

# Natural GaLore: A Memory Efficient Approach for LLM Training and Finetuning

Arijit Das

October 11, 2024

# Outline

- 1 Introduction
- 2 Parameter Optimization
  - Distributed Training Techniques
  - Approximation Techniques
- 3 Natural GaLore
- 4 Benchmarking Experiments
- 5 Application: Fine-Tuning TinyAgents
- 6 Conclusion

# Introduction

# Advanced Agentic Systems (AAS)

- Since GPT-3 (Brown et al., 2020), the concept of in-context learning, has made LLMs extremely flexible and powerful.
- This has opened the door to build AAS, which can respond to a wide variety of queries and solving complex problems, which may require a combination of: knowledge, reasoning, compute and action.
- However, building reliable AAS for practical applications remains challenging.

# Optimizing AAS

- Solving such a complex problem requires systemically breaking it down into modules.
- Each module or agent is a call to an LLM which performs a subtask, requiring its own set of prompts and parameters.
- These modules can then be combined together as a directed graph, which solves the problem.
- Building reliable and robust AAS hence requires good prompt and parameter optimization techniques.

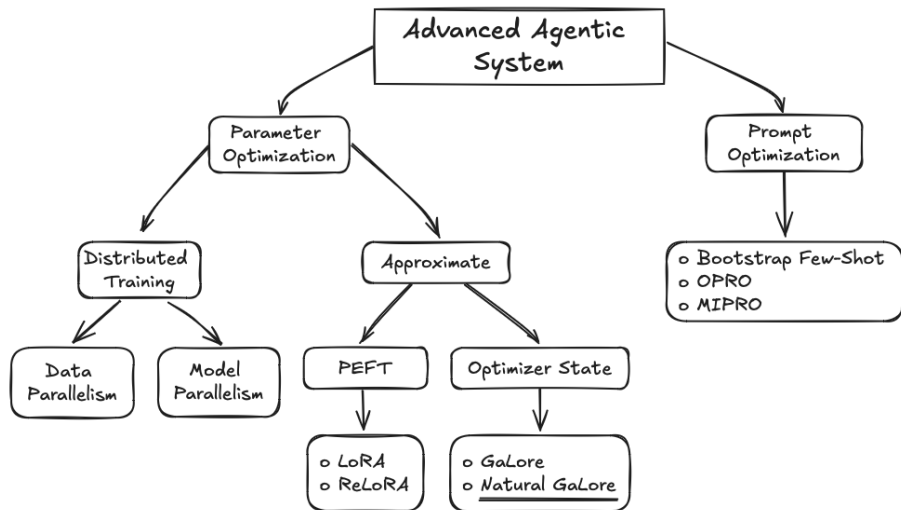
# Prompt Optimization

- Prompt optimization is the process of finding the optimal prompt for a given LLM to maximize performance on a specific task.
  - ▶ **Prompt Proposal:** The challenge comes from the fact that they are inherently fuzzy and have an exponentially large solution space.
  - ▶ **Credit Assignment:** Another issue is to find efficient ways of inferring how each prompt variable contributes to performance.
- Algorithms like OPRO (Yang et al., 2024) and MIPRO (Opsahl-Ong et al., 2024), approach this problem by building efficient prompt proposal sampling, and credit assignment mechanisms following ideas from Bayesian optimization.

# Parameter Optimization

- LLM training requires parameter optimization at three steps:
  - ▶ Pre-training: The model is optimized for the Next Token Prediction task on vast amounts of raw natural language text data.
  - ▶ Supervised Fine-tuning: Using high quality “Instruction Data”, the model is transformed from essentially an autocomplete model, into one which can perform in context learning.
  - ▶ Preference optimization: Here responses are sampled, ranked according to user preferences and then the model is updated to prefer those.

# AAS Optimization



- In this work, I focus on the parameter optimization problem



# Parameter Optimization

# Next Token Prediction

- LLMs are trained to predict the next token based on previously observed tokens (causal prediction).
- Given tokens  $x_{<t} = (x_1, x_2, \dots, x_{t-1})$  the model is trained to maximize the probability of the next token  $x_t$

$$\text{Prob}_{\theta}(x) = \prod_{t=1}^T \text{Prob}_{\theta}(x_t \mid x_{<t}) \quad (1)$$

# Objective: Negative Log-Likelihood (NLL)

- The training objective is to minimize the Negative Log-Likelihood (NLL):

$$\Phi(\theta) = - \sum_{t=1}^T \log \text{Prob}_{\theta}(x_t \mid x_{<t}) \quad (2)$$

- Penalizes low probability assignments to correct tokens.
- However is a high-dimensional, non-convex optimization problem.

# Adam Optimizer

- Adam (Kingma & Ba, 2014) is an iterative stochastic gradient descent algorithm with adaptive learning rates and momentum

$$\theta_{k+1} = \theta_k - \eta \mathbf{u}_k^*$$

where  $\mathbf{g}_k = \nabla_{\theta} \Phi(\theta_k)$  and  $\eta$  is the learning rate.

- The optimal update direction is determined using the momentum term  $\mathbf{m}_k \in \mathbb{R}^{r \times m}$  and the second moment estimate  $\mathbf{v}_k \in \mathbb{R}^{r \times m}$ . With all operations being elementwise, the update direction becomes:

$$\mathbf{m}_k = \beta_1 \mathbf{m}_{k-1} + (1 - \beta_1) \mathbf{g}_k \quad (3)$$

$$\mathbf{v}_k = \beta_2 \mathbf{v}_{k-1} + (1 - \beta_2) \mathbf{g}_k^2 \quad (4)$$

$$\mathbf{u}_k^* = \mathbf{m}_k / \sqrt{\mathbf{v}_k + \epsilon} \quad (5)$$

# Memory Challenges

- Training and fine-tuning LLMs demand enormous computational resources and are highly memory-intensive.
- Memory requirements:
  - ▶ Stem from storing billions of parameters, gradients, and optimizer states.
  - ▶ For example for pre-training a Llama-7B model using Adam, requires 72GB of memory: 14GB for parameters and gradients each, 42GB for optimizer states and 2GB for activations.
  - ▶ This limits the ability to train large models on hardware with limited memory capacity.
  - ▶ Increases training costs and environmental impact.
- **Objective:** Develop a memory-efficient approach for training and fine-tuning LLMs without sacrificing performance.

# Distributed Training Techniques

# Distributed Training Techniques

## • Data Parallelism

- ▶ DDP combines data parallelism with efficient gradient synchronization.
- ▶ **Pros:** Efficient gradient updates, good scalability.
- ▶ **Cons:** Memory bottlenecks persist when model size exceeds single GPU capacity.

## • Model Parallelism

- ▶ Partitions model across multiple devices.
- ▶ Techniques: Pipeline parallelism (Huang et al., 2019), Tensor parallelism (Shoeybi et al., 2019), Fully Sharded Data Parallel (FSDP) (Zhao et al., 2020).
- ▶ **Pros:** Allows training of models larger than a single GPU.
- ▶ **Cons:** Communication overhead, complex implementation.

# Distributed Training Techniques

- Data and Model Parallelism can be augmented with further techniques to reduce memory usage.
  - ▶ **Gradient Checkpointing** (Chen et al., 2016)
    - ★ Stores subset of activations during forward pass.
    - ★ Recomputes activations during backward pass.
    - ★ **Pros:** Reduces memory usage.
    - ★ **Cons:** Increases computational overhead.
  - ▶ **Memory Offloading**
    - ★ Moves optimizer states and gradients to CPU memory.
    - ★ Techniques: ZeRO-Offload (Rajbhandari et al., 2020).
    - ★ **Pros:** Significant memory reduction.
    - ★ **Cons:** Increased system complexity, operational costs.



# Approximation Techniques

# Parameter-Efficient Fine-Tuning (PEFT)

- **LoRA (Low-Rank Adaptation)** (Hu et al., 2022)

- ▶ Reparameterizes weight matrices using low-rank adapters

$$W = W_0 + BA, \quad B \in \mathbb{R}^{n \times r}, A \in \mathbb{R}^{r \times m} \quad (6)$$

- ▶ **Pros:** Reduces trainable parameters, lowers memory usage
- ▶ **Cons:** May not match full fine-tuning performance, especially on complex tasks (Xia et al., 2024).

- **ReLoRA** (Lialin & Schatz, 2023)

- ▶ Extends LoRA for pre-training.
- ▶ Periodically updates frozen weights using learned adapters.
- ▶ **Pros:** Enables continual learning with lower memory.
- ▶ **Cons:** Requires initial full-rank training phase.

# GaLore: Gradient Low-Rank Approximation

- Exploits low-rank structure of gradients to approximate optimizer states (Zhao et al., 2024).
- The gradients  $\mathbf{g} \in \mathbb{R}^{n \times m}$  are projected onto a low-rank form using SVD:  $\mathbf{g} = \mathbf{P}\Sigma\mathbf{Q}^T$

$$\mathbf{g}^{\text{low-rank}} = \mathbf{P}^T \mathbf{g}, \quad \mathbf{P} \in \mathbb{R}^{n \times r} \quad (7)$$

- Then after applying Adam to  $\mathbf{g}^{\text{low-rank}}$  the full parameter update is reconstructed by applying  $\mathbf{P}$ .
- **Pros:**
  - ▶ Significant memory reduction (up to 30% compared to LoRA).
  - ▶ Full-parameter updates, maintaining model capacity.
- **Cons:**
  - ▶ Performance may not match full optimizer state methods.
  - ▶ Low-rank approximation may not capture full optimization dynamics.

# Natural GaLore

# Natural GaLore: Proposed Approach

- **Goal:** Accelerating convergence of GaLore to bridge the gap with full state optimizers like Adam and AdamW.
- **Key Idea:** Incorporate second-order information into the low rank projected gradients using the Fisher Information Matrix (FIM) to efficiently estimate natural gradients.

# Low-Rank Gradient Descent in GaLore

- GaLore restricts updates to an affine subspace defined by its principle components  $\mathbf{P}_k \in \mathbb{R}^{n \times r}$ :

$$\mathbf{u}_k \in \theta_k + \text{Range}(\mathbf{P}_k)$$

- Then the second-order Taylor series expansion of the loss function along this subspace is:

$$\Phi(\theta_k + \mathbf{P}_k \mathbf{u}_k) \approx \Phi(\theta_k) + \mathbf{g}_k^{\text{low-rank}T} \mathbf{u}_k + \frac{1}{2} \mathbf{u}_k^T \mathbf{H}_k \mathbf{u}_k$$

where  $\mathbf{g}_k^{\text{low-rank}} = \mathbf{P}_k^T \nabla_{\theta} \Phi(\theta_k)$  is the projected gradient and  $\mathbf{H}_k = \mathbf{P}_k^T \nabla_{\theta}^2 \Phi(\theta_k) \mathbf{P}_k$  is the local Hessian matrix.

# Fisher Information Matrix (FIM)

- **Key idea:** When the loss function can be written in terms of the KL-divergence, the Fisher Information Matrix (FIM) approximates the Hessian matrix:

$$\mathbf{F}_k = \mathbb{E}_{x \sim p_{\text{data}}} [\mathbf{H}_k]$$

- It captures the curvature of the loss landscape, enabling more efficient optimization.
- Natural gradient descent is Fisher efficient and reduces variance in gradient updates (Amari, 1998):

$$\text{Var}[\theta_k] = \frac{1}{mk} \mathbf{F}_k^{-1}(\theta_k^*) + \mathcal{O}\left(\frac{1}{k^2}\right)$$

- Smaller variance translates to faster convergence.

# Natural Gradient Transform

- In practice, FIM cannot be calculated efficiently.
- We approximate the empirical FIM using a mini-batch of  $h \approx 20$  samples:

$$\hat{\mathbf{F}}_k = \frac{1}{h} \sum_{k=1}^h \mathbf{g}_k^{\text{low-rank}} \mathbf{g}_k^{\text{low-rank}T}$$

- The **natural gradient transform** can then be computed as:

$$\mathbf{g}_k^* = \hat{\mathbf{F}}_k^{-1} \mathbf{g}_k^{\text{low-rank}}$$

- As with GaLore, after applying Adam to  $\mathbf{g}_k^*$  the full parameter update is reconstructed by applying  $\mathbf{P}$ .



# Natural GaLore Algorithm

---

**Algorithm 1:** Natural GaLore, PyTorch-like

---

```
for weight in model.parameters():  
    grad = weight.grad  
    # original space -> low rank subspace + natural gradient transform  
    lor_grad = project(grad)  
    # update by Adam, AdamW, etc.  
    lor_update = update(lor_grad)  
    # low rank subspace -> original space  
    update = project.back(lor_update)  
    weight.data += update
```

---

The algorithm combines low-rank projection from GaLore with efficient second-order updates.

# Natural Gradient Computation

- Inverting the empirical FIM is computationally expensive.
- Here we use the Woodbury's Identity to compute the natural gradient efficiently:
  - ▶ By re-writing  $\hat{\mathbf{F}}_k = \lambda I + GG^T$ , we get:

$$(\lambda I + GG^T)^{-1} = \frac{1}{\lambda} I - \frac{1}{\lambda^2} G(I + \frac{1}{\lambda} G^T G)^{-1} G^T$$

where  $G = [\text{vec}(\mathbf{g}_k^{\text{low-rank}}), \dots, \text{vec}(\mathbf{g}_{k-s}^{\text{low-rank}})]$  is the gradient history.

- Can be computed using Cholesky Decomposition and matrix-vector products in  $\mathcal{O}(s^2)$  time:

$$Sz = y, \quad S = I + \frac{1}{\lambda} G^T G$$

- Hence enabling scalable low-rank second order optimization.

# Advantages of Natural GaLore

- **Curvature Information:**

- ▶ Accounts for the geometry of the loss landscape.
- ▶ Enables more informed optimization steps in high-dimensional, non-convex spaces.

- **Variance Reduction:**

- ▶ Natural gradients reduces variance in gradient estimates.
- ▶ Leads to faster convergence, especially in limited iterations.
- ▶ Further, when using a decaying learning rate schedule like with AdamW (Loshchilov & Hutter, 2017), the asymptotic convergence rate can be faster (Martens, 2020) by a significantly large constant factor.

- **Memory Efficiency:**

- ▶ Maintains low memory footprint, as with GaLore.
- ▶ Can be implemented without significant computational overhead.

# Benchmarking Experiments

# Pre-training Experiments on C4 Dataset

- **Models:** LLaMA variants with 60M, 130M, 350M, and 1.1B parameters.
- **Dataset:** C4 dataset.
- **Metrics:** Validation perplexity, memory consumption.
- **Results:**
  - ▶ Our method achieves lower perplexity than GaLore across all model sizes.
  - ▶ Closer performance to full optimizer state methods.
  - ▶ Maintains significant memory savings.
- **Conclusion:** Incorporating natural gradients accelerates convergence without significant additional memory overhead.

# Pre-training Experiments on C4 Dataset

**Table:** Comparison of Natural GaLore with other low-rank algorithms on pre-training various sizes of LLaMA models on the C4 dataset. Validation log perplexity is reported (averaged over 5 runs), along with a memory estimate (in gigabytes) of the total parameters and optimizer states based on BF16 format.

	60M	130M	350M	1.1B
Full-Rank	3.52 (0.36G)	3.22 (0.76G)	2.93 (2.06G)	2.72 (7.80G)
<i>Natural GaLore</i>	<b>3.53</b> (0.24G)	<b>3.22</b> (0.52G)	<b>2.93</b> (1.22G)	<b>2.80</b> (4.38G)
GaLore	3.56 (0.24G)	3.24 (0.52G)	2.95 (1.22G)	2.90 (4.38G)
Low-Rank	4.35 (0.26G)	3.82 (0.54G)	3.62 (1.08G)	4.96 (3.57G)
LoRA	3.55 (0.36G)	3.52 (0.80G)	3.24 (1.76G)	2.96 (6.17G)
ReLoRA	3.61 (0.36G)	3.38 (0.80G)	3.37 (1.76G)	2.91 (6.17G)
Rank $r/d_{\text{model}}$	128 / 256	256 / 768	256 / 1024	512 / 2048
Training Tokens	1.1B	2.2B	6.4B	13.1B

# Fine-Tuning on GLUE Benchmark

- **Model:** RoBERTa-Base.
- **Benchmark:** GLUE tasks (CoLA, MRPC, STS-B, etc.).
- **Comparison** with LoRA, GaLore and full fine-tuning.
- **Results:**
  - ▶ Comparable or better performance than LoRA.
  - ▶ Achieved an average score of 86.05, close to full fine-tuning baseline of 86.28.
  - ▶ Less memory consumption.
- **Conclusion:** Effective for memory-efficient fine-tuning without significantly sacrificing accuracy.

# Fine-Tuning on GLUE Benchmark

**Table:** Evaluating Natural GaLore for memory-efficient fine-tuning on the GLUE benchmark using pre-trained RoBERTa-Base. We report the average score of all tasks. Memory consumption is reported in millions of parameters (M).

	Memory	CoLA	STS-B	MRPC	RTE	SST-2	MNLI	QNLI	QQP	Avg
Full Fine-Tuning	747M	62.24	90.92	91.30	79.42	94.57	87.18	92.33	92.28	86.28
<b>Natural GaLore (rank=4)</b>	253M	61.50	<b>90.80</b>	<b>92.10</b>	<b>79.50</b>	<b>94.20</b>	<b>87.05</b>	<b>92.30</b>	91.15	<b>86.05</b>
GaLore (rank=4)	253M	60.35	90.73	92.25	79.42	94.04	87.00	92.24	91.06	85.89
LoRA (rank=4)	257M	<b>61.38</b>	90.57	91.07	78.70	92.89	86.82	92.18	<b>91.29</b>	85.61
<b>Natural GaLore (rank=8)</b>	257M	61.70	<b>90.90</b>	<b>92.25</b>	<b>79.80</b>	<b>94.40</b>	<b>87.20</b>	<b>92.35</b>	<b>91.25</b>	<b>86.23</b>
GaLore (rank=8)	257M	60.06	90.82	92.01	79.78	94.38	87.17	92.20	91.11	85.94
LoRA (rank=8)	264M	<b>61.83</b>	90.80	91.90	79.06	93.46	86.94	92.25	91.22	85.93



## Application: Fine-Tuning TinyAgents

# TinyAgent Framework

- TinyAgent framework (Erdogan et al., 2024) is an AAS for task-specific function-calling pipelines.
- **Goal:** Fulfill user queries through function-calling, respecting function interdependencies and passing correct arguments.
- **Key challenge:** Generating function-calling plans with correct functions, their arguments and interdependencies.

# LLMCompiler for Function Calling

- LLMCompiler (Kim et al., 2023) generates function-calling plans with required functions and arguments.
- It can then compile the plan into executable sequences.
- **Key challenge:** Original paper, implemented with 7B+ models. Could we fine tune smaller language models to generate a function plan with correct syntax, argument values, and respect for dependencies.

# Challenges with Off-the-Shelf TinyLlama 1.1B

- The off-the-shelf TinyLlama 1.1B (instruct-32k) performs poorly on function-calling tasks.
- Issues include:
  - ▶ Incorrect function sets.
  - ▶ Hallucinated function names.
  - ▶ Incorrect or missing dependencies.
  - ▶ Argument misplacement.

# Inneculation by Fine-Tuning

- To solve this, we fine-tune TinyLlama 1.1B using the curated TinyAgent dataset (Erdogan et al., 2024).
- Fine-tuning enables the model to learn:
  - ▶ Task-specific patterns for function-calling.
  - ▶ Precise understanding of task dependencies and arguments.
- Now there are three possibilities after fine-tuning:
  - ▶ Performance improvement  $\implies$  pre-training/instruction tuning data of the model was weak for this task.
  - ▶ No change in performance  $\implies$  the model doesn't have the capacity to learn this task.
  - ▶ Otherwise, annotation artifact.

# TinyAgent Dataset: Overview

- A meticulously curated dataset for building agentic systems on Apple MacBooks.
- Contains 40K examples of natural language queries and corresponding function-calling plans.
- Covers 16 distinct tasks such as:
  - ▶ Email, Contacts, SMS, Calendar, Notes, Reminders.
  - ▶ File Management, Zoom Meetings.
- Dataset split:
  - ▶ 38K training examples.
  - ▶ 1K validation examples.
  - ▶ 1K test examples.

# Fine-Tuning Strategy

- During fine-tuning, the prompt includes:
  - ▶ Descriptions of the ground-truth functions.
  - ▶ Irrelevant functions serving as negative samples.
- The model learns to select the correct functions rather than simply memorizing the ground truth.
- Several **in-context examples** are used to demonstrate how queries translate into function-calling plans.
- Examples are selected via RAG, leveraging a **DeBERTa-v3-small** model (He et al., 2021) fine-tuned for multi-label classification to retrieve relevant examples from the training set.

# Training Objective

- The training objective is to maximize the **accuracy of generated function-calling plans**.
- Success is defined as:
  - ▶ Correct function selection.
  - ▶ Correct arguments.
  - ▶ Correct order of function-calls.
- Verifying the selection of the correct function set is straightforward: set comparison.
- Ensuring correct argument handling and order requires constructing the associated **Directed Acyclic Graph (DAG)**.
- DAGs check for equality in the structure and dependencies of the function-calls.



# Fine-Tuning for Function Calling

- **Task:** Function calling using TinyAgent framework.
- **Model:** TinyLlama 1.1B fine-tuned for 3 epochs, batch size 32, learning rate  $7 \times 10^{-5}$ .
- **Dataset:** TinyAgent dataset with 40K examples.
- **Metrics:** Success rate in generating correct function-calling plans.
- **Results:**
  - ▶ Our method achieves a success rate of **83.09%**.
  - ▶ Outperforms 16-bit LoRA (80.06%) and GPT-4-turbo (79.08%).
  - ▶ Uses **30% less memory**.
- **Conclusion:** Enhances performance of smaller models, making them competitive with larger models.

# Fine-Tuning for Function Calling

**Table:** Latency, size, and success rate of TinyAgent models before and after quantization. Latency is the end-to-end latency of the function calling planner, including the prompt processing time and generation.

Model	Weight Precision	Latency (seconds)	Model Size (GB)	Success Rate (%)
GPT-3.5	Unknown	3.2	Unknown	65.04
GPT-4-Turbo	Unknown	3.9	Unknown	79.08
TinyAgent-1.1B	16-bit ( <i>Natural GaLore</i> )	3.9	2.2	<b>83.09</b>
	16-bit (LoRA)	3.9	2.2	80.06
TinyAgent-7B	16-bit (Erdogan et al., 2024)	19.5	14.5	84.95

# Conclusion

# Conclusion

- Presented **Natural GaLore**, a memory-efficient approach for LLM training and fine-tuning.
- Enhanced GaLore by incorporating **natural gradients** for better performance.
- Achieved significant memory savings without sacrificing accuracy.
- Validated effectiveness through extensive experiments on pre-training and fine-tuning tasks.
- Demonstrated practical benefits in advanced agentic systems.

# Future Work

- Explore **low-memory and structured projection matrices** for further memory efficiency.
- Conduct more extensive empirical evaluations on fine-tuning tasks in Advanced Agentic Systems.
- Inspire future research on **optimizer state approximation** for memory-efficient training.
- Make large-scale model training more accessible on consumer-grade hardware.

Questions?

# References I

- Shun-ichi Amari. Natural gradient works efficiently in learning. *Neural Computation*, 10(2):251–276, 1998. URL <https://ieeexplore.ieee.org/document/6790500>.
- Tom B Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, et al. Language models are few-shot learners. In *Advances in Neural Information Processing Systems*, volume 33, pp. 1877–1901, 2020. URL <https://arxiv.org/abs/2005.14165>.
- Tianqi Chen, Bing Xu, Chiyuan Zhang, and Carlos Guestrin. Training deep nets with sublinear memory cost. In *Proceedings of the 20th International Conference on Machine Learning (ICML)*, 2016. URL <https://arxiv.org/abs/1604.06174>.
- Lutfi Eren Erdogan, Nicholas Lee, Siddharth Jha, Sehoon Kim, Ryan Tabrizi, Suhong Moon, Coleman Hooper, Gopala Anumanchipalli, Kurt Keutzer, and Amir Gholami. TinyAgent: Function calling at the edge. *arXiv preprint arXiv:2409.00608*, 2024. URL <https://arxiv.org/abs/2409.00608>.

# References II

- Pengcheng He, Jianfeng Gao, and Weizhu Chen. DeBERTaV3: Improving DeBERTa using ELECTRA-style pre-training with gradient-disentangled embedding sharing. *arXiv preprint arXiv:2111.09543*, 2021. URL <https://arxiv.org/abs/2111.09543>.
- Edward J Hu, Yelong Shen, Phillip Wallis, Zeyuan Allen-Zhu, Yanzhi Li, Shean Wang, and Weizhu Chen. LoRA: Low-rank adaptation of large language models. In *International Conference on Learning Representations*, 2022. URL <https://arxiv.org/abs/2106.09685>.
- Yanping Huang, Youlong Cheng, Ankur Bapna, Orhan Firat, Menglong Chen, Denny Chen, Zhifeng Hu, Yuxin Shen, Maxim Krikun, Yonghui Wu, et al. GPipe: Efficient training of giant neural networks using pipeline parallelism. In *Advances in Neural Information Processing Systems*, volume 32, pp. 103–112, 2019. URL <https://arxiv.org/abs/1811.06965>.
- Sehoon Kim, Suhong Moon, Ryan Tabrizi, Nicholas Lee, Michael W Mahoney, Kurt Keutzer, and Amir Gholami. An LLM compiler for parallel function calling. *arXiv preprint arXiv:2312.04511*, 2023. URL <https://arxiv.org/abs/2312.04511>.



# References III

- Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. *arXiv preprint arXiv:1412.6980*, 2014. URL <https://arxiv.org/abs/1412.6980>.
- Vladimir Lialin and Arthur Schatz. ReLoRA: Low-rank fine-tuning reloaded. *arXiv preprint arXiv:2307.09769*, 2023. URL <https://arxiv.org/abs/2307.09769>.
- Ilya Loshchilov and Frank Hutter. Decoupled weight decay regularization. *arXiv preprint arXiv:1711.05101*, 2017. URL <https://arxiv.org/abs/1711.05101>.
- James Martens. New insights and perspectives on the natural gradient method. *Journal of Machine Learning Research*, 21:1–76, 2020. URL <https://www.jmlr.org/papers/volume21/17-678/17-678.pdf>.
- Krista Opsahl-Ong, Michael J Ryan, Josh Purtell, David Broman, Christopher Potts, Matei Zaharia, and Omar Khattab. Optimizing instructions and demonstrations for multi-stage language model programs, 2024. URL <https://arxiv.org/abs/2406.11695>.

## References IV

- Samyam Rajbhandari, Jeff Rasley, Olatunji Ruwase, and Yuxiong He. ZeRO: Memory optimizations toward training trillion parameter models. In *Proceedings of the International Conference for High Performance Computing, Networking, Storage and Analysis*, pp. 1–16, 2020. URL <https://arxiv.org/abs/1910.02054>.
- Mohammad Shoeybi, Mostofa Patwary, Rohan Puri, Patrick LeGresley, Jared Casper, and Bryan Catanzaro. Megatron-LM: Training multi-billion parameter language models using model parallelism. *arXiv preprint arXiv:1909.08053*, 2019. URL <https://arxiv.org/abs/1909.08053>.
- Tianxiang Xia, Hao Peng, Zheyu Chen, Lemao Li, Zhiyuan He, Zhen Yang, and Wei-Ying Ma. Chain-of-thought lora: Efficient adaptation of large language models. In *Proceedings of the 2024 Conference on Empirical Methods in Natural Language Processing (EMNLP)*, 2024. To appear.
- Chengrun Yang, Xuezhi Wang, Yifeng Lu, Hanxiao Liu, Quoc V. Le, Denny Zhou, and Xinyun Chen. Large language models as optimizers, 2024. URL <https://arxiv.org/abs/2309.03409>.

# References V

- Jiawei Zhao, Zhenyu Zhang, Beidi Chen, Zhangyang Wang, Anima Anandkumar, and Yuandong Tian. GaLore: Memory-efficient LLM training by gradient low-rank projection. *arXiv preprint arXiv:2403.03507*, 2024. URL <https://arxiv.org/abs/2403.03507>.
- Tianshi Zhao, Zhen Sun, Xiaodong Wang, Fei Zhou, Yang Guo, and Alexander J Smola. Extending torchelastic for stateful training jobs. *arXiv preprint arXiv:2006.06873*, 2020. URL <https://arxiv.org/abs/2006.06873>.