CSU33081: Computational Mathematics

Assignment 3 - Senán d'Art - 17329580

Question 1

Using the code:

```
a = InfinityNorm([-2 1 0; 1 -2 1; 0 1 -1.5])
b = InfinityNorm([4 -1 0 1 0; -1 4 -1 0 1; 0 -1 4 -1 0; 1 0 -1

function N = InfinityNorm (A)
    N = max(sum(abs(A)));
end
```

$a = 4$
$b = 7$

Answer: (i)

Question 2

Fourth order Lagrange polynomial:

$$f(x) = \frac{(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)}y_1 + \frac{(x-x_1)(x-x_3)(x-x_4)(x-x_5)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)}y_2 + \frac{(x-x_1)(x-x_2)(x-x_4)(x-x_5)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)(x_3-x_5)}y_3 + \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_5)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_5)}y_4 + \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_3)}y_5$$

Variables:

$$x_1 = 14, x_2 = 22, x_3 = 30, x_4 = 38, x_5 = 46$$

$$y_1 = 320, y_2 = 490, y_3 = 540, y_4 = 500, y_5 = 480$$

$$f(x) = \frac{(x-22)(x-30)(x-38)(x-46)}{(14-22)(14-30)(14-38)(14-46)}320+ \\ \frac{(x-14)(x-30)(x-38)(x-46)}{(22-14)(22-30)(22-38)(22-46)}490+ \\ \frac{(x-14)(x-22)(x-38)(x-46)}{(30-14)(30-22)(30-38)(30-46)}540+ \\ \frac{(x-14)(x-22)(x-30)(x-46)}{(38-14)(38-22)(38-30)(38-46)}500+ \\ \frac{(x-14)(x-22)(x-30)(x-38)}{(46-14)(46-22)(46-30)(46-30)}480$$

$$f(x) = \frac{5x^4 - 460x^3 + 9760x^2 + 11280x - 434640}{6144}$$

$x = 26$

Result:

$f(26) = 530$

Answer: $530W$

Question 3

3-point, central distance formula results in: $O(h^2)$

Answer:(ii)

Question 4

(a)

3-point, backwards distance:

$$f'(x_{i+2}) = \frac{x_{i+2} - x_{i+1}}{(x_i - x_{i+1})(x_i - x_{i+2})}y_i+ \\ \frac{x_{i+2} - x_i}{(x_{i+1} - x_i)(x_{i+1} - x_{i+2})}y_{i+1}+ \\ \frac{2x_{i+2} - x_i - x_{i+1}}{(x_{i+2} - x_i)(x_{i+2} - x_{i+1})}y_{i+2}$$

Male:
$x_{i+2} = 2006$

$x_{i+1} = 2003$
$x_i = 2002$

$y_{i+2} = 665,647$
$y_{i+1} = 646,493$
$y_i = 638,182$

$$f'(x_{i+2}) = \frac{(2006) - (2003)}{((2002) - (2003))((2002) - (2006))}(638182) + \frac{(2006) - (2002)}{((2003) - (2002))((2003) - (2006))}(646493) + \frac{2(2006) - (2002) - (2003)}{((2006) - (2002))((2006) - (2003))}(665647)$$

$$f'(x_{i+2}) = \frac{957273}{2} - \frac{2585972}{3} + \frac{4659529}{12} = 4940$$

Female:
$x_{i+2} = 2006$
$x_{i+1} = 2003$
$x_i = 2002$

$y_{i+2} = 256,257$
$y_{i+1} = 225,042$
$y_i = 215,005$

$$f'(x_{i+2}) = \frac{(2006) - (2003)}{((2002) - (2003))((2002) - (2006))}(215005) + \frac{(2006) - (2002)}{((2003) - (2002))((2003) - (2006))}(225042) + \frac{2(2006) - (2002) - (2003)}{((2006) - (2002))((2006) - (2003))}(256257)$$

$$f'(x_{i+2}) = 10,681$$

(b)

3-point, central distance:

$$f'(x_{i+1}) = \frac{x_{i+1} - x_{i+2}}{(x_i - x_{i+1})(x_i - x_{i+2})}y_i + \frac{2x_{i+1} - x_i - x_{i+2}}{(x_{i+1} - x_i)(x_{i+1} - x_{i+2})}y_{i+1} + \frac{x_{i+1} - x_i}{(x_{i+2} - x_i)(x_{i+2} - x_{i+1})}y_{i+2}$$

Males:

Using formula, prediction = 673601

$$\frac{673601 - 677807}{677807} = -0.0062 = 0.62\%$$

Females:

Using formula, prediction = 277987

$$\frac{277987 - 276417}{276417} = 0.00567 = 0.57\%$$

Answer: (ii)