New York University Shanghai

Reading for Session 12: Convex Optimization

BUSF-SHU 210: Business Analytics (Spring 2018)

The optimization models we have learned so far are all linear models, i.e., the objective function and the constraints are both linear in the decision variables. In practice, however, there are a lot of applications for which we need to develop non-linear programming models. In Session 11, we are going to introduce a notable application of non-linear programming: portfolio optimization

Portfolio Optimization

Suppose we are providing investment advice to Ms. Liu, who has some savings to invest. She has identified five stocks that she would be interested in investing in:

- Apple Inc. (AAPL)
- Amazon.com, Inc. (AMZN)
- Walt Disney Co. (DIS)
- Whole Foods Market, Inc. (WFM)
- Wal-Mart Stores, Inc. (WMT)

Ms. Liu is stumped by the question of how to allocate her investment among these five stocks. She would like to maximize the annual return and to minimize the risk. To help Ms. Liu, we collected data from Yahoo Finance on the daily prices of the five stocks over a ten-year period. Then, we do some initial calculations to summarize the monthly return rates of these 5 stocks. The monthly return rates of the 5 stocks over this 120-month horizon are reported in the file `Portfolio.csv`.

Ms. Liu would like to allocate her capital onto a portfolio of these five stocks for **one year**, with the hope that the annual return can be maximized whereas the risk can be effectively controlled. Assume that Ms. Liu now has $200,000 for investment. The price of each stock is normalized to $1. The risk-free investment (say a bond), has an annual return rate of 0.05. The price of the bond is also normalized to $1. Our goal is to help Ms. Liu optimize her investment portfolio.

Questions

To prepare for Session 12, please load the data "`Portfolio.csv`" into Pyton or *R*. This data set has the following five variables:

- $AAPL$: Each row means the return rate of Apple Inc. in that month
- $AMZN$: Each row means the return rate of Amazon.com, Inc. in that month

- DIS: Each row means the return rate of Walt Disney Co in that month
- WFM: Each row means the return rate of Whole Foods Market, Co in that month
- WMT: Each row means the return rate of Wal-Mart Stores, Inc. in that month

Please answer the following questions before the class starts, but no need to submit your solutions:

(a) What is the estimated annual return rate for each stock?

(b) What is the standard deviation of the estimated annual return rate for each stock?

(c) Let x_1 be the number of shares for $AAPL$, x_2 be the number of shares for $AMZN$, x_3 be the number of shares for DIS, x_4 be the number of shares for WFM, x_5 be the number of shares for WMT, and x_0 is the quantity of bonds in the portfolio. What is the expected annual return for this portfolio? What is the variance of the annual return for this portfolio?

(d) Suppose the objective is to maximize:

$$\text{Expected Annual Return } - \lambda * \text{Variance of Annual Return},$$

where $\lambda \geq 0$ is a parameter that captures how Ms. Liu weighs return and risk. Please try to formulate this problem as an optimization model with the portfolio $(x_1, x_2, x_3, x_4, x_5, x_0)$ as the decision variables.