Exercises 4

Fourth lecture

1. Conditional probability: approximately 1/125 of all births are fraternal twins and 1/300 of births are identical twins. Elvis Presley had a twin brother (who died at birth). What is the probability that Elvis was an identical twin? (You may approximate the probability of a boy or girl birth as 1/2.)

2. Suppose you have a Beta$(4, 4)$ prior distribution on the probability θ that a coin will yield a head. Then the coin is independently spun ten times. Heads appear 3 times. Calculate the posterior density of θ and give the Bayes estimate $\bar{\theta}$.

3. A popuation of measures is normal with mean θ and standard deviation $\sigma = 40$. Reconsider the example of the two physicists A and B in the notes, where A has a normal prior for θ with mean 900 and standard deviation 20, while B has a normal prior with mean 800 and standard deviation 80.

    - Assume the the experiment concerns a sample of size $n = 1$ and that the single observation is 850. Find the posteriors for A and B.
    - Then, suppose the the sample has size $n = 15$ with a sample mean $\bar{y} = 850$. Find the posteriors for A and B.
    - Finally, suppose that $n = 100$ and $\bar{y} = 850$. Compare the posteriors of A and B and the likelihood.

Solutions

1.