

Echelon Form and Systems of Equations

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Question 1

- a) Give an example of a linear system with a 0 row but no free variables
- b) Give an example of a linear system with a free variable but no 0-row.
- c) Give an example of a linear system with a unique solution.
- d) Give an example of a linear system with no solutions.

Question 2

Put the following matrices into Echelon Form and find the solution set of each.

- a) $\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & -3 & 4 \end{array}\right)$
- b) $\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 4 & 8 & -12 & 8 \\ 0 & 6 & -1 & 11 \end{array}\right)$
- c) $\left(\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 4 & -4 & -8 \end{array}\right)$

Question 3

Write the coefficient matrix for the following systems. In each case, state if the matrix is in echelon form.

a)

$$\begin{aligned}x_1 + 2x_2 - 3x_3 &= 5 \\x_2 - 100x_3 &= 1\end{aligned}$$

b)

$$x_1 - x_2 + x_3 = 0$$

$$x_2 + x_3 = 0$$

$$x_1 = 0$$

c)

$$x_1 - 4x_2 = 10$$

$$x_2 = 8$$

$$x_1 - 6x_2 = 2$$

c)

$$x_1 + x_2 + x_3 + x_4 = 6$$

$$x_2 - x_4 = 2$$

$$x_3 = 6$$

Challenge- Extra Credit Question

Can you exhibit a linear system with 3 equations and 2 unknowns (x_1, x_2)

$$a_{11}x_1 + a_{12}x_2 = b_1$$

$$a_{21}x_1 + a_{22}x_2 = b_2$$

$$a_{31}x_1 + a_{32}x_2 = b_3$$

that has a solution for any values b_1, b_2, b_3 ?