Yiddish word of the day

"bulbe" = בולבע

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Yiddish expression of the day

"zolst hobn tsen hayzer, yeder hoiz zol hobn tsen tsimern, un yeder tsimer zol zayn tsen betn, un zolst zikh kayklen fun eyn bet in der tsveyter mit kholere" =

זאָלסט האָבן צען הײַזער,
יעדער הויז זאָל האָבן
צען צימערן, און יעדער צימער
זאָל זײַן צען בעטן, און זאָלסט
זיך קײַקלען פֿון איין בעט אין
דער צווייטער מיט כאָלערע

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Determinants

Again - We know the equation $ax=b$ has a unique solution precisely when $a \neq 0$

- Want some condition like this

Def: Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Then

Recall: $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then A is invertible precisely when

Def: $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$. Thus

$\underline{\qquad\qquad} A =$

ex) $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ Find $\underline{\qquad} A$

ex) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 1 & 4 & 2 \end{pmatrix}$ Find ____ A

ex) $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 1 & 4 & 5 \end{pmatrix}$ Find ____ A

- We want to define ______ for any dimensional matrix

A $n \times n$ matrix

1) The ______________ of A is the $(n-1) \times (n-1)$ matrix obtained by deleting row i, column j from A

2) The ______________ A is the determinant of the (i,j) submatrix of A ($M_{ij}(A)$)

3) The ______________ of A is $(-1)^{i+j} M_{ij}(A)$

" $C_{ij}(A)$

OK, so with these 3 def, we can once and for all define the ______

$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$ Then

ex) $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 1 & 4 & 4 \end{pmatrix}$

- $M_{11}(A) =$ $\quad M_{12}(A) =$ $\quad M_{13} =$

- $C_{11}(A) =$ $\quad C_{12}(A) =$ $\quad C_{13} =$

$A =$

ex) $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$ Find $\det A$

$\det A =$

Gross! I said that determinants give us an easier way to see if matrix is invertible. As of now, looks like I lied.

Thm:

ex) i) $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ ii) $A = \begin{pmatrix} 1 & 6 & 9 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{pmatrix}$ iii) $A = \begin{pmatrix} 1 & 2 & 4 & 7 \\ 0 & 3 & 5 & 8 \\ 0 & 0 & 6 & 9 \\ 0 & 0 & 0 & 10 \end{pmatrix}$

$\det A =$ $\det A =$ $\det A =$

Fact: If an matrix is in Echelon Form,

$\det A =$

Q: How does

Fact: $\det(\quad) =$

Recall: Doing row operations $\longleftrightarrow$

- Start with matrix A. Then putting it in Echelon form is just multiplying by a few elementary matrices.
  - If B is the matrix obtained from A that's now in Echelon form

$B =$

________________________ .

$\Rightarrow \det B =$

$\det B =$

$\Rightarrow$

Recall the 3 Elementary matrices

1) $D_i^n(c)$: $\det(D_i^n(c)) =$

ex) $D_2^3(4) = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \Rightarrow \det D_2^3(4) =$

2) P_{ij}^n : $\det(P_{ij}^n) =$

ex) $\det\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix} =$

<u>Check</u>: $\det\begin{pmatrix} 0 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & 0 & 6 \end{pmatrix} =$

3) $T_{ij}^n(c)$: $\det(T_{ij}^n(c)) =$!!! **WOW !!!**

ex) $A = \begin{pmatrix} 1 & 3 & 4 & 5 \\ 2 & 1 & 6 & 3 \\ 0 & 0 & 4 & 6 \\ 1 & -2 & 2 & 1 \end{pmatrix}$ Find $\det A$

Recall - If A is an invertible matrix, then

the matrix equation $A\vec{x} = \vec{b}$ has unique solution for any $\vec{b}$

$\Rightarrow$ thus the columns of A

ex) Is there a vector $\vec{0} \neq \vec{x}$ such that

$$\begin{pmatrix} 1 & 3 & 4 & 5 \\ 2 & 1 & 6 & 3 \\ 0 & 0 & 4 & 6 \\ 1 & -2 & 2 & 1 \end{pmatrix} \vec{x} = \vec{0} \qquad ?$$

ex) $A = \begin{pmatrix} 1 & 2 & -12 & -8 & -12 \\ 0 & 3 & 2 & 2 & 8 \\ 0 & 0 & -4 & 1 & 3 \\ 0 & 6 & 4 & 4 & 6 \\ 1 & 8 & -8 & -4 & 9 \end{pmatrix}$

Q A invertible?

We've shown these last few weeks

A $n \times n$ matrix. Then the following are equivalent

1) $(A \mid \vec{b})$ has a ________ for any vector $\vec{b}$

2) The columns of A ______ $\mathbb{R}^n$

3) The columns of A are ______ in $\mathbb{R}^n$

4) The columns of A are ____________ for $\mathbb{R}^n$

5) A is ____________

6) det A ____________