MA 341 Practice Test 1 (Dr. Park)

Problem 1.

(1) Prove that for all $n \in \mathbb{N}$

$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

(2) Prove that for all $n \geq 5$

$$2n - 3 \leq 2^{n-2}.$$

Problem 2. Let $S = \{1/n : n \in \mathbb{N}\}$. Find the supremum and infimum of S and prove your answers.

Problem 3. If $x > 0$, prove that there exists $n \in \mathbb{N}$ such that $1/2^n < x$.

Problem 4. Let $J_n = (0, 1/n]$ for $n \in \mathbb{N}$. Find the number of elements in the set, $\cap_{n=1}^{\infty} J_n$.

Problem 5. Suppose that (x_n) is bounded and that $\lim_{n\to\infty} y_n = 0$. Find $\lim_{n\to\infty} x_n y_n$.

Problem 6. Suppose that $\lim_{n\to\infty} x_n = x$ and that $x_n \le 0$. Show that $x \le 0$.

Problem 7. Suppose that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$. Show that $\lim_{n\to\infty} x_n y_n = xy$.

Problem 8. Show that $\lim_{n\to\infty} y_n = y$ and that $|y| > 0$. Show that there is a natural number K so that if $n \ge K$,

$$|y_n| > |y|/2.$$

Problem 9. Show that

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0.$$

Problem 10. Let $x_1 \ge 2$ and $x_{n+1} = 1 + \sqrt{x_n - 1}$ for $n \in \mathbb{N}$.

(1) Show that (x_n) is decreasing and bounded below.

(2) Find the limit: $\lim_{n\to\infty} x_n$.

Problem 11. Suppose that (x_n) is a positive sequence with $\lim_{n\to\infty} x_n = x$, where $x > 0$. Show that

$$\lim_{n\to\infty} \sqrt{x_n} = \sqrt{x}.$$

Problem 12. Use the Ratio Test to prove that

$$\lim_{n\to\infty} \frac{3^n}{n!} = 0.$$