Problem Set 4

[Your Full Name Here]

MATH 100 — Introduction to Proof and Problem Solving — Summer 2023

Problem 4.1.

(a) Let $x \in \mathbb{Z}$. Prove that $3x + 1$ is even if and only if $5x - 2$ is odd.

Solution. □

(b) Let $a, b \in \mathbb{Z}$. Prove that if $a + b$ and ab are of the same parity, then a and b are even.

Solution. □

Problem 4.2.

(a) Let $a, b \in \mathbb{Z}$, where $a \neq 0$ and $b \neq 0$. Prove that if $a \mid b$ and $b \mid a$, then $a = b$ or $a = -b$.

Solution. □

(b) Let x and y be *even* integers. Prove that $x^2 \equiv y^2 \bmod 16$ if and only if either

(1) $x \equiv 0 \bmod 4$ and $y \equiv 0 \bmod 4$; or
(2) $x \equiv 2 \bmod 4$ and $y \equiv 2 \bmod 4$

Solution. □

(c) Prove for every two real numbers x and y we have $|x + y| \geqslant |x| - |y|$.

Solution. □

Problem 4.3.

(a) Prove that for every two sets A and B, the sets $A \setminus B$, $B \setminus A$ and $A \cap B$ are pairwise disjoint. Give an element-wise proof.

Solution. □

(b) Show, using set operation laws, that for every three sets A, B and C that

$$A \setminus (B \setminus C) = (A \cap C) \cup (A \setminus B).$$

Solution. □

Collaborators:

References:

- [Book(s): Title, Author]
- [Online: Link]
- [Notes: Link]

Fin.