Force Calculation – z direction displ.

This section deals with calculation of force acting on spheroid when kept inside the magnetic field in different configurations.

This section deals with configuration of applied magnetic field where it is displaced only in z-direction.

→ force acting on a spheroid kept inside a magnetic field can be evaluated using maxwell stress Tensor.

Normal Vector Calculation → (Spherical co-ordinates)

This calculation is shown in detail in Section B(b) or 'Normal Vector to surface of Spheroid.nb'.

normal

$$\vec{n} = \left((R - R_s\cos(2\theta))^2 \sin\theta,\ -4R_s\cos\theta\sin^2\theta\,(R - R_s\cos(2\theta)),\ 0 \right) \quad \text{— ① e}$$

↳ referenced from eqn. 12b.

- now, we convert this normal vector $\vec{n}$ from spherical co-ordinates to cartesian co-ordinates. This is achieved by multiplying it with transformation matrix.

SphToCar

$$M_{s\to c} = \begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \quad \text{— ② e}$$

$\vec{n}_c \rightarrow$ normal in cartesian coordinates

$$\vec{n}_c = M_{S\to C} \cdot \vec{n}$$

$$\boxed{n_x} \quad \vec{n}_c^{\,x} = \left(R - 2R_S - 3R_S\cos(2\theta)\right)\left(R - R_S\cos(2\theta)\right)\cos\varphi\sin^2\theta$$

$$\boxed{n_y} \quad n_c^{y} = \left(R - 2R_S - 3R_S\cos(2\theta)\right)\left(R - R_S\cos(2\theta)\right)\sin^2\theta\sin\varphi \qquad \text{— ③e}$$

$$\boxed{n_z} \quad n_c^{z} = \cos\theta\left(R + 2R_S - 3R_S\cos(2\theta)\right)\left(R - R_S\cos(2\theta)\right)\sin\theta$$

Area element of Spheroid

$$dA_i = \hat{n}_i \, dS \qquad \text{— ④e}$$

where $\hat{n}_i$ = unit normal vector in i^{th} direction

$$dS = \|n\| \, d\theta \, d\varphi \rightarrow \text{differential area element for any surface.} \qquad \text{— ⑤e}$$

$$dA_i = \frac{n_i}{\|n\|} \|n\| \, d\theta \, d\varphi = n_i \, d\theta \, d\varphi \qquad \text{— ⑥e}$$

$$\left.\begin{aligned} \boxed{A_x} \quad dA_x &= n_x \, d\theta \, d\varphi \\ \boxed{A_y} \quad dA_y &= n_y \, d\theta \, d\varphi \\ \boxed{A_z} \quad dA_z &= n_z \, d\theta \, d\varphi \end{aligned}\right\} \text{— ⑦e}$$

Hence, initially we used a normal vector rather than a unit normal vector.

Magnetic field at the surface of Spheroid →

$B_{out}(r, \theta, \phi)\big|_{r=r(\theta)}$ calculated in eq 42d is used directly over here.

$\boxed{\text{Bout Spheroid}}$

$$B_{out,surf}(\theta, \phi) = B_{out}(r, \theta, \phi)\big|_{r=r(\theta)} \quad \text{— 8e}$$

Converting the magnetic field at the surface of spheroid in cartesian coordinates. →

Using the matrix defined in eq. 2e, it can be converted easily.

$$B_{out,cart,surf}(\theta, \phi) = M_{s\to c} \cdot B_{out,surf}(\theta, \phi) \quad \text{— 9e}$$

$\boxed{\text{Bout Cart}}$

Now, we define

$\boxed{B_{out}^2}$

$$B_{out,cart,surf,sq}(\theta, \phi) = \left(B_{out,cart,surf}(\theta, \phi)\right)^2 \quad \text{— 10e}$$

$\boxed{B_x}$ $\boxed{B_y}$ $\boxed{B_z}$

$$\left.\begin{aligned} B_x &= B^x_{out,cart,surf}(\theta, \phi) \\ B_y &= B^y_{out,cart,surf}(\theta, \phi) \\ B_z &= B^z_{out,cart,surf}(\theta, \phi) \end{aligned}\right\} \text{— 11e}$$

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$$f = \int \nabla \cdot \overleftrightarrow{T}\, dV \qquad \text{— (12) e}$$

$$= \oint \overleftrightarrow{T} \cdot \hat{n}\, ds = \oint \overleftrightarrow{T} \cdot \vec{n}\, d\theta\, d\varphi \qquad \text{— (13) e}$$

$$T_{ij} = \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij} B^2\right) + \frac{1}{\mu_0}\left(E_i E_j - \frac{1}{2}\delta_{ij} E^2\right) \qquad \text{— (14) e}$$

Since, there is no electric field

$$T_{ij} = \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij} B^2\right) \qquad \text{— (15) e}$$

$$T_{xx} = \frac{1}{\mu_0}\left(B_x^2 - \frac{1}{2}B^2\right) = \frac{1}{2\mu_0}\left(B_x^2 - B_y^2 - B_z^2\right) \qquad \text{— (16) e}$$

$$T_{xy} = \frac{1}{\mu_0}\left(B_x B_y\right) \qquad \text{— (17) e}$$

$$T_{xz} = \frac{1}{\mu_0}\left(B_x B_z\right) \qquad \text{— (18) e}$$

$$f_x = \oint T_{xx}\, dA_x + \oint T_{xy}\, dA_y + \oint T_{xz}\, dA_z \qquad \text{— (19) e}$$

$$= \underbrace{\oint T_{xx}\, n_x\, d\theta\, d\varphi}_{(f_{x_1}) - 19e\text{-}1} + \underbrace{\oint T_{xy}\, n_y\, d\theta\, d\varphi}_{(f_{x_2})\ 19e\text{-}2} + \underbrace{\oint T_{xz}\, n_z\, d\theta\, d\varphi}_{(f_{x_3})\ 19e\text{-}3}$$

Similarly f_y direction can be written as

$$f_y = \int T_{yy}\, dA_y + \int T_{yx}\, dA_x + \int T_{yz}\, dA_z \qquad \text{— (20) e}$$

$$= \underbrace{\oint T_{yy}\, n_y\, d\theta\, d\varphi}_{(f_{y_1})} + \underbrace{\oint T_{yx}\, n_x\, d\theta\, d\varphi}_{(f_{y_2})} + \underbrace{\oint T_{yz}\, n_z\, d\theta\, d\varphi}_{(f_{y_3})}$$

$$f_z = \underbrace{\oint T_{zz}\, n_z\, d\theta\, d\varphi}_{(f_{z_1})} + \underbrace{\oint T_{zy}\, d\theta\, d\varphi\, n_y}_{(f_{z_2})} + \underbrace{\oint T_{zx}\, d\theta\, d\varphi\, n_x}_{(f_{z_3})} \qquad \text{(21) e}$$

$$F = (f_x, f_y, f_z) \qquad \text{— (22) e}$$

Teacher's Signature