Chapter 12

Question B.3

Reflexive: $\forall x \in R$, $\lceil x \rceil = \lceil x \rceil$. Thus reflexive.

Symmetric: $\forall x, y \in R$, if $\lceil x \rceil = \lceil y \rceil$, then $\lceil y \rceil = \lceil x \rceil$. Thus symmetric.

Transitive: Given that $x \sim y$, $y \sim z$. Then $\lceil x \rceil = \lceil y \rceil$, $\lceil y \rceil = \lceil z \rceil$. Therefore $\lceil x \rceil = \lceil z \rceil$. Thus $x \sim z$.

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The partition is $[i, i+1)\ \forall i \in \mathbb{Z}$.

Question B.4

Reflexive: Let $x = x$, then $x - x = 0$. 0 is a multiple of 10.

Symmetric: Let $x, y \in \mathbb{Z}$ and $x \sim y$. Thus $x - y = 10k$ where $k \in \mathbb{Z}$. $y - x = -10k$. $-10k$ is a multiple of 10.

Transitive: Given that $x \sim y$, $y \sim z$. Then $x - y = 10k$, $y - z = 10k'$ where $k, k' \in \mathbb{Z}$. $x - z = 10k + 10k' = 10(k + k')$. $10(k + k')$ is a multiple of 10.

The partition is $\{i + 10k : \forall k \in \mathbb{Z}\} \forall i \in [0, 10) \cap \mathbb{Z}$ 5/5

Question D.1

Reflexive: $aa^{-1} = e$ Since H is a subgroup $e \in H$.

Symmetric: $ab^{-1} \in H$. Since H is a subgroup which is closed under inverses. $(ab^{-1})^{-1} \in H$, $ba^{-1} \in H$.

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Transitive: Given that $x \sim y$, $y \sim z$. $xy^{-1} \in H$, $yz^{-1} \in H$. H is closed under multiplication, thus $xy^{-1}yz^{-1} \in H$, also $xz^{-1} \in H$.

The equivalence class of e is H. Since $x \in H$ implies $xe \in H$.

Question D.2

Reflexive: $a^{-1}a = e$ Since H is a subgroup $e \in H$.

Symmetric: $a^{-1}b \in H$. Since H is a subgroup which is closed under inverses. $(a^{-1}b)^{-1} \in H$, $b^{-1}a \in H$.

Transitive: Given that $x \sim y$, $y \sim z$. $x^{-1}y \in H$, $y^{-1}z \in H$. H is closed under multiplication, thus $x^{-1}yy^{-1}z \in H$, also $x^{-1}z \in H$.

The equivalence class of e is H. Since $x \in H$ implies $xe \in H$.

No they are not equivalent. 5/5

They are not the same.

Counter example:

Let G be S_3, $H = \{e, (1,2)\}$

$a = (1,3)$, $b = (1,3,2)$ $b^{-1} = (1,2,3)$

$ab^{-1} = (1,2) \in H$

$\therefore$ $a \sim b$. from D1's equivalence relation

$a^{-1} = (1,3)$

$a^{-1}b = (1,3)\cdot(1,3,2) = (2,3) \notin H$. 5|3

a and b are not equivalent in D2's equivalence relation.