

# BLM1612 - Circuit Theory

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# Impedance and Ohm's Law

- Objective of Lecture
  - Describe the mathematical relationships between ac voltage and ac current for a resistor, capacitor, and inductor .
    - Discuss the phase relationship between the ac voltage and current.
  - Explain how Ohm's Law can be adapted for inductors and capacitors when an ac signal is applied to the components.
    - Derive the mathematical formulas for the impedance and admittance of a resistor, inductor, and capacitor.

# Resistors

- Ohm's Law

$$v(t) = Ri(t) = R I_m \cos(\omega t + \theta)$$

$$\mathbf{V} = RI_m \angle \theta = R\mathbf{I} \quad \text{where } \theta = \phi$$

- The voltage and current through a resistor are in phase as there is no change in the phase angle between them.

# Capacitors

$$i(t) = C \, dv(t)/dt \text{ where } v(t) = V_m \cos(\omega t)$$

$$i(t) = -C\omega V_m \sin(\omega t)$$

$$i(t) = \omega C V_m \sin(\omega t + 180^\circ)$$

$$i(t) = \omega C V_m \cos(\omega t + 180^\circ - 90^\circ)$$

$$i(t) = \omega C V_m \cos(\omega t + 90^\circ)$$

# Capacitors

$$\mathbf{V} = V_m \angle 0^\circ$$

$$\mathbf{I} = \omega C V_m \cos(\omega t + 90^\circ)$$

$$V_m \cos(\omega t + 90^\circ) = V e^{j90^\circ} = V \angle 90^\circ = jV$$

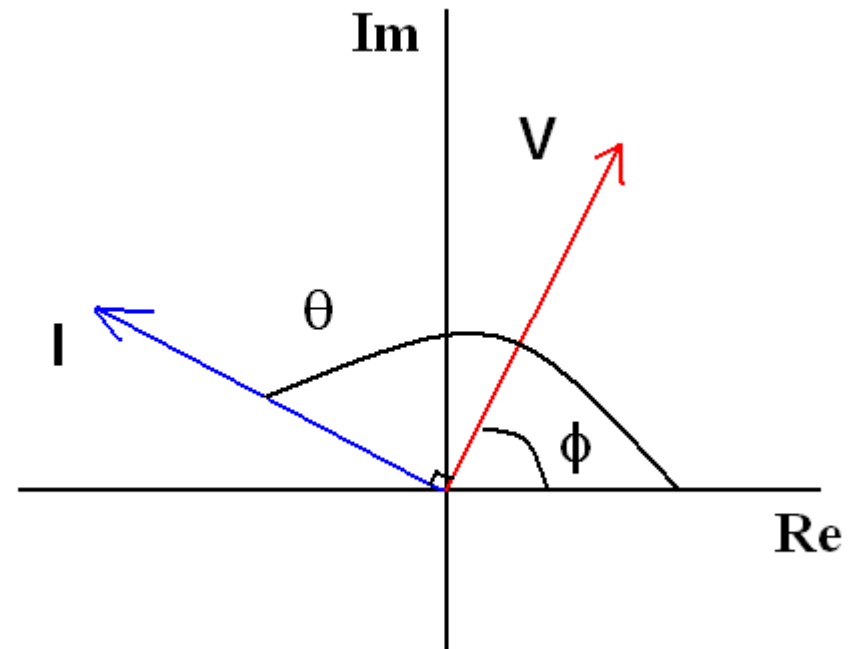
$$\mathbf{I} = j\omega C \mathbf{V}$$

or

$$\mathbf{V} = (1/j\omega C) \mathbf{I} = - (j/\omega C) \mathbf{I}$$

# Capacitors

- 90° phase difference between the voltage and current through a capacitor.
  - Current needs to flow first to place charge on the electrodes of a capacitor, which then induce a voltage across the capacitor
- Current leads the voltage (or the voltage lags the current) in a capacitor.



# Inductors

$$v(t) = L \, d i(t)/dt \quad \text{where } i(t) = I_m \cos(\omega t)$$

$$v(t) = -L\omega I_m \sin(\omega t) = \omega L I_m \cos(\omega t + 90^\circ)$$

$$\mathbf{V} = \omega L I_m \angle 90^\circ$$

$$\mathbf{I} = I_m \cos(\omega t)$$

$$I_m \cos(\omega t + 90^\circ) = \mathbf{I} e^{j90^\circ} = \mathbf{I} \angle 90^\circ = j\mathbf{I}$$

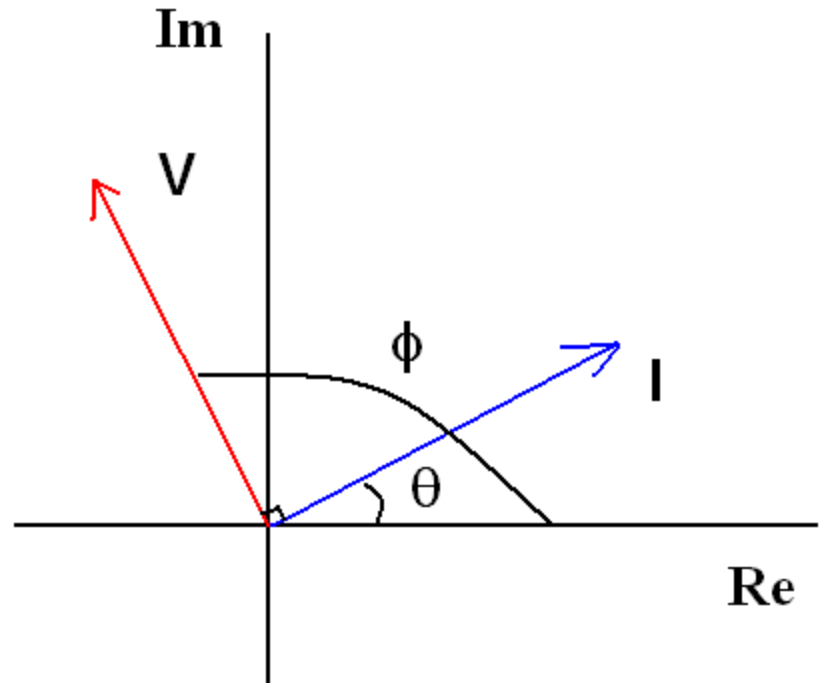
$$\mathbf{V} = j\omega L \mathbf{I}$$

or

$$\mathbf{I} = (1/j\omega L) \mathbf{V} = - (j/\omega L) \mathbf{V}$$

# Inductors

- $90^\circ$  phase difference between the voltage and current through an inductor.
- The voltage leads the current (or the current lags the voltage).





# Impedance

- If we try to force all components to following Ohm's Law,  $\mathbf{V} = \mathbf{Z} \mathbf{I}$ , where  $\mathbf{Z}$  is the impedance of the component.

$$\text{Resistor: } \mathbf{Z}_R = R \quad R \angle 0^\circ$$

$$\text{Capacitor: } \mathbf{Z}_C = -j/(\omega C) \quad 1/\omega C \angle -90^\circ$$

$$\text{Inductor: } \mathbf{Z}_L = j\omega L \quad \omega L \angle 90^\circ$$

# Admittance

- If we rewrite Ohm's Law:
- $\mathbf{I} = \mathbf{Y} \mathbf{V}$  ( $\mathbf{Y} = 1/\mathbf{Z}$ ), where  $\mathbf{Y}$  is admittance of the component

$$\text{Resistor: } \mathbf{Y}_R = 1/R = G \quad G \angle 0^\circ$$

$$\text{Capacitor: } \mathbf{Y}_C = j\omega C \quad \omega C \angle 90^\circ$$

$$\text{Inductor: } \mathbf{Y}_L = -j/(\omega L) \quad 1/\omega L \angle -90^\circ$$

# Impedances-Admittances

Impedances	Value at $\omega =$		Admittances	Value at $\omega =$	
	0 rad/s	$\infty$ rad/s		0 rad/s	$\infty$ rad/s
$Z_R = R = 1/G$	R	R	$Y_R = 1/R = G$	G	G
$Z_L = j\omega L$	0 $\Omega$	$\infty \Omega$	$Y_L = -j/(\omega L)$	$\infty \Omega$	0 $\Omega$
$Z_C = -j/(\omega C)$	$\infty \Omega$	0 $\Omega$	$Y_C = j\omega C$	0 $\Omega$	$\infty \Omega$

- Inductors act like short circuits under d.c. conditions and like open circuits at very high frequencies.
- Capacitors act like open circuits under d.c. conditions and like short circuits at very high frequencies.

# Impedance

Generic component  
that represents a  
resistor, inductor,  
or capacitor.

$$\mathbf{Z} = |Z| \angle \phi$$

$$\mathbf{Z} = R + jX$$

$$|Z| = \sqrt{R^2 + X^2}$$

$$\phi = \tan^{-1}(X/R)$$

$$R = |Z| \cos(\phi)$$

$$X = |Z| \sin(\phi)$$

# Admittance

$$\mathbf{Y} = \mathbf{1}/\mathbf{Z} = \frac{1}{R + jX}$$

$$G = \frac{R}{R^2 + X^2}$$

$$B = \frac{-X}{R^2 + X^2}$$

$$\mathbf{Y} = |Y| \angle \gamma$$

$$\mathbf{Y} = G + jB$$

$$|Y| = \sqrt{G^2 + B^2}$$

$$\gamma = \tan^{-1}(B/G)$$

$$G = |Y| \cos(\gamma)$$

$$B = |Y| \sin(\gamma)$$

# Summary

- Ohm's Law can be used to determine the ac voltages and currents in a circuit.
  - Voltage leads current through an inductor.
  - Current leads voltage through a capacitor.

Component		Impedance		Admittance	
Resistor	$Z_R$	$R$	$R \angle 0^\circ$	$G$	$G \angle 0^\circ$
Capacitor	$Z_C$	$-j/\omega C$	$1/\omega C \angle -90^\circ$	$j\omega C$	$\omega C \angle +90^\circ$
Inductor	$Z_L$	$j\omega L$	$\omega L \angle +90^\circ$	$-j/\omega L$	$1/\omega L \angle -90^\circ$

# Ohm's Law with Series and Parallel Combinations

- Objective of Lecture
  - Derive the equations for equivalent impedance and equivalent admittance for a series combination of components.
  - Derive the equations for equivalent impedance and equivalent admittance for a parallel combination of components.
- Ohm's Law in Phasor Notation

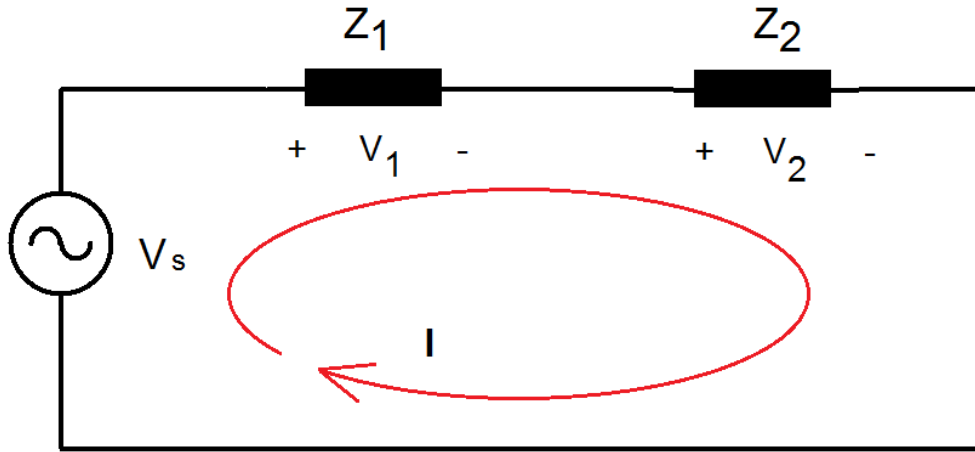
$$\mathbf{V} = \mathbf{I} \mathbf{Z}$$

$$\mathbf{I} = \mathbf{V} / \mathbf{Z}$$

$$\mathbf{V} = \mathbf{I} / \mathbf{Y}$$

$$\mathbf{I} = \mathbf{V} \mathbf{Y}$$

# Series Connections



Using Kirchhoff's  
Voltage Law:

$$V_1 + V_2 - V_s = 0$$

Since  $Z_1$ ,  $Z_2$ , and  $V_s$  are in series, the current flowing through each component is the same.

Using Ohm's Law:  $V_1 = I Z_1$       and       $V_2 = I Z_2$

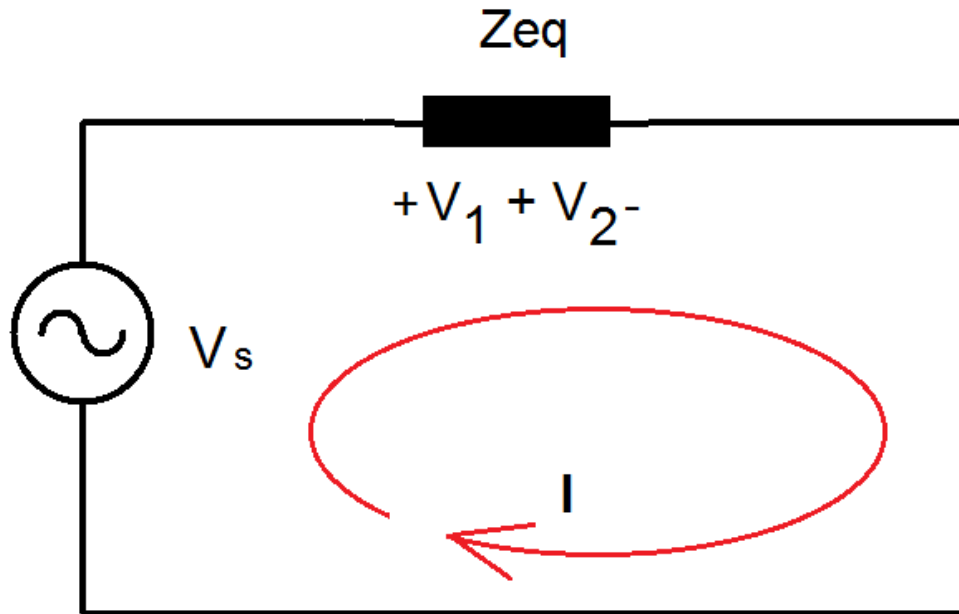
Substituting into the equation from KVL:

$$I Z_1 + I Z_2 - V_s = 0V$$

$$I (Z_1 + Z_2) = V_s$$



# Equivalent Impedance: Series Connections

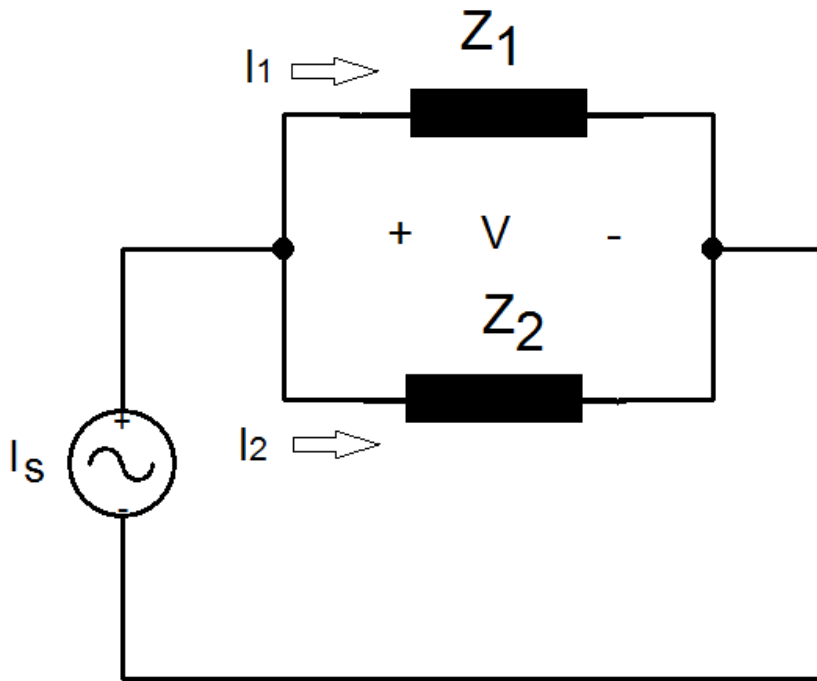


We can replace the two impedances in series with one equivalent impedance,  $Z_{eq}$ , which is equal to the sum of the impedances in series.

$$Z_{eq} = Z_1 + Z_2$$

$$V_s = Z_{eq} I$$

# Parallel Connections



Using Kirchoff's Current Law,  
 $I_1 + I_2 - I_s = 0$

Since  $Z_1$  and  $Z_2$  are in parallel,  
the voltage across each  
component,  $V$ , is the same.

Using Ohm's Law:

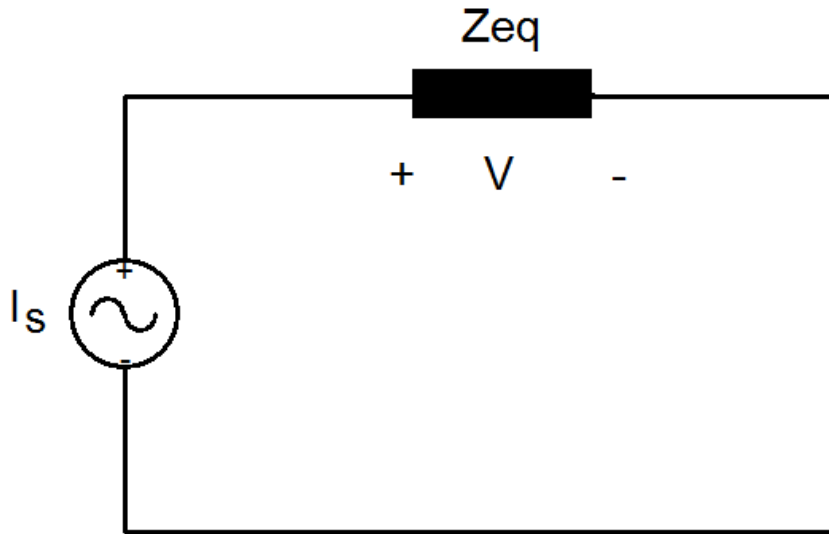
$$V = I_1 Z_1$$

$$V = I_2 Z_2$$

$$V/Z_1 + V/Z_2 = I_s$$

$$I_s (1/Z_1 + 1/Z_2)^{-1} = V$$

# Equivalent Impedance: Parallel Connections



We can replace the two impedances in series with one equivalent impedance,  $Z_{eq}$ , where  $1/Z_{eq}$  is equal to the sum of the inverse of each of the impedances in parallel.

$$1/Z_{eq} = 1/Z_1 + 1/Z_2$$

Simplifying

(only for 2 impedances in parallel)

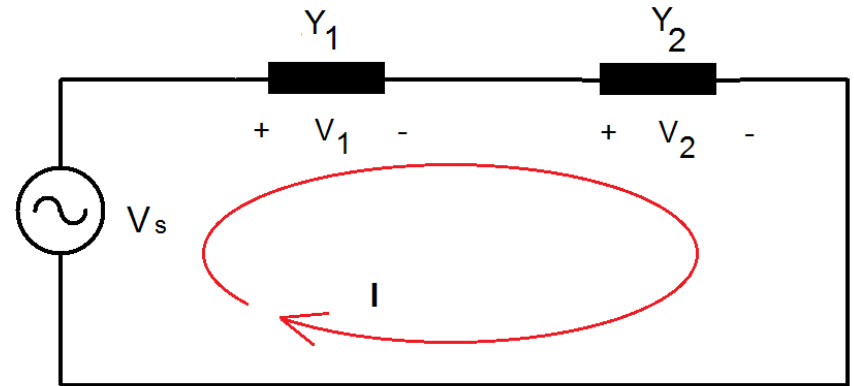
$$Z_{eq} = Z_1 Z_2 / (Z_1 + Z_2)$$

- An abbreviated means to show that  $Z_1$  is in parallel with  $Z_2$  is to write  $Z_1 \parallel Z_2$ .

# If you used Y instead of Z

In series:

The reciprocal of the equivalent admittance is equal to the sum of the reciprocal of each of the admittances in series



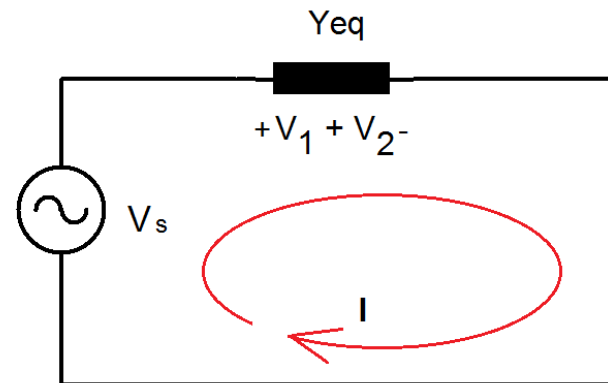
In this example

$$1/Y_{eq} = 1/Y_1 + 1/Y_2$$

Simplifying

(only for 2 admittances in series)

$$Y_{eq} = Y_1 Y_2 / (Y_1 + Y_2)$$



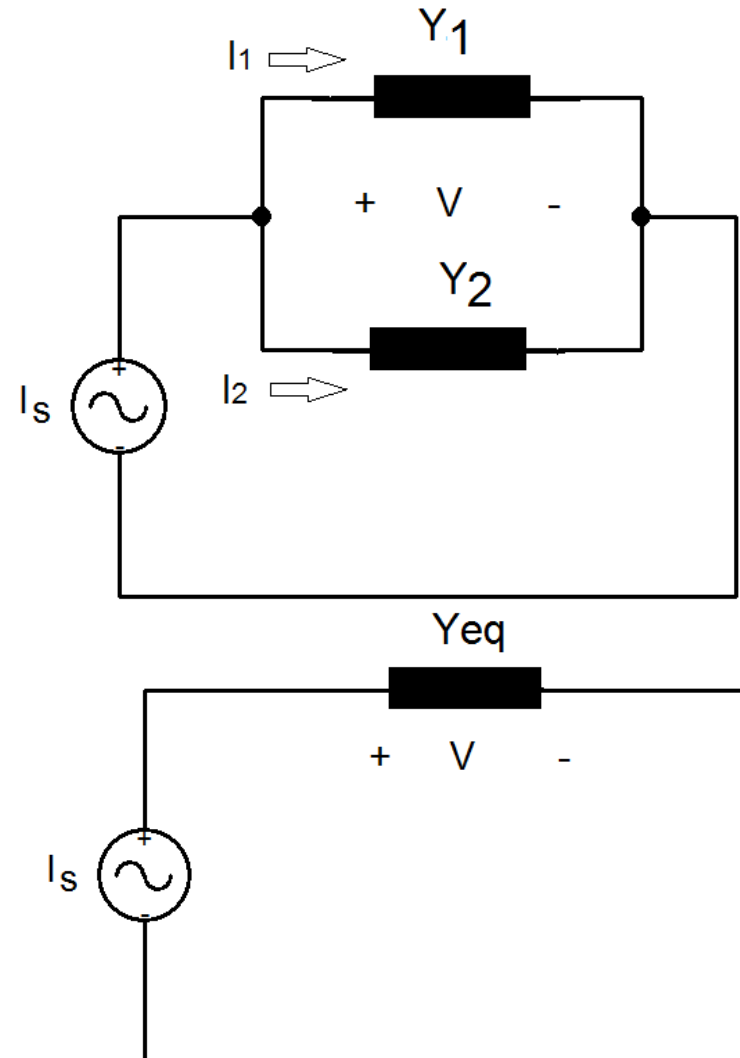
# If you used Y instead of Z

- In parallel:

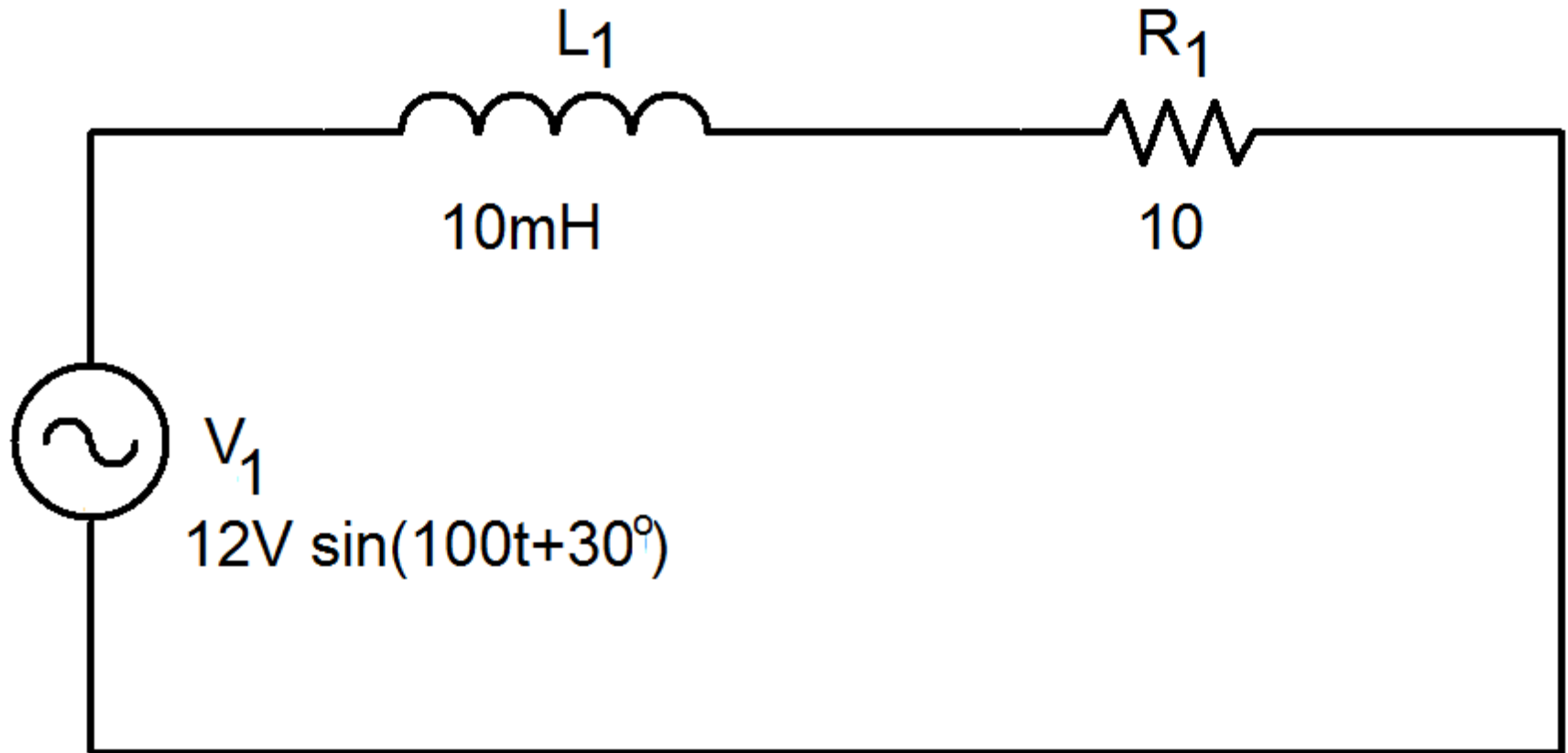
The equivalent admittance is equal to the sum of all of the admittance in parallel

In this example:

$$Y_{eq} = Y_1 + Y_2$$



# Example 03...



# ...Example 03...

- Impedance

$$Z_R = 10 \, \Omega$$

$$Z_L = j\omega L = j(100)(10\text{mH}) = 1j \, \Omega$$

$$Z_{\text{eq}} = Z_R + Z_L = 10 + 1j \, \Omega \quad (\text{rectangular coordinates})$$

In Phasor notation:

$$\mathbf{Z}_{\text{eq}} = (Z_R^2 + Z_L^2)^{1/2} \angle \tan^{-1}(\text{Im}/\text{Re})$$

$$\mathbf{Z}_{\text{eq}} = (100 + 1)^{1/2} \angle \tan^{-1}(1/10) = 10.05 \angle 5.7^\circ \, \Omega$$

$$\mathbf{Z}_{\text{eq}} = 10.1 \angle 5.7^\circ \, \Omega$$

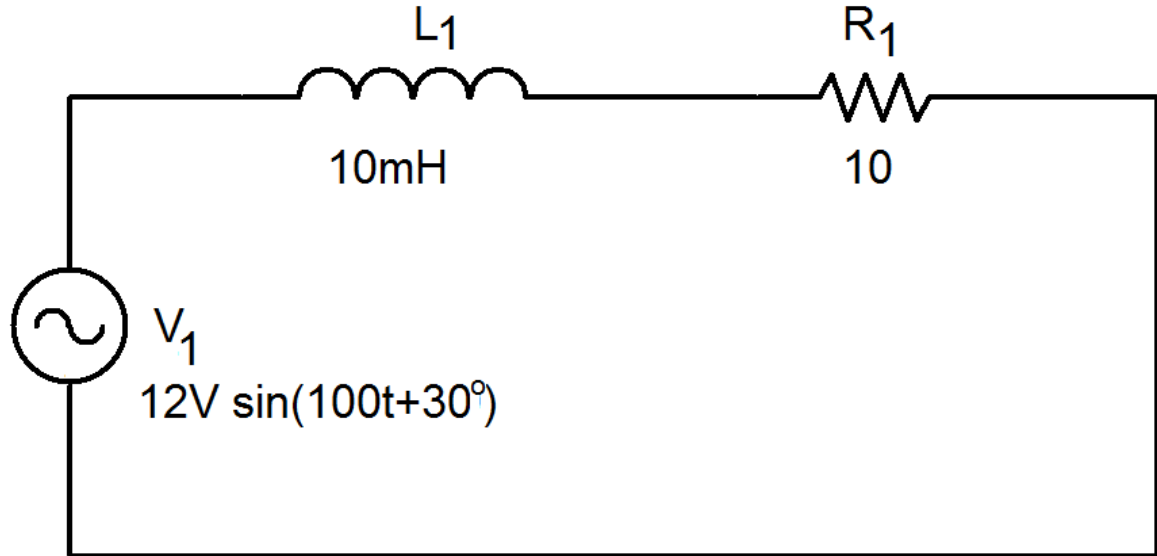
Impedances are easier than admittances to use when combining components in series.

# ...Example 03...

- Solve for Current
  - Express voltage into cosine and then convert a phasor.

$$V_1 = 12V \cos(100t + 30^\circ - 90^\circ) = 12V \cos(100t - 60^\circ)$$

$$\mathbf{V_1} = 12 \angle -60^\circ \text{ V}$$





## ...Example 03...

- Solve for Current

$$\mathbf{I} = \mathbf{V}/\mathbf{Z}_{\text{eq}} = (12 \angle -60^\circ \text{ V}) / (10.1 \angle 5.7^\circ \Omega)$$

$$\mathbf{V} = 12 \angle -60^\circ \text{ V} = 12\text{V } e^{-j60} \text{ (exponential form)}$$

$$\mathbf{Z}_{\text{eq}} = 10.1 \angle 5.7^\circ \Omega = 10.1 \Omega e^{j5.7} \text{ (exponential form)}$$

$$\mathbf{I} = \mathbf{V}/\mathbf{Z}_{\text{eq}} = 12\text{V } e^{-j60} / (10.1 e^{j5.7}) = 1.19\text{A } e^{-j65.7}$$

$$\mathbf{I} = 1.19\text{A } \angle -65.7^\circ$$

$$\mathbf{I} = \mathbf{V}_m/\mathbf{Z}_m \angle (\theta_V - \theta_Z)$$

## ...Example 03

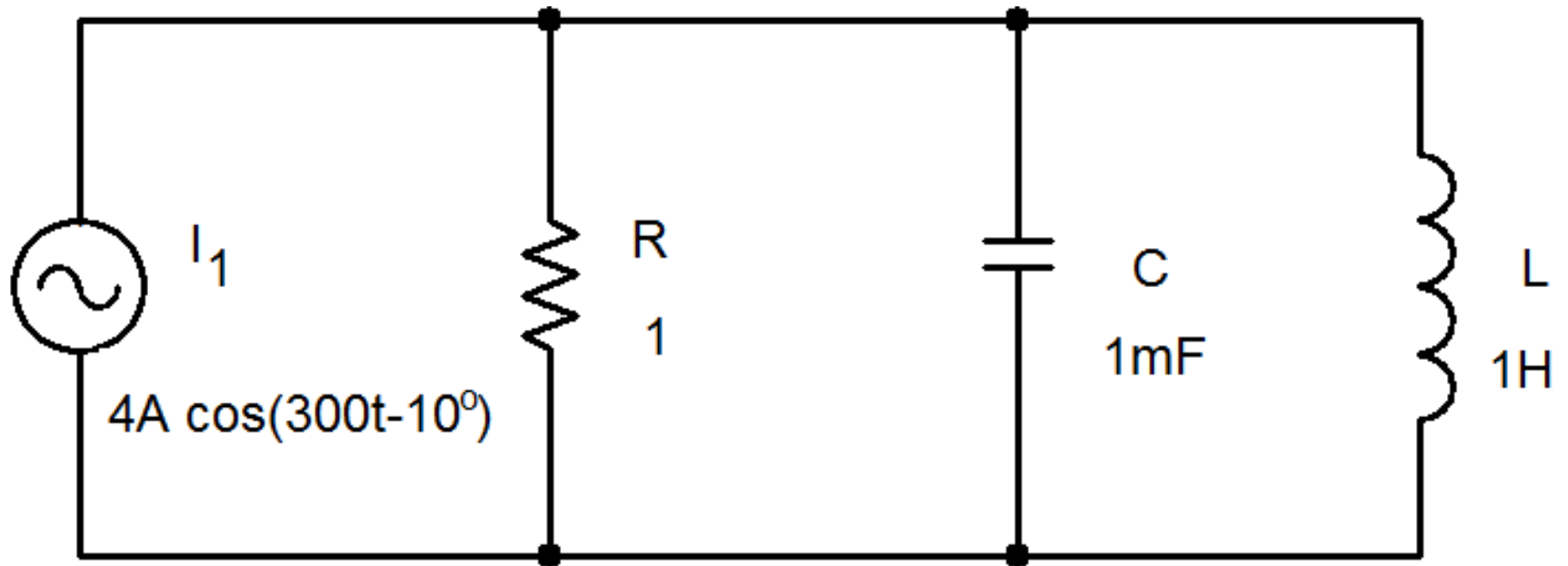
- Leading/Lagging

$$\mathbf{I} = 1.19\text{A } e^{-j65.7} = 1.19 \angle -65.7^\circ \text{ A}$$

$$\mathbf{V} = 12\text{V } e^{-j60} = 12 \angle -60^\circ \text{ V}$$

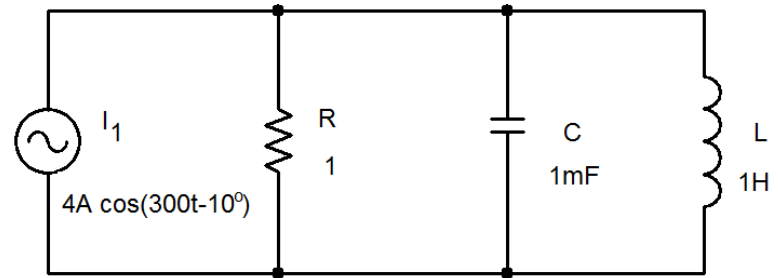
The voltage has a more positive angle, voltage leads the current.

# Example 04...



# ...Example 04...

- Admittance



$$Y_R = 1/R = 1 \Omega^{-1}$$

$$Y_L = -j/(\omega L) = -j/[(300)(1H)] = -j 3.33 \text{ m}\Omega^{-1}$$

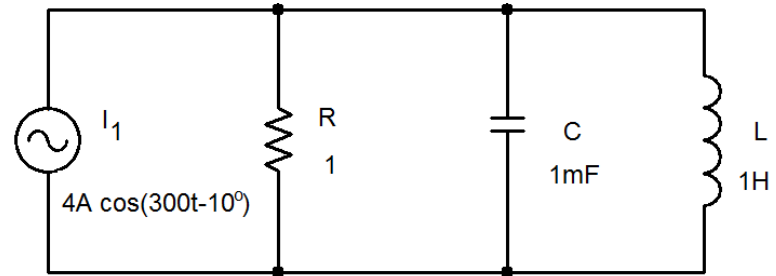
$$Y_C = j\omega C = j(300)(1mF) = 0.3j \Omega^{-1}$$

$$Y_{eq} = Y_R + Y_L + Y_C = 1 + 0.297j \Omega^{-1}$$

Admittances are easier than impedances to use when combining components in parallel.

# ...Example 04...

- Admittances:
  - In Phasor notation:



$$Y_{eq} = (Y_{Re}^2 + Y_{Im}^2)^{1/2} \angle \tan^{-1}(Im/Re)$$

$$Y_{eq} = (1^2 + (.297)^2)^{1/2} \angle \tan^{-1}(.297/1)$$

$$Y_{eq} = 1.04 \angle 16.5^\circ \Omega^{-1}$$

It is relatively easy to calculate the equivalent impedance of the components in parallel at this point as  $Z_{eq} = Y_{eq}^{-1}$ .

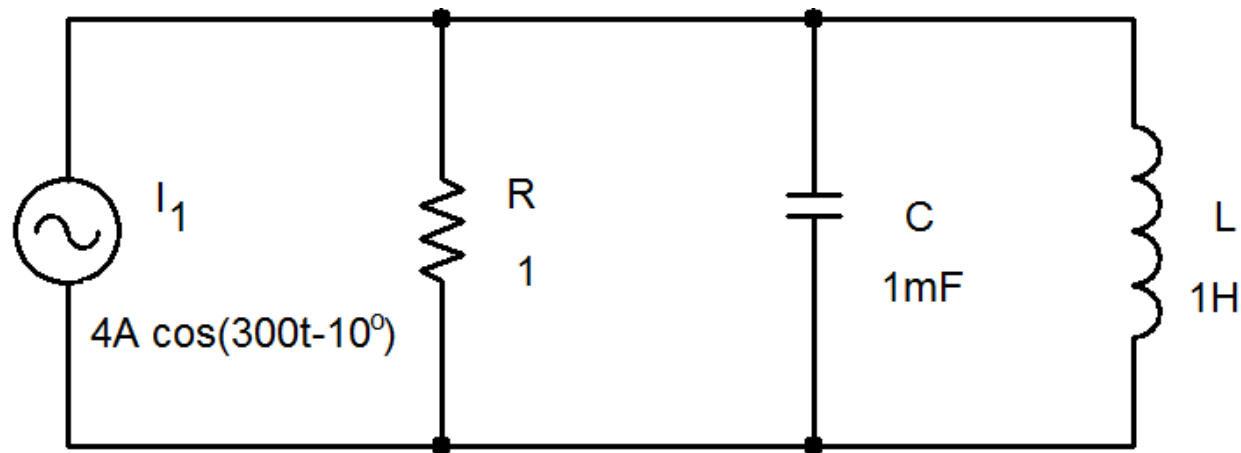
$$Z_{eq} = Y_{eq}^{-1} = 1/1.04 \angle 0-16.5^\circ \Omega = 0.959 \angle -16.5^\circ \Omega$$

# ...Example 04...

- Solve for Voltage
  - Convert a phasor since it is already expressed as a cosine.

$$I = 4\text{A} \cos(300t - 10^\circ)$$

$$\mathbf{I} = 4 \angle -10^\circ \text{ A}$$



# ...Example 04...

- Solve for Voltage

$$\mathbf{V} = \mathbf{I}/\mathbf{Y}_{\text{eq}}$$

$$\mathbf{V} = \mathbf{I}_m/\mathbf{Y}_m \angle (\theta_I - \theta_Y)$$

$$\mathbf{V} = (4 \angle -10^\circ \text{ A}) / (1.04 \angle 16.5^\circ \Omega^{-1})$$

$$\mathbf{V} = 3.84\text{V} \angle -26.5^\circ$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_{\text{eq}}$$

$$\mathbf{V} = \mathbf{I}_m\mathbf{Z}_m \angle (\theta_I + \theta_Z)$$

$$\mathbf{V} = (4 \angle -10^\circ \text{ A})(0.959 \angle -16.5^\circ \Omega^{-1})$$

$$\mathbf{V} = 3.84\text{V} \angle -26.5^\circ$$

## ...Example 04

- Leading/Lagging

$$\mathbf{I} = 4 \angle -10^\circ \text{ A}$$

$$\mathbf{V} = 3.84\text{V} \angle -26.5^\circ$$

Current has a more positive angle than voltage  
so current leads the voltage.



# Equations

Equivalent Impedances	Equivalent Admittances
In Series:	In Series:
$Z_{eq} = Z_1 + Z_2 + Z_3 \dots + Z_n$	$Y_{eq} = [1/Y_1 + 1/Y_2 + 1/Y_3 \dots + 1/Y_n]^{-1}$
In Parallel:	In Parallel:
$Z_{eq} = [1/Z_1 + 1/Z_2 + 1/Z_3 \dots + 1/Z_n]^{-1}$	$Y_{eq} = Y_1 + Y_2 + Y_3 \dots + Y_n$

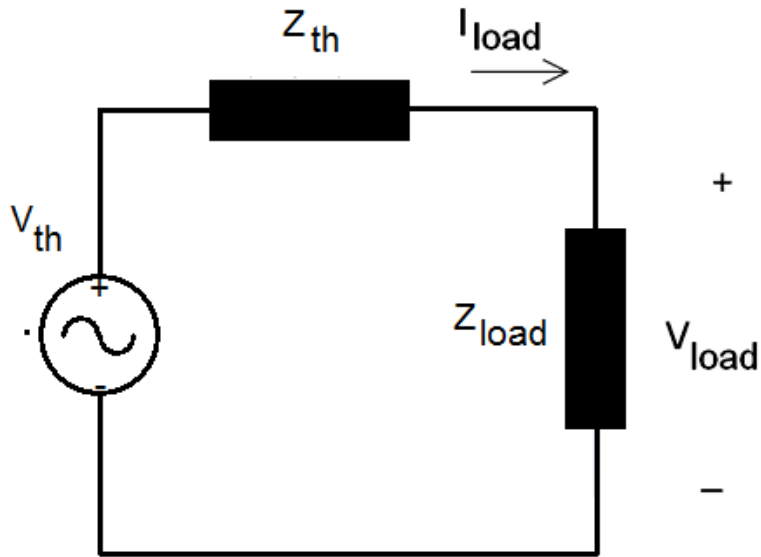
# Summary

- The equations for equivalent impedance are similar in form to those used to calculate equivalent resistance and the equations for equivalent admittance are similar to the equations for equivalent conductance.
  - The equations for the equivalent impedance for components in series and the equations for the equivalent admittance of components in parallel tend to be easier to use.
  - The equivalent impedance is the inverse of the equivalent admittance.

# Thévenin and Norton Transformation

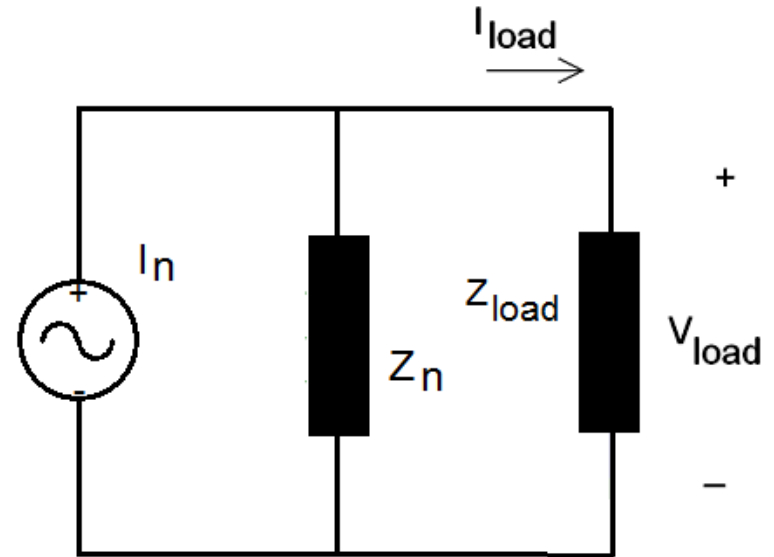
- Objective of Lecture
  - Demonstrate how to apply Thévenin and Norton transformations to simplify circuits that contain one or more ac sources, resistors, capacitors, and/or inductors.
- Source Transformation
  - A voltage source plus one impedance in series is said to be equivalent to a current source plus one impedance in parallel when the current into the load and the voltage across the load are the same.

# Equivalent Circuits



Thévenin

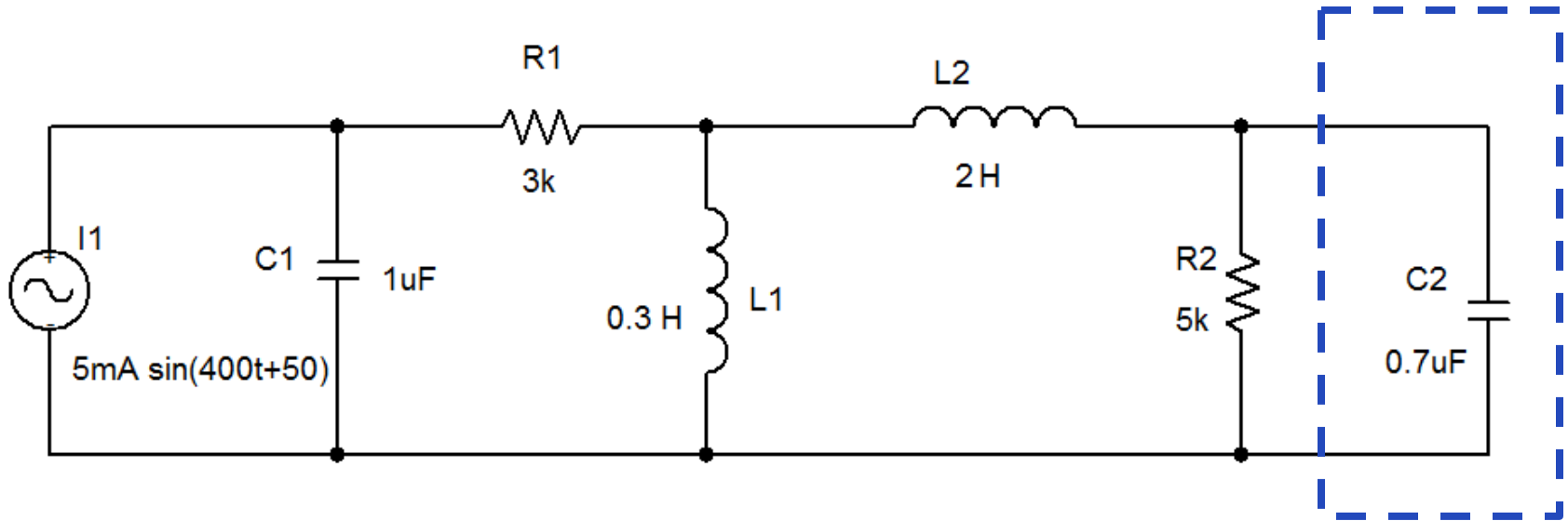
$$V_{th} = I_n Z_n$$



Norton

$$I_n = V_{th}/Z_{th}$$

# Example 05...



First, convert the current source to a cosine function and then to a phasor.

$$I_1 = 5\text{mA} \sin(400t+50^\circ) = 5\text{mA} \cos(400t+50^\circ-90^\circ) = 5\text{mA} \cos(400t-40^\circ)$$

$$I_1 = 5\text{mA} \angle -40^\circ$$

## ...Example 05...

- Determine the impedance of all of the components when  $\omega = 400 \text{ rad/s}$ .

– In rectangular coordinates

$$Z_{C_1} = -j / (\omega C_1) = -j / [(400 \text{ rad/s}) 1 \mu\text{F}] = -j 2.5 \text{ k}\Omega$$

$$Z_{R_1} = R_1 = 3 \text{ k}\Omega$$

$$Z_{L_1} = j\omega L_1 = j(400 \text{ rad/s})(0.3 \text{ H}) = j 120 \Omega$$

$$Z_{L_2} = j\omega L_2 = j(400 \text{ rad/s})(2 \text{ H}) = j 800 \Omega$$

$$Z_{R_2} = R_2 = 5 \text{ k}\Omega$$

$$Z_{C_2} = -j / (\omega C_2) = -j / [(400 \text{ rad/s}) 0.7 \mu\text{F}] = -j 3.57 \text{ k}\Omega$$

# ...Example 05...

- Convert to phasor notation

$$Z_{C_1} = 2.5k\Omega \angle -90^\circ$$

$$Z_{R_1} = 3k\Omega \angle 0^\circ$$

$$Z_{L_1} = 120\Omega \angle 90^\circ$$

$$Z_{L_2} = 800\Omega \angle 90^\circ$$

$$Z_{R_2} = 5k\Omega \angle 0^\circ$$

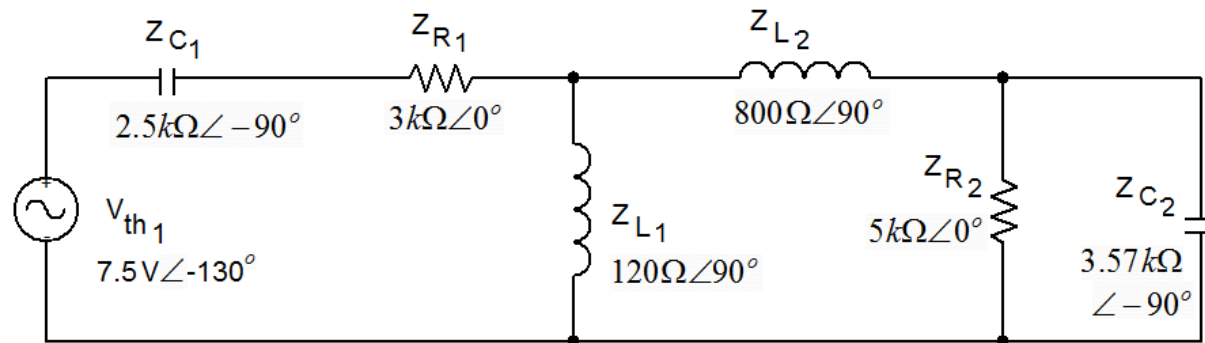
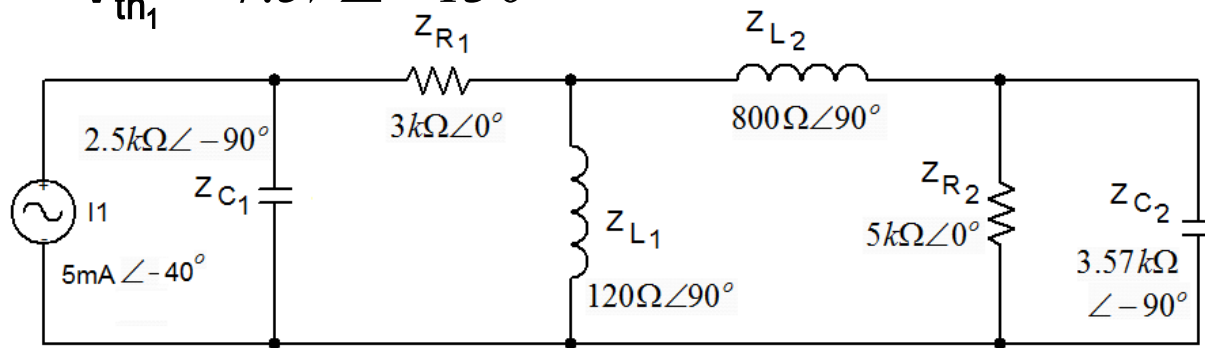
$$Z_{C_2} = 3.57k\Omega \angle -90^\circ$$

# ...Example 05...

$$V_{th_1} = I_1 Z_{C_1} = (5mA \angle -40^\circ)(2.5k\Omega \angle -90^\circ)$$

$$V_{th_1} = (5mA)(2.5k\Omega) \angle (-40^\circ + (-90^\circ))$$

$$V_{th_1} = 7.5V \angle -130^\circ$$





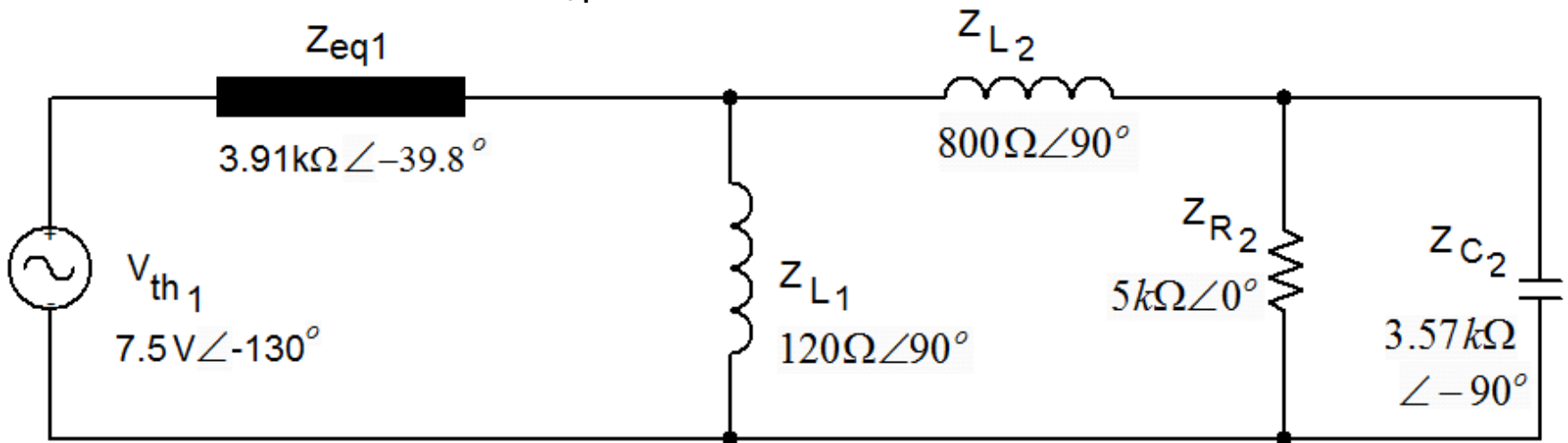
# ...Example 05...

- Find the equivalent impedance for  $Z_{C1}$  and  $Z_{R1}$  in series.
  - This is best done by using rectangular coordinates for the impedances.

$$Z_{eq1} = Z_{R1} + Z_{C1} = 3k\Omega - j2.5k\Omega$$

$$Z_{eq1} = \sqrt{(3k\Omega)^2 + (-2.5k\Omega)^2} \angle \tan^{-1}(-2.5k/3k)$$

$$Z_{eq1} = 3.91k\Omega \angle -39.8^\circ$$



# ....Example 05...

- Perform a Norton transformation.

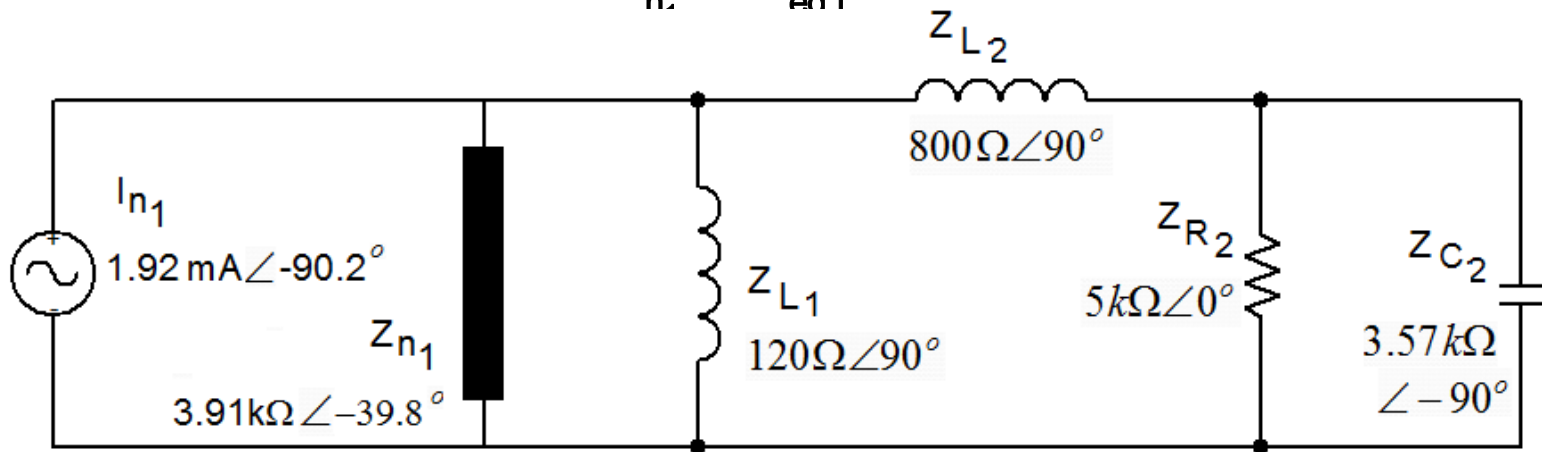
$$I_{n_1} = V_{th_1} / Z_{eq1}$$

$$I_{n_1} = (7.5V \angle -130^\circ) / (3.91k\Omega \angle -39.8^\circ)$$

$$I_{n_1} = (7.5V / 3.91k\Omega) \angle [-130^\circ - (-39.8^\circ)]$$

$$I_{n_1} = 1.92mA \angle -90.2^\circ$$

$$Z_{n_1} = Z_{eq1}$$



## ...Example 05...

- Since it is easier to combine admittances in parallel than impedances, convert  $Z_{n1}$  to  $Y_{n1}$  and  $Z_{L1}$  to  $Y_{L1}$ .
- As  $Y_{eq2}$  is equal to  $Y_{L1} + Y_{n1}$ , the admittances should be written in rectangular coordinates, added together, and then the result should be converted to phasor notation.

# ...Example 05...

$$\mathbf{Y}_{n1} = 1/\mathbf{Z}_{n1} = 0.256m\Omega^{-1} \angle 39.8^\circ$$

$$Y_{n1} = 0.256m\Omega^{-1} [\cos(39.8^\circ) + j \sin(39.8^\circ)]$$

$$Y_{n1} = (0.198 + j0.164)m\Omega^{-1}$$

$$\mathbf{Y}_{L1} = 1/\mathbf{Z}_{L1} = 8.33m\Omega^{-1} \angle -90^\circ$$

$$Y_{L1} = -j8.33m\Omega^{-1}$$

$$Y_{eq2} = (0.198 + j0.164)m\Omega^{-1} - j8.33m\Omega^{-1}$$

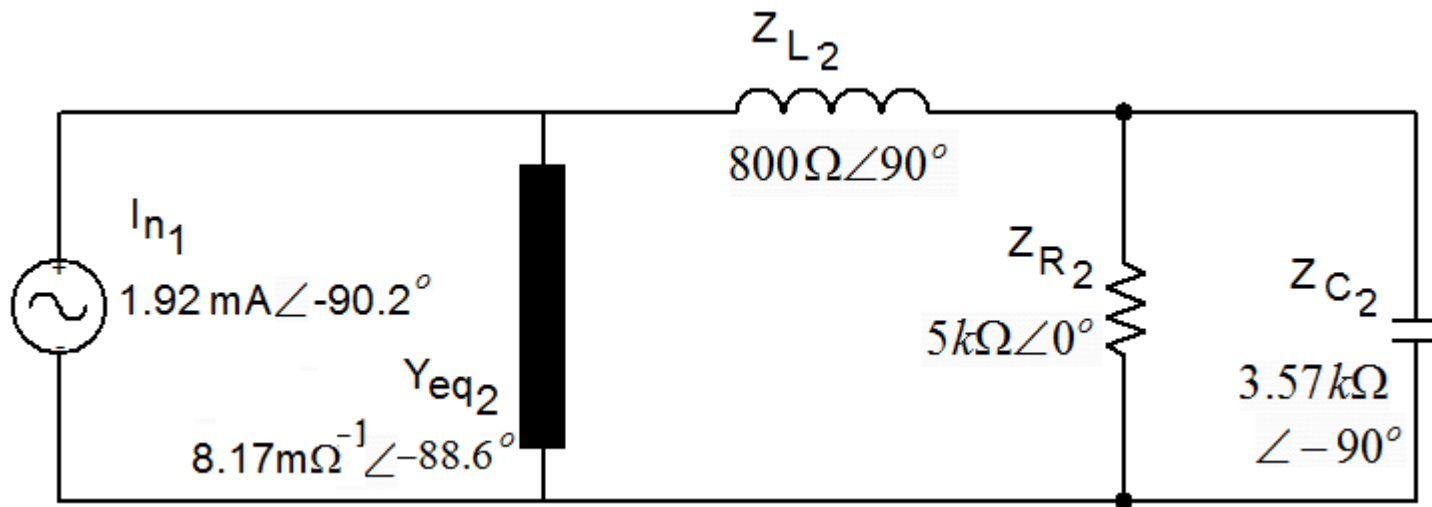
$$Y_{eq2} = (0.198 - j8.17)m\Omega^{-1}$$

$$\mathbf{Y}_{eq2} = \sqrt{(0.198)^2 + (-8.17)^2} m\Omega^{-1} \angle \tan^{-1}(-8.17/0.198)$$

$$\mathbf{Y}_{eq2} = 0.817m\Omega^{-1} \angle -88.6^\circ$$

## ...Example 05...

- Next, a Thévenin transformation will allow  $Y_{eq2}$  to be combined with  $Z_{L2}$ .

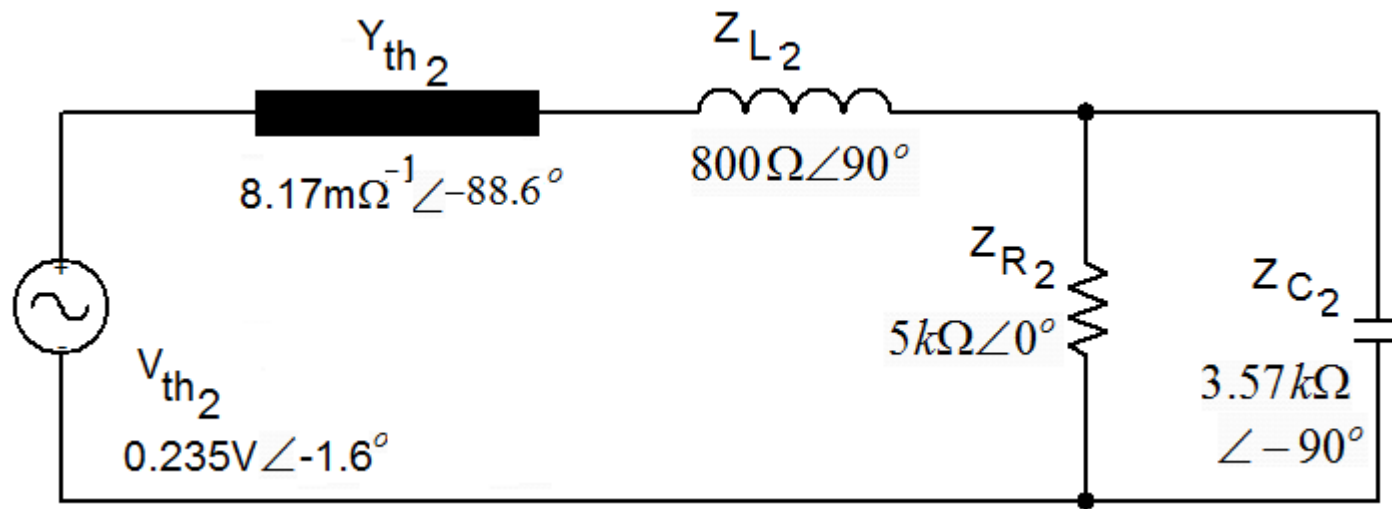


# ...Example 05...

$$V_{th_2} = I_{n_1} / Y_{th_2}$$

$$V_{th_2} = \frac{1.92mA \angle -90.2^\circ}{8.17m\Omega^{-1} \angle -88.6^\circ}$$

$$V_{th_2} = 0.235V \angle -1.6^\circ$$



# ...Example 05...

$$\mathbf{Z}_{th_2} = 1 / \mathbf{Y}_{th_2} = 122\Omega \angle 88.6^\circ$$

$$Z_{th_2} = 122\Omega [\cos(88.6^\circ) + j \sin(88.6^\circ)]$$

$$Z_{th_2} = (2.98 + j122)\Omega$$

$$Z_{L_2} = j800\Omega$$

$$Z_{eq_3} = Z_{th_2} + Z_{L_2}$$

$$Z_{eq_3} = (2.98 + j122)\Omega + j800\Omega$$

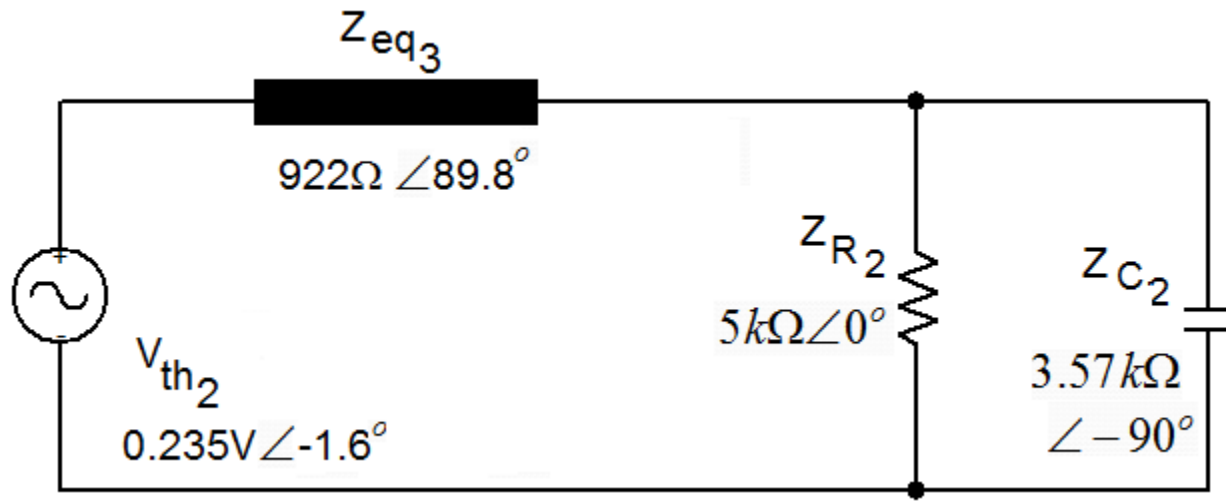
$$Z_{eq_3} = (2.98 + j922)\Omega$$

$$\mathbf{Z}_{eq_3} = \sqrt{(2.98)^2 + (922)^2} \Omega \angle [\tan^{-1}(922/2.98)]$$

$$\mathbf{Z}_{eq_3} = 922\Omega \angle 89.8^\circ$$

## ...Example 05...

- Perform a Norton transformation after which  $Z_{eq3}$  can be combined with  $Z_{R2}$ .



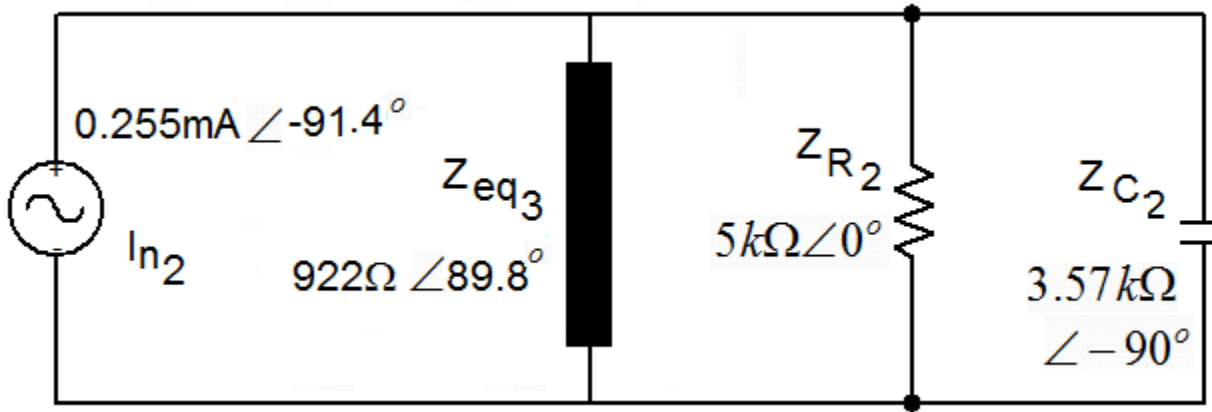


## ...Example 05...

$$I_{n2} = V_{th2} / Z_{eq3}$$

$$I_{n2} = \frac{0.235V \angle -1.6^{\circ}}{922\Omega \angle 89.8^{\circ}}$$

$$I_{n2} = 0.255mA \angle -91.4^{\circ}$$



# ...Example 05...

$$Y_{eq4} = 1/Z_{eq3} + 1/Z_{R_2}$$

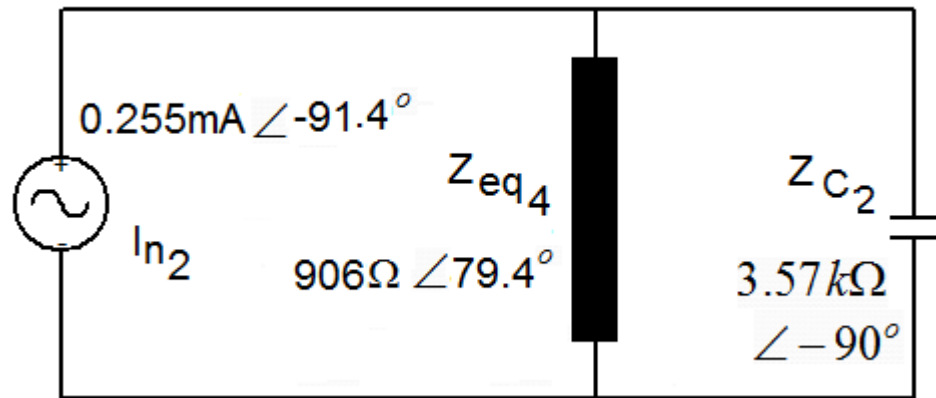
$$Y_{eq4} = 1.08m\Omega^{-1} \angle -89.8^\circ + 0.2m\Omega^{-1} \angle 0^\circ$$

$$Y_{eq4} = 1.08m\Omega^{-1} [\cos(-89.8^\circ) + j \sin(-89.8^\circ)] + 0.2m\Omega^{-1}$$

$$Y_{eq4} = (0.204 - j1.08)m\Omega^{-1}$$

$$Y_{eq4} = 1.10m\Omega^{-1} \angle -79.4^\circ$$

$$Z_{eq4} = 906\Omega \angle 79.4^\circ$$



# ...Example 05...

Use the equation for current division to find the current flowing through  $Z_{C_2}$  and  $Z_{eq4}$ .

$$I_{C_2} = \frac{Y_{C_2}}{Y_{C_2} + Y_{eq4}} I_{n2}$$

$$Y_{C_2} = 1/Z_{C_2} = 0.280m\Omega \angle 90^\circ$$

$$Y_{C_2} = j0.280m\Omega$$

$$Y_{eq4} = (0.204 - j1.08)m\Omega$$

$$I_{C_2} = \frac{0.280m\Omega \angle 90^\circ}{j0.280m\Omega + (0.204 - j1.08)m\Omega} (0.255mA \angle -91.4^\circ)$$

$$I_{C_2} = \frac{0.280m\Omega \angle 90^\circ}{(0.204 - j0.8)m\Omega} (0.255mA \angle -91.4^\circ)$$

$$I_{C_2} = \frac{0.280m\Omega \angle 90^\circ}{0.826m\Omega \angle -75.7^\circ} (0.255mA \angle -91.4^\circ)$$

$$I_{C_2} = 86.0\mu A \angle 74.3^\circ$$

## ...Example 05...

- Then, use Ohm's Law to find the voltage across  $\mathbf{Z_{C2}}$  and then the current through  $\mathbf{Z_{eq4}}$ .

$$\mathbf{V_{C_2}} = \mathbf{I_{C_2} Z_{C_2}} = (86.0 \mu A \angle 74.3^\circ)(3.57 k\Omega \angle -90^\circ)$$

$$\mathbf{V_{C_2}} = 0.309 V \angle -15.7^\circ$$

$$\mathbf{V_{C_2}} = \mathbf{V_{eq_4}}$$

$$\mathbf{I_{eq_4}} = \frac{\mathbf{V_{eq_4}}}{\mathbf{Z_{eq_4}}} = \frac{0.309 V \angle -15.7^\circ}{906 \Omega \angle 79.4^\circ}$$

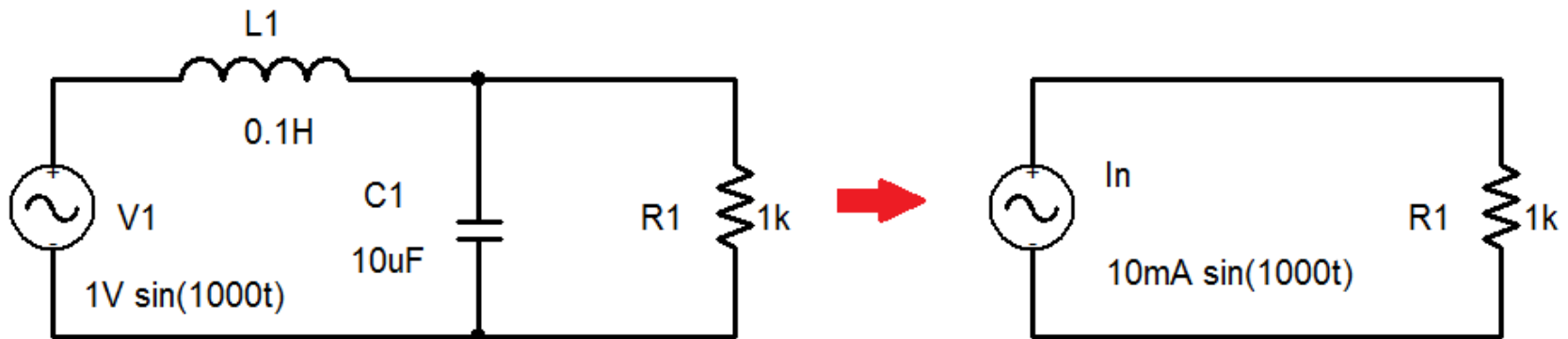
$$\mathbf{I_{eq_4}} = 0.341 mA \angle -95.1^\circ$$

## ...Example 05

- Note that the phase angles of  $I_{n2}$ ,  $I_{eq4}$ , and  $I_{C2}$  are all different because of the imaginary components of  $Z_{eq4}$  and  $Z_{C2}$ .
  - The current through  $Z_{C2}$  leads the voltage, which is as expected for a capacitor.
  - The voltage through  $Z_{eq4}$  leads the current.
  - Since the phase angle of  $Z_{eq4}$  is positive, it has an inductive part to its impedance.
    - Thus, it should be expected that the voltage would lead the current.

# Example

- Explain why the circuit on the right is the result of a Norton transformation of the circuit on the left.
- Also, calculate the natural frequency  $\omega_o$  of the RLC network.



# Summary

- Circuits containing resistors, inductors, and/or capacitors can be simplified by applying the Thévenin and Norton Theorems.
  - Transformations can easily be performed using currents, voltages, impedances, and admittances written in phasor notation.
  - Calculation of equivalent impedances and admittances requires the conversion of phasors into rectangular coordinates.
  - Use of the current and voltage division equations also requires the conversion of phasors into rectangular coordinates.

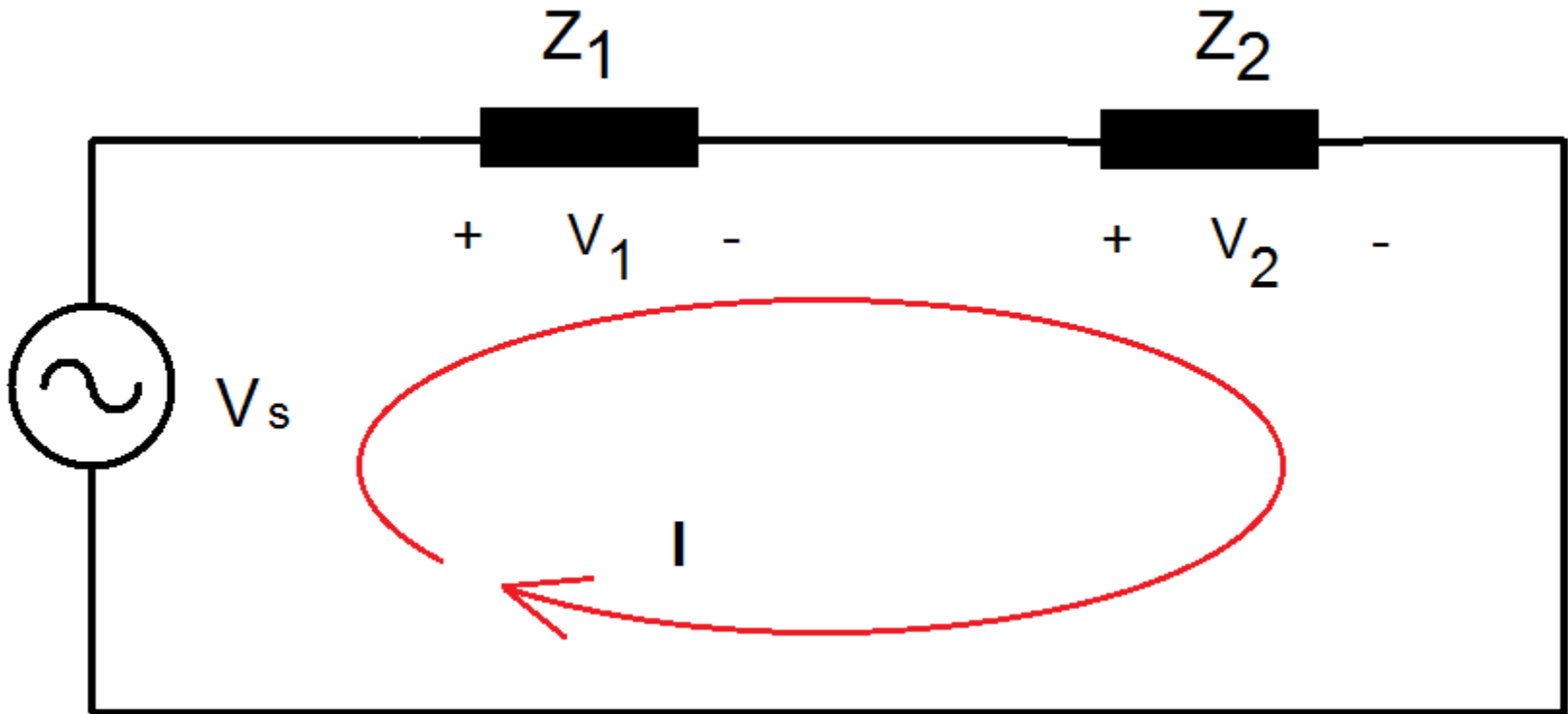
# Voltage and Current Division

- Objective of Lecture
  - Explain mathematically how a voltage that is applied to components in series and how a current that enters the a node shared by components in parallel are distributed among the components.



# Voltage Dividers

- Impedances in series share the same current



# Voltage Dividers

- From Kirchhoff's Voltage Law and Ohm's Law

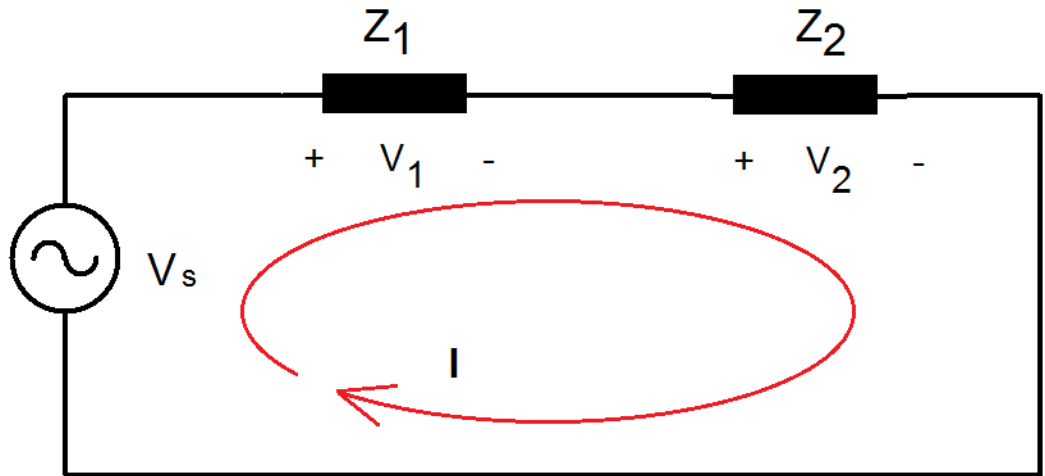
$$0 = -V_s + V_1 + V_2$$

$$V_1 = IZ_1 \quad \text{and} \quad V_2 = IZ_2$$

$$\text{Therefore, } V_2 = \frac{V_1}{Z_1} Z_2$$

$$V_1 = \frac{Z_1}{Z_1 + Z_2} V_s$$

$$V_2 = \frac{Z_2}{Z_1 + Z_2} V_s$$



# Voltage Division

The voltage associated with one impedance  $Z_n$  in a chain of multiple impedances in series is:

$$V_n = \left[ \frac{Z_n}{\sum_{s=1}^S Z_s} \right] V_{\text{total}} \quad \text{or} \quad V_n = \left[ \frac{Z_n}{Z_{\text{eq}}} \right] V_{\text{total}}$$

where  $V_{\text{total}}$  is the total of the voltages applied across the impedances.

# Voltage Division

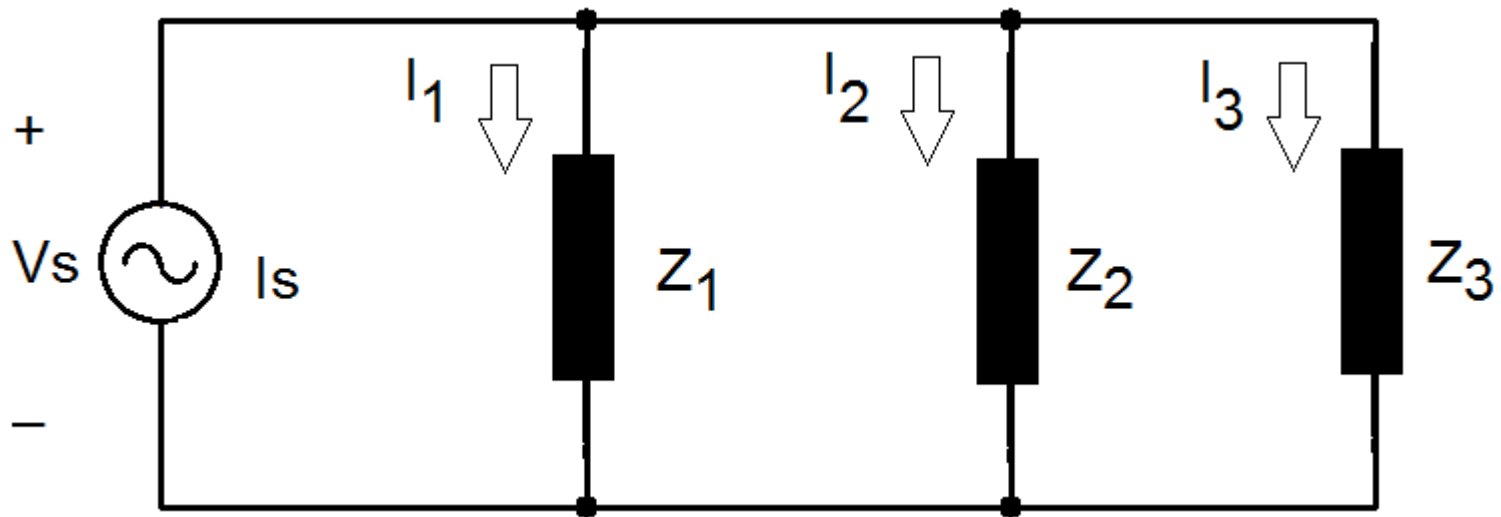
- Because of changes in phase angle of the voltage that occur with inductors and capacitors, the calculation of the percentage of the total voltage associated with a particular impedance,  $\mathbf{Z_n}$ , is not directly related to the percentage of the magnitude of that particular impedance,  $\mathbf{Z_n}$ , relative to the total equivalent impedance,  $\mathbf{Z_{eq}}$ .

$$\mathbf{Z_n} = Z_n \angle \varphi_n$$

$$\mathbf{Z_{eq}} = Z_{eq} \angle \varphi_{eq}$$

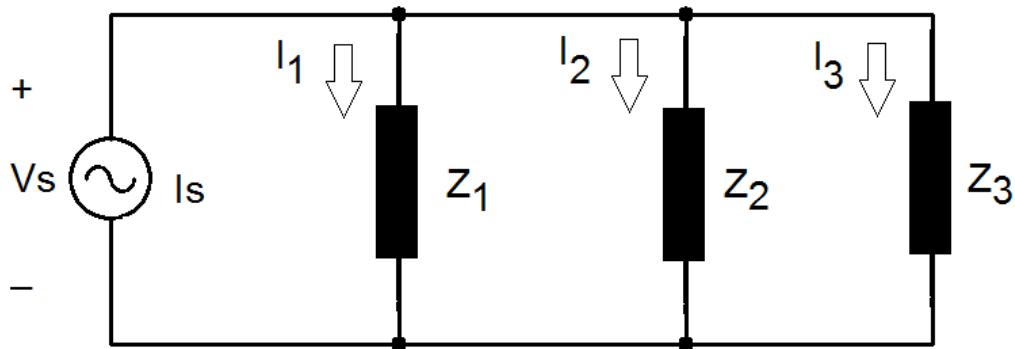
# Current Division

- All components in parallel share the same voltage



# Current Division

- From Kirchhoff's Current Law and Ohm's Law



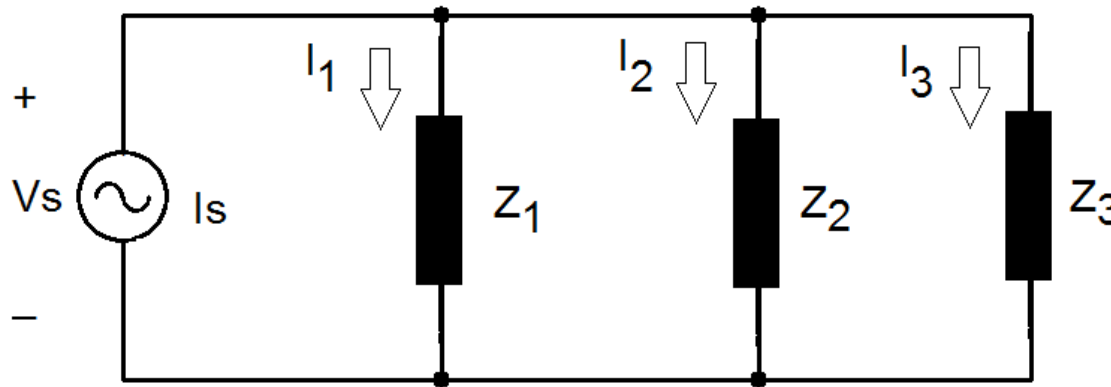
$$0 = -I_s + I_1 + I_2 + I_3$$

$$V_s = I_1 Z_1$$

$$V_s = I_2 Z_2$$

$$V_s = I_3 Z_3$$

# Current Division

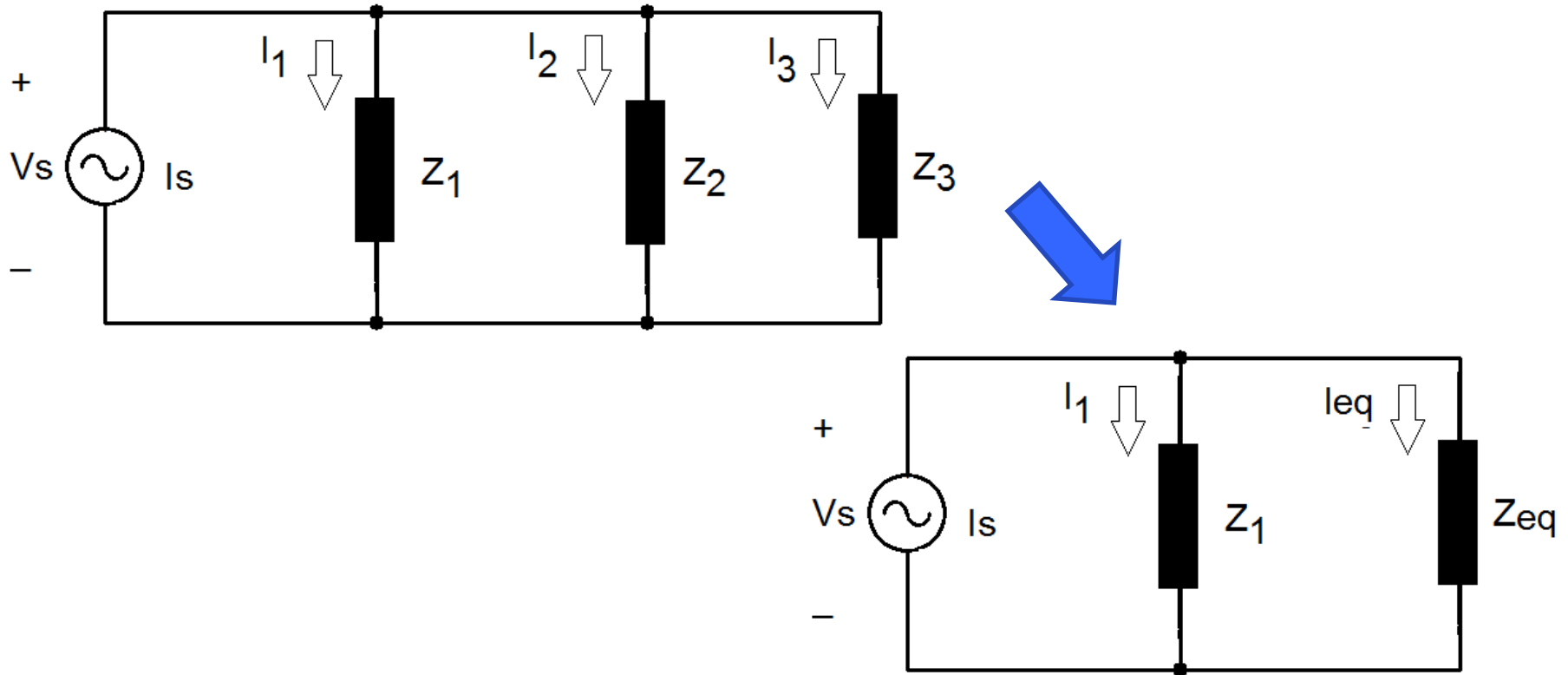


$$I_1 = \frac{Z_2 \parallel Z_3}{Z_1 + Z_2 \parallel Z_3} I_s$$

$$I_2 = \frac{Z_1 \parallel Z_3}{Z_2 + Z_1 \parallel Z_3} I_s$$

$$I_3 = \frac{Z_1 \parallel Z_2}{Z_3 + Z_1 \parallel Z_2} I_s$$

# Current Division



where  $Z_{eq} = Z_2 \parallel Z_3 = \frac{Z_2 Z_3}{Z_2 + Z_3}$  and  $I_1 = \frac{Z_{eq}}{Z_1 + Z_{eq}} I_s$



# Current Division

The current associated with one component  $Z_1$  in parallel with one other component is:

$$I_1 = \left[ \frac{Z_2}{Z_1 + Z_2} \right] I_{\text{total}}$$

The current associated with one component  $Z_m$  in parallel with two or more components is:

$$I_m = \left[ \frac{Z_{\text{eq}}}{Z_m} \right] I_{\text{total}}$$

where  $I_{\text{total}}$  is the total of the currents entering the node shared by the components in parallel.

# Summary

- The equations used to calculate the voltage across a specific component  $Z_n$  in a set of components in series are:

$$V_n = \left[ \frac{Z_n}{Z_{eq}} \right] V_{total}$$

$$V_n = \left[ \frac{Y_{eq}}{Y_n} \right] V_{total}$$

- The equations used to calculate the current flowing through a specific component  $Z_m$  in a set of components in parallel are:

$$I_m = \frac{Z_{eq}}{Z_m} I_{total}$$

$$I_m = \frac{Y_m}{Y_{eq}} I_{total}$$