Section-C(c)

draw a diagram with spheroid at center and applied field on inf boundry.

B_{out} CALCULATION → (General procedure)

Calculation of B_{out} for spheroid →

make sure the you correctlly state the problem,
"a superocunting, spheroid with zero penetration depth at the cneter of cordiante system
while applyinhg a quarople field

Considering the surface of the spheroid as the super-conductor. The magnetic field perpendicular to the surface must vanish at the surface.

$$\vec{B}_{out}(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta,\varphi)} = 0 \quad \text{— (1)c}$$

Given that there are no external currents around the sphere, and by Ampère's Law, we can introduce a scalar potential, Φ,

$$\vec{B}_{out}(r,\theta,\varphi) = B_0(r,\theta,\varphi) - \nabla\Phi_{out}(r,\theta,\varphi) \quad \text{— (2)c}$$

where $B_0(r,\theta,\varphi)$ → applied magnetic field in spherical co-ordinates.

Φ_{out} → induced potential outside the spheroid.
$\hat{n}(\theta,\varphi)$ → unit normal vector to surface of spheroid.

Further, applying Gauss's law for magnetism, it follows that:
ΔΦout = 0.

why phi has this form?

$$\Phi = \sum_{n=0}^{N_{max}} \left(r^{-(n+1)}\right)\sum_{m=-n}^{n} a_{n,m} Y_n^m(\theta,\varphi) + \sum_{n=0}^{N_{max}} r^n \sum_{m=-n}^{n} b_{n,m} Y_n^m(\theta,\varphi) \quad \text{— (3)c}$$

the general solution of scalar potential for the Laplacian equation in spherical coordinates is then given by summation of spherical harmonics

Since, the potential associated with induced magnetic field must vanish at boundaries. Eqn. 3c can be written as

at infinity instead of boundry

$$\Phi = \sum_{n=0}^{N_{max}} r^{-(n+1)} \sum_{m=-n}^{n} a_{n,m} Y_n^m(\theta,\varphi) \quad \text{— (4)c}$$

Our main aim is to calculate $\vec{B}_{out}(r,\theta,\varphi)$.
It can be evaluated by applying the boundary condition given by the eqn. 1c

$$\nabla\Phi = -\sum r^{-(n+2)}(n+1)\sum a_{n,m}Y_n^m(\theta,\varphi)\,\hat{r}$$

$$+\sum \frac{r^{-(n+1)}}{r}\sum a_{n,m}\frac{\partial Y_n^m(\theta,\varphi)}{\partial\theta}\,\hat{\theta} \qquad -(5c)$$

$$+\sum \frac{r^{-(n+1)}}{r\sin\theta}\sum a_{n,m}\frac{\partial Y_n^m(\theta,\varphi)}{\partial\varphi}\,\hat{\varphi}$$

$$\vec{B}_{out}(r,\theta,\varphi) = (B_0-\nabla\varphi)(r,\theta,\varphi)$$

Applying the boundary condition to obtain the coefficients $a_{n,m}$ present in $\nabla\Phi$ term.

$$\vec{B}_{out}(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta,\varphi)=r(\theta)\ \text{(for spheroid)}} = 0$$

$$B_0(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta)} = \nabla\Phi(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta)} \qquad (6c)$$

put 6c in red box

Now, I name this equation 6c as my primary equation. This equation would be used by me throughout the notes.

→ Say, left hand side part of eq. 6c as LHS and Right hand side part of eq. 6c as RHS

This convention is used throughout the notes and mathematica files.

Teacher's Signature..............

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Any arbitrary function $f(\theta, \varphi)$ which is defined in the interval $\theta \in (0, \pi)$ & $\varphi \in (0, 2\pi)$ and satisfies the condition

$$\int_0^{\pi}\int_0^{2\pi} f(\theta,\varphi) \sin\theta \, d\theta \, d\varphi < \infty \quad - \text{(7)c}$$

can be expressed as expansion of spherical harmonics.

Therefore, we try to express both LHS & RHS in terms of spherical harmonics expansion.

As, we know spherical harmonics forms the orthonormal basis for the angular function, every angular function can be expressed in its terms provided the above condition of eqn. 7C satisfies!

Then, we can compare its coefficients to obtain $a_{n,m}$'s.

mention why the condition 7-c is always true for your case !