

Syntax

$i \in \text{Int}$
 $b \in \text{Bool}$
 $x, y \in \text{Var}$
 $op \in \text{Prim} ::= + \mid - \mid * \mid <$
 $\tau \in \text{Types} ::= \text{bool} \mid \text{int} \mid \tau \rightarrow \tau \mid \tau \text{ list}$
 $\Gamma \in \text{Env} ::= \bullet \mid \Gamma, x : \tau$
 $e \in \text{Exp} ::= i \mid b \mid x \mid e \text{ op } e \mid \text{if } e \text{ then } e \text{ else } e \mid \text{let } x = e \text{ in } e$
 $\quad \mid \text{fun } x \rightarrow e \mid e \ e \mid \text{let rec } x = \text{fun } y \rightarrow e \text{ in } e$
 $\quad \mid [] \mid e :: e \mid \text{match } e \text{ with } [] \rightarrow e \mid x :: y \rightarrow e$

Derivation Rules

$$\begin{array}{c}
\frac{}{\Gamma \vdash i : \text{int}} \text{(T-INT)} \quad \frac{}{\Gamma \vdash b : \text{bool}} \text{(T-BOOL)} \quad \frac{(\Gamma(x) = \tau)}{\Gamma \vdash x : \tau} \text{(T-VAR)} \\
\\
\frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \tau \quad \Gamma \vdash e_3 : \tau}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : \tau} \text{(T-IF)} \\
\\
\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}} \text{(T-PLUS)} \quad \frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 - e_2 : \text{int}} \text{(T-MINUS)} \\
\\
\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 * e_2 : \text{int}} \text{(T-TIMES)} \quad \frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 < e_2 : \text{bool}} \text{(T-LT)} \\
\\
\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \text{fun } x \rightarrow e : \tau_1 \rightarrow \tau_2} \text{(T-FUN)} \quad \frac{\Gamma \vdash e_1 : \tau_1 \rightarrow \tau_2 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1 e_2 : \tau_2} \text{(T-APP)} \\
\\
\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : \tau_2} \text{(T-LET)} \quad \frac{\Gamma, x : \tau_1 \rightarrow \tau_2, y : \tau_1 \vdash e_1 : \tau_2 \quad \Gamma, x : \tau_1 \rightarrow \tau_2 \vdash e_2 : \tau}{\Gamma \vdash \text{let rec } x = \text{fun } y \rightarrow e_1 \text{ in } e_2 : \tau} \text{(T-LETREC)} \\
\\
\frac{}{\Gamma \vdash [] : \tau \text{ list}} \text{(T-NIL)} \quad \frac{\Gamma \vdash e_1 : \tau \quad \Gamma \vdash e_2 : \tau \text{ list}}{\Gamma \vdash e_1 :: e_2 : \tau \text{ list}} \text{(T-CONS)} \\
\\
\frac{\Gamma \vdash e_1 : \tau' \text{ list} \quad \Gamma \vdash e_2 : \tau \quad \Gamma, x : \tau', y : \tau' \text{ list} \vdash e_3 : \tau}{\Gamma \vdash \text{match } e_1 \text{ with } [] \rightarrow e_2 \mid x :: y \rightarrow e_3 : \tau} \text{(T-MATCH)}
\end{array}$$