Spanning, Linear Independence, and Bases

July 14, 2020

Question 1

a)Is the vector $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ a linear combination of the vectors $\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \end{pmatrix} \right)$

b)Is the vector $\begin{pmatrix} 2 \\ 8 \\ 11 \end{pmatrix}$ a linear combination of the vectors $\left(\begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 8 \\ 6 \end{pmatrix}, \begin{pmatrix} -1 \\ -12 \\ -1 \end{pmatrix} \right)$

c) Is the vector $\begin{pmatrix} 1 \\ -1 \\ -8 \end{pmatrix}$ a linear combination of the vectors $\left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ -4 \end{pmatrix} \right)$

Question 2
Answer the Challenge question from HW 1. (You can now answer it quickly, once you rephrase it in terms of spanning vectors)

Question 3
For the following, give an example if one exists, or state it is not possible. If it is not possible, explain why.
a) A sequence of 3 vectors in $\mathbb{R}^3$ that are LI.
b) A sequence of 2 vectors in $\mathbb{R}^3$ that are spanning vectors.

c) A sequence of 4 vectors in $\mathbb{R}^2$ that are spanning vectors.
d) A sequence of 3 vectors that are a basis for $\mathbb{R}^3$
e) A sequence of 3 vectors that are a basis for $\mathbb{R}^4$

Question 4
State whether the following are true or false. For those that are false, give a counter example.
a) Every list of two vectors in $\mathbb{R}^3$ are LI.
b) Every list of four vectors in $\mathbb{R}^3$ are spanning.
c) Every list of four linearly independent vectors in $\mathbb{R}^4$ are a basis.
d) Every list of five vectors in $\mathbb{R}^5$ that span $\mathbb{R}^5$ are a basis.