453 **Lecture 6**

Principle of mathematical induction

Thm If S is a subset of $\mathbb{N}$ satisfying

(i) $0 \in S$

(ii) For all $n \in \mathbb{N}$, $n \in S \Rightarrow n+1 \in S$.

Then, $S = \mathbb{N}$.

Pf: Let $T = \mathbb{N} \setminus S$. We want to show that T is empty. Assume not. Then using the well ordering principle there exists a smallest element $n_0 \in T$. Since $0 \in S$, $0 \notin T$. Hence, $n_0 > 0$. And since n_0 is the smallest element in T, $n_0 - 1$ (which is in $\mathbb{N}$ since $n_0 > 0$) does not belong to T. So $n_0 - 1 \in S$. But using property (ii) $n_0 - 1 + 1 = n_0$ also belong to S. Hence, n_0 belongs to both T and S which is impossible. So T is empty and $S = \mathbb{N}$. $\square$

Strong principle of induction

Thm If S is a subset of $\mathbb{N}$ satisfying

(i) $a \in S$

(ii) For all $n \in \mathbb{N}$, $n \geq a$, if $m \in S$ for all m satisfying $a \leq m \leq n$, then $n+1 \in S$.

Then, $S = \{ n \in \mathbb{N} \mid n \geq a \}$.

Pf: Modify the previous proof appropriately.

In practice in "proofs using induction" be sure to state what variable you are "inducting" on. Then, state the "Base case", the "induction hypothesis" and finally prove the "inductive step".

It is in the "inductive step" that you need to do some "proving".