Problem Set 2

[Your Full Name Here]

MATH 100 — Introduction to Proof and Problem Solving — Summer 2023

Problem 2.1. For the sets $A = \{1, 2, \ldots, 10\}$ and $B = \{2, 4, 6, 9, 12, 25\}$, consider the statements

$$P : A \subseteq B. \quad \text{and} \quad Q : |A \setminus B| = 6.$$

Determine which of the following statements are true, with justification.

(a) $P \vee Q$

Solution. □

(b) $P \vee \neg Q$

Solution. □

(c) $P \wedge Q$

Solution. □

(d) $\neg P \wedge \neg Q$

Solution. □

(e) $\neg P \vee \neg Q$

Solution. □

Problem 2.2. Consider the open sentences:

$$P(x,y) : x + y = -2. \quad \text{and} \quad Q(x,y) : x^2 + y^2 = 4.$$

where the domain of both x and y is $S = \{-2, 0, 2\}$.

State each of the following in words and determine all values of $x, y \in S$ for which the resulting statements are true, with justification.

(a) $\neg P(x,y)$

Solution. □

(b) $P(x,y) \vee Q(x,y)$

Solution. □

(c) $P(x,y) \wedge Q(x,y)$

Solution. □

(d) $P(x,y) \implies Q(x,y)$

Solution. □

(e) $Q(x,y) \implies P(x,y)$

Solution. □

(f) $P(x,y) \iff Q(x,y)$

Solution. □

Problem 2.3.

(a) For statements P, Q and R, show that

$$((P \vee Q) \Rightarrow R) \equiv (P \Rightarrow R) \wedge (Q \Rightarrow R)$$

Solution. □

(b) For statements P and Q, the implication $\neg P \implies \neg Q$ is called the *inverse* of the implication $P \implies Q$ which it is *not* equivalent to. Find another implication that is logically equivalent to $\neg P \implies \neg Q$ and verify your answer.

Solution. □

Collaborators:

References:

- [Book(s): Title, Author]
- [Online: Link]
- [Notes: Link]

Fin.