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Uber Pulley Problem

Section L

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Problem Statement:

"Jerky" Jerry decided to make a jabberwocky jumper using a pulley system (see diagram). His method was to attach one end of a chain to a barrel of rocks, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform away from the pulley to point A (the full length of the chain). When he sat in the jumper, he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pulley. He flew through the air as a projectile to point C, transitioning 75% of his speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the chain. Calculate the coefficient of kinetic friction between the jumper and ground.

Givens and Equations:

$m_J = 68$ kg
$m_B = 164$ kg

$m_C = 68$ kg
$L_C = 12$ m

$h_P = 17$ m
$\mu_P = 0.14$
$\Delta x_{BD} = 87$ m
$\mu_G = ?$

$$EQ\ 1: a\,dx = v\,dv$$

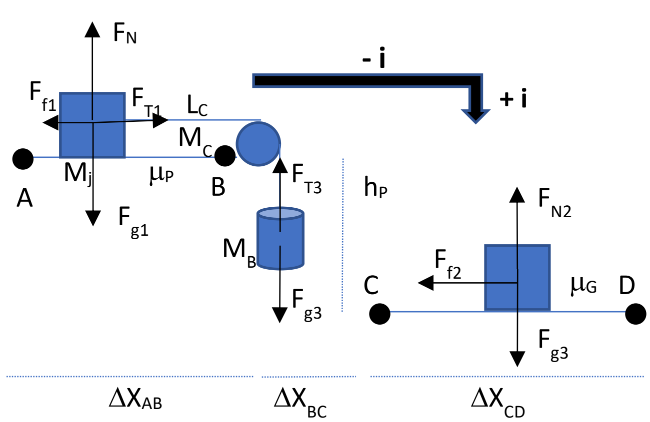
$$EQ\ 2: a = \frac{\sum F}{m}$$

$$EQ\ 3: {v_f}^2 = {v_i}^2 + 2a\Delta x$$

$$\text{EQ 4: } \Delta x = v_0 t + \frac{1}{2}at^2$$

$$EQ5: F_f = \mu F_N$$

Diagram:



Strategy/Solution:

A strategy for solving this problem involves marking 4 points on the jumper's path: points A, B, C, and D. Point A is where the jumper starts off. Point B is the location of the jumper at the end of the platform. Point C is where the jumper lands on the ground. Point D is where the jumper comes to a complete stop. The problem can be broken up into 3 stages: stage AB, stage BC, and stage CD, where each stage is the time in between two consecutive labeled points.

During stage AB, equation 5 can be used to find the force of friction acting on the jumper. The system of the jumper, the chain, and the barrel can be treated as a single entity, and the net forces acting on this system are the force of friction on the jumper and the force of gravity on the barrel and the chain (the system is considered to travel along the bent axis "i", as shown in the diagram). Equation 2 can be used to find the force of gravity on the system. Equation 2 can be used again to find the acceleration of the system as a function of the position of the jumper. After this, equation 1 can be used to find the velocity of the jumper when it reaches point B.

During stage BC, equation 3 can be used to find the vertical velocity of the jumper at point C. The Pythagorean theorem can be used to find the jumper's speed at point C. Equation 4 can be used to find the distance that the jumper 1travels from point B before it hits the ground.

During stage CD, equation 3 can be used to find the acceleration of the jumper while it is on the ground. This value can be used in equation 2 to find the net friction force acting on the object. This can be used in equation 5 in order to find the coefficient of friction between the jumper and the ground.

Stage AB:

$$F_f = \mu F_N$$

$$\sum F_y: F_{N1} - F_{g1} = ma_{y1}$$

$$F_{N1} - F_{g1} = 0$$

$$F_{N1} = F_{g1}$$

$$F_{N1} = 9.8 * 68$$

$$\underline{F_{N1} = 666.4\ N}$$

$$F_{f1} = \mu_P F_{N1}$$

$$F_{f1} = 0.14 * 666.4$$

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$$\underline{F_{f1} = 93.296\ N}$$

$$F_{g3} = m_B g + \frac{x}{L_c} * m_c g$$

$$F_{g3} = 1607.2 + \frac{x}{12} * 666.4$$

$$F_{g3} = 1607.2 + 55.533x$$

$$F_{net} = F_{g3} - F_{f1}$$

$$F_{net} = 1607.2 + 55.533x - 93.296$$

$$F_{net} = 1513.904 + 55.533x$$

$$a = \frac{F_{net}}{m_{Total}}$$

$$a = \frac{1513.904 + 55.533x}{300}$$

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$$\underline{a = 5.046 + 0.1851x}$$

$$\int a\,dx = \int v\,dv$$

$$\int v\,dv = \int_0^{12} (5.046 + 0.1851x)\,dx$$

$$\frac{v_B^2}{2} = 73.8792$$

$$\underline{v_B = 12.156\ m/s}$$

Stage BC:

$$v_{Cy}^2 = v_i^2 + 2a\Delta y$$

$$v_{Cy}^2 = 0^2 + 2*(-9.8)*17$$

$$v_{Cy}^2 = 0^2 + 2*(-9.8)*(-17)$$

$$\underline{v_{Cy} = 18.2538\ m/s}$$

$$v_{Cx} = 12.156\ m/s$$

$$v_c = \sqrt{v_{Cx}^2 + v_{Cy}^2}$$

$$v_c = \sqrt{18.2538^2 + 12.156^2}$$

$$v_c = \sqrt{18.2538^2 + 12.156^2}$$

$$\underline{v_c = 21.931\ m/s}$$

$$\Delta y = v_0 t + \frac{1}{2} a t^2$$

$$-17 = 0t_{BC} + \frac{1}{2}(-9.8)t_{BC}^2$$

$$\underline{t_{BC} = 1.8626\ s}$$

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

$$\Delta x = 12.156*1.8626 + \frac{1}{2}*0*1.8626^2$$

$$\underline{\Delta x_{BC} = 22.642\ m}$$

Stage CD:

$$v_{Cx} = \frac{3}{4}*v_c$$

$$v_{Cx} = \frac{3}{4}*21.931$$

$$\underline{v_{Cx} = 16.448\ m/s}$$

$$\Delta x_{BD} = \Delta x_{BC} + \Delta x_{CD}$$

$$87 = 22.642 + \Delta x_{CD}$$

$$\Delta x_{BD} = \Delta x_{BC} + \Delta x_{CD}$$

$$\underline{\Delta x_{CD} = 64.356\ m}$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$0^2 = 16.448^2 + 2a*64.356$$

$$\underline{a = 2.1019\ m/s^2}$$

$$a = \frac{F_{net}}{m}$$

$$2.1019 = \frac{F_{net}}{68}$$

$$F_{net} = 142.93\ N$$

$$F_{net} = F_f$$

$$F_f = 142.93\ N$$

$$\sum F_y: F_{N2} - F_{g3} = ma_y$$

$$F_{N2} - F_{g3} = 0$$

$$F_{N2} = F_{g3}$$

$$F_{N2} = 68*9.8$$

$$\underline{F_{N2} = 666.4\ N}$$

$$F_f = \mu F_N$$

$$142.93N = \mu_G*666.4N$$

$$\boxed{\mu = 0.2145}$$