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Singular Value Decomposition (SVD) Image Coding

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***Abstract*—The numerical techniques of transform image coding are well known in the image bandwidth compression literature. This concise paper presents a new transform method in which the singular values and singular vectors of an image are computed and transmitted instead of transform coefficients. The singular value decomposition (SVD) method is known to be the deterministically optimal transform for energy compaction [2]. A systems implementation is hypothesized, and a variety of coding strategies is developed. Statistical properties of the SVD are discussed and a self adaptive set of experimental results is presented. Imagery compressed to 1, 1.5, and 2.5 bits per pixel with less than 1.6, 1, and 1/3 percent, respective mean-square error is displayed. Finally, additional image coding scenarios are postulated for further consideration.**

I. INTRODUCTION

A numerical technique used to diagonalize matrices in numerical analysis known as singular value decomposition (SVD) is applied to imagery in an attempt to achieve a data compression. The SVD is an algorithm developed for a variety of applications including matrix diagonalization, regression, and pseudoinversion [1], [2]. The algorithm has been suggested for image coding [3] and provides basic insight into the optimal decomposition of an image into its unique eigenimages [4]. By approaching the image coding task from a viewpoint of numerical analysis, it is possible to formulate a solution which results in deterministically best truncation errors over all other unitary transforms.

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II. TRANSFORM METHODS

Transform methods have long been used for image compression techniques by providing a domain in which data are more appropriate to memoryless quantization and coding. If we take a linear algebra viewpoint, we can observe that a discrete image is an array of nonnegative scalar entries which we may refer to as a matrix. Let such an image matrix be designated $[G]$. Without loss of generality we will assume $[G]$ is square and can be represented by

$$[G] = [A][\alpha][B]^t \tag{1}$$

where t indicates conjugate transpose and where the $[A]$ and $[B]$ matrices are assumed unitary. Solving for $[\alpha]$ we observe that

$$[\alpha] = [A]^t[G][B]. \tag{2}$$

The $[\alpha]$ matrix is seen to be the "transform" of the image matrix where $[A]^t$ transforms the columns of the image and $[B]$ transforms the rows of the image. In terms of potentially independent samples or degrees of freedom of the picture, there exist $N^2 = N \times N$ brightness values in either $[G]$ or $[\alpha]$. If $[A]$ and $[B]$ are the Fourier matrices, then $[\alpha]$ is the matrix of coefficients associated with the two-dimensional Fourier transform of an image. We notice that when $[A]$ and $[B]$ are deterministic (i.e. not computed or "adapted" from $[G]$ such as Fourier, cosine, Walsh, etc.) then the N^2 potential degrees of freedom of the image map into N^2 coefficients in the transform domain $[\alpha]$. Even for the Karhunen-Loève separable transform, where the $[A]$ and $[B]$ matrices become the eigenvector matrices of the column and row covariance matrices, respectively, they become fixed for the class of imagery characterized by such statistics. A list of traditional transform techniques is presented in Table I indicating some of the properties of the individual transform methods.

The first entry in the table is the one of interest in this concise paper and has decidedly different transform properties from the remaining. The SVD method has the unique property that the coefficient matrix $[\alpha]$ is diagonal with at most N nonzero entries. The definition of this transform is given by

$$[\alpha] = [\Lambda]^{1/2} = [U]^t[G][V] \tag{3}$$

where

$$[G][G]^t = [U][\Lambda][U]^t \tag{4a}$$

and

$$[G]^t[G] = [V][\Lambda][V]^t. \tag{4b}$$

The $[\Lambda]$ matrix is diagonal and comprises the singular values of the picture matrix $[G]$. The matrices $[U]$ and $[V]$ are the respective singular vector matrices of $[G][G]^t$ and $[G]^t[G]$, and are orthogonal due to the nonnegative definiteness of $[G][G]^t$ and $[G]^t[G]$. Because of these properties of $[U]$ and $[V]$ we can solve for the image matrix such that

$$[G] = [U][\Lambda]^{1/2}[V]^t \tag{5a}$$

or, equivalently,

TABLE I
TRANSFORM DOMAINS FOR IMAGE REPRESENTATIONS

Entry	Transform Name	Representation	Unknown Parameters to be computed)	Algorithmic Implementation (on the order of:)	Reference
1	Singular Value Decomposition	$[G][G]^t = [U][\Lambda][U]^t$ $[G]^t[G] = [V][\Lambda][V]^t$ $[\alpha] = [\Lambda^{\frac{1}{2}}] = [U]^t[G][V]$	basis vectors and singular values	N^3 N^3 $2N^3$ } computed for each image	Golub (1970) Albert (1972)
2	Karhunen-Loève Hotelling, Principal Components, Factor Analysis	$[\phi_x] = [E_x][\Lambda_x][E_x]^t$ $[\phi_y] = [E_y][\Lambda_y][E_y]^t$ $[\alpha] = [E_x]^t[G][E_y]$	transform coefficients	N^3 N^3 } computed only once $2N^3$	Wintz (1972) Huang (1971)
3	Cosine	$[\alpha] = [\cos]^t[G][\cos]$	transform coefficients	$4N^2 \log_2 2N$	Ahmed (1974)
4	Fourier	$[\alpha] = [\mathcal{F}][G][\mathcal{F}]$	transform coefficients	$2N^2 \log_2 N$ (complex)	Andrews (1970)
5	Slant	$[\alpha] = [\text{Slant}]^t[G][\text{Slant}]$	transform coefficients	$2N^2 \log_2 N$	Pratt (1974)
6	DLB	$[\alpha] = [\text{DLB}]^t[G][\text{DLB}]$	transform coefficients	$2N^2 \log_2 N$ (integer arith.)	Haralick (1974)
7	Walsh	$[\alpha] = [W][G][W]$	transform coefficients	$2N^2 \log_2 N$ (additions)	Pratt (1969)
8	Haar	$[\alpha] = [\text{HAAR}]^t[G][\text{HAAR}]$	transform coefficients	$2(N-1)$	Andrews (1970)

$$[G] = \sum_{i=1}^{R} \lambda_i^{1/2} u_i v_i^t \tag{5b}$$

where $R \leqslant N$ and represents the rank or number of nonzero singular values λ_i and u_i, v_i are column vectors of $[U]$ and $[V]$. The unique property of the SVD transform is that the potential N^2 degrees of freedom or samples in the original image now get mapped into

	$[\Lambda^{1/2}] \Rightarrow N$	degrees of freedom
	$[U] \Rightarrow (N^2 - N)/2$	degrees of freedom
	$[V] \Rightarrow (N^2 - N)/2$	degrees of freedom
totaling	N^2	degrees of freedom.

Note also that if we provide only partial sums in (5b) and if we order the singular values in monotonic decreasing order, we have

$$[G]_K = \sum_{i=1}^{K} \lambda_i^{1/2} u_i v_i^t \tag{6a}$$

$$\| [G] - [G]_K \| = \sum_{i=K+1}^{R} \lambda_i. \tag{6b}$$

It can be shown that $\|[G] - [G]_K\|$ is minimal for all K over any other expansion of the form of (1). This means that in a *deterministic* sense there is no better orthogonal transform for image energy compression than the SVD [3]. The Karhunen–Loève transform is optimal only in a statistical or *mean*-square sense while the SVD is optimal in a *least*-square sense [6]. Fig. 1 presents a pictorial representation of the singular value decomposition.

III. SYSTEMS IMPLEMENTATION

In this section we discuss the systems implementation of an image coding scheme utilizing the SVD of imagery. Traditional image transform methods usually break an image up into subblocks for ease in hardware implementations. This approach is used here because the computational expense for large SVD's is great. For $N \times N$ subblock image partitions the computational increase over $2N^2 \log_2 N$ transforms is given by $N/\log_2 N$ which for $N = 16$ causes a factor of 4 increase over Fourier and similar transforms. This computational increase occurs in the SVD transformer of Fig. 2.

Fig. 2 provides a block diagram for an SVD coding scheme. The major components at the transmitter consist of the SVD transformer, coefficient selector (denoted as "truncation" in the figure) and parallel singular-value and singular-vector coders. The large dynamic range of the singular values necesitates care in coding, and the singular-vector coding algorithm treats the first singular plane (or eigenimage) and the remaining eigenimage planes differently. Because each singular vector has length unity, the scalar entries in the singular vectors lend themselves to efficient quantization.

Using the system of Fig. 2, the number of bits necessary for transmission of a subblock depends upon the truncation (if any), the bit distribution over the singular values, and the bit distribution over the singular vectors. In addition because the singular vectors are orthonormal, one need not transmit N scalar values per vector but only $N - i$ such values for the ith vector. Orthonormality reduces the degrees of freedom on the vectors such that they can be completed on the receiver.[1]

IV. CODING STRATEGIES

There are a variety of coding strategies available to compress data in the SVD domain. Generally these techniques can be lumped into two categories of algorithms: simple and complex. A simple technique is one that does not explicitly change its structure due to image content whereas the complex algorithms do. Normally such algorithmic distinction is re-

[1] An observant reviewer points out that there may exist a sign confusion for the completed vectors, thereby necessitating the addition of an overhead sign bit to each vector.

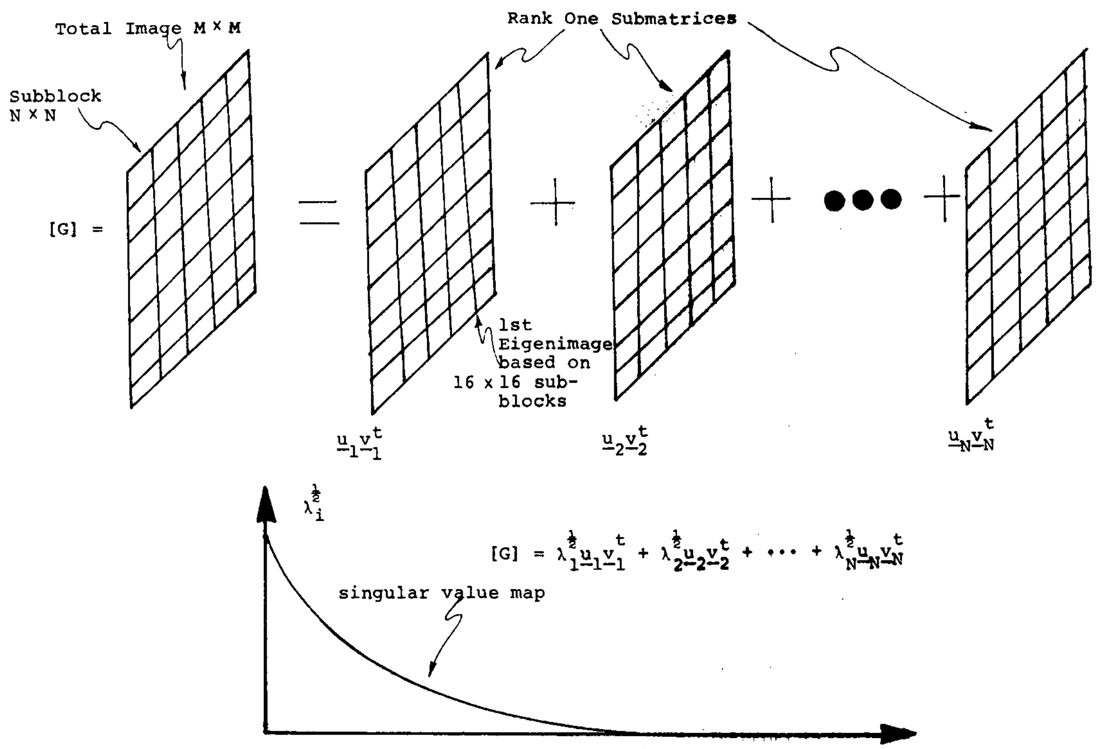

Fig. 1. Subblock SVD algorithm.

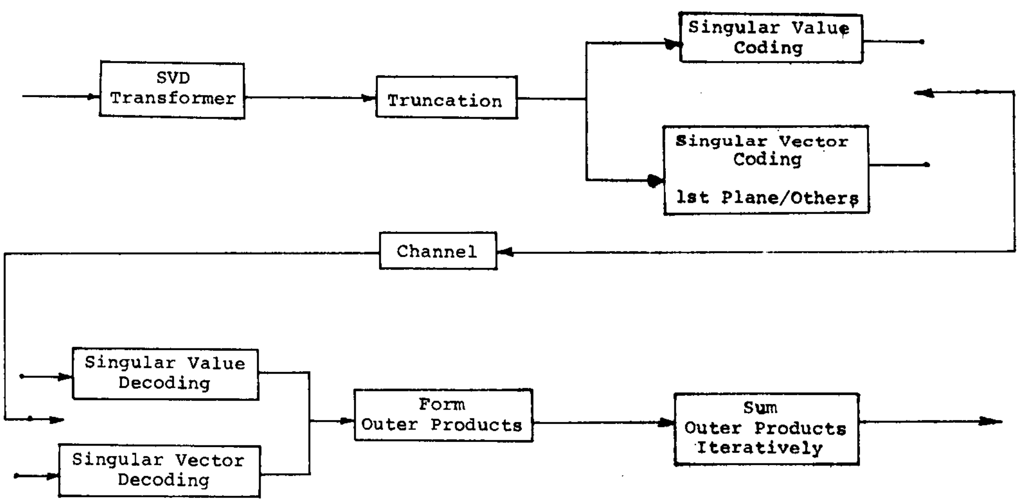

Fig. 2. SVD coding scheme.

ferred to as "nonadaptive" and "adaptive," respectively. However, the simple SVD algorithms are implicitly adaptive by their nature. This is true because by decomposing subblocks of an image into their respective singular values and vectors and by always indexing according to decreasing value, one can then use fixed coding techniques without worry about missing large energy components. In addition, no bookkeeping is necessary for this ordering as there is no preconceived notion as to the fixed or necessary ordering of the singular vectors (as there is in frequency or sequency concepts).

The simplest algorithm one might conceive of is to PCM code the singular values and singular vectors with a linear quantizer and fixed bit assignment. However, upon investigating the distribution of scalar entries that comprise the singular vectors, a max quantizer becomes more appropriate [5]. The importance (in a least-squares sense) of individual singular values warrants variable bit assignment. Because the singular vectors are really one-dimensional sampled waveforms, one might guess that there exists a certain amount of correlation between adjacent scalar entries. Thus a DPCM coding of the scalar components forming the vectors might be in order. For this case an exponential quantizer with variable bit assignments becomes appropriate.

The complex coding strategies are motivated by a general viewpoint that if a subblock of an image is in a region with similar such subblocks (i.e., common textures, means, variances, etc. in image segmentation parlance) then the SVD of one subblock may be quite similar to that of a neighboring region. Thus one might consider describing classes of SVD's such that a different coding scheme resulted depending on whether the subblock represented "busy," "low contrast," "high contrast," etc. regions of an image. (This approach was used with Markov modeling of imagery for traditional transform domain coding [6].) Another intriguing complex algorithm would be to envision using the singular vectors obtained from one subblock of a neighborhood as the assumed singular vectors of other neighboring subblocks and then transmitting only the new singular values of the new blocks. This may

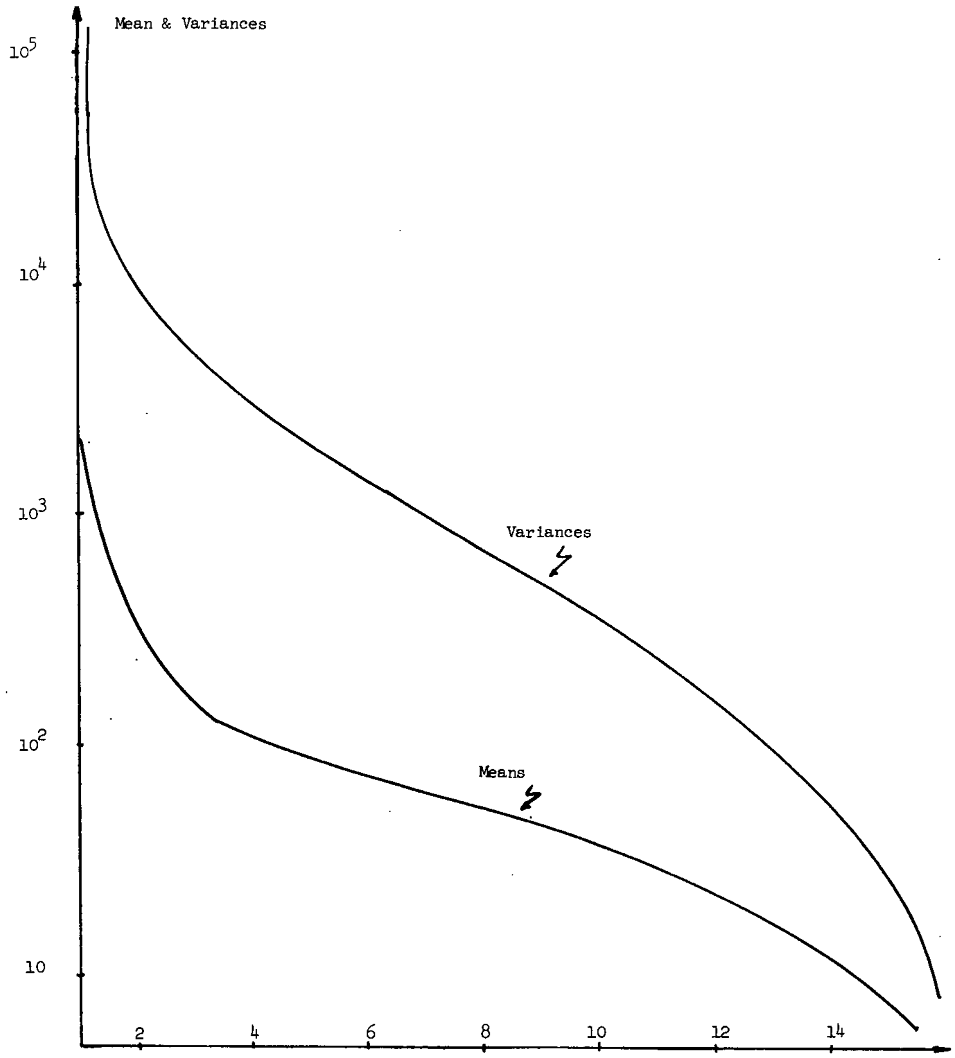

Fig. 3. Singular value mean and variance for "baboon."

have considerable potential in frame to frame situations where adjacent frames in time have a large degree of similarity in singular value decomposition.

V. STATISTICAL PROPERTIES OF THE SVD DOMAIN

In order to develop efficient quantizers and coders for the SVD domain, a test image (512 X 512) was broken into 16 X 16 subblocks for comparison with [6], and data were gathered over each subblock of the entire frame. For 16 X 16 subblocks, one obtains 16 monotonically ordered singular values whose means and variances are presented in Fig. 3. The exceedingly large dynamic range of between 4 and 5 orders of magnitude indicates the need for variable bit coding as a function of the singular value index.

The statistics describing the singular vectors are much better behaved. Figs. 4 and 5 present two specific singular vectors from a particular subblock as an illustration of the shape of these parameters. The singular vectors tend to be well behaved in their range and tend to have an increasing number of zero crossings as a function of increasing index. In fact it is known that the first singular vector always has no sign changes when the subblock is nonnegative (as it always is for imagery) [7]. Thus the lower indexed vectors tend to have a great deal of adjacent sample correlation.

Because the first vectors for $[U]$ and $[V]$ are guaranteed to have no sign changes (similar to the dc vectors of Fourier, Walsh, cosine, etc. transforms), these vectors have separate statistics from the remaining set. The mean of the components of the first singular vector over all subblocks in the test image is $+0.250$ with a very tight variance of 10^{-3}. Naturally a given first singular vector will not, in fact, be a constant of $+\frac{1}{4}$ (the appropriate normalized value to guarantee orthonormality) but was experimentally determined to be very close to Gaussian (normal) with parameters $N(0.25, 10^{-3})$.

The remaining eigenvectors are also quite well behaved with entries being normally distributed with $N(0, 10^{-1})$ which is more or less an expected result of the fact that they are of length one. Because of the difference in the statistics of the first singular vector and those of the remaining singular vectors, they are coded separately as indicated in the block diagram of Fig. 2.

VI. EXPERIMENTAL RESULTS

Two images are used for experimental purposes in this paper. They are referred to as the "baboon" and "site" and their SVD structure is revealed in Figs. 6 and 7. In each of these figures the respective images are broken into 16 X 16 subblocks and each subblock is decomposed into its 16 sing-

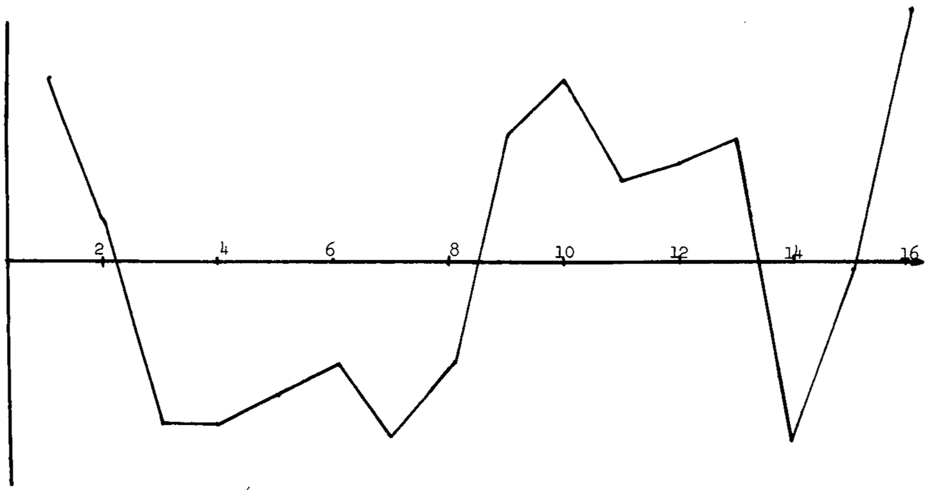

Fig. 4. Fifth singular vector.

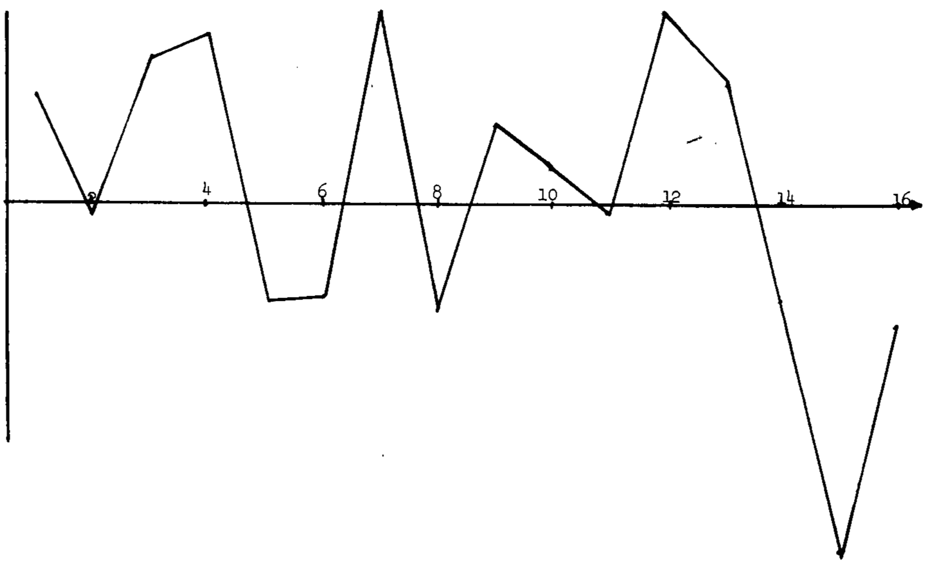

Fig. 5. Fifteenth singular vector.

ular values and associated 32 singular vectors. The first, second, third, and fourth eigenimages obtained from the corresponding singular vectors are displayed in the figures. The first plane has no zero crossings and consequently the display of negative data is not necessary. In the three remaining planes a linear display is presented with negative values being dark and positive values being light.

A complete SVD coding system has been simulated according to the block diagram for Fig. 2. Two coding strategies were investigated. Both used a linear PCM quantizer with variable bit assignment on the singular values and one used a linear while the other used a Max quantizer [5] with variable bit assignment on the PCM values of the entries in the singular vectors. An optimization routine in terms of mean-square error measured the best bit assignment and Fig. 8 provides some performance curves developed during the optimization process. The two lower curves indicate the truncation effects as the number of singular values are increased. The uppermost curve illustrates the mean square error using a linear quantizer on the singular values. The Max quantizer curve indicates about a 0.20 percent mean-square error improvement over the linear curve and is only about 0.20 percent worse (or introduces 0.20 percent more mean-square error) than the truncated but uncoded curves. Pictorial results, from which the upper two curves are derived, are presented in Fig. 9. Here the percentage mean-square errors and bit rates per pixel are listed under the respective coded images for both linear and Max quantization on the singular vectors. This particular image (the "baboon") has the distinct disadvantage for coding of containing considerable high frequency as well as low frequency subblocks. Thus a simple algorithm, as used here, will contain relatively large errors, as indicated by the truncation (no coding) curve of Fig. 8.

The "site" image has much less truncation error as indicated from Fig. 8. Consequently, it would be expected that such an image would lend itself to better results with the above algorithm. Another point is developed using this second test image. Specifically, it is advantageous that the transmitter of an image coding algorithm not have to compute new statistics for each image to be transmitted. Consequently, the "site" image has been coded with "baboon" statistics. This means that the quantizers and bit assignments defined by the "baboon" are forced on the "site." The remaining curve in Fig. 8 demonstrates the results of this experiment. Here the Max quantizer is about 0.50 percent worse than simple truncation (compared to 0.20 percent for the "baboon") because of the difference in statistics of the "site" and "baboon." The pictorial results associated with this curve are presented in

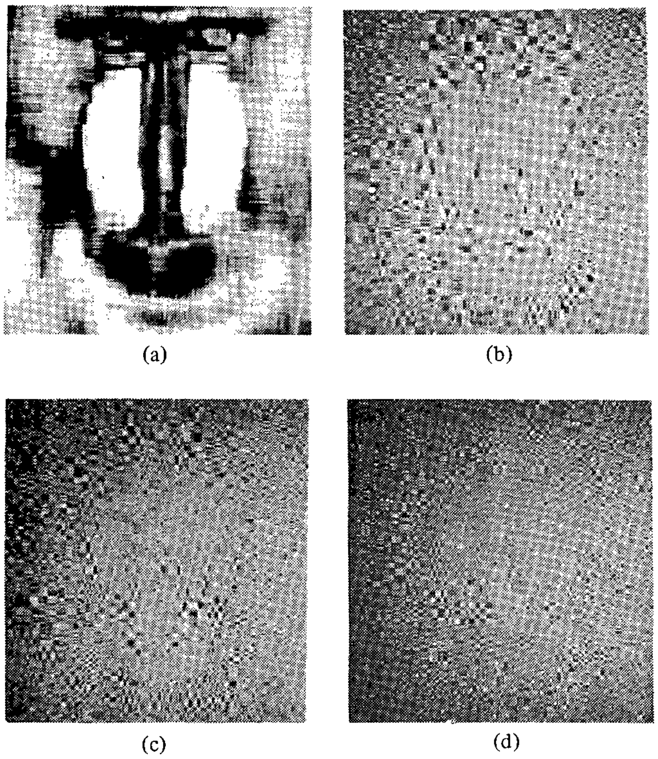

Fig. 6. "Baboon" image with SVD on 16 × 16 subblocks. (a) 1st eigenimage; (b) 2nd eigenimage; (c) 3rd eigenimage; and (d) 4th eigenimage.

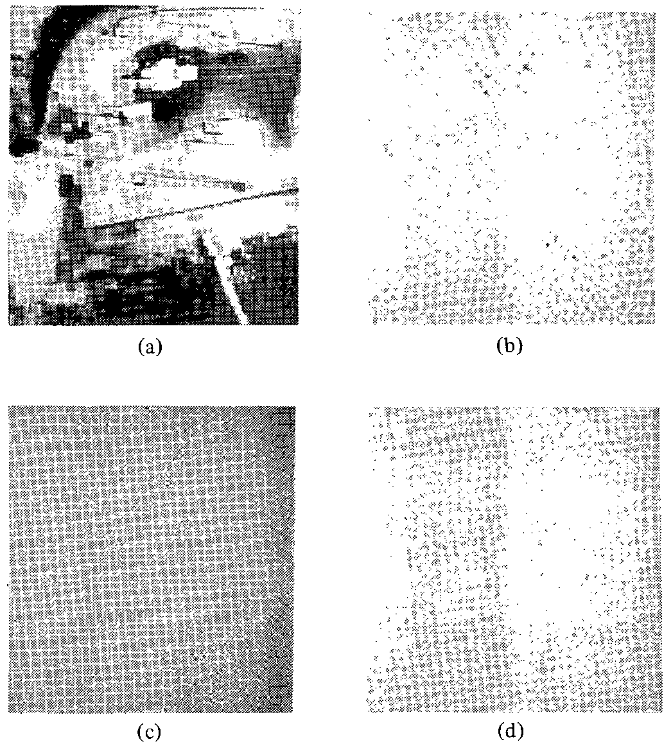

Fig. 7. "Site" image with SVD on 16 × 16 subblocks. (a) 1st eigenimage; (b) 2nd eigenimage; (c) 3rd eigenimage; and (d) 4th eigenimage.

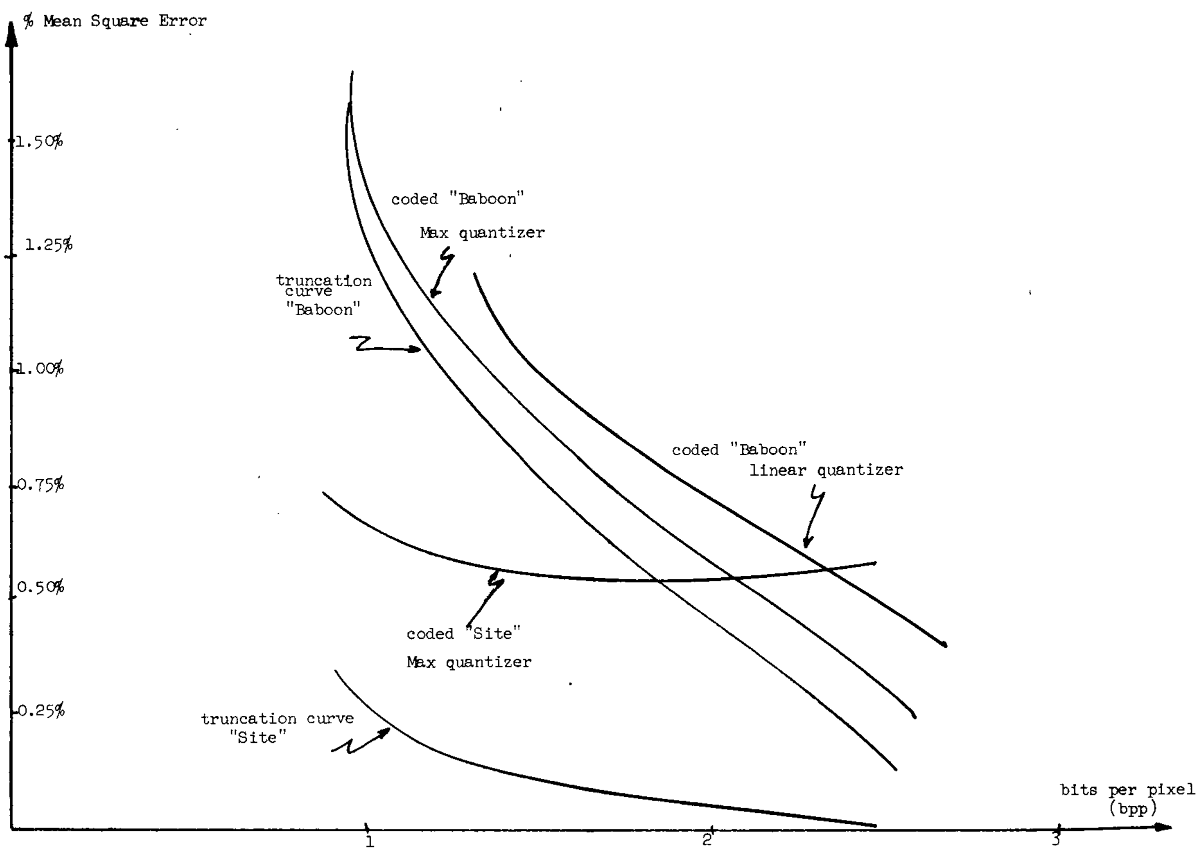

Fig. 8. Percentage mean-square error versus bits per pixel.

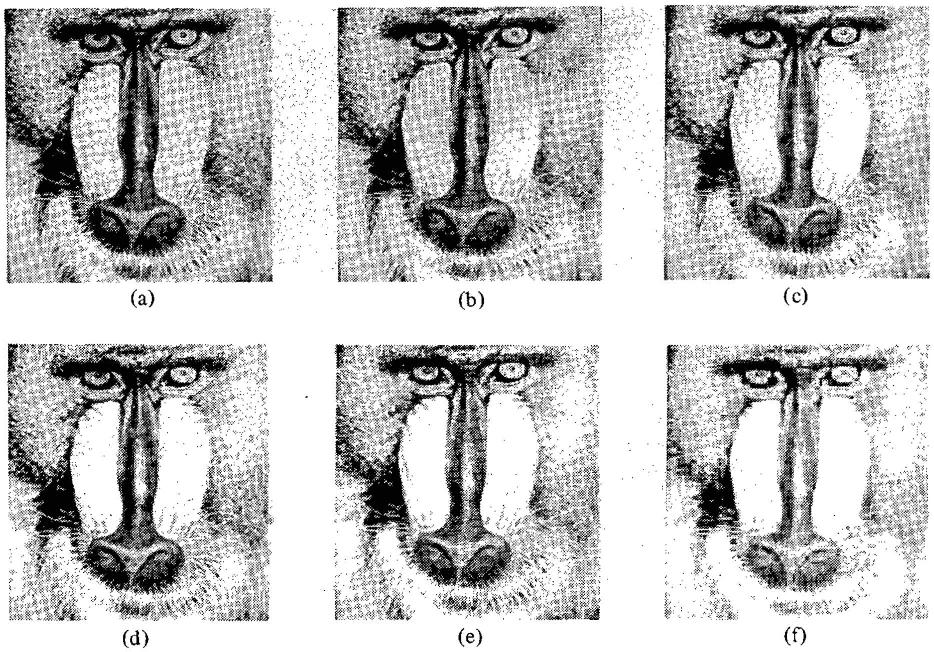

Fig. 9. "Baboon" image with SVD coding using "Baboon" statistics. (a) Linear quantizer: 2.67 bpp (0.3988 percent MSE). (b) Linear quantizer: 1.67 bpp (0.909 percent MSE). (c) Linear quantizer: 1.36 bpp (1.20 percent MSE). (d) Max quantizer: 2.50 bpp (0.3097 percent MSE). (e) Max quantizer: 1.57 bpp (0.836 percent MSE). (f) Max quantizer: 0.953 bpp (1.64 percent MSE).

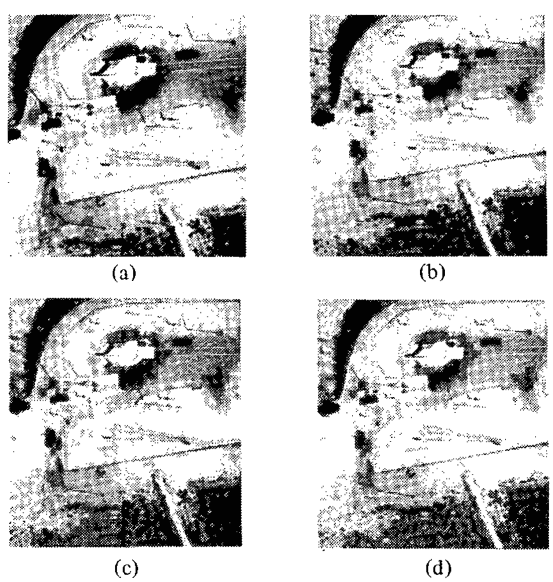

Fig. 10. "Site" image with SVD coding using "baboon" statistics. (a) Original. (b) 0.953 bpp (0.717 percent MSE). (c) 1.57 bpp (0.567 percent MSE). (d) 2.50 bpp (0.523 percent MSE).

Fig. 10. The quantization and coding details are listed in Table II for the pictures in Figs. 9 and 10.

VII. CONCLUSIONS

This concise paper has attempted to present the novel use of SVD's as a means for image data compression. The SVD techniques are introduced in the context of traditional transform methods for image coding. A systems implementation is developed and various critical parameters needed for coding are identified. A variety of coding strategies are presented for use in the SVD domain of an image. The statistical properties of the key parameters in the SVD domain of imagery are presented to help develop the optimal quantizers and coders therein. Finally, experimental results are provided on two test scenes for visual as well as mean-square error comparisons.

In concluding this concise paper it is important to emphasize the following. First, the work is incomplete and it is premature to base any conclusions on the viability of SVD coding in competition with other existing techniques. It is

TABLE II
QUANTIZER AND BIT ASSIGNMENTS

Figure	Truncation No. of Eigen-Images Retained	bpp	mse	Quantizer	Bit Assignments	Singular Values Singular Vectors
9a	8	2.67	.3988%	linear linear	20,8,8,8,7,7,7,7 5,3,3,3,3,3,3,3	Singular Values Singular Vectors
9b	4	1.67	.909%	linear linear	20,8,8,8 5,3,3,3	Singular Values Singular Vectors
9c	3	1.36	1.20%	linear linear	20,8,8 5,3,3	Singular Values Singular Vectors
9d	8	2.50	.3097%	linear max	6,4,4,4,3,3,3,3 5,3,3,3,3,3,3,3	Singular Values Singular Vectors
9e	4	1.57	.836%	linear max	6,4,4,4 5,3,3,3	Singular Values Singular Vectors
9f	2	.953	1.64%	linear max	6,4 5,3	Singular Values Singular Vectors
10b	2	.953	.717%	linear max	6,4, 5,3,	Singular Values Singular Vectors
10c	4	1.57	.523%	linear max	6,4,4,4 5,3,3,3	Singular Values Singular Vectors
10d	8	2.50	.567%	linear max	6,4,4,4,3,3,3,3 5,3,3,3,3,3,3,3	Singular Values Singular Vectors

fair to say that if as much effort is put into investigating the potential for SVD coding as has been put into traditional transform methods, then considerable improvement over the results presented here can be expected. However, algorithmic implementation might become quite complex. Consequently, only time and future study will tell whether SVD coding becomes a practical reality.

ACKNOWLEDGMENT

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Target Tracking over Fading Channels

F. S. CHANG AND M. D. SRINATH

***Abstract*—The problem of tracking a target in a multitarget environment when the observations are received over a fading channel is considered. The optimal Bayes solution to the tracking problem in such cases involves growing memory and hence is not feasible. A particularly effective suboptimal scheme uses a probabilistic judgment at each stage of the observations to overcome this problem. This concise paper presents an evaluation of the scheme in terms of mean-square error performance when the observations are received over fading channels.**

I. INTRODUCTION

Recently there has been an increasing interest in the synthesis of communication systems for space applications. The communication channel modifies the transmitted signal so that the desired message may arrive at the receiver terminal distorted, attenuated, and delayed. The message is usually observed in the presence of additive noise. The distortion introduced by the channel can be characterized by multiplicative

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