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Planning Home Assistance for AIDS Patients in the City of Rome, Italy

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In allocating resources to health-care services, the main difficulties derive from uncertainty concerning the number of patients who need the service and the level of care required by each patient who is entitled to receive it. I deal with the problem of allocating resources to the home-care service provided to AIDS patients in the city of Rome, Italy, which include medical assistance provided by nurses and doctors and social assistance and accompaniment of patients provided by social workers. A simple linear programming model solves the problem for single organizations, which provide assistance within a given budget, and for public-health authorities, which have to evaluate the results of tentative budgets assigned to home care.

Health-service planners have difficulty in allocating resources between services because the number of patients who need services is uncertain and the level of care each entitled patient requires is uncertain. I developed a simple linear programming model to solve the problem of allocating resources to the home-care services supplied to AIDS patients in the city of Rome, Italy.

Home care, introduced in Italy in 1990 by a law on interventions to prevent and fight AIDS, and better defined by a public health act in 1991, includes three types of assistance: medical assistance, social assistance, and accompaniment of patients. Medical care is provided by nurses and doctors and is funded by regional authorities; the second and third types of care are carried out by social workers and funded

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by town authorities.

Organizations that provide home care cover the three forms of assistance with their medical personnel (I make no distinction between nurses and doctors in the model) and social workers. These organizations work with a limited budget and must provide a minimum standard of service. Lack of balance between patient needs and available resources leads to a low standard of service or to excessive work for the staff or to both, which cannot be sustained for long.

Public-health authorities must set overall health policies, subject to social and political issues, which can also influence decisions on assistance to AIDS patients.

Thus, a basic piece of information for planning home health care for AIDS patients is the number of potential users of the service. This requires an accurate forecast of the HIV/AIDS epidemic, which can be supplied by an epidemiological model, such as Rossi's [1991]. The model I developed is, in fact, linked to Rossi's epidemiological model as used in a project concerning the city of Rome [Rossi, Schinaia, and De Angelis 1996; Schinaia et al. 1993].

Based on such information, public-health authorities decide how to assign the overall budget for health care to the various forms of assistance, such as hospital, day hospital, and home care.

I distinguish between a *local problem*, the resource allocation problem a single assistance organization faces, and a *global problem*, which is of interest to public-health authorities. For the single organization, the solution to the resource-allocation problem lies in a scheduled admission of new patients, since its task consists of logistically

organizing the actual assistance within the assigned budget limits.

With regard to public-health authorities, associated with each budget for home care is an optimal admission schedule, which gives the total number of patients the organization can assist in the planning period and still meet the minimum standard of service. This schedule quantifies the financial effort put into home care and is fundamental in supporting the decision on budget allocation.

I constructed a model that produces an optimal schedule for admitting new patients to the home-health-care system, subject to constraints on available resources. The model takes into account the minimum standard of service, as well as uncertainty regarding the level of service each patient requires.

This model solves both the local problem, the resource allocation problem a single organization faces, and the global problem public-health authorities face. Public-health authorities can use it, in fact, in an interactive way to evaluate the consequences of a tentative budget before approving or modifying it. I implemented the model with data collected from Operatori Sanitari Associati (OSA), one of the organizations in charge of home care in Rome, and the results are very satisfactory.

Assistance Requirements and Data Description

Patients requiring home care are subdivided into five classes of dependency, each characterized by a different degree of need. The classes are, in increasing order of dependence,

(1) self-sufficient,

(2) leaves home unaccompanied,

(3) leaves home accompanied,

(4) does not leave home, and

(5) permanently in bed.

In addition, a patient may leave the system due to being

(6) temporarily in a hospital, or

(7) dead.

Patients in a given class do not have a constant need for the service; weekly demand is a random variable, whose mean and standard deviation (Tables 1–3) I derived from data drawn from the records of the needs of 88 patients assisted by OSA for three months, totalling 538 patient-weeks. I measured the resource nurses and doctors as the number of interventions (with no distinction among visits, phlebo-

Class	Mean (μ)	Standard deviation (σ)
1	3.920	2.013
2	3.520	2.243
3	3.070	2.286
4	2.760	2.542
5	3.440	1.663

Table 1: The number of medical interventions per week is highest for AIDS patients who are still self-sufficient.

Class	Mean (μ)	Standard deviation (σ)
1	3.690	4.254
2	2.390	3.757
3	5.650	6.809
4	7.310	7.899
5	6.860	6.883

Table 2: As patients' conditions decline, they require more hours of social assistance per week, with the highest requirements for those who are permanently at home but not yet completely bedridden.

Class	Mean (μ)	Standard deviation (σ)
1	0.850	0.876
2	0.990	1.277
3	1.164	1.030
4	0.840	0.869
5	1.075	0.974

Table 3: The highest number of patient accompaniments per week are required by those who leave home only if accompanied.

clysis, injection, and so forth) per week and the resource social workers in hours per week.

Each patient accompaniment is equivalent to three hours of social assistance.

I calculated transition rates, according to which patients stay in the same class or move to another one, in weekly time steps (Table 4). I drew data from the records of the same 88 patients, looking back through their medical histories for up to two years.

The classification of patients into classes of dependency and the use of transition rates from one class to another is not an original feature of my model; it is shared, in fact, by most models that deal with planning of long-term assistance [Lagergren 1994; Leff, Dada, and Graves 1986]. The use of dependency classes, combined with transition rates, enables me to plan the use of resources for a number of weeks into the future by projecting the conditions of patients over time.

Evaluating Home Assistance

I relied on some approximations to construct simple linear constraints that relate the variable demand to the available quantity of each resource:

(1) a central-limit-theorem-based approximation that represents the total service de-

Class	1	2	3	4	5	6
1	0.9575	0.0160				0.0260
2		0.9540	0.0210			0.0230
3			0.9190	0.0300		0.0500
4				0.9150	0.0330	0.0360
5					0.8050	0.1100
6	0.0510	0.0530	0.0680	0.0200	0.0310	0.7480

Table 4: Transition rates within the same classes are the highest.

mand of the patients in each class; (2) a linear approximation for the standard deviation of the total service demand of the patients in any one class.

Approximation (1) is very reasonable, since the needs of the patients are independent and the number of patients in each class is sufficiently large in a single organization. Because of it, the total weekly need of n patients in the same class can be assumed to be normally distributed with expected value $n\mu$ and standard deviation $\sqrt{n}\sigma$.

Such approximation allows me to use quantiles of the cumulative normal distribution in dealing with the variability in weekly demand for a given type of assistance arising from n patients in the same class, within the framework of the chance-constrained approach to stochastic programming. The choice of the quantile depends on the minimum standard set for the service, expressed as the probability that the service can satisfy the needs of its patients. The higher the value of the probability, the higher the value of the quantile.

The mathematical expression of the service demand from n patients in the same class as a function of the chosen quantile is a difficult one to work with in an optimization model. The linear approximation (2), presented in Figure 1, enables me to simplify this expression. This approximation requires only that I fix lower (l) and upper (u) limits on the potential number of patients in each class of dependency.

A conservative estimate of the relative error is provided by $a/q(l)$, where a is the maximum difference between the two functions and is, in general, quite negligible. The approximated expression of the service demand is reported in the appendix.

Such expression is calculated for each class, and the sum of the results is used to construct the resource availability constraint. Multiplying the probabilities assigned to each class, we get an underestimate of the probability of satisfying the resource-availability constraint; a larger service demand from a class may be, in fact, compensated by a smaller demand
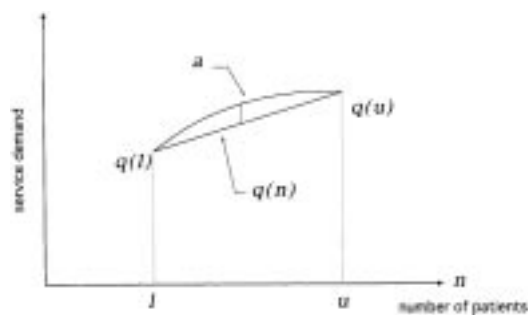
Figure 1: The service demand allows for a good linear approximation.

from the other classes. The exact probability, though, cannot be calculated analytically.

The movement of patients from one class to another, in weekly time steps, is obtained by applying the transition rates.

The Model

The goal of the model is to produce an optimized admission schedule for a given planning period. I used a 12-week planning period. I give the mathematical details of the stochastic linear programming model in the appendix.

The objective is to maximize the sum of the number of patients that can be admitted to home care each week; patients in different classes may be given different weights to express priority.

The model incorporates two kinds of constraints:

—constraints that relate the number of assisted patients in week t to the number of old assisted patients in week $t + 1$ for each class of dependence;

—constraints that relate the total quantity needed of each resource by the old and new assisted patients in week t to the available quantity, using the approximations previously introduced.

Numerical Application and Results

As applied to the city-of-Rome case, home assistance is restricted only to patients in classes 4 and 5, according to the policy implemented by OSA at the moment. The number of medical interventions that can be carried out by the assistance unit is 555 per week, and the hours of social work available are 1,033 per week. I utilize a probability value of .95 that the service can satisfy the needs of its patients for each class and service. I initial-

ize the model with 22 class 4 patients, 50 class 5 patients, and four temporary inpatients. The lower and upper bounds, respectively, are 18 and 30 for class 4, and 40 and 60 for class 5.

The objective is to maximize the total number of patients admitted to home care in the 12-week period.

As an example of a constraint of the first type, the number of existing class 5 patients to be assisted in week 3 is given by the sum of the number of existing class 4 and class 5 patients assisted in week 2 multiplied, respectively, by the transition rates 0.033 and 0.805 and of the number of new class 4 and class 5 patients admitted in week 2 multiplied, respectively, by 0.033 and 0.805 and of the number of temporary inpatients in week 2 multiplied by 0.031.

Constraints of the second type depend on the lower and upper bounds fixed for the number of patients of each class. A conservative estimate of the relative error introduced by the linear approximation of the standard deviation of the service demand amounts, in the case of the number of medical interventions, to 0.2 percent for class 4 and 0.06 percent for class 5, and in the case of hours of social work, to 0.2 percent for class 4 and to 0.09 percent for class 5. In all cases, the error is acceptable. Figures 2 through 5 show the results I obtained in this application (rounded to the closest integer).

New admissions are distributed rather evenly among the 12 weeks. They amount to 15 new patients in class 4 and 118 new patients in class 5, for a total of 133.

The total number of assisted patients is stable because the quantities of available
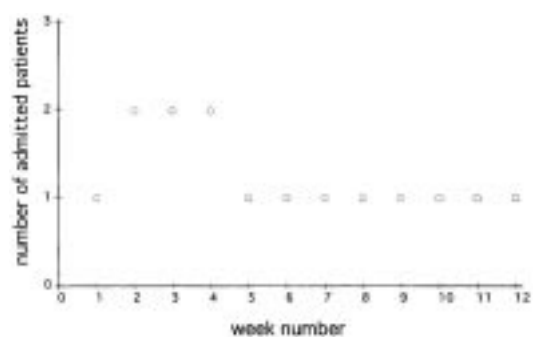
Figure 2: The number of new admissions of class 4 patients ranges between one and two per week.
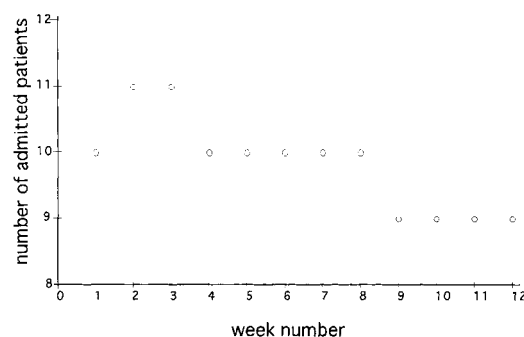

Figure 3: The number of new admissions of class 5 patients ranges between nine and 11 per week.
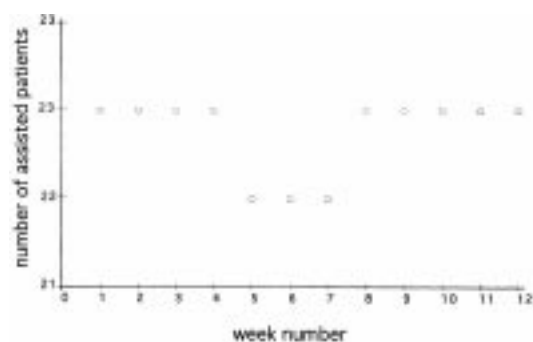

Figure 4: The total number of assisted class 4 patients ranges between 22 and 23.
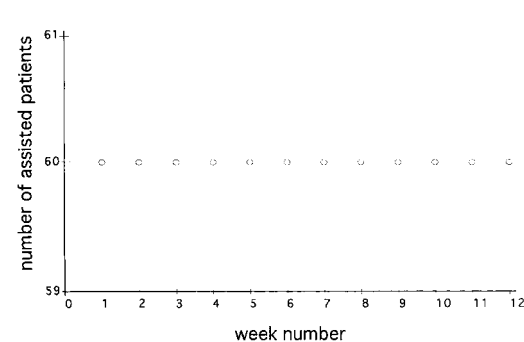

Figure 5: The total number of class 5 assisted patients is 60 constantly.

resources were constant in the various weeks.

The output of the linear programming problem solves the local problem: the single assistance organization is supplied with an admission schedule that takes into account the variability of the weekly demand for assistance and the transitions among classes.

The model can also be used to solve the global problem of setting resource levels. The linear programming output corresponding to a tentative resource budget, considered in light of the forecast from an epidemiological model, can give public-health authorities insight into the suitabil-

ity of the resource budget. Then, interactively, they can evaluate other choices: for example, they can change the weights in the objective function, they can restrict home care to fewer classes of dependence (as in the city-of-Rome application), they can try different quantiles, or they can increase the home-care budget.

Conclusions

I formulated a linear programming model that optimizes the admission schedule of new patients to home-care service. The model takes into account the variability of the quantity of assistance needed by any patient and takes steps to guarantee a minimum service standard during periods

of peak demand.

The model is easy to construct. It solves the problem facing home-care organizations, which work with fixed budgets. Public-health authorities can use the model to support decision making by evaluating the effects of different budget assignments. The model is useful at both local and global levels and is a practical and flexible tool for public-health planners.

Work is being carried on at present to introduce the model in the global planning of resources for home care to AIDS patients in the city of Rome, utilizing data collected centrally and the experience of a great number of organizations that provide such assistance.

Acknowledgment

APPENDIX

The Transition Rates

The formula I used to calculate the transition rates p_{ij} from class i to class j from one week to the next is

$$p_{ij} = \frac{\sum_{t=1}^{W} o_{ijt}}{\sum_{t=1}^{W} o_{it}},$$

where $W + 1$ is the number of weeks covered by the available data, o_{it} the number of patients in class i observed in week t, and o_{ijt} the number of patients moving from class i to class j between t and $t + 1$.

The Linear Approximation

The linear approximation for the quan-
tity of assistance $q(n)$ needed by n patients is expressed by the set of constraints:

$$q(n) = q(l)\lambda + q(u)\nu$$
$$n - l\lambda - u\nu = 0$$
$$\lambda + \nu = 1$$
$$\lambda \geq 0, \nu \geq 0$$

where

$$q(l) = l\mu + \sigma \sqrt{l}z_\alpha$$
$$q(u) = u\mu + \sigma \sqrt{u}z_\alpha$$

being z_α is the quantile of the cumulative normal distribution corresponding to the probability α.

The General Model

My linear programming model [De Angelis 1996] is based on the following assumptions:

—Patients are classified into C classes, of which $C-1$ refer to levels of dependency, while the last class refers to patients who are temporarily discharged from home care because they are temporary inpatients and therefore require no home care.

—There are A types of assistance.

—There are R resources.

—The planning horizon amounts to T time units;

—For each class i of patients and each type a of assistance, I define a probability α_{ia} ($0 \leq \alpha_{ia} \leq 1$); this is the probability that the quantity of type a assistance needed by the whole class i does not exceed the quantity available. The values α_{ia} chosen define the standard of the service offered.

—The transition of patients from one class to another is ruled by transition rates, with a matrix P of transition rates. The variables represent the number of old and new patients at the beginning of the time units. For simplicity, they are not restricted to be integer; the solution to the LP can be rounded off to the nearest integer values. De Angelis and Milanese [1995] describe a version of the problem with integer variables, but the increase in calculations required does not produce enough benefits to justify its use.

The model incorporates two types of constraints:

(1) Resource availability constraints guarantee that the total amount of each resource required in each time unit does not exceed the total amount available. The quantity of resource r needed to supply type a assistance to n_i patients of class i, calculated on the basis of the probability level α_{ia}, is given by $k_{ar}(n_i\mu_{ia} + \sigma_{ia}\sqrt{n_i}z_{\alpha_{ia}})$ (where k_{ar} represents the quantity of resource r required to supply a unit of type a assistance and $z_{\alpha_{ia}}$ is the quantile of the cumulated normal distribution; in my application, $k_{ar} = 3$ for the number of social-work hours required for an accompaniment, $k_{ar} = 1$ otherwise) and is approximated by a linear function.

(2) Intertemporal constraints relate the number of old patients in each class in time period $t + 1$ to the numbers of patients in each class in time period t.

I define the following constants:

$x_{i1}, i = 1, \ldots, C$ = the number of patients in class i at the beginning of the planning period (in my application, restricted to the classes 4, 5, and 6, $x_{41} = 22$, $x_{51} = 50$, $x_{61} = 4$);

$l_i, i = 1, \ldots, C - 1$ = lower bound on the number of patients of class i in any time period;

$u_i, i = 1, \ldots, C - 1$ = upper bound on the number of patients of class i in any time period;

$k_{ar}, a = 1, \ldots, A; r = 1, \ldots, R$ = quantity of resource r required to supply a unit of type a of assistance;

$z_{\alpha_{ia}}, i = 1, \ldots, C - 1; a = 1, \ldots, A$ = quantile of the cumulated normal distribution corresponding to the probability level α_{ia};

$f_{iar1} = k_{ar}(l_i\mu_{ia} + \sigma_{ia}\sqrt{l_i}z_{\alpha_{ia}}), i = 1, \ldots, C - 1; a = 1, \ldots, A$;

$r = 1, \ldots, R$ = amount of resource r required to supply type a assistance to l_i patients of class i in a time period, for the chosen probability level α_{ia};

$f_{iar2} = k_{ar}(u_i\mu_{ia} + \sigma_{ia}\sqrt{u_i}z_{\alpha_{ia}}), i = 1, \ldots, C - 1; a = 1, \ldots, A$;

$r = 1, \ldots, R$ = amount of resource r required to supply type a assistance to u_i patients of the class i in a time unit, for the chosen probability level α_{ia};

$b_{rt}, r = 1, \ldots, R; t = 1, \ldots, T$ = quantity of resource r available in the time unit t;

$w_{it}, i = 1, \ldots, C - 1; t = 1, \ldots, T$ = suitable weights (in my application, all weights are equal to 1);

and the following variables:

$x_{it}, i = 1, \ldots, C; t = 2, \ldots, T$ = number of old patients of class i in the time unit t;

$y_{it}, i = 1, \ldots, C; t = 1, \ldots, T$ (with $y_{Ct} = 0, t = 1, \ldots, T$ because a patient cannot enter the assistance scheme while in hospital) = number of new patients of class i accepted in the home-care scheme in time unit t;

$\lambda_{it1}, i = 1, \ldots, C - 1; t = 1, \ldots, T$ = coefficient used to express the number of patients of class i in the time unit t by means of l_i;

$\lambda_{it2}, i = 1, \ldots, C - 1; t = 1, \ldots, T$ = coefficient used to express the number of patients of class i in time unit t by means of u_i.

The objective function of the LP is

$$z = \sum_{i=1}^{C-1} \sum_{t=1}^{T} w_{it}y_{it} \quad \text{(to be maximized)}$$
(1)

and the constraints are

$$\sum_{i=1}^{C} (x_{it} + y_{it})p_{ij} - x_{j,t+1} = 0$$
$$t = 1, \ldots, T - 1; j = 1, \ldots, C; \quad (2)$$

$$x_{it} + y_{it} - l_i\lambda_{it1} - u_i\lambda_{it2} = 0$$
$$i = 1, \ldots, C - 1; t = 1, \ldots, T; \quad (3)$$

$$\lambda_{it1} + \lambda_{it2} = 1 \quad i = 1, \ldots, C - 1; t = 1, \ldots, T; \quad (4)$$

$$\sum_{i=1}^{C-1} \sum_{a=1}^{A} f_{iar1}\lambda_{it1} + \sum_{i=1}^{C-1} \sum_{a=1}^{A} f_{iar2}\lambda_{it2} \leq b_{rt}$$
$$r = 1, \ldots, R; t = 1, \ldots, T; \quad (5)$$

$$x_{it} \geq 0 \quad i = 1, \ldots, C; \, t = 1, \ldots, T; \quad (6)$$

$$\lambda_{it1} \geq 0, \, \lambda_{it2} \geq 0 \quad i = 1, \ldots, C - 1;$$
$$t = 1, \ldots, T; \quad (7)$$

$$y_{it} \geq 0, \, y_{Ct} = 0 \quad i = 1, \ldots, C - 1;$$
$$t = 1, \ldots, T. \quad (8)$$

Constraint (2) relates the number of old patients of class j in time period $t + 1$ to the number of old and new patients in the various classes in time period t; constraints (6), (7), and (8) are obvious; constraints (3), (4), and (5) are explained by the linear approximation introduced.

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