B_{OUT} CALCULATION → (General procedure)

Calculation of B_{out} for spheroid →

Considering the surface of the spheroid as the super-conductor. The magnetic field perpendicular to the surface must vanish at the surface.

$$\vec{B}_{out}(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta,\varphi)} = 0 \quad \text{— (1) c}$$

$$\vec{B}_{out}(r,\theta,\varphi) = B_0(r,\theta,\varphi) - \nabla\Phi_{out}(r,\theta,\varphi) \quad \text{— (2) c}$$

where $B_0(r,\theta,\varphi)$ → applied magnetic field in spherical co-ordinates.

Φ_{out} → induced potential outside the spheroid.

$\hat{n}(\theta,\varphi)$ → unit normal vector to surface of spheroid.

$$\Phi = \sum_{n=0}^{N_{max}} (r^{-(n+1)}) \sum_{m=-n}^{n} a_{n,m} Y_n^m(\theta,\varphi) + \sum_{n=0}^{N_{max}} r^n \sum_{m=-n}^{n} b_{n,m} Y_n^m(\theta,\varphi) \quad \text{— (3) c}$$

Since, the potential associated with induced magnetic field must vanish at boundaries. Eqn. 3c can be written as

$$\Phi = \sum_{n=0}^{N_{max}} r^{-(n+1)} \sum_{m=-n}^{n} a_{n,m} Y_n^m(\theta,\varphi) \quad \text{— (4) c}$$

PAGE NO.:
DATE: / /

Our main aim is to calculate $\vec{B}_{out}(r,\theta,\varphi)$. It can be evaluated by applying the boundary condition given by the eqn. 1c

$$\nabla\Phi = -\sum r^{-(n+2)}(n+1)\sum a_{n,m} Y_n^m(\theta,\varphi)\,\hat{r}$$

$$+\sum \frac{r^{-(n+1)}}{r}\sum a_{n,m}\frac{\partial Y_n^m(\theta,\varphi)}{\partial\theta}\,\hat{\theta} \qquad -⑤c$$

$$+\sum \frac{r^{-(n+1)}}{r\sin\theta}\sum a_{n,m}\frac{\partial Y_n^m(\theta,\varphi)}{\partial\varphi}\,\hat{\varphi}$$

$$\vec{B}_{out}(r,\theta,\varphi) = (B_0 - \nabla\Phi)(r,\theta,\varphi)$$

Applying the boundary condition to obtain the coefficients $a_{n,m}$ present in $\nabla\Phi$ term.

$$\vec{B}_{out}(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta,\varphi)=r(\theta)\ \text{(for spheroid)}} = 0$$

$$B_0(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta)} = \nabla\Phi(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta)}$$

⑥c

Now, I name this equation 6c as my primary equation. This equation would be used by me throughout the notes.

→ Say, left hand side part of eq. 6c as LHS and Right hand side part of eq. 6c as RHS

This convention is used throughout the notes and mathematica files.

Teacher's Signature..........

Any arbitrary function $f(\theta, \phi)$ which is defined in the interval $\theta \in (0, \pi)$ & $\varphi \in (0, 2\pi)$ and satisfies the condition

$$\int_0^{\pi}\int_0^{2\pi} f(\theta,\varphi) \sin\theta \, d\theta \, d\varphi < \infty \quad - \text{⑦c}$$

can be expressed as expansion of spherical harmonics.

Therefore, we try to express both LHS & RHS in terms of spherical harmonics expansion.

As, we know spherical harmonics forms the orthonormal basis for the angular function, every angular function can be expressed in its terms provided the above condition of eqn. 7c satisfies.

Then, we can compare its coefficients to obtain $a_{n,m}$'s.