Eigenvalues For Linear Operators

- Reminder to fill out EVALS !!!

- We are currently at . completion :(
- They are due SOON! (Tuesday 8/24) !!

Now to answer question - how to compute eigenvalues/vectors for linear transformations?

Thm: $T: V \to V$ be linear transformation. $\mathcal{B}$ a basis for V.

Then v is an eigenvector (with eigenvalue λ) for T if and only if

$[v]_{\mathcal{B}}$ is an eigenvector (with eigenvalue λ) for $A_{T,\mathcal{B}}$

ex) Find eigenvalues/vectors for

$T: \mathbb{R}_2[x] \to \mathbb{R}_2[x]$ by

$$T(a_0 + a_1 x + a_2 x^2) = (5a_0 + 2a_1 - 4a_2) + (6a_0 + 3a_1 - 5a_2)x + (10a_0 + 4a_1 - 8a_2)x^2$$

By thm: Form the matrix $A_{T,B}$ where $B = (1, x, x^2)$

$\longrightarrow A_{T,B} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$

$\Rightarrow A_T - \lambda I = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$

$0 = \det(A - \lambda I) =$

λ [illegible] :

$\Rightarrow$ Thm tells us that ______________________ is eigenvector for T with eigenvalue λ:

Check: $T(\underbrace{\qquad\qquad\qquad}_{w,})$ =

$\lambda_2 = $:

Thrm tells us that

is eigenvector of T

$T(\qquad) =$

$\underline{\lambda_3^2}$:

Fact: Eigenvectors coming from distinct eigenvalues are LI!

Then in this case $\mathcal{B}' = (w_1, w_2, w_3)$ are a basis for $\mathbb{R}_2[x]$ (since they are LI). The matrix of T wrt this basis is

$A_{T,B'} = \begin{pmatrix} \quad \end{pmatrix}$

Q: Is T an isomorphism?

→ this is the $\underbrace{\qquad\qquad}$ of $A_{T,B}$

Ex 2) $T: \mathbb{R}_2[x] \to \mathbb{R}_2[x]$ with "standard" matrix

$$A_{T,\mathcal{B}} = \begin{pmatrix} 7 & 0 & 12 \\ -12 & -5 & -12 \\ -8 & 0 & -13 \end{pmatrix} \quad (\mathcal{B} = (1, x, x^2))$$

$\Rightarrow$ 2nd column tells us ______

So __ is eigenvector with eigenvalue ______

Long way to find any other eigenvalues

$\underline{\lambda_i =}$ i

$\underline{\lambda_i}$:

Only have 2 eigenvectors for T

- we cannot diagnolize this matrix $A_{T,B}$

2 fun facts

A $n \times n$ matrix

1) $tr(A) =$

2) $det(A) =$

ex) $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 \\ 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & -36 \end{pmatrix}$

What are the eigenvalues/vectors of A?