Reading Guide for Explaining the Gibbs Sampler

Casella and George (1992)[1]

1. [Section 2] How does Gelfand and Smith (1990) suggest to obtain an approximate sample from $f(x)$? How is it different from or similar to the approach we talked about in class? What are the advantages and disadvantages of each approach?

2. [Section 2] The authors claim "Gibbs sampling can be used to estimate the density itself by averaging the final conditional densities from each Gibbs sequence." What is the theory behind this claim? How does Figure 3 support this claim?

3. [Section 2] Two simulations in Figure 1 and Figure 3: What are the similarities and differences? Why the conditional carries more information than the marginal?

4. [Section 3] Write down marginal distribution of y, and verify the conditional probabilities $A_{y|x}$ and $A_{x|y}$. Also, verify $A_{x|x} = A_{y|x}A_{x|y}$ and $f_x A_{x|x} = f_x A_{y|x} A_{x|y} = f_x$.

5. [Section 4] What is a fixed point integral equation in the bivariate case? How does it help illustrate how sampling from conditionals produces a marginal distribution? Hint: check equations (3.5), (4.1), and (4.2).

6. [Section 4] The authors claimed "a defining characteristic of the Gibbs sampler is that it always uses the full set of univariate conditionals to define the iteration." Explain this claim by illustrating how a Gibbs sampler works with k parameters $(\theta_1, \theta_2, \cdots, \theta_k)$.

7. [Section 5] Summarize different approaches to sampling the Gibbs sequence.

[1] Casella, G. and George, E. I. (1992), "Explaining the Gibbs Sampler", *The American Statistician*, **46**(3), 167-174.