Lecture 12 - Spectral Thms

Last time

Def: V innerproduct space $T: V \to V$ linear.

The ________ of T denoted ____

is the ! operator __ $: V \to V$ st

$$\langle \quad , \quad \rangle = \langle \quad , \quad \rangle$$

We are interested in special transformations

<u>Def</u>: $T: V \to V$ linear. Then we say T is

1) ____________ if $T \circ T^* =$

2) ____________ if $T =$ ____________

<u>Prop</u>: For any $T: V \to V$ the operators $T \circ T^*$ and $T^* \circ T$ are ____________

Prop: Let $T: V \to V$ be ______ then any ______ of T are ______

Pf)

Prop: $T: V \to V$ ______________ and $\mathcal{B}$ an _______ basis. Then

$$[\qquad]_{\mathcal{B}} =$$

Pf) Exercise :)

Prop: $T, S : V \to V$ __________. Then

__________ is true

Q: Is $(T \circ S)$?

Pf)

Thm: Spectral Thm for ______________ operators.

Let T be ______________ operator. Then $\exists$

ON basis B of ______________

Cor: Spectral Thm for ______ operators.

Let $T: V \to V$ be ______ then there exists

a basis of ON ______ for T,

with all real ______

(So, wrt this basis is $[T]_B = \left(\quad \right)$)

Rmk: True in ∞-dim

$\leadsto$ much harder proof

$\leadsto$ essential in QM.

Operators are "Hermitian" (= ________)

The basis of ________ are the

<u>observables</u> and the ________

are the measurements

ex) $\hat{H}\psi = E\psi$

$\left(\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V \right)$ E = energy

Def: We call an operator ________ if (usually over $\mathbb{R}$)

$T^* =$ ________

(sometimes called ________ - usually over $\mathbb{C}$)

Thm: TFAE (the following are equivalent)

1) T is ______

2) $\langle Tx, Ty \rangle = \langle \quad \rangle$ $\forall\, x, y \in V$

3) $\|Tx\| = \|x\|$

$$\left(\begin{array}{l} (2) = \text{preserves } \text{"} \qquad\qquad \text{"} \\ (3) = \text{preserves } \text{"} \qquad\qquad \text{"} \end{array} \right)$$

Prop: $T : V \to V$ ________. Then

any ________ of T has ________ with

$|\lambda| =$

____

Pf)

<u>Def</u>: A $n \times n$ real matrix. Say A is ______ if any of the equivalent conditions hold

1) The rows of A ____________________

2) The collumns of A ____________________

3) A invertible and $A^{-1} =$

[Call $O(n) = \{A \in M \mid AA^t = I\}$ the ________ group]

Def: A $n \times n$ $\mathbb{C}$-matrix is called

if any of the following hold

1) The rows of A

2) The collumns of A

3) A invertible and $A^{-1} =$

Cor: Spectral Thm for ______ transformations

$T: V \to V$ ______

Then $\exists$ basis of ______ with

______ with $|\lambda| =$ ______

Pf)

Def: $U(n) := \{ A \in M_{n \times n}(\mathbb{C}) \mid \underline{\hspace{4cm}} \}$

called ________________ group

Note if $A \in U(n)$

$\Rightarrow \det(AA^*) = \det(\mathbb{1}_n) =$

$\shortparallel$

Def: $SU(n) = \{A \in U(n) \mid \det A = \_\_\_\_\_\_ \}$

the "______________________________"

WE DID
IT!

פלסטיק

ז' פיל!