ME 312 Project Proposal

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1 Introduction

Our project is about truss optimization

2 Problem Formulation

A truss consists of N nodes and M member('links'), a link is connected between 2 nodes, it can only transmit forces along the direction of the link. We consider upward and rightward forces to be positive and negative otherwise, all angles to be taken from the horizontal, and compression forces to be positive and tension forces negative. We assume that fixed joints can exert arbitrarily large forces on the truss.

Given a truss and the various forces acting on it we want to find the optimal arrangement that can minimize the axial strain in the links

3 Methodology

We represent our truss as a set of linear equations which are in turn represented as a matrix multiplication of a column vector of link forces and a matrix representing the orientation of the links. For the sake of simplicity consider the case of a 2D truss as shown in Figure 1. The points represented by green are fixed joints while those with blue are revolute joints

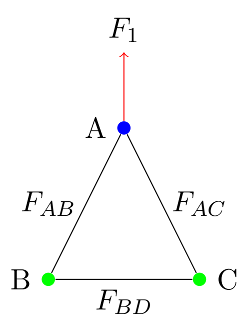

Figure 1: A simple 2D planar truss

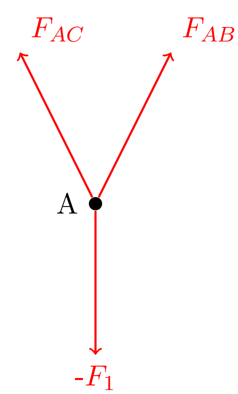

Figure 2: FBD of point A

From the FBD of point A in Figure 2 we can derive the equations of equilibrium from newton's 2'nd law as

$$F_{AB}\cos(\theta_{BA}) + F_{AC}\cos(\theta_{CA}) = -F_1\cos(\theta_1)$$

$$F_{AB}\sin(\theta_{BA}) + F_{AC}\sin(\theta_{CA}) = -F_1\sin(\theta_1)$$

The set of such equations for the entire truss can be represented in matrix form as

$$\begin{bmatrix} \cos(\theta_{BA}) & \cos(\theta_{CA}) & 0 \\ \sin(\theta_{BA}) & \sin(\theta_{CA}) & 0 \\ \cos(\theta_{AB}) & 0 & \cos(\theta_{CB}) \\ \sin(\theta_{AB}) & 0 & \sin(\theta_{CB}) \\ 0 & \cos(\theta_{AC}) & \cos(\theta_{BC}) \\ 0 & \sin(\theta_{AC}) & \sin(\theta_{BC}) \end{bmatrix} \begin{bmatrix} F_{AB} \\ F_{AC} \\ F_{BD} \end{bmatrix} = \begin{bmatrix} -F_1\cos(\theta_1) \\ -F_1\sin(\theta_1) \\ -F_B\cos(\theta_B) \\ -F_B\sin(\theta_B) \\ -F_C\cos(\theta_C) \\ -F_C\sin(\theta_C) \end{bmatrix}$$

$$AF = L$$

Here A is a $2N \times M$ matrix dependent only on the directions of the links, F is column vector of size M representing the forces in the links and L is load vector of size $2N$ reprsenting the loads and joints at the nodes.

If there is a joint at any of the nodes we simply remove that row without any change to the system. If the system if underactuated ($2N < M$) we simply remove such trusses from our analysis for now but we do have plans for solving them in the future see 4. If the system is overconstrained ($2N \geq M$ but $rank(A) < M$) we possess multiple solutions for the vector F. Again for now we simply fix a vector norm(mostly L_2 norm) and find a solution that minimizes it. Let us denote the solution of these equations as $F^{(0)}$

Now we do something very untrivial, for each of the links we find the axial strain in that link, and define the emperical loss function as the sum of all the strains in the link

$$\hat{\mathcal{L}}_\theta = \sum_{i=1}^{M} \frac{l_i |F_i(L, A)|}{\sigma_s} = \frac{1}{\sigma_s} l^T (A^T L (A^T A)^{-1})$$

We then compute the partial derivative of our loss function w.t.r to θ_{ij} and update the θ's as

$$\theta_{ij}^{t+1} = \theta_{ij}^{t} - \alpha \frac{\partial \hat{\mathcal{L}}_\theta}{\partial \theta_{ij}}|_{\theta=\theta^t}$$

We continue calculating the force from the updated θ^t as

$$F^{t+1} = A^T L (A^T A)^{-1}|_{\theta=\theta^t}$$

We keep on alternating between calculating F^t and updating θ^t until convergence

Note: for overconstrained systems the loss function needs to be modified to take the L_2 norm of F into consideration, but we ignore such subtleties at this stage of the project

4 Possible Future Directions

- Introduce stiffness in links for solving underactuated systems
- Create a formulation for 3D trusses
- Introduce torque in the formulation
- Introduce sliding joints, joints that can only restrict one degree of freedom
- Create a formulation of A in terms of the coordinates of nodes instead of the angle between links