Relations

Let A, B be sets

- For visual intuition, lets depict as

Def: A _____ from _ to _ is a

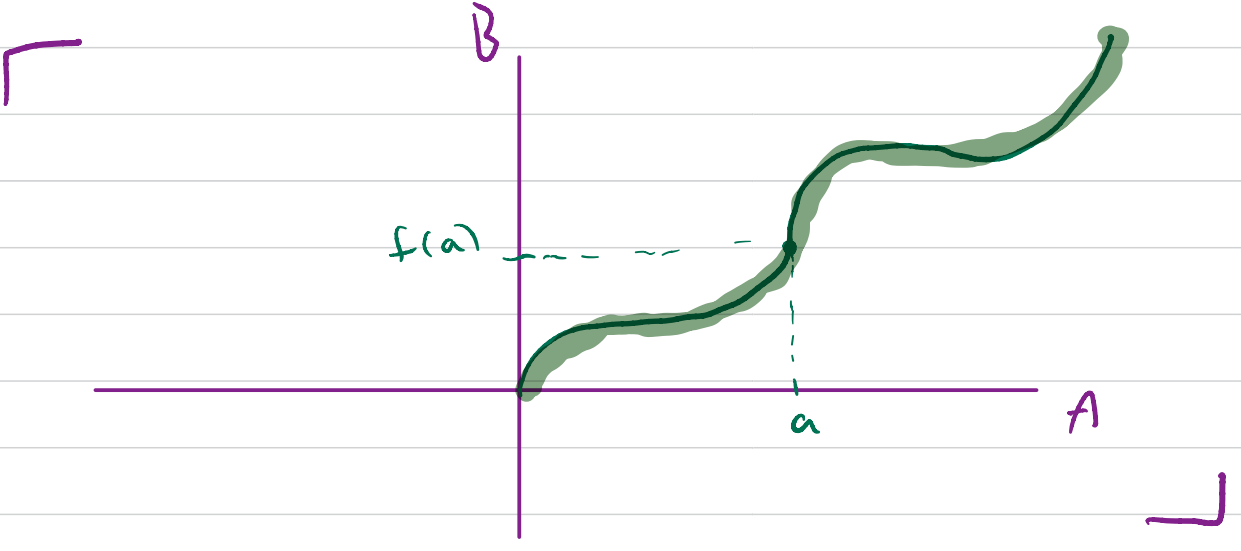
_____

- For a _____, given an element $(a, b) \in$ ___, we will write ___ and read it as "_____"
- If $(a, b) \notin$ ___ we will write ___ and say that _____

Examples

a) Let $f: A \longrightarrow B$ be a function. Define the _____

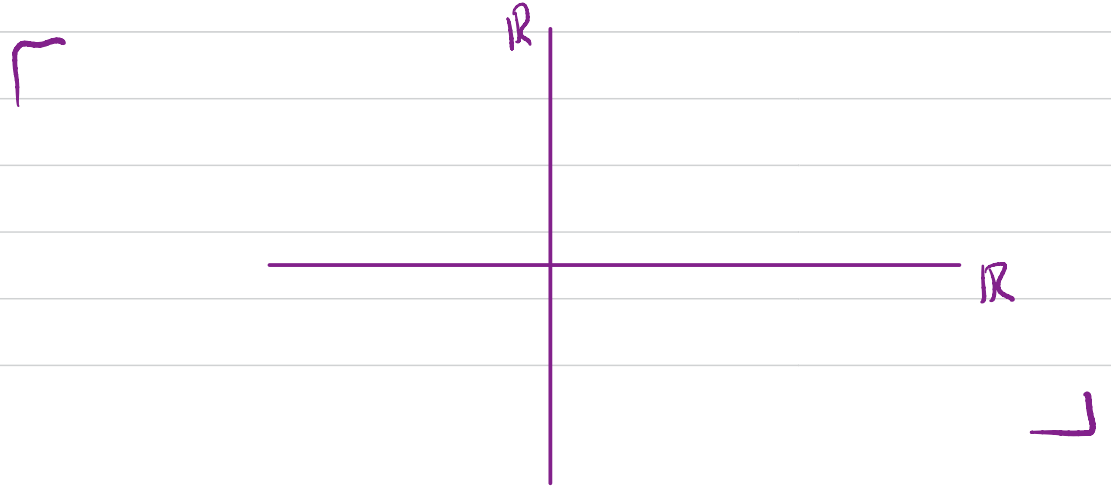
_____ $= \{ (\ ,\ ) \mid \qquad \} \subseteq A \times B$



this is a relation from A to B

⟶ In this way a function is just a special case of a relation!

b) Let $S = \{(x, y) \in \mathbb{R}^2 \mid \_\_\_\_\_\} \subseteq \mathbb{R} \times \mathbb{R}$



- Note that we can solve for x:

$$\_\_\_\_\_ = 1 \longrightarrow y = \_\_\_\_\_$$

this is not a _____, but it is still a

_____

Def: Let _____ be a _____ from A to B

Then the

i) _____ is the set

_____ $= \{ a \in A \mid$ _____ $\}$

$\rightsquigarrow$ this is the set of all _____

that occur in _____ of R.

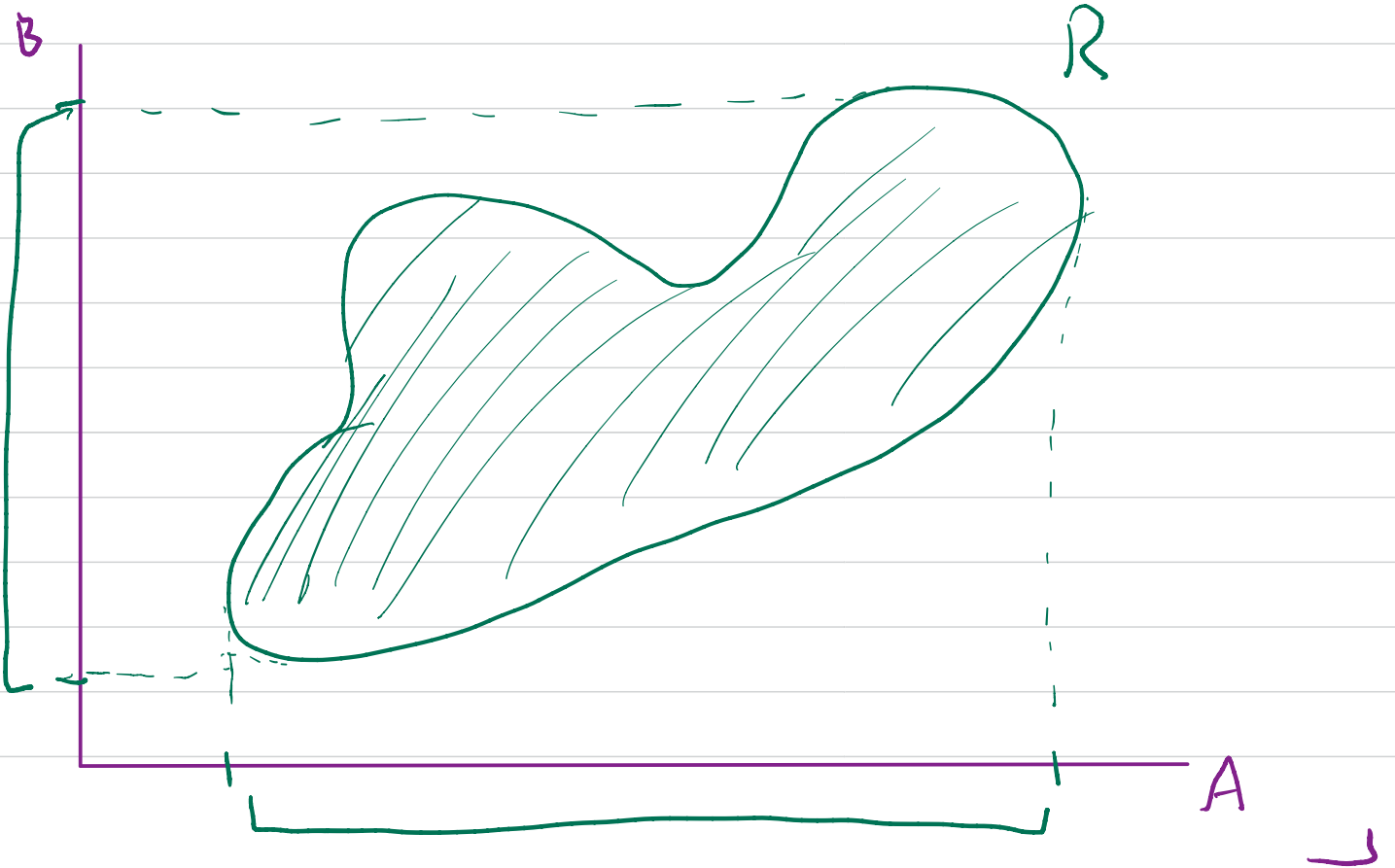
ii) The _____ of R (or the _____ of R)

is the set

_____ = $\{ b \in B \mid \quad \}$

⟶ this is the set of all _____ that occur as _____ of R

Visually we can think of it this way



ex) Again consider the relation

$$S = \{ (x,y) \in \mathbb{R}^2 \mid \_\_\_\_ \}$$

Then $\text{dom}(S) = \{ \_\_\_ \mid \_\_\_\_ \text{ for some } y \in \mathbb{R} \}$

$= [\ ,\ ]$ (from picture)

$\text{range}(S) = \{ \_\_\_ \mid \_\_\_\_ \text{ for some } x \in \mathbb{R} \}$

$= [\ \ \ ]$ (from picture)

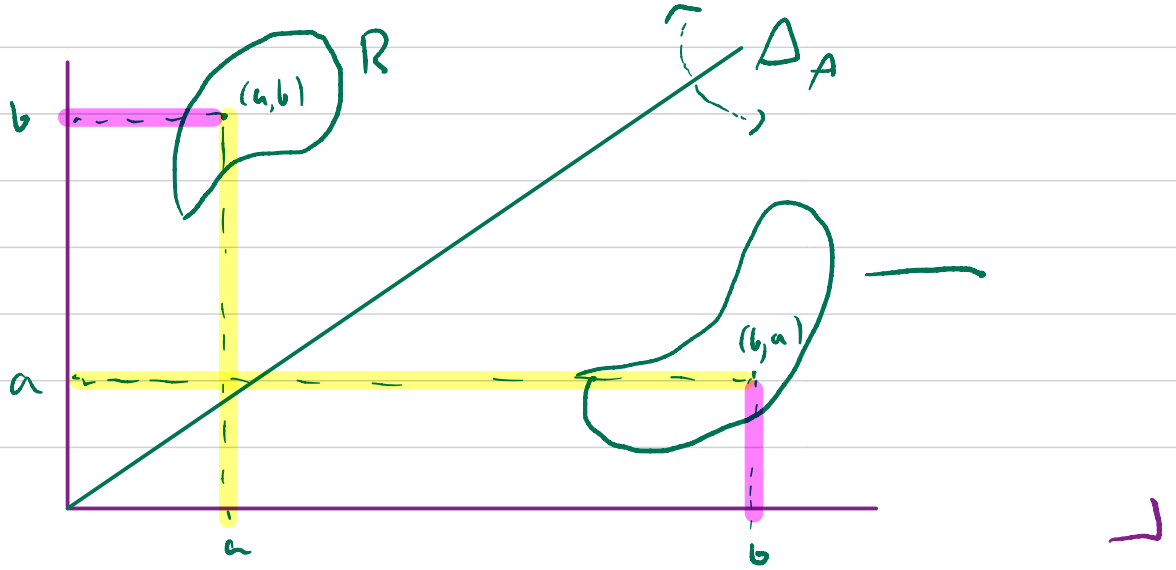
Def: Let $R \subseteq A \times B$ be a relation from A to B. Then the _____, denoted _____ is _____ $= \{(\ ,\ ) \in B \times A \mid \_\_\_ \in R\} \subseteq B \times A$

_____ is a relation from B to A

· If ____ = ____ then we obtain ____ by ____

· this is the subset defined as

$$\Delta_A = \{ (\quad) \mid a \in A \} \subseteq A \times A$$



ex) Consider the relation $R \subseteq \mathbb{R} \times \mathbb{R}$.

defined by $R = \{(a,b) \mid a \leq b\}$

- What is R^{-1}:

ex2) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{u, v, w, x, y, z\}$.

Consider the relation from A to B given by

$R = \{(2,x), (1,z), (2,v), (4,x), (4,u), (5,w)\}$

→ · $dom(R) =$

· $range(R) =$

→ $R^{-1} = \{$ $\}$

Special terminology when $A = B$

- From now on, assume $A = B$. Then a _____ is just called a _____ on A

ex) Equality

- Recall the relation on A

$R = \Delta_A = \{$ _____ $\mid$ _____ $\} \subseteq A \times A$

$\rightsquigarrow$ have aRb iff _____

$\rightsquigarrow$ this relation is just _____

We will generalize this concept of equality to "$\sim$" - relation

- For this, lets note the fundamental properties of $\sim$

1) for every $a \in A$ $a \sim a$

2) If $a \sim b$ then $b \sim a$

3) If $a \sim b$ and $b \sim c$ then $a \sim c$

Def: Let R be a relation on A.

1) R is called _____ if _____ $\forall a \in A$

⌜This is equiv to saying $\Delta_A \subseteq R$⌟

2) R is called _____ if, whenever _____ then _____

⌜This is equiv to saying $R =$ _____⌟

3) R is called _____ if whenever _____ and _____ then _____

Previously seen examples

a) Let $n \in \mathbb{N}$. Then define R relation on $\mathbb{Z}$ by

$a R b$ iff $a \equiv b \bmod n$.

- We saw that

1)

2)

3)

b) R on $\mathbb{Z}$ defined by aRb iff $a \leq b$

What does R satisfy?

Can also give "basic" examples too

Let $A = \{a, b, c\}$. Define the relation on A by

$R = \{(a,a), (b,b), (c,c), (a,b), (b,c), (a,c)\}$

- Is R _____?

- Is R _____?

- Is R _____ ?

Geometric Example

Let $A = \mathbb{R}$ and consider the relation R on $\mathbb{R}$.

$$R = \{(x,y) \in \mathbb{R}^2 \mid |x-y| \leq 1\}$$

- So if xRy then $|x-y| \leq 1 \longleftrightarrow \_\_ \leq x-y \leq \_\_$

  $\rightsquigarrow$ So R is bounded by lines

  - $y = x+1$
  - $y = x-1$

- R ____? (Note, ____ $\subseteq R$ so should be yes!)
  - Let $x \in \mathbb{R}$. Then $|\quad\quad| = \quad\quad \leq 1$

  So yes _____

- R _____
  - Let $x, y \in \mathbb{R}$ and suppose _____

    Then

• R _____

⌜Goal: Given $a, b, c \in \mathbb{R}$ with _____ and _____

determine whether or not _____

→ i.e. if _____ ≤ 1 and _____ ≤ 1

is _____ ≤ 1 ⌟

Let $x = 1.5$, $y = 1$ and $z = 0$

Then _____

but 

so R is