New York University Shanghai

Problem Set 10

Solutions

BUSF-SHU 210: Business Analytics (Spring 2019)

1. Radiation Therapy

Radiation therapy is a major treatment method for cancer patients. In radiation therapy, beams of high energy photons (γ-ray) are fired into the patient to kill cancerous cells. An important breakthrough in the history of radiation therapy is the so-called intensity modulated radiation therapy (IMRT), invented in early 1980s. To reach and effective destroy the tumor, radiation passes through healthy tissues, and damages healthy cells as well. This will seriously affect the post-treatment quality of life for the patient. The goal is thus to have the tumor cells receiving the radiation as much as possible, and the healthy tissues as little as possible. In IMRT, we achieve this goal by adjusting the intensity profile of each beam of $\gamma-$ray.

Specifically, based on CT scan of the patient, a radiation oncologist contours the tumor and the structures of the nearby healthy tissues. Then, each structure is discretized into voxels, typically $4mm \times 4mm \times 4mm$. Let's see a simple example as shown in Figure 1. This is a simplification for a tumor on the neck near the spinal cord. Tumors in this area are hard to perform surgery, so Radiation Therapy is the main treatment. The voxels in pink represent tumor tissues, Spinal Cord is represented by dark green voxels, and other healthy tissues are represented in light green tissues. The radiation oncologist plans to use 2 $\gamma-$ray beams, each with 3 beamlets, to perform the radiation therapy for this patient. We can control the intensity of each breamlet to minimize the total dose of healthy tissues (spinal cord and others), and to have the tumor tissues receiving the effective radiation doses. More specifically, the dose of radiation received by each spinal cord voxel is at most 7Gy (Gray). Otherwise, the spinal cord may receive some inevitable damages. Likewise, the dose of radiation received by each voxel of other healthy tissues should be at most 10Gy. On the other hand, the dose of radiation received by each tumor tissue voxel must be at least 7Gy, in order to effectively damage cancerous cells. When you design an IMRT treatment plan, you have to satisfy these requirements.
Different voxels have different absorption rates, depending on the nature of the tissue and the location, as shown in Figures 2 and 3. For example, if the intensity of Beamlet 4 is 2Gy, the dose of radiation received by Voxel 1 will be $2 \times 1 = 2$Gy, the dose of radiation received by Voxel 4 will be $2 \times 2 = 4$Gy, and the dose received by Voxel 7 will be $2 \times 1 = 1$Gy. The total dose received by a voxel will be the sum of doses received by this voxel from all the beamlets that pass this voxel. For example, the dose received by Voxel 1 should be the sum of the dose from Beamlet 4 and that from Beamlet 1.

Suppose you are trying to assist the radiation oncologist to design an IMRT plan for this patient. Specifically, you need to develop optimization models to decide the intensity for each $\gamma-$ray beamlets shown in Figures 1, 2, and 3.

(a) Suppose your goal is to minimize the total doses of healthy tissues. Formulate the IMRT de-

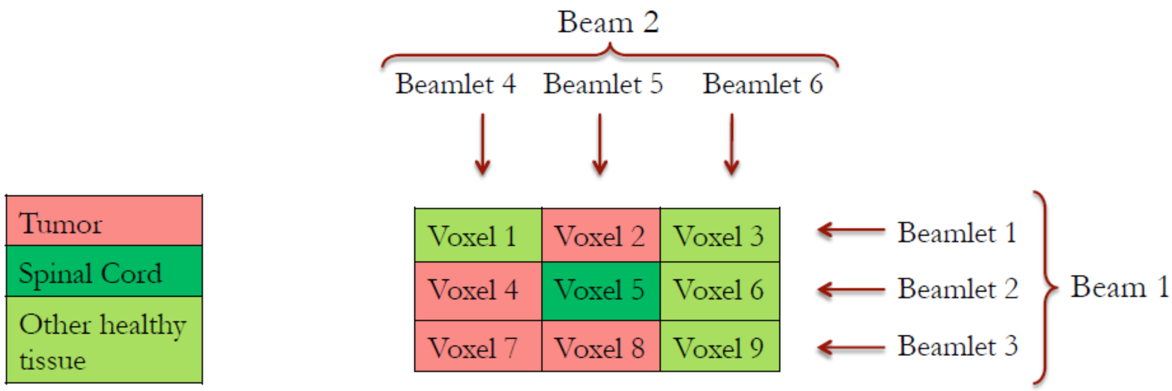

Figure 1: Radiation Therapy

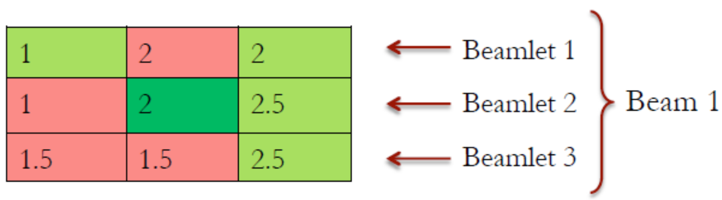

Figure 2: Absorption Rate: Beam 1

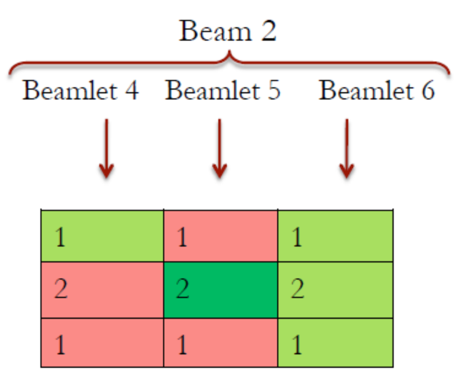

Figure 3: Absorption Rate: Beam 2

sign problem as a linear program. Clearly state the decision variables, the objective function, and constraints. What is the optimal intensity for each beamlet?

We define the following decision variables:

- x_1 = radiation intensity for beamlet 1
- x_2 = radiation intensity for beamlet 2
- x_3 = radiation intensity for beamlet 3
- x_4 = radiation intensity for beamlet 4
- x_5 = radiation intensity for beamlet 5
- x_6 = radiation intensity for beamlet 6

The objective function is to minimize the total doses received by healthy tissues:

$$\begin{aligned} \min\ & (1+2)x_1 + (2+2.5)x_2 + 2.5x_3 + x_4 + 2x_5 + (1+2+1)x_6 \\ =& 3x_1 + 4.5x_2 + 2.5x_3 + x_4 + 2x_5 + 4x_6 \end{aligned}$$

We establish the following constraints for the this problem:

- $x_1 + x_4 \leq 10$: Radiation dose of Voxel 1 is at most 10
- $2x_1 + x_5 \geq 7$: Radiation dose of Voxel 2 is at least 7
- $2x_1 + x_6 \leq 10$: Radiation dose of Voxel 3 is at most 10
- $x_2 + 2x_4 \geq 7$: Radiation dose of Voxel 4 is at least 7
- $2x_2 + 2x_5 \leq 7$: Radiation dose of Voxel 5 is at most 7
- $2.5x_2 + 2x_6 \leq 10$: Radiation dose of Voxel 6 is at most 10
- $1.5x_3 + x_4 \geq 7$: Radiation dose of Voxel 7 is at least 7
- $1.5x_3 + x_5 \geq 7$: Radiation dose of Voxel 8 is at least 7
- $2.5x_3 + x_6 \leq 10$: Radiation dose of Voxel 9 is at most 10
- $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$: Non-negativity constraint

Putting everything together, we formulate the problem as the following linear program:

$$\begin{aligned} & \min\ 3x_1 + 4.5x_2 + 2.5x_3 + x_4 + 2x_5 + 4x_6 \\ \text{Subject to} \quad & \\ & x_1 + x_4 \leq 10 \\ & 2x_1 + x_5 \geq 7 \\ & 2x_1 + x_6 \leq 10 \\ & x_2 + 2x_4 \geq 7 \\ & 2x_2 + 2x_5 \leq 7 \\ & 2.5x_2 + 2x_6 \leq 10 \\ & 1.5x_3 + x_4 \geq 7 \\ & 1.5x_3 + x_5 \geq 7 \\ & 2.5x_3 + x_6 \leq 10 \\ & x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \end{aligned}$$

We use Python to solve this linear program. We obtain the optimal solution as: $x_1^* = 1.75$, $x_2^* = 0$, $x_3^* = 2.33$, $x_4^* = 3.5$, $x_5^* = 3.5$, $x_6^* = 0$.

(b) Suppose your goal is to maximize the total doses of tumor tissues. Formulate the IMRT design problem as a linear program. Clearly state the decision variables, the objective function, and constraints. What is the optimal intensity for each beamlet?

We define the following decision variables:

- x_1 = radiation intensity for beamlet 1
- x_2 = radiation intensity for beamlet 2
- x_3 = radiation intensity for beamlet 3
- x_4 = radiation intensity for beamlet 4
- x_5 = radiation intensity for beamlet 5
- x_6 = radiation intensity for beamlet 6

The objective function is to maximize the total doses received by tumor tissues:

$$\max \ 2x_1 + x_2 + 3x_3 + 3x_4 + 2x_5$$

We establish the following constraints for the this problem:

- $x_1 + x_4 \leq 10$: Radiation dose of Voxel 1 is at most 10
- $2x_1 + x_5 \geq 7$: Radiation dose of Voxel 2 is at least 7
- $2x_1 + x_6 \leq 10$: Radiation dose of Voxel 3 is at most 10
- $x_2 + 2x_4 \geq 7$: Radiation dose of Voxel 4 is at least 7
- $2x_2 + 2x_5 \leq 7$: Radiation dose of Voxel 5 is at most 7
- $2.5x_2 + 2x_6 \leq 10$: Radiation dose of Voxel 6 is at most 10
- $1.5x_3 + x_4 \geq 7$: Radiation dose of Voxel 7 is at least 7
- $1.5x_3 + x_5 \geq 7$: Radiation dose of Voxel 8 is at least 7
- $2.5x_3 + x_6 \leq 10$: Radiation dose of Voxel 9 is at most 10
- $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$: Non-negativity constraint

Putting everything together, we formulate the problem as the following linear program:

$$
\begin{aligned}
&\max\ 2x_1 + x_2 + 3x_3 + 3x_4 + 2x_5 \\
&\text{Subject to} \\
&\quad x_1 + x_4 \leq 10 \\
&\quad 2x_1 + x_5 \geq 7 \\
&\quad 2x_1 + x_6 \leq 10 \\
&\quad x_2 + 2x_4 \geq 7 \\
&\quad 2x_2 + 2x_5 \leq 7 \\
&\quad 2.5x_2 + 2x_6 \leq 10 \\
&\quad 1.5x_3 + x_4 \geq 7 \\
&\quad 1.5x_3 + x_5 \geq 7 \\
&\quad 2.5x_3 + x_6 \leq 10 \\
&\quad x_1, x_2, x_3, x_4, x_5, x_6 \geq 0
\end{aligned}
$$

We use Python to solve this linear program. We obtain the optimal solution as: $x_1^* = 1.75$, $x_2^* = 0$, $x_3^* = 4$, $x_4^* = 8.25$, $x_5^* = 3.5$, $x_6^* = 0$.

(c) Now we take a more comprehensive perspective of designing the IMRT treatment plan. Specifically, we try to balance the benefit of destroying cancerous cells and the harm of damaging healthy tissues. For a given IMRT plan, we use D_T to denote the total dose of radiation received by the tumor, and D_H as the total dose of radiation received by healthy tissues. For a given weighting parameter α, the goal is to maximize

$$\alpha D_T - (1-\alpha) D_H$$

Formulate the IMRT design problem as a linear program if we set $\alpha = 0.7$. Clearly state the decision variables, the objective function, and constraints. What is the optimal intensity for each beamlet? What are the total doses of radiation received by tumor, and the total doses of radiation received by healthy tissues, respectively, associated with your prescribed IMRT plan?

We define the following decision variables:

- x_1 = radiation intensity for beamlet 1
- x_2 = radiation intensity for beamlet 2
- x_3 = radiation intensity for beamlet 3
- x_4 = radiation intensity for beamlet 4
- x_5 = radiation intensity for beamlet 5
- x_6 = radiation intensity for beamlet 6

The objective function is to maximize the total weighted doses received by tumor and healthy tissues:

$$
\begin{aligned}
\max\ &\alpha(2x_1 + x_2 + 3x_3 + 3x_4 + 2x_5) - (1-\alpha)(3x_1 + 4.5x_2 + 2.5x_3 + x_4 + 2x_5 + 4x_6) \\
=&(5\alpha - 3)x_1 + (5.5\alpha - 4.5)x_2 + (5.5\alpha - 2.5)x_3 + (4\alpha - 1)x_4 + (4\alpha - 2)x_5 + (4\alpha - 4)x_6
\end{aligned}
$$

We establish the following constraints for the this problem:

- $x_1 + x_4 \leq 10$: Radiation dose of Voxel 1 is at most 10
- $2x_1 + x_5 \geq 7$: Radiation dose of Voxel 2 is at least 7
- $2x_1 + x_6 \leq 10$: Radiation dose of Voxel 3 is at most 10
- $x_2 + 2x_4 \geq 7$: Radiation dose of Voxel 4 is at least 7
- $2x_2 + 2x_5 \leq 7$: Radiation dose of Voxel 5 is at most 7
- $2.5x_2 + 2x_6 \leq 10$: Radiation dose of Voxel 6 is at most 10
- $1.5x_3 + x_4 \geq 7$: Radiation dose of Voxel 7 is at least 7
- $1.5x_3 + x_5 \geq 7$: Radiation dose of Voxel 8 is at least 7
- $2.5x_3 + x_6 \leq 10$: Radiation dose of Voxel 9 is at most 10
- $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$: Non-negativity constraint

Putting everything together, we formulate the problem as the following linear program:

$$\max\ (5\alpha - 3)x_1 + (5.5\alpha - 4.5)x_2 + (5.5\alpha - 2.5)x_3 + (4\alpha - 1)x_4 + (4\alpha - 2)x_5 + (4\alpha - 4)x_6$$

Subject to

$$\begin{aligned}
&x_1 + x_4 \leq 10\\
&2x_1 + x_5 \geq 7\\
&2x_1 + x_6 \leq 10\\
&x_2 + 2x_4 \geq 7\\
&2x_2 + 2x_5 \leq 7\\
&2.5x_2 + 2x_6 \leq 10\\
&1.5x_3 + x_4 \geq 7\\
&1.5x_3 + x_5 \geq 7\\
&2.5x_3 + x_6 \leq 10\\
&x_1, x_2, x_3, x_4, x_5, x_6 \geq 0
\end{aligned}$$

We use Python to solve this linear program for $\alpha = 0.7$. We obtain the optimal solution as: $x_1^* = 1.75$, $x_2^* = 0$, $x_3^* = 4$, $x_4^* = 8.25$, $x_5^* = 3.5$, $x_6^* = 0$. The associated dose received by the tumor tissues is 47.25Gy, and the associated dose received by healthy tissues is 30.5Gy.

(d) As in part (c), we still consider maximizing

$$\mathrm{Opt}(\alpha) = \alpha D_T - (1 - \alpha) D_H$$

Now take $\alpha \in \mathcal{S} = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$. For each $\alpha \in \mathcal{S}$, denote $T(\alpha)$ as the total doses of radiation received by the tumor, and $H(\alpha)$ as the total doses of radiation received by healthy tissues, respectively, associated with your IMRT plan that maximizes $\mathrm{Opt}(\alpha)$. Please calculate $T(\alpha)$ and $H(\alpha)$ for each $\alpha \in \mathcal{S}$. Also plot the points $(T(\alpha), H(\alpha))$ for all the values of $\alpha \in \mathcal{S}$. This figure is usually called the Pareto frontier in the literature. What observations do you have from the scatter plot figure? Can you give an interpretation for the parameter α?

The model is the same as Part (c), with different α's. We summarize the total doses of radiation received by tumor and healthy tissues in the following table:

α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$T(\alpha)$	28	28	28	42.25	42.25	47.25	47.25	47.25	47.25	47.25	47.25
$H(\alpha)$	21.58	21.58	21.58	26.33	26.33	30.5	30.5	30.5	30.5	30.5	30.5

If α increases, the total dose of tumor tissues and that of healthy tissues are both higher. The parameter α captures the trade-off between destroying tumor tissues and the protecting healthy tissues. If α is larger, we put more weight on destroying tumor tissues. If α is smaller, we put more weight on protecting healthy tissues.

2. Assigning Medical Sales Representatives (1 Point)

You are responsible for assigning 22 hospitals to 4 medical sales representatives (SRs). As the SRs need to visit the hospitals under their responsibility quite often, the goal of assignment is to minimize the total travel distance of all 4 SRs. We summarize the distance between each hospital and each SR's office in the following table (in KM). Note that SR 1 is based in Hospital 4, SR 2 is based in Hospital 14, SR 3 is based in Hospital 16, and SR 4 is based in Hospital 22.

Hospital Index	Distance to SR 1	Distance to SR 2	Distance to SR 3	Distance to SR 4	Workload
1	16.16	24.08	24.32	21.12	0.1609
2	19	26.47	27.24	17.33	0.1164
3	25.29	32.49	33.42	12.25	0.1026
4	0	7.93	8.31	36.12	0.1516
5	3.07	6.44	7.56	37.37	0.0939
6	1.22	7.51	8.19	36.29	0.132
7	2.8	10.31	10.95	33.5	0.0687
8	2.87	5.07	5.67	38.8	0.093
9	3.8	8.01	7.41	38.16	0.2116
10	12.35	4.52	4.35	48.27	0.2529
11	11.11	3.48	2.97	47.14	0.0868
12	21.99	22.02	24.07	39.86	0.0828
13	8.82	3.3	5.36	43.31	0.0975
14	7.93	0	2.07	43.75	0.8177
15	9.34	2.25	1.11	45.43	0.4115
16	8.31	2.07	0	44.43	0.3795
17	7.31	2.44	1.11	43.43	0.071
18	7.55	0.75	1.53	43.52	0.0427
19	11.13	18.41	19.26	25.4	0.1043
20	17.49	23.44	24.76	23.21	0.0997
21	11.03	18.93	19.28	25.43	0.1698
22	36.12	43.75	44.43	0	0.2531

Each hospital must be assigned to exactly one SR. Furthermore, the total workload of each SR should be between 0.8 and 1.2. The workload of each hospital is given in the last column of the above table. The workload of one SR is the sum of the workloads of all the hospitals assigned to him/her.

(a) Formulate the SR assignment problem as an integer program model. What is the optimal SR assignment policy?
Define $x_{ij} \in \{0, 1\}$ as the decision variables, where $x_{ij} = 1$ refers to that SR i is assigned to Hospital j, and $x_{ij} = 0$ otherwise. The objective function is

$$\min \sum_{i=1}^{4} \sum_{j=1}^{22} d_{ij} x_{ij},$$

where d_{ij} is the distance between SR i and Hospital j, as shown in the Table.

There are several constraints for this integer programming problem:

- $\sum_{i=1}^{4} x_{ij} = 1$ ($j = 1, 2, \cdots, 22$): Each hospital is assigned to exactly one SR.
- $0.8 \leq \sum_{j=1}^{22} w_j x_{ij} \leq 1.2$ ($i = 1, 2, \cdots, 4$, w_j is the workload of hospital j): The total work load of each SR i is between 0.8 and 1.2 (w_j is the workload of Hospital j).

We solve the model using Python. Solving the model, we find that SR1 is assigned to hospitals $\{4, 5, 6, 7, 8, 9, 12, 19, 20\}$, SR2 is assigned to hospitals $\{11, 13, 14, 18\}$, and SR3 is assigned to hospitals $\{10, 15, 16, 17\}$, whereas SR4 is assigned to hospitals $\{1, 2, 3, 21, 22\}$.

(b) We would like to restrict the workload of each SR to the range between 0.9 and 1.1. Without resolving the model, can you discuss whether the total travel distance of all SRs will increase or decrease compared with the solution in part (a)? What is the new optimal SR assignment policy if the workload of each SR must be between 0.9 and 1.1?
If the workload for each SR is between 0.9 and 1.1, the constraints are more restrictive. Therefore, the optimal total travel distance will increase compared with the optimal solution in part (a).

We resolve the model using Python, and find that SR1 is assigned to hospitals $\{4, 5, 6, 7, 8, 9, 12, 19\}$, SR2 is assigned to hospitals $\{11, 13, 14, 18\}$, and SR3 is assigned to hospitals $\{10, 15, 16, 18\}$, whereas SR4 is assigned to hospitals $\{1, 2, 3, 20, 21, 22\}$.

3. Operating Room Scheduling

Hospitals have a limited number of Operating Rooms (ORs). Operating room managers must determine a weekly schedule assigning ORs to different departments in the hospital. Creating an acceptable schedule is a highly political process within the hospital. Surgeons are frequently paid on a fee-for-service basis, so changing allocated OR hours directly affects their income. The operating room manager's proposed schedule must strike a delicate balance between all the surgical departments in the hospital. Your job is to use integer programming to help the OR manager of Mount Sinai Hospital to schedule ORs to Ophthalmology, Gynecology, Oral Surgery, Otolaryngology, and General Surgery departments.

There are some logistical issues associated with the ORs of Mount Sinai Hospital. Specifically, the ORs are staffed in 8-hour blocks on each work day (Monday - Friday) of a week. At most 10 ORs are assigned every day. Furthermore, the number of ORs allocated to a department on a given day cannot exceed the number of surgery teams that department has available that day. You should also meet each department daily and weakly minimums and maximums.

The objective is to maximize the percentage of target allocation hours each department is actually allocated. If target allocation hours are t_j for department j, then we want to maximize the sum of $8x_{jk}/t_j$ over all departments and days of the week. Here, x_{jk} is the number of ORs assigned to department j on day k. For example, If otolaryngology has a target of 37.3 hours per week and we allocate them 4 ORs then their percentage of target allocation hours is $(8 \times 4)/37.3 = 85.8\%$. The following table summarizes the weekly target allocation hours of each department:

Department	Weekly Target Allocation Hours
Ophthalmology	39.4
Gynecology	117.4
Oral Sugery	19.9
Otolaryngology	26.3
General Surgery	189.0

The following table summarizes the number of surgery teams from each department available each day:

Department	Mon	Tue	Wed	Thur	Fri
Ophthalmology	2	2	2	2	2
Gynecology	3	3	3	3	3
Oral Sugery	0	1	0	1	0
Otolaryngology	1	1	1	1	1
General Surgery	6	6	6	6	6

The following table summarizes the Maximum number of ORs required by each department each day:

Department	Mon	Tue	Wed	Thur	Fri
Ophthalmology	2	2	2	2	2
Gynecology	3	3	3	3	3
Oral Sugery	1	1	1	1	1
Otolaryngology	1	1	1	1	1
General Surgery	6	6	6	6	6

The following table summarizes the weekly requirement on the total number of ORs:

Department	Min	Max
Ophthalmology	3	6
Gynecology	12	18
Oral Sugery	2	3
Otolaryngology	2	4
General Surgery	18	25

Please formulate the OR scheduling problem as an integer program and obtain the optimal OR assignment using Python.

We define $x_{i,j}$ as the number of ORs assigned to department i on day j, where $i = Op, Gy, Or, Ot, Ge$, and $j = M, Tu, W, Th, F$. The objective function is

$$\sum_{i,j} \frac{8x_{ij}}{t_i},$$

where t_i is the target allocation of department i, $t_{Op} = 39.4$, $t_{Gy} = 117.4$, $t_{Or} = 19.9$, $t_{Ot} = 26.3$, and $t_{Ge} = 189.0$.

The first set of constraints are that at most 10 ORs are assigned every day, i.e.,

$$x_{OP,M} + x_{GY,M} + x_{OS,M} + x_{OT,M} + x_{GS,M} \leq 10$$

$$x_{OP,Tu} + x_{GY,Tu} + x_{OS,Tu} + x_{OT,Tu} + x_{GS,Tu} \leq 10$$

$$x_{OP,W} + x_{GY,W} + x_{OS,W} + x_{OT,W} + x_{GS,W} \leq 10$$

$$x_{OP,Th} + x_{GY,Th} + x_{OS,Th} + x_{OT,Th} + x_{GS,Th} \leq 10$$

$$x_{OP,F} + x_{GY,F} + x_{OS,F} + x_{OT,F} + x_{GS,F} \leq 10$$

The number of ORs allocated to a department on a given day cannot exceed the number of surgery teams that department has available on that day. Thus, we have:

Department	Mon	Tue	Wed	Thur	Fri
Ophthalmology	$0 \leq x_{Op,M} \leq 2$	$0 \leq x_{Op,Tu} \leq 2$	$0 \leq x_{Op,W} \leq 2$	$0 \leq x_{Op,Th} \leq 2$	$0 \leq x_{Op,F} \leq 2$
Gynecology	$0 \leq x_{Gy,M} \leq 3$	$0 \leq x_{Gy,Tu} \leq 3$	$0 \leq x_{Gy,W} \leq 3$	$0 \leq x_{Gy,Th} \leq 3$	$0 \leq x_{Gy,F} \leq 3$
Oral Sugery	$0 \leq x_{Or,M} \leq 0$	$0 \leq x_{Or,Tu} \leq 1$	$0 \leq x_{Or,W} \leq 0$	$0 \leq x_{Or,Th} \leq 1$	$0 \leq x_{Or,F} \leq 0$
Otolaryngology	$0 \leq x_{Ot,M} \leq 1$	$0 \leq x_{Ot,Tu} \leq 1$	$0 \leq x_{Ot,W} \leq 1$	$0 \leq x_{Ot,Th} \leq 1$	$0 \leq x_{Ot,F} \leq 1$
General Surgery	$0 \leq x_{Ge,M} \leq 6$	$0 \leq x_{Ge,Tu} \leq 6$	$0 \leq x_{Ge,W} \leq 6$	$0 \leq x_{Ge,Th} \leq 6$	$0 \leq x_{Ge,F} \leq 6$

Since the number of surgery teams is lower than the maximum number of ORs required by each department and each day, the maximum number requirement is automatically satisfied once we establish the constraints for the number of surgery teams from each department available each day.

The next set of constraints are that the total number of ORs allocated to each department is between its maximum and minimum range.

$$3 \leq x_{OP,M} + x_{OP,Tu} + x_{OP,W} + x_{OP,Th} + x_{OP,F} \leq 6$$

$$12 \leq x_{Gy,M} + x_{Gy,Tu} + x_{Gy,W} + x_{Gy,Th} + x_{Gy,F} \leq 18$$

$$2 \leq x_{Or,M} + x_{Or,Tu} + x_{Or,W} + x_{Or,Th} + x_{Or,F} \leq 3$$

$$2 \leq x_{Ot,M} + x_{Ot,Tu} + x_{Ot,W} + x_{Ot,Th} + x_{Ot,F} \leq 4$$

$$18 \leq x_{Ge,M} + x_{Ge,Tu} + x_{Ge,W} + x_{Ge,Th} + x_{Ge,F} \leq 25$$

Finally, we have the integer constraint: $x_{i,j}$ are all integer-valued.