

# BLM1612 Circuit Theory

## Basic Concepts

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# Units

- numbers & units: used to express some measurable quantity
  - numbers: we typically use base-10 (numerals 0 through 9)
  - units: we typically use either English or Metric (SI)

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**TABLE 1.1**

Six basic SI units and one derived unit relevant to this text.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Charge	coulomb	C

- SI units are used by all modern engineering textbooks

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**TABLE 1.2**

The SI prefixes.

Multiplier	Prefix	Symbol
$10^{18}$	exa	E
$10^{15}$	peta	P
$10^{12}$	tera	T
$10^9$	giga	G
$10^6$	mega	M
$10^3$	kilo	k
$10^2$	hecto	h
10	deka	da
$10^{-1}$	deci	d
$10^{-2}$	centi	c
$10^{-3}$	milli	m
$10^{-6}$	micro	$\mu$
$10^{-9}$	nano	n
$10^{-12}$	pico	p
$10^{-15}$	femto	f
$10^{-18}$	atto	a

# Units

Name	Symbol	Quantity	Expressed in terms of other SI units	Expressed in terms of SI base units
hertz	Hz	frequency		$s^{-1}$
newton	N	force		$kg \cdot m \cdot s^{-2}$
joule	J	energy, work	$N \cdot m$	$kg \cdot m^2 \cdot s^{-2}$
watt	W	power	$J/s$	$kg \cdot m^2 \cdot s^{-3}$
coulomb	C	electric charge		$s \cdot A$
volt	V	voltage	$W/A$ or $J/C$	$kg \cdot m^2 \cdot s^{-3} \cdot A^{-1}$
ohm	$\Omega$	electric resistance	$V/A$	$kg \cdot m^2 \cdot s^{-3} \cdot A^{-2}$
farad	F	electric capacitance	$C/V$	$kg^{-1} \cdot m^{-2} \cdot s^4 \cdot A^2$
henry	H	inductance	$Wb/A$	$kg \cdot m^2 \cdot s^{-2} \cdot A^{-2}$
weber	Wb	magnetic flux	$V \cdot s$	$kg \cdot m^2 \cdot s^{-2} \cdot A^{-1}$

**joule:** fundamental unit of ***work or energy***

$$1 \text{ J} = 2.78 \times 10^{-7} \text{ kW} \cdot \text{h} = 2.39 \times 10^{-4} \text{ kcal}$$

- Since the **joule** is also a **watt-second** and the common unit for electricity sales to homes is the kW·h (kilowatt-hour), a kW·h is thus 1000 (kilo) × 3600 seconds = 3.6 MJ (megajoules).

# Examples

- A 1.2-kWh toaster takes roughly 4 minutes to heat four slices of bread. Find the cost of operating the toaster once per day for 1 month (30 days). Assume energy costs 9 cents/kWh. (answer = 21.6 cents)

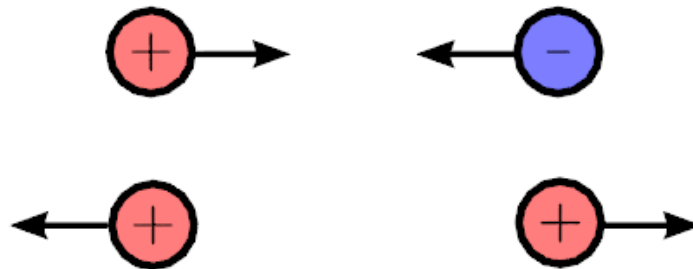
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- The clock speed of your computer is 2.6 GHz. This clock speed is equal to...
  - .....MHz
  - .....KHz
  - .....HZ

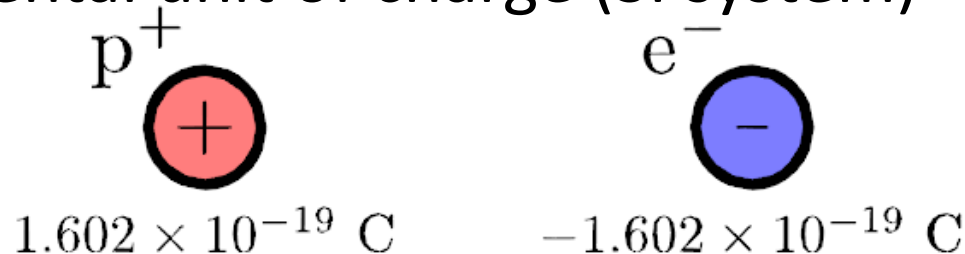
# Charge

Charge	
Units	coulomb (C)
Variable	$q, Q$

- is the fundamental property of matter that causes it to experience a force when placed in electro magnetic field and refers to electrons & protons
  - particles that attract each other (opposite “charge”) or repel each other (same “charge”)



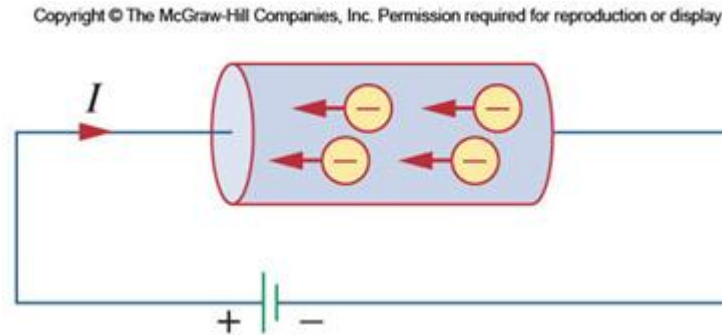
- fundamental unit of charge (SI system) = coulomb



# Current

Current	
Units	ampere ( $A = \frac{C}{s}$ )
Variable	$i$

- The movement of charge is called a current
- Historically the moving charges were thought to be positive



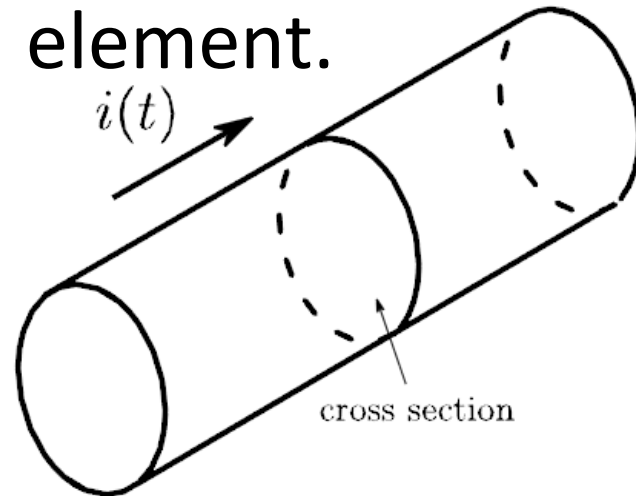
- The mechanism by which electrical energy is transferred
  - Send power from generation point to consumption point
  - Send signals from source to sink



# Current

- Current,  $i$ , is measured as charge moved per unit time through an element.

$$i = \frac{dq}{dt}$$



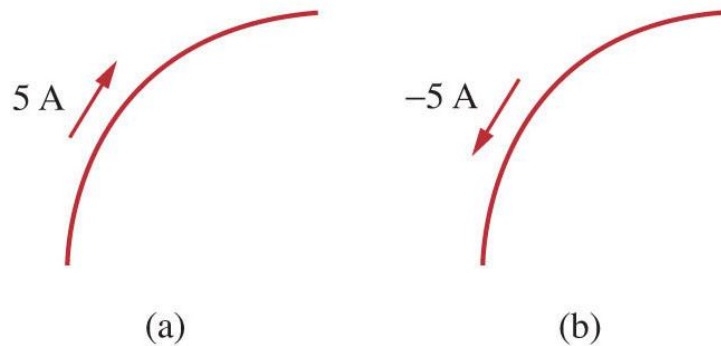
- Amount of charge that has passed a given point:

$$q(t) = \int_{t_0}^t i(\tau) d\tau + q(t_0)$$

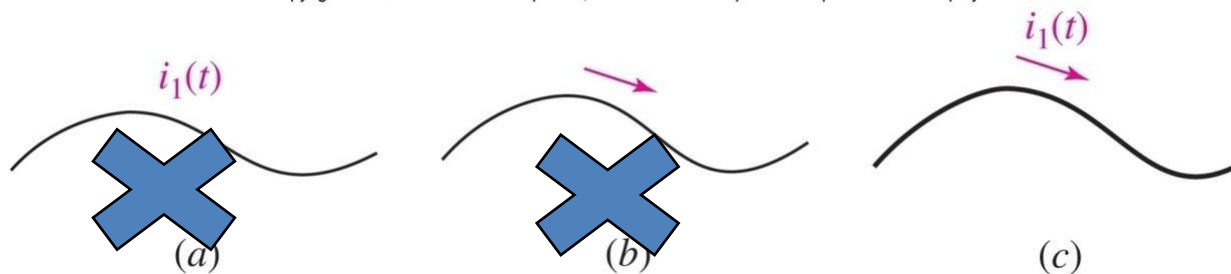
# Direction of Current

- defined as the flow of *positive* charge in a conductor (i.e. in reality, a positive forward current means the electrons are flowing backwards)
- when written, current must be labeled with **direction** & **value**:

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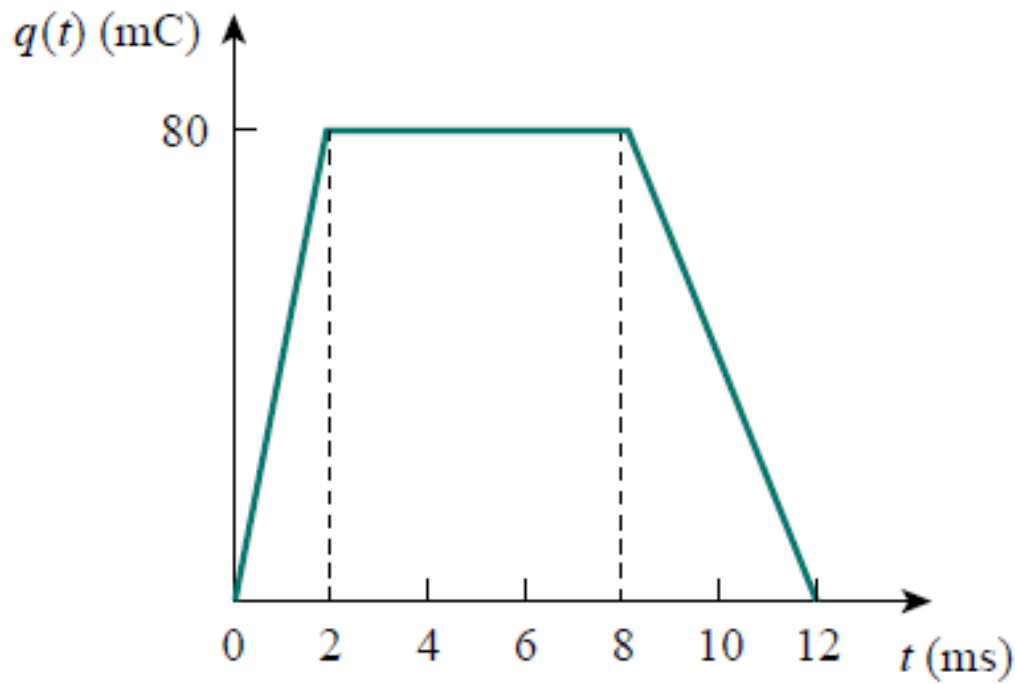




# Examples

- The charge entering a certain element is shown in below Figure. Find the current at:

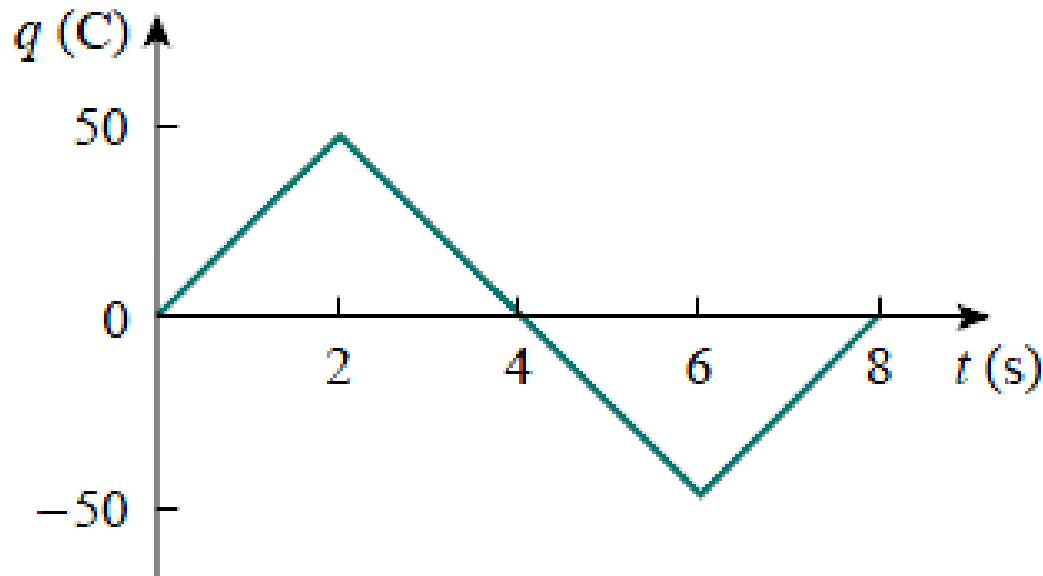
(a)  $t = 1 \text{ ms}$       (b)  $t = 6 \text{ ms}$       (c)  $t = 10 \text{ ms}$



- The slope is defined as the ratio of the vertical change between two points, to the horizontal change between the same two points.

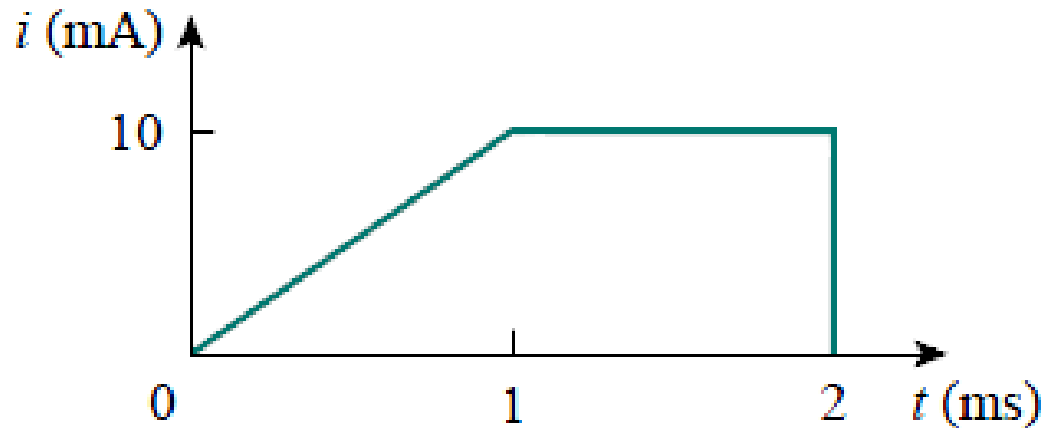
# Examples

- The charge flowing in a wire is plotted in below Figure. Sketch the corresponding current.



# Examples

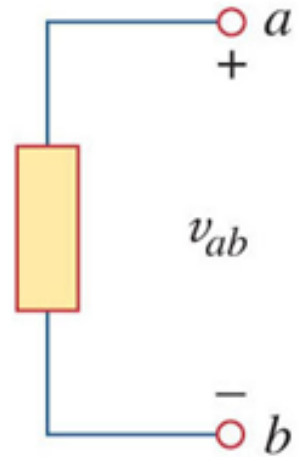
- The current flowing past a point in a device is shown in below Figure. Calculate the total charge through the point.



# Voltage

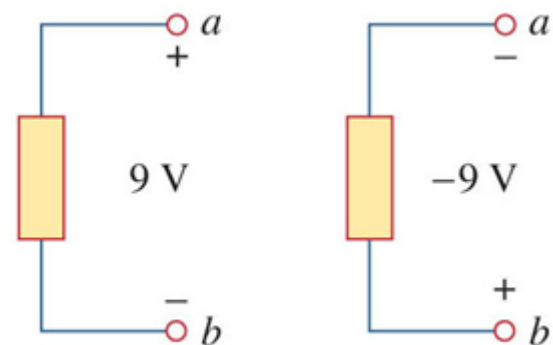
Voltage	
Units	volt ( $V = \frac{J}{C}$ )
Variable	$v$

- Electrons move when there is a difference in charge between two locations.
- Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in **volts** (V) (from  $a$  to  $b$ ).



- $V_{ab}$ 
  - point  $a$  is at a potential of  $v_{ab}$  volts higher than point  $b$

$$V_{ab} = -V_{ba}.$$



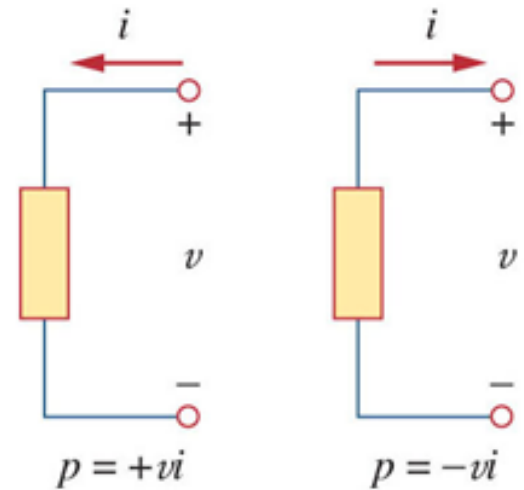
# Power

Power	
Units	watt ( $W = \frac{J}{s}$ )
Variable	$p$

- Power is the product of voltage and current

$$p = vi$$

- It is equal to the rate of energy provided or consumed per unit time.
- It is measured in Watts (W)
- Passive sign convention
  - current into positive terminal
  - positive for power *absorbed*
  - negative for power *supplied*

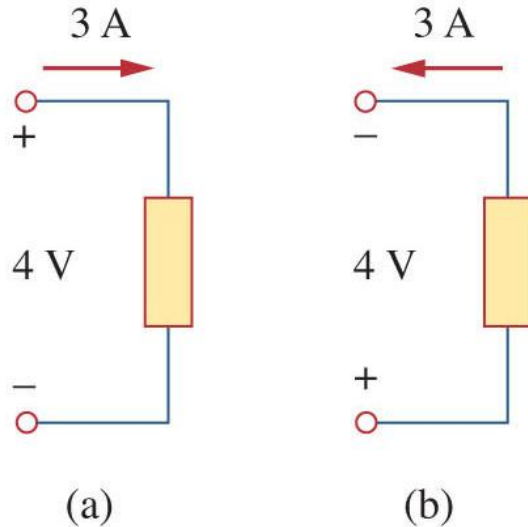


(a)

(b)

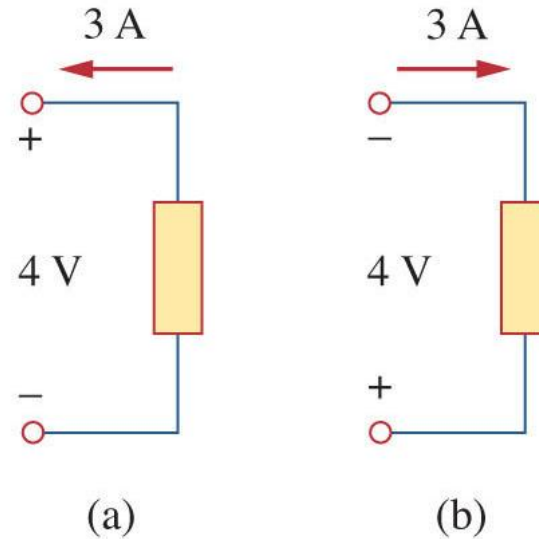
# Power

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Two cases of an element  
with an absorbing power of  
12 W

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Two cases of an element  
with an supplying power of  
12 W

# Conservation of Energy (Tellegen's Theorem )

- The sum of all power supplied must be absorbed by the other elements.
- For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero

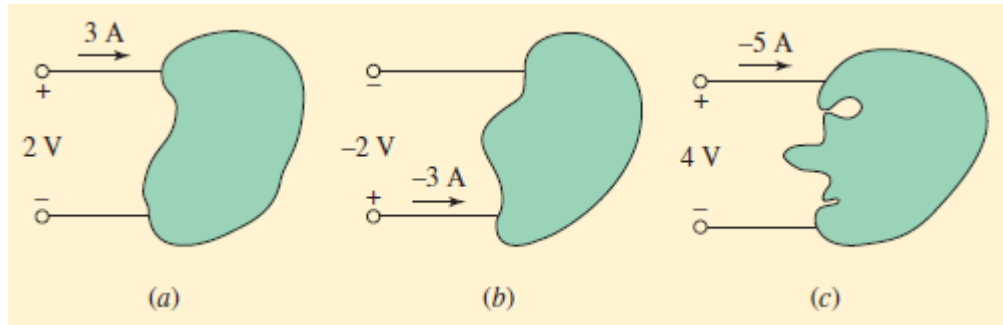
$$\sum p = 0$$

- The energy absorbed or supplied by an element from time  $t_0$  to time  $t$  is

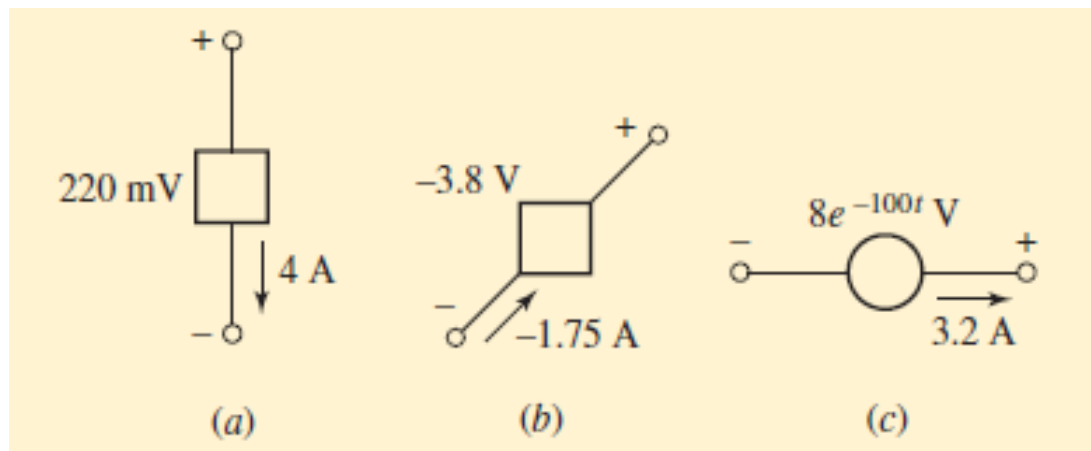
$$w = \int_{t_0}^t p \, dt = \int_{t_0}^t vi \, dt$$

# Examples

- Compute the power absorbed by each part



- Determine the power absorbed by each element (a,b,c). ( $t = 5$  ms for the element in(c))





# Circuit Elements

- circuit element: mathematical model for the voltage/current behavior of a component (or collection of components)

$$V = I \cdot R$$



$$V = L \frac{dI}{dt}$$



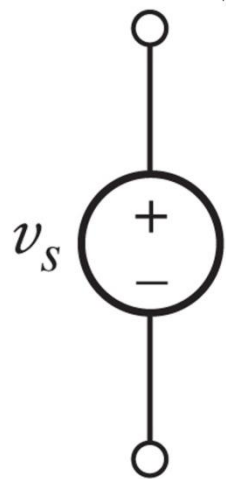
$$I = C \frac{dV}{dt}$$



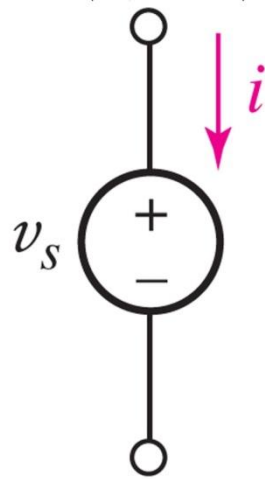
- sources-- independent vs. dependent
  - **independent**: value of the source voltage/current is *not* affected by the rest of the circuit
  - **dependent**: value of the source voltage/current is *controlled* by the rest of the circuit

# Independent Voltage Source

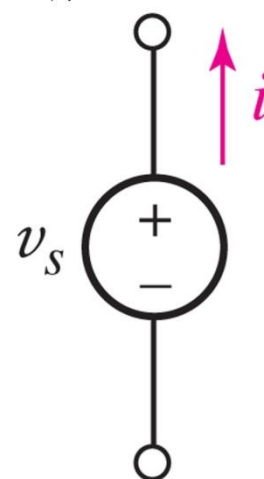
- An ideal voltage source is a circuit element that will maintain the specified voltage  $v_s$  across its terminals.
- The current will be determined by other circuit elements.



(a)



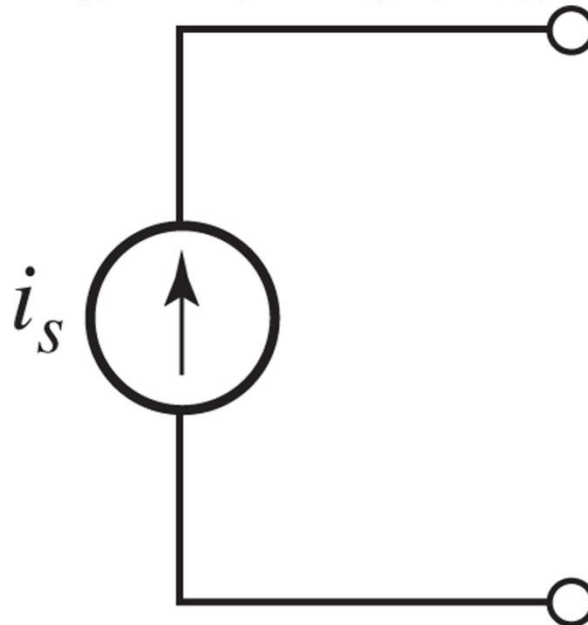
(b)



(c)

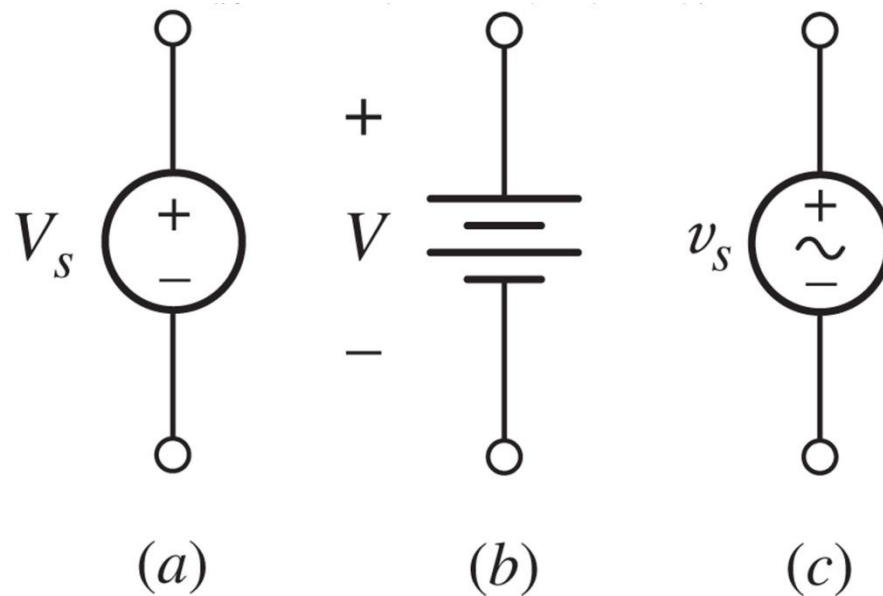
# Independent Current Source

- An ideal current source is a circuit element that maintains the specified current flow  $i_s$  through its terminals.
- The voltage is determined by other circuit elements.



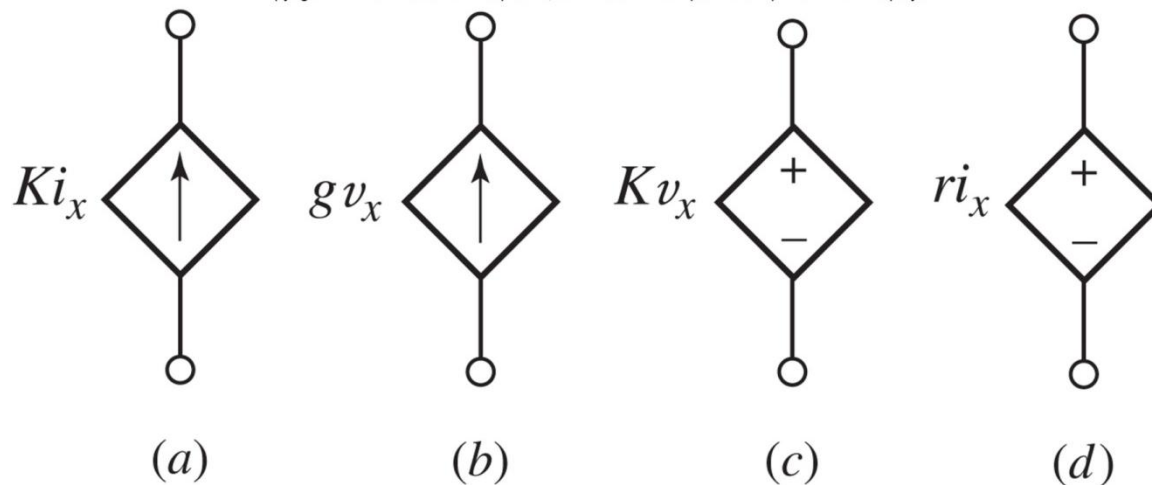
# Battery as Voltage Source

- A voltage source is an idealization (no limit on current) and generalization (voltage can be time-varying) of a battery.



# Dependent Sources

- **dependent V/C source:** source whose V/C is dependent upon or *controlled by* some V/C at another point

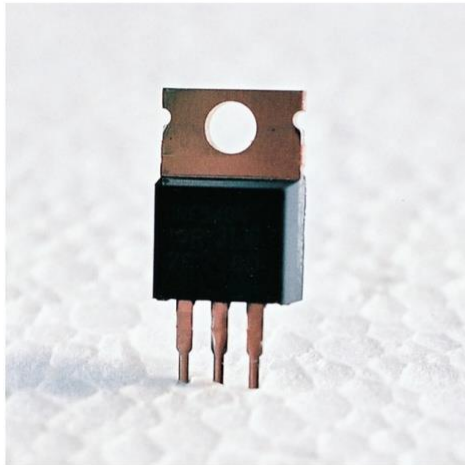


The four different types of dependent sources: (a) current-controlled current source; (b) voltage-controlled current source; (c) voltage-controlled voltage source; (d) current-controlled voltage source.

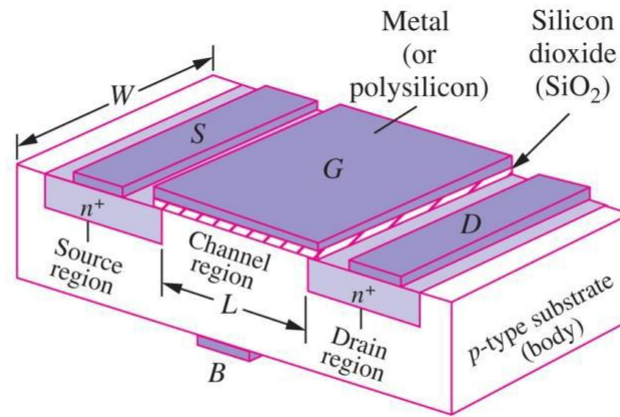
# Dependent Sources

- useful for analyzing (simplifying) the behavior of complicated circuit elements (e.g. transistors, operational amplifiers)

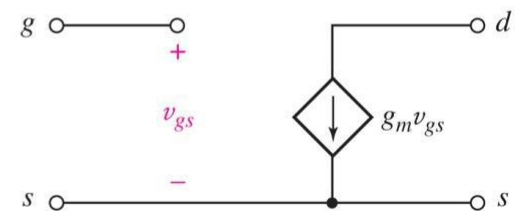
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(a)



(b)



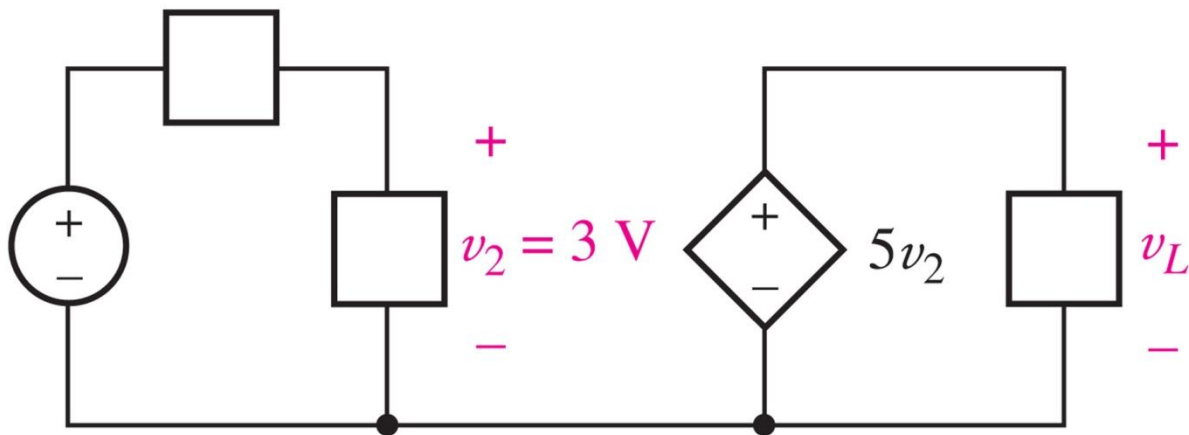
(c)

Metal Oxide Semiconductor Field Effect Transistor (MOSFET)

# Example Dependent Sources

- If  $v_2 = 3 \text{ V}$ , Find the voltage  $v_L$  in the circuit below.

Assume that all points along a wire that do not cross a circuit element have the same voltage.



$$v_2 = 3 \text{ V}$$

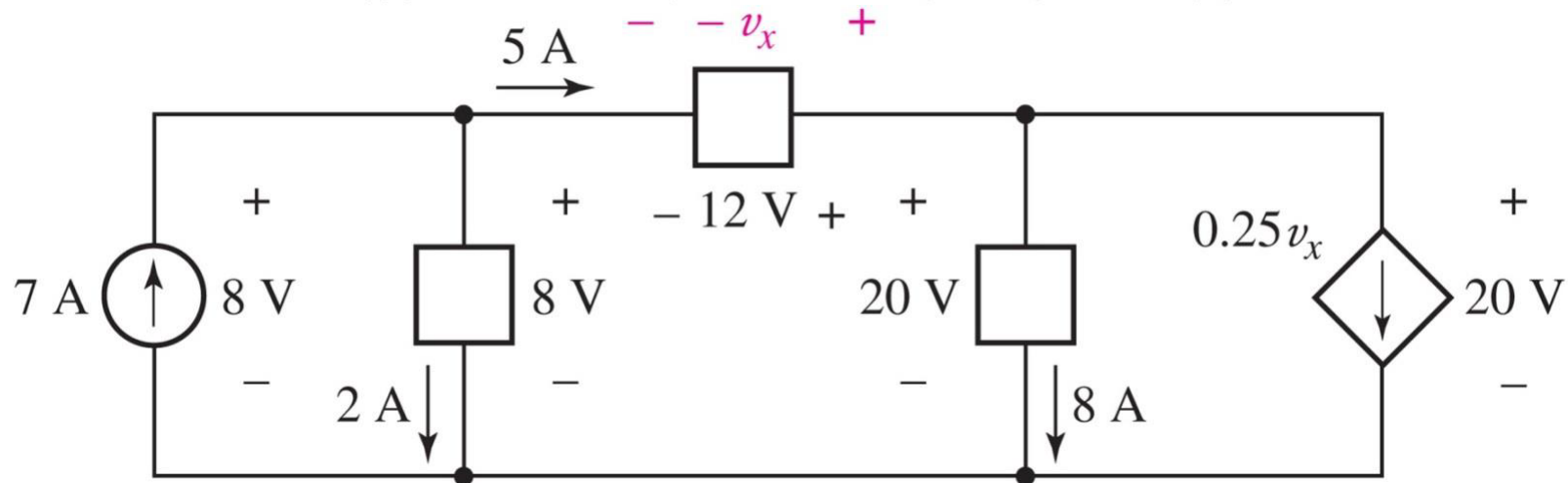
$$v_L = 5v_2$$

$$v_L = 15 \text{ V}$$

# Example Dependent Sources

- Determine the power *absorbed* by each element in the circuit

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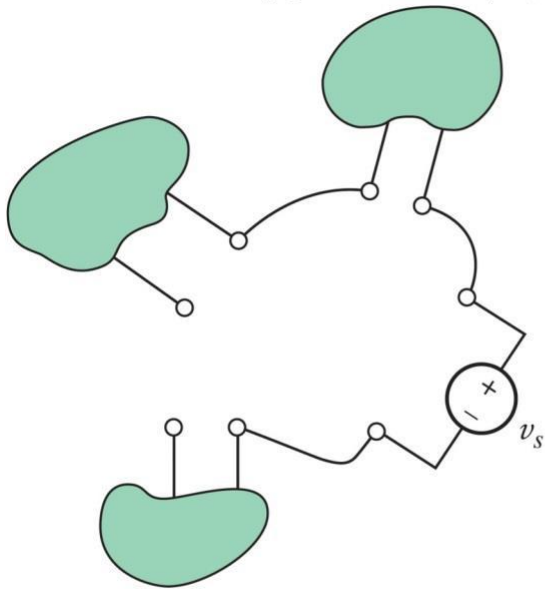




# Networks vs. Circuits

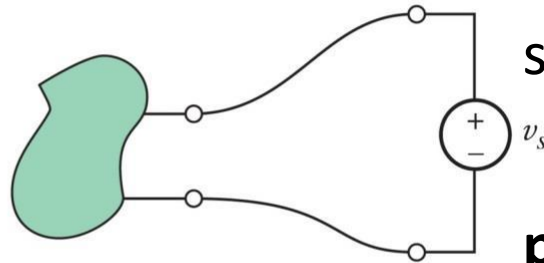
- **network:** interconnection of 2+ elements
- **circuit:** a network that contains at least 1 closed path

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(a)

network that is  
**not** a circuit



(b)

network that **is** a  
circuit

**active network:** contains at least 1 active device (that supplies energy)

**passive network:** contains no active devices (only consumes energy)

# Ohm's Law

- first discussed by Georg Simon Ohm (German physicist) in a pamphlet describing voltage & current measurements

$$V = I \cdot R \qquad v = i \cdot r$$

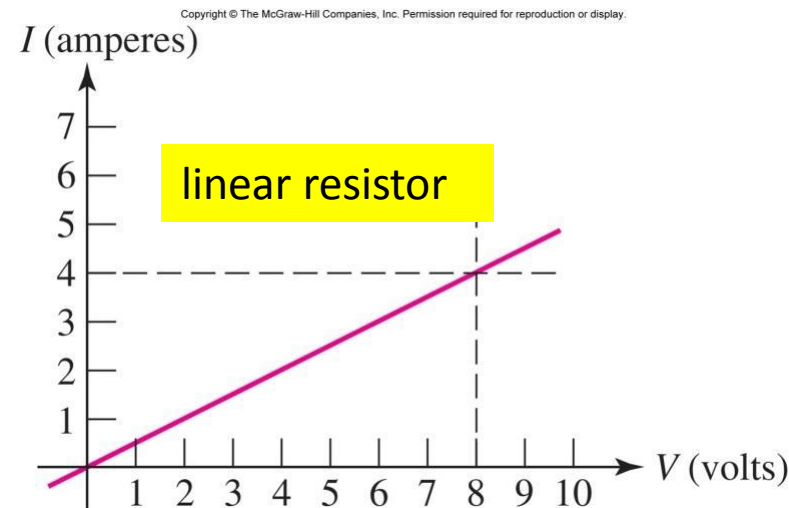
- the *voltage* across a conducting material is *linearly proportional* to the *current* flowing through that material

- constant of proportionality  
= the *resistance* of the material

- unit of resistance = the ***ohm***

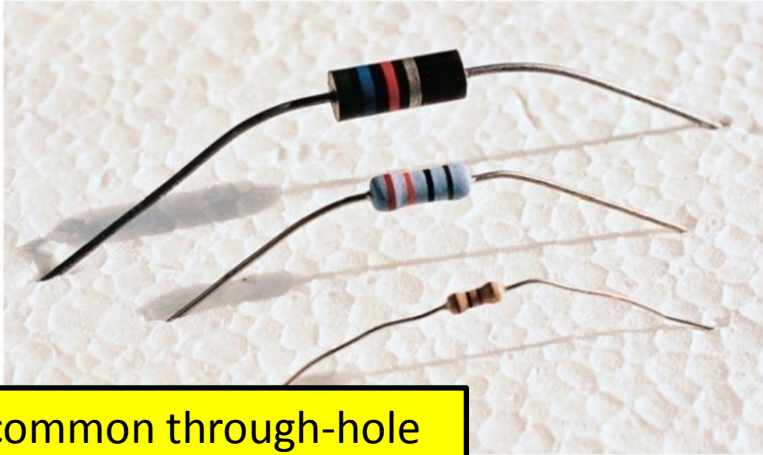
$$1 \, \Omega = 1 \, \text{V/A}$$

another idealization, but still a good approximation for many elements (over certain ranges of voltage, current)

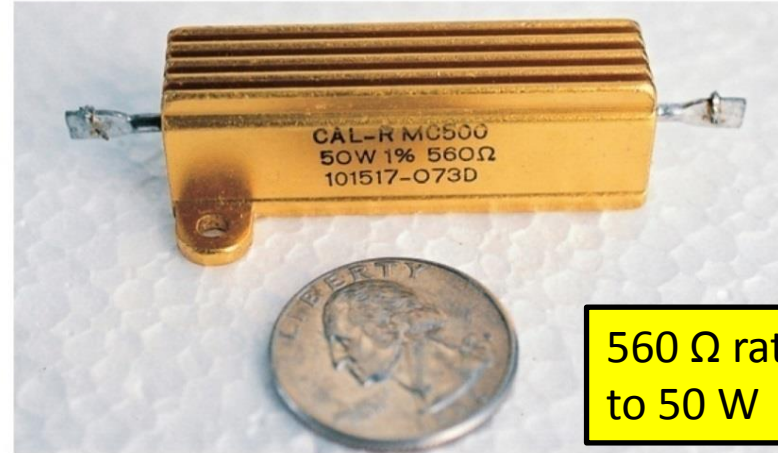


# Resistors

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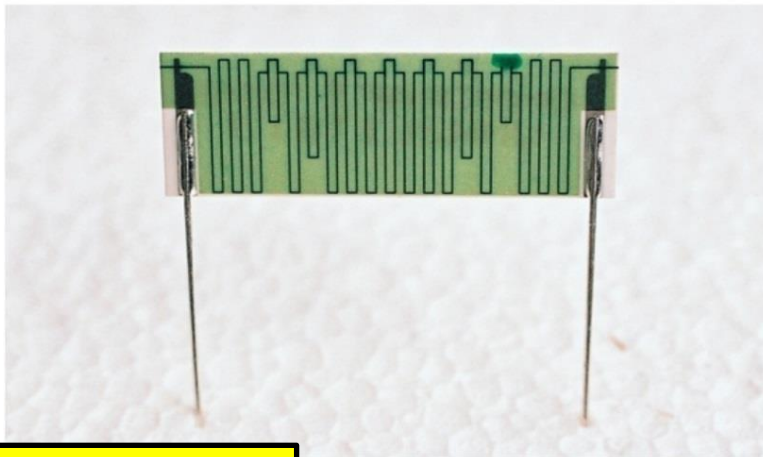


common through-hole  
components (for  
breadboarding)



560  $\Omega$  rated up  
to 50 W

(b)



10 teraohms ( $10 \times 10^{12} \Omega$ )

(c)

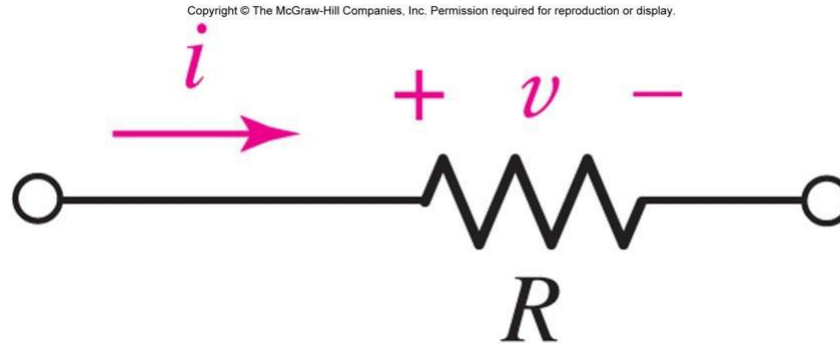


circuit symbol for  
a resistor

(d)

# Example

- Let  $R = 560\ \Omega$  and  $i = 42.4\ \text{mA}$ . Calculate the voltage across the resistor and power it is dissipating?



# Conductance & Open/Short Circuits

- for a linear resistor, the ratio of current : voltage is also a constant

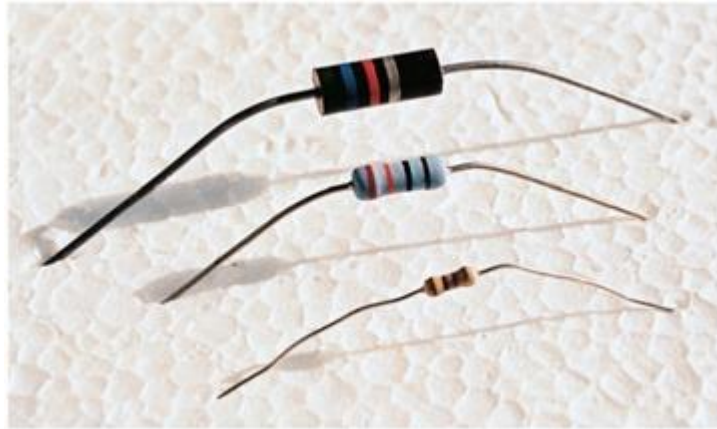
$$\frac{i}{v} = \frac{1}{R} = G$$

where  $G$  is called the *conductance*  
units = **siemens** (S),  $1 \text{ S} = 1 \text{ A/V} = \Omega^{-1}$

- 
- “**open circuit**”:  $R = \infty$ , and  $i = 0$  for *any* voltage across the open terminals
  - “**short circuit**”:  $R = 0$ , and  $v = 0$  for *any* current through the short
    - For all of our circuits, wires are assumed to be perfect short circuits.  
i.e. Since  $v = i R$  and wires have  $R = 0$ , all neighboring points along a wire have the same voltage.

# Resistors & Tolerance

- Real resistors are manufactured within a specific *tolerance* (5%, 10%, 20%).
- Given a  $50\ \Omega$  resistor with a tolerance of 10%, what is the maximum voltage across the resistor when a current of exactly 2 mA flows through it?

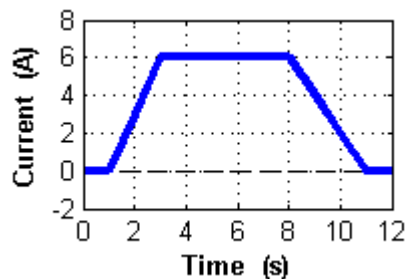


# Chapter 2 Summary & Review

- metric units for electricity: coulomb, volt, amp(ere), joule, Watt, ohm ( $\Omega$ )
- **charge** = polarization of particles; current = flow of charge
- **current** is defined with *value* and *direction* **voltage** is defined with value and polarity (+ terminal, – terminal)
- electrical **power**: *consumed* (current *enters* the + terminal; voltage drops) or *supplied* (current *leaves* the + terminal; voltage rises) is equal to  $V \times I$  and is an instantaneous or average quantity
- sources: **independent** vs. **dependent** (controlled or not controlled by the rest of the circuit)
- **Ohm's Law**:  $V = I R$ , a linear relationship that holds for resistors and other circuit elements for certain ranges of  $V$ ,  $I$

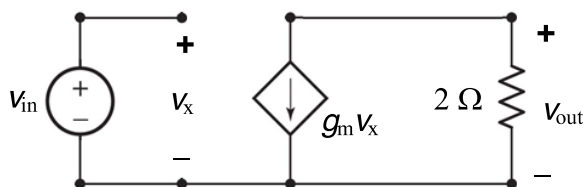
# Examples

- (a) The current through an element is given in the figure below (bold line).  
What is the net charge that passes through the element between  $t = 4$  and  $t = 8$  seconds?



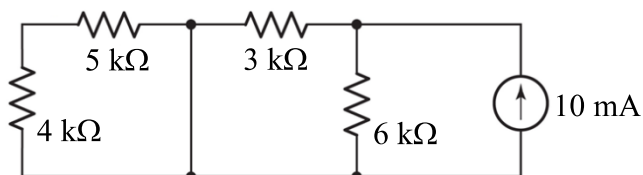
$$\begin{aligned}
 q &= \int i \, dt \\
 &= i \cdot \Delta t \text{ for constant current} \\
 &= (6 \text{ A})(8 - 4 \text{ s}) = \boxed{24 \text{ C}}
 \end{aligned}$$

- (b) In the circuit below,  $v_{in} = 3 \sin(\omega \cdot t) \text{ mV}$  and  $g_m = 10 \text{ A/V}$ . Determine  $v_{out}$ .



$$\begin{aligned}
 v_{out} &= -g_m v_x \cdot 2 \\
 &= -(10)(3 \sin(\omega t))(2) \\
 &= \boxed{-60 \sin(\omega t) \text{ mV}}
 \end{aligned}$$

- (c) In the circuit below, determine the power absorbed by the 5-k $\Omega$  resistor.

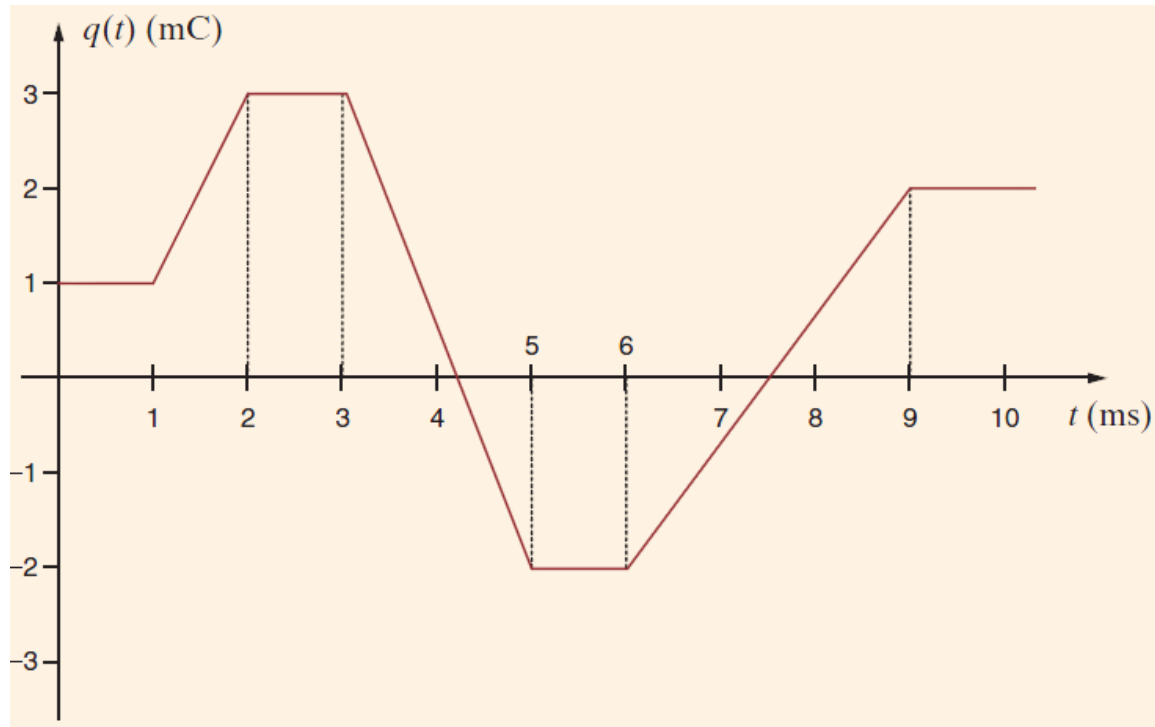
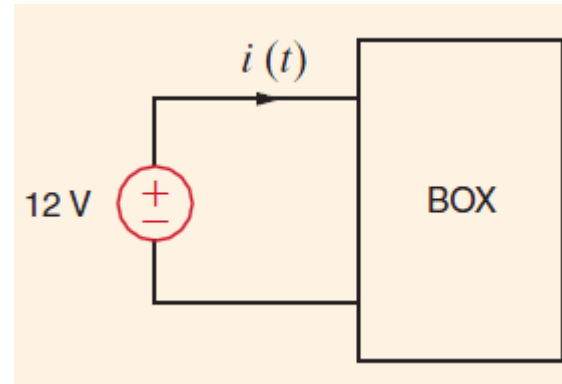


$$\begin{aligned}
 &\text{short circuit across} \\
 &\quad 4 \text{ k}\Omega \text{ and } 5 \text{ k}\Omega \\
 &\Rightarrow \text{no current through } 5 \text{ k}\Omega \\
 &\Rightarrow \boxed{P_{abs} = 0}
 \end{aligned}$$



# Examples

- The charge that enters the BOX is shown in below Figure. Calculate and sketch the current flowing into and the power absorbed by the BOX between 0 and 10 milliseconds.



# Solution

Recall that current is related to charge by  $i(t) = \frac{dq(t)}{dt}$ . The current is equal to the slope of the charge waveform.

$$i(t) = 0 \qquad 0 \leq t \leq 1 \text{ ms}$$

$$i(t) = \frac{3 \times 10^{-3} - 1 \times 10^{-3}}{2 \times 10^{-3} - 1 \times 10^{-3}} = 2 \text{ A} \qquad 1 \leq t \leq 2 \text{ ms}$$

$$i(t) = 0 \qquad 2 \leq t \leq 3 \text{ ms}$$

$$i(t) = \frac{-2 \times 10^{-3} - 3 \times 10^{-3}}{5 \times 10^{-3} - 3 \times 10^{-3}} = -2.5 \text{ A} \qquad 3 \leq t \leq 5 \text{ ms}$$

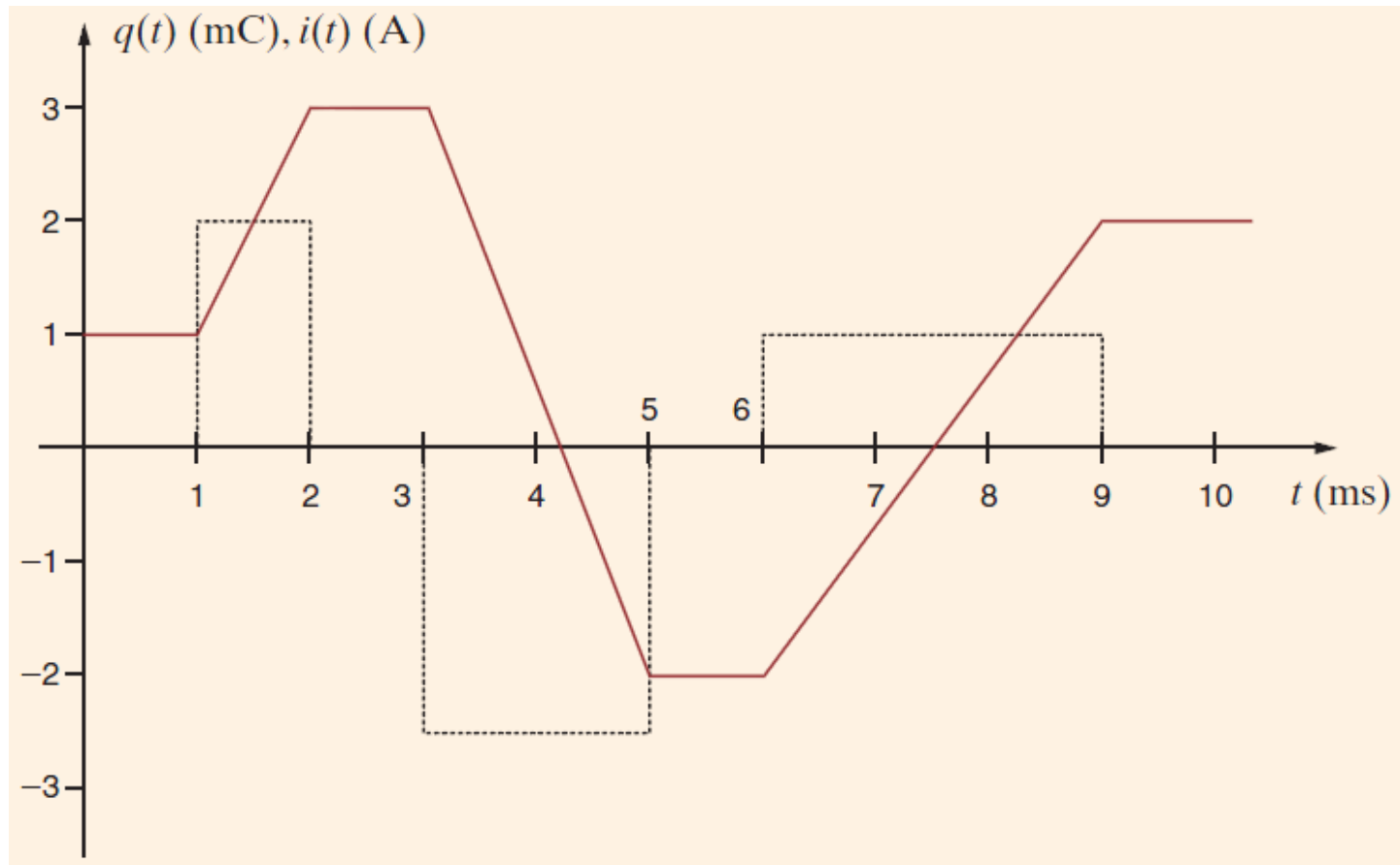
$$i(t) = 0 \qquad 5 \leq t \leq 6 \text{ ms}$$

$$i(t) = \frac{2 \times 10^{-3} - (-2 \times 10^{-3})}{9 \times 10^{-3} - 6 \times 10^{-3}} = 1.33 \text{ A} \qquad 6 \leq t \leq 9 \text{ ms}$$

$$i(t) = 0 \qquad t \geq 9 \text{ ms}$$

The current is plotted with the charge waveform in Fig. 1.21. Note that the current is zero during times when the charge is a constant value. When the charge is increasing, the current is positive, and when the charge is decreasing, the current is negative.

# Solution



# Solution

The power absorbed by the BOX is  $12 \cdot i(t)$ .

$$p(t) = 12 \cdot 0 = 0 \quad 0 \leq t \leq 1 \text{ ms}$$

$$p(t) = 12 \cdot 2 = 24 \text{ W} \quad 1 \leq t \leq 2 \text{ ms}$$

$$p(t) = 12 \cdot 0 = 0 \quad 2 \leq t \leq 3 \text{ ms}$$

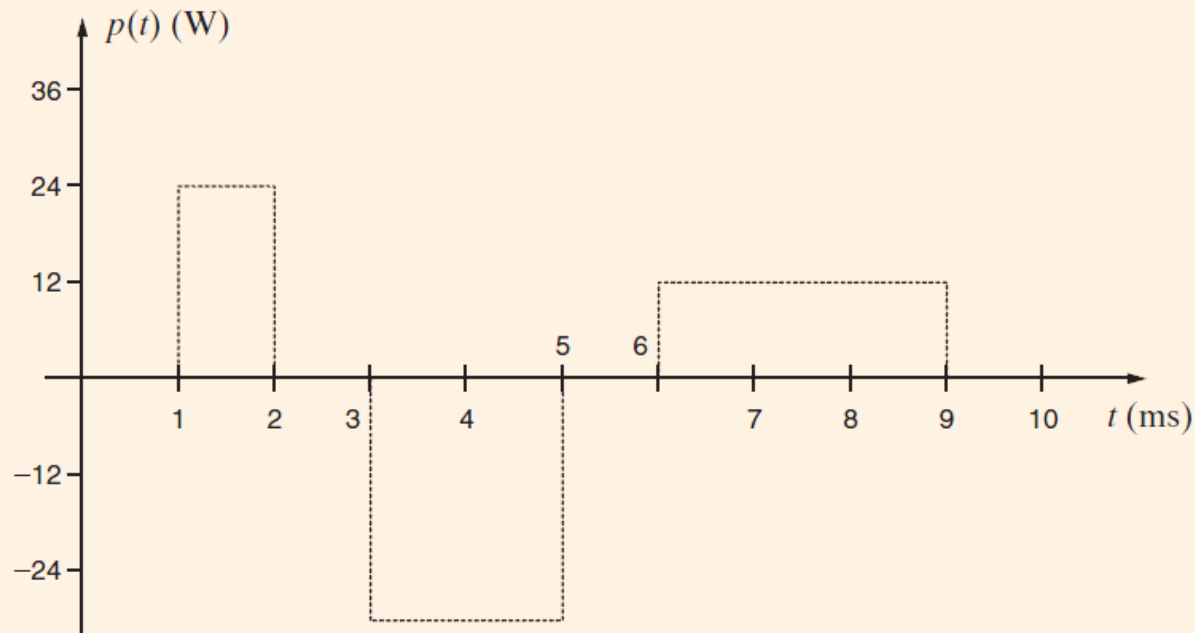
$$p(t) = 12 \cdot (-2.5) = -30 \text{ W} \quad 3 \leq t \leq 5 \text{ ms}$$

$$p(t) = 12 \cdot 0 = 0 \quad 5 \leq t \leq 6 \text{ ms}$$

$$p(t) = 12 \cdot 1.33 = 16 \text{ W} \quad 6 \leq t \leq 9 \text{ ms}$$

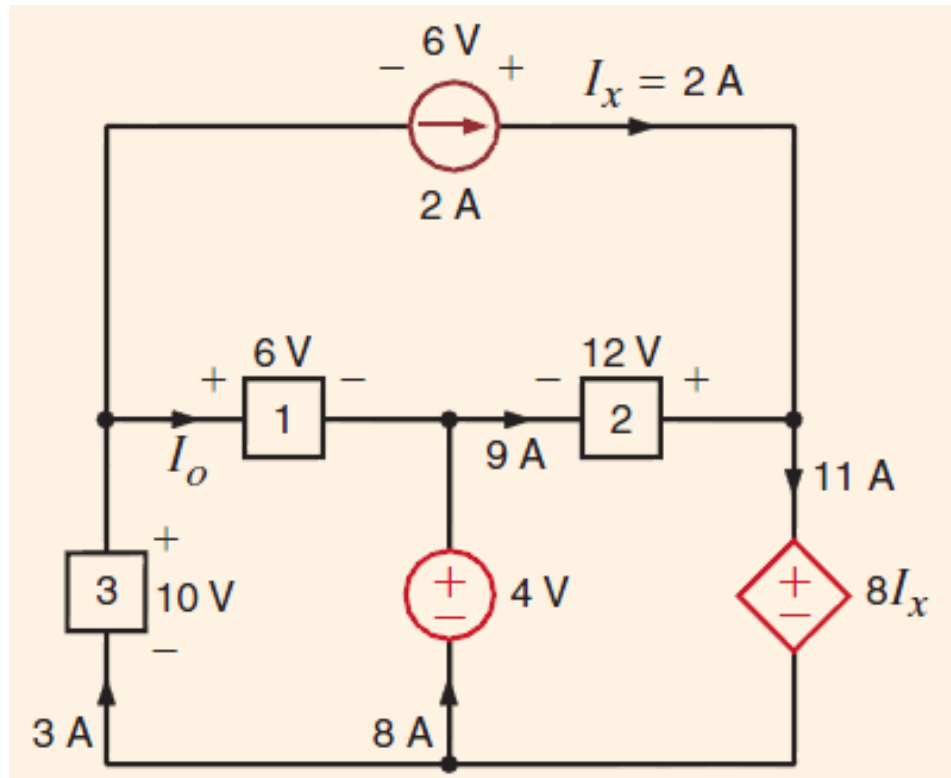
$$p(t) = 12 \cdot 0 = 0 \quad t \geq 9 \text{ ms}$$

The power absorbed by the BOX is plotted in Fig. 1.22. For the time intervals,  $1 \leq t \leq 2 \text{ ms}$  and  $6 \leq t \leq 9 \text{ ms}$ , the BOX is absorbing power. During the time interval  $3 \leq t \leq 5 \text{ ms}$ , the power absorbed by the BOX is negative, which indicates that the BOX is supplying power to the 12-V source.



# Example

- Use Tellegen's theorem to find the current  $I_o$  in the network in below Figure.



# Solution

First, we must determine the power absorbed by each element in the network. Using the sign convention for power, we find

$$P_{2A} = (6)(-2) = -12 \text{ W}$$

$$P_1 = (6)(I_o) = 6I_o \text{ W}$$

$$P_2 = (12)(-9) = -108 \text{ W}$$

$$P_3 = (10)(-3) = -30 \text{ W}$$

$$P_{4V} = (4)(-8) = -32 \text{ W}$$

$$P_{DS} = (8I_x)(11) = (16)(11) = 176 \text{ W}$$

Applying Tellegen's theorem yields

$$-12 + 6I_o - 108 - 30 - 32 + 176 = 0$$

or

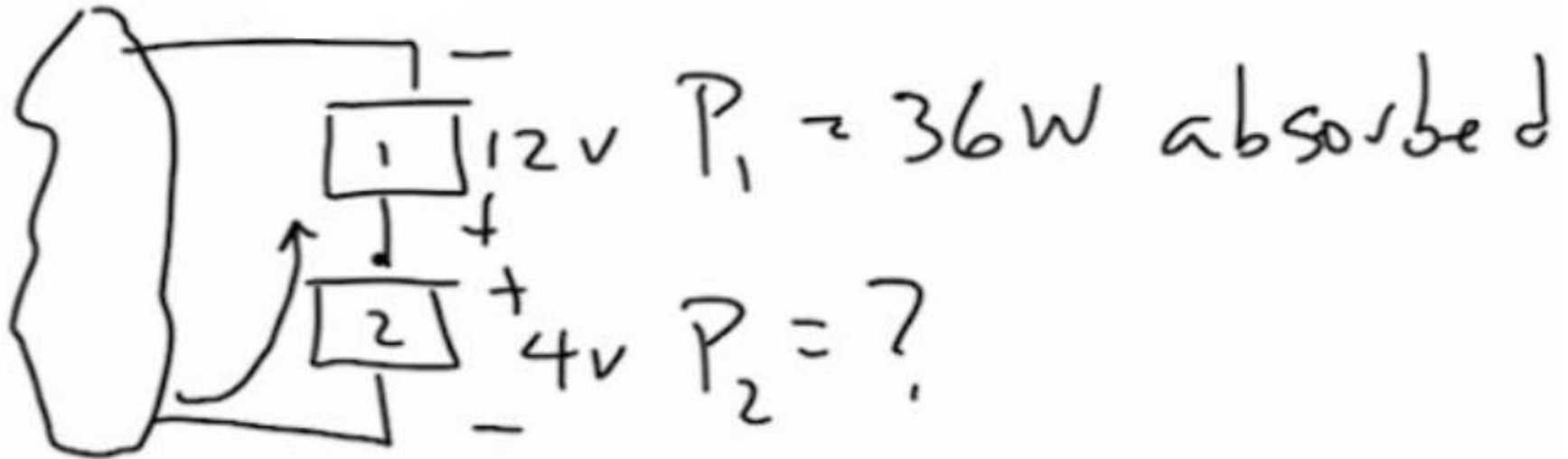
$$6I_o + 176 = 12 + 108 + 30 + 32$$

Hence,

$$I_o = 1\text{A}$$

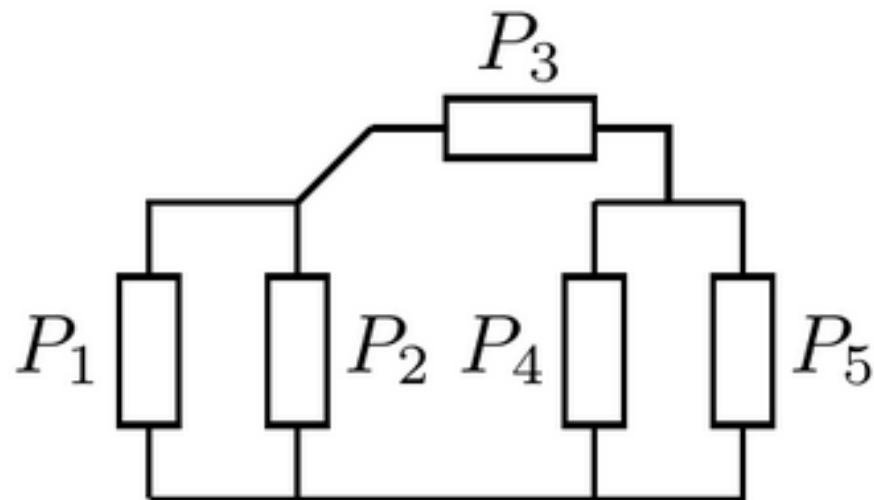
# Example

- Determine the missing quantity (power).



# Example

Below is a schematic of unknown elements.



We measure the following powers:

$$P_1 = -50W$$

$$P_2 = 40W$$

$$P_3 = -10W$$

$$P_4 = 5W$$

What is  $P_5$ ?

$$P_1 + P_2 + P_3 + P_4 + P_5 = 0$$

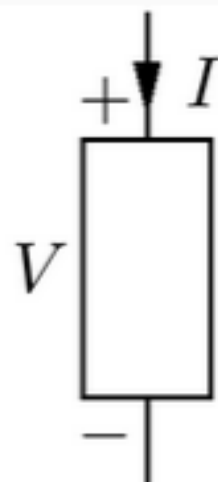
$$\Rightarrow P_5 = -(-50 + 40 - 10 + 5)W = \boxed{15W}$$



# Example

What is the power in Watts across the element if  $V = 0.4V$  and  $I = 2A$ ?

$$\begin{aligned} P &= +IV \\ &= +(2A)(0.4V) \\ &= \boxed{0.8W} \end{aligned}$$



What is the power across the element if  $V = 5V$  and  $I = 0.1A$ ?

*Opposite of convention:*

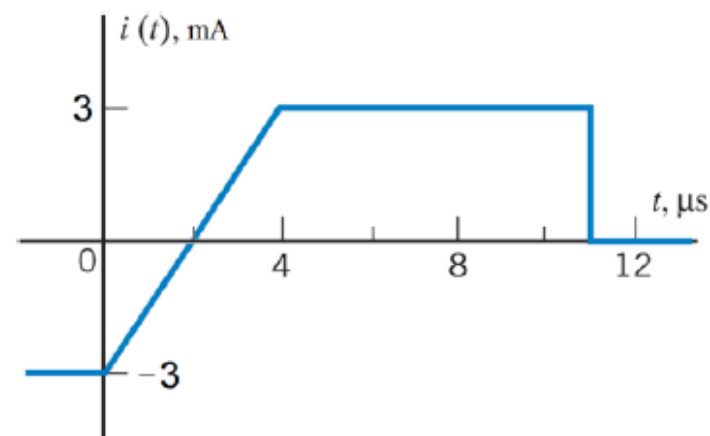
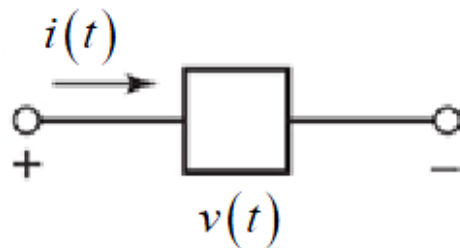
$$\begin{aligned} P &= -IV \\ &= -(0.1A)(5V) = \boxed{-0.5W} \end{aligned}$$



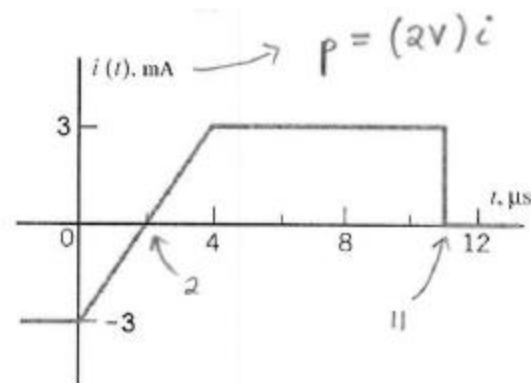
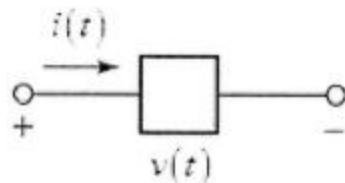
# Example

The current through a circuit element,  $i(t)$ , is plotted below. Assume  $v(t) = 2 \text{ V}$ .

- (a) Compute the net charge that passes through the element between  $t = 2 \text{ }\mu\text{s}$  and  $t = 12 \text{ }\mu\text{s}$ .
- (b) Determine if the element absorbs and/or delivers electrical energy.  
If the element absorbs electrical energy, when does this happen?  
If the element delivers electrical energy, when does this happen?
- (c) Compute the net energy absorbed by the element between  $t = 0 \text{ }\mu\text{s}$  and  $t = 9 \text{ }\mu\text{s}$ .



# Solution



$$(a) \quad q = \int_{2 \mu s}^{12 \mu s} i(t) d\tau = \left( \frac{1}{2} \right) (2 \mu s) (3 \text{ mA}) + (7 \mu s) (3 \text{ mA})$$

$$= 3 \text{ nC} + 21 \text{ nC} = \boxed{24 \text{ nC}}$$

$$(b) \quad \left. \begin{array}{l} \text{if } v \cdot i > 0 \Rightarrow \text{absorbs} \\ v \cdot i < 0 \Rightarrow \text{delivers} \end{array} \right\} \begin{array}{l} \text{absorbs from } t = 2 \rightarrow t = 11 \mu s \\ \text{delivers before } t = 2 \mu s \end{array}$$

$$(c) \quad w = \int_{0 \mu s}^{8 \mu s} p(t) d\tau, \quad p(t) = i(t) v(t)$$

$$= \left( \frac{1}{2} \right) (2 \mu s) (-6 \text{ mW}) + \left( \frac{1}{2} \right) (2 \mu s) (+6 \text{ mW})$$

$$+ (5 \mu s) (6 \text{ mW}) = \boxed{30 \text{ nJ}}$$