

**ENSEMBLE-BASED DATA
ASSIMILATION WITH MAPPING
DATASETS OF THE MARTIAN
ATMOSPHERE**

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Caltech

“As it is in terrestrial atmospheric science, so it goes in planetary atmospheric science.”

Gridded Climate Datasets

- great success in terrestrial atm. science
- have nearly replaced use of traditional data
- subsume data whence they came (to within the original data's uncertainty)
- the product of **Data Assimilation**

Data Assimilation is NOT just modeling arcana!

Tradition of “As it goes ...” – we are leveraging their experimentation and adopting *successful* ideas.

Terrestrial data analysis often begins with downloading a GCD (even CO2 now)

DA is a COMPLETE and POWERFUL treatment of observations.

<http://www.cdc.noaa.gov/PublicData/>

The screenshot shows a website for the NOAA Earth System Research Laboratory Physical Sciences Division. The header includes the NOAA logo and the text "U.S. Department of Commerce | National Oceanic & Atmospheric Administration | NOAA Research". Below the header, there is a navigation bar with links for "Physical Sciences Division", "About", "Contact", "Research", "Data", "Products", "Outreach", and "Intranet". On the left, a sidebar titled "Climate Datasets" lists various timescales: All, Sub-daily, Daily, Monthly, Surface, Multi-level, Land, Ocean, Radiation, Climate Indices, and a search link for "Search Datasets". The main content area is titled "PSD Gridded Climate Datasets: All" and includes links for "Descriptions" and "Summary Attributes". A table lists four datasets: CMAP Precipitation, CPC .25x.25 Daily US Unified Precipitation, CPC Hourly Precipitation, and CPC Soil Moisture. At the bottom of the main content area, it says "[29 total datasets]".

Datasets	Description
CMAP Precipitation	Monthly and pentad global gridded precipitation means. It includes a subset of NCEP Reanalysis) from 1979 to near the present.
CPC .25x.25 Daily US Unified Precipitation	US high resolution gridded precipitation (from station data) for 1948 to present.
CPC Hourly Precipitation	NCEP's hourly gridded US station precipitation from 1948.
CPC Soil Moisture	Monthly Gridded CPC Soil Moisture from a model from 1948 to present.

One of many different websites to find and download Gridded Climate Datasets

29 options here: Timescales go from monthly to sub-daily; Note options of dataset focus.

NCEP/NCAR Reanalysis 1: Summary

Go To: Temporal Coverage | Spatial Coverage | Levels | Update Schedule | Download/PDF Data | Restrictions | Details | Caveats | File Naming | Citation | References | Cite Source | Contact

One-Line Description:

- NCEP/NCAR Reanalysis 1

Temporal Coverage:

- 4-times daily, daily and monthly values for 1948/01/01 to present
- Long term monthly means, derived from data for years 1968 - 1996

Spatial Coverage:

- Global Grids

Levels:

- 17 Pressure level and 28 sigma levels. N/A

Update Schedule:

- Daily

We have separated the data documentation into seven sections:

- Pressure level
- Surface
- Surface Fluxes
- Other Fluxes
- Tropopause
- Derived Data
- Spectral Coefficients

Kalnay et al., *BAMS*, 1996

The screenshot shows the NCEP/NCAR Reanalysis 1: Summary page. The 'Levels' section contains the text '17 Pressure level and 28 sigma levels. N/A'. A red arrow points from this text to the first item in the bulleted list below: '• Pressure level'. The page also includes sections for Temporal Coverage, Spatial Coverage, One-Line Description, and Update Schedule, along with a reference to a paper by Kalnay et al. (1996).

So-called Reanalysis is very popular

Several different options to choose from – Pressure level is very popular



<p>NCEP/NCAR Reanalysis 1: Summary</p> <p>Reanalysis 1 is a global climate dataset that provides monthly mean and daily values for atmospheric variables from 1948 to the present.</p> <p>One-Line Description:</p> <ul style="list-style-type: none"> • NCEP/NCAR Reanalysis 1 <p>Temporal Coverage:</p> <ul style="list-style-type: none"> • 4 times daily, daily and monthly values for 1948/01/01 to present • Long term monthly means, derived from data for years 1968 - 1996 <p>Spatial Coverage:</p> <ul style="list-style-type: none"> • Global Grids <p>Levels:</p> <ul style="list-style-type: none"> • 17 Pressure levels and 28 sigma levels. N/A <p>Update Schedule:</p> <ul style="list-style-type: none"> • Daily <p>We have separated the data documentation into seven sections:</p>	<p>Spatial Coverage:</p> <ul style="list-style-type: none"> • 2.5 degree x 2.5 degree global grids (144x73) • 0.0E to 357.5E, 90.0N to 90.0S <p>Levels:</p> <ul style="list-style-type: none"> • 17 Pressure levels (mb): 1000, 925, 850, 700, 600, 500, 400, 300, 250, 200, 150, 100, 70, 50, 30, 20, 10 • Some variables have less: omega (to 100mb) and Humidities (to 300mb) <p>Update Schedule:</p> <ul style="list-style-type: none"> • Daily <p>Download/Plot Data:</p> <table border="1"> <thead> <tr> <th>Variable</th> <th>Startdate</th> <th>Level</th> <th>Download File</th> <th>Create Plot/Subset</th> </tr> </thead> <tbody> <tr><td>Air Temperature</td><td>4-times Daily</td><td>Pressure</td><td>see list</td><td></td></tr> <tr><td>Air Temperature</td><td>Daily</td><td>Pressure</td><td>see list</td><td></td></tr> <tr><td>Air Temperature</td><td>Monthly Mean</td><td>Pressure</td><td>air.mon.mean.nc</td><td></td></tr> <tr><td>Geopotential Height</td><td>4-times Daily</td><td>Pressure</td><td>see list</td><td></td></tr> <tr><td>Geopotential Height</td><td>Daily</td><td>Pressure</td><td>see list</td><td></td></tr> <tr><td>Geopotential Height</td><td>Monthly Mean</td><td>Pressure</td><td>height.mon.mean.nc</td><td></td></tr> <tr><td>Relative Humidity</td><td>4-times Daily</td><td>Pressure</td><td>see list</td><td></td></tr> <tr><td>Relative Humidity</td><td>Daily</td><td>Pressure</td><td>see list</td><td></td></tr> <tr><td>Relative Humidity</td><td>Monthly Mean</td><td>Pressure</td><td>rh.mon.mean.nc</td><td></td></tr> <tr><td>Specific Humidity</td><td>4-times Daily</td><td>Pressure</td><td>see list</td><td></td></tr> <tr><td>Specific Humidity</td><td>Daily</td><td>Pressure</td><td>see list</td><td></td></tr> <tr><td>Specific Humidity</td><td>Monthly Mean</td><td>Pressure</td><td>shl.mon.mean.nc</td><td></td></tr> <tr><td>Omega (Vertical Velocity)</td><td>4-times Daily</td><td>Pressure</td><td>see list</td><td></td></tr> <tr><td>Omega (Vertical Velocity)</td><td>Daily</td><td>Pressure</td><td>see list</td><td></td></tr> <tr><td>Omega (Vertical Velocity)</td><td>Monthly Mean</td><td>Pressure</td><td>omega.mon.mean.nc</td><td></td></tr> <tr><td>J-Wind</td><td>4-times Daily</td><td>Pressure</td><td>see list</td><td></td></tr> <tr><td>J-Wind</td><td>Daily</td><td>Pressure</td><td>see list</td><td></td></tr> <tr><td>J-Wind</td><td>Monthly Mean</td><td>Pressure</td><td>jwind.mon.mean.nc</td><td></td></tr> <tr><td>V-Wind</td><td>4-times Daily</td><td>Pressure</td><td>see list</td><td></td></tr> </tbody> </table>	Variable	Startdate	Level	Download File	Create Plot/Subset	Air Temperature	4-times Daily	Pressure	see list		Air Temperature	Daily	Pressure	see list		Air Temperature	Monthly Mean	Pressure	air.mon.mean.nc		Geopotential Height	4-times Daily	Pressure	see list		Geopotential Height	Daily	Pressure	see list		Geopotential Height	Monthly Mean	Pressure	height.mon.mean.nc		Relative Humidity	4-times Daily	Pressure	see list		Relative Humidity	Daily	Pressure	see list		Relative Humidity	Monthly Mean	Pressure	rh.mon.mean.nc		Specific Humidity	4-times Daily	Pressure	see list		Specific Humidity	Daily	Pressure	see list		Specific Humidity	Monthly Mean	Pressure	shl.mon.mean.nc		Omega (Vertical Velocity)	4-times Daily	Pressure	see list		Omega (Vertical Velocity)	Daily	Pressure	see list		Omega (Vertical Velocity)	Monthly Mean	Pressure	omega.mon.mean.nc		J-Wind	4-times Daily	Pressure	see list		J-Wind	Daily	Pressure	see list		J-Wind	Monthly Mean	Pressure	jwind.mon.mean.nc		V-Wind	4-times Daily	Pressure	see list		<p>Intranet</p> <p>PSD Gridded Climate Datasets: All Descriptions & Summary Attributes</p> <table border="1"> <thead> <tr> <th>Dataset</th> <th>Description</th> </tr> </thead> <tbody> <tr><td>GMAP Precipitation</td><td>Monthly and annual global gridded precipitation means. 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Gridded analyses of T, Z, R, Q, W, U, V

Mandatory pressure levels

2.5 deg by 2.5 deg

The data you requested are contained in these files:

uwnd.1948.nc
uwnd.1949.nc
uwnd.1950.nc
uwnd.1951.nc
uwnd.1952.nc
uwnd.1953.nc
uwnd.1954.nc
uwnd.1955.nc
uwnd.1956.nc
uwnd.1957.nc
uwnd.1958.nc
uwnd.1959.nc
uwnd.1960.nc
uwnd.1961.nc
uwnd.1962.nc
uwnd.1963.nc
uwnd.1964.nc
uwnd.1965.nc

PSD Gridded Climate Datasets: All Descriptions | Summary Attributes

Dataset	Description
CMAP Precipitation	Monthly and seasonal global gridded precipitation means. It includes a NCEP Reanalysis from 1948 to near the present.
GPCP v2c 3B43 Daily US Unified Precipitation	US High resolution gridded precipitation (three station data) for 1948 to present.
GPCP 3B43 Daily Precipitation	NCEP's daily global US station precipitation from 1948 to present.



Spatial Coverage:

- 2.5 degree x 2.5 degree global grids (144x72)
- 0.0E to 360.0E, 90.0N to 90.0S

Levels:

- 17 Pressure levels (avg: 1000, 998, 850, 700, 600, 500, 400, 300, 250, 200, 150, 100, 70, 50, 30, 20, 10) and Humidity (n: 30)

Update Schedule:

- Daily

Downloads/Pict Data:

File Type	Format	Size	Last Update	Preview
All Temperature	41x72x120	~100 MB	2013-01-01	
All Temperature	41x72x1	~100 MB	2013-01-01	
All Temperature	41x72	~100 MB	2013-01-01	
All Temperature	41x72x120	~100 MB	2013-01-01	
Geopotential Height	41x72x120	~100 MB	2013-01-01	
Geopotential Height	41x72x1	~100 MB	2013-01-01	
Geopotential Height	41x72	~100 MB	2013-01-01	
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Specific Humidity	41x72	~100 MB	2013-01-01	
Specific Humidity	41x72x120	~100 MB	2013-01-01	
Wind Speed	41x72x120	~100 MB	2013-01-01	
Wind Speed	41x72x1	~100 MB	2013-01-01	
Wind Speed	41x72	~100 MB	2013-01-01	
Wind Speed	41x72x120	~100 MB	2013-01-01	
Wind Velocity	41x72x120	~100 MB	2013-01-01	
Wind Velocity	41x72x1	~100 MB	2013-01-01	
Wind Velocity	41x72	~100 MB	2013-01-01	
Wind Velocity	41x72x120	~100 MB	2013-01-01	

NCEP/NCAR Reanalysis 1: Summary
 On the NCEP/NCAR Reanalysis 1 page, there is a link to the "Documentation" section. This section contains detailed information about the data, including its history, methodology, and various applications.

One-Line Description:

- NCEP/NCAR Reanalysis 1

Temporal Coverage:

- 4 times daily, daily and monthly values for 1948/01/01 to present.
- Long term monthly means, derived from data for years 1968 - 1996

Spatial Coverage:

- Global Grids

Levels:

- 17 Pressure level and 28 sigma levels. N/A

Update Schedule:

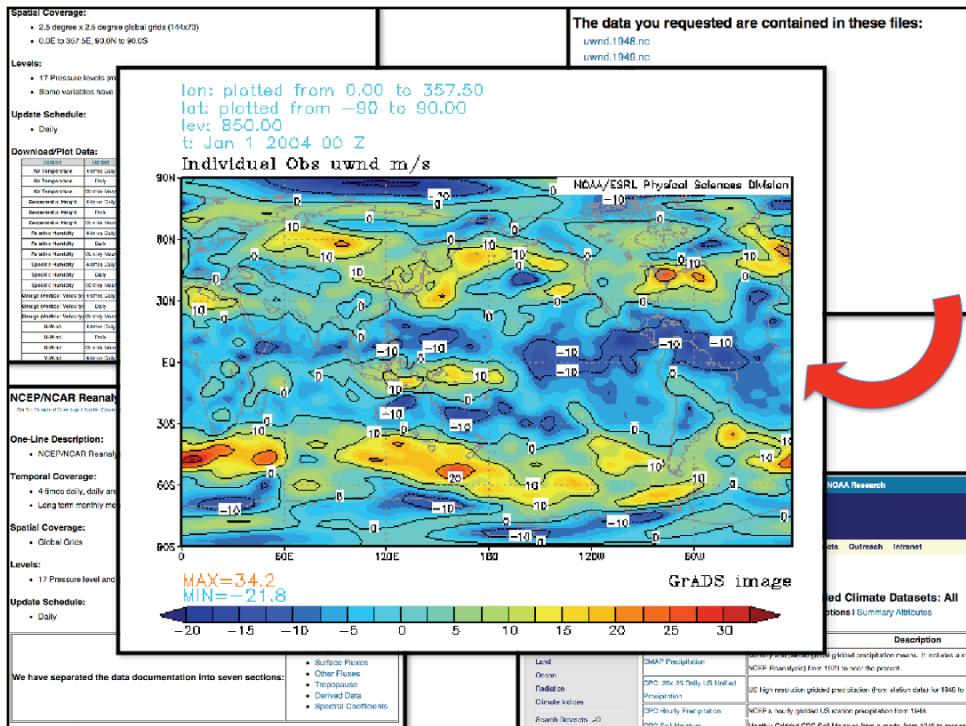
- Daily

We have separated the data documentation into seven sections:

4 times daily

netCDF files for each year from 1948 to present

Each file is about 500 MB



Zonal Wind (m/s) on 850 mbar level for 1 Jan 2004 with midnight on prime meridian

This is “data” – official record of U at this time in history.

Subsumes the wind obs whence this came.

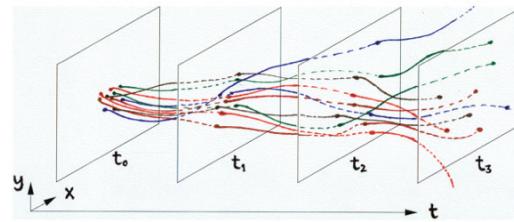
NOTE that this is very similar to the Mars Climate Database, except that instead of just being model output, this is the best estimate of what actually happened at this time on Earth!

More DA for the Mars atmosphere

* Please see abstract *

A little different than previous efforts:

- We employ an **Ensemble** of GCM integrations -- Monte Carlo
- Ensemble approximates the real-time relationships -- covariances -- between observations and the GCM's variables
- Information is spread across space, time, and other variables according to physics encoded in covariances



Several groups have performed DA for MGS/TES nadir retrievals

PLEASE see my abstract – much more information there than I can present here in 10 minutes

Next two talks about science results from the Oxford effort.



If **ensemble mean and variance** are close to **actual atmosphere's mean and variance**, then DA can estimate the “trajectory” through climatology that best agrees with the observations (to within their uncertainty)

====> **Gridded Climate Dataset**



If **ensemble mean and variance** are NOT close to **actual atmosphere's mean and variance**, then DA can help identify what model parameters are likely in error

====> **Model Improvement**

Treating climate as PDF, DA's ability to generate Gridded Climate Dataset depends on model being skillful

If not skillful, DA can instead be used to improve model

Evidence that DA can do both simultaneously!

Some benefits of using an ensemble

1. Can easily assimilate any observations that can be **Forward Modeled**, including Radiances:
 - this is what instruments like TES and MCS measure
 - terrestrial community found a clear benefit from assimilating radiances instead of retrievals
2. Ensemble DA automatically produces a compendium dataset of time-dependent, self-consistent error bars – ensemble spread
 - ... Could download U and δU

1. Retrievals inevitably have generally unknown correlated error structures

Really, DA and retrievals are one and the same -- not wise to assimilate an assimilation product

2. Treat sample standard deviation as uncertainty of state estimate

The pieces are in place (1)

* Please see abstract *



- **DART** is framework within which we are working
<http://www.image.ucar.edu/DARes/DART>



- **MarsWRF** is GCM with which we are working (Richardson et al., *JGR*, 2007)
<http://www.planetwrf.com>

Framework is important; choice of GCM is not! Could use CAM or FMS or other.

Both DART and WRF are examples of planetary science leveraging successful ideas from terrestrial atmospheric science

Ensemble DA is explicitly modular

DART already includes hooks for the Earth versions of CAM and FMS

The pieces are in place (2)

* Please see abstract *

- We have received **TES nadir Forward Operator** from TES team (big thanks to M. Smith, M. Kaelberer, and B. Conrath)
- TES Forward Operator is now “DART compliant”
- We have received **MCS limb Forward Operator** from MCS team (big thanks to A. Kleinböhl, T. Schofield, D. Kass, W. Abdou, and D. McCleese)
- Still have work to do with MCS Forward Operator

Can add Forward Operators for other mapping datasets!

A demonstration of ensemble DA

1. A “Cartoon” of ensemble DA – pseudo-meteorological variable

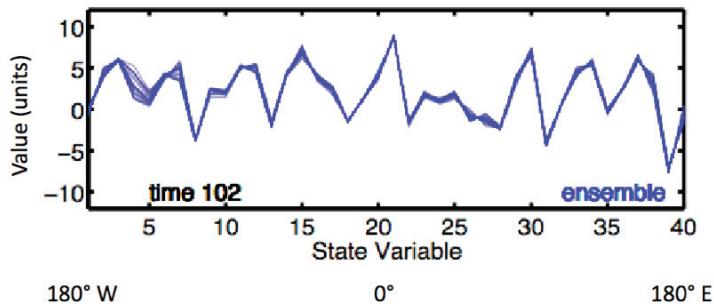
2. Some glimpses of a DA experiment using MarsWRF and synthetic remote sensing observations from a TES nadir-like instrument in an MGS-like orbit

Necessary, but not sufficient, test to pass

Before delving into a Large, 3D, Multivariate example, we first consider a cartoon of how this works

An ensemble DA cartoon

Get a “climatological” ensemble



Here we see output from an ensemble of model states for a pseudo-meteorological variable that will evolve under some specified dynamics.

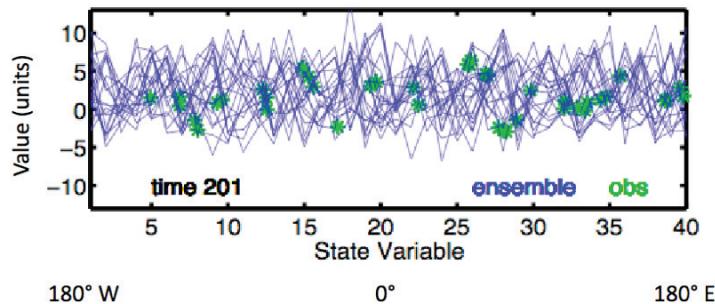
DON'T WORRY ABOUT DETAILS!!

Note periodic BCs and state is mainly comprised of waves

Here we allow our ensemble members to freely evolve into a climatological ensemble

An ensemble DA cartoon

Collapse ensemble to estimate of “truth”

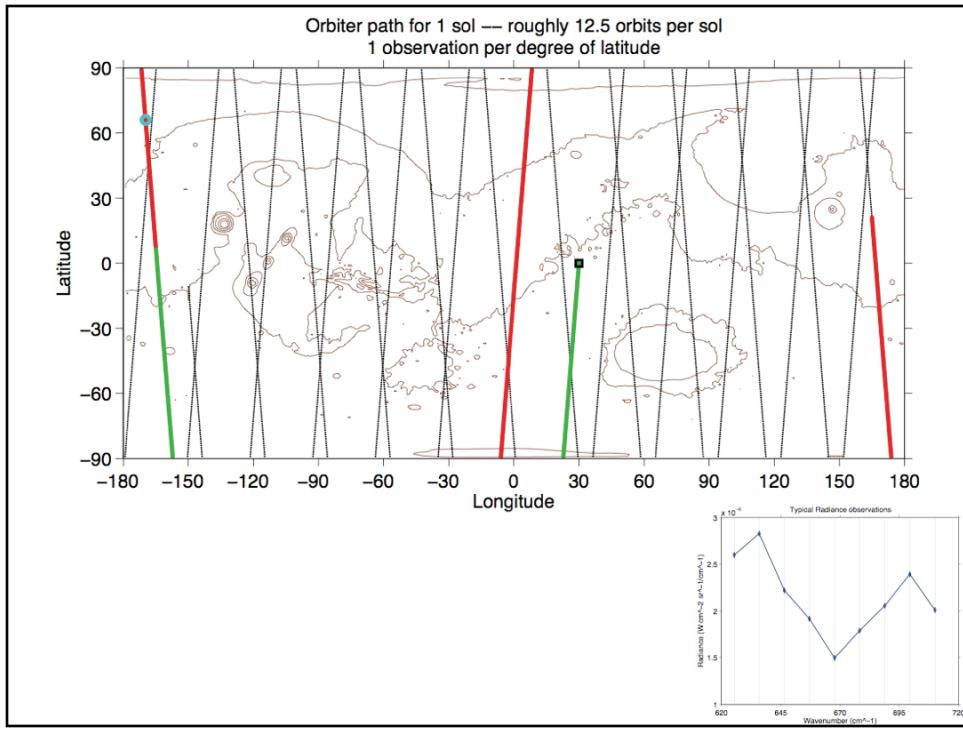


Now ensemble DA will collapse the climo ensemble to an estimate of Truth – only knowledge of truth is from observations (green *)’s

Truth evolves under same dynamics as ensemble members, so model and truth have same mean and variance

Gridded Climate Dataset is time sequence of ensemble mean;
Self-Consistent Error Bars is time sequence of ensemble spread;

Note state estimate is more certain than observations are -- Consider usefulness of Gridded Climate Dataset vs. record of values of green *’s



For our MarsGCM DA experiment: MGS-like orbit (~12.5 orbits per sol)

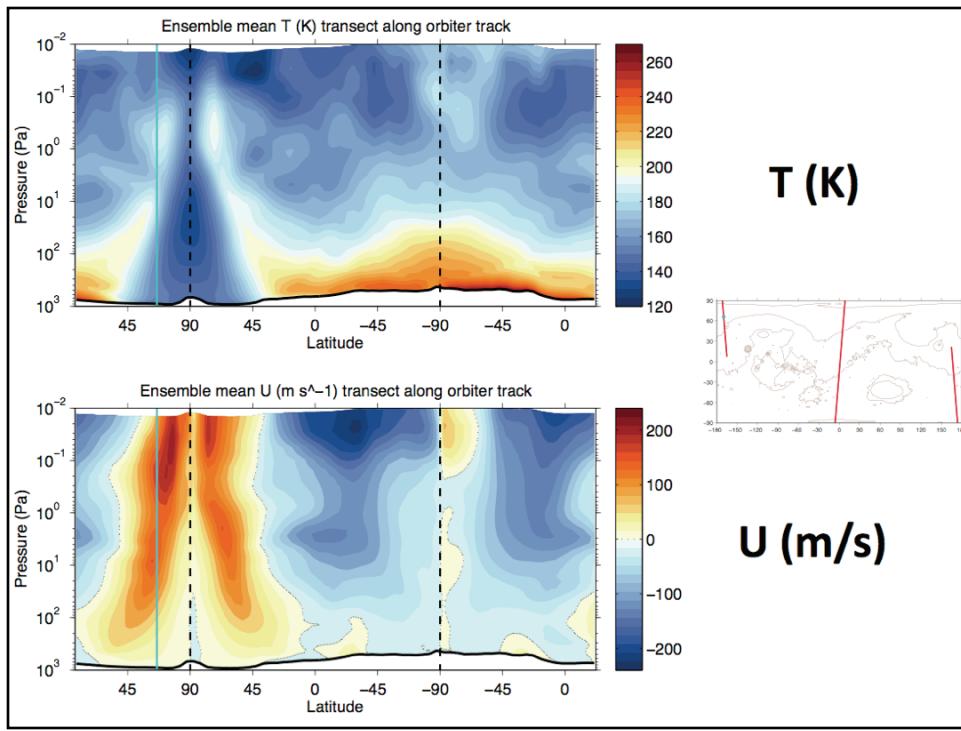
Green segment goes over South Pole;

Red segment defines a transect of observations from about 1 complete orbit – I will be showing model output along this transect;

Cyan circle shows location whence I'll be showing individual vertical profiles

Observations are TES nadir-like – synthetic radiances over the 9 central wavenumbers of the 15 micron feature (low-res scan);

These 9 are the main observations used for T(p) retrievals.

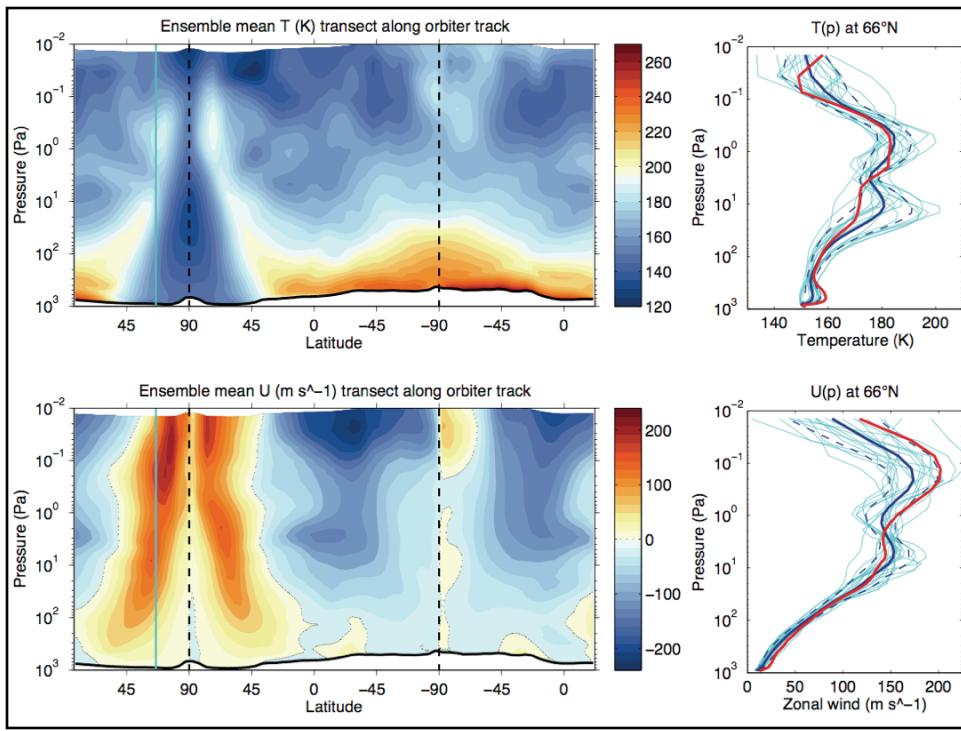


$L_{\text{sub_s}} \sim 260$; around sol 500 of Mars year;
 specified Dust;
 No H₂O cycle or clouds
 5 deg by 5 deg, 40 levels

Here is the ensemble mean along the red transect in the previous image: Latitude vs. Pressure (about 90 km)

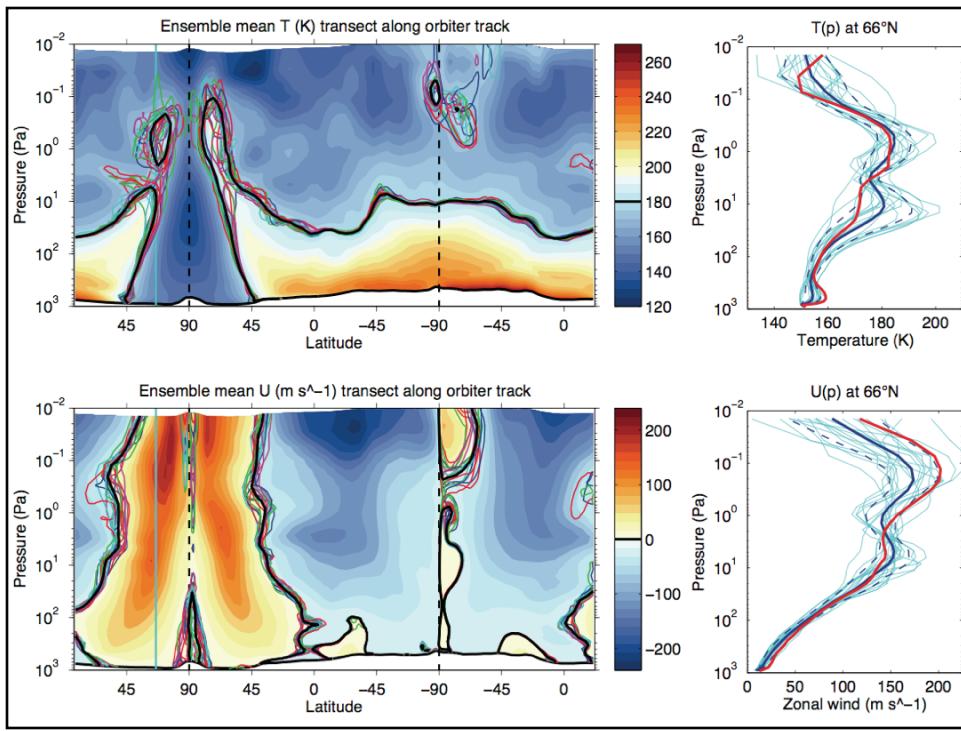
Ascending over the NP, Descending over the Equator and SP, and Ascending over the Equator

Note the northern Polar Vortex with temperatures high aloft and a strong jet stream



In effort to visualize climo spread, here are 20 individual $T(p)$ and $U(p)$ profiles at the cyan line

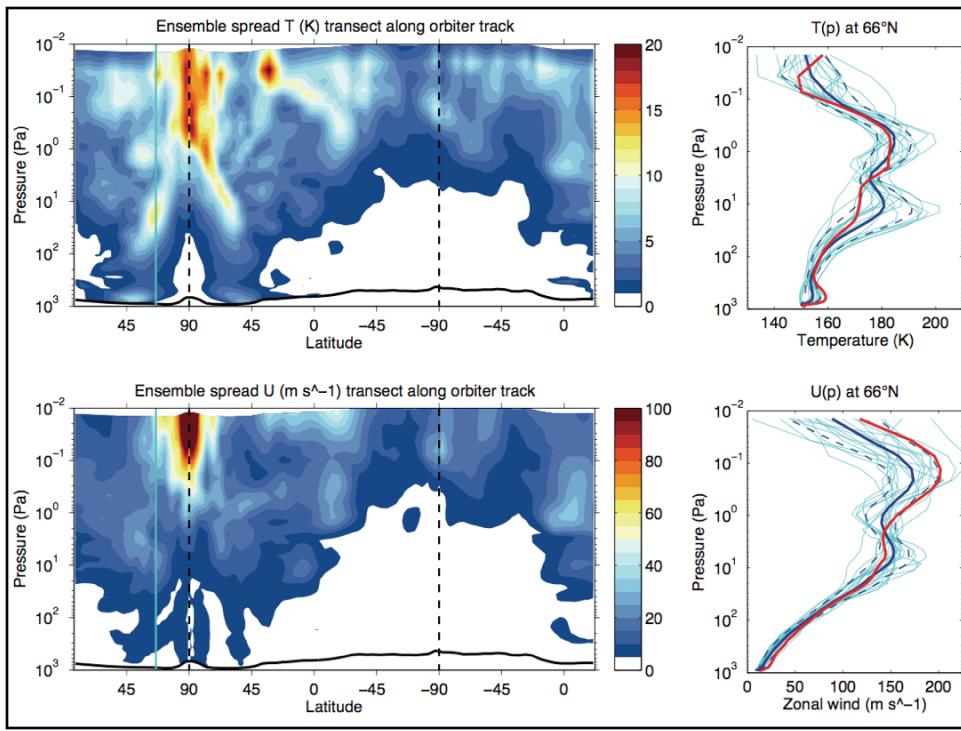
Cyan lines show members, Blue line show ensemble mean, Blue dashed lines show +/- 1-sigma ensemble spread, and Red line shows Truth



Another way to visualize climo spread – so-called Spaghetti Diagrams

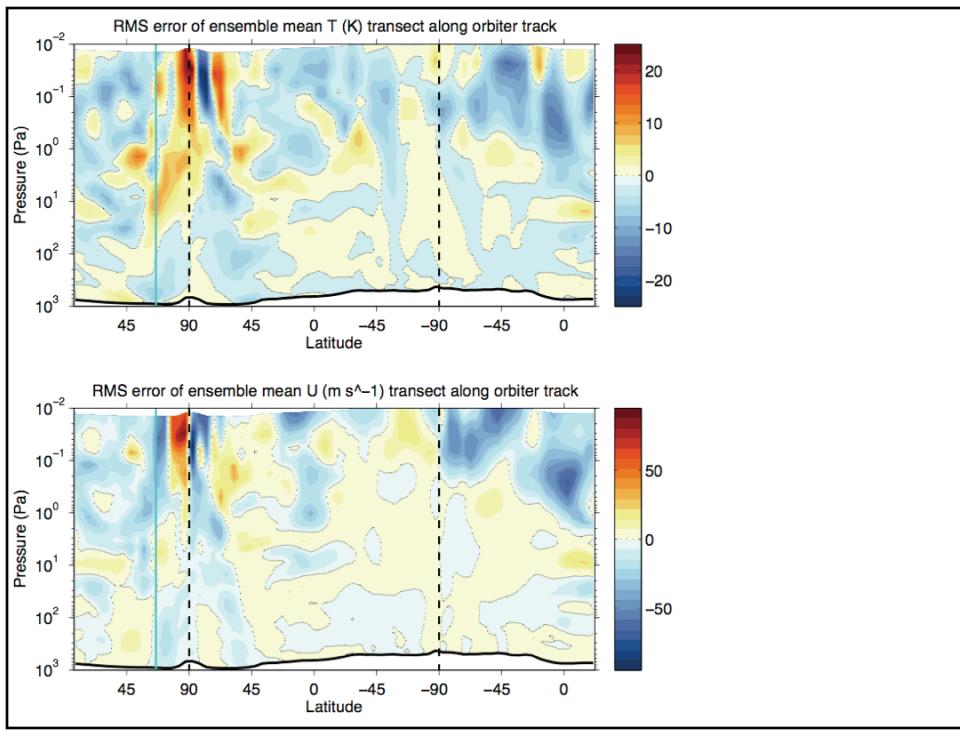
Ensemble mean 180 K contour is in thick black; all ensemble members' 180 K contour also plotted

Same with U 0 m/s contour



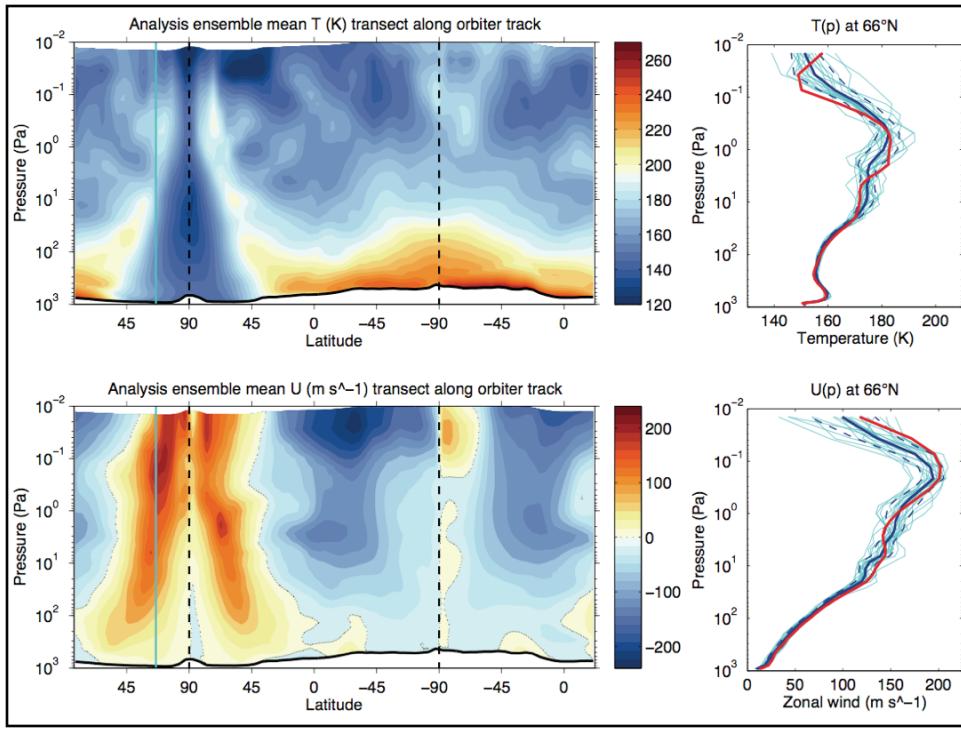
Yet another way to visualize climo spread – ensemble spread at all locations.

Greater than zero everywhere, but lowest values are plotted white: ensemble seems very certain of the S.H. lower atmosphere temperatures and that the air is condensing CO₂ over the winter pole.



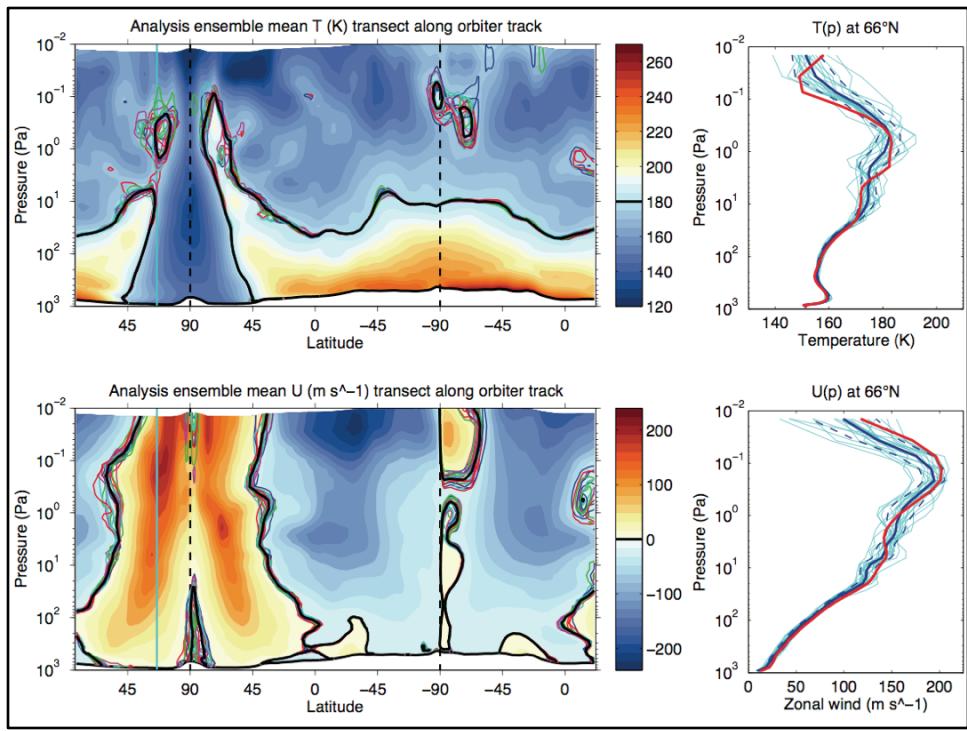
Since we have Truth available, we can actually find the RMS error value of the climo mean

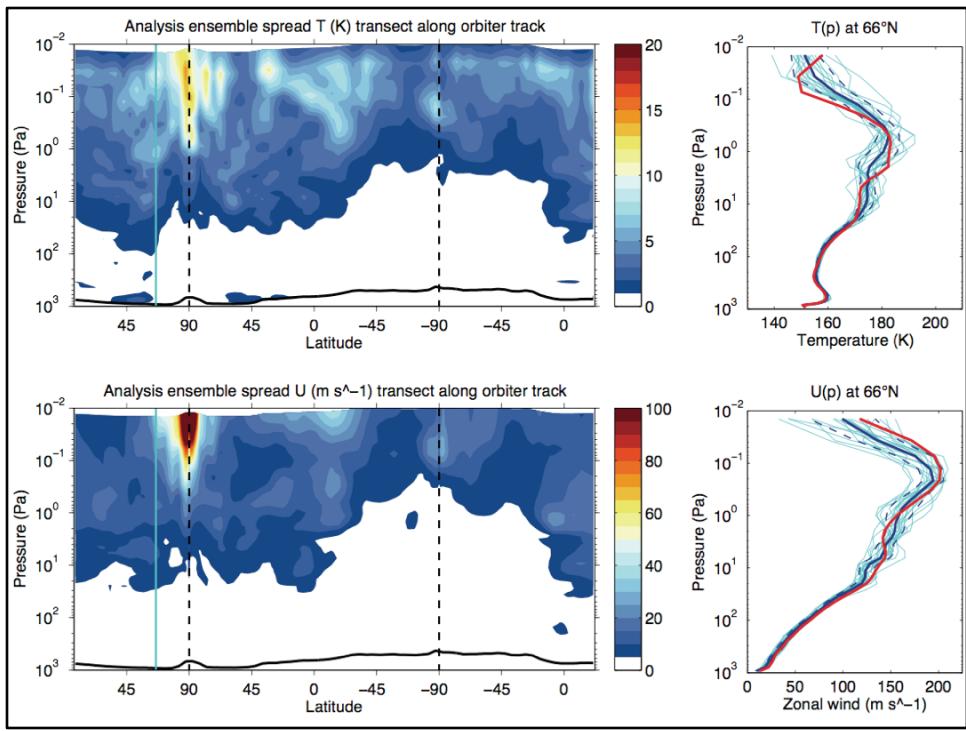
Consistent with mispositioned polar vortex

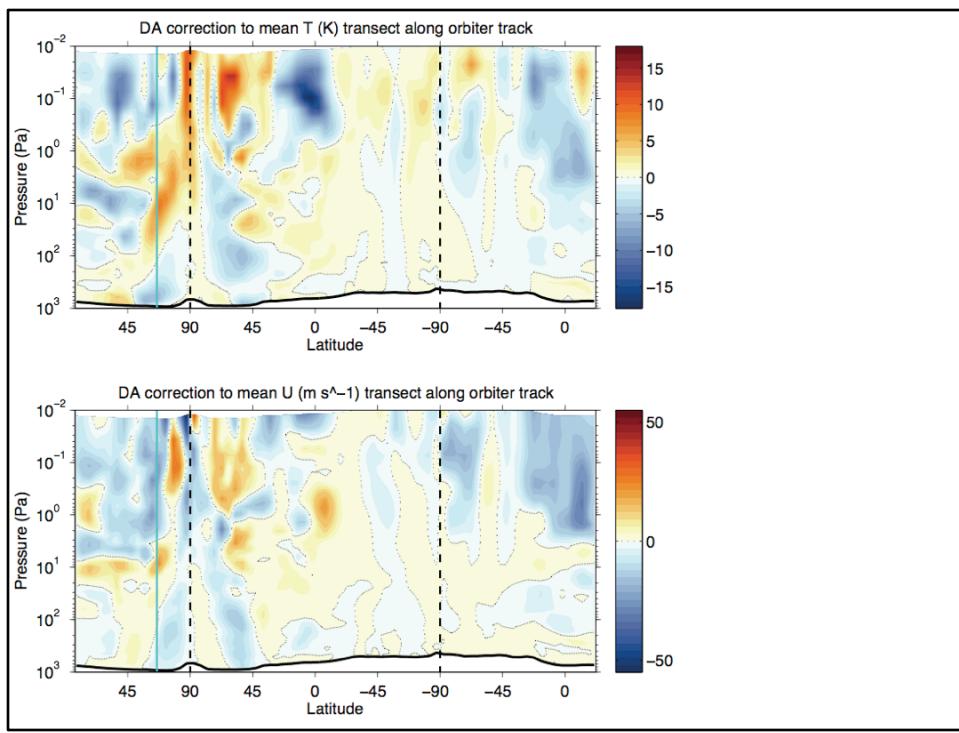


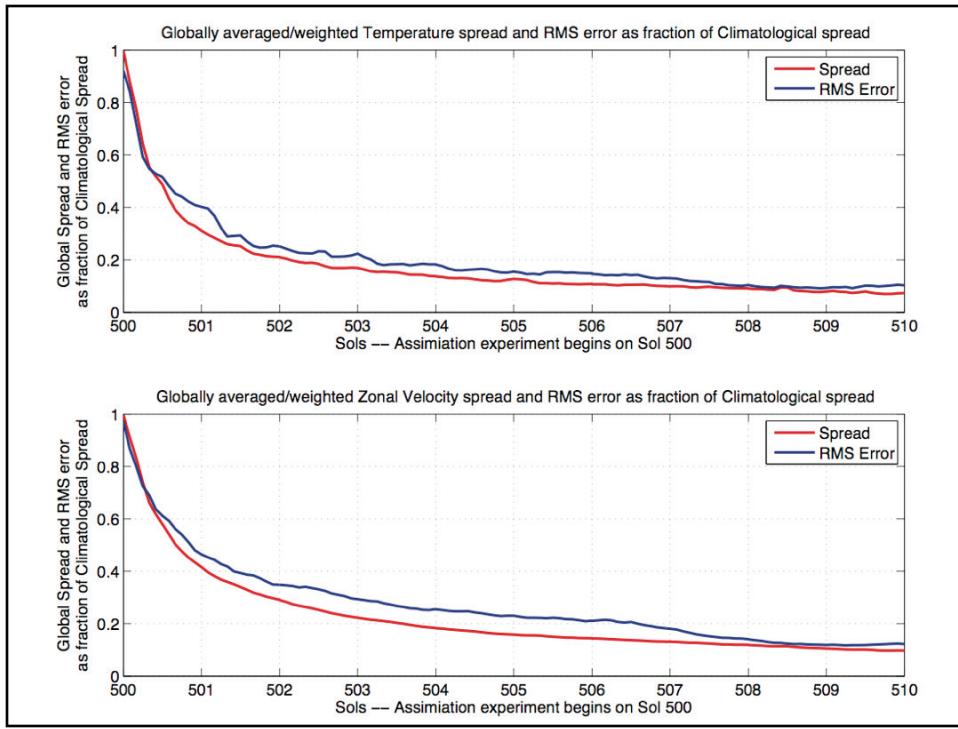
After we do the assimilation, all the ensemble members have been changed, hence so have the ensemble mean and spread.

Much tighter spread in lower atmosphere, but also tightened aloft









Defining a globally averaged value for the ensemble spread and for the RMS error of the ensemble mean compared to truth, we can watch a 10-sol time series of our ensemble estimate converge toward truth as we continue to assimilate TES-like observations.

Normalized as fraction of climo spread.

It's working! And it is correcting both temperature and zonal wind – other variables too!

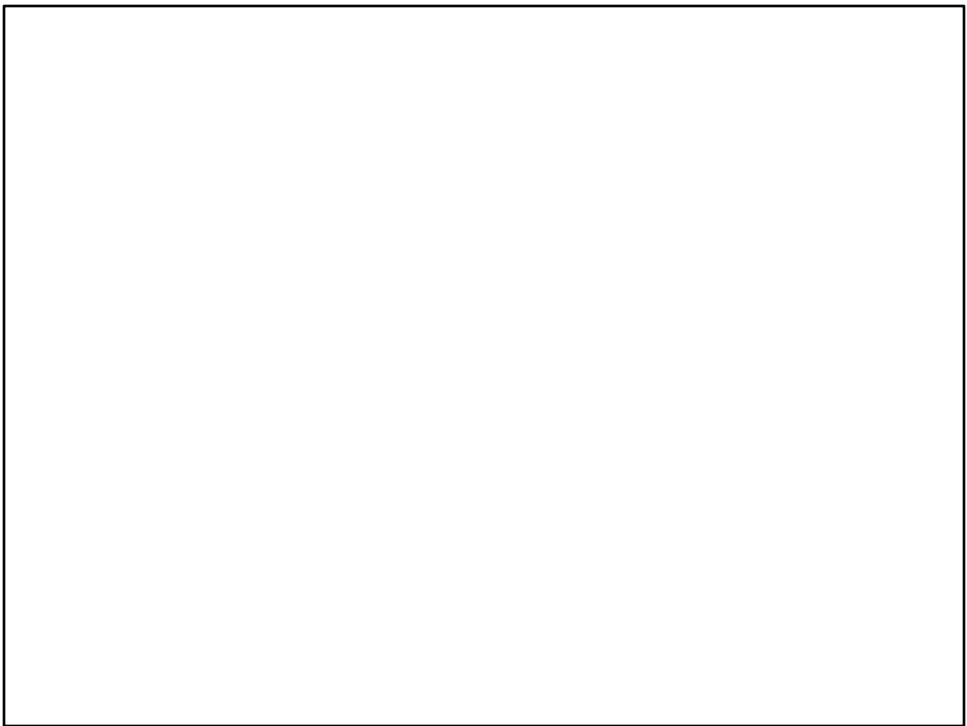
Last slide

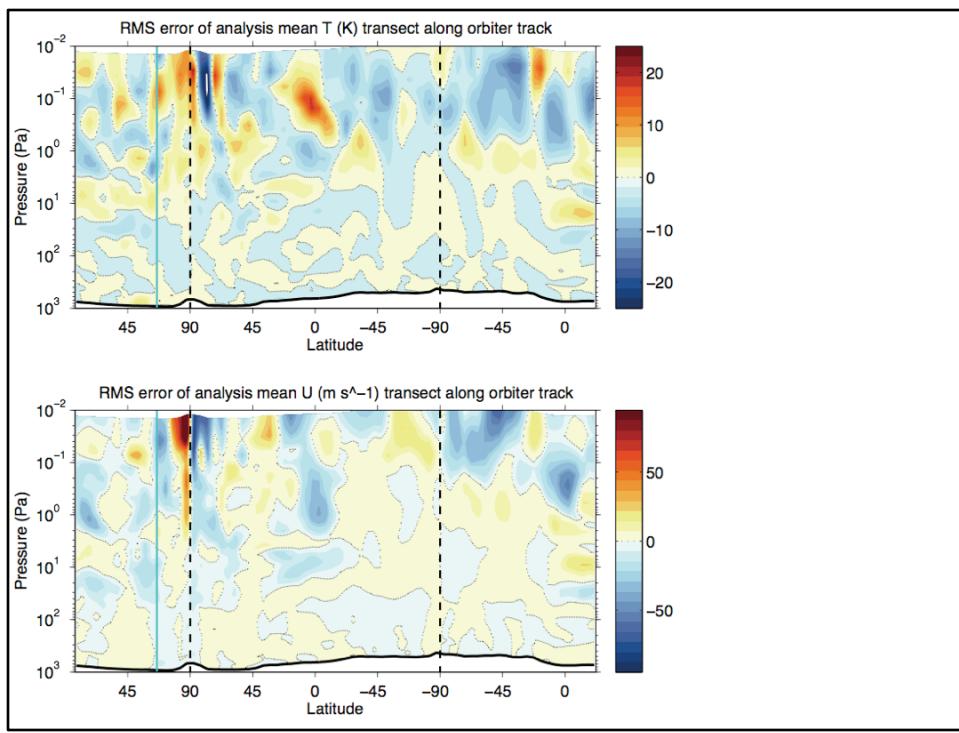
* Please see abstract *

- We are getting close to using actual data
- Need to explore more synthetic observation experiments with radiatively active dust and a water cycle in the model

Again, big thanks to many who are helping us:

- **DART** :: J. Anderson, N. Collins, and T. Hoar
- **MarsWRF** :: A. Toigo, C. Newman, and C. Lee
- **TES** :: M. Smith, M. Kaelberer, B. Conrath, and J. Bandfield
- **MCS** :: A. Kleinböhl, T. Schofield, D. Kass, W. Abdou, D. McCleese, and N. Heavens





Dynamics imparts covariance

What is the covariance between the pressure at the center of the Low and the zonal wind north of the Low (at "X")?

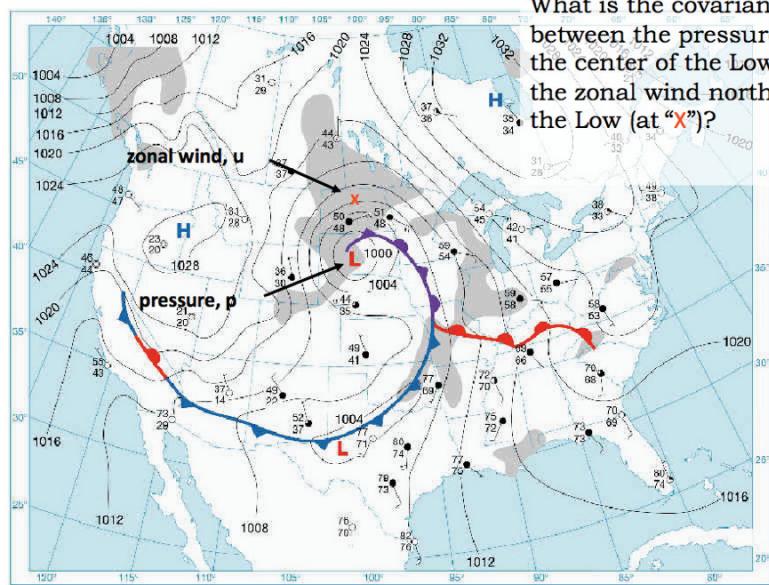


Image from CUNY

Geostrophic balance

$$u_{\text{north}} \approx -\frac{1}{\rho f} \frac{(p_{\text{north}} - p_{\text{center}})}{\Delta y}$$

$$u_{\text{north}} + \varepsilon_u \approx -\frac{1}{\rho f} \frac{(p_{\text{north}} - (p_{\text{center}} + \varepsilon_p))}{\Delta y}$$

$$\varepsilon_u = +\frac{1}{\rho f \Delta y} \varepsilon_p$$

Error expectations:

$$\langle \varepsilon_p^2 \rangle = \sigma_p^2$$

$$\langle \varepsilon_u^2 \rangle = \sigma_u^2 = \left(\frac{1}{\rho f \Delta y} \right)^2 \sigma_p^2$$

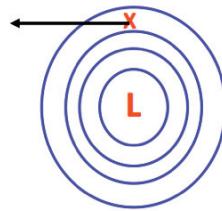
$$\langle \varepsilon_p \varepsilon_u \rangle = \text{cov}(p_{\text{center}}, u_{\text{north}}) = +\frac{1}{\rho f \Delta y} \sigma_p^2$$

Don't be scared by math!

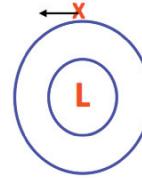
Geostrophic covariance

$$\mathbf{X}_{\text{geostrophic}} = \begin{bmatrix} p_{\text{center}} \\ u_{\text{north}} \end{bmatrix} \quad \mathbf{P}_{\text{geostrophic}} = \begin{bmatrix} \sigma_p^2 & \text{cov} \\ \text{cov} & \sigma_u^2 \end{bmatrix}$$

$$\varepsilon_u = + \frac{1}{\rho f \Delta y} \varepsilon_p$$



Stronger cyclone \Rightarrow
Stronger easterly wind



Weaker cyclone \Rightarrow
Weaker easterly wind

Next slide is full multi-dimension extension, so lead into it with how covariance allows correction of u .