MSDABC Network Analytics - Homework 0 (written) — Math diagnostics

Due: Sunday, November 3, 9:00 PM, Shanghai Time

General Instructions:

- These questions check your level of mathematical preparedness for this course. You can check your programming preparedness by doing Homework 0 (coding).
- You should print out this homework and write answers in the space left for you, then scan the homework into pdf format and submit it through Gradescope. For Gradescope, the entry code for this course is 97B277.
- Show your deductions, i.e., how you reached your answer for each of the problems. Answers without deductions, even if they are accurate, will be considered incorrect.

Problem 1 Define a two-variable function as follows: $f(x, y) = x^2 + 2xy + y^2$. Compute its partial derivative with respect to x.

Problem 2 If w_1, w_2, w_3 are independent vectors, show that the differences $v_1 = w_2 - w_3$ and $v_2 = w_1 - w_3$ and $v_3 = w_1 - w_2$ are dependent.

Problem 3 For each of the transformations below, show which is linear and which is not linear. The input is the vector $v = (v_1, v_2)$. Justify your answer.

1. $T(v) = (v_2, v_1)$
2. $T(v) = (v_1, v_1)$
3. $T(v) = (0, v_1)$
4. $T(v) = (0, 1)$

Problem 4 Suppose a linear transformation T transforms $(1, 1)$ to $(2, 2)$ and $(2, 0)$ to $(0, 0)$. Find $T(v)$ when:

1. v = (2, 2)
2. v = (3, 1)
3. v = (1, 1)

Problem 5 You know that $x = 12y + 12(x + 1)$ and $y = 13y + 13(x + 2)$. What are x and y?

Problem 6 Fifty-two percent of the students at a certain college are females. Five percent of the students in this college are majoring in computer science. Two percent of the students are women majoring in computer science. If a student is selected at random, find the conditional probability that

1. the student is female given that the student is majoring in computer science;
2. this student is majoring in computer science given that the student is female.

Problem 7 Someone rolls a fair six-sided die and you win points equal to the number shown. What is the expected number of points after one roll? After 2 rolls? After 100 rolls?

Problem 8 Prostate cancer is the most common type of cancer found in males. As an indicator of whether a male has prostate cancer, doctors often perform a test that measures the level of the prostate-specific antigen (PSA) that is produced only by the prostate gland. Although PSA levels are indicative of cancer, the test is notoriously unreliable. Indeed, the probability that a noncancerous man will have an elevated PSA level is approximately .135, increasing to approximately .268 if the man does have cancer. If, on the basis of other factors, a physician is 70 percent certain that a male has prostate cancer, what is the conditional probability that he has the cancer given that

1. the test indicated an elevated PSA level
2. the test did not indicate an elevated PSA level

Problem 9 Prove by induction that for any positive integer number n , $n^3 + 2n$ is divisible by 3.

Problem 10 What critical operation is generally faster in a hashtable than in a linked list, and how fast is it typically in each? Use the big-oh notation to justify your answer. When will a hashtable degrade to the speed of a list?

Problem 11 The Centers for Disease Control (CDC) reported on trends in weight, height and body mass index from the 1960's through 2002.1 The general trend was that Americans were much heavier and slightly taller in 2002 as compared to 1960; both men and women gained approximately 24 pounds, on average, between 1960 and 2002. In 2002, the mean weight for men was reported at 191 pounds. Suppose that an investigator hypothesizes that weights are even higher in 2006 (i.e., that the trend continued over the subsequent 4 years). We recruit 100 random men into our sample of American males in 2006 and measure their weights. We weigh each participant and compute summary statistics on the sample data. Suppose in the sample we determine the following: mean weight 197.1, sample standard deviation 25.6. Do the sample data support the investigator's hypothesis?