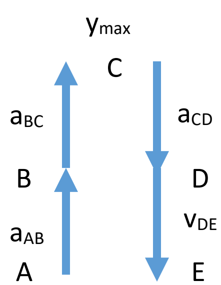
Uber Rocket Problem

Darshan Krishnaswamy September 30, 2019 Section L

Problem Statement:

One calm afternoon Calculus Cam decides to launch Hamster Huey into the air using a model rocket. The rocket is launched straight up off the ground, from rest. The rocket engine is designed to burn for a specified time while producing non-constant net acceleration given by the equations below. After the engine stops the rocket continues upward in free-fall. A parachute opens after the rocket falls a specified distance from its maximum height. When the parachute opens, assume the rocket instantly stops, and then increases speed to a terminal velocity given by the equation below. Assume the air resistance affects the rocket only during the parachute stage. Calculate the total time the rocket is in the air

Diagram



Givens

A = -0.9
B = 0
C = 15
t_E = 4.7 s
h = 99 m
v_t = -13 m/s
D = 7
g = -9.81 m/s^2

$$\boldsymbol{a_y[t] = At^2 + B + C\ m/s^2}$$

$$\boldsymbol{v_p[t] = v_T(1 - e^{\frac{t}{D}})\ m/s}$$

Strategy

A strategy for solving this problem is to mark five points along the path of the rocket. Point A represents the starting point of the rocket. Point B represents the point at which the rocket's engine stops and the rocket continues in free fall. Point C represents the point at which the rocket reaches its maximum height. Point D is when the rocket's parachute deploys. Point E represents when the rocket finally reaches the ground. Different stages can be used to represent the times in between the different points. Stage AB represents the time in between points A and B, stage BC represents the time between points B and C, and so on.

The velocity function of the rocket during stage AB can be found by integrating the acceleration equation given. This can be integrated once more to obtain the position equation for stage AB. By plugging in the end time of stage AB, the final position and velocity of the rocket during stage AB can be obtained.

During stage BC, the rocket is in free fall and is only affected by the force of gravity. Thus, the final position and velocity of the rocket at the end of stage AB can be used to find the maximum height reached by the rocket and the time at which it occurs through the use of the official kinematic equation number four. Kinematic equation number one can then be used to find the amount of time that the rocket will take to reach this maximum height.

During stage CD, the rocket remains in free fall and continues to be affected only by gravity. The official kinematic equation number three can be used to find out how long the rocket will take to fall 99 meters, which is when the parachute first deploys.

During stage DE, the parachute is open and the rocket approaches a terminal velocity. The equation for the rocket's velocity can be integrated to obtain the position equation, which can be used to find out the amount of time that the rocket will fall for until it reaches the ground.

Stage AB:

$$v_{AB}[t] = \int a[t]\,dt$$

$$v_{AB}[t] = \int (At^2 + Bt + C)\,dt$$

$$v_{AB}[t] = \int (-0.9t^2 + 15)\,dt$$

$$v_{AB}[t] = -0.3t^3 + 15t + C$$

$$\text{set } v_{AB}[0] = 0\text{ m/s}$$

$$0 = C$$

$$\underline{v_{AB}[t] = -0.3t^3 + 15t}$$

$$y[t] = \int v[t]\,dt$$

$$y[t] = \int (-0.3t^3 + 15t)\,dt$$

$$y[t] = -0.075t^4 + 7.5t^2 + C$$

$$\text{set } y_{AB}[0] = 0$$

$$0 = C$$

$$\underline{y_{AB}[t] = -0.075t^4 + 7.5t^2}$$

$$set\ t_B = 4.7\text{ s}$$

$$v_B = -0.3 * 4.7^3 + 15 * 4.7$$

$$\underline{v_B = 39.353\text{ m/s}}$$

$$y = -0.075 * 4.7^4 + 7.5 * 4.7^2$$

$$\underline{y_B = 129.077\text{ m}}$$

Stage BC:

$${v_c}^2 = {v_b}^2 + 2a\Delta y$$

$$v_f = 0\text{ m/s}$$

$$v_B = 39.353\text{ m/s}$$

$$a = -9.81\text{ m/s}^2$$

$$0^2 = 39.353^2 + 2*(-9.81)*\Delta y$$

$$\underline{\Delta y = 79.013\ \mathrm{m}}$$

$$\Delta y = y_C - y_B$$

$$\Delta y = 79.013\ \mathrm{m}$$

$$y_B = 129.077\ \mathrm{m}$$

$$79.013\ \mathrm{m} = y_C - 129.077\ \mathrm{m}$$

$$\underline{y_c = 210.585\ \mathrm{m}}$$

$$\Delta y = \frac{1}{2}(v_B + v_c) * t_C$$

$$\Delta y = 79.013\ \mathrm{m}$$

$$v_B = 39.353\ \mathrm{m/s}$$

$$v_c = 0\ \mathrm{m/s}$$

$$79.013 = \frac{1}{2}(39.353 + 0) * t_C$$

$$\underline{t_{BC} = 4.016\ \mathrm{s}}$$

Stage CD:

$$\Delta y = \frac{1}{2}at^2 + v_c t$$

$$a = -9.81\ \mathrm{m/s^2}$$

$$v_c = 0\ \mathrm{m/s}$$

$$\text{set } \Delta y = -99\ \mathrm{m}$$

$$-99 = -4.9{t_D}^2$$

$$\underline{t_{CD} = 4.495\ \mathrm{s}}$$

$$\Delta y = y_D - y_C$$

$$-99\ \mathrm{m} = y_D - 210.585\ \mathrm{m}$$

$$\underline{y_D = 111.585\ \mathrm{m}}$$

Stage DE:

$$v_{DE}[t] = v_T\left(1 - e^{-\frac{t}{D}}\right)$$

$$v_{DE}[t] = -13(1 - e^{-\frac{t}{7}})$$

$$y[t] = \int v_{DE}[t]\,dt$$

$$y_{DE}[t] = -13\int (1 - e^{-\frac{t}{7}})\,dt$$

$$y[t] = -13(t + 7e^{-\frac{t}{7}}) + C$$

$$\text{set } y_{DE}[0] = 111.585\ \mathrm{m}$$

$$111.585 = -13(t + 7e^{-\frac{0}{7}}) + C$$

$$C = 202.585$$

$$y[t] = -13\left(t + 7e^{-\frac{t}{7}}\right) + 200.078$$

$$\text{set } y[t] = 0$$

$$0 = -13\left(t + 7e^{-\frac{t}{7}}\right) + 200.078$$

$$0 = -13t + 91e^{-\frac{t}{7}} + 200.078$$

Solver

$$\underline{t_{DE} = 14.51\ \mathrm{S}}$$

Overall:

$$t_{Tptal} = t_{AB} + t_{BC} + t_{CD} + t_{DE}$$

$$t_{AB} = 4.7\ \mathrm{s}$$

$$t_{BC} = 4.016\ \mathrm{s}$$

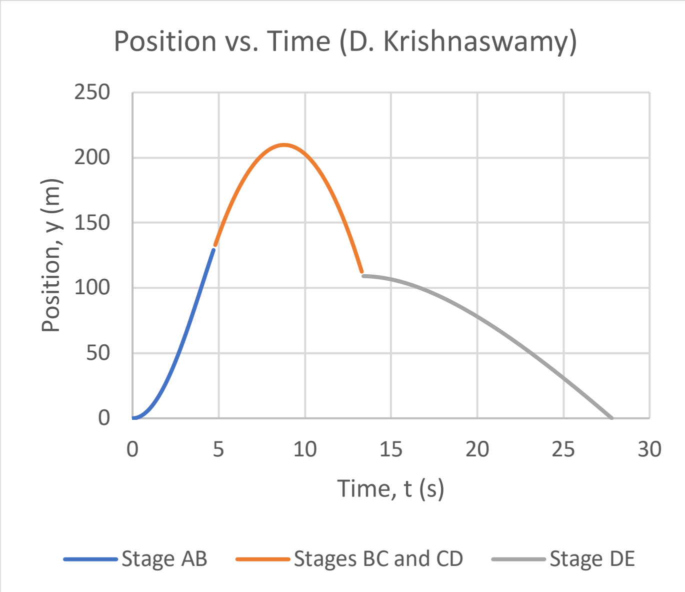
$$t_{CD} = 4.495\ \mathrm{s}$$

$$t_{DE} = 14.51\ s$$

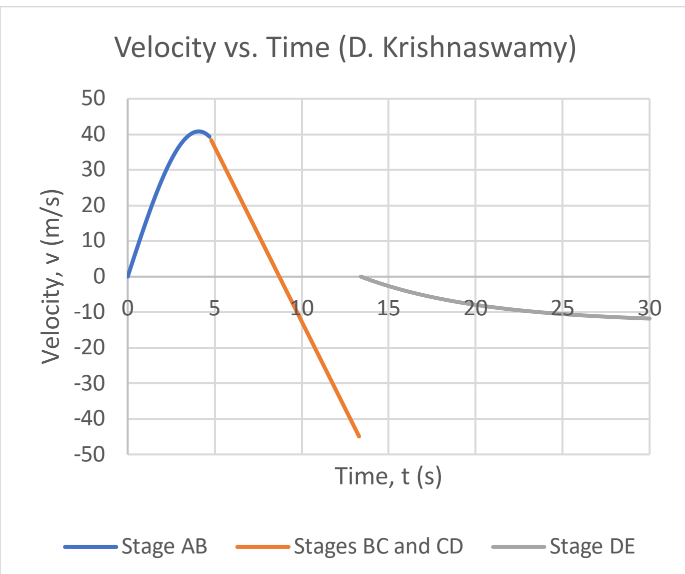
$$t_{Tptal} = 4.7\mathrm{s} + 4.016\mathrm{s} + 4.495\mathrm{s} + 14.51\mathrm{s}$$

$$\boxed{t_{Total} = 2\dot{7}.7\ \text{seconds}}$$

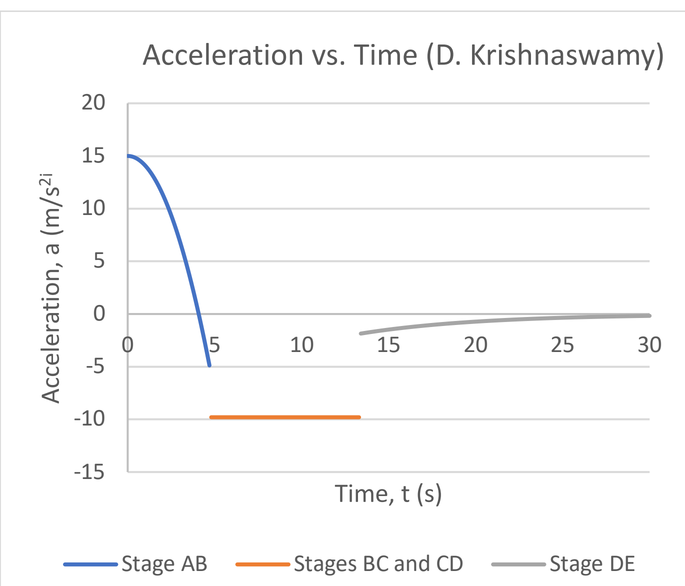
Graphs



Position vs. Time (D. Krishnaswamy)
Position, y (m)
250
200
150
100
50
0
0
5
10
15
20
25
30
Time, t (s)
Stage AB
Stages BC and CD
Stage DE



Velocity vs. Time (D. Krishnaswamy)
Velocity, v (m/s)
50
40
30
20
10
0
-10
-20
-30
-40
-50
0
5
10
15
20
25
30
Time, t (s)
Stage AB
Stages BC and CD
Stage DE



Acceleration vs. Time (D. Krishnaswamy)
Acceleration, a (m/s2i
20
15
10
5
0
-5
-10
-15
0
5
10
15
20
25
30
Time, t (s)
Stage AB
Stages BC and CD
Stage DE