Section 2(a)

SPHEROID →

The surface of ellipsoid centred at the origin of a cartesian co-ordinate system is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad \text{— (1)a}$$

Consider, $a = b$, the shape of surface describes a spheroid given by equation

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{c^2} = 1 \quad \text{— (2)a}$$

Since, we are interested to study about the non uniformity of a sphere,

we put $a = R+\epsilon$, $c = R-\epsilon$

$$\Rightarrow \frac{x^2+y^2}{(R+\epsilon)^2} + \frac{z^2}{(R-\epsilon)^2} = 1 \quad \text{— (3)a}$$

Converting eqn 3a in spherical co-ordinates

$$\left.\begin{aligned} x &= r\sin\theta\cos\varphi \\ y &= r\sin\theta\sin\varphi \\ z &= r\cos\theta \end{aligned}\right\} \quad \text{— (4)a}$$

Putting eq 4a in eq 3a,

$$\frac{r^2\sin^2\theta}{(R+\epsilon)^2} + \frac{r^2\cos^2\theta}{(R-\epsilon)^2} = 1 \quad \text{— (5)a}$$

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$$\Rightarrow \frac{r^2 \sin^2\theta (R-\epsilon)^2 + r^2 \cos^2\theta (R+\epsilon)^2}{(R-\epsilon)^2 (R+\epsilon)^2} = 1 \quad \text{— (6)a}$$

$$r^2 = \frac{(R+\epsilon)^2 (R-\epsilon)^2}{\sin^2\theta (R-\epsilon)^2 + \cos^2\theta (R+\epsilon)^2} \quad \text{— (7)a}$$

Taking square root both the sides.

$$r = \frac{\pm (R^2-\epsilon^2)}{\sqrt{\sin^2\theta (R^2+\epsilon^2-2\epsilon R) + \cos^2\theta (R^2+\epsilon^2+2\epsilon R)}} \quad \text{— (8)a}$$

$$r = \frac{\pm (R^2-\epsilon^2)}{\sqrt{R^2+\epsilon^2-2\epsilon R(\sin^2\theta - \cos^2\theta)}} \quad \text{— (9)a}$$

$$r = \frac{\pm (R^2-\epsilon^2)}{\sqrt{R^2+\epsilon^2+2\epsilon R\cos(2\theta)}} \quad \text{— (10)a}$$

$$r = \frac{\pm R^2 (1-\epsilon^2/R^2)}{\sqrt{R^2 (1+\epsilon^2/R^2 + 2\epsilon/R \cos(2\theta))}} \quad \text{— (11)a}$$

say $\frac{\epsilon}{R} = s$.

$$r = \frac{\pm R (1-s^2)}{\sqrt{(1+s^2+2s\cos(2\theta))}} \quad \text{— (12)a}$$

Now, taylor expanding the RHS about s upto first order of s can be given as,

$$r = f(s) = \pm \frac{R(1-s^2)}{\sqrt{1+s^2+2s\cos(2\theta)}} \quad \text{— (13)a}$$

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Taylor expansion of a function of s about 0 is written as,

$$f(s) = f(0) + f'(0)s + f''(0)\frac{s^2}{2} \quad \text{— (14)a}$$

$$f(0) = R$$

$$f'(s)\Big|_{s=0} = \frac{-2Rs\left(\sqrt{1+s^2+2s\cos(2\theta)}\right) - \dfrac{R(2s+2\cos(2\theta))}{2\sqrt{1+s^2+2s\cos(2\theta)}}}{(1+s^2+2s\cos(2\theta))}$$

$$f'(s)\Big|_{s=0} = -R\cos(2\theta)$$

$$f(s) = R - Rs\cos(2\theta)$$

$$\boxed{r(\theta) = f(s) = R - Rs\cos(2\theta)} \quad \text{— (15)a}$$

→ parametric equation of spheroid.