Lecture 1

- OH :

- Email :

	M	T	W	Th	F	
6/26			PS 1 Due		PS 2 Due	6/30
7/3			PS 3 Due		• PS 4 due • Glossary (1)	7/7
7/10			PS 5 Due		• Proof Part (1) due • PS 6 Due	7/14
7/17					PS 7 Due	7/21
7/24			PS 8 Due		• Proof Part (2) Due • Glossary (2)	7/28

Sets!

- **Def**: A ________ is a ________ of ________

⇝ **Notation**:
- ______-______ letters (   ) will denote ________ typically
- ______-______ letters (   ) will denote ________ of sets
- the symbol ______ denotes that an ________ belongs to a set (Latex: \in)

We read this as

- ________ : "a is an element of A"

- ________ = "a is **not** an element of A"

  (Latex: \notin)

How to describe a set?

1) We know all the ________ of the set and we ________ them all out within { }

ex) $A = \{-3, -2, 2, 3\} = \{2, -3, 3, -2\}$

(ie, ________ doesn't matter)

$A = \{$blue, green, purple, peach$\}$

2) We have a ______ that describes a ______-______ in the set

- We call this ______-______ notation, and it takes the form

$$\{\ \_\_\_\_\_\_\_\_ \mid \text{properties of this element}\ \}$$

We read this as
"the set of all elements such that this property holds"

First some important sets

$\mathbb{N}$ = the set of natural #'s = $\{ \qquad \}$ (\mathbb{N})

$\mathbb{Z}$ = set of integers = $\{. \qquad \}$ (\mathbb{Z})

$\mathbb{Q}$ = set of rational #'s = $\{ \qquad \mid a, b \in \mathbb{Z}, b \neq 0 \}$ (\mathbb{Q})

$\mathbb{R}$ = set of real #'s (\mathbb{R})

ex) ii) $A = \{ n \in \mathbb{Z} \mid n \text{ even and } |n| \leq 4 \}$

$= \{ n \in \mathbb{Z} \mid n = 2k, \text{ for } k \in \mathbb{Z} \text{ and } |k| \leq 2 \}$

$= \{ \qquad \}$

$=$ " "

A weird set

Def: The set with ________ is called the ________ and is denoted ____

- There are different ways of describing this set

ex) $\{x \in \mathbb{Z} \mid x^2 < 0\}$ or

$\{$ days of the week that end in r $\}$

WARNING !!

• the ________ has no elements, but the set $X = \{\quad\}$ is **NOT** the ________

It has an element!

Def: The ________ of a set is the # of ________ in the set. For a set X we denote the cardinality by

ex) $A = \{x \in \mathbb{Z} \mid x \text{ even and } |x| \leq 4\}$

$\rightsquigarrow$ ________ = ________

Rmk: This notion of __________ is straightforward when the set is ______

For ______ sets it is tricky (but super interesting)

ex) We will see that the following all have the same __________

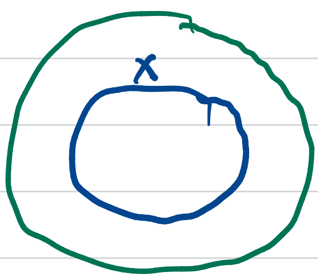
(   )

but that ____ has a larger ______

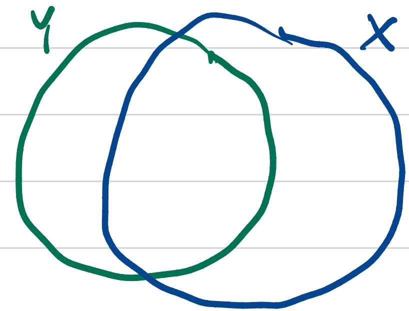
Subsets

Def: Given 2 sets X, Y we say that X is a ______ of Y (denoted ________ / ______) if every element of ______ is also an element of ______

ex) Y X



Non ex: Y X



Q: Does a set always have at least one subset?

$\underline{A}$

ex) Let $B = \{\phi, \{\phi\}, 1, 2, \{1, 2\}\}$

• note $\phi \Rightarrow \{\phi\}$

$\{\phi\} \quad \{\{\phi\}\}$

$1, 2 \Rightarrow \{1, 2\}$

$\{1, 2\} \Rightarrow \{\{1, 2\}\}$

And <u>always</u> ϕ

B

• Is $\{\{2\}3\}$? ______ ! because

$\{1\} \in$? ______ !

Def: We say two sets A, B are ______ if both

1)

2)

We will write this as

• A set X is said to be a ______ of a set Y

if 1)

2)

We will write this as $X \quad Y$ (\subset)

(or sometimes $X \quad Y$) (\subsetneq)

Def: The ______ of a set X is the set of all ______ of X, including ____ and ____.

- We denote this set ______

ex) For the following, find $|X|$, $P(X)$, and $|P(X)|$

a) $X = \{1, 2, 3\} \rightsquigarrow |X| =$

- $\phi \quad X \rightsquigarrow \phi \quad P(X)$
- $1 \,\_\, X \rightsquigarrow \{1\} \quad X \rightsquigarrow \{1\} \quad P(X)$

$\cdot\ 1,2 \quad X \rightsquigarrow \{1,2\} \quad X \longrightarrow \{1,2\} \quad \mathcal{P}(X)$

$\cdot\ 1,2,3 \quad X \rightsquigarrow \{1,2,3\} \quad X \rightsquigarrow \{1,2,3\} \quad \mathcal{P}(X)$

$\vdots$

$\mathcal{P}(X) = \{ \qquad \qquad \}$

$\rightsquigarrow \quad |\mathcal{P}(X)| = \qquad =$

b) $X = \{1, \{2\}\} \longrightarrow |X| =$

$\mathcal{P}(X) = \{ \qquad \qquad \}$

$|\mathcal{P}(X)| = \qquad =$

c) $X = \phi \rightsquigarrow |X| =$

$\mathcal{P}(X)$

$|\mathcal{P}(X)| = \_\_\_ =$

Rmk: We shall indeed prove later that $|\mathcal{P}(X)| = \_\_\_$

For now, we will accept it as fact

Set Operations

For this section, we are going to assume all ______ are ______ of some set $\mathcal{U}$ which we call the ______ set

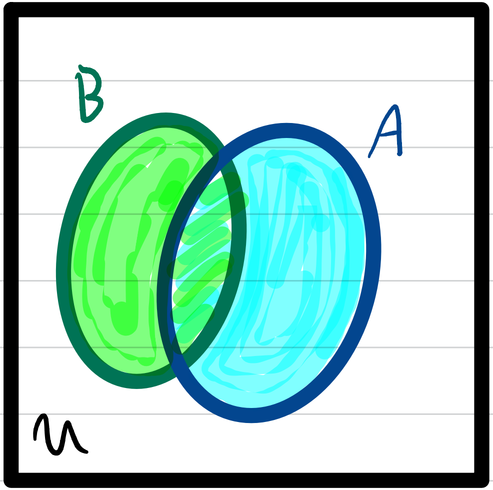
Def: Let $A, B \subseteq \mathcal{U}$. Then

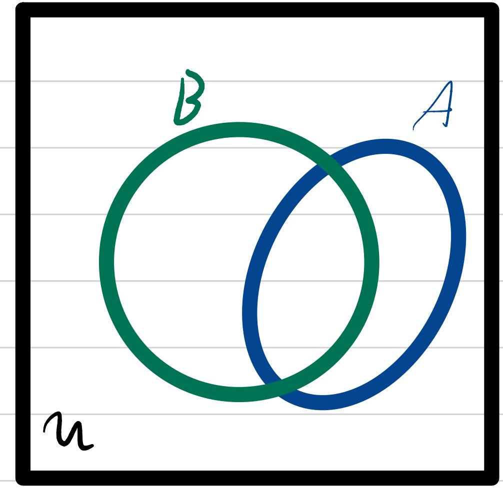
1) The ______ of A, B, denoted ______ (\cup) is the set

$$\underline{\qquad\qquad} = \{ x \in \mathcal{U} \mid \qquad\qquad\qquad \}$$

2) The ______ of A, B, denoted ______ (\cap) is the set

$$\underline{\qquad\qquad} = \{ x \in \mathcal{U} \mid \qquad\qquad\qquad \}$$





ex) $A = \{1, 2, 3, \bigstar, \square\}$ $B = \{\bigstar, 3, b, 1\}$

$\rightsquigarrow A$ $B =$

A B

[Note that we did not ______ elements in ______ that were in both ____, ____. We only write them once]

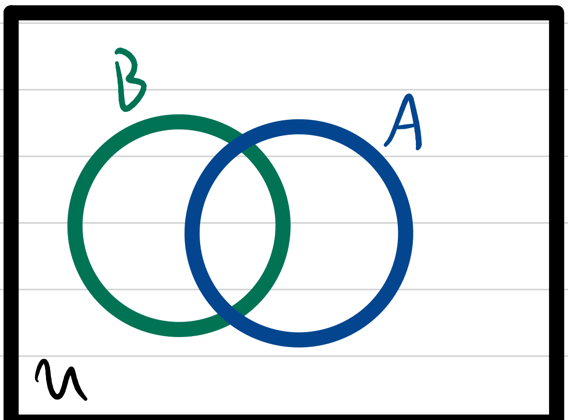
3) The ______ of A and B is the set

$$\_\_\_\_\_\_ = \_\_\_\_\_\_ = \{x \in \mathcal{U} \mid \qquad \}$$

ie: We ______ the part of ______ that is in ______

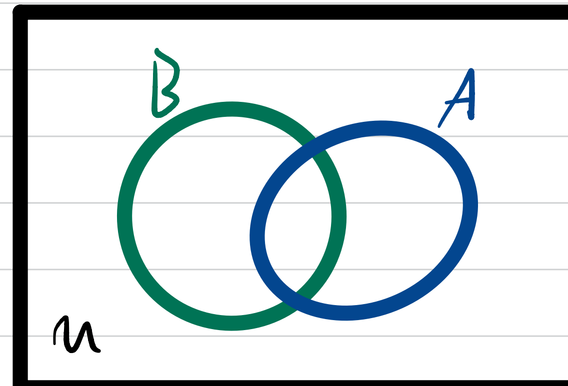
~> also have the set

______ $= \{ x \in \mathcal{U} \mid \qquad \}$



A B

______



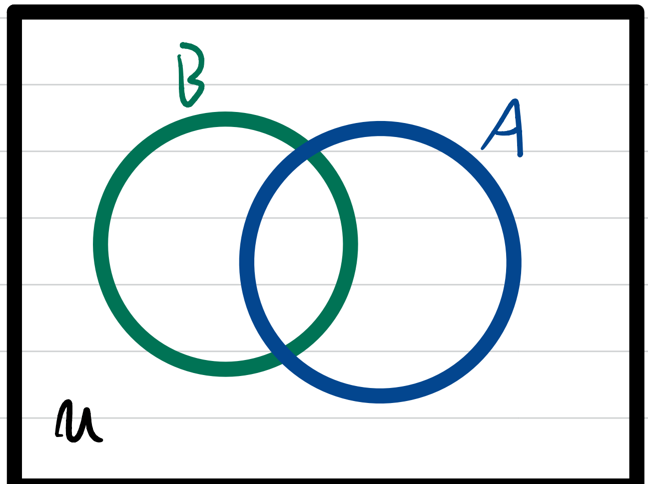
B A

______

$\rightsquigarrow$ related to this is the so called ________ - ________ of A, B

________ = $\cup$

(\triangle)



ex) $A = \{1, 2, 3, A, \square\}$ $\quad B = \{*, 3, 6, 1\}$

$\rightsquigarrow$ $A \quad B =$

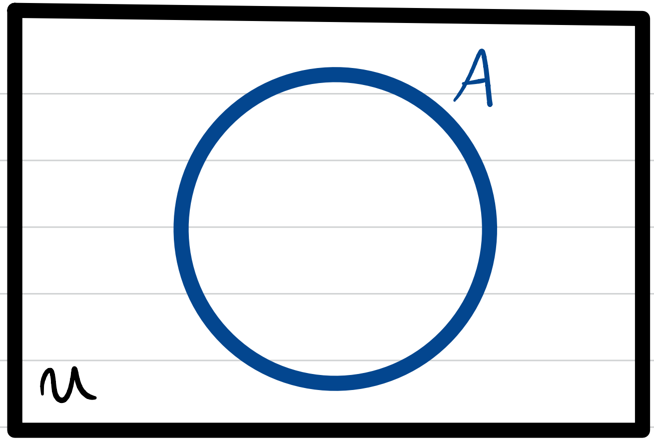
$B \quad A =$

$A \quad B =$

4) The ________ of A in $\mathcal{U}$ is the set

______ $=$ _______ $= \{x \in \mathcal{U} \mid$

= "the set of elements of $\mathcal{U}$ that ________________ "



____

ex) $\mathcal{U} = \mathbb{Z}$ and A: the set of ________ #'s.

$\rightsquigarrow$ then ________ = the set of ________ #'s

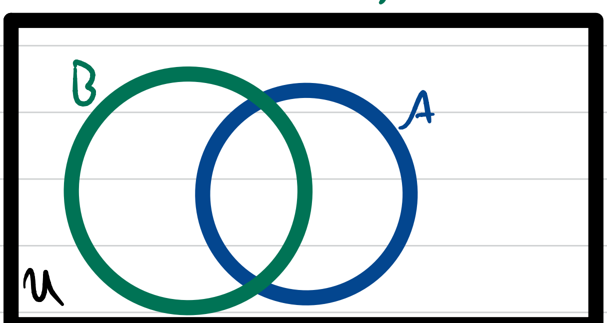
Prop: We have the following identities.

1) $A \setminus B = A \cap B^c$

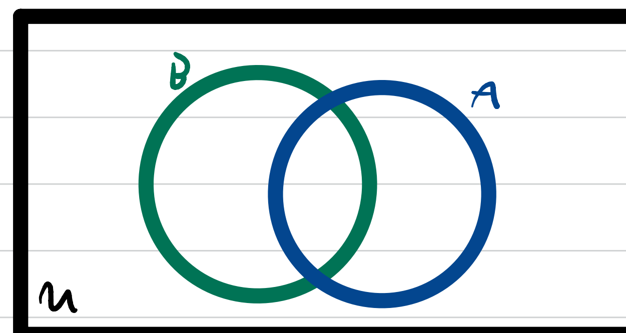
2) $(A \cap B)^c = A^c \cup B^c$

3) $(A \cup B)^c =$

We will prove (1) by a picture. We will learn how to prove such statements without pictures soon.



$A \setminus B$



$A \cap B^c$

Indexed Sets

$\rightsquigarrow$ Want a way to concisely express elements of sets

ex) $\{a_1, a_2, a_3, \ldots, a_n\} = \{a_i\}_{i=1}^{n}$

$= \{a_i\}_{i \in I}$

where $I = \{ \qquad \}$

ex) $A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i = \bigcap_{i \in I} A_i$ (\bigcap)

with $I = \{ \qquad \}$

$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i = \bigcup_{i \in I} A_i$ (\bigcup)

for $I = \{ \qquad \}$

Why? - We will be interested in collections of sets that aren't enumerable (can't list them all out)

$\longrightarrow$ this motivates us to consider ________ collection of sets, which is ________ by some ________ set I,

$\rightsquigarrow$ $\{a_\alpha, a_\beta, a_\gamma, \dots\} =$

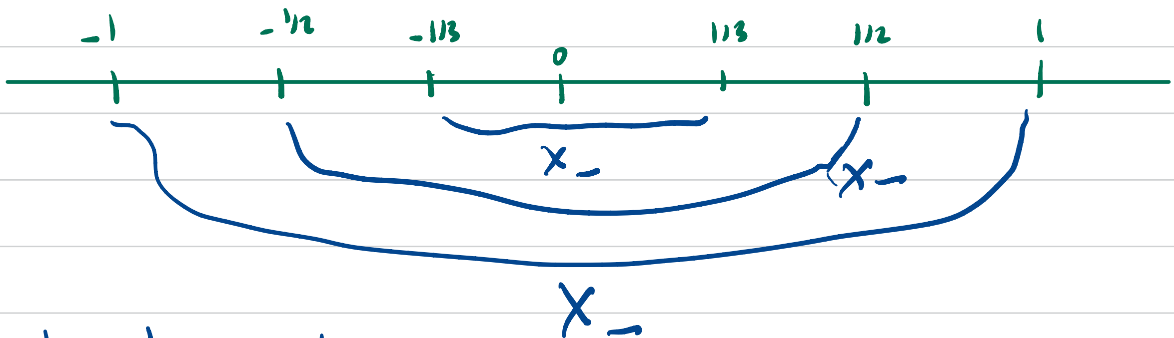
$X_\alpha \cap X_\beta \cap \dots =$ $\qquad\qquad$ $I = \{\alpha, \beta, \gamma, \dots\}$

$X_\alpha \cup X_\beta \cup \dots =$

ex) Let $I = [1, \infty)$

⤳ for each $r \in I$ consider the set

$$X_r = \left(-\frac{1}{r}, \frac{1}{r}\right)$$



⤳ also have sets like $X_\pi = (\quad , \quad)$

⟶ One can show that

1) $\displaystyle\bigcap_{r \in I} X_r =$

2) $\bigcup_{r \in I} X_r =$ ______ $= (\quad , \quad)$

Partions of Sets

Let A be a set, and let $\mathcal{S} = \{X_\alpha\}_{\alpha \in I}$ be a collection of non - ______ subsets of A for some indexing set I

[ie, $\mathcal{S} \subseteq$ ______]

Def: We say that $\mathcal{S}$ is a ______ of A if

1) $X_\alpha \cap X_\beta =$ when $\alpha \neq \beta$

2) $\bigcup_{g \in I} X_g =$ ______

ex) $A = \mathbb{Z}$. Can someone tell me a ______ of A?

ex) Let's generalize this. Let $A = \mathbb{Z}$ and let $n \in \mathbb{Z}$.

<u>Goal</u>: Find a partition of A wrt ______

$\rightsquigarrow$ For any integer $0 \leq r < n$ consider the subset

$$X_r = [r]_n = \{k \in \mathbb{Z} \mid \begin{array}{l} k \text{ has } \_\_\_\_\_\_ \; r \\ \text{when } \_\_\_\_\_\_ \text{ by } n \end{array}\}$$

ex: When $n=2$. What is

$$[1]_2 = \{k \in \mathbb{Z} \mid \begin{array}{l} k \text{ has } \_\_\_\_\_\_ \; 1 \\ \text{when } \_\_\_\_\_\_ \text{ by } 2 \end{array}\} =$$

$$[0]_2 = \{k \in \mathbb{Z} \mid \begin{array}{l} k \text{ has } \_\_\_\_ \text{remainder} \\ \text{when } \_\_\_\_ \text{ by } 2 \end{array}\} =$$

ex: When $n=3$

$[0] = \{ \quad \}$

$[1] = \{ \quad \}$

$[2] = \{ \quad \}$

- Note $[0] \cap [1] = [0] \cap [2] = [1] \cap [2] = \_\_\_\_$

• Also we have

$$[0]_3 \cup [1]_3 \cup [2]_3 = \underline{\qquad\qquad}$$

$\leadsto$ $S = \{[0]_3, [1]_3, [2]_3\}$ is a $\underline{\qquad\qquad}$ for $\mathbb{Z}$

$\leadsto$ This turns out to be true for any n. This partition is called $\underline{\qquad\qquad}$ arithmetic and we will return to it often!

ex) $A = \mathbb{R}$. Let's find a partition of A.

$\leadsto$ for each $m \in \underline{\qquad}$ consider the subset

$$X_m = [\qquad)$$

~> Note that $X_n \cap X_m = [\quad\quad) \cap [\quad\quad)$

$= \quad\quad$ if $n \neq m$

Moreover $\bigcup_{n \in \_\_} X_n =$

~> the set $\mathcal{S} = \{X_n \mid n \in \_\_\}$ is a partition for $\mathbb{R}$.

Cartesian Products

Def: For sets A and B, we define the __________, denoted ________ as the set of __________

$$\_\_\_\_\_\_ = \{ (\quad , \quad) \mid \qquad \}$$

ex 1) Let $A = B =$ _____. Then

$$\_\_\_\_\_\_\_\_ = \{ \qquad \mid \qquad \}$$

(this is often denoted by _____)

ex2) $A = \{1, 2, 3\}$ and $B = \{dog, cat\}$

______ $= \{ \qquad \}$

- Note that we can also find

______ $= \{ \qquad \}$

These are ______ in general!

- Indeed, if ______ $=$ ______ then

1) ______ $\subseteq$ ______ and

2) ______ $\subseteq$

[In our case ( ) $\in$ ____ but

( ) $\notin$ ____ so ____ $\neq$ ____]

<u>Lemma</u>: If A, B are finite sets then

$$\left| \_\_\_\_ \right| = | \quad | \cdot | \quad |$$