Exercises 2

Second lecture

The procedure to find sampling distribution of the mean of $n = 100$ i.i.d. observations from a normal with mean $\mu = 170$ and standard deviation $\sigma = 20$ is as follows:

```
n <- 100
mu <- 170
sigma <- 20
N <- 10000  # number of samples
ybar <- rep(0, length = N)
for(i in 1:N){
    y <- rnorm(n, mean = mu, sd = sigma)
    ybar[i] <- mean(y)
}
hist(ybar, 100, probability = TRUE)
```

1. Experiment 1: suppose that you have 1 sample of size $n = 100$ from $N(\mu = 170, \sigma = 20)$.
   - Check using simulation that the mean and the standard deviation of the sampling distribution of the mean correspond to the theoretical values.
   - Check that the sampling distribution of the mean is normal.
   - Study using simulation the sampling distribution of the variance $S^2 = \sum_i (Y_i - \bar{Y})^2/(n-1)$. Note that in R the function for S^2 is `var(y)`
   - Check if the found distribution has a mean equal to σ^2.
2. Experiment 2: suppose that you have 1 sample of size $n = 5$ from an exponential distribution with rate $\lambda = 3$. To simulate a sample of size n from the exponential distribution use `rexp(n, lambda)`.
   - Check using simulation the sampling distribution of the mean.
   - Check using simulation the sampling distribution of the MLE of the failure rate.
   - Compute using simulation the bias^2 and the variance of the MLE estimator.
3. Find using R the following quantiles
   - (standard normal, `qnorm`) $z_{0.9}$
   - (t with 3 degrees of freedom, `qt`) $t_{0.9}(3)$
   - (t with 10 degrees of freedom) $t_{0.9}(10)$
   - (t with 3 degrees of freedom) $-t_{0.1}(10)$