Geotopia University

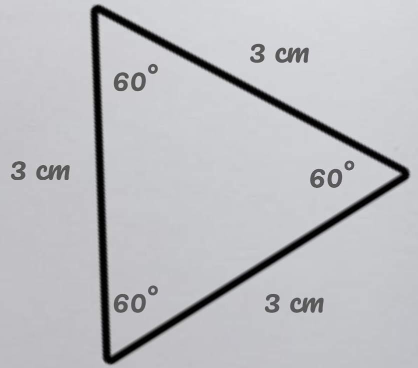
Professor’s Note

"Imagination is more important than knowledge. For knowledge is limited, whereas imagination embraces the entire world, stimulating progress, giving birth to evolution."

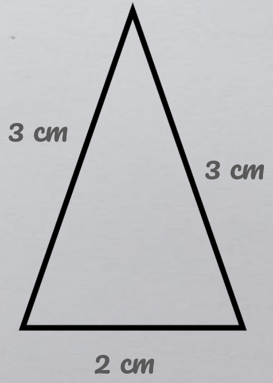
- Albert Einstein

Note #137 Special Triangles

Equilateral Triangle

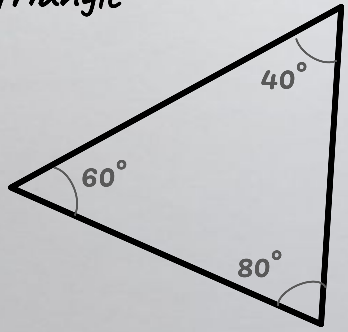


Isosceles Triangle

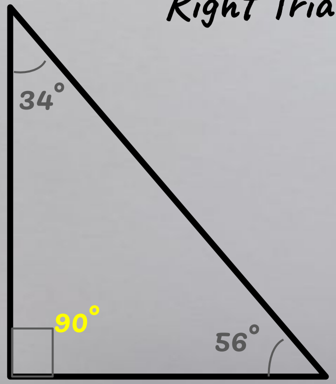


Note #138 3 Types of Triangles

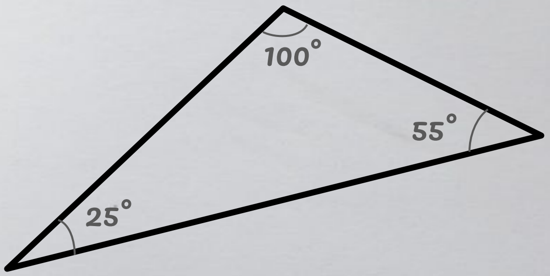
Acute Triangle



Right Triangle



Obtuse Triangle



Facts:

1) Angle sum of a triangle is always 180°

2) The base of a triangle is any of the 3 sides

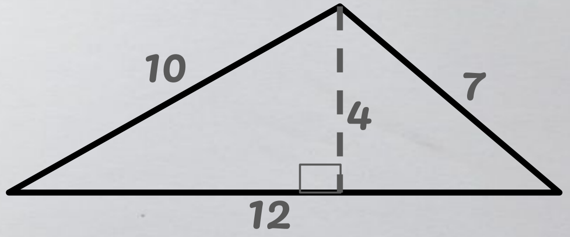
3) The height of a triangle is always forming a right angle with the base and ends at one vertex (angle).

Note #139 Area of triangles

The formula for the area of a triangle is ½ times the base times the height.

$$\text{Area} = \frac{1}{2}bh$$

Example:



$$A = \frac{1}{2} \cdot 12 \cdot 4$$

$$A = 6 \cdot 4$$

$$A = 24$$

The area of the triangle is 24 square units.

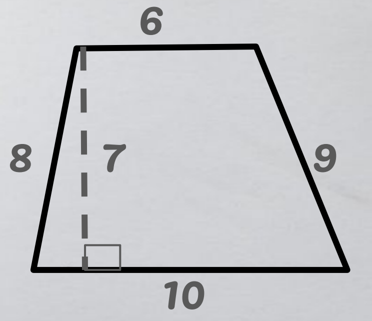
Note #140 Area of Trapezoid

The formula for the area of a trapezoid is ½ the sum of the two bases times the height.

$$A = \frac{1}{2}(b_1 + b_2)h$$

The perimeter is 6+8+10+9=33

Example:



$$A = \frac{1}{2}(6 + 10)(7)$$

$$A = \frac{1}{2}(16)(7)$$

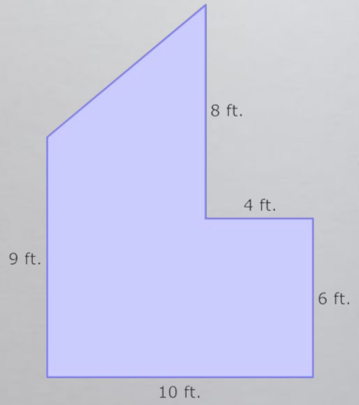
$$A = (8)(7)$$

$$A = 56$$

The area of the triangle is 24 square units.

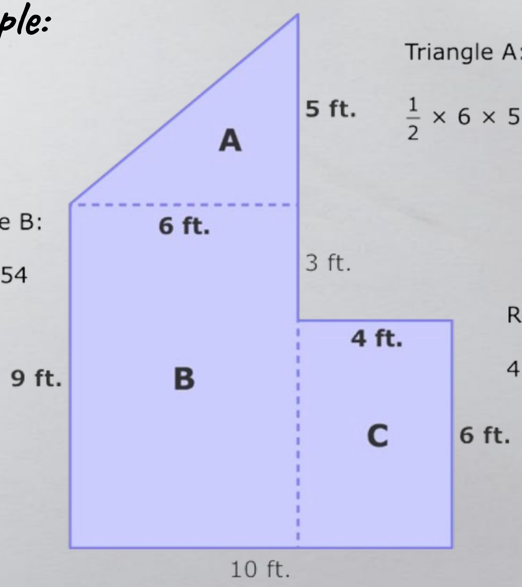
Note #140 Composite Figure

This composite figure (also known as polygon) is made up of basic shapes put together.



To find the area of a composite figure, **break** the composite figure into basic shapes, **find** the area of each basic shape, and then **add** the areas!

Example:



Triangle A:

$\frac{1}{2} \times 6 \times 5 = 15$

Rectangle B:

$9 \times 6 = 54$

Rectangle C:

$4 \times 6 = 24$

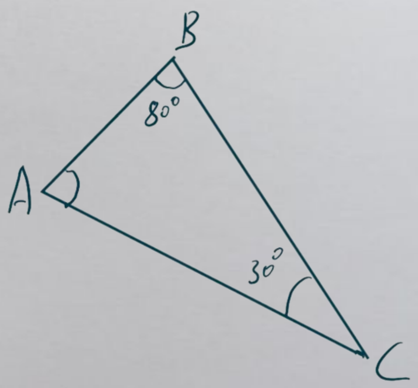
Now, add the areas of the basic shapes.

$15+54+24=93$

So, the area of the compound shape is 93 square feet!

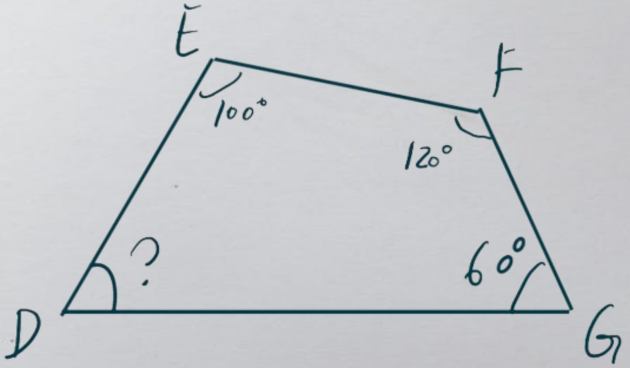
Note #158 Angle Measure

To find the measure of an angle, one way is to use a protractor. However, we can calculate the angle measure as well.



The angle sum of triangle is always 180°

Angle A = 180° - 30° - 80° = 70°

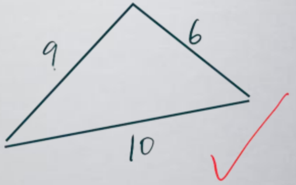


The angle sum of quadrilateral is always 360°

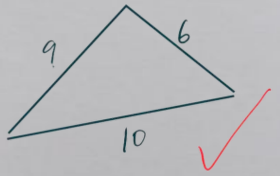
Angle D = 360° - 100° - 120° - 60° = 80

Note #159 Triangle Inequality

Any side of a triangle must be shorter than the **sum** of the other two sides.



$6+9 > 10$



$10-9 < 6$

Also, any side of a triangle must be longer than the **difference** between other two sides.

If you have two lengths of 3 and 5, then the possible length for the third side is:

3, 4, 5, 6, 7 (between 5-3 and 5+3)

If you have two lengths of 6 and 10, then the possible length for the third side is:

5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

(between 10-6 and 10+6)

If you have two lengths of 2 and 9, then the possible length for the third side is:

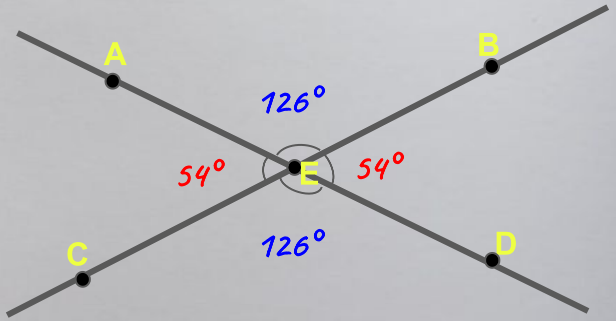
8, 9, 10

(between 9-2 and 9+2)

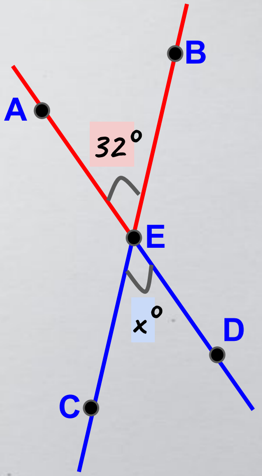
Note #170 Vertical Angles

If two lines are crossing each other, there will be two pairs of vertical angles.

Vertical angles are opposite to each others and equal in measure.

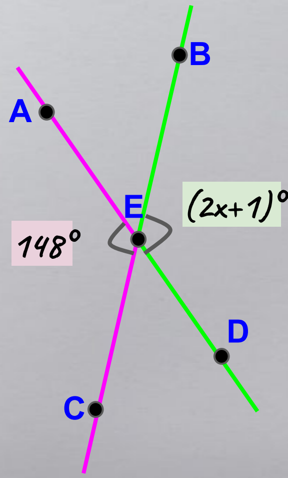


$\measuredangle AEC = \measuredangle DEB = 126^\circ$



$\measuredangle AEB = \measuredangle DEC$

$32^\circ = x^\circ$



$\measuredangle AEC = \measuredangle DEB$

$148^\circ = (2x+1)^\circ$

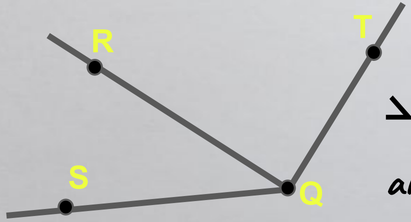
$148 = 2x+1$

$147 = 2x$

$73.5 = x$

Note #171 Supplementary Angles

If two angles are side by side, they are **adjacent angles.**



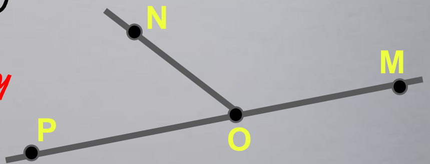
$\angle RQT$ and $\angle RQS$

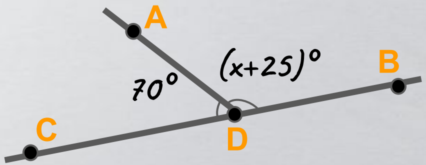
are adjacent angles

If two angle measures add up to 180°, they are **supplementary angles.**

$\angle NOP + \angle NOM = 180^\circ$

They are supplementary





$\angle ADB$ and $\angle ADC$

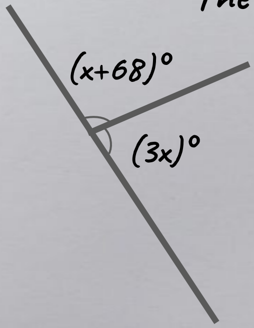
are supplementary

$70 + x + 25 = 180$

$95 + x = 180$

$x = 85$

The two angles are supplementary



$x + 68 + 3x = 180$

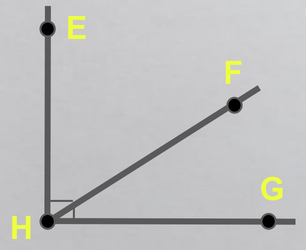
$68 + 4x = 180$

$4x = 112$

$x = 28$

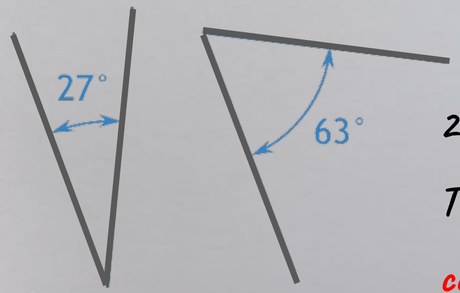
Note #171 Complementary Angles

Two angles are called **complementary** when their measures add up to 90°.



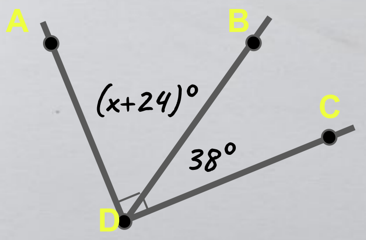
$\angle EHF + \angle FHG = 90^\circ$

They are **complementary**



$27^\circ + 63^\circ = 90^\circ$

The two angles are **complementary** but not adjacent

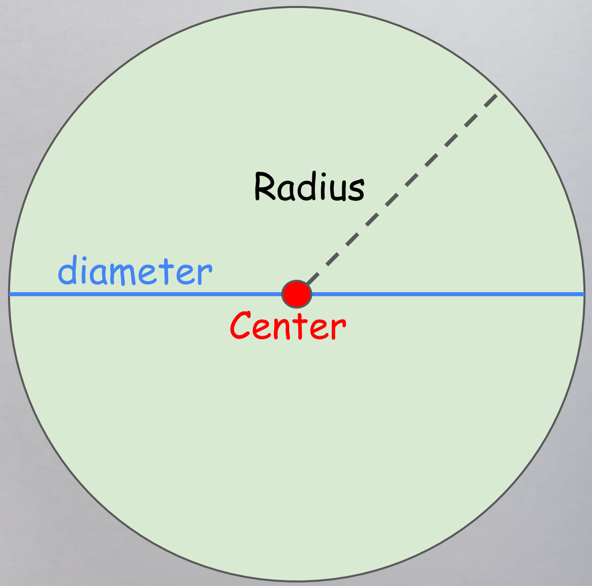


$\angle ADB$ and $\angle BDC$ are **complementary**

$$x + 24 + 38 = 90$$
$$x + 62 = 90$$
$$x = 28$$

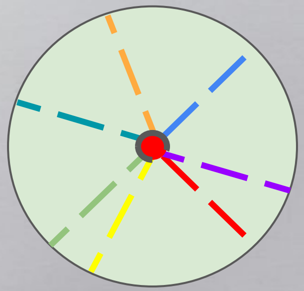
Note #200 Circle

Circle is a curve that is a radius away from the center.

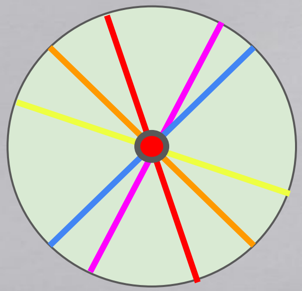


Diameter is always twice the length of **radius.**

In a circle, you can draw as many radius as you want.

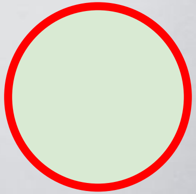


And you can draw as many diameter as you want too.



Note #201 Circumference

The Circumference is the distance once around the circle.



Circumference!

$$\underset{\text{Circumference}}{C} = \underset{\text{Pi}}{\pi} \cdot \underset{\text{diameter}}{d} = \underset{\text{Double}}{2} \cdot \underset{\text{Pi}}{\pi} \cdot \underset{\text{Radius}}{r}$$

Circumference is 3.14 of the diameter, or 3.14 of twice the radius

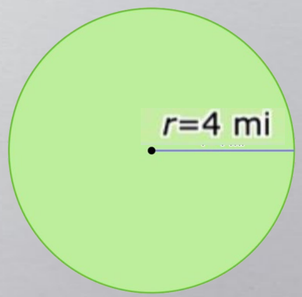
We know the diameter is 2 cm

$$C = \pi d = \pi \cdot 2 = 2\pi \ cm$$

(in terms of π)

$$2\pi = 2 \cdot 3.14 = 6.28 \ cm$$

(in decimal)



We know the diameter is 2 time the radius. So diameter is 8 mi.

$$C = 2\pi r = 2 \cdot \pi \cdot 4 = 8\pi \ mi$$

(in terms of π)

$$8\pi = 8 \cdot 3.14 = 25.12 \ mi$$

(in decimal)

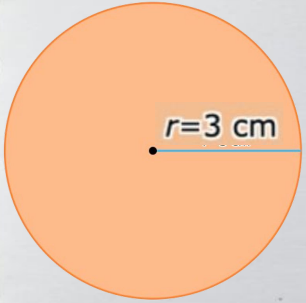
Note #202 Area

The area of a circle is π times the radius squared.

Area

$$\underset{\text{Area}}{A} = \underset{\text{Pi}}{\pi} \cdot \underset{\text{Radius Square}}{r^2}$$

Area is 3.14 of the radius squares. Multiply radius by itself and then 3.14 for area.



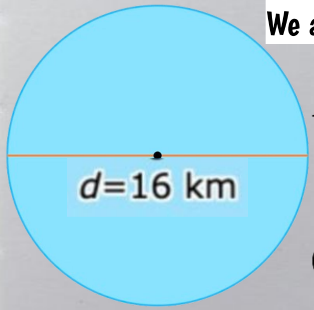
We know the radius is 3 cm.

$$A = \pi r^2 = \pi \cdot 3^2 = 9\pi \ mi^2$$

(in terms of π)

$$9\pi = 9 \cdot 3.14 = 28.26 \ mi^2$$

(in decimal)



Half the diameter is 16 km is the radius 8 km.
We always need radius for area!

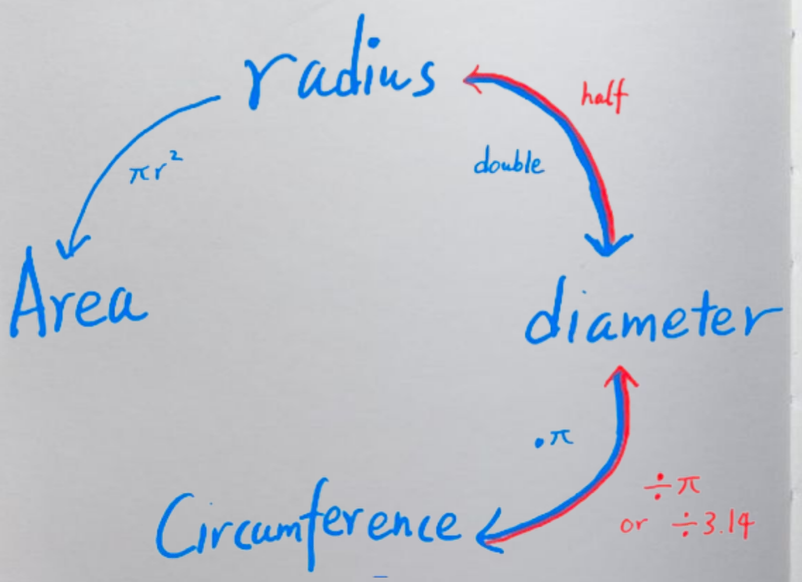
$$A = \pi r^2 = \pi \cdot 8^2 = 64\pi \ mi^2$$

(in terms of π)

$$64\pi = 64 \cdot 3.14 = 200.96 \ mi^2$$

(in decimal)

Note #203 Circumference to Area



If circumference is 15.7 cm,

then diameter is 15.7 ÷ π = 5 cm

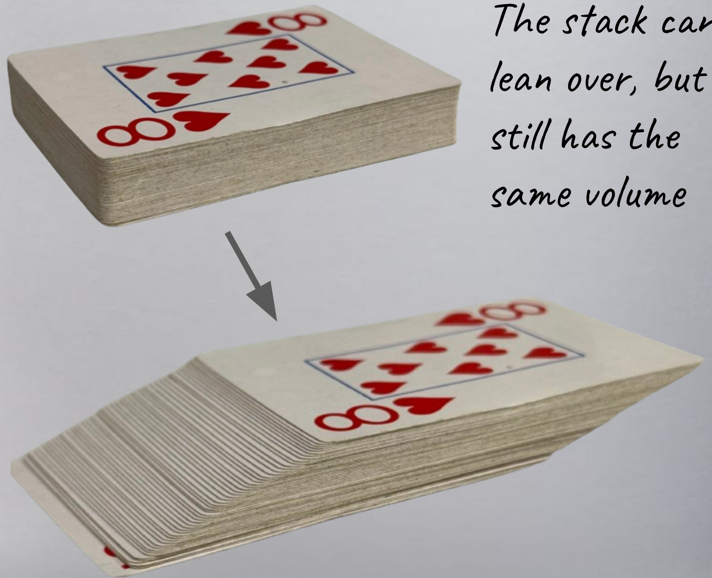
then radius is half of 5 which is 2.5 cm

then area is $\pi(2.5^2) = 6.25\pi\ cm^2$

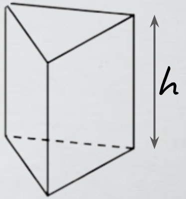
or $19.63\ cm^2$

Note #225 Volume

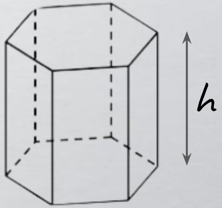
The volume of a prism is calculated by multiplying the area of its base by its height.



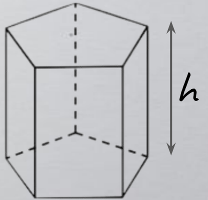
The stack can lean over, but still has the same volume



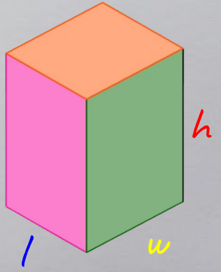
Volume
=base triangle * height



Volume
=base hexagon * height



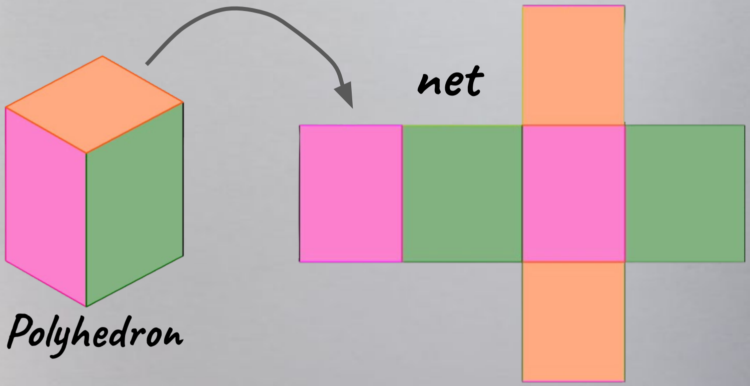
Volume
=base pentagon * height



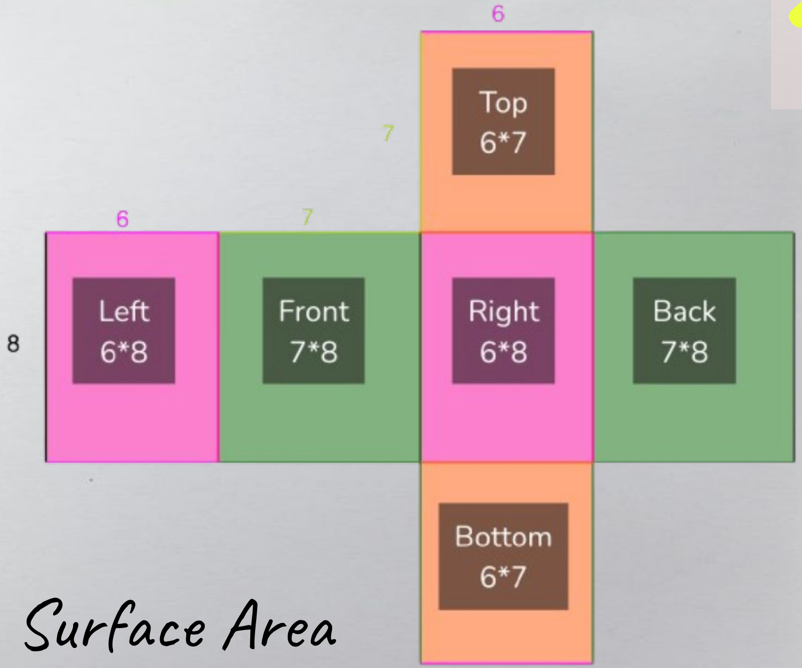
Volume = base * height
=length * width * height

Note #226 Surface Area of Rectangular Prism (Box)

To find the surface area, we need to "open" the prism. In other words, have the **net** of the prism.



Polyhedron



Surface Area

$=6*8+7*8+6*7+6*8+7*8+6*7$

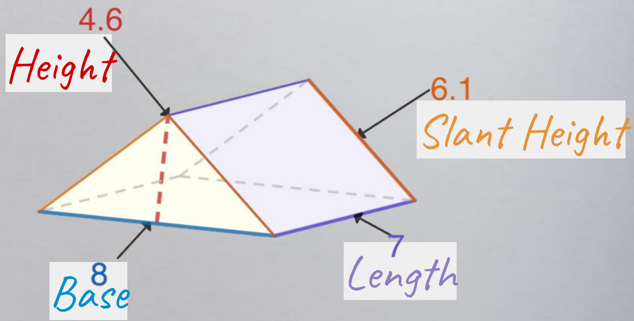
$=292 \text{ unit}^2$

Or is there a shortcut?

Note #227 Triangular Prism

Volume of triangular prism is

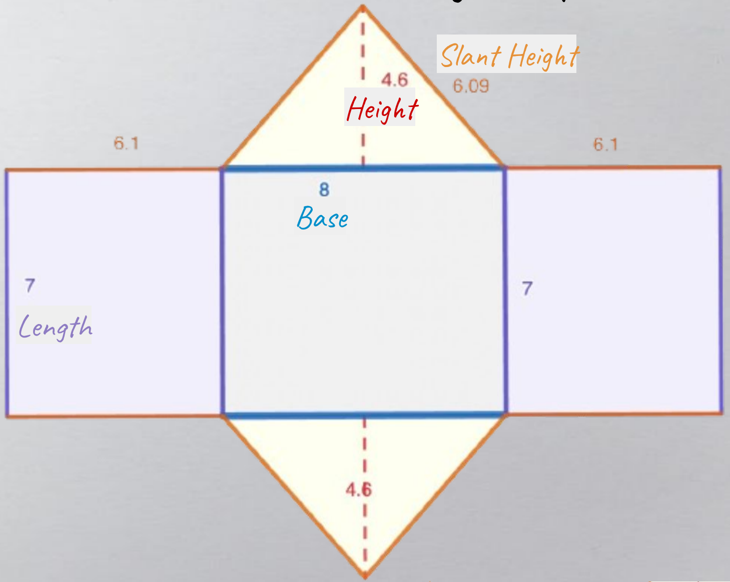
$$\boxed{V = \underbrace{A}_{\text{Base Area}} \cdot \underbrace{l}_{\text{Length}}}$$ or $$\boxed{V = (\frac{1}{2}bh) \cdot l}$$

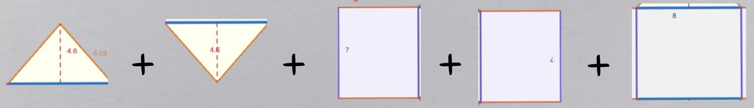


$$V = (\frac{1}{2}bh) \cdot l = (\frac{1}{2} \times 8 \times 4.6) \times 7$$
$$= 128.8\ unit^3$$

(Volume is area of triangle times length)

Surface Area of triangular prism:





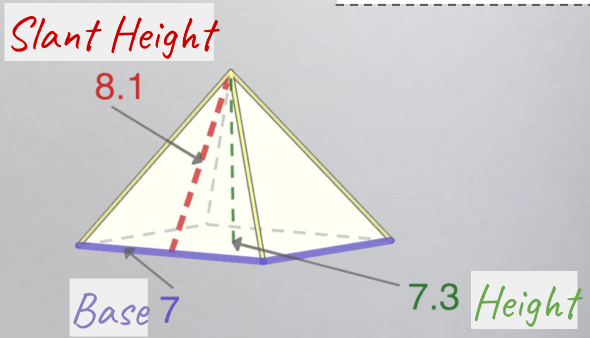
½*4.6*8 + ½*4.6*8 + 6.1*7 + 6.1*7 + 8*7

= 178.2 unit^2

(Surface Area is the sum of 5 faces)

Note #228 Square Pyramid

Volume of a Pyramid is $V = \frac{1}{3} A \cdot h$ (Height; Base Area)

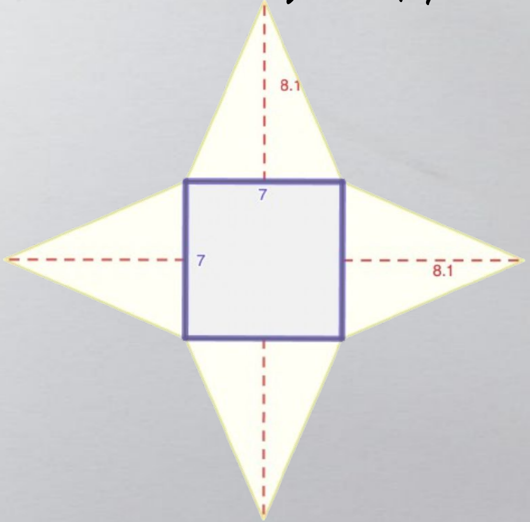


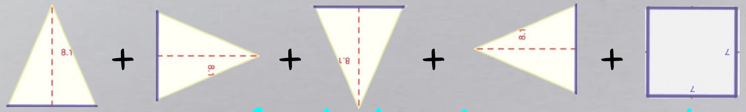
Volume of a square pyramid is
one third the base square times the height.

$$V = \frac{1}{3} A \cdot h = \frac{1}{3}(7 \times 7) \times 7.3$$

$$= 119.23\ unit^3$$

Surface Area of square pyramid:





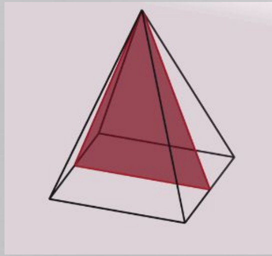
4(½*7*8.1) + 7*7

$= 162.4\ unit^2$

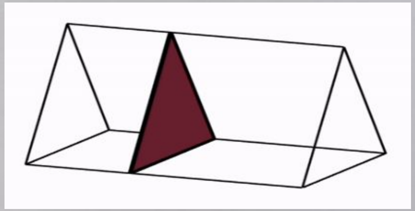
(Surface Area is the sum of 5 faces)

Note #250 Cross Section

A **cross section** is the shape we get when cutting straight through an object.

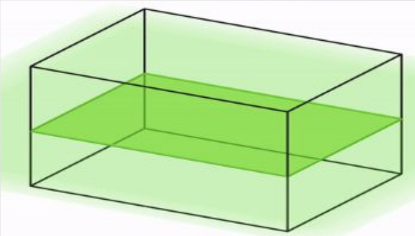


Vertical cut of a square pyramid

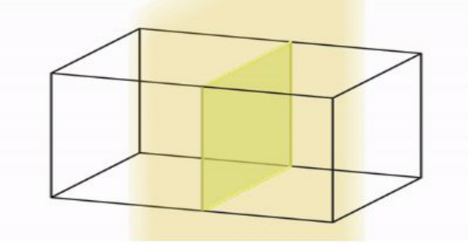


Vertical cut of a triangular prism

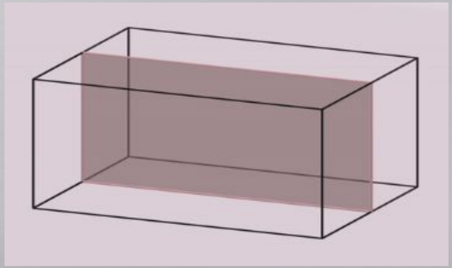
Rectangular Prism



Horizontal cut



Vertical cut from the front



Vertical cut from the right