Section G(g)

[Mathematica file
→ Force General Translation.nb]

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FORCE CALCULATION (General Translation)
(General Procedure)

This section deals with calculating force acting on spheroid for the general displacement of applied magnetic field.

Spheroid is described as

$$r(\theta) = R - Rs\cos(2\theta) \quad \text{— (1)g}$$

and the position vector to its surface, can be written as

$\boxed{\text{vsph}}$

$$\vec{v}(\theta,\varphi) = (R - Rs\cos(2\theta))\,\hat{r} = r(\theta)\,\hat{r} \quad \text{— (2)g}$$

the normal vector to the surface of spheroid can be written as:-

$\boxed{\text{normal}}$

$$\vec{n}(\theta,\varphi) = (R - Rs\cos(2\theta))^2\sin\theta\,\hat{r} - 4Rs\cos\theta\sin^2\theta\,(R - Rs\cos(2\theta))\,\hat{\theta} \quad \text{— (3)g}$$

Since, we aim to calculate F_x, F_y, F_z (force acting on spheroid) we need to work in cartesian co-ordinate system.

$$\vec{n}_{cart.}(\theta,\varphi) = M_{S\to C}\cdot\vec{n}(\theta,\varphi) \quad \text{— (4)g}$$

$\vec{n}_{cart}$ is normal vector in cartesian coordinate system. $M_{S\to C}$ is the transformation matrix from spherical to cartesian co-ordinate system.

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$\boxed{\text{Sph To Cart}}$

$$M_{S\to C} = \begin{pmatrix} \cos\varphi\sin\theta & \cos\theta\cos\varphi & -\sin\varphi \\ \sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \quad \text{— (5) g}$$

$$\vec{n}_{cart}(\theta,\varphi) = \boxed{\text{normalcart}}$$

$$\big((R-2R_S-3R_S\cos(2\theta))(R-R_S\cos(2\theta))\sin^2\theta\cos\varphi,$$
$$(R-2R_S-3R_S\cos(2\theta))(R-R_S\cos(2\theta))\sin^2\theta\sin\varphi, \quad \text{— (6) g}$$
$$(R+2R_S-3R_S\cos(2\theta))(R-R_S\cos(2\theta))\sin\theta\cos\theta\big)$$

$\boxed{nx}$ $$n^{x}_{cart}(\theta,\varphi) = (R-2R_S-3R_S\cos(2\theta))(R-R_S\cos(2\theta))\sin^2\theta \quad \text{— (7) g}$$

$\boxed{ny}$ $$n^{y}_{cart}(\theta,\varphi) = (R-2R_S-3R_S\cos(2\theta))(R-R_S\cos(2\theta))\sin^2\theta\sin\varphi$$

$\boxed{nz}$ $$n^{z}_{cart}(\theta,\varphi) = (R+2R_S-3R_S\cos(2\theta))(R-R_S\cos(2\theta))\sin\theta\cos\theta \quad \text{— (9) g}$$

Area element of Spheroid →

As explained in Section E (e) previously,

$$dA_i = \hat{n}_i \cdot dS \quad \text{— (10) g}$$

where $\hat{n}_i$ → unit normal vectors' i^{th} component.

$$dS = \|n\|\, d\theta\, d\varphi \quad \text{— (11) g}$$

$$dA_i = n_i\, d\theta\, d\varphi \quad \text{— (12) g}$$

[Ref → Marsden (Vector Calculus) 6th Edition
Pg. 384 (Area of Parametrized surface)]

Magnetic field at surface of spheroid →

Eq. 38 f given in Mathematica file is used for the calculation of force acting on spheroid. ⟶ $B_{out}(r(\theta),\theta,\phi)$ — 13 g

Eq 38 f passes the test case of sphere (meaning it tends to the expression for sphere when $s \to 0$) —— 14 g

We also need to convert magnetic field at surface of spheroid into Cartesian co-ordinate system.

$\boxed{\text{Bout Cart}}$ $$B^{cart}_{out}(r(\theta),\theta,\phi) = M_{s\to c} \cdot B_{out}(r(\theta),\theta,\phi) \quad \text{—— (15) g-1}$$

$$\lim_{s\to 0} B_{out}(r(\theta),\theta,\phi) = \Big(0,\ \tfrac{1}{4} b_z \big(6 d_z \sin(\theta) + \cos\theta\,(3 d_x \cos\phi + 10 R \sin\theta + 3 d_y \sin\phi)\big),$$
$$\tfrac{3}{4} b_z (d_y \cos\phi - d_x \sin\phi)\Big) \quad \text{—— (14) g}$$

$$\boxed{\text{Bout2}[\theta,\phi]} = (B^{cart}_{out}(r(\theta),\theta,\phi))^2$$
$$= (B^{x}_{out})^2 + (B^{y}_{out})^2 + (B^{z}_{out})^2 \quad \text{—— (15) g-2}$$

$$\left.\begin{aligned} \boxed{B_x} &= B^{cart,x}_{out}(\theta,\phi) \\ \boxed{B_y} &= B^{cart,y}_{out}(\theta,\phi) \\ \boxed{B_z} &= B^{cart}_{out,z}(\theta,\phi) \end{aligned}\right\} \text{eqn. (16) g}$$

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$$\left.\begin{aligned} Bx2 &= (B_{out}^{cart,x})^2 \\ By2 &= (B_{out}^{cart,y})^2 \\ Bz2 &= (B_{out}^{cart,z})^2 \end{aligned}\right\} \quad \text{— (17) g}$$

Maxwell Stress Tensor is defined as

$$T_{ij} = \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij} B^2\right) \quad \text{— (18) g}$$

force from maxwell stress Tensor can be given as

$$F = \sum_j \oint T_{ij}\, dA_j$$

$$F = \int (\nabla \cdot T)\, dV \quad \text{— (19) g}$$

$$= \oint \overleftrightarrow{T} \cdot \hat{n}\, dS \quad \text{— (20) g}$$

$$= \oint \overleftrightarrow{T} \cdot \vec{n}\, d\theta\, d\varphi \quad \text{— (21) g}$$

$$T_{xx} = \frac{1}{\mu_0}\left((B_{out}^{cart,x})^2 - \frac{1}{2}(B_{out}^{cart})^2\right) \quad \text{— (22) g}$$

$$T_{xy} = \frac{1}{\mu_0}\left(B_{out}^{cart,x} \cdot B_{out}^{cart,y}\right) \quad \text{— (23) g}$$

$$T_{xz} = \frac{1}{\mu_0}\left(B_{out}^{cart,x} \cdot B_{out}^{cart,z}\right) \quad \text{— (24) g}$$

$$fx_1 = \oint T_{xx}\, dA_x = \oint T_{xx}\, n_x\, d\theta\, d\varphi \quad \text{— (25) g}$$

$$fx_2 = \oint T_{xy}\, dA_y = \oint T_{xy}\, n_y\, d\theta\, d\varphi \quad \text{— (26) g}$$

$$fx_3 = \oint T_{xz}\, dA_z = \oint T_{xz}\, n_z\, d\theta\, d\varphi \quad \text{— (27) g}$$

mention which fx1,fx2,fx3 be

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$F_x = f_{x_1} + f_{x_2} + f_{x_3}$ —— 28 g

Similarly, we can calculate force in y & z dir^n (F_y & F_z)

$T_{yy} = \frac{1}{\mu_0}\left((B_{out}^{cart,y})^2 - \frac{1}{2}(B_{out}^{cart})^2\right)$ —— 29 g

$T_{yx} = \frac{1}{\mu_0}(B_{out}^{cart,y})(B_{out}^{cart,x}) = T_{xy}$ —— 30 g

$T_{yz} = \frac{1}{\mu_0}(B_{out}^{cart,y})(B_{out}^{cart,z})$ —— 31 g

$f_{y_1} = \oint T_{yy}\, n_y\, d\theta\, d\varphi$ — 32 g

add final result of Fx, Fy, Fz here since it is not bulcky

$f_{y_2} = \oint T_{yx}\, n_x\, d\theta\, d\varphi$ — 33 g

$f_{y_3} = \oint T_{yz}\, n_z\, d\theta\, d\varphi$ — 34 g

$F_y = f_{y_1} + f_{y_2} + f_{y_3}$ —— 35 g

$T_{zz} = \frac{1}{\mu_0}\left((B_{out}^{cart,z})^2 - \frac{1}{2}(B_{out}^{cart})^2\right)$ —— 36 g

$T_{zx} = T_{xz}$ —— 37 g

$T_{zy} = T_{yz}$ —— 38 g

$f_{z_1} = \oint T_{zz}\, n_z\, d\theta\, d\varphi$ —— 39 g

$f_{z_2} = \oint T_{zx}\, n_x\, d\theta\, d\varphi$ —— 40 g

$f_{z_3} = \oint T_{zy}\, n_y\, d\theta\, d\varphi$ —— 41 g

$F_z = f_{z_1} + f_{z_2} + f_{z_3}$ —— 42 g

$F = (F_x, F_y, F_z)$ —— 43 g