MA 341 Practice Test 2 (Dr. Park)

Chapter 3: Topics are the Bolzano-Weierstrass theorem, the Cauchy sequences, infinite series with Comparison test and Limit comparison test.

Problem 1. Show that if (x_n) is unbounded, then there exists a subsequence (x_{n_k}) such that

$$\lim_{k\to\infty} \frac{1}{x_{n_k}} = 0.$$

Problem 2.

(1) If $x_n = \sqrt{n}$, prove that (x_n) satisfies $\lim |x_{n+1} - x_n| = 0$, but that it is not a Cauchy sequence.

(2) Calculate the value of $\sum_{n=1}^{\infty} (1/2)^{3n}$.

(3) If $\sum a_n$ with $a_n > 0$ is convergent, then is $\sum \sqrt{a_n a_{n+1}}$ always convergent? Either prove it or give a counterexample.

Problem 3. Evaluate the limits of the following sequences and infinite series

(1) $\displaystyle \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{5n}$ (2) $\displaystyle \sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

Chapter 4: We proved function limits and Limit theorems. You need to understand $\epsilon - \delta$ and $\epsilon - N$ argument.

Problem 4. Use either the $\epsilon - \delta$ definition of limit or the Sequential Criterion for limits to establish the limit:

$$\lim_{x\to 1} \frac{x^2 - x + 1}{x+1} = \frac{1}{2}$$

Problem 5. Evaluate the following limits, or prove that they do not exist.

(1) $\displaystyle\lim_{x\to\infty} \frac{2+3x}{\sqrt{5x+1}}$ (2) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{6x^2+2x}}{4x+3}$

Problem 6.

(1) Does $\lim_{x\to 0+} \sin(1/x)$ exist? You must justify your answer.

(2) Does $\lim_{x\to 0+} x\sin(1/x)$ exist? You must justify your answer.

Chapter 5: We proved function continuity, Maximum-minimum theorem, Root-finding theorem, Intermediate value theorem, Uniform continuity, and Lipschitz functions.

Problem 7. Prove that $f(x) = |x|$ is continuous at every real number c.

Problem 8. Prove that every polynomial of odd degree with real coefficients has at least one real root.

Problem 9. Prove that $g(x) = 1/x^2,\ x \in (0,1)$ is not uniformly continuous on $(0,1)$.

Problem 10. Prove that $f(x) = x^3 + 1$ is a Lipschitz function on $I = [1,3]$.