Subspaces

Last time

- Recall that for $X \subseteq \mathbb{F}^n$ we defined

$$\mathrm{Fct}(X, \mathbb{F}) =$$

$$\mathrm{Cts}(X, \mathbb{F}) =$$

$$\mathrm{Diff}(X, \mathbb{F}) =$$

These are all subsets, but they have more structure,
They are themselves ______________________

Def: Let V be a $\mathbb{F}$-vs, $W \subseteq V$ subset,
We say W is a ______________ if

1)

2)

3)

- it will often be usefull to "______"
  the vector space V into ______
  (we will return to this idea)

- Common occurance of ______
  - <u>Def</u>: Let V be an $\mathbb{F}$-vs and $(v_1, \ldots, v_k)$ in V. Then we say w is a ______

of these vectors if

"generation"

<u>Def</u>: Let $S \subseteq V$ be a subset of V.

The ________ of S is the set

<u>Lemma</u>: ________ is a subspace of V.

Pf)

HW: Show ________ is the smallest subspace containing ________

- We often pay particular attention to how ________ is a ________ of a list of vectors.

Def: Say a list of vectors are ______

if ______

Why care? "Uniqueness claims"

HW: Suppose $S \subseteq V$ is ______

Then any vector $w \in$ ______

has a ______

Put the 2 notions together and get...

<u>Def</u>: A ________________ of an $\mathbb{F}$-vs V

is a set $\mathcal{B} = (\qquad\qquad)$

such that

1)

2)

ex) i) $V = \mathbb{F}^n$ then "standard ______"

is ______________________

ii) $S = \{x_1 \dots x_n\}$. $V = \mathrm{Fct}(S, \mathbb{F})$

then the "dirac delta" basis

$\underline{\delta_i}$ defined as

iii) $V = \mathbb{F}[t]_{\leq n}$ has "standard basis"

$\mathcal{B} =$

Prop: let $\mathcal{B} = (v_1 \ldots v_n)$ be set in V.

Then $\mathcal{B}$ is a

$\iff$ every $w \in V$

How to get ________ ?

Lemma: Let $S = (v_1, \ldots, v_n)$ subset, $w \in V$.

Let $\tilde{S} = (v_1, \ldots, v_n, w)$. Then

i) $\text{Span}(S) = \text{Span}(\tilde{S}) \iff$ ________

ii) If S is LI, then so is $\tilde{S} \iff$ ________

Pf) (i)

This will help us construct ________

Def: Say V is ________ if there is a finite subset that ________

ex) i) $\mathbb{F}^n$

ii) $M_{m \times n}(\mathbb{F})$

iii) $\mathbb{F}[t]_{\leq n}$

iv) S finite set, $\mathrm{Fct}(S, \mathbb{F})$

HW: v) Show that $V = \mathrm{Fct}(\mathbb{Z}, \mathbb{F})$ not __________

vi) $\mathbb{F}[t]$ not __________

Prop: Let $S = (v_1, \ldots, v_n)$ be set that __________ V.

a) Given L a __________ subset of V,

we obtain a __________ for V by

adjoining elements of S to L

b) Obtain a __________ for V by

excluding elements in S.

Pf)

Cor: Every fd vector space has a basis.

Rmk: true for general VS, harder to prove. Uses "Zorns Lemma"

Towards ________

Prop: S, L finite subsets in V.

Assume i) S ________ V

ii) L is ________

Then

P7)

Cor: V fd VS and B a basis. Then

Exercise

a) Any other basis B' ________

b) If S is finite subset spanning V then ________

c) If L is finite LI set then ________

$\Rightarrow$ Def: The ______________ of a finite dim VS

V is defined to be ______________

Next time: Linear transformations