

BLM1612 - Circuit Theory

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Ohm's Law
Kirchhoff's Current Law (KCL)
Kirchhoff's Voltage Law (KVL)

Objectives of the Lecture

- Present Kirchhoff's Current and Voltage Laws.
- Demonstrate how these laws can be used to find currents and voltages in a circuit.
- Explain how these laws can be used in conjunction with Ohm's Law.

Resistivity, ρ

- Resistivity is a material property
 - Dependent on the number of free or mobile charges (usually electrons) in the material.
 - In a metal, this is the number of electrons from the outer shell that are ionized and become part of the ‘sea of electrons’
 - Dependent on the mobility of the charges
 - Mobility is related to the velocity of the charges.
 - It is a function of the material, the frequency and magnitude of the voltage applied to make the charges move, and temperature.

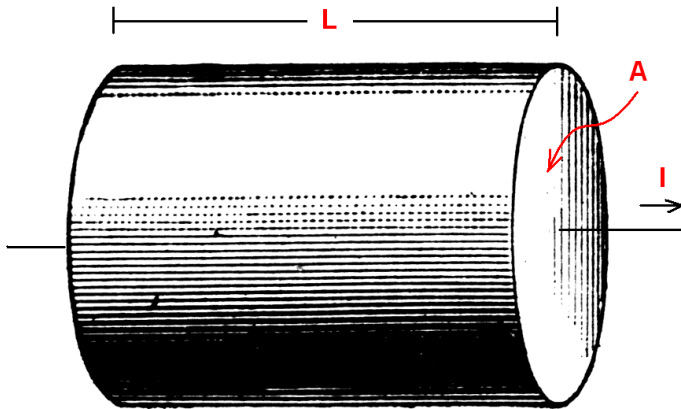
Resistivity of Common Materials at Room Temperature (300K)

Material	Resistivity ($\Omega\text{-cm}$)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon (Graphite)	4×10^{-5}	Conductor
Germanium	0.47	Semiconductor
Silicon	640	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

Resistance, R

- Resistance takes into account the physical dimensions of the material

$$R = \rho \frac{L}{A}$$



— where:

- L is the length along which the carriers are moving
- A is the cross sectional area that the free charges move through.

Ohm's Law

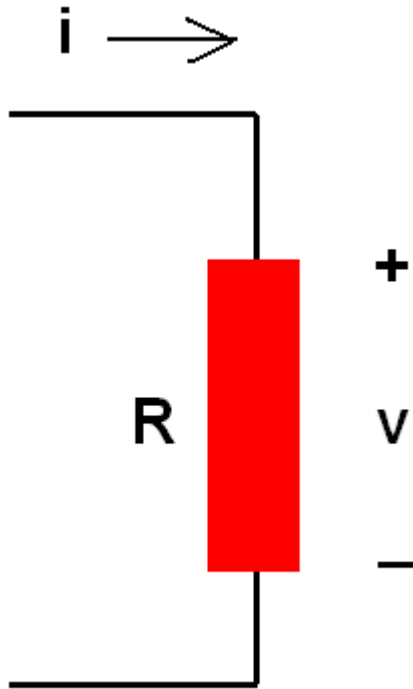
- Voltage drop across a resistor is proportional to the current flowing through the resistor

$$V = iR$$

Units: $V = A\Omega$

where $A = C/s$

Short Circuit



- If the resistor is a perfect conductor (or a short circuit)

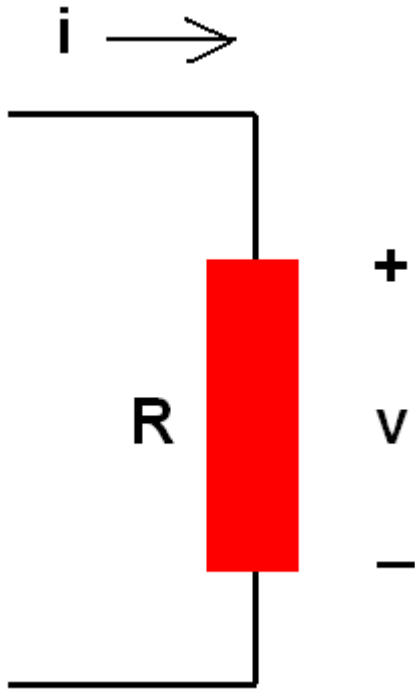
$$R = 0 \, \Omega,$$

- then

$$v = iR = 0 \, \text{V}$$

- no matter how much current is flowing through the resistor

Open Circuit



- If the resistor is a perfect insulator, $R = \infty \Omega$

- then

$$i = \lim_{R \rightarrow \infty} \frac{v}{R} = 0$$

- no matter how much voltage is applied to (or dropped across) the resistor.

Conductance, G

- Conductance is the reciprocal of resistance

$$G = R^{-1} = i/v$$

– Unit for conductance is S (siemens) or (mhos, Ω^{-1})

$$G = A\sigma/L$$

where σ is conductivity,

which is the inverse of resistivity, ρ

Power Dissipated by a Resistor

$$p = iv = i(iR) = i^2R$$

$$p = iv = (v/R)v = v^2/R$$

$$p = iv = i(i/G) = i^2/G$$

$$p = iv = (vG)v = v^2G$$

Power (con't)

- Since R and G are always real positive numbers
 - Power dissipated by a resistor is always positive
- The power consumed by the resistor is not linear with respect to either the current flowing through the resistor or the voltage dropped across the resistor
 - This power is released as heat. Thus, resistors get hot as they absorb power (or dissipate power) from the circuit.

Short and Open Circuits

- There is no power dissipated in a short circuit.

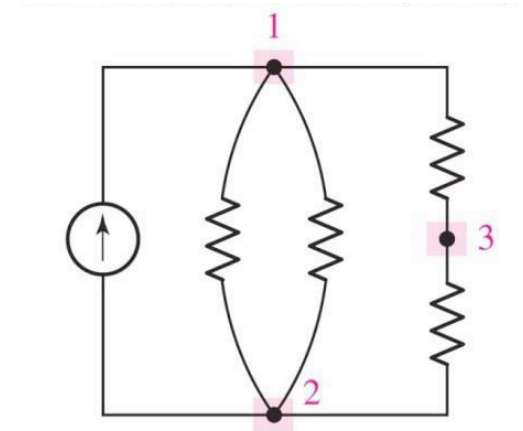
$$p_{sc} = v^2 R = (0V)^2 (0\Omega) = 0W$$

- There is no power dissipated in an open circuit.

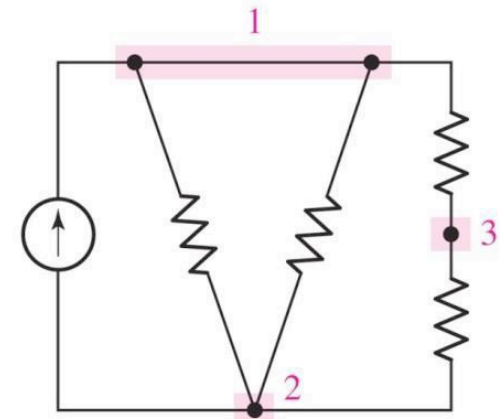
$$p_{oc} = i^2 / R = (0A)^2 / (\infty \Omega) = 0W$$

Circuit Terminology

- Node
 - point at which 2+ elements have a common connection
 - e.g., node 1, node 2, node 3
- Path
 - a route through a network, through nodes that never repeat
 - e.g., $1 \rightarrow 3 \rightarrow 2$, $1 \rightarrow 2 \rightarrow 3$
- Loop
 - a path that starts & ends on the same node
 - e.g., $3 \rightarrow 1 \rightarrow 2 \rightarrow 3$
- Branch
 - a single path in a network; contains one element and the nodes at the 2 ends
 - e.g., $1 \rightarrow 2$, $1 \rightarrow 3$, $3 \rightarrow 2$



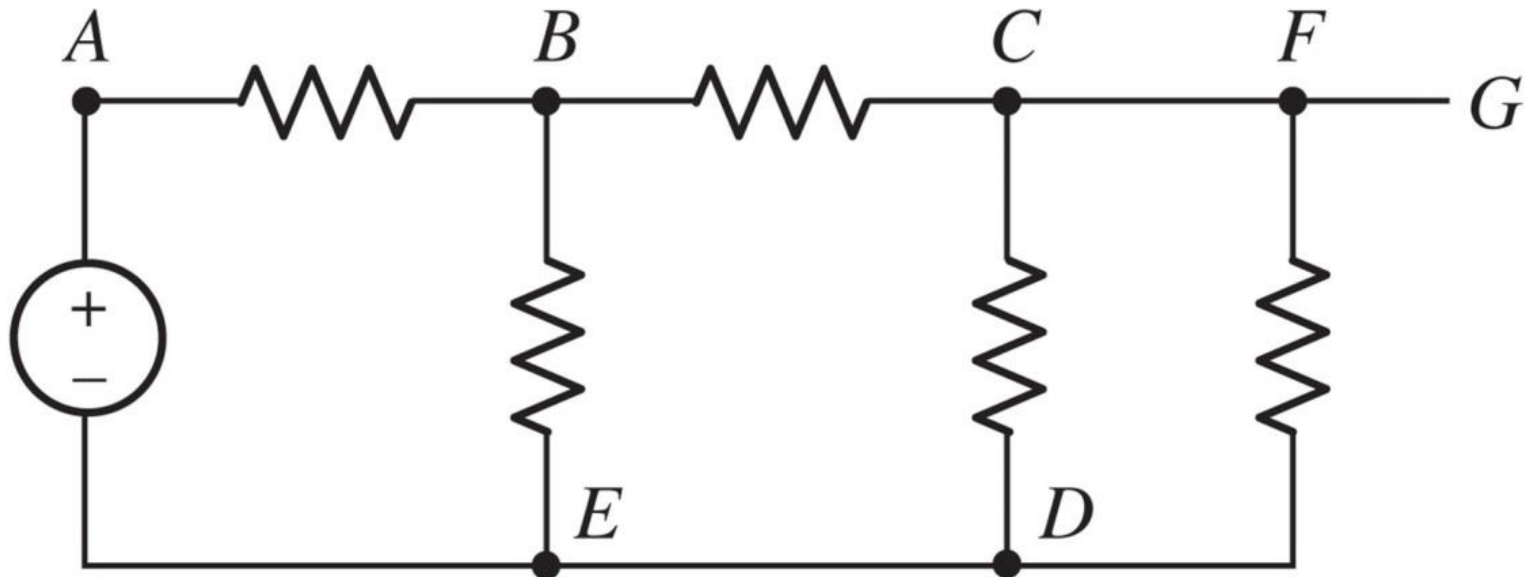
(a)



(b)

Exercise

- For the circuit below:
 - Count the number of circuit elements.
 - If we move from B to C to D , have we formed a path and/or a loop?
 - If we move from E to D to C to B to E , have we formed a path and/or a loop?



Kirchhoff's Current Law (KCL)

- Gustav Robert Kirchhoff: German university professor, born while Ohm was experimenting
- Based upon conservation of charge

$$\sum_{n=1}^N i_n = 0$$

Where N is the total number of branches connected to a node.

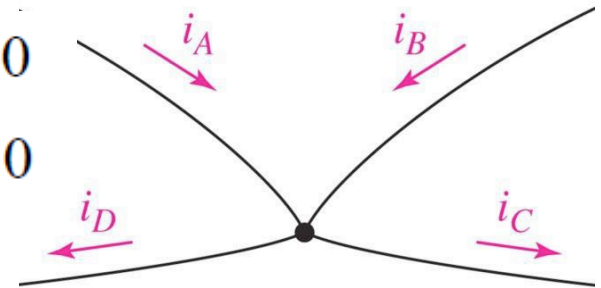
– the algebraic sum of the charge within a system can not change.

– the algebraic sum of the currents entering any node is zero.

$$\sum_{\text{node}} i_{\text{enter}} = \sum_{\text{node}} i_{\text{leave}}$$

$$i_A + i_B - i_C - i_D = 0$$

$$-i_A - i_B + i_C + i_D = 0$$



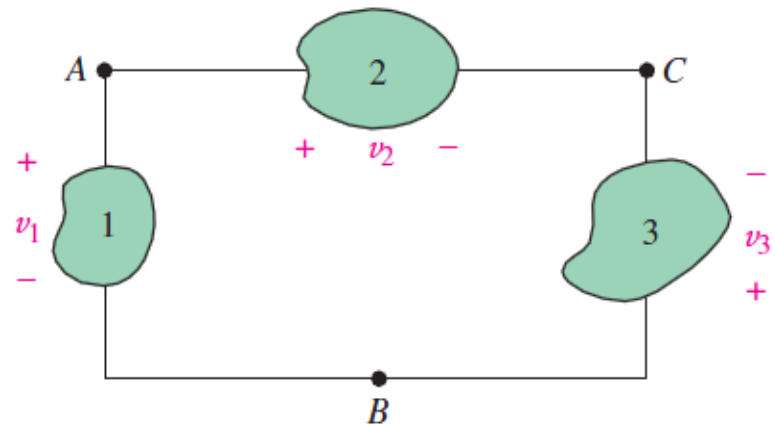
Kirchhoff's Voltage Law (KVL)

- Based upon conservation of energy
 - the algebraic sum of voltages dropped across components around a loop is zero.
 - The energy required to move a charge from point A to point B must have a value independent of the path chosen.

$$\sum_{m=1}^M v = 0$$

Where M is the total number of branches in the loop.

$$\sum v_{\text{drops}} = \sum v_{\text{rises}}$$

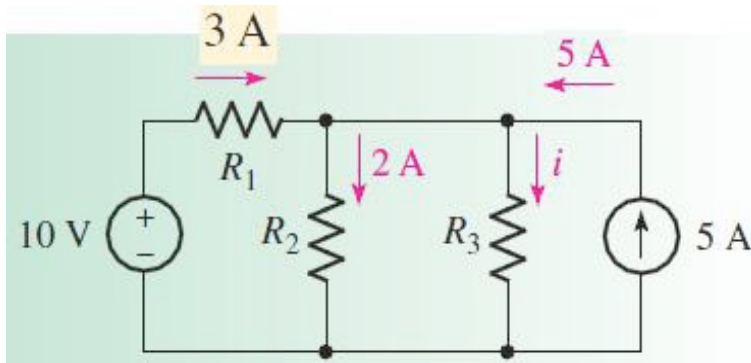
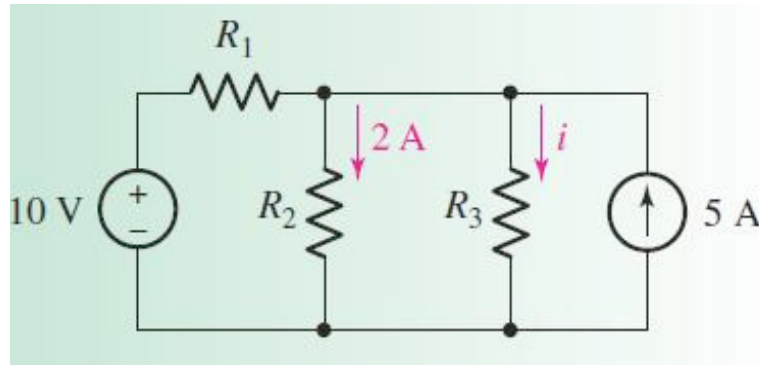


$$-v_1 + v_2 - v_3 = 0$$

$$v_1 - v_2 + v_3 = 0$$

Example-01

- For the circuit, compute the current through R_3 if it is known that the voltage source supplies a current of 3 A.
- Use KCL



$$3 - 2 - i + 5 = 0$$

$$i = 3 - 2 + 5 = 6 \text{ A}$$

Example-02

- Referring to the single node below, compute:

a. i_B , given $i_A = 1$ A, $i_D = -2$ A, $i_C = 3$ A, and $i_E = 4$ A

b. i_E , given $i_A = -1$ A, $i_B = -1$ A, $i_C = -1$ A, and $i_D = -1$ A

- Use KCL

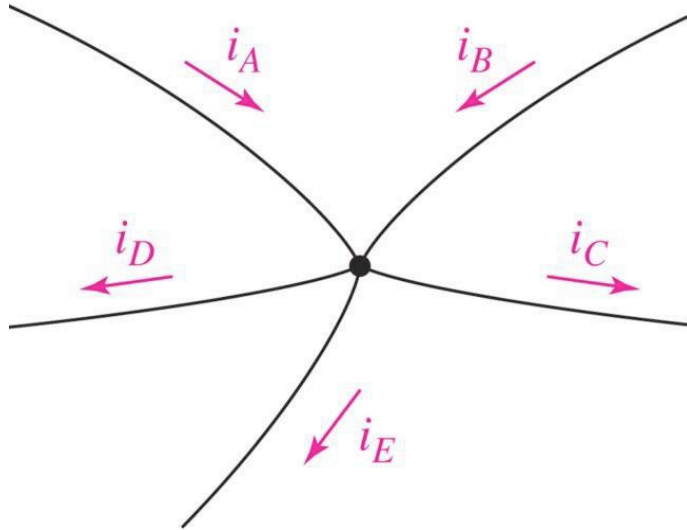
$$i_A + i_B - i_C - i_D - i_E = 0$$

a. $i_B = -i_A + i_C + i_D + i_E$

$$i_B = -1 + 3 - 2 + 4 = 4 \text{ A}$$

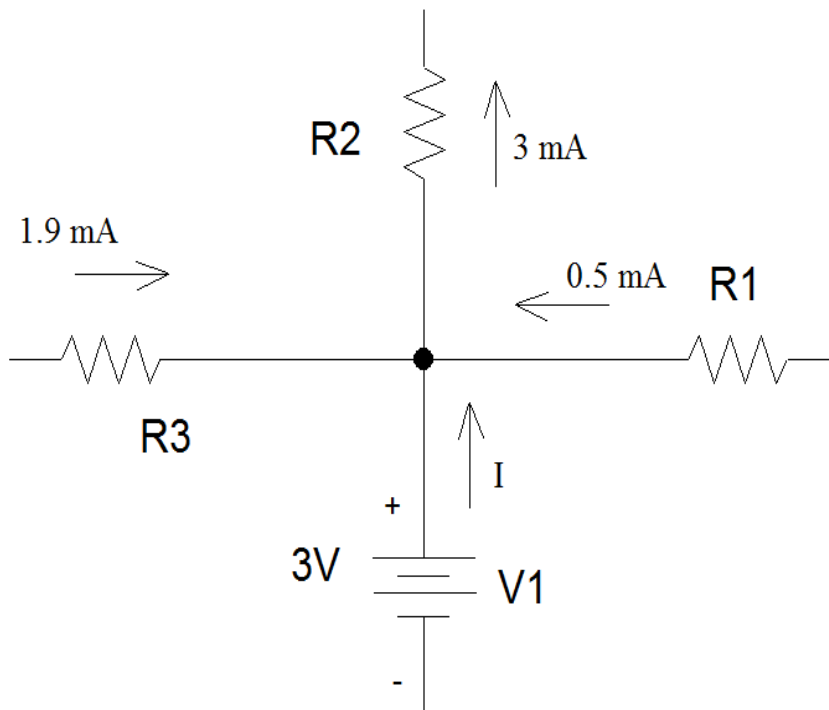
b. $i_E = i_A + i_B - i_C - i_D$

$$i_E = -1 - 1 + 1 + 1 = 0 \text{ A}$$



Example-03

- Determine I , the current flowing out of the voltage source.



– Use KCL

- $1.9 \text{ mA} + 0.5 \text{ mA} + I$ are entering the node.
- 3 mA is leaving the node.

$$1.9 \text{ mA} + 0.5 \text{ mA} + I = 3 \text{ mA}$$

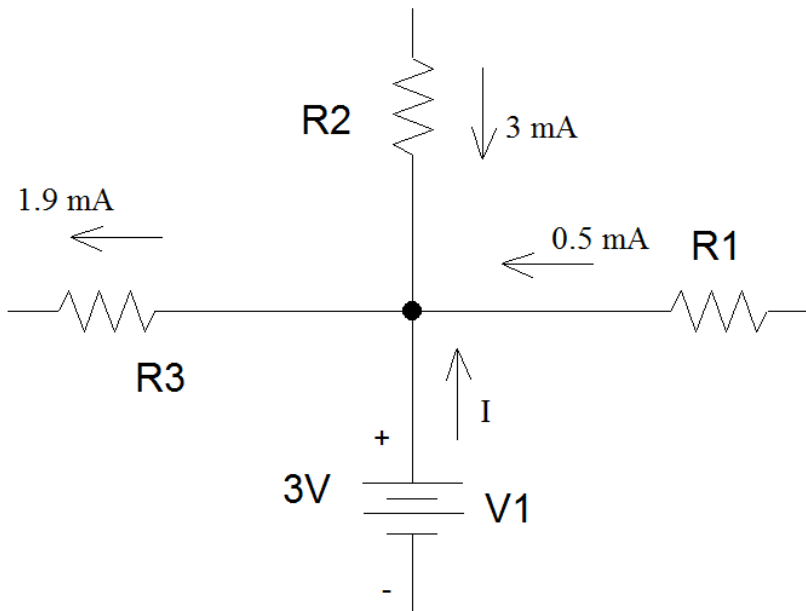
$$I = 3 \text{ mA} - (1.9 \text{ mA} + 0.5 \text{ mA})$$

$$I = 0.6 \text{ mA}$$

$V1$ is generating power.

Example-04

- Suppose the current through R2 was entering the node and the current through R3 was leaving the node.



– Use KCL

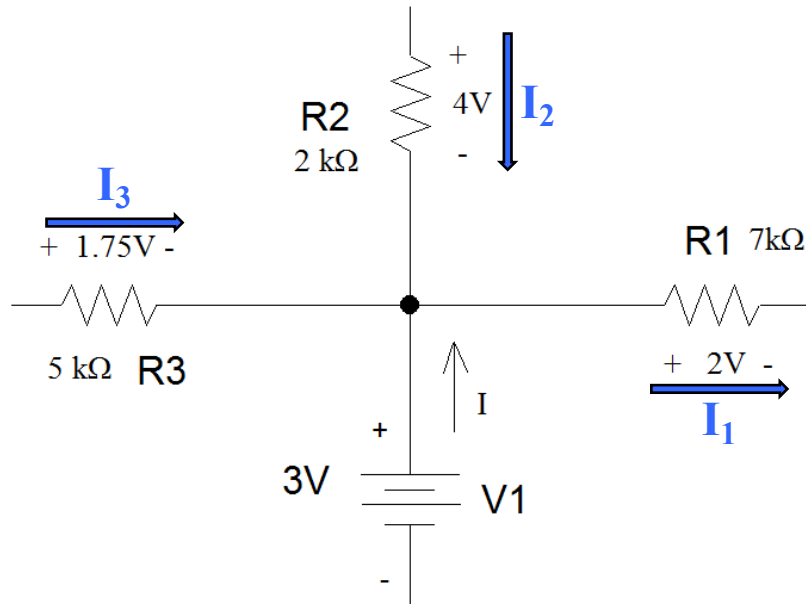
- 3 mA + 0.5 mA + I are entering the node.
- 1.9 mA is leaving the node.

$$\begin{aligned}3mA + 0.5mA + I &= 1.9mA \\ I &= 1.9mA - (3mA + 0.5mA) \\ I &= -1.6mA\end{aligned}$$

V1 is dissipating power.

Example-05

- If voltage drops are given instead of currents,



- you need to apply Ohm's Law to determine the current flowing through each of the resistors before you can find the current flowing out of the voltage supply.

- I_1 is leaving the node.
- I_2 is entering the node.
- I_3 is entering the node.
- I is entering the node.

$$I_1 = 2V / 7k\Omega = 0.286mA$$

$$I_2 = 4V / 2k\Omega = 2mA$$

$$I_3 = 1.75V / 5k\Omega = 0.35mA$$

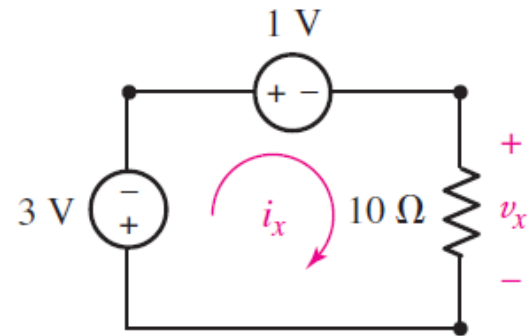
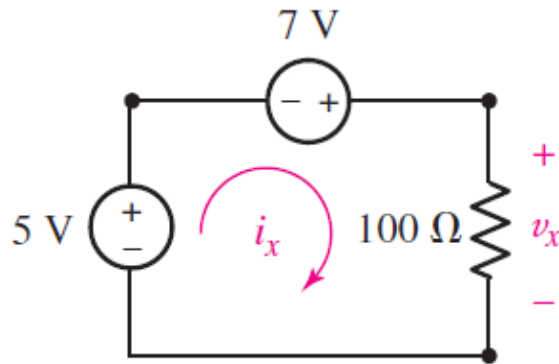
$$I_2 + I_3 + I = I_1$$

$$2mA + 0.35mA + I = 0.286mA$$

$$I = 0.286mA - 2.35mA = -2.06mA$$

Example-06

- For each of the circuits in the figure below, determine the voltage v_x and the current i_x .



– Applying KVL clockwise around the loop and Ohm's law

$$-5 - 7 + v_x = 0$$

$$v_x = 12 \text{ V}$$

$$i_x = \frac{v_x}{100} = \frac{12}{100} \text{ A} = 120 \text{ mA}$$

$$+3 + 1 + v_x = 0$$

$$v_x = \underline{-4 \text{ V}}$$

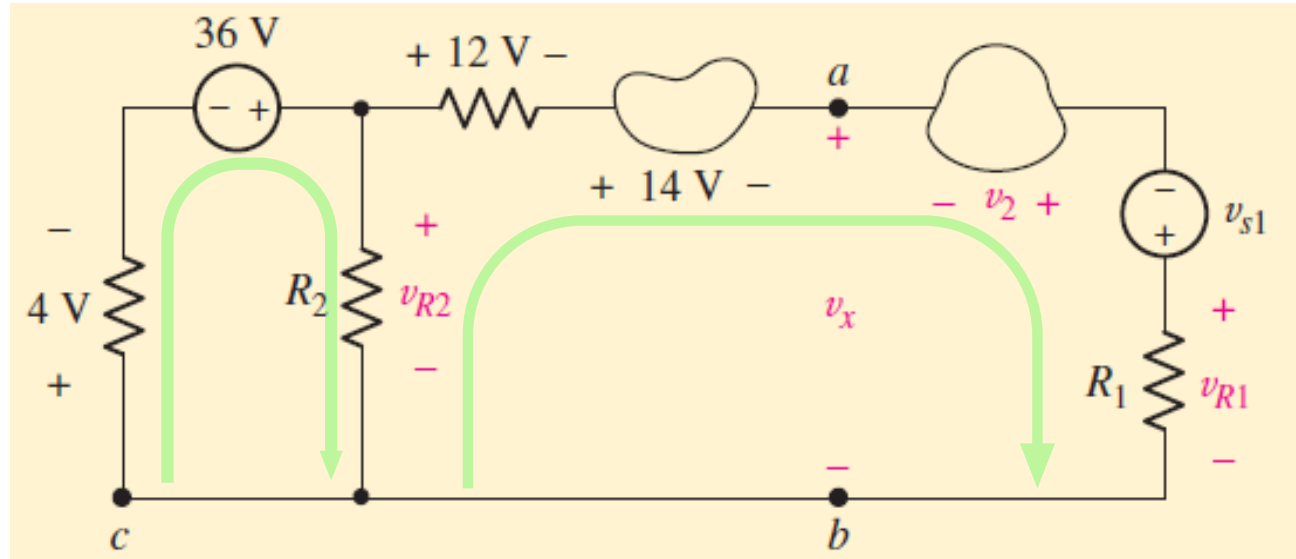
$$i_x = \frac{v_x}{10} = \underline{-400 \text{ mA}}$$

Example-07

- For the circuit below, determine

a. v_{R2}

b. v_x



a. $4 - 36 + v_{R2} = 0$

$v_{R2} = 32 \text{ V}$

b. $-32 + 12 + 14 + v_x = 0$

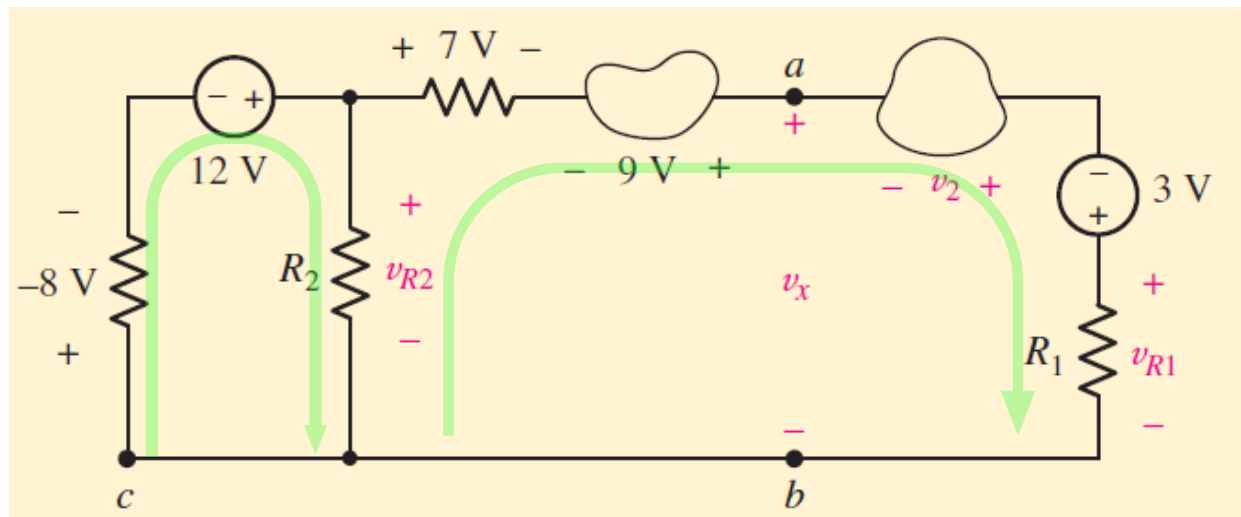
$v_x = 6 \text{ V}$

Example-08

- For the circuit below, determine

a. v_{R2}

b. v_x if $v_{R1} = 1$ V.



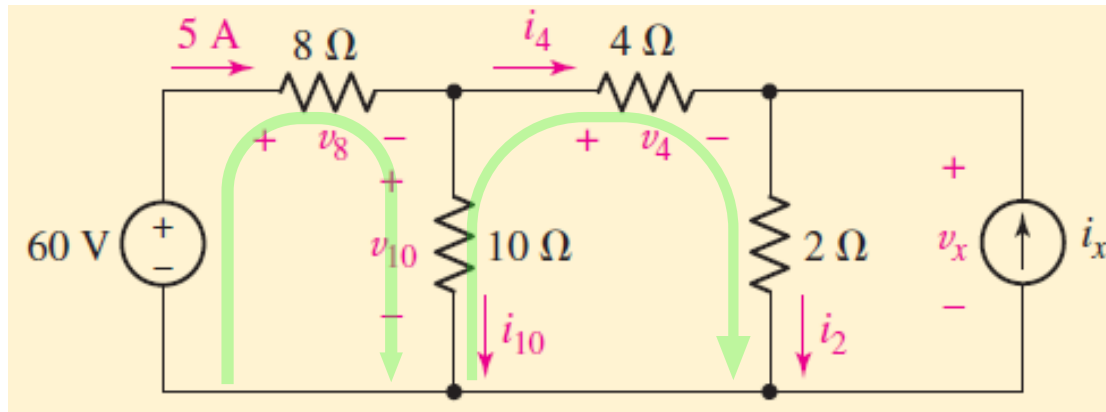
a. KVL yields $-8 - 12 + v_{R2} = 0$ $v_{R2} = 20$ V

b. KVL yields $-20 + 7 - 9 - v_2 - 3 + v_{R1}$

where $v_{R1} = 1$ V. Thus, $v_2 = -24$ V

Example-09

- For the circuit below, determine v_x



$$-60 + v_8 + v_{10} = 0$$

$$v_{10} = 0 + 60 - 40 = 20 \text{ V}$$

$$-v_{10} + v_4 + v_x = 0$$

$$v_x = 20 - v_4$$

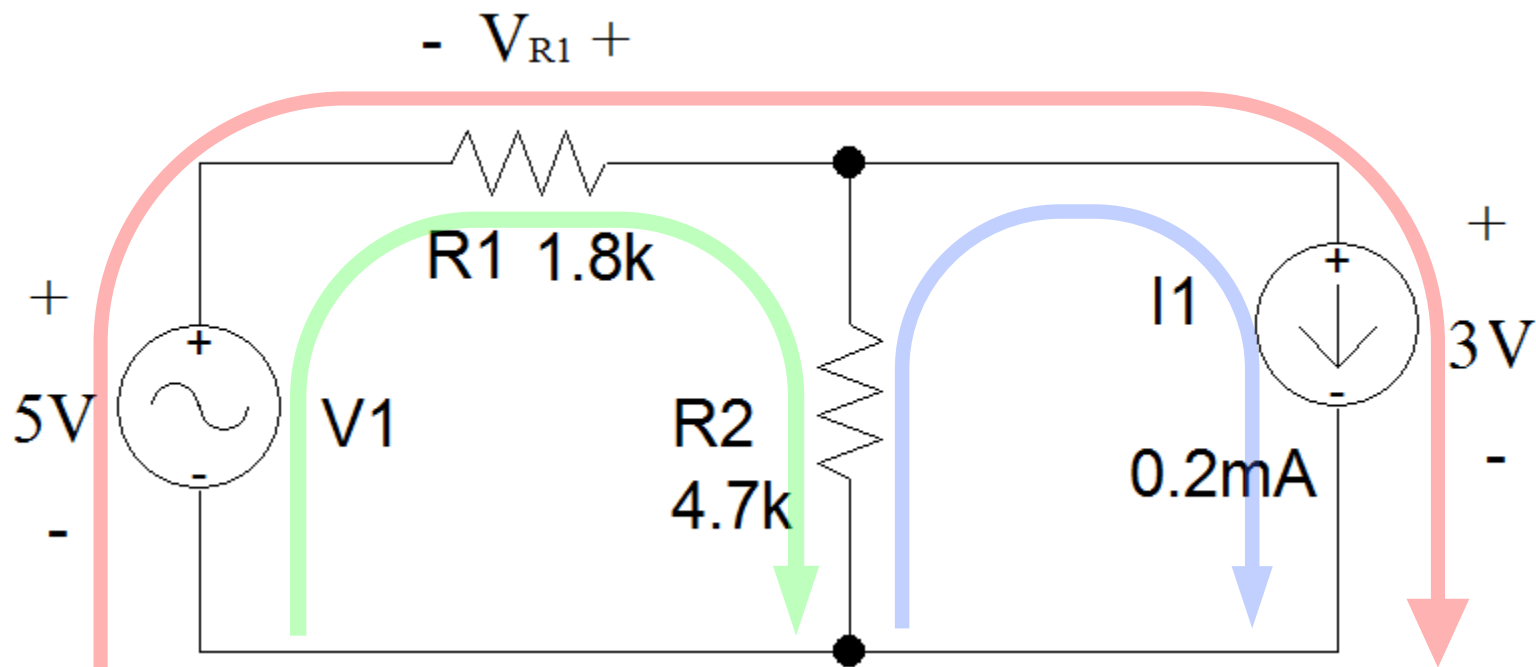
$$i_4 = 5 - i_{10} = 5 - \frac{v_{10}}{10} = 5 - \frac{20}{10} = 3$$

$$v_4 = (4)(3) = 12 \text{ V}$$

$$v_x = 20 - 12 = 8 \text{ V}$$

Example-10...

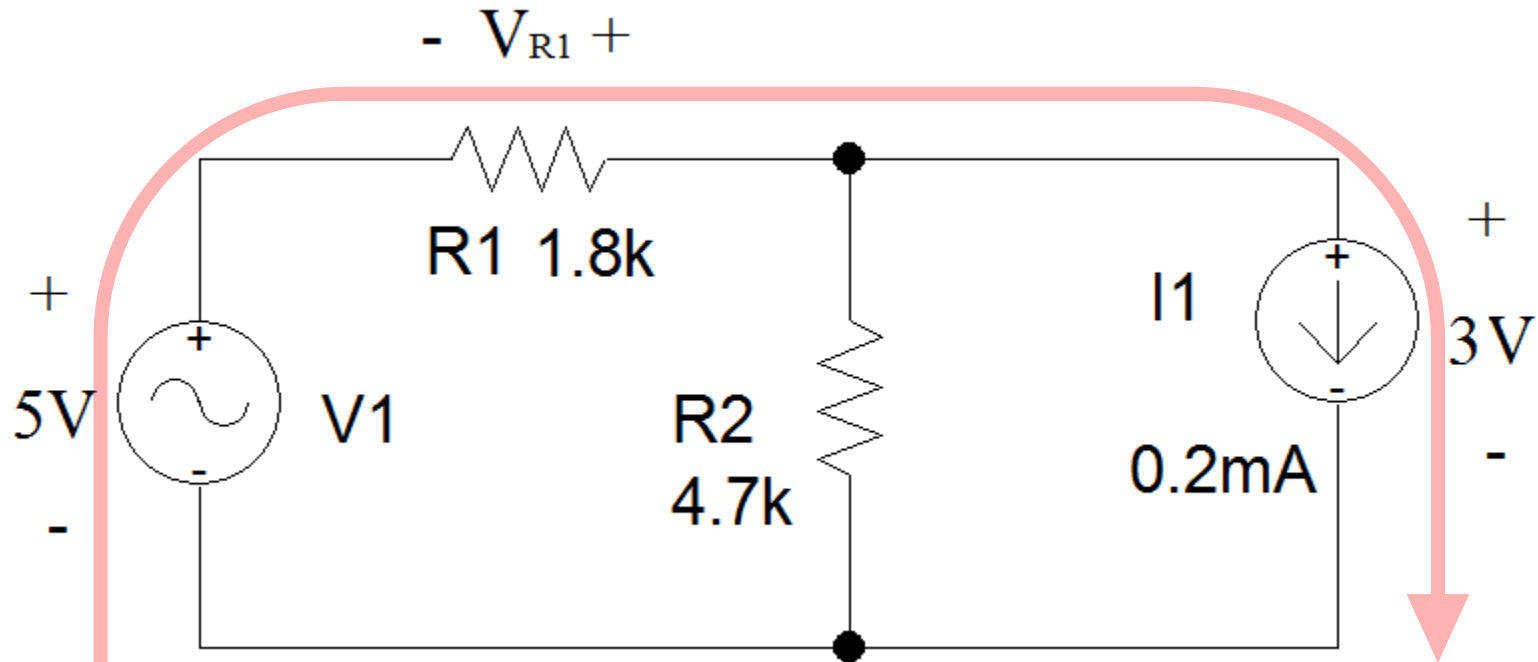
- Find the voltage across R1.
 - Note that the polarity of the voltage has been assigned in the circuit schematic.



– First, define a loop that include R1.

...Example-10...

- If the red loop is considered



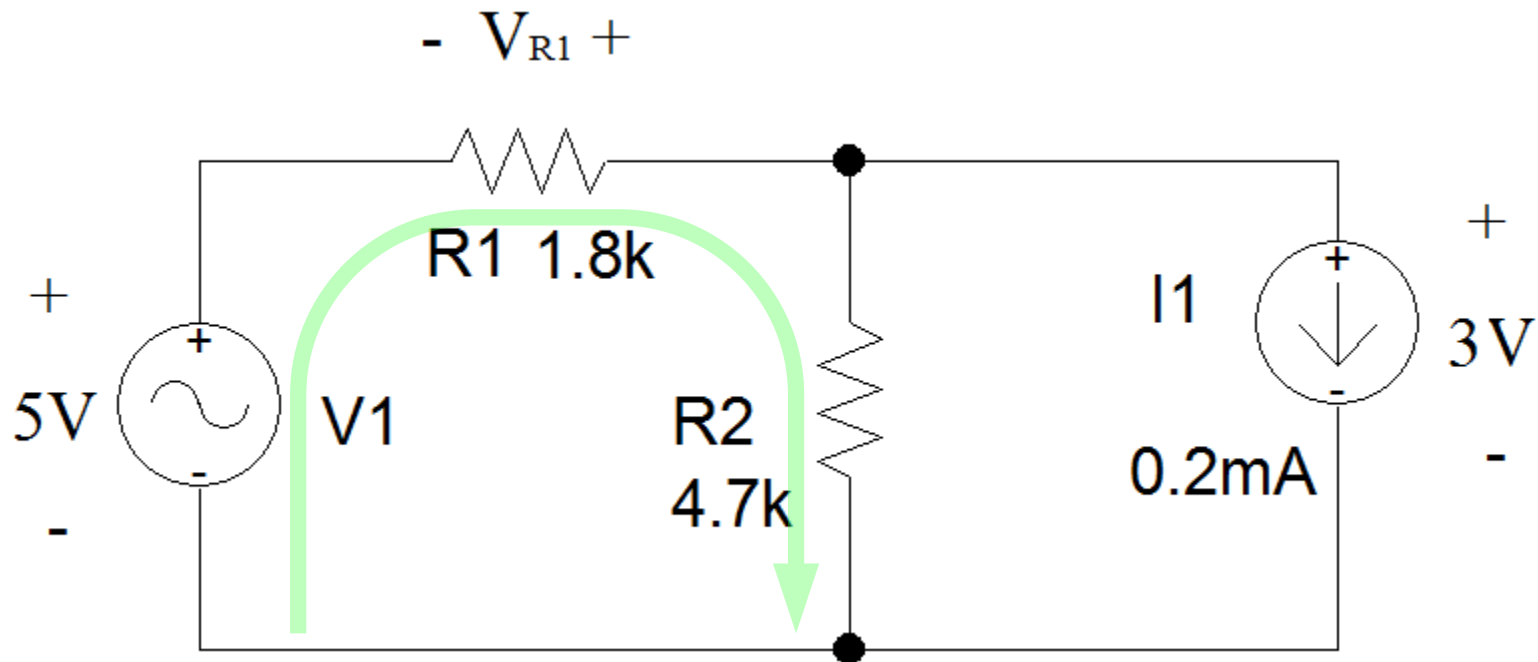
- By convention, voltage drops are added and voltage rises are subtracted in KVL.

$$-5 \text{ V} - V_{R1} + 3 \text{ V} = 0$$

$$V_{R1} = 2 \text{ V}$$

...Example-10

- Suppose you chose the green loop instead.
 - Since R_2 is in parallel with I_1 , the voltage drop across R_2 is also 3V.

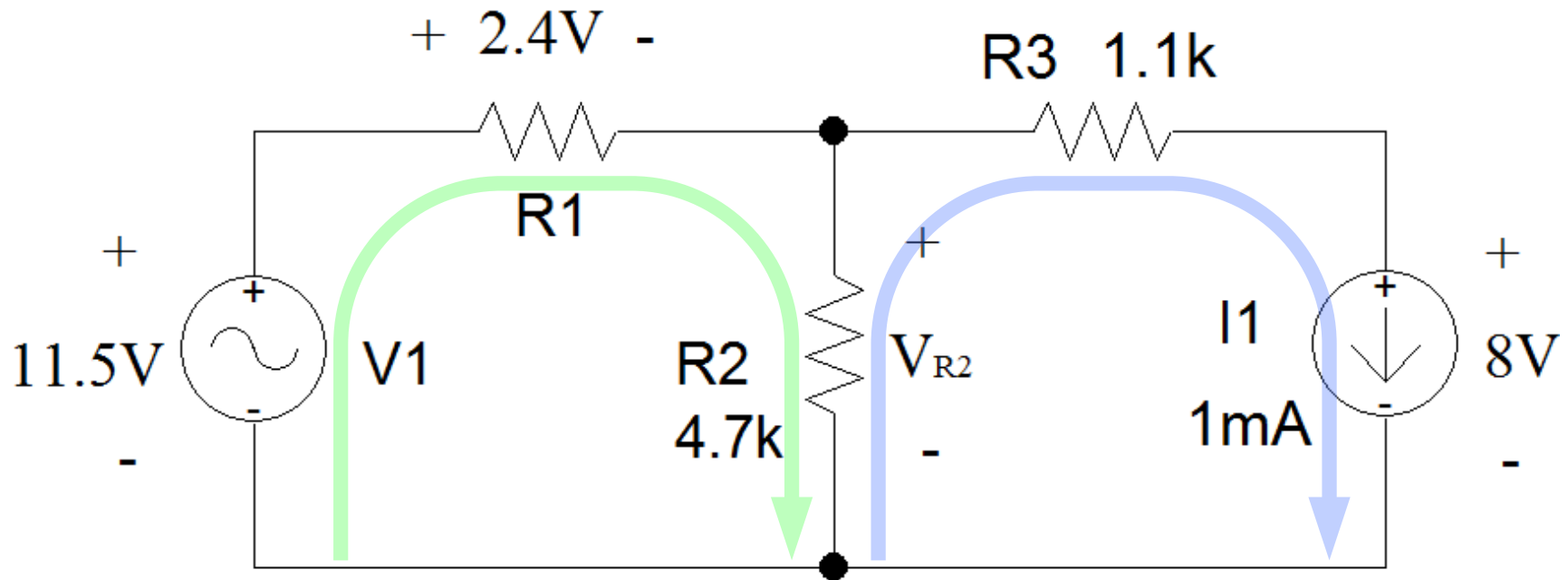


$$-5 \text{ V} - V_{R1} + 3 \text{ V} = 0$$

$$V_{R1} = 2 \text{ V}$$

Example-11...

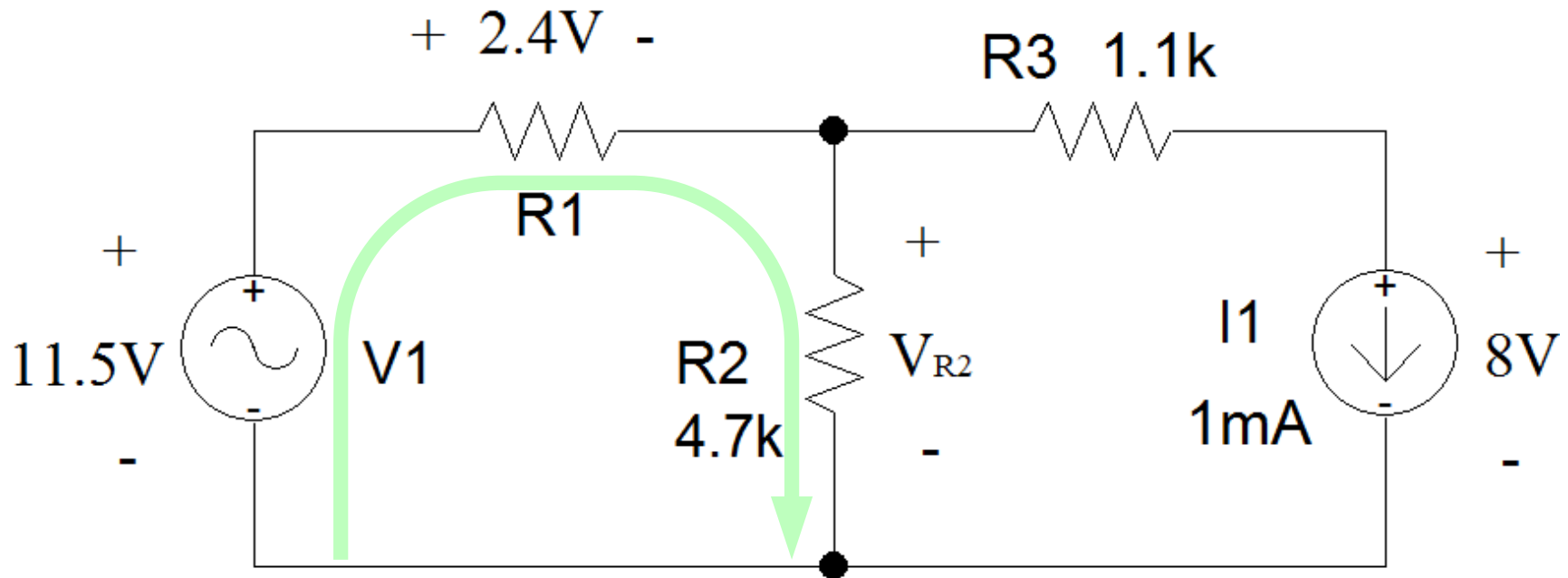
- Find the voltage across R_2 and the current flowing through it.



– First, draw a loop that includes R_2 .

...Example-11...

- If the green loop is used:

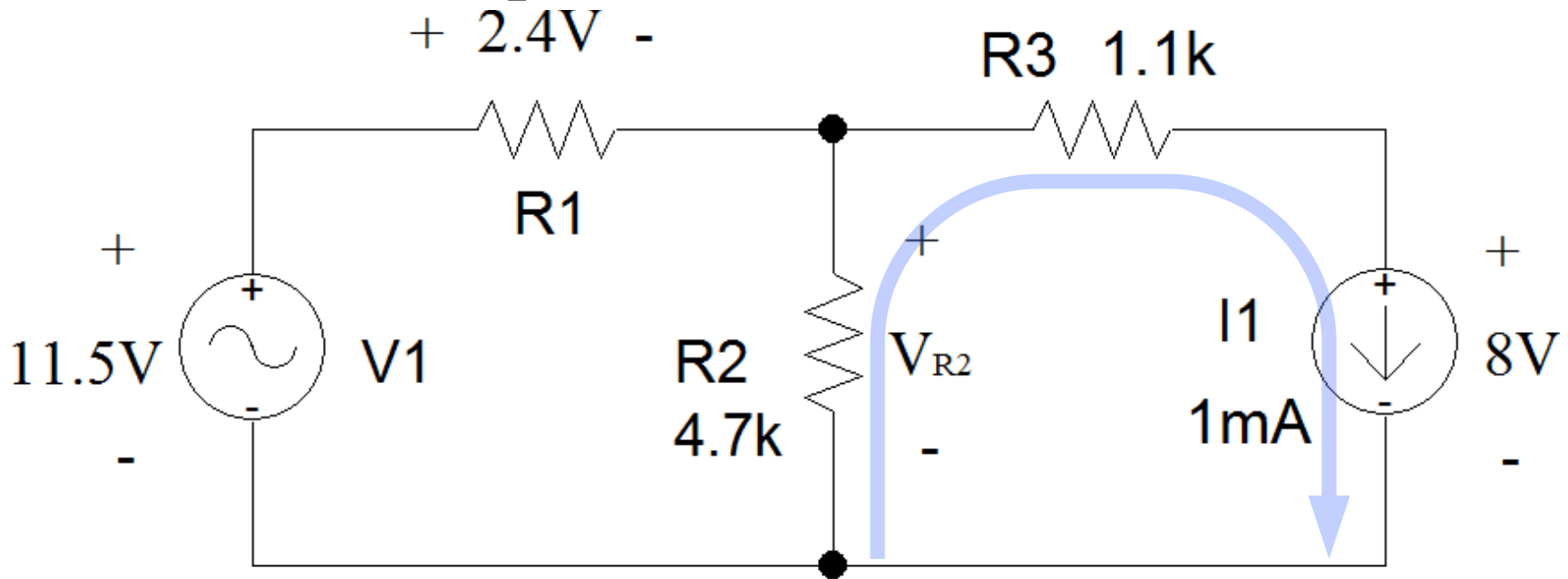


$$-11.5 \text{ V} + 2.4 \text{ V} + V_{R2} = 0$$

$$V_{R2} = 9.1 \text{ V}$$

...Example-11...

- If the blue loop is used:



- First, find the voltage drop across R3

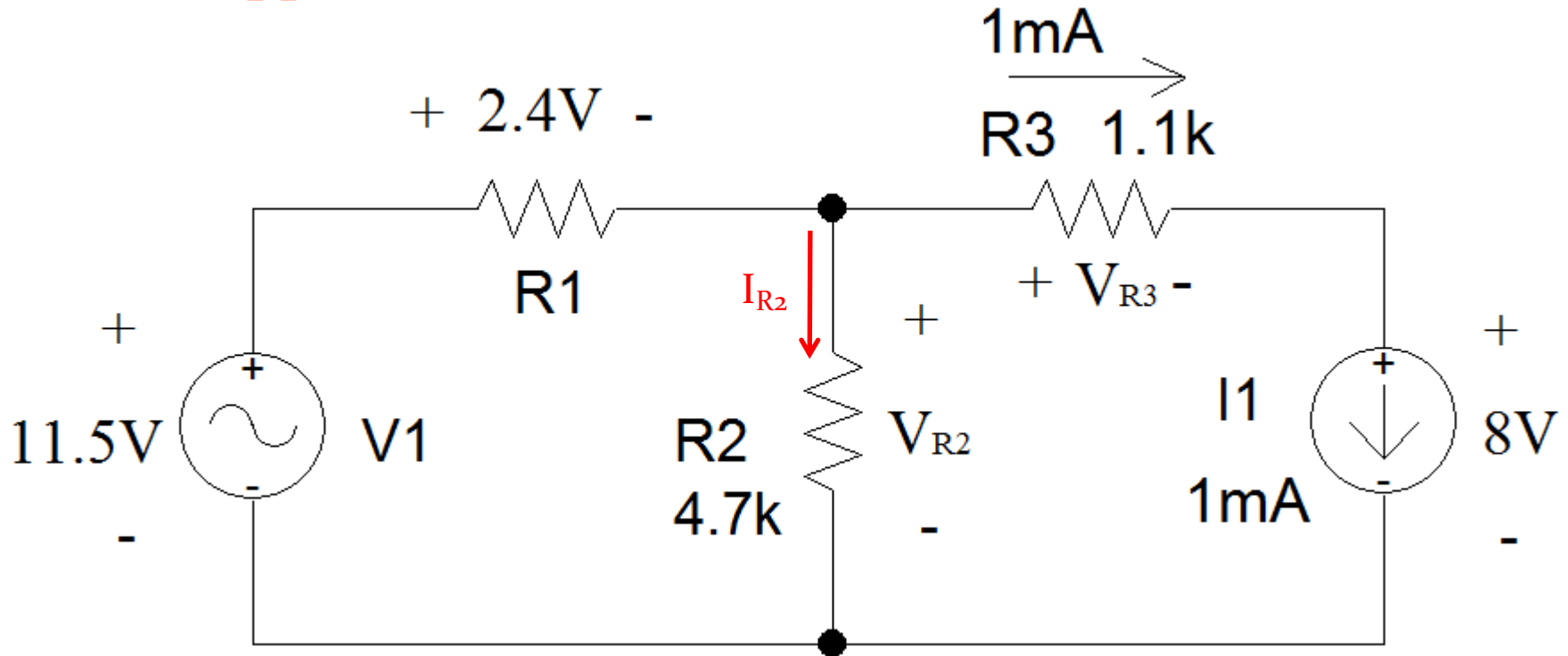
$$1 \text{ mA} \times 1.1 \text{ k}\Omega = 1 \times 10^{-3} \text{ A} \times 1.1 \times 10^3 \Omega = 1.1 \text{ V}$$

$$1.1 \text{ V} + 8 \text{ V} - V_{R2} = 0$$

$$V_{R2} = 9.1 \text{ V}$$

...Example-11

- Once the voltage across R2 is known, Ohm's Law is applied to determine the current.



$$I_{R2} = 9.1 \text{ V} / 4.7 \text{ k}\Omega = 9.1 \text{ V} / (4.7 \times 10^3 \Omega)$$

$$I_{R2} = 1.94 \times 10^{-3} \text{ A} = 1.94 \text{ mA}$$