ASSIGNMENT 2. DUE IN CLASS FRI, SEP 8, 2017.

1. Recall from class that if $f : X \to X$ be a bijective map, then there exists a map $f^{-1} : X \to X$ (the inverse map to f) such that $f \circ f^{-1} = f^{-1} \circ f = \mathrm{Id}_X$.
   (a) Now suppose that $f, g : X \to X$ are two bijective maps. Prove that
   $$(f \circ g)^{-1} = g^{-1} \circ f^{-1}.$$
   (b) Give an example of a finite set X with three elements and two bijective maps $f, g : X \to X$ such that
   $$(f \circ g)^{-1} \neq f^{-1} \circ g^{-1}.$$
   Give an example of a finite set X with three elements and two bijective maps $f, g : X \to X$ such that
   $$f \circ g \neq g \circ f$$
   (so composition of bijections is NOT a *commutative* operation).
   (c) Is there an example (like the one in the previous problem) with X a set with two elements ?
2. Let $f : A \to B, g : B \to C, h : C \to D$ be maps. Let $F = g \circ f$ and $G = h \circ g$. First make sure you understand what the domain and the codomain (i.e. the source and the target) of the maps F, G are). Now, prove that
   $$h \circ F = G \circ f.$$
   (i.e. $h \circ (g \circ f) = (h \circ g) \circ f$. So the composition operation is at least *associative* – compare to the last problem.)
3. Let $X = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid b \neq 0\}$, and let $\sim$ be the equivalence relation defined by $(a, b) \sim (a', b')$ if and only if $ab' = a'b$. Prove that the binary operation $\oplus$ on $X/\sim$ defined by
   $$[(a, b)] \oplus [(c, d)] = [(ad + bc, bd)]$$
   is well defined.
4. Let X be a set and $X_1, \ldots, X_n$ subsets of X satisfying:
   (i)
   $$X = \bigcup_{1 \leq i \leq n} X_i,$$
   and
   (ii) $X_i \cap X_j = \emptyset$ if $i \neq j$, for all $i, j, 1 \leq i, j \leq n$.

Let $\sim$ be the relation on X defined by: $x \sim x'$ if and only if there exists $i, 1 \leq i \leq n$, such that $x, x' \in X_i$. Prove that $\sim$ is an equivalence relation. What are the different equivalence classes of $\sim$.

5. Let X be a finite set. Describe the equivalence relation having the *greatest* number of distinct equivalence classes, and the one with the *smallest* number of equivalence classes.