Recall from last time

1) Given linear system

$$\begin{aligned} a_{11}x_1 + \cdots &+ a_{1n}x_n = b_1 \\ &\vdots \\ a_{m1}x_1 + \cdots &+ a_{mn}x_n = b_m \end{aligned} \quad \longleftrightarrow \quad A\vec{x} = \vec{b}$$

2) We saw we could mult matrices, but they "behave" diff than mult of #'s.

- We had two non-zero matrices A, B such that ____
- We had 3 matrices A, B, C with $B \neq C$ and ____

1) Note the similarities of $A\vec{x} = \vec{b}$ with $ax = b$

- note $ax = b$ has a unique solution precisely when

Q: Can we by matrix A?

Def: Let A $n \times n$ matrix. Then A is if there exists some other matrix, denoted such that

Note: If matrix A is ______ then the linear system $A\vec{x} = \vec{b}$ ______ for any $\vec{b}$

Some consequences of being an invertible matrix

Have 2 old results. Let A $n \times n$ matrix.

1) the columns of A span $\mathbb{R}^n$ iff the matrix eqn $A\vec{x} = \vec{b}$ ______

2) the columns of A are LI iff
null(A) = ________
(also equivalently, whenever $A\vec{x} = \vec{b}$ has a solution,
____________)

Thus, since we saw $A\vec{x} = \vec{b}$ has unique solution for any $\vec{b}$ if A is __________ then,
________________ !!!

A couple of results

1) If A is ________, so is ____, and ______ = ________

2) If A, B are 2 ________ $n \times n$ matrices, then AB is ________ too, with ______________________

3) Suppose A is ______ Then multiplication behaves more like how we are familiar.

ex) Suppose A ______ and we have 2 matrices B, C with $AB = AC$

then ....

ex) Similar statement about "zero divisors"

See HW

Q2: How to determine if matrix is invertible, and how to find the inverse if it is.

A) First in case A is 2×2 matrix.

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Then A is invertible when and $A^{-1} =$

ex) $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$. Find A^{-1} if it exists.

Now more generally

Let A be an $n \times n$ matrix

1) Augment A by ________

$$\left(\begin{array}{c|c} \quad & \quad \\ \quad & \quad \end{array}\right)$$

2) Put A into ____________

- if at any point, you get a 0 row on left

  ________ , A is ____________ !!

- else finish putting it into reduced row ech form

This matrix is A^{-1} !!

ex) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 1 & 4 & 2 \end{pmatrix}$ Find A^{-1} if it exists.

Claim $A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -1/2 & -1/2 & 1/2 \\ 1/2 & 1 & -1/2 \end{pmatrix}$ — Skip the checking

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ex) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$ **Find A^{-1} if it exists.**

Elementrary Matrices

- These matrices will do the following
  - Justify this invertible matrix algorithm
  - Justify our row operations
    - lead to some new ideas (see HW)
  - Provide a bridge to ______

Recall the 3 elementary row operations

1)

2)

3)

Associated to these 3 operations are the following 3 matrices

1) $D_i^n(c) =$

ex) $D_2^3(112) =$

$D_3^4(-12) =$

$D_2^5(5) =$

2) $P_{ij}^n =$

ex) $P_{12}^3 =$

ex) $P^5_{24} =$

ex) $P^4_{14} =$

3) $T^n_{ij}(c) =$

ex) $T^2_{12}(2) =$

$T_{23}^{3}(-1) =$

$T_{24}^{4}(-6) =$

$T_{35}^{5}(4) =$

Q - Who the hell cares?

ex) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$

$\quad \parallel$

P^2_{12}

ex) $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$

$\quad \parallel$

$D^2_1(3)$

ex) $\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$

$\quad\quad \parallel$

$T_{12}^{2}(-2)$

Doing row operations to our matrix

$\parallel$

Note:
- Doing row operations to a matrix is a reversible process
- So thinking of row operations as matrix mult, the fact that they are reversible means ______

1) $\left(D_i^n(c)\right)^{-1} =$

ex) $D_i^2(2) = \left(\qquad \right)$

$$(D^{4}_{i}(u))^{-1} =$$

2) $(P^{n}_{ij})^{-1} =$

3) $(\Gamma^{n}_{ij}(u))^{-1} =$