Problem Set 8

[Your Full Name Here]

MATH 100 — Introduction to Proof and Problem Solving — Summer 2023

Problem 8.1. Give an example, with an explanation, of functions for the following if you think examples exist. If you think no such example exists, prove why

(a) An injective but not surjective function

Solution. □

(b) Let $A = \{a, b, c\}$. A surjective function $f : A \to \mathcal{P}(A)$

Solution. □

(c) A function that is neither surjective nor injective

Solution. □

(d) A surjective but not injective injective

Solution. □

(e) Let $A = \{a, b, c\}$. An injective function $f : A \to \mathcal{P}(A)$

Solution. □

Problem 8.2. Let A, B be finite sets such that $|A| = |B| = n$. Prove by induction that there are $n!$ bijective functions from A to B.

Solution. $\square$

Problem 8.3. Let $f : A \to B$ be a function and let $X \subseteq A$ and $Y \subseteq B$. Recall we defined the sets

$$f(X) = \{y \in Y : y = f(x) \text{ for some } x \in X\} \subseteq B$$
$$f^{-1}(Y) = \{x \in X : f(x) \in Y\} \subseteq A$$

(a) Prove that $X \subseteq f^{-1}(f(X))$. Give an example to show that this containment can sometimes be strict (ie $X \subsetneq f^{-1}(f(X))$)

Solution. □

(b) Make a similar conjecture and then prove it about the relationship between Y and $f(f^{-1}(Y))$ (is one contained in the other? If so, which one?)

Solution. □

(c) Prove that $f : A \to B$ is injective iff for all subsets $X \subseteq A$ we have $X = f^{-1}(f(X))$

Solution. □

(d) Make a similar conjecture as in part c and prove it about f being surjective.

Solution. □

Collaborators:

References:

- [Book(s): Title, Author]
- [Online: Link]
- [Notes: Link]

Fin.