# Optimization Techniques Section 4

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#### Convergence

- We have tried to find some x<sup>k</sup> that converge to the desired point x\*(most often a local minimizer). The fundamental question is how fast the convergence is.
- Let's define errors at each iteration step as  $|e_k| = |x^k x^*|$  and  $|e_{k+1}| = |x^{k+1} x^*|$

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#### Convergence

• Linear Convergance:

There exist a constant  $0< c_1<1$ where  $|e_{k+1}| \le c_1^* |e_k|$ 

- If c<sub>1</sub> is very close to 0 then superlinear convergence.
- Quadratic Convergance :

There exist a constant  $0 < c_2 < 1$ where  $|e_{k+1}| \le c_2 * |e_k|^2$ 

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#### Convergence Example

- Two methods are given; one of them showing linear convergence and the other one showing quadratic convergence.
- After certain number of steps errors reach 3 digit precision for both of the methods ( $|e_k|$  < 0,001).
- How many more steps (iterations) will be necessary if we require 12 digits of precision?
- $c_1 = c_2 = 1/2$

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## Convergence Example

- In case of Linear Convergence: c1=0.5
- $|e_{k+1}| \le 0.5* |e_k|$
- $|e_{k+2}| \le 0.5* |e_{k+1}|$
- $|e_{k+2}| \le 0.5*0.5*|e_k|$
- ...
- $|e_{k+n}| \le 0.5^{n*} |e_k|$
- $|e_k| \le 0.001$  (It is given.)
- $|e_{k+n}| \le 10^{-12}$  (It is required.)
- $|e_{k+n}| \le 0.5^n * 10^{-3}$
- 0.5<sup>n</sup> ≈10<sup>-9</sup>
- $\log_{10} 2^{-n} = \log_{10} 10^{-9}$
- $-n * log_{10}2 = -9 (log_{10}2 \approx 0.301)$

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## Convergence Example

- In case of quadratic Convergence: c2=0.5
- $|e_{k+1}| \le 0.5^* |e_k|^2$
- $|e_{k+2}| \le 0.5^* |e_{k+1}|^2$
- $|e_{k+2}| \le 0.5*(0.5*|e_k|^2)^2$
- $|e_{k+3}| \le 0.5*(0.5*(0.5*|e_k|^2)^2)^2$
- $|e_k| \le 10^{-3}$  (It is given.)
- $|e_{k+n}| \le 10^{-12}$  (It is required.)
- $|e_k|^2 = (10^{-3})^2 = 10^{-6}$
- $(|e_k|^2)^2 = 10^{-12}$
- n = 2

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### with N dimensions

- $f(x)=(1/2) * x^T * q * x + b^T * x + c$
- f: R<sup>n</sup>--> R<sup>1</sup>
- q--> n\*n
- b--> n\*1
- c--> 1\*1
- x--> n\*1
- •
- df=q\*x+b
- ddf=q
- opt\_Ndim\_v2.m

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### To avoid local minimums

- Random restart
- Non Derivative Techniques

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