ST3009: Statistical Methods for Computer Science

Week 1 Assignment - Senán d'Art - 17329580

Question 1

(a)

No restrictions so letters can be in any order. To generate all possible results:

$$\binom{10}{1}\binom{9}{1}\binom{8}{1}\binom{7}{1}\binom{6}{1}\binom{5}{1}\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{1}{1}$$

As every time we choose a letter it is removed from the pool of available letters. So for the first letter we can choose 1 of the available 10, for the second there are only 9 letters remaining so we can choose 1 of 9 and so on.
This becomes:

$$10 * 9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1$$

Which is:

$$10! = 3,628,800$$

(b)

Restrictions of *E* and *F* being next to each other but in any order means we can treat it as a pool of 9 characters. Following the reasoning of part (a) we start at choosing 1 of 9 possible letters followed by 1 of 8 and so forth.

$$9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1 = 9!$$

But since *E* and *F* can be in the order *EF* or *FE* we need to multiply this by 2.

$$9! * 2 = 725,760$$

(c)

Word 'BANANA' contains 6 letters. Ignoring the fact that *A* and *N* are repeated we have a total number of possibilities of:

$$6!$$

But '*A*' *is* repeated 3 times and '*N*' 2 times. Due to this we must discount these duplicates resulting in:

$$\frac{6!}{(3!)(2!)} = 60$$

(d)

There are 5 letters to choose from. We must choose 3.

$$\binom{5}{3} = 10$$

Question 2

(a)

For each roll of the die there are 6 possible results, 4 rolls:

$$6 * 6 * 6 * 6 = 6^4 = 1,296$$

(b)

Exactly two *3*s. This means we only need to consider the two that aren't *3*. Each of these can have 1 of 5 values (1,2,4,5,6). They occupy 2 of the 4 rolls meaning that the number of locations is 4 choose 2.

$$\binom{4}{2} * \binom{5}{1} * \binom{5}{1} = \binom{4}{2} * 5^2 = 150$$

(c)

To get all possible scenarios of at least two *3*s:

- two *3*s - as above:

$$\binom{4}{2} * \binom{5}{1} * \binom{5}{1} = \binom{4}{2} * 5^2 = 150$$

- three *3*s - similar to above but now 3 of 4 locations are *3*s and only one can be 1,2,4,5,6:

$$\binom{4}{3} * \binom{5}{1} = \binom{4}{3} * 5 = 20$$

- four *3*s - all *3*s:

$$\binom{4}{4} = 1$$

The sum of these options is:

$$150 + 20 + 1 = 171$$

Question 3

(a)

Similar to Q1(c). There are 8 total but each of the 4 distinct cards are repeated twice.

$$\frac{8!}{2! * 2! * 2! * 2!} = \frac{8!}{(2!)^4} = 2,520$$

(b)

Assuming order is important, the number of possible combinations is gotten by choosing 1 of 4 for the first card and 1 of the remaining 3 for the second.

$$\binom{4}{1} * \binom{3}{1}$$

But since the order is irrelevant this must be divided by 2 to remove duplicates:

$$\frac{\binom{4}{3} * \binom{3}{1}}{2} = 6$$

(c)

Each card can have 2 values:

$$2^2$$

But one of these combinations is duplicated: {hearts,diamonds} = {diamonds, hearts}

$$2^2 - 1 = 3$$