Section B (b) [Mathematica file → "Normal vector to surface of spheroid."]

PAGE NO.
DATE: / /

Normal Vector Calculation To Surface Of Spheroid →

→ The position vector to surface of spheroid can be written as,

$$\vec{v}(\theta, \varphi) = r(\theta, \varphi)\,\hat{r} \quad \text{—— ① b}$$

where $r(\theta, \varphi)$ is parametrization of the surface.

for spheroid, $r(\theta, \varphi) = r(\theta) = R - R s \cos(2\theta)$
which is given in eq 15a.

→ normal vector to any arbitrary surface with position vector $\vec{v}(\theta, \varphi)$ is given by

$$\vec{n} = \frac{\partial \vec{v}(\theta, \varphi)}{\partial \theta} \times \frac{\partial \vec{v}(\theta, \varphi)}{\partial \varphi} \quad \text{— ② b}$$

$$\|\vec{n}\| = \left| \frac{\partial \vec{v}(\theta, \varphi)}{\partial \theta} \times \frac{\partial \vec{v}(\theta, \varphi)}{\partial \varphi} \right| \quad \text{—— ③ b}$$

as we know, $\hat{n} = \dfrac{\vec{n}}{\|\vec{n}\|}$

[Ref :- Vector Calculus, 6th Edition by Jarrold E. Marsden
Pg - 379 (Parametrized surfaces)]

Calculating normal vector,

$$\frac{\partial \vec{v}(\theta,\varphi)}{\partial \theta} = \frac{\partial r(\theta,\varphi)}{\partial \theta}\hat{r} + r(\theta,\varphi)\frac{\partial \hat{r}}{\partial \theta} \quad \text{— (4) b}$$

$$\frac{\partial \vec{v}(\theta,\varphi)}{\partial \varphi} = \frac{\partial r(\theta,\varphi)}{\partial \varphi}\hat{r} + r(\theta,\varphi)\frac{\partial \hat{r}}{\partial \varphi} \quad \text{— (5) b}$$

as we know,

$$\frac{\partial \hat{r}}{\partial \theta} = \hat{\theta} \ , \quad \frac{\partial \hat{r}}{\partial \varphi} = \sin\theta\,\hat{\varphi} \quad \text{— (6) b}$$

$$\frac{\partial \vec{v}(\theta,\varphi)}{\partial \theta} = \frac{\partial r(\theta,\varphi)}{\partial \theta}\hat{r} + r(\theta,\varphi)\hat{\theta} \quad \text{— (7) b}$$

$$\frac{\partial \vec{v}(\theta,\varphi)}{\partial \varphi} = \frac{\partial r(\theta,\varphi)}{\partial \varphi}\hat{r} + r(\theta,\varphi)\sin\theta\,\hat{\varphi} \quad \text{— (8) b}$$

$$\vec{n} = \frac{\partial \vec{v}(\theta,\varphi)}{\partial \theta} \times \frac{\partial \vec{v}(\theta,\varphi)}{\partial \varphi}$$

$$\left[\begin{aligned} \vec{n} = & -r(\theta,\varphi)\sin\theta\frac{\partial r(\theta,\varphi)}{\partial \theta}\hat{\theta} - r(\theta,\varphi)\frac{\partial r(\theta,\varphi)}{\partial \varphi}\hat{\varphi} \\ & + (r(\theta,\varphi))^2\sin\theta\,\hat{r} \end{aligned}\right] \text{— (9) b}$$

for Spheroid, $r(\theta,\varphi) = r(\theta) = R - Rs\cos(2\theta)$

$$\left[\begin{aligned} \frac{\partial r(\theta,\varphi)}{\partial \theta} &= \frac{\partial r(\theta)}{\partial \theta} = +Rs\sin(2\theta)(2) \\ \frac{\partial r(\theta,\varphi)}{\partial \varphi} &= \frac{\partial r(\theta)}{\partial \varphi} = 0 \end{aligned}\right] \text{— (10) b}$$

PAGE NO.:
DATE: / /

Substituting the variables in eq 10 b to eq 3b, normal vector can be given as,

$$\vec{n}_{spheroid} = (r(\theta))^2 \sin\theta\, \hat{r} - r(\theta)\, 2\sin\theta \sin(2\theta)\, Rs\, \hat{\theta} + 0\,\hat{\phi} \quad \text{—— (11) b}$$

$$\vec{n}_{spheroid} = (R - Rs\cos(2\theta))^2 \sin\theta\, \hat{r} - 4Rs\sin^2\theta\cos\theta\,(R - Rs\cos(2\theta))\,\hat{\theta} \quad \text{—— (12) b}$$

$$\hat{n} = \frac{\vec{n}}{\|n\|} = \frac{(R - Rs\cos(2\theta))^2 \sin\theta\, \hat{r} - 4Rs\cos\theta\sin^2\theta\,(R - Rs\cos(2\theta))\,\hat{\theta}}{\sqrt{(R - Rs\cos(2\theta))^4 \sin^2\theta + 16R^2s^2\cos^2\theta\sin^4\theta\,(R - Rs\cos(2\theta))^2}} \quad \text{—— (13) b}$$

$$\hat{n} = \frac{(R - Rs\cos(2\theta))^2 \sin\theta\, \hat{r} - 4Rs\cos\theta\sin^2\theta\,(R - Rs\cos(2\theta))\,\hat{\theta}}{\sqrt{(R - Rs\cos(2\theta))^4 \sin^2\theta + 16R^2s^2\cos^2\theta\sin^4\theta\,(R - Rs\cos(2\theta))^2}} \quad \text{—— (14) b}$$

$$\hat{n} = \frac{(R - Rs\cos(2\theta))^2 \sin\theta\, \hat{r} - 4Rs\cos\theta\sin^2\theta\,(R - Rs\cos(2\theta))^2\,\hat{\theta}}{(R - Rs\cos(2\theta))\sin\theta\sqrt{(R - Rs\cos(2\theta))^2 + 16R^2s^2\cos^2\theta\sin^2\theta}} \quad \text{—— (15) b}$$

Teacher's Signature..........

$$\boxed{\hat{n} = \frac{(R - Rs\cos(2\theta))\,\hat{r} - 4Rs\cos\theta\sin\theta\,\hat{\theta}}{\sqrt{(R-Rs\cos(2\theta))^2 + 4R^2s^2\sin^2(2\theta)}} \quad \text{— (16) b}}$$

$\hat{n}$ represents the unit normal vector.