

# NORMAL VECTOR CALCULATION

## TO SURFACE OF SPHEROID →

→ The position vector to surface of spheroid can be written as,

$$\vec{r}(\theta, \varphi) = r(\theta, \varphi) \hat{r} \quad \text{--- (1b)}$$

where  $r(\theta, \varphi)$  is parametrization of the surface.

for spheroid,  $r(\theta, \varphi) = r(\theta) = R - R_s \cos(2\theta)$   
which is given in eq 15a.

→ normal vector to any arbitrary surface with position vector  $\vec{r}(\theta, \varphi)$  is given by

$$\vec{n} = \frac{\partial \vec{r}(\theta, \varphi)}{\partial \theta} \times \frac{\partial \vec{r}(\theta, \varphi)}{\partial \varphi} \quad \text{--- (2b)}$$

$$\|\vec{n}\| = \left| \frac{\partial \vec{r}(\theta, \varphi)}{\partial \theta} \times \frac{\partial \vec{r}(\theta, \varphi)}{\partial \varphi} \right| \quad \text{--- (2b)}$$

as we know,  $\hat{n} = \frac{\vec{n}}{\|\vec{n}\|}$

mention page also

[Ref :- Vector Calculus, 6<sup>th</sup> Edition  
by J. J. Stoker]



Calculating normal vector,

$$\frac{\partial \vec{r}(\theta, \phi)}{\partial \theta} = \frac{\partial r(\theta, \phi)}{\partial \theta} \hat{r} + r(\theta, \phi) \frac{\partial \hat{r}}{\partial \theta} \quad \text{--- (4) b}$$

$$\frac{\partial \vec{r}(\theta, \phi)}{\partial \phi} = \frac{\partial r(\theta, \phi)}{\partial \phi} \hat{r} + r(\theta, \phi) \frac{\partial \hat{r}}{\partial \phi} \quad \text{--- (5) b}$$

as we know,

$$\frac{\partial \hat{r}}{\partial \theta} = \hat{\theta}, \quad \frac{\partial \hat{r}}{\partial \phi} = \sin \theta \hat{\phi} \quad \text{--- (6) b}$$

$$\frac{\partial \vec{r}(\theta, \phi)}{\partial \theta} = \frac{\partial r(\theta, \phi)}{\partial \theta} \hat{r} + r(\theta, \phi) \hat{\theta} \quad \text{--- (7) b}$$

$$\frac{\partial \vec{r}(\theta, \phi)}{\partial \phi} = \frac{\partial r(\theta, \phi)}{\partial \phi} \hat{r} + r(\theta, \phi) \sin \theta \hat{\phi} \quad \text{--- (8) b}$$

$$\vec{n} = \frac{\partial \vec{r}(\theta, \phi)}{\partial \theta} \times \frac{\partial \vec{r}(\theta, \phi)}{\partial \phi}$$

$$\left[ \vec{n} = -r(\theta, \phi) \sin \theta \frac{\partial r(\theta, \phi)}{\partial \theta} \hat{\theta} + r(\theta, \phi) \frac{\partial r(\theta, \phi)}{\partial \phi} \hat{\phi} + (r(\theta, \phi))^2 \sin \theta \hat{r} \right] \quad \text{--- (9) b}$$

for Spheroid,  $r(\theta, \phi) = r(\theta) = R - R_s \cos(2\theta)$

$$\left[ \begin{aligned} \frac{\partial r(\theta, \phi)}{\partial \theta} &= \frac{\partial r(\theta)}{\partial \theta} = +R_s \sin(2\theta)(2) \\ \frac{\partial r(\theta, \phi)}{\partial \phi} &= \frac{\partial r(\theta)}{\partial \phi} = 0 \end{aligned} \right] \quad \text{--- (10) b}$$



Substituting the variables in eq 10 b to eq 9 b,  
normal vector can be given as,

$$\vec{n}_{\text{spheroid}} = (r(\theta))^2 \sin \theta \hat{r} - r(\theta)^2 \sin \theta \sin(2\theta) R s \hat{\theta} + 0 \hat{\phi} \quad \text{--- (11) b}$$

$$\vec{n}_{\text{spheroid}} = \frac{(R - R s \cos(2\theta))^2 \sin \theta \hat{r} - 4 R s \sin^2 \theta \cos \theta (R - R s \cos(2\theta)) \hat{\theta}}{(R - R s \cos(2\theta))} \quad \text{--- (12) b}$$

$$\hat{n} = \frac{\vec{n}}{\|\vec{n}\|} = \frac{(R - R s \cos(2\theta))^2 \sin \theta \hat{r} - 4 R s \cos \theta \sin^2 \theta (R - R s \cos(2\theta)) \hat{\theta}}{\sqrt{(R - R s \cos(2\theta))^4 \sin^2 \theta + 16 R^2 s^2 \cos^2 \theta \sin^4 \theta} (R - R s \cos(2\theta))} \quad \text{--- (13) b}$$

$$\hat{n} = \frac{(R - R s \cos(2\theta))^2 \sin \theta \hat{r} - 4 R s \cos \theta \sin^2 \theta (R - R s \cos(2\theta)) \hat{\theta}}{\sqrt{(R - R s \cos(2\theta))^4 \sin^2 \theta + 16 R^2 s^2 \cos^2 \theta \sin^4 \theta} (R - R s \cos(2\theta))} \quad \text{--- (14) b}$$

$$\hat{n} = \frac{(R - R s \cos(2\theta))^2 \sin \theta \hat{r} - 4 R s \cos \theta \sin^2 \theta (R - R s \cos(2\theta)) \hat{\theta}}{(R - R s \cos(2\theta)) \sin \theta \sqrt{(R - R s \cos(2\theta))^2 + 16 R^2 s^2 \cos^2 \theta \sin^2 \theta}} \quad \text{--- (15) b}$$



$$\hat{n} = \frac{(R - Rs \cos(2\theta)) \hat{r} - 4Rs \cos\theta \sin\theta \hat{\theta}}{\sqrt{(R - Rs \cos(2\theta))^2 + 4R^2 s^2 \sin^2(2\theta)}} \quad \text{--- (16) b}$$

$\hat{n}$  represents the unit normal vector.

put the final  $n^\wedge$  in a red box