Since our original goal was to maximize $f^T C f$ and $f^T C f = \lambda_1$,

$\Rightarrow$ we must maximize λ_1,

Ideally we should have λ_1 equal to the largest eigenvalue of C. But since e is the eigenvector corresponding to that, it would mean that e and f are equal (upto the ~~±~~ sign). Since, we are given that e and f should be $\perp$, we will have to choose the second-largest eigenvalue for λ_1.

$\Rightarrow$ f is the eigenvector of C corresponding to the second largest eigenvalue of C.

□