Recall: Given 2 linear maps

$f_1 : V \to W$

$f_2 : V \to W$ $\xrightarrow{\text{get}}$ linear ____ $: V \otimes V \to W \otimes W$

such that ____ (   ) = __ $\otimes$ ____

Prop: Let $f : V \to W$ linear. Then there exists a ! linear map

__ : ______ $\longrightarrow$ ________

such that ____ (   ) = __ $\wedge$ ____

(Rmk: True for more general K.)

Pf)

Just like before, given

$$V \xrightarrow{g_1} W \xrightarrow{g_2} Z$$

we have ______ = ______ : ______ $\longrightarrow$ ______

and for the identity map $V \xrightarrow{id} V$ we have

______ = : ______ $\longrightarrow$ ______

ex) $V = \mathbb{C}[t]_{\leq 2}$ with basis $\mathcal{B} = (1, t, t^2)$

$g : V \to V$ be

$g(f(t)) = f'(t) + 3f(t)$

i) Find $[g]_{\mathcal{B}}$

ii) Compute the matrix of $\Lambda^2 g : \Lambda^2(V) \to \Lambda^2(V)$ with respect to basis

$\mathcal{C} = (\underline{\qquad\qquad\qquad\qquad})$

ex2) What about the map $\Lambda^3(\varphi) : \Lambda^3(V) \to \Lambda^3(V)$.

Note $\dim V = 3$ so $\dim(\Lambda^3(V)) =$ ______

So this map $\Lambda^3(\varphi)$ will just be scaling by a #.

<u>What #?</u> Have basis

$e = (\qquad\qquad)$

<u>Compute</u>: $\Lambda^3(\varphi)(\qquad\qquad) =$

Def: Let V be n-dim vs, and let

$\varphi: V \to V$ be a linear map.

We define the ________ of φ to be

the scalar multiple on which $\Lambda^n(\varphi)$ acts.

(that is ______ is the ! number st

$\Lambda^n(\varphi)(v_1 \wedge \dots \wedge v_n) = \_\_\ v_1 \wedge \dots \wedge v_n$)

Thm: Let $\varphi: V \to V$ and $\gamma: V \to V$ be 2 linear maps.

Then ______ $=$ ______ $\cdot$ ______

Pf)

Cor: i) We can compute the ______ of φ using any basis.

HW → ii) If φ is an isomorphism then ______ $= (\_\_\_\_\_\_)^{-1}$

Pf)

ex) $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ viewed as a linear map $\mathbb{R}^2 \to \mathbb{R}^2$

Then $\Lambda^2(A)(e_1 \wedge e_2) :=$

Back to Reality

Some (hopefully) familliar definitions

<u>Def</u>: Let $T: V \to V$ be linear map.

We say $0 \neq v \in V$ is an ______ if

______ (call this ______ an ______)

Q: How to find ________

→ If $v \in V$ is an ________ the

$$\underline{(\qquad)v} = 0$$

$\Longleftrightarrow$ $(\qquad\qquad)$ has nontrivial ________

$\Longleftrightarrow$ $(\qquad\qquad)$ is not ________

(HW)

$\Longleftrightarrow$ $\underline{\quad\quad}(\qquad) = 0$

$\Rightarrow$ **Prop/Def:** Let $T : V \to V$ be linear. Define

$$c_T(x) :=$$

Then the ________ of T are the ________ of $c_T(x)$

Def: Let $\lambda \in \mathbb{F}$ be an ________ of T. Write

$$E_\lambda := \ker(\qquad)$$

• We call $\dim(E_\lambda) :=$

Rmk: There is another notion of ____________

The "____________"

If λ is ______ of $c_T(x)$ then

$$c_T(x) = (x-\lambda)^d \, p(x)$$

This d is the ____________

Fact: ________________ $\leq$ ________________

There is a whole story here about

- minimal polynomials
- characteristic polynomials

} and their interplay

- Diagonalizability (some more on this Wed)
- Jordan Form (with "generalized eigenvectors")

- "Cayley - Hamilton" thrm

- All of these are really important, and should be looked into if one is serious about learning linear algebra.

- Next time $\longrightarrow$ inner product spaces / adjoint (M)
  Spectral Thrm (W)