```

Homework : Sort 0 1 2.

Approach : Take 3 pointing variables  $i$ ,  $j$  and  $k$  where  $i$ ,  $j$  and  $k$  are such that :

$$arr[0 \dots i-1] = 0$$

$$arr[k+1 \dots n-1] = 2$$

$i=0$ ,  $j=0$  and  $k=n-1$ .

$$arr[]: \{0, 1, 1, 0, 2, 1, 0\}$$

$i$   $j$   $k$

If  $arr[j] == 0$ ,  $\text{swap}(arr[i], arr[j])$  and then  $i++$ ,  $j++$ .

If  $arr[j] == 1$ , increment  $j$ .

If  $arr[j] == 2$ ,  $\text{swap}(arr[j], arr[k])$  and then  $k--$

$$\{0, 1, 1, 0, 2, 1, 0\} \rightarrow \text{swapping } arr[i], arr[j] \& i++, j++.$$

$i$   $j$   $k$

$$\{0, 1, 1, 0, 2, 1, 0\} \rightarrow j++$$

$i$   $j$   $k$

$$\{0, 1, 1, 0, 2, 1, 0\} \rightarrow j++$$

$i$   $i$   $k$

$\{0, 1, 1, 0, 2, 1, 0\} \rightarrow j++$

$\{0, 1, 1, 0, 2, 1, 0\} \rightarrow$  swapping arr[i], arr[j] & i++, j++.

$\{0, 0, 1, 1, 2, 1, 0\} \rightarrow$  Swapping arr[j], arr[k] & k--

$\{0, 0, 1, 1, 0, 1, 2\} \rightarrow$  swapping arr[i], arr[j] & i++, j++.

$\{0, 0, 0, 1, 1, 1, 2\} \rightarrow j++$

$\{0, 0, 0, 1, 1, 1, 2\} \rightarrow$



Observe how we are maintaining  $\text{arr}[0 \dots i-1] = 0$  and  $\text{arr}[k+1 \dots n-1] = 2$ .

Why are we not incrementing j when swapping (arr[j], arr[k])?

arr[k] might be 0. If I swap arr[j] with arr[k], then we will get arr[j] = 0, we want swap this with arr[i] in the next step so we should not omit arr[j] by incrementing j.

Code :

```
void sort012(int *arr, int n)
{
    int i = 0, j = 0, k = n-1;
    while(j <= k) {
        if(arr[j] == 1)
            j++;
        else if(arr[j] == 0) {
            swap(arr[i++], arr[j++]);
        }
        else {
            swap(arr[j], arr[k--]);
        }
    }
}
```



## What is time complexity?

Ans: Time complexity gives the "idea" of the amount of time taken by an algorithm as a function of the input size.

It does not necessarily depict the exact execution-time taken by an algorithm, because run-time of a program depends on the following factors:

- ① Computer Hardware / Architecture
  - ② Background Processes and load on the system during execution.
  - ③ The exact location in memory where the program is stored.
- , etc.

Thus, it's impossible to determine the execution time of a program that is applicable in all possible cases.

To compare different algorithms, we can easily calculate the time (and space) complexity of a program and judge which is faster and in general, more optimized.

## How to Calculate?

Big Oh is used to denote the worst-case time complexity (upper-bound) of an algorithm.

Eg: I take at most 15 minutes to finish my lunch.

Constant Time:  $O(1)$

Linear Time:  $O(n)$ .

Eg: `for(int i=0; i<10; i++){}`

```

    cout << "Hello" << endl;
}

```

This will always run the loop 10 times irrespective of any input given. Thus, constant time complexity -  $\mathcal{O}(1)$ .

```

Eg: for(int i=0 ; i<n ; i++){
    cout << "Hello" << endl;
}

```

This will always run the loop  $n$  times which depends on  $n$  'linearly'. Thus, linear time complexity -  $\mathcal{O}(n)$ .

## Order of Time Complexity :

$$\mathcal{O}(1) < \mathcal{O}(\log n) < \mathcal{O}(\sqrt{n}) < \mathcal{O}(n) < \mathcal{O}(n \log n) < \mathcal{O}(n^2) < \mathcal{O}(n^3) < \mathcal{O}(2^n) < \mathcal{O}(n!)$$



Faster / Efficient

## Finding Upper Bound :

$$\text{Q1: } f(n) = 4n^4 + 3n^2 + 2$$

- ① Remove all terms of lower power/precedence (refer to the order above).
- ② Remove constants multiplied/divided with/from the highest precedence term.

$$\textcircled{1} \quad \mathcal{O}(f(n)) = 4n^4 + 3n^2 + 2 \approx 4n^4$$

$$\textcircled{2} \quad \mathcal{O}(f(n)) = 4 \cdot n^4 = \boxed{n^4}$$

$\Rightarrow f(n) = \mathcal{O}(n^4)$  which means that  $f(n)$  will always be upper-bounded/smaller than  $n^4$  after some value of  $n = n_0$ .

$$\varnothing 2 \quad f(n) = 4\log n + 5\sqrt{n} + 17$$

$$O(f(n)) = 4\log n + 5\sqrt{n} + 17 = \boxed{\sqrt{n}}$$

$$f(n) = O(\sqrt{n})$$

$\varnothing 3$  Reverse an array.

→ We swap  $\text{arr}[i]$  with  $\text{arr}[j]$  starting with  $i=0$  &  $j=n-1$  and increment & decrement  $i$  &  $j$  respectively until  $i < j$ .

Thus, we make approx.  $\frac{n}{2}$  swaps for an array of length  $n$ .

$$f(n) = \frac{n}{2} \Rightarrow O(f(n)) = n$$

$$f(n) = O(n)$$

$\varnothing 4$  Linear Search.

We traverse the whole array, so :

$$f(n) = n \Rightarrow O(f(n)) = n$$

$$f(n) = O(n)$$

```
∅5. int a = 0, b = 0;
    for (i = 0; i < N; i++) {
        for (j = 0; j < N; j++) {
            a = a + j;
        }
    }
    for (k = 0; k < N; k++) {
        b = b + k;
    }
```

} Runs  $N$  times for each iteration  
i: 0 to  $N-1$

} Runs  $N$  times.

$$\text{Time Complexity} = O(f(N)) = O(N^2 + N) = O(N^2)$$

```
∅6. int a = 0;
    for (i = 0; i < N; i++) {
```

Careful!

```

96. int a = 0;
    for (i = 0; i < N; i++) {
        for (j = N; j > i; j--) {
            a = a + i + j;
        }
    }

```

Careful!

} Runs from N to  $i+1$  times  
for  $i: 0$  to  $N-1$ .

Detailed Soln:

For  $i=0$ : Inner for loop runs N times ( $j=N$ ;  $j>0$ ;  $j--$ )

For  $i=1$ : Inner for loop runs  $N-1$  times ( $j=N$ ;  $j>1$ ;  $j--$ )

For  $i=2$ : Inner for loop runs  $N-2$  times ( $j=N$ ;  $j>2$ ;  $j--$ )

For  $i=N-1$ : Inner for loop runs 1 time.

$$\begin{aligned}
 \text{Total: } & N + (N-1) + (N-2) + \dots + 1 \\
 &= 1 + 2 + 3 + \dots + N \\
 &= \frac{N(N+1)}{2} \\
 &= \frac{N^2}{2} + \frac{N}{2}
 \end{aligned}$$

$$f(n) = \frac{N^2}{2} + \frac{N}{2} = \boxed{O(N^2)}$$

$10^8$  Operation Rule:

Most modern machines can perform  $10^8$  operations/second.

We use the constraints given in a question to determine the maximum T.C. I can have in my solution.

| input size    | required time complexity |
|---------------|--------------------------|
| $n \leq 10$   | $O(n!)$                  |
| $n \leq 20$   | $O(2^n)$                 |
| $n \leq 500$  | $O(n^3)$                 |
| $n \leq 5000$ | $O(n^2)$                 |
| $n \leq 10^6$ | $O(n \log n)$ or $O(n)$  |
| $n$ is large  | $O(1)$ or $O(\log n)$    |

Source : CodeForces.

**Space Complexity :** Gives the 'idea' of the amount of space required by a program "with respect to" the input.

**$O(1)$  Space Complexity -**

- ① int a;
- ② int a, b, c, d, z;
- ③ int arr[1000];

**$O(n)$  Space Complexity :**

- ① int arr[n]; // Bad practice
- ② vector<int> v(n);



Binary Search is a searching algorithm that works in  $O(\log n)$  time complexity.

## IMPORTANT :



Binary Search is only applicable for monotonic search spaces. For example, B.S. can be applied for searching an element in a SORTED ARRAY only.

A sorted array where elements are arranged in a non-decreasing order is a monotonically increasing search space. Similarly, if elements are arranged in a non-increasing order, then it's a monotonically decreasing search space.

Eg : arr[ ] : {1, 3, 3, 6, 7, 10} → monotonically increasing search space

# APPLYING BINARY SEARCH

arr[ ] : { 1, 3, 3, 6, 7, 10 } search = 7

① Fix a low and high index for the array where you want to search.

② Find the mid index for the given low & high index.

$$\text{mid} = (\text{low} + \text{high}) / 2 ; \quad \rightarrow (0+5) / 2 = 2$$

arr[ ] : { 1, 3, 3, 6, 7, 10 }  
low                  mid                  high

③ Check the following :

- (i) If  $\text{arr}[\text{mid}] == \text{search}$ , return  $\text{mid}$ , you found it!
- (ii) If  $\text{arr}[\text{mid}] < \text{search}$ , then all elements  $\text{arr}[0 \dots \text{mid}]$  will definitely be smaller than  $\text{search}$ . Thus, update your search space :  $\text{low} = \text{mid} + 1$ .
- (You have discarded  $0 \dots \text{mid}$  indices)
- (iii) If  $\text{arr}[\text{mid}] > \text{search}$ , then all elements  $\text{arr}[\text{mid} \dots n-1]$  will definitely be greater than  $\text{search}$ . Thus, update your search space :  $\text{high} = \text{mid} - 1$ .
- (You have discarded  $\text{mid} \dots n-1$  indices)

$\text{arr}[] : \{1, 3, 3, 6, 7, 10\}$

$\text{arr}[\text{mid}] = 3$  and  $3 < 7$   
 $\text{low} = \text{mid} + 1 = 3$ .



$\text{arr}[] : \{1, 3, 3, 6, 7, 10\}$

④ Repeat steps ② and ③ until  $\text{low} \leq \text{high}$ .

$\text{arr}[] : \{1, 3, 3, 6, 7, 10\}$

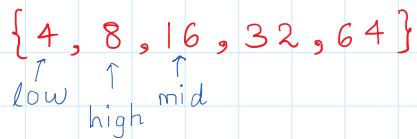
$\text{arr}[\text{mid}] == 7$ , return  $\text{mid}$ ,

return 4;

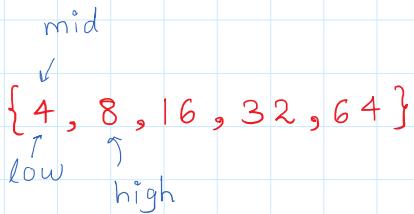
EXAMPLE :  $\text{arr}[] : \{4, 8, 16, 32, 64\}$        $K = 4$

①  $\{4, 8, 16, 32, 64\}$

②  $(\text{arr}[\text{mid}] = 16) > 4 \Rightarrow \text{high} = \text{mid} - 1$ .



③  $(\text{arr}[\text{mid}] = 8) > 4 \Rightarrow \text{high} = \text{mid} - 1$



④  $(\text{arr}[\text{mid}] = 4)$  We found it !

Note: If  $\text{low} > \text{high}$  and we haven't found the element, then the element doesn't exist at all.

Code :

```
int binarySearch(int arr[], int size, int key) {
    int start = 0;
    int end = size-1;

    int mid = (start + end)/2;

    while(start <= end) {
        if(arr[mid] == key) {
            return mid;
        }

        // go to right wala part
        if(key > arr[mid]) {
            start = mid + 1;
        }
        else{
            end = mid - 1;
        }

        mid = (start+end)/2;
    }
    return -1;
}
```

```

int even[6] = {2,4,6,8,12,18};
int odd[5] = {3, 8, 11, 14, 16};

int evenIndex = binarySearch(even, 6, 20);

cout << " Index of 18 is " << evenIndex << endl;

int oddIndex = binarySearch(odd, 5, 8);

cout << " Index of 8 is " << oddIndex << endl;

```

```

lovebabbar@192 ~ % cd "/Users/lovebabbar/" && g++ binarySearch.cpp -o binarySearch && "/Users/lovebabbar/"binarySearch
Index of 18 is -1
Index of 8 is 1

```

## CAUTION :

What if you had an array of size  $2^{31} - 1$ . and during some iteration :

$$\text{low} = 2^{31} - 2 \quad \text{and} \quad \text{high} = 2^{31} - 1,$$

$$\text{mid} = \frac{(\text{low} + \text{high})}{2}$$

will throw integer overflow error because  $2^{31} + 2^{31} - 1 = 2^{32} - 1$

which is greater than INT\_MAX. Thus, we can either use long long or even better,

$$\text{mid} = \frac{\text{low} + (\text{high} - \text{low})}{2}$$

## New Code :

```

int binarySearch(int arr[], int size, int key) {
    int start = 0;
    int end = size-1;

    int mid = start + (end-start)/2;

    while(start <= end) {
        if(arr[mid] == key) {
            return mid;
        }

        // go to right wala part
        if(key > arr[mid]) {
            start = mid + 1;
        }
        else{ //key < arr[mid]
            end = mid - 1;
        }

        mid = start + (end-start)/2;
    }
    return -1;
}

```

## Linear Search v/s Binary Search :

Linear Search's Time Complexity =  $O(n)$ .

In Binary Search, we are essentially halving our search space in every iteration.

$$\Rightarrow n \rightarrow \frac{n}{2} \rightarrow \frac{n}{4} \rightarrow \frac{n}{8} \rightarrow \dots \rightarrow 1.$$

Applying Geometric Progression :

$$a = n$$

$$r = \frac{1}{2}$$

we want to know the no. of elements passed by the time we reach 1.

Say 1 is the  $z^{\text{th}}$  element.

$$ar^{z-1} = 1 \\ \rightarrow n \left(\frac{1}{2}\right)^{z-1} = 1$$

$$\rightarrow 2^{z-1} = n$$

$$\rightarrow 2^z = 2n$$

→ Taking  $\log_2$  on both sides :

$$\log 2^z = \log 2n$$

$$\rightarrow z = \log(2n) \approx \boxed{O(\log n)}$$



Q1. Find the first and last occurrence of an element in a sorted array.

**Approach: Finding First Occurrence.**

→ Applying binary search, if at any instance mid is such that  $\text{arr}[\text{mid}] == \text{key}$ , then :

- ① Either this is the first occurrence itself.
- ② Or there are more 'key' towards the left of mid.

So we'll temporarily store mid and then search in the left half by making  $\text{high} = \text{mid} - 1$ .

If  $\text{arr}[\text{mid}] < \text{key}$ , then search in right half by making  $\text{low} = \text{mid} + 1$ .

If  $\text{arr}[\text{mid}] > \text{key}$ , then search in left half by making  $\text{high} = \text{mid} - 1$ .

**Example:**  $\text{arr}[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$      $\text{key} = 4$ .

①  $\text{arr}[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$

$\begin{matrix} \uparrow \\ l \end{matrix}$        $\begin{matrix} \uparrow \\ \text{mid} \end{matrix}$        $\begin{matrix} \uparrow \\ h \end{matrix}$

$\text{arr}[\text{mid}] < \text{key} \Rightarrow$  Definitely in the right half.

$\text{low} = \text{mid} + 1$ .    //  $\text{low} = 4$

②  $\text{arr}[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$

$\begin{matrix} \uparrow \\ l \end{matrix}$        $\begin{matrix} \uparrow \\ \text{mid} \end{matrix}$        $\begin{matrix} \uparrow \\ h \end{matrix}$

$\text{arr}[\text{mid}] == \text{key} \Rightarrow$  Either mid or before mid.

store mid in a variable ans. //  $\text{ans} = 5$ .

Then make  $\text{high} = \text{mid} - 1$ . //  $\text{high} = 4$

$\begin{matrix} \downarrow \\ \text{mid} \end{matrix}$

③  $\text{arr}[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$

③  $\text{arr[]} = \{1, 1, 2, 3, 4, 4, 4, 4\}$

$\text{arr}[mid] == \text{key} \Rightarrow$  Either mid or before mid.

store mid in a variable ans. //  $\text{ans} = 4$

Then make high = mid - 1. //  $\text{high} = 3$

④  $\text{arr[]} = \{1, 1, 2, 3, 4, 4, 4, 4\}$

$\text{low} > \text{high} \Rightarrow$  break out. Last value of  $\boxed{\text{ans} = 4}$

Note: low, mid, high and ans are storing the indices and not values.

### Finding Last Occurrence.

→ Applying binary search, if at any instance mid is such that  $\text{arr}[mid] == \text{key}$ , then :

- ① Either this is the last occurrence itself.
- ② Or there are more 'key' towards the right of mid.

So we'll temporarily store mid and then search in the right half by making  $\text{low} = \text{mid} + 1$ .

If  $\text{arr}[mid] < \text{key}$ , then search in right half by making  $\text{low} = \text{mid} + 1$ .

If  $\text{arr}[mid] > \text{key}$ , then search in left half by making  $\text{high} = \text{mid} - 1$ .

Example:  $\text{arr[]} = \{1, 1, 2, 3, 4, 4, 4, 4\}$  key = 4.

①  $\text{arr[]} = \{1, 1, 2, 3, 4, 4, 4, 4\}$

$\text{arr}[\text{mid}] < \text{key} \Rightarrow$  Definitely in the right half.

$\text{low} = \text{mid} + 1$ . //  $\text{low} = 4$

②  $\text{arr}[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$

$\begin{matrix} \uparrow & \uparrow & \uparrow \\ l & \text{mid} & h \end{matrix}$

$\text{arr}[\text{mid}] == \text{key} \Rightarrow$  Either mid or after mid.

store mid in a variable ans. //  $\text{ans} = 5$ .

Then make  $\text{low} = \text{mid} + 1$ . //  $\text{low} = 6$

③  $\text{arr}[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$

$\begin{matrix} \downarrow \\ \text{mid} \\ \uparrow & \uparrow \\ l & h \end{matrix}$

$\text{arr}[\text{mid}] == \text{key} \Rightarrow$  Either mid or after mid.

store mid in a variable ans. //  $\text{ans} = 6$

Then make  $\text{low} = \text{mid} + 1$ . //  $\text{low} = 7$

④  $\text{arr}[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$

$\begin{matrix} \downarrow \\ \text{mid} \\ \uparrow & \uparrow \\ l & h \end{matrix}$

$\text{arr}[\text{mid}] == \text{key} \Rightarrow$  Either the last occurrence is at index mid or to the right of mid.

Store mid in a variable ans. //  $\text{ans} = 7$

Then make  $\text{low} = \text{mid} + 1$ .

⑤  $\text{arr}[] = \{1, 1, 2, 3, 4, 4, 4, 4\}$

$\begin{matrix} \uparrow & \uparrow \\ h & l \end{matrix}$

$\text{low} > \text{high} \Rightarrow$  break out. Last value of  $\boxed{\text{ans} = 7}$

## Code

```

int firstOcc(int arr[], int n, int key) {
    int s = 0, e = n-1;
    int mid = s + (e-s)/2;
    int ans = -1;
    while(s<=e) {
        if(arr[mid] == key){
            ans = mid;
            e = mid - 1;
        }
        else if(key > arr[mid]) { //Right me jao
            s = mid + 1;
        }
        else if(key < arr[mid]) { //left me jao
            e = mid - 1;
        }
        mid = s + (e-s)/2;
    }
    return ans;
}

```

```

int lastOcc(int arr[], int n, int key) {
    int s = 0, e = n-1;
    int mid = s + (e-s)/2;
    int ans = -1;
    while(s<=e) {
        if(arr[mid] == key){
            ans = mid;
            s = mid + 1;
        }
        else if(key > arr[mid]) { //Right me jao
            s = mid + 1;
        }
        else if(key < arr[mid]) { //left me jao
            e = mid - 1;
        }
        mid = s + (e-s)/2;
    }
    return ans;
}

```

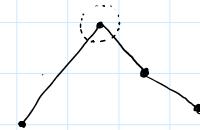
Follow-up question:

Q. Count the number of occurrences of an element in a sorted array.

→ (Last Occurrence Index) - (First Occurrence Index) + 1

Q. Peak in mountain array.

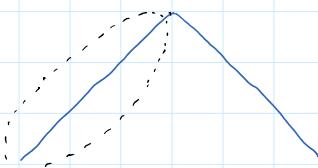
Eg: arr[] : {0, 10, 5, 2} →  
ans = 10.



Approach: If for 'mid' index,

1.  $\text{arr}[\text{mid}] < \text{arr}[\text{mid}+1]$ :

This element lies in the increasing slope / ascend. Search in the right half.  $\Rightarrow \text{low} = \text{mid} + 1$

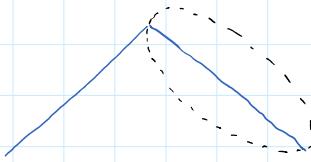
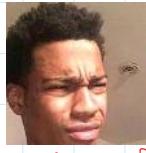


2. In all other cases.

This element lies in the decreasing slope / descend. or is the peak element itself. Since mid can be the peak element, search in the left half including mid.

element, search in the left half including mid.

$\Rightarrow \text{high} = \text{mid}$ .



Confused?

Example:  $\text{arr[]} : \{1, 10, 5, 2, 0\}$

①  $\text{low} = 0$ ,  $\text{high} = 4$ ,  $\text{mid} = 2$

$\text{arr[]} : \{1, 10, 5, 2, 0\}$

l            mid            h

$\text{arr}[\text{mid}]$  is not smaller than  $\text{arr}[\text{mid}+1]$ .  $\Rightarrow \text{high} = \text{mid}$

②  $\text{low} = 0$ ,  $\text{high} = 2$ ,  $\text{mid} = 1$

$\text{arr[]} : \{1, 10, 5, 2, 0\}$

l            mid            h

$\text{arr}[\text{mid}]$  is not smaller than  $\text{arr}[\text{mid}+1]$ .  $\Rightarrow \text{high} = \text{mid}$

③  $\text{low} = 0$ ,  $\text{high} = 1$ ,  $\text{mid} = 0$

$\text{arr[]} : \{1, 10, 5, 2, 0\}$

l            h

$\text{arr}[\text{mid}]$  is smaller than  $\text{arr}[\text{mid}+1]$   $\Rightarrow \text{low} = \text{mid}+1$ .

④  $\text{low} = 1$ ,  $\text{high} = 1$ ,  $\text{mid} = 1$

$\text{arr[]} : \{1, 10, 5, 2, 0\}$

l            h

Do this till low < high

Code:

```
int peakIndexInMountainArray(vector<int>& arr) {  
    int s = 0;  
    int e = arr.size() - 1;  
  
    int mid = s + (e-s)/2;  
  
    while(s<e) {  
        if(arr[mid] < arr[mid+1]){  
            s = mid + 1;  
        }  
        else  
        {  
            e = mid;  
        }  
        mid = s + (e-s)/2;  
    }  
    return s;  
}
```

Homework: Find pivot in a rotated sorted array.

Discussed in the next Lecture. 😊



What is sorting ?

$\text{arr}[] : \{1, 7, 9, 2, 3, 0\}$

$\downarrow$  sort

$\text{arr}[] : \{0, 1, 2, 3, 7, 9\}$

Arranging elements in a non-decreasing order is called sorting. Sorting can also be done in a non-increasing order.

What is Selection Sort ?

- ① You will make rounds/passes through the array.
- ② In each pass, you have to bring the smallest element to its correct place in the array.
- ③ You will then only consider the unsorted array excluding the smallest element you dealt with.
- ④ Repeat this until your array becomes sorted. (The array to sort will have only 1 element left).

Example:  $\text{arr}[] : \{64, 25, 12, 22, 11\}$

1.  $\begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \\ \{64 & 25 & 12 & 22 & 11\} \end{array}$   
 $i = 0$

Search for the smallest element in this array and put it at  $i=0$ .

2.  $\begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \\ \{11 & 25 & 12 & 22 & 64\} \end{array}$   
 $i = 1$

Sorted      Search for the smallest element in this array and put it at  $i = 1$ .

3. { 11 12 25 22 64 }

Sorted      Search for the smallest element in this array and put it at  $i = 2$ .

4. { 11 12 22 25 64 }

Sorted      Search for the smallest element in this array and put it at  $i = 3$ .

5. { 11 12 22 25 64 }

Sorted      1 element left so it is definitely sorted.

Note: I made 4 passes for an array of length 5.



For each pass we will have to run a for loop from  $i = \text{no. of elements}$  put in their correct places to  $n-1$  to get the smallest element from the unsorted array.

There will be  $n-1$  passes

Code :

```
void selectionSort(vector<int>& arr, int n)
{
    for(int i = 0; i < n-1; i++ ) {
        int minIndex = i;

        for(int j = i+1; j<n; j++) {
            if(arr[j] < arr[minIndex])
                minIndex = j;
        }

        swap(arr[minIndex], arr[i]);
    }
}
```

## Time Complexity :

$(n-i)$  passes :

Each pass has  $(n-i-1)$  comparisons.  
and 1 swap.

(so  $(n-i)$  operations that are  $O(1)$ )

For  $i=0 : n$

$i=1 : n-1$

$i=2 : n-2$

:

:

$i=n-2 : 2$

$$\Rightarrow f(n) = n + (n-1) + (n-2) + \dots + 2$$

$$\Rightarrow f(n) = 2 + 3 + \dots + (n-1) + n.$$

$$\Rightarrow f(n) = \frac{n(n+1)}{2} - 1$$

$$\Rightarrow f(n) = \frac{n^2}{2} + \frac{n}{2} - 1$$

$$\Rightarrow f(n) = \boxed{O(n^2)}$$

## Space Complexity :

No extra memory/space has been used. Thus

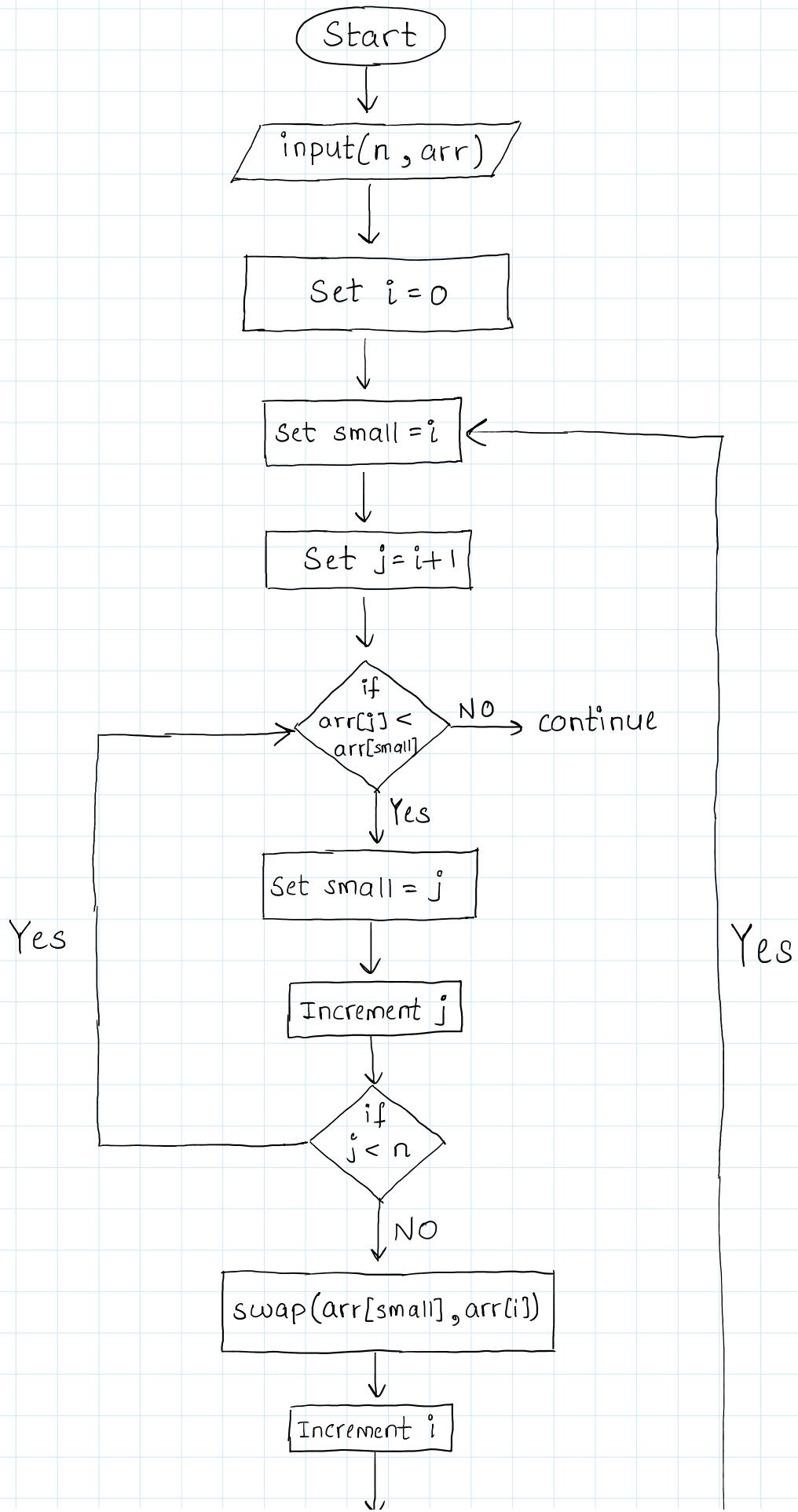
$$\boxed{O(1)}$$

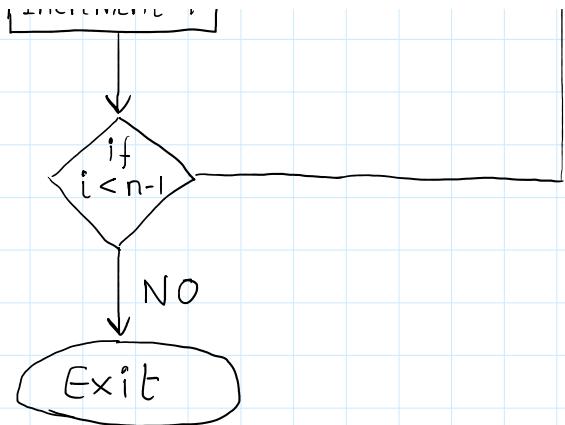
## Use Cases :

① Works nicely with smaller arrays.

② When there are strict memory constraints.

## Homework: Flow Chart for Selection Sort.





**Homework:** Is selection sort stable?

Ans: Since we make swaps after each pass in the unsorted array, we can have an array like

$\text{arr[]} = \{4, 2, 3, 1, 4\}$  where the two 4's have  
 $\begin{matrix} \uparrow \\ 0 \end{matrix}$        $\begin{matrix} \uparrow \\ 1 \end{matrix}$  an order which might get

changed after sorting i.e. the (zero) 4 might appear after the (one) 4 in the sorted array.

Link : [Is selection sort stable?](#)



## What is Recursion ?

→ When a function calls itself directly or indirectly, the process is called recursion and this call is called a recursive call.

Eg: `int fun(int n) {  
 ~~  
 fun(n);  
}`

If the solution of a problem depends on a smaller problem (subproblem) of the same type, then we will use recursion.

Eg: Find  $2^n$ .

We can say that  $2^n = \underbrace{2 \times 2 \times 2 \times \dots \times 2}_{n \text{ times}}$

$$\Rightarrow 2^n = 2 \times 2^{n-1}$$

If we make a function that gives us  $2^m$  when it's called like :

`fun(m);`

Then  $\text{fun}(n) = 2 * \text{fun}(n-1)$ ; ↪ Recurrence Relation

Eg: Find factorial. ( $n!$ )

We know that  $5! = 5 \times 4!$

$$\Rightarrow \text{fact}(5) = 5 \times \text{fact}(4)$$

$$\Rightarrow \text{fact}(n) = n * \text{fact}(n-1)$$

Given that we stop at a condition, example  $0! = 1$ , this is called base condition. Without this, our function will go bonkers !

$$\text{fact}(3) = 3 \times \text{fact}(2)$$

$$\text{fact}(2) = 2 \times \text{fact}(1)$$

$$\text{fact}(1) = 1 \times \text{fact}(0)$$

$$\begin{aligned}
 \text{fact}(0) &= 0 \times \text{fact}(-1) \\
 \text{fact}(-1) &= -1 \times \text{fact}(-2) \\
 &\vdots \quad \vdots \quad \vdots \\
 &\infty
 \end{aligned}$$

Step 1: Specify a base case where we return.

Step 2: Solve for the bigger problem using the smaller problem.

```

1 #include<iostream>
2 using namespace std;
3
4 int factorial(int n) {
5
6     //base case
7     if(n == 0)
8         return 1;
9
10    return n * factorial(n-1);
11 }
12
13 int main() {
14
15     int n;
16     cin >> n;
17
18     int ans = factorial(n);
19
20     cout << ans << endl;
21
22     return 0;

```

```

lovebabbar@192 Lecture31: Recursion Day1 % cd "/Users/lovebabbar/Documents/CodeHelp-OSA-Busted-Series/Lecture31: Recursion Day1/" && g++ factorial.cpp -o factorial && "/Users/lovebabbar/Documents/CodeHelp-OSA-Busted-Series/Lecture31: Recursion Day1/"factorial
5
128
lovebabbar@192 Lecture31: Recursion Day1 % cd "/Users/lovebabbar/Documents/CodeHelp-OSA-Busted-Series/Lecture31: Recursion Day1/" && g++ factorial.cpp -o factorial && "/Users/lovebabbar/Documents/CodeHelp-OSA-Busted-Series/Lecture31: Recursion Day1/"factorial
4
24
lovebabbar@192 Lecture31: Recursion Day1 % cd "/Users/lovebabbar/Documents/CodeHelp-OSA-Busted-Series/Lecture31: Recursion Day1/" && g++ factorial.cpp -o factorial && "/Users/lovebabbar/Documents/CodeHelp-OSA-Busted-Series/Lecture31: Recursion Day1/"factorial
6
728
lovebabbar@192 Lecture31: Recursion Day1 %

```

Without a base case, we will get 'Segmentation Fault' which means you have exhausted your function call stack (the memory where function calls are accumulated when called).

```

int factorial(int n) {
    //base case

    return n * factorial(n-1);
}

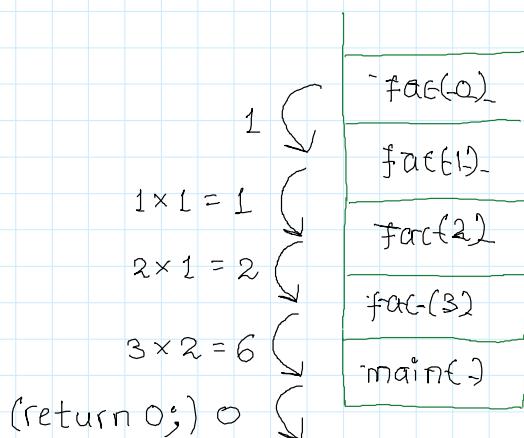
```

```

6
zsh: segmentation fault
lovebabbar@192 Lecture31: Recursion Day1 %

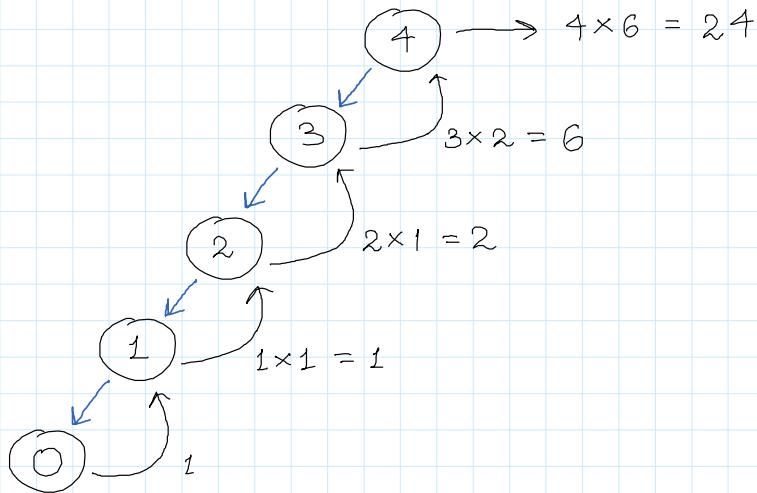
```

For  $\text{fac}(3)$ , function call stack will look like this :



## Recursion Tree :

Let's make a recursion tree for  $\text{fac}(4)$  where  $n = 4$ .



## Final Structure of a Recursive Function :

Fun( ) {

    Base Case

    Processing

    Recurrence Relation

}

Tail Recursion

Fun( ) {

    Base Case

    Recurrence Relation

    Processing

}

Head Recursion

```

3
4 int power(int n) {
5
6     //base case
7     if(n == 0)
8         return 1;
9
10    //recurrence relation
11    int smallerProblem = power(n-1);
12    int biggerProblem = 2 * smallerProblem;
13
14    return biggerProblem;
15}
  
```

lovebabbar@192 Lecture31: Recursion Day1 % cd "/Users/lovebabbar/Documents/CodeHelp-DSA-Busted-Series/Lecture31: Recursion Day1/" && g++ power.cpp -o power && "/Users/lovebabbar/Documents/CodeHelp-DSA-Busted-Series/Lecture31: Recursion Day1/power"
5
32
lovebabbar@192 Lecture31: Recursion Day1 %

OR

```
4 int power(int n) {
5     //base case
6     if(n == 0)
7         return 1;
8
9     //recursive relation
10    return 2 * power[n-1];
11 }
12 }
```

10  
1024  
lovebabbar@192 Lecture31: Recursion Day1 % cd "/Users/lovebabbar/Documents/CodeHelp-DSA-Busted-Series/Lecture31: Recursion Day1" && g++ power.cpp -o power && "/Users/lovebabbar/Documents/CodeHelp-DSA-Busted-Series/Lecture31: Recursion Day1"/power  
5  
32  
lovebabbar@192 Lecture31: Recursion Day1 % cd "/Users/lovebabbar/Documents/CodeHelp-DSA-Busted-Series/Lecture31: Recursion Day1" && g++ power.cpp -o power && "/Users/lovebabbar/Documents/CodeHelp-DSA-Busted-Series/Lecture31: Recursion Day1"/power  
6  
64  
lovebabbar@192 Lecture31: Recursion Day1 %

Example : Print n to 1.

Approach: Base case is when  $n=0$ , else we will print the number and then call for  $n-1$ .

```
3
4 void print(int n) {
5     //base case
6     if(n == 0) {
7         return ;
8     }
9
10    cout << n << endl;
11
12    print(n-1);
13
14 }
```

Macros, Global Variable etc./ Counting  
lovebabbar@192 ~ %

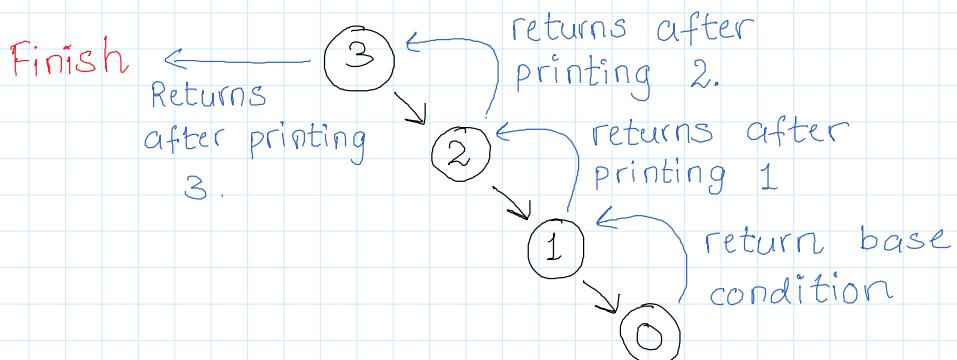
(Tail Recursion)  $\rightarrow$  n to 1.

```
3
4 void print(int n) {
5     //base case
6     if(n == 0) {
7         return ;
8     }
9
10    //Recursive relation
11    print(n-1);
12
13    cout << n << endl;
14 }
```

lovebabbar@192 Lecture30: Macros, Glob Var etc. %

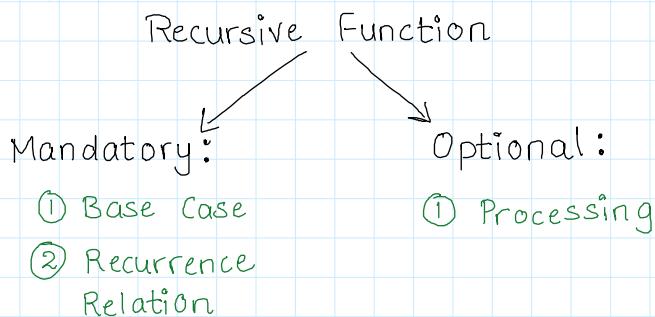
(Head Recursion)  $\rightarrow$  1 to n.

Recursion tree for 1 to n. ( $n=3$ )





## RECAP :



Solving a recursive problem involves 'leap of faith'

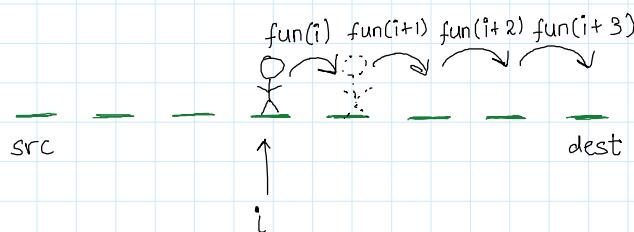
wherein you solve one part/sub-problem and then expect your function to do the rest by calling it recursively.

Example: Go from source to destination.

src = 1 , dest = 10. (Use recursion)

Soln: Base case is when src == dest.

At any given position, I will move one step ahead and then call myself again (Provided src != dest)



```
void reachHome(int src, int dest) {  
    cout << "source " << src << " destination " << dest << endl;  
    //base case  
    if(src == dest) {  
        cout << "pahuch gya " << endl;  
        return ;  
    }  
  
    //processing - ek step aage badhjao  
    src++;  
  
    //recursive call  
    reachHome(src, dest);  
}
```

Example: Fibonacci Series

0, 1, 1, 2, 3, 5, 8, 13, ...

Print  $n^{\text{th}}$  term of Fibonacci Series.

Soln:  $f(n) = f(n-1) + f(n-2)$

But,  $f(n) = 0$  where  $n \leq 1$

and  $f(2) = 1$

Baaki,  $f(n) = f(n-1) + f(n-2)$  holds true.

```
class Solution {
public:
    int fib(int n) {
        //base case
        if(n == 0)
            return 0;

        if(n == 1)
            return 1;

        int ans = fib(n-1) + fib(n-2);

        return ans;
    }
};
```

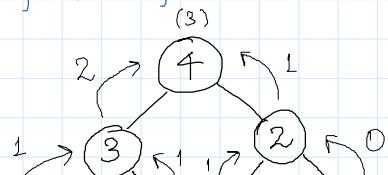
Homework: Solve using for loop.

```
1 #include <iostream>
2 using namespace std;
3
4 int fib(int n) {
5     if(n == 1 || n == 2) return n-1;
6     int a = 0, b = 1;
7     int ans;
8     for(int i=3;i<=n;i++) {
9         ans = a + b;
10        a = b;
11        b = ans;
12    }
13    return ans;
14 }
15
16 int main(void)
17 {
18     int n;
19     cin >> n;
20     cout << "nth Fibonacci number is " << fib(n) << endl;
21 }
22 }
```

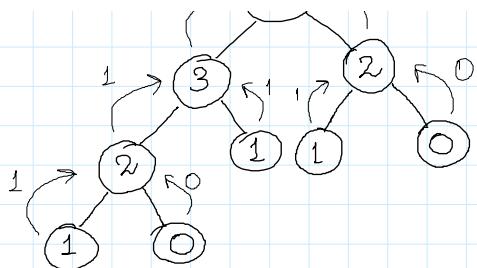
PROBLEMS    OUTPUT    TERMINAL    DEBUG CONSOLE

```
10
nth Fibonacci number is 34
```

Recursion Tree for fibonacci :



(Zero-based index)



Example : Count ways to reach  $n^{\text{th}}$  stair.

Soln :

```
#include <iostream>
using namespace std;

int climbStairs(int n) {
    if(n < 0) return 0;
    // only 1 way to reach the first stair
    // as you are already standing on it
    if(n == 0) return 1;

    // you came from the previous stair
    int penultimate = climbStairs(n-1);
    // you came from the stair before the previous stair
    int antepenultimate = climbStairs(n-2);
    return penultimate + antepenultimate;
}

int main(void)
{
    int n;
    cin >> n;
    cout << climbStairs(n) << endl;
    return 0;
}
```

Recursion tree is similar to Fibonacci.

Example : Say digits

I/P - 412

O/P - "four" "one" "two"

```

#include <iostream>
#include <vector>
using namespace std;

void breakDigits(int n, vector<int>& ans) {
    if(n == 0) return;
    breakDigits(n/10, ans);
    ans.push_back(n%10);
}

void sayDigits(vector<int>& digits, string arr[]) {
    for(int i=0;i<digits.size();i++) {
        cout << arr[digits[i] - 1] << " ";
    }
    cout << endl;
}

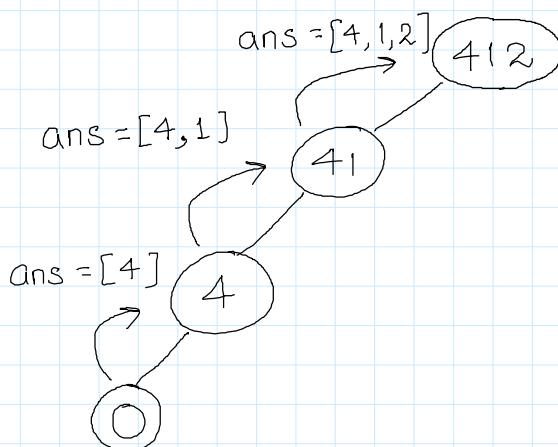
int main(void)
{
    int n;
    cin >> n;
    vector<int> digits;
    breakDigits(n, digits);
    string arr[10] = {"one", "two", "three", "four", "five", "six", "seven", "eight",
    "nine"};
    sayDigits(digits, arr);
    return 0;
}

```

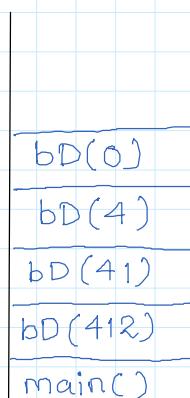
p\Work\Coding\Recursion\" ; if (\$?) { g++ SayDigits.cpp -o SayDigits } ; if (\$?) { .\SayDigits }

145236  
one four five two three six

## Recursion Tree : (Homework)



## Function Call Stack :





Q1. Is Sorted : Check if array is sorted using Recursion.

Logic: Assume `isSorted(v, i)` checks if the array `v[0...i]` is sorted. So, at a given step, we have to check  $v[i] \geq v[i-1]$  & & `isSorted(v, i-1)` must be true.

Base Case -  $i=0$ , one element is always sorted.

```
bool isSorted(vector<int>& v, int i) {
    if(v.size() == 0 || v.size() == 1)
        return true;
    if(i <= 0)
        return true;

    return v[i] >= v[i-1] && isSorted(v, i-1);
}
```

```
bool isSorted(int arr[], int size) {

    //base case
    if(size == 0 || size == 1){
        return true;
    }

    if(arr[0] > arr[1])
        return false;
    else {
        bool remainingPart = isSorted(arr + 1, size - 1);
        return remainingPart;
    }
}
```

Babbar Code

Homework: Find sum of the elements of an array using Recursion.

Logic: Assume `getSum(arr, n)` gives the sum of the array `arr` containing `n` elements. Base case -  $n=0$  or  $n=1$ .

```
int getSum(int arr[], int n) {
    if(n == 0) return 0;
    int sum = arr[0];
    sum += getSum(arr+1, n-1);
    return sum;
}
```

Call from  
`main()`

$\downarrow$   
`getSum(arr, n-1)`

```
int sumOfArray(vector<int> v, int ind, int sum) {
    if(ind == v.size()) return 0;
    sum = v[ind];
    sum += sumOfArray(v, ind+1, sum);
    return sum;
}
```

Call from  
`main()`  $\downarrow$   
`sumOfArray(v, 0, 0)`

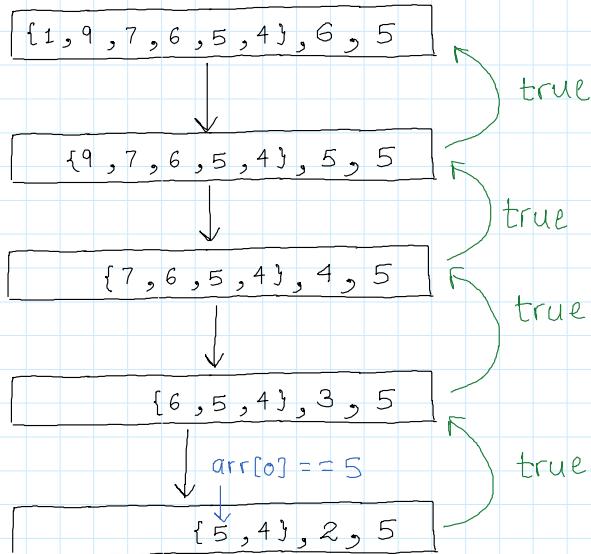
Q2. Search Element in array using Recursion. (Easy)

```
bool linearSearch(int arr[], int n, int val) {
    if(n < 0) return false;
    if(arr[0] == val)
        return true;
    // if the element is found even once, then we return true
    return linearSearch(arr+1, n-1, val);
}
```

Recursion Tree

Let `arr[] = {1, 9, 7, 6, 5, 4}`

val = 5, n = 6



## BINARY SEARCH USING RECURSION :

```
void printArr(int arr[], int l, int r) {
    cout << "Size of array is " << r-l+1 << endl;
    for(int i=l; i<=r; i++) {
        cout << arr[i] << " ";
    }
    cout << endl;
}

bool binarySearch(int arr[], int val, int l, int r) {
    printArr(arr, l, r);
    if(l>r) return false;

    int mid = l + (r-l)/2;

    if(arr[mid] == val)
        return true;
    else if(arr[mid] > val)
        return binarySearch(arr, val, l, mid-1);
    else if(arr[mid] < val)
        return binarySearch(arr, val, mid+1, r);

    return false;
}
```

```
1 10
2 1 3 4 5 7 8 11 12 13 14
3
4 14
```

```
Output.txt ...
1 Size of array is 10
2 1 3 4 5 7 8 11 12 13 14
3 Size of array is 5
4 8 11 12 13 14
5 Size of array is 2
6 13 14
7 Size of array is 1
8 14
9 Found
```

If you are not able to understand this, watch the video numbered 12 to understand Binary Search.

Homework : first and last Position using Recursion.

### Iterative Approach

```
// return the index of the first occurrence of val in vector v
pair<int, int> firstAndLastPosition(vector<int>& v, int val) {
    int n = v.size();
    int l = 0, r = n-1, mid;
    pair<int, int> ans(-1, -1);
    while(l <= r) {
        mid = (l+r)/2;
        if(v[mid] == val) {
            ans.first = mid;
            r = mid - 1;
        }
        else if(v[mid] > val)
            r = mid - 1;
        else
            l = mid + 1;
    }
    return ans;
}
```

We can also break this down into 2 functions.

```

// return the index of the first occurrence of val in vector v
pair<int, int> firstAndLastPosition(vector<int>& v, int val) {
    int n = v.size();
    int l = 0, r = n-1, mid;
    pair<int, int> ans(-1, -1);
    while(l <= r) {
        mid = (l+r)/2;
        if(v[mid] == val) {
            ans.first = mid;
            r = mid - 1;
        }
        else if(v[mid] > val)
            r = mid - 1;

        else
            l = mid + 1;
    }
    if(ans.first == -1)
        return ans;

    l = ans.first, r = n-1;
    while(l <= r) {
        mid = (l+r)/2;
        if(v[mid] == val) {
            l = mid + 1;
            ans.second = mid;
        }
        else if(v[mid] > val)
            r = mid - 1;
    }
    return ans;
}

```

we can also break this down into 2 functions.

## Recursive Approach ↴

```

int firstOcc(vector<int>& v, int l, int r, int val) {
    if(l > r) return -1;
    int n = v.size(), ans = -1;
    int mid = l + (r-l)/2;
    if(v[mid] == val) {
        ans = mid;
        int t = firstOcc(v, l, mid-1, val);
        if(t != -1)
            ans = t;
    }
    else if(v[mid] > val) {
        ans = firstOcc(v, l, mid-1, val);
    }
    else {
        ans = firstOcc(v, mid+1, r, val);
    }
    return ans;
}

int lastOcc(vector<int>& v, int l, int r, int val) {
    if(l > r) return -1;
    int n = v.size(), ans = -1;
    int mid = l + (r-l)/2;
    if(v[mid] == val) {
        ans = mid;
        int t = lastOcc(v, mid+1, r, val);
        if(t != -1)
            ans = t;
    }
    else if(v[mid] > val) {
        ans = lastOcc(v, l, mid-1, val);
    }
    else {
        ans = lastOcc(v, mid+1, r, val);
    }
    return ans;
}

```

Homework: Find total no. of occurrences of an element in a sorted array.

Solution : ( last occurrence - first Occurance + 1 )

Homework : Find the peak in the mountain array.

Solution :

```
int findPeak(int arr[], int l, int r) {
    if(l > r) return -1;
    int ans = -1;
    int mid = (l+r) / 2;
    if(arr[mid] > arr[mid-1] && arr[mid] > arr[mid+1])
        return mid;

    else if(arr[mid] < arr[mid+1])
        return findPeak(arr, mid+1, r);

    return findPeak(arr, l, mid-1);
}
```

Homework : Find Pivot in a rotated sorted array.

Solution :

```
int findPivotRec(vector<int>& nums, int l, int r) {
    if(l > r) return 0;
    int mid = l + (r - l)/2;
    // given that we don't rotate the array k times which is a
    // multiple of n.
    if(mid>0 && nums[mid] < nums[mid-1])
        return mid;

    else if(nums[mid] >= nums[0])
        return findPivotRec(nums, mid+1, r);

    return findPivotRec(nums, l, mid);
}
```

Homework : Search in rotated sorted array. (easy)

Solution : Find pivot and then use binary search in the half according to the value to be found.

Homework : Find square root using Binary Search.

```
long long int squareRoot(int n, int l, int r) {
    if(l > r) return 0;
    long long int mid = (l+r)/2;

    if(mid*mid <= n && (mid+1)*(mid+1)>n)
        return mid;

    else if(mid*mid > n) {
        return squareRoot(n, l, mid-1);
    }

    return squareRoot(n, mid+1, r);
}
```



# RECURSION WITH STRINGS

Q1. Reverse a string using recursion.

I/P : "abcdc"  
O/P : "cdcba"

I/P : "babbar"  
O/P : "rabbab"

I/P : "abccba"  
O/P : "abccba"

Logic: Solve 1 case and identify base cases.

Our function will look something like this :

```
void reverse (string s , int i , int j) {  
    ~~~~~  
    ~~~~~  
}
```

where i & j are the 2 indices to be swapped.

Thus, base case will be when  $i > j$ .

We solve the given sub-task by swapping elements at i and at j, and then incrementing & decrementing i and j respectively. Call the function again till we hit base case.

Code :

```
#include<iostream>  
using namespace std;  
  
void reverse(string& str, int i, int j) {  
  
    //base case  
    if(i>j)  
        return ;  
  
    swap(str[i], str[j]);  
    i++;  
    j--;  
  
    //Recursive call  
    reverse(str,i,j);  
}  
  
int main() {  
  
    string name = "babbar";  
  
    reverse(name, 0 , name.length()-1 );  
  
    cout << name << endl;  
  
    return 0;  
}
```



Don't forget to pass the string by reference or else we will not be modifying the original string name

Usecase: Let name = "abcde"

① reverse ("abcde", 0 , 4 )

$\hookrightarrow$  abcde  $\Rightarrow$  ebcda  
 $i \rightleftarrows j$

② reverse("ebcda", 1, 3)

↳ ebcda  $\Rightarrow$  edcba  
 $i \leftrightarrow j$

③ reverse("edcba", 2, 2)

↳ edcba  $\Rightarrow$  edcba  
 $i$   
 $j$

④ reverse("edcba", 3, 1)

↳ Base Case hits. We can also make our base case hit for  $i \geq j$

**Homework:** Improve the code by using up just one extra variable i instead of 2 ( $i & j$ ).

**Logic:** Same as above just use i to get the element to be swapped. We can use the string's size to get the element.

First Element :  $i$

Second Element :  $n - i - 1$ .

**Code :**

```
1 #include <iostream>
2 using namespace std;
3
4 void reverse(string& s, int i) {
5     if(i >= s.size()/2)
6         return;
7     // swap the i and n-i-1 indexed nodes
8     swap(s[i], s[s.size() - i - 1]);
9     // call for the next index
10    reverse(s, i+1);
11 }
12
13 int main(void)
14 {
15     string s;
16     cin >> s;
17     reverse(s, 0);
18     cout << s << endl;
19     return 0;
20 }
```

PROBLEMS OUTPUT TERMINAL DEBUG CONSOLE  
ork\Coding\Recursion\" ; if (\$?) { g++ ReverseString.cpp  
abcdefg  
gfedcba

Q2 Check Palindrome. (If the reversed string is same as the original string)

I/P : "abccba"

O/P : true

I/P : "abcba"

O/P : false

Approach 1: Reverse string and check if its equal to the original string.

Time Complexity :  $O(n)$

Space Complexity :  $O(n)$

Approach 2: Take 2 pointers i & j pointing to the first and last element. Check if they are equal & then increment and decrement i & j respectively till  $i < j$ . If at some point  $str[i] \neq str[j]$ , return false.

Code :

```
bool checkPalindrome(string str, int i, int j) {  
    //base case  
    if(i>j)  
        return true;  
  
    if(str[i] != str[j])  
        return false;  
    else{  
        //Recursive call  
        return checkPalindrome(str, i+1, j-1);  
    }  
}
```

Homework: Write the recursive function using just one extra pointing variable.

Code :

```
1 #include <iostream>  
2 using namespace std;  
3  
4 bool isPalindrome(string& s, int i) {  
5     if(i >= s.size()/2) return true;  
6  
7     if(s[i] != s[s.size()-i-1])  
8         return false;  
9  
10    return isPalindrome(s, i+1);  
11}  
12  
13 int main(void)  
14 {  
15     string s;  
16     cin >> s;  
17     if(isPalindrome(s, 0)) {  
18         cout << "Palindrome\n";  
19     }  
20     else {  
21         cout << "Not Palindrome\n";  
22     }  
23     return 0;  
24 }
```

PROBLEMS    OUTPUT    TERMINAL    DEBUG CONSOLE

abccba  
Palindrome

(Similar to reverse)  
string

**Note:** The time complexity of accessing the size of a `std::string` in C++ is  $O(1)$ .

Q3. Find power(a,b) using recursion.

I/P : 2, 4

O/P : 16

Approach: We can represent  $a^b$  as follows:

$$a^b = \begin{cases} a \cdot a^{b-1} & \text{if } b \text{ is odd} \\ a^{b/2} \cdot a^{b/2} & \text{if } b \text{ is even} \\ a \cdot a & \end{cases}$$

We can use a recursive function with base case being  $b=0$  (or  $b=1$ ), because  $a^0 = 1$  (or  $a^1 = a$ )

For a given call, if  $b \% 2 == 0$  then return  $\text{func}(a, b/2) * \text{func}(a, b/2)$ ,  
else return  $a * \text{func}(a, b-1)$  which equals

$$a * \text{func}(a, b/2) * \text{func}(a, b/2).$$

Confused?



Example: power(2,5)

$$\begin{aligned} 2^5 &= 2 \times (2^4) \\ 2^4 &= 2 \times (2^2) \\ 2^2 &= 2^1 \times (2^1) \\ 2^1 &= 2 \end{aligned}$$

When  $\text{power}(2,5)$  is called, we will perform 2 steps in one call to improve efficiency.  
Instead of giving back  $2 * \text{power}(2,4)$  we will calculate  $\text{pow}(2,2)$  before-hand by using variable =  $\text{power}(2, 5/2)$ ;  
Then return  $2 * \text{variable} * \text{variable}$ .

For even power,  
return variable \* variable.

Code :

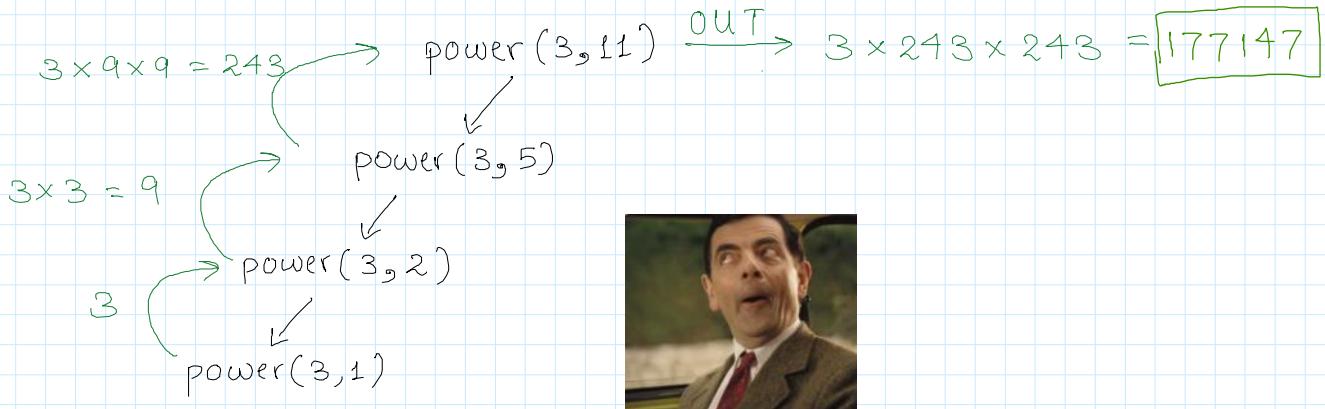
```
int power(int a, int b) {
    //base case
    if( b == 0 )
        return 1;

    if(b == 1)
        return a;

    //RECURSIVE CALL
    int ans = power(a, b/2);

    //if b is even
    if(b%2 == 0) {
        return ans * ans;
    }
    else {
        //if b is odd
        return a * ans * ans;
    }
}
```

Recursion Tree :  $\text{power}(3, 11)$



q4 Bubble Sort using Recursion.

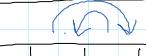
Approach : The recursive function will be called  $n$  times, each time it will correctly position the  $i$ th largest element correctly. Base case hits when only 1 or no elements are left.

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 6 | 1 | 9 | 5 |
|---|---|---|---|---|



bubbleSort(arr, 0) :

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 6 | 1 | 9 | 5 |
|---|---|---|---|---|



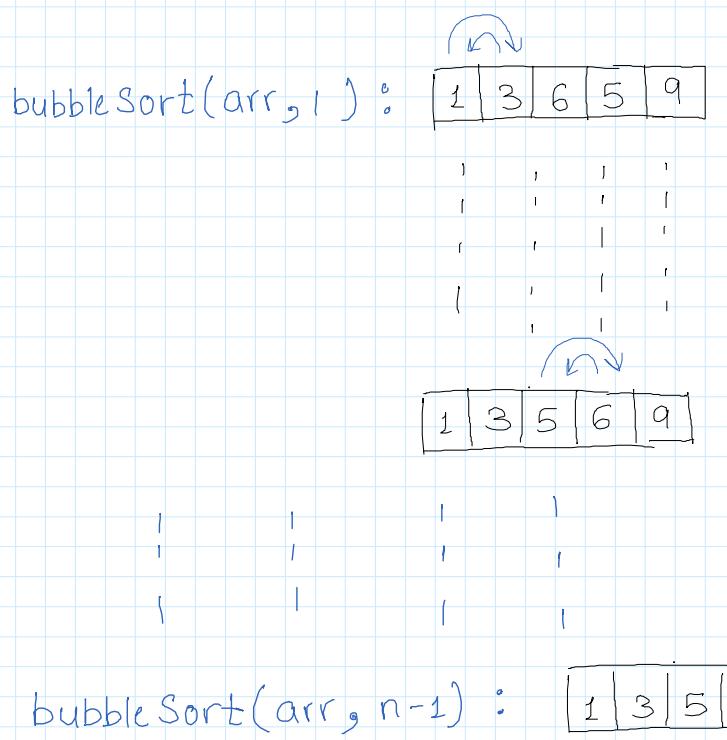
|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 1 | 6 | 9 | 5 |
|---|---|---|---|---|



|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 1 | 6 | 9 | 5 |
|---|---|---|---|---|



|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 1 | 6 | 5 | 9 |
|---|---|---|---|---|



## Homework:

- ① Selection sort using recursion.

**Code:**

```
4  int minIndex(int a[], int i, int j)
5  {
6      if (i == j)
7          |  return i;
8      // minimum element's index from a[(i+1)....j]
9      int k = minIndex(a, i + 1, j);
10     // Comparing with current element and updating answer
11     return (a[i] < a[k])? i : k;
12 }
13 |
14 void recurSelectionSort(int a[], int n, int index = 0)
15 {
16     // Return when starting and size are same
17     if (index == n)
18         |  return;
19
20     // minimum element's index in a[index....(n-1)]
21     int k = minIndex(a, index, n-1);
22
23     swap(a[k], a[index]);
24
25     // Recursively calling selection sort function
26     recurSelectionSort(a, n, index + 1);
```

```
23     swap(a[k], a[index]);
24
25     // Recursively calling selection sort function
26     recurSelectionSort(a, n, index + 1);
27 }
28
```

PROBLEMS    OUTPUT    TERMINAL    DEBUG CONSOLE

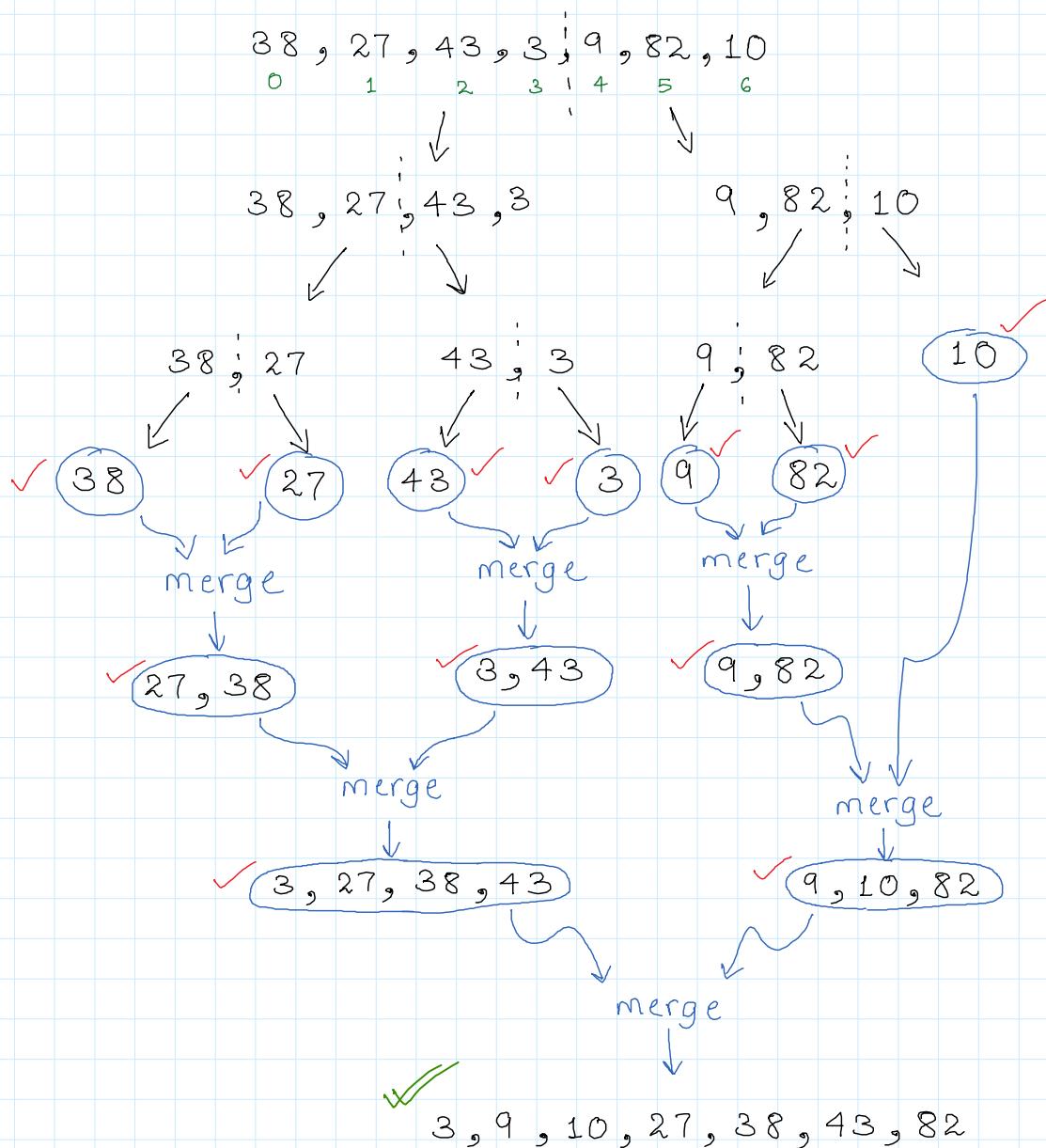
```
5
3 1 4 5 2
1 2 3 4 5
```

[Refer Here](#) for detailed explanation of Homework Questions.



Merge Sort is a sorting algorithm that uses the concept of divide and conquer.

Eg:



✓ - Sorted Array.

Visualization (Source: Hacker Earth)

Approach 1 :

Logic - We will divide / split the array into 2 parts, call the function for the 2 parts recursively (with the

hope that this will return two sorted sub-parts).

We then merge the 2 sorted arrays and return the new sorted array.

Our base case is when we have 1 element in the array to be sorted, i.e.  $\text{start} \geq \text{end}$ , we should return without doing anything.

Process : arr : 38, 27, 43, 3, 9, 82, 10  
              0   1   2   3   4   5   6

`mergeSort(arr, 0, 6)` is called for the entire array.

This checks for the base case,  $(6-0+1) = \text{length of the array} = 7 \neq 1$ , thus, base condition not satisfied.

We then call `mergeSort(arr, 0, 3)` & `mergeSort(arr, 4, 6)` with the faith that our 2 halves will get sorted.

We will then merge these 2 sorted arrays using `merge(arr, 0, 6)`.

Code : `mergeSort` function.

```
void mergeSort(int *arr, int s, int e) {  
    //base case  
    if(s >= e) {  
        return;  
    }  
  
    int mid = (s+e)/2;  
  
    //left part sort karna h  
    mergeSort(arr, s, mid);  
  
    //right part sort karna h  
    mergeSort(arr, mid+1, e);  
  
    //merge  
    merge(arr, s, e);  
}
```

helper `merge` function

```
void merge(int *arr, int s, int e) {  
  
    int mid = (s+e)/2, len1 = mid - s + 1, len2 = e - mid;  
    int *first = new int[len1];  
    int *second = new int[len2];  
    //copy values  
    int mainArrayIndex = s;  
    for(int i=0; i<len1; i++) {  
        first[i] = arr[mainArrayIndex++];  
    }  
  
    mainArrayIndex = mid+1;  
    for(int i=0; i<len2; i++) {  
        second[i] = arr[mainArrayIndex++];  
    }  
    //merge 2 sorted arrays  
    int index1 = 0;  
    int index2 = 0;  
    mainArrayIndex = s;  
    while(index1 < len1 && index2 < len2) {  
        if(first[index1] < second[index2]) {  
            arr[mainArrayIndex++] = first[index1++];  
        }  
        else{  
            arr[mainArrayIndex++] = second[index2++];  
        }  
    }  
    while(index1 < len1) {  
        arr[mainArrayIndex++] = first[index1++];  
    }  
    while(index2 < len2) {  
        arr[mainArrayIndex++] = second[index2++];  
    }  
}
```

```

    while(index1 < len1) {
        arr[mainArrayIndex++] = first[index1++];
    }

    while(index2 < len2) {
        arr[mainArrayIndex++] = second[index2++];
    }
    delete []first;
    delete []second;
}

```

Time Complexity :  $O(n \log n)$   $\rightarrow O(n)$  for every  $O(\log n)$  partitions.

Space Complexity :  $O(n)$



We are using 2 auxilliary arrays to merge the 2 sorted arrays.

**Homework:** Count inversions in an array.

**Explanation -** An inversion is when  $arr[i] > arr[j]$  for some  $i < j$ .

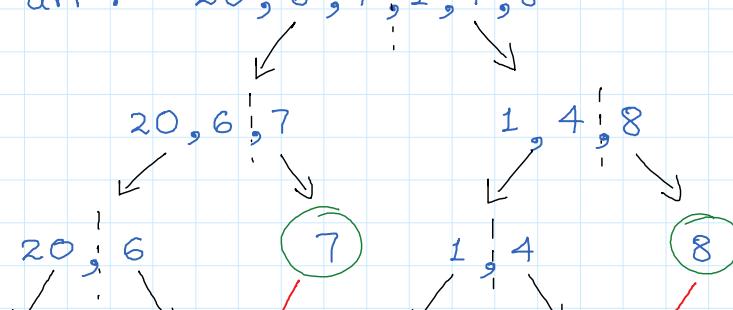
**Example :** 8 3 6 4 2

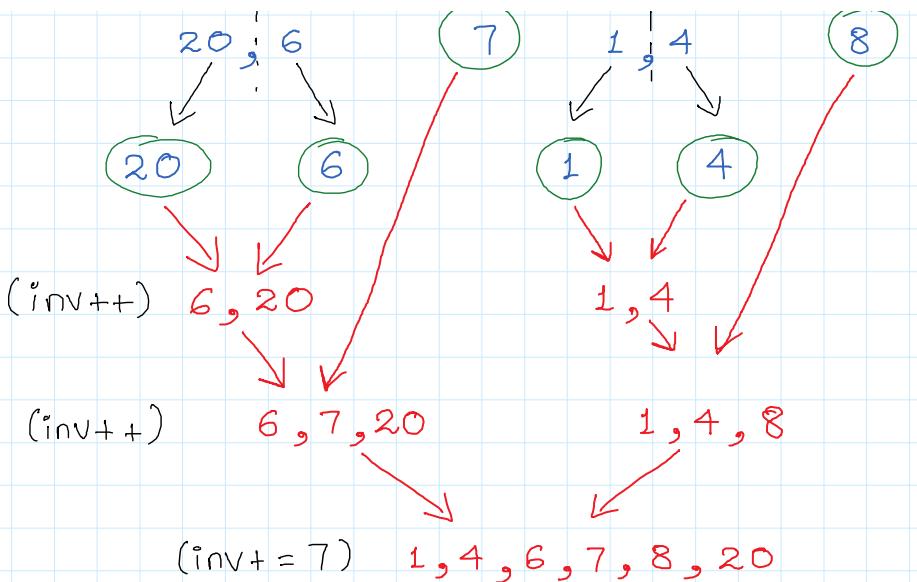
|              |         |         |
|--------------|---------|---------|
| Inversions — | ① 8 - 3 | ⑤ 3 - 2 |
|              | ② 8 - 6 | ⑥ 6 - 4 |
|              | ③ 8 - 4 | ⑦ 6 - 2 |
|              | ④ 8 - 2 | ⑧ 4 - 2 |

Answer = 8 (Elements occurring before, which are greater than element(s) after them.)

**Logic :** When we use the Merge Sort's partitioning approach, we merge the sorted arrays. We use this merging to count the number of inversions in the array.

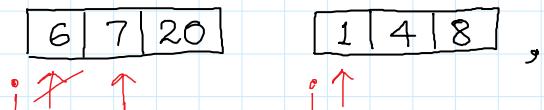
**Use case:** arr : 20, 6, 7, 1, 4, 8





answer = 9

For two sorted arrays



①  $6 > 1$ , so all elements in arr1  $> 1$ . (arr1 is sorted)  
 $\Rightarrow$  inversion += 3 (length of arr1)

②  $6 > 4$ , so again, all elements in arr1  $> 4$ . (arr1 is sorted)  
 $\Rightarrow$  inversion += 3

③  $6 < 8$ , increment i.  $7 < 8$ , increment i.  
 $20 > 8$ , inversion += 1.

Logic is to fix j for all i and count the number of inversions. Repeat for all j.

Code:

inversionCount:

Counts the inversions for the split arrays and then for the merged arrays.

```

int inversionCount(vector<int>& nums, int l, int r) {
    if(l >= r) return 0;
    vector<int> temp(r - l + 1);
    int mid = l + (r - l)/2, inv = 0;

    inv += inversionCount(nums, 0, mid);
    inv += inversionCount(nums, mid+1, r);
    inv += merge(nums, l, mid, r, temp);

    return inv;
}

```

```
int merge(vector<int>& nums, int l, int mid, int r, vector<int>& temp) {
    int i = l, j = mid+1, k = l;
    int inv = 0;
    while(i <= mid && j <= r) {
        if(nums[i] <= nums[j]) {
            temp[k++] = nums[i++];
        } else {
            temp[k++] = nums[j++];
            // all elements from nums[i] to nums[mid] will be greater than nums[j]
            inv += (mid - i + 1);
        }
    }
    while(i <= mid) {
        temp[k++] = nums[i++];
    }
    while(j <= r) {
        temp[k++] = nums[j++];
    }

    for(i=l;i<=r;i++) {
        nums[i] = temp[i];
    }
    return inv;
}
```

merge function :

counts the inversions while merging the 2 sub-arrays.



**Intuition:** Given an array  $\text{arr}[]: \{3\ 6\ 1\ 4\ 2\ 5\}$ , we choose an element (let's call it pivot) and place it at its correct position. We recursively call this function for the left and right parts where the elements are/maybe in wrong positions (The faith that our function will solve for the left and right sub-parts is the leap of faith).

**Example:**  $\text{arr}[]: \{3\ 6\ 1\ 4\ 2\ 5\}$

pivot

Let's say we keep the first element as our pivot.

We must place 3 at its correct position. Find out

how many elements are smaller than pivot. In our

case, there are 2 elements 1 and 2 that are

smaller than 3, so the correct position of 3 is

index 2.

**Note:** Placing the pivot at its correct position also means that all elements to its left must be smaller and all elements to its right must be greater than the pivot (in no particular order).

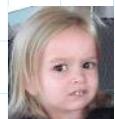
$\text{arr}[]: \{3\ 6\ 1\ 4\ 2\ 5\}$

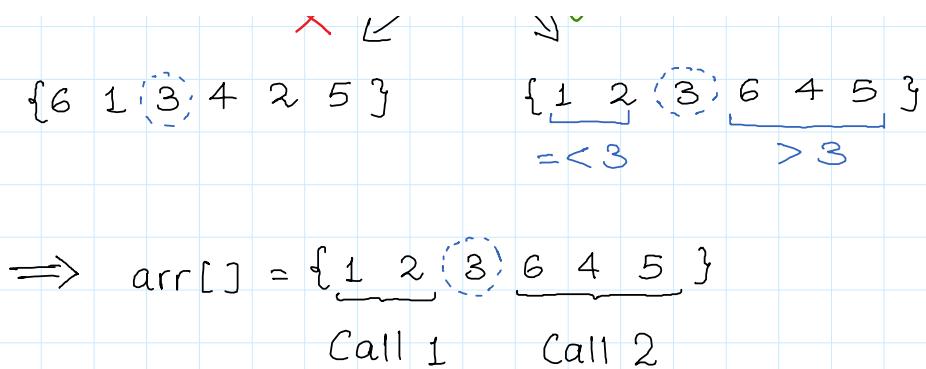
X

✓

$\{6\ 1\ 3\ 4\ 2\ 5\}$

$\{1\ 2\ 3\ 6\ 4\ 5\}$





**Code :** Main code (Simple Recursive Calls)

- ① Base Case : if low exceeds high.
- ② Partition the array and return the correct index of the pivot element.
- ③ Repeat for the left half.
- ④ Repeat for the right half.

Partitioning Code

- ① For a given array in the range  $arr[s \dots e]$ .
- ② Take the pivot element as  $arr[s]$ .
- ③ Count all elements smaller than pivot.
- ④ Place/Swap pivot element with  $arr[s + count]$
- ⑤ Start checking all elements with  $i=s$  &  $j=e$  till  $i < \text{pivot}$  &  $j > \text{pivot}$ . If  $arr[i] > \text{pivot}$  and  $arr[j] < \text{pivot}$  then swap them.

```

void quickSort(int arr[], int s, int e) {
    if(s >= e) {
        return;
    }

    // The arr[partIndex] will now be at its correct position
    int partIndex = partition(arr, s, e);
    cout << "partIndex = " << partIndex << endl;

    // call the function for the left part
    quickSort(arr, s, partIndex-1);
    // call it for the right part
    quickSort(arr, partIndex+1, e);
}

```

```

int partition(int arr[], int s, int e) {
    int pivot = arr[s];
    // count elements smaller than pivot
    int count = 0;
    for(int i=s+1;i<=e;i++) {
        if(arr[i] <= pivot)
            count++;
    }

    int pivotIndex = s + count;
    swap(arr[s], arr[pivotIndex]);
    // now arr[pivotIndex] is correctly placed
    int i = s, j = e;
    while(i < pivotIndex && j > pivotIndex) {
        if(arr[i] > arr[pivotIndex] && arr[j] <= arr[pivotIndex])
            swap(arr[i++], arr[j--]);
        else if(arr[i] < arr[pivotIndex]) {
            i++;
        }
        else {
            j--;
        }
    }
    return pivotIndex;
}

```

## Homework :

① Is quick sort, an in-place sorting algorithm ?

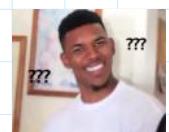
Ans: Yes. We are not creating secondary arrays and performing swaps in the original array.

② Is quick sort a stable sorting algorithm ?

Ans : Refer [Here](#) for detailed explanation

## Time and Space Complexity

Time Complexity - Average Case :  $O(n \log n)$   
 Worst Case :  $O(n^2)$



Space Complexity -  $O(n)$



Time Complexity of Recursive functions  
 baadme padhaenge.

Hamari baat maan lo.



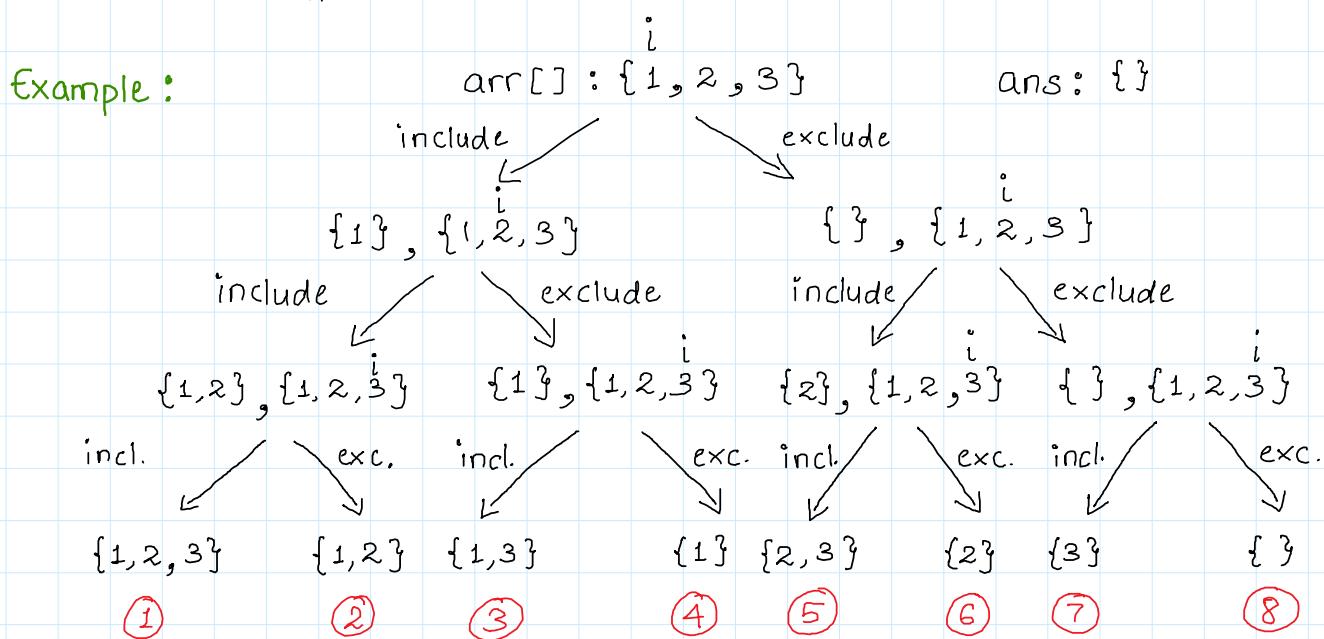
# 1. Generate all subsets (Power Set)

I/P : [1, 2, 3]

O/P : {[], {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}}

Note - We can generate these subsets in any order as long as we cover all the subsets.

**Approach :** We can either include or exclude an element to generate a subset. Thus, we have 2 options for every element, either include it in the subset or exclude it.



What do you need?

- ① Array
- ② Index
- ③ Answer array.

Code :

```
class Solution {
private:
    void solve(vector<int>& nums, vector<int> output, int i, vector<vector<int>> &ans) {
        if(i >= nums.size()) {
            ans.push_back(output);
            return;
        }
        // exclude this element and move on to the next index
        solve(nums, output, i+1, ans);
        // include this element
        output.push_back(nums[i]);
        solve(nums, output, i+1, ans);
    }
public:
    vector<vector<int>> subsets(vector<int>& nums) {
        vector<vector<int>> ans;
        vector<int> output;
        int ind = 0;
        solve(nums, output, ind, ans);
        return ans;
    }
};
```

Checking output array for each recursive call

```
#include <iostream>
#include <vector>
using namespace std;

void solve(vector<int>& nums, vector<int> output, int i, vector<vector<int>> &ans) {
    cout << "output: ";
    for(auto x: output) {
        cout << x << " ";
    }
    cout << endl;
    if (i == nums.size()) {
        ans.push_back(output);
        return;
    }
    // exclude this element and move on to the next index
    solve(nums, output, i + 1, ans);
    // include this element
    output.push_back(nums[i]);
    solve(nums, output, i + 1, ans);
}

vector<vector<int>> subsets(vector<int>& nums) {
    vector<vector<int>> ans;
    vector<int> output;
    int ind = 0;
    solve(nums, output, ind, ans);
    return ans;
}
```

The terminal window shows the following output:

```
1 3
2 1 2 3
1 output:
2 output:
3 output:
4 output:
5 output: 3
6 output: 2
7 output: 2
8 output: 2 3
9 output: 1
10 output: 1
11 output: 1
12 output: 1 3
13 output: 1 2
14 output: 1 2
15 output: 1 2 3
16
17 3
18 2
19 2 3
20 1
21 1 3
22 1 2
23 1 2 3
24
```

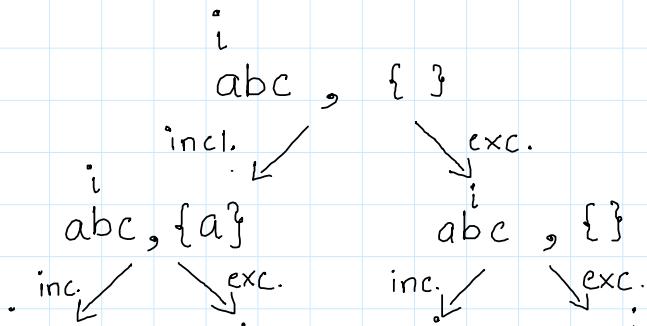
## 2. Subsequences of String. (slight change)

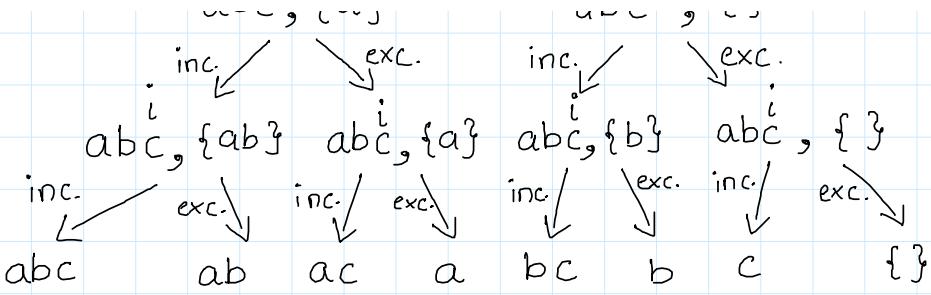
I/P : "abc"

O/P : {"", "a", "b", "c", "ab", "ac", "bc", "abc"}

Same Approach.

Recursion Tree :





Code :

```

void solve(vector<string>& ans, string str, string output, int i) {
    //base case
    if(i>=str.length()) {
        if(output.length()>0)
            ans.push_back(output);
        return;
    }

    //exclude
    solve(ans, str, output, i+1);
    //include
    output.push_back(str[i]);
    solve(ans, str, output, i+1);
}

vector<string> subsequences(string str){

    vector<string> ans;
    string output = "";
    solve(ans,str,output,0);
    return ans;
}

```

Note : In the question, we have to exclude the empty substring / subsequence.

Homework : Solve the above questions using bits .



Concept : If we have a string like "abc", then we know that 3-bit numbers 000, 001, ..., 111 represent all  $2^3 = 8$  subsequences.

|         |     |               |      |
|---------|-----|---------------|------|
| "abc" : | 000 | $\Rightarrow$ | ""   |
|         | 001 | $\Rightarrow$ | "c"  |
|         | 010 | $\Rightarrow$ | "b"  |
|         | 011 | $\Rightarrow$ | "bc" |
|         | 100 | $\Rightarrow$ | "a"  |
|         | 101 | $\Rightarrow$ | "ac" |
|         |     | -             | .    |

We 'mask' the binary number on the string.

Ex: "abc"

011

$1\ 1\ 0 \Rightarrow "ab"$   
 $1\ 1\ 1 \Rightarrow "abc"$ 
"bc"

So, for a string of length  $n$ , we will take an  $n$ -bit binary number and iterate through all the  $2^n$  ranging from  $\underbrace{000\dots 00}_n$  to  $\underbrace{111\dots 11}_n$ .

Code :

```

string maskedString(string str, int n) {
    string ans;
    int ind = str.length()-1;
    for(int j=0;j<str.length();j++) {
        if((1<<j) & n) {
            ans.push_back(str[ind]);
        }
        ind--;
    }
    return ans;
}

vector<string> subsequences(string str){
    vector<string> ans;
    for(int i=0;i<(1<<str.length());i++) {
        string temp = maskedString(str, i);
        if(!temp.empty())
            ans.push_back(temp);
    }
    return ans;
}

```

```

1 abc
2 Output.txt x +
3
4 1 c
5 2 b
6 3 cb
7 4 a
8 5 ca
9 6 ba
10 7 cba
11 8

```



## Letter Combinations of a Phone Number



Pressing 2 can give "a", "b" or "c"

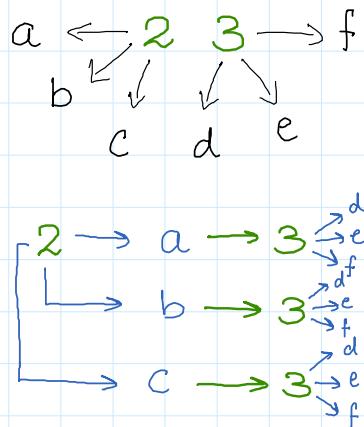
Pressing 23 can give: ad, ae, af, bd, be, bf, cd, ce, cf.

We have to return all possible combinations that can be formed by pressing a given phone number.

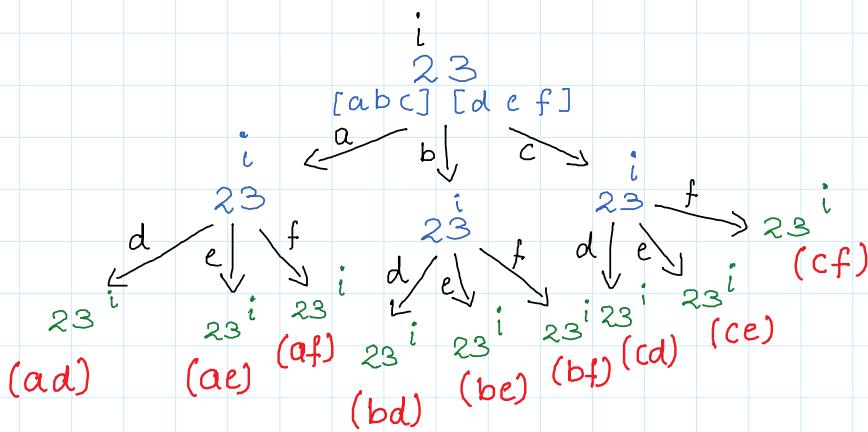
Note: Mapping exists only for numbers 2 - 9.

Approach: Just like subsets / subsequences (watch Lecture 37),

we will iterate each digit in the phone number and for each alphabet belonging to the digit, we will choose it and move to the next digit recursively.



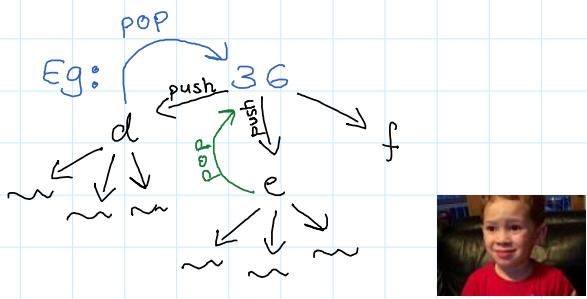
Example: Phone Number - 23  
[a,b,c], [d,e,f]



Code :

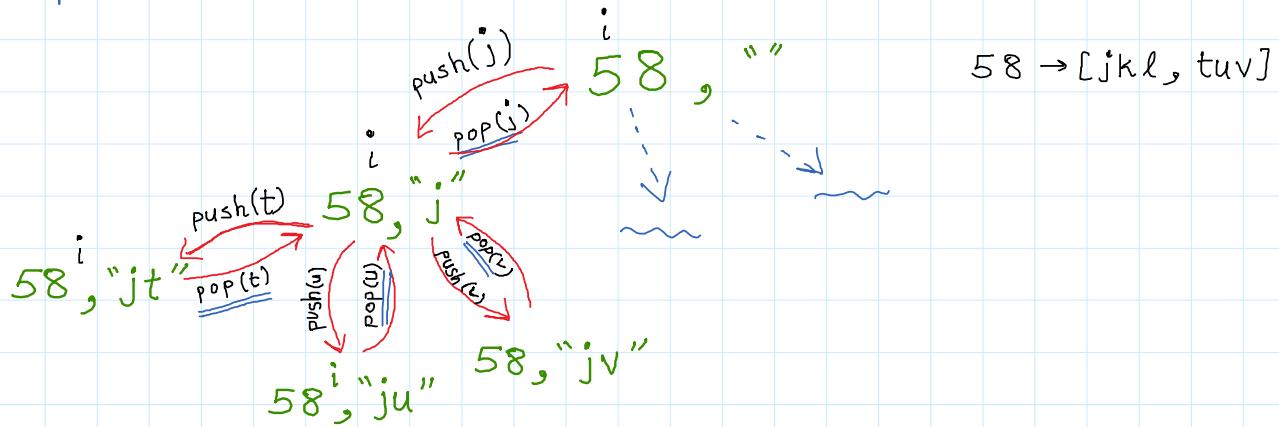
```
private:
    void solve(string digits, string mapping[], vector<string> &ans, string output, int i) {
        if(i >= digits.length()) {
            ans.push_back(output);
            return;
        }
        int number = digits[i] - '0';
        for(int j=0;j<mapping[number].length();j++) {
            output.push_back(mapping[number][j]);
            solve(digits, mapping, ans, output, i+1);
            output.pop_back();
        }
    }
public:
    vector<string> letterCombinations(string digits) {
        vector<string> ans;
        if(digits.length()==0)
            return ans;
        string output;
        int index = 0;
        string mapping[10] = {"", "", "abc", "def", "ghi", "jkl", "mno", "pqrs", "tuv", "wxyz"};
        solve(digits, mapping, ans, output, index);
        return ans;
    }
}
```

`digits[i]` will give me a character like "2" and not 2.



For all 'char' digits in the mapping of number, push it in our output string, get all combinations for this digit and then pop it to make way for the next digit.

## Pop's Importance:





## PROBLEM STATEMENT :

We are given a string str, say "abc", we have to return all the permutations of the string in lexicographically increasing order.

A string str1 is lexicographically less than a string str2 if:

- ①  $\text{str1} \neq \text{str2}$
- ② str1 is a prefix of str2

OR

- ② There exists some  $i$  ( $0 \leq i \leq \min(|\text{str1}|, |\text{str2}|)$ ) such that  $\text{str1}[i] < \text{str2}[i]$  (ASCII values) and for all  $0 \leq j < i$ ,  $\text{str1}[j] = \text{str2}[j]$ .

Example : str1 = "abcd<sup>0 1 2 3 4 5 6</sup>efg"  
str2 = "ab<sup>0 1 2 3 4 5</sup>cxyz"

Equal ← First Difference where  
 $\text{str1}[i] < \text{str2}[i]$

↳  $\text{str1} < \text{str2}$  lexicographically

## APPROACH :

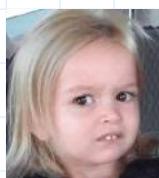
Say our string is "abc"

We want to generate the following permutations:

abc, acb, bac, bca, cab, cba

So, we want to swap elements throughout the string to generate the permutations.

Confused? No worries.



EXAMPLE : str = "abc"

Starting with  $i=0$ , abc

0 1 2

① Swap a with a, increment i.

abc  
0 1 2  
i  
bac  
0 1 2  
i  
cba  
0 1 2  
i

② Swap a with b, increment i.

③ Swap a with c, increment i.

Call the function again for  $i=1$  for each output in steps ① to ③.

④ abc  $\Rightarrow$  Swap b with b  $\Rightarrow$  abc ①

⑤ abc  $\Rightarrow$  Swap b with c  $\Rightarrow$  acb ②

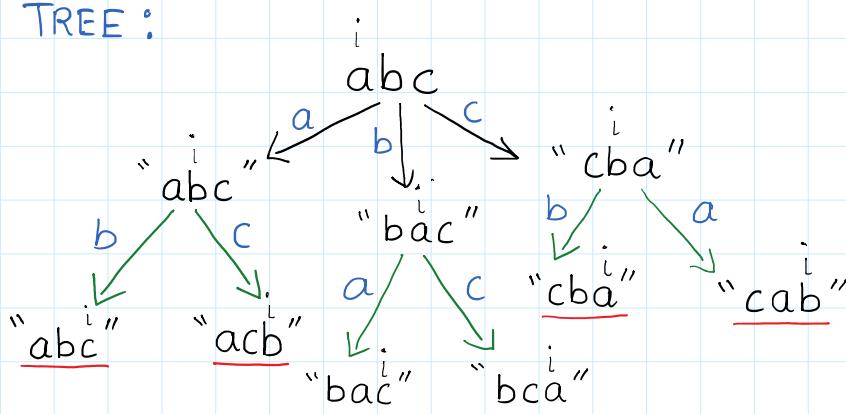
⑥ bac  $\Rightarrow$  Swap a with a  $\Rightarrow$  bac ③

⑦ bac  $\Rightarrow$  Swap a with c  $\Rightarrow$  bca ④

⑧ cba  $\Rightarrow$  Swap b with b  $\Rightarrow$  cba ⑤

⑨ cba  $\Rightarrow$  Swap b with a  $\Rightarrow$  cab ⑥

RECURSION TREE :



Code :

```
class Solution {
private:
    void solve(vector<int> nums, vector<vector<int>>& ans, int i) {
        if(i >= nums.size()) {
            ans.push_back(nums);
            return;
        }
        for(int j=i;j<nums.size();j++) {
            swap(nums[i], nums[j]);
            solve(nums, ans, i+1);
            swap(nums[i], nums[j]);
        }
    }
public:
    vector<vector<int>> permute(vector<int>& nums) {
        vector<vector<int>> ans;
        int index = 0;
        solve(nums, ans, index);
        return ans;
    }
}
```

When you do a swap & generate all permutations for

the given index, you need to swap them again to bring the array back to its original state.

EXAMPLE :

str: "w<sup>i</sup>x<sup>j</sup>y<sup>i+1</sup>z<sup>n-i-1</sup>"  
while returning, we must swap x and z again  
suppose we are swapping with j  
"w<sup>i</sup>z<sup>j</sup>y<sup>i+1</sup>x<sup>n-i-1</sup>"  
Some recursive calls we don't care about.



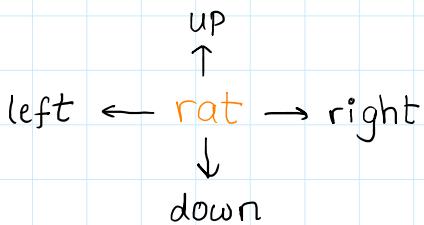
## PROBLEM LINK : [Click Here](#)

Given : →

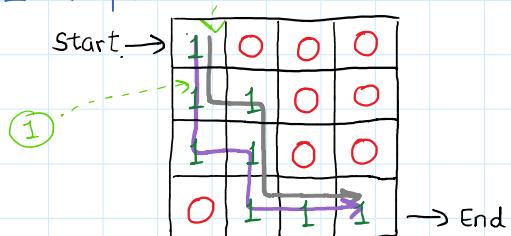
|       |   |   |   |   |
|-------|---|---|---|---|
| Start | 1 | 0 | 0 | 0 |
|       | 1 | 1 | 0 | 0 |
|       | 1 | 1 | 0 | 0 |
| End   | 0 | 1 | 1 | 1 |

where : 1 - we can come to this cell  
0 - we cannot come to this cell.

Aim is to output all possible ways to reach the End  $(n-1, n-1)$  from the Start  $(0, 0)$ . The rat can move in all the 4 directions up, down, left, right.



Example :



1. DDRDRR
2. DRDDRR

## APPROACH :



At any point, we have 4 choices - up, down, left or right.

For each position, check if you can go U/D/L/R keeping in mind the following constraints :

- ① Don't go out of bounds. ( $0 \leq i < n, 0 \leq j < n$ )
- ② Don't go to an already visited position.
- ③ Don't go to a position marked 0.

③ Don't go to a position marked 0.

For point ②, maintain a 2-D array 'visited' which keeps track of all the visited positions. ( $n \times n$  size)

Once you reach  $(n-1, n-1)$ , you can print the string showing the path you took and then make all cells of the 'visited' matrix as False.

Thus, when you move, you will make  $\text{visited}[i][j]$  (if you moved to  $m[i][j]$ ) and you will append your string with D, U, L or R according to your movement.

CODE: To check if a position is legit, we discussed some conditions above.

```
bool isSafe(int x, int y, int n, vector<vector<int>> visited, vector<vector<int>> &m) {
    if( (x>=0 && x<n) && (y>=0 && y<n) && visited[x][y] == 0 && m[x][y] == 1) {
        return true;
    } else {
        return false;
    }
}
```

In our main 'solve' function, we will first check if we have reached the end of the maze.

```
void solve(vector<vector<int>> &m, int n, vector<string>& ans, int x,
           int y, vector<vector<int>> visited, string path) {
    //you have reached x,y here

    //base case
    if( x == n-1 && y == n-1){
        ans.push_back(path);
        return;
    }
}
```

For the position  $(x, y)$ , mark  $\text{visited}[i][j] = 1$  (or true)

```
visited[x][y] = 1;
```

Now, for each direction (U, D, L, R) make newx & newy and make a move if going in that direction is safe.

if path is safe, then append the move in the path string.

```
if(isSafe(newx, newy, n, visited, m)) {  
    path.push_back('D');  
    solve(m,n,ans,newx,newy,visited,path);  
    path.pop_back();
```

Call for the new position

After the recursion

call returns, remove this move to make way for the next option.  
(U/D/L/R)

This gets called 4 times :

- ① newx = x , newy = y - 1
- ② newx = x - 1 , newy = y
- ③ newx = x , newy = y + 1
- ④ newx = x + 1 , newy = y .

```
int newx = x+1;  
int newy = y;  
if(isSafe(newx, newy, n, visited, m)) {  
    path.push_back('D');  
    solve(m,n,ans,newx,newy,visited,path);  
    path.pop_back();  
  
    //left  
    newx = x;  
    newy = y-1;  
    if(isSafe(newx, newy, n, visited, m)) {  
        path.push_back('L');  
        solve(m,n,ans,newx,newy,visited,path);  
        path.pop_back();  
    }  
  
    //Right  
    newx = x;  
    newy = y+1;  
    if(isSafe(newx, newy, n, visited, m)) {  
        path.push_back('R');  
        solve(m,n,ans,newx,newy,visited,path);  
        path.pop_back();  
    }  
  
    //UP  
    newx = x-1;  
    newy = y;  
    if(isSafe(newx, newy, n, visited, m)) {  
        path.push_back('U');  
        solve(m,n,ans,newx,newy,visited,path);  
        path.pop_back();  
    }
```

After this, we have covered all possible paths from (x,y) to (n-1, n-1). So, we will now backtrack and make visited[x][y] = 0 (or False)

```
visited[x][y] = 0;
```



What is time complexity? (Discussed in Lecture 11)

→ Time complexity is the order of growth of a function measured against the input size.

## \* Recursive Function Examples :

### ① Factorial:

```
int factorial(int n) {
    if(n <= 1)
        return 1;
    return n * factorial(n-1);
}
```

Time Complexity = ??

Step 1: Write the recurrence relation as a function.

$$f(n) = n * f(n-1)$$

Step 2: Break down the function into components and include their respective Time complexities in a new function  $T(n)$ , which is the total Time Complexity.

$$T(n) = \underbrace{\quad}_{\text{Part 1}} + \underbrace{\quad}_{\text{Part 2}} + \dots$$

$$T(n) = k_1 + k_2 + T(n-1)$$

```
int factorial(int n) {
    if(n <= 1)
        return 1;
    return n * factorial(n-1);
```

Part 1 : if {                                                  Part 3 : Recursive Call  
                     ↑  
                     Part 2 (Multiplication operation)

We will take all constant-time components together ( $K$ ) and write the recurrence relation as a function of  $n$ .

$$T(n) = K + T(n-1) \quad \text{--- (1)}$$

Repeat this for the smaller input(s) in eq (1)

$$T(n-1) = K + T(n-2)$$

$$T(n-2) = K + T(n-3)$$

⋮ ⋮ ⋮ ⋮ ⋮

⋮ ⋮ ⋮ ⋮ ⋮

$$T(2) = K + T(1)$$

$$T(1) = 1 \quad (\text{Base Case})$$

If you add up all these  $n$  equations, you get:

$$\begin{aligned} T(n) &= K + T(n-1) \\ T(n-1) &= K + T(n-2) \\ + T(n-2) &= K + T(n-3) \\ + T(n-3) &= K + T(n-4) \\ + T(n-4) &= K + T(n-5) \\ \vdots &\quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ + T(2) &= K + T(1) \\ + T(1) &= 1 \end{aligned}$$


---


$$T(n) = \underbrace{K + K + \dots + K}_{n \text{ times}} + 1$$

$$T(n) = nk + 1 \approx nk \quad (\text{For large values of } n)$$

$T(n) = O(n)$

(Order of  $n$ )

## ② Binary Search :

Constant  $\left\{ \begin{array}{l} \text{Part 1} \\ \text{Part 2} \end{array} \right. \{$

```
bool binarySearch(int arr[], int val, int l, int r) {
    if(l > r) return false;
    int mid = l + (r-l)/2;
```

```

Constant { Part 1  if(l > r) return false;
            Part 2  {
                Part 3  {
                    Part 4  {
                        Part 5  {
                            Part 6  {
                                Part 7  int mid = l + (r-l)/2;
                                Part 8  if(arr[mid] == val)
                                         return true;
                                Part 9  else if(arr[mid] > val)
                                         return binarySearch(arr, val, l, mid-1);
                                Part 10 else if(arr[mid] < val)
                                         return binarySearch(arr, val, mid+1, r);
                                Part 11
                                Part 12 return false;
                            }
                        }
                    }
                }
            }
        }
    }
}

```

Clearly,

$$f(n) = C + f(n/2)$$

(Only one of the 2 recursive functions will be called for each call)

$$\begin{aligned}
 T(n) &= K + T(n/2) \\
 + T(n/2) &= K + T(n/4) \\
 + T(n/4) &= K + T(n/8) \\
 + \vdots & \vdots \quad \vdots \quad \vdots \quad \vdots \\
 + T(2) &= K + T(1) \\
 + T(1) &= 1
 \end{aligned}$$

$$\underline{\underline{T(n) = a * K + 1}}$$

[where a is the no. of times we halved n to reach to the base case]

What is a?

$$n \rightarrow n/2 \rightarrow n/4 \rightarrow n/8 \rightarrow \dots \rightarrow 1$$

a times.

Using G.P :

$$\text{first term} = n$$

$$\text{last term} = 1$$

$$\text{common ratio} = 1/2$$

$$a^{\text{th}} \text{ term} = n \left(\frac{1}{2}\right)^{a-1} = 1$$

$$\Rightarrow n = \frac{(a-1)}{2}$$

$$\Rightarrow \log n = a - 1$$

$$\Rightarrow a \approx \log n \quad (\text{For large values of } n)$$

$$T(n) = (\log n) * k + 1$$

$$T(n) \approx k * \log n$$

$$T(n) = O(\log n)$$

### ③ Merge Sort :

```

void MergeSort(vector<int> &v, int l, int r)
{
    if(l >= r) return;
    int mid = l + (r - l)/2;
    MergeSort(v, l, mid);
    MergeSort(v, mid+1, r);
    merge(v, l, mid, r);
}

Constant T.C. {
    Part 2 {
        Part 3 {
            Part 4 {
        }
    }
}

```

$$T(n) = k + T(n/2) + T(n/2) + \underbrace{nk'}_{\text{Part 4}}$$

Copying the merged array in the original array

$$T(n) = \underbrace{k}_{\text{Ignore constant}} + nk' + 2T(n/2)$$

as it's insignificant as compared to  $nk'$ .

$$\begin{aligned}
T(n) &= nk' + 2T(n/2) \\
&+ [T(n/2) = \frac{nk'}{2} + 2T(n/4)] \times 2 \\
&+ [T(n/4) = \frac{nk'}{4} + 2T(n/8)] \times 4 \\
&\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
&+ [T(2) = 2k' + 2T(1)] \times (\frac{\log n}{2}) \\
&+ [T(1) = 1] \times (\log n)
\end{aligned}$$

$$\Rightarrow T(n) = nk' + 2T(\frac{n}{2})$$

$$2 \times T(\frac{n}{2}) = nk' + 4T(\frac{n}{4})$$

$$4 \times T(\frac{n}{4}) = nk' + 8T(\frac{n}{8})$$

$$8 \times T(\frac{n}{8}) = nk' + 16T(\frac{n}{16})$$

|      |      |      |      |

$$T(n) = \log_2 n \times nk' = O(n \log n)$$

#### ④ Fibonacci Number :

$$f(n) = k + f(n-1) + f(n-2)$$

$$T(n) = k + T(n-1) + T(n-2)$$

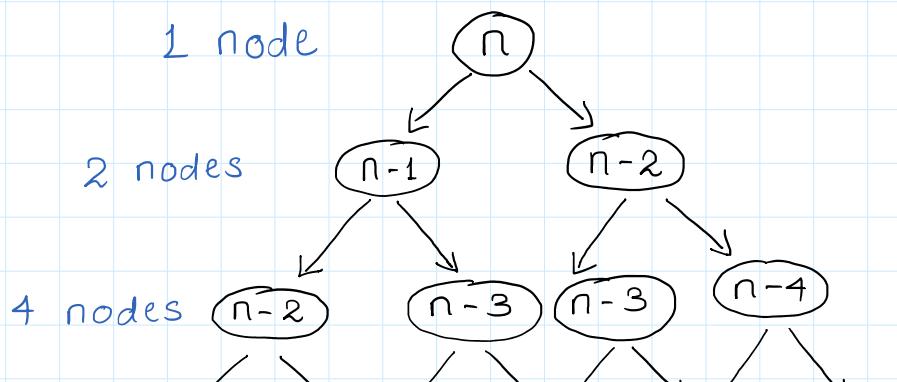
$$T(n-1) = k + T(n-2) + T(n-3)$$

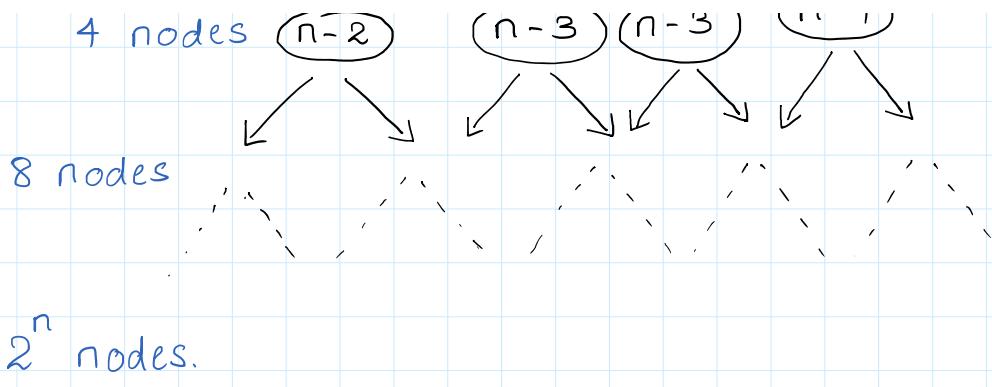
$$T(n-2) = k + T(n-3) + T(n-4)$$

$$T(n-3) = k + T(n-4) + T(n-5)$$

|      |      |      |

Use a recursion tree for calculating Time Complexity.





Total number of nodes =  $1 + 2 + 4 + 8 + \dots + 2^n$

$$= 2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^n = 1 \left( \frac{2^{n+1} - 1}{2 - 1} \right)$$

$$= (2^{n+1} - 1)$$

If each node takes  $k$  amount of time,

$$T.C = k \times 2 \times 2^n - k \approx 2k * 2^n \approx O(2^n)$$

A few recursive questions:

① Climb Stairs -

```
int countDistinctWayToClimbStair(long long nStairs)
{
    //base case
    if(nStairs < 0)
        return 0;

    if(nStairs == 0)
        return 1;

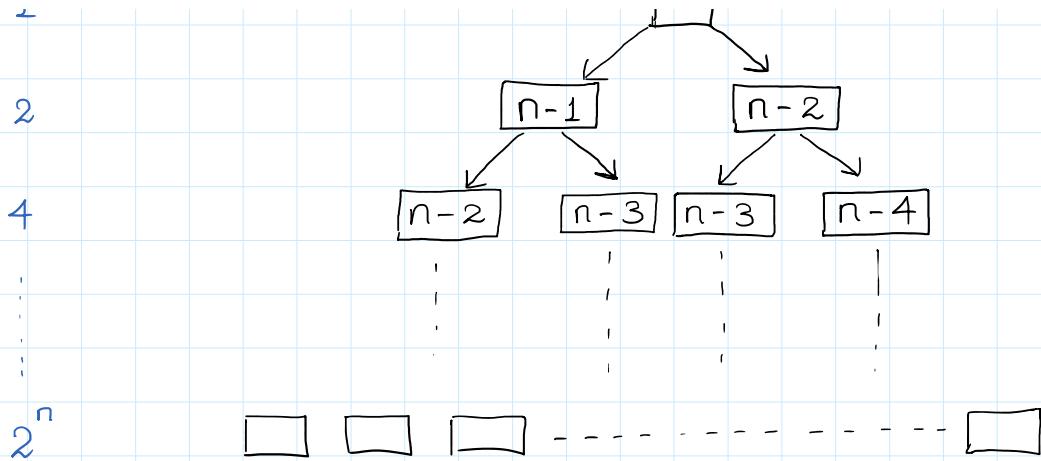
    //R.C
    int ans = countDistinctWayToClimbStair(nStairs-1)
              + countDistinctWayToClimbStair(nStairs-2);

    return ans;
}
```

Const {

$$T(n) = k + T(n-1) + T(n-2)$$





Height of the tree =  $n$ .

For depth =  $i$ , no.of nodes =  $2^i$

$$\begin{aligned}
 \text{Total no.of nodes} &= 1 + 2 + 4 + 8 + \dots + 2^n \\
 &= 2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^n \\
 &= 1 \left[ \frac{2^{n+1} - 1}{2 - 1} \right] = (2^{n+1} - 1)
 \end{aligned}$$

Time Complexity =  $\mathcal{O}(2^n)$  (Same as Fibonacci)

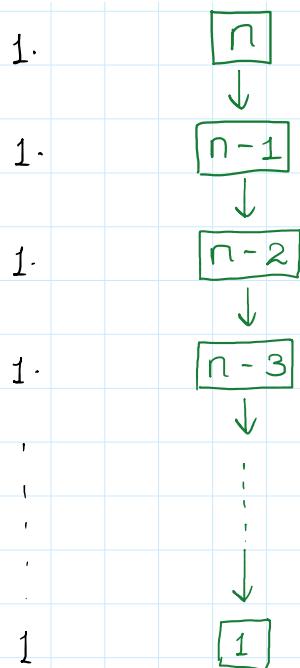
## ② Linear Search :

```

bool linearSearch(int arr[], int size, int k) {
    //base case
    if(size == 0)
        return false;
    if(arr[0] == k) {
        return true;
    }
    else {
        bool remainingPart = linearSearch(arr+1, size-1, k);
        return remainingPart;
    }
}
    
```

Applies for 'is array sorted' and other linear questions.

$$T(n) = k + T(n-1)$$



Total no. of nodes =  $\underbrace{1+1+1+1+\dots+1}_{n \text{ times}}$

$$T.C. = n = O(n)$$

### ③ Finding Power :

Constant      {  
 Constant      {  

```

int power(int a, int b) {
    //base case
    if( b == 0 )
        return 1;

    if(b == 1)
        return a;

    //RECURSIVE CALL
    int ans = power(a, b/2);

    //if b is even
    if(b%2 == 0) {
        return ans * ans;
    }
    else {
        //if b is odd
        return a * ans * ans;
    }
}
    
```

$$T(n) = k + T(n/2) \quad (\text{Same as Binary Search})$$

$$T.C. = O(\log n)$$

## ④ Quick Sort :

```

void solve(vector<int>& arr, int s, int e) {

    //base case
    if(s >= e)
        return ;

    //partition karenfe
    int p = partition(arr, s, e);

    //left part sort karo
    solve(arr, s, p-1);

    //right wala part sort karo
    solve(arr, p+1, e);

}

```

In the best case, the pivot element will get placed in the middle after partitioning. This will become similar to merge sort T.C.

$$T.C. = nK + 2T(n/2)$$

$$\text{Time Complexity} = O(n \log n)$$

## ⑤ Subsequences / Subsets :

```

void solve(vector<int> nums, vector<int> output, int index, vector<vector<int> & ans) {
    //base case
    if(index >= nums.size()) {
        ans.push_back(output);
        return ;
    }

    //exclude
    solve(nums, output, index+1, ans);

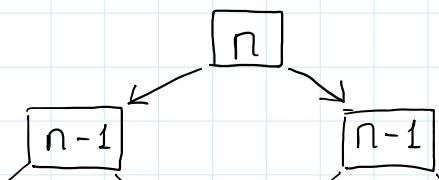
    //include
    int element = nums[index];
    output.push_back(element);
    solve(nums, output, index+1, ans);
}

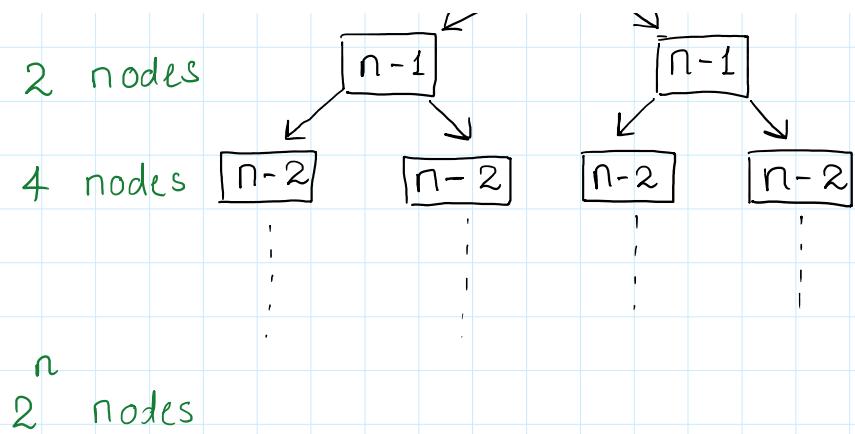
```

$$T(n) = 2T(n-1) + K$$

1. node.

2 nodes





$$\text{Total nodes} = 1 + 2 + 4 + \dots + 2^n$$

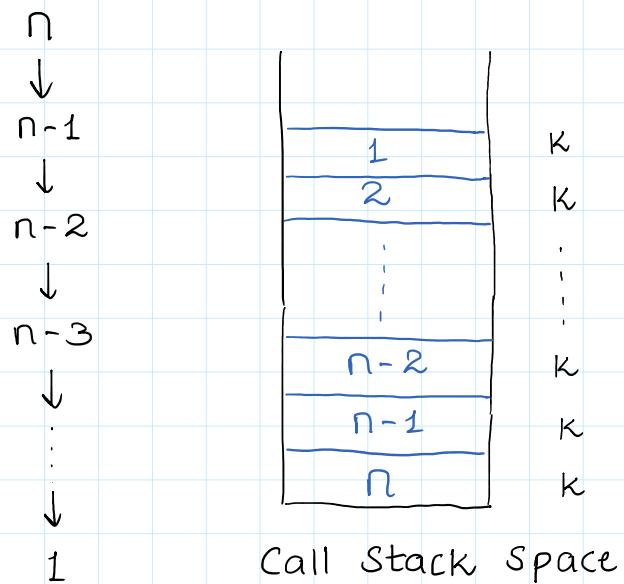
$$= 1 \left[ \frac{2^{n+1} - 1}{2 - 1} \right] = (2^{n+1} - 1)$$

$$T.C. = O(2^n)$$

If you are printing / storing the elements of every subsequence, then  $T.C. = O(2^n \times n)$

**Space Complexity :** Maximum space required at any instance.

① Factorial :



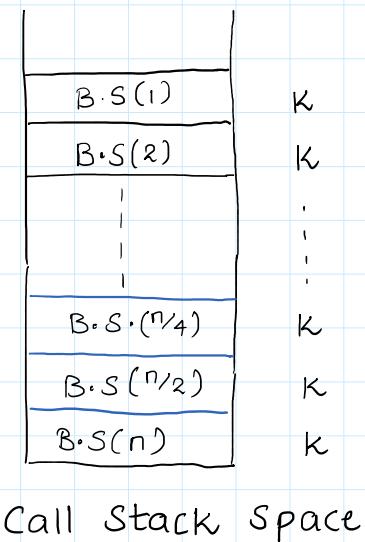
∴ Unless we reach base case  $n=1$ , all the calls will remain in the stack space, thus when we are calling for  $n=1$ , all  $n$  calls will be in the stack.

$$S.C. = k * n = O(n)$$

## ② Binary Search :

$$S.C. = k * \log(n)$$

$$S.C. = O(\log n)$$



## ③ Merge Sort :

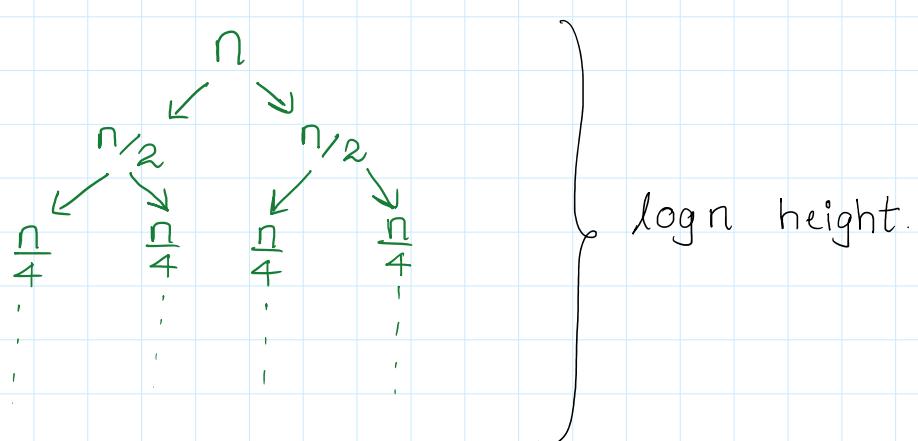
We are creating  $2 - \frac{n}{2}$  sized arrays for every  $n$  sized array that is used in a call.

$$\text{Total} = n$$

$$\text{Total} = n$$

$$\text{Total} = n$$

$$\text{Total} = n$$



$$\text{Space Complexity} = O(n \log n)$$