

# ST3009: Statistical Methods for Computer Science

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## Week 3 Assignment - Senán d'Art - 17329580

### Question 1

#### (a)

Probability of rolling an exact number is:  $\frac{1}{6}$

For 6 consecutive rolls:

$$\frac{1}{6} * \frac{1}{6} * \frac{1}{6} * \frac{1}{6} * \frac{1}{6} * \frac{1}{6} = \left(\frac{1}{6}\right)^6 = \frac{1}{42,656} = 0.00002344336084$$

#### (b)

Probability of rolling three 3's:

$$\left(\frac{1}{6}\right)^3$$

Probability of rolling any number other than 3 twice:

$$\left(\frac{5}{6}\right)^2$$

Probability of both. However we must take into account the two rolls that do not produce a 3 can be in any 2 of 6 locations:

$$\binom{6}{2} * \left(\frac{5}{6}\right)^2 * \left(\frac{1}{6}\right)^4 = 0.00803755144$$

#### (c)

The 1 can be in any of six locations (similar to above):

$$\binom{6}{1} * \frac{1}{6} * \left(\frac{5}{6}\right)^5 = 0.401877572$$

#### (d)

1 - (Probability of not rolling a 1, six times in a row)

$$1 - \left(\frac{5}{6}\right)^6 = 0.6651020233$$

### Question 2

Probability of rolling a 1 in the case of the first die:  $\frac{1}{6}$

Probability of rolling a 1 in the case of the second die:  $\frac{1}{20}$

$$P(A) = \frac{1}{6}$$

$$P(B) = \frac{1}{6} * \frac{1}{20} = \frac{1}{120}$$

Definition of independence:  $P(A \cap B) = P(A)P(B)$

$$P(A)P(B) = \frac{1}{6} * \frac{1}{120} = \frac{1}{720}$$

$$P(A \cap B) = P(A|B)P(B) = 1 * \frac{1}{120}$$

$$P(A \cap B) \neq P(A)P(B)$$

The events are **not independent**.

### Question 3

(a)

Probability of picking the correct password:

$$P(wrong) * P(wrong) * P(wrong) * \dots * P(right)$$

This becomes:

$$P(wrong)^{k-1} * P(right)$$

As we are removing one each time it is incorrect:

$$\frac{n-1}{n} * \frac{n-2}{n-1} * \frac{n-3}{n-2} * \dots * \frac{1}{n-(k-1)}$$

(b)

When  $n = 6$  and  $k = 3$

$$\frac{6-1}{6} * \frac{6-2}{6-1} * \frac{1}{6-(3-1)} = 0.1666666667$$

(c)

In this case we are not removing passwords when they are tested. Again:

$$P(wrong) * P(wrong) * P(wrong) * \dots * P(right)$$

As in part (a):

$$P(wrong)^{k-1} * P(right)$$

But because they are not being removed we can write it as:

$$\left(\frac{n-1}{n}\right)^{k-1} * \frac{1}{n}$$

**(d)**

$$\left(\frac{6-1}{6}\right)^{3-1} * \frac{1}{6} = 0.1157407407$$

#### Question 4

**(a)**

1 - (Probability of not getting flagged 3 times)

$$1 - (0.3)^3 = 0.973$$

**(b)**

1 - (Probability of not being flagged 3 times)

$$1 - (0.95)^3 = 0.142625$$

**(c)**

$P(R) = 0.1$ , probability that a visitor is a robot

$P(R^c) = 0.9$ , probability that a visitor is not a robot

$P(F|R) = 0.973$ , probability that a visitor is flagged, given they are a robot

$P(F|R^c) = 0.142625$ , probability that a visitor is flagged, given they are not a robot

$$P(R|F) = \frac{P(F|R)P(R)}{P(F|R)P(R) + P(F|R^c)P(R^c)}$$

When we include the values:

$$P(R|F) = \frac{(0.973)(0.1)}{(0.973)(0.1) + (0.142625)(0.9)} = 0.431174874$$