Theory of the Airlift Pump: Two Approaches

Abdul Basit Ahmad
Research Scholar,
Mechanical Engineering,
Indian Institute of Technology, Patna
Email: basit@iitp.ac.in

ABSTRACT

Two alternative models for predicting the volumetric flow rate of liquid as a function of the volumetric flow rate of air are presented here. The first one relies on the unit cell formulation as presented by deCachard and Delhaye. The second approach is an original approach, based on the dynamics of a Taylor bubble rising in the airlift pump. The main limitation of both the models is that they assume that there is only a single Taylor bubble in the airlift pump at a time - a sort of restriction on the fully developed slug flow regime. This restriction will be removed in future works.

Nomenclature

ε Void Fraction
α Submergence ratio
j Superficial velocity
u Velocity
Bo Bond Number
ρ Density
σ Surface tension coefficient
W Mass flow rate
Q Volumetric flow rate
V_m Mean velocity of flow
V_o Velocity of Taylor bubble in still fluid
C_o Velocity profile coefficient
ΔP_f Frictional pressure drop
Z_s Submerged Length
Z_l Lift of pump,**known**
β Fraction of unit cell transit time corresponding to the Taylor bubble
l_b Length of Taylor bubble
δ Wall film thickness

1 Introduction

The following analysis is only valid for slug flow regime assuming only one Taylor bubble in the airlift pump at any time. This restriction will be relaxed in future work.

1.1 Basic Definitions

Void fraction can be defined in many different ways. It can be defined as time-averaged or line-averaged void fraction. However, we will be using the area(or volume) averaged void fraction which is defined as the ratio of the cross section area (or volume) of channel occupied by the gas divided by the total cross section area (or volume) of the channel. This assumes a two-phase flow system, one gas and one liquid.

Thus we have:

$$\varepsilon = \frac{A_g}{A} \tag{1}$$
$$1-\varepsilon = \frac{A_f}{A} \tag{2}$$

where, A_g is the cross sectional area occupied by the gas and A_f is the cross sectional area occupied by the liquid.

Let the total mass flow rate be denoted by W. W_g and W_f denote the gas mass flow rate and liquid mass flow rate respectively. The volumetric flow rate is represented by the symbol Q.

The quality, x is defined as:

$$x = \frac{W_g}{W_g + W_f} \tag{3}$$
$$(1-x) = \frac{W_f}{W_g + W_f} \tag{4}$$

The gas velocity:

$$u_g = \frac{W_g}{\rho_g A_g} \tag{5}$$

The liquid velocity:

$$u_f = \frac{W_f}{\rho_f A_f} \tag{6}$$

These can also be expressed in terms of the volumetric flow rates as:

$$u_g = \frac{Q_g}{A_g} \tag{7}$$
$$u_f = \frac{Q_f}{A_f} \tag{8}$$

There is the concept of superficial velocity, j defined as the rate of volumetric flow per unit area.

Thus, we have:

$$j_g = \frac{W_g}{\rho_g A} = \varepsilon u_g \tag{9}$$
$$j_f = \frac{W_f}{\rho_f A} = (1-\varepsilon)\, u_l \tag{10}$$
$$j = j_g + j_f \tag{11}$$

2 Unit Cell Model

This study assumes non-aerated liquid slugs. This implies that all of the gas phase is in the Taylor bubble and the liquid slugs have virtually zero gas fraction.

According to Ros(1961), the condition for non-aerated slug is:

$$Bo = \frac{(\rho_f - \rho_g)\, gD^2}{\sigma} < 140 \tag{12}$$

which at STP corresponds to $D < 32mm$, which is what is assumed here.
The fraction of the unit cell transit time corresponding to the Taylor bubble is given as:

$$\beta = \frac{l_b}{l} \tag{13}$$

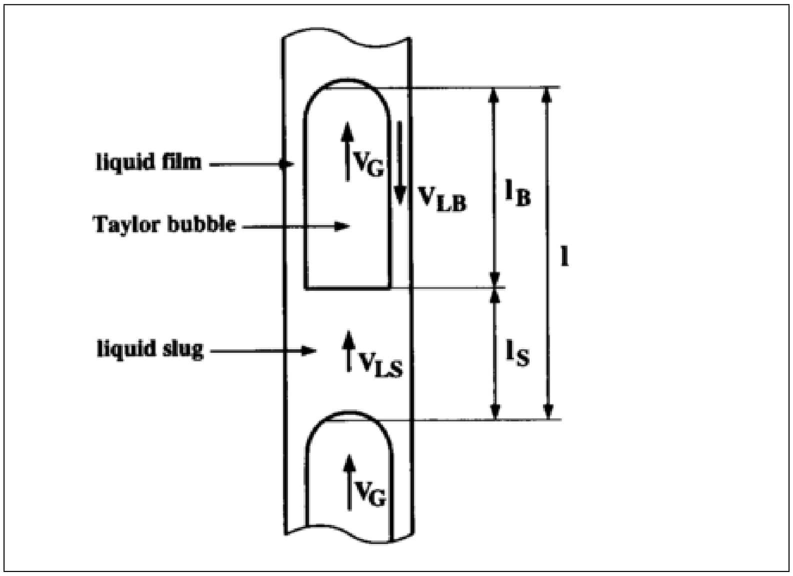

Figure 1. Unit Cell as considered for theory development

For a Taylor bubble, the following relation was proposed by Nicklin *et al* (1962):

$$u_g = C_o J + V_o = C_o V_m + V_o = C_o \left(\frac{Q_g + Q_f}{A} \right) + V_o \tag{14}$$

C_o is called the velocity profile coefficient and we have:

$$C_o = 1.2 \tag{15}$$

and V_o is called the velocity of Taylor bubble in still fluid. It is given, as per Kim *et al.* (2014)

$$V_o = 0.352\left(1 - 3.18Bo^{-1} - 14.77Bo^{-2}\right)\sqrt{gD} \tag{16}$$

Kim *et al.*(2014) performed a momentum balance for the air-lift pump as:

$$\rho_f g Z_s = \rho_f g (1-\varepsilon)(Z_s + Z_l) + (1-\varepsilon)\Delta P_f \tag{17}$$

They used a single phase expression for the frictional pressure drop.

$$\Delta P_f = \frac{f(Z_s + Z_l)\rho_l V_m^2}{2D} \tag{18}$$

A more correct expression would be to use a two-phase expression for the frictional pressure drop. Such an expression has been presented by deCachard and Delhaye(1996):

$$\Delta P_f = (1-\beta)\frac{f(Z_s + Z_l)\rho_l V_m^2}{2D} + \beta\rho_f g(Z_s + Z_l) \tag{19}$$

Substituting in eqn.(17) the expression for ΔP_f, as obtained in eqn.(19) and dividing throughout by $\rho_f g(Z_s + Z_l)$, we have:

$$\alpha = (1-\varepsilon)\left[1 + \left[(1-\beta)\frac{fV_m^2}{2gD} + \beta\right]\right] \tag{20}$$

Now, we have from geometrical considerations and definition of void fraction:

$$\varepsilon = \frac{\pi(R-\delta)^2(l_b - R) + (2/3)\pi(R-\delta)^3}{\pi R^2(Z_s + Z_l)} \tag{21}$$

Now, according to deCachard and Delhaye, we also have:

$$\delta = R(1-\sqrt{\varepsilon}) \tag{22}$$

Thus, by substituting eqn.(22) into eqn.(21), we have:

$$\beta = 1 - \frac{2}{3}\frac{R}{(Z_s + Z_l}\sqrt{\varepsilon} + \frac{R}{(Z_s + Z_l} \tag{23}$$

Also from the definition of u_g, we have another relation for ε as:

$$\varepsilon = \frac{Q_g}{(C_o V_m + V_o)A} \tag{24}$$

Now, eqn.(24) and eqn.(refeqn:beta) can be substituted into eqn.(20) and a relation between Q_g and Q_f may be obtained. This has been done using the symbolic math library of the Matlab software and the resultant plot is shown here.

3 Per Bubble Theory

Let V_w denote the total volume of the liquid in the riser of an airlift pump which has a Taylor bubble just entering from the inlet. The situation is plotted in fig.3. Now, assume that the bubble moves infinitesimally upwards by a length dh. Then, assume that the amount of liquid that flows out of the riser into the container is given by dV_w.

Some liquid flows backwards through the falling film and the rest is transmitted upwards. Ignoring higher order terms of dh in the expression, we have the following eqn:

$$dV_w = \pi R^2 dh - 2\pi R\delta dh \tag{25}$$

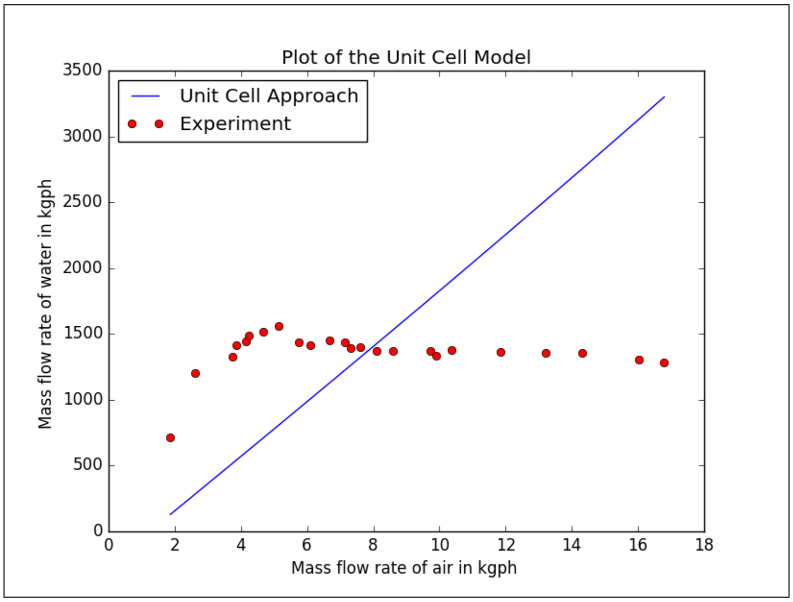

Figure 2. Comparison of Unit Cell approach with experiment

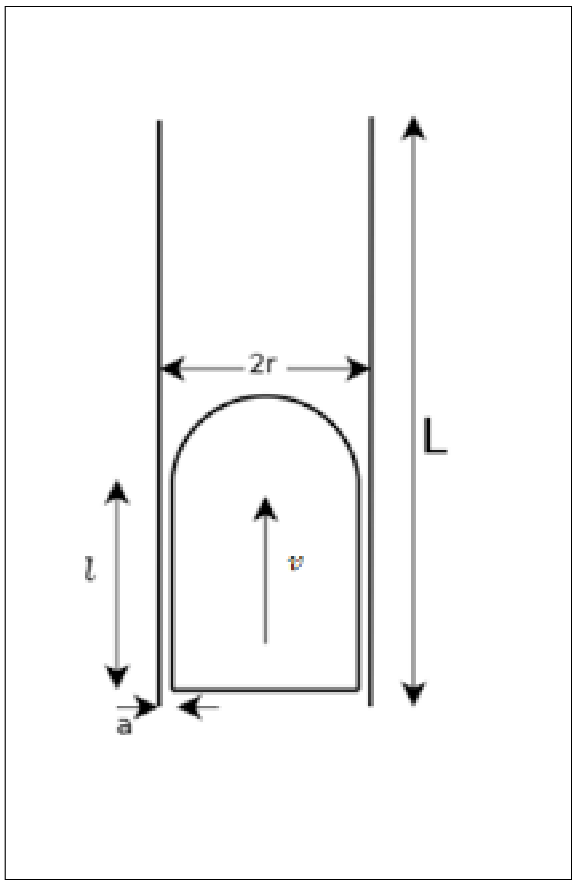

Figure 3. Instantaneous position of bubble

taking the time derivative of this expression, we have:

$$\frac{dV_w}{dt} = \pi R^2 \frac{dh}{dt} - 2\pi R \delta \frac{dh}{dt} \tag{26}$$

which may be written as:

$$Q_f = \pi R^2 u_b - 2\pi R \delta u_b \tag{27}$$

Now substituting in eqn.(27), the expression from eqn(22) and eqn(14), we have:

$$Q_f = \pi R^2 \left(C_o V_m + V_o\right) \left[2\sqrt{\varepsilon} - 1\right] \tag{28}$$

This is also solved symbolically with the symbolic math library of Matlab. The plot of comparison of all models is shown here.

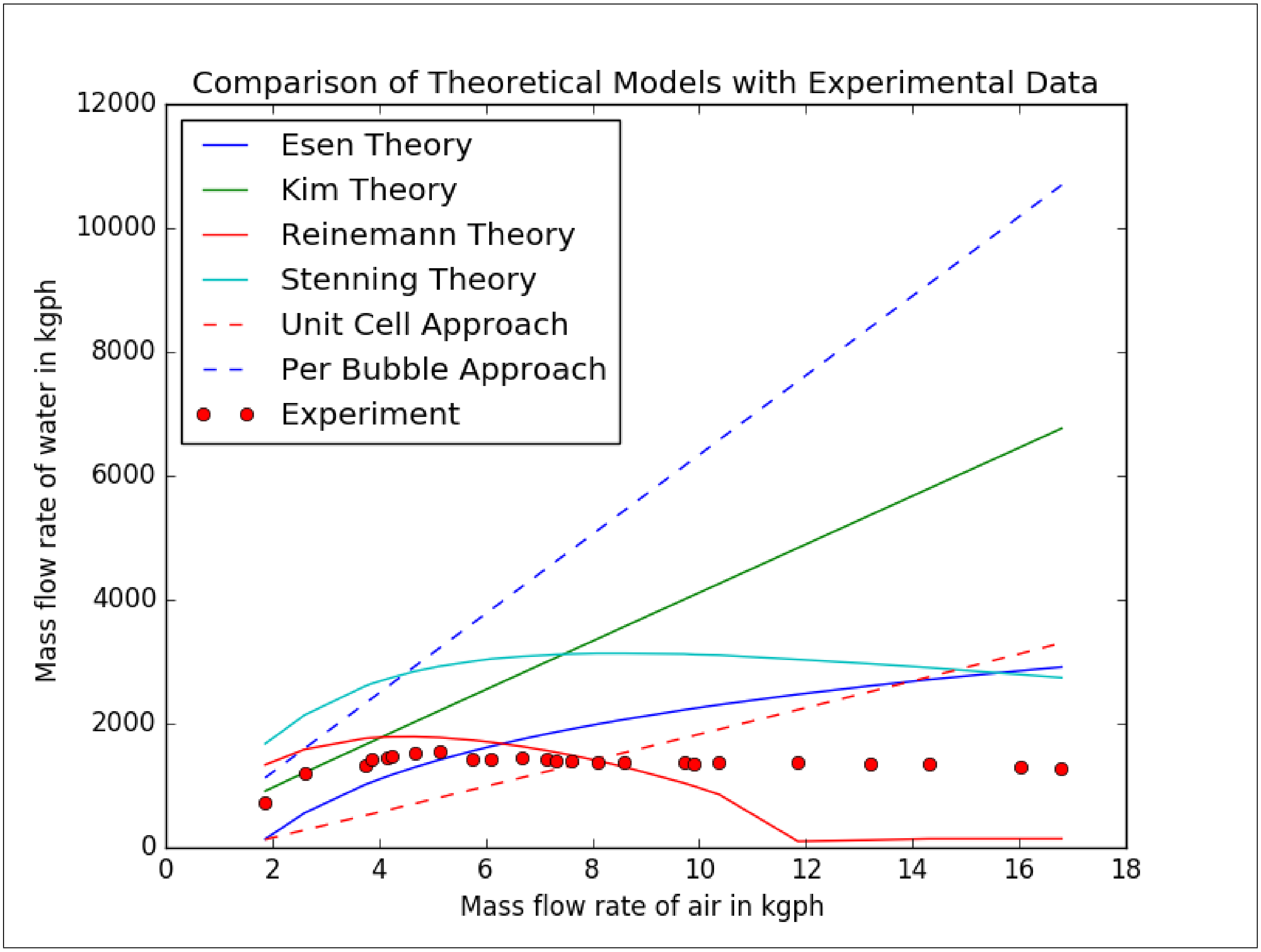

Figure 4. Comparison of models with experiment