CS763 (Computer Vision)
Spring 2019
Assignment 1

Rajat Rathi	160050015
Nikhil Tummidi	160050096
Gurparkash Singh	160050112

January 27, 2019

Contents

Answer 2

Given non linear distortion $\mathbf{x_d} = \mathbf{x_u}(1 + q_1 r + q_2 r^2)$
Therefore,

$$\begin{bmatrix} x_d \\ y_d \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_u(q_1 r + q_2 r^2) \\ 0 & 1 & y_u(q_1 r + q_2 r^2) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_u \\ y_u \\ 1 \end{bmatrix},$$

where r $= \sqrt{x_u^2 + y_u^2}$
We know that general mapping yields ${}^{\mathbf{a}}\mathbf{x} = {}^{\mathbf{a}}\mathbf{H_s}(\mathbf{x})\, {}^{\mathbf{s}}\mathbf{x}$,

so here ${}^{\mathbf{a}}\mathbf{H_s}(\mathbf{x}) = \begin{bmatrix} 1 & 0 & x(q_1 r + q_2 r^2) \\ 0 & 1 & y(q_1 r + q_2 r^2) \\ 0 & 0 & 1 \end{bmatrix}$ with r $= \sqrt{x^2 + y^2}$

Now, given ${}^{\mathbf{a}}\mathbf{x}$ $(i.e., \mathbf{x_d})$, ${}^{\mathbf{a}}\mathbf{H_s}(\mathbf{x})$ we need to solve for ${}^{\mathbf{s}}\mathbf{x}$ $(i.e., \mathbf{x_u})$

So we iterate on $\mathbf{x^{(i)}}$ with an initial guess of $\mathbf{x^{(0)}} = {}^{\mathbf{a}}\mathbf{x}$,

for the equation $\mathbf{x^{(i+1)}} = [{}^{\mathbf{a}}\mathbf{H_s}(\mathbf{x^{(i)}})]^{-1}\, {}^{\mathbf{a}}\mathbf{x}$ until it converges.

We get $[{}^{\mathbf{a}}\mathbf{H_s}(\mathbf{x^{(i)}})]^{-1} = \begin{bmatrix} 1 & 0 & -x^{(i)}(q_1 r + q_2 r^2) \\ 0 & 1 & -y^{(i)}(q_1 r + q_2 r^2) \\ 0 & 0 & 1 \end{bmatrix}$ where r $= \sqrt{\left(x^{(i)}\right)^2 + \left(y^{(i)}\right)^2}$

Therefore, this reduces to iteratively solving

$$\begin{bmatrix} x^{(i+1)} \\ y^{(i+1)} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -x^{(i)}(q_1 r + q_2 r^2) \\ 0 & 1 & -y^{(i)}(q_1 r + q_2 r^2) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_d \\ y_d \\ 1 \end{bmatrix}$$

$$\begin{aligned} \implies x^{(i+1)} &= x_d - x^{(i)}(q_1 r + q_2 r^2) \\ y^{(i+1)} &= y_d - y^{(i)}(q_1 r + q_2 r^2) \end{aligned}$$

where r if $i = k$ is $\sqrt{\left(x^{(k)}\right)^2 + \left(y^{(k)}\right)^2}$ and $x^{(0)} = x_d$, $y^{(0)} = y_d$ until it converges to get x_u and y_u.
The implementation of iteration is in code.

Answer 3

Given,

A Projective Transformation Matrix, $M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

A Hyperbola, $y = 1/x$.

We can represent the above hyperbola in parametric form as $X = \begin{bmatrix} t \\ 1/t \\ 1 \end{bmatrix}$ $(t \neq 0)$.

Our aim is to calculate $Y = MX$.

Substituting the values of M and X in the above expression, we get

$$\begin{aligned} Y = & \; MX \\ = & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} t \\ 1/t \\ 1 \end{bmatrix} \\ = & \begin{bmatrix} 1 \\ 1/t \\ t \end{bmatrix} \\ \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = & \begin{bmatrix} 1/t \\ 1/t^2 \\ 1 \end{bmatrix} (t \neq 0) \end{aligned}$$

$\therefore$, Final image of the hyperbola obtained under given transform is $y = x^2$ $(x, y \neq 0)$.

The above curve obtained after transformation corresponds to a Parabola, which is defined at all points except $(0, 0)$.

Answer 4

Any $3D$ vanishing point in $\mathbb{P}^3$ P can be projected to $\mathbb{P}^2$ P' using the relation

$$P' = KR[I_3| - X_O]P$$

Let L_1, L_2, L_3 be the vanishing points corresponding to the mutually perpendicular directions l_1, l_2, l_3 respectively. In case of vanishing points, we see that as the fourth coordinate (the homogeneous coordinate) is zero, the translation of the camera plays no part in the computation.

$$\begin{aligned}
L &= KR \begin{bmatrix} 1 & 0 & 0 & -t_x \\ 0 & 1 & 0 & -t_y \\ 0 & 0 & 1 & -t_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix} \\
&= KR \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\
&= KR\, l
\end{aligned}$$

Now, we have the following equations for both the cameras.

$$\begin{aligned}
P_1 &= K_1 R_1 l_1 & \qquad Q_1 &= K_2 R_2 l_1 \\
P_2 &= K_1 R_1 l_2 & \qquad Q_2 &= K_2 R_2 l_2 \\
P_3 &= K_1 R_1 l_3 & \qquad Q_3 &= K_2 R_2 l_3
\end{aligned}$$

Since l_1, l_2, l_3 are mutually perpendicular, from above, we get the following equations corresponding to camera P.

$$\begin{aligned}
& l_1^T\, l_2 = 0 \\
\implies & (R_1^{-1}\, K_1^{-1}\, P_1)^T\, (R_1^{-1}\, K_1^{-1}\, P_2) = 0 \\
\implies & (P_1^T\, K_1^{-T}\, R_1^{-T}\, R_1^{-1}\, K_1^{-1}\, P_2) = 0 \\
\implies & (P_1^T\, K_1^{-T}\, K_1^{-1}\, P_2) = 0 \quad (R_1\ is\ ortho-normal)
\end{aligned}$$

We model $K_1 = \begin{bmatrix} f_p/s_p & 0 & x_h \\ 0 & f_p/s_p & y_h \\ 0 & 0 & 1 \end{bmatrix}$ and $P_i = \begin{bmatrix} p_{ix} \\ p_{iy} \\ 1 \end{bmatrix}$

Substituting values for P_1, P_2, we get an equation in $x'_h, y'_h, f_p/s_p$, where $x'_h, y'_h = (f_p/s_p)x_h, (f_p/s_p)y_h$.

Using the other two equations $l_2^T l_3 = 0$ and $l_1^T l_3 = 0$, we get a total of three equations which we can solve to get the values of $x_h, y_h, f_p/s_p$ and thus K_1. Similar operations for camera 2 will give us K_2 as well.

From the above expressions, it is clear that even though we can compute value of f_p/s_p, we **cannot** compute the respective values of f_p and s_p. Similarly we cannot compute individual values for f_q and s_q.

Now we observe that

$$l_1 = R_1^{-1} K_1^{-1} P_1 = R_2^{-1} K_2^{-1} Q_1$$
$$\therefore K_2^{-1} Q_1 = R_2 R_1^{-1} K_1^{-1} P_1$$

Here $R_2 R_1^{-1}$ is the R which we need to infer as per the problem statement (relative rotations between two cameras).
Let $Q_1' = K_2^{-1} Q_1$ and $P_1' = K_1^{-1} P_1$ So,

$$\begin{aligned} Q_1' &= R\, P_1' \\ \textit{Similarly,} \quad Q_2' &= R\, P_2' \\ Q_3' &= R\, P_3' \end{aligned}$$

So,

$$\begin{bmatrix} Q_1' & Q_2' & Q_3' \end{bmatrix} = R \begin{bmatrix} P_1' & P_2' & P_3' \end{bmatrix}$$
$$\begin{bmatrix} Q_1' & Q_2' & Q_3' \end{bmatrix} \begin{bmatrix} P_1' & P_2' & P_3' \end{bmatrix}^{-1} = R$$

Where all P_i' and Q_i' are 3x1 column vectors and are calculated using the K_1, K_2 we derived and the P_i, Q_i provided.
So, we **can** calculate R from the above equation.

Since the translation does not play any role in the calculations (as we are dealing with vanishing points), there is no way to infer t from the given information.
This also makes sense intuitively because any translation of the camera should not result in change in the coordinates of vanishing points of a set of parallel lines.
In conclusion, with the given information,

1. we **can** infer **R**.
2. we **can not** infer **t**.
3. we **can** infer the ratio $\mathbf{f_p/s_p}$, $\mathbf{f_q/s_q}$, but we can not infer respective values of $\mathbf{f_p}$, $\mathbf{f_q}$, $\mathbf{s_p}$, $\mathbf{s_q}$.

Answer 5

(a) Let us consider a pair of parallel lines L_1, L_2 in $\mathbb{P}^3$ coordinate system.

$$L_1 = A + rP$$
$$L_2 = B + sP$$

Here, L_1 and L_2 are two parallel lines parameterized by r and s respectively and are in the direction P. Projection Matrix for a system with camera constant c is

$$C = \begin{bmatrix} c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Let us project the line L_1 onto the image plane. Let L_1' be the new line obtained.

$$\begin{aligned} L_1' = &\quad CL_1 \\ = &\begin{bmatrix} c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_x + rp_x \\ a_y + rp_y \\ a_z + rp_z \\ 1 \end{bmatrix} \\ = &\begin{bmatrix} c(a_x + rp_x) \\ c(a_y + rp_y) \\ c(a_z + rp_z) \\ a_z + rp_z \end{bmatrix} \\ = &\begin{bmatrix} \frac{c(a_x+rp_x)}{a_z+rp_z} \\ \frac{c(a_y+rp_y)}{a_z+rp_z} \\ c \\ 1 \end{bmatrix} \end{aligned}$$

Now, since we are considering vanishing points, we will see the point where the points at infinity are projected onto the image plane. Taking the limiting case of $r \to \infty$. Therefore the point in image plane where the point at infinity in world coordinates is projected is

$$L_1' = \begin{bmatrix} \frac{cp_x}{p_z} \\ \frac{cp_y}{p_z} \\ c \\ 1 \end{bmatrix}$$

Further, since we know the position of image plane, we can drop the third coordinate for convenience.

$$L_1' = \begin{bmatrix} \frac{cp_x}{p_z} \\ \frac{cp_y}{p_z} \\ 1 \end{bmatrix}$$

Similarly, doing the calculations for L_2', we get

$$L_2' = \begin{bmatrix} \frac{cp_x}{p_z} \\ \frac{cp_y}{p_z} \\ 1 \end{bmatrix}$$

From the above calculations, we see that the points of projections for both the lines are same in the image plane for the limiting case. Therefore we conclude that that the projections (in image plane) of any two parallel lines L_1 , L_2 in $\mathbb{R}^3$ have an intersection point, the vanishing point.

(b) Let the three sets of parallel lines have direction vectors P, Q and R respectively. Since we are given that these vectors are co-planar, we get $P\ .\ (\ Q \times R\ ) = 0$.

$$\begin{vmatrix} p_x & q_x & r_x \\ p_y & q_y & r_y \\ p_z & q_z & r_z \end{vmatrix} = 0$$

Multiplying by c^2, on both sides and using properties of determinants, we get

$$\begin{vmatrix} c\,p_x & c\,q_x & c\,r_x \\ c\,p_y & c\,q_y & c\,r_y \\ p_z & q_z & r_z \end{vmatrix} = 0$$

Now, from part(a), the above expression can be written as $|\,CP \quad CQ \quad CR\,| = 0$. Further simplifying, $CP\ .\ (\ CQ \times CR\ ) = 0$. Again, from part(a), we know that CP, CQ and CR are the points of intersection of pairs of parallel lines in the image plane. If L_1', L_2' and L_3' are the three points of projection, we have, $L_1'\ .\ (\ L_2' \times L_3'\ ) = 0$.

Now, if we have 3 points and we want to prove that they are co-linear, we will prove that one of the points lies on the line that passes from the other two points.

Here, we have 3 points, L_1', L_2' and L_3'. The line that passes through L_2' and L_3' is given by $(\ L_2' \times L_3'\ )$. If L_1' were to pass through this line, then $L_1'\ .\ (\ L_2' \times L_3'$

$) = 0$. But this is true as proved earlier in the answer.

Therefore it follows that he vanishing points corresponding to three (different) sets of parallel lines on a 3D plane are co-linear in the image plane.

Answer 6

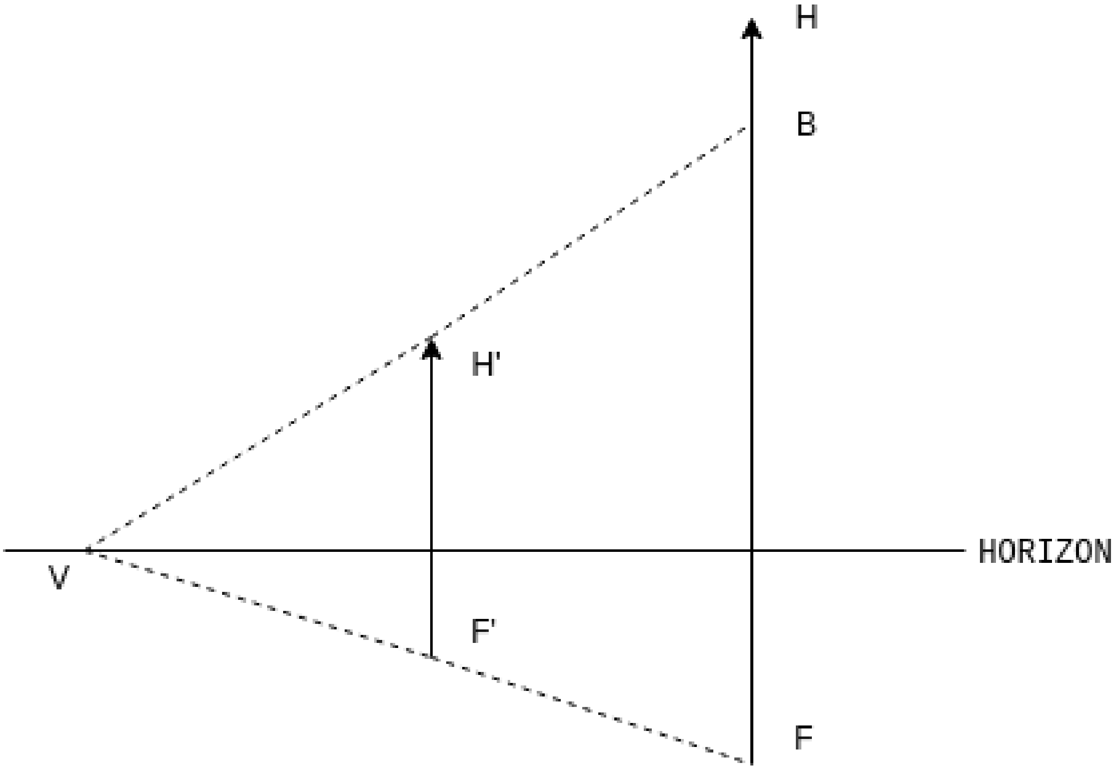

Figure 1: Simplified view of the image. H'F' is Christ and FH is the required height.

Let $Real\,(XY)$ be the distance between points X and Y of the image in real world.

Given, height of Christ $Real\,(F'H') = 180\,cm$. We know that F and F', i.e. the feet of two persons will lie on the ground. The line joining these two points will intersect horizon at the vanishing point V. By using $impixelinfo$ in MATLAB, we got the pixel coordinates for points $F'\,(424, 682)$, $F\,(746, 798)$, $H'\,(424, 434)$, $H\,(746, 219)$. We also got equation of Horizon as $y = 579$ from $impixelinfo$.

Let the point V will be $(x, 579)$. From the line equation of VF,

$$\frac{y_F - y_V}{x_F - x_V} = \frac{y_{F'} - y_V}{x_{F'} - x_V}$$

$$\implies \frac{798 - 579}{746 - x} = \frac{682 - 579}{424 - x}$$

$$\begin{aligned} 116x &= 424 * 219 - 103 * 746 \\ x &\approx 138 \end{aligned}$$

Therefore pixel coordinates of V are $(138, 579)$. Now, from V, we join H', and extend it to intersect the person at B. Now, since lines FF' and BH' are parallel to each other in real world, (as they have same vanishing point) $\textit{Real}\,(FB) = \textit{Real}\,(F'H') = 180\,cm$.

Since equation of line FH is $x = 746$, let the point B be $(746, y)$. From line equation of VB,

$$\begin{aligned}
\frac{y_B - y_{H'}}{x_B - x_{H'}} &= \frac{y_B - y_V}{x_B - x_V} \\
\implies \frac{y - 434}{746 - 424} &= \frac{y - 579}{746 - 138} \\
286y &= 434 * 608 - 322 * 579 \\
y &\approx 271
\end{aligned}$$

Therefore, the pixel coordinates of B are $(746, 271)$.

We can observe that the vertical set of parallel lines like H'F' and HF have vanishing point (say, V') at *infinity* (i.e., they do not converge) and are perpendicular to the normal of image plane, we can directly apply the following formula for computing the cross ratio of {F, B, H, V'}.

$$\begin{aligned}
\frac{\textit{Real}\,(FH)}{\textit{Real}\,(FB)} &= \frac{FH}{FB} \\
\frac{\textit{Real}\,(FH)}{180} &= \frac{798 - 219}{798 - 271} \\
\textit{Real}\,(FH) &\approx 198\,cm
\end{aligned}$$

So, the estimated height of person on the right is $198\,cm$.