Tutorial - 1

1. Define the graph state and prove first theoram of graph theory.

Ans. Graph : A graph is a pair of sets(V,E) where V is the set of vertices and E is the set of edges,

Theoram 1 : In a graph G, the sum of the degree of all vertices of G is equal to the twice the number of edges of G.

In other words : If G is a graph with n vertices and e edges then

$$deg(V_1) + deg(V_2) + deg(V_3) \ldots + deg(V_n) = 2e$$

▷ proof :

let f be any edge of graph G

• case - 1

if f is a loop incident on vertex V_1 then f is count twice when we count degree of vertex V1.

• Case - 2

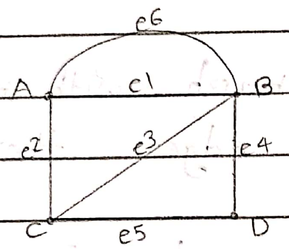
if f is incident on V_1 and V_2 then f is count in both $deg(V_1)$ and $deg(V_2)$ so of is count twice if we add $deg(V_1)$ and $deg(V_2)$

- So that adding the degree of all vertices involves counting twice of each edge of G.

- Therefor addition of degree of all vertexes is is equal to twice of number of edges.

Note : The sum of degree of all vertices in graph G is always even number.

Ex.:



$= deg(A) + deg(B) + deg(C) + deg(D)$

$= 3 + 4 + 3 + 2$

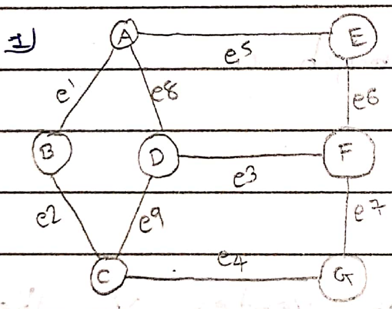
$= 12$

$= 2 * 6$ (number of edges)

$= 12$

2. Find the degree of all vertex of graph

1)



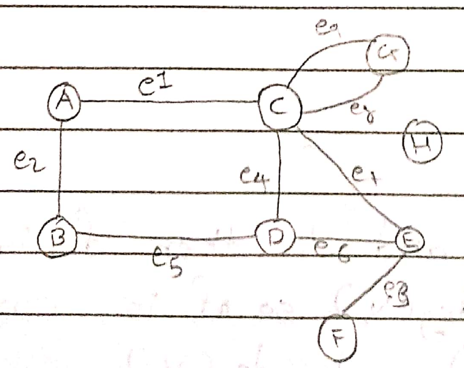
let $G(V, E)$ be a graph

$V = \{A, B, C, D, E, F, G\}$

$E = \{e_1, e_2, \ldots, e_9\}$

$deg(A) = 3$, $deg(B) = 2$, $deg(C) = 3$,
$deg(D) = 3$, $deg(E) = 2$, $deg(F) = 3$,
$deg(G) = 2$, Total = 15

2.



$deg(A) = 2$, $deg(B) = 2$, $deg(C) = 5$,
$deg(D) = 3$, $deg(E) = 3$, $deg(F) = 1$,
$deg(G) = 2$, $deg(H) = 0$, Total = 18

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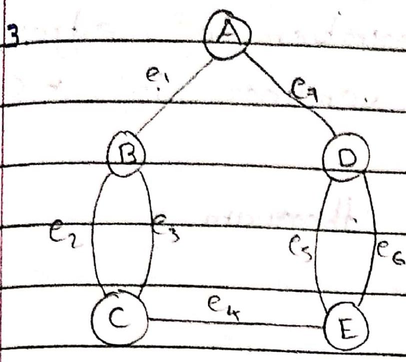
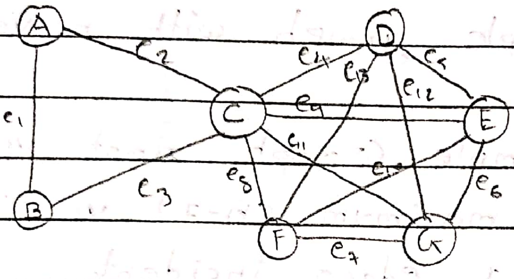
3.



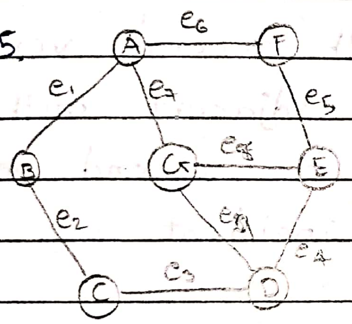
deg(A) = 2, deg(B) = 3, deg(C) = 3,
deg(D) = 3, deg(E) = 3, Total = 14

4.



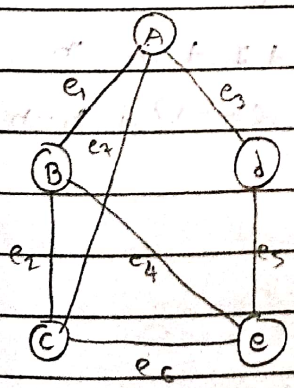
deg(A) = 2, deg(B) = 2, deg(C) = 6,
deg(D) = 4, deg(E) = 4, deg(F) = 4,
deg(G) = 4, Total = 26

5.



deg(A) = 3, deg(B) = 2, deg(C) = 2,
deg(D) = 3, deg(E) = 3, deg(F) = 2
deg(G) = 3, Total = 18

3) Draw a graph with five vertices a, b, c, d, e such that deg(a) = 3, b is an odd vertex, deg(c) = 2 and e and d are adjacent.



deg(A) = 3, deg(B) = 3, deg(C) = 2,
deg(D) = 2, deg(E) = 2, Total = 12

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4) Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$

Ans The question it self is a theoream 3.

proof :

Let G is a simple graph with n vertices

Sinse G is a simple Graph first vertex can be adjacent with maximum $(n-1)$ vertices. so on first vertex $n-1$ edges incident.

Now first vertex adjacent with second vertex so second vertex can be adjacent with maximum $(n-2)$ vertices. So on second vertex maximum $(n-2)$ new edges incident.

Continuing.
On $(n-1)$th vertex maximum 1 new edge incident
So that the maximum number of edges in G

$= (n-1) + (n-2) + (n-3) + \ldots + 1 + 0$

$= (n-1) + (n-2) + (n-3) + \ldots + 1 + n - n$

$= 1 + 2 + 3 + \ldots + (n-3) + (n-2) + (n-1) + n - n$

$= \frac{n(n+1)}{2} - n$

$= \frac{n(n-1)}{2}$

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5) prove that in a graph the number of the vertices with odd degree is even.

Ans By the first theoram if G is a graph with n vertices and e edges then

$$\sum_{i=1}^{n} deg(V_i) = 2*e = \text{even number} \quad \ldots\ldots (1)$$

The quantity in the left side of above equation can be expressed as the sum of even degree vertices and odd degree vertices as follow

$$\sum_{i=1}^{n} deg(V_i) = \sum_{even} deg(V_i) + \sum_{odd} deg(V_i)$$

Since the left hand side of the above equation is even by by (1) and the first expression on the right hand side is also even as sum of even number is even. so second expression must be even.

$$\sum_{odd} deg(V_i) = \text{even number}$$

In above equation deg (Vi) is odd so that the total number of the terms in the sum must be even to make a sum an even number.

Therefor the number of vertices of odd degree in a graph is always even.

6) A graph has five vertices of degree 4 and two vertices of degree 2. How many edges does it have?

Ans By the first theorem

$$\sum_{i=1}^{N} \deg(v_i) = 2e$$

$$5 \times 4 + 2 \times 2 = 2e$$

$$20 + 4 = 2e$$

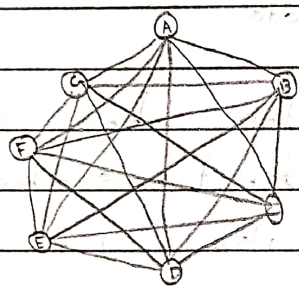
$$24 = 2e$$

$$\boxed{e = 12}$$

7) Draw K_7, $K_{3,5}$, $K_{2,6}$ and find number of edges for each

Ans

K_7



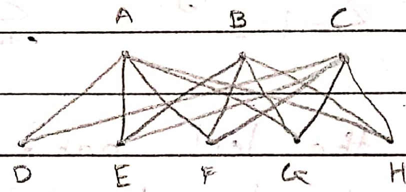
$\deg(A)=6$, $\deg(B)=6$, $\deg(C)=6$
$\deg(D)=6$, $\deg(D)=6$, $\deg(E)=6$
$\deg(F)=6$, Total $= 42$

$\deg(A) \ldots + \deg(F) = 2e$

$42 = 2e$

$e = 21$

$K_{3,5}$



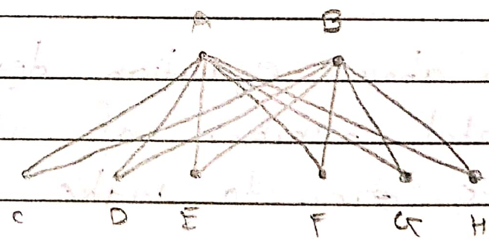
$\deg(A) + \deg(B) \ldots + \deg(H) = 2e$

$5+5+5+3+3+3+3+3 = 2e$

$30 = 2e$

$e = 15$

$K_{2,6}$



$\deg(A) + \deg(B) \ldots + \deg(H) = 2e$

$6+6+2+2+2+2+2+2 = 2e$

$2e = 24$

$e = 12$

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