Principles of Induction

- Notation: $\mathbb{Z}_{>0} = \{1, 2, 3, 4, \ldots\}$

- The ______________________ is a method of proof for statements like ______________

- To prove these statements, we do the following

  1) (Base Step):

  2a) (Inductive hypothesis):

2b) (Inductive leap) :

⌈ Rmks : • An intuition for this can be gotten from comparing to pushing over dominos

• The logical foundation is that

$$P \wedge (\qquad) \Rightarrow \underline{\qquad}$$ is a tautology ⌋

ex) Show that for $n \geq 1$

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

Pf) Let us first identify the open sentance $P(n)$.

$P(n)$:

ex) Let $n \geq 1$. Prove that $\frac{d^n}{dx^n}\left(e^{x^2}\right) = P_n(x)e^{x^2}$ where $P_n(x)$ is a degree n polynomial.

Pf)

It can happen that our ____________, that is, our first instance where the statement is ____________ does ____________ at $n=1$, but instead ____________ ____________

→ In these cases we will still ____________ by "____________"

<u>One then proves</u>

1) <u>Base Case</u>:

2a) <u>Inductive hypothesis</u> :

2b) <u>Inductive leap</u> :

This is referred to as the ___ (or ______)
principle of induction

ex) Recall last class we defined, for $n \geq 2$ integer.

$$P_n = \left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\cdots\left(1-\frac{1}{n^2}\right)$$

We conjectured it is also of form

$$P_n = \frac{n+1}{2n}$$ Lets prove it.

Pf)

ex) If $A_1 \ldots A_n$ are sets, for $n \geq 2$ then

$$(A_1 \cup \cdots \cup A_n)^c = A_1^c \cap A_2^c \cap \cdots \cap A_n^c$$

ex) Let $n \geq 2$, and let $f_1 \dots f_n$ be differentiable functions. Prove that

$$\frac{(f_1 \cdots f_n)'}{f_1 \cdots f_n} = \frac{f_1'}{f_1} + \frac{f_2'}{f_2} + \cdots + \frac{f_n'}{f_n}$$

Pf)

Strong Principle of Induction

The ________________________ is a variant of the first principle of ____________ that we saw

$\leadsto$ this will be usefull for __________ problems

Still trying to prove statements like

$\forall n \geq m,$ ______________ for some fixed $m \in \mathbb{Z}$

This time though, our steps will be

1) Base Case

2a) Inductive hypothesis

2b) Inductive leap

Thm: Prime Factorization of the integers

Every positive integer $n \geq 2$ is a product of prime #'s.

Pf)

Corollary: Every integer $n \geq 2$ has at least one prime # dividing it.

Corollary: There are ∞ many prime #'s.

Pf)

Recall: Last class we defined the sequence

$$a_1 = 1$$
$$a_2 = 4$$
$$a_n = 2a_{n-1} - a_{n-2} \quad \text{for } n \geq 3.$$

$\rightsquigarrow$ Conjectured that $a_n = n^2$. Let's prove it!