

a) Is $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ a LC of $\underbrace{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{\vec{v}_1}, \underbrace{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{\vec{v}_2}, \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\vec{v}_3}$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{array} \right) \xrightarrow{R_2 \rightarrow R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array} \right)$$

Note there is a solution, but C_2 is a free variable.

$$\bullet C_3 = -1 \quad \text{and} \quad C_1 = 2 - C_2$$

$$\boxed{C_2 = 6}$$

$$C_1 = 2 - 6 = -4$$

$$-4\vec{v}_1 + 6\vec{v}_2 - \vec{v}_3 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\underbrace{(-4\vec{v}_1 + 6\vec{v}_2 - \vec{v}_3)}_{\begin{pmatrix} -4 \\ 4 \end{pmatrix} + \begin{pmatrix} 6 \\ 6 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix}} = \begin{pmatrix} -4+6+0 \\ -4+6-1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \checkmark$$

ex) Is $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ a LC of $\begin{matrix} \vec{v}_1 \\ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \end{matrix}, \begin{matrix} \vec{v}_2 \\ \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \end{matrix}, \begin{matrix} \vec{v}_3 \\ \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \end{matrix}$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 2 & 0 & 2 \\ 2 & 0 & 1 & 3 \end{array} \right) \xrightarrow{R_3 \rightarrow R_3 - 2R_1} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & -3 & -1 \end{array} \right) \quad \ddots$$

No solution, so No! It is not a LC of them.