Recap

To prove ________ we have 3 options

1) (________)

2) ($\neg$___ ___ $\neg$___)

3) (___ ___ $\neg$___ ___ $\bot$)

ex) Let $x \in \mathbb{R} \setminus \{0\}$. Show that if $x + \frac{1}{x} < 2$ then $x < 0$

Pf 1)

Pt 2)

pt3)

Existence Proofs

<u>Goal</u>: Prove statements of the form

____

$\rightsquigarrow$ Show that __________ an $x \in S$ making

_____ true

Methods

1)

2) Use other ________

3) Use proof by ________

ex) Show that $\exists\, a, b \in \mathbb{Q}$ such that a^b is irrational.

ex2) Show $\exists\, a \in \mathbb{Q}$, b irrational such that

1) $a^b \in \mathbb{Q}$

2) a^b irrational

ex 3) Show $\exists\, a, b$ irrational such that

$a^b \in \mathbb{Q}$

These were all direct - How about Method $\underline{2}$

ex) Show that $x^2 + 2x - 5 = 0$ has a solution on $[1, 2]$

Pf)

Now a method $\underline{3}$ example

<u>Thrm</u>: (Pigeonhole Principle)

Suppose n objects are placed in m boxes. If $n > m$ then ______________________________

____

Pf)

ex) Let S be a set of 3 integers. For $\phi \neq A \subseteq S$ let

$\sigma_A =$ ______________

Prove there exists two distinct non-empty subsets $B \neq C \subseteq S$ such that

σ_B ___ σ_C __________

Scratch work: $S = \{2, 5, 7\}$

A	$\{2\}$	$\{5\}$	$\{7\}$	$\{2,5\}$	$\{2,7\}$	$\{5,7\}$
σ_A						

$\rightsquigarrow$ For $B = \{ \quad \}$ and $C = \{ \quad \}$

have σ_B σ_C

Pf)

Uniqueness

Rather than just asking for ______________

we can ask for a ___________ element such that

______________ is true.

⇝ We write ___________ , ___________

In this case, there are ______ steps to proving such a statement

1) Show ______________

2) Show ____________________

$\rightarrow$ typically goes like this:

- Suppose $x, y \in S$ such that $R(x)$, $R(y)$ ______ then ______

<u>or</u>

- Suppose $x \neq y \in S$ with $R(x)$, $R(y)$ ______

$\rightsquigarrow$ we get a ____________ ex) Show $x^5+2x-5=0$ has unique solution in $[1, 2]$.

pf)

Disproving Existance Statements

To ______ , ______ we will prove it's ______ is true

⇝ Remember: $\neg($ ______ $) \equiv$ ______

ex) Disprove that there exists integers $a \geq 2$ and $n \geq 1$ such that $a^2+1=2^n$

Goal: Prove ______ we have ______

Pf) For the sake of contradiction,

Prove or Disprove

General tip for ______ quantified statements

- if you think they are ______, PROVE THEM!
- if you think they are ______, find a ______

ex) Prove or disprove: If ab, bc, ac are all even then a, b, c are even.

ex) Prove or disprove: Suppose A is a set such that

$A \cap B = \phi$ for every set B. then

$A = \phi$.

ex) Prove or disprove: If A, B, C are sets such that

$$A \times C = B \times C \quad \text{then} \quad A = B$$

Strategy: Elements of $A \times C$ are ordered pairs

$(\quad\quad)$ with $\_ \in A$, $\_ \in C$.

If $A \times C = B \times C$ then given an

$(\quad\quad) \in A \times C$ it is also in $B \times C$

so $\_\_\_ \in B$. This makes it seem plausible.

HOWEVER. Is there a hidden assumption

we made?

$A =$ $B =$ $C =$

$\longrightarrow$ Modify the claim

ex) Let A, B, C sets with $C \neq$ ____ .

If $A \times C = B \times C$ then $A = B$

Forming Conjectures (basically __________ guesses)

ex) Consider the following Product, for $n \geq 2$

$$P_n = \left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right) \cdots\cdots \left(1-\frac{1}{n^2}\right)$$

Let's compute a few terms and make a conjecture about the value

$P_2 =$

$P_3 =$

$P_4 =$

$P_5 =$

$\rightsquigarrow$ Conjecture : For $n \geq 2$ $P_n =$ ______

- We will see how to prove this next class.

Similar example

Sequence defined by $a_1 = 1$, $a_2 = 4$ and

$$a_n = 2a_{n-1} - a_{n-2} + 2 \quad \text{for } n \geq 3.$$

$\rightsquigarrow$ Lets conjecture a formula.

$a_3 =$

$a_4 =$

$a_5 =$

$\rightsquigarrow$ $a_n =$ ______

We'll prove next class