Problem Set 3

[Your Full Name Here]

MATH 100 — Introduction to Proof and Problem Solving — Summer 2023

Problem 3.1. For a triangle T, let $r(T)$ denote the ratio of the length of the longest side of T to the length of the smallest side of T. Let $\triangle$ denote the set of all triangles and let

$$P(T_1,\ T_2) : r(T_2) \geqslant r(T_1).$$

be an open sentence where the domain of both T_1 and T_2 is $\triangle$.

(a) Express the following quantified statement in words:

$$\exists T_1 \in \triangle,\ \forall T_2 \in \triangle,\ P(T_1, T_2). \tag{$*$}$$

Solution. □

(b) Express the negation of the quantified statement in $(*)$ in symbols.

Solution. □

(c) Express the negation of the quantified statement in $(*)$ in words.

Solution. □

Problem 3.2.

(a) Let $x \in \mathbb{R}$. Prove that if $0 < x < 1$, then $x^2 - 2x + 2 \neq 0$.

Solution. $\square$

(b) For two sets A and B, prove $B \subseteq A$ is the same as proving the implication

$$x \in B \implies x \in A$$

Prove that for any set X, we have $\varnothing \subseteq X$.

Solution. $\square$

Problem 3.3.

(a) Prove that if a and c are odd integers, then $ab + bc$ is even for every integer b.

Solution. $\square$

(b) Let $x \in \mathbb{Z}$. Prove that if 2^{2x} is an odd integer, then 2^{-2x} is an odd integer.

Solution. $\square$

Collaborators:

References:

- [Book(s): Title, Author]
- [Online: Link]
- [Notes: Link]

Fin.