

# BLM1612 - Circuit Theory

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# **1<sup>st</sup> Order Op Amp Circuits**

# Objectives of Lecture

- Discuss analog computing and the application of 1<sup>st</sup> order operational amplifier circuits.
- Derive the equations that relate the output voltage to the input voltage for a differentiator and integrator.
- Explain the source of the phase shift between the output and input voltages.

# Subsystems

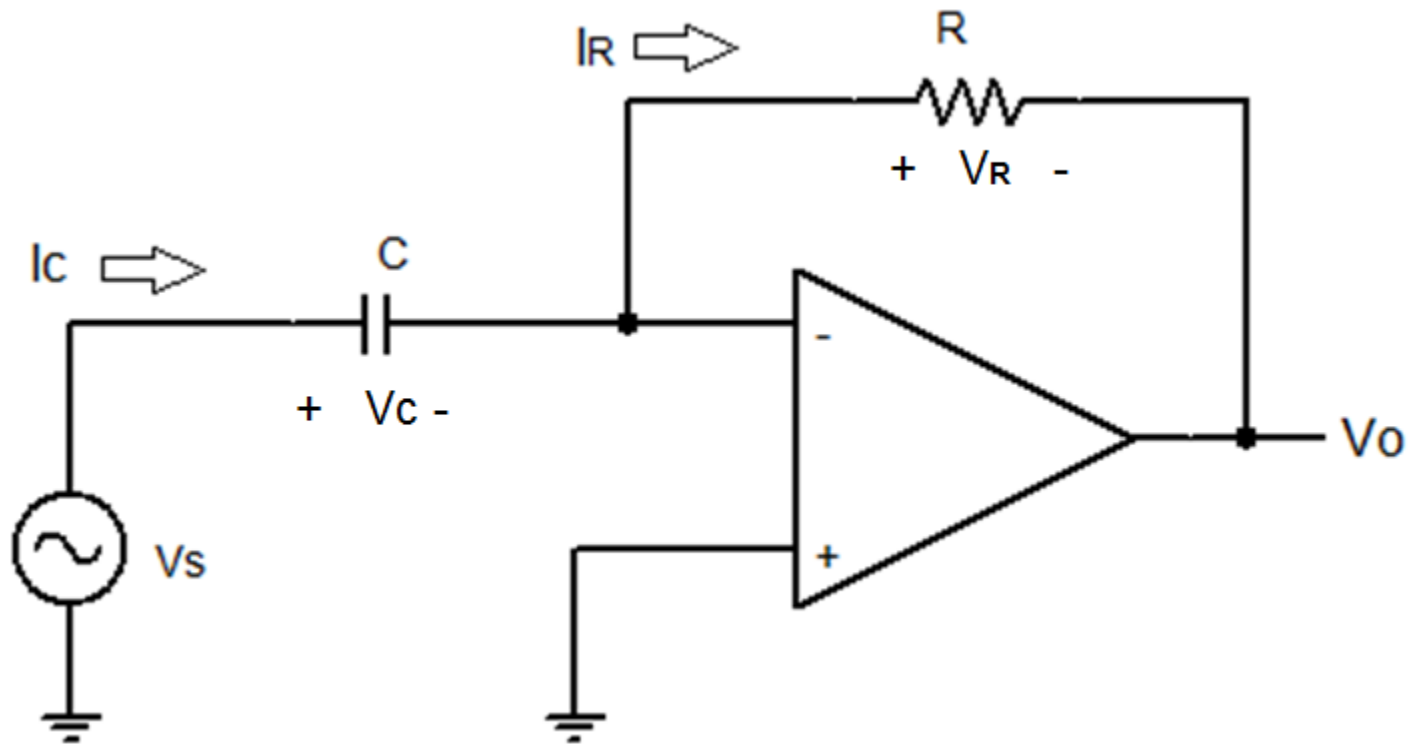
- Multipliers
    - Inverting and non-inverting amplifiers
      - Typically fixed number, which means fixed resistor values in amplifiers
  - Adders and Subtractors
    - Summing and difference amplifiers
  - Differentiators
  - Integrators
- } 1<sup>st</sup> order op amp circuits

# Capacitors

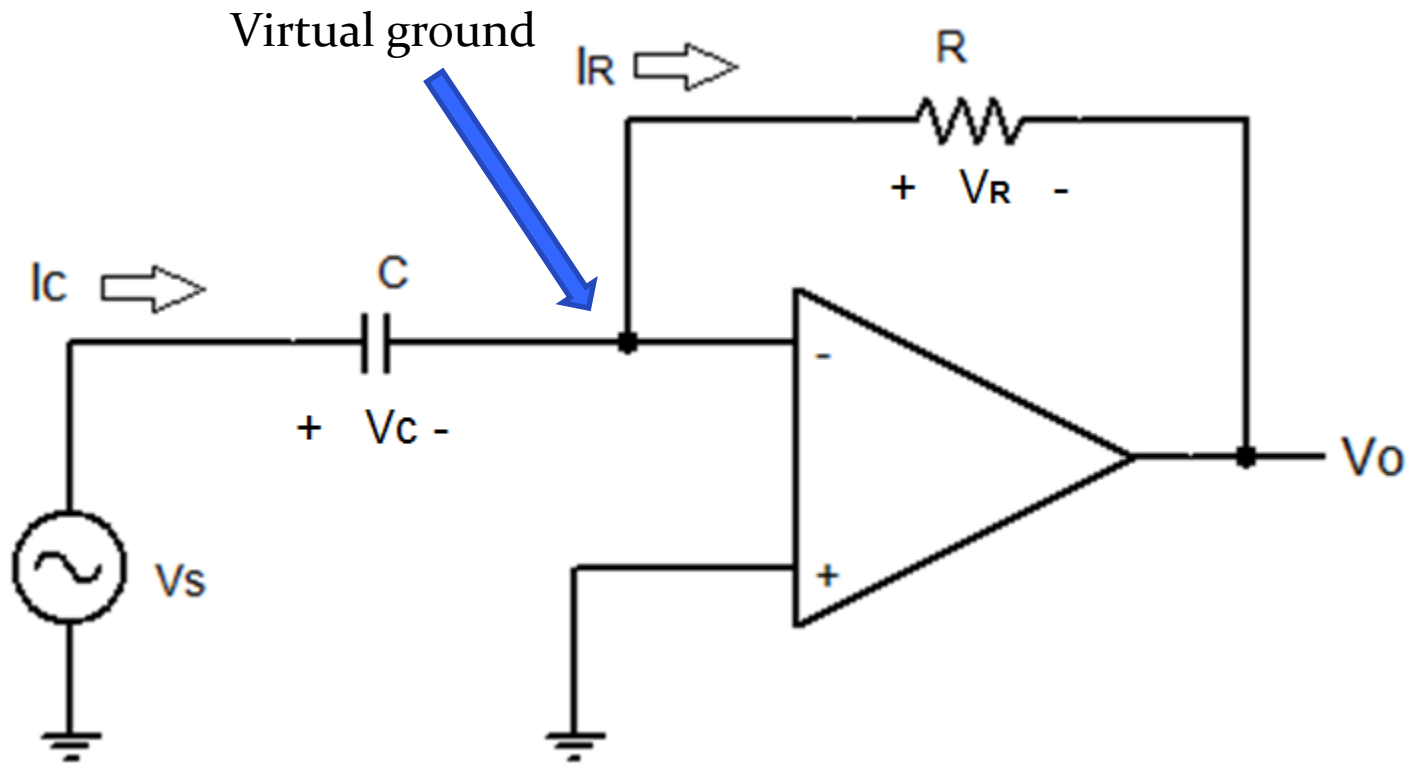
$$i_C(t) = C \frac{dv_C}{dt}$$

$$v_C(t) = \frac{1}{C} \int_{t_o}^{t_1} i_C(t) dt + v_C(t_o)$$

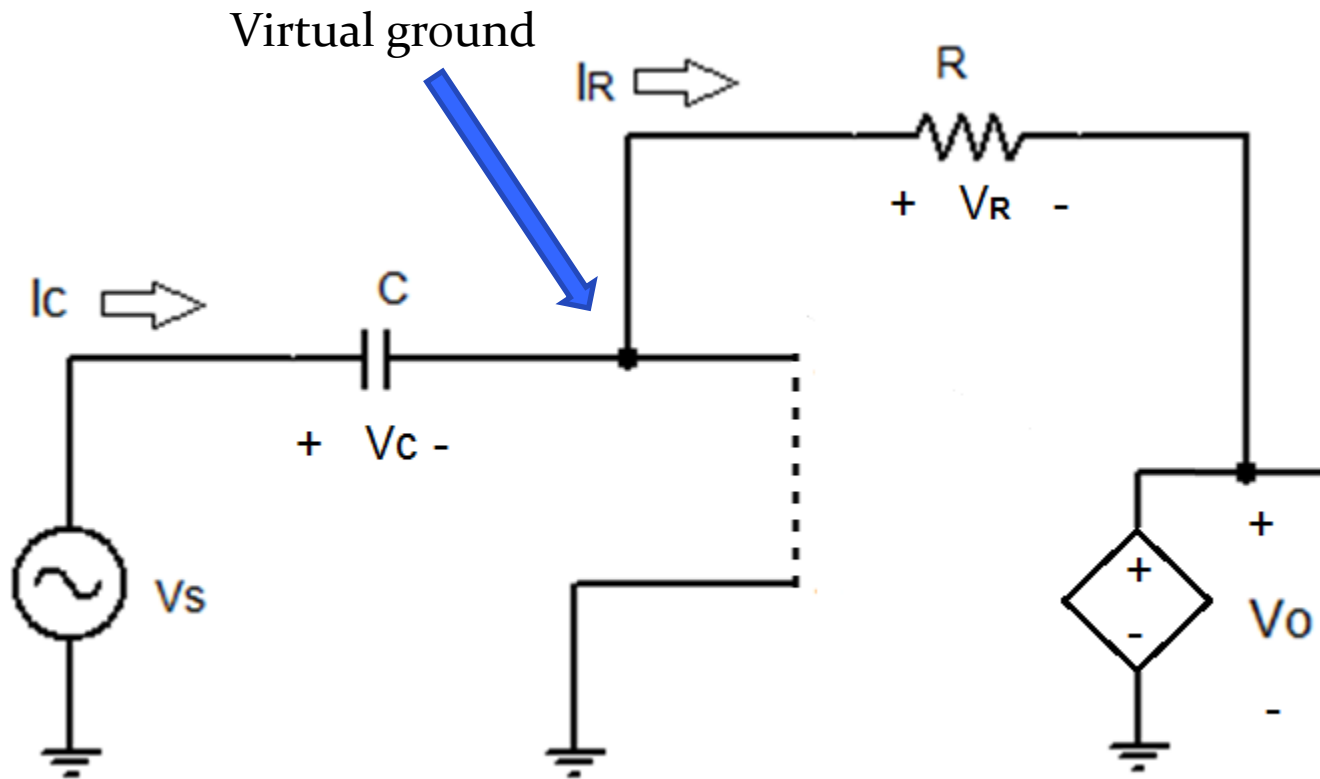
# Differentiator



# Ideal Op Amp Model



# Op Amp Model





# Analysis

- Since current is not allowed to enter the input terminals of an ideal op amp.

$$i_C(t) = i_R(t)$$

$$v_C(t) = v_S(t)$$

$$i_C(t) = C \frac{dv_C}{dt} = C \frac{dv_S}{dt}$$

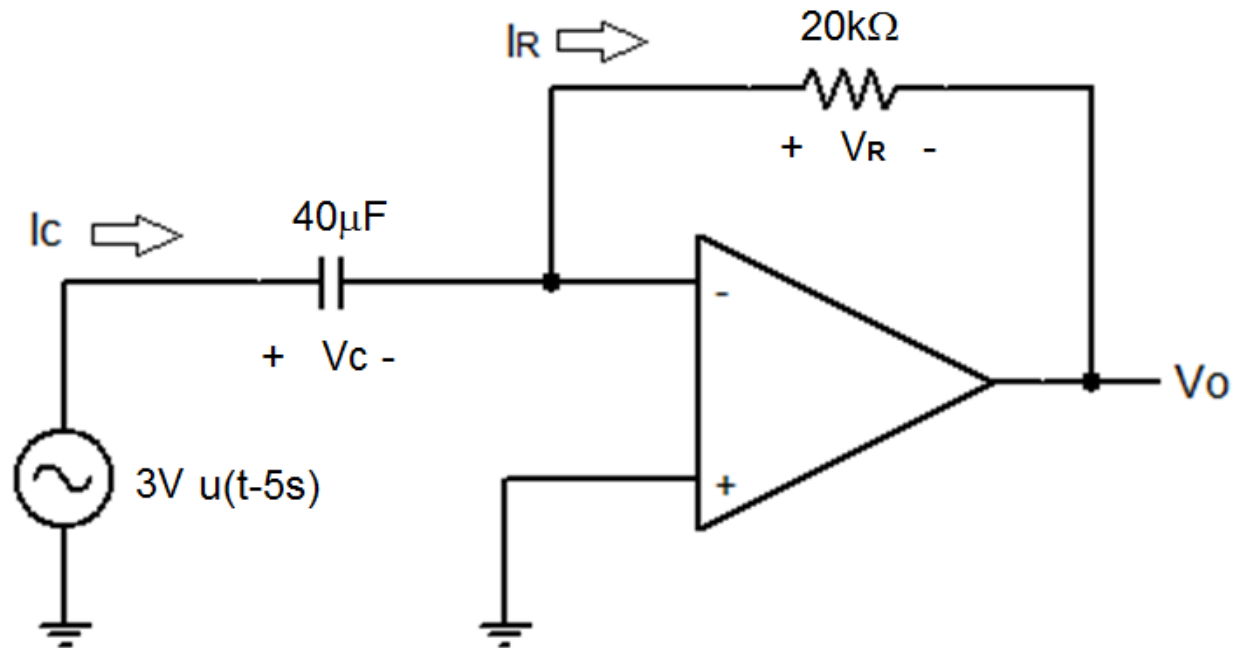
$$i_R(t) = -\frac{v_o}{R}$$

$$-\frac{v_o}{R} = C \frac{dv_S}{dt}$$

$$v_o(t) = -RC \frac{dv_S(t)}{dt}$$

# Example 01...

- Suppose  $v_s(t) = 3V u(t-5s)$ 
  - The voltage source changes from 0V to 3V at  $t = 5s$ .
    - Initial condition of  $V_C = 0V$  when  $t < 5s$ .
    - Final condition of  $V_C = 3V$  when  $t > 5RC$ .



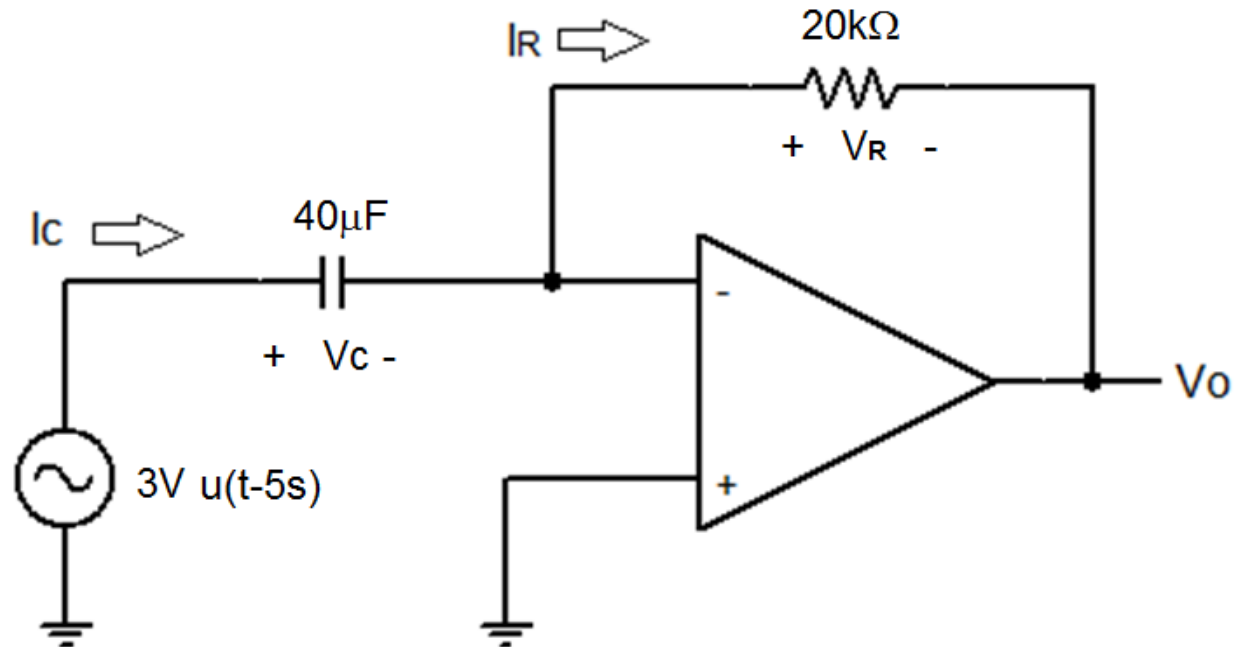
# ...Example 01...

$$v_C(t) = 0V \quad \text{when } t < t_o$$

$$v_C(t) = V_{C_{initial}} + (V_{C_{final}} - V_{C_{initial}}) e^{-(t-t_o)/\tau} \quad \text{when } t > t_o$$

$$v_C(t) = 0V + (3V - 0V) e^{-(t-5s)/0.8s} \quad \text{when } t > t_o$$

$$v_C(t) = 3V e^{-(t-5s)/0.8s} \quad \text{when } t > 5s$$



# ...Example 01

$$v_o(t) = -RC \frac{dv_C(t)}{dt}$$

$$v_o(t) = 0V \quad \text{when } t < 5s$$

$$v_o(t) = 0V \quad \text{when } t > t_o + 5\tau, \text{ where } \tau = RC$$

$$v_o(t) = 0V \quad \text{when } t > 5s + 5(20k\Omega)(40\mu F) = 9s$$

$$v_o(t) = \frac{-1}{0.8s} (-20 \times 10^3 \Omega)(40 \times 10^{-6} F)(3V) e^{-(t-5s)/0.8s}$$

$$v_o(t) = 3V e^{-(t-5s)/0.8s}$$

## Example 02

- Let  $R = 2 \text{ k}\Omega$ ,  $C = 0.1 \mu\text{F}$ , and  $v_s(t) = 2V \sin(500t)$  at  $t = 0\text{s}$

Since  $v_C(t) = v_s(t)$

$$v_o(t) = -RC \frac{dv_s}{dt}$$

$$v_o(t) = -(2000\Omega)(10^{-7} \text{ F}) \frac{d[2V \sin(500t)]}{dt}$$

$$v_o(t) = (-0.2\text{ms})(2V)(500)\cos(500t)$$

$$v_o(t) = -0.2V \cos(500t) \quad \text{when } t > 0\text{s}$$

$$v_o(t) = 0V \quad \text{when } t < 0\text{s}$$

# Cosine to Sine Conversion

$$v_o(t) = -0.2V \cos(500t)$$

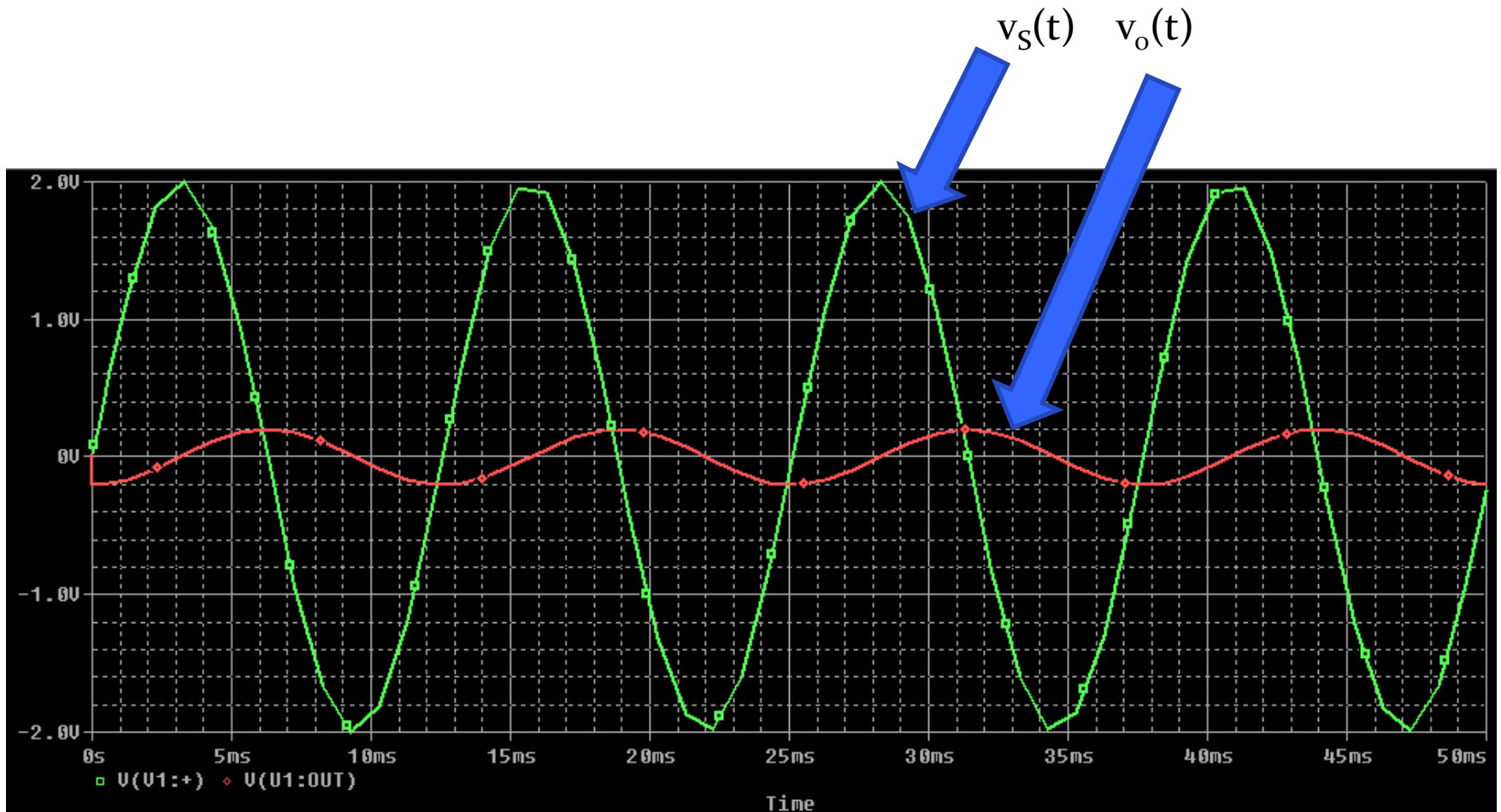
$$v_o(t) = -0.2V \sin(500t + 90^\circ)$$

$$v_o(t) = 0.2V \sin(500t + 90^\circ - 180^\circ)$$

$$v_o(t) = 0.2V \sin(500t - 90^\circ)$$

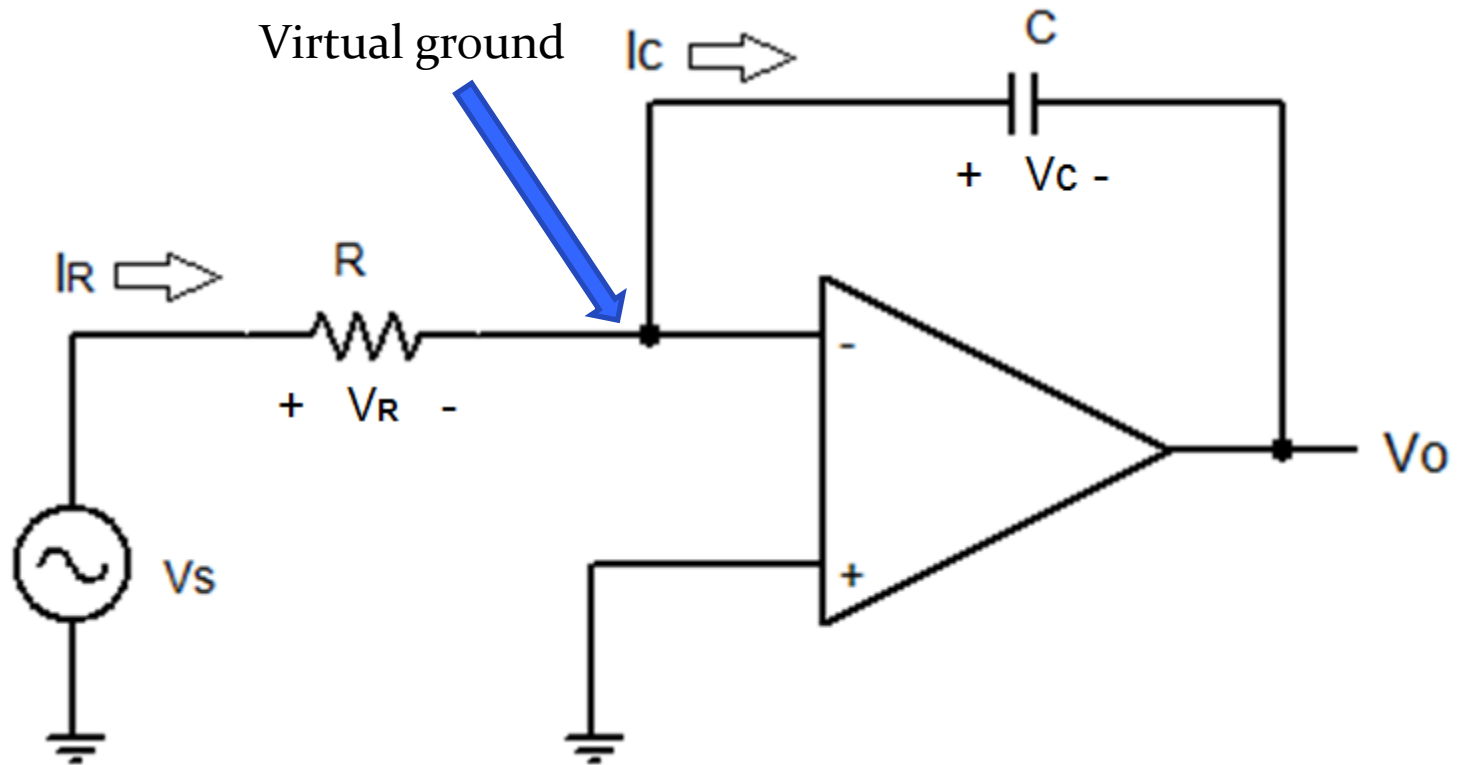
As  $v_s(t) = 2V \sin(500t)$ , the output voltage lags the input voltage by 90 degrees.

# PSpice Simulation



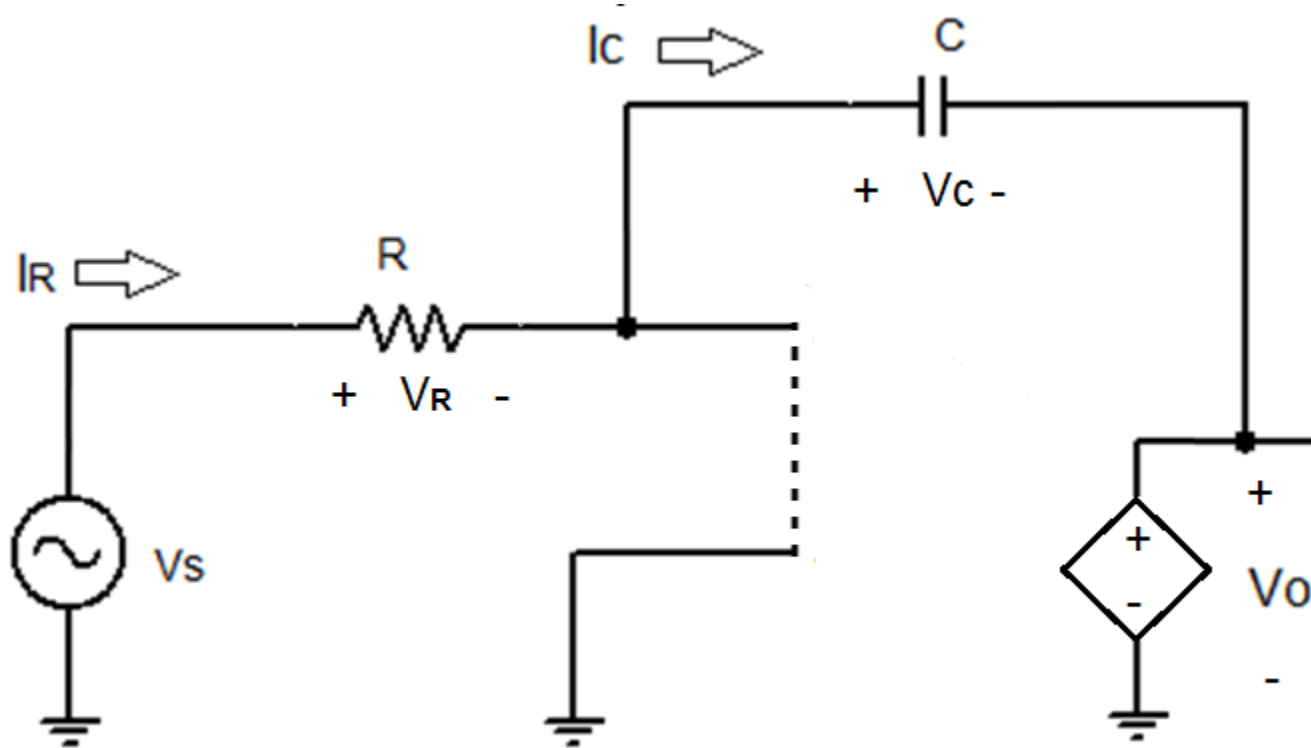
Shows the 90 degree phase shift as well as the attenuation.

# Integrator



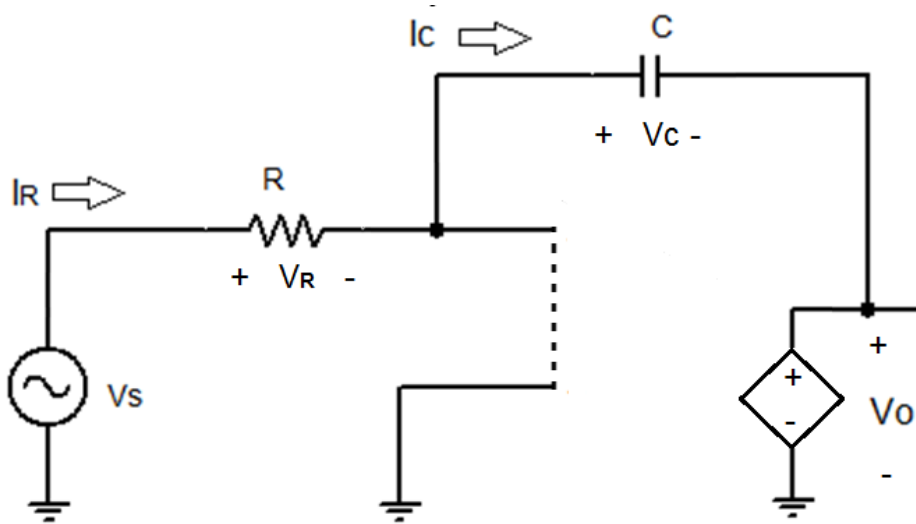


# Op Amp Model



# Integrator

- Op-Amp Model:



$$i_R = \frac{v_S(t) - v_1}{R} = \frac{v_S(t)}{R}$$

$$i_C = C \frac{dv_C}{dt}$$

$$v_C(t) = v_1 - v_o(t) = -v_o(t)$$

$$i_R - i_C = 0 \text{ mA}$$

$$\frac{v_S(t)}{R} - C \frac{d[-v_o(t)]}{dt} = 0$$

$$\frac{dv_o(t)}{dt} + \frac{v_S(t)}{RC} = 0$$

$$v_o(t_2) = \frac{-1}{RC} \int_{t_1}^{t_2} v_S(t) dt + v_o(t_1)$$

# Example 03

- Let  $R = 25 \text{ k}\Omega$ ,  $C = 5 \text{ nF}$ ,  $v_s(t) = 3V \sin\left(6.24k \frac{\text{rad}}{\text{s}} t\right)$  at  $t=0\text{s}$

$$V_o(t_2) = \frac{-1}{RC} \int_{t_1}^{t_2} V_{in}(t) dt + V_o(t_1)$$

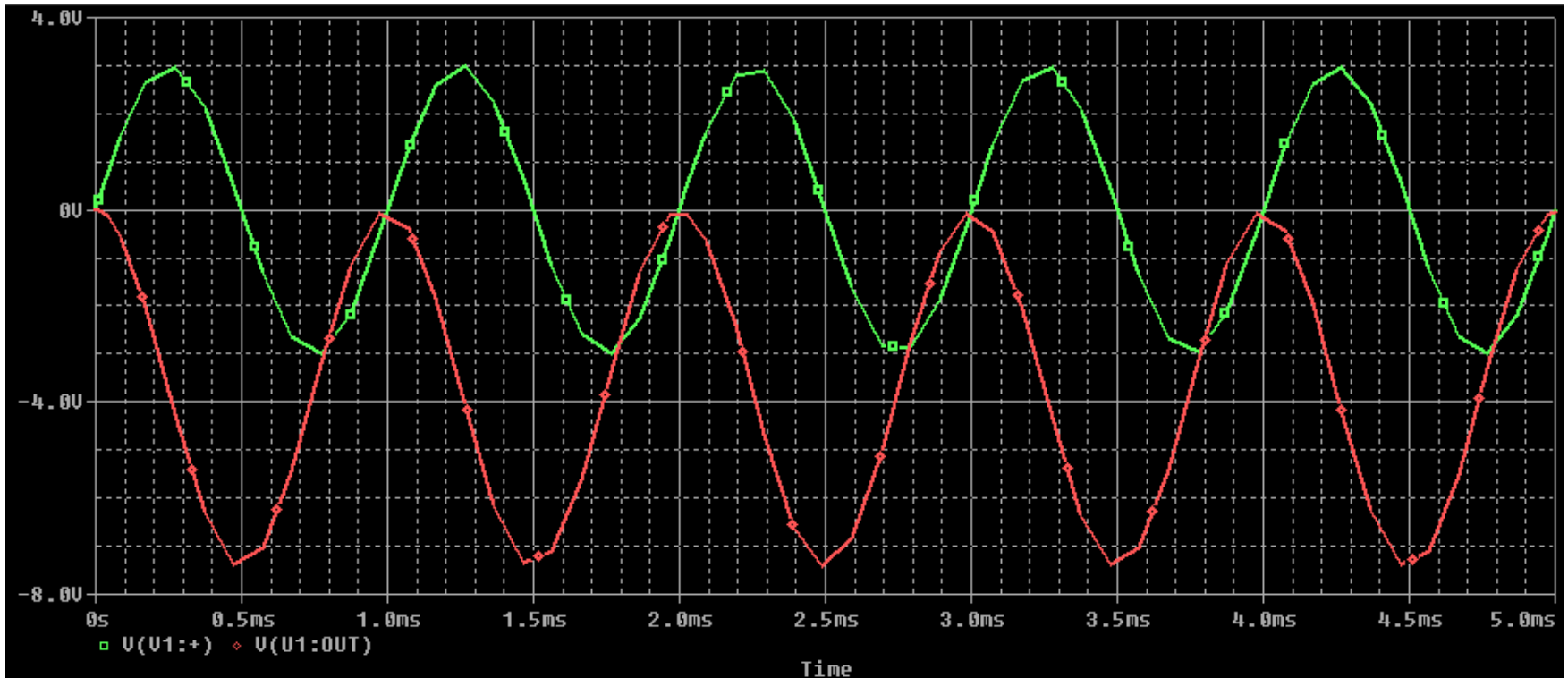
$$V_o(t_2) = \frac{-1}{25k\Omega(5nF)} \int_{t_1}^{t_2} 3V \sin\left(6.24k \frac{\text{rad}}{\text{s}} t\right) dt$$

$$V_o(t_2) = 3.85V \cos\left(6.24k \frac{\text{rad}}{\text{s}} t\right) \Big|_{t_1}^{t_2} + V_o(t_1)$$

$$V_o(t_2) = 3.85V \sin\left(6.24k \frac{\text{rad}}{\text{s}} t_2 + 90^\circ\right) - 3.85V \text{ when } t_1 = 0\text{s}$$

since  $v_o(t) = -v_C(t)$  and the voltage across a capacitor can't be discontinuous.

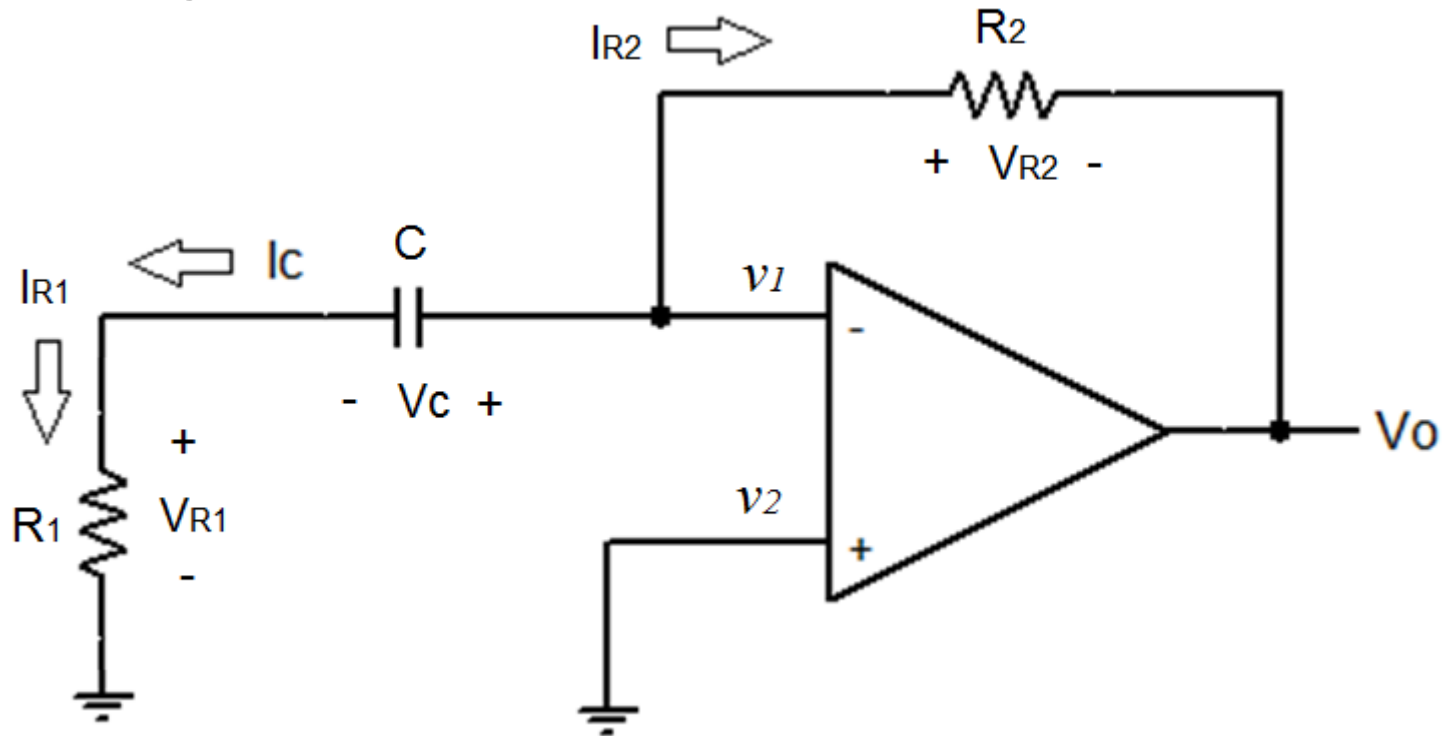
# PSpice Simulation



Shows that the output voltage leads the input voltage by +90 degree and the voltage offset due to the  $V_o(t_1)$  term.

# Example 04...

Initial Charge on Capacitor



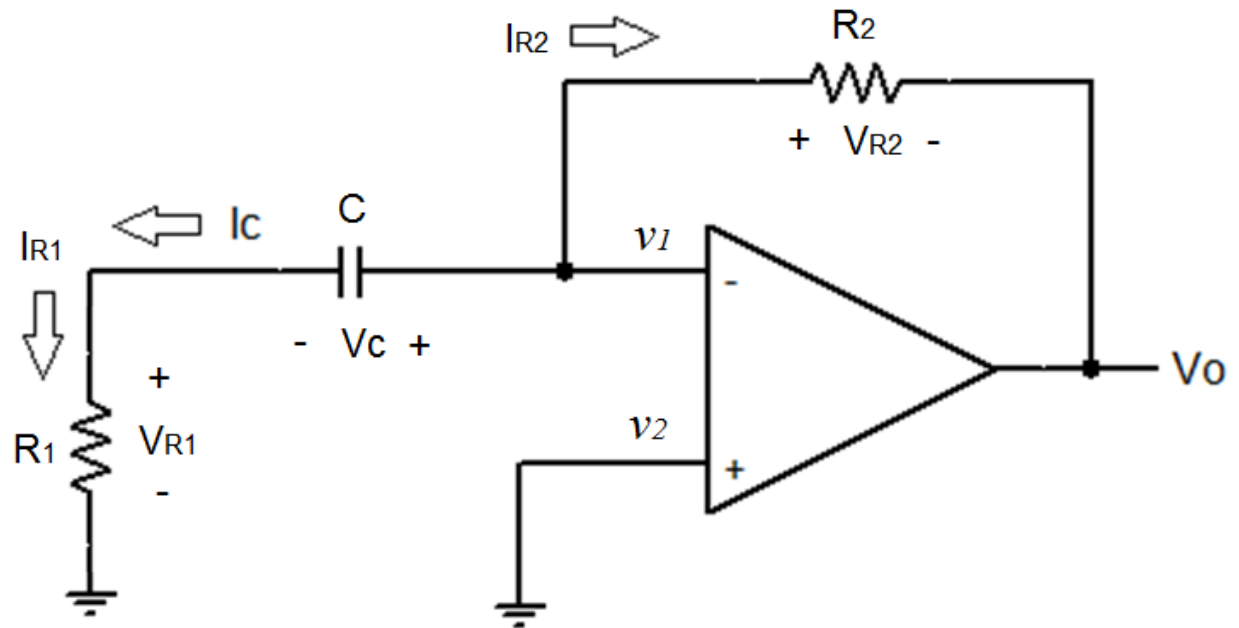
# ...Example 04...

If there is an initial charge that produces a voltage on the capacitor at some time,  $t_0$ :

The voltage on the negative input of the op amp is:

$$V_1 = V_C + V_{R1}$$

$$V_1 = V_2 = 0V$$



## ...Example 04...

The current flowing through  $R_1$  is the same current flowing through C.

$$i_C(t) = C \frac{dv_C(t)}{dt}$$

$$i_{R1}(t) = \frac{V_{R1}}{R_1} = \frac{[v_1 - v_C(t)]}{R_1} = \frac{[0V - v_C(t)]}{R_1} = -\frac{v_C(t)}{R_1}$$

$$\text{at } t = t_o, i_R(t_o) = -\frac{v_C(t_o)}{R_1}$$

$$\text{as } t \rightarrow \infty, v_C(t) \rightarrow 0V, i_C(t) \rightarrow 0mA$$

# ...Example 04...

$$i_C(t) - i_{R1}(t) = 0$$

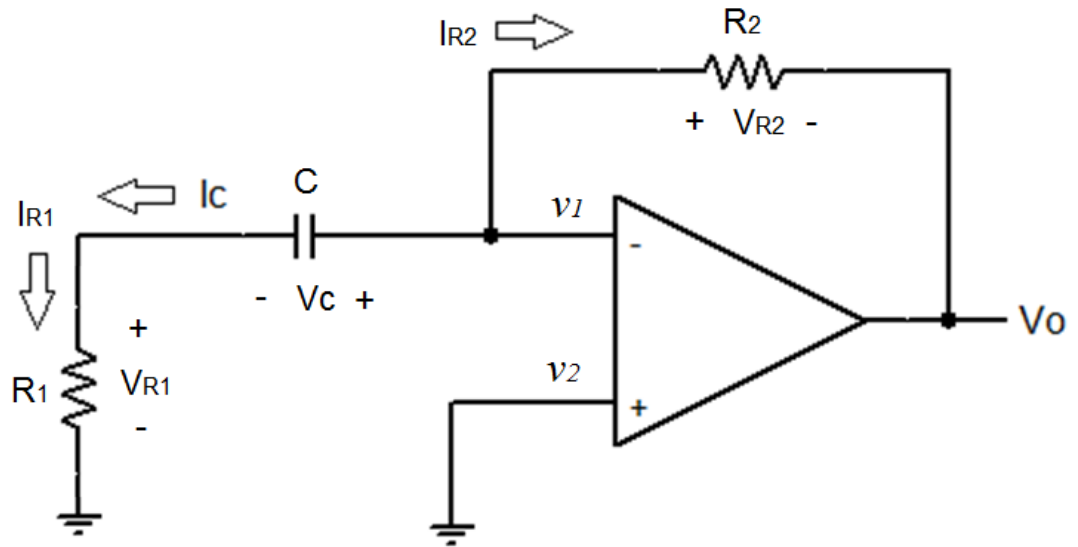
$$C \frac{dv_C(t)}{dt} + \frac{v_C(t)}{R_1} = 0$$

$$\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R_1 C} = 0$$

$$v_C(t) = v_C(t_o) e^{-\frac{t-t_o}{R_1 C}}$$

$$i_C = C \frac{dv_C(t)}{dt} = -\frac{1}{R_1} v_C(t_o) e^{-\frac{t-t_o}{R_1 C}}$$

$R_1 C$  is the time constant,  $\tau$ .



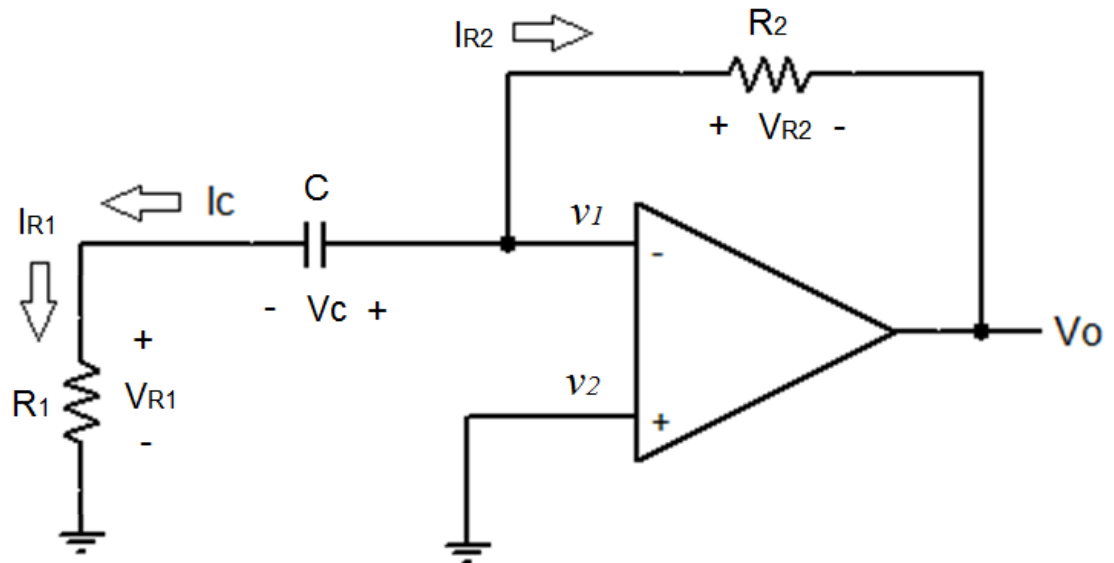


# ...Example 04

$$i_{R2} = -i_C$$

$$i_{R2} = \frac{0V - v_o(t)}{R_2} = -\frac{v_o(t)}{R_2}$$

$$v_o(t) = \frac{R_2}{R_1} v_C(t_o) e^{-\frac{t-t_o}{R_1 C}}$$



# Summary

- Differentiator and integrator circuits are 1<sup>st</sup> order op amp circuits.
  - When the C is connected to the input of the op amp, the circuit is a differentiator.
    - If the input voltage is a sinusoid, the output voltage lags the input voltage by 90 degrees.
    - The output voltage may be discontinuous.
  - When the C is connected between the input and output of the op amp, the circuit is an integrator.
    - If the input voltage is a sinusoid, the output voltage leads the input voltage by 90 degrees.
    - The output voltage must be continuous.