Mathematical Programming Approach for Routing Home Care Nurses

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Abstract - **Nowadays more patients need to receive medical treatments at home. Sequencing home care nurses visits shows similarities with the multiple vehicle routing problem with time windows (MVRPTW). Additional constraints make this problem much more complex. These refer to medical requirements and to the importance of continuity of care. In this paper, we present a mathematical model describing the problem of determining routes for home care nurses which takes into account constraints from the MVRPTW along with medical and continuity of care constraints. We also provide some numerical results.**

Keywords - **home care nurses; mathematical programming; metaheuristics (Tabu); routing.**

I. INTRODUCTION

The ageing population and the urge to reduce costs entail health care services to be provided at home more often than they used to. Health care managers must solve new types of problems such as nurses scheduling, districting or routing problems. Routing home care nurses is a difficult task that we address by solving a mathematical programming model using a metaheuristic approach. In this paper, we model a real problem faced by a public medical clinic.

The literature includes only a few articles on routing home care nurses. In [1] the authors develop a computerised system which solves the nurse scheduling and the routing home care nurses problems. In [2] the routing home care nurses problem is formulated as a vehicle routing problem with time windows and multiple depots. In [4] the authors introduce a scheduling problem for a variety of home care providers which is modeled as a set partitioning problem and solved with a repeated matching algorithm.

This paper is organized as follows. Section 2 depicts the problem faced by the CLSC Les Forges in Trois-Rivières. Section 3 is devoted to the mathematical programming model, and in Section 4 we briefly describe the solution approach based on a Tabu search method. A comparison between our results and the manual results is provided in Section 5.

II. PROBLEM OVERVIEW

Nowadays, more patients receive their medical treatments at home. The medical public clinic CLSC Les Forges in Trois-Rivières (covering a territory divided into sectors) is accountable for planning nurse visits to patients. Each patient is assigned to a sector according to his home address (note that the masculine gender is used throughout the paper). Each nurse is also assigned to one sector but more than one nurse may be assigned to a sector. Even if whenever possible a nurse visits only patients from his sector, he may however have to visit patients from other sectors to balance nurse workloads. If a patient has to be visited by a nurse from another sector, we try to choose the one from the nearest sector to reduce the total traveling time. The impact of this flexibility is that the problem is not separable by sector.

In home care services it is important to consider the human aspects such as the continuity of care. The patients prefer receiving their treatments by the same nurse, and nurses also think that it is preferable to become better acquainted with their patients. Because of holidays and days off, it is not always possible to maintain continuity of care since the follow-up nurse of some patients may not be available. Therefore the administrators must rely on nurses from the recall lists. In our model, we account for the continuity of care requirements as soft constraints.

At the CLSC Les Forges, the rules to specify the time frames for returning the blood samples to the clinic are formulated as follows. If a blood sample is performed before 10h00 AM, the nurse must turn back the blood sample at the clinic no later than 10h00 AM (strictly speaking). If a blood sample is performed between 10h00 AM and 11h00 AM, the nurse must then turn back the blood sample no later than 11h00 AM. In our approach, we model these returns to the clinic as fictitious destination nodes to be visited before 10h00 AM and 11h00 AM, respectively. The complexity of these constraints is due to the fact that, even if we know before hand that a nurse has to perform blood samples on some patients, we do not know in advance the times when they will be performed, and hence we do not know returning to the clinic is necessary.

When solving the problem we assume that all patients are visited, and we do not allow routes to end after 12h00 PM to preserve the usual working schedule even though in practice some flexibility is allowed to a nurse in planning his visits to patients.

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III. MATHEMATICAL MODEL

The home care nurse routing problem is formulated as a vehicle routing problem with time windows (VRPTW) including additional constraints. In this model, we denote $P = \{1, 2,.., N\}$: the set of all patients.

P^+: the set of patients requiring a blood sampling, $P^+ \subset P$.

$I = I^r \cup I^l \cup I^f$: the set of all nurses including the sets I^r, I^l and I^f of regular nurses, of nurses from the recall list, and of fictitious nurses, respectively.

C_r, C_l, and C_f : the daily costs for the different nurse categories where $C_r < C_l << C_f$.

Note that in addition to the regular nurses and to the nurses from the recall lists categories, we introduce a third category of fictitious nurses that are assigned to patients in last resort indicating that these patients are not visited.

To specify the underlying network, we define the set of nodes V as follows:

- a node i is associated with each patient in P;
- two different nodes 0 and D are associated with the clinic where 0 and D are used to denote the origin and the ending node of each route, respectively;
- two fictitious nodes p_{10} and p_{11} are also associated with the clinic; these nodes have to be visited whenever some nurse has to take back blood samples before 10h00 AM or 11h00 AM, respectively.

With each node $i \in V$ we associate

r_i: the time required to complete the treatment of patient $i \in P$.

$r_O = r_D = r_{p_{10}} = r_{p_{11}} = 0.$

$[e_i, f_i]$: the time window of the arrival time at node $i \in V$.

More explicitly, the time windows for the different nodes are specified as follows:

$$[e_i, f_i] = \begin{cases} [H8, H11] & \text{if } i \in P^+ \\ [H8, H12] & \text{if } i \in V - P^+ \cup \{p_{10}, p_{11}\} \\ [H8, H10] & \text{if } i = p_{10} \\ [H10, H11] & \text{if } i = p_{11} \end{cases}$$

where $H8<H10<H11<H12$ are constant values associated with 8h00 AM, 10h00 AM, 11h00 AM, and 12h00 AM, respectively.

The set $A = \{(i, j) \in V \times V \mid e_i + r_i + t_{ij} \le f_j\}$ includes the admissible arcs; i.e. $(i, j) \in A$ if the time windows of nodes i and j allows sufficient time to move from i to j.

The variables are denoted as follows:

$$x_{ij}^k = \begin{cases} 1 & \text{if nurse } k \text{ visits successively nodes } i \text{ and } j \\ 0 & \text{otherwise} \end{cases}$$

b_i^k: the time when nurse k arrives at node $i \in V$.

$b_{p_{10}}^k$: the time when nurse k takes blood samples to the clinic before 10h00 AM (corresponding to the time when nurse k arrives at the fictitious node p_{10}).

$b_{p_{11}}^k$: the time when nurse k arrives at the clinic for turning back blood samples before 11h00 AM but after 10h00 AM (corresponding to the time when nurse k arrives at the fictitious node p_{11}).

y_i^k: the additional modeling binary variables, $\forall i \in P^+, k \in I$

Note that for all nurses $k \in I$, the variables b_i^k are initialized at the following values:

$H8 < b_i^k < H11, \quad i \in P^+$

$H8 < b_i^k < H12, \quad i \in V - P^+ \cup \{p_{10}, p_{11}\}$

$b_{p_{10}}^k = H10$

$b_{p_{11}}^k = H11$

The value of b_i^k may be modified during the solution procedure whenever the nurse k visits the node i.

To specify the objective function, denote by t_{ij}, the traveling time from node i to node j. The cost associated with nurse k moving from i to j is specified as follows:

$$c_{ij}^k = \begin{cases} t_{ij} & \text{if } k \in I^r \cup I^l \cup I^f \text{ and } i \ne 0 \\ t_{ij} + C_r & \text{if } k \in I^r \text{ and } i = 0 \\ t_{ij} + C_l & \text{if } k \in I^l \text{ and } i = 0 \\ t_{ij} + C_f & \text{if } k \in I^f \text{ and } i = 0 \end{cases}$$

According to this notation, the traveling time is considered as a cost. Also when the nurse k is leaving the clinic, we add the daily cost corresponding to his category. The cost structure is also modified in order to account for the requirement that a patient should be visited by a nurse of his sector whenever possible. Denote by $s(k)$ and $\varsigma(j)$ the sectors of the nurse k and of the node j, respectively. Note that 0, D, p_{10} and p_{11} are nodes for every sector. For any pair of nodes i and j and for any nurse k, the cost is specified as follows:

$$\bar{c}_{ij}^k = \begin{cases} c_{ij}^k & \text{if } \varsigma(j) = s(k) \text{ or if } j = D, p_{10}, p_{11} \\ c_{ij}^k + C_a & \text{if } \varsigma(j) \ne s(k) \text{ and } \varsigma(j) \text{ is an adjacent neighbour sector of } s(k) \\ c_{ij}^k + C_{na} & \text{if } \varsigma(j) \ne s(k) \text{ and } \varsigma(j) \text{ is not an adjacent neighbour sector of } s(k) \end{cases}$$

where C_a and C_{na} are positive parameters of the model. Note that the penalty cost increases when a nurse visits a node in other sectors farther than the adjacent neighbour ones.

The standard constraints for the VRPTW constraints (see Solomon (1987)) are summarized as follows:

$$\sum_{k \in I} \sum_{j \in V} x_{ij}^k = 1, \ \forall i \in P \tag{1}$$

$$\sum_{j \in V} x_{ij}^k - \sum_{j \in V} x_{ji}^k = 0, \ \forall i \in V \setminus \{0, D\}, \ k \in I \tag{2}$$

$$\sum_{j \in P} x_{0j}^k \le 1, \ \forall k \in I \tag{3}$$

$$\sum_{j \in V_s} x_{jD}^k = \sum_{j \in V_s} x_{0j}^k, \ \ \forall k \in I_s \tag{4}$$

$$b_i^k + r_i + t_{ij} - b_j^k \le M \times \left(1 - x_{ij}^k\right), \ \forall (i,j) \in A, \ k \in I \tag{5}$$

$$e_i \le b_i^k \le f_i, \ \ \forall i \in V, \ k \in I \tag{6}$$

Now additional constraints are required for the blood samples and the continuity of care requirements.

3.1 Blood sample related constraints

The blood sample constraints are more complex to formulate. The constraints (7) specify that whenever a nurse k performs a blood sample on at least one patient i before 10h00 AM, then he must visit the fictitious destination node p_{10} (i.e. he has to return to the clinic).

$$1 + \left(H10 - b_i^k\right) \le M \times \left(\sum_{j \in P} x_{jp_{10}}^k + 1 - \sum_{j \in P \cup \{p_{10}\}} x_{ij}^k \right), \tag{7}$$
$$\forall i \in P^+, \ \forall k \in I$$

Here M is a very large scalar.

To understand this constraint, first note that, if any patient $i \in P^+$ is visited by nurse k before 10h00 AM, then it follows that $\sum_{j \in P \cup \{p_{10}\}} x_{ij}^k = 1$ and $b_i^k \le H10$.

Hence $\left(H10 - b_i^k\right) \ge 0$ and $1 + \left(H10 - b_i^k\right) > 0$. Also, since $\left(1 - \sum_{j \in P \cup \{p_{10}\}} x_{ij}^k\right) = 0$, it follows that

$$M \times \left(\sum_{j \in P} x_{jp_{10}}^k + 1 - \sum_{j \in P \cup \{p_{10}\}} x_{ij}^k \right) = M \times \left(\sum_{j \in P} x_{jp_{10}}^k \right).$$

Thus constraint (7) reduces to

$$0 < 1 + \left(H10 - b_i^k\right) \le M \times \sum_{j \in P} x_{jp_{10}}^k$$

inducing that $\sum_{j \in P} x_{jp_{10}}^k > 0$ (i.e., forcing a visit to the fictitious destination node p_{10}).

Note that if nurse k does not visit any patient $i \in P^+$ before 10h00 AM, then $b_i^k > H10$ for all $i \in P^+$. Hence the constraints (7) are inactive for all $i \in P^+$ in the sense that they are satisfied for any value of $\sum_{j \in P} x_{jp_{10}}^k$.

Thus $\sum_{j \in P} x_{jp_{10}}^k = 0$ since the costs associated with all arcs (j, p_{10}) are positive.

Similar arguments can be used to verify that the constraints (8) and (9) force a nurse k performing any blood sample after 10h00 to visit the fictitious node p_{11}.

$$1 + \left(b_i^k - b_{p_{10}}^k\right) \le M \times \left(\sum_{j \in P} x_{jp_{11}}^k + 1 - \sum_{j \in P \cup \{p_{11}\}} x_{ij}^k \right), \tag{8}$$
$$\forall i \in P^+, \ \forall k \in I$$

$$1 + \left(b_i^k - H10\right) \le M \times \left(\sum_{j \in P} x_{jp_{11}}^k + 1 - \sum_{j \in P \cup \{p_{11}\}} x_{ij}^k \right), \tag{9}$$
$$\forall i \in P^+, \ \forall k \in I$$

The constraints (8) apply when the nurse k had to turn back to the clinic before 10h00, and constraints (9) when he did not.

Now even if the constraints (7) guarantee that the nurse k visit the fictitious destination node p_{10} whenever he performs any blood sample before 10h00, nevertheless these constraints do not prevent the visit to p_{10} taking place later than 10h00 AM (i.e., having $b_i^k > H10$), nor they prevent this visit taking place before any blood sample has been performed. For these reasons, we include the following constraints requiring also additional binary modeling variables y_i^k, $\forall i \in P^+$, to be formulated. Note that these variables are useful to formulate the model but they have no particular interpretation.

First we introduce a set of constraints guaranteeing that $y_i^k = 1$ for exactly one patient $i \in P^+$ visited by nurse k before 10h00 AM, if any. The constraints (10)

$$y_i^k \le \sum_{j \in V - D} x_{ji}^k, \ \forall i \in P^+, \forall k \in I \tag{10}$$

guarantee that y_i^k takes value 1 only if nurse k performs a blood sample on some patient $i \in P^+$, i.e. only if

$$\sum_{j \in V_S - D} x_{ji}^k = 1$$

The constraints (11)

$$H10 - b_i^k \le M \times \left(\sum_{j \in P^+} y_j^k \right), \forall i \in P^+, \forall k \in I \tag{11}$$

guarantee that whenever a blood sample is performed on some patient $i \in P^+$ before 10h00 AM (i.e., $b_i^k \le H10$), then $\sum_{j \in P^+} y_j^k > 0$.

Moreover, in order to guarantee that $\sum_{j \in P^+} y_j^k = 1$ whenever $\sum_{j \in P^+} y_j^k > 0$, we introduce the term $\sum_{k \in I} \sum_{j \in P^+} y_j^k$ in the objective function to be minimized.

Finally, it follows that the constraints

$$b_{p10}^{k} \le M + (H10 - M) \times \sum_{j \in P^{+}} y_j^k,\ \forall\, k \in I \qquad (12)$$

guarantee that $b_{p_{10}}^{k} \le H10$, inducing that nurse k returns to the clinic no later than 10h00 AM whenever $\sum_{j \in P^{+}} y_j^k = 1$. Furthermore, it is easy to see that the constraints (12) are inactive when nurse k is not performing any blood sample before 10h00.

Finally to guarantee that nurse k returns to the clinic no later than 10h00 AM but only after performing at least one blood sample on a patient, constraints (13) are added to the model.

$$b_i^k - b_{p10}^{k} \le M \times \left(1 - y_i^k\right), \forall\, i \in P^{+}, \forall\, k \in I \qquad (13)$$

Indeed, since $y_i^k = 1$ for exactly one patient $i \in P^{+}$ visited by nurse k before 10h00 AM, it follows that the corresponding constraint (13) is active. Hence $b_i^k - b_{p_{10}}^{k} \le 0$, and $b_i^k \le b_{p_{10}}^{k}$.

It is easier to deal with the case where the nurse k has to visit p_{11} (i.e., returning to the clinic before 11h00 AM) since this is the last return. On the one hand, to guarantee that p_{11} is visited no later than 11h00 AM, it is sufficient to fix properly the upper bound of the time window associated with $b_{p_{11}}^{k}$ to the value 11h00. On the other hand, the constraints (14) guarantee that p_{11} is visited after the last blood sample has been performed.

$$b_{p11}^{k} \ge b_i^k, \forall i \in P^{+}, k \in I \qquad (14)$$

3.2 Continuity of care constraints

In order to account for the continuity of care requirement, a set L_k of patients is assigned to each regular nurse $k \in I^r$ such that $L_k \cap L_h = \phi$ for each pair of nurses k and h. Also the set $\bigcup_{k \in I^r} L_k$ includes only patients requiring a follow-up because of medical decision based on the kind of the patient treatment or because of his general health state. Since the continuity of care requirement is considered as a soft constraint, then it is modeled by including a penalty cost each time a nurse k visits a patient j requiring follow-up by nurse h. The costs in the objective function are then modified accordingly: for all pairs of nodes $i, j \in P$, and for all regular nurses $k \in I^r$.

$$\tilde{c}_{ij}^{k} = \begin{cases} \bar{c}_{ij}^{k} + C_{cc} & \text{if } j \in L_h \text{ with } k \neq h \text{ and } k \in I^r \\ \bar{c}_{ij}^{k} & \text{otherwise} \end{cases}$$

where C_{cc} is the penalty cost for bypassing the continuity of care requirement.

Consequently, the objective function to be minimized is specified as follows $\left(\sum_{k \in I} \sum_{(i,j) \in A} \tilde{c}_{ij}^{k} x_{ij}^{k} + \sum_{k \in I} \sum_{j \in P^{+}} y_j^k \right)$ including two terms related to the nurses and to the modeling binary variables y_j^k, respectively.

IV. A MULTI SOLUTIONS APPROACH FOR THE GLOBAL PROBLEM

We use a metaheuristic approach based on Tabu search to solve the problem. To generate an initial solution of the global problem, we proceed as follows. First we obtain a solution for each sector individually using a Tabu search procedure to solve the preceding model restricted to each sector (including only the patients and the nurses of the sector, and where the nurses can only visit patients of the sector). This procedure is initialized with routes generated with Lau *et al.* method ([6]) aiming at reducing the number of nurses working. The initial solution for the global problem is generated from those for the sectors.

The solution approach is a variant of the scatter search approach ([5]) relying on an adaptive memory ([7]) including pools of solutions. One pool includes the best solutions generated so far in order to preserve the quality of the new solutions to be generated. The second pool contains solutions being as different as possible from those in the first pool in order to explore more extensively the feasible domain. A new initial solution for the global problem can be generated by combining solutions from both pools. In order to account more properly for the continuity of care requirements and the blood sample constraints, the new solution is initialized by selecting for each nurse a workload assigned to his in some solution of the pools. The resulting combination may require a repair process since some patients may be assigned to several nurses (in which case they have to be removed from some nurse workloads according to some strategy), or others may not be assigned to any nurse. In this case, each of these patients is sequentially included in some nurse workload according to a process where we consider first his follow-up nurse (if any), then nurses from his sector, then nurses from other sector, and finally in last resort we introduce a new fictitious nurse in the solution.

Then this solution is improved as follows. First, each sector is redefined in order to specify a new sector including all the patients visited by the nurses of the original sector. Then the problem for each new sector is solved using a Tabu search procedure. Finally, a new solution is generated. The pools are updated with the solution generated. This process is repeated until some stopping criterion is satisfied.

The method is implemented in Java version 1.5 running in the Redhat-Linux-Gnu environment, and the tests are completed on a PC machine equipped with an AMD 2.002 GHz processor having a 2048 Kilobytes central memory and a 1024 Kilobytes memory cache.

V. NUMERICAL RESULTS

To determine proper values for the different parameters, a first numerical experimentation was completed with a set of randomly generated problems according to a process based on Solomon approach ([8]) and including 100 patients where the percentage of visits with blood sample is equal to 25% or 50 % and where the size of the follow-up set includes 2 to 4 patients. Since the global problem cannot be solved to optimality with CPLEX 9.0 due to its complexity and its size, therefore we could not evaluate the optimality gap of our solutions. Nevertheless, the numerical results indicate the numerical efficiency of our approach requiring an average CPU time of less than 5 minutes ([3]).

The second numerical experimentation was completed with real value data provided by the CLSC Les Forges. The nurses visit between 80 and 100 patients every day. The territory covered by the clinic is divided into 4 sectors. The personnel include 14 regular nurses (full and part time) and 12 nurses from the recall list. A set of patients requiring continuity of care is assigned to each regular nurse. We performed tests on data obtained for 3 different days. The characteristics of these data are summarized in Table 1. The clinic provided us with an estimated service time for each patient depending on the type of treatment.

Problem	# of patients	# of regular nurses	# of nurses from the recall list	% of visits with blood sample	# patients with follow-up
Day 1	84	9	4	34.5	34
Day 2	92	13	3	45.6	38
Day 3	82	9	6	23.1	34

Table 1. Data provided by the CLSC

The numerical results indicate that the costs (evaluated according to the objective function) of the CLSC solutions are 10.9% higher on average than the costs of our solutions. Furthermore the CLSC solutions require more nurses (12.8% on average) than our solutions. But the mean number of patients without follow-up is equal to 0.6 in our solutions while this number is equal to 0 in the CLSC solutions. This can be explained by the fact that we deal with this soft constraint using a penalty cost in our model.

Problem		Mean cost	Mean # nurses	Mean # patients without follow-up	Mean time (sec.)
Day 1	Our sol.	9050.6	12.0	0.8	589.1
	CLSC	9623.0	13.0	0.0	-
Day 2	Our sol.	9545.0	14.0	1.0	651.7
	CLSC	10673.0	16.0	0.0	-
Day 3	Our sol.	9998.2	13.0	0.0	408.5
	CLSC	11413.0	15.0	0.0	-

Table 2. Numerical results

However, these comparisons must be analyzed with care since we used weights to represent the relative priority with regards to total traveling time, to the visits completed by nurses from other sectors and to the violations in the continuity of care. The values that we used in our implementation may be different than those that the nurses have in mind when they complete their planning. However we think that our numerical results are very encouraging and that they may indicate some advantages for managers to rely on computer tools of this type in their planning process. Furthermore, the values of the different parameters in the global model can be adjusted to the satisfaction of the head nurse in order to obtain proper results for his specific context.

VI. CONCLUSIONS

We introduce a mathematical programming model to determine home care nurse route for the real problem of CLSC Les Forges accounting for the continuity of care requirements and for the blood sampling constraints. The numerical results indicate that the solutions generated with our approach compare favourably with those of the CLSC and that they can be obtained rapidly using a regular PC. Even if each medical public clinic has its own characteristics, we are convinced that our approach can be implemented with small adaptations.

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