Section G(g)

[Mathematica file
→ Force General Translation.nb]

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FORCE CALCULATION (General Translation)
(General Procedure)

This section deals with calculating force acting on spheroid for the general displacement of applied magnetic field.

Spheroid is described as

$$r(\theta) = R - Rs \cos(2\theta) \quad \text{— ① g}$$

and the position vector to its surface, can be written as

vsph

$$\vec{v}(\theta,\varphi) = (R - Rs\cos(2\theta))\,\hat{r} = r(\theta)\,\hat{r} \quad \text{— ② g}$$

the normal vector to the surface of spheroid can be written as:-

normal

$$\vec{n}(\theta,\varphi) = (R - Rs\cos(2\theta))^2 \sin\theta\,\hat{r} - 4Rs\cos\theta\sin^2\theta\,(R - Rs\cos(2\theta))\,\hat{\theta} \quad \text{— ③ g}$$

Since, we aim to calculate F_x, F_y, F_z (force acting on spheroid) we need to work in cartesian co-ordinate system.

$$\vec{n}_{cart.}(\theta,\varphi) = M_{s\to c}\cdot\vec{n}(\theta,\varphi) \quad \text{— ④ g}$$

$\vec{n}_{cart}$ is normal vector in cartesian coordinate system. $M_{s\to c}$ is the transformation matrix from spherical to cartesian co-ordinate system.

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$\boxed{\text{SphToCart}}$ $M_{s\to c} = \begin{pmatrix} \cos\varphi \sin\theta & \cos\theta \cos\varphi & -\sin\varphi \\ \sin\theta \sin\varphi & \cos\theta \sin\varphi & \cos\varphi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix}$ — 5g

$\vec{n}_{cart}(\theta, \varphi) = \boxed{\text{normalcart}}$

$$\Big((R-2R_s-3R_s\cos(2\theta))(R-R_s\cos(2\theta))\sin^2\theta\cos\varphi,\ (R-2R_s-3R_s\cos(2\theta))(R-R_s\cos(2\theta))\sin^2\theta\sin\varphi,\ (R+2R_s-3R_s\cos(2\theta))(R-R_s\cos(2\theta))\sin\theta\cos\theta\Big)$$ — 6g

$\boxed{nx}$ $\hat{n}^{x}_{cart}(\theta, \varphi) = (R-2R_s-3R_s\cos(2\theta))(R-R_s\cos(2\theta))\sin^2\theta\cos\varphi$ — 7g

$\boxed{ny}$ $n^{y}_{cart}(\theta, \varphi) = (R-2R_s-3R_s\cos(2\theta))(R-R_s\cos(2\theta))\sin^2\theta\sin\varphi$ — 8g

$\boxed{nz}$ $n^{z}_{cart}(\theta, \varphi) = (R+2R_s-3R_s\cos(2\theta))(R-R_s\cos(2\theta))\sin\theta\cos\theta$ — 9g

Area element of Spheroid →

As explained in Section E (e) previously,

$dA_i = \hat{n}_i \cdot dS$ — 10g

where $\hat{n}_i$ → unit normal vectors' i^{th} component

$dS = \|n\|\, d\theta\, d\varphi$ — 11g

$dA_i = n_i\, d\theta\, d\varphi$ — 12g

[Ref → Marsden (Vector Calculus) 6th Edition
Pg-384 (Area of Parametrized surface)]

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Magnetic field at surface of spheroid →

Eq 38 f given in Mathematica file is used for the calculation of force acting on spheroid. ⟶ $B_{out}(r(\theta), \theta, \varphi)$ — (13) g

Eq 38 f passes the test case of spheroid (meaning it tends to the expression for sphere when $s \to 0$) — (14) g

We also need to convert magnetic field at surface of spheroid into cartesian co-ordinate system.

$\boxed{\text{Bout Cart}}$ $B_{out}^{cart}(r(\theta), \theta, \varphi) = M_{s \to c} \cdot B_{out}(r(\theta), \theta, \varphi)$ — (15) g − 1

$$\lim_{s \to 0} B_{out}(r(\theta), \theta, \varphi) = \Big(0, \tfrac{1}{4} b_z \big(6 d_z \sin(\theta) + \cos\theta\,(3 d_x \cos\varphi + 10 R \sin\theta + 3 d_y \sin\varphi)\big),$$
$$\tfrac{3}{4} b_z (d_y \cos\varphi - d_x \sin\varphi)\Big)$$
— (14) g

$$\boxed{\text{Bout2}[\theta, \varphi]} = \left(B_{out}^{cart}(r(\theta), \theta, \varphi)\right)^2 = (B_{out}^{x})^2 + (B_{out}^{y})^2 + (B_{out}^{z})^2 \quad \text{— (15) g − 2}$$

$\boxed{B_x} = B_{out}^{cart,x}(\theta, \varphi)$

$\boxed{B_y} = B_{out}^{cart,y}(\theta, \varphi)$

$\boxed{B_z} = B_{out,z}^{cart}(\theta, \varphi)$

} eqn. (16) g

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$$\left.\begin{aligned} Bx2 &= \left(B_{out}^{cart,x}\right)^2 \\ By2 &= \left(B_{out}^{cart,y}\right)^2 \\ Bz2 &= \left(B_{out}^{cart,z}\right)^2 \end{aligned}\right\} \text{— (17) g}$$

Maxwell Stress Tensor is defined as

$$T_{ij} = \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij} B^2\right) \quad \text{— (18) g}$$

force from maxwell stress Tensor can be given as

$$F = \sum_j \oint T_{ij}\, dA_j$$

$$F = \int (\nabla \cdot T)\, dV \quad \text{— (19) g}$$

$$= \oint \overleftrightarrow{T} \cdot \hat{n}\, ds \quad \text{— (20) g}$$

$$= \oint \overleftrightarrow{T} \cdot \vec{n}\, d\theta\, d\varphi \quad \text{— (21) g}$$

$$T_{xx} = \frac{1}{\mu_0}\left(\left(B_{out}^{cart,x}\right)^2 - \frac{1}{2}\left(B_{out}^{cart}\right)^2\right) \quad \text{— (22) g}$$

$$T_{xy} = \frac{1}{\mu_0}\left(B_{out}^{cart,x} \cdot B_{out}^{cart,y}\right) \quad \text{— (23) g}$$

$$T_{xz} = \frac{1}{\mu_0}\left(B_{out}^{cart,x} \cdot B_{out}^{cart,z}\right) \quad \text{— (24) g}$$

$$f_{x_1} = \oint T_{xx}\, dA_x = \oint T_{xx}\, n_x\, d\theta\, d\varphi \quad \text{— (25) g}$$

$$f_{x_2} = \oint T_{xy}\, dA_y = \oint T_{xy}\, n_y\, d\theta\, d\varphi \quad \text{— (26) g}$$

$$f_{x_3} = \oint T_{xz}\, dA_z = \oint T_{xz}\, n_z\, d\theta\, d\varphi \quad \text{— (27) g}$$

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$$F_x = f_{x_1} + f_{x_2} + f_{x_3} \quad \text{—— (28) g}$$

Similarly, we can calculate force in y & z dirⁿ (F_y & F_z)

$$T_{yy} = \frac{1}{\mu_0}\left(\left(B_{out}^{cart,y}\right)^2 - \frac{1}{2}\left(B_{out}^{cart}\right)^2\right) \quad \text{—— (29) g}$$

$$T_{yx} = \frac{1}{\mu_0}\left(B_{out}^{cart,y}\right)\left(B_{out}^{cart,x}\right) = T_{xy} \quad \text{—— (30) g}$$

$$T_{yz} = \frac{1}{\mu_0}\left(B_{out}^{cart,y}\right)\left(B_{out}^{cart,z}\right) \quad \text{—— (31) g}$$

$$f_{y_1} = \oint T_{yy}\, n_y \, d\theta \, d\varphi \quad \text{— (32) g}$$

$$f_{y_2} = \oint T_{yx}\, n_x \, d\theta \, d\varphi \quad \text{— (33) g}$$

$$f_{y_3} = \oint T_{yz}\, n_z \, d\theta \, d\varphi \quad \text{— (34) g}$$

$$F_y = f_{y_1} + f_{y_2} + f_{y_3} \quad \text{—— (35) g}$$

$$T_{zz} = \frac{1}{\mu_0}\left(\left(B_{out}^{cart,z}\right)^2 - \frac{1}{2}\left(B_{out}^{cart}\right)^2\right) \quad \text{—— (36) g}$$

$$T_{zx} = T_{xz} \quad \text{—— (37) g}$$

$$T_{zy} = T_{yz} \quad \text{— (38) g}$$

$$f_{z_1} = \oint T_{zz}\, n_z \, d\theta \, d\varphi \quad \text{—— (39) g}$$

$$f_{z_2} = \oint T_{zx}\, n_x \, d\theta \, d\varphi \quad \text{— (40) g}$$

$$f_{z_3} = \oint T_{zy}\, n_y \, d\theta \, d\varphi \quad \text{— (41) g}$$

$$F_z = f_{z_1} + f_{z_2} + f_{z_3} \quad \text{— (42) g}$$

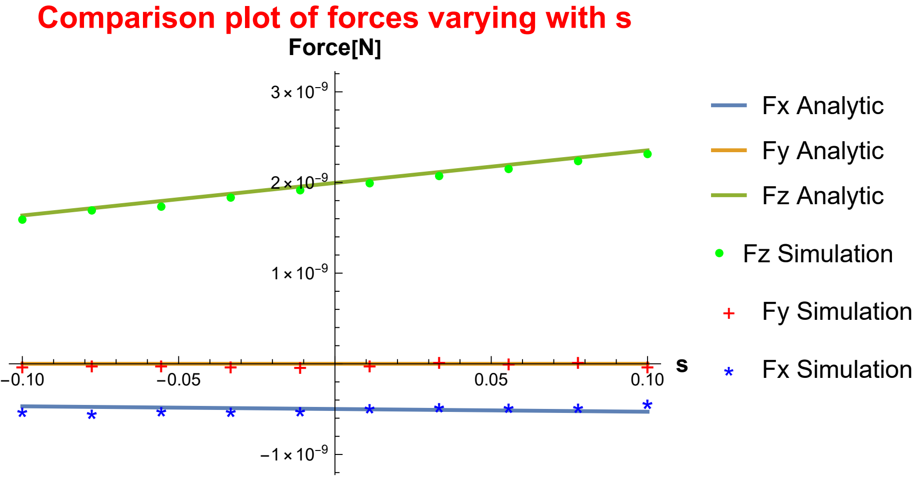
$$F = (F_x, F_y, F_z) \quad \text{— (43) g}$$

$$F_x = -\frac{b_z^2 d x \pi R^3 (5+3s)}{10\mu}$$

$$F_y = -\frac{b_z^2 d y \pi R^3 (5+3s)}{10\mu}$$

$$F_z = -\frac{2 b_z^2 d z \pi R^3 (5+9s)}{5\mu}$$

— Eqn. 44 g



Comparison plot of forces varying with s
Force[N]
3 × 10⁻⁹
2 × 10⁻⁹
1 × 10⁻⁹
−1 × 10⁻⁹
−0.10
−0.05
0.05
0.10
s
Fx Analytic
Fy Analytic
Fz Analytic
Fz Simulation
Fy Simulation
Fx Simulation