

Welcome All!

• All admin stuff is on canvas

• See the video I posted

Yiddish word of the day

"oysegtskeykhnt" = גַּזְדָּוִילַכְ"

" " =

Yiddish curse / blessing / expression of the day

"likht vi esn bagl" = סַדְ" ? יְמָנָה כֹּסֶגֶשׁ

" "

" :

OK

So why Linear Algebra?

- This is the question we will explore in the class
- Moreover, there is the final project you all will complete about this

- Turns out, a lot of questions in "nature" can be expressed as a system of equations.
- Want some better way of solving these.

Some notation

- Will use x_1, x_2, x_3, \dots as variables.

First thing to Note

• If a system of equations looks like this

$$x_1 + 3x_2 - 4x_3 = 1$$

$$2x_2 - x_3 = 0$$

$$x_3 = 4$$

we can

Def: 1) The leading variable in an equation is _____

2) A free variable in an equation _____

3) A linear system of equations is in Echelon Forms (EF)
if _____

ex 1) $3x_1 - 4x_2 + 6x_3 = 8$ In / Not in EF?
 $x_2 - x_3 = 4$
 $x_2 + 10x_3 = 6$

$$2) x_1 - 4x_2 - x_3 + x_4 = 1$$

$$-10x_3 - 3x_4 = 2$$

$$x_4 = 1$$

In / Not in EF?

$$3) 5x_1 - 2x_2 + 6x_3 = 6$$

$$\begin{matrix} x_2 - x_3 \\ x_1 \end{matrix} = 0$$

In / Not in EF?

Goal Want to convert a "general" system like this

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \quad (\text{A})$$

into echelon form!

There are 3
"do" to our system to put it into Echelon Form
that we can

$$\text{ex. } 1) \quad x_1 + 2x_2 - x_3 = 2$$

$$-2x_1 + x_2 + 3x_3 = 2$$

$$-x_1 + 5x_2 + 3x_3 = 5$$

Goal: Put this into Row echelon Form

$$\text{ex2) } \begin{array}{l} x_1 + x_2 + x_3 = 1 \\ x_1 + 2x_2 + 3x_3 = 2 \\ 2x_1 + x_2 = 1 \end{array} \quad \begin{array}{l} R_2 \rightarrow R_2 - R_1 \\ \longrightarrow \\ R_3 \rightarrow R_3 - 2R_1 \end{array}$$

In These 2 examples

- 1) No "zero rows" and no free variables = _____
- 2) A zero row and a free variable = _____

ex3: $x_1 - 3x_2 + 2x_3 = 0$

$$2x_1 - 5x_2 + 4x_3 = 1$$

$$x_1 - 4x_2 + 2x_3 = -2$$

Note: If your system looks like

$$a_{11}x_1 + \dots + a_{1n}x_n = 0$$

:

:

$$a_{m1}x_1 + \dots + a_{mn}x_n = 0$$

then

We call a system like this

In Summary

Start With Linear System

- Put into Echelon Form

1 unique solution if _____

∞-many solutions if _____

no solution if _____

Section 1.2 - Matrices

In these linear systems usually write about the coefficients
in front of the variables.

$$a_{11}x_1 + \dots + a_{1m}x_m = b_1$$

$$a_{m1}x_1 + \dots + a_{mn}x_n = b_m$$

this _____ is called the _____ of the linear system.

ex) Write coefficient matrix for

$$3x_1 - x_2 + x_3 = 4$$

$$-2x_1 - x_2 + 2x_3 = 1$$

$$x_1 - 4x_2 + 6x_3 = 3$$

Def: A matrix is in Echelon Form (EF) if

1)

2)

3)

ex) i)
$$\left(\begin{array}{ccc|c} 3 & -3 & 6 & 9 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 10 & 6 \end{array} \right)$$
 In / Not in EF?

ii)
$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right)$$
 In / Not in EF?

Def: The positions of the leading terms in a row
are called pivot positions

The columns of the matrix that have a pivot
positions are called pivot columns

Q: How to put a matrix in EF? A:

ex)
$$\left(\begin{array}{ccc|cc} 3 & -3 & 6 & 1 & 9 \\ 0 & 1 & -1 & 4 & 6 \\ 0 & 1 & 10 & 1 & 6 \end{array} \right)$$

ex)
$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 0 & -8 \end{array} \right)$$

Is there an issue with this coefficient matrix?

What You Can Do Now

- Write the coefficient matrix for Linear Systems
- Put Matrix into Echelon Form
- Tell if system has a solution

