oliveboard

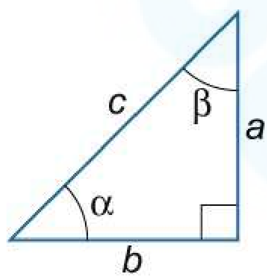
GEOMETRY FORMULAE

For Bank, Insurance, SSC, Railways & Other Government Exams

Geometry Formula

1. Right Triangle

Legs of a right triangle: a, b
Hypotenuse: c
Altitude: h
Medians: m_a , m_b , m_c
Angles: α , β
Radius of circumscribed circle: R
Radius of inscribed circle: r
Area: S



$\alpha + \beta = 90°$

$$\sin\alpha = \frac{a}{c} = \cos\beta$$

$$\cos\alpha = \frac{b}{c} = \sin\beta$$

$$\tan\alpha = \frac{a}{b} = \cot\beta$$

$$\cot\alpha = \frac{b}{a} = \tan\beta$$

$$\sec\alpha = \frac{c}{b} = \operatorname{cosec}\beta$$

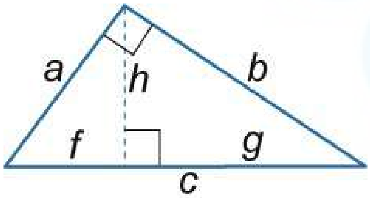
$$\operatorname{cosec}\alpha = \frac{c}{a} = \sec\beta$$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

$$a^2 = fc,\ b^2 = gc,$$

where f and c are projections of the legs a and b, respectively, onto the hypotenuse c.

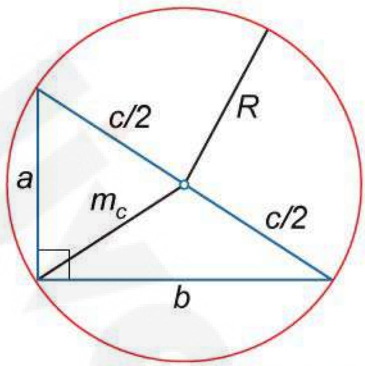


$$h^2 = fg,$$

where h is the altitude from the right angle.

$$m_a^2 = b^2 - \frac{a^2}{4},\ m_b^2 = a^2 - \frac{b^2}{4},$$

where m_a and m_b are the medians to the legs a and b.



$$m_c = \frac{c}{2},$$

where m_c is the median to the hypotenuse c.

$$R = \frac{c}{2} = m_c$$

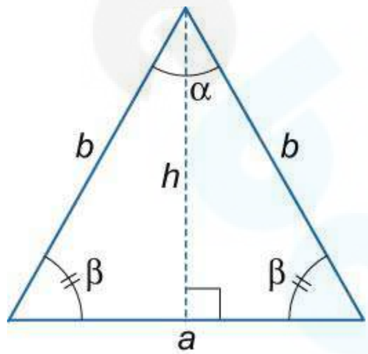
$$r = \frac{a+b-c}{2} = \frac{ab}{a+b+c}$$

$$ab = ch$$

$$S = \frac{ab}{2} = \frac{ch}{2}$$

2. Isosceles Triangle

Base: a
Legs: b
Base angle: β
Vertex angle: α
Altitude to the base: h
Perimeter: L
Area: S



$$\beta = 90^\circ - \frac{\alpha}{2}$$

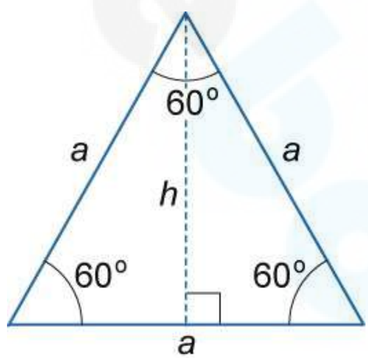
$$h^2 = b^2 - \frac{a^2}{4}$$

$$L = a + 2b$$

$$S = \frac{ah}{2} = \frac{b^2}{2}\sin\alpha$$

3. Equilateral Triangle

Side of a equilateral triangle: a
Altitude: h
Radius of circumscribed circle: R
Radius of inscribed circle: r
Perimeter: L
Area: S



$$h = \frac{a\sqrt{3}}{2}$$

$$R = \frac{2}{3}h = \frac{a\sqrt{3}}{3}$$

$$r = \frac{1}{3}h = \frac{a\sqrt{3}}{6} = \frac{R}{2}$$

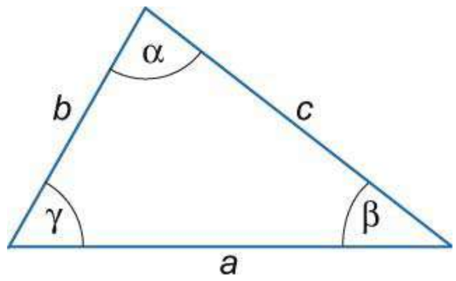
$$L = 3a$$

$$S = \frac{ah}{2} = \frac{a^2\sqrt{3}}{4}$$

4. Scalene Triangle

(A triangle with no two sides equal)

Sides of a triangle: a, b, c
Semiperimeter: $p = \frac{a+b+c}{2}$
Angles of a triangle: α, β, γ
Altitudes to the sides a, b, c: h_a, h_b, h_c
Medians to the sides a, b, c: m_a, m_b, m_c
Bisectors of the angles α, β, γ: t_a, t_b, t_c
Radius of circumscribed circle: R
Radius of inscribed circle: r
Area: S

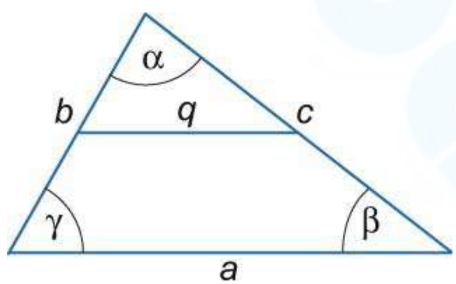


$$\alpha + \beta + \gamma = 180^\circ$$

$$a + b > c\,,$$
$$b + c > a\,,$$
$$a + c > b\,.$$
$$|a - b| < c\,,$$
$$|b - c| < a\,,$$
$$|a - c| < b\,.$$

Midline

$$q = \frac{a}{2},\ q \parallel a\,.$$



Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos\alpha$,

$b^2 = a^2 + c^2 - 2ac\cos\beta$,

$c^2 = a^2 + b^2 - 2ab\cos\gamma$.

Law of Sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R,$$

where R is the radius of the circumscribed circle.

$$R = \frac{a}{2\sin\alpha} = \frac{b}{2\sin\beta} = \frac{c}{2\sin\gamma} = \frac{bc}{2h_a} = \frac{ac}{2h_b} = \frac{ab}{2h_c} = \frac{abc}{4S}$$

$$r^2 = \frac{(p-a)(p-b)(p-c)}{p},$$

$$\frac{1}{r} = \frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c}.$$

$$\sin\frac{\alpha}{2} = \sqrt{\frac{(p-b)(p-c)}{bc}},$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{p(p-a)}{bc}},$$

$$\tan\frac{\alpha}{2} = \sqrt{\frac{(p-b)(p-c)}{p(p-a)}}.$$

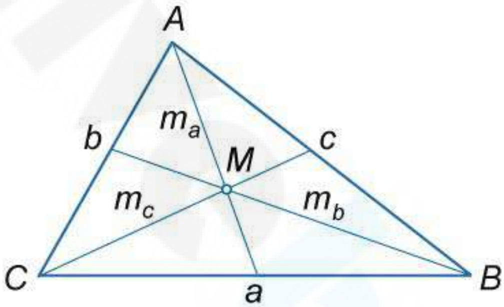
$$h_a = \frac{2}{a}\sqrt{p(p-a)(p-b)(p-c)},$$

$$h_b = \frac{2}{b}\sqrt{p(p-a)(p-b)(p-c)},$$

$$h_c = \frac{2}{c}\sqrt{p(p-a)(p-b)(p-c)}.$$

$$h_a = b\sin\gamma = c\sin\beta\ ,$$
$$h_b = a\sin\gamma = c\sin\alpha\ ,$$
$$h_c = a\sin\beta = b\sin\alpha\ .$$

$$m_a^2 = \frac{b^2+c^2}{2} - \frac{a^2}{4},$$
$$m_b^2 = \frac{a^2+c^2}{2} - \frac{b^2}{4},$$
$$m_c^2 = \frac{a^2+b^2}{2} - \frac{c^2}{4}.$$



$$AM = \frac{2}{3}m_a,\ BM = \frac{2}{3}m_b,\ CM = \frac{2}{3}m_c\ \text{(Fig.15).}$$

$$t_a^2 = \frac{4bcp(p-a)}{(b+c)^2},$$
$$t_b^2 = \frac{4acp(p-b)}{(a+c)^2},$$
$$t_c^2 = \frac{4abp(p-c)}{(a+b)^2}.$$

$$S = \frac{ah_a}{2} = \frac{bh_b}{2} = \frac{ch_c}{2},$$

$$S = \frac{ab \sin\gamma}{2} = \frac{ac \sin\beta}{2} = \frac{bc \sin\alpha}{2},$$

$$S = \sqrt{p(p-a)(p-b)(p-c)} \text{ (Heron's Formula)},$$

$$S = pr,$$

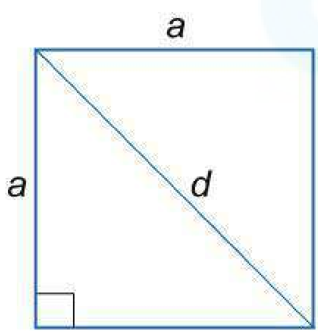
$$S = \frac{abc}{4R},$$

$$S = 2R^2 \sin\alpha \sin\beta \sin\gamma,$$

$$S = p^2 \tan\frac{\alpha}{2} \tan\frac{\beta}{2} \tan\frac{\gamma}{2}.$$

5. Square

Side of a square: a
Diagonal: d
Radius of circumscribed circle: R
Radius of inscribed circle: r
Perimeter: L
Area: S



$$d = a\sqrt{2}$$

$$R = \frac{d}{2} = \frac{a\sqrt{2}}{2}$$

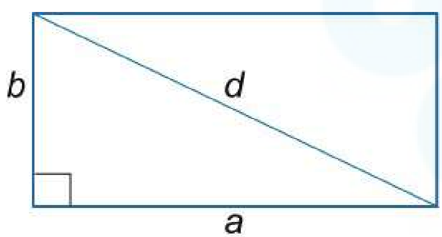
$$r = \frac{a}{2}$$

$$L = 4a$$

$$S = a^2$$

6. Rectangle

Sides of a rectangle: a, b
Diagonal: d
Radius of circumscribed circle: R
Perimeter: L
Area: S



$$d = \sqrt{a^2 + b^2}$$

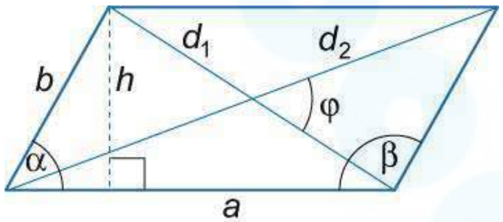
$$R = \frac{d}{2}$$

$$L = 2(a+b)$$

$$S = ab$$

7. Parallelogram

Sides of a parallelogram: a, b
Diagonals: d_1, d_2
Consecutive angles: α, β
Angle between the diagonals: φ
Altitude: h
Perimeter: L
Area: S



$$\alpha + \beta = 180^\circ$$

$$d_1^2 + d_2^2 = 2\left(a^2 + b^2\right)$$

$$h = b \sin \alpha = b \sin \beta$$

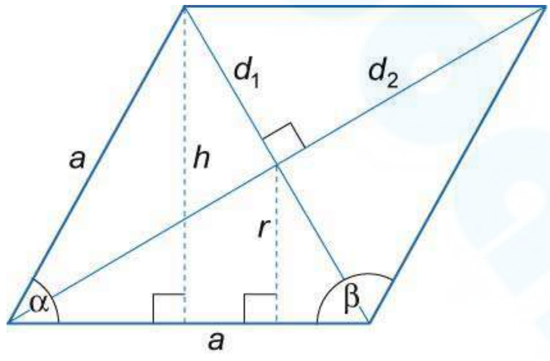
$$L = 2(a+b)$$

$$S = ah = ab \sin \alpha$$

$$S = \frac{1}{2} d_1 d_2 \sin \varphi .$$

8. Rhombus

Side of a rhombus: a
Diagonals: d_1, d_2
Consecutive angles: α, β
Altitude: H
Radius of inscribed circle: r
Perimeter: L
Area: S



$$\alpha + \beta = 180°$$

$$d_1^2 + d_2^2 = 4a^2$$

$$h = a \sin \alpha = \frac{d_1 d_2}{2a}$$

$$r = \frac{h}{2} = \frac{d_1 d_2}{4a} = \frac{a \sin \alpha}{2}$$

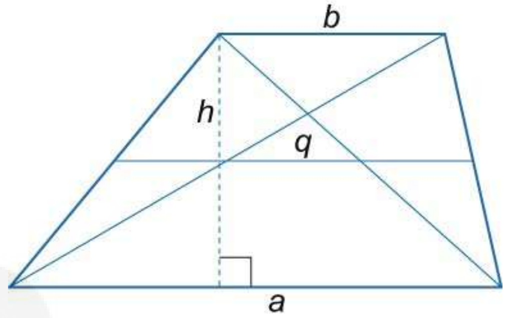
$$L = 4a$$

$$S = ah = a^2 \sin \alpha$$,

$$S = \frac{1}{2} d_1 d_2 .$$

9. Trapezoid

Bases of a trapezoid: a, b
Midline: q
Altitude: h
Area: S

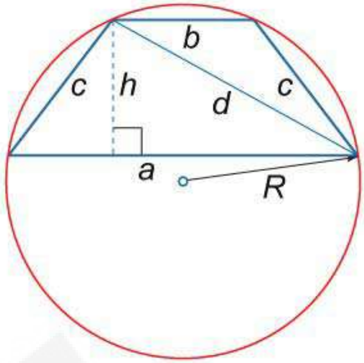


$$q = \frac{a+b}{2}$$

$$S = \frac{a+b}{2} \cdot h = qh$$

10. Isosceles Trapezoid

Bases of a trapezoid: a, b
Leg: c
Midline: q
Altitude: h
Diagonal: d
Radius of circumscribed circle: R
Area: S



$$q = \frac{a+b}{2}$$

$$d = \sqrt{ab + c^2}$$

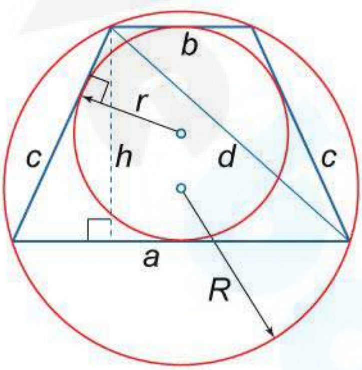
$$h = \sqrt{c^2 - \frac{1}{4}(b-a)^2}$$

$$R = \frac{c\sqrt{ab + c^2}}{\sqrt{(2c - a + b)(2c + a - b)}}$$

$$S = \frac{a+b}{2} \cdot h = qh$$

11. Isosceles Trapezoid with Inscribed Circle

Bases of a trapezoid: a, b
Leg: c
Midline: q
Altitude: h
Diagonal: d
Radius of inscribed circle: R
Radius of circumscribed circle: r
Perimeter: L
Area: S



$$a + b = 2c$$

$$q = \frac{a + b}{2} = c$$

$$d^2 = h^2 + c^2$$

$$r = \frac{h}{2} = \frac{\sqrt{ab}}{2}$$

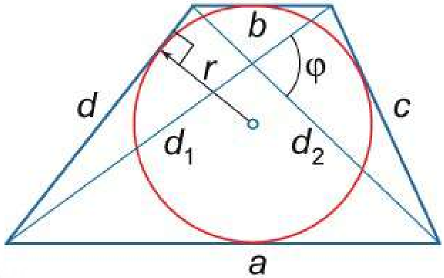
$$R = \frac{cd}{2h} = \frac{cd}{4r} = \frac{c}{2}\sqrt{1 + \frac{c^2}{ab}} = \frac{c}{2h}\sqrt{h^2 + c^2} = \frac{a+b}{8}\sqrt{\frac{a}{b} + 6 + \frac{b}{a}}$$

$$L = 2(a+b) = 4c$$

$$S = \frac{a+b}{2} \cdot h = \frac{(a+b)\sqrt{ab}}{2} = qh = ch = \frac{Lr}{2}$$

12. Trapezoid with Inscribed Circle

Bases of a trapezoid: a, b
Lateral sides: c, d
Midline: q
Altitude: h
Diagonals: d_1, d_2
Angle between the diagonals: φ
Radius of inscribed circle: r
Radius of circumscribed circle: R
Perimeter: L
Area: S



$$a + b = c + d$$

$$q = \frac{a+b}{2} = \frac{c+d}{2}$$

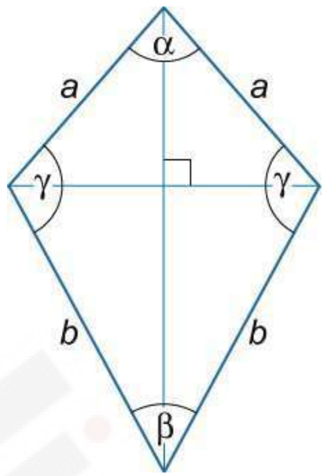
$$L = 2(a + b) = 2(c + d)$$

$$S = \frac{a+b}{2} \cdot h = \frac{c+d}{2} \cdot h = qh,$$

$$S = \frac{1}{2} d_1 d_2 \sin \varphi .$$

13. Kite

Sides of a kite: a, b
Diagonals: d_1, d_2
Angles: α, β, γ
Perimeter: L
Area: S



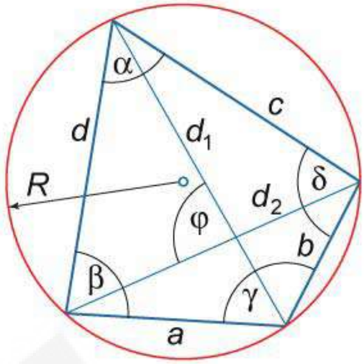
$$\alpha+\beta+2\gamma=360^{o}$$

$$L=2(a+b)$$

$$S=\frac{d_1 d_2}{2}$$

14. Cyclic Quadrilateral

Sides of a quadrilateral: a, b, c, d
Diagonals: d_1, d_2
Angle between the diagonals: φ
Internal angles: $\alpha, \beta, \gamma, \delta$
Radius of circumscribed circle: R
Perimeter: L
Semiperimeter: p
Area: S



$\alpha + \gamma = \beta + \delta = 180^{o}$

Ptolemy's Theorem
ac + bd = d1 d2

L = a + b + c + d

$$R = \frac{1}{4}\sqrt{\frac{(ac+bd)(ad+bc)(ab+cd)}{(p-a)(p-b)(p-c)(p-d)}},$$

where $p = \frac{L}{2}$.

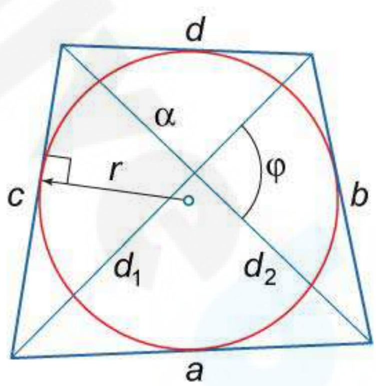
$$S = \frac{1}{2} d_1 d_2 \sin\varphi,$$

$$S = \sqrt{(p-a)(p-b)(p-c)(p-d)},$$

where $p = \frac{L}{2}$.

15. Tangential Quadrilateral

Sides of a quadrilateral: a, b, c, d
Diagonals: d_1, d_2
Angle between the diagonals: φ
Radius of inscribed circle: r
Perimeter: L
Semiperimeter: p
Area: S



$$a + c = b + d$$

$$L = a + b + c + d = 2(a + c) = 2(b + d)$$

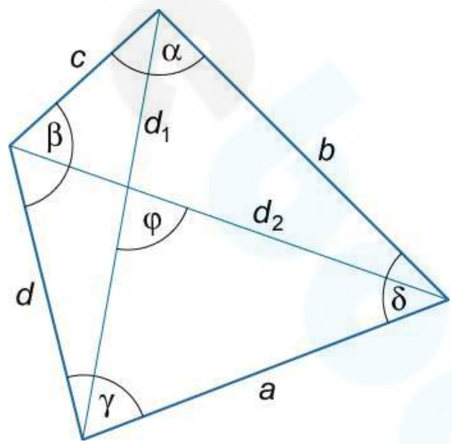
$$r = \frac{\sqrt{d_1^2 d_2^2 - (a-b)^2 (a+b-p)^2}}{2p},$$

where $p = \frac{L}{2}$.

$$S = pr = \frac{1}{2} d_1 d_2 \sin\varphi$$

16. General Quadrilateral

Sides of a quadrilateral: a, b, c, d
Diagonals: d_1, d_2
Angle between the diagonals: φ
Internal angles: $\alpha, \beta, \gamma, \delta$
Perimeter: L
Area: S



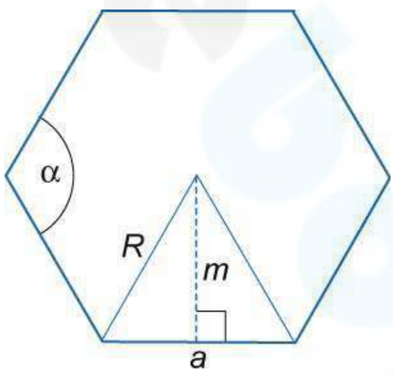
$$\alpha + \beta + \gamma + \delta = 360^\circ$$

$$L = a + b + c + d$$

$$S = \frac{1}{2} d_1 d_2 \sin \varphi$$

17. Regular Hexagon

Side: a
Internal angle: α
Slant height: m
Radius of inscribed circle: r
Radius of circumscribed circle: R
Perimeter: L
Semiperimeter: p
Area: S



$$\alpha = 120°$$

$$r = m = \frac{a\sqrt{3}}{2}$$

$R = a$

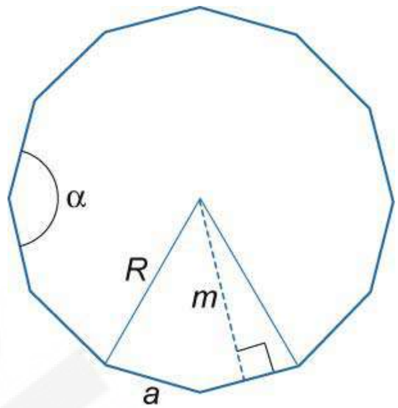
$L = 6a$

$$S = pr = \frac{a^2 3\sqrt{3}}{2},$$

where $p = \dfrac{L}{2}$.

18. Regular Polygon

Side: a
Number of sides: n
Internal angle: α
Slant height: m
Radius of inscribed circle: r
Radius of circumscribed circle: R
Perimeter: L
Semiperimeter: p
Area: S



$$\alpha = \frac{n-2}{2} \cdot 180^\circ$$

$$\alpha = \frac{n-2}{2} \cdot 180^\circ$$

$$R = \frac{a}{2\sin\frac{\pi}{n}}$$

$$r = m = \frac{a}{2\tan\frac{\pi}{n}} = \sqrt{R^2 - \frac{a^2}{4}}$$

$$L = na$$

$$S = \frac{nR^2}{2}\sin\frac{2\pi}{n},$$

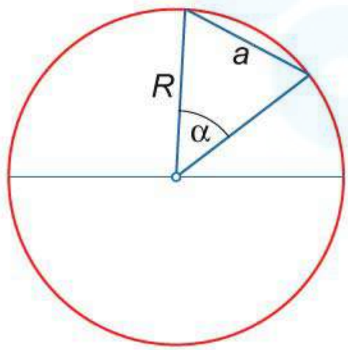
$$S = pr = p\sqrt{R^2 - \frac{a^2}{4}},$$

where $p = \frac{L}{2}$.

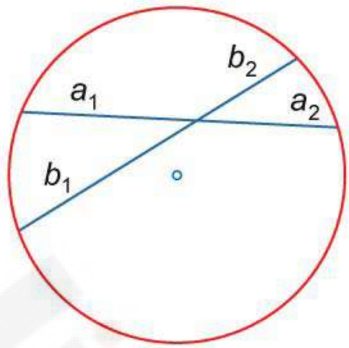
19. Circle

Radius: R
Diameter: d
Chord: a
Secant segments: e, f
Tangent segment: g
Central angle: α
Inscribed angle: β
Perimeter: L
Area: S

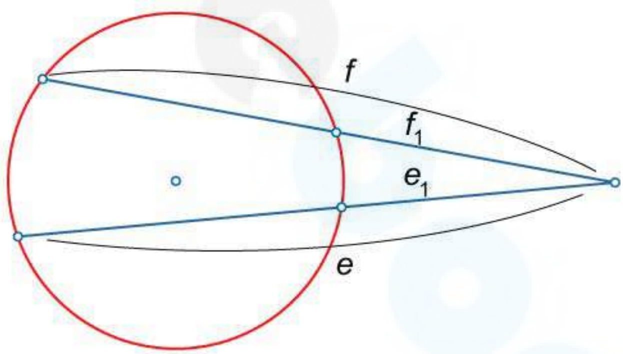
$$a = 2R\sin\frac{\alpha}{2}$$



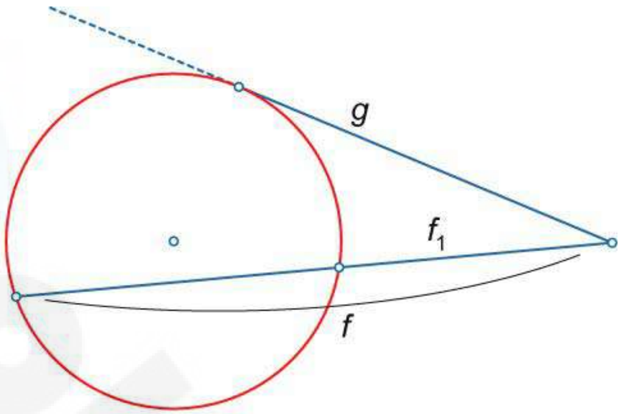
$a_1a_2 = b_1b_2$



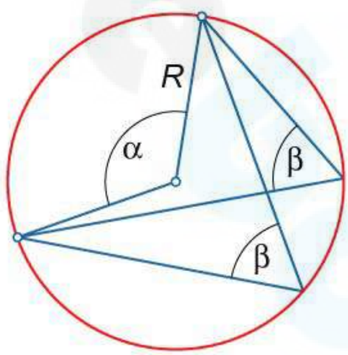
$ee_1 = ff_1$



$g^2 = ff_1$



$$\beta = \frac{\alpha}{2}$$

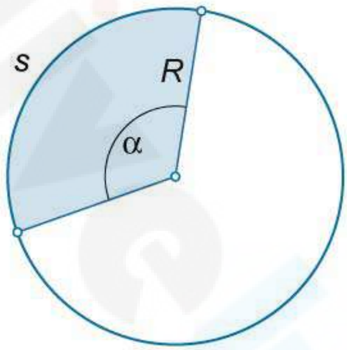


$$L = 2\pi R = \pi d$$

$$S = \pi R^2 = \frac{\pi d^2}{4} = \frac{LR}{2}$$

20. Sector of a Circle

Radius of a circle: R
Arc length: s
Central angle (in radians): x
Central angle (in degrees): α
Perimeter: L
Area: S



$$s = Rx$$

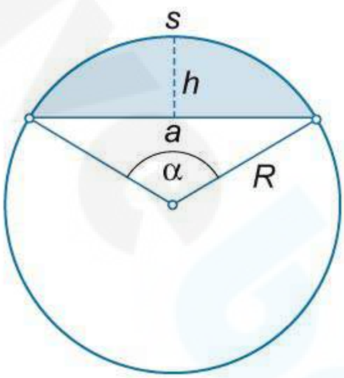
$$s = \frac{\pi R \alpha}{180^\circ}$$

$$L = s + 2R$$

$$S = \frac{Rs}{2} = \frac{R^2 x}{2} = \frac{\pi R^2 \alpha}{360^\circ}$$

21. Segment of a Circle

Radius of a circle: R
Arc length: s
Chord: a
Central angle (in radians): x
Central angle (in degrees): α
Height of the segment: h
Perimeter: L
Area: S



$$a = 2\sqrt{2hR - h^2}$$

$$h = R - \frac{1}{2}\sqrt{4R^2 - a^2}\,,\ h < R$$

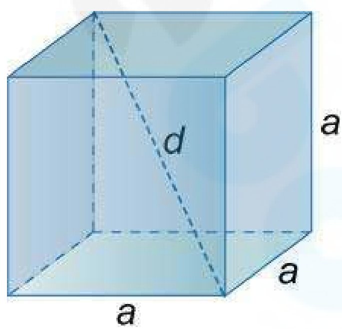
$$L = s + a$$

$$S = \frac{1}{2}\left[sR - a(R-h)\right] = \frac{R^2}{2}\left(\frac{\alpha\pi}{180°} - \sin\alpha\right) = \frac{R^2}{2}(x - \sin x),$$

$$S \approx \frac{2}{3}ha.$$

22. Cube

Edge: a
Diagonal: d
Radius of inscribed sphere: r
Radius of circumscribed sphere: r
Surface area: S
Volume: V



$$d = a\sqrt{3}$$

$$r = \frac{a}{2}$$

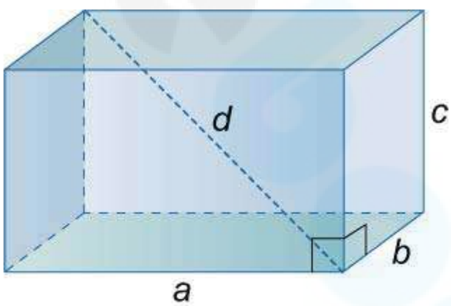
$$R = \frac{a\sqrt{3}}{2}$$

$$S = 6a^2$$

$$V = a^3$$

23. Rectangular Parallelepiped

Edges: a, b, c
Diagonal: d
Surface area: S
Volume: V



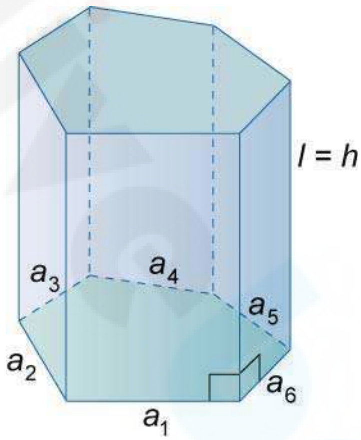
$$d = \sqrt{a^2 + b^2 + c^2}$$

$$S = 2(ab + ac + bc)$$

$$V = abc$$

24. Prism

Lateral edge: l
Height: h
Lateral area: S_L
Area of base: S_B
Total surface area: S
Volume: V



$S = S_L + 2S_B$.

Lateral Area of a Right Prism

$$S_L = (a_1 + a_2 + a_3 + \ldots + a_n)l$$

Lateral Area of an Oblique Prism

$S_L = pl$,

where p is the perimeter of the cross section.

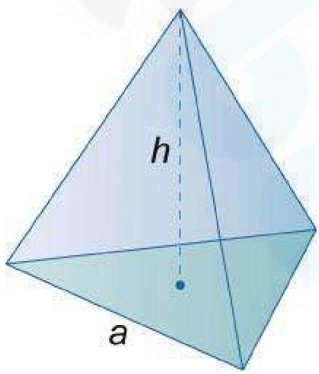
$$V = S_B h$$

Cavalieri's Principle

Given two solids included between parallel planes. If every plane cross section parallel to the given planes has the same area in both solids, then the volumes of the solids are equal.

25. Regular Tetrahedron

Triangle side length: a
Height: h
Area of base: S_B
Surface area: S
Volume: V



$$h = \sqrt{\frac{2}{3}}\, a$$

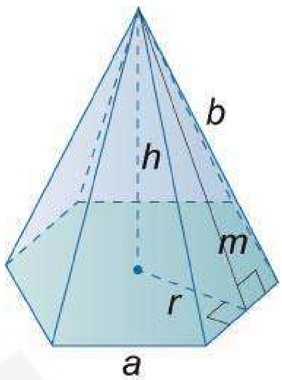
$$S_B = \frac{\sqrt{3}a^2}{4}$$

$$S = \sqrt{3}a^2$$

$$V = \frac{1}{3}S_B h = \frac{a^3}{6\sqrt{2}}.$$

26. Regular Pyramid

Side of base: a
Lateral edge: b
Height: h
Slant height: m
Number of sides: n
Semiperimeter of base: p
Radius of inscribed sphere of base: r
Area of base: S_B
Lateral surface area: S_L
Total surface area: S
Volume: V



$$m = \sqrt{b^2 - \frac{a^2}{4}}$$

$$h = \frac{\sqrt{4b^2 \sin^2 \frac{\pi}{n} - a^2}}{2\sin \frac{\pi}{n}}$$

$$S_L = \frac{1}{2} nam = \frac{1}{4} na\sqrt{4b^2 - a^2} = pm$$

$$S_B = pr$$

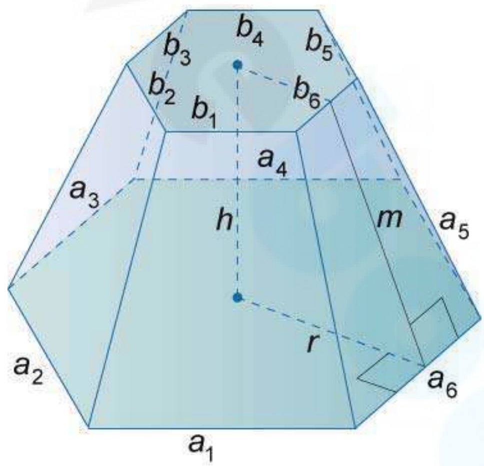
$$S = S_B + S_L$$

$$V = \frac{1}{3} S_B h = \frac{1}{3} prh$$

27. Frustum of a Regular Pyramid

Base and top side lengths: $\begin{cases} a_1, a_2, a_3, \ldots, a_n \\ b_1, b_2, b_3, \ldots, b_n \end{cases}$

Height: h
Slant height: m
Area of bases: S_1, S_2
Lateral surface area: S_L
Perimeter of bases: P_1, P_2
Scale factor: k
Total surface area: S
Volume: V



$$\frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \ldots = \frac{b_n}{a_n} = \frac{b}{a} = k$$

$$\frac{S_2}{S_1} = k^2$$

$$S_L = \frac{m(P_1 + P_2)}{2}$$

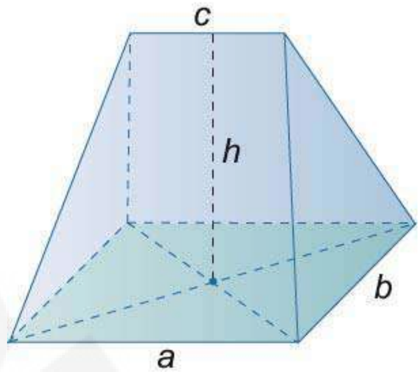
$$S = S_L + S_1 + S_2$$

$$V = \frac{h}{3}\left(S_1 + \sqrt{S_1 S_2} + S_2\right)$$

$$V = \frac{hS_1}{3}\left[1 + \frac{b}{a} + \left(\frac{b}{a}\right)^2\right] = \frac{hS_1}{3}\left[1 + k + k^2\right]$$

28. Rectangular Right Wedge

Sides of base: a, b
Top edge: c
Height: h
Lateral surface area: S_L
Area of base: S_B
Total surface area: S
Volume: V



$$S_L = \frac{1}{2}(a+c)\sqrt{4h^2+b^2} + b\sqrt{h^2+(a-c)^2}$$

$$S_B = ab$$

$$S = S_B + S_L$$

$$V = \frac{bh}{6}(2a+c)$$

29. Platonic Solids

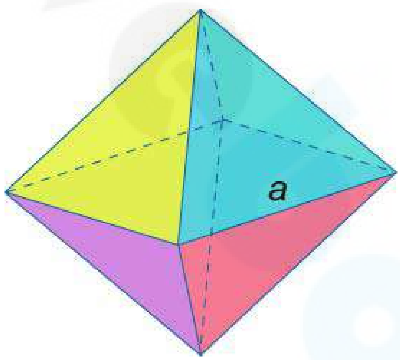
Edge: a
Radius of inscribed circle: r
Radius of circumscribed circle: R
Surface area: S
Volume: V

Five Platonic Solids

The platonic solids are convex polyhedra with equivalent faces composed of congruent convex regular polygons.

Solid	Number of Vertices	Number of Edges	Number of Faces	Section
Tetrahedron	4	6	4	3.25
Cube	8	12	6	3.22
Octahedron	6	12	8	3.27
Icosahedron	12	30	20	3.27
Dodecahedron	20	30	12	3.27

Octahedron



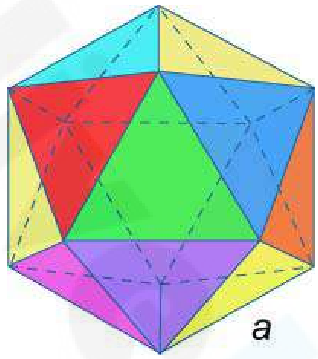
$$r = \frac{a\sqrt{6}}{6}$$

$$R = \frac{a\sqrt{2}}{2}$$

$$S = 2a^2\sqrt{3}$$

$$V = \frac{a^3\sqrt{2}}{3}$$

Icosahedron



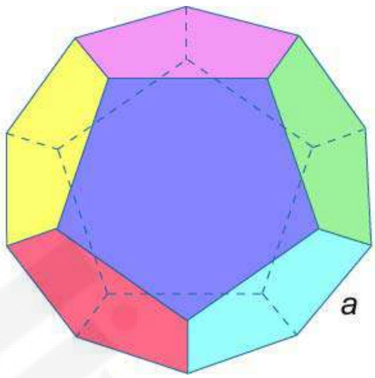
$$r = \frac{a\sqrt{3}\left(3+\sqrt{5}\right)}{12}$$

$$R = \frac{a}{4}\sqrt{2\left(5+\sqrt{5}\right)}$$

$$S = 5a^2\sqrt{3}$$

$$V = \frac{5a^3\left(3+\sqrt{5}\right)}{12}$$

Dodecahedron



$$r = \frac{a\sqrt{10\left(25 + 11\sqrt{5}\right)}}{2}$$

$$R = \frac{a\sqrt{3}\left(1 + \sqrt{5}\right)}{4}$$

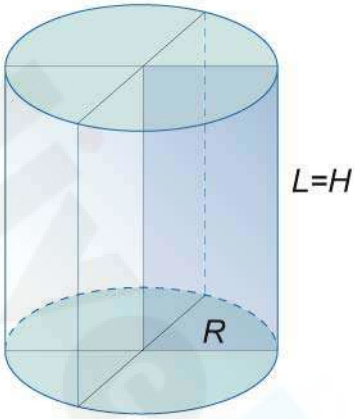
$$S = 3a^2\sqrt{5\left(5 + 2\sqrt{5}\right)}$$

$$V = \frac{a^3\left(15 + 7\sqrt{5}\right)}{4}$$

30. Right Circular Cylinder

Radius of base: R
Diameter of base: d

Height: H
Lateral surface area: S_L
Area of base: S_B
Total surface area: S
Volume: V



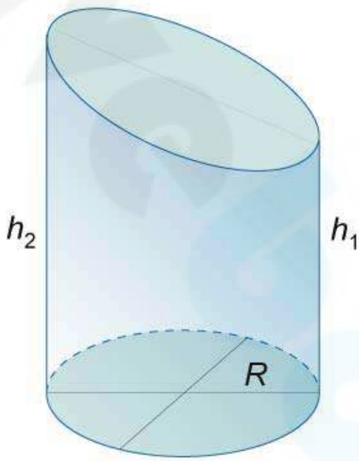
$$S_L = 2\pi RH$$

$$S = S_L + 2S_B = 2\pi R(H+R) = \pi d\left(H + \frac{d}{2}\right)$$

$$S_B H = \pi R^2 H$$

31. Right Circular Cylinder with an Oblique Plane Face

Radius of base: R
The greatest height of a side: h_1
The shortest height of a side: h_2
Lateral surface area: S_L
Area of plane end faces: S_B
Total surface area: S
Volume: V



$$S_L = \pi R(h_1 + h_2)$$

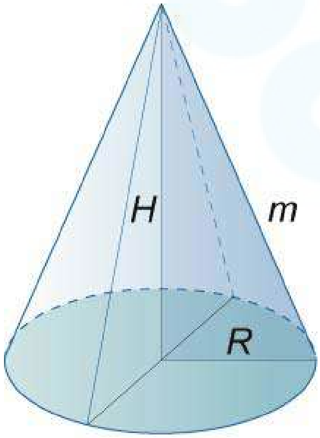
$$S_B = \pi R^2 + \pi R\sqrt{R^2 + \left(\frac{h_1 - h_2}{2}\right)^2}$$

$$S = S_L + S_B = \pi R\left[h_1 + h_2 + R + \sqrt{R^2 + \left(\frac{h_1 - h_2}{2}\right)^2}\right]$$

$$V = \frac{\pi R^2}{2}\left(h_1 + h_2\right)$$

32. Right Circular Cone

Radius of base: R
Diameter of base: d
Height: H
Slant height: m
Lateral surface area: S_L
Area of base: S_B
Total surface area: S
Volume: V



$$H = \sqrt{m^2 - R^2}$$

$$S_L = \pi R m = \frac{\pi m d}{2}$$

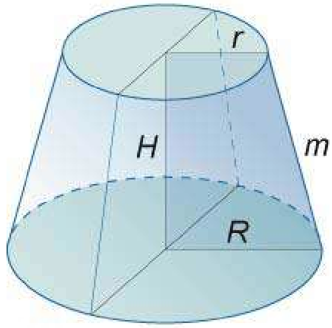
$$S_B = \pi R^2$$

$$S = S_L + S_B = \pi R(m + R) = \frac{1}{2}\pi d\left(m + \frac{d}{2}\right)$$

$$V = \frac{1}{3}S_B H = \frac{1}{3}\pi R^2 H$$

33. Frustum of a Right Circular Cone

Radius of bases: R, r
Height: H
Slant height: m
Scale factor: k
Area of bases: S_1, S_2
Lateral surface area: S_L
Total surface area: S
Volume: V



$$H = \sqrt{m^2 - (R - r)^2}$$

$$\frac{R}{r} = k$$

$$\frac{S_2}{S_1} = \frac{R^2}{r^2} = k^2$$

$$S_L = \pi m(R + r)$$

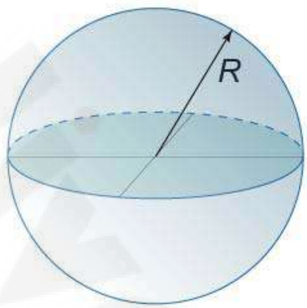
$$S = S_1 + S_2 + S_L = \pi\left[R^2 + r^2 + m(R + r)\right]$$

$$V = \frac{h}{3}\left(S_1 + \sqrt{S_1 S_2} + S_2\right)$$

$$V = \frac{hS_1}{3}\left[1 + \frac{R}{r} + \left(\frac{R}{r}\right)^2\right] = \frac{hS_1}{3}\left[1 + k + k^2\right]$$

34. Sphere

Radius: R
Diameter: d
Surface area: S
Volume: V

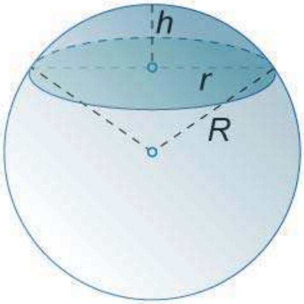


$$S = 4\pi R^2$$

$$V = \frac{4}{3}\pi R^3 H = \frac{1}{6}\pi d^3 = \frac{1}{3}SR$$

35. Spherical Cap

Radius of sphere: R
Radius of base: r
Height: h
Area of plane face: S_B
Area of spherical cap: S_C
Total surface area: S
Volume: V



$$R = \frac{r^2 + h^2}{2h}$$

$$S_B = \pi r^2$$

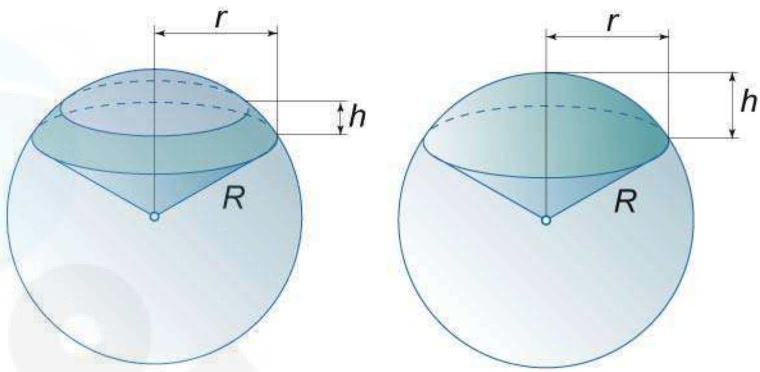
$$S_C = \pi\left(h^2 + r^2\right)$$

$$S = S_B + S_C = \pi\left(h^2 + 2r^2\right) = \pi\left(2Rh + r^2\right)$$

$$V = \frac{\pi}{6} h^2 (3R - h) = \frac{\pi}{6} h\left(3r^2 + h^2\right)$$

36. Spherical Sector

Radius of sphere: R
Radius of base of spherical cap: r
Height: h
Total surface area: S
Volume: V



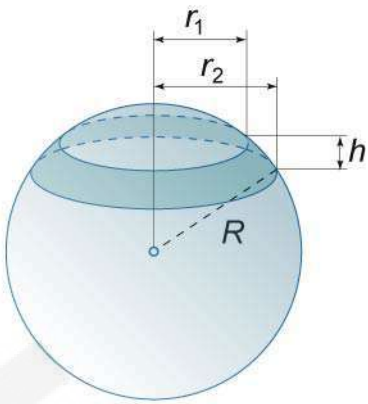
$$S = \pi R(2h + r)$$

$$V = \frac{2}{3}\pi R^2 h$$

Note: The given formulas are correct both for "open" and "closed" spherical sector.

37. Spherical Segment

Radius of sphere: R
Radius of bases: r_1, r_2
Height: h
Area of spherical surface: S_S
Area of plane end faces: S_1, S_2
Total surface area: S
Volume: V



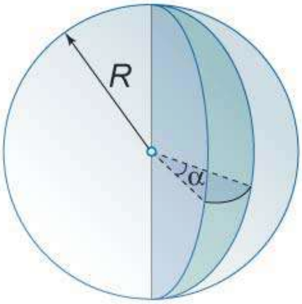
$$S_S = 2\pi Rh$$

$$S = S_S + S_1 + S_2 = \pi\left(2Rh + r_1^2 + r_2^2\right)$$

$$V = \frac{1}{6}\pi h\left(3r_1^2 + 3r_2^2 + h^2\right)$$

38. Spherical Wedge

Radius: R
Dihedral angle in degrees: x
Dihedral angle in radians: α
Area of spherical lune: S_L
Total surface area: S
Volume: V



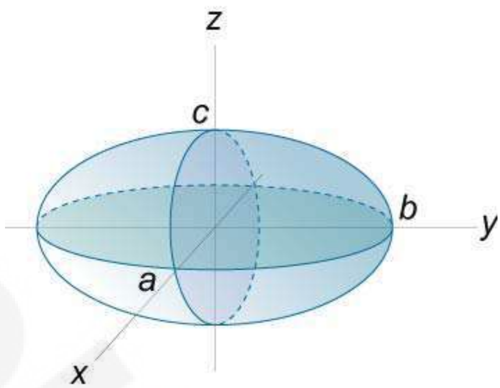
$$S_L = \frac{\pi R^2}{90}\alpha = 2R^2x$$

$$S = \pi R^2 + \frac{\pi R^2}{90}\alpha = \pi R^2 + 2R^2x$$

$$V = \frac{\pi R^3}{270}\alpha = \frac{2}{3}R^3x$$

39. Ellipsoid

Semi-axes: a, b, c
Volume: V



$$V=\frac{4}{3}\pi abc$$

Prolate Spheroid

Semi-axes: a, b, b (a > b)
Surface area: S
Volume: V

$$S=2\pi b\left(b+\frac{a\arcsin e}{e}\right),$$

where $e=\frac{\sqrt{a^2-b^2}}{a}$.

$$V=\frac{4}{3}\pi b^2 a$$

Oblate Spheroid

Semi-axes: a, b, b ($a < b$)
Surface area: S
Volume: V

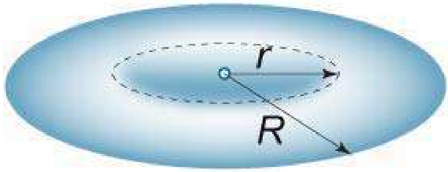
$$S = 2\pi b\left(b + \frac{a \operatorname{arcsinh}\left(\frac{be}{a}\right)}{be/a} \right),$$

where $e = \frac{\sqrt{b^2 - a^2}}{b}$.

$$V = \frac{4}{3}\pi b^2 a$$

40. Circular Torus

Major radius: R
Minor radius: r
Surface area: S
Volume: V



$S = 4\pi^2 Rr$

$V = 2\pi^2 Rr^2$