Problem Set 5

[Your Full Name Here]

MATH 100 — Introduction to Proof and Problem Solving — Summer 2023

Problem 5.1. Let $a, b \in \mathbb{Z}$. Disprove the statement:

If ab and $(a+b)^2$ are of opposite parity, then a^2b^2 and $a + ab + b$ are of opposite parity.

Solution. $\square$

Problem 5.2. Following are the steps to prove the number $\sqrt{2}$ is irrational, this is a classic example of a proof by contradiction. Using these notes, write down a formal proof of this fact.

- Suppose $\sqrt{2}$ is rational, then $\sqrt{2} = q/p$ for some integers p, q.
- One can assume that p and q have no common factors (that is, they are coprime).
- Squaring, we get $q^2 = 2p^2$. Therefore $2 \mid q^2$.
- Argue that this gives us that $2 \mid q$ (Hint: check old lecture notes).
- Plug a new expression for q back in
- Argue that we know something then about p that contradicts some assumption of p and q

Solution. □

Problem 5.3.

(a) Show that there exist *no* non-zero real numbers a and b such that

$$\sqrt{a^2+b^2} = \sqrt[3]{a^3+b^3}$$

Solution. $\square$

(b) Disprove the statement:

There exist *odd* integers a and b such that $4 \mid (3a^2 + 7b^2)$.

(Hint: use a lemma we proved last week)

Solution. $\square$

Collaborators:

References:

- [Book(s): Title, Author]
- [Online: Link]
- [Notes: Link]

Fin.