ASSIGNMENT 4. DUE IN CLASS FRI, SEP 22, 2017.

1. Consider the law of composition
$$x \circ y = (x^3 + y^3)^{1/3}$$
on the set $\mathbb{R}$ of the real numbers. Prove that this law satisfies the group axioms.
2. Recall the definition of the dihedral group
$$D_8 = \{e, \sigma, \sigma^2, \sigma^3, \rho, \sigma\rho, \sigma^2\rho, \sigma^3\rho\}$$
discussed in class on Fri, Sept 15. List all subgroups of D_8 (you do *not* need to prove that these are all the subgroups).
3. Two elements x, y of a group G are said to be *conjugate* in G if there exists an element $s \in G$ such that $y = sxs^{-1}$. Show that the relation " x and y are conjugate" is an equivalence relation on the set G.
4. Let G be the set of all 2×2 real matrices
$$\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$$
with $ad \neq 0$. Prove that G forms a group under matrix multiplication. Is G abelian ?
5. Recall the definition of subgroups from class. "A subset H of G is a subgroup if it contains the identity element, and is closed under taking products and inverses." Prove that a *non-empty* subset H of a group G is a subgroup of G if for all $a, b \in H$, $ab^{-1} \in H$.
6. Prove that if in a group G, every element is its own inverse, then G is abelian.