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MATH 453

LECTURE 1

Aug 21, 2017

1. SETS as collection of objects including the empty collection.

We will denote the empty set by  $\phi$ .

2. SUBSETS

A SET  $Y$  is a SUBSET of a set  $X$  (denoted  $Y \subset X$ ) if  $y \in Y \Rightarrow y \in X$ .

NOTICE THAT the empty set and  $X$  itself are subsets of  $X$ .

Two sets  $X$  and  $Y$  are equal if and only if

$$X \subset Y \text{ and } Y \subset X.$$

This will be our standard method of proving that two sets are equal.

YOU WILL NEED TO PROVE BOTH INCLUSIONS!



3. POWER SET The power set of a set  $X$  (denoted  $2^X$ ) is the set whose elements are all subsets of  $X$ .

If  $X$  is a finite set with  $n$  elements then  $2^X$  is a finite set with  $2^n$  elements.

#### 4. MAPS BETWEEN SETS

A map  $f: X \rightarrow Y$  is an association of an element of  $Y$  to each element of  $X$ . We will denote the element of  $Y$  associated to the element  $x$  of  $X$  by  $f(x)$ .

We will sometimes use the "maps to" notation and instead of saying " $x$  maps to  $f(x)$ " write " $x \xrightarrow{f} f(x)$ ".



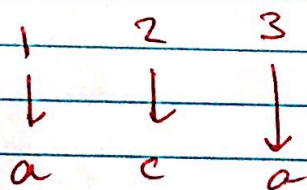
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MAPS NEED NOT BE GIVEN BY "NICE" formulas like maps you are used to such

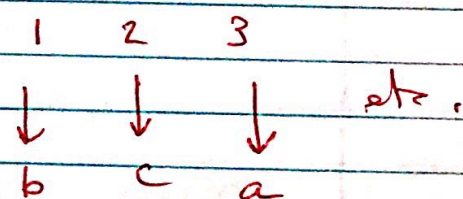
$$\sin : \mathbb{R} \longrightarrow \mathbb{R} \quad \text{or} \quad \exp : \mathbb{R} \longrightarrow \mathbb{R}$$

For example following are maps from

Eg 1  $X = \{1, 2, 3\}$  to  $Y = \{a, b, c\}$



or



Two maps are equal if they take the same value at each element of  $X$  i.e.

$$f = g \quad \text{iff} \quad f(x) = g(x) \quad \text{for all } x \in X.$$

Ex

How many distinct maps are there between  $X$  and  $Y$  in Eg 1 ?

(HINT. There are  $3^3 = 27$  distinct maps.

Do you understand why ? )



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5. GIVEN SETS  $X$  and  $Y$  we will denote the set of all maps from  $X$  to  $Y$  by  $Y^X$ .

If  $X$  and  $Y$  are finite sets then  $\text{card}(Y^X) = \text{card}(Y)^{\text{card}(X)}$ .

(STOP and SEE IF YOU UNDERSTAND WHY THIS IS TRUE).

WHAT DOES THIS DO WITH THE FACT THAT  $\text{card}(2^X) = 2^{\text{card}(X)}$ ?

## 6. COMPOSITION OF MAPS

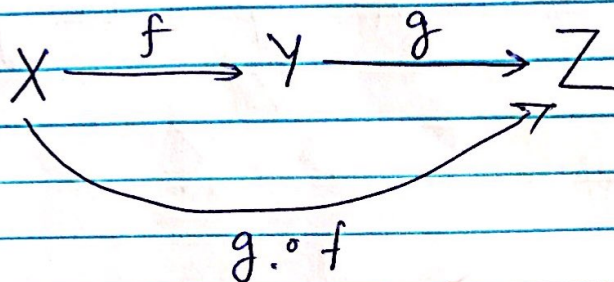
If  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  are maps then their composition  $g \circ f: X \rightarrow Z$  is defined by

$$g \circ f(x) = g(f(x)).$$



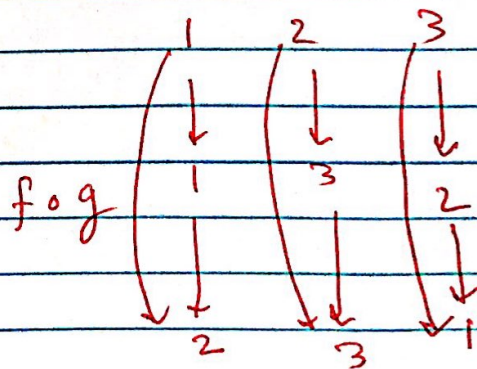
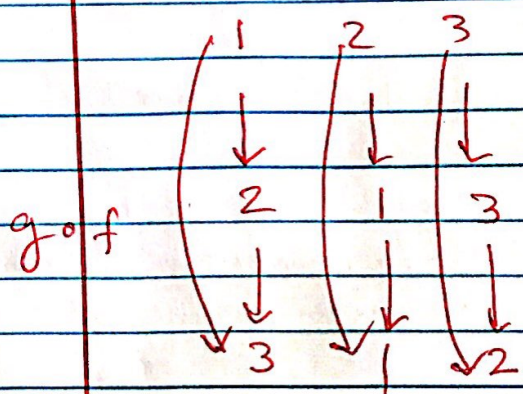
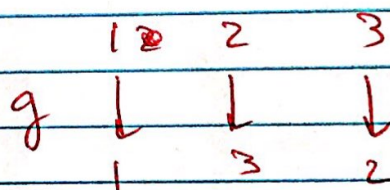
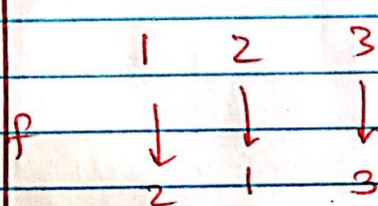
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PICTORIALLY :



EXAMPLE

$$X = Y = \{1, 2, 3\}$$



NOTICE THAT  $g \circ f \neq f \circ g$

(TWO ~~FUNCTIONS~~ MAPS ARE EQUAL IF THEY TAKE THE SAME VALUES AT ALL POINTS)



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## PROPERTIES OF MAPS

### INJECTIVE (or 1-1) MAPS.

A MAP  $f : X \rightarrow Y$  is SAID TO BE INJECTIVE if  $f(x) = f(x') \Rightarrow x = x'$  for all  $x, x' \in X$ .

IN OTHER WORDS INJECTIVE MAPS SEND DISTINCT ELEMENTS OF  $X$  to distinct elements of  $Y$ .

### L. CANCELLATION PROPERTY OF INJECTIVE MAPS.

$$\begin{array}{ccccc} X & \xrightarrow{g} & Y & \xrightarrow{f} & Z \\ & \xrightarrow{h} & & & \end{array}$$

If  $f$  is injective and  $f \circ g = f \circ h$  then  $g = h$ . (Why?)

(It is as if you can "cancel"  $f$  from the equality  $f \circ g = f \circ h$ ).