Section - C (c)

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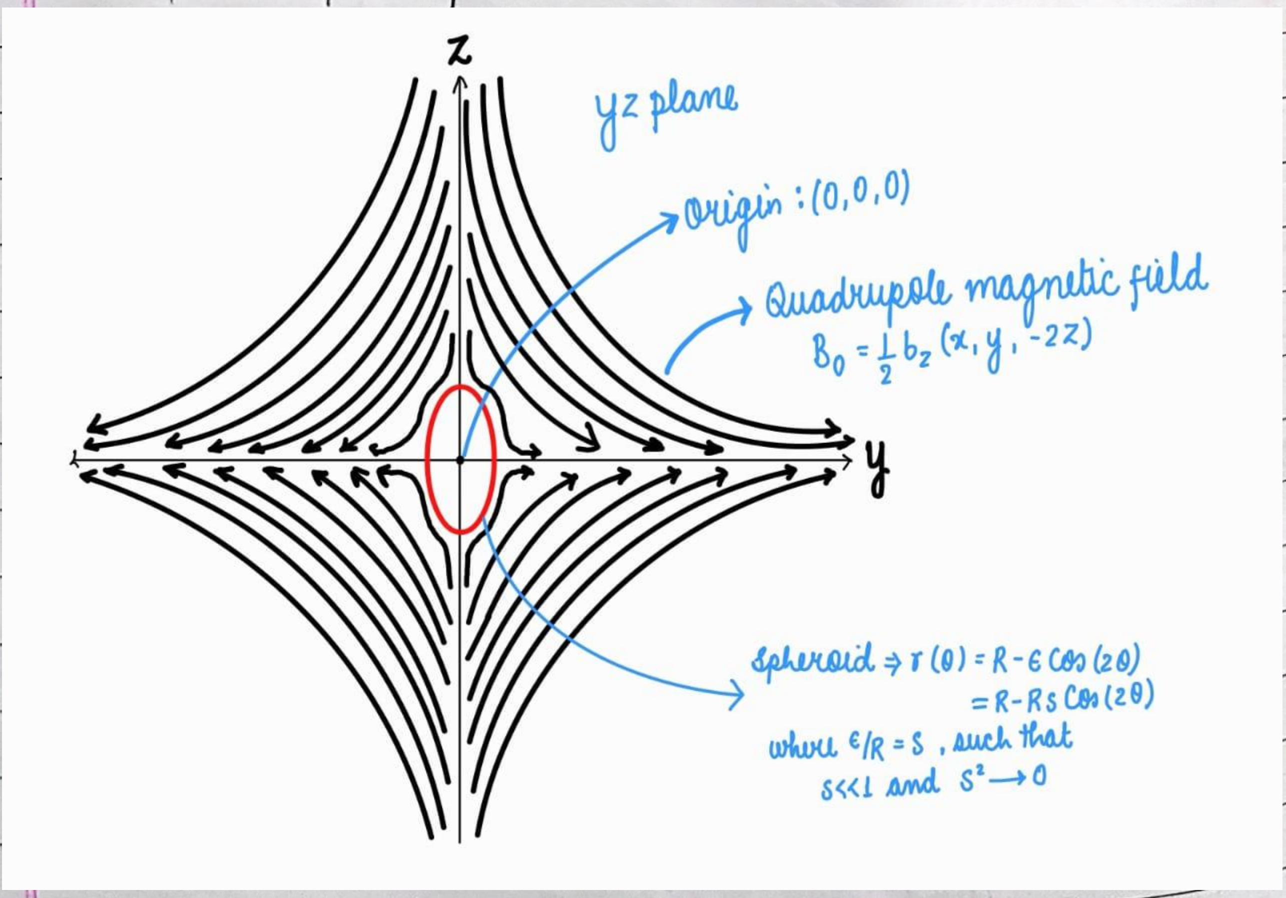
B_{out} CALCULATION → (GENERAL PROCEDURE)

Calculation of B_{out} for spheroid →

Consider the superconducting spheroid trapped inside a quadrupole magnetic field. The spheroid is trapped at the centre of co-ordinate system. We assume zero penetration depth for the spheroid. Due to which, the magnetic field perpendicular to surface must vanish at the surface of superconducting spheroid.

Quadrupole field is given by $B_0 = \frac{1}{2} b_z (x, y, -2z)$. The figure below demonstrates the spheroid trapped inside the quadrupole field.



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$$\vec{B}_{out}(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta,\varphi)} = 0 \quad \text{— (1) c}$$

given that there are no external currents around the spheroid by ampere's law, $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$ (Ampere's law) — (1.1) c

Since $I_{enclosed} = 0$

$$\oint \vec{B}\cdot dl = 0 \Rightarrow \int_S (\nabla\times B)\, dA = 0 \quad \text{— (1.2) c}$$

$$B = -\nabla\Phi_{out}(r,\theta,\varphi) \quad \text{— (1.3) c}$$

Since $\nabla\cdot B = 0$ — (1.4) c

$$-\nabla\cdot(\nabla\Phi) = 0 \Rightarrow -\nabla^2\Phi = 0 \quad \text{— (1.5) c}$$

$$\vec{B}_{out}(r,\theta,\varphi) = B_0(r,\theta,\varphi) - \nabla\Phi_{out}(r,\theta,\varphi) \quad \text{— (2) c}$$

where $B_0(r,\theta,\varphi) \rightarrow$ applied magnetic field in spherical co-ordinates.

$\Phi_{out} \rightarrow$ induced potential outside the spheroid.

$\hat{n}(\theta,\varphi) \rightarrow$ unit normal vector to surface of spheroid.

$$\Phi = \sum_{n=0}^{N_{max}} \left(r^{-(n+1)}\right) \sum_{m=-n}^{n} a_{n,m} Y_n^m(\theta,\varphi) + \sum_{n=0}^{N_{max}} r^n \sum_{m=-n}^{n} b_{n,m} Y_n^m(\theta,\varphi) \quad \text{— (3) c}$$

solution of laplacian is given by this eqn

Since, the potential associated with induced magnetic field must vanish at infinity. ∴ Eqn. 3c can be written as

$$\Phi = \sum_{n=0}^{N_{max}} r^{-(n+1)} \sum_{m=-n}^{n} a_{n,m} Y_n^m(\theta,\varphi) \quad \text{— (4) c}$$

Our main aim is to calculate $\vec{B}_{out}(r,\theta,\varphi)$.
It can be evaluated by applying the boundary condition given by the eqn. 1c

$$\nabla\Phi = -\sum r^{-(n+2)}(n+1)\sum a_{n,m} Y_n^m(\theta,\varphi)\,\hat{r}$$
$$+\sum \frac{r^{-(n+1)}}{r}\sum a_{n,m}\frac{\partial Y_n^m(\theta,\varphi)}{\partial\theta}\,\hat{\theta} \qquad -5c$$
$$+\sum \frac{r^{-(n+1)}}{r\sin\theta}\sum a_{n,m}\frac{\partial Y_n^m(\theta,\varphi)}{\partial\varphi}\,\hat{\varphi}$$

$$\vec{B}_{out}(r,\theta,\varphi) = (B_0 - \nabla\varphi)(r,\theta,\varphi)$$

Applying the boundary condition to obtain the coefficients $a_{n,m}$ present in $\nabla\Phi$ term.

$$\vec{B}_{out}(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta,\varphi)=r(\theta)\ (\text{for spheroid})} = 0$$

$$\boxed{B_0(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta)} = \nabla\Phi(r,\theta,\varphi)\cdot\hat{n}(\theta,\varphi)\Big|_{r=r(\theta)}} \quad 6c$$

Now, I name this equation 6c as my primary equation. This equation would be used by me throughout the notes.

→ Say, left hand side part of eq. 6c as LHS and Right hand side part of eq. 6c as RHS.

This convention is used throughout the notes and mathematica files.

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Any arbitrary function $f(\theta, \phi)$ which is defined in the interval $\theta \in (0, \pi)$ & $\varphi \in (0, 2\pi)$ and satisfies the condition

$$\int_0^{\pi}\int_0^{2\pi} f(\theta,\varphi) \sin\theta \, d\theta \, d\varphi < \infty \quad - \text{⑦c}$$

can be expressed as expansion of spherical harmonics.

Therefore, we try to express both LHS & RHS in terms of spherical harmonics expansion.

As, we know spherical harmonics forms the orthonormal basis for the angular function, every angular function can be expressed in its terms provided the above condition of eqn. 7c satisfies.

Then, we can compare its coefficients to obtain $a_{n,m}$'s.

Eqn. ⑦c is checked every time for the every case.