

Syntax

$i \in \text{Int}$
 $b \in \text{Bool}$
 $x, y \in \text{Var}$
 $op \in \text{Prim} ::= + \mid - \mid * \mid <$
 $v \in \text{Value} ::= i \mid b \mid (\mathcal{E})[\text{fun } x \rightarrow e] \mid (\mathcal{E})[\text{rec } x = \text{fun } y \rightarrow e] \mid [] \mid v :: v$
 $\mathcal{E} \in \text{Env} ::= \bullet \mid \mathcal{E}, x = v$
 $e \in \text{Exp} ::= i \mid b \mid x \mid e \text{ op } e \mid \text{if } e \text{ then } e \text{ else } e \mid \text{let } x = e \text{ in } e$
 $\quad \mid \text{fun } x \rightarrow e \mid e \text{ } e \mid \text{let rec } x = \text{fun } y \rightarrow e \text{ in } e$
 $\quad \mid [] \mid e :: e \mid \text{match } e \text{ with } [] \rightarrow e \mid x :: y \rightarrow e$

Derivation Rules

$$\begin{array}{c}
\frac{}{\mathcal{E} \vdash i \Downarrow i} \text{(E-INT)} \quad \frac{}{\mathcal{E} \vdash b \Downarrow b} \text{(E-BOOL)} \quad \frac{(\mathcal{E}(x) = v)}{\mathcal{E} \vdash x \Downarrow v} \text{(E-VAR)} \\
\\
\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \quad \mathcal{E} \vdash e_2 \Downarrow i_2 \quad i_1 \text{ plus } i_2 \text{ is } i_3}{\mathcal{E} \vdash e_1 + e_2 \Downarrow i_3} \text{(E-PLUS)} \quad \frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \quad \mathcal{E} \vdash e_2 \Downarrow i_2 \quad i_1 \text{ minus } i_2 \text{ is } i_3}{\mathcal{E} \vdash e_1 - e_2 \Downarrow i_3} \text{(E-MINUS)} \\
\\
\frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \quad \mathcal{E} \vdash e_2 \Downarrow i_2 \quad i_1 \text{ times } i_2 \text{ is } i_3}{\mathcal{E} \vdash e_1 * e_2 \Downarrow i_3} \text{(E-TIMES)} \quad \frac{\mathcal{E} \vdash e_1 \Downarrow i_1 \quad \mathcal{E} \vdash e_2 \Downarrow i_2 \quad i_1 \text{ less than } i_2 \text{ is } b_3}{\mathcal{E} \vdash e_1 < e_2 \Downarrow b_3} \text{(E-LT)} \\
\\
\frac{\mathcal{E} \vdash e_1 \Downarrow \text{true} \quad \mathcal{E} \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow v} \text{(E-IFT)} \quad \frac{\mathcal{E} \vdash e_1 \Downarrow \text{false} \quad \mathcal{E} \vdash e_3 \Downarrow v}{\mathcal{E} \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow v} \text{(E-IFF)} \\
\\
\frac{\mathcal{E} \vdash e_1 \Downarrow v_1 \quad \mathcal{E}, x = v_1 \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \text{let } x = e_1 \text{ in } e_2 \Downarrow v} \text{(E-LET)} \\
\\
\frac{}{\mathcal{E} \vdash \text{fun } x \rightarrow e \Downarrow (\mathcal{E})[\text{fun } x \rightarrow e]} \text{(E-FUN)} \\
\\
\frac{\mathcal{E} \vdash e_1 \Downarrow (\mathcal{E}_2)[\text{fun } x \rightarrow e_0] \quad \mathcal{E} \vdash e_2 \Downarrow v_2 \quad \mathcal{E}_2, x = v_2 \vdash e_0 \Downarrow v}{\mathcal{E} \vdash e_1 \ e_2 \Downarrow v} \text{(E-APP)} \\
\\
\frac{\mathcal{E}, x = (\mathcal{E})[\text{rec } x = \text{fun } y \rightarrow e_1] \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \text{let rec } x = \text{fun } y \rightarrow e_1 \text{ in } e_2 \Downarrow v} \text{(E-LETREC)} \\
\\
\frac{\mathcal{E} \vdash e_1 \Downarrow (\mathcal{E}_2)[\text{rec } x = \text{fun } y \rightarrow e_0] \quad \mathcal{E} \vdash e_2 \Downarrow v_2 \quad \mathcal{E}_2, x = (\mathcal{E}_2)[\text{rec } x = \text{fun } y \rightarrow e_0], y = v_2 \vdash e_0 \Downarrow v}{\mathcal{E} \vdash e_1 \ e_2 \Downarrow v} \text{(E-APPREC)} \\
\\
\frac{}{\mathcal{E} \vdash [] \Downarrow []} \text{(E-NIL)} \quad \frac{\mathcal{E} \vdash e_1 \Downarrow v_1 \quad \mathcal{E} \vdash e_2 \Downarrow v_2}{\mathcal{E} \vdash e_1 :: e_2 \Downarrow v_1 :: v_2} \text{(E-CONS)} \\
\\
\frac{\mathcal{E} \vdash e_1 \Downarrow [] \quad \mathcal{E} \vdash e_2 \Downarrow v}{\mathcal{E} \vdash \text{match } e_1 \text{ with } [] \rightarrow e_2 \mid x :: y \rightarrow e_3 \Downarrow v} \text{(E-MATCHNIL)} \quad \frac{\mathcal{E} \vdash e_1 \Downarrow v_1 :: v_2 \quad \mathcal{E}, x = v_1, y = v_2 \vdash e_3 \Downarrow v}{\mathcal{E} \vdash \text{match } e_1 \text{ with } [] \rightarrow e_2 \mid x :: y \rightarrow e_3 \Downarrow v} \text{(E-MATCHCONS)} \\
\\
\frac{(i_3 = i_1 + i_2)}{i_1 \text{ plus } i_2 \text{ is } i_3} \text{(B-PLUS)} \quad \frac{(i_3 = i_1 - i_2)}{i_1 \text{ minus } i_2 \text{ is } i_3} \text{(B-MINUS)} \quad \frac{(i_3 = i_1 * i_2)}{i_1 \text{ times } i_2 \text{ is } i_3} \text{(B-TIMES)} \quad \frac{(b_3 = (i_1 < i_2))}{i_1 \text{ less than } i_2 \text{ is } b_3} \text{(B-LT)}
\end{array}$$