Problem Set 7

[Your Full Name Here]

MATH 100 — Introduction to Proof and Problem Solving — Summer 2023

Problem 7.1. Let $A = \{1, 2, 3, 4\}$. Give an example, with reasoning, of a relation on A that is:

(a) reflexive and symmetric but not transitive.

Solution. $\square$

(b) reflexive and transitive but not symmetric.

Solution. $\square$

(c) symmetric and transitive but not reflexive.

Solution. $\square$

(d) reflexive but neither symmetric nor transitive.

Solution. $\square$

(e) symmetric but neither reflexive nor transitive.

Solution. $\square$

(f) transitive but neither reflexive nor symmetric.

Solution. $\square$

Problem 7.2. Suppose $H \subseteq \mathbb{Z}$ is a subset that satisfies

(a) If $x \in H$ then $-x \in H$

(b) If $x, y \in H$ then $x + y \in H$

Show that the relation $xRy \iff x - y \in H$ is an equivalence relation. (Hint: first show that $0 \in H$)

Unimportant Remark: We denote the set of equivalences classes $\mathbb{Z}/H$ and read it as '$\mathbb{Z}$ mod H'. This is called the space of cosets in group theory. This problem works much more generally: replace $\mathbb{Z}$ by any group G, and then H is called a subgroup of G.

Solution. □

Problem 7.3. Let $n\mathbb{Z} := \{nx : x \in \mathbb{Z}\} \subset \mathbb{Z}$ be the subset of all multiples of n. Show that $n\mathbb{Z}$ satisfies conditions (a) and (b) from the above problem. What is the equivalence relation defined above in this case? What are the space of all cosets?

Solution. $\square$

Problem 7.4. Let $H = \{2^m \mid m \in \mathbb{Z}\}$. A relation R is defined on $\mathbb{Q}_{>0}$, the set of positive rational numbers by:

$$aRb \quad \text{if and only if} \quad \frac{a}{b} \in H.$$

(a) Show that R is an equivalence relation

(b) Describe the equivalence class $[3]$

(c) Prove $[2] = H$.

Solution. $\square$

Collaborators:

References:

- [Book(s): Title, Author]
- [Online: Link]
- [Notes: Link]

Fin.