HW #2, Sep. 6th

CH4:

$\frac{20}{20}$

A.4) $\begin{cases} ax^2 = b \\ x^3 = e \end{cases}$

$ax^3 = bx$

$a = bx$

$b^{-1}a = x$.

$x = b^{-1}a$

B.5) for $G = \mathbb{Q}^*$ (exclude 0)

and $*$ = multiplication

$2 \in G$, there is no y for

$2 = y^2$.

C.5) $xax^{-1} \cdot xbx^{-1} = xabx^{-1}$

$xbx^{-1} \cdot xax^{-1} = xbax^{-1} = xabx^{-1}$ 2/2

$\therefore$ xax^{-1} and xbx^{-1} commutes

Since $ab = ba$ by assumption

D.4) $xay = a^{-1}$

$xayax = a^{-1}ax = x$

$ayax = e$ 2/2

$yax = a^{-1}$

E.1) Let the set of all elements that does not equal to itself's inverse be S. Since S is a subset of G, S is finite.
Since each $x \in S$, there is $y = x^{-1}$, and inverse is unique, divide S into subsets $A_1, A_2 \dots A_n$ such that A_1 has a_1 and its inverse. A_2 has a_2 and its inverse, until $\{A_n\}$ is a partition of S.

This is doable since S is finite.

Thus S has $2n$ elements, a even number.

F.2) Every row of a group table has a slot for each element of the group, i.e. the nuber of slots = the number of elements.

If there is repetition, then $ay_1 = ay_2$ $\xrightarrow{\text{by cancellation law}}$ then $y_1 = y_2$, contradict to definition of group table.

Thus each element must show exactly once, or there is not enough elements to fill the slots. 4/4

F.5)

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	a	e
c	c	b	e	a

4/4

CH5:

A.3) Let $X = \frac{\pi}{4}$, $y = \frac{5}{4}\pi$

$\tan(x+y) = ?$ then $\tan X = 1$, $\tan y = 1$

but $\tan(X+y) \notin Q$

4/4

$\therefore$ H is not closed under the operation, not a subgroup

A.4) for any $X \in H$, $X = 2^a 3^b$. $y \in H$, $y = 2^c 3^d$

$X^{-1} = 2^{-a} \cdot 3^{-b} \in H$.

$X \cdot y = 2^{a+c} \cdot 3^{b+d} \in H$ ($(\mathbb{Z}, +)$ is a group)

$\therefore$ H is a subgroup of G

B.4) Let $f, g \in H$

$[f+g](x) = \int_0^1 f(x) + g(x)\,dx = \int_0^1 f(x)\,dx + \int_0^1 g(x)\,dx$

$= 0 + 0 = 0 \Rightarrow [f+g] \in H.$

$[-f](x) = \int_0^1 -f(x)\,dx = -\int_0^1 f(x)\,dx = -0 = 0$

$\Rightarrow -f \in H$

$\therefore H$ is a subgroup of G.

4/4

C.3) Let $x_1, x_2 \in H$, then $\exists\, y_1, y_2 \in G$, $x_1 = y_1^2$, $x_2 = y_2^2$.

then $x_1 * x_2 = y_1^2 * y_2^2$, since G is abelian,

$x_1 * x_2 = y_1 y_2 y_1 y_2 = (y_1 y_2)^2$

$\therefore x_1 * x_2 \in H.$

Also, $x_1^{-1} = y_1^2 x^{-1} x^{-1} = (y_1 x^{-1})^2$

$\therefore x_1^{-1} \in H$

$\therefore H$ is a subgroup of G

D.3) Let $a, b \in C$,

then $ax = xa$ for $\forall x \in G$, $bx = xa$ for $\forall x \in G$.

$xab = axb = abx$, $\therefore ab \in C$.

$a^{-1}x = a^{-1}xaa^{-1} = a^{-1}axa^{-1} = xa^{-1}$, $\therefore a^{-1} \in C$.

$\therefore C$ is a subgroup of G.