Image compression using SVD

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Abstract

It is well known that the images, often used in variety of computer applications, are difficult to store and transmit. One possible solution to overcome this problem is to use a data compression technique where an image is viewed as a matrix and then the operations are performed on the matrix. Image compression is achieved by using Singular Value Decomposition (SVD) technique on the image matrix. The advantage of using the SVD is the property of energy compaction and its ability to adapt to the local statistical variations of an image. Further, the SVD can be performed on any arbitrary, square, reversible and non reversible matrix of m x n size. In this paper, SVD is utilized to compress and reduce the storage space of an image. In addition, the paper investigates the effect of rank in SVD decomposition to measure the quality in terms of MSE and PSNR.

1. INTRODUCTION:

Images, used in Image Processing application, require big storage space and a considerable bandwidth for transmission. One of the possible solutions to this problem is data compression whose main goal is to reduce the quantity of data used to represent a digitized image. The data compression techniques always eliminate the redundant data. The process of compressing (or decompression) images can be classified into two categories, lossless compression and lossy compression. Lossless methods produce exact copy of the original data and have a limit of reduction in terms of the entropy. Lossy techniques usually work in terms of the incapacity of the human visual system to detect small details and variations in images. The elimination of those details reduces the amount of space required to store the image. Under certain circumstances, lossy compression can be used to achieve the requirements of storage space, although the image quality is reduced by the compression process. A gray scale image can be seen like a matrix of m x n size. A digital color image has the size of m x n x 3. For a CIF image that has a size of 352 X 288 if each pixel is represented in 8 bits for R, G and B components, the storage requirement is 2.4 MB. This establishes the need for compression.

The paper is organized as follows. Section 2 presents the singular value decomposition and section 3 discusses the implementation details. The test results are given in section 4 while section 5 indicates the conclusions.

2. SINGULAR VALUE DECOMPOSITION:

The method refactors matrix A in three new matrices U, S, and V in such way that, U and V are orthogonal matrices and S is a diagonal matrix

$$A = U\,S\,V^T$$

$$A = [Ur \quad Um-r] \begin{bmatrix} \in & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Vr^T \\ V^T{}_{n-r} \end{bmatrix}$$

$$A = U_r\, S\, V_r^T = A_r$$

3. IMPLEMENTATION DETAILS

Three different sets of image of size 291 x 240, 214 x 244 and 256 x 256 are considered for the analysis. Validation of the proposed project is done using MATLAB version 7.0 at PESIT R & D

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Centre of Telecommunication Engineering department. For each image S matrix is found and graph is plotted for different ranks. For each image reconstructed matrix is found from U, S and V matrix and reconstructed image is displayed. In all the cases, differences between the original and the reconstructed pixel intensities are plotted. In each case, the MSE and PSNR are evaluated for different ranks. The compression ratio is calculated and plotted as a function of rank of the image matrix. A graph is drawn for different ranks versus MSE and PSNR.

4. TEST RESULTS:

To demonstrate the effect of SVD on compression three set of sample images are taken. The sizes of the images are of approximately same. The sample images are as shown in Fig 1. The exhaustive surveys are conducted for analyzing the scheme. A sample of the result is reported below.

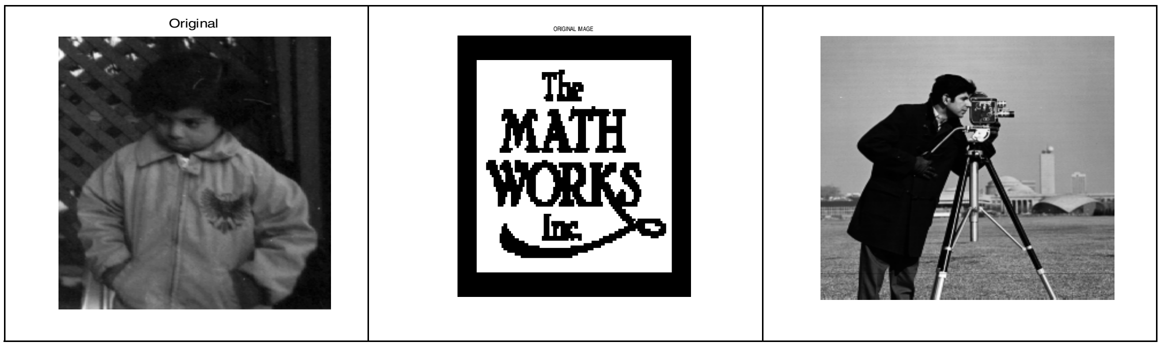


Fig. 1 Input Images that are considered correspond to boy, text and camera man

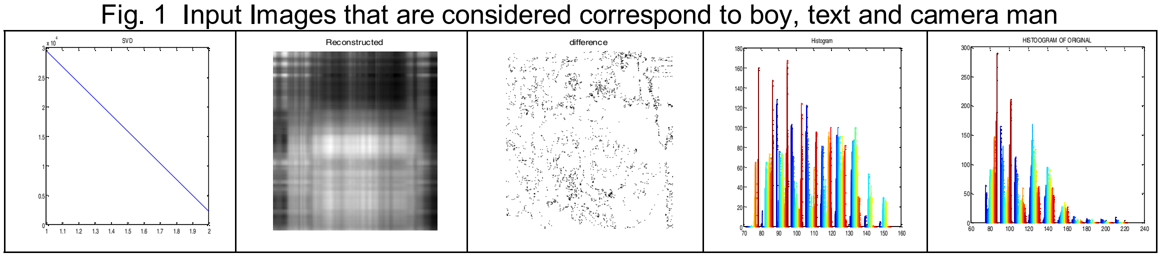


Fig. 1a) Energy compaction, reconstructed image, difference between original image and reconstructed image, histograms of reconstructed and original for rank 2.

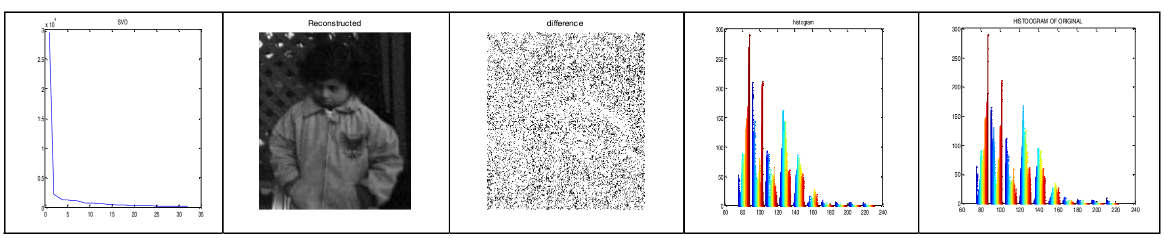


Fig. 1b) Energy compaction, reconstructed image, difference between original image and reconstructed image, histograms of reconstructed and original for rank 32.

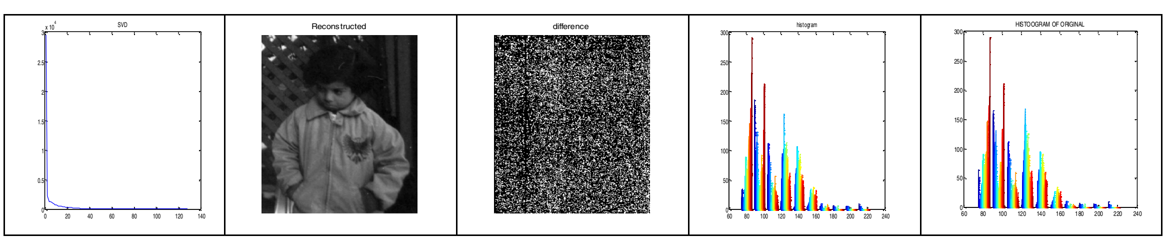
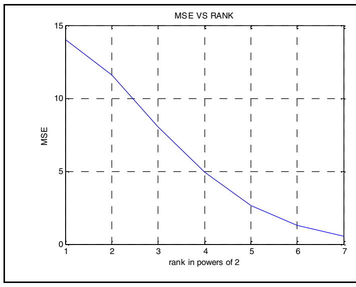
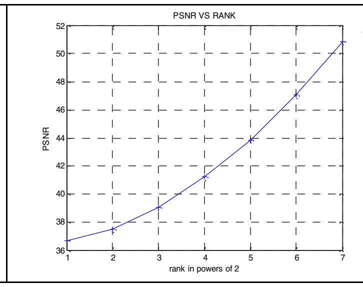


Fig. 1c) Energy compaction, reconstructed image, difference between original image and reconstructed image, histograms of reconstructed and original for rank 128.





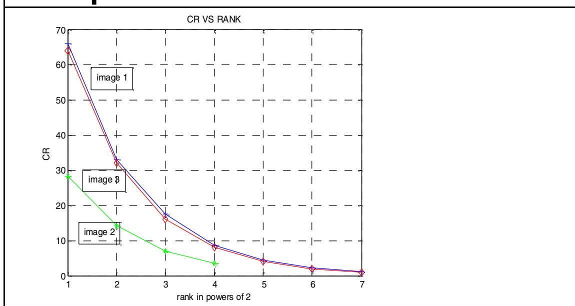
Rank	MSE	PSNR	CR
2	14.01	36.66	66:1
4	11.61	37.48	33:1
8	8.035	39.08	16.5:1
16	4.931	41.20	8.25:1
32	2.674	43.85	4.12:1
64	1.2816	47.05	2.06:1
128	0.5327	50.86	1.03:1

Fig. 1d) Graph of MSE and PSNR versus rank

Table 1: Variation of PSNR, MSE and CR versus rank

GRAPHS:

Compression ratio versus rank



PSNR versus Compression ratio

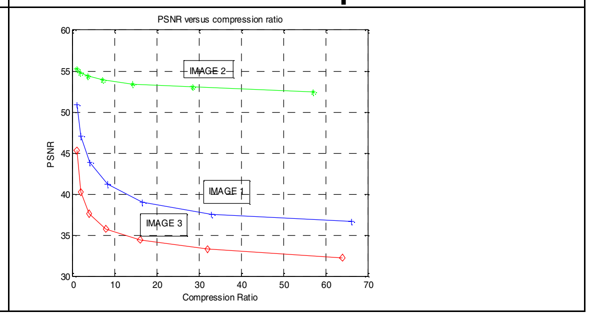


Fig. 4: Variation of compression ratio versus variation in rank and PSNR against compression ratio.

5. CONCLUSIONS

The paper presented the process of image compression using SVD. It is observed that the matrix coefficients move towards Y-axis as the rank of the image matrix increases which indicates that the maximum energy is concentrated with only first few coefficients. It is also noticed that with the increase in the rank of the reconstructed image matrix, the number of entries in the matrix would increase which in turn improve the perceptual picture quality correspondingly. The perceptual quality of the textual image is better compared to the picture image even for the smaller rank. It can also be observed that SVD technique provides much better quality for text data even at lower ranks compared to image objects. More compression ratio can be achieved for smaller ranks. For text data perceptual quality of the picture is almost same as original for smaller rank and there by increase the compression ratio. It can be observed from Fig 4 that PSNR falls as compression ratio increases.

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