MATH 453 LECTURE 1 Aug 21, 2017

1. SETS as collection of objects including the empty collection.

We will denote the empty set by ϕ.

2. SUBSETS

A SET Y is a SUBSET of a set X (denoted $Y \subset X$) if $y \in Y \Rightarrow y \in X$.

NOTICE THAT the empty set and X itself are subsets of X.

Two sets X and Y are equal if and only if $X \subset Y$ and $Y \subset X$.

This will be our standard method of proving that two sets are equal.

YOU WILL NEED TO PROVE BOTH INCLUSIONS!

3. POWER SET The power set of a set X (denoted 2^X) is the set whose elements are all subsets of X.

If X is a finite set with n elements then 2^X is a finite set with 2^n elements.

4. MAPS BETWEEN SETS

A map $f : X \longrightarrow Y$ is an association of an element of Y to each element of X.
We will denote the element of Y associated to the element x of X by $f(x)$.
We will sometimes use the "maps to" notation and instead of saying "x maps to f(x)" write "$x \xmapsto{f} f(x)$".

MAPS NEED NOT BE GIVEN BY "NICE" formulas like maps you are used to such

$$\sin : \mathbb{R} \longrightarrow \mathbb{R} \quad \text{or} \quad \exp : \mathbb{R} \longrightarrow \mathbb{R}$$

For example following are maps from

Eg 1 $X = \{1, 2, 3\}$ to $Y = \{a, b, c\}$

$$\begin{array}{ccc} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ a & c & a \end{array} \quad \text{or} \quad \begin{array}{ccc} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ b & c & a \end{array} \quad \text{etc.}$$

Two maps are equal if they take the same value at each element of X i.e.

$$f = g \quad \text{iff} \quad f(x) = g(x) \text{ for all } x \in X.$$

$(f: X \longrightarrow Y)$

Ex How many distinct maps are there between X and Y in Eg 1 ?

(HINT. There are $3^3 = 27$ distinct maps. Do you understand why?).

5. GIVEN SETS X and Y we will denote the set of all maps from X to Y by

$$Y^X.$$

If X and Y are finite sets then

$$\mathrm{card}(Y^X) = \mathrm{card}(Y)^{\mathrm{card}(X)}.$$

(STOP and SEE IF YOU UNDERSTAND WHY THIS is TRUE).

WHAT DOES THIS TO DO WITH the FACT THAT $\mathrm{card}(2^X) = 2^{\mathrm{card}(X)}$?

6. <u>COMPOSITION OF MAPS</u>.

If $f: X \to Y$ and $g: Y \to Z$ are maps then their composition $g \circ f: X \longrightarrow Z$ is defined by

$$g \circ f(x) = g(f(x)).$$

PICTORIALLY :

$$\begin{tikzcd} X \arrow[r, "f"] \arrow[rr, bend right, "g \circ f"'] & Y \arrow[r, "g"] & Z \end{tikzcd}$$

EXAMPLE $X = Y = \{1, 2, 3\}$

$$f \quad \begin{array}{ccc} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ 2 & 1 & 3 \end{array} \qquad\qquad g \quad \begin{array}{ccc} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ 1 & 3 & 2 \end{array}$$

$$g \circ f \quad \begin{array}{ccc} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ 2 & 1 & 3 \\ \downarrow & \downarrow & \downarrow \\ 3 & 1 & 2 \end{array} \qquad\qquad f \circ g \quad \begin{array}{ccc} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ 1 & 3 & 2 \\ \downarrow & \downarrow & \downarrow \\ 2 & 3 & 1 \end{array}$$

NOTICE THAT $g \circ f \neq f \circ g$

(TWO ~~FUNCTIONS~~ MAPS ARE EQUAL IF THEY TAKE THE SAME VALUES AT ALL POINTS)

PROPERTIES OF MAPS

INJECTIVE (or 1-1) MAPS.

A MAP $f : X \to Y$ is SAID TO BE INJECTIVE if $f(x) = f(x') \implies x = x'$ for all $x, x' \in X$.

IN OTHER WORDS INJECTIVE MAPS SEND DISTINCT ELEMENTS OF X to distinct elements of Y.

L: CANCELLATION PROPERTY OF INJECTIVE MAPS.

$$X \underset{h}{\overset{g}{\rightrightarrows}} Y \xrightarrow{f} Z$$

If f is injective and $f \circ g = f \circ h$ then $g = h$. (Why?)

(It is as if you can "cancel" f from the equality $f \circ g = f \circ h$).