Exercises 3

Third lecture

1. In a random sample of size 100 from a Bernoulli with success probability θ we get 2 successes.

   - Find the 95% confidence interval for θ using the normal approximation
   $$Z = \frac{\hat{p} - p}{\sqrt{\hat{p}(1-\hat{p})/n}} \approx N(0,1).$$
   - (*) Find the approximate 95% likelihood interval using the likelihood ratio statistic.

   Comment.

2. It is suggested that the time to breakdown of an insulating fluid between electrodes at a particular voltage has an exponential distribution with parameter λ. A random sample of $n = 10$ breakdown times yields the following sample data (in minutes):
   $$41.53, 18.73, 2.99, 30.34, 12.33, 117.52, 73.02, 223.63, 4.00, 26.78$$
   We want to obtain a 95% confidence interval for the expected breakdown time $\mu = 1/\lambda$. It is known that the random variable
   $$\frac{n\bar{Y}}{\mu} \sim \text{Gamma}\,(\text{shape} = n, \text{scale} = 1)\,.$$
   The quantiles of order c of the gamma distribution can be found in R with `qgamma(c, shape = , scale = )`.

   - Try to find the exact confidence limits L and U such that $P(L < \mu < U) = 0.95$.
   - Compare with the asymptotic confidence interval using a normal approximation, i.e.,
   $$Z = \frac{\bar{Y} - \mu}{\bar{Y}/\sqrt{n}} \approx N(0,1)$$

3. The following data are relative to gestational age (weeks) and birthweight (kg) of 12 male babies.

Age	Birthweight
40	2.968
38	2.795
40	3.163
35	2.925
36	2.625
37	2.847
41	3.292
40	3.473
37	2.628
38	3.176
40	3.421
38	2.975

Assume a linear regression model

$$Y_i = \alpha + \beta z_i + \epsilon_i$$

with $\epsilon_i \sim N(0, \sigma^2)$ independent.

- Find the MLE of β and interpret.
- Find the 95% confidence limits for β.
- Test the hypothesis $H : \beta = 0$.