**PRACTICE MIDTERM 1, 2017**

*Please write your name in the top left corner. Attempt all questions. Time 50 min.*

1. (20 pts)
   (a) How many injective maps are there from the set $\{1, 2, 3, \ldots, n\}$ to itself ?
   (b) How many surjective maps are there from the set $\{1, 2, 3, \ldots, n\}$ to itself ?
   (c) Can a map $f : X \to X$ be injective but not surjective ?
   (d) Is the relation on $\mathbb{N}$ defined by "$a \sim b$ if and only $a + b$ is an odd number" an equivalence relation ?
   (e) Is the relation on $\mathbb{N}$ defined by "$a \sim b$ if and only $a + b$ is an even number" an equivalence relation ?
   (f) What is the order of the group $Z_{10}^*$ ?
   (g) Is the group $Z_5^*$ isomorphic to a cyclic group ?
   (h) How many elements of the dihedral group $D_8$ have order 4 ?
   (i) How many elements of the dihedral group $D_{50}$ (of order 50) have order 4 ?
   (j) Let $G$ be a group and $\phi : G \to G$ the map defined by $\phi(g) = g^{-1}$. Is $\phi$ always a group homomorphism ?
2. (10 pts) Prove that the set $Z_n^*$ of congruence classes mod $n$ of numbers which are coprime to $n$ is a group under multiplication, by showing that this set is closed under multiplication, taking inverses, and that it contains the multiplicative identity.
3. (10 pts) If a non-trivial group $G$ has no non-trivial proper subgroups, then prove that $G$ is a finite group of prime order. (Do *not* assume that the group $G$ is finite to begin with.)