

## Yiddish word of the day



שְׁוִים שְׁבָתָן  
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## Yiddish phrase of the day

"int shnai ken men nicht  
markhn galm o'khos"

=  
פֶּרֶם "יְהֹוָה כָּן  
פֶּרֶם כָּן יְהֹוָה  
! אֱלֹהִים בָּרוּךְ

TAKE THE EVALS !!!

Currently at 73% !!!

# Orthogonal Stuff

First in  $\mathbb{R}^n$

- How to use LA to talk about "distance" and "length"?

Def: Let  $\vec{u} = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix}$ ,  $\vec{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$  in  $\mathbb{R}^n$ .

Define  $\vec{u} \cdot \vec{v} :=$

Def. 1) Let  $\vec{u}$  in  $\mathbb{R}^n$ . Then define the  of

$\vec{u}$  to be

2) Let  $\vec{u}, \vec{v}$  in  $\mathbb{R}^n$ . The 

is defined to be 

Now let  $V$  be  $\mathbb{V}^n$ .

Can define "distance" between arbitrary vectors

Let  $B$  be basis. Then taking coordinate vectors  
gives vectors in  $\mathbb{R}^n$

Def: Let  $v, w$  in  $V$ . Then the

$$\text{dist}(v, w) :=$$

Def 2: Let  $\vec{u}, \vec{v}$  in  $\mathbb{R}^n$ . Then we say  $\vec{u}, \vec{v}$  are

if  $\vec{u} \cdot \vec{v} =$

- Say  $(\vec{u}_1, \dots, \vec{u}_r)$  are

if

- Fact: If  $(\vec{u}_1, \dots, \vec{u}_r)$  are

then they are

Recall: If  $\vec{b}$  in span  $(\vec{v}_1, \dots, \vec{v}_n)$  then

$$\vec{b} =$$

- However, to find these it's kind

annoying.

• Unless

In this case

$$=$$

Having a denominator is always annoying. Hence the following def

Def: An orthogonal sequence  $(\vec{v}_1, \dots, \vec{v}_n)$  is said to  
be

Goal: Have a

Why?

• Def! We say  $Q = \begin{pmatrix} \vec{v}_1 & \vec{v}_n \\ \downarrow & \downarrow \end{pmatrix}_{\text{nan}}$

is an

if

Facts: Let  $Q = \begin{pmatrix} \vec{v}_1 & \vec{v}_n \\ \downarrow & \downarrow \end{pmatrix}_{\text{nan}}$ . Then  
the following are equivalent

1)  $Q$  is a \_\_\_\_\_ matrix

2) For any  $v \in \mathbb{R}^n$

3) For any  $\vec{v}, \vec{w} \in \mathbb{R}^n$

What this means ? Let  $Q$  be an

- $Q$  preserves (by 2)

- $Q$  preserves (by 3)

{ Google "conformal maps" and  
"isometries" }

## Practically

If you want to change a vector without changing it (move it around but not deform it)

then an Orthogonal matrix is the way to go.

## Facts

- Can always make a basis into ON basis



- Let  $A = \begin{pmatrix} \vec{v}_1 & \vec{v}_n \\ J & J \end{pmatrix}_{m \times n}$  with  $\vec{v}_1, \dots, \vec{v}_n \in \mathbb{R}^m$ . Then  
(can decompose A as)

$$A = QR$$

with

Q

R

"upper triangular"

("QR" factorization)

(8.6 has applications of this)

Sections

8.4 / 8.5 in book are really cool

(especially 8.5) but we won't discuss them. Look

at them though !!! (cool stuff !!!!!)