BLM1612 Circuit Theory

Nodal and Mesh Analysis

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Nodal (or "Node-Voltage") Analysis

- a general, powerful method for methodical linear circuit analysis
- based on **Kirchhoff's Current Law**
- allows us to analyze circuits for **any number of nodes**, *N*
- requires us to solve a system of (at least) *N* – 1 simultaneous equations

<u>Analysis Steps</u>

(1) Choose a reference node (usually ground or the bottom node) to have a voltage of *zero*.

(2) Assign a unique voltage variable to each node that is *not* the reference (v_1, v_2, v_3, … v_{N-1}).

(3) For voltage sources, assign a current (i_1, i_2, …) through each and write the value of the source in terms of node voltages.

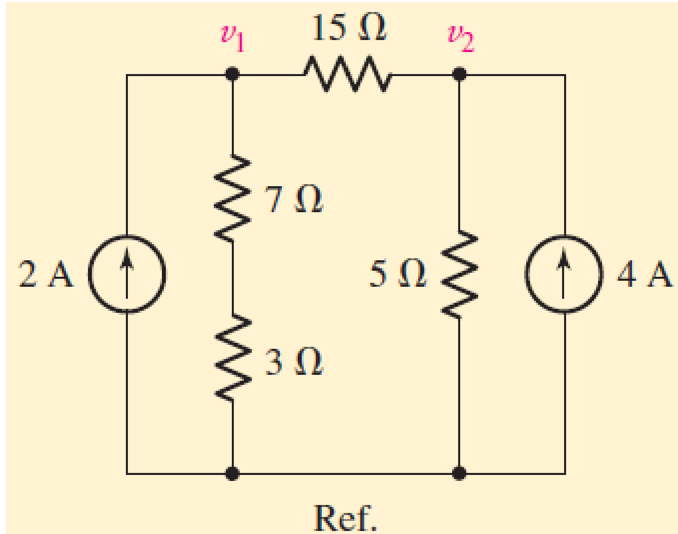
Write a KCL equation *at every node* (except for the reference) in terms of voltage differences (divided by $R_1 \ldots R_N$) and all V or I sources.

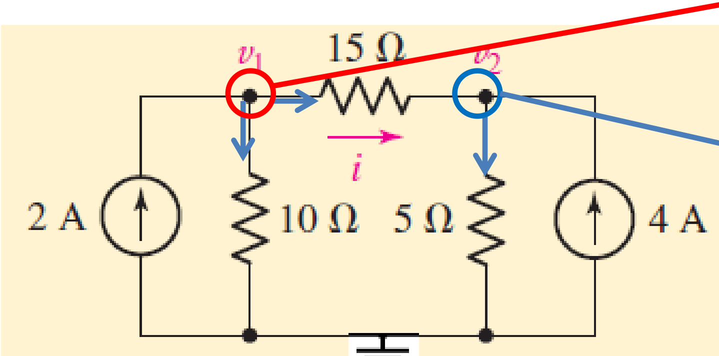
For dependent sources, write an equation that governs each in terms of node voltages.

(4) Solve the $N - 1$ node equations + source equations simultaneously.

Example (pg 82, #4.1)

- Determine the current flowing left to right through the 15 ohms resistor





$$2 = \frac{v_1}{10} + \frac{v_1 - v_2}{15}$$

$$4 = \frac{v_2}{5} + \frac{v_2 - v_1}{15}$$

$$5v_1 - 2v_2 = 60$$
$$-v_1 + 4v_2 = 60$$

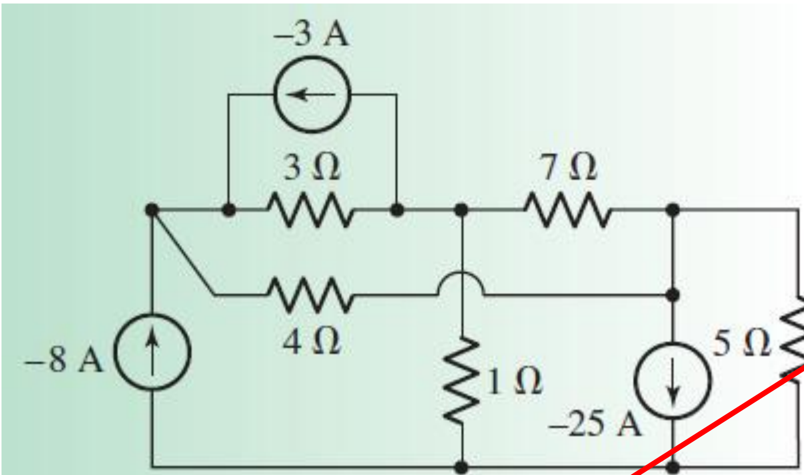
$v_1 = 20\text{ V}$ $v_2 = 20\text{ V}$

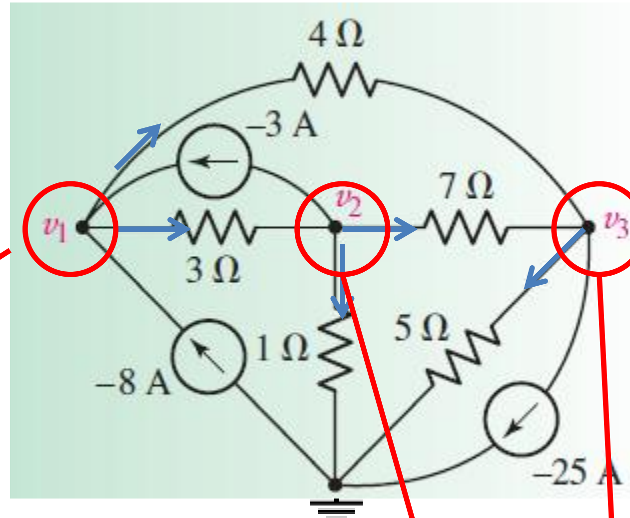
$v_1 - v_2 = 0$

zero current is flowing through the 15 Ω

Example (pg 83, #4.2)

- Determine the nodal voltages for the circuit.





$$-8 - 3 = \frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4}$$

$$0.5833v_1 - 0.3333v_2 - 0.25v_3 = -11$$

$$-(-3) = \frac{v_2 - v_1}{3} + \frac{v_2}{1} + \frac{v_2 - v_3}{7}$$

$$-0.3333v_1 + 1.4762v_2 - 0.1429v_3 = 3$$

$$-(-25) = \frac{v_3}{5} + \frac{v_3 - v_2}{7} + \frac{v_3 - v_1}{4}$$

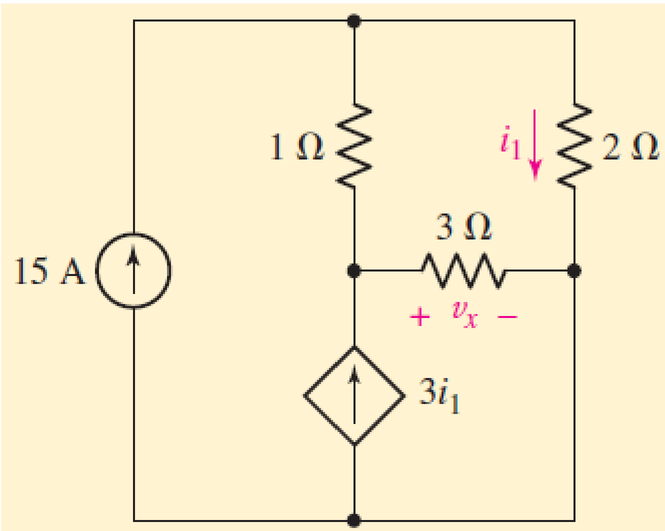
$$-0.25v_1 - 0.1429v_2 + 0.5929v_3 = 25$$

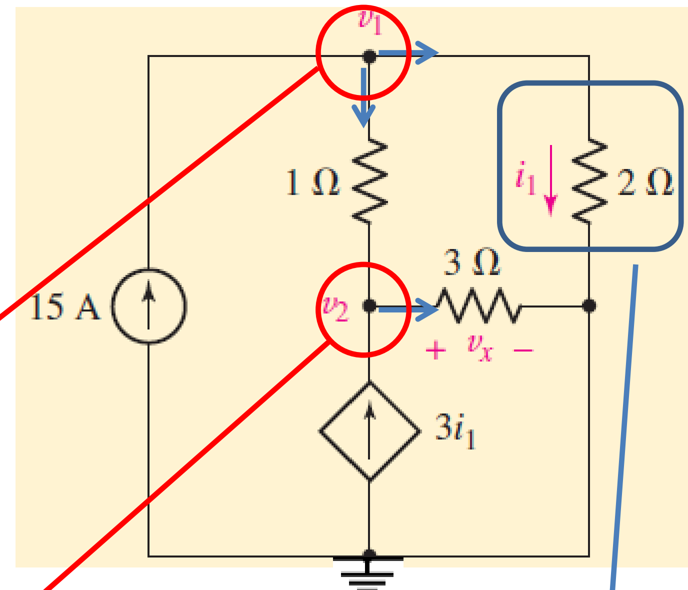
$$\begin{bmatrix} 0.5833 & -0.3333 & -0.25 \\ -0.3333 & 1.4762 & -0.1429 \\ -0.25 & -0.1429 & 0.5929 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 3 \\ 25 \end{bmatrix}$$

$v_1 = 5.412$ V $\quad v_2 = 7.736$ V $\quad v_3 = 46.32$ V

Example (pg 86, #4.3)

- Determine the power supplied by the dependent source.





$$15 = \frac{v_1 - v_2}{1} + \frac{v_1}{2}$$

$$3i_1 = \frac{v_2 - v_1}{1} + \frac{v_2}{3}$$

$$i_1 = \frac{v_1}{2}$$

$$3v_1 - 2v_2 = 30$$

$$-15v_1 + 8v_2 = 0$$

we need an additional equation that relates i_1 *to one or more nodal voltages*

$$v_1 = -40\text{ V} \quad v_2 = -75\text{ V}$$

$$i_1 = 0.5v_1 = -20\text{ A}$$

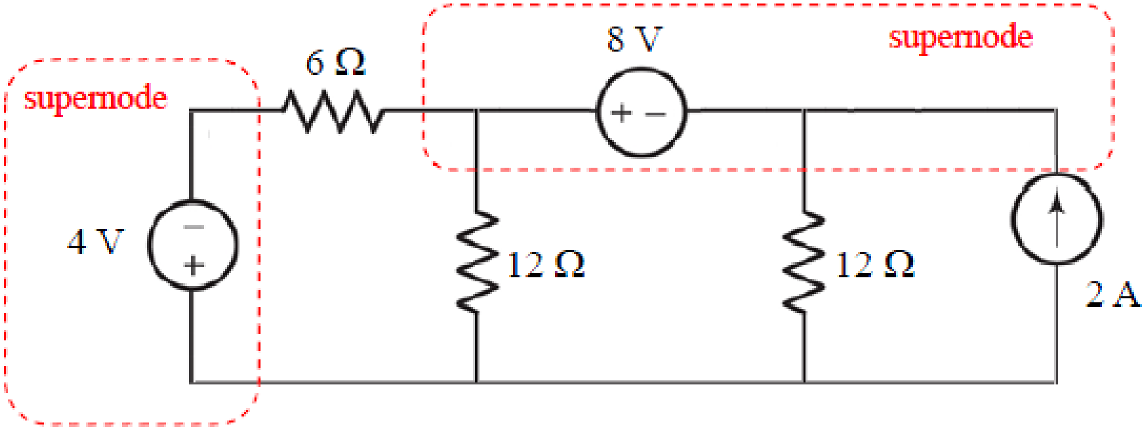
power absorbed by the dependent source

$$(3i_1)(v_2) = -(-60)(-75) = -4.5\text{ kW}$$

Actually 4.5 kW is supplied

Nodal Analysis with Supernodes

supernode:
a collection of multiple nodes separated by voltage sources



Analysis Steps

(1) Choose a reference node (usually ground or the bottom node) to have a voltage of *zero*.

(2) Assign a unique voltage variable to each node that is *not* the reference (v_1, v_2, v_3, … v_{N-1}).

(3) For independent & dependent voltage sources, identify a *supernode* and write the voltage across the supernode in terms of node voltages.

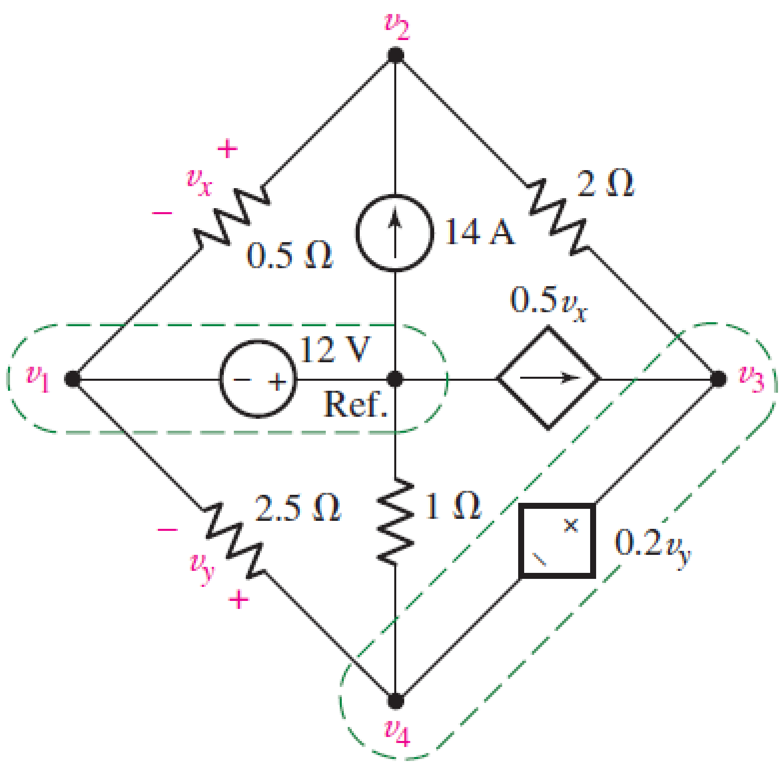
Write a KCL equation *at all N – 1 nodes **including the supernode*** (and not the reference, or a supernode which includes the reference).

(4) Solve the $N - 1$ node equations + source equations simultaneously.

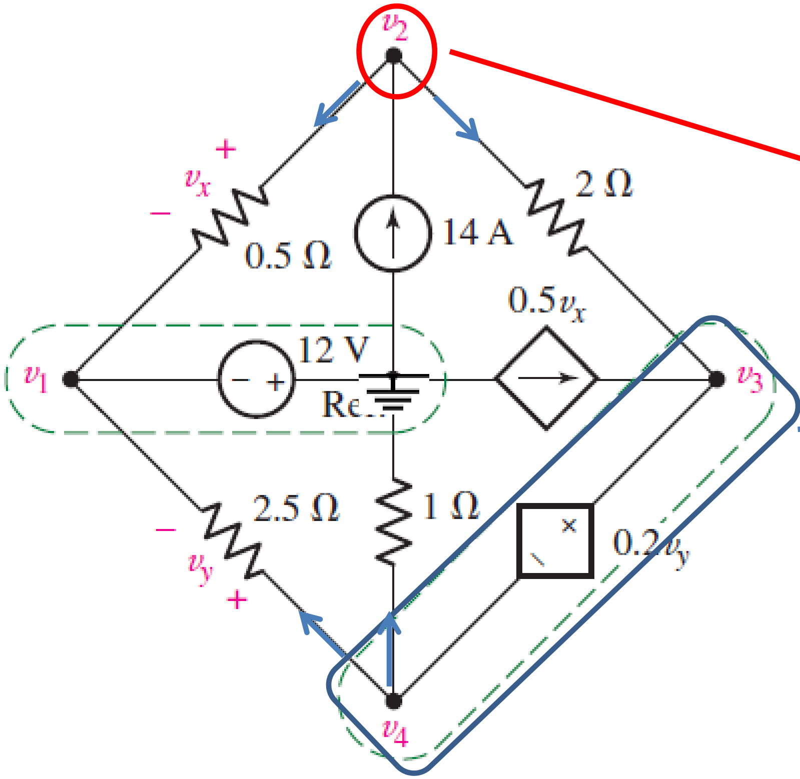
Example (page 91, #4.6)

Determine the node-to-reference voltages in the circuit provided.

- identify the nodes & supernodes
- write KCL at each node (except the reference)



Example (page 91, #4.6)



$$v_1 = -12 \text{ V.}$$

$$\frac{v_2 - v_1}{0.5} + \frac{v_2 - v_3}{2} = 14$$

$$0.5v_x = \frac{v_3 - v_2}{2} + \frac{v_4}{1} + \frac{v_4 - v_1}{2.5}$$

When we relate the source voltages to the node voltages

$$v_3 - v_4 = 0.2v_y$$

$$0.2v_y = 0.2(v_4 - v_1)$$

When we express the dependent current source in terms of the assigned variables

$$0.5v_x = 0.5(v_2 - v_1)$$

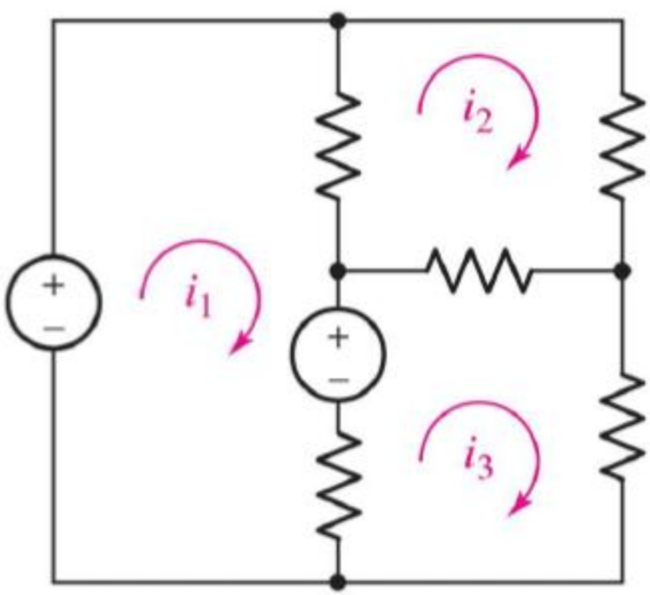
$$\begin{aligned}
-2v_1 + 2.5v_2 - 0.5v_3 \quad &= \quad 14 \\
0.1v_1 - \quad v_2 + 0.5v_3 + 1.4v_4 &= \quad 0 \\
v_1 \quad &= -12 \\
0.2v_1 \quad + \quad v_3 - 1.2v_4 &= \quad 0
\end{aligned}$$

$v_1 = -12$ V, $v_2 = -4$ V, $v_3 = 0$ V, and $v_4 = -2$ V.

Mesh (Current) Analysis

- another powerful method for methodical linear circuit analysis
- based on **Kirchhoff's Voltage Law**
- allows us to analyze circuits for **any number of mesh currents**, *M*
- **mesh** = a loop that does not contain any other loops

  mesh current = flows only around the *perimeter* of a mesh



3 meshes,
3 mesh currents

Mesh (Current) Analysis

- another powerful method for methodical linear circuit analysis
- based on **Kirchhoff's Voltage Law**
- allows us to analyze circuits for **any number of mesh currents**, *M*
- **mesh** = a loop that does not contain any other loops

  mesh current = flows only around the *perimeter* of a mesh

Analysis Steps

(1) Draw a mesh current for each mesh. (Clockwise is standard but not required.)

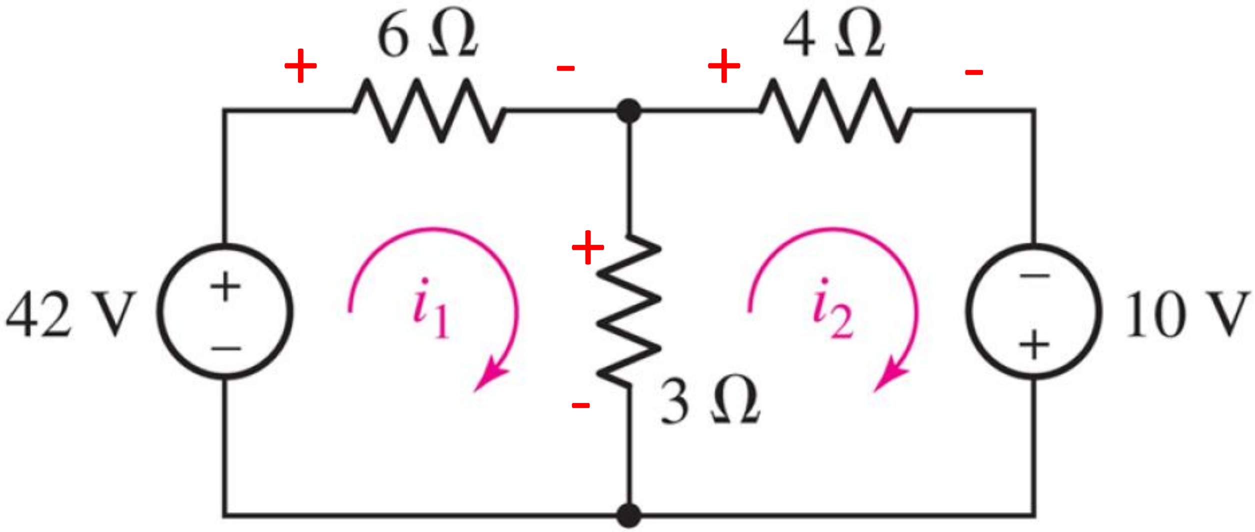
(2) Write a KVL equation for each mesh. Employ all necessary currents for each term.

(3) Introduce a voltage variable for each independent or dependent *current* source.

(4) Express additional unknowns (e.g. dependent V/I) in terms of mesh currents.

(5) Solve the simultaneous equations (*M* meshes + dependent source equations).

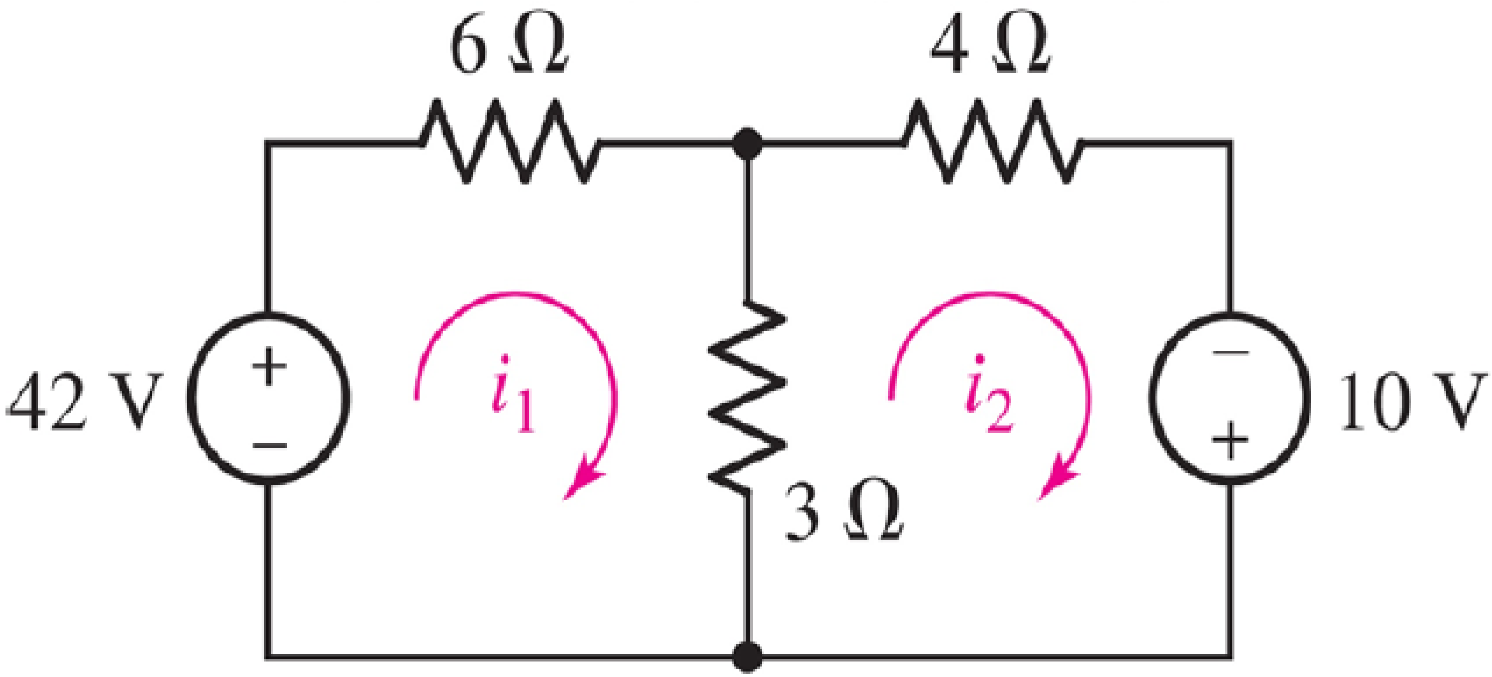
Writing Mesh Equations



$$-42 + 6i_1 + 3(i_1 - i_2) = 0$$

$$3(i_2 - i_1) + 4i_2 - 10 = 0$$

Writing Mesh Equations



$$9i_1 - 3i_2 = 42$$

$$-3i_1 + 7i_2 = 10$$

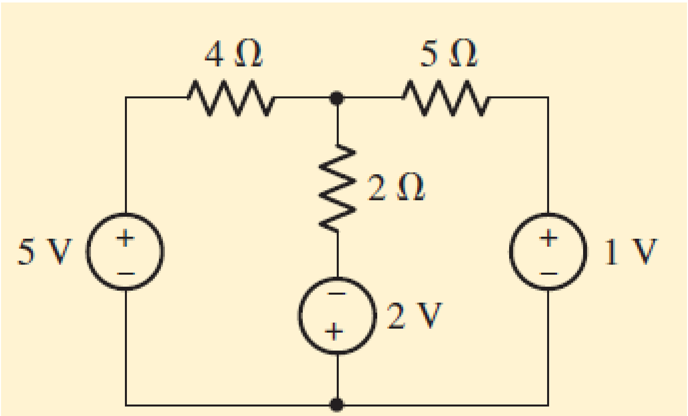
$$\begin{bmatrix} 9 & -3 \\ -3 & 7 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 42 \\ 10 \end{bmatrix} \qquad \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

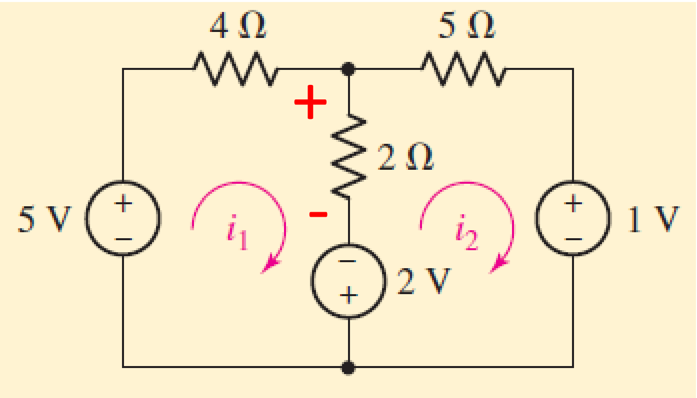
The current through the 6-Ω resistor is 6 A.

The current through the 3-Ω resistor is $(i_1 - i_2) = 2$ A

Example (page 94, #4.7)

- Determine the power supplied by the 2 V source





mesh 1

$$-5 + 4i_1 + 2(i_1 - i_2) - 2 = 0$$

mesh 2

$$+2 + 2(i_2 - i_1) + 5i_2 + 1 = 0$$

$$6i_1 - 2i_2 = 7$$

$$-2i_1 + 7i_2 = -3$$

$$i_1 = \frac{43}{38} = 1.132 \text{ A}$$

$$i_2 = -\frac{2}{19} = -0.1053 \text{ A}$$

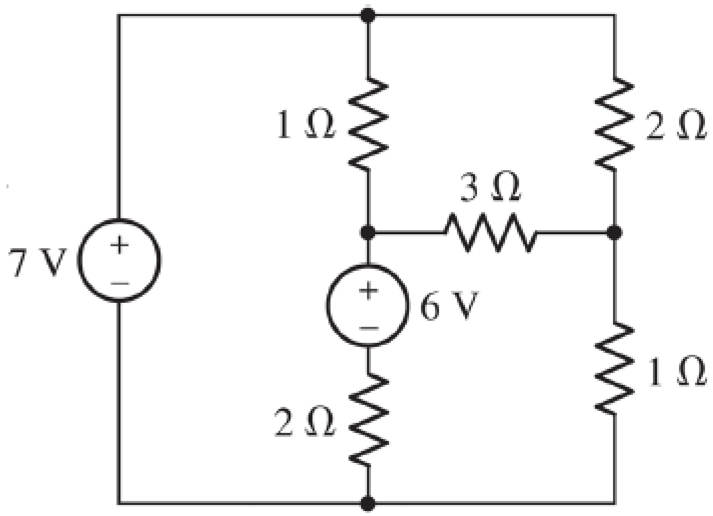
Power absorbed by the 2 V source

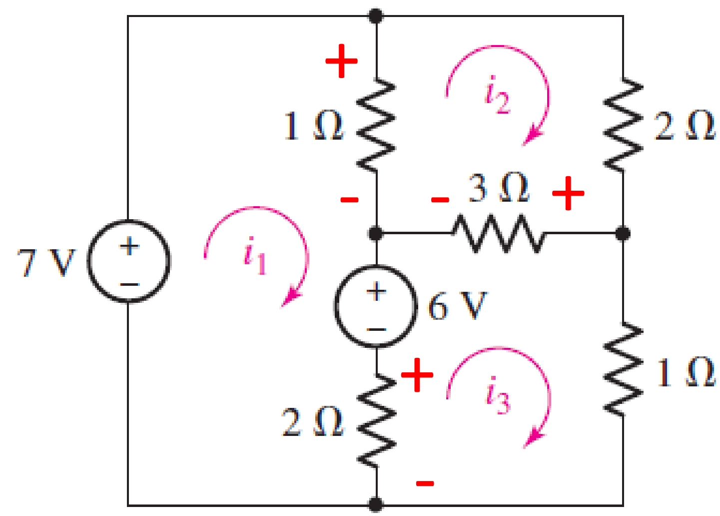
$$-(2)(1.237) = -2.474 \text{ W}$$

Actually 2.474 W is supplied

Example (page 95, #4.8)

- Use mesh analysis to determine the three mesh currents in the circuit





Mesh 1 $-7 + 1(i_1 - i_2) + 6 + 2(i_1 - i_3) = 0$

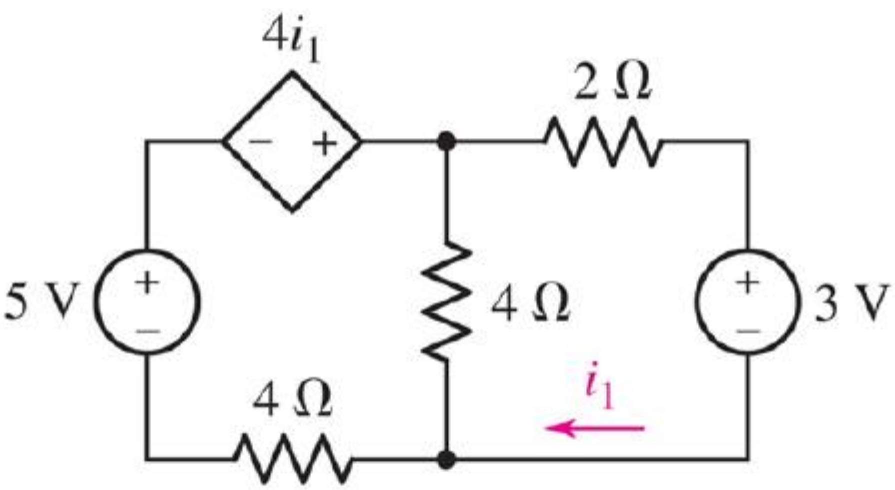
Mesh 2 $1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$

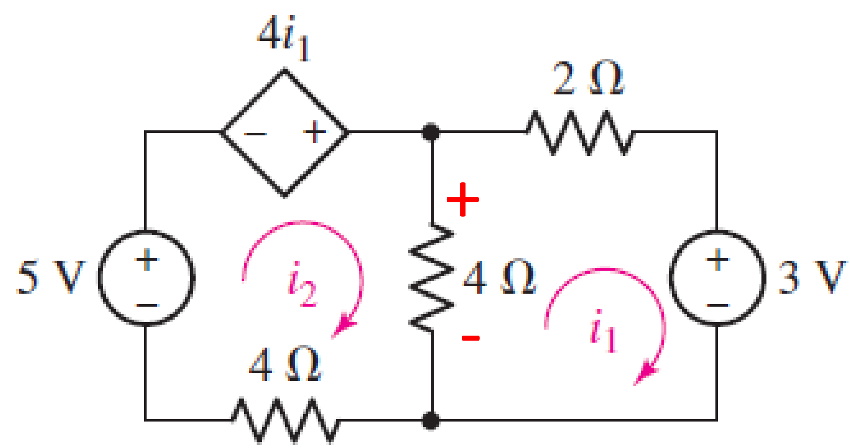
Mesh 3 $2(i_3 - i_1) - 6 + 3(i_3 - i_2) + 1i_3 = 0$

$i_1 = 3$ A, $i_2 = 2$ A, and $i_3 = 3$ A.

Example (page 96, #4.9)

- Determine the current i_1 in the circuit





Left Mesh

$$-5 - 4i_1 + 4(i_2 - i_1) + 4i_2 = 0$$

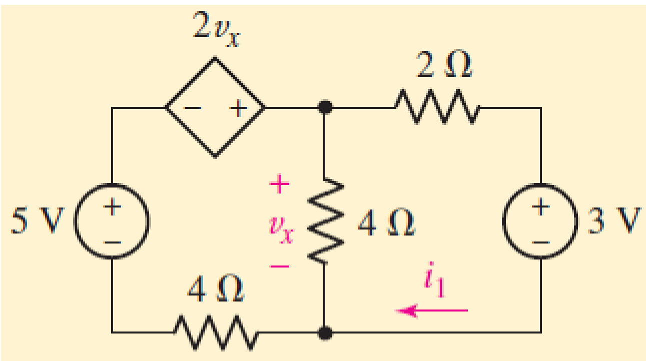
Right Mesh

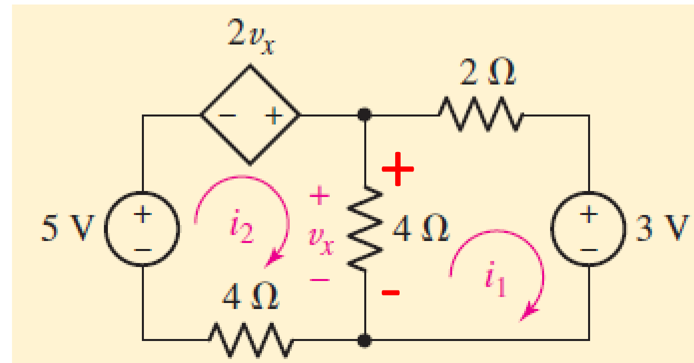
$$4(i_1 - i_2) + 2i_1 + 3 = 0$$

$$i_2 = 375 \text{ mA, so } i_1 = -250 \text{ mA}$$

Example (page 97, #4.10)

- Determine the current i_1 in the circuit





Left Mesh $-5 - 2v_x + 4(i_2 - i_1) + 4i_2 = 0$

Right Mesh $4(i_1 - i_2) + 2i_1 + 3 = 0$

We need to construct an equation for v_x *in terms of mesh currents*

$$v_x = 4(i_2 - i_1)$$

$4i_1 = 5$ $\quad$ $i_1 = 1.25\ \text{A}$

Mesh Analysis with Supermeshes

supermesh = a mesh that contains multiple meshes with a shared current source

For **nodal** analysis, we joined nodes near a **voltage** source. → supernode
For **mesh** analysis, we join meshes near a **current** source. → supermesh

→ Reduces the number of simultaneous equations by the number of current sources.

Analysis Steps

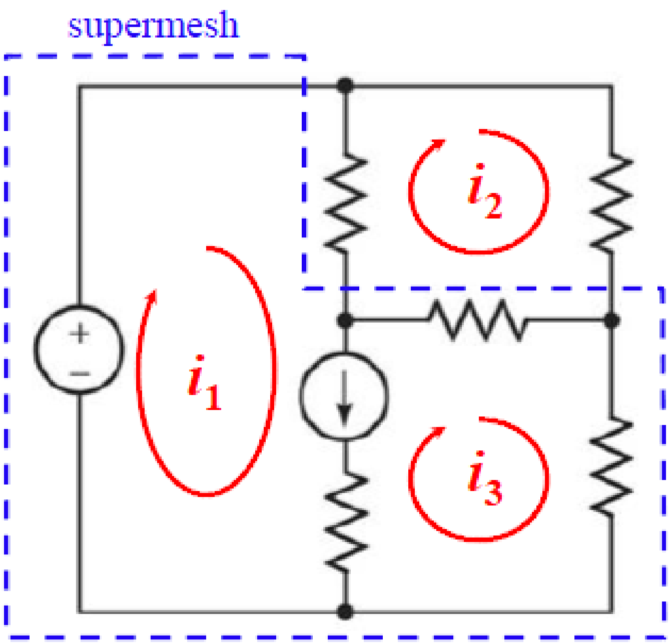
(1) Draw a mesh current for each mesh.

(2) Identify supermeshes.

(3) Write KVL around each supermesh, then KVL for each mesh that is not part of a supermesh.

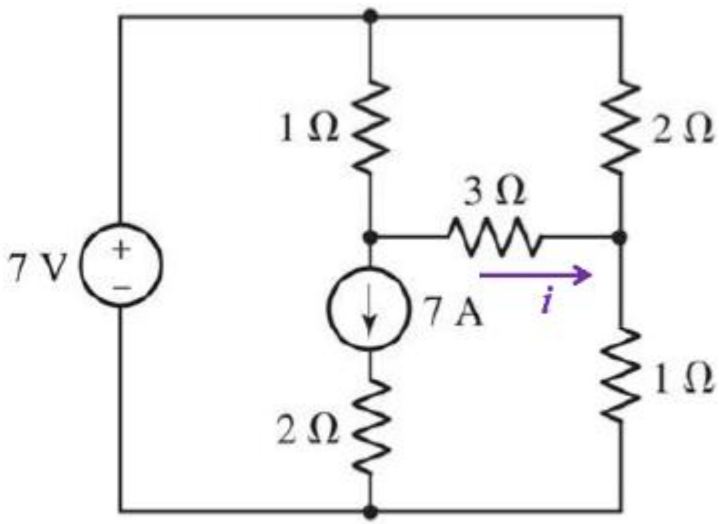
(4) Express additional unknowns (dependent V/I) in terms of mesh currents.

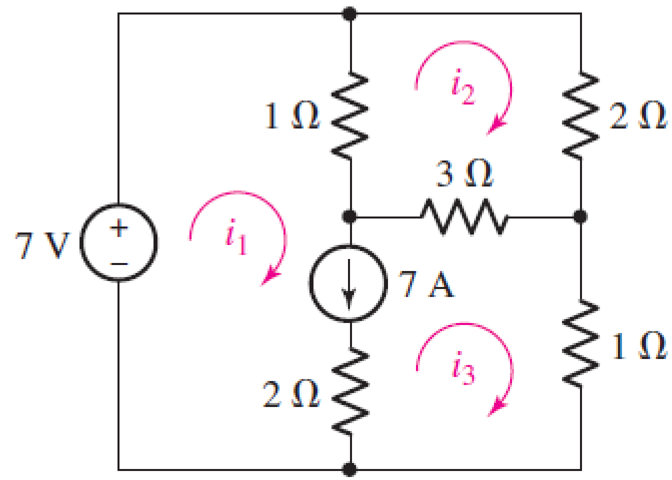
(5) Solve the simultaneous equations.



Example (page 98, #4.11)

Determine the current *i as labeled in the circuit.*





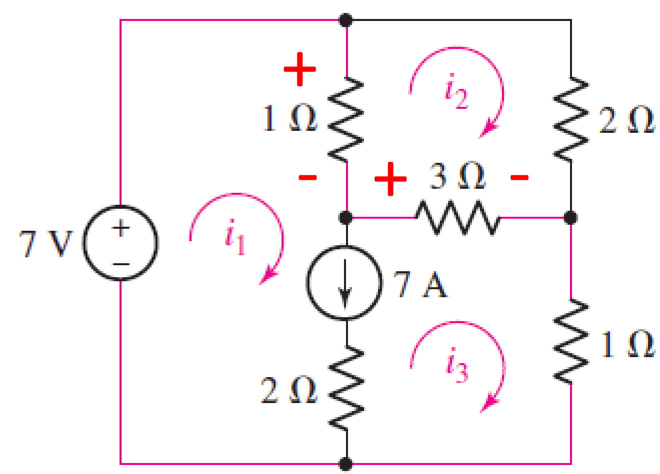
Supermesh

$$-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3 = 0$$

Mesh 2

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

$$-i_1 + 6i_2 - 3i_3 = 0$$



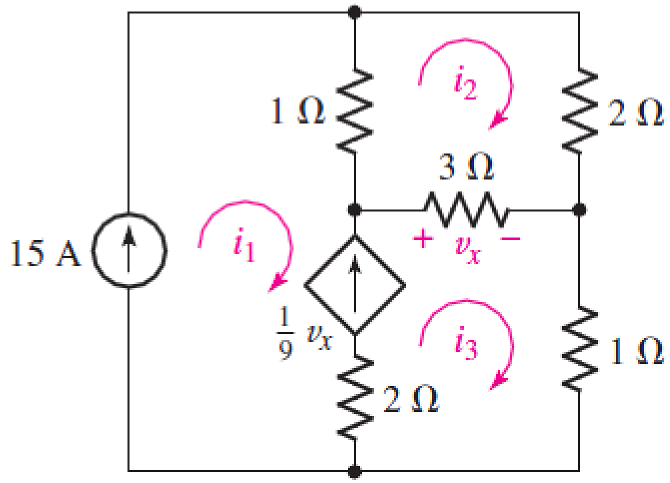
independent source current is related to the mesh currents

$$i_1 - i_3 = 7$$

$i_1 = 9$ A, $i_2 = 2.5$ A, $i_3 = 2$ A

Example (page 99, #4.12)

- Evaluate the three unknown currents in the circuit



Mesh 1 $\quad i_1 = 15 \text{ A}$

- one of the two mesh currents relevant to the dependent current source, there is no need to write a supermesh equation about meshes 1 and 3

$$\frac{v_x}{9} = i_3 - i_1 = \frac{3(i_3 - i_2)}{9}$$

$$-i_1 + \frac{1}{3}i_2 + \frac{2}{3}i_3 = 0 \quad \text{or} \quad \frac{1}{3}i_2 + \frac{2}{3}i_3 = 15$$

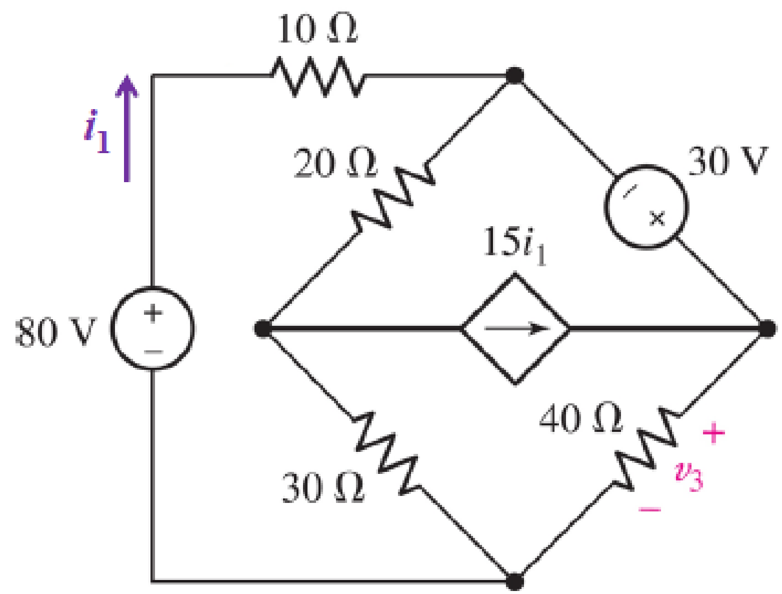
Mesh 2 $\quad 1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$

$$6i_2 - 3i_3 = 15$$

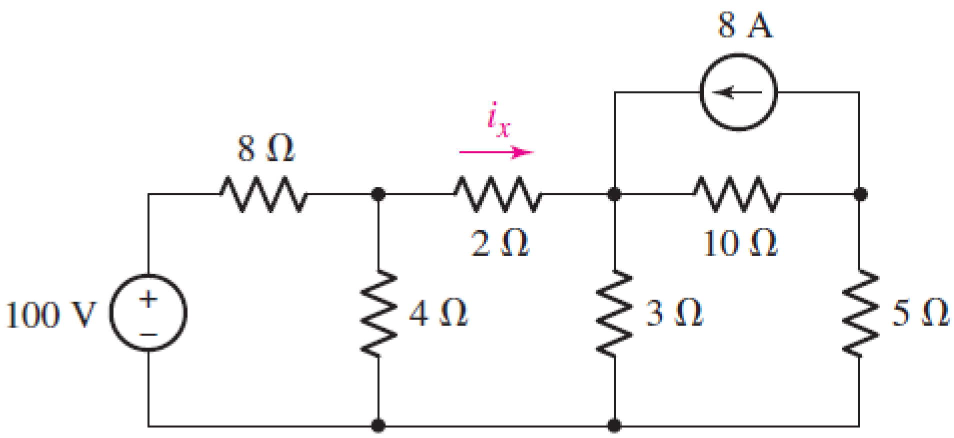
$i_1 = 15 \text{ A} \quad i_2 = 11 \quad i_3 = 17 \text{ A}$

Practice (page 100, #4.10)

- Determine v_3 *in the circuit*



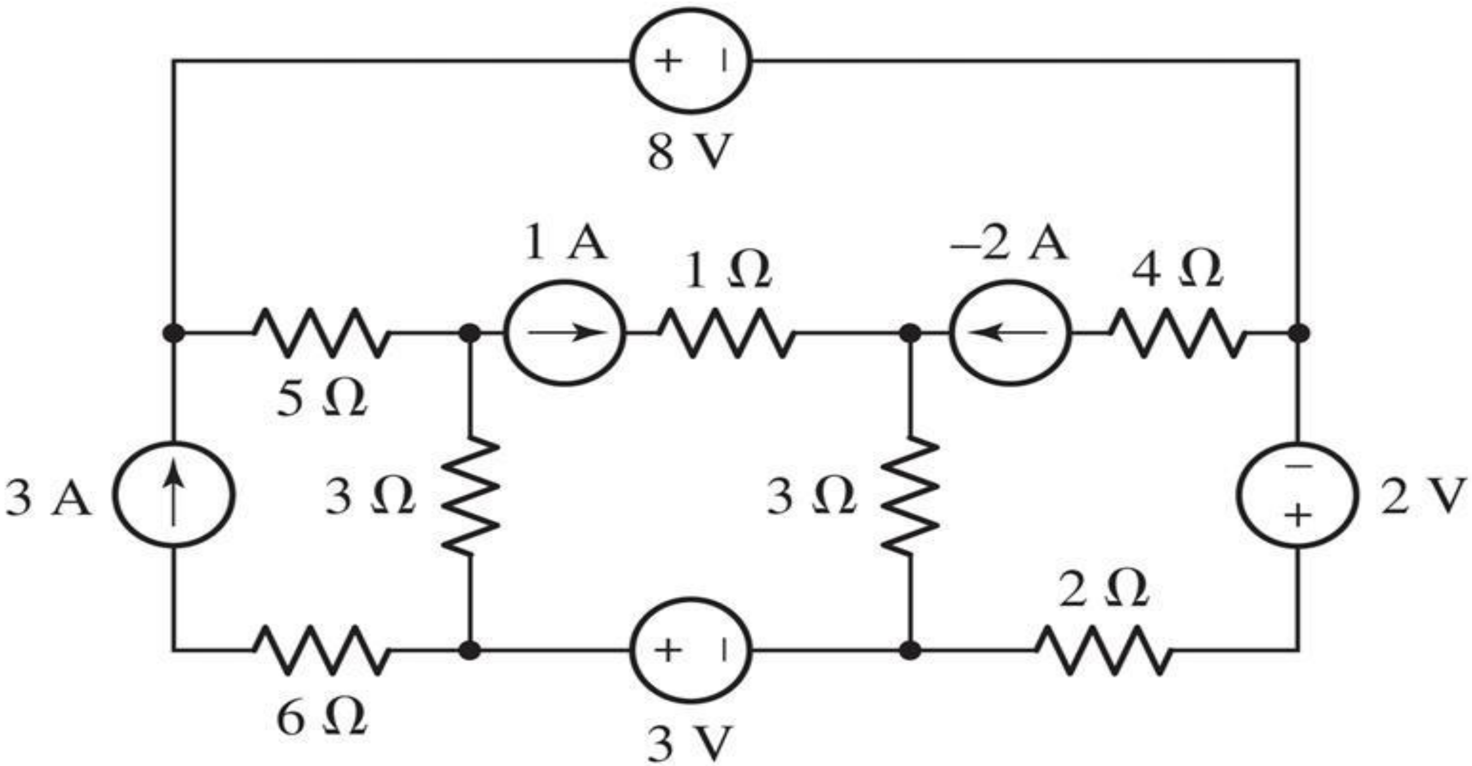
Nodal vs. Mesh Analysis: A Comparison



- A planar circuit with five nodes and four meshes. Determine the current i_x .

Nodal & Mesh Analysis

• Set up a complete, valid set of simultaneous equations to solve for the power absorbed by the 5-Ω resistor. You are not required to solve these equations.



Nodal & Mesh Analysis

• Set up a complete, valid set of simultaneous equations to solve for the power absorbed by the 3-Ω resistor. You are not required to solve these equations.