Last time

- **quantified statements**

    _____ :

    _____ :

- **Proofs**

    - _____ proofs and _____ proofs
    - _____ proofs (how to prove that $P \Rightarrow Q$)

Contrapositive

Thm: Let P and Q be statements. Then

$$P \Rightarrow Q \equiv \underline{\hspace{6cm}}$$

Pf)

A proof by contrapositive of $P \Rightarrow Q$ is giving a direct proof of ______

ex1) Let $x \in \mathbb{Z}$. Show that if $5x-7$ is even, then x is odd.

Pf) Rather than proving directly, which could be done, but is much harder (see pg 24 on typed notes), we will use the method of proof by contrapositive.

Re-write the following as a contrapositive statement.

ex1) Suppose $x, y \in \mathbb{R}$. If $y^3 + yx^2 \leq x^3 + xy^2$ then $y \leq x$.

- We will use proof by contrapositive. That is we want to show that if ____________ then ________________
  So let us assume __________ .

ex2) Suppose $x, y \in \mathbb{Z}$. If xy is not divisible by 5 then x is not divisible by 5 and y is not divisible by 5.

- Lets say $P(x,y)$:
  $Q(x,y)$:

⟶ So a proof by contraposition would be showing:

$$\neg Q(x,y) \Rightarrow \neg P(x,y)$$

- $\neg Q(x,y)$:

- $\neg P(x,y)$:

⇝ So we want to show: If

WARNING!!!

- Proving ______ is equivalent to proving the ______ ______.
- This is **NOT** the same as proving ______

Biconditional Statements

- **Recall**: We have

$$P \underline{\hspace{3cm}} Q \equiv (\hspace{4cm}) \wedge (\hspace{3cm})$$

and we say P _____ Q

$\rightsquigarrow$ To prove _____ need to prove two things

1)

2)

Describe what needs to be proved for the following

ex1) Let $a \in \mathbb{Z}$. Then $a^3 + a^2 + a$ is even iff a is even

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ex2) Suppose $x, y \in \mathbb{R}$. Then $(x+y)^2 = x^2 + y^2$ iff $x = 0$ or $y = 0$

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Now lets prove something

ex) An integer n is even iff n^2 is even.

Pf)

Proof by Cases

ex) Let $x, y \in \mathbb{Z}$. Show that x and y have the same parity iff $x+y$ is even.

- $P(x,y) =$ ______

- $Q(x,y) =$ ______

$\rightsquigarrow$ $P \Rightarrow Q$: If ______
then ______

$Q \Rightarrow P$: If ______ then ______

____

Pf) First we will prove that if ________________

then $x+y$ is even. We can consider 2 cases.

- Case 1: We assume x, y are ________.

  Then we can write $x =$ ________ and

  $y =$ ________ for some ________ $\in \mathbb{Z}$.

  We then compute

  $x+y =$

  and hence is ________

- Case 2: We assume that x and y are ______

Then we can write

$x =$ ______ for ______ $\in \mathbb{Z}$

$y =$ ______

Again we compute

$x+y =$

which is again ______

Now we prove that if ______ is ______ then x and y have the same parity.