Yiddish word(s) of the day (ײִדיש װאָרט אויפֿן טאָג)

	freylekh	=	פֿריילעך
=	tsufridn	=	צופֿרידן
	gliklekh	=	גליקלעך

Yiddish expression of the day

"Zol men trinken af simkhes" = זאָל מען טרינקען אויף שׂמחות

Chapter 3 - Matrix Algebra

Quickly Recall

A basis $\vec{v}_1 \ldots \vec{v}_n$ of $\mathbb{R}^n$ are a list of vectors that

1)

2)

Recall: The standard basis for $\mathbb{R}^n$ was the following

$$\vec{e}_1 = \begin{pmatrix} \\ \\ \\ \end{pmatrix}, \quad \vec{e}_2 = \begin{pmatrix} \\ \\ \\ \end{pmatrix}, \ldots, \quad \vec{e}_n = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

Remember that any $\vec{v}$ in $\mathbb{R}^n$ can be expressed as

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} = \_\_\vec{e}_1 + \_\_\vec{e}_2 + \_\_\vec{e}_3 + \cdots + \_\_\vec{e}_n$$

Recall 2: A subspace $W \subseteq \mathbb{R}^n$ is a subset that is

1) closed under

2) Closed under

ex) $A = \begin{pmatrix} \vec{v}_1 & \cdots & \vec{v}_k \\ \downarrow & & \downarrow \end{pmatrix}_{n \times k}$ be this $n \times k$ matrix

Then the column space of A, denoted $col(A)$

is ______________________________

Chapter 3 - Matrices / Linear Transformations

Section 3.1 - Linear Transformations.

Def: A function $f: A \to B$ is

ii) A $T: \mathbb{R}^n \to \mathbb{R}^m$ is a function such that

1) $T($ $)$
2) $T($ $)$

ex) $T: \mathbb{R}^2 \to \mathbb{R}^2$ by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x-y \end{pmatrix}$

- Is T a linear transf?

1)

2)

ex) $T: \mathbb{R}^3 \to \mathbb{R}^2$ by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ Is this linear?

1)

2)

ex3) $T: \mathbb{R}^2 \to \mathbb{R}$ by $T\begin{pmatrix} x \\ y \end{pmatrix} = x + y + 2$

ex) $T: \mathbb{R}^4 \to \mathbb{R}^2$ by $T\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} xy \\ zw \end{pmatrix}$

Q: Is this linear.

Note: If T is a linear transformation

then $T\left(\vec{x}_1 + \cdots + \vec{x}_n\right) =$

Now let $\vec{e}_1, \ldots, \vec{e}_n$ be the standard basis of $\mathbb{R}^n$

Then $\vec{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = v_1\vec{e}_1 + v_2\vec{e}_2 + \cdots + v_n\vec{e}_n$

Then $T(\vec{v}) = T\Big(\qquad\qquad \Big)$

$= T(\qquad) + T(\qquad) + \cdots + T(\qquad)$

$=$

That is: A linear transf is uniquely defined by what it "does" to the standard basis!

ex) Suppose $T: \mathbb{R}^3 \to \mathbb{R}^2$ and

$$T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}, \quad T\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}1\\6\end{pmatrix}, \quad T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}-3\\1\end{pmatrix}$$

Q: What is $T\begin{pmatrix}2\\3\\4\end{pmatrix}$

Def: Again let $\vec{e}_1, \dots, \vec{e}_n$ be the standard basis for $\mathbb{R}^n$
and

$$T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$$

The standard matrix associated to T is the $m \times n$ matrix

$$A_T = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}_{m \times n}$$

ex) i) $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x-y \end{pmatrix}$

$\longrightarrow A_T = \begin{pmatrix} \quad \end{pmatrix}$

ii) $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\qquad T: \mathbb{R}^3 \to \mathbb{R}^2$

$A_T = \begin{pmatrix} \quad \end{pmatrix}$

iii) Again consider the transf T where I just told you what T does to a basis

$$T\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}2\\3\end{pmatrix},\quad T\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}1\\6\end{pmatrix},\quad T\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}-2\\1\end{pmatrix}$$

$$\longrightarrow A_T = \begin{pmatrix} \quad & \quad & \quad \end{pmatrix}$$

<u>Def.</u> Let $\vec{x}$ be a vector in $\mathbb{R}^n$, $\vec{x}=\begin{pmatrix}x_1\\ \vdots \\ x_n\end{pmatrix}$

Let $T:\mathbb{R}^n\to\mathbb{R}^m$ be linear $\longrightarrow A_T$

$$A_T\vec{x} \stackrel{def}{=}$$

ex) Let A be as in example 3

Q: What is $A\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \overset{\text{def}}{=}$

$=$

$=$

$=$!!

1) Defining matrix mult with a vector this way "works" because

2) Consider the following system of equations

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 4 \\ x_1 - 2x_2 + 3x_3 &= 8 \\ x_1 + 6x_2 - x_3 &= 1 \end{aligned} \quad \xleftrightarrow{\text{vector eqn}} \quad \begin{pmatrix} x_1 + 2x_2 - x_3 \\ x_1 - 2x_2 + 3x_3 \\ x_1 + 6x_2 - x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 1 \end{pmatrix}$$

$$\underbrace{x_1 \begin{pmatrix} \\ \\ \end{pmatrix} + x_2 \begin{pmatrix} \\ \\ \end{pmatrix} + x_3 \begin{pmatrix} \\ \\ \end{pmatrix}} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

$$\|$$

$$\begin{pmatrix} \\ \\ \\ \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

!!

- That is, solving the linear system A is the same as finding some $\vec{x}$ such

We have the following. The 3 ideas are equivalent

- A linear system with coefficient matrix $(A|\vec{b})$ is consistent

$\Updownarrow$ (last week)

$\Updownarrow$ (from today)

★ That is, this equivalence tells us that solving systems of equations is really the same thing as understanding $T(\vec{x})$

<u>Def</u>: Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be linear transformation.

1) The range of T $\quad R(T) = \{ \quad : \quad \}$

2) The null space of the associated standard matrix A_T
$\quad \mathrm{null}(A) = \{ \quad : \quad = \quad \}$ conceptual

- by the above connection the collection of all vectors $\vec{x}$ such that ______ is the
______

ex) Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 6 \end{pmatrix}$ Find null(A).

ex) $A = \begin{pmatrix} 1 & -2 & 3 \\ -1 & 4 & -6 \end{pmatrix}$ Find null(A)

Check:

In Summary: The following are equivalent

1) the columns of A are LI
2) there's a pivot in ________
3) null(A) = ________

last week (items 1–2)

new characterization (item 3)

More generally

We saw that the linear system

$(A \mid \vec{b})$ has an answer precisely when

I.e, a vector $\vec{b}$ is a LC of the collumns of matrix A
precisly when $A\vec{x} = \vec{b}$ for some $\vec{x}$ in $\mathbb{R}^n$

Hence, the following are equivalent

1) $[A \mid \vec{b}]$ is consistent for any vector $\vec{b}$
2) the columns of A ________
3) A has a pivot in ________

old

4)

new

ex) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ x-y \end{pmatrix}$

Is the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ in the $R(T)$?