What is Being Proved?

Given below are proofs for results: State what is being proved:

(a) *Proof.* Assume for the sake of contradiction that there exists an odd integer r that is the sum of three even integers x, y, z. Then we can write $x = 2l, y = 2k, z = 2m$ for integers l, k, m. But then we would have

$$\begin{aligned} r &= x + y + z \\ &= 2l + 2k + 2m \\ &= 2(l + k + m) \text{ where } l + k + m \in \mathbb{Z} \end{aligned}$$

However, this contradicts the fact that r is odd. □

Proof Explanation. We see this is a proof by contradiction because the proof starts out by saying "For the sake of contradiction...." The claim they are assuming for the sake of contradiction is that there does exist an odd integer that can be expressed as the sum of three even integers. **Hence the claim they are proving is the following:** "**There does not exist an odd integer that can be expressed as the sum of three even integers**"
We can see that the proof succeeds in doing so, because their assumption that r could be expressed as a sum of three even integers lead to the conclusion that r is an even number. However, r cannot be both an even number and an odd number, which was the contradiction

(b) *Proof.* Let A, B, C be sets. By contrapositive, we will assume that $x \in A \cup C$. Then we know that $x \in A$ or $x \in C$. If $x \in A$ then $x \in A \cup B$ as desired. If $x \in C$ then $x \in B \cup C$ as desired.
Either way, we see that if $x \in A \cup C$ then $x \in A \cup B$ or $x \in B \cup C$ □

Proof Explanation. We notice that the method of proof is contrapositve, since they mention "by contrapositve..." at the beginning of the proof. The assumption at the beginning says that $x \in A \cup C$. We know therefore that this will be the negation of the original conclusion we wish to draw. Hence the conclusion of the proof should be that $x \notin A \cup C$.

Moreover, the conclusion of the contrapositive proof is that $x \in A \cup B$ or $x \in B \cup C$ which is the negation of the hypothesis of the original claim. We have that the negation is $x \notin A \cup B$ **AND** $x \notin B \cup C$. Hence the original claim that this proof is proving is:

"If $x \notin A \cup B$ **and** $x \notin B \cup C$ **then** $x \notin A \cup C$"