Section A (a)

SPHEROID →

The surface of ellipsoid centred at the origin of a cartesian co-ordinate system is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad \text{— (1)a}$$

Consider, $a = b$, the shape of surface describes a spheroid given by equation

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{c^2} = 1 \quad \text{— (2)a}$$

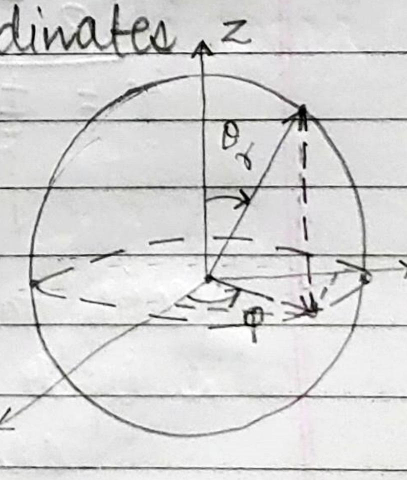
Since, we are interested to study about the non uniformity of a sphere.

we put $a = R+\epsilon$, $c = R-\epsilon$, where $\epsilon \ll R$.

$$\Rightarrow \frac{x^2 + y^2}{(R+\epsilon)^2} + \frac{z^2}{(R-\epsilon)^2} = 1 \quad \text{— (3)a}$$

Converting eqn 3a in spherical co-ordinates,

$$\left.\begin{aligned} x &= r \sin\theta \cos\varphi \\ y &= r \sin\theta \sin\varphi \\ z &= r \cos\theta \end{aligned}\right\} \quad \text{— (4)a}$$



Putting eq 4a in eq 3a,

$$\frac{r^2 \sin^2\theta}{(R+\epsilon)^2} + \frac{r^2 \cos^2\theta}{(R-\epsilon)^2} = 1 \quad \text{— (5)a}$$

$$\Rightarrow \frac{r^2 \sin^2\theta (R-\epsilon)^2 + r^2 \cos^2\theta (R+\epsilon)^2}{(R-\epsilon)^2 (R+\epsilon)^2} = 1 \quad \text{— (6)a}$$

$$r^2 = \frac{(R+\epsilon)^2 (R-\epsilon)^2}{\sin^2\theta (R-\epsilon)^2 + \cos^2\theta (R+\epsilon)^2} \quad \text{— (7)a}$$

Taking square root both the sides.

$$r = \frac{\pm (R^2-\epsilon^2)}{\sqrt{\sin^2\theta (R^2+\epsilon^2-2\epsilon R) + \cos^2\theta (R^2+\epsilon^2+2\epsilon R)}} \quad \text{— (8)a}$$

$$r = \frac{\pm (R^2-\epsilon^2)}{\sqrt{R^2+\epsilon^2-2\epsilon R(\sin^2\theta - \cos^2\theta)}} \quad \text{— (9)a}$$

$$r = \frac{\pm (R^2-\epsilon^2)}{\sqrt{R^2+\epsilon^2+2\epsilon R\cos(2\theta)}} \quad \text{— (10)a}$$

$$r = \frac{\pm R^2(1-\epsilon^2/R^2)}{\sqrt{R^2(1+\epsilon^2/R^2+2\epsilon/R\cos(2\theta))}} \quad \text{— (11)a}$$

say $\boxed{\frac{\epsilon}{R} = s}$, and $\boxed{s \ll 1 \text{ such that } s^2 \to 0}$

$$r = \frac{\pm R(1-s^2)}{\sqrt{(1+s^2+2s\cos(2\theta))}} \quad \text{— (12)a}$$

Now, taylor expanding the RHS about s upto first order of s can be given as,

$$r = f(s) = \frac{\pm R(1-s^2)}{\sqrt{1+s^2+2s\cos(2\theta)}} \quad \text{— (13)a}$$

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Taylor expansion of a function of s about 0 is written as,

$$f(s) = f(0) + f'(0)s + f''(0)\frac{s^2}{2} \quad \text{— (14a)}$$

$$f(0) = R$$

$$f'(s)\Big|_{s=0} = \frac{-2Rs\left(\sqrt{1+s^2+2s\cos(2\theta)}\right) - \frac{R(2s+2\cos(2\theta))}{2\sqrt{1+s^2+2s\cos(2\theta)}}}{(1+s^2+2s\cos(2\theta))}$$

$$f'(s)\Big|_{s=0} = -R\cos(2\theta)$$

$$f(s) = R - Rs\cos(2\theta)$$

$$\boxed{r(\theta) = f(s) = R - Rs\cos(2\theta)} \quad \text{— (15a)}$$

↳ parametric equation of spheroid.