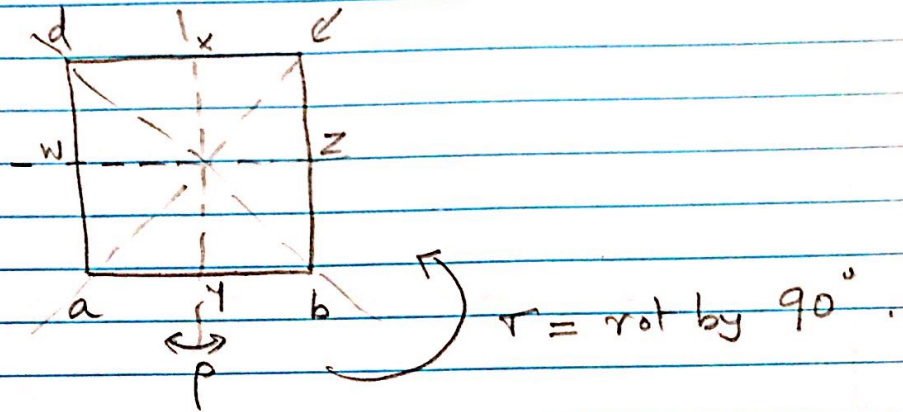


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Lecture 1

1. The dihedral groups.

The group D_8 (symmetries of the square)

$$D_8 = \{e, r, r^2, r^3, p, rp, r^2p, r^3p\}$$

 r = rot counter clockwise by 90°

r	a	b	c	d
	\downarrow	\downarrow	\downarrow	\downarrow
	b	c	d	a

 p = reflection about the line xy .

p	a	b	c	d
	\downarrow	\downarrow	\downarrow	\downarrow
	b	a	d	c

Notice $r^4 = p^2 = e$.Also $(rp)^2 = e \Rightarrow rp \neq r^{-1}$.

The multiplication table can be computed from the above relations.

Dihedral group of order $2n$ is the group of symmetries of regular n -gon.

$$D_{2n} = \{e, r, \dots, r^{n-1}, p, rp, r^2p, \dots, r^{n-1}p\}$$

$$r^n = p^2 = e$$

$$p r p = r^{-1}.$$

The groups D_{2n} , $n > 2$ are all non-abelian and finite.

$$|D_{2n}| = 2n.$$