## Syntax

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\begin{array}{rcl}
\boldsymbol{i} \in \textbf{Int} & & \\
\boldsymbol{b} \in \textbf{Bool} & & \\
\boldsymbol{x}, \boldsymbol{y} \in \textbf{Var} & & \\
\boldsymbol{op} \in \textbf{Prim} & ::= & + \mid - \mid * \mid < \\
\boldsymbol{\tau} \in \textbf{Types} & ::= & \texttt{bool} \mid \texttt{int} \mid \boldsymbol{\tau} \rightarrow \boldsymbol{\tau} \mid \boldsymbol{\tau}\ \texttt{list} \\
\boldsymbol{\Gamma} \in \textbf{Env} & ::= & \bullet \mid \boldsymbol{\Gamma}, \boldsymbol{x} : \boldsymbol{\tau} \\
\boldsymbol{e} \in \textbf{Exp} & ::= & \boldsymbol{i} \mid \boldsymbol{b} \mid \boldsymbol{x} \mid \boldsymbol{e}\ \boldsymbol{op}\ \boldsymbol{e} \mid \texttt{if}\ \boldsymbol{e}\ \texttt{then}\ \boldsymbol{e}\ \texttt{else}\ \boldsymbol{e} \mid \texttt{let}\ \boldsymbol{x} = \boldsymbol{e}\ \texttt{in}\ \boldsymbol{e} \\
& & \mid \texttt{fun}\ \boldsymbol{x} \rightarrow \boldsymbol{e} \mid \boldsymbol{e}\ \boldsymbol{e} \mid \texttt{let rec}\ \boldsymbol{x} = \texttt{fun}\ \boldsymbol{y} \rightarrow \boldsymbol{e}\ \texttt{in}\ \boldsymbol{e} \\
& & \mid [] \mid \boldsymbol{e} :: \boldsymbol{e} \mid \texttt{match}\ \boldsymbol{e}\ \texttt{with}\ [] \rightarrow \boldsymbol{e} \mid \boldsymbol{x} :: \boldsymbol{y} \rightarrow \boldsymbol{e}
\end{array}
$$

## Derivation Rules

$$\frac{}{\Gamma \vdash i : \texttt{int}}\ (\text{T-Int}) \qquad \frac{}{\Gamma \vdash b : \texttt{bool}}\ (\text{T-Bool}) \qquad \frac{(\Gamma(x) = \tau)}{\Gamma \vdash x : \tau}\ (\text{T-Var})$$

$$\frac{\Gamma \vdash e_1 : \texttt{bool} \qquad \Gamma \vdash e_2 : \tau \qquad \Gamma \vdash e_3 : \tau}{\Gamma \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 : \tau}\ (\text{T-If})$$

$$\frac{\Gamma \vdash e_1 : \texttt{int} \qquad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 + e_2 : \texttt{int}}\ (\text{T-Plus}) \qquad \frac{\Gamma \vdash e_1 : \texttt{int} \qquad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 - e_2 : \texttt{int}}\ (\text{T-Minus})$$

$$\frac{\Gamma \vdash e_1 : \texttt{int} \qquad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 * e_2 : \texttt{int}}\ (\text{T-Times}) \qquad \frac{\Gamma \vdash e_1 : \texttt{int} \qquad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 < e_2 : \texttt{bool}}\ (\text{T-Lt})$$

$$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \texttt{fun } x \to e : \tau_1 \to \tau_2}\ (\text{T-Fun}) \qquad \frac{\Gamma \vdash e_1 : \tau_1 \to \tau_2 \qquad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1\ e_2 : \tau_2}\ (\text{T-App})$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \texttt{let } x = e_1 \texttt{ in } e_2 : \tau_2}\ (\text{T-Let}) \qquad \frac{\Gamma, x : \tau_1 \to \tau_2, y : \tau_1 \vdash e_1 : \tau_2 \qquad \Gamma, x : \tau_1 \to \tau_2 \vdash e_2 : \tau}{\Gamma \vdash \texttt{let rec } x = \texttt{fun } y \to e_1 \texttt{ in } e_2 : \tau}\ (\text{T-LetRec})$$

$$\frac{}{\Gamma \vdash [] : \tau\ \texttt{list}}\ (\text{T-Nil}) \qquad \frac{\Gamma \vdash e_1 : \tau \qquad \Gamma \vdash e_2 : \tau\ \texttt{list}}{\Gamma \vdash e_1 :: e_2 : \tau\ \texttt{list}}\ (\text{T-Cons})$$

$$\frac{\Gamma \vdash e_1 : \tau'\ \texttt{list} \qquad \Gamma \vdash e_2 : \tau \qquad \Gamma, x : \tau', y : \tau'\ \texttt{list} \vdash e_3 : \tau}{\Gamma \vdash \texttt{match } e_1 \texttt{ with } [] \to e_2 \mid x :: y \to e_3 : \tau}\ (\text{T-Match})$$