Effect of Missing Ranks in Instant Runoff Voting
CS&SS 552

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Abstract

Instant runoff voting (IRV), is used in various public elections around the United States. This voting system allows voters to give a ranked preference of multiple candidates, as opposed to ranking only one candidate such as in the first-past-the-post voting system. In IRV elections, voters are not required to rank all of the candidates, so natural missingness arises when some voters forgo providing a rank for all of the candidates. Additionally, strategic missing tactics are sometimes utilized by voters, rather than simply providing a ranking based on their true candidate preferences, to influence the outcome of an election. The effect of such missingness on the outcome of the election is explored here through a simulation study, with a couple of the simulations motivated by real IRV elections in Alaska and Minneapolis, Minnesota. The results of the simulations show that generally the more voters that participate in an election, the less effect missingness has on the election outcome.

1 Introduction

Instant runoff voting (IRV) is a form of rank choice voting in which a single winner is elected. It is used for some public elections in the United States (*Ranked Choice Voting Information*, 2024). For example, Alaska held an IRV gubernatorial election to elect a governor in 2022 and Minneapolis, Minnesota held an IRV municipal election to elect a mayor in 2021 (*Alaska gubernatorial and lieutenant gubernatorial election*, 2022; Minneapolis, 2021). We'll take a closer look at these two elections in Section 2.2 and 3.2. In rank choice voting, voters are allowed to rank candidates in order of their preferences on their election ballot, but they are not required to rank every single candidate. A realistic missingness scenario that can occur is that some voters may rank some, but not all, of the candidates because the voters may feel impartial about many of the candidates, and hence only rank their top few candidates. Alternatively, a voter may try to vote strategically based on how they believe others will vote. This report explores the effect of missing ranks on IRV election outcomes through a simulation study. In particular, section 2 will address a simulated example, and a real data example to explore the effect of missing ranks on IRV election outcomes, while section 3 explores the effect of strategic voting which induces missing ranks in a simulated and real data IRV example as well.

2 Missingness

2.1 Missingness at Random

To start off, consider an election with 3 candidates and 1 winner, where the the candidates are unequally preferred in the voting population. For each voter, I simulate a ranked preference for all 3 of the candidates using the built-in `sample` function in R to sample 3 candidates without replacement and with unequal probability. For example, if the 3 candidates are A,B, and C, then $A > B > C$ is a possible voter preference.

I simulate high probabilities for 2 or 3 of the candidates, perturb the probabilities with noise, and then assign equally low probabilities to the rest of the candidates. For the case where there are only 3 candidates in the election and 3 high probabilities, I assign all 3 candidates equal probabilities and then slightly perturb them using a Uniform(0.01,0.03) distribution. For example, if there are 3 total candidates, all of which have high probability, then $(0.33, 0.36, 0.31)$ is a possible probability combination for those 3 candidates. For more than 3 candidates, I ensure that the 3 candidates with high probability are liked by 75% of the voter population. For example, for 6 candidates, a possible probability combination is

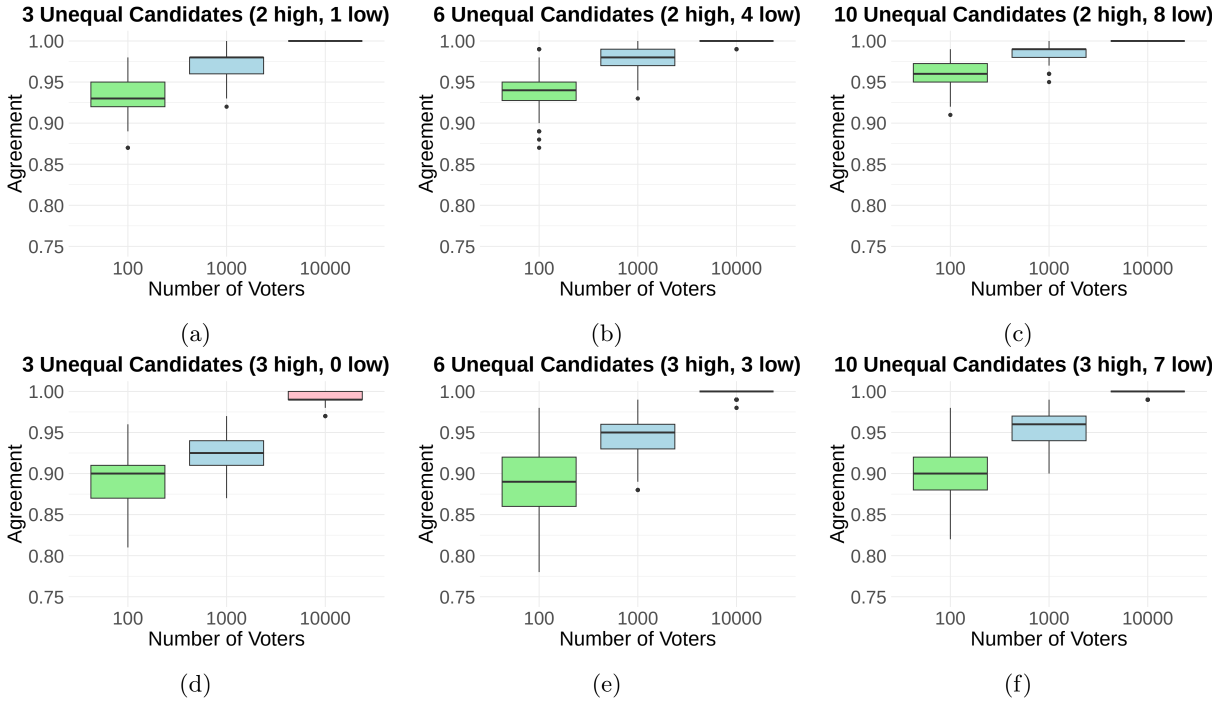


Figure 1: Boxplots of the distribution of agreement between election results for simulated data with complete voter ranking and data with incomplete voter rankings, varying in the number of voters and the number of candidates. All candidates have unequal probability, and the missingness mechanism for the incomplete rankings is uniformly random. The plots (a)-(c) show the distribution of the agreement proportion for 3,6,and 10 candidates, when 2 of the candidates have high probabilities, and the rest have low probabilities.The bottom plots (d)-(f) shows the distribution of the agreement proportion for 3,6,and 10 candidates, when 3 of the candidates have high probabilities, and the rest have low probabilities.

$(0.25, 0.28, 0.22, 0.083, 0.083, 0.083)$. Note that $0.25 + 0.28 + 0.22 = 0.75$ and the rest of the candidates have equal probability of $\frac{1-0.25}{3} \approx 0.083$. I use the `count.votes` function from the library `vote` (Raftery et al., 2021) in R to determine the winner of the IRV election. Then, for each voter I randomly sample a number of missing ranks, which are all equally likely. For example, if a voters preference was $A > B > C$ and they had 1 missing vote, their vote would become $A > B$. Similarly, if they had 2 missing ranks, their vote would become A. I again determine the winner of the IRV election using the `count.votes` function, this time with the voter preferences with missing ranks. Lastly, I check whether the results of the election, with and without missing ranks, agree. I simulate 100 such simulations, with varying numbers of voters and candidates, and calculate an agreement proportion using the formula

$$\text{Agreement} = \frac{\text{\# of simulations that had matching election results}}{100}$$

I repeat this 100 times to get 100 agreement proportions, and plotted the results in Figure 1. If missingness had no effect, the agreement proportion would be 1 between the election results from the data with a complete ranking from each voter, and the data with incomplete voter rankings. Figure 1 shows that as the number of voters increasing, missing data has less of an effect since the agreement proportion increases between the election with and without missing ranks As the number of candidates increases, it appears the effect of missing slightly decreases as well, since the mean agreement proportion increases with the increase in the number of candidates. As the number of highly probable candidates increases, it appears the effect of missingness increases, since the agreement proportion decreases between the election with and without missing ranks.

2.2 Real Data: Alaska

Since there are many factors that can vary in a simulated election, it may be more realistic to explore a missingness scenario from a real life IRV election. The 2022 Alaska gubernatorial election was held on November 8th, 2022 to elect the governor of Alaska (*Alaska gubernatorial and lieutenant gubernatorial* *election*, 2022). There were 4 candidates: Mike Dunleavy (R), Les Gara (D), Charlie Pierce (R), and Bill Walker (I), with a 5th option to write-in a candidate. The data was accessed from Atsusaka (2024), and contains 266,693 ballots.

To explore the role missingness played in this Alaskan election, the first step is to extract a couple summary metrics of the data in order to mimic the election through simulation. Some data cleaning was necessary before these summary metrics could be computed. This data cleaning involved removing the ballot for voters that overvote, meaning they cast more votes than allowed. In these cases, the ballot is considered spoiled and is not counted in the election (*Overvote*, n.d.). Additionally, the `count.votes` function identified 6,271 invalid ballots, so those were removed during data cleaning as well. Invalid ballots consist of ballots that do not have consequtive votes. For example, if a voter ranked candidates 2-5, but did not provide a rank 1 choice, their vote would be considered invalid.

One summary metric is the probability that a voter in the population will vote for a particular candidate. The `count.votes` function was used to identify the number of rank 1 votes for each candidate. Then, the probabilities for each candidate were estimated as

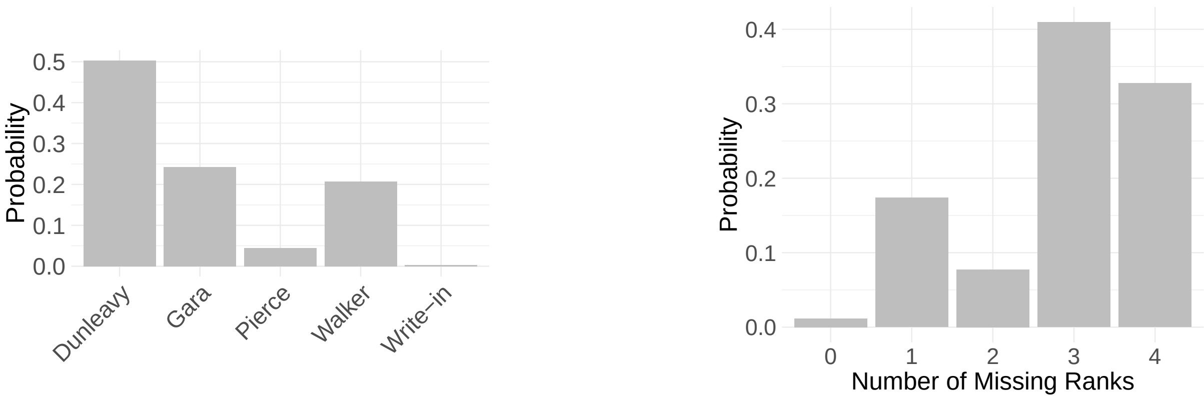
$$P(\text{candidate } i) \approx \frac{\#\text{ voters that ranked candidate } i \text{ 1st}}{\text{total number of voters}}$$

Refer to Figure 2(a) to see this summary metric calculated from the data.

Another summary metric summarizes the occurrence of missingness in the dataset. This will be measured as the probability that a voter will have j missing ranks where $j \in \{0, 1, 2, 3, 4\}$, since a voter can have at least 0 ranks missing, or at most 4 ranks missing. For example, if a voter has 2 missing ranks, then they did not rank any candidates 4th or 5th. A voter cannot have 5 missing ranks because their vote would then be considered invalid. We can formalize this as

$$P(j \text{ missing ranks}) \approx \frac{\#\text{ of voters with j missing ranks}}{\text{total number of voters}}$$

Refer to Figure 2(b) to see this summary metric calculated from the data.



(a) Probability of a voter voting for each candidate.

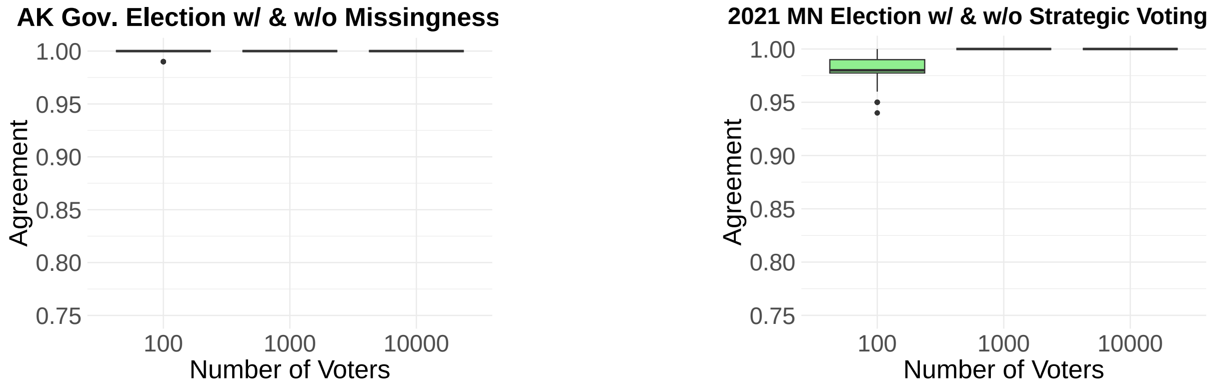
(b) Probability that a voter's ballot is missing 0-4 ranks.

Figure 2: Summary metrics from the 2022 Alaska gubernatorial election.

These summary metrics were used to simulate artificial data for this election with and without missingness, similar to the simulation process from 2.1. Refer to Figure 3(a) for the results of the simulation, which shows that missingness did not impact the results of the election. This could be due to the fact that Dunleavy was so well liked, with a 50% chance that a voter would vote for them.

3 Strategic Missingness

In this section, we explore the effect of strategic missingness, i.e. missing ranks that are a result of strategic voing, in IRV elections on election outcomes.



(a) Probability of a voter voting for each candidate.

(b) Probability that a voter's ballot is missing 0-4 ranks.

Figure 3: (a) Boxplots of the distribution of agreement between election results for simulated data mimicking the 2022 Alaska gubernatorial election with and without missing rankings, varying the number of voters. (b) Boxplots of the distribution of agreement between election results for simulated data mimicking the 2021 Minneapolis Municipal election and a simulated election in which strategic voting via the "Don't Vote Frey" campaign was successfully implemented, varying the number of voters.

3.1 Dilution

Consider a population that consists of two groups, Group 1 and Group 2. There is an election with 4 candidates, A,B,C, and D. In Group 1, the general preference of candidates is $A > B > C > D$, with the probabilities 0.55, 0.2, 0.15, and 0.1 that a voter will vote for the respective candidates. In Group 2, the preference is reversed, $A < B < C < D$, with the respective probabilities being 0.1, 0.15, 0.2, and 0.55. The bottom plot of Figure 4 shows the fraction of 1,000 simulated elections that candidate D won with varying sizes of Group 2 (orange bar plot). As we would expect, Group 2 in this case needs to be about at least half of the population to win the election, i.e. candidate D wins, half of the time. Now, suppose that Group 2 thinks that their vote gets *diluted*, or in other words, the influence of their vote is reduced if they provide a full rank, and so everybody in Group 2 only provides one ranking. The bottom plot of Figure 4 shows that in this case Group 2 needs to be over 60% of the population in order to win the election at least half of the time (purple bar plot). This simulation emphasizes the benefit of rank choice voting, in particular it's ability to use a voters ranking of candidates, rather than just their top choice. In the scenario where each voter in Group 2 only voted for one candidate, they lost a lot of power because voters who ranked D 2nd, 3rd, or 4th did not get to contribute to helping candidate D win.

Now suppose Group 2 was able to convince every voter to only vote for candidate D, so the probability of a voter in Group 2 voting for D was 1, and the probability of voting for C, B and A were all 0. If we only care about when candidate D wins, this is equivalent to if everyone in Group 2 ranked D first, and C,B,and A in any order after that. The top plot of Figure 4 shows that in this case Group 2 needs to be only above 40% of the population in order to win the election (blue bar plot). While this is the ideal situation, it is unrealistic that a group will unanimously vote for the same candidate.

Similar conclusions could be made with a population of size 10,000, which can be seen by the histograms on the right in Figure 4. The main difference between the plots from a population of size 1,000 and 10,000 is that the left tails of the histograms have less weight in the bigger population of 10,000.

Overall, this simulation shows that providing a full rank may be better in some situations than a partial rank, in terms of electing a candidate that best represents the preference of the voting population. This is because realistically the population will not be unanimous in their candidate preference, and so the additional rankings (besides the first rank) can influence the election. Unfortunately, this is not quite enough yet to show that RCV does not *dilute* a voter's vote. Rank choice voting does occasionally give a voter more than one chance to vote for a candidate, as seen by the results in Figure 4 when Group 2 only need to be about 50% of the population to elect candidate D at least 50% of the time when a rank choice ballot is used rather then 60% when a first-past-the-post ballot is used which only allows the voter to vote for a single candidate. In general, this can occur when a voter ranks an unpopular candidate as their first choice which gets eliminated due to low popularity, so the voters second choice is counted instead. In this case, the voter did not "waste" their vote by ranking the unpopular candidate because the voter got to show support for the unpopular candidate but ultimately their second choice

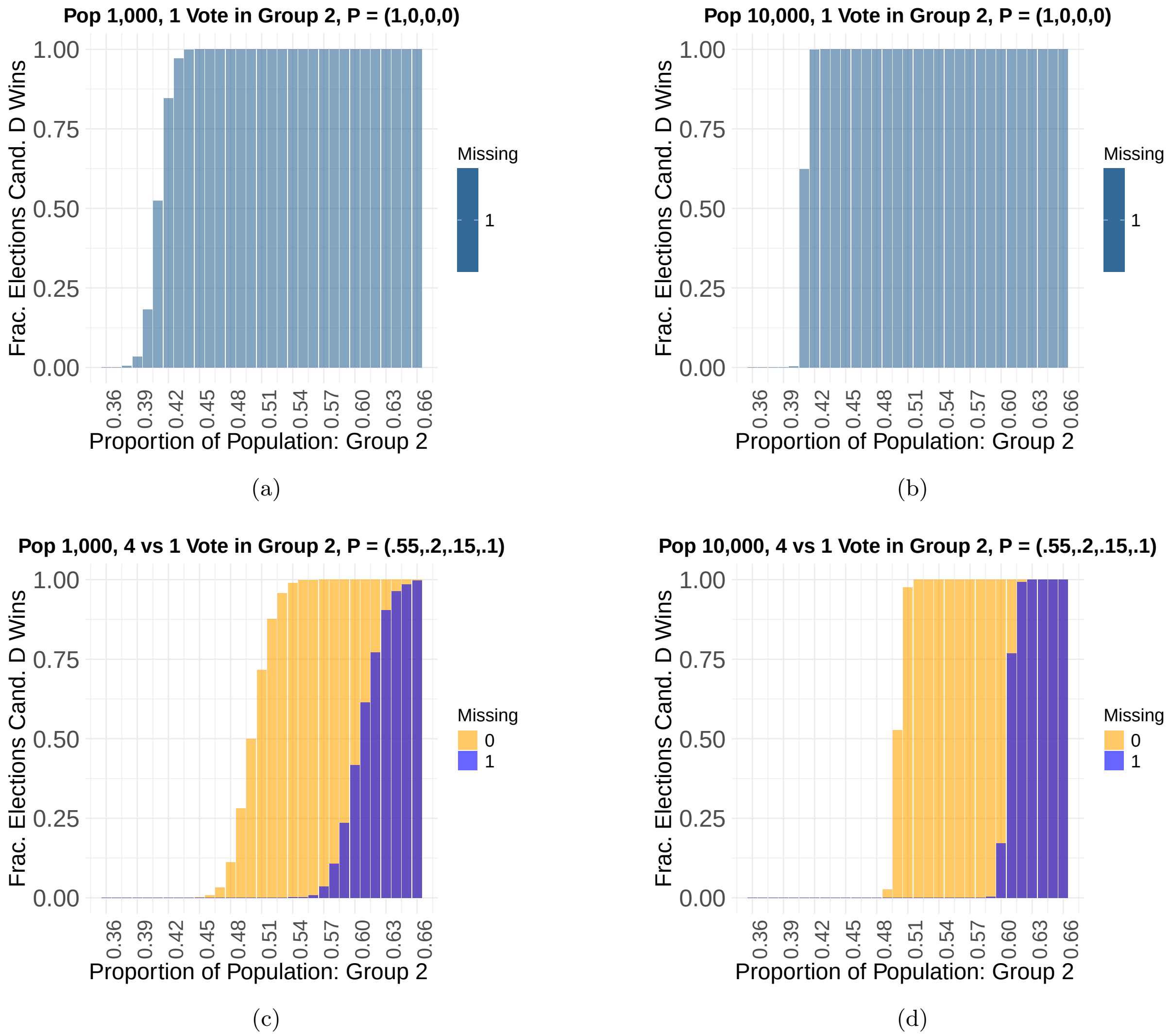


Figure 4: Example of strategic voting in an IRV election with 4 candidates and 2 groups of voters with group 1 having the general candidate preference $A > B > C > D$ and group 2 having the general candidate preference $D > C > B > A$. In the top plots, everyone in the second group ranks the same candidate, D, first (Missing=1). In the bottom plots, every voter in the second group votes ranks all candidates (Missing=0) or only one candidate (Missing=1) but not necessarily the same candidate. The plots on the left have a population of size 1,000, while the plots on the right have a population of size 10,000.

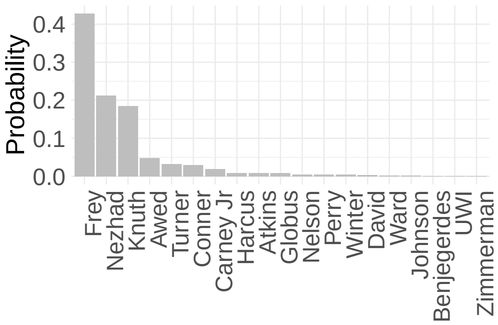
ends up getting counted. This property of rank choice voting is what makes it less prone to "spoiler" candidates and lowers the barrier of entry for women and candidates of color (*Ranked Choice Voting*, 2024). However, if the unpopular candidate wasn't eliminated, and the election was close between the voters second choice and another candidate they disliked and the disliked candidate ended up winning, then it would have been better if the voter ranked their second choice as their first choice so that their vote had more influence on the election. For this reason, I cannot conclude from the simulation in Section 3.1 alone that rank choice voting doesn't dilute votes.

Note that two groups having the preferences $A > B > C > D$ and $D < C < B < A$ occurs when the population is voting by the political spectrum with for example A being an extreme left candidate, B being a moderate left candidate, C being a moderate right candidate, and D being an extreme right candidate. As a result, these candidate preferences are bimodal with the modes occurring at the extreme ends of the political spectrum. It would be interesting to see how the results of this simulation would change if the candidate preferences were bimodal with the modes occurring at the moderate parts of the political spectrum. For example, this could look like the preferences for the two groups being $A > B > C > D$ and $C > D > A > B$ where B and D are the extreme left and extreme right candidates respectively, and

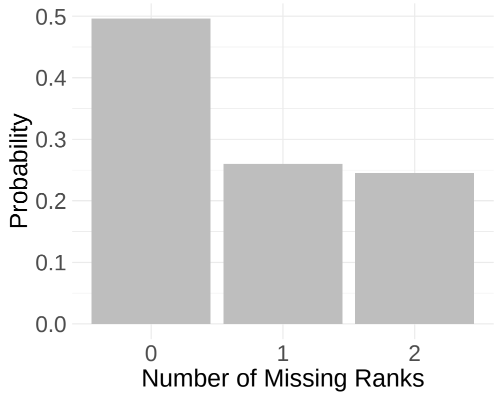
A and C are the moderately left and moderately right candidates respectively.

3.2 Real Data: Minneapolis

Next we explore the effect of strategic missingness in a real life IRV election. Minneapolis, Minnesota held an IRV municipal election to elect a mayor in 2021. There were a total of 145,337 ballots, but only 143,974 valid ones. The data was obtained from Minneapolis (2021), and there were 19 candidates with each voter allowed to rank up to 3 candidates. Frey, the incumbent mayor at the time, was reelected. During the election, there was a strategic voting campaign targeted at Knuth and Nezhad supporters, with the slogan "Don't Vote Frey", to discourage voters from ranking the incumbent mayor (Navratil, 2021). This section aims to explore whether this strategic voting strategy would have changed the results of the election, or if Frey would've still won. Two summary metrics, the probabilities that a voter in the voting population will vote for each candidate and the probability of missing ranks, were calculated from the election data. Refer to Figure 5 to visualize these two summary metrics, and to Section 2.2 for an explanation of how these metrics were calculated.



(a) Probability of a voter voting for each candidate.



(b) Probability that a voter's ballot is missing 0-2 ranks.

Figure 5: Summary metrics from the 2021 Minnneapolis Municipal election.

These summary metrics were used to simulate artificial data for this election, and the election winner was determined. Additionally, if a candidate ranked Knuth or Nezhad first, and Frey second or third, their vote for Frey was erased. If a voter ranked Frey second, their third ranked candidate became their second ranked candidate. The election winner was recalculated with this additional missingness in the data, and compared to the results without this additional missingness. Note that the "Don't Vote Frey" campaign may have influenced some of the voters in this 2021 municpial election to vote strategically and not rank Frey, but this simulation verified if the results of the election would have changed if *every* Knuth and Nezhad supporter voted strategically by not ranking Frey. Refer to Figure 3(b) for the results of this simulation, which shows that with a large enough number of voters (≥ 1000), the results of the election would not have changed if *every* supporter of Knuth and Nezhad voted strategically.

4 Conclusion

From the simulations in Sections 2.1, 2.2, and 3.2, it appears that as the number of voters increases, the effect of missing ranks in IRV elections has less of an effect on the election outcome, if any effect at all. For computational purposes, the simulations have a maximum of 10,000 voters, but even then the agreement between elections with and without missingness was essentially 1. Reproducible code and data can be found on Github. Other avenues that would be worth exploring is running these simulations with at least 100,000 voters, as this is more realistic of public elections around the United States that use instant runoff voting. Additionally, it would be interesting to explore how the effect of limiting the number of candidates the voters are allowed to rank influence the election. For example, in the 2021 Minneapolis municipal election, each voter was only allowed to rank up to 3 candidates, even though there were 19. A simulation study that explores varying the number of allowed ranks could show whether 3 is a good choice for an election, or perhaps another number like 4 would be better.

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