**MATH08067 Probability with Applications** **Spring, 2020**

**Tutorial 1 – Questions**

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**1)**. Let $f(x) = 3x^4 - 2x^2 + 4\sin(x)$, $g(x) = e^{2x^3-4x}$ and $h(x) = e^x/\cos(x)$. Compute the derivatives $f'(x), g'(x)$ and $h'(x)$ of these functions.

**2)**. A student can select one of 6 different mathematics books, one of 3 different chemistry books and one of 4 different science books. In how many different ways can a student select a book of mathematics, a book of chemistry and a book of science?

**3)**. Let $f(x) = 4x^7 - 2x^3 + 1$, $g(x) = \sin(3x)$ and $h(x) = x/(1+x^2)$. Compute the integrals

$$\int_0^2 f(x)\,dx, \quad \int_0^\pi g(x)\,dx, \quad \text{and} \quad \int_0^4 h(x)\,dx.$$

**4)**. There are 3 different roads from city $A$ to city $B$ and 2 different roads from city $B$ to city $C$. In how many ways can someone go from city $A$ to city $C$ passing by city $B$?

**5)**. Let

$$f(x) = x^2\cos(2x + x^3),\ g(x) = xe^{-x^2+\sin(2x)},\ h(x) = \frac{\sin(e^{3x-x^4})}{3x^5 - 2x^3 + 2}.$$

In each case, find the derivatives $f'(x), g'(x)$ and $h'(x)$ of these functions.

**6)**. In a company, ID cards have 5 digit numbers.
a) How many ID cards can be formed if repetition of the digit is allowed?
b) How many ID cards can be formed if repetition of the digit is not allowed?

**7)**. Let $f(x) = e^{-2x}$, $g(x) = x\sin(2\pi x^2)$, and $h(x) = x\cos(2\pi x)$. Compute

$$\int_0^1 f(x)dx, \ \int_0^1 g(x)dx, \quad \text{and} \quad \int_0^1 h(x)dx.$$

**8)**. Three coins are tossed and a die is rolled. What is the total number of all possible outcomes?

**9)***. (*Extra question; please try to solve it at home before looking at the solutions.*) Consider a function $f(x, y)$ given by

$$f(x,y) = \begin{cases} e^{-x/2}\, y & \text{if } \ 0 \leqslant x \ \text{ and } \ 1 \leqslant y \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Compute $\int\int_{\mathbb{R}^2} f(x,y)dxdy$.

(b) Compute the integral of $f(x, y)$ over the domain $x < 2y$.

**Hint**: *You may find that one order of integration is better than the other.*