

PROBLEM 2

2.1

$$\begin{aligned}
 (a) \quad & P(q_{t+1}=k \mid q_t=j, o_1, o_2, \dots, o_T) \\
 &= P(q_{t+1}=k, q_t=j, o_1, \dots, o_T) / P(q_t=j, o_1, \dots, o_T) \\
 &= \alpha_t(j) a_{jk} b_j(o_{t+1}) \beta_{t+1}(k) / \alpha_t(j) \beta_t(j)
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad & P(q_{t-1}=i, q_t=j, q_{t+1}=k \mid o_1, o_2, \dots, o_T) \\
 &= P(q_{t-1}=i, q_t=j, q_{t+1}=k, o_1, \dots, o_T) / P(o_1, \dots, o_T) \\
 &= \alpha_{t-1}(i) a_{ij} b_j(o_t) a_{jk} b_k(o_{t+1}) \beta_{t+1}(k) / \sum_{i=1}^n \alpha_{t-1}(i) \beta_{t-1}(i)
 \end{aligned}$$

2.2

$$\begin{aligned}
 V_t(j) &= \max_{i=1}^n V_{t-1}(i) a_{ij} b_j(o_t) \\
 &= b_j(o_t) \left[\max_{i=1}^n V_{t-1}(i) a_{ij} \right] \\
 &= b_j(o_t) \left[\max_{l \neq j} \left(q \max_{i=1}^n V_{t-1}(i) \right), p V_{t-1}(j) \right] \\
 &= b_j(o_t) \left[\max \left(q \max_{i=1}^n V_{t-1}(i), p V_{t-1}(j) \right) \right]
 \end{aligned}$$

[This is possible only because $p > q$]

\Rightarrow At each time t , $n \ll O(T)$
 calculate $V_t = \max_{i=1}^n V_{t-1}(i) \ll O(N)$

for each state j , $\ll O(N)$

$$V_t(j) = \max(q \times V_t, p \times V_{t-1}(j)) b_j(o_t)$$

$$\Rightarrow O(T \times (N+N)) = O(NT)$$

2.3

Let $V_t(j)$ be the original quantity as in the viterbi algorithm. Let $W_t(j)$ be the quantity required in the problem.

$$V_1(j) = \pi_j b_j(o_1)$$

$$V_t(j) = \max_{i=1}^n V_{t-1}(i) a_{ij} b_j(o_t)$$

$$W_k(j) = \pi_j a_{jj}^{k-1} \prod_{i=1}^k b_j(o_i)$$

$$W_t(j) = \max \left(\max_{i=1}^n W_{t-1}(i) a_{ij} b_j(o_t), \right.$$

$$\left. V_{t-k+1}(j) a_{jj}^{k-1} \prod_{i=t-k+2}^t b_j(o_i) \right)$$

Intuition \Rightarrow You can either take t steps by choosing a path of $t-1$ steps with at least k repetitions and take 1 optimal step, or choosing a path of $t-k+1$ steps till state j and staying at state j for $k-1$ more steps.