Yiddish word of the day

" " = שרעק

" " =

Yiddish phrase of the day

"A moshel iz nisht keyne rai'ch" = אַ מאָשעל איז נישט קיין רײַעה

" " =

Lecture 3 - Linear Independance

<u>Recall</u>: Let $A = \begin{pmatrix} \vec{v}_1 & \cdots & \vec{v}_r \\ \downarrow & & \downarrow \end{pmatrix}_{n \times r}$ matrix.

Then we saw last class that the vectors $\vec{v}_1, \ldots, \vec{v}_r$ <u>span</u> $\mathbb{R}^n$ if and only if the

- <u>Note</u>: We don't **require the** solution to be unique.

<u>Recall - Free Variables</u>

- In an equation, the leading variable is

· A free variable in a system of equations is

ex) $\begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} \longleftrightarrow$

Linear Independence

(want some "sense" of uniqueness of solutions)

Def: Let $\vec{v}_1, \ldots, \vec{v}_k$ be vectors in $\mathbb{R}^n$. Then we say $\vec{v}_1 \ldots \vec{v}_k$ are ______ if

2 questions

1) How to tell if any given vectors are ______

2) Who cares? What's the utility if they are?

ex) Determine if the vectors $\vec{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$, $\vec{v}_3 = \begin{pmatrix} -7 \\ 3 \\ 1 \end{pmatrix}$ are LI ?

ex2: Are the vectors $\overbrace{\begin{pmatrix}1\\2\\0\end{pmatrix}}^{v_1}, \overbrace{\begin{pmatrix}2\\-1\\0\end{pmatrix}}^{v_2}, \overbrace{\begin{pmatrix}-1\\3\\1\end{pmatrix}}^{v_3}, \overbrace{\begin{pmatrix}0\\0\\1\end{pmatrix}}^{v_4}$ LI?

ex) Are the vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ LI?

A pattern Emerges

- To answer Q1 - (How to tell) we do the following
  - Put our vectors ______
    - Put ______ into ______
      - Check if there are ______
        - If there are ______
          ⇒ they are ___ LI
        - If there are no ______
          ⇒ then they ___ LI!

- Thus if vectors are LI then their matrix has a ________ !
____

- This tells us the following: If $\vec{v}_1 \dots \vec{v}_k$ are LI vectors in $\mathbb{R}^n$ then ________

Q2 - Why do we care?

Suppose that $\vec{v}_1 \dots \vec{v}_k$ are LI vectors in $\mathbb{R}^n$
- Now let $\vec{b}$ in span$(\vec{v}_1, \dots, \vec{v}_k)$

(that is $\vec{b} =$ ________ ...)

- Then if $\vec{b} =$ ________ for some other

Then

Proof :

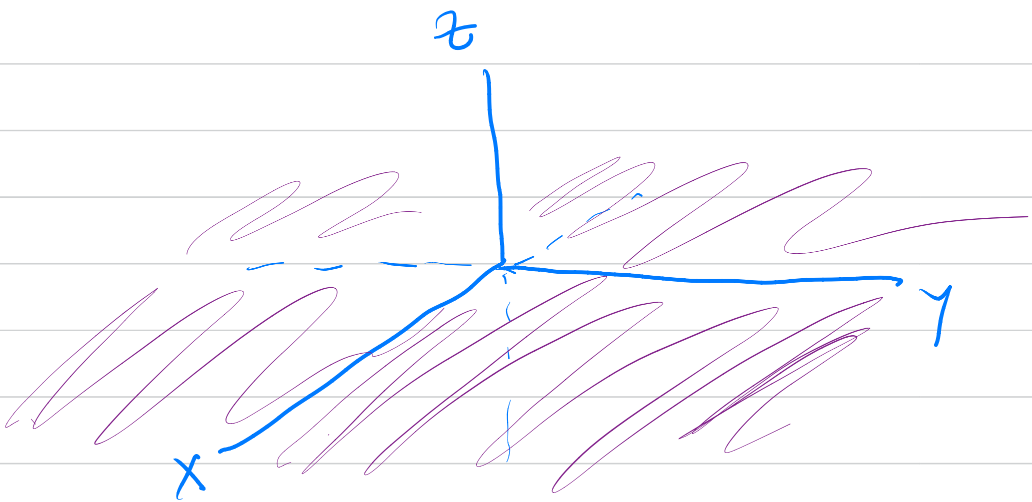
Subspaces and Basis

Subspace - A subspace $W \subseteq \mathbb{R}^n$ is

- If $\vec{v}_1, \vec{v}_2$ are in W then
- If c any real #, and $\vec{v}_1$ in W then

ex: Consider $\text{span}(\vec{v}_1 \ldots \vec{v}_k)$ for $\vec{v}_i$ in $\mathbb{R}^n$

ex) $W = \left\{ \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} : x, y \text{ are real \#'s} \right\}$

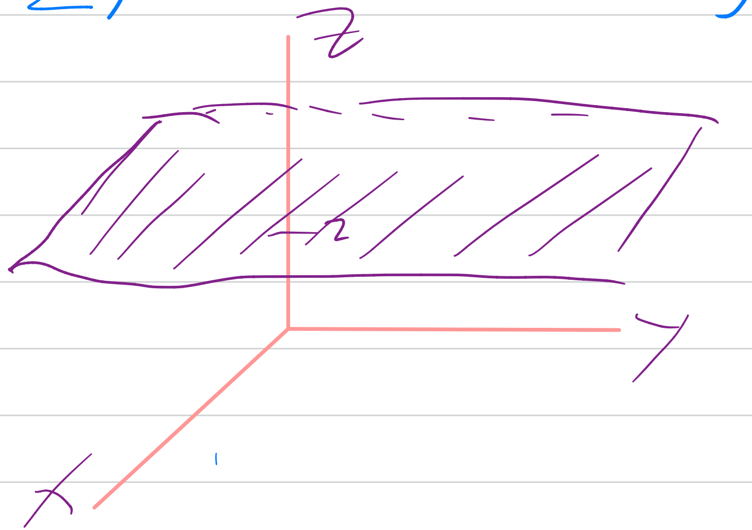


Q: Is this a subspace?

1) Closed under addition:

2) Closed under multiplication by a #.

ex) $W = \left\{ \begin{pmatrix} x \\ y \\ 2 \end{pmatrix} : x, y \text{ are real \#'s} \right\}$



Q: Is W a subspace?

ex) $W = \left\{ \begin{pmatrix} x \\ x+y \\ x-y \\ x^2 \end{pmatrix} : x, y \text{ are real \#'s} \right\} \subseteq \mathbb{R}^4$

Is this a subspace?

ex) $W = \left\{ \begin{pmatrix} x \\ x+y \\ x-y \end{pmatrix} : x, y \text{ are real \#'s} \right\}$

Verify that W is a subspace!

- Closed under addition.

NOT Correct Way!

Basis - Combine the ideas of LI and spanning.

Def: Let $W \subseteq \mathbb{R}^n$ be a subspace of $\mathbb{R}^n$ (could be $\mathbb{R}^n$ itself)
Then the vectors $\vec{v}_1 \dots \vec{v}_k$ of W are called ______
for W if

1)

2)

Q: What's the meaning of this?

- $2 \Rightarrow$

- $1 \Rightarrow$

ex: $W = \mathbb{R}^n$. Suppose $\vec{v}_1 \dots \vec{v}_k$ is a basis for $\mathbb{R}^n$

- then since $\vec{v}_1 \dots \vec{v}_n$ span $\mathbb{R}^n$, we must have

- Also since $\vec{v}_1 \dots \vec{v}_n$ are LI we must have

$\Rightarrow$ thus any basis has

Also note: As mentioned before, if $\vec{v}_1 \dots \vec{v}_n$ is a basis for $\mathbb{R}^n$ the linear system

$$\left(\quad \vdots \quad \right)$$ has

Thm: Every subspace $W \subseteq \mathbb{R}^n$ has a basis!

ex: Are the vectors $\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$, $\vec{v}_3 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ a basis for $\mathbb{R}^3$?

ex) Consider the subspace $W = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} : x_1 + x_2 + x_3 = 0 \right\}$

(hyperplane)

Are the vectors $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ a basis for W?

1) Check they are LI as "usual"

2) Now we show that any vector in W can be expressed as a linear combo of these two vectors.