Logic

Logic is the ______ of math, i.e. the rules of the ______

Def: A ______ is a ______ sentance which can be ______ determined to be ______ or ______

ex) (and non examples)

a) The integer 11 is divisible by ______

______

b) The integer 11 is a ______ #

______

c) Is $10^{10} \in \mathbb{Z}$

d) The integer 10^{10} is biggggg

Def: Given a __________ P, the ________ of P is the statement ________ denoted ___P (or __P) (\neg)

⇝ It is characterized by the following __________

P	
T	
F	

ex) a) P: The # 11 is odd

_____ = The # 11 is _____ odd

= the # 11 is _____

b) P: The day of the week is Wednesday

_____ : The day of the week is _____

Def: Given statements P and Q, the _____ of P and Q is the statement

_____ denoted _____ (\lor)

· It is defined via the following _____ table

P	Q	
T	T	
T	F	
F	T	
F	F	

$\rightsquigarrow$ ie, ______ is ______ if ______ one of P or Q is ______

ex) a) P: 5 is odd ______
Q: 10 is prime ______
$\implies P \vee Q$ is ______

b) Let P be any statement. Then ______ is always true.

P		
T		
F		

⇝ we call such statements ______ (ie always true)

<u>Def</u>: Given statements P and Q the ______ of P and Q is the statement ______ denoted ______ (\land)

It is defined via the truth table

P	Q	
T	T	

→ is only true when ______

T	F	
F	T	
F	F	

P and Q is

______

Def: Given statements P and Q the ______ is the statement

____ P ____ Q, denoted ________ (implies)

• We call P the ________ and Q the ________

Its truth table is defined as

P	Q	
T	T	
T	F	
F	T	
F	F	

Note that when the
____ P is false the
______ Q may be
T or F, but _____ is
always ____

ex) P: It is raining
Q: I will stay at home

~> ________ : If it is raining,

⌜If it doesn't rain and I go out, did I lie?

— —

→ $P \Rightarrow Q$ = ___ ⌟

Some terminology: ________ is read in diff ways

- if ___ then ___
- ___ implies ___
- ___ if ___
- ___ only if ___
- P is ___ for Q
- Q is ______ for P

all these mean

____

We will be considering the truth value of ________ that depend on a ________.

Def: An ________ sentence is a ________ sentence which contains ________, where each ________ can assume ________ value in a given ________, called the ________ of the ________, which becomes a statement if the ________ are replaced with specific values.

ex) For $x \in \mathbb{R}$ consider the statement

$P(x): |x| = 3$

$\underline{x=1}$: $P(1)$ ________

$x=-2$: $P(-2)$ ______

$x=-3$: $P(-3)$ ______

- $P(x)$: x is odd for $\underbrace{x \in \mathbb{Z}}_{\text{this is the \_\_\_\_\_\_}}$

We can combine open sentences using ____________ to form new open sentences

ex) Domain: $x \in \mathbb{R}$ $\quad P(x)$: $|x|=3$

$Q(x)$: $x=-3$

$\rightsquigarrow \neg P(x)$: $\quad \neg P(1)$: ___

$P(x) \wedge Q(x)$: $P(3) \wedge Q(3)$

$Q(x) \Rightarrow P(x)$:

$P(x) \vee Q(x)$:

<u>Def</u>: The implication ______ is called the ______ of $P \Rightarrow Q$

- Given statements P and Q the ______ of P and Q is the statement

  ______ $\wedge$ ______

  denoted ______ (\iff)

- We read ________ as
  - P ___ and only ____ Q
  - P is ______ to Q
  - P is ______ and ________ for Q

    ($Q \Rightarrow P$) ($P \Rightarrow Q$)

The truth table is given as

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$P \Rightarrow Q \wedge Q \Rightarrow P$
T	T			
T	F			
F	T			

F	F			

⇝ ______ is true when ______ P and Q are either ______ or ______

Compound Statements

<u>Def</u>: A ______ statement is a statement consisting of ______ one statement involving ______ one ______ connective ($\neg, \wedge, \vee, \Rightarrow, \Leftarrow$)

- Each statement in a ______ statement is called a ______ statement ex) $P \Longleftrightarrow Q$ is a compound statement with ________

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Def) 1) A compound statement is called a ________ if it is ________ for all possible ________ of truth values for its ________ statements

2) A compound statement is called a ________ if it is ________ for all possible ________ of truth values for its ________ statements

ex) i) $P \vee (\neg P)$ is a ________

ii) $P \wedge (\neg P)$ is a ________

P	$\neg P$	$P \wedge (\neg P)$

iii) $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$ is a ________

P	Q	$P \Rightarrow Q$	$P \wedge (P \Rightarrow Q)$	$(P \wedge (P \Rightarrow Q)) \Rightarrow Q$
T	T			
T	F			
F	T			
F	F			

iv) $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$ is a ______

P	Q	R	$P \Rightarrow Q$	$Q \Rightarrow R$	$(P \Rightarrow Q) \wedge (Q \Rightarrow R)$	$P \Rightarrow R$	$((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

notice how there's more options now

Logical Equivalences

Def: Two compound statements R and S are ____________ if they have the ______________ for all possible combinations of ________ values for its component statements

⇝ We denote this as R   S (\equiv)

Thrm: Let P and Q be statements. Then

$$P \Rightarrow Q \qquad ((\neg P) \vee Q))$$

Pf) We will build the truth table

P	Q	¬P	P=>Q	(¬P)∨Q
T	T			
T	F			
F	T			
F	F			

Thm: Let P, Q, R be statements. Let ____ and ____ be a ________ and ________. Then

1) Identity laws: $P \vee \quad \equiv$

$P \wedge \quad \equiv$

2) Domination laws: $P \vee \quad \equiv$

$P \wedge \quad \equiv$

3) Double negation: $\neg(\neg P) \equiv$

4) Commutative laws: $P \vee Q \equiv$

$P \wedge Q \equiv$

5) Associative laws: $(P \vee Q) \vee R \equiv$

$(P \wedge Q) \wedge R \equiv$

6) Distributive laws: $P \vee (Q \wedge R) \equiv$

$P \wedge (Q \vee R) \equiv$

7) De-Morgan's Laws: $\neg(P \vee Q) \equiv$

$\neg(P \wedge Q) \equiv$

Pf) We will prove De-Morgan's Laws (the first part)

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg(P \vee Q)$	$(\neg P) \wedge (\neg Q)$
T	T	F	F			
T	F	F	T			
F	T	T	F			
F	F	T	T			

Application: Prove that

1) $\neg(P \Rightarrow Q) \equiv P \wedge (\neg Q)$

Pf)

2) $((\neg Q) \Rightarrow (P \wedge \neg P)) \equiv Q$

Pf)

3) $((P \land \neg Q) \Rightarrow (R \land \neg R)) \equiv P \Rightarrow Q$