

BLM1612 - Circuit Theory

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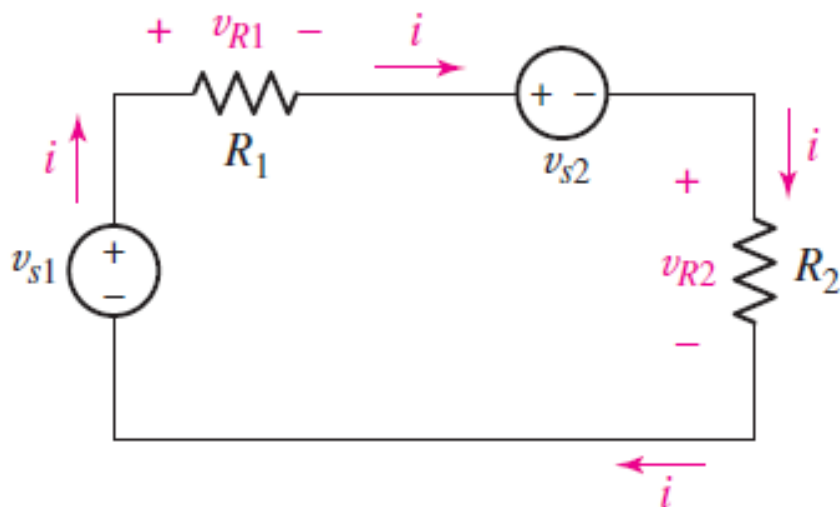
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Kirchhoff's Current Law (KCL)
Kirchhoff's Voltage Law (KVL)
Serial / Parallel Circuits

Objectives of the Lecture

- Explain mathematically how resistors in series are combined and their equivalent resistance.
- Explain mathematically how resistors in parallel are combined and their equivalent resistance.
- Rewrite the equations for conductances.
- Explain mathematically how a voltage that is applied to resistors in series is distributed among the resistors.
- Explain mathematically how a current that enters the a node shared by resistors in parallel is distributed among the resistors.

The Single-Loop Circuit



$$-v_{s1} + v_{R1} + v_{s2} + v_{R2} = 0$$

$$v_{R1} = R_1 i \quad \text{and} \quad v_{R2} = R_2 i$$

$$-v_{s1} + R_1 i + v_{s2} + R_2 i = 0$$

$$i = \frac{v_{s1} - v_{s2}}{R_1 + R_2}$$

- First step in the analysis is the assumption of reference directions for the unknown currents.
- Second step in the analysis is a choice of the voltage reference for each of the two resistors.
- The third step is the application of Kirchhoff's voltage law to the only closed path.

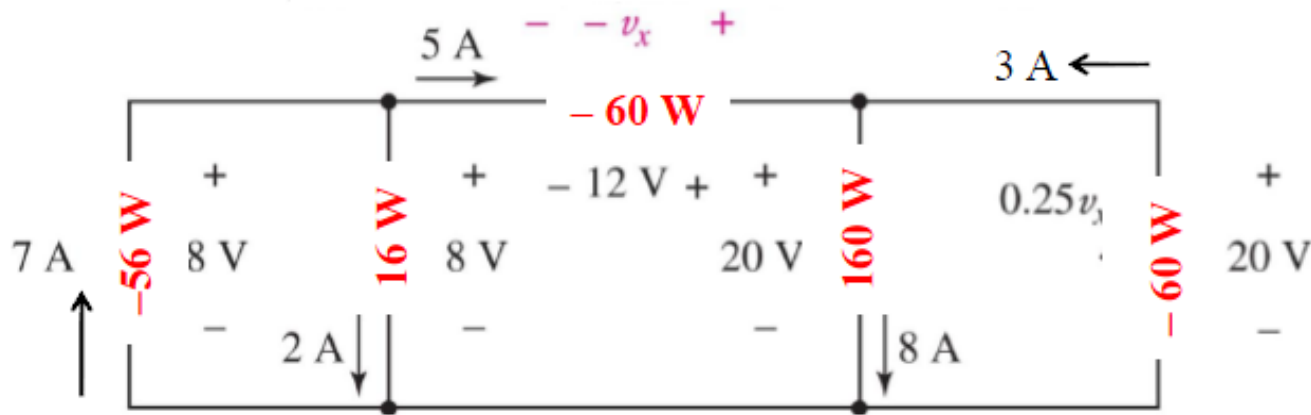
Conservation of Energy

- The sum of the absorbed power for each element of a circuit is zero.

$$\sum_{\text{all elements}} P_{\text{absorbed}} = 0$$

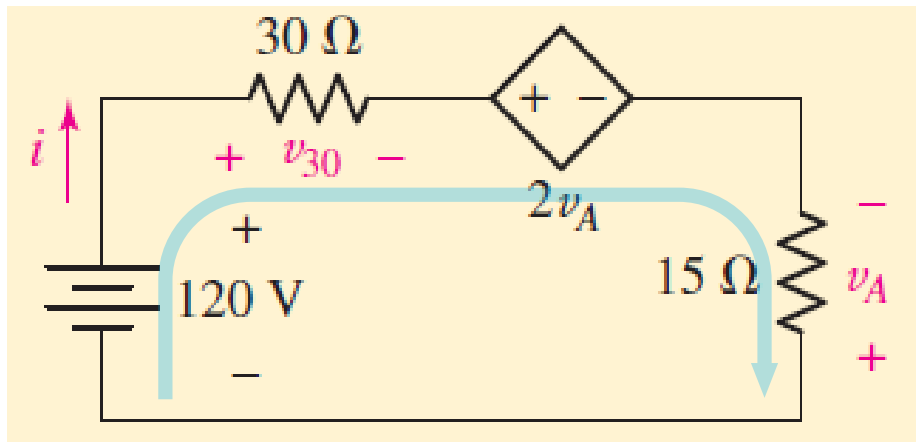
- The sum of the absorbed power equals the sum of the supplied power

$$\sum P_{\text{absorbed}} = \sum P_{\text{supplied}}$$



$$\sum P_{\text{abs}} = -56 + 16 - 60 + 160 - 60 = -176 \text{ W} + 176 \text{ W} = 0$$

Example-01



- Compute the power absorbed in each element for the circuit shown in the Figure.

– power absorbed by each element:

$$-120 + v_{30} + 2v_A - v_A = 0$$

$$v_{30} = 30i \quad \text{and} \quad v_A = -15i$$

$$-120 + 30i - 30i + 15i = 0$$

$$i = 8 \text{ A}$$

$$p_{120\text{V}} = (120)(-8) = -960 \text{ W}$$

$$p_{30\Omega} = (8)^2(30) = 1920 \text{ W}$$

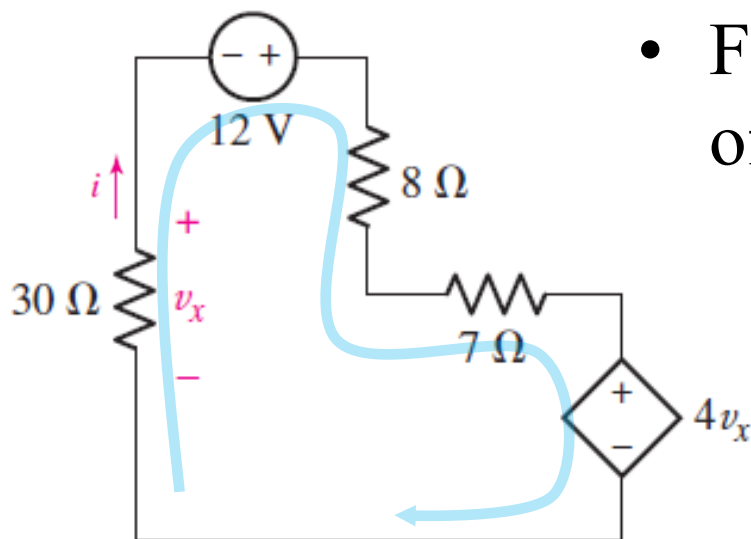
$$\begin{aligned} p_{\text{dep}} &= (2v_A)(8) = 2[(-15)(8)](8) \\ &= -1920 \text{ W} \end{aligned}$$

$$p_{15\Omega} = (8)^2(15) = 960 \text{ W}$$

Example-02

- Find the power absorbed by each of the five elements in the circuit.

– power absorbed by each element:



$$P_{abs}|_{30\Omega} = \frac{24^2}{5} \times \frac{1}{30} = \underline{768 \text{ mW}}$$

$$P_{abs}|_{12V} = +\frac{4}{25} \times 12 = \underline{1.92 \text{ W}}$$

$$P_{abs}|_{8\Omega} = -\frac{4^2}{25} \times 8 = \underline{204.8 \text{ mW}}$$

$$P_{abs}|_{7\Omega} = -\frac{4^2}{25} \times 7 = \underline{179.2 \text{ mW}}$$

$$P_{abs}|_{4v_x} = -\frac{4}{25} \times 4v_x = \frac{-4}{25} \times 4 \times \frac{24}{5} = \underline{-3.072 \text{ W}}$$

$$-v_x - 12 + (8 + 7)i + 4v_x = 0$$

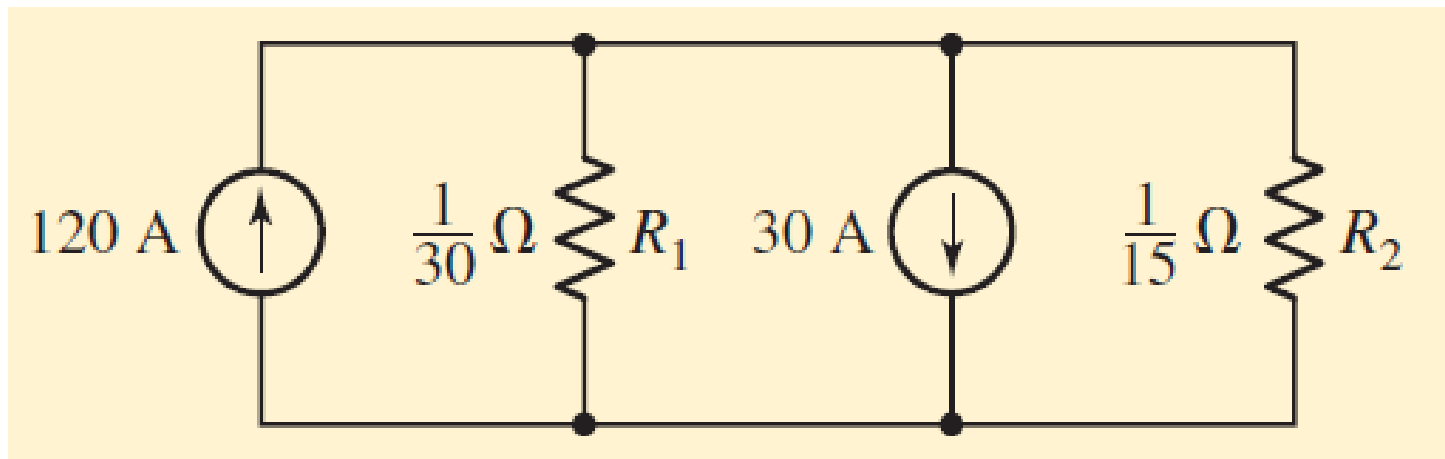
$$i = \frac{-v_x}{30} \quad v_x = \frac{24}{5} \text{ V}$$

$$i = -\frac{4}{25} \text{ A}$$

(Check: $768 + 1920 + 204.8 + 179.2 - 3072 = 0 \text{ mW}$)

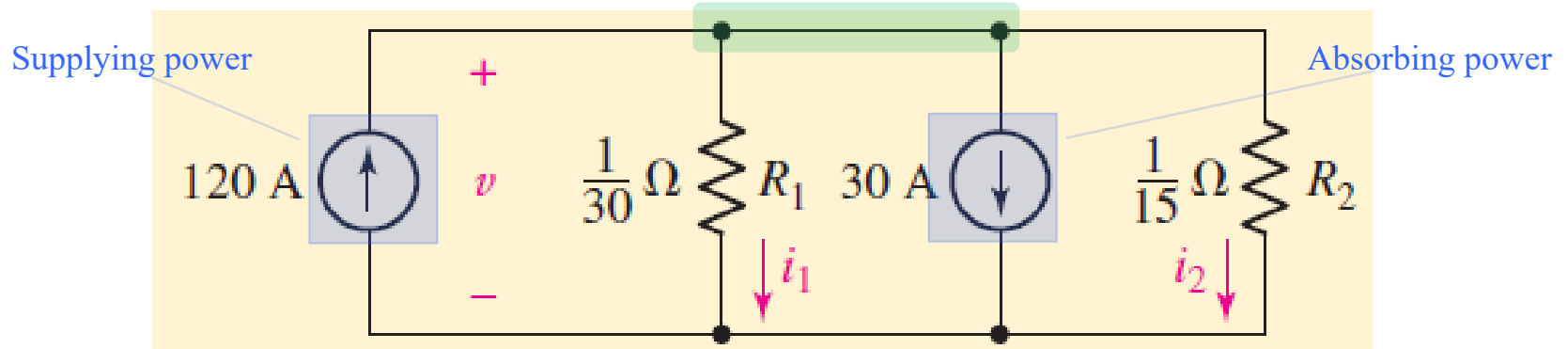
The Single-Node-Pair Circuit

- KVL forces us to recognize that the **voltage across each branch** is the same as that across any other branch.
- Elements in a circuit having a common voltage across them are said to be connected in **parallel**.



Example-03

- Find the voltage, current, and power associated with each element in the following circuit.



– power absorbed by each element:

$$-120 + i_1 + 30 + i_2 = 0$$

$$i_1 = 30v \quad \text{and} \quad i_2 = 15v$$

$$-120 + 30v + 30 + 15v = 0$$

$$v = 2 \text{ V}$$

$$i_1 = 60 \text{ A} \quad \text{and} \quad i_2 = 30 \text{ A}$$

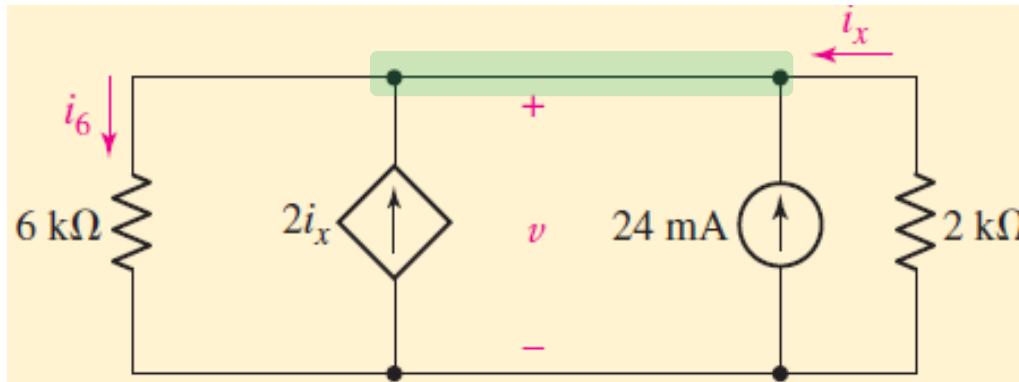
$$p_{R1} = 30(2)^2 = 120 \text{ W}$$

$$p_{R2} = 15(2)^2 = 60 \text{ W}$$

$$p_{120\text{A}} = 120(-2) = -240 \text{ W}$$

$$p_{30\text{A}} = 30(2) = 60 \text{ W}$$

Example-04



- Determine the value of v and the power absorbed by the independent current source in the circuit.

$$i_6 - 2i_x - 0.024 - i_x = 0$$

$$i_6 = \frac{v}{6000} \quad \text{and} \quad i_x = \frac{-v}{2000}$$

$$\frac{v}{6000} - 2 \left(\frac{-v}{2000} \right) - 0.024 - \left(\frac{-v}{2000} \right) = 0$$

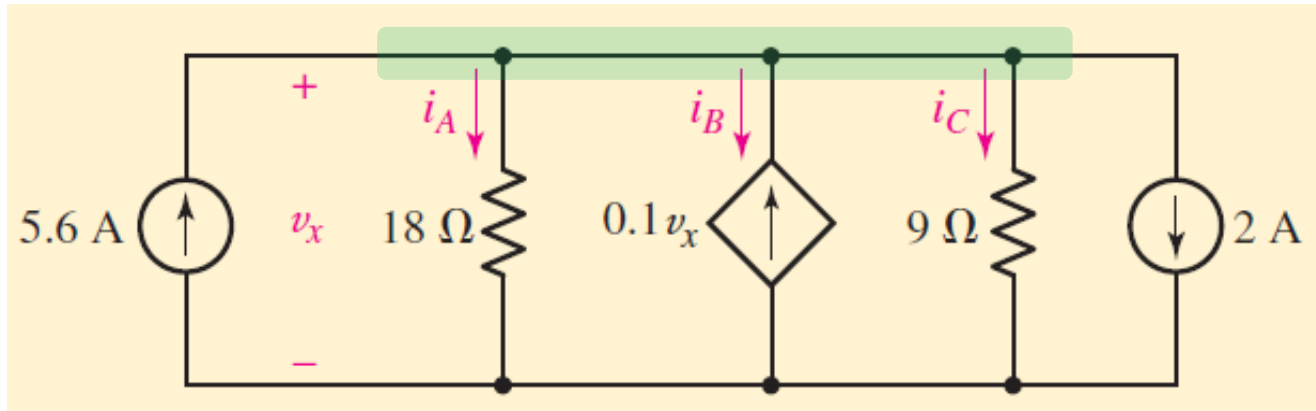
$$v = (600)(0.024) = 14.4 \text{ V}$$

$$p_{24} = -14.4(0.024) = -0.3456 \text{ W } (-345.6 \text{ mW})$$

- Actually 345.6 mW is supplied

Example-05

- For the single-node-pair circuit, find i_A , i_B and i_C .



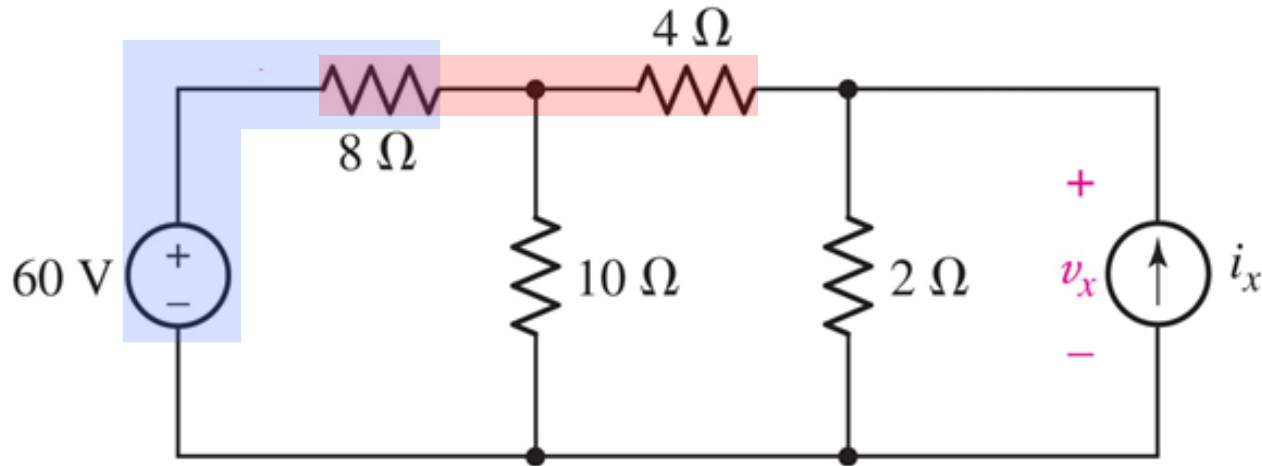
$$5.6 - \frac{v_x}{18} + 0.1v_x - \frac{v_x}{9} - 2 = 0 \quad v_x = 54 \text{ V.}$$

$$i_A = \frac{v_x}{18} = \underline{3 \text{ A}}, \quad i_B = -0.1v_x = \underline{-5.4 \text{ A}}, \quad i_C = \frac{v_x}{9} = \underline{6 \text{ A}}$$

$$5.6 = i_A + i_B + i_C + 2 = 3 - 5.4 + 6 + 2 = 5.6$$

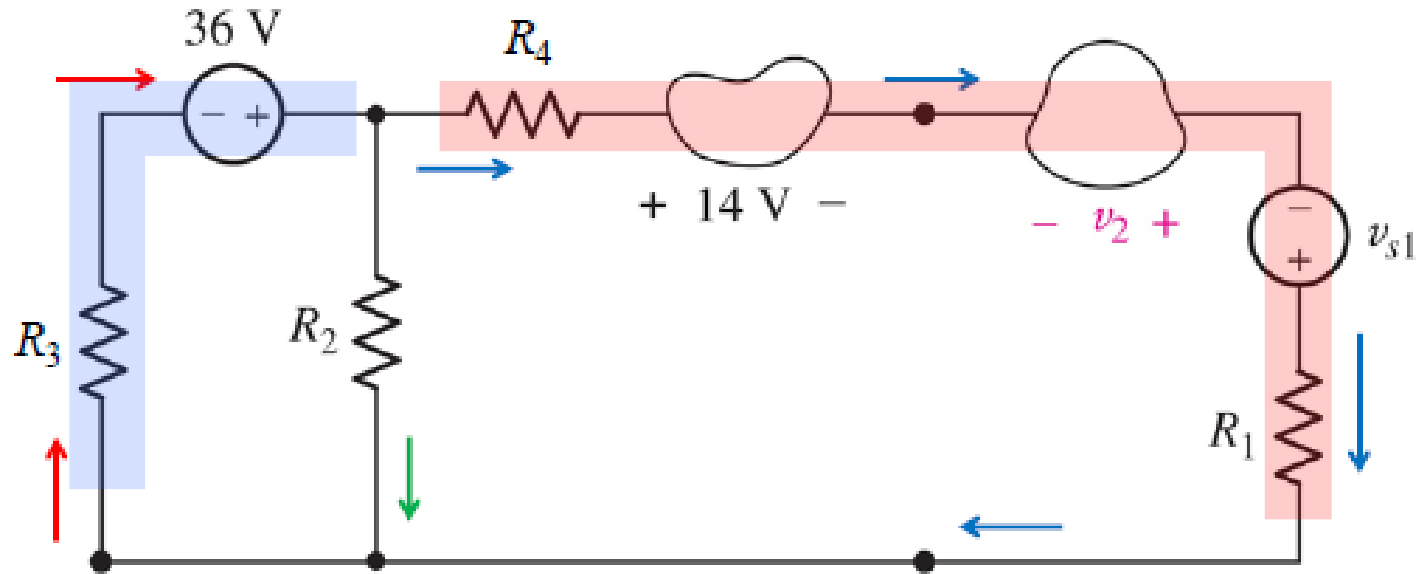
Series Circuits

- Series
 - all elements in a circuit (loop) that carry the same current



- The 60 V source and the 8 Ω resistor are in series.
- The 8 Ω resistor and 4 Ω resistor are **not** in series.

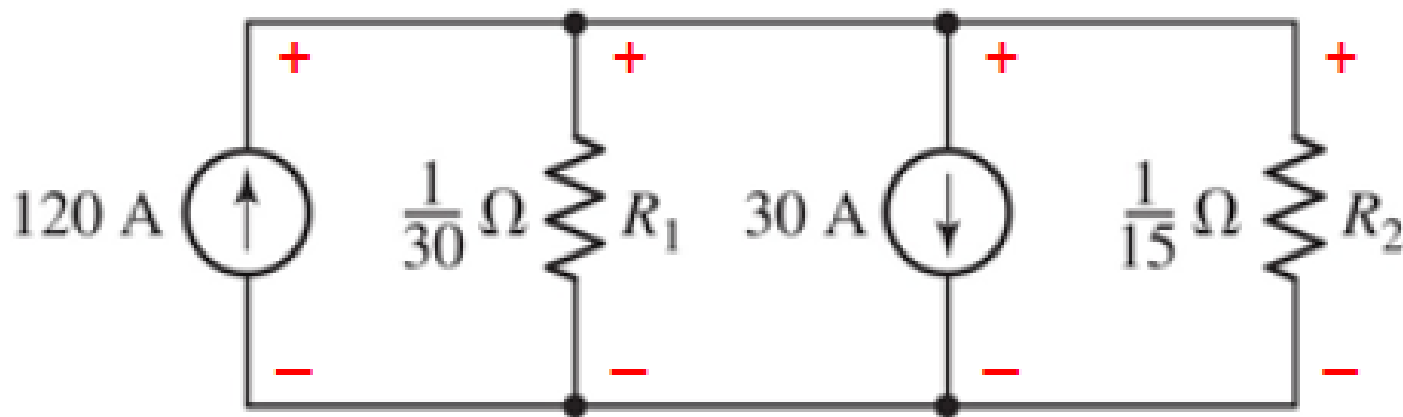
Series Circuits



- R_3 is in series with the 36 V source.
- R_4 , the 14 V element, the v_2 element, the v_{s1} source, and R_1 are in series.
- No element is in series with R_2 .

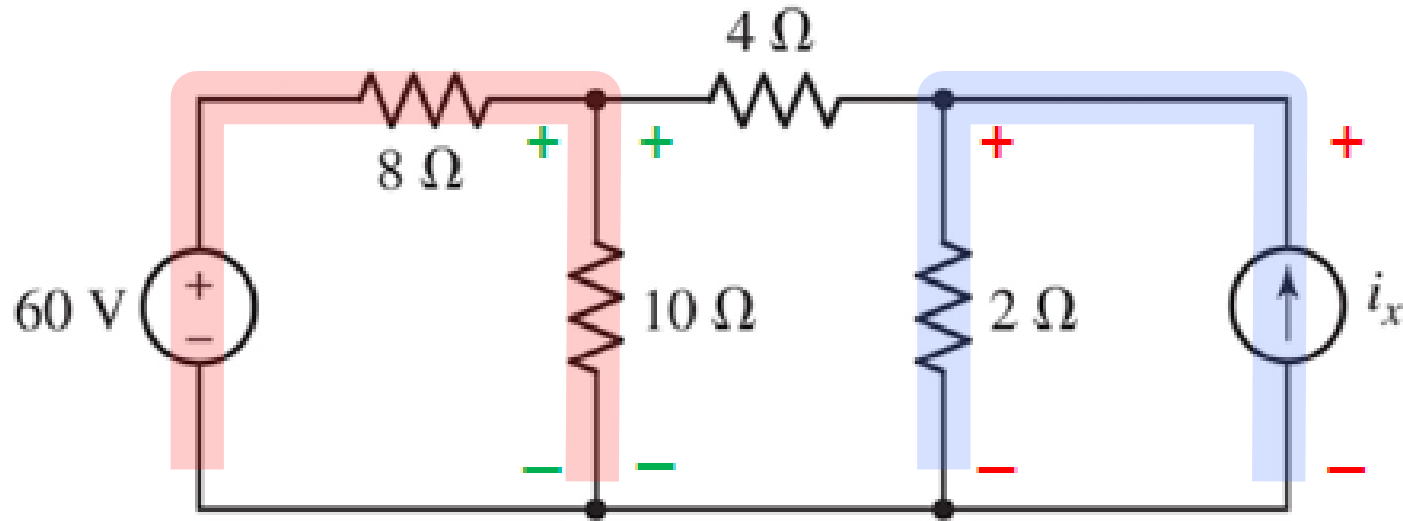
Parallel Circuits

- Parallel
 - all elements in a circuit that have a common voltage across them (elements that share the same 2 nodes)



- The 120 A source, $\frac{1}{30} \Omega$ resistor, 30 A source, and $\frac{1}{15} \Omega$ resistor are in parallel.

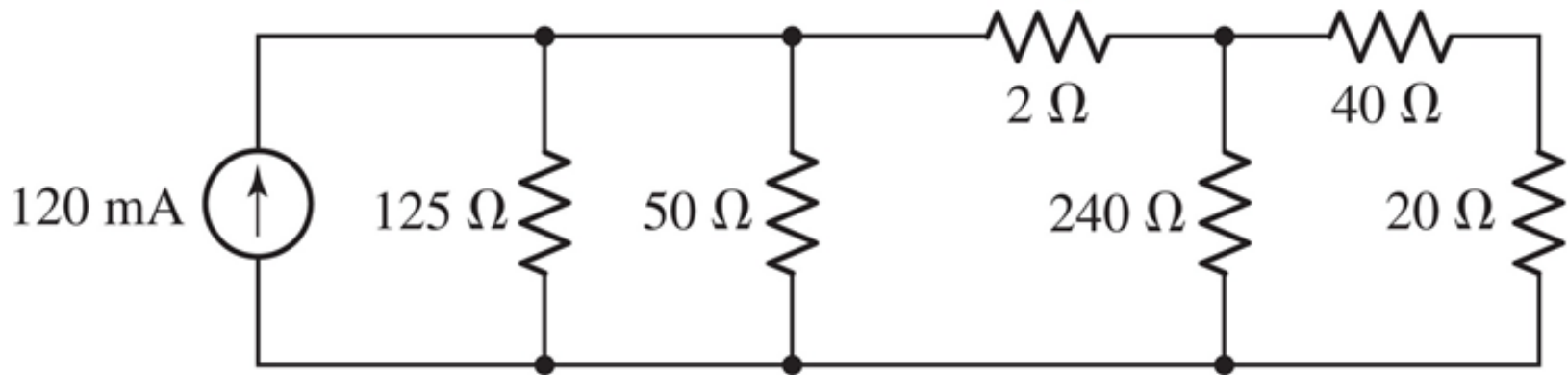
Parallel Circuits



- The current source and the 2 Ω resistor are in parallel.
 - No other single elements are in parallel with each other.
- The 60 V source and 8 Ω resistor branch is in parallel with the 10 Ω resistor.

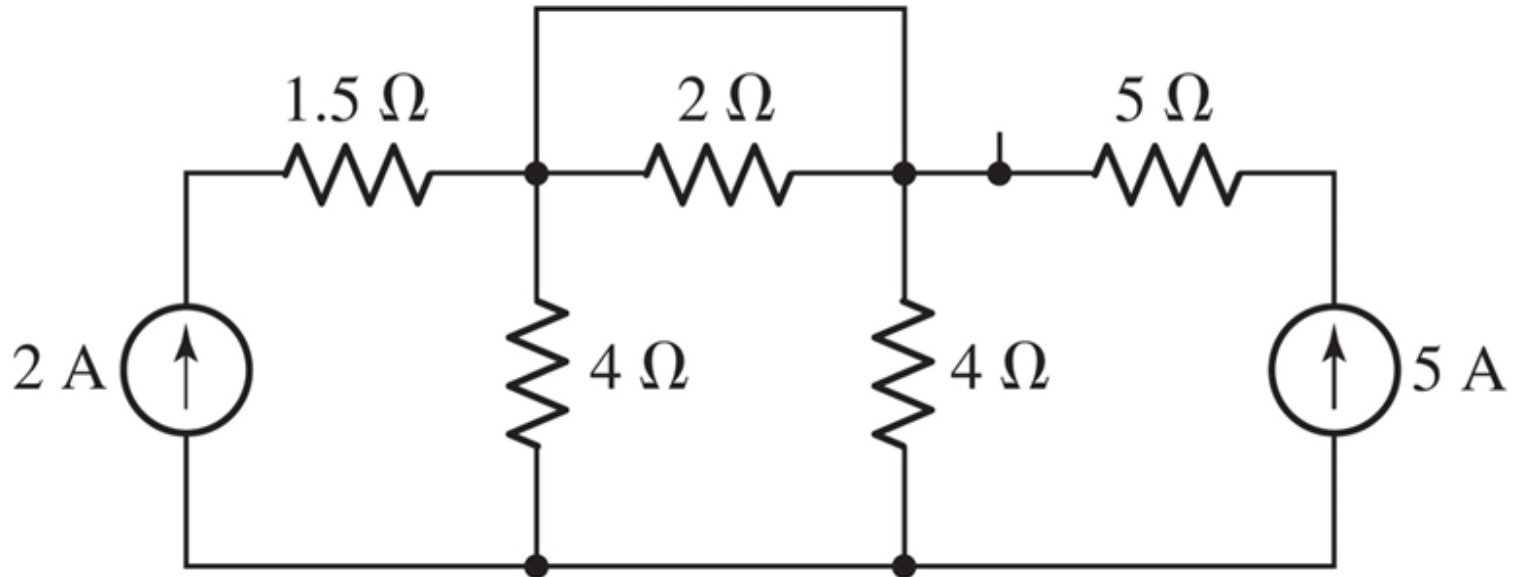
Example-06

- In the following circuit;
 - a. which individual elements are in series/in parallel?
 - b. which groups of elements are in series/in parallel?



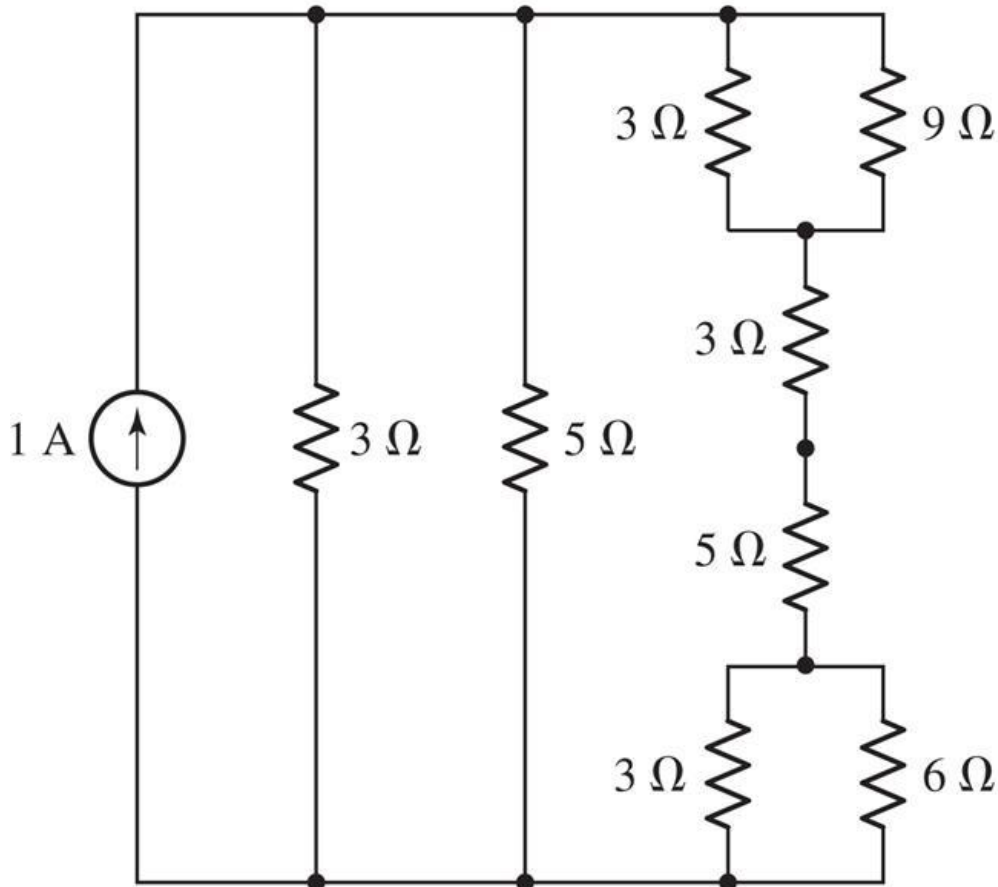
Example-07

- In the following circuit;
 - a. which individual elements are in series/in parallel?
 - b. which groups of elements are in series/in parallel?



Example-08

- In the following circuit;

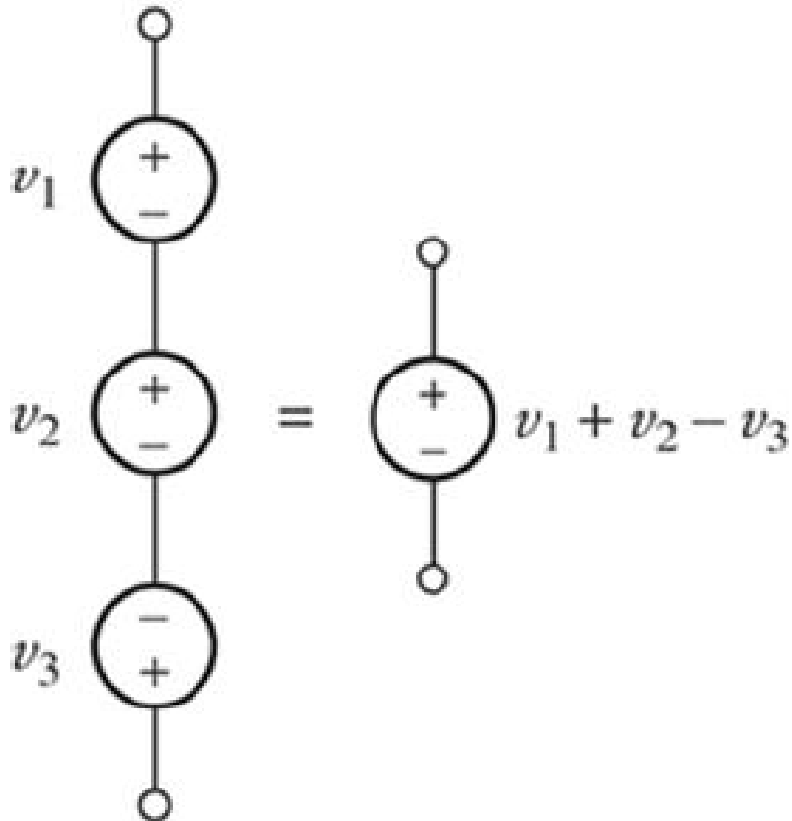


- a. which individual elements are in series/in parallel?
- b. which groups of elements are in series/in parallel?

Voltage Sources in Series

- can replace **voltage** sources in series with a **single equivalent source**

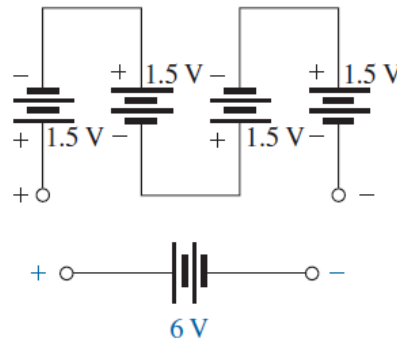
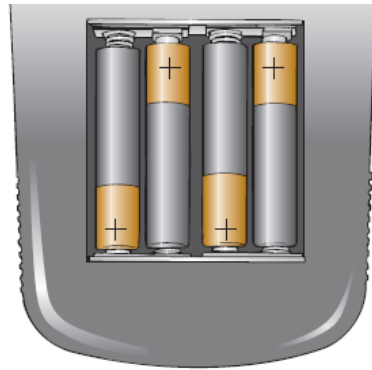
$$v_{\text{equivalent}}^{\text{series}} = \sum_{n=1}^N v_n$$



- all other voltage, current, & power relationships in the circuit remain **unchanged**
- might greatly simplify analysis of an otherwise complicated circuit

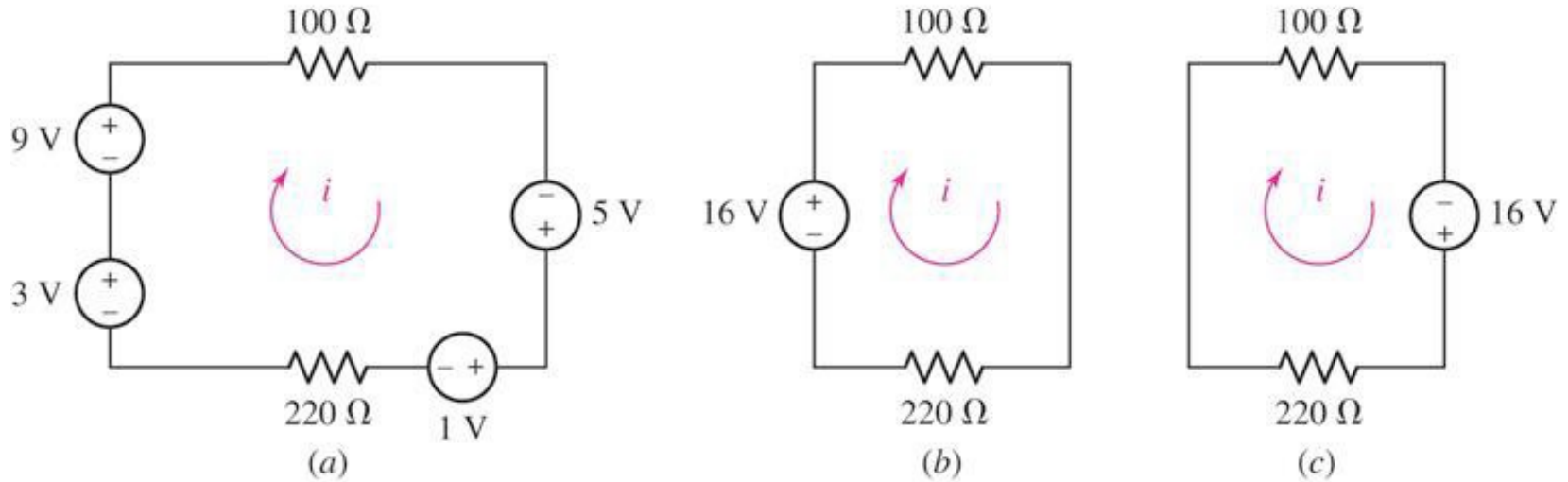
Voltage Sources in Series

- The connection of batteries in series to obtain a higher voltage is common in much of today's portable electronic equipment.



- Four 1.5V AAA batteries have been connected in series to obtain a source voltage of 6V.
 - The voltage has increased, but the maximum current for each AAA battery and for the 6V supply is the same.
 - The power available has increased by a factor of 4 due to the increase in terminal voltage.

Example-09

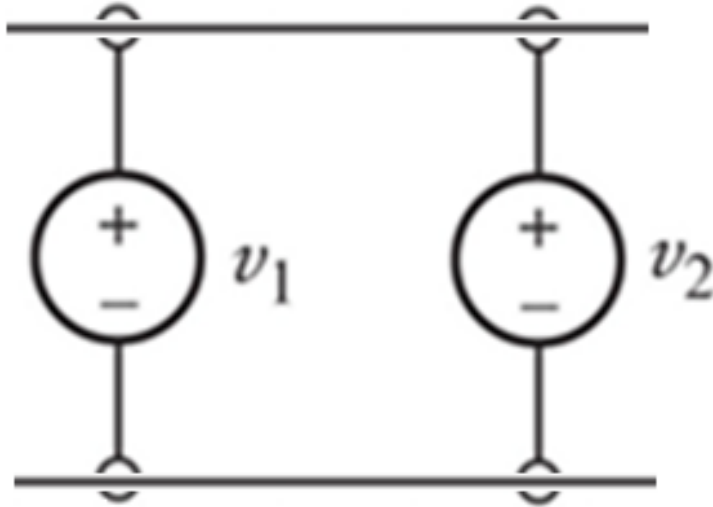


$$(a) \quad -3 - 9 + 100i - 5 + 1 + 220i = 0 \Rightarrow i = 16/320 = 50 \text{ mA}$$

$$(b,c) \quad -16 + 100i + 220i = 0 \Rightarrow i = 16/320 = 50 \text{ mA}$$

- The current and the power consumed by the resistors is the same in (a,b,c).
- However, the voltage sources must be broken out from the equivalent to solve for their individual powers delivered.

Voltage Sources in Parallel

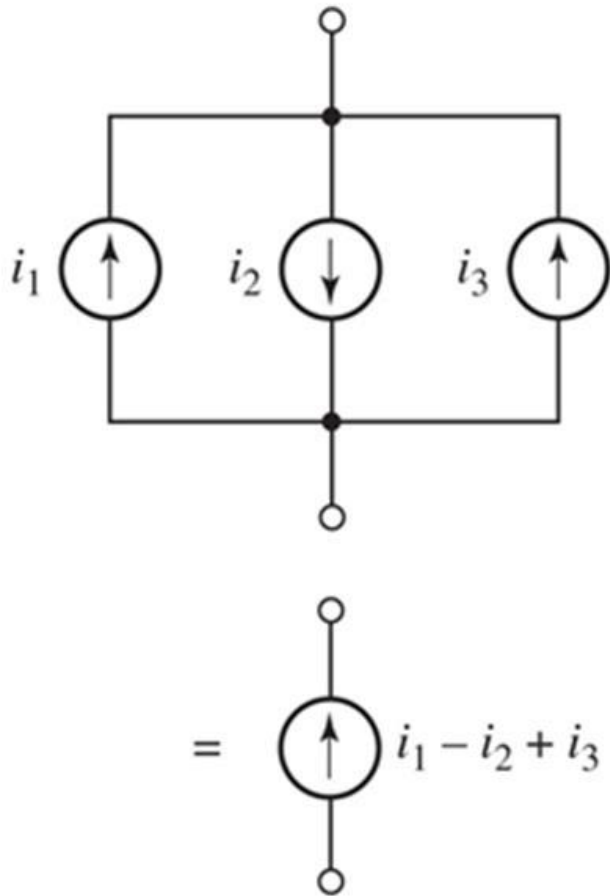


- Unless $v_1 = v_2 = \dots$, this circuit is not valid for ideal sources.
- All real voltage sources have internal resistance and are usually not exactly equal.
- Current will flow from the higher source to the lower source until equilibrium is reached (e.g. dangerously).
- Properly designed, a bank of equal voltage sources can deliver many times the current of a single source.

Current Sources in Parallel

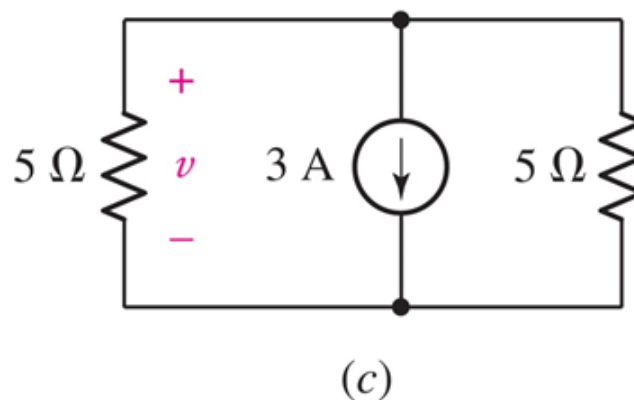
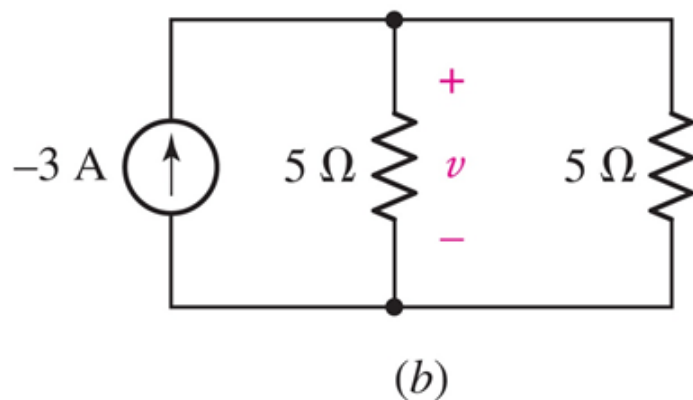
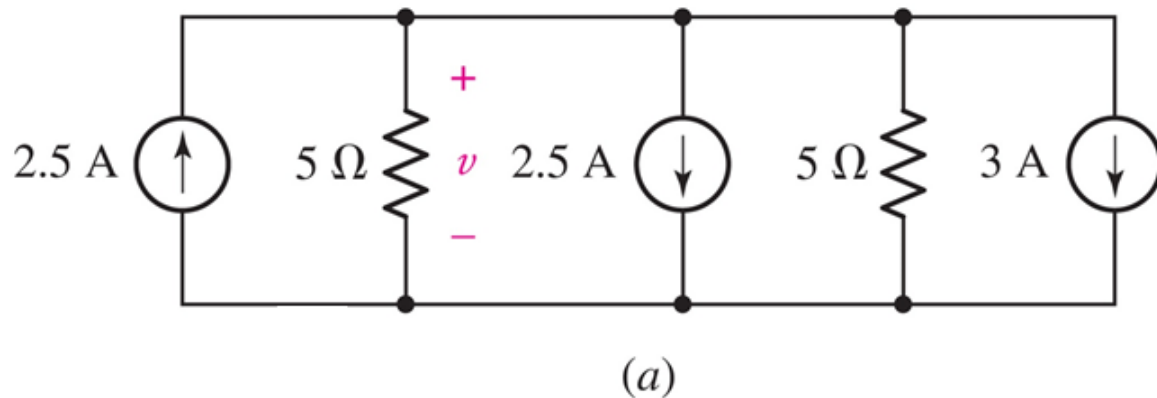
- can replace **current** sources in parallel with a **single equivalent source**

$$i_{\text{equivalent}}^{\text{parallel}} = \sum_{n=1}^N i_n$$



- all other voltage, current, & power relationships in the circuit remain **unchanged**
- as with voltage sources, this technique may simplify circuit analyses

Example-10

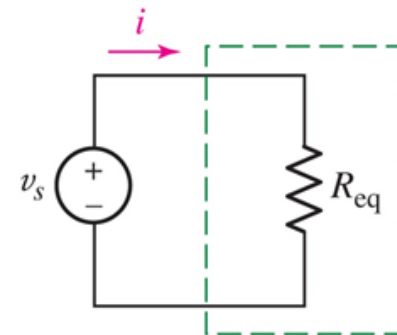
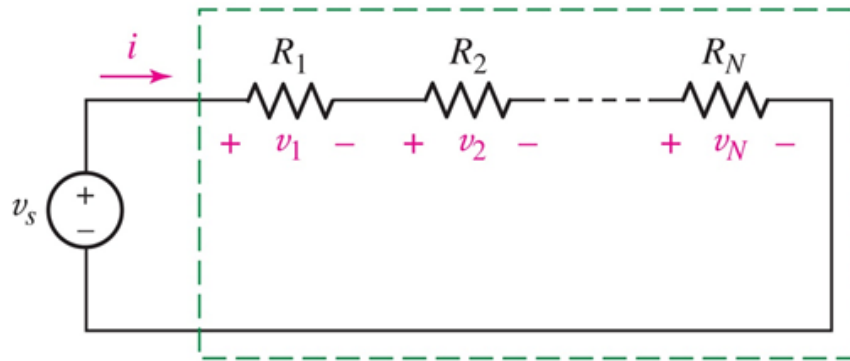


$$(a) \quad 2.5 - v/5 - 2.5 - v/5 - 3 = 0 \Rightarrow v = -7.5 \text{ V}$$

$$(b,c) \quad -3 - v/5 - v/5 = 0 \Rightarrow v = -7.5 \text{ V}$$

Resistors in Series

- As with voltage/current sources, resistors may also be replaced with equivalents.
 - In series, resistances are added.
 - the total resistance of series resistors is always larger than the value of the largest resistor.



$$-v_s + v_1 + v_2 + \dots + v_N = 0$$

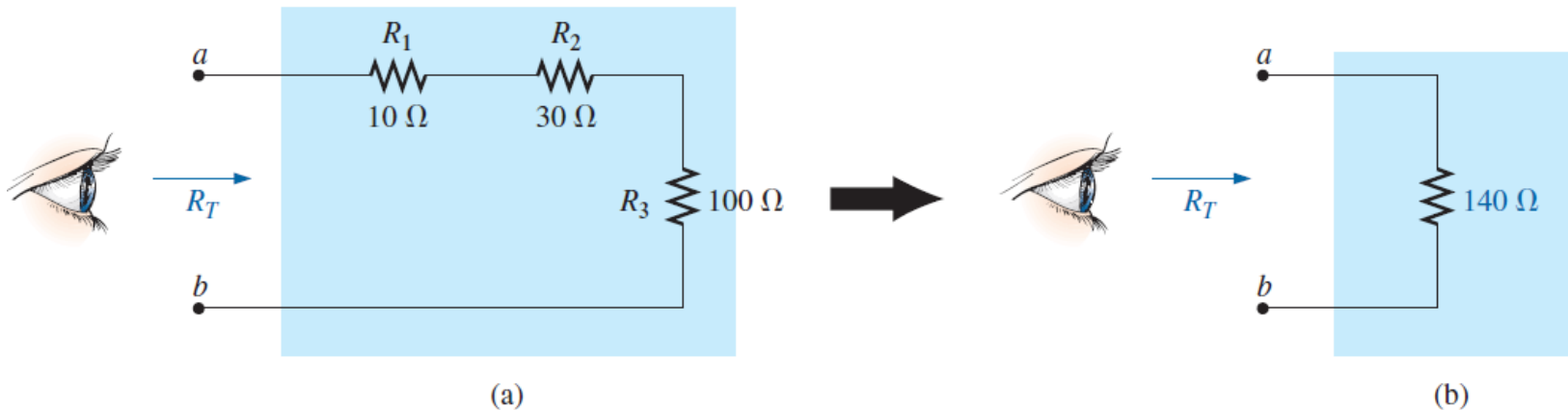
$$-v_s + iR_1 + iR_2 + \dots + iR_N = 0$$

$$-v_s + i[R_1 + R_2 + \dots + R_N] = 0$$

$$R_{\text{equivalent}}^{\text{series}} = \sum_{n=1}^N R_n$$

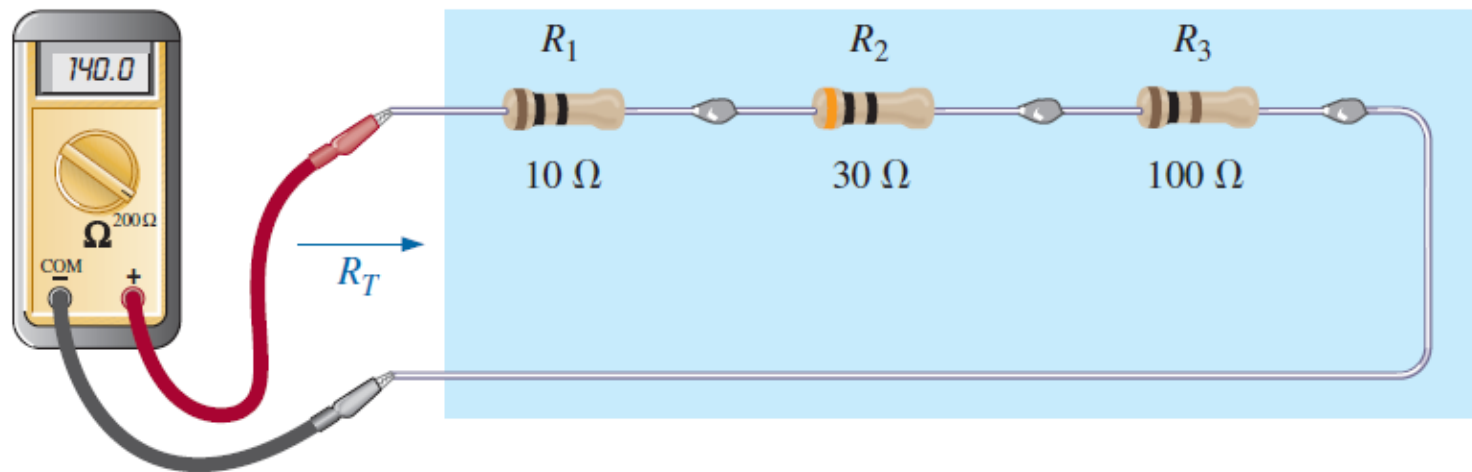
Resistors in Series

- It is important to realize that when a **dc supply** is connected, it does **not see** the individual connection of elements but simply the total resistance **seen** at the connection terminals
- Resistance **seen** at the terminals of a series circuit:



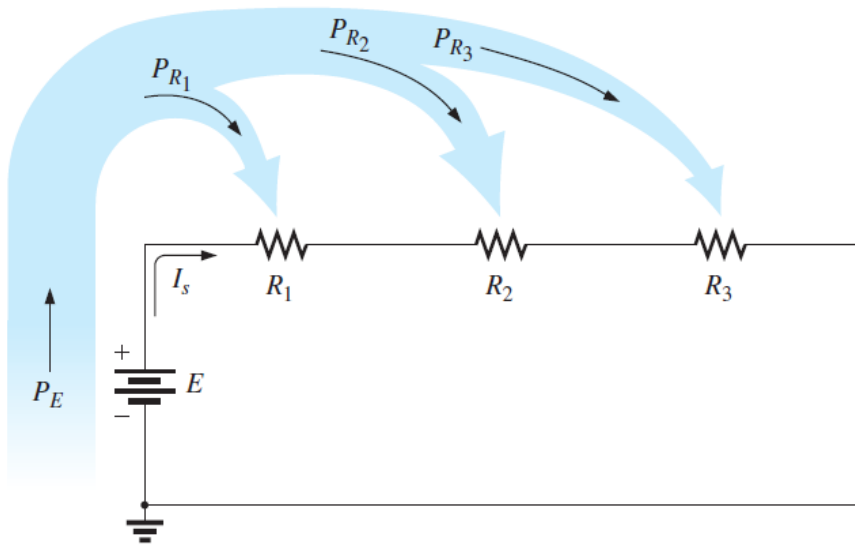
Resistors in Series

- The **total resistance** of any configuration can be measured by simply connecting an **ohmmeter** across the access terminals as shown below.



- Since there is no polarity associated with resistance, either lead can be connected to point a , with the other lead connected to point b .

Power Distribution in Series Circuit



$$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$

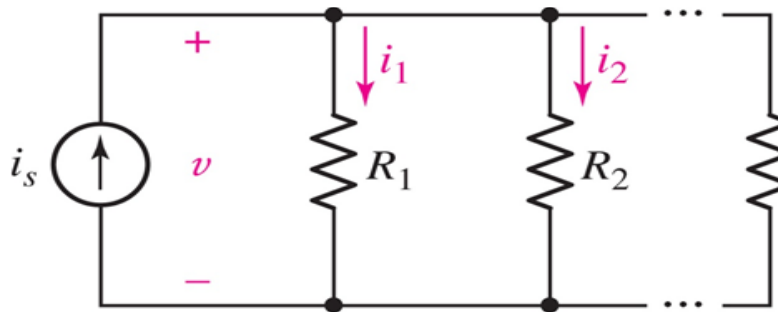
- For any network composed of resistive elements, the power applied by the battery will equal that dissipated by the resistive elements

- For R_1
$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1} \quad (\text{watts, W})$$

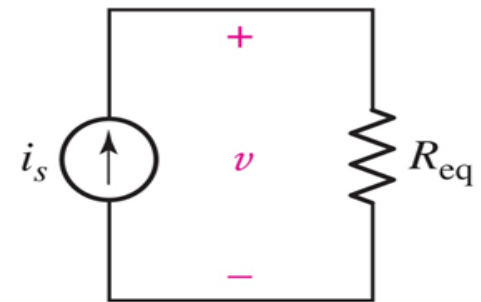
- In a series resistive network, the larger the resistor, the more the power absorbed.

Resistors in Parallel

- For resistors in parallel, the **reciprocals** of the resistances sum to **1 / (the equivalent)**.
 - the total resistance of parallel resistors is always less than the value of the smallest resistor.



(a)



(b)

$$-i_s + i_1 + i_2 + \dots + i_N = 0$$

$$-i_s + v/R_1 + v/R_2 + \dots + v/R_N = 0$$

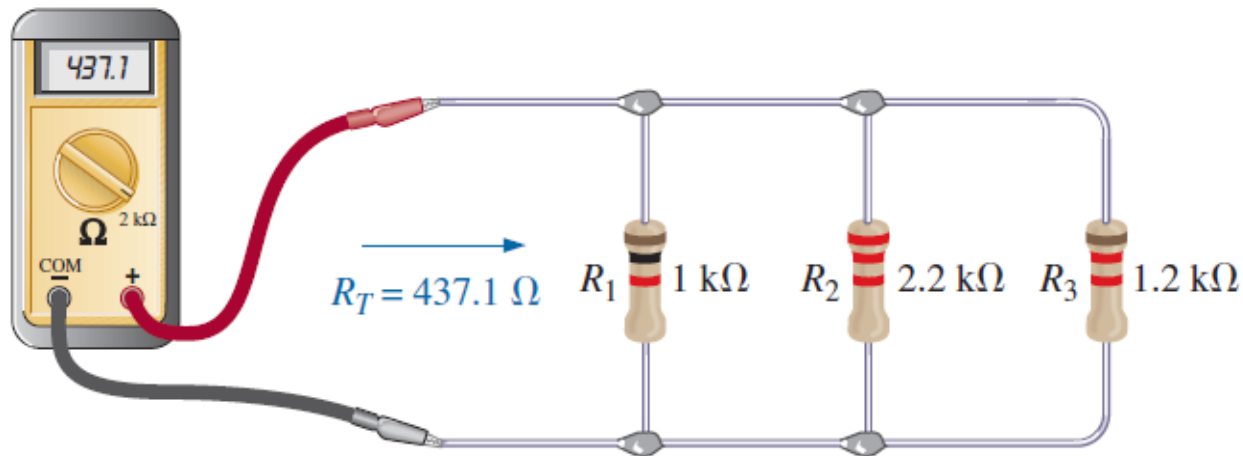
$$-i_s + v[1/R_1 + 1/R_2 + \dots + 1/R_N] = 0$$

$$-i_s + v[1/R_{\text{parallel equivalent}}] = 0$$

$$1/R_{\text{parallel equivalent}} = \sum_{n=1}^N 1/R_n$$

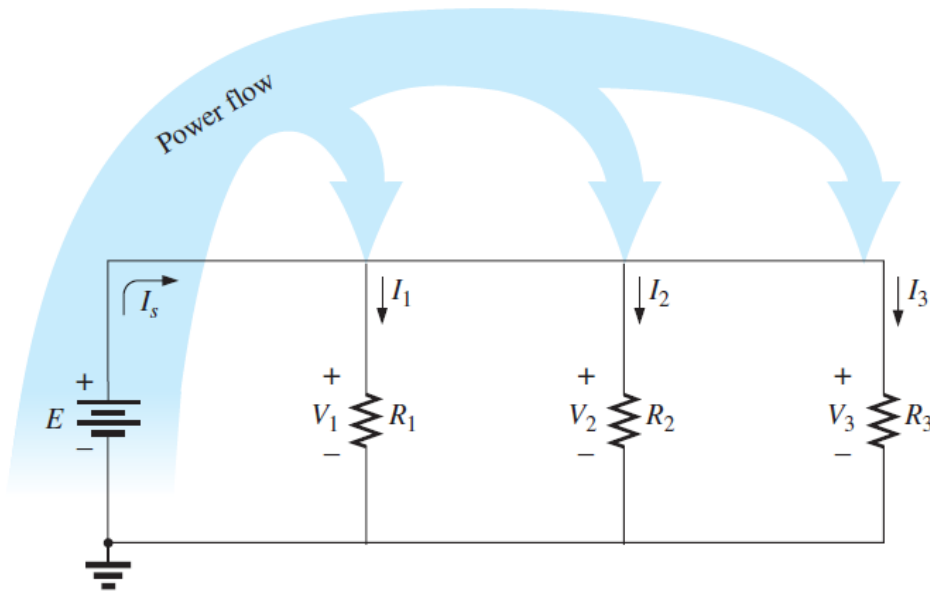
Resistors in Parallel

- The **total resistance** of any configuration can be measured by simply connecting an **ohmmeter** across the access terminals as shown below.



- There is no polarity to resistance, so either lead of the ohmmeter can be connected to either side of the network.
- Always keep in mind that ohmmeters can never be applied to a **live** circuit.

Power Distribution in Parallel Circuit



$$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$

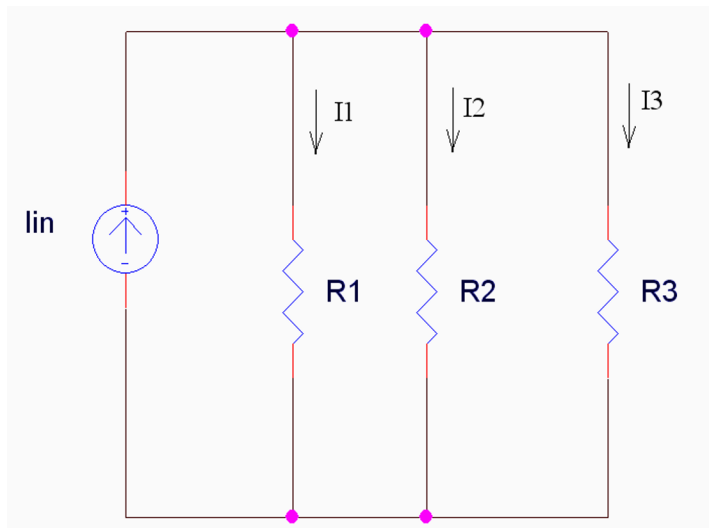
- For any network composed of resistive elements, the power applied by the battery will equal that dissipated by the resistive elements

- For R_1
$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1} \quad (\text{watts, W})$$

- In a parallel resistive network, the larger the resistor, the less the power absorbed.

Symbol for Parallel Resistors

- To make writing equations simpler, we use a symbol to indicate that a certain set of resistors are in parallel.



– Here, we would write

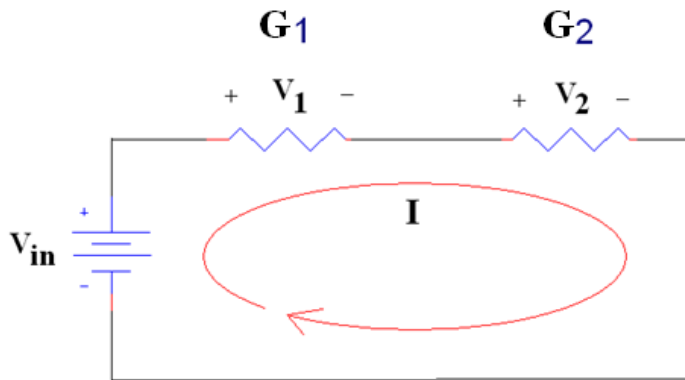
$$R1 \parallel R2 \parallel R3$$

to show that R1 is in parallel with R2 and R3.

- This also means that we should use the equation for equivalent resistance if this symbol is included in a mathematical equation.

If G is used instead of R

- In series:
 - The reciprocal of the equivalent conductance is equal to the sum of the reciprocal of each of the conductors in series



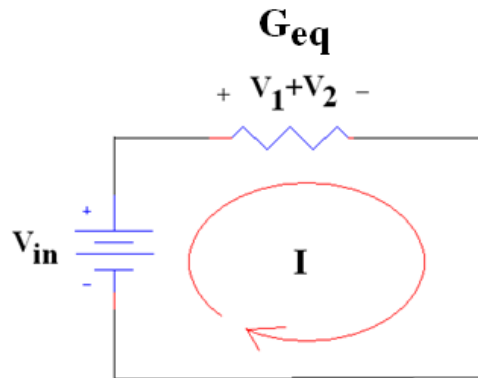
- In this example

$$1/G_{eq} = 1/G_1 + 1/G_2$$

- Simplifying

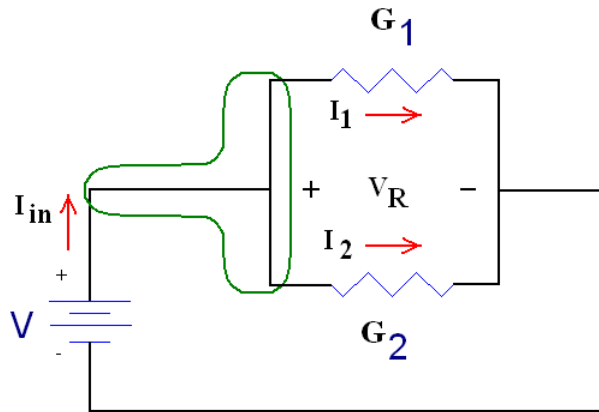
(only for 2 conductors in series)

$$G_{eq} = G_1 G_2 / (G_1 + G_2)$$



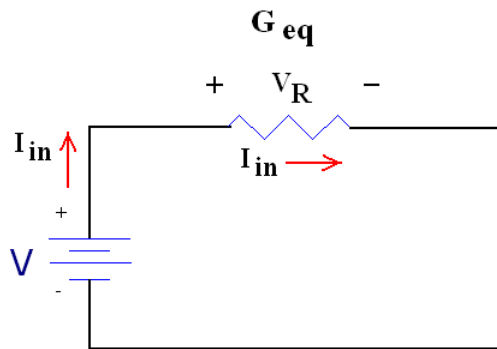
If G is used instead of R

- In parallel :
 - The equivalent conductance is equal to the sum of all of the conductors in parallel



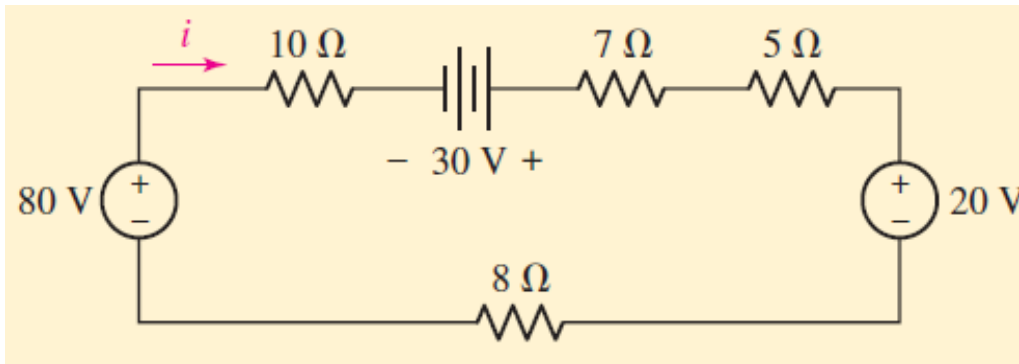
- In this example

$$G_{eq} = G_1 + G_2$$

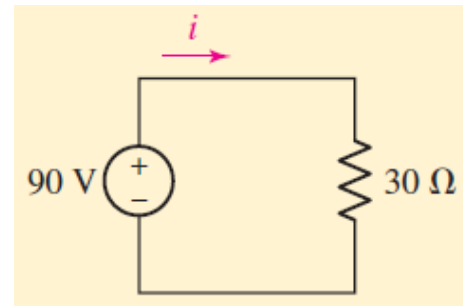
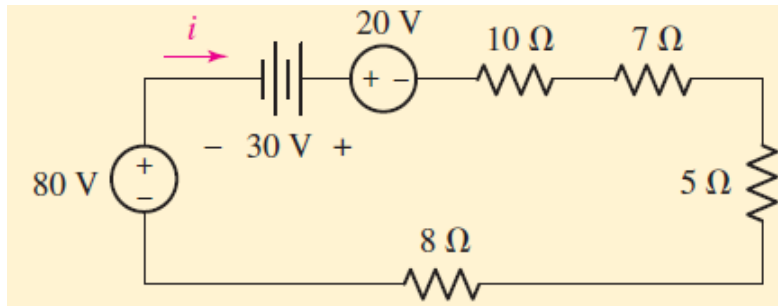


Example-11

- Use resistance and source combinations to determine the



current i and the power delivered by the 80 V source in this circuit .



$$-90 + 30i = 0$$

$$i = 3 \text{ A}$$

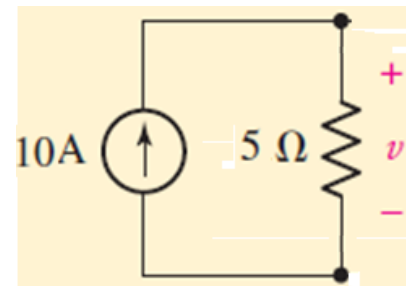
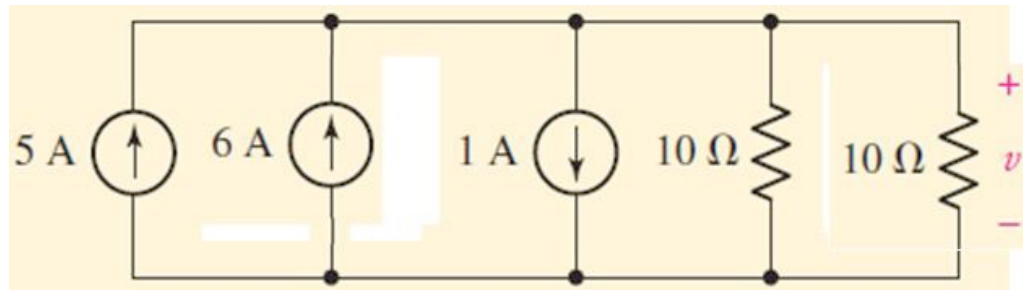
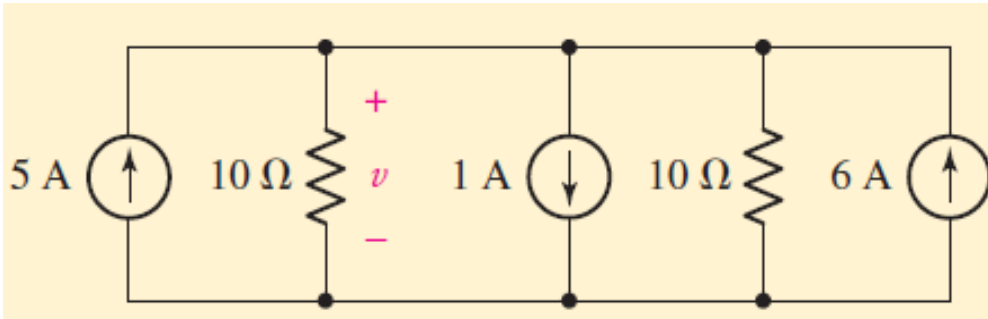
$$-80 \text{ V} \times 3 \text{ A} = -240 \text{ W}$$

Actually 240 W is supplied

Example-12

- Determine v in this circuit by first combining

the three current sources, and then the two 10 ohm resistors.



$$v = (5 - 1 + 6)10 // 10 = 10 \times 5 = \underline{50 \text{ V}}$$

For the same value resistors

- a. As you increase the number of resistors in series
 - Does R_{eq} increase or decrease?

- b. As you increase the number of resistors in parallel
 - Does R_{eq} increase or decrease?

Summary

Series and Parallel Circuits		
Series	Duality	Parallel
$R_T = R_1 + R_2 + R_3 + \cdots + R_N$	$R \nleftrightarrow G$	$G_T = G_1 + G_2 + G_3 + \cdots + G_N$
R_T increases (G_T decreases) if additional resistors are added in series	$R \nleftrightarrow G$	G_T increases (R_T decreases) if additional resistors are added in parallel
Special case: two elements	$R \nleftrightarrow G$	$G_T = G_1 + G_2$
$R_T = R_1 + R_2$		and $R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$
I the same through series elements	$I \nleftrightarrow V$	V the same across parallel elements
$E = V_1 + V_2 + V_3$	$E, V \nleftrightarrow I$	$I_T = I_1 + I_2 + I_3$
Largest V across largest R	$V \nleftrightarrow I$ and $R \nleftrightarrow G$	Greatest I through largest G (smallest R)
$V_x = \frac{R_x E}{R_T}$	$E, V \nleftrightarrow I$ and $R \nleftrightarrow G$	$I_x = \frac{G_x I_T}{G_T} = \frac{R_T I_T}{R_x}$
		with $I_1 = \frac{R_2 I_T}{R_1 + R_2}$ and $I_2 = \frac{R_1 I_T}{R_1 + R_2}$
$P = EI_T$	$E \nleftrightarrow I$ and $I \nleftrightarrow E$	$P = I_T E$
$P = I^2 R$	$I \nleftrightarrow V$ and $R \nleftrightarrow G$	$P = V^2 G = V^2 / R$
$P = V^2 / R$	$V \nleftrightarrow I$ and $R \nleftrightarrow G$	$P = I^2 / G = I^2 R$

