Normal subgroups

Def. A subgp $N \subset G$ is called a normal subgp of G if it satisfies
$$gNg^{-1} \subset N$$
for each $g \in G$.

This is equivalent to saying that for each $g \in G$ and $n \in N$
$$gng^{-1} \in N.$$

Thm. N is a normal subgroup of G
$$\Updownarrow$$
$$gNg^{-1} = N \text{ for every } g \in G$$
$$\Updownarrow$$
$$gN = Ng \text{ for every } g \in G$$
$$\Updownarrow$$
for every $g \in G$, $n \in N$, there exists $n' \in N$ s.t
$$gn = n'g.$$

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Pf: Exercise.

2. Quotient gp.

Def Given a normal subgp $N \subset G$, the set of cosets of N (left cosets, and right cosets are equal in this case) form a group with the operations.

$$(g_1 N) \cdot (g_2 N) = g_1 g_2 N$$

$$(gN)^{-1} = g^{-1} N.$$

(One needs to check that these operations are well-defined since they are defined using representatives).

This group is called the quotient group of G by N (denoted G/N).

Note that the identity element of G/N is the coset N.

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