BLM1612 Circuit Theory

Linearity & Superposition

Source Transformations

Thevenin & Norton Equivalents

Maximum Power Transfer

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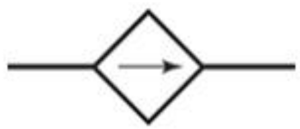
Linearity

linear element = circuit element whose voltage & current are related by a constant of proportionality

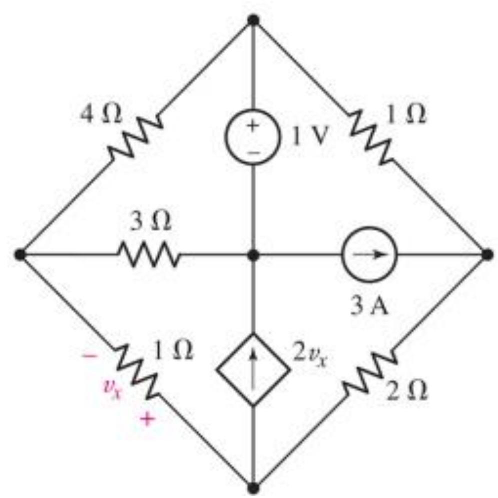
<u>Examples:</u>

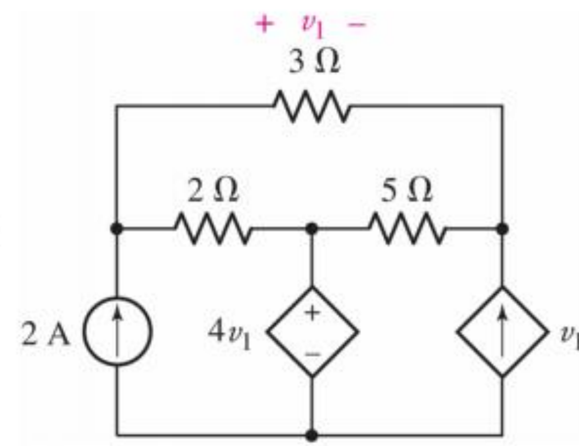

resistor, $V = I\,R \rightarrow V\,/\,I = \boldsymbol{R}$


voltage-controlled current source, $I = g_m V \rightarrow V\,/\,I = \mathbf{1/g_m}$

<u>linear circuit</u>

composed of only
independent sources,
linear dependent sources,
& other linear elements





Superposition

Theorem. In any linear network, the voltage across or the current through any element may be calculated by adding algebraically all the individual voltages or currents caused by the separate independent sources alone, with all other independent **voltage sources** replaced by **short circuits** and all other independent **current sources** replaced by **open circuits**.

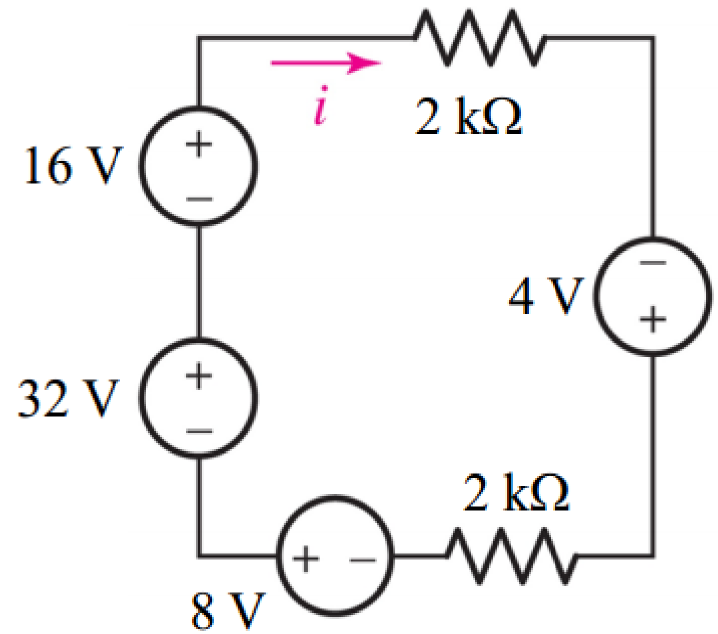
Translation. To solve for voltage across or current through an element, we may deactivate all sources but one, then solve for v/i, then deactivate all but the next and solve for v/i, and so on, and sum the individual v/i contributions to obtain the total v/i.

Notes: (1) N independent sources → N contributions to v/i

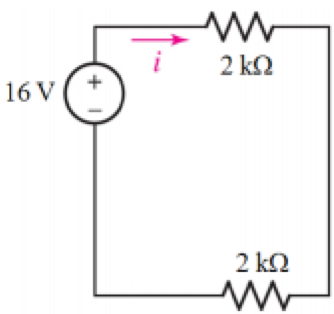
(2) dependent sources stay active

Superposition: Voltage Sources

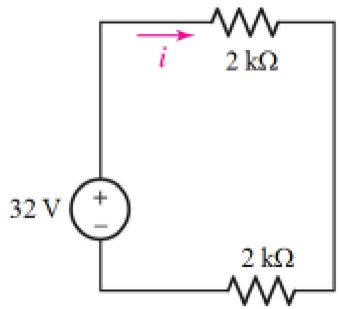
Determine the current *i* using superposition.



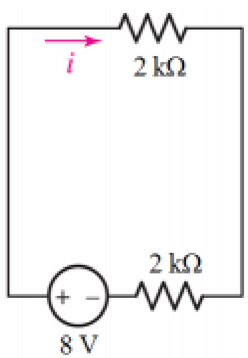
deactivate all but 1 → solve → sum



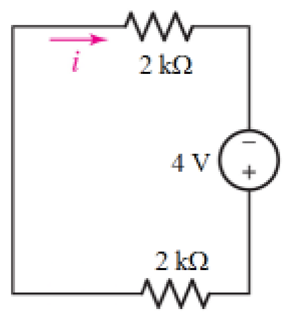
$i' = 16/4 = 4 \text{ mA}$



$i'' = 32/4 = 8 \text{ mA}$

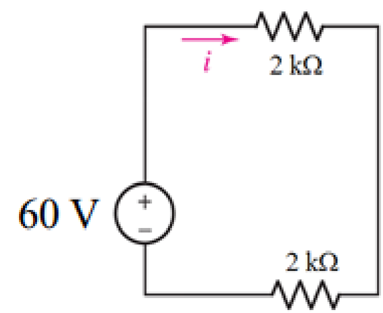


$i''' = 8/4 = 2 \text{ mA}$



$i'''' = 4/4 = 1 \text{ mA}$

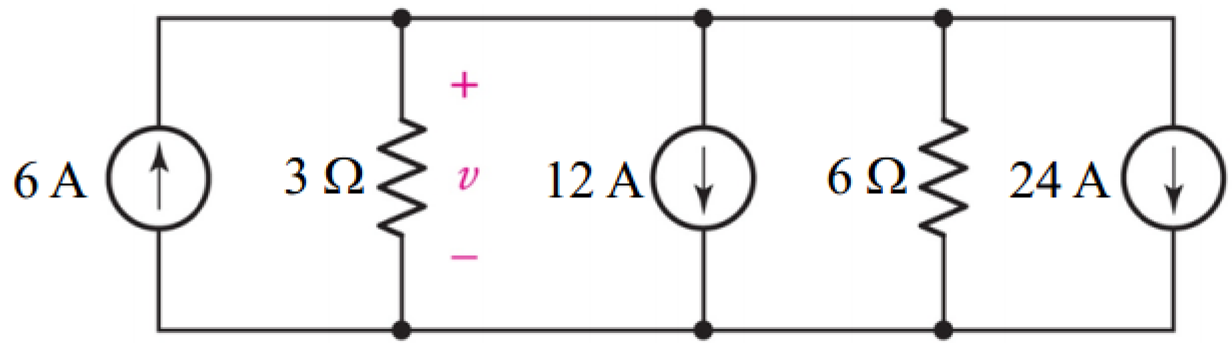
$$\begin{aligned} i &= i' + i'' + i''' + i'''' \\ &= 4 + 8 + 2 + 1 \\ &= \boxed{15 \text{ mA}} \end{aligned}$$



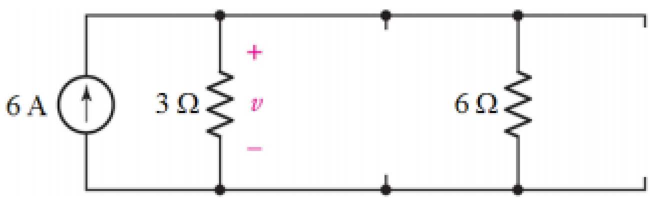
$i = 60/4 = 15 \text{ mA}$

Superposition: Current Sources

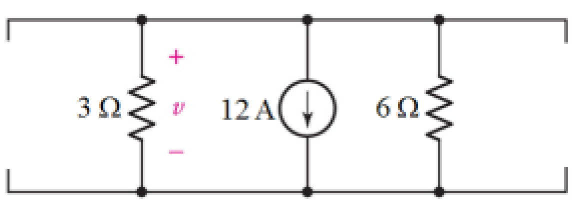
Determine the voltage v using superposition.



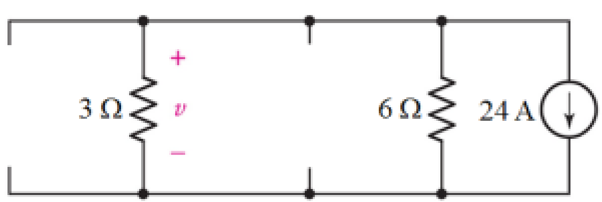
deactivate all but 1
→ solve → sum



$$6-\frac{v'}{3}-\frac{v'}{6}=0 \qquad v'=12\text{ V}$$



$$-12-\frac{v''}{3}-\frac{v''}{6}=0 \qquad v''=-24\text{ V}$$



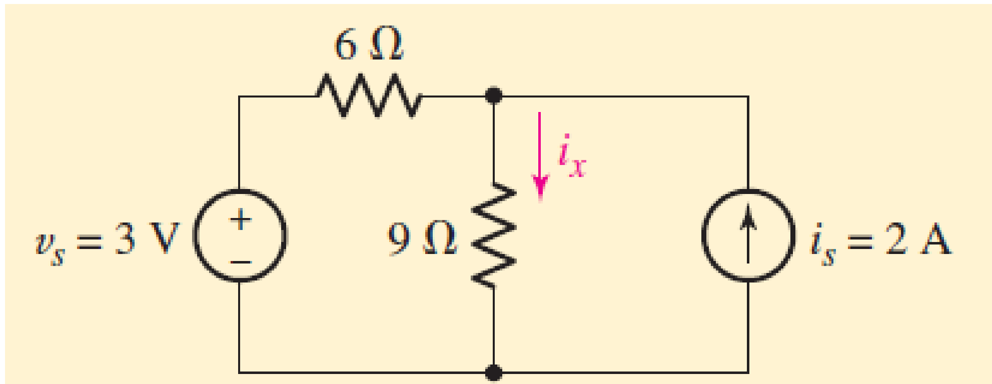
$$-24-\frac{v'''}{3}-\frac{v'''}{6}=0 \qquad v'''=-48\text{ V}$$

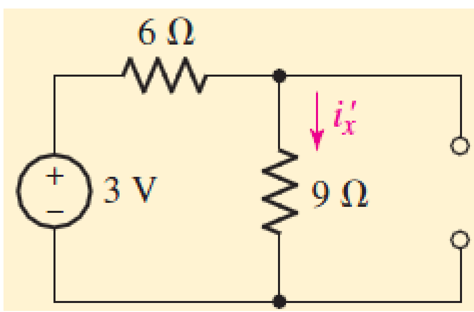
$$\sum \; = 12-24-48$$

$$v=\boxed{-60\text{ V}}$$

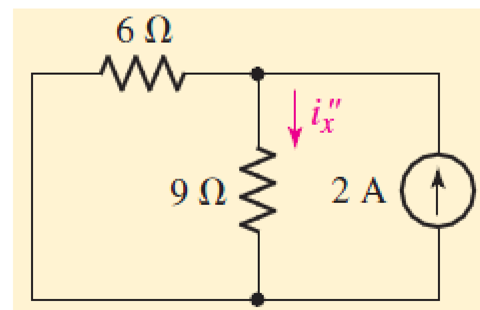
Example 5.1

- For the circuit, use superposition to determine the unknown branch current i_x.





Current source open-circuited



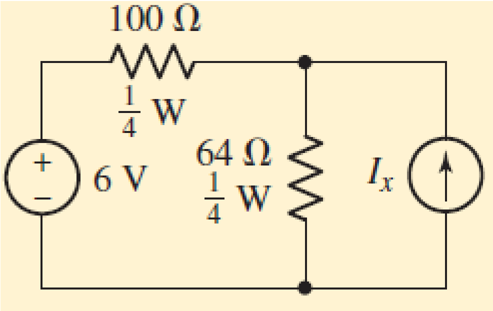
Voltage source short-circuited

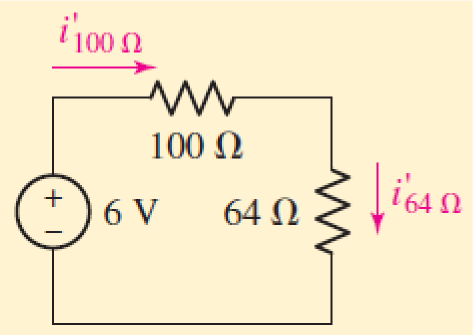
$$i_x = i_{x|_{3\,\text{V}}} + i_{x|_{2\,\text{A}}} = i_x' + i_x''$$

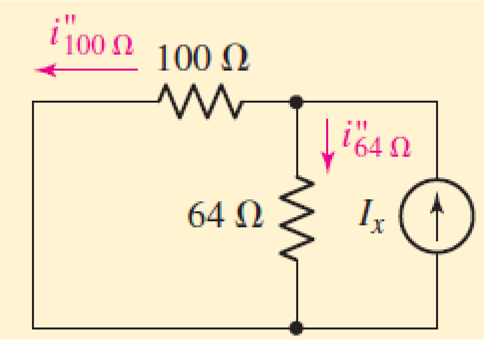
$$i_x = \frac{3}{6+9} + 2\left(\frac{6}{6+9}\right) = 0.2 + 0.8 = 1.0 \text{ A}$$

Example 5.2

- Referring to the circuit, determine the maximum positive current to which the source I_x can be set before any resistor exceeds its power rating and overheats.

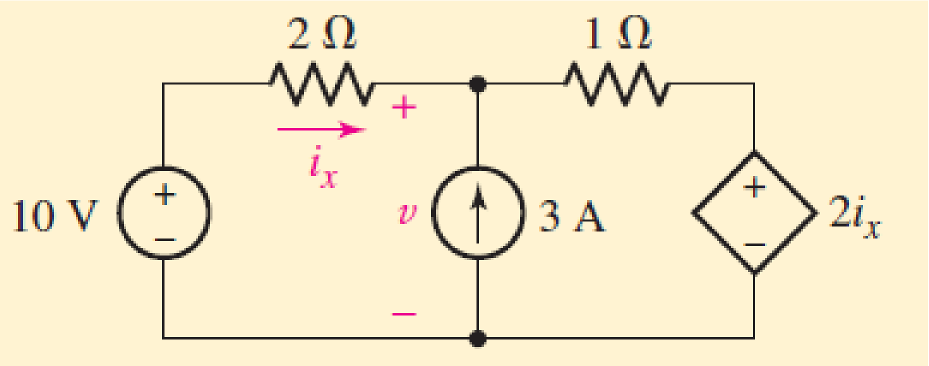


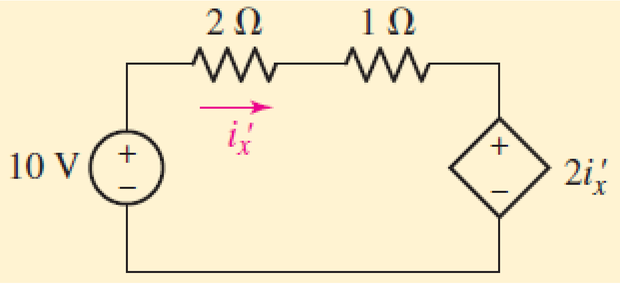


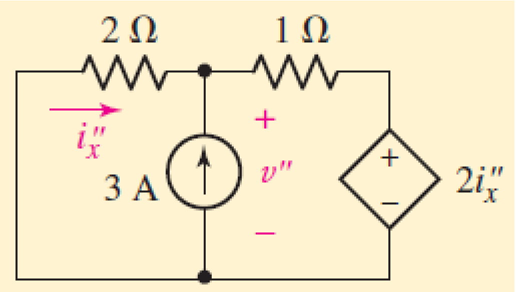


Example 5.3

- In the circuit, use the superposition principle to determine the value of i_x.







$$-10 + 2i'_x + i'_x + 2i'_x = 0$$

$$i'_x = 2\text{ A}$$

$$\frac{v''}{2} + \frac{v'' - 2i''_x}{1} = 3$$

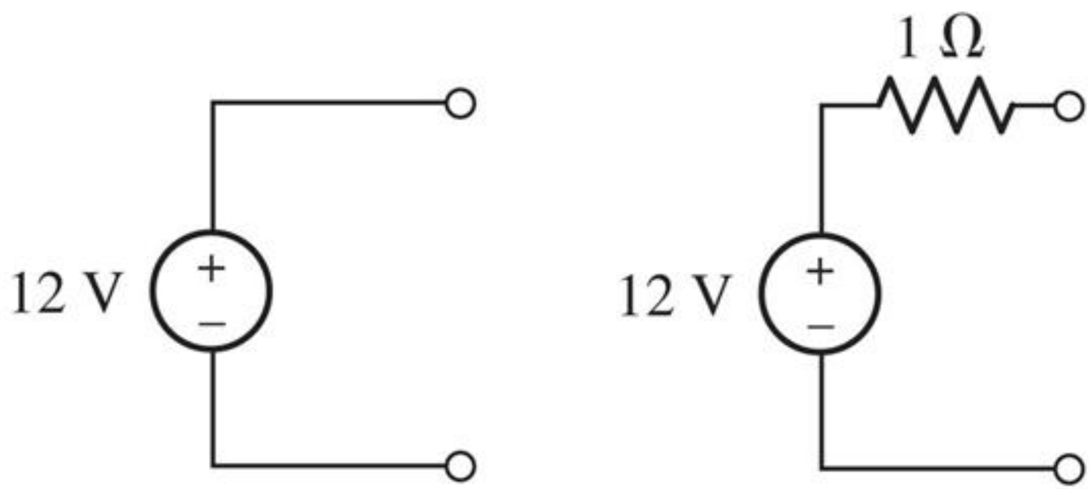
$$v'' = 2(-i''_x) \qquad i''_x = -0.6\text{ A}$$

$$i_x = i'_x + i''_x = 2 + (-0.6) = 1.4\text{ A}$$

Practical Voltage Sources

Ideal voltage sources have **zero series** resistance.

→ Real (practical) voltage sources have a **small, nonzero** resistance.



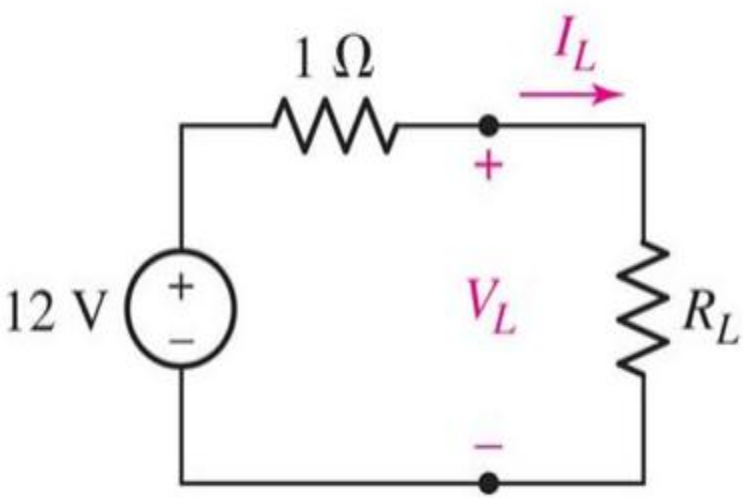
ideal vs. **practical**

current output is unlimited

current output is limited to $12\ V / 1\ \Omega = 12\ A$

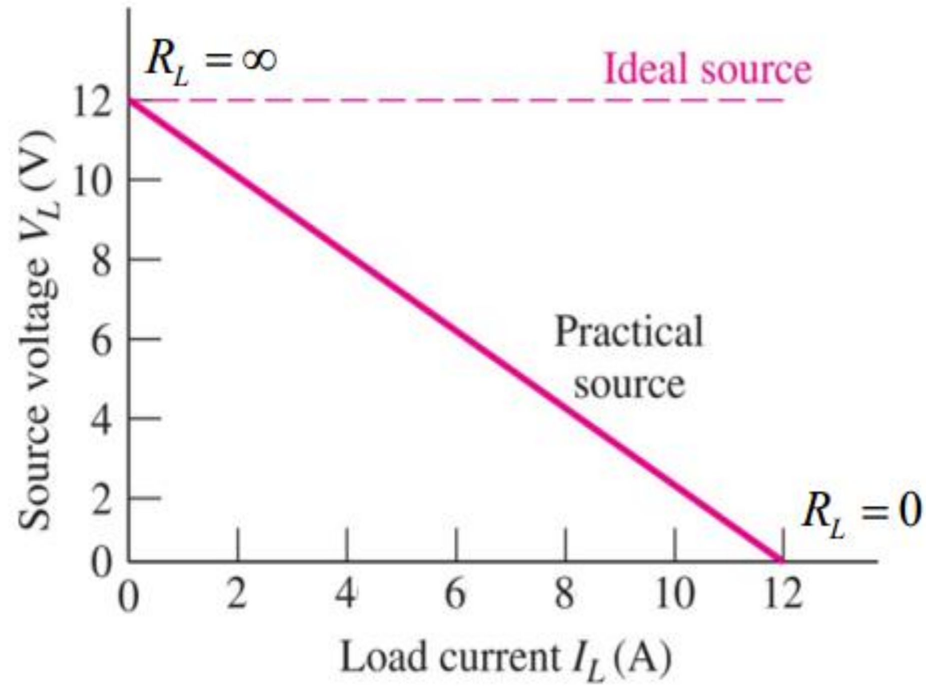
Practical Voltage Sources

The small, nonzero resistance of the voltage source acts to *limit* the current produced by the source.



$$V_s = 12\text{ V}$$

$$R_s = 1\,\Omega$$

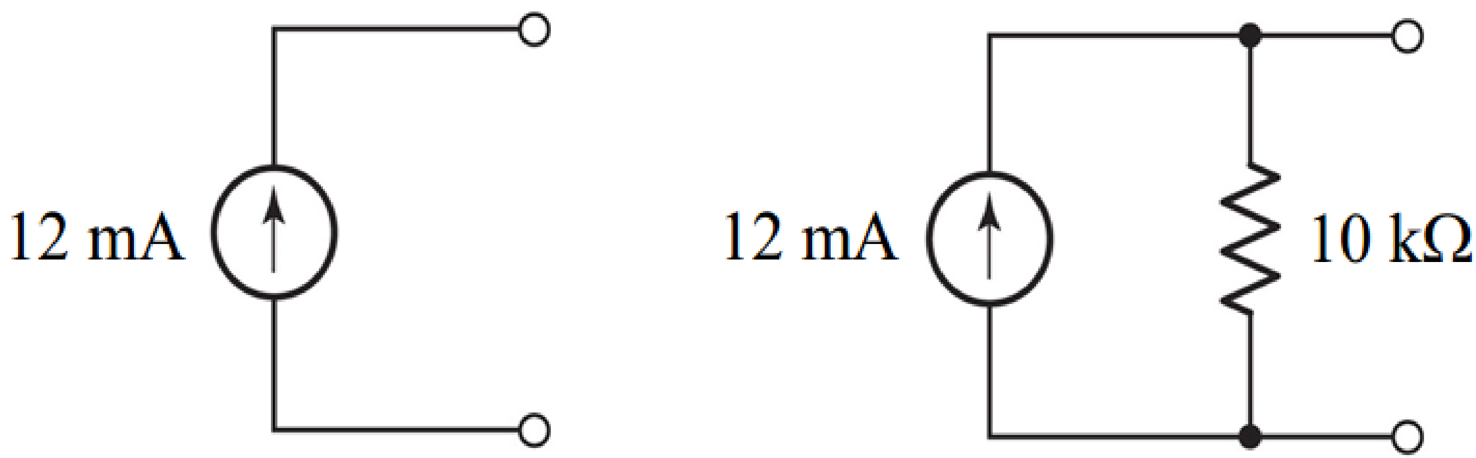


$$I_{\max} = V_s / R_s$$

Practical Current Sources

Ideal current sources have **infinite parallel resistance.**

→ Real (practical) current sources have a **large, finite** resistance.



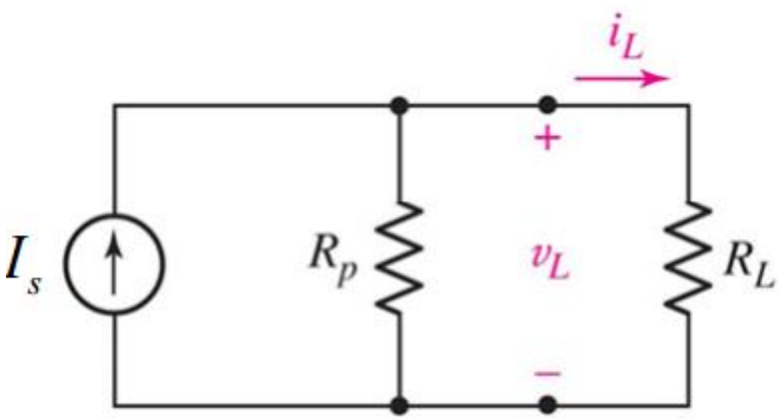
ideal vs. **practical**

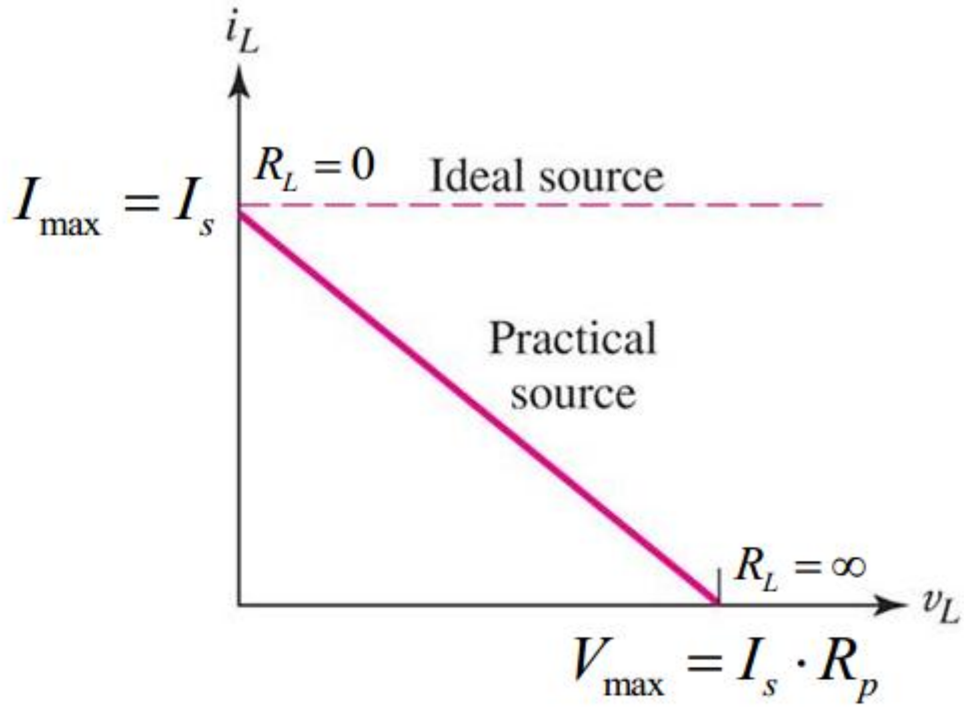
voltage output is unlimited

voltage output is limited to 12 mA x 10 kΩ = 120 V

Practical Current Sources

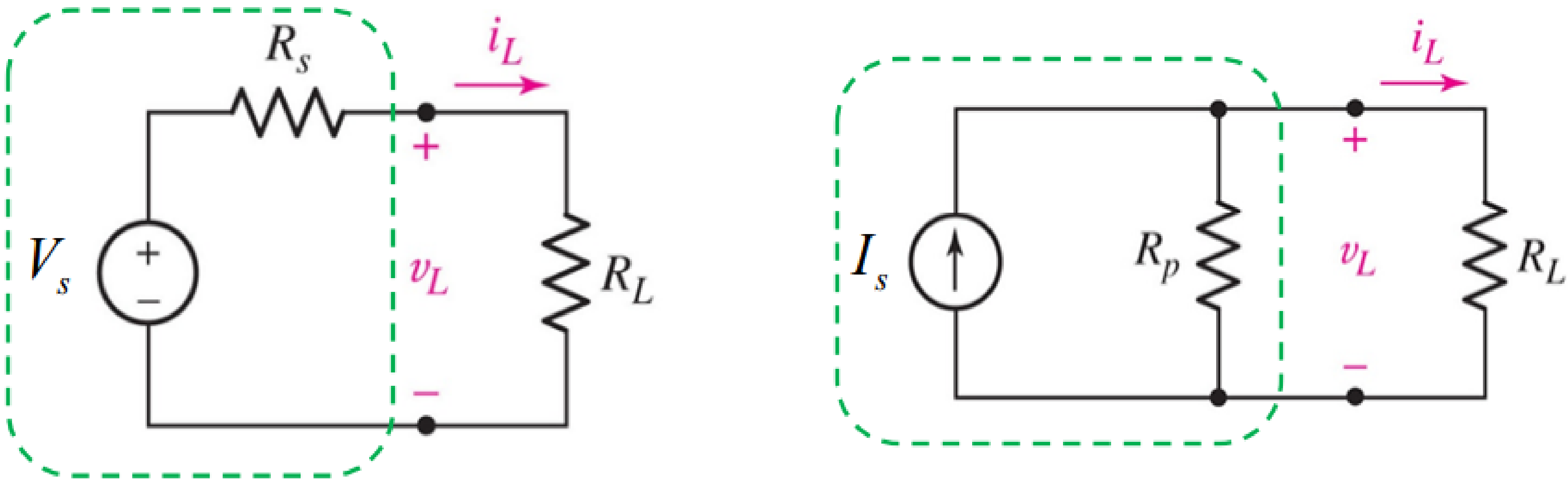
- The large, finite resistance of the current source acts to *limit* the voltage produced by the source.





Equivalent Practical Sources

If these two circuits provide the same *v*/*i* characteristics at their outputs (v_L, i_L), the two circuits are <u>equivalent</u>.



$$I_{\max} = V_s / R_s$$

$$V_{\max} = V_s$$

$$R_s = R_p = \frac{V_s}{I_s}$$

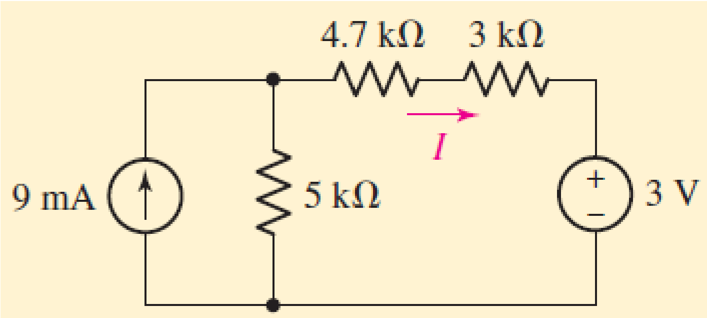
condition
for equivalence

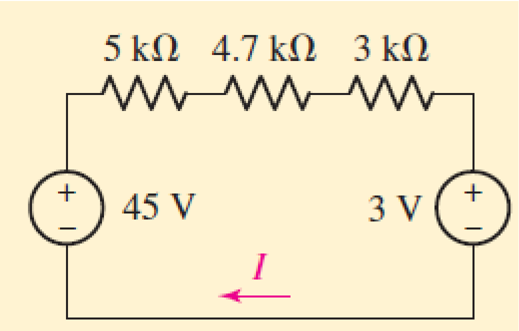
$$I_{\max} = I_s$$

$$V_{\max} = I_s \cdot R_p$$

Example 5.4

- Compute the current through the 4.7 kΩ resistor after transforming the 9 mA source into an equivalent voltage source.



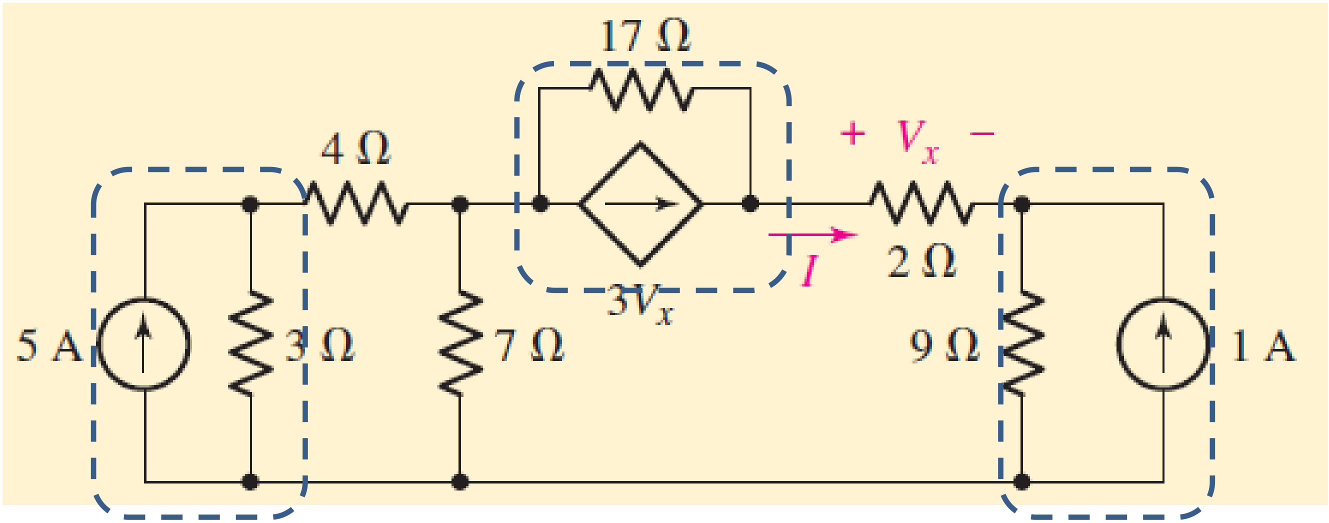


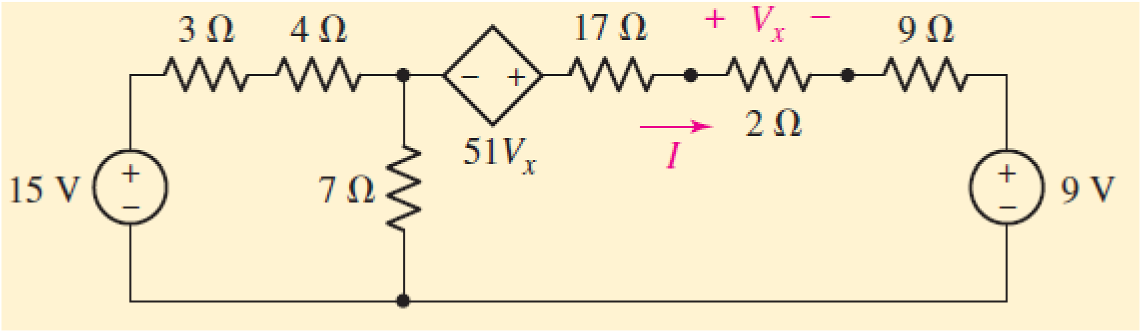
$$-45 + 5000I + 4700I + 3000I + 3 = 0$$

$$I = 3.307 \text{ mA.}$$

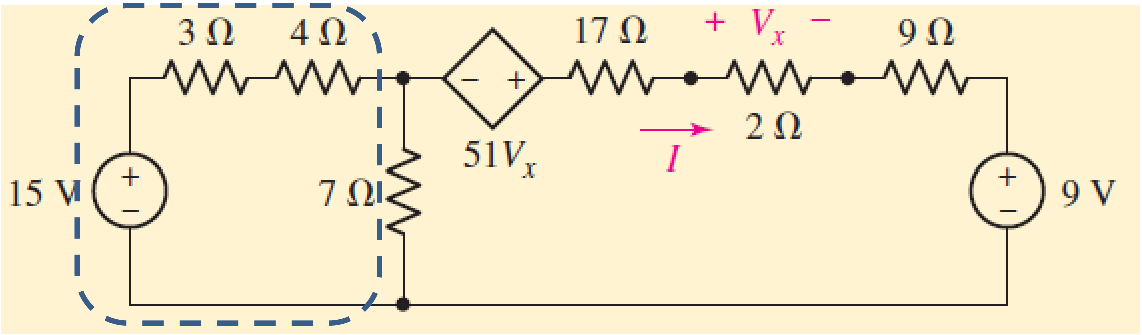
Example 5.5

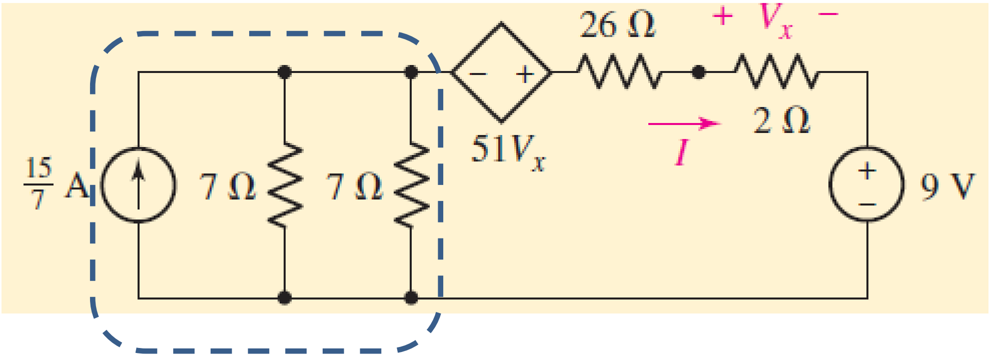
- Calculate the current through the 2 Ω resistor by making use of source transformations to first simplify the circuit.

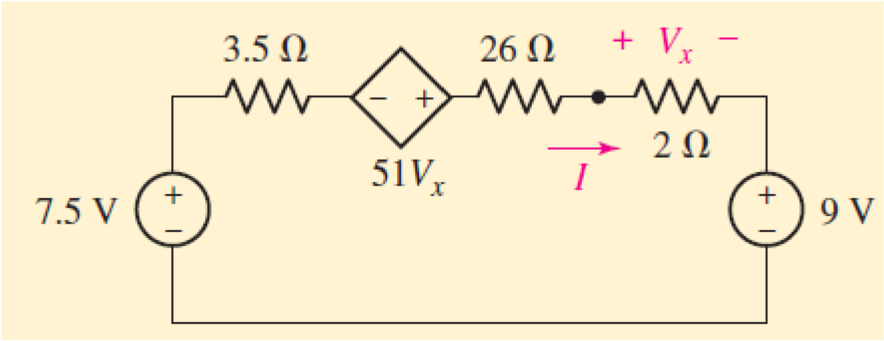




Example 5.5







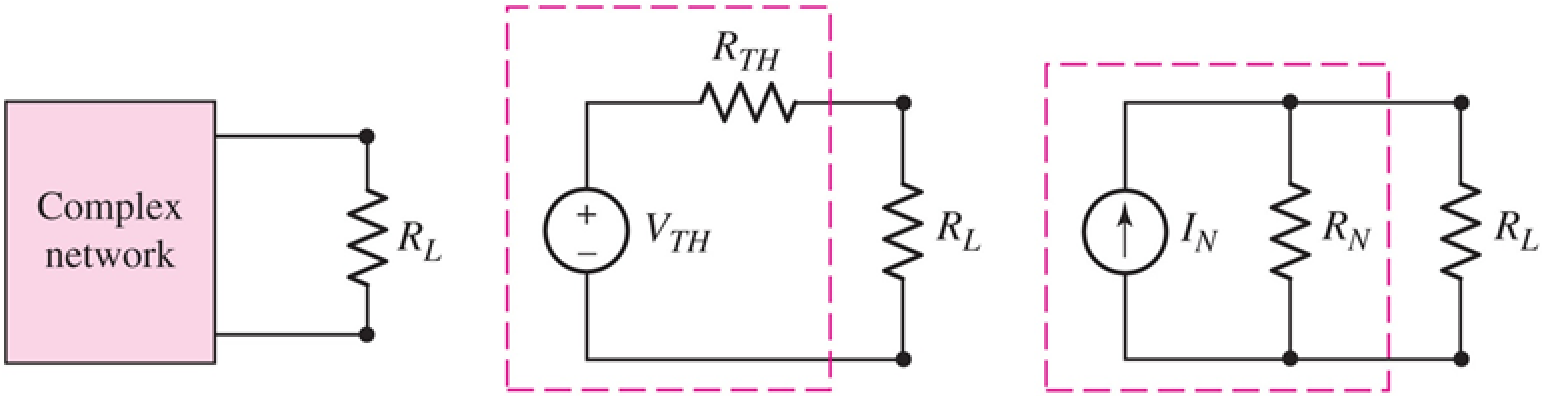
$$-7.5 + 3.5I - 51V_x + 28I + 9 = 0$$

$$V_x = 2I$$

$$I = 21.28 \text{ mA}$$

Thévenin & Norton Equivalents

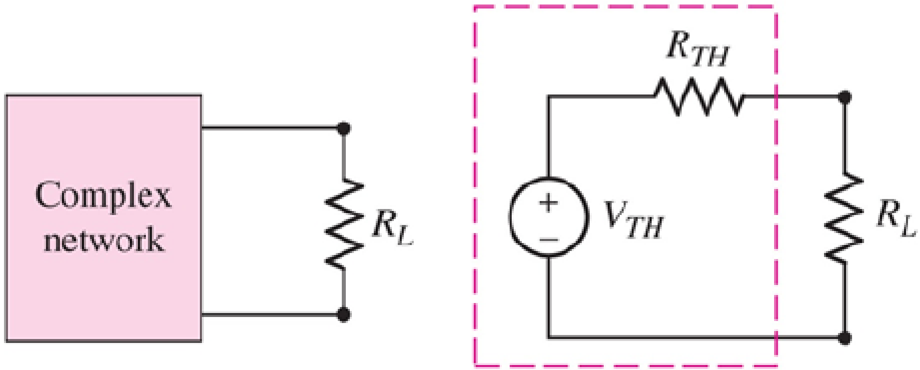
L. C. Thevenin -- French engineer; published his theorem in 1883
E. L. Norton -- scientist with Bell Telephone Laboratories



(a) (b) (c)

- Any linear circuit network at two terminals may be replaced with a Thevenin equivalent (V_{TH}, R_{TH}) or a Norton equivalent *(I_N, R_N).*
- The equivalent will behave the same as the original network (v_L, i_L) with respect to those two terminals.

Thévenin & Norton Equivalents

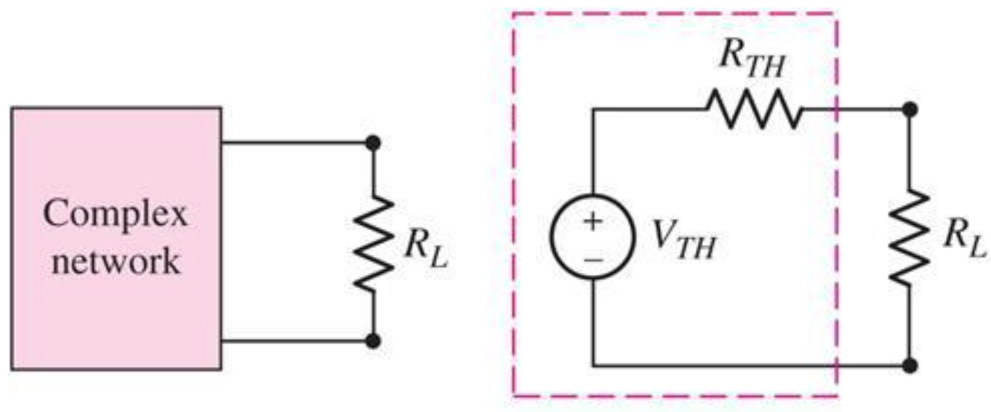


The Thevenin equivalent is more commonly seen in practice.

- allows us to replace a large, complicated circuit with a much simpler 2-element series/parallel circuit
- the simpler circuit allows for rapid calculations of *V, I, P* that the original circuit can deliver to a load
- helps us to choose the best value of load resistance to maximize the power delivered (e.g. from an amplifier, to a speaker)

Thevenin Equivalent, Method 1

How to determine V_{TH} *and* R_{TH} *with respect to two terminals:*

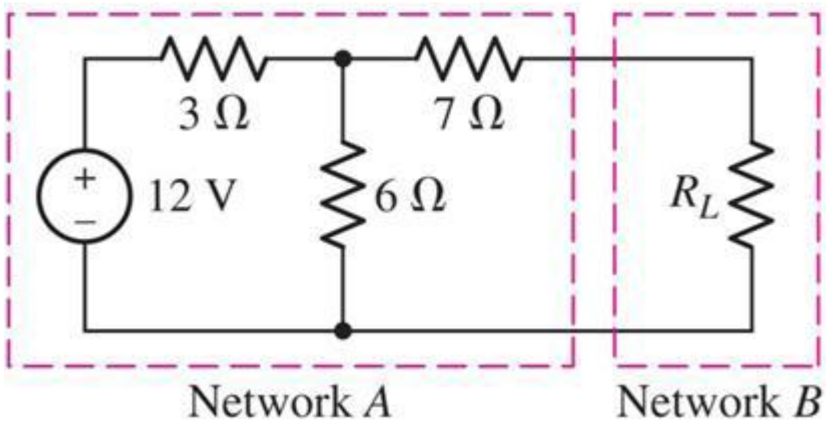


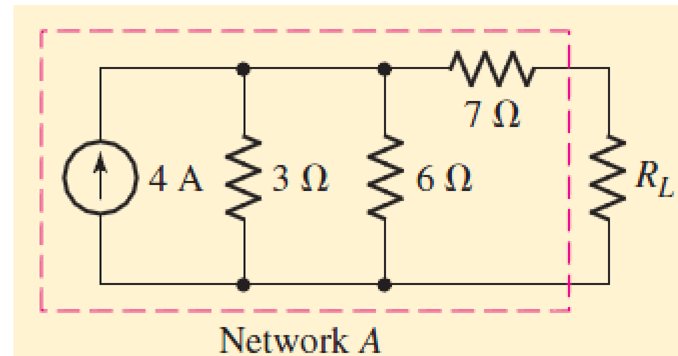
(a) (b)

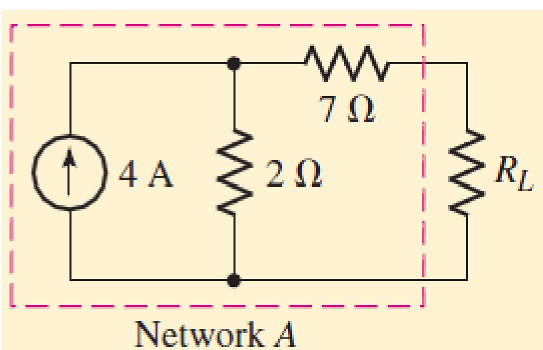
(1) Use **repeated source transformations** to arrive at a single voltage source in series with a single series resistance.

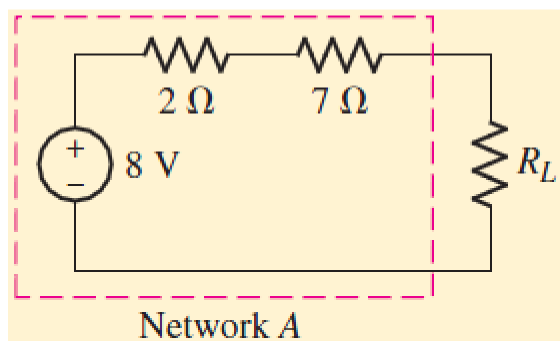
Example 5.6

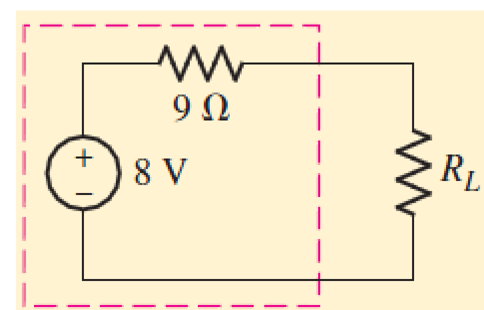
- Determine the Thévenin equivalent of network *A*, and compute the power delivered to the load resistor R_L.









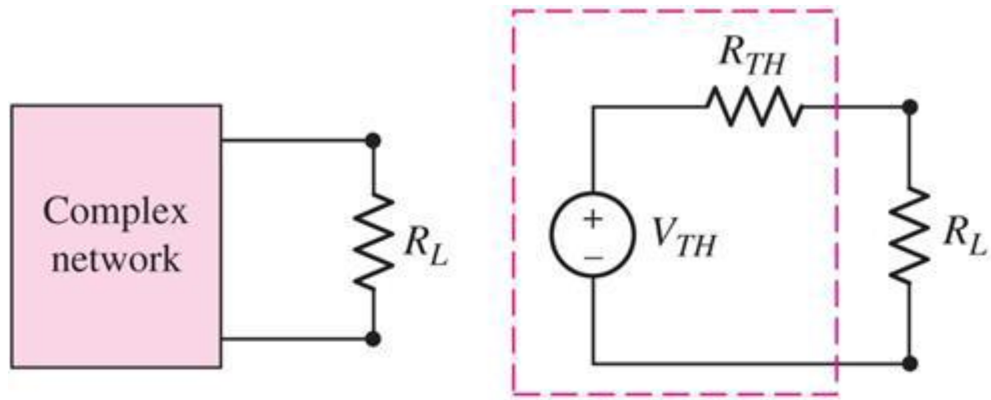


power delivered to the load

$$P_L = \left(\frac{8}{9 + R_L}\right)^2 R_L$$

Thevenin Equivalent, Method 2

How to determine V_{TH} *and* R_{TH} *with respect to two terminals:*



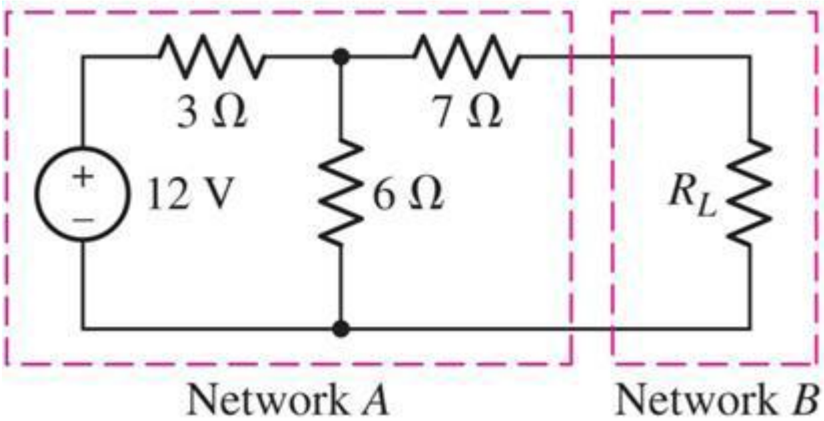
(*a*) (*b*)

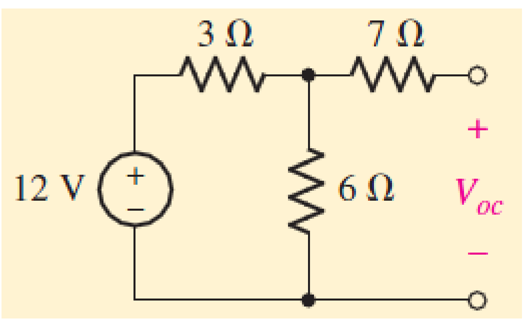
(2) *Open* the load and determine the **open-circuit voltage** (V_{OC}), then *short* the load and determine the **short-circuit current** (I_{SC}):

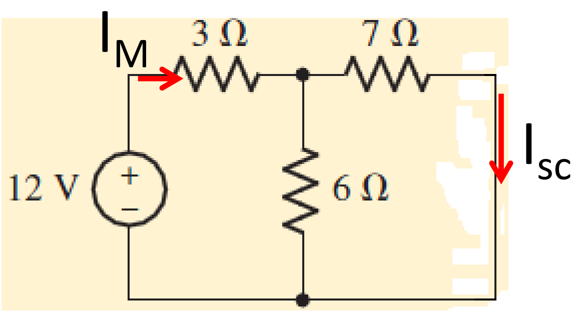
$$V_{TH} = V_{OC} \qquad R_{TH} = \frac{V_{OC}}{I_{SC}}$$

Example

- Determine the Thevenin equivalent of Network A using *open-circuit voltage* and *short-circuit current*.







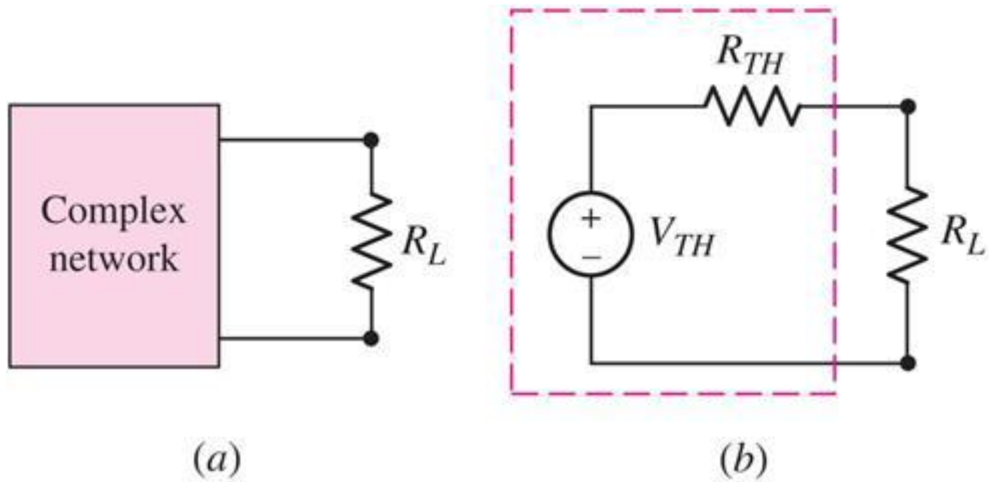
$$V_{oc} = 12\left(\frac{6}{3+6}\right) = 8\text{ V}$$

I_M=12/(3+7||6)=1.9259 A
I_{SC}=(1.9259×6)/13=0.8889 A

$R_{TH}=V_{OC}/I_{SC}$=8/0.8889=9 Ω
$V_{TH}=V_{OC}$=8 V

Thevenin Equivalent, Method 3

How to determine V_{TH} *and* R_{TH} *with respect to two terminals:*

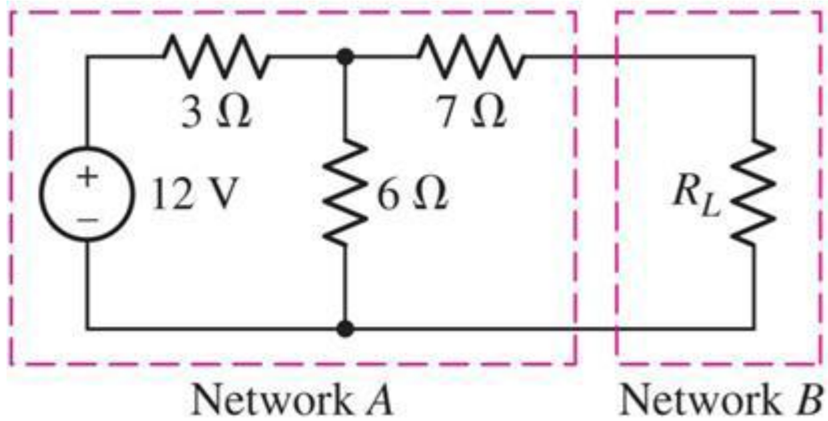


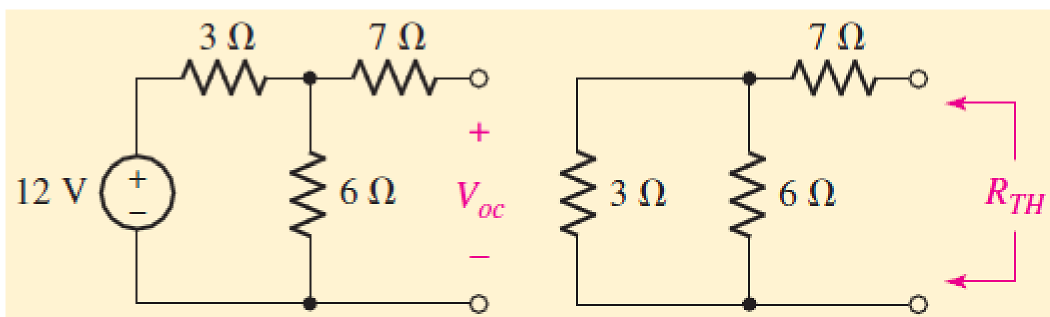
(3) Open the load and determine the **open-circuit voltage (V_{OC}),** then **deactivate all independent sources** (short-circuit the V sources and open-circuit the I sources) and find the **equivalent resistance (R_{eq}):**

$$V_{TH} = V_{OC} \qquad R_{TH} = R_{eq}$$

Example

- Determine the Thevenin equivalent of Network A by deactivating the independent sources.



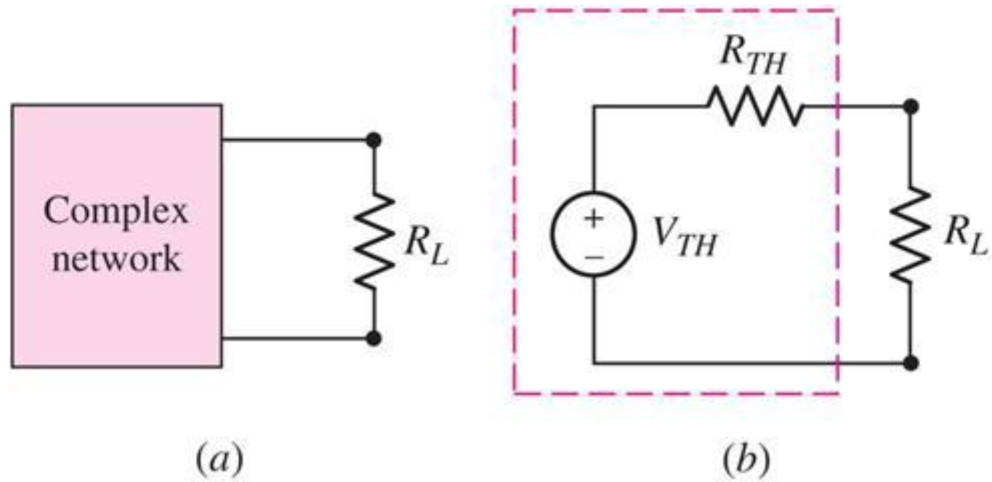


$$V_{oc} = 12\left(\frac{6}{3+6}\right) = 8\text{ V}$$

R_{TH}=7+(6||3)=9 Ω

Thevenin Equivalent, Method 4

How to determine V_{TH} *and* R_{TH} *with respect to two terminals:*



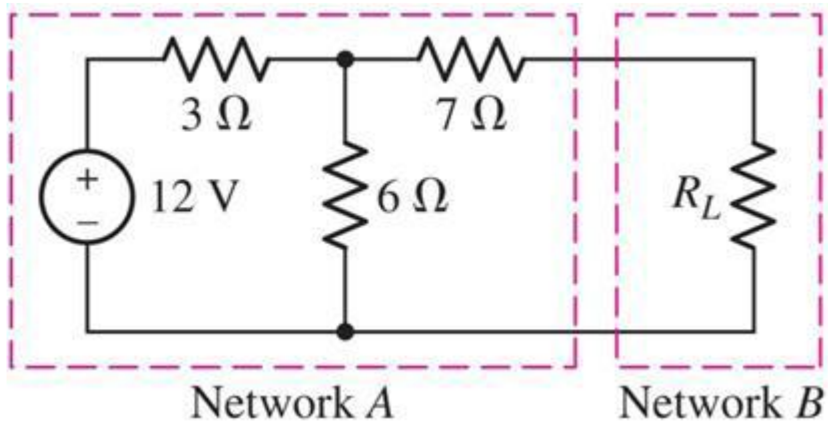
4) *Open* the load and determine the **open-circuit voltage** (VOC), then *deactivate all independent sources* and apply **a test source**:

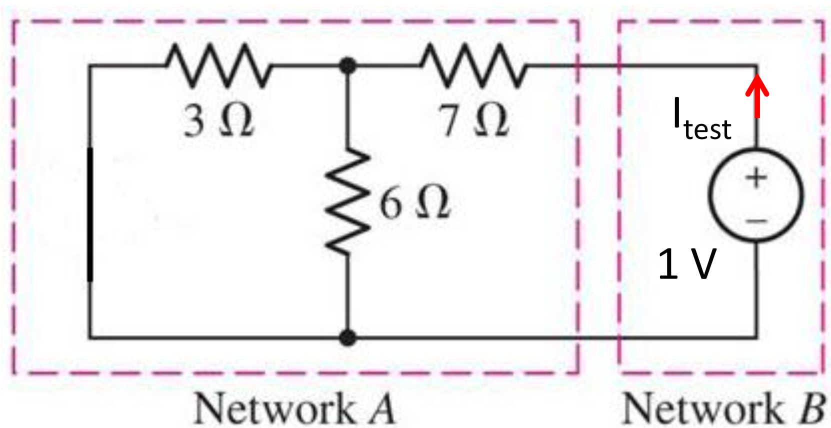
$$V_{\mathrm{TH}} = V_{\mathrm{OC}} \qquad R_{\mathrm{TH}} = \frac{V_{\mathrm{test}}}{I_{\mathrm{test}}}$$

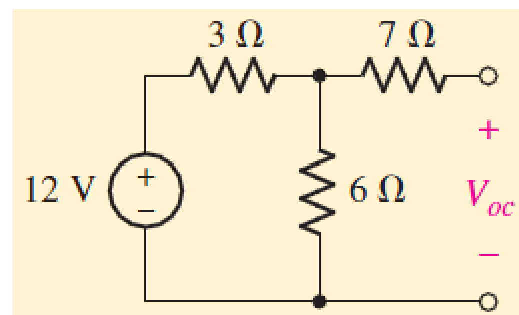
- The only solution method for finding V_{TH} and R_{TH} (of the 4 presented in the prior slides) that is guaranteed to work when the circuit includes dependent sources is the test-source method.

Example

- Determine the Thevenin equivalent of Network A by using a test source.





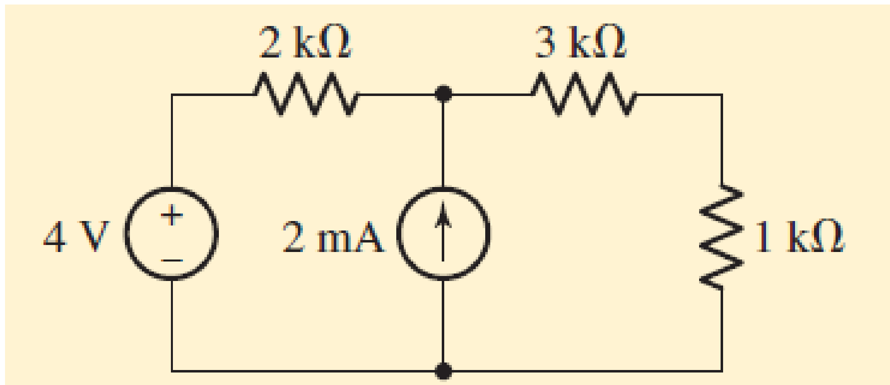


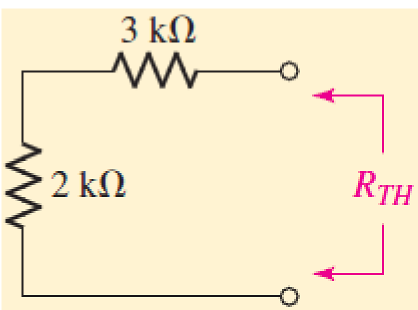
$$V_{oc} = 12\left(\frac{6}{3+6}\right) = 8\text{ V}$$

I_{test}=1/(7+6||3)=0.111 A
$R_{test} = V_{test}/I_{test}$ = 1/0.111 =9 Ω

Example 5.8

- Find the Thévenin and Norton equivalent circuits for the network faced by the 1 kΩ resistor



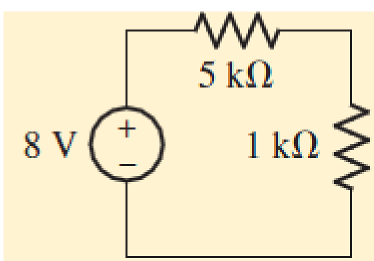


R_{TH}=5 kΩ

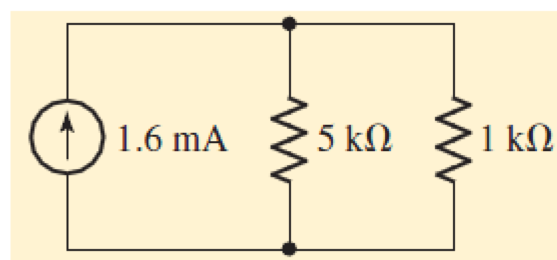
Superposition can be applied to find V_{OC}

$V_{oc|_{4\,V}} = 4\text{ V}.$ $\quad V_{oc|_{2\,mA}} = (0.002)(2000) = 4\text{ V}.$

$V_{oc} = 4 + 4 = 8\text{ V}.$



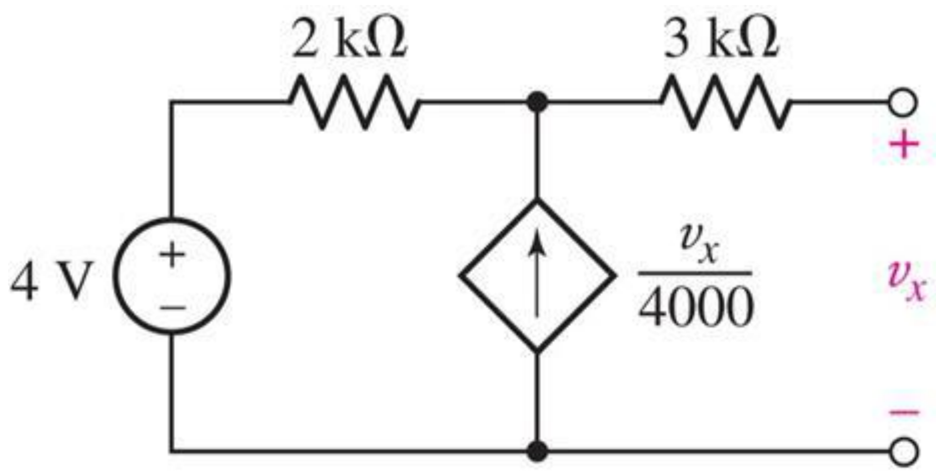
Thevenin equivalent

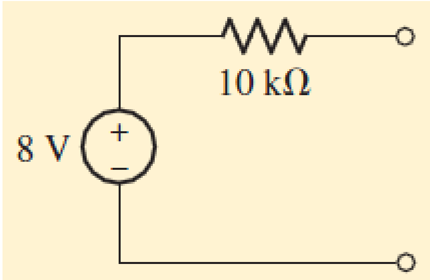


Norton equivalent

Example 5.9

- Determine the Thevenin equivalent of this network at the open-circuit terminals:





Thevenin equivalent

To find V_{OC} we note that $v_X = V_{OC}$ and that the dependent source current must pass through the 2 k resistor, since no current can flow through the 3 k resistor.

$$-4 + 2 \times 10^3 \left(-\frac{v_x}{4000}\right) + 3 \times 10^3 (0) + v_x = 0 \qquad v_x = 8\text{ V} = V_{oc}$$

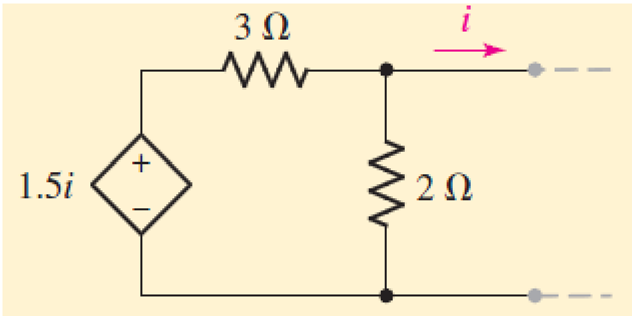
The dependent source prevents us from determining R_{TH} *directly for* the inactive network through resistance combination; we therefore seek I_{SC}.

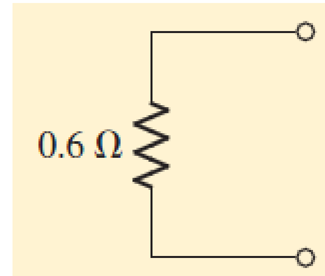
Upon short-circuiting the output terminals, it is apparent that Vx = 0 and the dependent current source is not active.

$$I_{sc} = 4/(5 \times 10^3) = 0.8\text{ mA}. \qquad R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{8}{0.8 \times 10^{-3}} = 10\text{ k}\Omega$$

Example 5.10

- Find the Thevenin equivalent of this circuit.



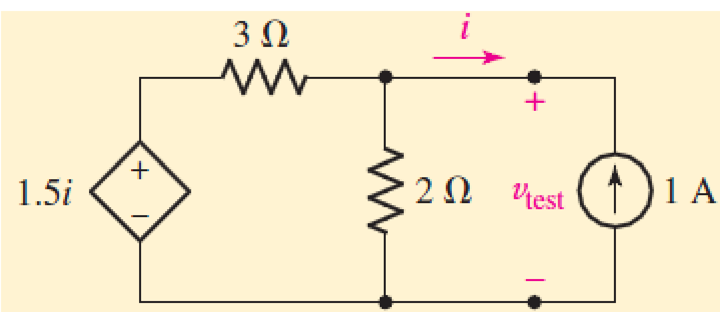


Thevenin equivalent

The rightmost terminals are already open-circuited, hence i = 0. Consequently, the dependent source is inactive, so $V_{OC} = 0$.

We apply a 1 A source externally, measure the voltage V_{test}

$$R_{TH} = v_{\text{test}}/1$$

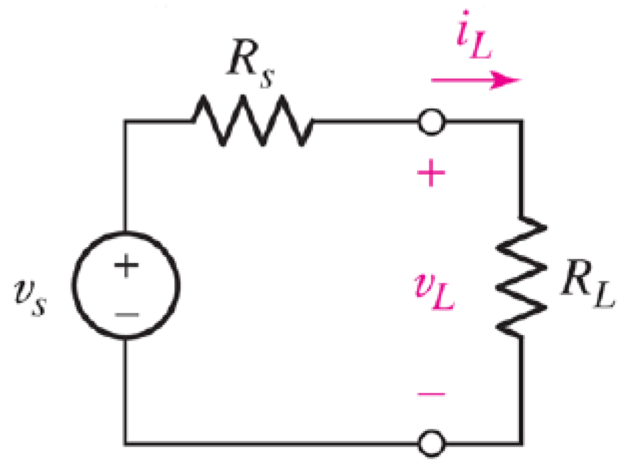


$$\frac{v_{\text{test}} - 1.5(-1)}{3} + \frac{v_{\text{test}}}{2} = 1$$

$$v_{\text{test}} = 0.6 \text{ V}$$

$$R_{TH} = 0.6\ \Omega$$

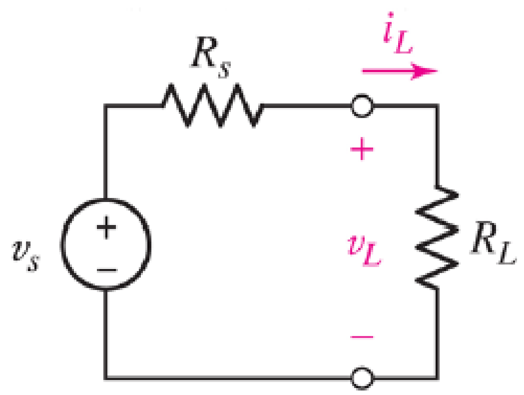
Power from a Practical Source



- The power delivered to a load from a practical voltage source is

$$p_L = i_L \cdot v_L = \frac{v_L^{\,2}}{R_L} = \frac{1}{R_L}\left[v_s \cdot \frac{R_L}{R_s + R_L}\right]^2 = \frac{v_s^{\,2} R_L}{(R_s + R_L)^2}$$

Maximum Power Transfer



$$p_L = \frac{v_s^{\,2} R_L}{(R_s + R_L)^2}$$

The maximum value of p_L vs. R_L occurs when $\dfrac{d}{dR_L} p_L = 0$

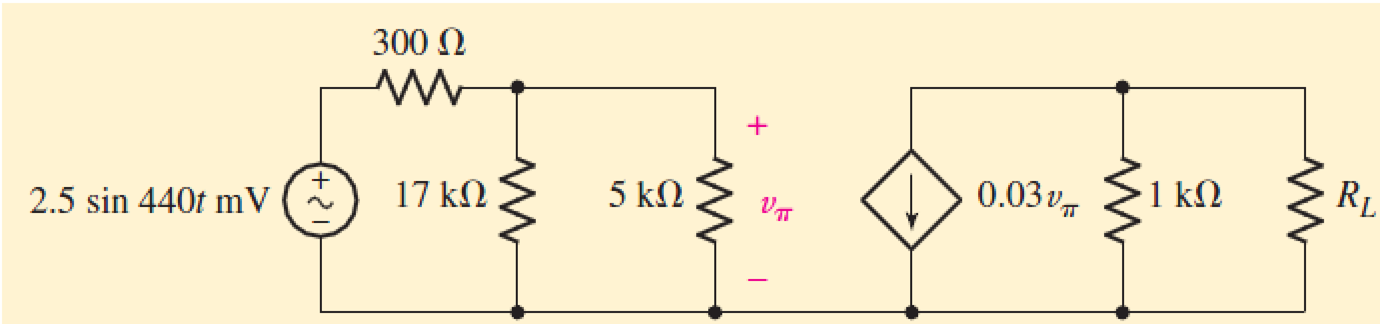
$$\frac{d}{dR_L} p_L = \frac{(R_s + R_L)^2 v_s^{\,2} - 2 v_s^{\,2} R_L (R_s + R_L)}{(R_s + R_L)^4}$$

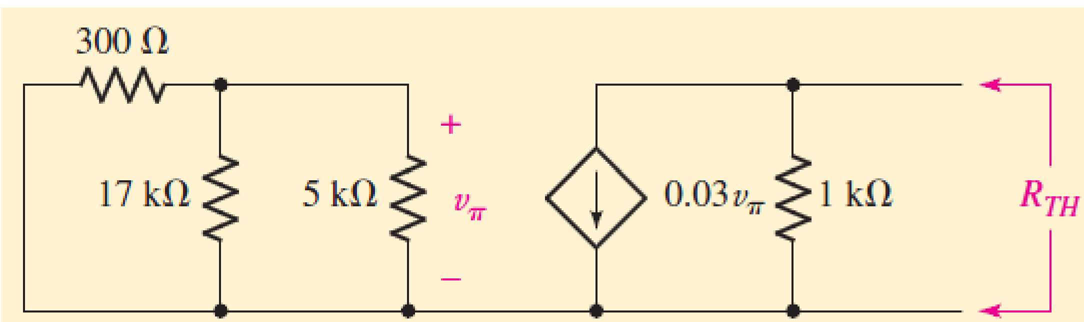
if $\boxed{R_L = R_s}$, $\dfrac{d}{dR_L} p_L = 0$

Maximum power is delivered to the load when the **load resistance is equal to the Thevenin resistance of the source**.

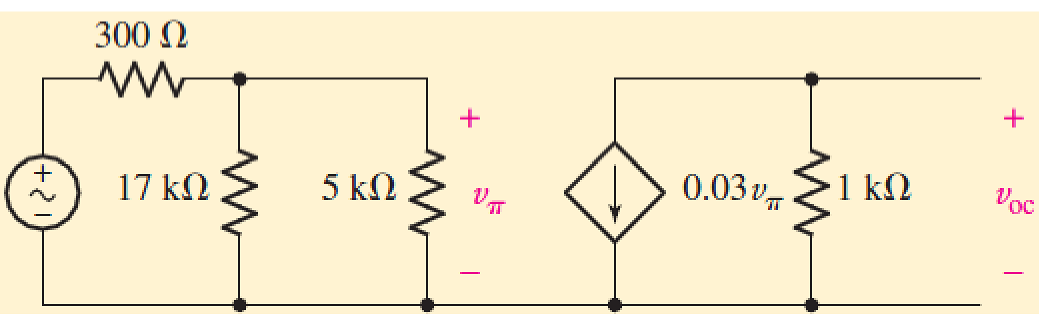
Example 5.11

- The circuit shown in below Figure is a model for the common-emitter bipolar junction transistor amplifier. Choose a load resistance so that maximum power is transferred to it from the amplifier, and calculate the actual power absorbed.





$$R_{TH} = 1\ \mathrm{k}\Omega.$$



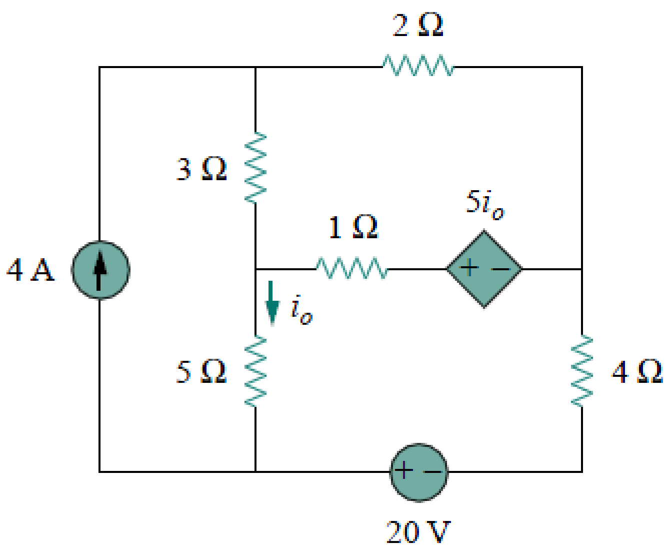
$$v_{oc} = -0.03v_\pi(1000) = -30v_\pi$$

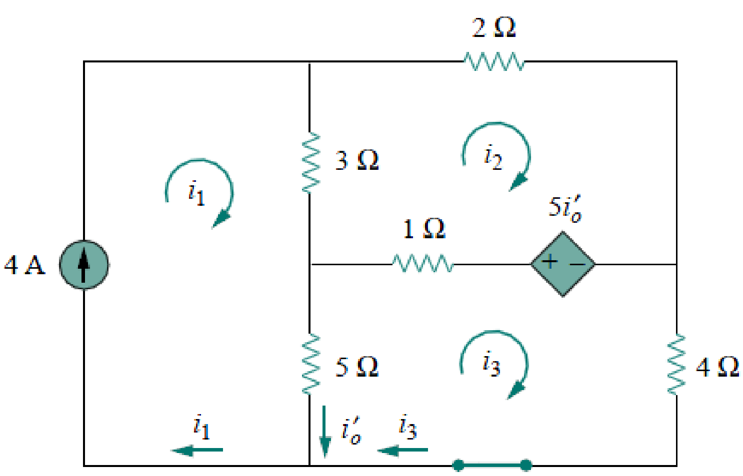
$$v_\pi = (2.5 \times 10^{-3} \sin 440t)\left(\frac{3864}{300 + 3864}\right)$$

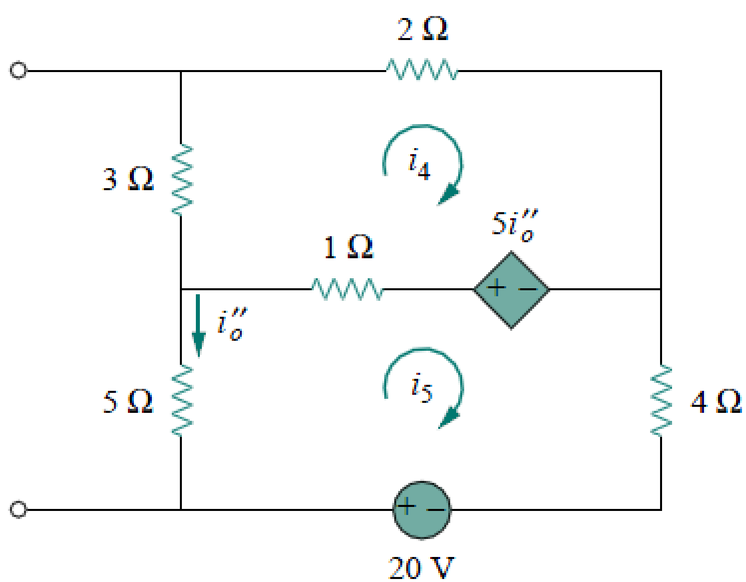
$$p_{max} = \frac{v_{TH}^2}{4R_{TH}} = \boxed{1.211 \sin^2 440t\ \mu\mathrm{W}}$$

Example

Find i_o in the circuit using superposition.

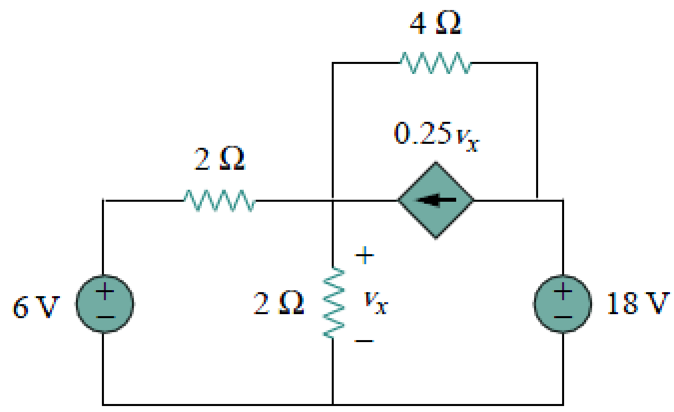


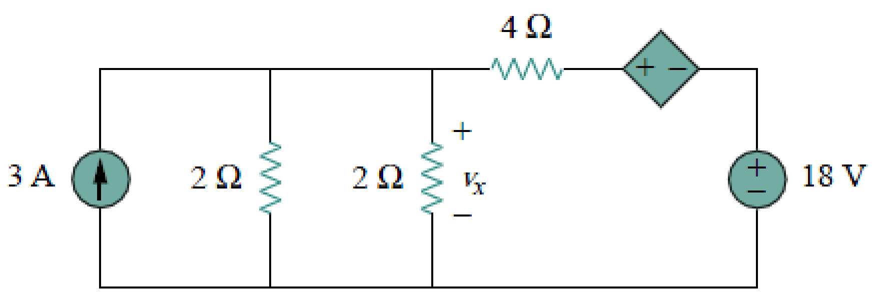


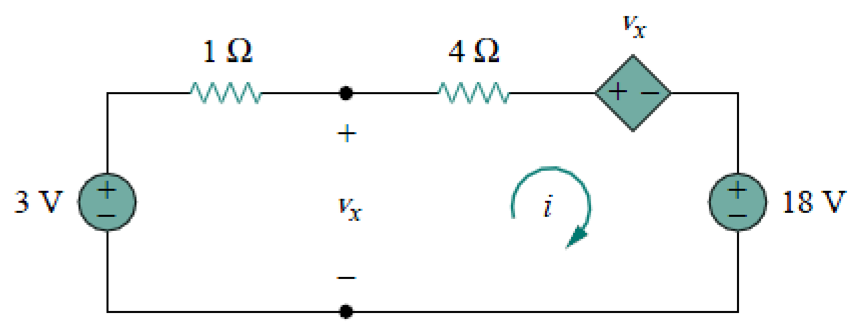


Example

Find v_x *in Figure using source transformation.*

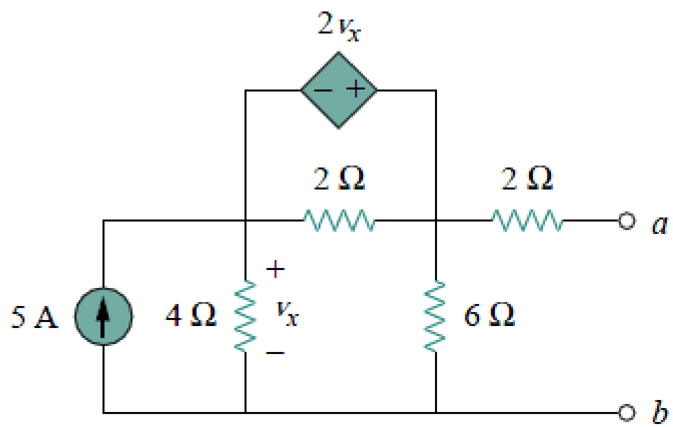


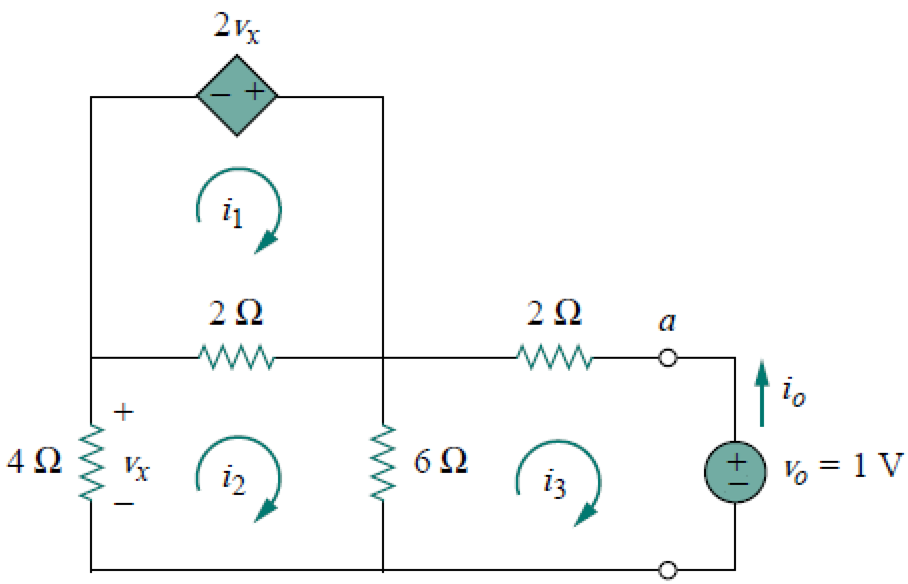


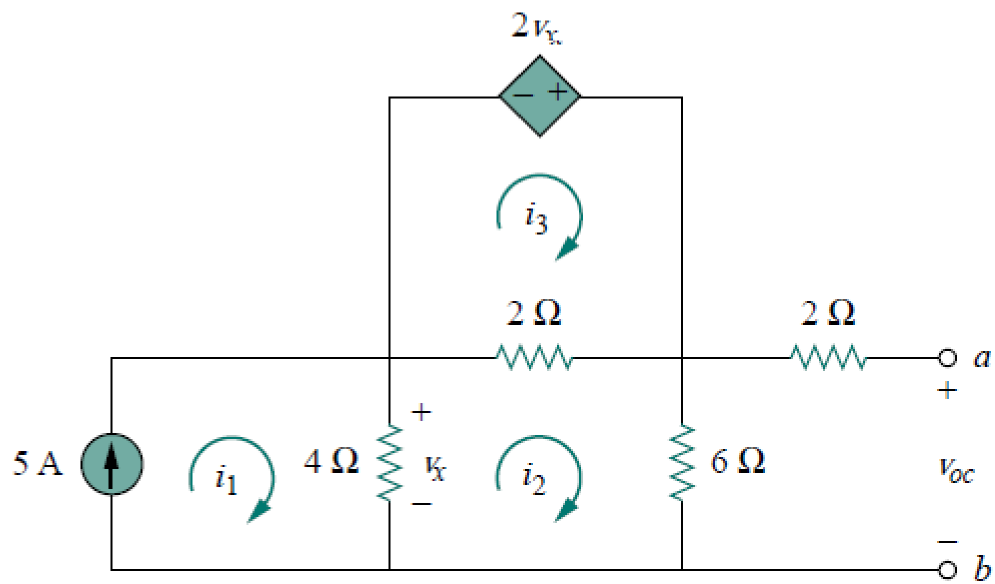


Example

Find the Thevenin equivalent of the circuit.







Example

Find the value of R_L *for maximum power transfer in the circuit.*
Find the maximum power.