Linear Transformations, and their Standard Matrices

August 2, 2020

Question 1

a)Let A be the matrix A= $\begin{pmatrix} 1 & 1 \\ 1 & -3 \end{pmatrix}$. Does there exist a vector $\vec{x} \in \mathbb{R}^2$ such that $A\vec{x} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

b)Let A be the matrix A= $\begin{pmatrix} 1 & 1 & -1 \\ 4 & 8 & -12 \\ 0 & 6 & -1 \end{pmatrix}$. Does there exist a vector $\vec{x} \in \mathbb{R}^3$ such that $A\vec{x} = \begin{pmatrix} 2 \\ 8 \\ 11 \end{pmatrix}$

c) Let A be the matrix A= $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 4 & -4 \end{pmatrix}$. Does there exist a vector $\vec{x} \in \mathbb{R}^3$ such that $A\vec{x} = \begin{pmatrix} 1 \\ -1 \\ -8 \end{pmatrix}$

Question 2

Write the corresponding standard matrix for the following linear transformations.

a) $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x - 2y \\ 3z \end{pmatrix}$

b) $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + 3z \\ x - y \\ 0 \end{pmatrix}$

c) $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x - y \\ x + y \\ 3y \end{pmatrix}$

d) $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - 2y \\ x + 6y \\ 0 \end{pmatrix}$

Question 3

Consider the following three matrices.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 4 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 0 & 3 \end{pmatrix}$$

Decide what products of these matrices exist, and compute all those that do. For example; BA exists but AB does not, so you would need to compute BA. (Hint; there will be 5 products that exist overall, don't figure you can "square" matrices sometimes)