5. **Alternating Series Test** If the series is of the form $\Sigma(-1)^{n-1}b_n$ or $\Sigma(-1)^n b_n$, then the Alternating Series Test is an obvious possibility. Note that if Σb_n converges, then the given series is absolutely convergent and therefore convergent.
6. **Ratio Test** Series that involve factorials or other products (including a constant raised to the nth power) are often conveniently tested using the Ratio Test. Bear in mind that $|a_{n+1}/a_n| \to 1$ as $n \to \infty$ for all p-series and therefore all rational or algebraic functions of n. Thus the Ratio Test should not be used for such series.
7. **Root Test** If a_n is of the form $(b_n)^n$, then the Root Test may be useful.
8. **Integral Test** If $a_n = f(n)$, where $\int_1^\infty f(x)\,dx$ is easily evaluated, then the Integral Test is effective (assuming the hypotheses of this test are satisfied).

In the following examples we don't work out all the details but simply indicate which tests should be used.

EXAMPLE 1 $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{2n+1}$

Since $a_n \to \frac{1}{2} \neq 0$ as $n \to \infty$, we should use the Test for Divergence. ■

EXAMPLE 2 $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{3n^3+4n^2+2}$

Since a_n is an algebraic function of n, we compare the given series with a p-series. The comparison series for the Limit Comparison Test is Σb_n, where

$$b_n = \frac{\sqrt{n^3}}{3n^3} = \frac{n^{3/2}}{3n^3} = \frac{1}{3n^{3/2}}$$ ■

EXAMPLE 3 $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$

Since the integral $\int_1^\infty xe^{-x^2}\,dx$ is easily evaluated, we use the Integral Test. The Ratio Test also works. ■

EXAMPLE 4 $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^4+1}$

Since the series is alternating, we use the Alternating Series Test. We can also observe that $\Sigma|a_n|$ converges (compare to $\Sigma 1/n^2$) so the given series converges absolutely and hence converges. ■

EXAMPLE 5 $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{k!}$

Since the series involves $k!$, we use the Ratio Test. ■

EXAMPLE 6 $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2+3^n}$

Since the series is closely related to the geometric series $\Sigma 1/3^n$, we use the Direct Comparison Test or the Limit Comparison Test. ■