ASSIGNMENT 3. DUE IN CLASS FRI, SEP 15, 2017.

1. (a) If $p_1, \ldots, p_t$ are prime numbers, prove that $p_1 \cdots p_t + 1$ is not divisible by any one of the primes $p_1, \ldots, p_t$.
   (b) Prove using contradiction and part (a) that the number of primes is infinite.
2. Recall from class the equivalence relation on integers defined by $m \sim n$ if and only if 4 divides $m - n$. Denote by $[n]$ the equivalence class of an integer n under this equivalence relation.
   (a) Prove that all primes greater than 2 belong either to [1] or [3].
   (b) Prove that the equivalence class [1] is closed under multiplication (i.e. the product of two elements of [1] belongs to [1]).
   (c) Prove that the number of primes in [3] (i.e. the number of primes that can be written in the form $4k + 3$) is infinite.
3. Using Euclidean division compute the gcd of 1024 and 560 and express the gcd as $1024x + 560y$ where x, y are integers (Bezout identity).
4. Prove using the principle of mathematical induction that
$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2.$$
5. Using the binomial formula and the principle of induction prove that $n^p - n$ is divisible by p for any natural number n and prime p.