

# Hanford Washington Problem

## Given Data, Table, and Plot

In[1]:= **Indexes** = {2.5, 2.6, 3.4, 1.3, 1.6, 3.8, 11.6, 6.4, 8.3}

Out[1]= {2.5, 2.6, 3.4, 1.3, 1.6, 3.8, 11.6, 6.4, 8.3}

In[2]:= **Deaths** = {147, 130, 130, 114, 138, 162, 208, 178, 210}

Out[2]= {147, 130, 130, 114, 138, 162, 208, 178, 210}

In[3]:= **Locations** = {"Umatilla", "Morrow", "Gilliam", "Sherman",  
"Wasco", "Hood River", "Portland", "Columbia", "Clatsop"}

Out[3]= {Umatilla, Morrow, Gilliam, Sherman, Wasco, Hood River, Portland, Columbia, Clatsop}

In[4]:= **labeledData** = Transpose[{Locations, Indexes, Deaths}]

Out[4]= {{Umatilla, 2.5, 147}, {Morrow, 2.6, 130}, {Gilliam, 3.4, 130},  
{Sherman, 1.3, 114}, {Wasco, 1.6, 138}, {Hood River, 3.8, 162},  
{Portland, 11.6, 208}, {Columbia, 6.4, 178}, {Clatsop, 8.3, 210}}

In[5]:= {"Umatilla", 2.5`, 147}, {"Morrow", 2.6`, 130}, {"Gilliam", 3.4`, 130},  
{"Sherman", 1.3`, 114}, {"Wasco", 1.6`, 138}, {"Hood River", 3.8`, 162},  
{"Portland", 11.6`, 208}, {"Columbia", 6.4`, 178}, {"Clatsop", 8.3`, 210}

Out[5]= {{Umatilla, 2.5, 147}, {Morrow, 2.6, 130}, {Gilliam, 3.4, 130},  
{Sherman, 1.3, 114}, {Wasco, 1.6, 138}, {Hood River, 3.8, 162},  
{Portland, 11.6, 208}, {Columbia, 6.4, 178}, {Clatsop, 8.3, 210}}

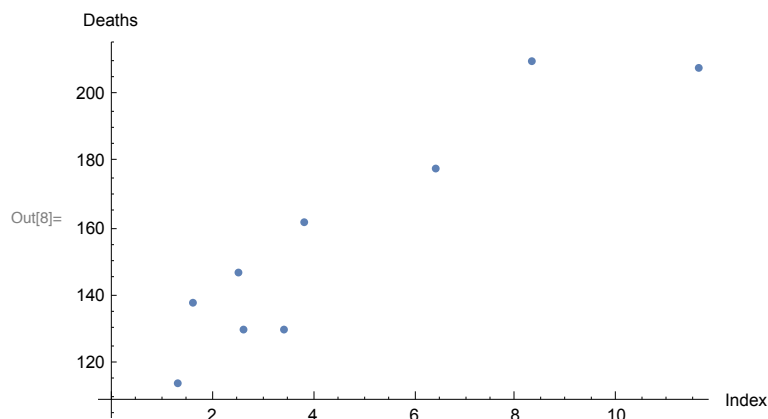
In[6]:= **Text[Grid[Prepend[labeledData, {"Location", "Index", "Deaths"}],  
Alignment → Center, Dividers → {2 → True, 2 → True}, Spacings → {1, 1}]]**

Location	Index	Deaths
Umatilla	2.5	147
Morrow	2.6	130
Gilliam	3.4	130
Sherman	1.3	114
Wasco	1.6	138
Hood River	3.8	162
Portland	11.6	208
Columbia	6.4	178
Clatsop	8.3	210

```
In[7]:= UData = Transpose[{Indexes, Deaths}] (*Unlabeled Data*)
```

```
Out[7]= {{2.5, 147}, {2.6, 130}, {3.4, 130}, {1.3, 114},
        {1.6, 138}, {3.8, 162}, {11.6, 208}, {6.4, 178}, {8.3, 210}}
```

```
In[8]:= plot = ListPlot[UData, AxesLabel → {"Index", "Deaths"}]
```



## 2-Point Line

```
In[9]:= point1 = {Indexes[[5]], Deaths[[5]]}
```

```
Out[9]= {1.6, 138}
```

```
In[10]:= point2 = {Indexes[[7]], Deaths[[7]]}
```

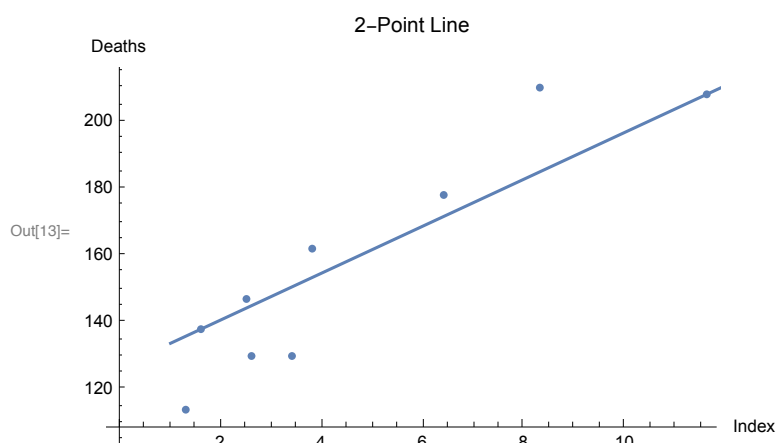
```
Out[10]= {11.6, 208}
```

```
In[11]:= twopoint = Fit[{point1, point2}, {1, x}, x]
```

```
Out[11]= 126.8 + 7. x
```

```
In[12]:= twopointplot = Plot[twopoint, {x, 1, 12}, PlotLabel → "2-Point Line"];
```

```
In[13]:= Show[plot, twopointplot, PlotLabel → "2-Point Line"]
```



## 2-Point Line Residuals

```

In[14]:= twopointline[x_] := 126.79999999999993 + 6.999999999999999 x

In[15]:= tpDeaths = twopointline[Indexes]

Out[15]= {144.3, 145., 150.6, 135.9, 138., 153.4, 208., 171.6, 184.9}

In[16]:= tpresiduals = Deaths - tpDeaths

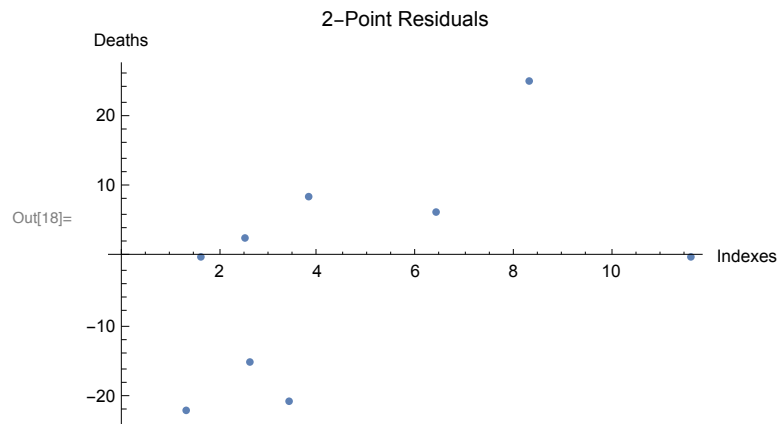
Out[16]= {2.7, -15., -20.6, -21.9, 8.52651 × 10-14, 8.6, 8.52651 × 10-14, 6.4, 25.1}

In[17]:= enumResiduals = Transpose[{Indexes, tpresiduals}]

Out[17]= {{2.5, 2.7}, {2.6, -15.}, {3.4, -20.6}, {1.3, -21.9}, {1.6, 8.52651 × 10-14},
          {3.8, 8.6}, {11.6, 8.52651 × 10-14}, {6.4, 6.4}, {8.3, 25.1}}

In[18]:= tpresidualsPlot = ListPlot[enumResiduals,
  AxesLabel → {"Indexes", "Deaths"}, PlotLabel → "2-Point Residuals"]

```



```

In[19]:=

In[20]:= sumResiduals = Total[residuals]

Out[20]= Total[residuals]

```

## Median-Median Line

```

In[21]:= SIndexes = Sort[Indexes] (*Sorted Indexes*)

Out[21]= {1.3, 1.6, 2.5, 2.6, 3.4, 3.8, 6.4, 8.3, 11.6}

In[22]:= iFirst = SIndexes[[1 ;; 3]]

Out[22]= {1.3, 1.6, 2.5}

In[23]:= iSecond = SIndexes[[4 ;; 6]]

Out[23]= {2.6, 3.4, 3.8}

```

```

In[24]:= iThird = SIndexes[[7 ;; 9]]
Out[24]= {6.4, 8.3, 11.6}

In[25]:= SDeaths = {114, 138, 147, 130, 130, 162, 178, 210, 208} (*Sorted Deaths*)
Out[25]= {114, 138, 147, 130, 130, 162, 178, 210, 208}

In[26]:= dFirst = SDeaths[[1 ;; 3]]
Out[26]= {114, 138, 147}

In[27]:= dSecond = SDeaths[[4 ;; 6]]
Out[27]= {130, 130, 162}

In[28]:= dThird = SDeaths[[7 ;; 9]]
Out[28]= {178, 210, 208}

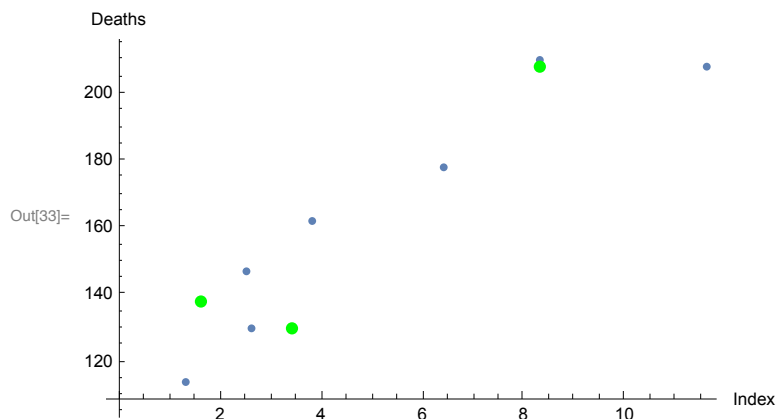
In[29]:= iMed = {Median[iFirst], Median[iSecond], Median[iThird]}
Out[29]= {1.6, 3.4, 8.3}

In[30]:= dMed = {Median[dFirst], Median[dSecond], Median[dThird]}
Out[30]= {138, 130, 208}

In[31]:= Med = Transpose[{iMed, dMed}]
Out[31]= {{1.6, 138}, {3.4, 130}, {8.3, 208}}

In[32]:= Medplot = ListPlot[Med, AxesLabel → {"Index", "Deaths"},
    PlotStyle → {RGBColor[0, 1, 0], PointSize[0.02]};
Show[plot, Medplot]

```



```

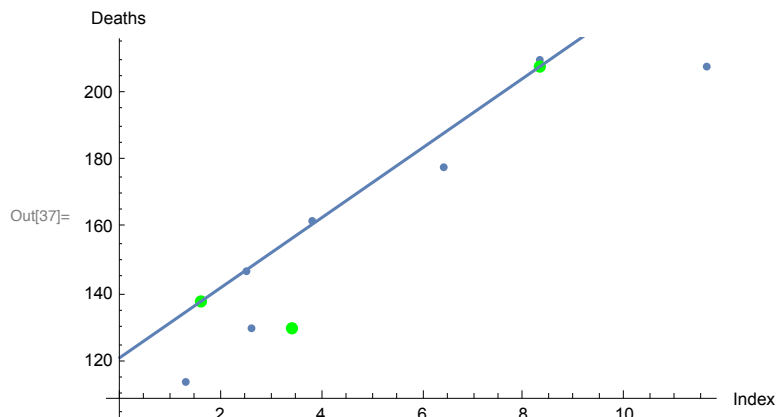
In[34]:= MedEndpoints = {Med[[1]], Med[[3]]}
Out[34]= {{1.6, 138}, {8.3, 208}}

In[35]:= MedFit = Fit[MedEndpoints, {1, x}, x]
Out[35]= 121.284 + 10.4478 x

```

```
In[36]:= line = Plot[MedFit, {x, 0, 12}];
```

```
In[37]:= Show[plot, Medplot, line]
```



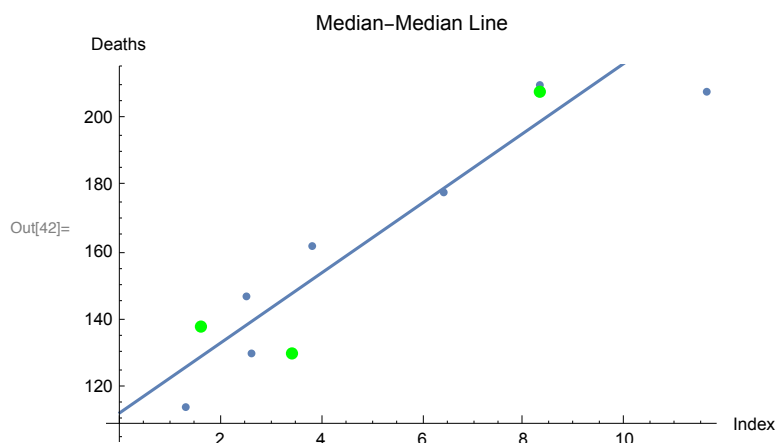
```
In[38]:= fit[x_] := 121.284 + 10.4478 x
```

```
In[39]:=
```

```
In[40]:= MM[x_] := fit[x] - (fit[3.4] - 130) / 3
```

```
In[41]:= MMLine = Plot[MM[x], {x, 0, 12}, PlotLabel -> "Median-Median Line"];
```

```
In[42]:= Show[plot, Medplot, MMLine, PlotLabel -> "Median-Median Line"]
```



## Median-Median Line Residuals

```
In[43]:= MM[x]
```

```
Out[43]= 112.348 + 10.4478 x
```

```
In[44]:= MM[x_] := 112.348 + 10.4478 x
```

```
In[45]:= MMDeaths = MM[Indexes]
```

```
Out[45]= {138.468, 139.512, 147.871, 125.93, 129.064, 152.05, 233.542, 179.214, 199.065}
```

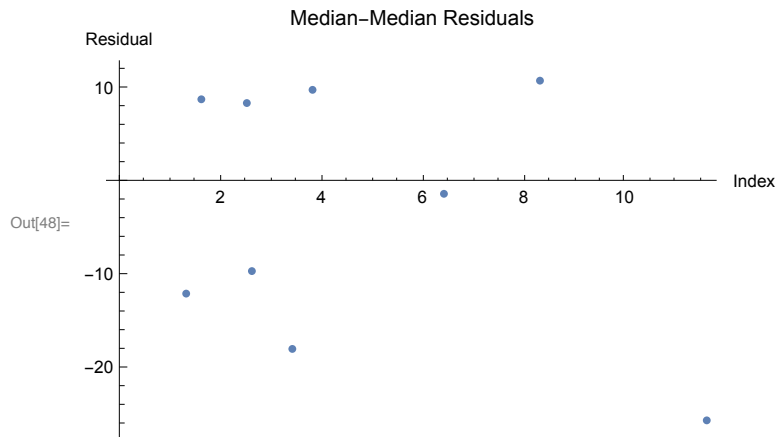
```

In[46]:= MMResiduals = Deaths - MMDeaths
Out[46]= {8.5325, -9.51228, -17.8705, -11.9301, 8.93552, 9.95036, -25.5425, -1.21392, 10.9353}

In[47]:= enumMMResiduals = Transpose[{Indexes, MMResiduals}]
Out[47]= {{2.5, 8.5325}, {2.6, -9.51228}, {3.4, -17.8705}, {1.3, -11.9301}, {1.6, 8.93552},
          {3.8, 9.95036}, {11.6, -25.5425}, {6.4, -1.21392}, {8.3, 10.9353}}

In[48]:= MMResidualsPlot = ListPlot[enumMMResiduals,
  AxesLabel → {"Index", "Residual"}, PlotLabel → "Median-Median Residuals"]

```



```

In[49]:= sumMMResiduals = Total[MMResiduals]
Out[49]= -27.7157

```

## Least-Squares Line

```

In[50]:= myA = Total[Deaths * Deaths]
Out[50]= 232741

In[51]:= myB = Total[Indexes * Indexes]
Out[51]= 287.67

In[52]:= myC = Total[Indexes]
Out[52]= 41.5

In[53]:= myD = Total[Deaths * Indexes]
Out[53]= 7427.1

In[54]:= myE = Total[Deaths]
Out[54]= 1417

In[55]:= myN = Length[Deaths]
Out[55]= 9

```

```
In[56]:= m = (myD * myN - myE * myC) / (myB * myN - myC * myC)
```

```
Out[56]= 9.27386
```

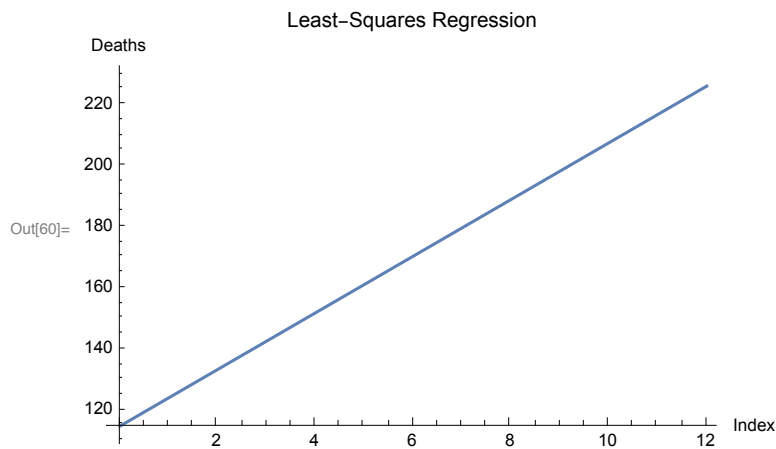
```
In[57]:= lowercaseB = (myE - m * myC) / myN
```

```
Out[57]= 114.682
```

```
In[58]:=
```

```
In[59]:= LeastSquare[x_] := m * x + lowercaseB
```

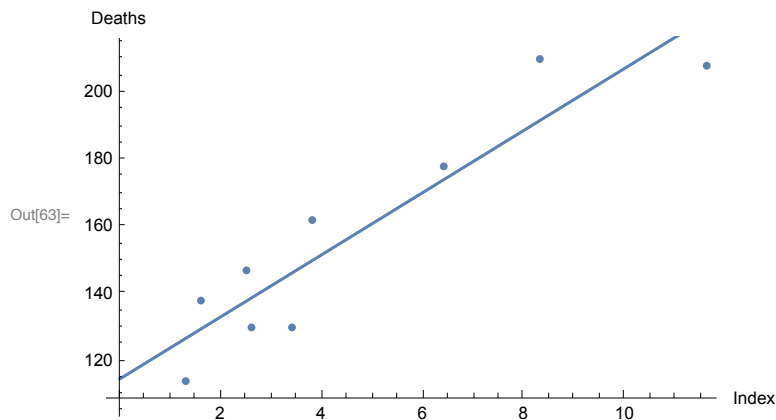
```
In[60]:= lsPlot = Plot[LeastSquare[x], {x, 0, 12},  
  AxesLabel -> {"Index", "Deaths"}, PlotLabel -> "Least-Squares Regression"]
```



```
In[61]:=
```

```
In[62]:=
```

```
In[63]:= Show[plot, lsPlot]
```



## Least-Squares Residuals

```
In[64]:= lqDeaths = LeastSquare[Indexes]
```

```
Out[64]= {137.866, 138.794, 146.213, 126.738, 129.52, 149.922, 222.258, 174.034, 191.655}
```

In[65]:= **Deaths**

Out[65]:= {147, 130, 130, 114, 138, 162, 208, 178, 210}

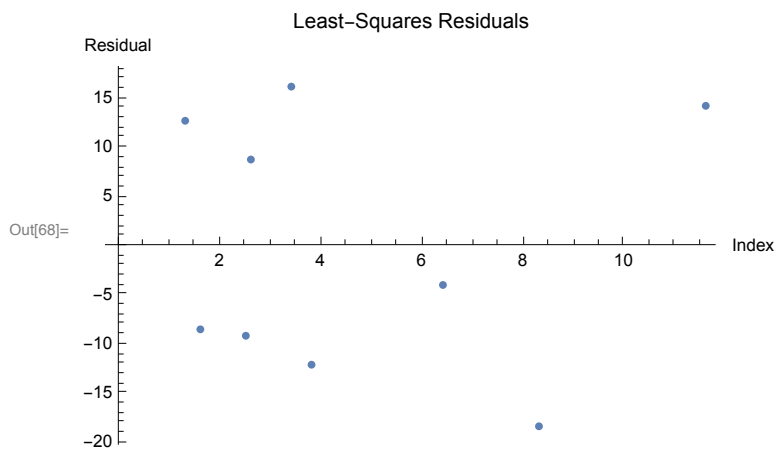
In[66]:= **lqResiduals = lqDeaths - Deaths**

Out[66]:= {-9.13371, 8.79367, 16.2128, 12.7376,  
-8.48019, -12.0777, 14.2585, -3.96564, -18.3453}

In[67]:= **EnumlqResiduals = Transpose[{Indexes, lqResiduals}]**

Out[67]:= {{2.5, -9.13371}, {2.6, 8.79367}, {3.4, 16.2128}, {1.3, 12.7376}, {1.6, -8.48019},  
{3.8, -12.0777}, {11.6, 14.2585}, {6.4, -3.96564}, {8.3, -18.3453}}

In[68]:= **lqResidualsPlot = ListPlot[EnumlqResiduals,  
AxesLabel → {"Index", "Residual"}, PlotLabel → "Least-Squares Residuals"]**



## Side By Side Comparison of Residuals for each line

In[69]:= **GraphicsRow[{tpresidualsPlot, lqResidualsPlot, MMResidualsPlot}, PlotLabel →  
{ "2 Point Residuals", "Median-Median Residuals", "Least-Squares Residuals"}]**

