Yiddish word of the day

hilke pilke shpiler = הילקע פּילקע שפּילער

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Yiddish expression/curse ect

"Zolst vaksn vi a tsibile, mit der kop in der erd" = זאָלסט וואַקסן ווי אַ ציבעלע מיטן קאָפּ אין דער ערד!

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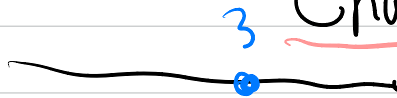
Chapter 2: $\mathbb{R}^n$

ex: $\mathbb{R}$ 

real line

$\mathbb{R}^2$:

$\mathbb{R}^3$!

Def: $\mathbb{R}^n$ is

- an element in $\mathbb{R}^n$ is called an ex) $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ is a ________ in

ii) $\begin{pmatrix} -2 \\ \sqrt{10} \end{pmatrix}$ is a ________ in

Addition of two vectors in $\mathbb{R}^n$ is defined "component wise"

$\vec{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$ $\vec{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$ $\Rightarrow$

Multiplication by a "scalar" (i.e. a real #) is defined componentwise,

- if c is a real # $\vec{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$ is an n-vector, then

$c\vec{v} =$

Def: The for $\mathbb{R}^n$ are the following vectors.

$\vec{e}_1 = \begin{pmatrix} \quad \end{pmatrix}, \quad \vec{e}_2 = \begin{pmatrix} \quad \end{pmatrix}, \quad \dots, \quad \vec{e}_n = \begin{pmatrix} \quad \end{pmatrix}$

ex: $\mathbb{R}^2$ the standard basis is

$\vec{e}_1 = \begin{pmatrix} \quad \end{pmatrix}, \quad \vec{e}_2 = \begin{pmatrix} \quad \end{pmatrix}$

$\begin{pmatrix} 2 \\ 3 \end{pmatrix} =$

Def: Let $\vec{u}_1, \vec{u}_2, \dots, \vec{u}_m$ be vectors in $\mathbb{R}^n$. Then a linear combination of these vectors is

ex) Let $\vec{u}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\vec{u}_2 = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$. Then is a linear combination of $\vec{u}_1, \vec{u}_2$

ex: Is the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ a linear combination of the vectors $\vec{u}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\vec{u}_2 = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$?

ex2: Is the vector $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ a LC of

$\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$, $\vec{v}_3 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$?

ex: Is $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ a LC of $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$

Thrm: A vector $\vec{b}$ in $\mathbb{R}^n$ is a LC of the vectors $\vec{v}_1, \dots, \vec{v}_m$ if and only if

(We're saying that a vector $\vec{b}$ in $\mathbb{R}^n$ is LC of $\vec{v}_1, \dots, \vec{v}_m$
||
)

ex) Is the vector $\begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$ a LC of $\vec{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} -3 \\ -5 \\ -4 \end{pmatrix}$

$\vec{v}_3 = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}$

$\Rightarrow \left(\quad \middle| \quad \right)$

Check:

Chapter 2.3 - Span

Def: Let $\vec{v}_1, \ldots, \vec{v}_k$ be vectors in $\mathbb{R}^n$.
Then the **span** of $\vec{v}_1, \ldots, \vec{v}_k$ is

Denote this collection by ____________________

Verb: If span$(\vec{v}_1, \dots, \vec{v}_k) = \mathbb{R}^n$ then we say that the vectors $\vec{v}_1, \dots, \vec{v}_k$ ______________ $\mathbb{R}^n$

Recall: A vector $\vec{b}$ is a LC of vectors $\vec{v}_1, \dots, \vec{v}_k$ is the same thing as

$$\left(\quad \quad \quad \middle| \quad \right)$$

ex) Is the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ in span$\left(\overset{\vec{v}_1}{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}, \overset{\vec{v}_2}{\begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}}, \overset{\vec{v}_3}{\begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}} \right)$

- the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is in span(v_1, v_2, v_3)

  if and only if

ex) Is the vector $\begin{pmatrix}1\\2\\3\end{pmatrix}$ in $\text{span}\left(\begin{pmatrix}1\\0\\2\end{pmatrix}, \begin{pmatrix}0\\2\\0\end{pmatrix}, \begin{pmatrix}2\\0\\4\end{pmatrix}\right)$?

Note: Only way A system is inconsistent is if

- So if $\vec{v}_1, \ldots, \vec{v}_k$ $\mathbb{R}^n$ then for any vector $\vec{b}$ in $\mathbb{R}^n$ the linear system

$$\left(\begin{array}{ccc|c} \vec{v}_1 & \cdots & \vec{v}_k & \vec{b} \\ \downarrow & & \downarrow & \downarrow \end{array}\right)$$ is

$\Rightarrow$ there won't be

$\Rightarrow$ there is a

This implies that if $\vec{v}_1 \dots \vec{v}_k$ span $\mathbb{R}^n$

then k ______ n

ex:

A line can't span $\mathbb{R}^2$

Summary: Let $A = \begin{pmatrix} \vec{v}_1 & \cdots & \vec{v}_k \\ \downarrow & & \downarrow \end{pmatrix}_{n \times k}$ be a $n \times k$ matrix

Then the following are equivalent

1) For every vector $\vec{b}$ in $\mathbb{R}^n$

2) The vectors $\vec{v}_1, \ldots, \vec{v}_k$

3) There is a

ex) Use "language of spanning" to describe solution sets to equations.

Find the solution set to the equations

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 0 \\ -x_1 + 4x_2 - 6x_3 &= 0 \end{aligned} \Longrightarrow \begin{aligned} x_1 - 2x_2 + 3x_3 &= 0 \\ 2x_2 - 3x_3 &= 0 \end{aligned}$$

ex) Find the solution set to the homogeneous system with coefficient matrix $A = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 2 & 3 & 3 & 1 \\ 2 & 6 & 0 & 4 \\ 1 & -1 & 4 & -1 \end{pmatrix}$

has EF $\longrightarrow \begin{pmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$