ST3009: Statistical Methods for Computer Science

Week 3 Assignment - Senán d'Art - 17329580

Question 1

(a)

Probability of rolling an exact number is: $\frac{1}{6}$

For 6 consecutive rolls:

$$\frac{1}{6} * \frac{1}{6} * \frac{1}{6} * \frac{1}{6} * \frac{1}{6} * \frac{1}{6} = (\frac{1}{6})^6 = \frac{1}{42,656} = 0.00002344336084$$

(b)

Probability of rolling three 3's:

$$(\frac{1}{6})^3$$

Probability of rolling any number other than 3 twice:

$$(\frac{5}{6})^2$$

Probability of both. However we must take into account the two rolls that do not produce a 3 can be in any 2 of 6 locations:

$$\binom{6}{2} * (\frac{5}{6})^2 * (\frac{1}{6})^4 = 0.00803755144$$

(c)

The 1 can be in any of six locations (similar to above):

$$\binom{6}{1} * \frac{1}{6} * (\frac{5}{6})^5 = 0.401877572$$

(d)

1 - (Probability of not rolling a 1, six times in a row)

$$1 - (\frac{5}{6})^6 = 0.6651020233$$

Question 2

Probability of rolling a 1 in the case of the first die: $\frac{1}{6}$
Probability of rolling a 1 in the case of the second die: $\frac{1}{20}$
$P(A) = \frac{1}{6}$
$P(B) = \frac{1}{6} * \frac{1}{20} = \frac{1}{120}$
Definition of independence: $P(A \bigcap B) = P(A)P(B)$

$$P(A)P(B) = \frac{1}{6} * \frac{1}{120} = \frac{1}{720}$$

$$P(A \bigcap B) = P(A|B)P(B) = 1 * \frac{1}{120}$$

$P(A \bigcap B) \neq P(A)P(B)$

The events are **not independent**.

Question 3

(a)

Probability of picking the correct password:

$$P(wrong) * P(wrong) * P(wrong) * ... * P(right)$$

This becomes:

$$P(wrong)^{k-1} * P(right)$$

As we are removing one each time it is incorrect:

$$\frac{n-1}{n} * \frac{n-2}{n-1} * \frac{n-3}{n-2} * ... * \frac{1}{n-(k-1)}$$

(b)

When $n = 6$ and $k = 3$

$$\frac{6-1}{6} * \frac{6-2}{6-1} * \frac{1}{6-(3-1)} = 0.1666666667$$

(c)

In this case we are not removing passwords when they are tested. Again:

$$P(wrong) * P(wrong) * P(wrong) * ... * P(right)$$

As in part (a):

$$P(wrong)^{k-1} * P(right)$$

But because they are not being removed we can write it as:

$$(\frac{n-1}{n})^{k-1} * \frac{1}{n}$$

(d)

$$(\frac{6-1}{6})^{3-1} * \frac{1}{6} = 0.1157407407$$

Question 4

(a)

1 - (Probability of not getting flagged 3 times)

$$1 - (0.3)^3 = 0.973$$

(b)

1 - (Probability of not being flagged 3 times)

$$1 - (0.95)^3 = 0.142625$$

(c)

$P(R) = 0.1$, probability that a visitor is a robot
$P(R^c) = 0.9$, probability that a visitor is not a robot
$P(F|R) = 0.973$, probability that a visitor is flagged, given they are a robot
$P(F|R^c) = 0.142625$, probability that a visitor is flagged, given they are not a robot

$$P(R|F) = \frac{P(F|R)P(R)}{P(F|R)P(R) * P(F|R^c)P(R^c)}$$

When we include the values:

$$P(R|F) = \frac{(0.973)(0.1)}{(0.973)(0.1) * (0.142625)(0.9)} = 0.431174874$$