Section F(f) [Mathematica file → General Translation .nb]

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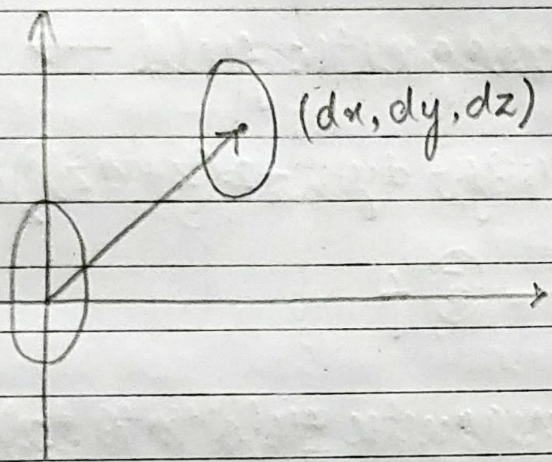
Bout CALCULATION (General Translation)



Demonstration of configuration of spheroid.

This configuration can be obtained via two mechanism
1. Translation of spheroid.
2. Translation of applied magnetic field.

We use 2nd mechanism throughout to acheive the above specified configurations.

Using the same name of the parameters as was used in section D(d).

rSpheroid

$$\left.\begin{aligned} r(\theta) &= R - \epsilon \cos(2\theta) \\ &= R - Rs\cos(2\theta) \end{aligned}\right\} \longrightarrow (1)f$$

Position vector to surface of spheroid :-

$$r(\theta,\phi) = r(\theta)\hat{r} = \left(R - Rs\cos(2\theta)\right)\hat{r} \quad \text{—} \quad (2)f$$

Normal vector to surface of spheroid :-

$$\hat{n} = \frac{R - Rs\cos(2\theta)\ \hat{r} - 4R\cos\theta\sin(\theta)s\ \hat{\theta}}{\sqrt{(R - Rs\cos(2\theta))^2 + 4R^2s^2\sin^2(2\theta)}} \quad - (3)f$$

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Note → Detailed explaination of eqn 1f is given in Section
explanation of eqn. 2f, 3f given in section B(b)

Defining the applied magnetic field →

$$B_0(x,y,z) = \frac{1}{2} b_z \left(x+dx,\ y+dy,\ -2(z+dz)\right) \quad — (4)f$$

$$\nabla \cdot B_0(x,y,z) = 0 \quad — (5)f$$

converting the applied magnetic field into spherical co-ordinates system. →

$$M_{s\to c} \cdot B_0(x,y,z) = \boxed{B_0 \text{ spherical}}\ B_0(r,\theta,\varphi) \quad — (6)f$$

$$B_0^r(r,\theta,\varphi)$$

$$= \frac{1}{2} b_z \left(-2dz\cos(\theta) - 2r\cos^2(\theta) + \sin\theta\,(dx\cos\varphi + r\sin\theta + dy\sin\right.$$

$$B_0^\theta(r,\theta,\varphi) = b_z dz\sin\theta + \frac{1}{2} b_z \cos\theta\,(dx\cos\varphi + 3r\sin\theta + dy\sin\varphi).$$

$$B_0^\varphi(r,\theta,\varphi) = \frac{1}{2} b_z (dy\cos\varphi - dx\sin\varphi) \quad — (7)f$$

Applying the boundary condition →

→ This part involves applying the boundary condition given in the equation eqn: 6C. Let us first focus on solving the LHS part of eqn 6C.

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LHS $\rightarrow$

$$B_0(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta)} \longrightarrow LHS$$

$$\boxed{LHSBCVsph[\theta,\varphi]} = \underbrace{B_0(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta)}}_{f_{LHS}(s)} \quad \text{—— (8) f}$$

We now, simplify the eqn 8 f by taylor expanding w.r.t s about s=0 upto first order. This is given as,

$$\boxed{AppLHSBCvsph[\theta,\varphi]} = f_{LHS}(0) + f'_{LHS}(0)\,s + O(s^2) \quad \text{—— (9) f}$$

This eq. 9 f is simplified and as we know $\left(B_0(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\big|_{r=r(\theta)}\right)$ is a function of θ,φ and can be expressed in terms of spherical harmonic expansion.

We first verify whether eq 9 f can be expressed in spherical harmonics or not.

$$\int_0^{2\pi}\int_0^{\pi} AppLHSBCvsph(\theta,\varphi)\sin(\theta)\,d\theta\,d\varphi < \infty \quad \text{—— (10) f}$$

Since the integral is finite, it can be expressed in terms of spherical harmonics expansion.

$\boxed{b_{l,k}}$

$$b_{l,k} = \int_0^{2\pi}\int_0^{\pi} Y_l^{*k}(\theta,\varphi)\sin\theta\left(AppLHSBCVsph(\theta,\varphi)\right)d\theta\,d\varphi \quad \text{—— (11) f}$$

$$= \int_0^{2\pi}\int_0^{\pi} Y_l^{*k}(\theta,\varphi)\sin\theta\left(\underbrace{B_0(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta)}}_{\text{Simplified version of this.}}\right)d\theta\,d\varphi$$

$$\left[Y_n^{-m}(\theta,\phi) = (-1)^m Y_n^{m*}(\theta,\phi) \right]$$

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$b_{l,k}$ are obtained from eqn 11f. All the non zero $b_{l,k}$'s are stated in the eqn 12 f given below.

$$(12)f \begin{cases} b_{0,0} = -\frac{16}{15} b_z \sqrt{\pi} Rs \\ b_{1,0} = -\frac{2}{5} b_z dz \sqrt{\frac{\pi}{3}} (5+8s), \quad b_{1,1} = \frac{1}{5}(dx-idy)\sqrt{\frac{\pi}{6}}(4s-5) \\ b_{2,0} = \frac{-2}{21} b_z \sqrt{\frac{\pi}{5}} R(21+11s) \\ b_{3,0} = \frac{16}{5} b_z dz \sqrt{\frac{\pi}{7}} s, \quad b_{3,1} = \frac{8}{5} b_z (dx-idy) \sqrt{\frac{\pi}{21}} s \\ b_{4,0} = \frac{48}{35} b_z \sqrt{\pi} Rs \end{cases}$$

Note that, $b_{l,-k} = (-1)^k b_{l,k}^*$ —— (13)f

therefore eqn. 13f is derived as below

$$b_{l,k} = \iint \sin\theta \, f(\theta,\phi) \, Y_l^{*k}(\theta,\phi) \, d\theta \, d\phi$$

$$b_{l,-k} = \iint \sin\theta \, f(\theta,\phi) \, Y_l^{*-k}(\theta,\phi) \, d\theta \, d\phi$$

as we know, $Y_l^{*-k}(\theta,\phi) = (-1)^k Y_l^{k}(\theta,\phi)$ —— (14)f

$$\boxed{b_{l,-k} = (-1)^k b_{l,k}^*} \quad \text{—— (15) f}$$

We can even verify the all nonzero coefficients are determined or not by eqn. 16 f given below

$$\sum_{l=0}^{L} \sum_{k=-l}^{l} b_{l,k} Y_l^k(\theta,\phi) - \left(B_0(r,\theta,\phi) \cdot \hat{n}(\theta,\phi) \big|_{r=r(\theta)} \right) = 0$$

(16) f

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Now, we are done evaluating the LHS part. Let us move on to calculate the RHS part and express it as sum of spherical harmonics terms.

$$\Phi_{out}(r,\theta,\varphi) = \sum_{n}^{N_{max}} r^{-(n+1)} \sum_{m=-n}^{n} g_{n,m} Y_n^m(\theta,\varphi) \quad \text{— (19) f}$$

$\boxed{\text{PhiOut}(r,\theta,\varphi,N_{max})}$

Taking the gradient of the eqn 19 f we obtain the eqn 5c.
Note → The RHS part of the eqn. always remains same as we are only shifting B_0 and keeping other things const. RHS consists of $\nabla\Phi \cdot \hat{n}$ which alway remains same.

However, for completion of calculation, I am stating all the calculations explicitly.

$$\boxed{\text{gradph}(r,\theta,\varphi,N_{max})} = \nabla\Phi_{out}(r,\theta,\varphi) \quad \text{— (20) f / (5) c}$$

RHS part of eqn 6c can be written as,

$$\nabla\Phi_{out}(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta)} = \text{——— (21) f / (19) d + (20) d}$$

Eqn. 19d & 20d are illustrated in three steps in mathematica file

$\boxed{\text{rhsBCvsph}}$
↓
$\boxed{\text{Simplified RHS BC}}$
↓
$\boxed{\text{moresimplified RHS BC}}$

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Since, we know the coefficients of LHS, we can predict which components of RHS will be zero.

from eqn. 12f we can say that only non zero coefficients are $m = 0$ or $m = \pm 1$ for other m's we do not have any non zero coefficient. Hence RHS should also contain only $m = 0$ & $m = \pm 1$ terms.

We expand RHS (more simplified RHSBC) upto $N_{max} = 10$, although we know that $N_{max} = 8$ is enough to encounter the expression & coefficients to be correct.

Then we taylor expand this above evaluated expression w.r.t s about $s = 0$ up to first order.

$$\boxed{\text{AppRHSBC}[\theta, \varphi]} = \underbrace{\left(\nabla \Phi_{out}(r, \theta, \varphi) \cdot \hat{n}(\theta, \varphi)\Big|_{r = r(\theta)}\right)}_{\text{its taylor series expanded form}} \quad \text{— (22) f}$$

Also note that while expanding the eqn. 22f we removed all the terms other than $m = 0$ and $m = \pm 1$ since we already knew that their contribution is going to be zero to equate with LHS terms.

We evaluate the coefficients of RHS which are written as

$\boxed{qq_{l,k}}$

$$qq_{l,k} = \int_0^{2\pi}\int_0^{\pi} \sin\theta \, Y_l^{k}(\theta, \varphi) \, (\text{AppRHSBC}) \, d\theta \, d\varphi \quad \text{— (23) f}$$

$$\text{RHS} = \sum qq_{l,k} \, Y_l^{k}(\theta, \varphi) \quad \text{—— (24) f}$$

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The coefficients are given in Mathematica file.

$\boxed{\text{tablegd}}$ = table of the coefficients of RHS. — (25) f

Now we compare the corresponding coefficients of LHS & RHS.

$$\sum_{l,k} b_{l,k} Y_l^k(\theta,\phi) = \sum_{l',k'} gg_{l',k'} Y_{l'}^{k'}(\theta,\phi)$$

$$\left[b_{l,k} = gg_{l',k'} \right] \quad (\text{for } l = l' \;\&\; k = k')$$

$\boxed{\text{bcoffLHS}}$ $\boxed{\text{gcoeffRHS}}$ — (26) f → It is divided in several parts.

first of all we compare the coefficients with $k=0$.
As, we have already encountered a problem in Section for solving this.

$$\left.\begin{aligned}
b_{0,0} &= h_0(g_{00}, g_{20}) \\
b_{1,0} &= h_1(g_{1,0}, g_{3,0}) \\
b_{2,0} &= h_2(g_{0,0}, g_{2,0}, g_{4,0}) \\
b_{3,0} &= h_3(g_{1,0}, g_{3,0}, g_{5,0}) \\
b_{4,0} &= h_4(g_{2,0}, g_{4,0}, g_{6,0}) \\
b_{5,0} &= h_5(g_{3,0}, g_{5,0}, g_{7,0}) \\
b_{6,0} &= h_6(g_{4,0}, g_{6,0}, g_{8,0})
\end{aligned}\right\} \text{— (27) f (Divided in parts)}$$

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we can see that there are 7 eqn's and 9 variables again. To rectify this problem. we ignore $g_{7,0}$ & $g_{8,0}$ because those terms contained S/R^9 & S/R^{10} terms as their coefficients. We consider these terms to be $S/R^9 \to 0$, $S/R^{10} \to 0$ hence, now we have

7 eq^{ns} and 7 variables which

gives the solution for each g's.

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Similarly, $K = 1$,

$$\left.\begin{aligned}
b_{1,1} &= P_1(g_{1,1}, g_{3,1}) \\
b_{2,1} &= P_2(g_{2,1}, g_{4,1}) \\
b_{3,1} &= P_3(g_{1,1}, g_{3,1}, g_{5,1}) \\
b_{4,1} &= P_4(g_{2,1}, g_{4,1}, g_{6,1}) \\
b_{5,1} &= P_5(g_{3,1}, g_{5,1}, g_{7,1}) \\
b_{6,1} &= P_6(g_{4,1}, g_{6,1}, g_{8,1})
\end{aligned}\right\} \quad - (28)f$$

(Divided in parts)

again we ignore $g_{7,1}$ & $g_{8,1}$ because of their coefficients containing s/R^9 & s/R^{10} term which tends to zero.

$\boxed{\text{glist}}$ → list of non simplified g's — (29) f

$\boxed{\text{g0}}$ → list of g's with $k=0$ and non zero value. — (30) f

$\boxed{\text{g1}}$ → list of non zero g's with $k=1$. — (31) f

$\boxed{\text{gg}}$ → list of all non zero g's after simplification — (32) f

The non zero simplified coefficients of Φ_{out} term are →

$$\left.\begin{aligned}
g_{1,-1} &= -\frac{1}{10} b_z (dx + i dy) \sqrt{\frac{\pi}{6}} R^3 (5+3s) \\
g_{1,0} &= \frac{1}{5} b_z dz \sqrt{\frac{\pi}{3}} R^3 (5+9s) \\
g_{1,1} &= \frac{1}{10} b_z (dx - i dy) \sqrt{\frac{\pi}{6}} R^3 (5+3s) \\
g_{2,0} &= \frac{2}{21} b_z \sqrt{\frac{\pi}{5}} R^5 (7+5s)
\end{aligned}\right\}$$

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$$g_{3,-1} = \frac{2}{5} b_z (dx + i dy) \sqrt{\frac{3\pi}{7}} R^5 s$$

$$g_{3,0} = -\frac{12}{5} b_z dz \sqrt{\frac{\pi}{7}} R^5 s$$

$$g_{3,1} = -\frac{2}{5} b_z (dx - i dy) \sqrt{\frac{3\pi}{7}} R^5 s$$

$$g_{4,0} = -\frac{16}{21} b_z \sqrt{\pi} R^7 s$$

⟶ (33) f

Calculating B_{out} from the evaluated coefficients →

$$\Phi_{out}(r,\theta,\varphi) = \sum_{n=0}^{N} r^{-(n+1)} \sum_{m=-n}^{n} g_{n,m} Y_n^m(\theta,\varphi) \quad \text{— (34) f}$$

$$\nabla \Phi_{out}(r,\theta,\varphi) \quad \text{— (35) f}$$

$$B_{out} = B_0(r,\theta,\varphi) - \nabla\Phi_{out}(r,\theta,\varphi) \quad \text{— (36) f}$$

final simplified equation of $B_{out}(\theta,\varphi)$ at $r = r(\theta)$ is given in eq. 38 f in the mathematica file.

This equation is further used for calculating force acting of spheroid using maxwell stress tensor.