 Ref No: Ex/CSE/PC/B/T/225/2022

Bachelor in Computer Science and Engineering
2^{nd} Year, 2^{nd} Semester Exam 2022
Graph Theory and Combinatorics

Full Marks : 100 Time : 3 Hrs

Write answers to the point. Make and state all the assumptions(wherever made).
ALL PARTS OF A QUESTION SHOULD BE ANSWERED TOGETHER.
WRITE YOUR NAME, CLASS ROLL NUMBER, EXAMINATION ROLL NUMBER AND REGISTRATION NUMBER ON THE TOP SHEET OF YOUR ANSWER SCRIPT.

Section A Answer all questions [$10 \times 2.5 = 25$]

(1) Prove : "If a graph has exactly two vertices of odd degree, there must be a path joining these two vertices."

(2) Prove that the largest number of edges in a planar graph is $3n - 6$, where n is the number of vertices in, the graph

(3) Show that every simple graph has two vertices of the same degree.

(4) The degree of the vertices of a certain tree T of order 13 are 1, 2 and 5. If T has exactly 3 vertices of deree 2, how many terminals it has ?

(5) What are cut-set and cut-vertices? Why are they useful?

(6) How many solutions are there of $x + y + z = 17$ in **positive integers**

(7) In a race with 30 runners where 8 trophies will be given to the top 8 runners (the trophies are distinct: first place, second place, etc), how many ways can this be done?

(8) How many ways can you do the above problem if a certain person, Ram, must be one of the top 3 winners?

(9) We have k different boxes and r different objects. We want to distribute the objects into the boxes such that at no box is empty. In how many ways can this be done?

(10) There are five people of different height. In how many ways can they stand in a line, so there is no 3 consecutive people with increasing height?

Section B Answer any Five(5) Questions. [$5 \times 6 = 30$]

You can keep your answer in the "Combinatorial Form"

(1) Solve the following difference equation for the given initial conditions

$$a_n = -4a_{n-1} - 4a_{n-2}, n \geq 2, a_0 = 4, a_1 = 8$$

(2) Find a recurrence relation and initial condition for the number of n-bit strings that do not contain two consecutive 1s. How many such 6-bit strings are there.

(3) Suppose we are to select k balls of n different sizes and we must select at least one ball of each size. Use generating functions to find the number of ways to select the balls.

(4) In how many ways can we distribute 10 identical mabls among 4 children if each child must get at least 2 marbles and not more than 3 marbles ?

(5) Find the numbe of solutions of the equation $a+b+c = 7$ where a, b, c are non-negative integers satisfying the conditions: a must be one of $0, 1, 3$, $1 \leq b \leq 3$ and c must be 4 or 6.

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(6) How many non-negative integer solutions are there of the equation $x + y + z = 20$ satisfying the conditions $x \leq 10, y \leq 5, z \leq 15$? **Use the principle of Inclusion-Exclusion to solve the problem.**

Section C Answer any Five(5) Questions [$5 \times 6 = 30$]

(1) How can one ensure that Kruskal's Algorithm for finding the minimum spanning tree gives us the optimal solution.

(2) Let $G(V, E)$ be a simple graph with at most $2n$ vertices. If the degree of each vertex is at least n, then show that the graph is connected.

(3) Let $G(V, E)$ be a connected graph with at least two vertices. If the number of edges in $G(V, E)$ is less than the number of vertices, then prove that $G(V, E)$ has a vertex of degree one.

(4) Given the adjacency matrix of a connected graph, how do you determine the diameter of the graph

(5) Let $G(V, E, \gamma)$ be a graph, $V = \{1, 2, \ldots, 10\}$, $E = e_1, e_2 \ldots, e_n$ and $\gamma(E) = \{\{x, y\} | x, y \in V, x \neq y$ and x divides y or y divides $x\}$. Find n. Draw this graph. Is this a simple graph?

(6) In a directed graph, prove that the following are equal

(i) the sum of indegrees of the vertices

(ii) the sum of outdegrees of the vertices

(iii) the number of arcs

Section D Answer any one question [15]

(1) Suppose that n objects are divided into classes $c_1, c_2, \ldots, c_k$, with $c_1 \leq c_2 \ldots \leq c_k$.

(i) Determine the number of distinct arrangements of r objects chosen from these n, when $r \leq c_1$.

(ii) Determine the number of distinct arrangements of r objects, when $r \leq n$.

(2) If G_1 is a complete r-partite graph and G_2 is a tree, show that $G_1 \times G_2$ does not contain an odd induced cycle of length of at least 5.

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