ASSIGNMENT 1. DUE IN CLASS FRI, SEP 1, 2017.

1. Let $f : X \to Y, g : Y \to Z$ be two maps, and let $h = g \circ f$ be their composition. Prove that:
   (a) if h is injective, then f is injective; if *additionally* f is surjective, then g is injective.
   (b) if h is surjective, then g is surjective; if *additionally* g is injective, then f is surjective.
2. (a) Let X be a finite set and $f : X \to X$ a map. Prove that f is injective if and only if f is surjective. (You need to prove both implications.)
   (b) Does the fact "f is injective if and only if f is surjective" continue to hold if X is not finite. Justify your answer (with an example if need be).
3. Let X and Y be finite sets having m and n elements respectively. For the last three parts consider all the three cases, $m > n, m = n, m < n$.
   (a) What is the cardinality of Y^X ?
   (b) How many distinct injective maps are there from X to Y ?
   (c) How many distinct surjective maps are there from X to Y ?
   (d) How many bijections are there from X to Y ?
4. Let X be the set of all maps from $\mathbb{R}$ to $\mathbb{R}$ (i.e. each element $x \in X$ is a real valued function on $\mathbb{R}$). Let R denote the relation on X defined by: xRy if and only if there exists some $c > 0$ such that $x(t) = y(t)$ for all t with $|t| < c$. Prove that R is an equivalence relation on X.
5. Let $X = \mathbb{R}^2$ and consider the relation R on X defined by $(x', y')R(x'', y'')$ if and only if $x'y' = x''y''$.
   (a) Prove that R is an equivalence relation.
   (b) Describe the equivalence classes of R (feel free to draw them if you think it helps you to describe them).
   (c) What is the equivalence class of the origin $(0, 0)$?