

ST3009: Statistical Methods for Computer Science

Week 5 Assignment - Senán d'Art - 17329580

Question 1

(a)

There are 5 marbles of each colour. Therefore if we pick any marble (with probability 1), the chances of picking another marble of the same colour are $\frac{4}{9}$.

If the colours are the same we win \$1.10, otherwise we lose \$1.00.

$$E[X] = (1.1 * \frac{4}{9}) - (1 * \frac{5}{9}) = -0.06667$$

(b)

Variance can be described as:

$$Var[X] = E[X^2] - (E[X])^2$$

Using the results from **(a)**:

$$Var[X] = (\frac{4}{9} * 1.1^2) + (\frac{5}{9} * 1^2) - (-0.06667)^2 = 1.08889$$

Question 2

(a)

Again we can use the formulas as above:

$$E[X] = (1 * 0.6) - (0 * 0.4) = 0.6$$

$$Var[X] = (0.6 * 1^2) + (0.4 * 0^2) - 0.6^2 = 0.24$$

(b)

Assuming $E[X] = E[X_1] + E[X_2] + \dots + E[X_n]$.

Yes because of the linearity of expected value:

$$E[aX + bY] = aE[X] + bE[Y]$$

(c)

$$E[Y] = E[X_1] + E[X_2] + \dots + E[X_n] = n * \mu$$

$$E\left[\frac{1}{n}Y\right] = \frac{1}{n} * (E[X_1] + E[X_2] + \dots + E[X_n]) = \mu$$

(d)

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Question 3

(a)

Since each time we remove a ball, it is not replaced, it has an influence on the next ball. In this case $1 = \text{white}$, $0 = \text{red}$.

$$X_1 = 0, X_2 = 0$$

$$\frac{8}{13} * \frac{7}{12} = 0.35897$$

$$X_1 = 0, X_2 = 1$$

$$\frac{8}{13} * \frac{5}{12} = 0.25641$$

$$X_1 = 1, X_2 = 0$$

$$\frac{5}{13} * \frac{8}{12} = 0.25641$$

$$X_1 = 1, X_2 = 1$$

$$\frac{5}{13} * \frac{4}{12} = 0.12821$$

(b)

Definition of independence: $P(X)P(Y) = P(X \cup Y)$

Take event: $X_1 = 0, X_2 = 0$

$P(X \cup Y)$, can be written as: $P(Y) * (P(X) \text{ given } Y)$

$$\frac{8}{13} * \frac{8}{13} \neq \frac{8}{13} * \frac{7}{13}$$

They are not independent.

(c)

$E[X_2]$ can be calculated using the values from **(a)** and the fact that if X_2 is white, its value is 1. Otherwise 0.

$$E[X_2] = (0.25641 * 1) + (0.12821 * 1) + (0.35897 * 0) + (0.25641 * 0) = 0.38462$$

(d)

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