Math 117

Overview / Admin Stuff

- Look at syllabus for grades / hw details
    - HW
        - Small HW due usually
        - Larger HW due usually

    think of these as "discussions"

    - Glossary

- Final Problem Set

Email

darubins@ucsc.edu

OH:

M: 3:45 - 4:45

Th: 11:45 - 12:45

Overview of Course

- More general vector spaces and fields
  $\rightsquigarrow \mathbb{R}, \mathbb{C}, \mathbb{F}_p$
- More advanced topics / in depth
  $\sim$ quotient spaces, tensor products, spectral theorem
- Prove things!!!!!

Vector Spaces

- **Math 21**

Study the set $\mathbb{R}^n$: $\{ \qquad \}$

- In this set we could

1) "vectors"

2) "vectors"

in some <u>coherent</u> way

ex) $(a+b)\vec{x} =$

$a(\vec{x}+\vec{y}) =$

$(ab)\vec{x} =$

$0(\vec{x}) =$

$1\vec{x} =$

Such a structure can be axiomitized

Q) What are the "essential notions" needed in describing the structure of $\mathbb{R}^n$ above?

First focus on these "________" (or "_______")

- how to generalize $\mathbb{R}$? What can we do in $\mathbb{R}$?

  1) We can

  2) We can

Def: A ________ is a set $\mathbb{F}$ with 2

"__________"

$\mathbb{F} \times \mathbb{F} \longrightarrow \mathbb{F}$ $\qquad\qquad$ $\mathbb{F} \times \mathbb{F} \longrightarrow \mathbb{F}$

$(a, b) \longmapsto$ $\qquad\qquad$ $(a, b) \longmapsto$

<u>Such that</u>

(A1) Associativity of ____ : For $a, b, c \in \mathbb{F}$ have

(A2) Commutativity of ____ : For $a, b \in \mathbb{F}$ have

(A3) $\exists!$ element ____ such that, $\forall a \in \mathbb{F}$

(A4) For $a \in \mathbb{F}$, $\exists!$ ________ such that

(M1) Associativity of ________ : For $a, b, c \in \mathbb{F}$

(M2) Commutativity of ________ : For $a, b \in \mathbb{F}$

(M3) $\exists!$ element ________ called the ________

such that

(M4) $\forall$ $\underline{a \neq \qquad}$ $\in \mathbb{F}$ $\exists!$ ______ called the ______

such that

(D) <u>Distributive law</u> : For all $a, b, c \in \mathbb{F}$

ex) 1) $\mathbb{R}$

2)

3)

new one!

$\longrightarrow$ 4)

Non-examples:

1)

2)

Def: Something "different" about $\mathbb{R}$ vs ______

- Note that $1_{\mathbb{R}} + 1_{\mathbb{R}} + \cdots + 1_{\mathbb{R}} \neq$

- However!

$\Rightarrow$ True in general: Only 1 of two things happen

1) Either $\exists n$ st $n \cdot 1_{\mathbb{F}}$

2) $n \cdot 1_{\mathbb{F}} \neq$

<u>Case 1</u>: The <u>smallest</u> n st ______ is called the ______

<u>Case 2</u>: $\mathbb{F}$ said to be of ______

ex i) $\mathbb{F} =$

ii) $\mathbb{F} =$

<u>Lemma</u>: For any field $\mathbb{F}$, $0_{\mathbb{F}}\, a =$ $\forall a$

Pf)

HW Q: For $a, b \in \mathbb{F}$, if $ab = 0_{\mathbb{F}}$ then $a = 0$ or $b = 0$

Quick Vector on ________________

- we will see, many questions in this class really boil down to the existance of a

______ for some ______ p.

- Certain Fields are "better behaved" in this respect

Def: $\mathbb{F}$ a field. Say $\mathbb{F}$ is ______ if for any (monic) ______

p ______

$\exists\ \alpha \in \mathbb{F}$ called a ______ of p

such that

ex) $\mathbb{Q}$?

$\mathbb{R}$?

$\mathbb{C}$?

$\mathbb{F}_p$? —

2nd Generalization

- Now we can ask for a generalization of the "vectors"

Def: Let $\mathbb{F}$ be field. Then a ________________ over $\mathbb{F}$ is a set V with 2 operations

$$V \times V \longrightarrow V \qquad\qquad \mathbb{F} \times V \longrightarrow V$$

$$(v, w) \longmapsto \qquad\qquad (\alpha, v) \longmapsto$$

such that

(A1) For $u, v, w \in V$

(A2) For $u, v \in V$

(A3) $\exists$ element ______ such that, $\forall v \in V$

(A4) $\forall v$, $\exists$ ______ such that

(S1) $\forall v \in V$, $1_{\mathbb{F}} v =$

(S2) $\forall a, b \in \mathbb{F}$, $v \in V$

$(a +_{\mathbb{F}} b) v =$

(S3) $\forall a \in \mathbb{F}$, $v, w \in V$

$a(\qquad) =$

Lemma: A __________ has a unique 0_v

Pf)

HW) 1) Additive inverses are unique

2) $0_{\mathbb{F}} \cdot v = 0_v \quad \forall v \in V$

3) $a 0_v = 0_v \quad \forall a \in \mathbb{F}$

Examples

1) Let $\mathbb{F}$ be any field, then the set

$$\underline{\mathbb{F}^n} = \{ \qquad \}$$

is a vector space over $\mathbb{F}$ with

a)

b)

2) Again, let $\mathbb{F}$ be field, then

$$M_{m \times n}(\mathbb{F}) = \left\{ \qquad \qquad \qquad \qquad \right\}$$

is a vector space over $\mathbb{F}$ with

a)

b)

3) Let $n \in \mathbb{N}$. The set

$$\mathbb{F}[t]_{\leq n} = \left\{ \qquad \right\}$$

is a vector space over $\mathbb{F}$

4) Let S be any set and $\mathbb{F}$ a field. The set

$$\mathrm{Fct}(S, \mathbb{F}) = \left\{ \qquad \right\}$$

is a vector space over $\mathbb{F}$ with a)

b)

5) Variations on (4). Let $X \subseteq \mathbb{F}^n$, consider $\mathrm{Fct}(X, \mathbb{F})$.

- cts functions:
- diff functions

⋮

- smooth functions

$\rightsquigarrow$ these are all subsets...

$\mathrm{Sm}(X, \mathbb{F})$ $\mathrm{Diff}(X, \mathbb{F})$ $\mathrm{Cts}(X, \mathbb{F})$ $\mathrm{Fct}(X, \mathbb{F})$

$\rightsquigarrow$ Leads to notion of ________

Next time!