Detecting the Tricritical Point of the $2d$ Blume–Capel Model

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The spin-1 two-dimensional classical Blume-Capel model on a square lattice is known to exhibit a tricritical point described by the tricritical Ising CFT with central charge $c = 7/10$. By using the Renyi entropies via a replica-trick on classical statistical mechanical systems, and calculating the Renyi Mutual Information (RMI) with Monte Carlo simulations, we can extract the value of the central charge at the tricritical point. We vary the parameters of Hamiltonian such that we obtain the tricritical point reproducing the correct central charge value predicted by CFT.

Introduction – The Hamiltonian of the spin-1 Blume-Capel model on a two-dimensional square lattice [1, 2] is given by

$$H = -J \sum_{\langle ij \rangle} S_i^z S_j^z + D \sum_{\langle i \rangle} (S_i^z)^2, \tag{1}$$

where $S_i^z = \pm 1, 0$. Though the model cannot be solved exactly, it has been shown to exhibit a tricritical point described by the tricritical Ising CFT with central charge $c = 7/10$ [3]. However, the position of the tricritical point can be found only numerically as this is not an exactly solvable model. There has been an extensive study in the literature using various sophisticated numerical techniques [4–17] to pin down the values of the parameters D, J and temperature T of the tricritical point. Here we endeavor to detect this phase transition point by using the quantity called Renyi Mutual Information (RMI), which is able to detect all correlations in a physical system, even those missed by traditional connected correlation functions. The method of detection of detection of classical phase transitions using RMI was developed by two of the authors [18, 19]. Since the RMI can detect finite-temperature critical points, and even identify their universality class, without the knowledge of an order parameter or other thermodynamic estimators, if we are sitting exactly at the tricirtical point, we expect to extract a value of c from our numerical simulations, matching with the CFT value of 0.7.

Results – Varying (D_c/J, kT_c/J) values, we find that the closest match to the actual $c = 0.7$ is obtained for ($D_c/J = 1.9$, $kT_c/J = 0.79$). We also provide a χ^2 estimate, which clearly indicates this as the best fit.

Discussion –

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