SC1007 Data Structures and Algorithms

Tutorial 5: Hash Table and Graph Representation

Q1 The type of a hash table H under closed addressing is an array of list references, and under open addressing is an array of keys. Assume a key requires one "word" of memory and a linked list node requires two words, one for the key and one for a list reference. Consider each of these load factors for closed addressing: 0.5, 1.0, 2.0.
Estimate the total space requirement, including space for lists, under closed addressing, and then, assuming that the same amount of space is used for an open addressing hash table, what are the corresponding load factors under open addressing?

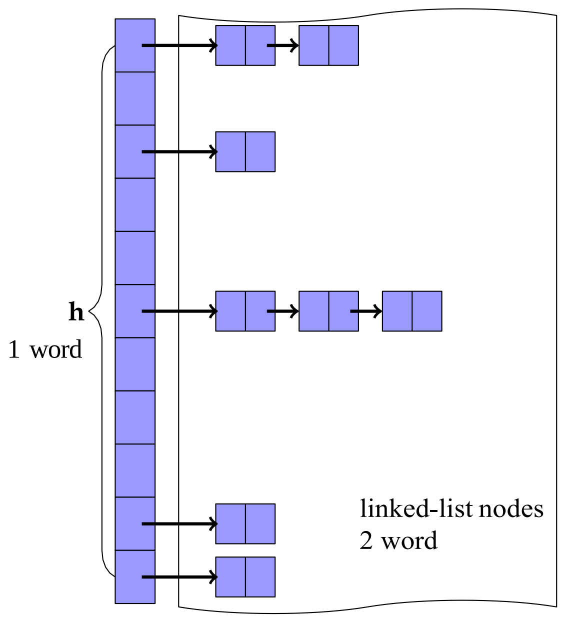

Figure 5.1: Closed Addressing Hash Table

Q2 Consider a hash table of size n using open address hashing and linear probing. Suppose that the hash table has a load factor of 0.5, describe with a diagram of the hash table, the best-case and the worst-case scenarios for the key distribution in the table. For each of the two scenario, compute the average-case time complexity in terms of the number of key comparisons when inserting a new key. You may assume equal probability for the new key to be hashed into each of the n slots.
[Note: Checking if a slot is empty is not a key comparison.]

Q3 Consider a hash function: $h(k) = k \: mod \: m$, where $m = 2^p - 1$ and k is a character string interpreted in radix 2^p. Show that if string x and be derived from string y by permuting its characters, then x and y hash to the same value.

Note: A character string interpreted in radix 2^p: Each character in the string is assigned a numeric value. The string is then interpreted as a number in base 2^p. For example, suppose $p = 3$, so that base $2^p = 8$ is used, and the character values are assigned as follows:

$\boldsymbol{A = 1, B = 2, C = 3}$

Then, the string "CAB" is interpreted as:

$$k = 3 \times 8^2 + 1 \times 8^1 + 2 \times 8^0 = 202$$

Similarly, the string "BCA" is interpreted as:

$$k = 2 \times 8^2 + 3 \times 8^1 + 1 \times 8^0 = 153$$

$h(202) = 202 \: mod \: (2^3 - 1) = 6$

$h(153) = 153 \: mod \: (2^3 - 1) = 6$