ST3009: Statistical Methods for Computer Science

Week 2 Assignment - Senán d'Art - 17329580

Question 1

(a)

The *sample space* is the set of all possible outcomes. Each roll has 6 possible outcomes:

$$6 * 6 * 6 = 6^3 = 216$$

(b)

Possibilities:

One 2, the 2 can be in any of 3 rolls, the remaining two rolls can have 5 values:

$$\binom{3}{1} * 5^2 = 75$$

Two 2s, 2 in two of 3 rolls, remaining roll can have 5 values:

$$\binom{3}{2} * 5 = 15$$

Three 2s:

$$\binom{3}{3} = 1$$

$Total = 91$

$$\frac{91}{216} = 0.4212962963$$

(c)

Methodology:

Roll a 6-sided die 3 times.

Repeat 10,000 times.

Count number of times at least one 2 was rolled in each set.

Return probability of rolling at least one 2.

```
S = 6; %6-sided die
R = 3; %roll 3 times
N = 1; %1 die
T = 10000; %run 10,000 times
Out = randi([1 S],[R N T]);
twoCount = 0;
for n = 1:T
    if any(Out(:,:,n)==2)
        twoCount = twoCount + 1;
    end
end

twoCount/T
```

(d)

To get 17, combination must be 6, 6, 5 (in any order).

To get the number of orderings, consider the location of the 5. There are 3.

$$Probability = \frac{3}{216} = 0.01388889$$

(e)

First roll is a 1 so we only consider the two remaining rolls.

Total rolls:

$$\binom{6}{1} * \binom{6}{1} = 6 * 6 = 6^2 = 36$$

Since the remaining two rolls must sum to 11, the two numbers rolled must be [5, 6].

There are two possible orderings: [5, 6], [6, 5]

$$\frac{2}{36} = 0.055555556$$

Question 2

(a)

Two parts:

Probability of rolling a 1 *and* rolling a 5 on a 6-sided die:

$$\frac{1}{6} * \frac{1}{6} = \frac{1}{36}$$

Probability of not rolling a 1 *and* rolling a 5 on a 20-sided die:

$$\frac{5}{6} * \frac{1}{20} = \frac{5}{120}$$

Total:

$$\frac{1}{36} * \frac{5}{120} = 0.069444444$$

(b)

Only 1 die can roll a 15 so we must roll anything *but* 1 on the first die:

$$\frac{5}{6} * \frac{1}{20} = 0.041666667$$

Question 3

$P(Guilt) = 0.6$

$P(Characteristic) = 0.2$

$$P(Guilt|Characteristic) = \frac{P(Characteristic|Guilt)P(Guilt)}{P(Characteristic)}$$

This becomes:

$$P(Guilt|Characteristic) = \frac{P(Characteristic|Guilt)P(Guilt)}{P(Characteristic|Guilt)P(Guilt) + P(Characteristic|Guilt^C)P(Guilt^C)}$$

Which when we include the required values becomes:

$$\frac{1 * 0.6}{(1 * 0.6) + (0.2 * 0.4)} = 0.8823$$

Question 4

Methodology similar to Q3 but with a matrix.

Known values: $P(Location)$, $P(Observing|Location)$

Based of these we can get: $P(Location^C)$, $P(Observing|Location^C)$

These let us calculate: $P(Location|Observing)$, using:

$$P(Location|Observing) = \frac{P(Observing|Location)P(Location)}{P(Observing|Location)P(Location) + P(Observing|Location^C)P(Location^C)}$$

For example, the cell highlighted in the question:

$P(Location) = 0.05$

$P(Location^C) = 0.95$

$P(Observing|Location) = 0.75$

$P(Observing|Location^C) = 0.25$

$$P(Location|Observing) = \frac{0.75 * 0.05}{(0.75 * 0.05) + (0.25 * 0.95)} = 0.1363$$

This is the implementation in Matlab:

Define the matrix of $PriorBeliefofLocation = PriorBelief$

Define the matrix of $P(OberserveTwoBarsofsignal|Location) = ProbabilityOfBars$

Instantiate a zero-filled matrix of size 4 * 4 $\rightarrow PostBelief$

For each location in $PostBelief$, perform the operation defined above where the inputs are the matching locations from the other two matrices.

Print the result.

```
PriorBelief = [0.05 0.1 0.05 0.05;
               0.05 0.1 0.05 0.05;
               0.05 0.05 0.1 0.05;
               0.05 0.05 0.1 0.05];
ProbabilityOfBars = [0.75 0.95 0.75 0.05;
                     0.05 0.75 0.95 0.75;
                     0.01 0.05 0.75 0.95;
                     0.01 0.01 0.05 0.75];
size = 4
PostBelief = zeros(size, size);
for n = 1:size
    for m = 1:size
        PostBelief(n,m) = (ProbabilityOfBars(n,m)*PriorBelief(n,m))/((ProbabilityOfB
    end
end

PostBelief
```

Result:

$$\begin{matrix} 0.1364 & 0.6786 & 0.1364 & 0.0028 \\ 0.0028 & 0.2500 & 0.5000 & 0.1364 \\ 0.0005 & 0.0028 & 0.2500 & 0.5000 \\ 0.0005 & 0.0005 & 0.0058 & 0.1364 \end{matrix}$$