ST3009: Statistical Methods for Computer Science

Week 1 Assignment - Senán d'Art - 17329580

Question 1

(a)

No restrictions so letters can be in any order. To generate all possible results: $$ \binom{10}{1} \binom{9}{1} \binom{8}{1} \binom{7}{1} \binom{6}{1} \binom{5}{1} \binom{4}{1} \binom{3}{1} \binom{2}{1} \binom{1}{1} $$ As every time we choose a letter it is removed from the pool of available letters. So for the first letter we can choose 1 of the available 10, for the second there are only 9 letters remaining so we can choose 1 of 9 and so on.
This becomes: $$ 1098765432*1 $$ Which is: $$ 10! = 3,628,800 $$

(b)

Restrictions of *E* and *F* being next to each other but in any order means we can treat it as a pool of 9 characters. Following the reasoning of part (a) we start at choosing 1 of 9 possible letters followed by 1 of 8 and so forth. $$ 987654321 = 9! $$ But since *E* and *F* can be in the order *EF* or *FE* we need to multiply this by 2. $$ 9! * 2 = 725,760 $$

(c)

Word 'BANANA' contains 6 letters. Ignoring the fact that *A* and *N* are repeated we have a total number of possibilities of: $$ 6! $$ But '*A*' *is* repeated 3 times and '*N*' 2 times. Due to this we must discount these duplicates resulting in: $$ \frac{6!}{(3!)(2!)} = 60 $$

(d)

There are 5 letters to choose from. We must choose 3. $$ \binom{5}{3} = 10 $$

Question 2

(a)

For each roll of the die there are 6 possible results, 4 rolls: $$ 6 * 6 * 6 * 6 = 6 ^ 4 = 1,296 $$

(b)

Exactly two *3*s. This means we only need to consider the two that aren't *3*. Each of these can have 1 of 5 values (1,2,4,5,6). They occupy 2 of the 4 rolls meaning that the number of locations is 4 choose 2. $$ \binom{4}{2} * \binom{5}{1} * \binom{5}{1} = \binom{4}{2} * 5^2 = 150 $$

(c)

To get all possible scenarios of at least two *3*s:

- two *3*s - as above: $$ \binom{4}{2} * \binom{5}{1} * \binom{5}{1} = \binom{4}{2} * 5^2 = 150 $$
- three *3*s - similar to above but now 3 of 4 locations are *3*s and only one can be 1,2,4,5,6: $$ \binom{4}{3} * \binom{5}{1} = \binom{4}{3} * 5 = 20 $$
- four *3*s - all *3*s: $$ \binom{4}{4} = 1 $$ The sum of these options is: $$ 150 + 20 + 1 = 171 $$

Question 3

(a)

Similar to Q1(c). There are 8 total but each of the 4 distinct cards are repeated twice.
$$ \frac{8!}{2!*2!*2!*2!} = \frac{8!}{(2!)^4} = 2,520 $$

(b)

Assuming order is important, the number of possible combinations is gotten by choosing 1 of 4 for the first card and 1 of the remaining 3 for the second.

$$ \binom{4}{1}\binom{3}{1} $$

But since the order is irrelevant this must be divided by 2 to remove duplicates:

$$ \frac{\binom{4}{3}\binom{3}{1}}{2} = 6 $$

(c)

Each card can have 2 values:

$$ 2^2 $$

But one of these combinations is duplicated: {hearts,diamonds} = {diamonds, hearts}

$$ 2^2 -1 = 3 $$