

ST3009: Statistical Methods for Computer Science

Week 1 Assignment - Senán d'Art - 17329580

Question 1

(a)

No restrictions so letters can be in any order. To generate all possible results: $\binom{10}{1} \binom{9}{1} \binom{8}{1} \binom{7}{1} \binom{6}{1} \binom{5}{1} \binom{4}{1} \binom{3}{1} \binom{2}{1} \binom{1}{1}$ As every time we choose a letter it is removed from the pool of available letters.

This becomes: $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Which is: $10! = 3,628,800$

(b)

Restrictions of E and F being next to each other but in any order means we can treat it as a pool of 9 characters. $9!$ But since E and F can be in the order EF or FE we need to multiply this by 2. $9! \cdot 2 = 725,760$

(c)

Word 'BANANA' contains 6 letters but 'A' is repeated 3 times and 'N' 2 times.

$$\frac{6!}{(3!)(2!)} = 60$$

(d)

There are 5 letters to choose from. We must choose 3. $\binom{5}{3} = 10$

Question 2

(a)

For each roll of the die there are 6 possible results, 4 rolls: $6 \cdot 6 \cdot 6 \cdot 6 = 6^4 = 1,296$

(b)

Exactly two 3s. Since there are 4 rolls, the number of locations for the 3s is 4 choose 2. The remaining two rolls must have 1 of 5 values (1,2,4,5,6). $\binom{4}{2} \cdot \binom{5}{1} \cdot \binom{5}{1} = \binom{4}{2} \cdot 5^2 = 150$

(c)

To get all possible scenarios of at least two 3s:

- two 3s - as above: $\binom{4}{2} \cdot \binom{5}{1} \cdot \binom{5}{1} = \binom{4}{2} \cdot 5^2 = 150$
- three 3s - similar to above but now 3 of 4 locations are 3s and only one can be 1,2,4,5,6: $\binom{4}{3} \cdot \binom{5}{1} = \binom{4}{3} \cdot 5 = 20$
- four 3s - all 3s: $\binom{4}{4} = 1$ The sum of these options is: $150 + 20 + 1 = 171$