What is Being Proved?

Given below are proofs for results: State what is being proved:

(a) *Proof.* Assume that x is even, so that $x = 2a$ for $a \in \mathbb{Z}$. Then

$$\begin{aligned} 3x^2 - 4x - 5 &= 3(2a)^2 - 4(2a) - 5 \\ &= 2(6a^2 - 4a - 3) + 1 \end{aligned}$$

so we have that $3x^2 - 4x - 5$ is odd
Now assume that x is odd, so that $x = 2b + 1$ for some $b \in \mathbb{Z}$. Then we compute

$$\begin{aligned} 3x^2 - 4x - 5 &= 3(2b+1)^2 - 4(2b+1) - 5 \\ &= 2(6b^2 + 2b - 3) \end{aligned}$$

So we see that $3x^2 - 4x - 5$ is even $\square$

Proof Explanation.

(b) *Proof.* By contrapositive, we will assume that $n = k^2$ for some $k \in \mathbb{Z}$. Then we have 4 cases:

(1) First, we assume that $k = 4l$ for some $l \in \mathbb{Z}$. Then

$$n = (4l)^2 = 16l^2 \equiv 0 \text{ mod } 4$$

(2) Next, we assume that $k = 4a + 1$ for an $a \in \mathbb{Z}$. Then

$$n = (4a+1)^2 = 16a^2 + 8a + 1 \equiv 1 \text{ mod } 4$$

(3) Now, we assume that $k = 4b + 2$ for an $b \in \mathbb{Z}$. Then

$$n = (4b + 2)^2 = 16b^2 + `16b + 4 \equiv 0 \text{ mod } 4$$

(4) Finally, we assume that $k = 4c + 3$ for an $c \in \mathbb{Z}$. Then

$$n = (4c + 3)^2 = 16c^2 + 24c9 \equiv 1 \text{ mod } 4$$

This concludes the proof, as we have exhausted all possibilities. $\square$

Proof Explanation.